# EXTENSIONS OF THE SPACE TRAJECTORIES ERROR ANALYSIS PROGRAMS 


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## Preface

The objectives of Contract NAS5-11873 are twofold. First, the computational capabilities of the Space Trajectories Error Analysis Programs (STEAP) are to be extended by incorporating the capability to perform generalized covariance analyses and to target the multiple probe and bus entry events for a Planetary Explorer type mission. The second objective is to modify the Lander Trajectory reconstructions (LTR) program developed for the Viking mission for use on a Planetary Explorer type mission.

During the reporting period the incorporation of the generalized covariance analysis capability was completed. Several example cases are presented to indicate the results that can be obtained from such an analysis. New control parameters and new target parameters have been incorporated into the STEAP targeting algorithm to facilitate targeting Planetary Explorer type missions. Also the logic to be used for mini-probe targeting is described. The conversion of the LTR program to handle Planetary Explorer type missions is progressing satisfactorily. An integrator instability problem is discussed and a solution is proposed. It is recommended that the integrator instability problem be circumvented by switching to a quasi-static model of the equations of motion when the entry vehicie reaches terminal velocity in the lower atmosphere of Venus.

## Introduction

During the reporting period work progressed on each of the three main contract tasks. Task one calls for the incorporation of a generalized covariance analysis capability into the existing STEAP programs. Task two is to provide the capability in STEAP for targeting the main-probe, the mini-probe and the bus for a Planetary Explorer type mission. The third task is to convert the LTR program developed for the Viking mission for use on Venus missions. The progress to date for each of the three tasks are described in the following sections.

## Task 1: Generalized Covariance Analysis

The generalized covariance analysis technique has been incorporated into the STEAP error analysis program ERRAN. The checkout phase is complete; all sample cases have been run. The Analytical Manual documentation for generalized covariance analysis has been completed. The primary generalized covariance subroutines GNAVM and MEAN, for propagating actual 2nd moment matrices and means, respectively, have also been documented.

The sample cases used to checkout the generalized covariance program are summarized below:
A. Viking Mars approach trajectory: mismatch between assumed and actual doppler noise statistics.
B. Venus approach trajectory (1977):

1. Mismatch between assumed and actual station location statistics. Station location errors treated as consider parameters. Actual statistics for all runs; spin axis distance, $\sigma_{\text {R }}^{\prime}=4.5 \mathrm{~m}$; z-height, $\sigma_{z}^{\prime}=0$; longitude, $\sigma i \lambda=9 \mathrm{~m}$.
a. assumed: $\sigma_{R S}=3 \sigma_{R S}^{\prime}, \sigma_{\lambda}=3 \sigma_{\lambda}^{\prime}, \rho_{\lambda}=0$
actual: longitude correlation, $\rho_{\lambda}^{\prime}=0$
b. assumed: $\sigma_{\mathrm{RS}}=\sigma_{\mathrm{RS}}^{\prime}, \sigma_{\lambda}=\sigma_{\lambda}{ }^{\prime}, \rho_{\lambda}=0$ actual: $\quad \rho_{\lambda}{ }^{\prime}=0$
c. assumed: $\sigma_{R S}=\frac{1}{3} \sigma_{R S}^{\prime}, \quad \sigma_{\lambda}=\frac{1}{3} \sigma_{\lambda}^{\prime}, \rho_{\lambda}=0$ actual: $\quad \rho_{\lambda}{ }^{\prime}=0$
d. assumed: $\quad \sigma_{R S}=\frac{1}{9} \sigma_{R S}^{\prime}, \sigma_{\lambda}=\frac{1}{9} \sigma_{\lambda}^{\prime}, \rho_{\lambda}=0$ actual: $\quad \rho_{\lambda}{ }^{\prime}=0$
e. assumed: $\sigma_{\mathrm{RS}}=\sigma_{\mathrm{RS}}, \quad \sigma_{\lambda}=\sigma_{\lambda}^{\prime}, \rho_{\lambda}=0$ actual: $\quad \rho_{\lambda}{ }^{\prime}=.99$
f. assumed: $\sigma_{\mathrm{RS}}=\sigma_{\mathrm{RS}}{ }^{\prime}, \sigma_{\lambda}=\sigma_{\lambda}{ }^{\prime}, \sigma_{\lambda}=.99$ actual: $\quad \rho_{\lambda}{ }^{\prime}=0$
2. Doppler bias uncertainty ignored by filter. Actual and assumed station location statistics identical. The actual standard deviations used for the ignored doppler bias in each of the cases are listed below:
a. $\sigma_{b}{ }^{\prime}=1 \mathrm{~mm} / \mathrm{sec}$
b. $\sigma_{b}^{\prime}=10 \mathrm{~mm} / \mathrm{sec}$

