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TECHNICAL MEMORANDUM

LUNAR ORBITAL PHOTOGRAPHIC PLANNING CHARTS FOR CANDIDATE APPOLO J-MISSIONS

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Groundtrack Separation

ABSTRACT

A technique is presented for minimizing Mapping Camera film usage by reducing redundant coverage while meeting the desired sidelap of >55%. The technique uses the normal groundtrack separation determined as a function of the number of revolutions between the respective tracks, of the initial and final nodal azimuths (or orbital inclination), and of the lunar latitude. In the typical case of a nodal azimuth of 145° (or 35°), the film usage would be reduced by 30% over that required if complete lightside (terminator-to-terminator) passes were made. The technique is also applicable for planning Panoramic Camera photography such that photographic contiguity is attained but redundant coverage is minimized.

Graphs are included for planning Mapping Camera (MC) and Panoramic Camera (PC) photographic passes for a specific mission (i.e., specific groundtracks) to Descartes (Apollo 16), for specific missions to potential Apollo 17 sites such as Alphonsus, Proclus, Gassendi, Davy, and Tycho, and for a potential Apollo orbit-only mission with a nodal azimuth of 85°. Graphs are also included for determining the maximum number of revolutions which can elapse between successive MC and PC passes, for >55% sidelap and rectified contiguity respectively, for nodal azimuths between 5° and 85°.

Data from the included graphs indicate that on the specific mission considered to Descartes, MC passes are required every 16th rev prior to and every 14th rev after the CSM lunar orbital plane change for rendezvous (LOPC) in order to acquire the desired >55% sidelap. Contiguity of the rectified PC photography is attained for photographic passes every 35th rev prior to and every 32nd rev after the LOPC.

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For an Apollo 17 mission to Alphonsus, the desired MC sidelap can be met with MC passes every 9th rev prior to and every 10th rev after the plane change; contiguity of rectified PC photography is attained with PC passes every 21st rev prior to and every 23rd rev after the LOPC. For a mission to Proclus, MC passes are required every 6th rev prior to and every 5th rev after the LOPC and PC passes every 14th rev prior to and every 11th rev after the LOPC; a mission to Tycho requires MC passes every third rev to meet the desired sidelap and PC passes every 7th rev prior to and every 8th rev after the LOPC to attain rectified panormaic contiguity.

For a potential Apollo orbit-only mission (near-polar) photographic passes are required every second rev for $\geq 55\%$ MC sidelap and every 5th rev for MC and PC rectified contiguity.

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TECHNICAL MEMORANDUM

I. INTRODUCTION

Photographic planning for the Mapping and Panoramic Cameras on the J-missions is concerned with maximizing the areal coverage while minimizing the total amount of redundant photography acquired for each camera.* Since the Panoramic Camera (PC) is film limited (sufficient film for 503° of arc in stereo), the problem becomes one of maximizing areal coverage with the available film. The Mapping Camera (MC) is not necessarily film limited for missions where the nodal azimuth is $\lesssim 25°$ (sufficient film for ~ 26 lightside terminator-to-terminator passes). However, MC film usage does become a concern when the camera is used to support other experiments such as the Laser Altimeter on the darkside, or photography is acquired for other than mapping purposes - such as obliques acquired for photogeologic studies.

Since successive spacecraft groundtracks intersect at a point near the highest latitude overflown (generally near the landing site), redundant photographic coverage of these regions is acquired with each camera if complete terminator-to-terminator passes are made. The primary cartographic requirement for MC photography in the SIM-down attitude is $\geq 55\%$ sidelap,** which, at a spacecraft altitude of 60 nmi, means a groundtrack separation of ≤ 74 km. Panoramic Camera photographic planning is primarily concerned with photography of specific targets and secondarily concerned with photographing all other lightside regions overflown. Therefore, rectified PC photography of these

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*Minimizing the redundant coverage due to overlap of coverage acquired on other J-missions is not considered here.

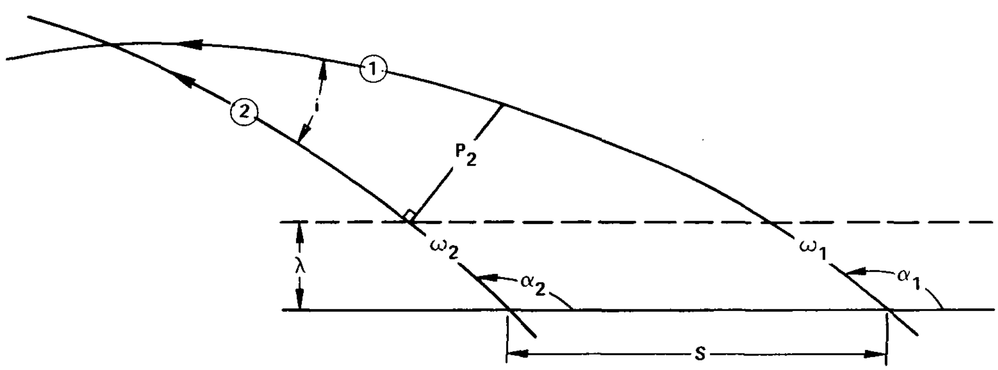
$$**\%\ \text{Sidelap} = \left(1 - \frac{|\text{groundtrack separation}|}{\text{width of area photographed}}\right) \times 100$$

regions should be contiguous but photographic sidelap is not necessary.

This memorandum provides a technique for planning MC vertical photography which satisfies the $\geq$55% sidelap requirement and minimizes redundancy and for planning PC regional photography which provides contiguity and maximizes usage of available film. Graphs are included for a specific mission (i.e., specific groundtracks) to Descartes (Apollo 16), for specific missions to potential Apollo 17 sites such as Proclus, Alphonsus, Gassendi, Davy and Tycho,* and for a potential Apollo orbit-only mission (near-polar).

II. GROUNDTRACK SEPARATION

The geometry of the intersection of the groundtracks is shown in the following illustration which depicts the reference configuration (see also Figure 1).



REFERENCE CONFIGURATION FOR INTERSECTION OF TWO GROUND TRACKS

The normal separation P_2 from groundtrack 2 to groundtrack 1 at latitude λ, with azimuths α_2 and α_1 at the node, respectively, and separated by a distance S on the lunar equator, is given by [See Appendix Equation (4)]:

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*The orbital parameters for these missions were obtained from S. C. Wynn of Bellcomm.

$$P_2(\lambda,S;\alpha_1,\alpha_2) = \text{arc sin} \frac{\cos(\omega_2+\varepsilon_2)\sin i}{\sqrt{1-\sin^2(\omega_2+\varepsilon_2)\sin^2 i}}$$

where

$$\omega_2 = \text{arc sin} \left(\frac{\sin\lambda}{\sin\alpha_2}\right)$$

$$\varepsilon_2 = \text{arc cos} \left(\sin\alpha_2 \frac{\sin S}{\sin i}\right)$$

$$i = \text{arc cos} [\cos\alpha_2 \cos\alpha_1 + \sin\alpha_2 \sin\alpha_1 \cos S]$$

Since, for the orbital altitudes considered, successive groundtracks precess to the west at 1°/revolution, the integer part of S, in degrees of arc, is equivalent to the number of revolutions between two groundtracks. The normal separation distance P_2 between two groundtracks as a function of lunar latitude can be plotted for each value of S. The resultant graph (e.g., Figure 2) is a family of curves which indicate the separation distance on the surface as a function of lunar latitude for various numbers of revolutions between groundtracks, i.e., plots of P_2 (λ, S) vs λ for different values of S when the parameters α_1, α_2 have been assigned.

