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ABSTRACT

By the superposition of the conical solution for the diffraction of a plane pulse by a three-dimensional corner, the solution for a general incident plane wave is constructed. A numerical program is presented for the computation of the pressure distribution on the surface due to an incident plane wave of any wave form and at any incident angle. Numerical examples are presented to show the pressure signature at several points on the surface due to incident wave with a front shock wave, two shock waves in succession or a compression wave with the same peak pressure. The examples show that when the distance of a point on the surface from the edges or the vertex is comparable to the distance for the front pressure raise to reach the maximum, the peak pressure at that point can be much less than that given by a regular reflection, because the diffracted wave front arrives at that point prior to the arrival of the peak incident wave.

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LIST OF FIGURES

FIGURE

1. INTRODUCTION

Although the pressure wave created by a supersonic airplane is three-dimensional in nature, the radius of curvature of the wave front is usually much larger than the length scale of a structure. Therefore, the incident waves can be approximated by progressing plane waves composed of compression, expansion and shock waves. By the decomposition of a plane wave to a succession of plane pulses, the basic problem is therefore the diffraction of a plane pulse by a three-dimensional corner.

After the incidence of a plane pulse on a three-dimensional corner at the instant $t = 0$, the disturbed regions behind the incident plane wave are either a simple reflection from the surface of the corner, a two-dimensional diffraction by an edge or the three-dimensional diffraction by the vertex as shown in Fig. 1. The last region is confined by a sonic sphere $r = Ct$, centered at the vertex. The solution for the diffraction by an edge is a two-dimensional conical solution obtained by Keller and Blank [1]. Once the appropriated two-dimensional solutions corresponding to the incident angles are constructed, the boundary data on the sonic sphere about the vertex are obtained, and the three-dimensional conical solution inside the sonic sphere is constructed by the determination of the eigenfunctions and their coefficients [2]. In the next section, the essential procedure and the equations required for the numerical program I are presented for the computation of the following items: i) the two-dimensional conical solution for each edge corresponding to the direction cosines of the incident wave, ii) the boundary data on the sonic sphere around the vertex and iii) the coefficients in the eigenfunction expansions of the solution inside the sonic sphere.

In section 3, the superposition of the solution of the diffraction of a plane pulse to that of a plane wave of a given wave form is described. The superposition is carried out by numerical program II with the coefficients of eigenfunctions expansions from program I and the incident wave form as input data. Numerical results are presented showing the pressure signatures received by several points on the surface of the corner corresponding to various types of incident wave forms.

Both the numerical program I and II are described and listed in the appendix.

2. INCIDENCE BY A PLANE PULSE

Fig. 1 shows a unit plane pulse incident on a corner of the cube. The three edges are chosen as the three coordinate axes x_j with $j = 2,3,4$.* The direction cosines of the normal to the incident pulse are designated as n_j with $n_2^2 + n_3^2 + n_4^2 = 1$. The equation for the plane of the incident pulse is

$$H_o = n_2 \bar{x}_2 + n_3 \bar{x}_3 + n_4 \bar{x}_4 = 1 \qquad (1)$$

$$\text{with } \bar{x}_j = x_j/(Ct)$$

where C is the speed of sound and t is the time after the passing of the plane pulse over the vertex.

If the plane pulse hit the j - th edge first before hitting the vertex, n_j will be negative. This happens when the incident pulse diffracted by the first corner of the cube is subsequently diffracted by the adjacent corners.

* They begin with 2 so that the index j-1 will never be zero which will not be accepted by the computing machine.

For the incidence of a plane wave with the first corner of a building, n_j's are nonnegative, i.e.,

$$n_j \geq 0 \quad j = 2, 3, 4 \tag{2}$$

In order to avoid the complicated discussion for various cases when one, two, or all of the n_j's are negative, the discussions in this report will be restricted to the case of Eq. (2).

The plane pulse is intercepted by the j-axis at $X_j = 1/n_j$, and intersects the x_j-x_{j-1} plane along the line

$$n_{j-1} \bar{x}_{j-1} + n_j \bar{x}_j = 1 \tag{3}$$

For the convenience of programming, quantities with subscript 1 and 5 are identified with those with subscript 4 and 2 respectively.

The diffraction due to the j-th edge is confined inside the Mach cone G_j with vertex at X_j on the $\bar{x}_j$ axis,

$$G_j: \ (1-n_j \bar{x}_j) > [(\bar{x}_{j-1})^2 + (\bar{x}_{j+1})^2]^{\frac{1}{2}} \ (1-n_j^2)^{\frac{1}{2}} \tag{4}$$

The diffraction by the vertex is confined inside the sonic sphere

$$S: \quad \bar{x}_2^2 + \bar{x}_3^2 + \bar{x}_4^2 < 1 \tag{5}$$

The plane pulse will be reflected by the face, $\bar{x}_j = 0$, (the $\bar{x}_{j-1}$- $\bar{x}_{j+1}$ plane) since $n_j > 0$ and the plane of the reflected wave is

$$P_j = n_{j-1} \bar{x}_{j-1} - n_j \bar{x}_j + n_{j+1} \bar{x}_j = 1 \tag{6}$$

Across the reflected wave i.e., from $P_j > 1$ to $P_j < 1$, the pressure rises from unity to 2.

Inside the cone G_j but outside and ahead of the sonic sphere S, the solution is a function of two conical coordinates ξ_j and η_j with

$$\xi_j = \frac{\bar{x}_{j-1}\,(1-n_j^2)^{\frac{1}{2}}}{1-n_j\bar{x}_j} \qquad \text{and } \eta_j = \frac{-x_{j+1}\,(1-n_j^2)^{\frac{1}{2}}}{1-n_j\,\bar{x}_j} \tag{7}$$

The cone G_j becomes the domain inside a unit circle,

$$\xi_j^2 + \eta_j^2 = 1 \quad .$$

The reflected wave P_{j-1} in ξ,η variables becomes

$$-n_{j-1}\quad \xi_j - n_{j+1}\,\eta_j = (1-n_j^2)^{\frac{1}{2}} \tag{8}$$

and it is tangential to the unit circle at the point

$$A_j^-:\quad \xi_j^- = -n_{j-1}/(1-n_j^2)^{\frac{1}{2}} \text{ and } \eta_j^- = -n_{j+1}/(1-n_j^2)^{\frac{1}{2}} \tag{9}$$

Similarly the reflected wave P_{j+1} becomes

$$n_{j-1}\,\xi_j + n_{j+1}\,\eta_j = (1-n_j^2)^{\frac{1}{2}}$$

and the point of contact to the unit circle is

$$A_j^+:\ \xi_j^+ = n_{j-1}/(1-n_j^2)^{\frac{1}{2}} \text{ and } \eta_j^+ = n_{j+1}/(1-n_j^2)^{\frac{1}{2}} \tag{10}$$

as shown in Fig. 2. The boundary condition on the unit circle, $\rho_j=1$, for the disturbance pressure p is

$p = 2$	$\omega_j^+ > \theta_j > 0$
$p = 1$	$\omega_j^- > \theta_j > \omega_j^+$
$p = 2$	$3\pi/2 > \theta_j > \omega_j^-$

where $\omega_j^+ = \arcsin \eta_j^+$

$$\omega_j^- = \pi + \omega^+_j$$

and ρ_j, θ_j are the polar coordinates in ξ_j, η_j plane.

The disturbance pressure which lies inside the sonic cone G_j but ahead of the sonic sphere, $x_{j+1}^2 + x_j^2 + x_{j-1}^2 > 1$ and $x_j > n_j$, is given by the two-dimensional conical solution [1],

$$p = 1 + p_j \, (\rho_j, \theta_j) \text{ for } 1 \geq \rho_j \geq 0 \text{ and } 3\pi/2 > \theta_j > 0 \tag{11}$$

with

$$p_j = p_j^- \, (\rho_j, \, \theta_j, \, \omega_j^-) - p_j^+ \, (\rho_j, \, \theta_j, \, \omega_j^+) + 1$$

and $p_j^{\pm} \, (\rho_j, \, \theta_j, \, \omega_j) = \dfrac{1}{\pi} \arctan \dfrac{(1-\tilde{\rho}^2) \sin (2\omega_j/3)}{2\tilde{\rho} \cos(2\theta_j/3) - (1+\tilde{\rho}^2) \cos (2\omega_j/3)}$ where the arctangent lies in the first and second quadrants and $\tilde{\rho} = \left\{\rho_j/[1+(1-\rho_j^2)^{\frac{1}{2}}]\right\}^{2/3}$.

In the domain common to the cones G_j and G_{j+1} from two edges j and j+1 but outside the sonic sphere S, the disturbance pressure is

$$p = 1 + p_j(\rho_j,\theta_j) + p_{j+1}(\rho_{j+1}, \, \theta_{j+1}) \tag{12}$$

In the domain ahead the two cones G_j and G_{j+1}, and behind the reflected shock P_{j-1}, the disturbance pressure is

$$p = 2 \tag{13}$$

In the remaining domain outside the sonic sphere and behind the incident shock, the distrubance pressure is

$$p = 1 \tag{14}$$

Ahead of the incident shock, the pressure is of course undisturbed,

$$p = 0 \tag{15}$$

Equations (11) to (15) define the disturbance pressure outside the sonic sphere S.

For the pressure distribution inside the sonic sphere S, the pressure distribution is represented by the eigenfunction expansions,

$$p(\zeta,\mu,\varphi) = \sum_{\lambda} K_{\lambda} Z_{\lambda} (\zeta) G_{\lambda} (\mu,\varphi) \tag{16}$$

where $\zeta = r/(Ct) = (\bar{x}_2^2 + \bar{x}_3^2 + \bar{x}_4^2)^{\frac{1}{2}}$

$$\bar{x}_4 = - \zeta\mu$$

$$\bar{x}_2 = \zeta (1-\mu^2)^{\frac{1}{2}} \sin (\varphi-3\pi/2),$$

and

$$\bar{x}_3 = \zeta (1-\mu^2)^{\frac{1}{2}} \cos (\varphi-3\pi/2).$$

The eigenvalues λ's, the eigenfunctions $G_{\lambda}(\mu,\varphi)$ and the associated function $Z_{\lambda}(\zeta)$ are defined and are determined by the first numerical program in Ref. 2. The constants $A_{m\lambda}$, $B_{m\lambda}$, $C_{j\lambda}$, $D_{j\lambda}$ characterizing the function $G_{\lambda}(\mu,\varphi)$, $a_{o\lambda}$ and the eigenvalues λ's are now introduced as the input data for the programs in this report.

For a given set of direction cosines (n_j) of the incident pulse, program I computes the following items: i) the pressure distributions in the various regions outside of the sonic sphere by the appropriate equation of Eqs. (11 to 15), ii) the boundary data, $F(\mu,\varphi)$ on the sonic sphere and iii) the coefficients K_{λ} in Eq. (16) for the solution inside the sonic sphere. The coefficients K_{λ} are related to the boundary data by the equation (see section 5 of [2])

$$K_\lambda = \tfrac{1}{2}\left\{ \int_0^1 d\mu \int_{-\pi}^{\pi} d\varphi\ F(\mu,\varphi)\ G(\mu,\varphi) \right.$$

$$\left. + \int_{-1}^{0} d\mu \int_{-3\pi/2}^{3\pi/2} d\varphi\ F(\mu,\varphi)\ G(\mu,\varphi) \right\} \tag{17}$$

Program I in this report is a generalization and an extension of the second program in Ref. 2 to compute the coefficients K_λ for any incident angle and to compute the pressure distribution outside the sonic sphere. With the knowledge of K_λ, the pressure distribution inside the sphere is given by Eq. (16) and are computed by the third program in Ref. 2. Figures 3, 4, and 5 show the pressure distribution on the surface due to the incidence of a unit plane pulse with direction cosines, 0.3, 0.4 and 0.83333. The discontinuities in the slope of the pressure distribution occur at the crossing of the sonic cones around an edge and that of the sonic sphere.

3. INCIDENCE OF A PLANE WAVE

The incident plane wave p_i can be represented in general as

$$p_i = \Psi(s)$$

with $s = Ct - (n_2x_2 + n_3x_3 + n_4x_4)$ where the wave form Ψ is a given function of its phase s and n_j's are the direction cosines with respect to the axes x_j.