$$
\begin{aligned}
& \text { c. } \sigma_{b}^{\prime}=30 \mathrm{~mm} / \mathrm{sec} \\
& \text { d. } \sigma_{b}^{\prime}=100 \mathrm{~mm} / \mathrm{sec}
\end{aligned}
$$

3. Earth/Vemus trajectory (1977 launch window): Demonstrates utility of generalized covariance analysis as applied to midcourse guidance. Three guidance events executed: fixed-time-of-arrival (5 days), two-variable B-plane ( 64 days), and three-variable B-plane ( 115.8 days). Mismatch between assumed and actual injection covariance, consider parameter statistics, ind execution error statistics. Actual execution error statistics were non-zero mean.

Typical generalized covariance analysis results will be presented and discussed next. Sample results for cases B.1.a, b, c, and dare presented in the first figure. Shown in the figure are the maximum actual velocity estimation error uncertainties during the latter phase of the Venus approach phase for the four different filter design. The top curve shows the actual estimation error uncertainties for the conservative design (case B.1.a). The next curve corresponds to the moderate filter design (case B.1.b). Actual estimation errors can be reduced even further if the filter is designed slightly optimistically (case B. 1.c). However, the downward trent is reversed if the filter design becomes overly optimistic (case B.1.d). This designt is less satisfactory than the slightly optimistic design for the entire approach phase and, after the sphere of inflwence (SOI) is pierced, generates actual estimation errors which exceed those generated with the moderate filter design. Although only the velocity estimation error uncertainties are shown here, the position estimation error uncertainties exhibit the same behavior, but to a less dramatic extent. The filter-generated estimation error statistics are not shown in this plot. As expected, these estimation error statistics always decrease as the filter design goes from conservative to overly optimistic.

In the second figure are shown both actual and filter maximum position estimation error statistics for the case when doppler bias uncertainties are completely ignored by the filter (case B.2.b). The behavior shown in the figure appears plausible. Prior to penetrating the sphere of influence, the spacecraft velocity is not changing rapidly and, as a result, (doppler) observability is reduced. Consequently, ignoring the doppler bias uncertainty during this phase can be detrimental. But after the sphere of influence has been pierced, the spacecraft velocity begins to change rapidly, both in magnitude and direction, so that (doppler) observability increases. In this situation, neglecting the doppler bias uncertainty appears to be of no consequence. Case B.2.a showed no signi-


ficant differences between actual and filter estimation error statistics. Cases B. 2.C and B.2.d showed increased separation between actual and filter statistics.

An additional gain generator has also been developed, programmed and checked out in both the generalized covariance and simulation programs. This additional gain generator is the equivalent recursive weighted-least-squares consider gain generator. The prediction event in both programs has also been extended to transform cartesian uncertainties to B-plane uncertainties whenever a prediction to a point inside the target planet sphere of influence is made. Finally, the azimuth and elevation of the spacecraft relative to the tracking station is also computed whenever a measurement is processed.

## Task 2: Entry Probe Separation Simulation

Modifications to the STEAP II interplanetary targeter, enabling it to target Planetary Explorer main probes, have been completed and are well into the checkout phase. The required changes were of two basic types. First, an option was added for using more expeditious controls. Rather than targeting with the three Cartesian components of inertial velocity, one may now use the magnitude, an in-plane rotation angle, and an out-of-plane rotation angle of the velocity relative to the launch planet. This latter set of controls has produced mich more rapid convergence in the targeting of typical Planetary Explorer missions at injection. Second, new target options had to be provided to facilitate targeting probes to prescribed radius, right ascension, and declination in either the subsolar-orbit-plane or equatorial planetocentric coordinate frames.

The executive structure of the miniprobe targeting algorithm has been layed out, and detailed work on its individual subroutines is underway. A two-stage targeting procedure has been decided upon. In the first stage, the minimum-miss probe release controls will be obtained for the conic approximation to the miniprobe trajectories. In the second and optional stage, the same controls are determined for the virtual mass approximation starting from the optimal conic controls. The optimization routine wi. 11 be either a quadratically-convergent descent routine or a pseudo-inverse least-squares routine due originally to Gauss. Choice between the two methods will be on the basis of actual computer time studies.

