ORBITAL OPERATIONS STUDIES

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PROPOSED GEO-ALTITUDE COORDINATES OF VERTICAL POSITIONS

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This information circular is one of a continuing series concerned with means of coordinating separately organized operations with respect to each other and with respect to the characteristic features of the Earth-centered Geosphere. This particular article is concerned with the coordination of the vertical positions of things in meteorological, aeronautical and aerospace-flight operations.

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PROPOSED GEO-ALTITUDE COORDINATES

OF VERTICAL POSITIONS

ABSTRACT

"Geo-Altitude" coordinates of geometric height, pressure, temperature, density and potential energy levels in and of the Earth's atmosphere are defined herein in terms of a generalized concept of an "Ideal Geo-Atmosphere". This simplified Standard Reference Atmosphere utilizes the newly formulated Geospheric System of Units for coordinating the space-time positions, sizes and amounts of things throughout the Earth-centered region of the universe. It is intended primarily to provide for coordinating meteorological and aeronautical practices throughout the world.

PROPOSED GEO-ALTITUDE COORDINATES OF VERTICAL POSITIONS

1. Introduction

A serious lack of coordination currently exists between meteorological and aeronautical practices. For example, the constant pressure levels of the atmosphere are identified with numbers of the metric system's millibar units of pressure in current meteorological practice, but with numbers of feet of the pressure-altitude measure of pressure in the aeronautical practices of even most of the otherwise "metric" countries of the world. As a consequence, meteorological charts are now drawn for levels such as at 700, 500 and 400 millibars, or for pressure-altitudes of about 9,882, 18,280, and 23,574 feet for which no flyer could or would wish to obtain a clearance to fly. Conversely, the effective utilization of aircraft observations of the state of the atmosphere is currently forestalled largely by the difficulty of relating them to those particular meteorological mapping levels.

It is especially noteworthy in this regard that the use of numbers of feet of pressure-altitude is of life and death importance to everyone who flys. If, for example, aircraft altimeters were to indicate numbers of millibars, it would be virtually impossible for pilots to relate them to the charted numbers of feet of elevation of the solid terrain over which they and their passengers wish to fly. Rather, the development of safe all-weather flying is attributable largely to the evolution of the pressure-altitude measure of atmospheric pressures:

- o Numbers of feet of which correspond to numbers of feet of geometric elevation above mean sea level with sufficient precision for most enroute flying; and
- o To which small residual "D-value" differences from geometric elevations are automatically added by "setting" pressure-altimeters with appropriate "altimeter settings" for specific places and times.

It was also shown in 1945[1] that the use of such residual D-value differences between the geometric and barometric kinds of "altitude" would be very advantageous from a meteorological point of view. As pointed out in 1963[2], however, neither the English foot nor the metric system's meter unit of length is suitable for evaluating such "altitude" coordinates of vertical positions. Rather, it was suggested in 1963[2], 1964[3] and 1967[4] that this coordinative problem might well be solved by utilizing a "nautical foot" unit of length — of which exactly 6,000 instead of about 6,078 English feet would be contained in the nautical mile unit of length.

Continued investigations of this units problem has resulted in a companion article[5] on the "Environmental Aspects of the Units Problem". The evolution of the metric units of measure into the International System of "SI Units" is seen there to have provided a medium of exchange for quantitative measures of any and all physical qualities; much as the dollar provides a medium of exchange for evaluating the worth of things. The International System of Units is thus also seen to serve primarily as a definitive means toward more specifically useful quantitative and qualitative ends — of which the coordination of the space-time positions, unit-amounts and unit-sizes of things in and of the Earth-centered or Geospheric region of the universe could very well be served by a newly formulated Geospheric System of Units.

This Geospheric System of "Geounits" essentially consists of a continued subdivision of the common Earth-related coordinates of time and angular positions down to the personally-related foot-like and thumb-width scale-sizes of things. As summarized in the appended fly-sheet, it also takes full advantage of the extremely fortunate circumstance that very nearly 100 times the length of anyone's and everyone's foot happens to be contained in the length of one second of great-circle arc anywhere and everywhere on the surface of the Earth.

The Geospheric System of Units thereby offers a smoothly evolutionary means of solving many of our current units problems - of which the coordination of the vertical positions of things in and of the atmosphere is one of the most critically important. The full advantage of its "geounits" for this purpose can be realized, however, only if they are also utilized to define the several different qualitative kinds of "Geo-Altitude" in terms of a corresponding "Ideal Geo-Atmosphere" counterpart of previous Standard Atmospheres.

2. Defining Equations

Such an Ideal Geo-Atmosphere can well be defined to be the gaseous part of an Ideal Geosphere as specified in the appended fly-sheet. Following the lead of previous Standard Atmospheres, it is simply and usefully conceived to consist of an Ideal Gas whose pressure, p, temperature, T, and density, ρ, are interrelated anywhere and everywhere by the equation

$$p = \rho RT \tag{1}$$

The invariable gas constant, R, in this equation is defined to be the ratio

$$R = p_o/\rho_o T_o \tag{2}$$

of suitably selected and assigned values p_o, ρ_o and T_o of its pressure, density and temperature at any and all horizontal positions of its zero-altitude level.

The rate of change, dp/dA, of pressure with respect to altitude, A, anywhere and everywhere in this Ideal Atmosphere is also conceived to be in hydrostatic equilibrium with an ideally defined field of gravitational acceleration, g, in accordance with the differential equation

$$dp/dA = -\rho g \tag{3}$$

Incremental changes of its pressure, dp, and altitude, dA, are thereby defined to be interrelated by the equation

$$dp/p = -(g/RT)dA \tag{4}$$

in which values of T and g are ideally conceived to vary only in the vertical direction — or hence to be constant throughout each of its constant-altitude and/or coincident constant-pressure, constant-temperature, constant-density, constant-potential-energy, etc. levels.

