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SINGLE-DEGREE-OF-FREEDOM ROLL RESPONSE DUE TO TWO-DIMENSIONAL VERTICAL GUSTS

By John C. Houbolt and Asim Sen

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Work carried out in part of NASA Contract NAS1-9200 by

Aeronautical Research Associates of Princeton, Inc.
50 Washington Road, Princeton, New Jersey 08540

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ABSTRACT

The single-degree-of-freedom of roll response due to encountering vertical gusts which are random in the spanwise direction as well as the flight direction is studied. Cross-spectral functions associated with the von Kármán spectral function are used to evaluate the roll response. Results analogous to the results that apply in the case of vertical motion response only are found. It is shown that the wing tip acceleration due to roll for the two-dimensional turbulence case can be greater than the vertical acceleration that is found for the vertical motion reference case. An interesting possible means for evaluation of gust severity σ_w and turbulence scale L from vertical and rolling accelerations only is developed.

INTRODUCTION

An assumption commonly made in treating the response of aircraft in a random gust environment is to assume the gusts are uniform in the spanwise direction. Some studies, however, references 1-5, have examined the effect of treating the gusts to be random in the spanwise direction as well as in the flight direction. The single-degree-of-freedom case of pure roll does not, however, seem to have been given explicit consideration. Conceptually, both vertical gust and lateral gust effects on the rudder can induce excitation of the pure roll degree of freedom. For the vertical gusts, roll cannot develop of course when the gusts are considered to be uniform in the spanwise direction. Treatment in terms of a two-dimensional vertical gust field is necessary.

The purpose of this report is to treat the single-degree-of-freedom roll case due to vertical gusts alone. The main objective is to establish the extent to which rolling motion is induced by the randomness of the gusts in the spanwise direction. A comparison of the vertical acceleration at the wing tip due to roll with the c.g. acceleration of the airplane that results in the case of single-degree-of-freedom of vertical motion only (uniform spanwise gusts) is the measure used herein to assess the rolling motion.

SYMBOLS

a	slope of the lift curve
b	aircraft span
c	wing chord
c_o	mean aerodynamic chord
$F_y(\omega)$	Fourier transform of function y
g	acceleration of gravity
I_x	rolling moment of inertia of airplane
k	reduced frequency $\frac{\omega c}{2V}$
K_ϕ	alleviation factor for vertical acceleration
K_r	alleviation factor for rolling acceleration
L	turbulence scale
m	airplane mass
r	radius of gyration of aircraft in roll
S	wing area
V	velocity of aircraft
w	vertical gust velocity
W	aircraft weight
y	spanwise coordinate
z_t	wing tip deflection due to roll
λ	gust wavelength
μ	mass parameter
μ_r	"mass parameter" for roll
ρ	air density
σ_w	rms value of vertical gust velocities

$\sigma_{\Delta n}$ rms value of c.g. vertical acceleration

$\sigma_{\Delta n_t}$ rms value of wing tip vertical acceleration due to roll

ϕ angle of roll

$\phi_{\Delta n}$ power spectrum of c.g. vertical acceleration

$\phi_{\Delta n_t}$ power spectrum of wing tip vertical acceleration due to roll

ϕ_w power spectrum of vertical gust velocities

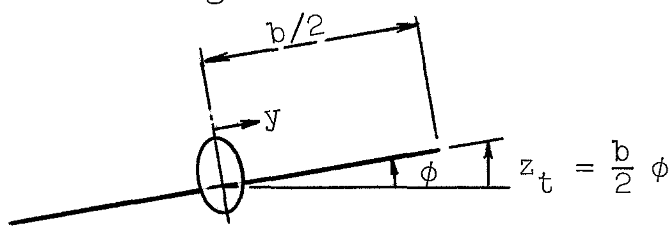
ϕ_{12} cross-spectral of vertical gust velocities between path 1 and path 2

ω circular frequency

Ω spatial frequency $\frac{\omega}{V}$

ROLLING EQUATION

Consider the airplane to have the single-degree-of-freedom of roll, as depicted in the following sketch