When the wave form is a Heaviside function, the diffraction due to the three-dimensional corner is given by the conical solution described in the preceeding section. It will be designated as $p^*\ (r/(Ct), \mu,\varphi)$. The solution corresponding to a plane wave of wave form $\Psi(\eta)$ is given by the Stieltjes integral

$$
\begin{aligned}
p(r,\theta,\varphi,t) &= \int p^{*}\left(\frac{r}{Ct-\eta}, \mu,\varphi\right) d\Psi(\eta) \\
&= \sum_{i} p^{*}\left[\frac{r}{Ct-\eta_{j}}\right]\left[(\Psi(\eta_{j+1}) - \Psi(\eta_{j})\right]
\end{aligned}
\tag{18}
$$

The second form is employed in the numerical program II for the computation of the pressure signature for points in the surface of the corner.

Numerical examples have been carried out for various wave forms with $n_j = 1/\sqrt{3}$. Fig. 6 shows the pressure signature received at points $r = 0$, $\frac{1}{2}$ and 1 along the line dividing the top surface of the corner ($\mu = 0$, $\varphi = \pi$). The incident wave is a simple N-wave in sonic boom problems with front shock strength ϵ. The length of the N-wave, which is 4, is nearly the length of an airplane. The unit length scale in the numerical examples is therefore of the order of hundreds of feet. As shown in the figure, the pressure signature at the vertex is 8/7 times the incident wave form in agreement with the theorem stated in [2]. At point $r = 0.5$, the front part of the pressure signature is equal to twice the incident wave form, i.e. the same as a regular reflection and then decreases from the value of a regular reflection after the arrival of the diffracted waves from the edges and corner. Similar phenomenon is observed for the pressure signature at $r = 1$ with a relative delay in the arrival of the diffracted waves.

Fig. 7 shows the pressure signature at $r = 0.5$, 1.0 and 2.0 along the same line $\mu = 0$, $\varphi = \pi$, while the pressure raise in the incident wave in Fig. 6 is now spread over a thickness of 0.3. The peak pressure received at $r = 0.5$ and at $r = 1.0$ is less than twice the total pressure raise, i.e., the value of regular reflection while, at $r = 2.0$ it is equal to the value of regular reflection. Fig. 8 shows the pressure signatures at the same set of points with the pressure raise in the incident wave spread over a thickness of 0.5. The peak pressure at all these points are less than the value given by a regular reflection.

The differences in the value of the peak pressure at a point on the surface from that of a regular reflection is due to the arrival at the point of the diffracted wave front prior to that of the peak incidence wave front. This is the case for all the points in Fig. 8. In Fig. 7 the pressure raise is faster, so that at $r = 2$, the peak incident wave arrives prior to the diffracted waves and the peak pressure at $r = 2$ is the same as the value of a regular reflection. In Fig. 6, the shock thickness is zero, therefore, the peak value at any surface point is the same as in a regular reflection with the exception of points along the edges and at the vertex which always lies inside the diffracted region.

Fig. 9 shows the pressure signatures at $r = 0.5$, 1.0, and 2.0 along the same line $\mu = 0$, $\varphi = \pi$ when the front shock in Fig. 6 is split to two shock waves joined by the expansion wave of thickness 0.3. Although the peak pressure is the same as that in the single shock, the peak pressure received at points $r = 0.5$ and 1.0 are nearly 25% less than that in the case of regular reflection. The peak pressure at point $r = 2.0$ does reach the value of regular reflection i.e. 2ϵ. Fig. 10 shows the pressure signature at the same three points when the front shock is split into two shocks separated by an expansion wave of thickness 0.6. The peak pressure for all three points are now nearly 25% less than the value 2ϵ in a regular reflection. Figs. 9 and 10 again demonstrate that when the diffracted wave front arrives prior to the arrival of the peak incident wave, the local peak pressure can be much less than the value given by a regular reflection.

The time interval between the arrival at point r along the line ($\mu = 0$, $\varphi = \pi$) of the leading incident wave and that of the leading diffracted wave from the edge (both edges in the present example with $n_2 = n_3 = n_4 = 1/\sqrt{3}$ is

$$\Delta T_1 = (d/C)\ [\sqrt{2}-1]/\sqrt{6} \tag{19}$$

The time interval between the former and the arrival of the diffracted wave from the vertex is

$$\Delta T_2 = (r/C)\ [\sqrt{3}-\sqrt{2}]/\sqrt{3} \tag{20}$$

When the distance between the two split front shock waves is d, the time interval between the arrival of the two shock waves at the point r is

$$\Delta T^* = (d/C) \tag{21}$$

The condition for the point r to receive a peak pressure less than that of a regular reflection is $\Delta T_1 < \Delta T^*$, that is,

$$r/d < \sqrt{6}/(\sqrt{2}-1) = 5.92 \tag{22}$$

In the problem of sonic boom, the length scale of the incident wave is about one quarter the length of an airplane. When the distance between two front shocks is 0.3, the maximum radial distance r allowed by the criterion (22) is nearly 1.8, therefore it is of the order of half an airplane length or hundreds of feet. In other words, when the front shock is split into two shock waves (Figs. 9, 10) or spread out to finite thickness, the area on the surface of the corner within a significant distance from the vertex (of hundreds of feet) will receive a peak pressure much less than the value given by a regular reflection due to the relief from diffraction by edges and the vertex.

CONCLUSION

In this report, numerical programs are presented for the computation of pressure distribution on the surface of a three-dimensional corner due to an incident plane wave of any wave form at any incident angle. Numerical examples show that the area on the surface of the corner within a significant distance from the vertex which can be (of the order of hundreds of feet) will receive a peak pressure much less than that of a regular reflection when the front shock is split to two shock waves or when the total pressure raise is spread to a finite thickness.

REFERENCES

1. Keller, J.B. and Blank, A., "Diffraction and Reflection of Pulses by Wedges and Corners," Communication on Pure and Applied Mechanics, Vol. 4, No. 1, pp. 75-94, June 1951.

2. Ting, L. and Kung, F., "Diffraction of a Pulse by a Three-Dimensional Corner," NASA CR-1728, March 1971.

APPENDIX

NUMERICAL PROGRAMS

Program I : determination of coefficients K_λ of the eigenfunction from the given eigenvalues of λ's at a given set of direction cosines (n_j) of the incident pulse

Input Definition

ETA(J) : n_2 n_3 n_4, the direction cosines of the normal to the incident pulse

II : control constant

II=1 ; for the calculation of K_λ for odd function

II equal to any integer other than one; for the calculation of K_λ for even function

NMAX: } number of terms in the eigenfunction
LMAX:

XLAM: λ, eigenvalue

BMIN (J): associated constants for odd function

DMIN (L): associated constants for even function

INPUT FORMAT

II = 2, NMAX, LMAX (3I5)

Set 1 { XLAM (F15.0)
BMIN(J) (5F15.0)
DMIN(L) } (5F15.0)

Set 2 { }

.
.
.

END FILE

II = 1, NMAX LMAX

{ }

{ }

END FILE

Note that the input data of XLAM, BMIN(J), DMIN(L) are determinated by the first program in Ref. [2], and the input data of NMAX, LMAX must be the same as that program. The calculation of K_λ at $\lambda = 0$ for the even function must be calculated first, and the end file cards are used to separate even function and odd function.

Output and Definition

XLAM : λ

BMIN(J) : coefficients

DMIN(L) :

These are input data listing for identification and verification

ETA(2) , ETA(3) , ETA(4) : n_2, n_3, n_4, a set of direction cosines (n_j)

K(LAMDA) : K_λ, the coefficients of the eigenfunction of incident pulse.

Program II: to determine the pressure distribution due to plane wave of a given wave form

Input Definition

R: r

THE: θ

PSI: φ

Spherical coordinates of pts.

ETA(2), ETA(3), ETA(4): n_2, n_3, n_4 a set of direction cosines (n_j)

RSTART: the first value of r for pressure calculation

TSTART: the initial value of T for pressure calculation at each value of r

RMAX: maximum value of r for the pressure calculation

TMAX: maximum value of t for the calculation of pressure distribution at each r

DELTT:	Δt, time increment $T = Ti - n \cdot \Delta t$
NR:	number of points r between RSTART and RMAX
NT:	number of points T between TSTART and TMAX
NUMBOG:	total number of λ for even or odd function
NFMAX:	total numbers of F(I)
F(I):	the increment of incident wave form function Ψ between phases S_i and S_{i-1}

INPUT FORMAT

```
II = 2,    NMAX,    LMAX                      (3I5)

NUMBOE                                        (I5)

           Set  { XLAM     }
            1   { BMIN(J)  }                  }(5F15.0)
                { DMIN(L)  }

           Set  {  }
            2   {  }
                .
                .
                .
           END FILE

II = 1,    NMAX     LMAX

NUMBOE     Set  {  }
            1   {  }

           Set  {  }
            2   {  }
                .
                .
                .
           END FILE
```

Output

The input data are printed in the first part of the output, and there are NR numbers of tables in the second part. Each table is for each value of r, and the pressure distributions are printed in the first column, and the second column are values of T from TSRART to TMAX.

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```
C     PROGRAM I
C     DETERMINATION OF THE COEFFICIENTS OF THE EIGENFUNCTION EXPANSION FROM
C     THE INITIAL DATA
      DIMENSION F(70),FUP(70,70),FBT(70,70),SFL(70,70)
      DIMENSION SUMJ(70),SUML(70),BMIN(70),DMIN(70)
      DIMENSION ETA(10),WF1(10),WF2(10),BWF1(10),BWF2(10)
      COMMON ETA,WF1,WF2,BWF1,BWF2,XXI
      PI=3.1415926
      XXI=1.
      READ(5,7000) (ETA(I),I=2,4)
 7000 FORMAT(3F15.8)
      ETA(1)=ETA(4)
      ETA(5)=ETA(2)
      DO 402 J=2,4
      SIGNEP=SIGN(1.,ETA(J+1))
      SIGNEM=SIGN(1.,ETA(J-1))
      ABSEM=ABS(ETA(J-1))
      ABSEP=ABS(ETA(J+1))
      EJPJM=ETA(J-1)**2+ETA(J+1)**2
      IF(SQRT(EJPJM)-(1.E-08)) 444,444,445
  444 WO=PI/4.
      BWO=(5.*PI)/4.
      GO TO 446
  445  WO=ASIN(ABSEM/SQRT(EJPJM))
      BWO=ASIN(ABSEP/SQRT(EJPJM))
  446 CONTINUE
      WF1(J)=(PI/2.-WO*SIGNEM)*(2./3.)
      WF2(J)=2.*PI-WF1(J)
      BWF1(J)=(PI+BWO*SIGNEP)*(2./3.)
      BWF2(J)=2.*PI-BWF1(J)
  402 CONTINUE
      LLMAX=50
      IDIM=70
      MAXU1=60
      MAXTOP=60
      MAXU2=60
      MAXBOT=60
 1000 READ(5,1001) II,NMAX,LMAX
 1001 FORMAT(3I5)
      IF(ENDFILE 5) 9999,1002
 1002 CONTINUE
      IF(II .EQ. 1) 101,103
C
C
C    ODD FUNCTION
```

```
  101 IIMAX=LMAX+NMAX
      WRITE(6,1003)
 1003 FORMAT(1H1*    ODD FUNCTION*)
      NCALL=NMAX
      LCALL=LMAX
      GO TO 104
C
C    EVEN FUNCTION
  103 IIMAX=LMAX+NMAX+2
      WRITE(6,1004)
 1004 FORMAT(1H1,*    EVEN FUNCTION*)
      NCALL=NMAX+1
      LCALL=LMAX+1
  104 CONTINUE
      WRITE(6,7532) ETA(2),ETA(3),ETA(4)
 7532 FORMAT(* ETA(2)=*,E12.5,3X,*ETA(3)=*,E12.5,3X,*ETA(4)=*,E12.5)
       PI=3.1415926
      ALPHA=PI/2.
      XMU=0.
      PPHI=2.*(PI-ALPHA/2.)
      XNU1=PI/PPHI
      HALPHI=PPHI/2.
      EPS    =0.000001
      WRITE(6,502) NMAX,LMAX,IIMAX
  502 FORMAT(      *   NMAX=*,I5,5X,*LMAX=*,I5,5X,*IIMAX=*,I5)
  400 READ(5,100)XLAM
      IF(ENDFILE 5) 1000,1111
 1111 CONTINUE
  100 FORMAT(5F15.0)
      READ(5,100)   (BMIN(I),I=1,NCALL)
      READ(5,100)   (DMIN(L),L=1,LCALL)
      U0=0.
      MAXPUS=MAXTOP+1
      INDEX=1
      WRITE(6,32) XLAM
   32 FORMAT(      *   XLAM=*,E15.8)
      WRITE(6,503)
  503 FORMAT(//*    COEFICIENTS OF EIGENFUNCTION*)
      WRITE(6,304) (BMIN(I),I=1,NCALL)
      WRITE(6,304) (DMIN(L),L=1,LCALL)
  304 FORMAT(5E20.8/(5X,5E20.8))
      DELU1=(1.-U0-2.*EPS   )/MAXU1
      MAXUST=MAXU1+1
      UK=U0+EPS
      DO 5  I=1,MAXUST
      TOTARG=0.
```