Also following the lead of previous Standard Atmospheres, the Ideal Geo-Atmosphere is usefully conceived to be stratified into layers — in each of which the temperature, T, is linearly related to altitude, A, and an assigned temperature T_i at a particular altitude A_i in accordance with

$$T = T_i + L(A-A_i) \tag{5}$$

The lapse rate L = dT/dA in this equation has a suitably selected and assigned constant numerical value for each such "i-th" layer as specified in Table 1. In addition, the Ideal Geosphere's field of gravitational acceleration

$$g = g_0 r_0^2/r^2 \tag{6}$$

is simply conceived and defined to vary inversely as the square of the radial distance

$$r = r_0+A \tag{7}$$

of any particular altitude, A, in and of the atmosphere above its zero-altitude level at a radius r_0 — throughout which g has a suitably selected and defined constant value of $g = g_0$.

These definitions provide for integrating Equation 4, which can thereby be written as

$$\frac{dp}{p} = -\frac{g_0 r_0^2}{R} \cdot \frac{dr}{r^2[T_i+L(r-r_i)]} \tag{8}$$

In the special case of an isothermal "i-th" layer in which L=0, the integration of Equation 8 from the level (p_i, r_i) to a level (p,r) provides the algebraic equations

$$\log p/p_i = -\frac{g_0 r_0^2 \log e}{RT_i} \cdot \frac{r-r_i}{rr_i} \tag{9}$$

or

$$\log p/p_i = -\frac{g_0 \log e}{RT_i(1+A_i/r_0)} \cdot \frac{A-A_i}{1+A/r_0} \tag{10}$$

For these layers in which L is not zero, it is convenient to introduce the altitude

$$h = (r_i - T_i/L) - r_o \tag{11}$$

at which the linearly extrapolated temperature would reach zero. Equation 8 then becomes

$$\frac{dp}{p} = -\frac{g_o r_o^2}{RL} \cdot \frac{dr}{r^2(r-r_o-h)} \tag{12}$$

Integrating from the level (p_i, r_i) at which $T=T_i$ to the level (p,r) at which $T=T_i+L(r-r_i)$ then yields the algebraic equations

$$\log\frac{p}{p_i} = -\frac{g_o r_o^2}{RL(r_o+h)^2}\log\frac{Tr_i}{T_i r} + \frac{g_o r_o^2 \log e}{RL(r_o+h)} \cdot \frac{r-r_i}{rr_i} \tag{13}$$

or

$$\log\frac{p}{p_i} = -\frac{g_o}{RL(r_o+h/r_o)^2}\log\frac{T(1+A_i/r_o)}{T_i(1+A/r_o)} + \frac{g_o \log e}{RL(1+h/r_o)(1+A_i/r_o)r_o} \cdot \frac{A-A_i}{1+A/r_o} \tag{14}$$

Equations 10 and 14, together with the assignment of numerical values of their constants as specified in Tables 1 and 2, provide the definitive equations listed in Table 2 for the pressures, p, at any and all altitudes, A in this ideally conceived Geo-Atmosphere. Equation 1 then also provides for computing the ideally defined density at any such altitude from those values of pressure and the defined vertical distribution of temperature.

Each such "horizontal" constant-altitude, constant-pressure and constant-density surface of this Ideal Geo-Atmosphere is also usefully conceived to be a constant potential-energy surface, the numerical values, Φ, of which can well be defined to be the work required to lift a unit amount of mass, ${}^1M$, from its zero-altitude level against the acceleration of gravity defined in Equations 6 and 7, or hence to be given by

$$\Phi = \int_0^A {}^1M g\,dA = {}^1M g_o r_o^2 \int_{r_o}^{r} \frac{dr}{r^2} = {}^1M g_o r_o \frac{r-r_o}{r} = {}^1M g_o \frac{A}{1+A/r_o} \tag{15}$$

3. Geounits of Altitude

These defining equations and numerical constants have been purposefully selected to make the coordination of vertical positions with respect to the resulting definitions of the several different kinds of "altitude" as usefully convenient as possible. Toward this coordinative end, they have been defined in terms of as simply and memorably conceived round-numbers as practicable of the "geomile" counterpart of the "nautical mile" with which horizontal space-time positions can well be and largely are coordinated throughout the world.

This "geomile" unit of length basically provides for maximum useful convenience of coordinating horizontal distances with respect to numbers of angular minutes of latitudes, longitudes and great-circle arcs on the surface of the Earth — and thereby also with respect to the numbers of the 4-second units of mean solar or sidereal time during which the Earth rotates one minute of angle with respect to the Sun or stars. It does so by virtue of having been defined to be the angular minute-like 360(60)th or 21,600th part of an ideally conceived circumference of the Earth of exactly 40,000,000 meters.

This simply conceived definition of the "geomile, M'L" unit of length makes 54 or $2 \cdot 3^3$ such geomiles exactly equivalent to 100 kilometers — and thereby provides for convenient conversions among corresponding numbers of geomiles and numbers of the basic SI meter unit of exchange among any and all such more specifically useful units of length. In comparison, the current definition of the International Nautical Mile of exactly 1,852 or $2^2 \cdot 463$ meters is essentially the nearest whole number approximation of the readily calculable inverse conversion factor of $10^5/2 \cdot 3^3$ or 1,851.851 ··· meters per geomile — and has thereby unnecessarily introduced the inconvenient prime-number factor of 463 into all such conversions between corresponding numbers of miles and of the basic SI meter unit of exchange.