Figure 2 is a plot of the normal groundtrack separation distance as a function of the lunar latitude for the indicated number of revolutions and the case where the inclination parameters $\alpha_1 = \alpha_2 = 145°$. Indicated on the figure by the two dashed horizontal lines are the constant groundtrack separation distance required for rectified-PC contiguity and that required for MC 55% sidelap. The plot indicates that, to achieve a sidelap >55% for the MC everywhere along the groundtrack, photographic passes must be made every 4th revolution in the equatorial region. In fact, if photographic passes are made every 4th revolution, the MC sidelap at the equator is 59%, increases to 75% at -28°S and at 26.5°N latitude, and to 100% where the groundtracks intersect at ±35°. If photographic passes are made every 5th revolution, the sidelap is >55% only at latitudes >16°N or <-18.5°S; for photographic passes made every 8th revolution the sidelap is >55% only at latitudes above 28°N or below -30.5°S.

III. PLANNING EXAMPLE

For the case of an azimuth at the node of 145° (orbital inclination of 35°), Figure 2 can be used to plan the MC vertical photographic sequences. The figure indicates that a sidelap of >55% is attained by having a photographic pass every 4th revolution. When an interval of 32 revolutions separates the first and last photographic passes, photography would be taken on revs 1, 5, 9, 13, 17, 21, 25, 29, and 33. (The number of revolutions between the first and last photographic passes depends on the total time in this orbit or on the desired width of the target strip.) Since these tracks overlap, especially near the point of highest latitude, an orderly sequence of steps is used to limit the redundant coverage by limiting the length of each pass. The steps to be taken to acquire the required

TABLE I

SUMMARY OF RESULTS OBTAINED USING PLANNING TECHNIQUE ON EXAMPLE SITE (MC COVERAGE ONLY)

Latitude of Coverage for Photo-Pass		Arc Length of Pass (See Fig. 3)	Photo-Pass Rev Number*									Rev Sep	Latitude Limits for >55% Sidelap (See Fig. 2)	
Start	Stop												Upper	Lower
-35.0°S	35.0°N	180°	1								33	32	-35.0°S	32.5°N
-35.0°S	32.5°N	160°					17					16	-34.3°S	32.0°N
-34.3°S	32.0°N	150°			9				25			8	-30.3°S	25.8°N
-30.3°S	25.8°N	111°		5		13		21		29		4	--	--

*Columns 1 and 2 indicate photo-pass start and stop latitudes for that rev.

sidelap while minimizing redundant coverage are summarized in Table I. These steps are:

1. on revs 1 and 33, passes from -35°S to 35°N (a 180° strip) would be acquired. The sidelap between these passes (a separation of 32 revs, Figure 2) would be >55% at latitudes below -35°S and above 32.5°N and <55% between -35°S and 32.5°N.

2. On rev 17, MC photography would be acquired between -35.0°S and 32.5°N. The sidelap with revs 1 and 33, respectively (rev separation of 16), would be >55% between -35.0° and -34.3°S and between 32.0° and 32.5°N; and would be <55% between -34.3°S and 32.0°N.

3. On revs 9 and 25, MC photography would be acquired between -34.3°S and 32.0°N. The sidelap with revs 1, 17, and 33, respectively (rev separation of 8), would be ≥55% between -34.3° and -30.3°S and between 25.8° and 32.0°N; and would be <55% between -30.3°S and 25.8°N.

4. On revs 5, 13, 21, and 29, photography would be acquired between -30.3° and 25.8°N. The sidelap with revs 1, 9, 17, 25, and 33 (rev separation of 4) would be ≥55% at all points of the strip.

The photographic sidelap is now ≥55% for the entire region between the groundtracks of revs 1 and 33.*

Figure 3 indicates the arc length along the groundtrack between the lunar equator and the indicated lunar latitude for groundtracks of various inclinations. The arc length of each photo-pass for the MC sequence suggested above is shown in Table I and totals 1264 degrees of arc (equivalent to 7.0 terminator-to-terminator passes), a reduction of 30% in film usage over that required if terminator-to-terminator passes were scheduled every 4 revolutions.

Planning for contiguity of rectified PC photographic coverage would be similar to that described above using the PC line of Figure 2 rather than the MC sidelap line.

IV. PHOTOGRAPHIC PLANNING CHARTS FOR POSSIBLE J-MISSIONS

Planning for the MC and PC photographic sequences for an Apollo mission using the technique described above will differ slightly from the ideal procedures discussed. In the typical case for an Apollo landing mission, the CSM lunar orbital plane change for rendezvous (LOPC) changes the orbital inclination (and hence α_2 of the illustration on page 2) thereby requiring three plots similar to Figure 2:

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*The photographed areas to the east of the groundtrack of rev 1 and to the west of the groundtrack of rev 33 have no sidelap. The sidelap, if desired, could be provided by a photographic pass on rev 2 (and rev 32) which would change the above sequence.

1. $\alpha_1 = \alpha_2$ = inclination pre-LOPC. This plot would be used to plan the photographic sequences between CSM circularization and the plane change.

2. α_1 = pre-LOPC inclination, α_2 = post-LOPC inclination. This plot would be used to plan the first photographic pass after the plane change.

3. $\alpha_1 = \alpha_2$ = inclination post-LOPC for planning the subsequent photographic sequences.

If a second plane change is performed, additional plots would be required (not provided here).

In the case where $\alpha_1 \neq \alpha_2$, as in step 2 above, it can be shown (see Appendix) that the $\geq$55% sidelap requirement cannot be satisfied at all points along the groundtrack when the change in nodal azimuth is greater than 2.44° (exclusive of node shift). Similarly, MC/rectified-PC contiguity cannot be satisfied when the change in nodal azimuth exceeds 5.40°.

The groundtrack separation as a function of lunar latitude has been determined for several possible J-mission landing sites and these groundtrack separations are summarized in Figures 4-9. The orbit-only mission is similarly summarized in Figure 10. The format in Figures 4-10 is similar to that of the planning example (Figure 2) except that the various curves are denoted by the actual "revs between passes" which, when $\alpha_1 \neq \alpha_2$ (across the LOPC), incorporates the nodal shift.* Figure 3 provides a curve of arc length versus latitude for each specific groundtrack inclination. Some preliminary conclusions for each mission considered are discussed below and are summarized in Table IV. The planning charts are provided for detailed analysis by the user when other mission constraints, such as lighting, are considered.