Development in terms of a strip theory approach is considered adequate for present purposes. Through use of the aerodynamic and response relations given in references 6-7, the equation for rolling motion may be shown to be

$$I\ddot{\phi} = -\frac{a}{2}\rho V(F + iG)\dot{\phi}\int_{-b/2}^{b/2} cy^2 dy + \frac{a}{2}\rho V(P + iQ)\int_{-b/2}^{b/2} cywdy \qquad (1)$$

where sinusoidal motion is implied. The first term on the right-hand side is the rolling moment due to roll rate, the second term is the rolling moment generated by the vertical gusts w ; $F + iG$ represents the Theodorsen lift function for an oscillating airfoil, $P + iQ$ is the corresponding type function for a sinusoidal gust.

We introduce the wing tip vertical acceleration

$$\Delta n_t = \frac{b}{2g} \ddot{\phi} \tag{2}$$

With equation (1) and this relation we can derive, after a little manipulation, the following relation for the frequency response function for Δn_t

$$F_{\Delta n_t}(\omega) = \frac{a\rho SV}{2W} \frac{4\mu_r(P + iQ)}{4\mu_r + \frac{2G}{k} - i\frac{2F}{k}} \frac{b^2}{8r^2} \int_{-1}^{1} \frac{c}{c_o} \frac{2y}{b} F_w(y,\omega)\, d\left(\frac{y}{b/2}\right) \tag{3}$$

where

$$\mu_r = \frac{8r^2}{b^2 I_c} \mu$$

in which r is the radius of gyration associated with the rolling moment of inertia $(I_x = mr^2)$, b is the wing span, μ the mass ratio $\frac{2W}{a\rho cgS}$, and I_c is an aerodynamic rolling integral defined as

$$I_c = \int_{-1}^{1} \frac{c}{c_o} \left(\frac{y}{b/2}\right)^2 d\left(\frac{y}{b/2}\right)$$

The chord c_o represents the mean aerodynamic chord of the wing. From equation (3) the spectrum for Δn_t follows as

$$\phi_{\Delta n_t} = \left(\frac{a\rho SV}{2W}\right)^2 f_1(k) f_2(k) f_3(k) \phi_w$$

$$= \left(\frac{a\rho SV}{2W}\right)^2 \sigma_w^2\, \phi_1(k) \tag{4}$$

where $\phi_1 = f_1 f_2 f_3 \frac{\phi_w}{\sigma_w^2}$; the specific f functions are given by

$$f_1(k) = \frac{16\mu_r^{\,2}}{\left(4\mu_r + \frac{2G}{k}\right)^2 + \left(\frac{2F}{k}\right)^2} \qquad (5)$$

$$f_2(k) = P^2 + Q^2 \qquad (6)$$

$$f_3(k) = \left(\frac{b^2}{8r^2}\right)^2 \int_{-1}^{1}\int_{-1}^{1} \frac{c(y)}{c_o}\,\frac{c(\eta)}{c_o}\,\frac{y}{b/2}\,\frac{\eta}{b/2}\,\frac{\phi_{12}(x-\eta,k)}{\phi_w(k)}\,d\left(\frac{y}{b/2}\right)d\left(\frac{\eta}{b/2}\right) \qquad (7)$$

in which ϕ_{12} is the cross-spectra for vertical gust velocities associated with a path separation distance of $x - \eta$ (reference 5), and ϕ_w is the point spectral function (references 5 and 8).

We wish to note the particular form that equation (4) has been expressed. Care has been taken here to write the equation so as to have a correspondence with the spectral equation that applies for c.g. vertical acceleration for an airplane with vertical motion only and with uniform spanwise gusts. Relative to this case, only two differences occur; the function f_3, associated with induced rolling power, appears as an additional function, and a modified mass parameter μ_r is found to take the place of the mass parameter μ. Thus equation (4) is precisely the vertical response equation if $f_3 = 1$ and μ is used in place of μ_r. Response results very similar in form to the vertical motion case can therefore be expected for the rolling case being treated.

