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GCA CORPORATION
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A STUDY OF PLANETARY METEOROLOGY

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SUMMARY

The concept is described of deducing the temperature profile of a planetary atmosphere from orbiter measurements of the planet's IR limb radiance profile. Expressions are derived for the weighting functions associated with the limb radiance profile for two infrared transmission models. Analysis of the weighting functions for the Martian atmosphere indicates that a limb radiance profile in the 15μ CO_2 band can be used to determine the Martian atmospheric temperature profile from 20 to 60 km.

1. INTRODUCTION

The primary objectives of the current research program are:

(1) To study the meteorology of the planet Jupiter, with particular emphasis on the deduction of the atmospheric temperature profile through analysis of available infrared and microwave observations of the planet's emission spectrum.

(2) To develop and apply inversion techniques for inferring atmospheric temperature and constituent profiles of planetary atmospheres.

During the first quarterly period, work was performed on the first objective listed above. This work involved a preliminary analysis of the Jovian infrared emission observations and an estimate of the temperature profile based upon this analysis. A complete discussion was presented in Quarterly Progress Report No. 1. During the second quarterly period, work was performed on the second objective listed above, and, in particular, on the concept of measuring the IR limb radiance profile of a planetary atmosphere from an orbiter to infer the temperature profile. This work is reviewed in this quarterly report.

The inference of atmospheric temperature and constituent profiles from measurements of the spectral distribution of the emitted thermal radiation from a planetary atmosphere has received a good deal of attention recently. Such experiments have already been successfully flown in the Earth orbiting NIMBUS satellites (see, for example, Conrath et al., 1970), are being flown on the current Mariner flight to Mars, and will be flown on future flights to Mars and Venus. Another technique, based upon scanning the limb of the planet rather than scanning in wavelength, can also provide valuable information on the vertical profiles of temperature and constituents of a planetary atmosphere. This concept is just beginning to receive attention (reference). This technique has certain advantages over the spectral scan, and can be used to complement spectral scans.

The basic geometry of the limb, or horizon, radiance observation is shown in Figure 1. A high angular resolution (~0.1 degrees for orbital altitude of 10^3 km) radiometer on a planetary orbiter intercepts radiation emanating from an atmospheric path that is tangent to the planetary surface at height h, which is termed the tangent height. In the figure, R represents altitude. The wavelength interval to which the radiometer is sensitive is determined by the nature of the experiment. If the temperature profile is the object of the experiment, the wavelength interval will lie in an atmospheric absorption band of a constituent whose distribution is known - e.g., the 15μ CO_2 band would be a good choice for Earth, and, possibly, Mars and Venus. If the water vapor profile is the object of the measurement, then an infrared water vapor band would be selected. The radiation received by the instrument is a function of the distribution of temperature and absorbing gas along the ray path. If the distribution of absorbing gas is known, the temperature above the tangent height can be inferred from the radiance observations. If the temperature distribution is known, we can infer the distribution