The way in which the geomile is defined also provides for simply and usefully identifying its 60th part or second-like "sile, S'L" as well as its 6,000th part or centisecond-like and foot-like "geofoot" or "file, F'L" geounit subdivisions of the ideally conceived 40,000,000 meter circumference of the Earth. Again, the conversion factors of 0.0324 or $2^2 \cdot 3^4/10^4$ siles per meter and 3.24 or $2^2 \cdot 3^4/10^2$ geofeet or files per meter contain no prime-number factors other than 1, 2, 3 or 5. In addition, these conveniently Earth-related and meter-related geounits of length are very little more than 1 part in 80 larger than the current 100-feet of pressure-altitude units of "flight altitude" and the nominal English-foot counterpart of the length of anyone's and everyone's own foot.

Consequently numbers of geomiles, siles and geofeet of pressure-altitude can very well provide for convenient coordination of particular levels in and of the atmosphere. For this purpose, uniform intervals of 1 geomile or 60 siles of pressure-altitude could well be designated as the mandatory reporting levels for standardized meteorological mapping levels as proposed in Table 4. An extremely wide range of choice would thereby be provided for whichever of

1 sixty sile,	2 thirty-sile,	3 twenty-sile,
4 fifteen-sile,	5 twelve-sile,	6 ten-sile,
10 six-sile,	12 five-sile,	15 four-sile,
20 three-sile,	30 two-sile or	60 one-sile

intermediate intervals and/or levels might best serve particular purposes in readily identifiable and fully coordinated ordinal-number relationship to whichever other such intervals might best serve other particular purposes.

4. Geounits of Atmospheric Temperatures

For example, convenient interpolations or extrapolations could thereby well be made to intermediate flight levels such as the current 1000-feet, 10-sile or 1/6-geomile intervals between Eastward and Westward flights or to the intermediate 500-foot, 5-sile, or 1/12-geomile intervals for Northward or Southward flights. It is especially important in this regard to be able to make precise hydrostatic computations of the basic pressure-height state of the atmosphere at any and all such flight levels in order to utilize the single-heading or altimetric-drift forms of the geostrophic wind equation[1],[6],[7]. The difficulty of such calculations in terms of either feet or meters of elevation and numbers of millibars of pressure has, however, restricted such wind determinations in current meteorological practice almost entirely to those few mandatory reporting levels for the round numbers of millibars listed in Table 3.

In contrast, the height-like pressure-altitude measure of atmospheric pressures provides[1] for converting Equation 4 into the extremely simplified residual form of the hydrostatic equation

$$\frac{dD}{dA} = S = \frac{T^{*}-T_A}{T_A} \qquad (16)$$

The "S-values" of this equation are usefully identified as being either the "Residual Altimetric Scale Factor"

$$S = \frac{dD}{dA} = \frac{d(E-A)}{dA} \qquad (17)$$

between the geometric measures of the elevation E and barometric measures of the pressure-altitude A in and of the atmosphere; or as the hydrostatically equivalent "Specific Temperature Anomaly"

$$S = \frac{T^{*}-T_A}{T_A} \qquad (18)$$

of the "Virtual Temperature, T^{*}" of the actual atmosphere with respect to the prescribed values of the "Ideal Atmospheric Reference Temperature, T_A, for particular pressure-altitudes, A (which is simply identified elsewhere in this discussion as being the Ideal Geo-Atmospheric Temperature, T).

The non-dimensional residual S-value measure of atmospheric temperatures has been found to be very useful both for navigational[8] and meteorological[9] purposes. Its full utility requires, however, that its numerical relationship to other measures of temperature — such as especially to numbers of degrees celsius — be as usefully convenient as possible. Toward this end, the symbol T_A in the denominator of Equation 18 can very well be evaluated as number $T_A/^\circ A$ of "Absolute Atmospheric Degrees, $^\circ A$"; defined in accordance with the equation

$$T_A/^\circ A = T_A/^\circ C + 273 \tag{19}$$

to be exactly equal to numbers of celsius or centigrade degrees, $T_A/^\circ C$, plus the integral number 273.

The use of the additive conversion factor of 273 instead of the factor 273.15 between degrees celsius and the kelvin measure of temperature explicitly recognizes the fact that the kelvin measure pertains to an extremely generalized Ideal Gas for which $p = R\rho T$. A more precise equation of state for actual gases can thereby well be expressed[10] as

$$p = cR\rho T \tag{20}$$

in which the coefficient c can be evaluated with an equation such as

$$c = 1 - \frac{Aaa}{RT}\,p - \frac{Aaaa}{RT}\,p^2 - \cdots \tag{21}$$

in terms of temperature dependent "virial coefficients, Aaa and Aaaa". For dry air the coefficient c has the numerical values

p/mb	-100°C	-50°C	0°C	50°C
500:	0.9958	0.9992	0.9997	0.9999
1000:	0.9917	0.9984	0.9994	0.9999

Consequently "Absolute Virial Temperatures, $T_V = cT$" for dry air are related to Degrees Celsius by the following additive conversion factors

p/mb	-100°C	-50°C	0°C	50°C
500	272.42	272.97	273.07	273.12
1000	271.72	272.79	272.99	273.12

The inclusion of fractions of degrees in any such generally applicable conversion factor is thereby seen to be unwarranted; and the number 273 is as typically representative as any one integral number of degrees can practicably be for all such conversions in which variations associated with specific temperatures, pressures, water-vapor content, etc. need not be taken into explicit account.