In addition, Figure 11 and 12 have been provided to cover mission changes (i.e. changes in azimuthal angles α_1 and/or α_2 or other orbital missions considered). Figure 11 indicates the maximum number of revolutions that can elapse between successive MC photo-passes in order to maintain the desired $\geq$55% sidelap.

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*When $\alpha_1 \neq \alpha_2$ actual "revs between passes" equals the algebraic sum of the nodal shift and the unshifted revs between passes (S in Figure 1). See page A7.

Figure 11 covers the case for a change in azimuthal angle of $\pm 2°$ across the LOPC. Of course, interpolation in this figure is delimited by the earlier statement that for an azimuthal change >2.44° a sidelap of >55% is not possible at all points along the groundtrack. Figure 12 is a similar chart for rectified-PC/MC contiguity and interpolation is delimited by an azimuthal change of 5.40°.

A. Descartes

Table II (Page 8) lists the Apollo 16 (Descartes) mission parameters of interest. The initial orbital inclination is 8.83°. A 2.56° plane change for rendezvous performed on rev 44 increases the orbital inclination to 10.06° and shifts the node ∿4° eastward. Figures 4a, b, and c indicate the distance between two groundtracks separated by the indicated number of revolutions as a function of lunar latitude for the cases pre-LOPC, post-LOPC, and immediately preceding and after LOPC, respectively. Figure 3 indicates the arc length along the groundtrack between the lunar equator and the indicated latitude for the two inclinations.

Figure 4a indicates that prior to LOPC, MC passes every 16th rev would provide a sidelap of ≥55% everywhere, while Figure 4b indicates that photographic passes every 14th rev post-LOPC would be required. Since the LOPC shifts the ascending node of the groundtrack 4° eastward in addition to changing the inclination, Figure 4c indicates that 17 revolutions can elapse between the last MC photographic pass prior to and the first pass after the plane change and yet still acquire the ≥55% sidelap.

The points at which each MC photographic pass should be initiated and where they should be terminated in order to minimize film usage can be determined by the method discussed earlier. The amount of film used can be determined with Figure 3.

Contiguity of rectified Panoramic Camera photography can be maintained if PC passes are made every 35th rev prior to and every 32nd rev after the plane change for rendezvous.

These results are summarized in Table IV.

B. Potential Apollo 17 Sites

Table III (Page 9) lists the parameters of interest for potential Apollo 17 missions to Alphonsus, Proclus, Davy, Gassendi, and Tycho.

TABLE II

J-2 Mission Parameters

Site	Descartes
Launch Date	3/17/72
Landing Point Coordinates	- 8.8°S, 14.6°E
Approach Azimuth	-90.0°
Sun Elevation at Landing	8.01°
LOPC-1	1.44°
inclination pre-PC	8.83°
inclination post-PC	10.06°
nodal shift (eastward)	+4°
Mission Events (elapsed time, hrs.)	
Liftoff-TLI	2.48
Translunar flight time	74.10
LOI - CSM Circularization	23.05
Circularization - LOPC	73.84
LOPC - TEI	58.63
Transearth flight time	115.19
Total Mission Duration	14.0 days

TABLE III

J-3 Mission Parameters

Site	Alphonsus	Proclus	Davy	Gassendi	Tycho
Launch Date	12/9/72	12/5/72	12/9/72	12/12/72	12/9/72
Landing Point Coordinates	-13.3°S,-4.2°W	16.2°,47.0°	-10.9°,-6.0°	-18.0°,-40.0°	-40.9°,-11.2°
Approach Azimuth	-83.0°	-74.5°	-82.0°	-75.0°	-73.0°
Sun Angle at Landing	13.5°	6.8°	17.58°	6.16°	9.41°
LOPC-1	1.62°	10.64°	2.56°	5.08°	2.43°
inclination Pre-LOPC	15.03°	157.81(22.19°)	13.45°	23.34°	43.60°
inclination Post-LOPC	13.40°	152.82(27.18°)	10.86°	18.19°	41.22°
nodal shift (eastward)	0	22°	1°	2.5°	- 0.5°
Mission Events (elapsed time, hrs.)					
Liftoff-TLI	2.72	2.96	2.69	2.11	2.75
Translunar flight time	90.42	79.76	101.43	87.07	78.69
LOI - CSM Circularization	23.03	22.88	22.98	21.07	45.91
Circularization - LOPC	63.84	63.85	63.84	65.83	63.87
LOPC - TEI	58.66	58.88	58.56	58.35	58.69
Transearth flight time	80.37	93.28	93.39	97.63	70.16
Total Mission Duration	13.3 days	13.4 days	14.3 days	13.8 days	13.3 days

1. Alphonsus

For an Apollo 17 landing at Alphonsus requiring an approach azimuth of -83.0° and a sun angle at landing of 13.5°, the initial 60 nmi circular orbit would be inclined 15.03° to the equatorial plane. On rev 44 the 1.62° plane change for rendezvous would change the orbital inclination to 13.40° without shifting the node.

Figure 5a indicates that prior to the plane change, MC passes every 9th revolution would provide a sidelap of ≥55% everywhere, while Figure 5b indicates that after the plane change MC photo-passes every 10th revolution would provide the required sidelap. Contiguity of the rectified panoramic photography would require photographic passes every 21st revolution prior to the plane change and every 23rd revolution after the plane change.

Since the plane change decreases the orbital inclination, the groundtrack separation between the pre- and post-LOPC passes is increased during the first several revolutions, especially in the south. Therefore, to acquire the needed ≥55% sidelap (especially between -5 and -11°S), only 7 revolutions should elapse between the last MC photo-pass prior to the plane change and the first photo-pass after (see Figure 5c). However, contiguity of the rectified panoramic photography can be retained if no more than 21 revolutions elapse between the last PC pass prior to and the first PC photo-pass after the plane change.

2. Proclus

On an Apollo 17 mission to Proclus (16.2°N, 47.0°E) an approach azimuth of approximately -74.5° is required, resulting in an initial 60 nmi circular orbit with an azimuth at the node of 157.81°. The CSM plane change for rendezvous of 10.64° shifts the node 22° eastward and changes the azimuth at the node to 152.82° (increases the inclination by 4.99°).

The plots of the groundtrack separation as a function of lunar latitude (Figures 6a, 6b, 6c) indicate that prior to the plane change MC photographic passes every 6th revolution will provide the required sidelap while after the plane change maneuver MC photo-passes every 5th revolution will be required. Contiguity of the rectified panoramic photography can be maintained with photographic passes every 14th revolution prior to and every 11th rev after the plane change.