```
      CALL FFF(UK,INDEX,MAXTOP,F,II,PPHI,IDIM)
      DO 4 J=1,MAXPUS
      FUP(I,J)=F(J)
    4 CONTINUE
      IROW=I
      DO 13 J=1,NCALL
      IF(II .EQ. 1) 111,113
C
C   ODD FUNCTION
  111 CONTINUE
      XVAL=J
      CALL SININT(FUP,XVAL,IROW,MAXTOP,PI,0,IDIM ,TRIINT)
      GO TO 114
C
C   EVEN FUNCTION
  113 CONTINUE
      XVAL=J-1
      CALL COSINT(FUP,XVAL,IROW,MAXTOP,PI,0,IDIM ,TRIINT)
  114 CONTINUE
      SS=XVAL
      CALL PPDD(SS,XLAM,UK,LLMAX,PP,DP)
      TOTARG=TOTARG+BMIN(J)*PP*TRIINT
   13 CONTINUE
      SUMJ(I)=TOTARG
      UK=UK+DELU1
    5 CONTINUE
      TOTEM1=SUMJ(1)
      DO 19 I=2,MAXU1
      COEF=3.+(-1.)**I
      TOTEM1=TOTEM1+COEF*SUMJ(I)
   19 CONTINUE
      TOTEM1=(TOTEM1+SUMJ(MAXUST))*DELU1/3.
      MAXPUS=MAXBOT+1
      INDEX=2
      DELU2=(1.+U0-2.*EPS   )/MAXU2
      MAXUSM=MAXU2+1
      UK=-1.+EPS
      DO 9 I=1,MAXUSM
      TOTARG=0.
      CALL FFF(UK,INDEX,MAXBOT,F,II,PPHI,IDIM)
      DO 8 J=1,MAXPUS
      FBT(I,J)=F(J)
    8 CONTINUE
      IROW=I
      DO 23 L=1,LCALL
      IF(II .EQ. 1) 121,123
```

```
C
C   ODD FUNCTION
  121 CONTINUE
      XVAL=(2*L-1)*XNU1
      CALL SININT(FBT,XVAL,IROW,MAXBOT,HALPHI,0,IDIM ,TRIINT)
      GO TO 124
C
C   EVEN FUNCTION
  123 CONTINUE
      XVAL=2*(L-1)*XNU1
      CALL COSINT(FBT,XVAL,IROW,MAXBOT,HALPHI,0,IDIM ,TRIINT)
  124 CONTINUE
      SS=XVAL
       UF=-UK
      CALL PPDD(SS,XLAM,UF,LLMAX,PP,DP)
      TOTARG=TOTARG+DMIN(L)*PP*TRIINT
   23 CONTINUE
      SUML(I)=TOTARG
      UK=UK+DELU2
    9 CONTINUE
      TOTEM2=SUML(1)
      DO 29 I=2,MAXU2
      COEF=3.+(-1.)**I
      TOTEM2=TOTEM2+COEF*SUML(I)
   29 CONTINUE
      TOTEM2=(TOTEM2+SUML(MAXUSM))*DELU2/3.
      EEEM=TOTEM1+TOTEM2
      WRITE(6,11)EEEM
   11 FORMAT( /20X,*K(LAMDA)=*,E15.8)
      WRITE(6,401)
  401 FORMAT(////)
      GO TO 400
 9999 STOP
      END
```

```
      SUBROUTINE SININT(FP,XVAL,IROW,MAXST,UPLIM,BOTLIM,IDIM,TRIINT)
C     FILON'S METHOD FOR THE NUMERICAL EVALUATION OF TRIGONAMETRICAL
C     INTEGRALS--(INTEGRAND=FP(P)*SIN(X*P))
      DIMENSION FP(IDIM,IDIM)
      HH=(UPLIM-BOTLIM)/MAXST
      S2S=0.5*FP(IROW,1)*SIN(XVAL*BOTLIM)
      DO 14 J=3,MAXST,2
      P=BOTLIM+(J-1)*HH
      S2S=S2S+FP(IROW,J)*SIN(XVAL*P)
   14 CONTINUE
      J=MAXST+1
      S2S=S2S+0.5*FP(IROW,MAXST+1)*SIN(XVAL*UPLIM)
      S2SM=0.
      DO 16 J=2,MAXST,2
      P=BOTLIM+(J-1)*HH
      S2SM=S2SM+FP(IROW,J)*SIN(XVAL*P)
   16 CONTINUE
      THE=XVAL*HH
      IF(THE-0.2) 25,21,21
   21 ALPHA=(THE**2+THE*SIN(THE)*COS(THE)-2.*SIN(THE)**2)/THE**3
      BETA=2.*(THE*(1.+COS(THE)**2)-2.*SIN(THE)*COS(THE))/THE**3
      GARM=4.*(SIN(THE)-THE*COS(THE))/THE**3
      GO TO 31
   25 ALPHA=2.*THE**3/45.-2.*THE**5/315.+2.*THE**7/4725.
      BETA=2./3.+2.*THE**2/15.-4.*THE**4/105.+2.*THE**6/567.
      GARM=4./3.-2.*THE**2/15.+THE**4/210.-THE**6/11340.
   31 FA=FP(IROW,1)
      FB=FP(IROW,MAXST+1)
      TRIINT=HH*(-ALPHA*(FB*COS(XVAL*UPLIM)-FA*COS(XVAL*BOTLIM))+BETA*
     1S2S+GARM*S2SM)
      RETURN
      END
```

```
      SUBROUTINE COSINT(FP,XVAL,IROW,MAXST,UPLIM,BOTLIM,IDIM,TRIINT)
C     FILON'S METHOD FOR THE NUMERICAL EVALUATION OF TRIGONAMETRICAL
C     INTEGRALS--(INTEGRAND=FP(P)*COS(X*P))
      DIMENSION FP(IDIM,IDIM)
      HH=(UPLIM-BOTLIM)/MAXST
      S2S=0.5*FP(IROW,1)*COS(XVAL*BOTLIM)
      DO 14 J=3,MAXST,2
      P=BOTLIM+(J-1)*HH
      S2S=S2S+FP(IROW,J)*COS(XVAL*P)
   14 CONTINUE
      J=MAXST+1
      S2S=S2S+0.5*FP(IROW,MAXST+1)*COS(XVAL*UPLIM)
      S2SM=0.
      DO 16 J=2,MAXST,2
      P=BOTLIM+(J-1)*HH
      S2SM=S2SM+FP(IROW,J)*COS(XVAL*P)
   16 CONTINUE
      THE=XVAL*HH
      IF(THE-0.2) 25,21,21
   21 ALPHA=(THE**2+THE*SIN(THE)*COS(THE)-2.*SIN(THE)**2)/THE**3
      BETA=2.*(THE*(1.+COS(THE)**2)-2.*SIN(THE)*COS(THE))/THE**3
      GARM=4.*(SIN(THE)-THE*COS(THE))/THE**3
      GO TO 31
   25 ALPHA=2.*THE**3/45.-2.*THE**5/315.+2.*THE**7/4725.
      BETA=2./3.+2.*THE**2/15.-4.*THE**4/105.+2.*THE**6/567.
      GARM=4./3.-2.*THE**2/15.+THE**4/210.-THE**6/11340.
   31 FA=FP(IROW,1)
      FB=FP(IROW,MAXST+1)
      TRIINT=HH*( ALPHA*(FB*SIN(XVAL*UPLIM)-FA*SIN(XVAL*BOTLIM))+BETA*
     1S2S+GARM*S2SM)
      RETURN
      END
```

```
      SUBROUTINE FFF(UU,INDEX,MAX,F,II,PPHI,IDIM)
      DIMENSION F(IDIM)
      PI=3.1415926
      IF(INDEX .EQ. 1) 4,6
    4 DELPSI=PI/FLOAT(MAX)
      GO TO 7
    6 DELPSI=PPHI/FLOAT(2*MAX)
    7 CONTINUE
      PSI=0.
      MAXPUS=MAX+1
      DO 19 N=1,MAXPUS
      BPSI=PSI-PPHI/2.
      CALL FSF(BPSI,UU,FS ,PPHI)
      FA=FS
      BPSI=2.*PI-PSI-PPHI/2.
      CALL FSF(BPSI,UU,FS ,PPHI)
      FB=FS
      IF(II .EQ. 1) 14,16
   14 F(N)=(FA-FB)/2.
      GO TO 17
   16 F(N)=(FA+FB)/2.
   17 CONTINUE
      PSI=PSI+DELPSI
   19 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE CON2D(RHO,TAU,W1,W2,SS,PC)
      PI=3.1415926
      EPSIL=0.000001
      CON1=1.-RHO**2
      IF(CON1+EPSIL) 10,10,14
   10 WRITE(6,11)
   11 FORMAT(*  1-RHO**2+EPSIL  .LE. ZERO, CHECK THE PROGRAM *)
      STOP
   14 IF(CON1 )15,15,17
   15 GG=1.
      GO TO 21
   17 GG=(RHO/(1.+SQRT(CON1)))**SS
   21 BTAU=SS*TAU
      DD=(1.-GG**2)*SIN(0.5*(W2-W1))
      CC=(1.+GG**2)*COS(0.5*(W2-W1)) -2.*GG*COS(BTAU-(W2+W1)*0.5)
      SQCD=SQRT(CC**2+DD**2)
      IF(SQCD-EPSIL) 23,25,25
   23 PC=0.5
      GO TO 30
   25 IF(CC) 28,26,26
   26 PC=ASIN(DD/SQCD)/PI
      GO TO 30
   28 PC=ASIN(DD/SQCD)/PI*(-1.)+1.
   30 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE FSF(BPSI,UU,FS,PPHI)
      DIMENSION ETA(10),X(10),F(10),WF1(10),WF2(10),BWF1(10),BWF2(10)
      COMMON ETA,WF1,WF2,BWF1,BWF2,XXI
      EPS=1.E-06
      PI=3.1415926
      CONS23=2./3.
      EEPS=1.E-05
  199 CONTINUE
      X(2)=XXI*SQRT(1.-UU**2)*SIN(BPSI)
      X(3)=XXI*SQRT(1.-UU**2)*COS(BPSI)
      X(4)=-XXI*UU
      TESTX2=ABS(X(2))
      TESTX3=ABS(X(3))
      TESTX4=ABS(X(4))
      IF(TESTX2 .LE. EPS) 191,192
  191 X(2)=0.
  192 IF(TESTX3 .LE. EPS) 193,194
  193 X(3)=0.
  194 CONTINUE
  198 CONTINUE
      X(1)=X(4)
      X(5)=X(2)
      IF(XXI-1.) 201,203,203
  201 WRITE(6,202)
  202 FORMAT(*  ERROR XXI .LT. 1---CASE A*)
      STOP
  203 IF(X(2) .GT. 0. .AND. X(3) .GT. 0. .AND. X(4) .GT. 0. )204,206
  204 THALPI=ABS(BPSI-1.57079632)
      IF(THALPI-EEPS) 208,208,209
  208 BPSI=BPSI+EEPS
      GO TO 199
  209 WRITE(6,205)
  205 FORMAT(*  ERROR X(2),X(3), AND X(4) .GT. 1E-06---CASE B*)
      STOP
  206 SUMXN=0.
      DO 207 J=2,4
  207 SUMXN=SUMXN+ETA(J)*X(J)
      IF(SUMXN-1.) 215,213,211
  211 FS=0.
      GO TO 300
  213 FS=0.5
      GO TO 300
  215 DO 258 J=2,4
      IF(X(J)-ETA(J)) 221,223,223
  221 F(J)=100.
      GO TO 259
```