This particular number 273 also happens to provide for extremely convenient hydrostatic computations with Equation 16 – in which the values of Ideal Geo-Atmospheric Temperature, T_A, need to be calculated as a defined function of pressure-altitude, A. Such computations have been greatly simplified by the fortunate circumstance that the Zero-Altitude temperature of this and previous Standard Atmospheres have been designated as being $15^{\circ}C$, or hence to be exactly equivalent to 15+273, 288 or $2^5 \cdot 3^2$ of the "Absolute Atmospheric Degrees, $^{\circ}A$". Consequently the computations of equations 16 and 18 for the Zero-Altitude level involves successive multiplications or divisions by and only by 2 and 3.

A similar convenience of multiplication or division by $T_A/^{\circ}A$ also happens to be provided by the vertical distribution of Ideal Geo-Atmospheric Temperatures specified in Table 1. This particular definition has purposefully been selected to provide as conveniently useful computations as possible between numbers $T_A/^{\circ}C$ of the idealized temperature at each such altitude – consistent with also being as typically representative as any one such numbers can practicably be of the temperatures[11] of the 1962 U.S. Standard Atmospheres and, thereby, of the temperature at a particular altitude at any and all horizontal positions and times in the actual atmosphere. It has proven to be possible to do so with only the extremely convenient numerical values of idealized lapse rates of exactly 0; $\pm 6^{\circ}C$ per geomile, $1^{\circ}C$ per thousand geofeet or $0.1^{\circ}C$ per sile; and $-12^{\circ}C$ per geomile, $2^{\circ}C$ per thousand geofeet or $0.2^{\circ}C$ per sile. As a consequence it also happens to provide for the convenience of having to multiply or divide in Equations 16 and 18 by the convenient constant conversion factors $T_A/^{\circ}A$ of:

$216=6^3=2^3\cdot 3^3$	throughout the ideal isothermal region of $-57^{\circ}C$ between 6 and 15 geomiles of altitude;
$270=3^3\cdot 10$	throughout the ideal isothermal region of $-3^{\circ}C$ between 24 and 30 geomiles of altitude; and
$180=2\cdot 3^2\cdot 10$	throughout the ideal isothermal region of $-93^{\circ}C$ between 45 and 48 geomiles of altitude.

5. Gravitational Geounits

The gravitational field of the Ideal Geosphere has also purposefully been defined to provide for as convenient end-use calculations that involve the weight-like geounits of force as practicable. Toward this end, the acceleration of gravity, g_o, throughout the Ideal Biosphere at the Zero Altitude level of the Ideal Atmosphere has been designated as being one geounit of acceleration, ^1g, of

$$g_o = 1\ ^1g = 10/1.0206 \text{ or } 10^5/2\cdot 3^6\cdot 7 \text{ m/s}^2.$$

This particular number of the SI meters per second squared medium of exchange for units of acceleration has been selected to contain as few large prime-number factors as practicable — consistent with its also being near the middle of the range of variability of the observed accelerations of gravity throughout the land and water surface of the Earth.

This definition of the "one gee" unit of acceleration provides especially for conveniently-useful exactly-defined equivalences such as among

$3^2\cdot 7/2\cdot 10$, 63/20 or 3.15 "geogees, ^1g" and
1 geofoot per second per centisecond,
1 geofoot per decisecond squared,
1 sile per second squared,
1 geomile per minute per second, or
1 dile per minute squared;

or $3 \cdot 7/2^2$, 21/4 or 5.25 "centigees" and
1 geomile per minute squared,
1 geomile per hour per second or
1 (geo)knot per second.

In comparison, the inconvenient prime-number factor of 28,019 is unnecessarily introduced into all such computations that involve units of force and Earth-related units of acceleration by the current usage of the Standard Acceleration of Gravity for mean sea level and 45^o of latitude of

$$g_o(45^o) = 9.80665 \text{ or } 5 \cdot 7 \cdot 28019/10^5 \text{ m/s}^2$$

or

$$g_o(45^o) = 1.000866699\ {}^1g$$

to define weight-like units of force.

The acceleration of gravity is also ideally conceived to vary inversely with the square of radial distances as defined in Equation 6 to provide a smooth transition to idealized computations of things such as the orbital periods of satellites in the airless but full-of-fiery-energy "Pyrospheric" portion of the Geosphere. The complications of considering deviations from the inverse square-law due to latitude-dependent effects of Earth-related centrifugal forces have been purposefully eliminated for this world-wide altitude-defining purpose, and perturbations due to the Sun's gravitational field have been shown(12) to be of the order of

1 part in 13,000,000	throughout the range of altitudes up to the nominal 48 geomile altitude of the top of the Ideal Atmosphere,
1 part in 43,000	at the 6.62 "radile" radius of synchronous Earth-satellites;
1 part in 60	at the very nearly 60 and 3/8 "radile" radius of the Moon's orbit, and
1 part in 1	at the nominal outer extent of the Earth-centered Geosphere of 235 "radiles" at very nearly 1/100 of the Astronomical Unit distance to the Sun.