Since the azimuthal change at LOPC is greater than 2.44° but less than 5.40° we expect to get a sidelap of <55% along part of the track but maintain MC and rectified-PC contiguity at all points. The effect of the plane change is to shift the node 22° eastward so that the ascending node of rev 44 is equivalent to the ascending node of rev 22. Since the groundtrack of rev 23 differs in inclination from that of rev 45, the separation is zero at the equator and >74 km at latitudes <-12°S. and at latitudes >12.5°N (Figure 6c, rev separation of 22). However, if only 5 revolutions elapse between the last MC photo-pass prior to and the first pass after the LOPC, and subsequent MC passes are made every 5th rev, the MC sidelap will be ≥55% everywhere except in that region northeast of the groundtrack between -24° and -27.2°S and between 24° and 27.2°N (arc lengths of 27°) photographed on the first post-LOPC photo-pass (MC contiguity will still be maintained in this region).

Contiguity of rectified panoramic photography can be maintained with PC passes every 14th rev prior to and every 11th rev after the LOPC.

3. <u>Davy</u>

An Apollo 17 mission to Davy will use an approach azimuth of -82.0°, resulting in an initial azimuth at the node of 13.45°. The CSM plane change of 2.56° changes the azimuth to 10.86° (decreases the inclination by 2.59°), and shifts the node 1° eastward.

The groundtrack separation and MC sidelap between possible successive photographic passes is shown in Figure 7a for the initial inclination, in Figure 7b for the lower inclination, and in Figure 7c for the photo-passes immediately pre- and post-LOPC. The figures indicate that a MC photographic pass every 10th rev prior to the plane change, the first rev after the plane change, and every 13th rev thereafter will provide the required sidelap except for a fraction of a degree at the maximum southern latitude. (Note that these passes need not be 180° of arc.)

Contiguity of the rectified panoramic photography can be maintained with a photo-pass every 23rd rev prior to and every 29th rev after the plane change. In addition, Figure 7c indicates that no more than 23 revs should elapse between the last PC photo-pass prior to and the first PC photo-pass after the plane change.

4. Gassendi

An Apollo 17 mission to Gassendi would employ an approach azimuth of approximately -75.0° with an initial azimuth at the node of 23.34°. The CSM plane change for rendezvous of 5.08° changes the azimuth at the node to 18.19° (decreases the inclination by 5.15°) and shifts the node eastward 2.5°.

The groundtrack-separation/MC-sidelap as a function of lunar latitude is shown in Figure 8a (initial inclination), Figure 8b (lower inclination) and Figure 8c (pre- and post-LOPC photo-passes). These charts indicate that MC passes every 6th rev prior to and every 7th rev after the plane change will provide the required sidelap of ≥55%. Since the azimuthal change at LOPC is >2.44° but <5.40° we expect to get a sidelap of <55% along part of the groundtrack but maintain MC and rectified-PC contiguity at all points. In order to provide the required MC sidelap over as much of the area overflown as possible, a photo-pass should be made just before and just after the plane change. Nevertheless, the region between the two groundtracks between -10°S and -18.2°S (an arc length of 57°) will have an MC sidelap <55% (MC contiguity will be maintained however).

Contiguity of the rectified panoramic photography can be maintained if a photo-pass is made every 13th rev prior to and every 17th rev after the plane change. Up to 7 revs can elapse between the last PC photo-pass prior to and the first PC photo-pass after the plane change.

The length of each MC and each PC photo-pass necessary to provide the required sidelap and the desired contiguity respectively can be determined by the method suggested in Section III.

5. Tycho

An Apollo 17 mission to Tycho would have an approach azimuth of approximately -73.0° with an initial azimuth at the node (orbital inclination) of 43.60°. The plane change of 2.43° would change the nodal azimuth by 2.38° to 41.22° and shift the node 0.5° westward.

The groundtrack separation as a function of the number of revolutions separating photo-passes and the lunar latitude is given in Figure 9a for the initial inclination, in Figure 9b for the decreased inclination, and in Figure 9c for the pre- and post-LOPC photo passes.

The charts indicate that >55% sidelap can be attained both pre- and post-LOPC if MC photo-passes are made every 3rd rev and >53% can be attained across the LOPC if no more than one revolution elapses between the last MC photo-pass prior to and the first MC photo-pass immediately after the LOPC. Even though the change in nodal azimuth at LOPC is <2.44°, the effect of the node shift is to reduce the sidelap to <55% only between -34° and -40°S (where the sidelap is ~53%, an arc length of 18°). However, photo-passes may not be possible every 3rd rev due to operational constraints (uninterrupted crew sleep periods >7 hours) in which case the sidelap in the equatorial region would be <55%. Mapping Camera and rectified Panoramic Camera contiguity would be attained for photo-passes every 7th rev prior to and every 8th rev after the LOPC and if no more than 7 revolutions were allowed to elapse between the last pre- and the first post-LOPC photo-passes. Contiguity of unrectified panoramic photography could be maintained if no more than 15 revolutions were allowed to elapse between successive PC photo-passes pre-LOPC and no more than 16 revolutions were allowed to elapse post-LOPC.

Since MC film usage would be crucial in the case of a Tycho mission, minimizing film usage while maximizing areal coverage using the method described in Section III would result in a significant decrease in film usage over that required in complete terminator-to-terminator photo-passes.

C. Apollo Orbit-Only Mission

An orbit-only mission with a nodal azimuth of 85° (near-polar) has been discussed as a follow-on to the Apollo landing missions. The purpose of such a flight would be remote sensing of the moon from orbit and high-resolution and mapping-quality photography of as much of the moon as possible. Since the orbit would be near-polar the groundtracks would be widely separated in the equatorial region and overlapping near the maximum latitudes reached.

Figure 10 indicates the distance between two groundtracks (with an inclination of 85°) separated by the indicated number of revolutions as a function of lunar latitude. The chart indicates that no more than two revolutions can elapse between successive MC photo-passes if a sidelap of >55% is desired at all points along the groundtrack. In fact, if three revolutions elapse between successive MC passes the sidelap is >55% only at latitudes above 34°N and below -33°S; if five revolutions elapse between successive MC passes the MC sidelap would be >55% only at latitudes above 60°N and below -60°S. Mapping Camera and rectified Panoramic Camera contiguity can be maintained with photographic passes every 5th revolution; unrectified-PC contiguity can be maintained with a photo-pass every 10th rev.

As an example of the use of Figure 11 it is noted that if the orbital inclination remains above 55°, MC photo-passes are required every second rev in order to obtain ≥55% sidelap, inclinations between 55° and 38° require MC passes every third rev for the desired sidelap, inclinations between 38° and 29° require MC passes every fourth rev, etc.