```
  223 VAL1=SQRT(ETA(J-1)**2+ETA(J+1)**2)
      VAL2=1.-ETA(J)*X(J)
      Y=(X(J-1)*VAL1)/VAL2
      Z=(-X(J+1)*VAL1)/VAL2
      RHO=SQRT(Y**2+Z**2)
      IF(RHO-1.) 225,225,251
  225 CONTINUE
      IF(RHO-EPS)226,226,228
  226 TAU=PI/2.
      GO TO 240
  228 CONTINUE
      TAU0=ASIN(ABS(Z/RHO))
      IF(Z) 235,231,231
  231 IF(Y) 233,232,232
  232 TAU=TAU0
      GO TO 240
  233 TAU=PI-TAU0
      GO TO 240
  235 IF(Y) 236,236,237
  236 TAU=PI+TAU0
      GO TO 240
  237 IF(X(J)) 1237,1237,1238
 1237 F(J)=100.
      GO TO 259
 1238 CONTINUE
      WRITE(6,238) TAU0,TAU,Y,Z
  238 FORMAT(*  ERROR FOR Y .GT. AND Z .LT. 0----CASE C*/4E20.8)
      WRITE(6,253) J,VAL1,VAL2,RHO,X(2),X(3),X(4),XXI,BPSI ,UU
      STOP
  240 CONTINUE
      W1=WF1(J)
      W2=WF2(J)
      BW1=BWF1(J)
      BW2=BWF2(J)
      CALL CON2D(RHO,TAU,W1,W2,CONS23,FIF)
      CALL CON2D(RHO,TAU,BW1,BW2,CONS23,FIIF)
      SIGNEP=SIGN(1.,ETA(J+1))
      SIGNEM=SIGN(1.,ETA(J-1))
  243 F(J)= ((1.-FIF)*SIGNEP+FIIF*SIGNEM)
      GO TO 259
  251 F(J)=100.
  259 CONTINUE
  258 CONTINUE
  253 FORMAT(I3,9E13.5)
      SUMF=0.
      DO 261 J=2,4
```

```
      SUMF=SUMF+F(J)
  261 CONTINUE
      IF(SUMF-EPS) 260,260,262
  260 FS=SUMF+1.
      GO TO 300
  262 CONTINUE
      IF(SUMF-90.) 263,263,265
C
C----INSIDE ALL THREE CONES
  263 WRITE(6,264) (F(J),J=1,4),FIF,FIIF
  264 FORMAT(*   ERROR SUMF .LE. 90---CASE D*/6E20.8)
      WRITE(6,200) VAL1,VAL2,Y,Z,RHO,TAU0,W1,W2,FIF,BW1,BW2,FIIF
      STOP
  265 IF(SUMF-190.) 267,267,269
C
C------INSIDE TWO CONES AND OUTSIDE THE THIRD CONE
  267 FS=SUMF-100.
      GO TO 300
  269 IF(SUMF-290.)271,271,272
C
C-----INSIDE ONE CONE  ONLY
  271 FS=SUMF-199.
      GO TO 300
  272 CONTINUE
C
C----OUTSIDE ALL THREE CONES
  273 DO 279 J=2,4
      TEST=ETA(J-1)*X(J-1)-ETA(J)*X(J)+ETA(J+1)*X(J+1)
      DJ=ETA(J+1)
      DM=ETA(J-1)
      IF(X(J) .LE. 0. .AND. TEST .LT. 1.) 275,279
  275 IF(X(J+1) .GE. DJ .OR. X(J-1) .GE. DM)281,279
  279 CONTINUE
      FS=1.
      GO TO 300
  281 FS=2.
  300 CONTINUE
  301 FORMAT(*  SUB FSF(NEW)* 5X, 5E20.8)
  200 FORMAT(6E20.8)
      RETURN
      END
```

```
      SUBROUTINE PPDD(SS,XLAM,XMU,LLMAX,PP,DP)
C     SUBROUTINE PPDD--WITHOUT DP
      XNU=SS
      EPS=1.E-06
      DD=1.
      TEM1=1.
      ZX=(1.-XMU)/2.
      IF(ABS(ZX) .LE. EPS.AND. ABS(XNU) .LE. EPS) 35,31
   31 CONTINUE
      DO 40 L=2,LLMAX
      ZL=L-1
      DD=DD*(ZL-XLAM     -1.)*(ZL+XLAM     )/(ZL**2+ZL*XNU)
      TEM1=TEM1+DD*ZX**(L-1)
   40 CONTINUE
       ARTFL=((1.-XMU)/(1.+XMU))**(XNU/2.)
      PP=ARTFL*TEM1
      GO TO 41
   35 PP=1.
   41 CONTINUE
      DP=0.
      RETURN
      END
```

```
C     PROGRAM II
      DIMENSION EDIM(20),BMIN(20),DMIN(20)
      DIMENSION ETA(10),WF1(10),WF2(10),BWF1(10),BWF2(10)
      DIMENSION ELAM(40),ZZZ(40,20),PP(400),F(400),GG(40)
      COMMON /BLOCK1/ETA,WF1,WF2,BWF1,BWF2,XXI
      READ(5,7000) (ETA(I),I=2,4)
      READ(5,10)   RMAX,TMAX,DELTT ,TSTART,RSTART
 7000 FORMAT(3F15.8)
      READ(5,7001) NFMAX,NT,NR
 7001 FORMAT(3I5)
      WRITE(6,7002)
 7002 FORMAT(1H1)
      ETA(1)=ETA(4)
      ETA(5)=ETA(2)
      READ(5,10) (F(I),I=1,NFMAX)
   10 FORMAT(5F10.0)
      READ(5,10)THE,PSI
      UU=COS(THE)
      LLMAX=50
       PI=3.1415926
      ALPHA=PI/2.
      PPHI=2.*(PI-ALPHA/2.)
      XNU1=PI/PPHI
      BPSI=PSI-PPHI/2.
      DELR=RMAX/NR
      DELT=TMAX/NT
      WRITE(6,1) (ETA(I),I=1,3),RMAX,TMAX,DELTT,DELR,DELT
    1 FORMAT(*    ETA(1)=*,E12.5,5X,*ETA(2)=*,E12.5,5X,*ETA(3)=*,E12.5/
     1*    RMAX=*,E12.5,5X,*TMAX=*,E12.5,5X,*DELTT=*,E12.5,5X,*DELR=*,
     2E12.5,5X,*DELT=*,E12.5)
      WRITE(6,7005)  UU,PSI
 7005 FORMAT(2X,*UU=*,E15.8,5X,*PSI=*,E15.8)
      WRITE(6,7003)
 7003 FORMAT(//3X,*F(I)*)
      WRITE(6,440) (F(I),I=1,NFMAX)
  440 FORMAT(5F20.5)
      DO 442 J=2,4
      SIGNEP=SIGN(1.,ETA(J+1))
      SIGNEM=SIGN(1.,ETA(J-1))
      ABSEM=ABS(ETA(J-1))
      ABSEP=ABS(ETA(J+1))
      EJPJM=ETA(J-1)**2+ETA(J+1)**2
      IF(SQRT(EJPJM)-(1.E-08)) 444,444,445
  444 W0=PI/4.
      BW0=(5.*PI)/4.
      GO TO 446
```

```
  445  WO=ASIN(ABSEM/SQRT(EJPJM))
       BWO=ASIN(ABSEP/SQRT(EJPJM))
  446  CONTINUE
       WF1(J)=(PI/2.-WO*SIGNEM)*(2./3.)
       WF2(J)=2.*PI-WF1(J)
       BWF1(J)=(PI+BWO*SIGNEP)*(2./3.)
       BWF2(J)=2.*PI-BWF1(J)
  442  CONTINUE
       NUMB=0.
 1000  READ(5,1001) II,NMAX,LMAX
 1001  FORMAT(3I5)
       IF(ENDFILE 5) 9999,1002
 1002  CONTINUE
       IF(II .EQ. 1) 101,103
C
C   ODD FUNCTION
  101  IIMAX=LMAX+NMAX
       READ(5,1001) NUMBOF
       NBBB=0
       WRITE(6,2)
    2  FORMAT(1H1,//*    ODD FUNCTION*)
       NCALL=NMAX
       LCALL=LMAX
       GO TO 104
C
C    EVEN FUNCTION
  103  IIMAX=LMAX+NMAX+2
       READ(5,1001) NUMBOE
       NBBB=0
       WRITE(6,3)
    3  FORMAT(1H1,//*    EVEN FUNCTION *)
       NCALL=NMAX+1
       LCALL=LMAX+1
  104  CONTINUE
       WRITE(6,502) NMAX,LMAX,IIMAX,ALPHA,PPHI,XNU1
  502  FORMAT(     *  NMAX=*,I5,5X,*LMAX=*,I5,5X,*IIMAX=*,I5/5X,              *
      1ALPHA=*,E12.5,5X,*PPHI=*,E12.5,5X,*XNU1=*,E12.5)
  110  READ(5,100) XLAM,EEEM
  100  FORMAT(5E15.8)
       IF(ENDFILE 5) 1000,1111
 1111  CONTINUE
 2000  NUMB=NUMB+1
       EOLD=EEEM
       NBBB=NBBB+1
       EEEM=(NUMBOE-(NBBB-1))*EEEM/NUMBOE
       READ(5,100)   (BMIN(I),I=1,NCALL)
```

```
      READ(5,100)   (DMIN(L),L=1,LCALL)
      CALL  T102(XLAM,1.,EDIM,BBBBBB)
      AAAAAA=BBBBBB
      WRITE(6,32) XLAM,AAAAAA,EOLD
   32 FORMAT(    *    XLAM=*,F14.9,5X,*AA0=*,F14.9,5X,*K(LAMDA)=*,F14.9)
      WRITE(6,503)
  503 FORMAT(//  *  COEFFICIENTS OF EIGEN-FUNCTIONS*)
      WRITE(6,304) (BMIN(I),I=1,NCALL)
      WRITE(6,304) (DMIN(L),L=1,LCALL)
  304 FORMAT(5E20.8/(5X,5E20.8))
      WRITE(6,401)
  401 FORMAT(////)
      ELAM(NUMB)=EEEM
      CALL  T102(XLAM,AAAAAA,EDIM,BBBBBB)
      DO 2001 I=1,10
 2001 ZZZ(NUMB,I)=EDIM(I)
      IF(UU) 2100,2002,2002
 2002 CONTINUE
      IF(II .EQ. 1) 200,203
C
C     ODD FUNCTION
  200 XMU=UU
      PHI=PSI
      PPUS=0.
      DO 201 J=1,NCALL
      SS=J
      CALL PPDD(SS,XLAM,XMU,LLMAX,PP,DP)
      SINFU=SIN(J*PHI)
      PPUS=PPUS+BMIN(J)*PP*SINFU
  201 CONTINUE
      GG(NUMB)=PPUS
      GO TO 110
C
C      EVEN FUNCTION
  203 CONTINUE
      XMU=UU
      PHI=PSI
      PPUS=0.
      DO 206 J=1,NCALL
      SS=J-1
      CALL PPDD(SS,XLAM,XMU,LLMAX,PP,DP)
      COSFU=COS(SS*PHI)
      PPUS=PPUS+BMIN(J)*PP*COSFU
  206 CONTINUE
      GG(NUMB)=PPUS
      GO TO 110
```

```
 2100 IF(II .EQ. 1) 301,400
C
C    ODD FUNCTION
  301 XMU=UU
      PHI=PSI
      PMIN=0.
      DO 302 L=1,LCALL
      SS=(2*L-1)*XNU1
      CALL PPDD(SS,XLAM,-XMU,LLMAX,PP,DP)
      SINFU=SIN(SS*PHI)
      PMIN=PMIN+DMIN(L)*PP*SINFU
  302 CONTINUE
      GG(NUMB)=PMIN
      GO TO 110
C
C    EVEN FUNCTION
  400 XMU=UU
      PHI=PSI
      PMIN=0.
      DO 402 L=1,LCALL
      SS=2*(L-1)*XNU1
      CALL PPDD(SS,XLAM,-XMU,LLMAX,PP,DP)
      COSFU=COS(SS*PHI)
      PMIN=PMIN+DMIN(L)*PP*COSFU
  402 CONTINUE
      GG(NUMB)=PMIN
      GO TO 110
 9999 MAXNUM =NUMB
      DO 2006 I=1,10
 2006 CONTINUE
       R=-DELR +RSTART
 2200 R=R+DELR
      IF(R-RMAX) 2202,2202,2201
 2201 STOP
 2202 CONTINUE
      WRITE(6,7004)  R
 7004 FORMAT(//5X,*R=*,E12.5,11X,*P*,17X,*T*)
       T=-DELT+TSTART
 2525 T=T+DELT
      IF(T-TMAX) 2602,2602,2200
 2602 CONTINUE
      DO 2400 N=1,NFMAX
      ST=T-N*DELTT +0.0001
      IF(ST) 2203,2203,2205
 2203 BIGP=0.
      GO TO 2400
```

```
 2205 XXI=R/ST
      IF(XXI-1.) 2300,2207,2207
 2207 CONTINUE
      CALL FSF(BPSI,UU,FS,PPHI)
      BIGP=FS
      GO TO 2400
 2300 BIGP=0.
      DO 2309 I=1,MAXNUM
      CALL INTER(XXI,I,ZZZ,ZXXI)
      BIGP=BIGP+ELAM(I)*ZXXI*GG(I)
 2309 CONTINUE
 2400 PP(N)=BIGP
      SUMFP=0.
      DO 2405 M=1,NFMAX
 2405 SUMFP=SUMFP+    PP(M)*F(M)
      WRITE(6,2407) SUMFP,T
 2407 FORMAT(19X,2E20.8)
      GO TO 2525
 2501 STOP
      END
```