In this regard, the "radile" geounit of length has been defined by simply conceiving of it as being the single $1/2\pi$ radial fraction of an infinitesimally thin Ideal Spherical Biosphere with an ideally defined "1 circile, C'L" circumference of exactly 40,000,000 SI meters. This definition that

1 radile, $R'L = 4 \cdot 10^7/2\pi$ or about 6,366,198 meters

or 1 radile, $R'L = 21,600/2\pi$ or about 3,437.75 geomiles

very well provides for evaluating, comparing and interpreting more detailed considerations of Earth-radii for specific horizontal positions. For example, the much more complicated mean sea level surface of the U.S. Standard Atmosphere, 1962[11] is defined to be an ellipsoid of revolution with

Equatorial Radii, a, of 6,378,178 meters,
A Flattening Factor, $f = (a-b)/a$ of 1/298.32,
Polar Radii, b, of 6,356,798 meters, and a
Nominal Zero-Altitude Radius, r_o, at 45_o of Latitude of
$r_o(45^o) = (a^4+b^4)/(a^2+b^2)$ of 6,367,532 meters

or, more informatively in terms of the simply conceived radial-reference or "radile" unit of length, as having

Equatorial Radii,	$r(0^o)$	= 1+.001882 radiles,
Polar Radii,	$r(90^o)$	= 1-.001477 radiles and
45^o Radii,	$r(45^o)$	= 1+.000210 radiles.

Significantly in this regard, the identification of potential-energy surfaces in the U.S. Standard Atmosphere, 1962 with height-like numbers of "geopotential altitude, H" defined by an equation such as

$$H = \Phi/g_o = \int_o^Z g dZ/g_o \tag{21}$$

in terms of "geometric altitudes, Z" tends strongly to confuse the meaning of the world "altitude". Rather, it is proposed here that "altitude" be utilized as a generic term for each of the geometric, pressure, density and/or potential-energy kinds of "altitude". Constant values of each such different kind of "altitude" can then well be simply conceived to coincide in the Ideal Atmosphere — and the slightly differing values of each of them at any one specific position in the actual atmosphere can very well serve individually to identify

- o Geometric measures of elevations above mean sea level that are equal to values of "geometric altitude";
- o Those pressures that are defined by the equations of Table 2 to correspond to particular values of "pressure altitude" in and of the Ideal Atmosphere;
- o Those densities that are defined by Equation 1 and the equations of Table 2 to correspond to particular values of "density altitude" in and of the Ideal Atmosphere, and

- Those potential energies, Φ, that are defined by

$$\Phi = {}^1Mg_oA/(1+A/r_o) \tag{15}$$

to correspond to particular values of "potential-energy altitude" in and of the Ideal Geosphere.

6. Geounits of Pressure and Density

The selection of the Zero-Altitude Pressure, p_o, of "1 atmosphere of pressure" also depends upon the nominal value of gravitational acceleration with which it is associated. Currently, the normal atmospheric pressure of 760 millimeters of mercury is associated with 1013.25 millibars or 101,325 of the SI newton per square meter unit of pressure when the mercury column is subject to a gravitational acceleration of 9.80665 meters per second squared. The inconvenient prime-number factors of this unit-defining number $101{,}325 = 3 \cdot 5^2 \cdot 7 \cdot 193$ can therefore well be eliminated in association with the change to the much more conveniently useful ^1g geounit of acceleration by defining the "1 atmosphere" geounit of pressure to be

$$p_o = 101{,}250 \text{ or } 3^4 \cdot 5^3 \cdot 10 \text{ SI newtons per square meter.}$$

This particular Zero-Altitude Pressure corresponds to 760.096 millimeters of mercury or only about a negligible 0.004 inches of mercury more than the aeronautically familiar 29.92 inches of mercury when it is subject to 1 geogee unit of acceleration. In addition, it is exactly equivalent to 1.00442348 times 400 of the T'p or "thile-scale" geounits of pressure. Therefore this "1 atmosphere" of pressure can also well be considered with sufficient precision for most purposes to be equivalent to the increment of pressure exerted by 400 geoinches of Ideal Water or by thirty-three and a third geofeet of Ideal Water subject to 1 gee of gravitational acceleration.

The Ideal Zero-Altitude Density, ρ_o, can in turn well be the same nominal value, namely

$$\rho_o = 1.225 \text{ or } 5^2 \cdot 7^2/10^3 \text{ SI kilograms per cubic meter,}$$

that was adopted for the 1962 U.S. Standard Atmosphere. These definitions of p_o and ρ_o in association with the assignment of 288 for the number of "Absolute Atmospheric Degrees, ^oA" of Zero-Altitude Temperatures, T_o, essentially specifies that the gas constant, R, throughout the Ideal Atmosphere has the numerical value of

$$R = p/\rho T = (2 \cdot 3^4 \cdot 5^4 \ N/m^2)/(7^2/2^3 \cdot 5 \ \ kg/m^3)(2^5 \cdot 3^2 \ ^oA)$$
$$= 3^2 \cdot 10^5/2^6 \cdot 7^2 \ m^2/^oAs^2;$$

and is only about 2.3 parts per 10,000 smaller than the gas constant adopted for the 1962 U.S. Standard Atmosphere. The hydrostatically useful "zero-altitude autoconvective lapse rate, g_o/R" thereby has ideally defined numerical values of

$$g_o/R = 2^5 \cdot 7 \cdot 10^3/3^8 \quad \text{or about } 34.1411 \ ^oC \text{ per kilometer}$$
$$= 2^4 \cdot 7 \cdot 10^5/3^{11} \quad \text{or about } 63.2243 \ ^oC \text{ per geomile}$$

and

$$R/g_o = 3^8/2^5 \cdot 7 \quad \text{or about } 29.2902 \text{ meters per } ^oC$$
$$= 3^{11}/2^4 \cdot 7 \cdot 10^5 \quad \text{or about } 0.0158167 \text{ geomiles per } ^oC.$$