V. SUMMARY

A technique is proposed for minimizing Mapping Camera film usage by reducing the redundant coverage while meeting the ≥55% sidelap requirement. The technique utilizes graphs of the separation of the groundtracks as a function of the number of revolutions between the tracks and as a function of the lunar latitude. For example, in the case of an orbital inclination of 145° (or 35°), the film usage is reduced by 30% over that required if complete 180° lightside passes are made. The technique is also applicable for planning Panoramic

TABLE IV

SUMMARY OF RESULTS OBTAINED USING PLANNING TECHNIQUE FOR CANDIDATE SITES

Maximum Number of Revs Between Photo-Passes to Achieve Constraint

Site	Mapping Camera (≥55% Sidelap)			Mapping Camera/ Rectified-Panoramic Camera Contiguity			Data From Figure
	Pre-LOPC	Across LOPC	Post-LOPC	Pre-LOPC	Across LOPC	Post-LOPC	
Apollo 16							
Descartes	16	17	14	35	36	32	4a, b, c
Apollo 17							
Alphonsus	9	7	10	21	21	23	5a, b, c
Proclus	6	-	5	14	-	11	6a, b, c
Davy	10	1	13	23	23	28	7a, b, c
Gassendi	6	1	7	13	7	17	8a, b, c
Tycho	3	~1	3	{ 7 15*	{ 7 15*	{ 8 16*	9a, b, c 9a, b, c
Orbit-Only							
85° Azimuth	2	-	-	{ 5 10*	- -	- -	10 10

*Unrectified Panoramic Camera (Contiguity)

Camera photography such that photographic contiguity is attained but redundant coverage is minimized.

Graphs are included for planning Mapping camera and Panoramic Camera photographic passes for a specific mission (i.e., groundtracks) to Descartes (Apollo 16); for specific missions to potential Apollo 17 sites such as Alphonsus, Proclus, Gassendi, Davy, and Tycho; and for a near-polar orbit-only mission.

Using the data from the included graphs, the maximum number of revolutions that can elapse between successive photo-passes and yet maintain a MC sidelap of >53% and rectified-PC contiguity respectively is summarized in Table IV.

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2015-PJH/WLP-du

Attachments

APPENDIX

This appendix explains the equations used to derive the planning charts. Since the spacecraft moves around the moon in a plane through the moon's center of mass, the intersection of the orbital plane with the moon (considered as a sphere) is a great circle referred to as the groundtrack. This great circle groundtrack (curve Z_2KYX_2 in Figure A1) intersects the lunar equator X_2X_1 with an azimuth of α_2 degrees at the ascending node. Y is a point on this great circle at lunar latitude λ degrees and at a distance of arc length ω_2 degrees along the track from the equator. Z_2 is the pole of the great circle, and X_2 its node so that distance Z_2X_2 is 90 degrees and KX_2 is $90-\varepsilon_2$ degrees. Arc KZ_1UX_1 is a similar track (a great circle) which intersects the lunar equator with an azimuth at the node of α_1 degrees and intersects the previous track at an angle of i degrees. The tracks are separated by S degrees along the equator. YU is a great circle arc of length P_2 degrees and is normal to track 2 at a latitude of λ degrees. We wish to calculate P_2 as a function $P_2(\lambda,S;\alpha_1,\alpha_2)$ of λ and S, given the azimuths α_1 and α_2.

Applying the cosine rule to the spherical triangle $KZ_1X_1X_2$, the angle of intersection i is obtained from

$$\cos i = -\cos \alpha_2 \cos (180-\alpha_1) + \sin \alpha_2 \sin (180-\alpha_1) \cos S \quad . \qquad \text{(1a)}$$

An equivalent form of this relation is

$$\cos i = \cos(\alpha_2-\alpha_1) - 2 \sin \alpha_1 \sin \alpha_2 \sin^2 \frac{S}{2} \qquad \text{(1b)}$$

from which it is evident that $i \geq |\alpha_1-\alpha_2|$ and that the minimum value occurs at S=0. A specialized form for the important case of $\alpha_2=\alpha_1$ is

$$\sin \left(\frac{i}{2}\right) = \sin \alpha_1 \sin\left(\frac{S}{2}\right) \qquad \text{when } \alpha_2=\alpha_1 \qquad \text{(1c)}$$

Arcs ε_1 and ε_2 are then given by the sine rule:

$$\sin(90-\varepsilon_2) = \sin(180-\alpha_1)\sin S/\sin i \qquad (2a)$$

$$\sin(90+\varepsilon_1) = \sin\alpha_2 \sin S/\sin i \quad . \qquad (2b)$$

Arc YX_2, of length ω_2 degrees, is found by applying the sine rule to the right spherical triangle YVX_2.

$$\sin\omega_2 = \sin\lambda/[\sin(180-\alpha_2)] \quad . \qquad (3)$$

The arc length YU, of P_2 degrees, is now found from the right spherical triangle $KYUZ_1$ using $KY = (90 - \varepsilon_2 - \omega_2)$ and the auxiliary angle γ_1. Since

$$\sin\gamma_1 = \sqrt{1 - \sin^2(\omega_2+\varepsilon_2)\sin^2 i}$$

the sine rule gives

$$P_2 = \text{arc sin}\left[\frac{\cos(\omega_2+\varepsilon_2)\cdot\sin i}{\sqrt{1-\sin^2(\omega_2+\varepsilon_2)\sin^2 i}}\right] \qquad (4)^*$$

P_2 is arbitrarily given a positive sign when track 2 is to the left of track 1 and a negative sign when track 2 is to the right of track 1 (i.e., to the left of point K). A normal separation from track 1, P_1, can similarly be defined.

The coordinates of the point of intersection, K, of tracks 1 and 2 are now determined. The latitude of K, λ_K, is found by applying the Law of Sines to spherical triangle KX_3X_2:

$$\lambda_K = \text{arc sin}\,[\sin(90-\varepsilon_2)\sin(180-\alpha_2)] \quad . \qquad (5)$$

---

*P_2 is also given by

$$P_2 = \text{arc sin}\,[\tan(90-\varepsilon_2-\omega_2)\cot\{\text{arc cos}(\sin i \sin(90-\varepsilon_2-\omega_2))\}]$$

By definition, of course, $P_2(\lambda_K,S;\alpha_1,\alpha_2) \equiv 0$, so that P_2 changes sign at crossing λ_K. The arc KX_3, of length λ_K, is the lunar meridian through K; the arc $X_3X_2 = S_K$ is then the separation in longitude of the intersection point K from the node of track 2. S_K is found from the right spherical triangle KX_2X_3:

$$\cos S_K = \cos(90-\varepsilon_2)/\cos\lambda_K \quad . \tag{6}$$

S_K is negative to the left of X_2, positive to the right. The arc X_3X_1, the longitudinal separation of K from the node of track 1, is $(S+S_K)$ degrees.