```
      SUBROUTINE INTER(XXI,ICOL,ZZZ,ZXXI)
      DIMENSION ZZZ(40,20)
      N=0
      X=0.
   10 N=N+1
      X=X+0.1
      IF(X-XXI) 10,20,30
   20 ZXXI=ZZZ(ICOL,N)
      GO TO 50
   30 IF(N .GT. 10) GO TO 40
      IF(N .LE. 1) GO TO 60
      ZXXI=ZZZ(ICOL,N-1)+(ZZZ(ICOL,N)-ZZZ(ICOL,N-1))*(XXI-X+0.1)/0.1
      GO TO 50
   40 WRITE(6,41) XXI
   41 FORMAT(//*    ERROR IN SUB. INTER --FOR XXI=*,E12.5,*  IS OUT OF RA
     1NGE*)
      STOP
   50 CONTINUE
C     WRITE(6,51) XXI,ZXXI,ZZZ(ICOL,N-1),ZZZ(ICOL,N)
   51 FORMAT(*    SUB  INTER*, 5E20.8)
      GO TO 65
   60 IF(ICOL .GT. 1) GO TO 61
      ZZZZZZ=1.
      GO TO 64
   61 ZZZZZZ=0.
   64 CONTINUE
      ZXXI=(ZZZ(ICOL,1)-ZZZZZZ)*XXI+ZZZZZZ
   65 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE FSF(BPSI,UU,FS,PPHI)
      DIMENSION ETA(10),X(10),F(10),WF1(10),WF2(10),BWF1(10),BWF2(10)
      COMMON /BLOCK1/ETA,WF1,WF2,BWF1,BWF2,XXI
      EPS=1.E-06
      PI=3.1415926
      CONS23=2./3.
      EEPS=1.E-05
  199 CONTINUE
      X(2)=XXI*SQRT(1.-UU**2)*SIN(BPSI)
      X(3)=XXI*SQRT(1.-UU**2)*COS(BPSI)
      X(4)=-XXI*UU
      TESTX2=ABS(X(2))
      TESTX3=ABS(X(3))
      TESTX4=ABS(X(4))
      IF(TESTX2 .LE. EPS) 191,192
  191 X(2)=0.
  192 IF(TESTX3 .LE. EPS) 193,194
  193 X(3)=0.
  194 CONTINUE
  198 CONTINUE
      X(1)=X(4)
      X(5)=X(2)
      IF(XXI-1.) 201,203,203
  201 WRITE(6,202)  XXI,X(2),X(3),X(4),UU,BPSI
  202 FORMAT(*  ERROR XXI .LT. 1---CASE A* 6E14.5)
      STOP
  203 IF(X(2) .GT. 0. .AND. X(3) .GT. 0. .AND. X(4) .GT. 0. )204,206
  204 THALPI=ABS(BPSI-1.57079632)
      IF(THALPI-EEPS) 208,208,209
  208 BPSI=BPSI+EEPS
      GO TO 199
  209 WRITE(6,205)
  205 FORMAT(*  ERROR X(2),X(3), AND X(4) .GT. 1E-06---CASE B*)
      STOP
  206 SUMXN=0.
      DO 207 J=2,4
  207 SUMXN=SUMXN+ETA(J)*X(J)
      IF(SUMXN-1.) 215,213,211
  211 FS=0.
      GO TO 300
  213 FS=0.5
      GO TO 300
  215 DO 258 J=2,4
      IF(X(J)-ETA(J)) 221,223,223
  221 F(J)=100.
      GO TO 259
```

```
  223 VAL1=SQRT(ETA(J-1)**2+ETA(J+1)**2)
      VAL2=1.-ETA(J)*X(J)
      Y=(X(J-1)*VAL1)/VAL2
      Z=(-X(J+1)*VAL1)/VAL2
      RHO=SQRT(Y**2+Z**2)
      IF(RHO-1.) 225,225,251
  225 CONTINUE
      IF(RHO-EPS)226,226,228
  226 TAU=PI/2.
      GO TO 240
  228 CONTINUE
      TAUO=ASIN(ABS(Z/RHO))
      IF(Z) 235,231,231
  231 IF(Y) 233,232,232
  232 TAU=TAUO
      GO TO 240
  233 TAU=PI-TAUO
      GO TO 240
  235 IF(Y) 236,236,237
  236 TAU=PI+TAUO
      GO TO 240
  237 IF(X(J)) 1237,1237,1238
 1237 F(J)=100.
      GO TO 259
 1238 CONTINUE
      WRITE(6,238) TAUO,TAU,Y,Z
  238 FORMAT(*  ERROR FOR Y .GT. AND Z .LT. C----CASE C*/4E20.8)
      WRITE(6,253) J,VAL1,VAL2,RHO,X(2),X(3),X(4),XXI,BPSI  ,UU
      STOP
  240 CONTINUE
      W1=WF1(J)
      W2=WF2(J)
      BW1=BWF1(J)
      BW2=BWF2(J)
      CALL  CON2D(RHO,TAU,W1,W2,CONS23,FIF)
      CALL  CON2D(RHO,TAU,BW1,BW2,CONS23,FIIF)
      SIGNEP=SIGN(1.,ETA(J+1))
      SIGNEM=SIGN(1.,ETA(J-1))
  243 F(J)=  ((1.-FIF)*SIGNEP+FIIF*SIGNEM)
      GO TO 259
  251 F(J)=100.
  259 CONTINUE
  258 CONTINUE
  253 FORMAT(I3,9E13.5)
      SUMF=0.
      DO 261  J=2,4
```

```
      SUMF=SUMF+F(J)
  261 CONTINUE
      IF(SUMF-EPS) 260,260,262
  260 FS=SUMF+1.
      GO TO 300
  262 CONTINUE
      IF(SUMF-90.) 263,263,265
C
C----INSIDE ALL THREE CONES
  263 WRITE(6,264) (F(J),J=1,4),FIF,FIIF
  264 FORMAT(*   ERROR SUMF .LE. 90---CASE D*/6E20.8)
      WRITE(6,200) VAL1,VAL2,Y,Z,RHO,TAU0,W1,W2,FIF,BW1,BW2,FIIF
      STOP
  265 IF(SUMF-190.) 267,267,269
C
C------INSIDE TWO CONES AND OUTSIDE THE THIRD CONE
  267 FS=SUMF-100.
      GO TO 300
  269 IF(SUMF-290.)271,271,272
C
C-----INSIDE ONE CONE  ONLY
  271 FS=SUMF-199.
      GO TO 300
  272 CONTINUE
C
C----OUTSIDE ALL THREE CONES
  273 DO 279 J=2,4
      TEST=ETA(J-1)*X(J-1)-ETA(J)*X(J)+ETA(J+1)*X(J+1)
      DJ=ETA(J+1)
      DM=ETA(J-1)
      IF(X(J) .LE. 0. .AND. TEST .LT. 1.) 275,279
  275 IF(X(J+1) .GE. DJ .OR. X(J-1) .GE. DM)281,279
  279 CONTINUE
      FS=1.
      GO TO 300
  281 FS=2.
  300 CONTINUE
  301 FORMAT(*  SUB FSF(NEW)* 5X, 5E20.8)
  200 FORMAT(6E20.8)
      RETURN
      END
```

```
      SUBROUTINE CON2D(RHO,TAU,W1,W2,SS,PC)
      EPSIL=0.000001
      CON1=1.-RHO**2
      IF(CON1+EPSIL) 10,10,14
   10 WRITE(6,11)
   11 FORMAT(*  1-RHO**2+EPSIL .LE. ZERO, CHECK THE PROGRAM *)
      STOP
   14 IF(CON1 )15,15,17
   15 GG=1.
      GO TO 21
   17 GG=(RHO/(1.+SQRT(CON1)))**SS
   21 BTAU=SS*TAU
      DD=(1.-GG**2)*SIN(0.5*(W2-W1))
      CC=(1.+GG**2)*COS(0.5*(W2-W1)) -2.*GG*COS(BTAU-(W2+W1)*0.5)
      SQCD=SQRT(CC**2+DD**2)
      IF(SQCD-EPSIL) 23,25,25
   23 PC=0.5
      GO TO 30
   25 IF(CC) 28,26,26
   26 PC=ASIN(DD/SQCD)/PI
      GO TO 30
   28 PC=ASIN(DD/SQCD)/PI*(-1.)+1.
   30 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE T102(XLAM,AAAAAA,EDIM,BBBBBB)
      DIMENSION EDIM(20)
      COMMON XXXXL
      XXXXL=XLAM
      XMAX=0.9989
      X0=0.001
      X0XL=F3(XLAM,X0)
      X0XLP=F3(XLAM+1.,X0)
      X0XLM=F3(XLAM-1.,X0)
      X0XLM2=F3(XLAM+2.,X0)
   10 F0=(X0XL+XLAM*(XLAM+1.)*X0XLM2/(4.*XLAM+6.))*AAAAAA
      G0=(XLAM*X0XLM+(XLAM+1.)*(XLAM+2.)*XLAM*X0XLP/(4.*XLAM+6.))*AAAAAA
      DELX=0.001
      X=X0
      F=F0
      G=G0
      I=1
      XTEST=0.0999
   20 XK1=F1(X,F,G)*DELX
      XL1=F2(X,F,G)*DELX
      XK2=F1(X+DELX/2., F+XK1/2., G+XL1/2.)*DELX
      XL2=F2(X+DELX/2., F+XK1/2., G+XL1/2.)*DELX
      XK3=F1(X+DELX/2., F+XK2/2., G+XL2/2.)*DELX
      XL3=F2(X+DELX/2., F+XK2/2., G+XL2/2.)*DELX
      XK4=F1(X+DELX, F+XK3, G+XL3)*DELX
      XL4=F2(X+DELX, F+XK3, G+XL3)*DELX
      DELF=1./6.*(XK1+2.*XK2+2.*XK3+XK4)
      DELG=1./6.*(XL1+2.*XL2+2.*XL3+XL4)
      X=X+DELX
      F=F+DELF
      G=G+DELG
      IF(X .GE. XTEST) 201,203
  201 EDIM(I)=F
      XTEST=XTEST+0.1
      I=I+1
  203 CONTINUE
      IF(X-0.0099999) 20,31,31
   31 IF(X-0.98999) 32,33,33
   32 DELX=0.01
      GO TO 20
   33 IF(X-0.9989) 34,40,40
   34 DELX=0.001
      GO TO 20
   40 ZETA=1.-X
      C1=-XLAM*(XLAM+1.)/2.
      GEND=(F+ZETA*G)/(1.+C1*ZETA)
```

```
41 BBBBBB=1./GEND
   FDIM(I)=GEND
   RETURN
   END
```

```
      FUNCTION F1(A,B,C)
      COMMON   XLAM
      F1=C
      RETURN
      END
```

```
      FUNCTION F2(A,B,C)
      COMMON  XLAM
      F2=(-2.*A*(A**2-1.)*C-XLAM*(XLAM+1.)*B)/( A**2*(A**2-1.))
      RETURN
      END
```