This value of R can well be conceived to pertain to "dry air", the deviations of actual moist air from which can well be accounted for in the "Virtual Temperature, T^*" of Equations 16 and 18. Such "virtual temperatures" can also be utilized(1) to account for deviations of actual accelerations of gravity, g^*, from the ideal values of g that are defined by Equation 6 for a particular altitude, A. That overall "virtual temperature, T^*" is essentially the temperature that the gas of the Ideal Atmosphere subject to the ideal acceleration of gravity g would need to have if its weight per unit volume were to be equal to that of an actual gas at the same pressure, with a temperature T, with a mixing ratio fraction w of water vapor per unit mass of dry air, and subject to a gravitational acceleration g^*. Accordingly(1)

$$T^* = T \, \frac{1+w/0.62917}{1+w} \cdot \frac{g}{g^*} \tag{23}$$

or, for convenient difference calculations,

$$T^* - = T \left[\frac{0.607w}{1+w} + \frac{g^*-g}{g^*} + \frac{0.6078w}{1+w} \cdot \frac{g^*-g}{g^*}\right] \tag{24}$$

in which 0.6078 = 1-1/0.62917 and the number 0.62917 is an ideally assumed ratio(10) of the molecular weights of water vapor and dry air.

7. Comparisons of Pressure-Altitudes

It is especially important for evaluations of the relative merits of this Ideal Geo-Atmosphere and previous Standard Atmospheres to recognize a subtle but very important difference of their basic purposes. On the one hand, for example, the basic purpose of the 1962 U.S. Standard Atmosphere is explicitly specified(11) as being the largely descriptive one of depicting "••• idealized middle-latitude year-round conditions •••" in terms of previously established units such as meters or feet and millibars. On the other hand, the Ideal Geo-Atmosphere is — in addition — also explicitly intended to serve as a unit-defining basis for as usefully convenient evaluations as possible of the state of the actual atmosphere at specific positions and times.

As indicated in the preceding discussion, this additional purpose is served primarily by defining a pressure-altitude coordinate for numbering the actual atmosphere's constant-pressure and very nearly constant-potential-energy levels(2) with numbers that correspond as closely as possible to their geometric altitudes above the lithosphere's and hydrosphere's mean sea level surface. The residual D-value differences between those geometric and barometric kinds of vertical coordinates at specific positions and times in the actual atmosphere thereby also provide for highly informative evaluations and depictions of:

- Atmospheric temperatures in terms of the vertical rates of change of D-values in accordance with the extremely simplified hydrostatic Equation 16; and
- The wind-velocity differences of the motions of the atmosphere and lithosphere with respect to the fixed stars that are associated with the differences of their constant potential-energy surfaces in accordance with the geostrophic and gradient wind equations.

The degree to which the Ideal Geo-Atmosphere and 1966 U.S. Standard Atmosphere correspond to each other and to the range of variability of actual mid-latitude atmospheric temperatures(11) is illustrated in Table 1. The degree to which corresponding values of pressure-altitudes differ are, in turn, proportional to the areas between the solid and dashed temperature traces and have been tabulated in the third and fifth "Δ" columns of Table 2. The Ideal Geo-Atmosphere is thereby seen to be as typically representative of specific thermodynamic states of the atmosphere as any one such idealization can practicably be expected to be — and which also provides for the extreme computational and coordinative utility of nominal lapse rates of

$-12^{o}C$ or $\pm 6^{o}C$ per geomile or
$- 2^{o}C$ or $\pm 1^{o}C$ per thousand geofeet

between isothermal layers with such usefully defined temperatures as

216 or 6^3 ^{o}A, 270 or $3^3 \cdot 10$ ^{o}A and 180 or $2 \cdot 3^2 \cdot 10$ ^{o}A

and between such easily remembered and usefully round altitude numbers as

0, 6, 15, 24, 30, 45 and 48

of the geomile subdivisions of the nominal 60-geomile "dile" length of 1-degree subdivisions of the Earth's great-circle circumferences.

In addition, the differences between the Ideal Geo-Atmosphere's and either of the 1962 Standard Atmosphere's geopotential or geometric altitudes for particular pressures are seen in the third and fifth columns of Table 3 to be less than the order of 100 feet throughout the range of altitudes up to the 20-kilometer or 65,000-foot level — and throughout which the 1954 Standard Atmosphere of the International Civil Aviation Organization and the 1962 U.S. Standard Atmosphere are practically equivalent and are currently used to calibrate pressure altimeters. Such differences are practically negligible for most current aeronautical purposes — and the much more conveniently useful Ideal Geo-Atmosphere could well be adopted for higher altitudes before supersonic and/or aerospaceplane flights at such higher altitudes become commonplace.

8. Conclusions

In concluding summary, the newly formulated Geospheric System of Units and the associated Ideal Geo-Atmosphere are thereby seen to offer a very promising practical way to coordinate the vertical positions of things in and of the atmosphere in and between meteorological and aeronautical operations throughout the world.

The evolution toward such fully coordinated world-wide practices could well be initiated by first utilizing them in meteorological practice. The basic meteorological advantage of doing so would be to provide for establishing mandatory reporting and mapping levels at the uniform and highly subdivisible 1-geomile intervals of pressure-altitude listed in Table 4 — instead of at the virtually random intervals listed in the last two columns of Table 3 between those few levels to which geostrophic and gradient wind considerations are currently restricted by the difficulty of hydrostatic computations in terms of numbers of millibars.

The conversion of the current world-wide aeronautical usage of English feet to the much more usefully and logically Earth-related geofeet, "siles" and geomiles could then well be deferred until flyers throughout the world have become thoroughly familiar with their meteorological usage. During that transition period, the differences of the vertical positions of round numbers of 100 English-foot flight-levels and of 100-geofoot or "sile" mapping levels listed in the last three columns of Table 4 would need to be taken into explicit account in only those relatively rare instances that precise comparisons at the higher altitudes might be required.