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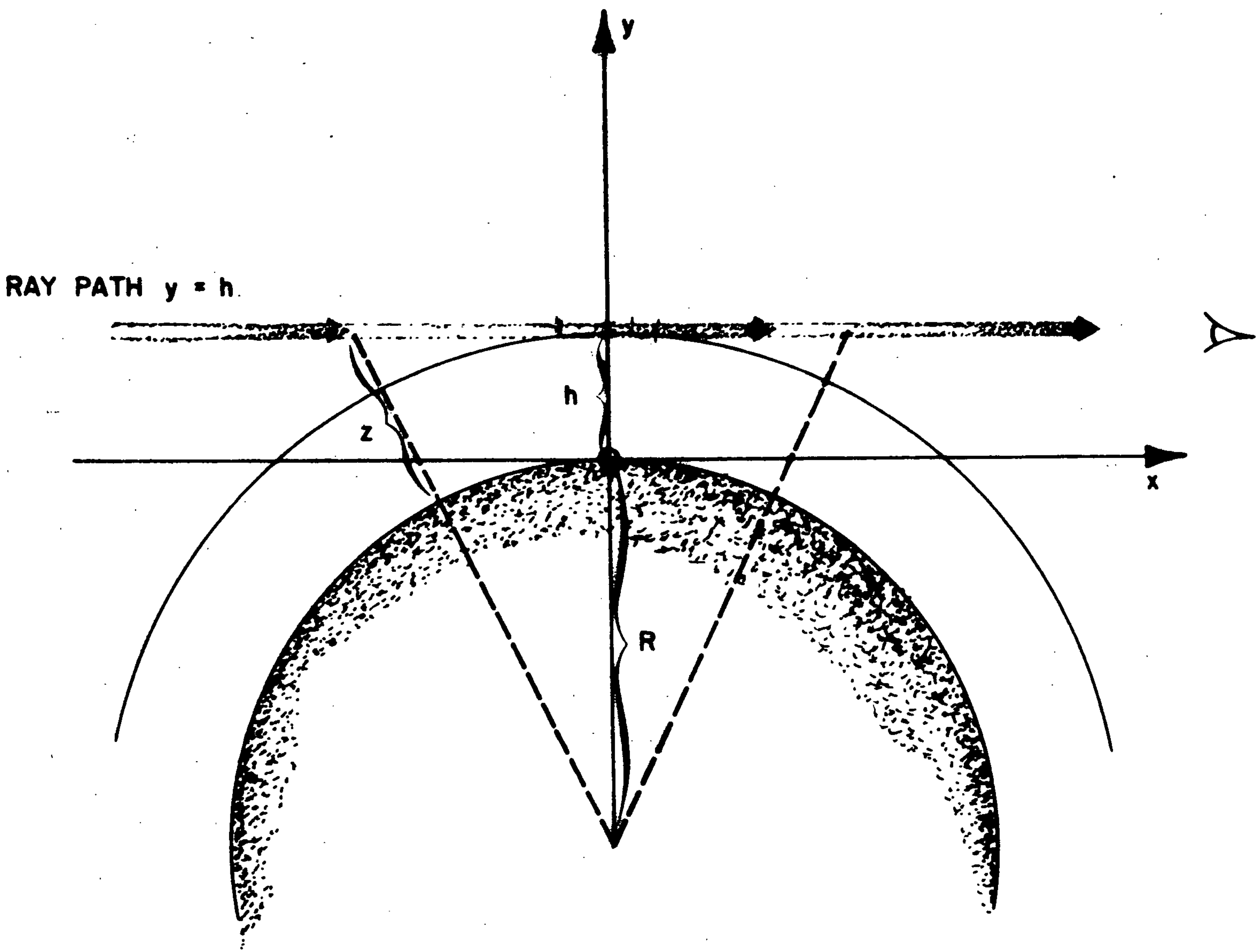

Figure 1. Limb Viewing Geometry.

of an absorbing gas. By scanning in tangent height from top to base of the atmosphere, it is thus possible to infer the vertical temperature structure or vertical distribution of absorbing gas. The radiance versus tangent height curve obtained through such a scan is called a horizon radiance profile. The geometry of the horizon radiance observation technique insures that one is receiving information on temperature, for example, from the atmosphere above a particular level — the tangent height. It will be shown below that for a gas with constant mixing ratio — such as CO_2 in the atmosphere of Earth, and probably in the atmospheres of Mars and Venus (at least to great heights) — the weighting functions for the horizon radiance observation are sharply peaked at the tangent height. This means that not only is the vertical slice of atmosphere under view limited at its lower bound by the tangent height being observed, but its upper bound is also effectively limited by the shape of the weighting function. In other words, for a particular tangent height h, most of the radiation comes from a rather narrow vertical interval between h and $h + \Delta Z$. Hence, the mean temperature of this interval could easily be inferred by an 'instant inversion' technique.

Below we develop the technique for two infrared transmission models: the Goody random model, and the transmission models used by Bartko and Hanel (1968) for calculations of temperature in the upper atmosphere of Venus. We then compute weighting functions for the atmosphere of Mars using the Goody random model.