```
      FUNCTION F3(D,E)
      F3=EXP(D*ALOG(E))
      RETURN
      END
```

```
      SUBROUTINE PPDD(SS,XLAM,XMU,LLMAX,PP,DP)
C     SUBROUTINE PPDD--WITHOUT DP
      XNU=SS
      EPS=1.E-06
      DD=1.
      TEM1=1.
      ZX=(1.-XMU)/2.
      IF(ABS(ZX) .LE. EPS.AND. ABS(XNU) .LE. EPS) 35,31
   31 CONTINUE
      DO 40 L=2,LLMAX
      ZL=L-1
      DD=DD*(ZL-XLAM     -1.)*(ZL+XLAM     )/(ZL**2+ZL*XNU)
      TEM1=TEM1+DD*ZX**(L-1)
   40 CONTINUE
       ARTFL=((1.-XMU)/(1.+XMU))**(XNU/2.)
      PP=ARTFL*TEM1
      GO TO 41
   35 PP=1.
   41 CONTINUE
      DP=0.
      RETURN
      END
```

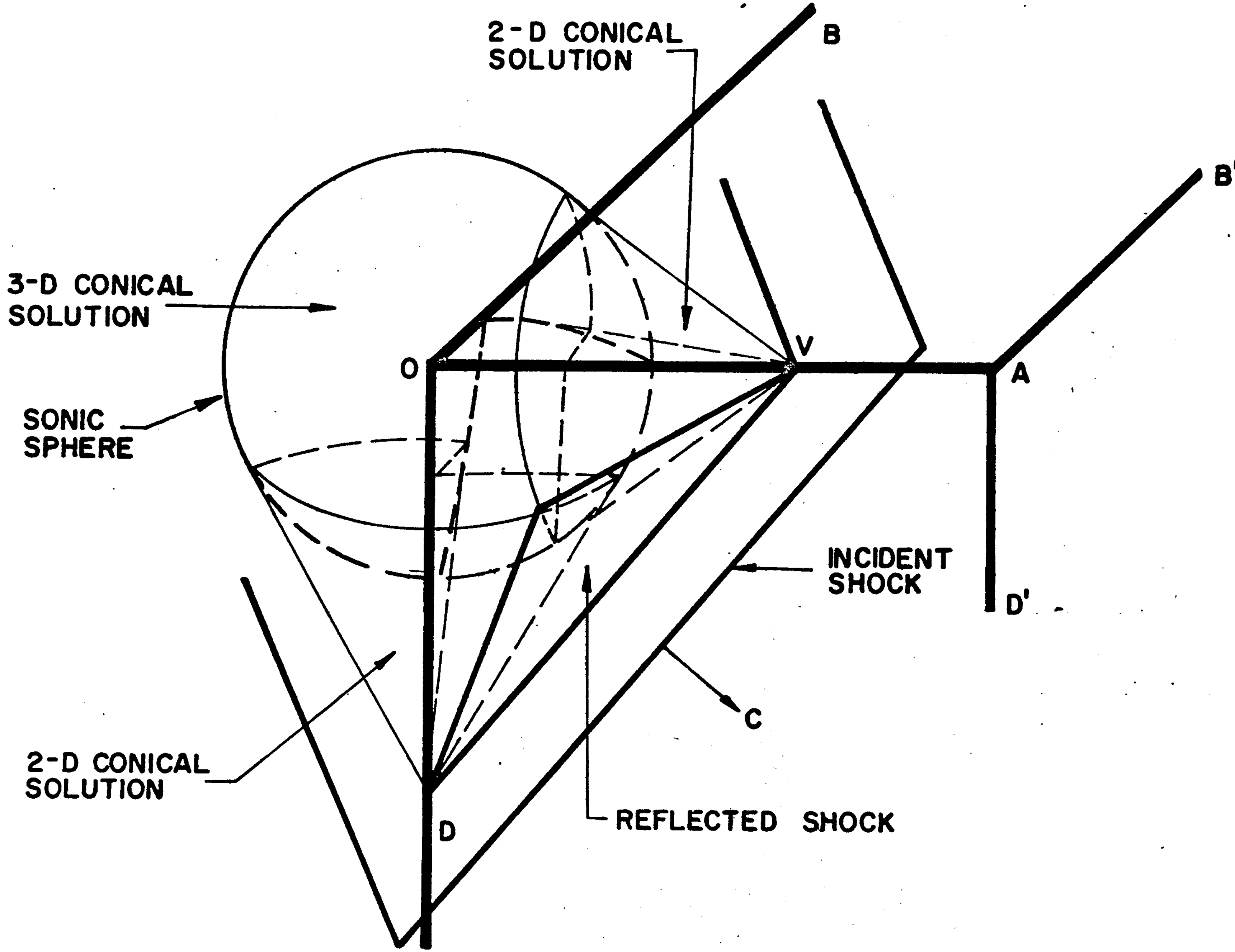

Fig. 1 Oblique incidence of a plane pulse on a three-dimensional corner

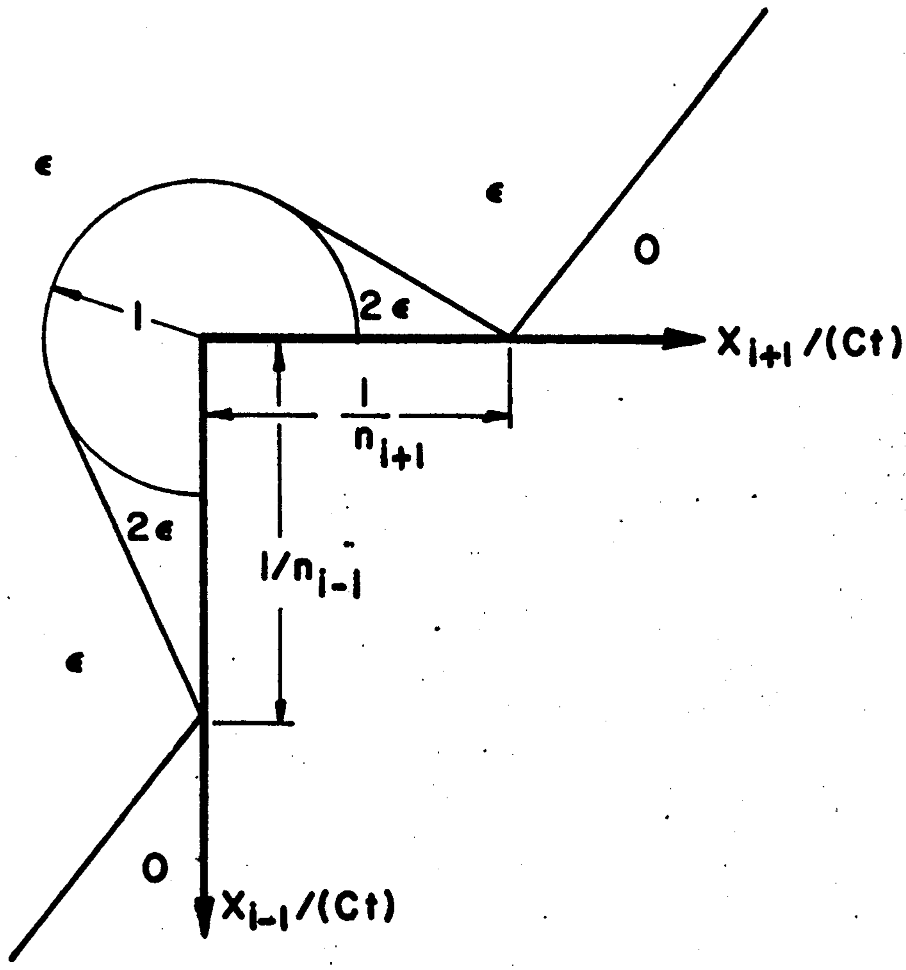

Fig. 2 Section normal to the i-th axis