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Table 1.

STANDARD & OBSERVED TEMPERATURES

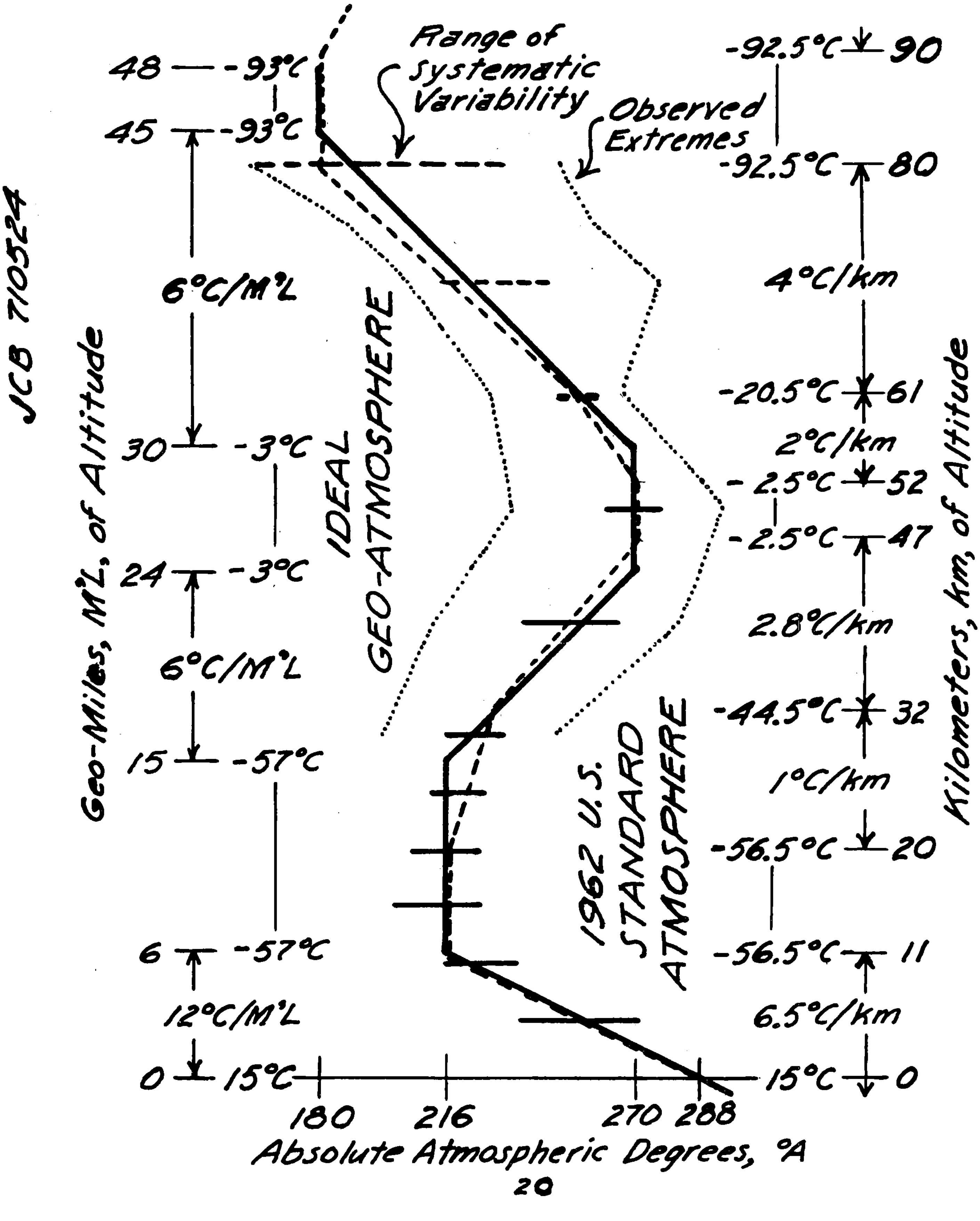

Table 2

NUMERICAL DEFINITION of the IDEAL GEO-ATMOSPHERE

$dp/dA = -\rho g$; $A = r - r_0$; $\rho = p/RT$; $g = g_0 r_0^2/r^2$ or $dp/p = -(g_0 r_0^2/R)\,dr/r^2T$

r_0 = 1 radile, R'L = $4 \cdot 10^7/2\pi$ meters $= 2^3 \cdot 3^3 \cdot 10^2/2\pi$ or 3437.75 geomiles, M'L

$g_0 = 10^5/2 \cdot 3^6 \cdot 7$ m/s^2 = 400/21 geoknots = 2000/63 geofeet per second squared

$R = p_0/\rho_0 T_0 = (2 \cdot 3^4 \cdot 5^4\ N/m^2)/(5^2 \cdot 7^2/10^3\ kg/m^3)(288)$ or $3^2 \cdot 10^5/2^6 \cdot 7^2\ m^2/s^2 \cdot {}^\circ A$

$L^* = g_0/R = 2^5 \cdot 7/3^8\ {}^\circ C/m = 2^4 \cdot 7 \cdot 10^5/3^{11}\ {}^\circ C/M'L$; $\log_{10} e = 0.43429\ 44819$

"Absolute Atmospheric Degrees" of Temperature, $T/{}^\circ A = T/{}^\circ C + 273$