2. DISCUSSION

The intensity of radiation received by an infrared radiometer with narrow field of view whose line of sight is tangent to the planetary atmosphere at the tangent height, h of Figure 1, is given by

$$I = \int_{-\infty}^{\infty} B \frac{\partial\tau}{\partial x} dx = \int_{0}^{\infty} B \frac{\partial\tau}{\partial x} dx + \int_{-\infty}^{0} B \frac{\partial\tau}{\partial x} dx \tag{1}$$

where B is the Planck intensity integrated over the spectral interval of the radiometer, and τ is the spectral transmittance of the atmosphere from the point x to the radiometer. From the geometry, the following relationship between x and height z exists

$$(R + H)^2 + x^2 = (R + z)^2 \tag{2}$$

The terms h^2 and z^2 can be neglected in comparison with the outer terms — since we are interested in the lower part of the atmosphere ($Z < 100$ km) — so that

$$x = \sqrt{2R(z - h)} \tag{3}$$

and

$$dx = \left(\frac{R}{2}\right)^{1/2} \frac{1}{(z - h)^{1/2}} dz \tag{4}$$

Thus Equation (1) can also be written as

$$I = \int_{h}^{\infty} B \frac{\partial\tau}{\partial z} dz + \int_{-\infty}^{h} B \frac{\partial\tau}{\partial z} dz \tag{5}$$

Expressions will be derived first for the weighting functions $\frac{\partial\tau}{\partial z}$ which determine where in the atmosphere the radiation received by the sensor originates. We assume that the observations are made in a CO_2 band for purposes of temperature inferences.

2.1 Goody Random Model

If it is assumed that the transmittance is given by the Goody random model, then

$$\tau = \exp\left[-\frac{k\bar{m}}{\delta}\left(1 + \frac{k\bar{m}}{\pi\bar{\alpha}}\right)^{-1/2}\right] \tag{6}$$

where k is the mean line intensity, δ is the mean line spacing, $\bar{m}$ is the effective amount of absorbing gas in g cm^{-2} along the path, and $\bar{\alpha}$ is the effective half-width along the path. For the Martian and Venusian CO_2 amounts and pressures, and the long paths of concern here, $k\bar{m}/\pi\bar{\alpha}$ is much greater than unity. Thus, Equation (6) can be written as

$$\tau = \exp\left[-\left(\frac{k\pi\,\bar{m}\bar{\alpha}}{\delta^2}\right)^{1/2}\right] \tag{7}$$

Following Rodgers and Walshaw (1966), the effective absorber amount and half-width are

$$\bar{m} = \int \Phi(\theta)dm \tag{8}$$

and

$$\bar{\alpha} = \frac{\int \alpha\ dm}{\int dm} \tag{9}$$

where $\Phi(\theta)$ is a correction for the effect of temperature on the line intensities and is given by

$$\log_e \Phi(\theta) = a(\theta - 260) + b(\theta - 260)^2, \tag{10}$$

where θ is temperature and a and b are constants for the band. For the present purpose of determining the CO_2 transmittances, an isothermal atmosphere is assumed so that

$$\bar{m}\,\bar{\alpha} = \Phi(\theta) \int \alpha \, dm \tag{11}$$

Since

$$\alpha = \alpha_s \left(\frac{p}{p_s}\right) \left(\frac{\theta_s}{\theta}\right)^{1/2} \tag{12}$$

where p is pressure and the subscript s refers to standard temperature and pressure, and $dm = \rho_{CO_2}\, dx = \rho\, c \left(\frac{R}{2}\right)^{1/2} \frac{1}{(z - h)^{1/2}}\, dz$, (13)

where ρ_{CO_2} is the carbon dioxide density, ρ is the atmospheric density, and c is the CO_2 mass fraction, Equation (11) can be written as

$$\bar{m}\,\bar{\alpha} = \Phi(\theta)\, \alpha_s \left(\frac{1}{p_s}\right)\left(\frac{\theta_s}{\theta}\right)^{1/2} \left(\frac{R}{2}\right)^{1/2} c \int_z^\infty p\, \rho \, \frac{dz}{(z - h)^{1/2}} \tag{14}$$

For an isothermal atmosphere both pressure and density have the same scale height H. Thus, Equation (14) becomes

$$\bar{m}\,\bar{\alpha} = \Phi(\theta)\, \alpha_s \left(\frac{p_o}{p_s}\right) \left(\frac{\theta_s}{\theta}\right)^{1/2} \left(\frac{R}{2}\right)^{1/2} c\, \rho_o \int_z^\infty e^{-2\, z/H} \frac{dz}{\sqrt{z - h}} \tag{15}$$

where the subscript o refers to surface values on the planet.