The rms value of acceleration at the wing tip due to roll follows from equation (4) as

$$\sigma_{\Delta n_t} = \frac{a\rho SV}{2W} K_r \, \sigma_w \qquad (8)$$

where

$$K_r = \left[\int_0^{k_c} \phi_1(k)\,dk\right]^{1/2} \qquad (9)$$

Equation (8) for wing tip acceleration due to roll is precisely the same form as has been developed for the vertical motion only case, reference 8. The only difference is in the factor K_r, which takes the place of the vertical alleviation factor K_ϕ. Results for K_r are developed in the next section.

EVALUATION OF K_r

To establish the magnitude of the wing tip acceleration that results from rolling response, specific application of equations (4) and (9) was made to the wing planform shown in figure 1(a). To evaluate the function f_3, equation (7), the wing was divided into equal spanwise intervals as shown in figure 1(b). Application of equation (7) in terms of this lumped area system yields the following result for the wing under consideration

$$f_3(k) = \left(\frac{b^2}{8r^2}\right)^2 \left[2(r_5^2 + r_6^2 + r_7^2) + 4(r_5 r_6 + r_6 r_7)\frac{\phi_{12}}{\phi_w} \right.$$

$$+ (4r_5r_7 - 2r_5^2)\frac{\phi_{13}}{\phi_w} - 4r_5r_6\frac{\phi_{14}}{\phi_w}$$

$$\left. - (4r_5r_7 + 2r_6^2)\frac{\phi_{15}}{\phi_w} - 4r_6r_7\frac{\phi_{16}}{\phi_w} - 2r_7^2\frac{\phi_{17}}{\phi_w}\right] \qquad (10)$$

where $r_n = \frac{2}{7}\,\frac{c_n}{c_o}\,\frac{y_n}{b/2}$, and where ϕ_{1n} is the cross-spectral function associated with a separation distance $s = (n-1)\,\frac{b}{7}$.

In the derivation of equation (10), use was made of the fact that

$$r_1 = -\,r_7$$

$$r_2 = -\,r_6$$

$$r_3 = -\,r_5$$

The equation represents a coarse interval numerical integration of equation (7), but is considered sufficiently accurate to yield representative results. The use of finite span intervals implies that the gust velocities are uniform over the span interval but that they differ from interval to interval as represented by the cross-spectral functions. In the evaluation of equation (10) the following reciprocal properties of the cross-spectral were used