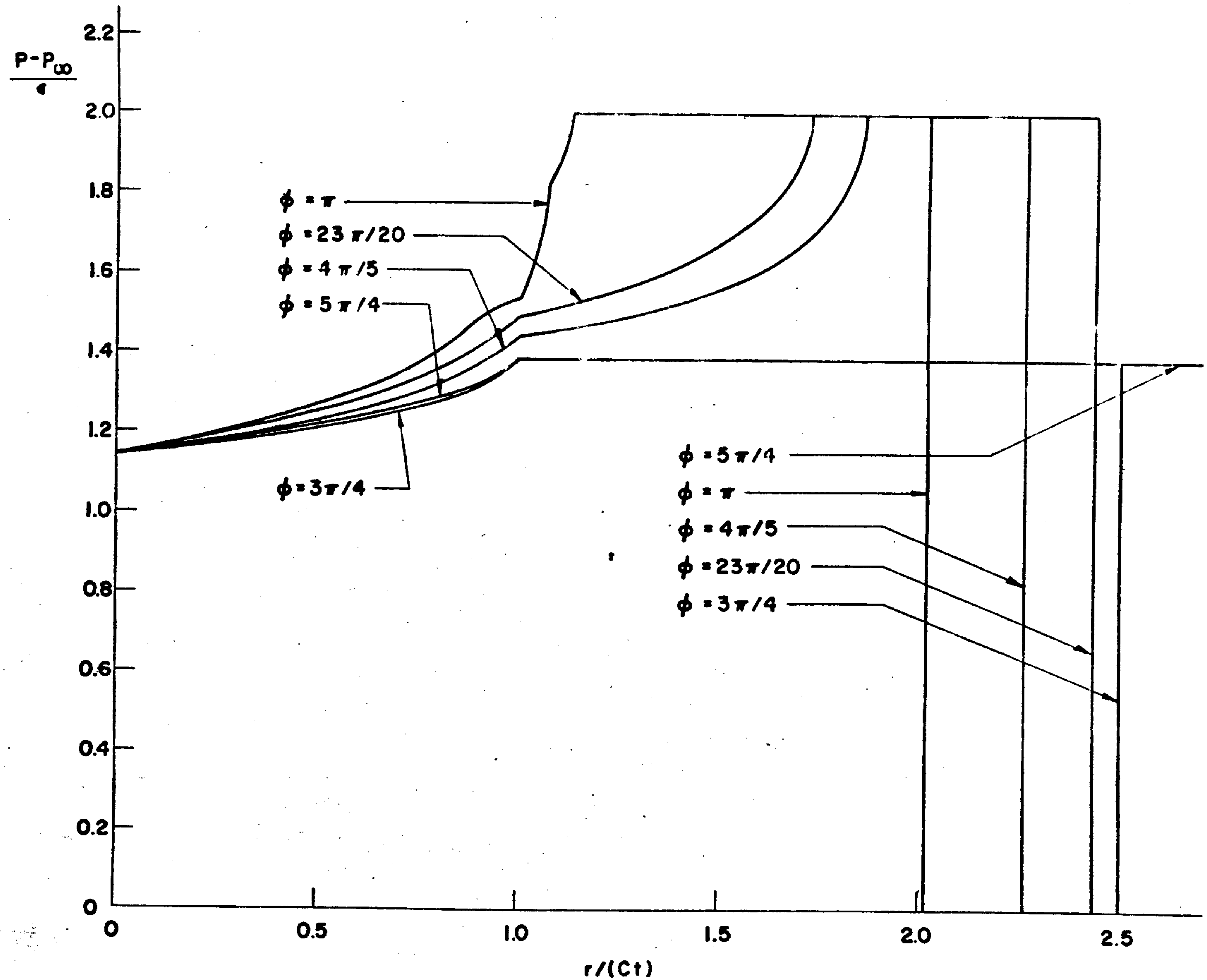

Fig. 3 Pressure distribution on the face $\theta = 0$ due to the incidence of a plane pulse with direction cosines 0.3, 0.4, 0.866

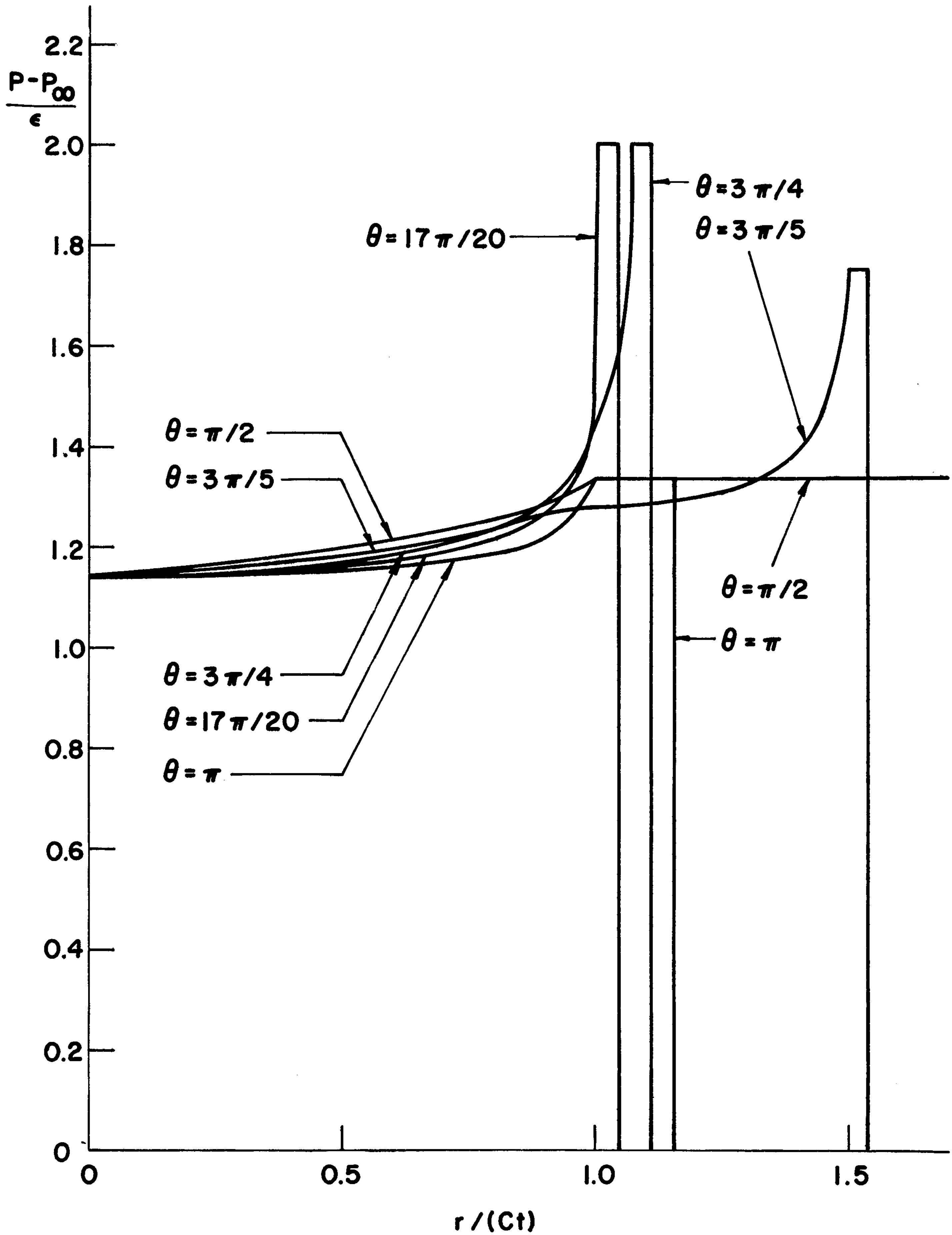

Fig. 4 Pressure distribution on the face $\varphi = -3\pi/4$ due to the incidence of a plane pulse with direction cosines 0.3, 0.4, 0.866

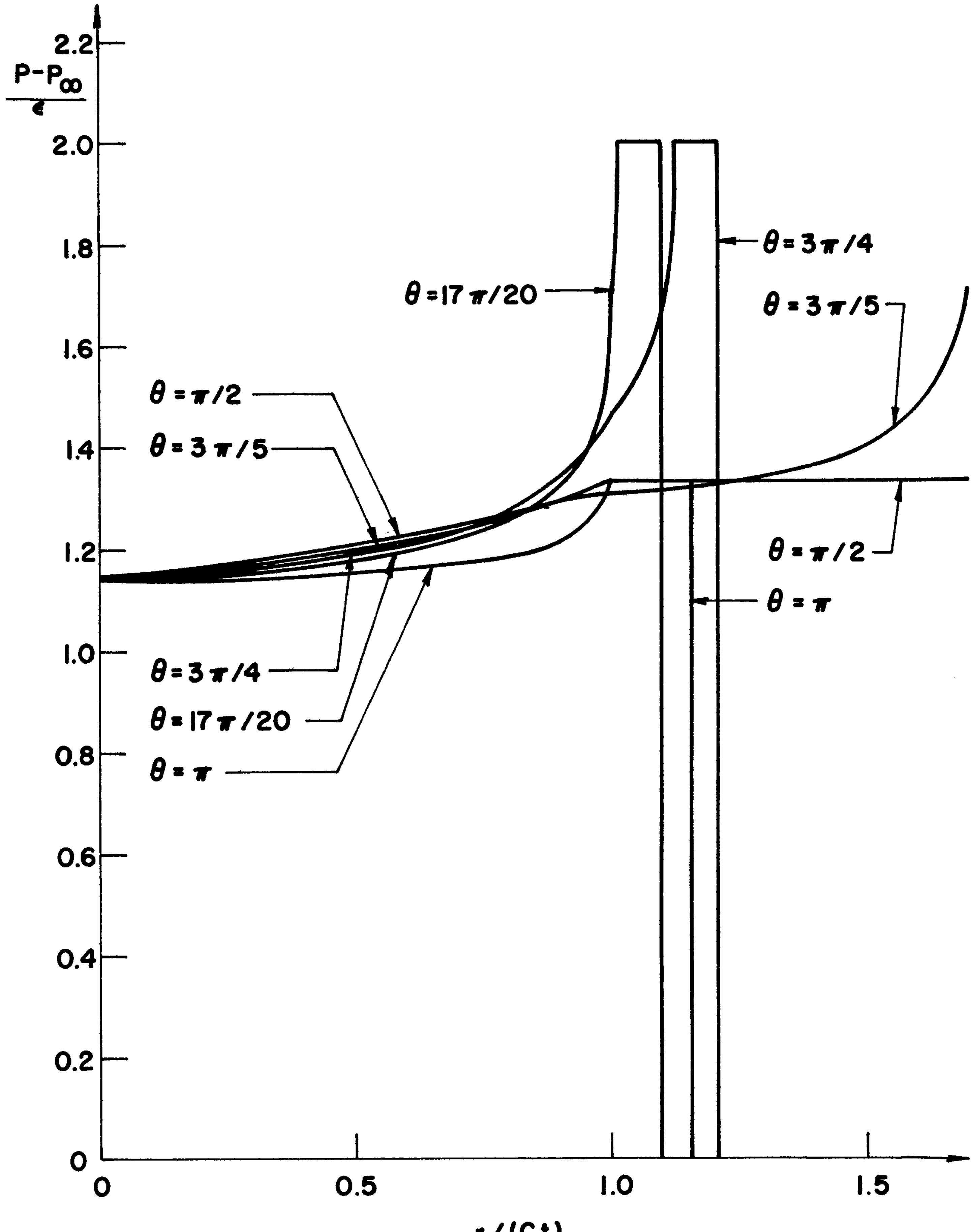

Fig. 5 Pressure distribution on the face $\varphi = 3\pi/4$ due to the incidence of a plane pulse with direction cosines 0.3, 0.4, 0.866

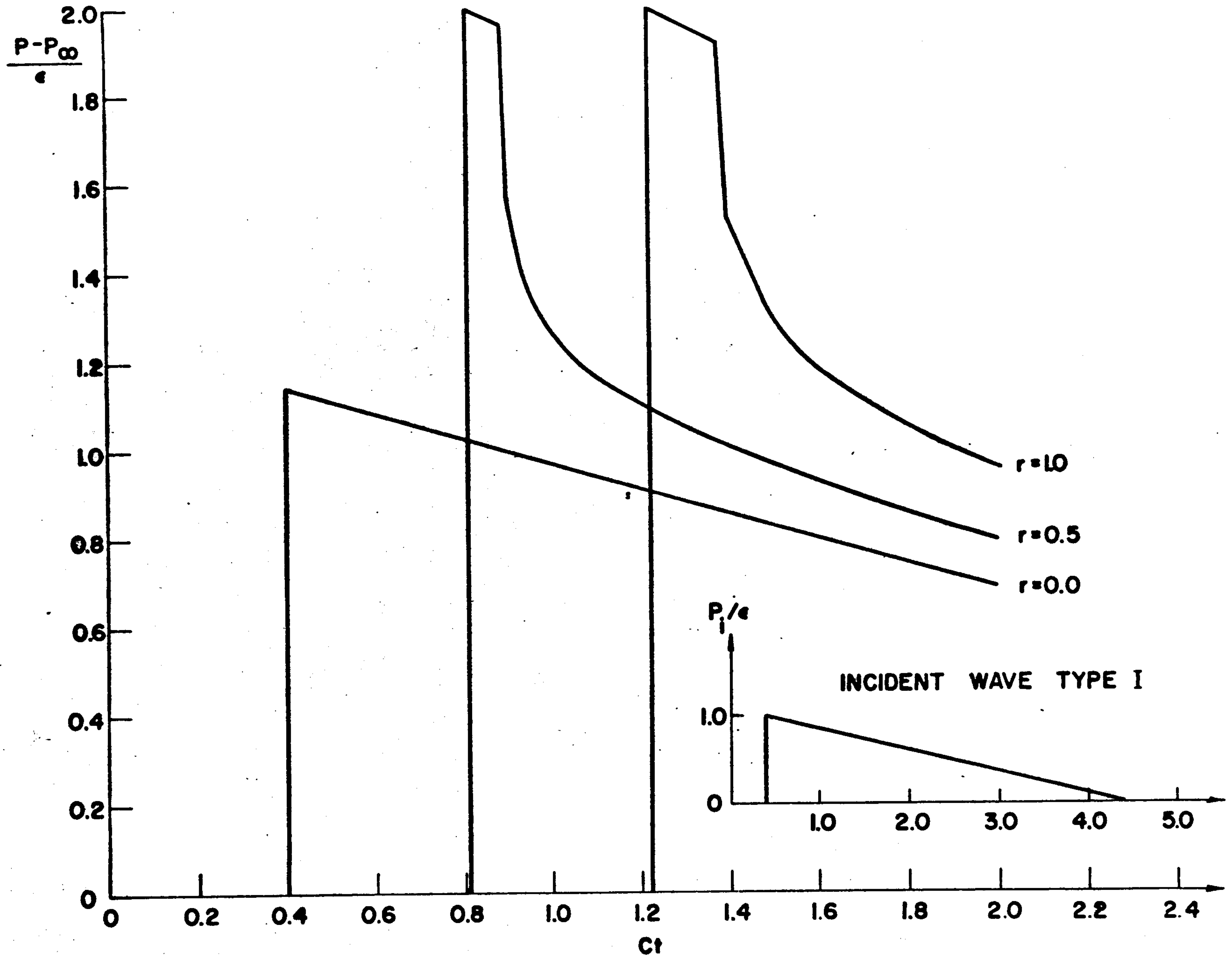

Fig. 6 Pressure signature received at points along the line $\theta = \pi/2$, $\varphi = \pi$ for plane wave incident at equal angles with the edges and with wave form of type I

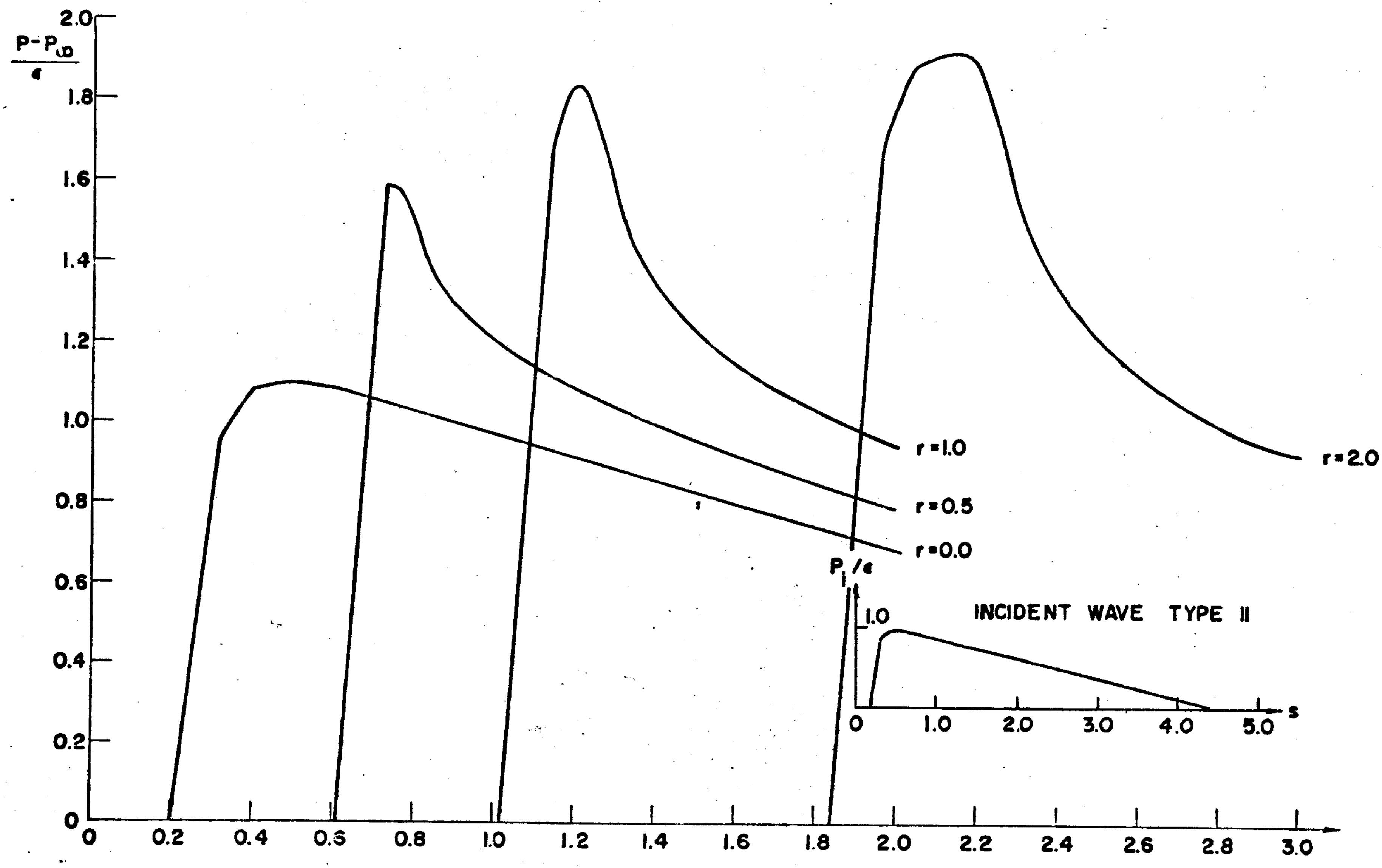