With the substitution, $t^2 = \frac{2\,(z - h)}{H}$, the integral in Equation (15) becomes

$$e^{-2h/H}\,(2H)^{1/2} \int_{\sqrt{\frac{2(z-h)}{H}}}^{\infty} e^{-t^2}\,dt \tag{16}$$

The integral in Equation (16) is equal to $\frac{\sqrt{\pi}}{2}\left[1 - \operatorname{erf} \int\right]$, where erf is the error function, and $\int = \sqrt{2(z-h)/H}$. Thus

$$\bar{m}\,\bar{\alpha} = \Phi(\theta)\,\alpha_s \left(\frac{P_o}{P_s}\right)\left(\frac{\theta_s}{\theta}\right)^{1/2} \frac{(\pi\,R\,H)^{1/2}}{2}\,c\rho_o\,e^{-2h/H}\left[1 \mp \operatorname{erf}\left(\frac{2(z-h)}{H}\right)^{1/2}\right] \tag{17}$$

where the negative sign in front of the error function is to be used for anterior values of z (to the right of the y axis in Figure 1) and the positive sign for posterior values of z. The expression for the transmittance from the point z to z = ∞ can now be written as

$$\tau(z,\,h) = \exp\left\{-\,A\,e^{-h/H}\left[1 \mp \operatorname{erf}\left(\frac{2(z-h)}{H}\right)^{1/2}\right]^{1/2}\right\} \tag{18}$$

where

$$A^2 = \frac{k\pi}{\delta^2}\,\Phi(\theta)\,\alpha_s\left(\frac{P_o}{P_s}\right)\left(\frac{\theta_s}{\theta}\right)^{1/2}\left(\frac{(\pi RH)}{2}\right)^{1/2} c\,\rho_o \tag{19}$$

Differentiating with respect to z yields

$$\frac{d\tau}{dz}(z, h) = \pm \frac{1}{\sqrt{2(z - h)H}} \frac{A}{\sqrt{\pi}} \exp\left[- Ae^{-h/H} \left(1 \mp \operatorname{erf}\sqrt{\frac{2(z - h)}{H}}\right)^{1/2} \right]$$

$$\left(1 \mp \operatorname{erf}\sqrt{\frac{2(z - h)}{H}}\right)^{-1/2} e^{-(2z-h)/H} \tag{20}$$

where the upper signs are to be used for anterior values of z and the lower signs for posterior values of z.

2.2 Bartko-Hanel Transmission Model

The Bartko-Hanel (1968) transmittance for a spectral interval can be written as

$$\tau = \exp\left[- (mu^*)^n \right] \tag{21}$$

where m and n are constants for the spectral interval, and u^*, the effective absorber amount, is given by

$$u^* = \left(\frac{T_s}{T}\right)^{3/2} \exp\left[\gamma\left(\frac{1}{T_s} - \frac{1}{T}\right)\right]\left(\frac{p}{p_s}\right)^k u \tag{22}$$

where u is path-length of absorbing gas in units of atmo-cm of CO_2, T_s and p_s are standard temperature and pressure, respectively, and γ is a constant for a spectral interval. Expression (22) is for a constant temperature and pressure path. We assume an isothermal atmosphere and, hence, a constant temperature for purposes of evaluating transmittances. The pressure does vary over the path and in this case expression (22) p^k is replaced by $\overline{p^k}$, where the bar represents an average over the atmospheric path.

$$\overline{p^k} = \int p^k \, du \Big/ \int du \tag{23}$$

From the definition of the path length,

$$du = \frac{\rho_{CO_2}}{\rho'_{CO_2}} \, dx = \frac{c\rho}{\rho'_{CO_2}} \left(\frac{R}{2}\right)^{1/2} \frac{1}{(z - h)^{1/2}} \, dz \tag{24}$$

where the prime represents conditions of standard temperature and pressure, and c is the CO_2 mass fraction.