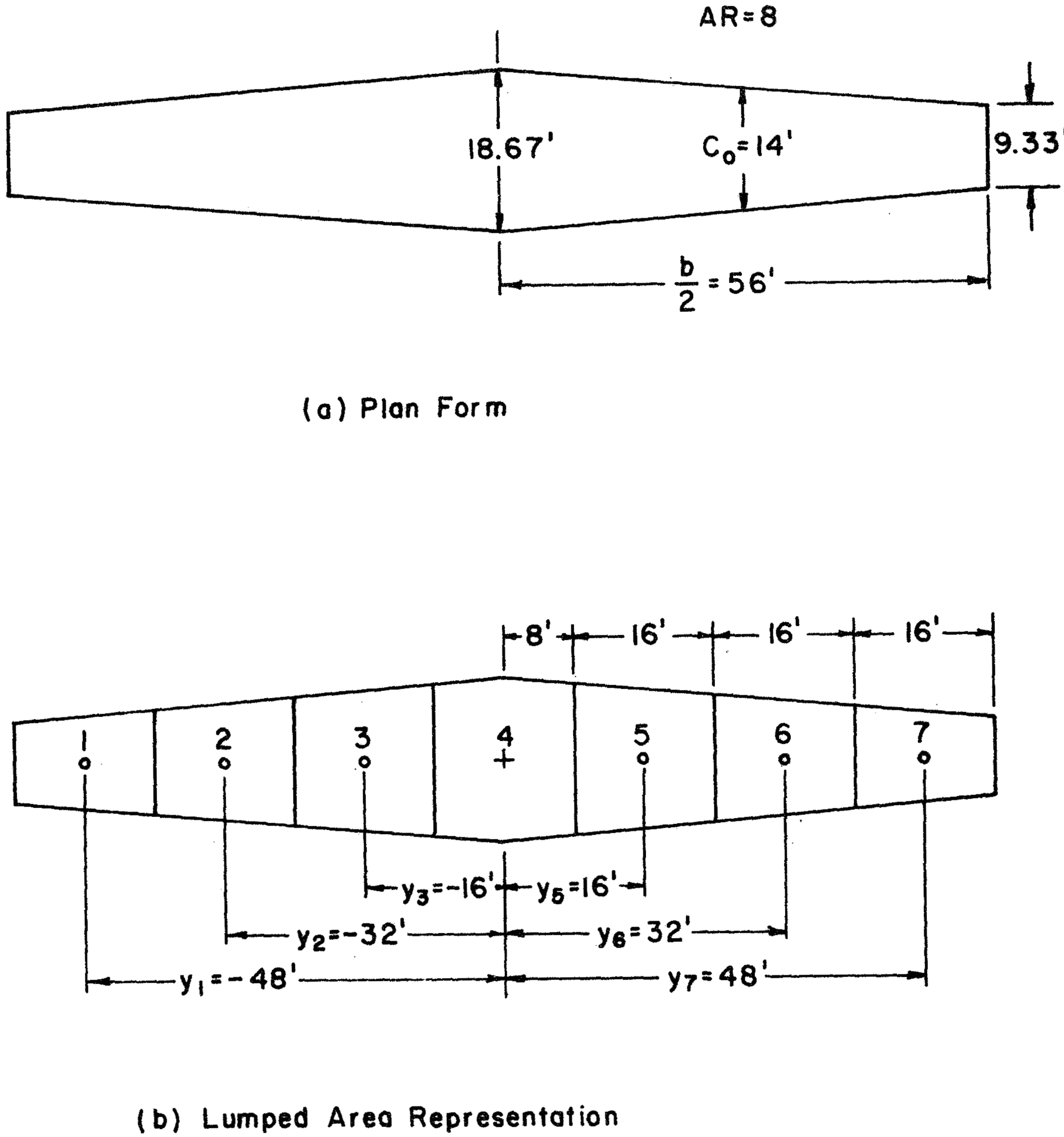


Figure 1. Example Wing Used in Analysis.

$$\phi_{12} = \phi_{21} = \phi_{23} = \phi_{32} = \phi_{56} = \phi_{65} = \phi_{67} = \phi_{76}$$

$$\phi_{13} = \phi_{31} = \phi_{35} = \phi_{53} = \phi_{57} = \phi_{75}$$

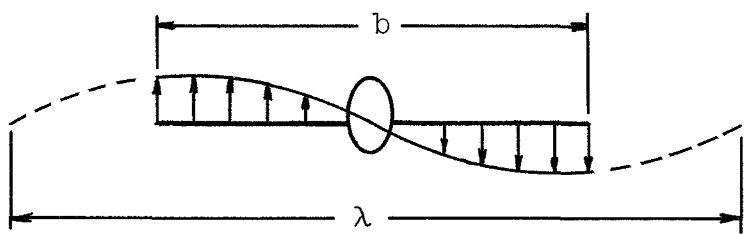
$$\phi_{14} = \phi_{41} = \phi_{25} = \phi_{52} = \phi_{36} = \phi_{63}$$

$$\phi_{15} = \phi_{51} = \phi_{26} = \phi_{62} = \phi_{37} = \phi_{73}$$

$$\phi_{16} = \phi_{61} = \phi_{27} = \phi_{72}$$

$$\phi_{17} = \phi_{71}$$

The evaluated result for f_3 for the wing of figure 1, using the cross-spectral results developed in reference 5, is shown in figure 2. Several interesting points are to be noted. The function is noted to be dependent on the scale value L at low value of k, but is independent of L at large k; for large L the function becomes independent of L over the whole range of k. The peak in the function at k of about .28 can be identified with the following physical interpretation that is associated with the nonuniformity of the gusts in the spanwise direction. Of the various frequency components in the spanwise direction, there are some that would tend to produce maximum rolling power, as the following sketch depicts



The gust wavelength λ is related to k according to the relations

$$k = \frac{\omega c}{2V} = \Omega \frac{c}{2} = \frac{\pi c}{\lambda}$$

For a $\frac{\lambda}{b} = 1.5$ (a representative choice), k becomes

$$k = \frac{\pi}{1.5A}$$

where A is the aspect ratio of the wing. For the $A = 8$ aspect ratio wing under consideration, this relation yields $k = .26$, which is noted to agree very well with the value of k at which the peak of f_3 occurs.