In Constant Lapse Rate, $L = dT/dr$, Regions above a level where $r = r_i = r_0 + A_i$; $T = T_i$ and $p = p_i$

$$T = T_i + L(r - r_i) = L[r - (r_0 + h)]; \qquad h = (r_i - T_i/L) - r_0$$

$$\log\frac{p}{p_i} = -\frac{L^*}{L}\frac{1}{(1+h/r_0)^2}\log\frac{T}{T_i}\frac{1+A_i/r_0}{1+A/r_0} + \frac{L^*}{L}\frac{\log e}{r_0(1+h/r_0)(1+A_i/r_0)}\frac{A-A_i}{1+A/r_0}$$

In Isothermal Regions with $T = T_i$ above a level where $r = r_i = r_0 + A_i$ and $p = p_i$

$$\log\frac{p}{p_i} = -\frac{L^*}{T_i}\frac{\log e}{1+A_i/r_0}\frac{A-A_i}{1+A/r_0}$$

For numbers: A/M'L of geomiles of Altitude; T/°A of Temperature; p/millibars of pressure:

$A \le 6$	$T = 288 - 12A$	$\log p = \log 1012.50$	$+ 5.19589 \log\frac{T}{288}\frac{1}{1+A/r_0}$	$- 0.000661\frac{A}{1+A/r_0}$
$6 \le A \le 15$	$T = 216$	$\log p = \log 223.014$		$- 0.126889\frac{A-6}{1+A/r_0}$
$15 \le A \le 24$	$T = 216 + 6A$	$\log p = \log 16.2632$	$- 10.6673 \log\frac{T}{216}\frac{1+15/r_0}{1+A/r_0}$	$+ 0.001334\frac{A-15}{1+A/r_0}$
$24 \le A \le 30$	$T = 270$	$\log p = \log 1.53951$		$- 0.100991\frac{A-24}{1+A/r_0}$
$30 \le A \le 45$	$T = 270 - 6A$	$\log p = \log 0.386079$	$+ 10.0922 \log\frac{T}{270}\frac{1+30/r_0}{1+A/r_0}$	$- 0.001292\frac{A-30}{1+A/r_0}$
$45 \le A \le 48$	$T = 180$	$\log p = \log 5.90859/10^3$		$- 0.150573\frac{A-45}{1+A/r_0}$

Table 3

COMPARISON OF PRESSURE-ALTITUDES

Millibars of Pressure	1962 Std. Geopotential Meters	Δ	Meters of Geo-Altitude	Δ	1962 Std. Geometric Meters	Geo-Miles of Geo-Altitude	Geo-Miles of Thickness
1000	111	6	105	6	111	0.057	0.727
850	1,457	5	1,452	5	1,457	0.784	0.940
700	3,012	4	3,008	5	3,013	1.624	1.386
500	5,574	1	5,575	4	5,579	3.010	0.872
400	7,185	-4	7,189	4	7,193	3.882	1.072
300	9,164	-10	9,174	3	9,177	4.954	0.650
250	10,363	-15	10,378	2	10,380	5.604	0.770
200	11,748	-20	11,804	2	11,806	6.374	0.986
150	13,608	-22	13,630	7	13,637	7.360	1.391
100	16,180	-26	16,206	15	16,211	8.751	1.226
70	18,442	-34	18,476	19	18,495	9.977	1.156
50	20,576	-41	20,617	26	20,643	11.133	0.768
40	22,000	-39	22,039	37	22,076	11.901	0.990
30	23,849	-23	23,872	66	23,938	12.891	0.627
25	25,029	-4	25,033	95	25,128	13.518	0.766
20	26,481	+29	26,452	140	26,592	14.284	
16.2632	27,834	+57	27,777.•••	180	27,957	15	
1.53951	44,437	- 7	44,444.•••	306	44,750	24	
0.386079	55,321	-234	55,555.•••	253	55,808	30	
0.00590859	81,881	-1452	83,333.•••	-281	83,052	45	
0.00211828	86,409	-2479	88,888.•••	-260	88,628	48	

Table 4

PROPOSED CONSTANT PRESSURE MAPPING LEVELS

Geo-Miles of Geo-Altitude M'L	Geo-Altitude Reference Temperature °C	°A	°A	Millibars of Pressure mb	Meters of Geo-Altitude m	1962 Std. Geopotential Flight Levels 100's of feet	Δ	Geo-Siles of Geo-Altitude S'L
0	15	288	$2^5 \cdot 3^2$	1012.5	0	0.2	0.2	0
1	3	276	$2^2 \cdot 3 \cdot 23$	809.171	1,851.851•••	60.9	0.9	60
2	- 9	264	$2^3 \cdot 3 \cdot 11$	640.348	3,703.703•••	121.6	1.6	120
3	-21	252	$2^2 \cdot 3^2 \cdot 7$	501.333	5,555.555•••	182.2	2.2	180
4	-33	240	$2^3 \cdot 3 \cdot 10$	387.894	7,407.407•••	242.8	2.8	240
5	-45	228	$2^2 \cdot 3 \cdot 19$	296.246	9,259.259•••	303.4	3.4	300
6	-57	216	$2^3 \cdot 3^3$	223.014	11,111.111•••	364.0	4.0	360
7	"	"	"	166.607	12,962.962•••	424.6	4.6	420
8	"	"	"	124.487	14,814.814•••	485.3	5.3	480
9	"	"	"	93.0316	16,666.666•••	545.9	5.9	540
10	"	"	"	69.5361	18,518.518•••	606.4	6.4	600
11	"	"	"	51.9832	20,370.370•••	667.0	7.0	660
12	"	"	"	38.8677	22,222.222•••	727.8	7.8	720
13	"	"	"	29.0662	24,074.074•••	789.1	9.1	780
14	"	"	"	21.7440	25,925.925•••	850.9	10.9	840
15	-57	216	$2^3 \cdot 3^3$	16.2632	27,777.777•••	913.3	13.3	900