Fig. 7 Pressure signature received at points along the line $\theta = \pi/2$ $\varphi = \pi$ for plane wave incident at equal angles with the edges and with wave form of type II

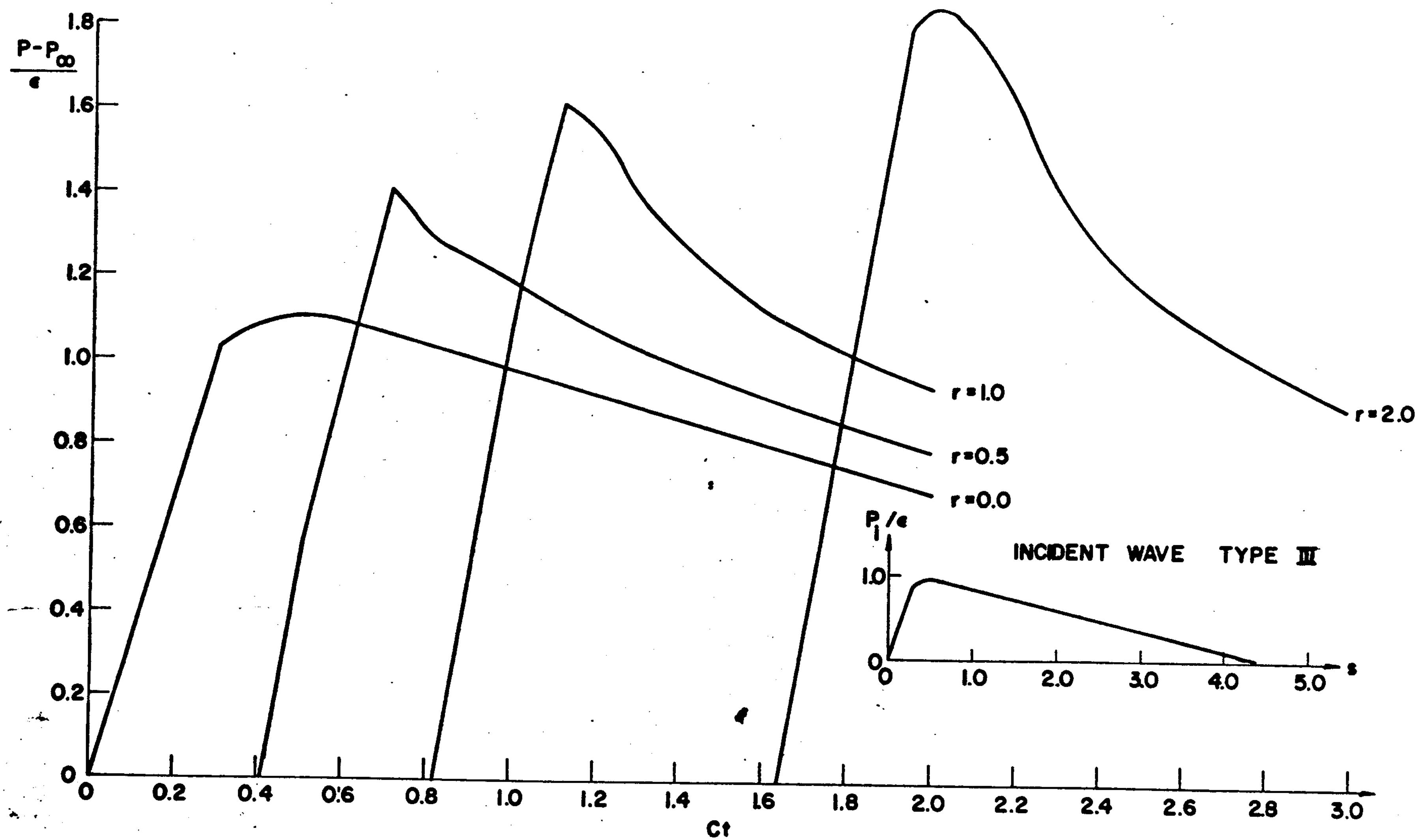

Fig. 8 Pressure signature received at points along the line $\theta = \pi/2$, $\varphi = \pi$ for plane wave incident at equal angles with the edges and with wave form of type III

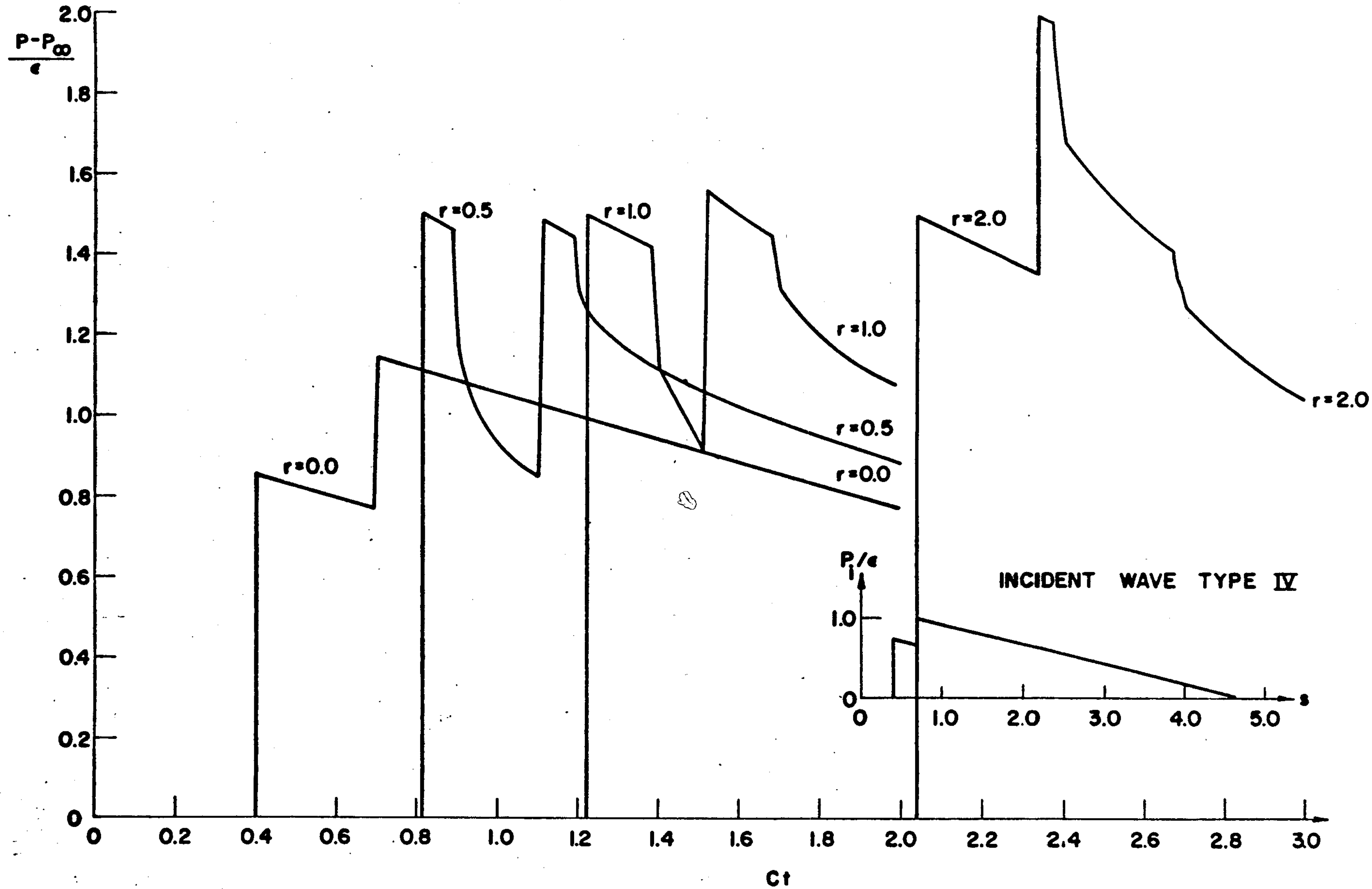

Fig. 9 Pressure signature received at points along the line $\theta = \pi/2$, $\phi = \pi$ for plane wave incident at equal angles with the edges and with wave form of type IV

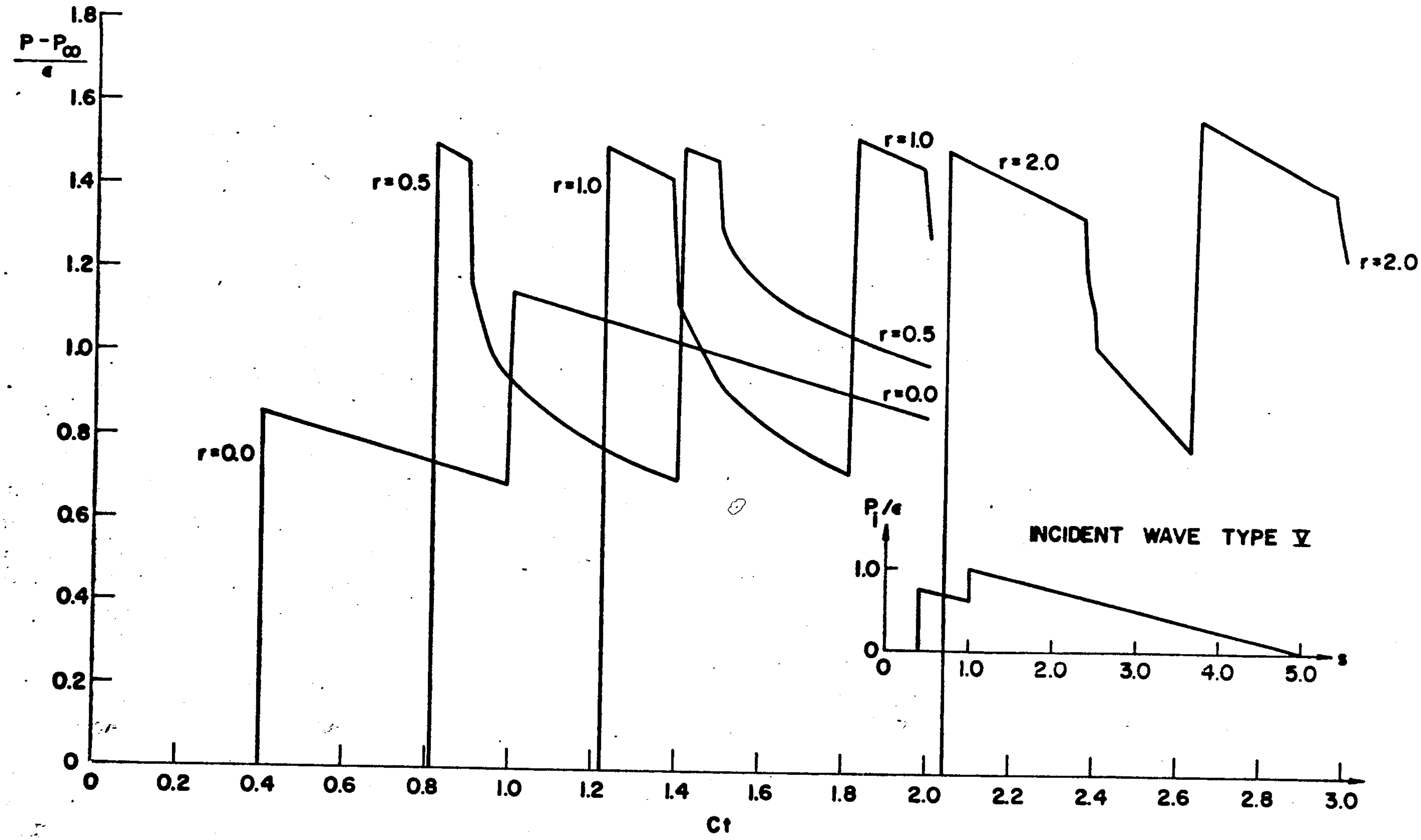

Fig. 10 Pressure signature received at points along line $\theta = \pi/2$, $\varphi = \pi$ for plane wave incident at equal angles with the edges and with wave form at type V