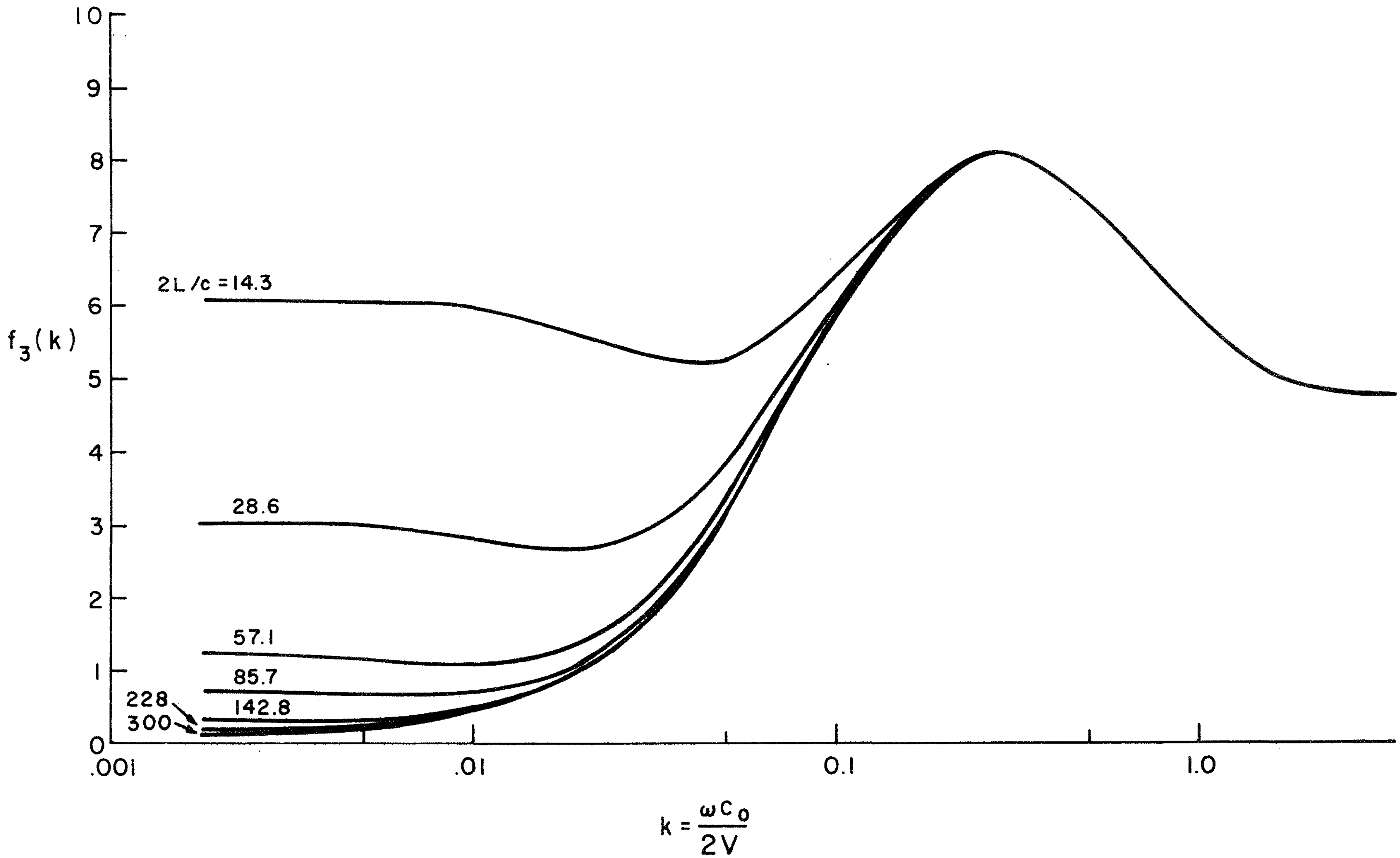


Figure 2. Function f_3 for Example Wing.