Figure A1 shows the reference geometry for the spherical triangle formed by two groundtracks, with azimuths α_1 and α_2 larger than 90°. Table A1 indicates the sign of ε_1 and ε_2 to be used for triangles with other configurations. The table also gives the longitude of the point of intersection K, LONG, in terms of the parameter S_K. These other configurations result when one of the sides of the reference triangle changes from less than to greater than 90°, or vice-versa, as S increases. They are distinguished by comparing the value of the intersection angle i to its value, i_{90}, when one side is 90°. i_{90} is given by

$$\cos i_{90} = \begin{cases} \cos(180-\alpha_1)/\cos(180-\alpha_2) & \alpha_2>\alpha_1 \\ \cos(180-\alpha_2)/\cos(180-\alpha_1) & \alpha_2<\alpha_1 \end{cases} \quad . \tag{7}$$

The value of the track separation along the equator S at which this change occurs, S_{90}, is given by

$$\sin S_{90} = \begin{cases} \sin i_{90}/\sin(180-\alpha_1) & \alpha_2>\alpha_1 \\ \sin i_{90}/\sin(180-\alpha_2) & \alpha_2<\alpha_1 \end{cases} \quad . \tag{8}$$

TABLE A1

Relation of other Triangle Configurations to the Reference Spherical Triangle KX_2X_1 in Figs. A1 and A2

	$\alpha_1,\alpha_2 < 90$		$\alpha_1,\alpha_2 > 90$	
	$i < i_{90}$	$i > i_{90}$	$i < i_{90}$	$i > i_{90}$
$\alpha_2>\alpha_1$	$\varepsilon_1>0$ $\varepsilon_2<0$ Long=$180-S_K$		$\varepsilon_1>0$ $\varepsilon_2<0$ Long=$180-S_K$	
$\alpha_2=\alpha_1$	$\varepsilon_1<0$ $\varepsilon_2<0$ Long=$180-S_K$	$\varepsilon_1<0$ $\varepsilon_2<0$ Long=$180-S_K$	$\varepsilon_1>0$ $\varepsilon_2>0$ Long=$S_K\leq 90$	$\varepsilon_1>0$ $\varepsilon_2>0$ Long=S_K
$\alpha_2<\alpha_1$	$\varepsilon_1<0$ $\varepsilon_2>0$ Long=S_K		$\varepsilon_1<0$ $\varepsilon_2>0$ Long=S_K	

We note that the intersection angle i increases monotonically with S. If $\alpha_1=\alpha_2$ the minimum value of i is zero, while for the case of $\alpha_2\neq\alpha_1$, $i \geq|\alpha_1-\alpha_2|$. If we now consider the intersection point K to be a "north pole" then the angle i is the angle between the two orbit planes or the separation in longitude of the two groundtracks, considered as new "meridians of longitude," at the "equator" of the pole K. Thus the angle i is the maximum normal separation of the groundtracks ($P_2 \leq i$ and $P_1 \leq i$) and this maximum normal separation occurs at a point 90° from the pole K along either groundtrack. Since the sides of the spherical triangle are $90+\varepsilon_1$ and $90-\varepsilon_2$ degrees, the points along the groundtracks 90° from K occur $+\varepsilon_1$ degrees north of the lunar equator along the groundtrack 1 and ε_2 degrees below the lunar equator along the groundtrack 2. The latitude of this point of maximum separation on groundtrack 2 is given by

$$\sin(\lambda_{\varepsilon_2}) = \sin \varepsilon_2 / \sin(180-\alpha_2) \quad . \tag{9}$$

In the plots, Figures 4 to 10, λ_{ε_2} is the latitude at which the peaks of the curves occur.

For a spacecraft altitude of 60 nmi and a MC frame width of 164 km the percent sidelap Ø is related to the groundtrack separation P_2 by

$$\text{Ø} = (1 - |P_2|/164) \times 100 \quad . \tag{10}$$

A MC sidelap of 55% corresponds to a groundtrack separation of 74 km or a surface arc of $i_o = 2.44°$. (MC contiguity and rectified-PC contiguity correspond to a separation of 164 km and $i_o' = 5.40°$). If all points along two groundtracks are to be separated by less than 74 km then the angle of intersection $i \leq i_o$ (and for groundtrack separations $\leq$ 164 km, $i \leq i_o'$). The corresponding track separation along the equator, S_o, is found from i_o by

$$\cos S_o = [\cos i_o + \cos \alpha_2 \cos(180-\alpha_1)]/(\sin \alpha_2 \sin(180-\alpha_1)) \quad . \tag{11}$$

The integral part of S_o is the smallest rev separation that need be plotted in Figures 4 to 10 since $\geq$55% sidelap is achieved on all rev separations smaller than the integral part of S_o.

Since $i \geq |\alpha_2 - \alpha_1|$, S_o is non-zero only if $i_o \geq |\alpha_2 - \alpha_1|$. Thus sidelap of $\geq$55% occurs only if $|\alpha_2 - \alpha_1| < 2.44°$ and MC and rectified-PC contiguity occurs on all parts of the track only if $|\alpha_2 - \alpha_1| < 5.4°$. General graphs of $S_o(\alpha_1 \alpha_2 i_o)$ and $S_o'(\alpha_1 \alpha_2 i_o')$ for these two cases are given in Figures 11 and 12.

The two latitudes at which the rev separation $P_2(\lambda, S; \alpha_1, \alpha_2)$ is equal to 74 km, λ_{74} and λ'_{74} are given by

$$\sin \lambda_{74} = \sin(180-\alpha_2)\sin \omega_{74} \qquad \text{if } i \geq i_o \quad . \qquad (12a)$$

$$\sin \lambda'_{74} = \sin(180-\alpha_2)\sin(\omega_{74}+2\varepsilon_2) \qquad \text{if } i \geq i_o . (12b)$$

where ω_{74} is given by

$$\omega_{74} = 90 - \varepsilon_2 - \arcsin\left[\frac{\sin i_o}{\cos i_o} \cdot \frac{\cos i}{\sin i}\right] \quad . \qquad (12c)$$

Note that $2(\omega_{74}+\varepsilon_2)$ is the length of arc along the groundtrack for which $P_2 > 74$ Km.

To summarize the algorithm for calculating P_2: given α_1,α_2, and S we first calculate $i_{90} = i_{90}(\alpha_1,\alpha_2)$ and $S_{90}(\alpha_1,\alpha_2)$ from Eqs. (7) and (8). $i(\alpha_1,\alpha_2,S)$ is then determined from Eq. (1) and $\varepsilon_1(i,\alpha_2)$ and $\varepsilon_2(i,\alpha_1)$ from Equation (2), ε_1 and ε_2 are given the sign required by Table A1 when i is compared to i_{90}. Given the latitude λ, $\omega_2(\lambda,\alpha_2)$ is then determined from Equation (3) and $P_2(\omega_2,\varepsilon_2,i)$ determined from Equation (4).