Miniprobe targeting will be carried out then in three basic subroutines. The first of these is the executive routine MPRTGR. Its purpose is to direct the targeting procedure. First is processed all the targeting
options such as the choice of coordinate systems, the spin axis orientation, and the mode of miniriobe trajectory simulation (2- or n-body). It then generates an initial guess at the probe release controls. Finally, it calls upon the second basic subroutine -- namely the least-squares optimization routine to iteratively refines the initial controls into the minimum miss valves. This latter routine known as LSOPT will be a generalized algorithm for minimizing the length of an arbitrary constraint error vector. It is the function of third basic subroutine, MPPROP to calculate this error vector.

In designing MPPROP two obvious complications arise. First, if impact latitude and longitude are taken directly as the constraints as suggested in the proposal, this does not define the error vector when a miniprobe misses the planet. The simplest solution to this problem is to define auxiliary constraints of $B \cdot T$ and $B \cdot R$ corresponding to the actual constraints of latitude and longitude and to minimize the length of the auxiliary constraint error vector. For miniprobe targeting the error vector will be 6 dimensional (two dimensions for each of the three miniprobes). Such is the approach currently being programmed in MPPROP. It is the exact analog to the Newton-Raphson scheme applied to $B \cdot T$ and $B \cdot R$ auxiliary constraints used in targeting the main probe to actual latitude and longitude constraints. The second difficulty is the presence of an engineering constraint on the tangential velocity of the miniprobes at release. Since the miniprobe booms will probably not exceed 1.5 m in length and the spin rate will probably be less than $100 \mathrm{rev} / \mathrm{min}$, this tangential velocity is bounded above by say $15 \mathrm{~m} / \mathrm{sec}$. The simplest way to handle this inequality constraiat is to use of slack variables. When the requirement is violated it will be treated as an equality constraint -- that is the tangential velocity at release will be targeted to its upper-bound value. This can readily be done by adding a seventh component to the constraint error vector. This ploy is the solution to the miniprobe tangential-velocity constraint adopted in MPPROP.

## Task 3: Atmospheric Entry Trajectory Reconstruction

The major LTR conversion problems have been solved. The converted LTR is currently being checked out by attempting to duplicate Viking entry trajectory results which were run on the. MMC version of LTR. Results have been duplicated for one case, thus far. At least one other Viking case will be duplicated as part of this conversion checkout phase. Preliminary main probe test cases have been defined for mode A Venusian entry. In these preliminary cases existing measurement types will be used to reconstruct the trajectory and atmosphere down to about 50 km altitude. Operation with and without a non-axial accelerometer and a gyro will be studied. Doppler tracking and parachute phase runs will be deferred until necessary models have been developed.

All modifications for deleting non-axial accelerometer or gyro measurements and for mode $B$ operation have been defined. Coordinate transformation and ephemeris subroutines required by range and doppler measurement processing have been developed and checked out.

In the process of attempting to run a Venus entry trajectory test case supplied to us by P. Argentiero, an integrator instability problem was encountered in the LTR program. The instability begins during the phase when terminal velocity is achieved. Two approaches have been studied for eliminating this instability problem. The first approach was to replace the existing 4 -th order two-step Runge-Kutta integrator with some other integrator. Three different integrators were tried (single-step 2nd and 4th order Runge-Kutta, Runge-Kutta-Ralston), but none proved successful. The regime of stability was doubled by using the single-step 4th order Runge-Kutta, but also went unstable and, in addition, increased the computation time. The second approach consisted of rewriting the equations of motion in a form which might lead to a stable integrator. An Encke formulation was studied, in which nonlinear perturbation equations, referenced to the terminal velocity solution, were integrated. However, this approach did not improve the situation at all; the integrator became unstable at essentially the same place as before. It appears that the only feasible solution, considering the available time remaining, is to use the existing exact equations of motion until the magnitude of $\dot{v}$ becomes sufficiently small ( $\sim 1 . \times 10^{-3} \mathrm{~m} / \mathrm{sec}^{2}$ ), and then delete the \& equation and assume quasi-static motion for the remainder of the trajectory. This approach is currently under study. Consideration of the results presented below indicates that this approach is justified.