From the f_3 function given by equation (10), the f_2 and ϕ_w expressions given in reference 8, and the ϕ_1 function as given in equation (4), the K_r function was evaluated by equation (9); the results, representing the basic results of the present paper, are shown in figure 3. This figure, in conjunction with equation (8), also shown on the figure, allows wing tip acceleration due to roll to be computed just as simply as vertical acceleration is computed through means of the alleviation factor K_ϕ .

An indication of the magnitude of the wing tip acceleration due to roll is afforded by the following example. The value of I_c for the wing shown in figure 1 is found to be $I_c = .552$. A representative value of r is $r = .15b$; the rolling mass parameter μ_r thus becomes $\mu_r = .326\mu$. If the airplane has a mass parameter $\mu = 40$ then $\mu_r = 13$. Assume $\frac{2L}{c} = 100$; then from figure 3 we find that $K_r = .95$. By contrast, for the same μ and $\frac{2L}{c}$ values, we find (reference 8) that the alleviation factor for the vertical motion case is $K_\phi = .72$. Thus we see that the rms wing tip acceleration due to the 2-d gust rolling effect is even slightly higher than the rms value for c.g. vertical acceleration for the vertical motion only case; specifically, in this case, in the ratio .95 over .72 .

The results shown in figure 3 are representative of wings of large aspect ratio (neighborhood of 8). For wings of small aspect ratio, such as delta wings, general trends may be expected to be similar, but numerically the results appear to decrease in value. With reference to figure 2, the peak of the f_3 curve shifts outward as the aspect ratio goes down. This would lead to K_r curves of the same type as in figure 3, but will lower values.

EVALUATION OF σ_w AND L

The striking analogy between the results developed herein and the results for the vertical motion only case suggests a rather simple way to deduce gust severity σ_w and turbulence scale value L directly from only the acceleration values that are measured in flights through rough air. Suppose that a more refined analysis than used herein, involving a better and more complete description of the lateral motion response of the airplane, were used to evaluate the K_r curves of figure 3. Suppose also that more realistic K_ϕ curves were also established for the vertical motion only case, using appropriate longitudinal dynamics. Figure 4 depicts, say, these more accurately established K_ϕ and K_r curves. In terms of these K values the c.g. vertical acceleration and wing tip acceleration due to roll would read