Southern Latitudes

Figure A2 illustrates the relationship between the reference spherical triangle KX_2X_1 and the triangle $X_2Z_2'K'Z_1'X_1$ which is used for the calculation of P_2 at southern latitudes. From Figure A2 it is clear that the southern triangle $X_2Z_2'K'Z_1'X_1$ is congruent to the supplementary triangle $X_{22}Z_2KZ_1X_{11}$. In the supplementary triangle the intersection angle i and the track separation S are the same as for the reference triangle while the two other sides and their opposite angles are the supplements of the corresponding sides and angles in the reference triangle i.e., α_1,α_2, $90+\varepsilon_1$, $90-\varepsilon_2$ are replaced by $180-\alpha_1$, $180-\alpha_2$, $90-\varepsilon_1$, $90+\varepsilon_2$ in the supplementary or southern triangle. The values of P_2 at negative values of λ are found from the solution of the reference triangle when α_1 and α_2 are replaced by $180-\alpha_1$ and $180-\alpha_2$. This can be written:

$$P_2(\lambda,S;\alpha_1,\alpha_2) = P_2(|\lambda|,S;\ 180-\alpha_1,\ 180-\alpha_2) \quad \text{for } \lambda < 0 \quad . \qquad (13)$$

Table A1 gives the changes in sign of ε_1 and ε_2 which attend the replacement of α_1 and α_2 by their supplements.

Nodal Shift

A LOPC often includes not only a change of azimuthal angle from α_1 to α_2 but also a shift of the node eastward by S_S degrees along the equator so that the reference triangle connecting the last rev before LOPC and the first rev after LOPC would have a separation parameter S of $-S_S+1$ degrees. It is easily shown that the normal groundtrack separation for these negative values of S are given by

$$P_2(\lambda,S;\alpha_1,\alpha_2) = -P_2(-\lambda,|S|;\alpha_1,\alpha_2) \quad \text{if } S < 0 \quad . \qquad (14)$$

In these cases the parameter "revs between passes" used to label the curves in Figures 4c to 9c is equal to $S + S_S$ degrees. Also, the separation parameter S may assume a zero value so that the intersection angle i may assume the minimum value given by Equation (1b).

Example of Computer Output

The above equations were programmed on a UNIVAC 1108 computer using the interactive processor called MATH. In applying this program we note that the reference point X_2 in Figure A1 is the node on the front face of the moon, i.e., the ascending node for northern sites and descending node for southern sites. The azimuths α_1 and α_2 are those for the northern triangle of each site. Thus for northern sites α_1 and α_2 are >90°, for southern sites <90°. (The azimuths at the descending node are the supplement of that of the ascending node).

Table A2 is a sample output for the Alphonsus site from which graph 5c was plotted. Table A2 is self-explanatory: $P_2(\lambda,S)$ is listed for $\lambda>0$ and $\lambda<0$, and the sides and angles of each triangle are listed as a check. Values of $\omega_1(\lambda)$ and $\omega_2(\lambda)$ are also given, as well as the latitude and relative longitude of the track intersection point, $\lambda_K(S)$, $\text{Long}_K(S)$, and the latitude at which P_2=74 km, λ_{74}. Angles i_{90}, S_{90} and S_o are given for each triangle. Tables of P_1 $(\lambda,S;\alpha_2,\alpha_1)$ were compared with these as a check since interchange of α_1 and α_2 implies interchange of P_1 and P_2.

Since the storage available in MATH is limited, some redundant calculation is necessary so that the computations for Table A2 (or Figure 5c) required about 1/5 of an hour on a UNIVAC 1108 computer.