Selecting a small enough integration step size ( $\Delta t=.25 \mathrm{sec}$ ) delays instability in the original integrator until the vehicle has nearly reached the planet surface. A comparison between the quasi-static solution and the exact solution is presented below. The quasi-static solution shown here has been updated: non-constant acceleration of gravity and the LTR-generated density profile were used in its computation.

| Altitude (km) | Quasi-static velocity (m/sec) | Time velocity (m/sec) |
| :---: | :---: | :---: |
| 70 | 80.3 | 83.4 |
| 60 | 32.7 | 32.9 |
| 50 | 15.55 | 15.87 |
| 40 | 9.14 | 9.15 |
| 30 | 6.06 | 6.07 |
| 20 | 4. 30 | 4. 30 |
| 10 | 3.17 | $\therefore \quad 3.21$ |

Data was also generated which permits a comparison between the GSFC No. 3609 Vemus model atmosphere, and the atmosphere modeled in LTR. The results are presented below:

| Altitude (km) | GSFC Density $\left(\mathrm{kg} / \mathrm{km}^{3}\right)$ | LTR Density $\left(\mathrm{kg} / \mathrm{km}^{3}\right)$ | GSFC Pressure (mb) | ITIR Pressure (mb) |
| :---: | :---: | :---: | :---: | :---: |
| 80 | 1.46E7 | Not available | 6.33 | Not available |
| 70 | 9. 34 E 7 | 8.08E7 | 4.34E1 | 3. 74 E 1 |
| 60 | 5. 24 E 8 | 4.87E8 | 2.64E2 | 2.40 E 2 |
| 50 | 1.98E9 | 1.85 E 9 | 1. 25E3 | 1.17 E 3 |
| 40 | 5.44E9 | 5. 34 E 9 | 4. 3E 3 | 4.17E3 |
| 30 | 1.22E10 | 1. 22E10 | 1.17E4 | 1.16E4 |
| 20 | 2.43E10 | 2.44 E 10 | 2.73E4 | 2. 73 [4 |
| 10 | 4.46 E 10 | 4.47E10 | 5. 7E4 | 5.72E4 |
| 0 | 7. 74E10 | Not available | 1.104E5 | 1.104E5 |

The LTR model assumed a constant molecular weight of 43.2 and the perfect gas law to generate atmospheric pressure and density.

## New Technology

A new technology disclosure for the generalized covariance analysis technique is in process. Complete documentation of this technique is not yet available. Briefly, the technique provides a means of studying the "actual" statistics of an estimate of the state of a dynamic system
when that estimate is generated by a linear recursive estimator, a Kalman filter being the best known example of such an estimator. The statistics of the estimate generated by the estimator are, of necessity, based on the a priori statistics assumed for the dynamic and measurement process when the estimator was designed. It is important to know how sensitive the state estimates are to differences between the "actual" statistics and the a priori statistics. The purpose of the generalized covariance analysis techniques is to compute the statistical properties of the difference between the actual and estimated state for some set of postulated "actual" statistics. The technique can be applied to an arbitrary linear recursive estimator.

## Program for Next Period

During the next period all contract tasks are to be completed.
Task 1 is complete now except for final documentation editing.
The mini-probe targeting algorithm for Task 2 wi 11 be coded and tested. The main probe/bus targeting algorithm is in the final phases of checkout. Once these targeting algorithms are checked out they will be incorporated into the NOMNAL, ERRAN and SIMUL programs.

The principal analysis effort left on Task 3 is the improved range/ doppler model which is yet to be completed. The remaining effort concerns the definition of test cases and the debug and checkout of mode $A$, mode $B$ and the sequential operation of modes $A$ and $B$.

The final task is the completion of all documentation. This effort will be paced by the completion of the above tasks. Rough draft documentation for each subtask will be generated as each is completed.

## Conclusions

1. Preliminary generalized covariance analysis results indicate that the generalized covariance program will be a useful tool for studying the sensitivity of a filter design to off-design conditions.
2. The integration of the exact entry equations of motion does not appear to be possible once terminal velocity is achieved using a Runge-Kutta type integrator. One feasible solution is to integrate the quasi-static equations once terminal velocity is achieved. Such an approach would also reduce the computation time for a Venusian entry trajectory.

## Recommendations

1. It is recommended that the integration instability problem be circumvented by switching to a quasi-static model of the equations of motion when the entry vehicle reaches terminal velocity in the lower atmosphere of Venus.