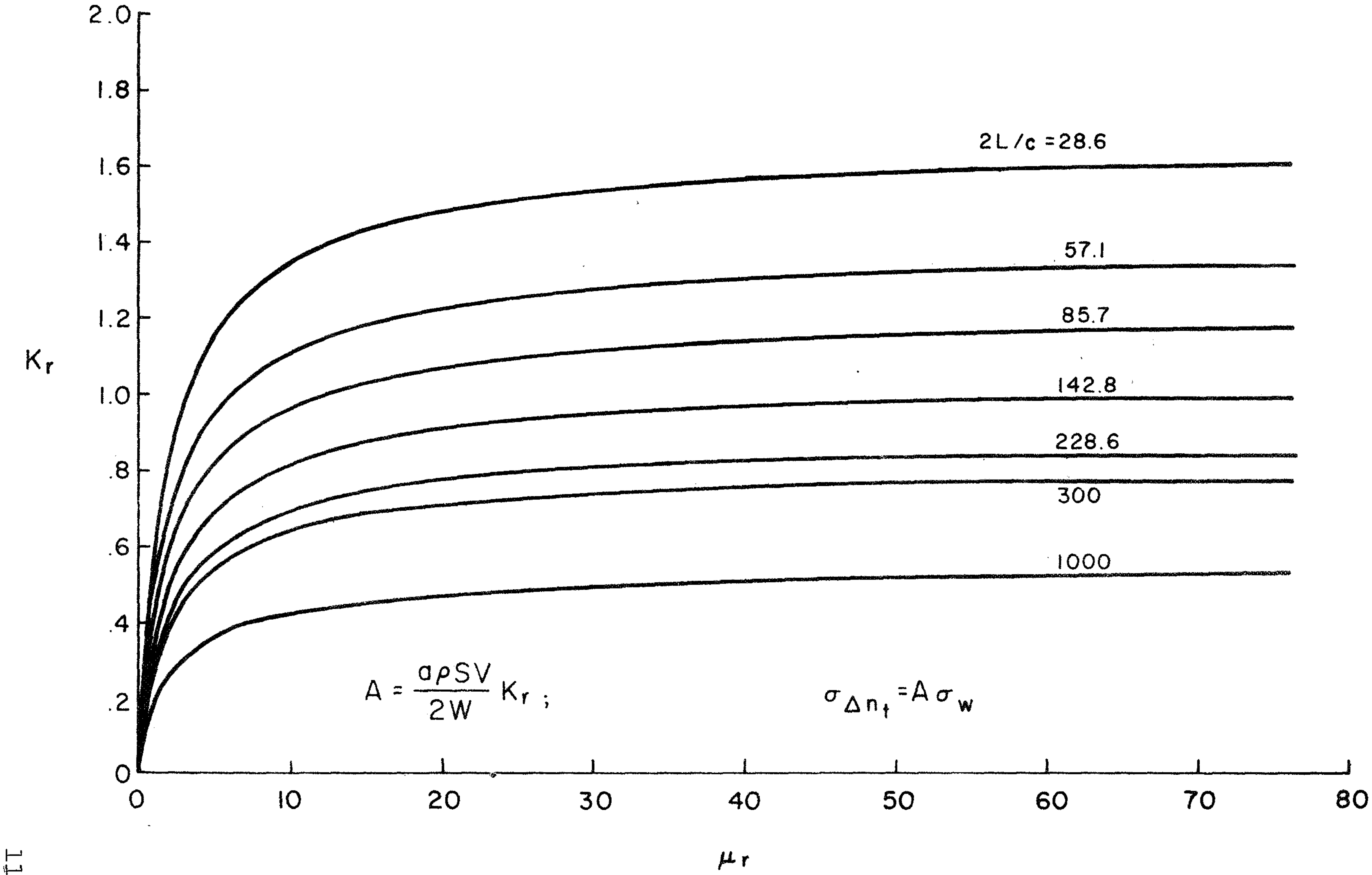


Figure 3. Alleviation Factor for Wing Tip Acceleration Due to Roll (nonuniform) spanwise gusts).

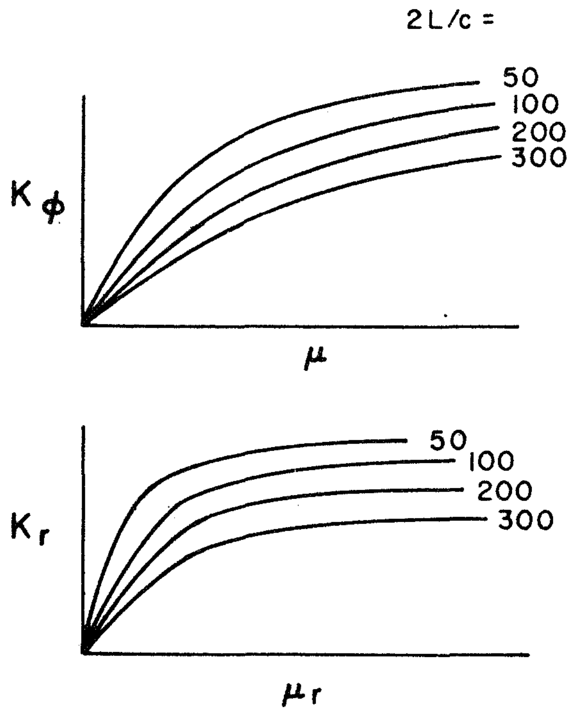


Figure 4. K_ϕ and K_r Values (Assumed Accurately Evaluated).