The effective absorber amount can then be written as

$$u^* = f(T) \, p_s^{-k} \int p^k \, du \tag{25}$$

where

$$f(T) = \left(\frac{T_o}{T}\right)^{3/2} \exp\left[\gamma \, \frac{1}{T_s} - \frac{1}{T}\right] \tag{26}$$

or

$$u^* = B \int_z^{\infty} p^k \rho \, \frac{dz}{(z - h)^{1/2}} \tag{27}$$

where

$$B = \frac{f(T) \, p_s^{-k} \, c}{\rho'_{CO_2}} \left(\frac{R}{2}\right)^{1/2} \tag{28}$$

Under the assumption of an isothermal atmosphere (for transmission calculations only), both pressure and density have the same scale height H. Thus,

$$u^* = B \, p_o^k \, \rho_o \int_x^{\infty} \exp\left[-(k + 1) \, z/H\right] \frac{dz}{(z - h)^{1/2}} \tag{29}$$

where the subscript o refers to the planet's surface.

If we let

$$t^2 = \frac{(k + 1)\ (z - h)}{H} \tag{30}$$

the integral in (29) becomes

$$2 \left(\frac{H}{k + 1}\right)^{1/2} \exp\ -(k + 1)h/H \int_{\sqrt{\frac{(z-h)(k+1)}{H}}}^{\infty} e^{-t^2}\, dt \tag{31}$$

The integral in Equation (31) is equal to $\sqrt{\frac{\pi}{2}}\,(1 - \text{erf}\ \alpha)$, where

$$\alpha = \sqrt{(z-h)(k+1)/H} \tag{32}$$

Thus,

$$u^* = B\ p_o{}^k \rho_o \left(\frac{\pi H}{k+1}\right)^{1/2} \exp\left[-(k+1)h/H\right] (1 \mp \text{erf}\ \alpha) \tag{33}$$

where the negative sign in front of the error function is to be used for anterior values of z, and the positive sign for posterior values of z. The expression for the transmittance from the point z to z = ∞ along the tangent path can be written as

$$\tau = \exp - \left[mE \exp\left[-(k+1)h/H\right] (1 \mp \text{erf}\ \alpha)\right]^n \tag{34}$$

where

$$E = B\ p_o{}^k \rho_o \left(\frac{\pi H}{k+1}\right)^{1/2} \tag{35}$$

Differentiating with respect to z yields

$$\frac{d\tau}{dz}(z,h) = \pm n \left[\frac{k+1}{\pi H(z-h)}\right]^{1/2} \exp - \left\{mE \exp - \left[(k+1)h/H\right] (1 \mp \text{erf } \alpha)^n\right\}$$

$$\left\{m E \exp\left[-(k+1)h/H\right] (1 \mp \text{erf } \alpha)\right\}^{n-1} e^{-\alpha^2} \tag{36}$$

2.3 Weighting Functions and Instant Inversion

The weighting function is defined as

$$W(z,h) = \left[\frac{d\tau}{dz}(z,h)\right]_A - \left[\frac{d\tau}{dz}(z,h)\right]_P \tag{37}$$

with A and P standing for anterior and posterior, respectively.

We have used the random transmission model expression (Equation 20) to compute weighting functions for the 15μ CO_2 band for the Martian atmosphere. The weighting functions for tangent heights of 0 to 60 km, at 10 km intervals in tangent height, are plotted in Figure 2. In these computations it has been assumed that the Martian scale height is 10 km, the surface pressure is 5 mb, the surface density is 1.2×10^{-5} g cm^{-3}, the temperature of the Martian atmosphere is 200^o and the band constants are those given by Rodgers and Walshaw (1966). It is obvious from Figure 2 that for tangent heights greater than 20 km most of the energy emanates from a rather narrow vertical thickness of atmosphere, of order 5 km. Thus, observations of 15 μ band radiance at 5 km or greater tangent height intervals are almost independent of each other and could be used to infer, quite directly, mean temperatures with 5 km vertical resolution. A rather simple technique, "instant inversion," is developed below.

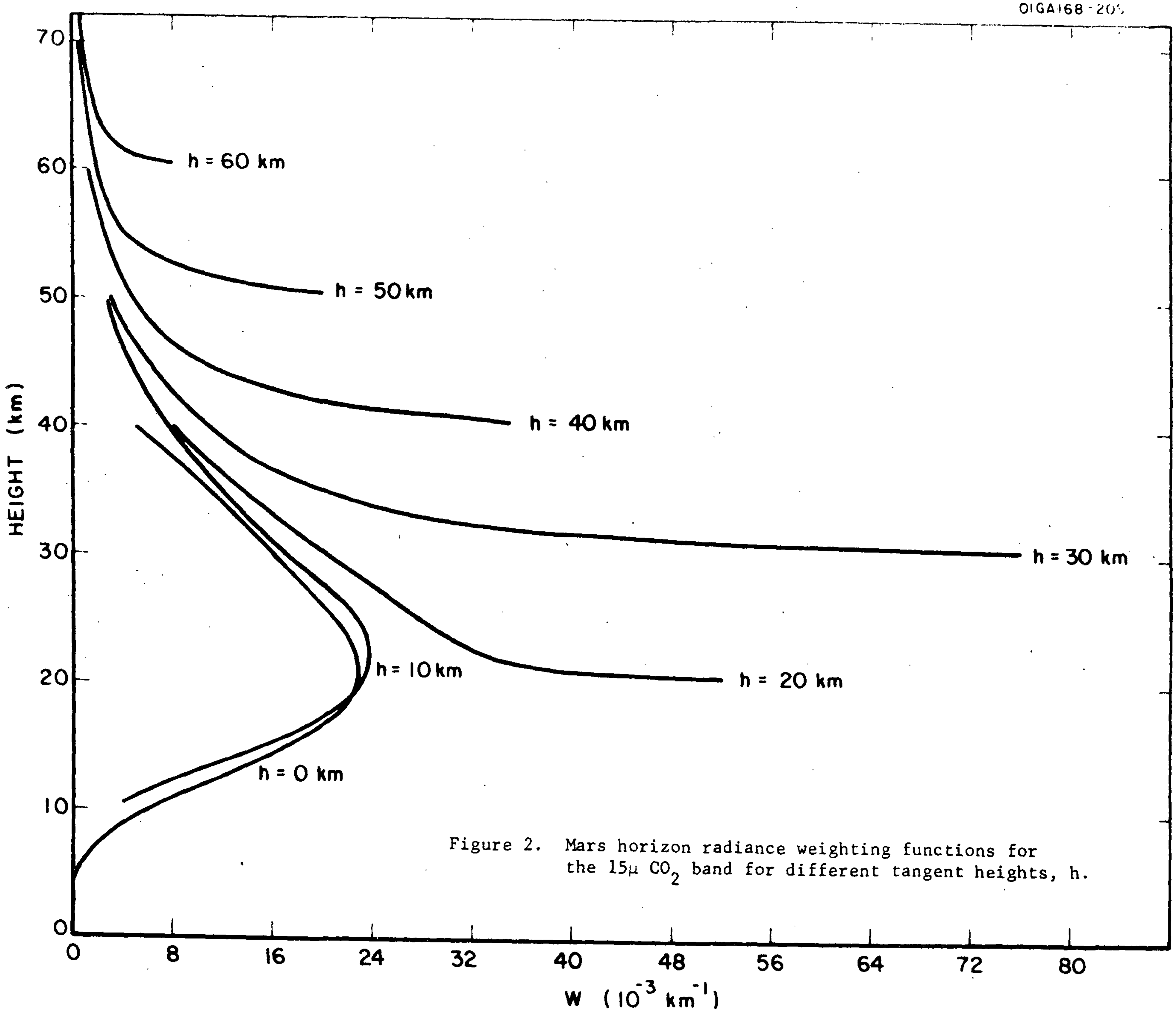

Figure 2. Mars horizon radiance weighting functions for the 15μ CO_2 band for different tangent heights, h.