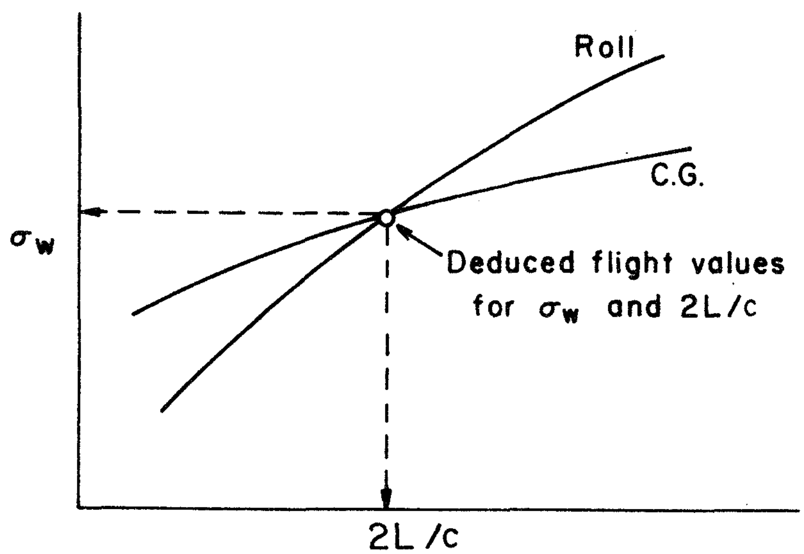


Figure 5. Deduction of σ_w and $\frac{2L}{c}$ from Vertical and Roll Accelerations Only.

$$\sigma_{\Delta n} = \frac{a\rho SV}{2W} K_{\phi}\left(\mu, \frac{2L}{c}\right) \sigma_w \qquad (11)$$

$$\sigma_{\Delta n_t} = \frac{a\rho SV}{2W} K_{r}\left(\mu, \frac{2L}{c}\right) \sigma_w \qquad (12)$$

For a given airplane and a given flight through rough air, σ_w and L as appearing in these equations must be the same. Division of equation (11) by equation (12) thus yields

$$\frac{\sigma_{\Delta n}}{\sigma_{\Delta n_t}} = \frac{K_{\phi}\left(\mu, \frac{2L}{c}\right)}{K_{r}\left(\mu, \frac{2L}{c}\right)}$$

Since $\sigma_{\Delta n}$ and $\sigma_{\Delta n_t}$ are the flight measured values, the only unknown appearing in this expression is $\frac{2L}{c}$. This value can be found simply by assuming various values until one is found that causes the expression to be satisfied. With $\frac{2L}{c}$ established, either of equations (11) or (12) may in turn be used to establish σ_w.

Another procedure is one that allows σ_w and L to be established simultaneously. With the measured value of $\sigma_{\Delta n}$, we use equation (11), assume several values of $\frac{2L}{c}$, and evaluate σ_w so as to satisfy the equation; from this evaluation we make a plot of σ_w vs. $\frac{2L}{c}$, such as the curve labelled c.g. in figure 5. The process is repeated using equation (12), yielding the curve marked roll. The intersection thus establishes the σ_w and $\frac{2L}{c}$ values for the gusts encountered.

CONCLUDING REMARKS

Gust encounter involving gusts which are considered random in the spanwise direction, as well as the flight direction, is considered further herein. The specific case treated is that of an airplane having the single-degree-of-freedom of roll only, encountering vertical gusts which are random in both the span and flight directions. It is shown that the wing tip acceleration due to rolls for the 2-d gust structure case is of the same order of magnitude as the c.g. vertical acceleration for the commonly considered case of vertical motion only (uniform spanwise gusts).

In connection with the use of complete lateral dynamic effects, as mentioned in the section dealing with the determination of L, it is considered worthwhile to establish how rolling acceleration is influenced by individual lateral dynamic effects. In particular, side slip and yaw, dihedral effect, roll due to rudder, lateral turbulence, and spanwise variation in longitudinal turbulence should be investigated to establish their relative importance.

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