From Equation (1) or (5) it can be seen that the intensity of radiation received by the radiometer can be written as

$$I(h) = \int_{\tau(-\infty,h)}^{1} B \, d\tau \tag{38}$$

where $\tau(-\infty,h)$ represents the transmittance of the entire atmospheric path tangent to the Martian atmosphere at height $z = h$. The weighting functions for tangent heights greater than 20 km are such that most of the energy emanates from a rather narrow vertical interval immediately above the tangent height. Thus, the temperatures above this narrow vertical interval do not contribute significantly to the observed emission. It may, therefore, be assumed that B in the integral of Equation (38) may be approximated by $\bar{B}(h)$, which represents the integrated value of the Planck function for the average temperature of the layer immediately above the tangent height. Thus,

$$I(h) = \bar{B}(h)[1 - \tau(-\infty, h)] \tag{39}$$

The term $[1 - \tau(-\infty, h)]$ may be called the emittance, $\epsilon(h)$. Thus, one can infer $\bar{B}(h)$ from

$$\bar{B}(h) = \frac{I(h)}{\epsilon(h)} \tag{40}$$

It is a simple matter to reclaim $\bar{T}(h)$ from the integrated Planck function. Hence, the vertical distribution of temperature of the Martian atmosphere can be obtained from observations of I as a function of tangent height h.

From Equation (18),

$$\tau(-\infty, h) = \exp\left(-\sqrt{2}\, Ae^{-h/H}\right) \tag{41}$$

The emittance of the Martian atmosphere as a function of tangent height is shown in Figure 3.

For purposes of deriving and illustrating the method, we have assumed a model isothermal atmosphere to calculate the variation of emittance with tangent height. Although the emittance varies with temperature, this is a second order effect compared to the variation of the Planck function with temperature. Hence, Equation (40) can be used to infer temperatures at different levels even though $\epsilon(h)$ is computed for an isothermal atmosphere. The effect of uncertainties in the emittance profile upon inferred temperatures can easily be examined.

Preliminary analysis of experiment feasibility indicates that it is within the state of the art. The desired vertical resolution at the tangent height is about 5 km. For a Martian orbiter at an altitude of 1000 km, this vertical resolution can be achieved with 0.1 degree vertical field of view. A similar successful measurement of the Earth's horizon radiance profile in the 15μ CO_2 band (615 cm^{-1} to 715 cm^{-1}) from a suborbital rocket flight utilized a radiometer with 0.025 degree vertical field of view (McKee et al, 1967). Expected Martian radiances in the 15μ band are of the order of 1/10 of Earth radiances (assuming stratospheric temperatures of 150K on Mars and 230K on Earth). The wider field of view suggested for the Martian experiment helps to compensate

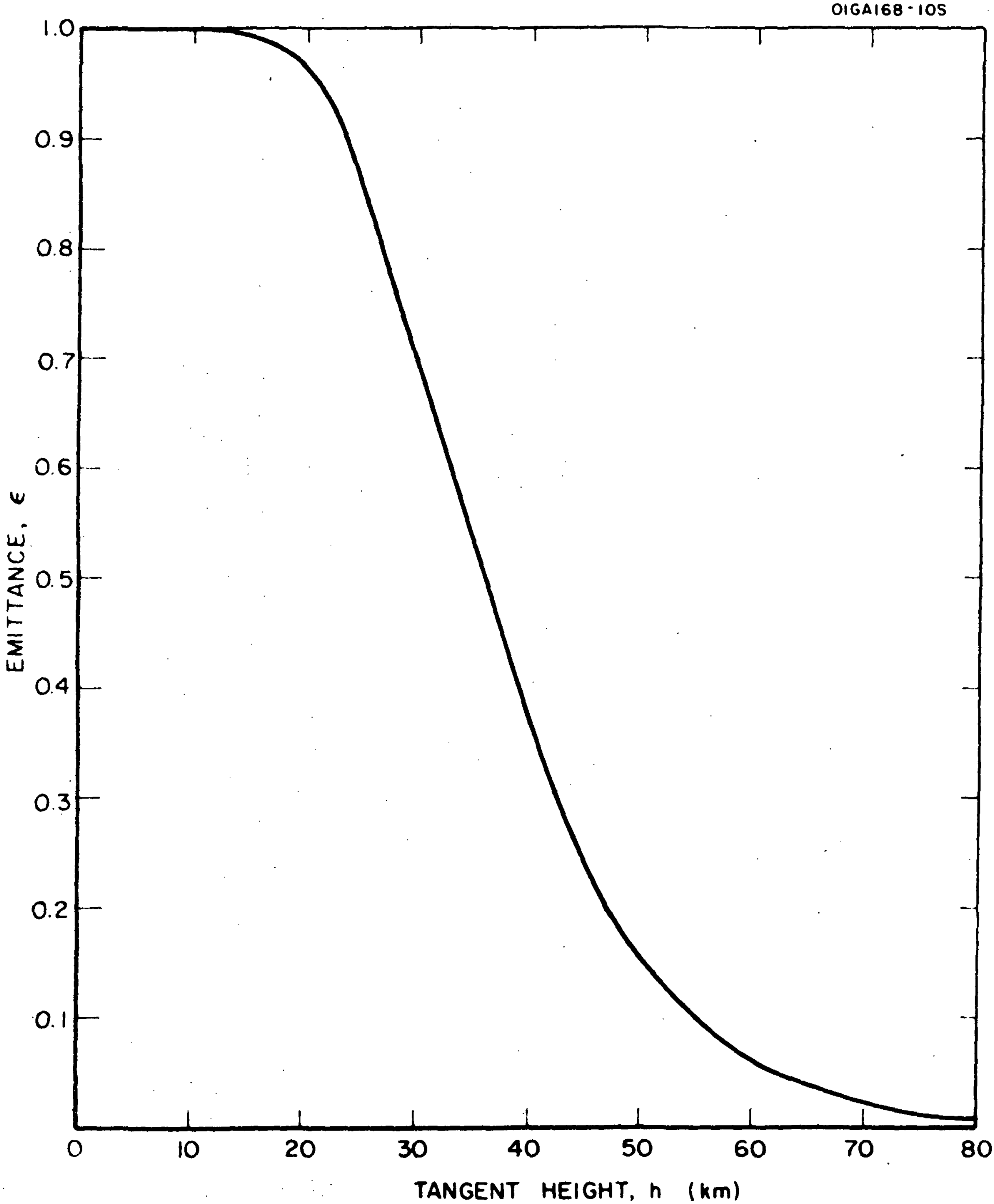

Figure 3. Emittance of the 15μ CO_2 band versus tangent height for the Martian atmosphere.

for the lower signal levels. Scan times of the order of 1 second for a complete vertical scan, as in the successful Earth experiment, are within the state of the art. The effective horizontal resolution of an individual temperature inference would be of the order of 200 km.

This analysis shows that the orbiter based horizon radiance technique is potentially a powerful tool for measuring the vertical temperature structure of the Martian atmosphere above heights of about 20 km. It would thus complement an orbiter infrared spectrometer experment at 4.3μ or at 15μ which could be used to infer temperature profiles for the lowest 20 km of the Martian atmosphere. If one wanted to observe the temperature profile of the lowest 20 km of the Martian atmosphere with the limb radiance technique, one has only to slide the spectral interval of the radiometer to a region of slightly lower CO_2 absorption. More sophisticated inversion techniques can also be used.

3. FUTURE PLANS

Plans for the next quarterly period include the following:

(1) Analysis of feasibility of deriving the temperature profile above the clouds of Venus with a limb radiance experiment.

(2) Analysis of the feasibility of deriving water vapor profiles for Mars and Venus (above the clouds) with a limb radiance experiment.

(3) Continuation of analyses of earth-based observations of the Jovian emission spectrum for the purpose of deducing the Jovian atmosphere temperature profile.

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