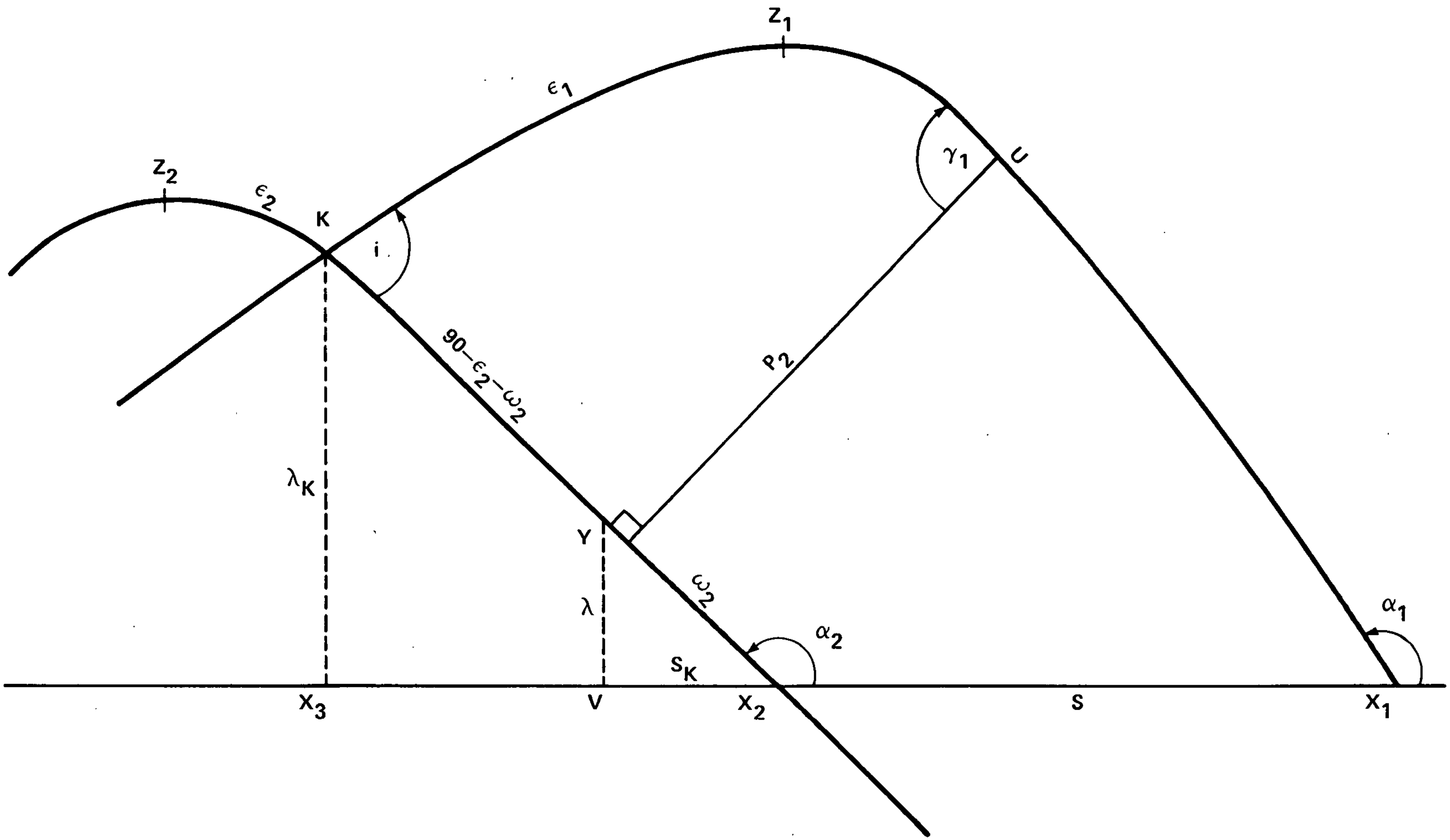
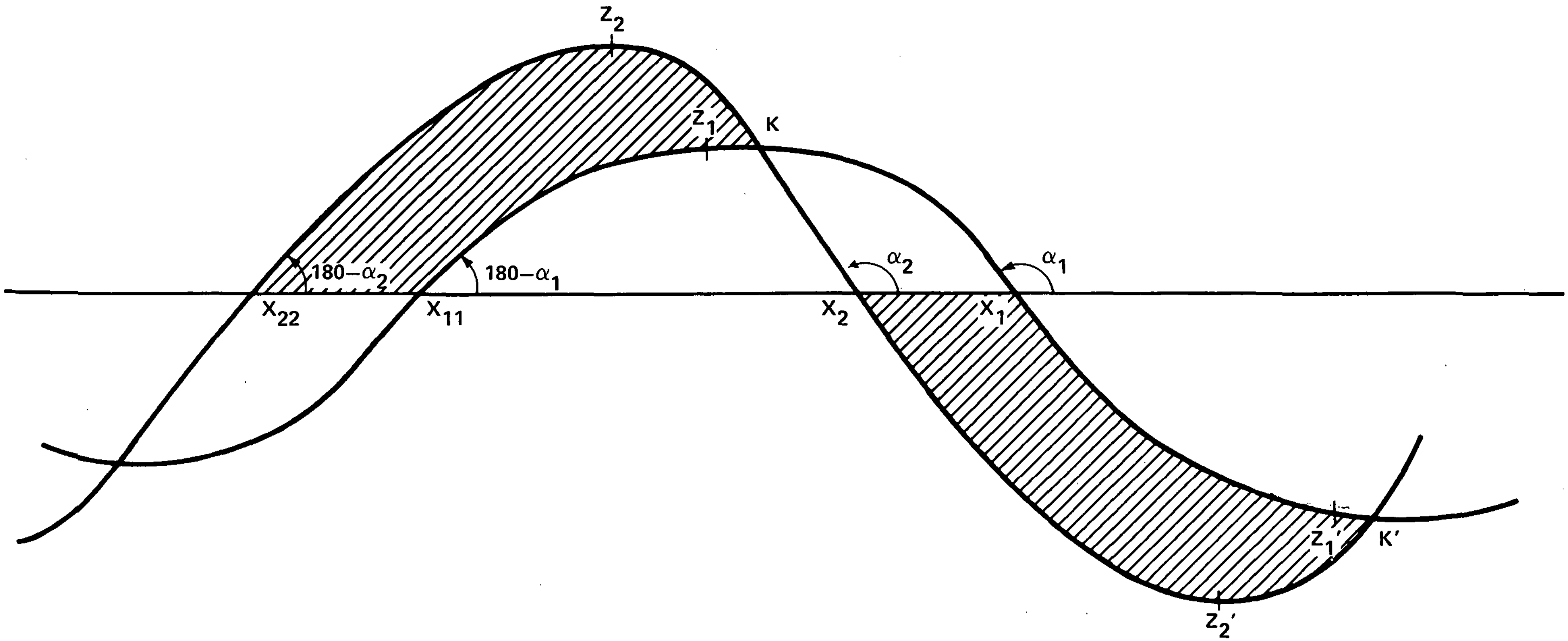


FIGURE A1 - REFERENCE GEOMETRY FOR THE SPHERICAL TRIANGLE FORMED BY TWO GROUND TRACKS



SPHERICAL TRIANGLE $X_{22}\ Z_2\ K\ Z_1\ X_{11}$ IS CONGRUENT TO TRIANGLE $X_2\ Z_2'\ K'\ Z_1'\ X_1$.

SPHERICAL TRIANGLE $K\ X_2\ X_1$ IS THE REFERENCE TRIANGLE

FIGURE A2 - RELATION OF SOUTHERN LATITUDE TRIANGLE TO REFERENCE GEOMETRY TRIANGLE

TABLE A2

Example of the Computer Output Used to Construct Figure 5c of the text, a plot of the groundtrack separation of tracks 1 and 2 measured normal to groundtrack 2, $P_2(\lambda,S;\alpha_1,\alpha_2)$ in km, as a function of lunar latitude λ in degrees, for selected values of the separation measured along the equator S, in degrees or revs, for values of the orbit plane azimuth at the node of α_1 = 15.03 and α_2 = 13.40 (J-mission to Alphonsus, 12/9/72).

```
  A(1)=  15.03            180-A(1)= 164.97
  A(2)=  13.40            180-A(2)= 166.60
I(SIDE=90)=   6.87    S(SIDE=90)=  27.47    S(P=74 KM)=   7.41
```

ANGLES AND SIDES OF SHERICAL TRIANGLE ABOVE EQUATOR (λ >0)

S	I	90+E1	A2	90-E2	180-A1	(INTERSECTION) LAT	LONG	(P=74 KM) LAT
1.00	1.65	9.08	13.40	9.05	164.97	2.09	8.81	
2.00	1.70	15.80	13.40	17.74	164.97	4.05	17.29	
4.00	1.90	29.15	13.40	33.02	164.97	7.26	32.30	
6.00	2.20	39.23	13.40	45.05	164.97	9.44	44.26	
8.00	2.55	46.48	13.40	54.24	164.97	10.94	53.49	-4.30
10.00	2.94	51.64	13.40	61.34	164.97	11.73	60.67	1.23
12.00	3.36	55.32	13.40	66.96	164.97	12.31	66.38	4.64
14.00	3.79	57.96	13.40	71.53	164.97	12.70	71.05	6.96
16.00	4.24	59.84	13.40	75.36	164.97	12.96	74.97	8.61
18.00	4.69	61.18	13.40	78.64	164.97	13.13	78.33	9.81
20.00	5.14	62.11	13.40	81.52	164.97	13.25	81.28	10.70
22.00	5.60	62.73	13.40	84.06	164.97	13.33	83.91	11.38
24.00	6.07	63.11	13.40	86.40	164.97	13.37	86.30	11.89
26.00	6.53	63.30	13.40	88.53	164.97	13.40	88.49	12.29
27.47	6.87	63.34	13.40	90.00	164.97	13.40	90.00	12.52
28.00	6.99	63.33	13.40	90.51	164.97	13.40	90.52	12.59
30.00	7.46	63.24	13.40	92.36	164.97	13.39	92.43	12.83
32.00	7.92	63.04	13.40	94.12	164.97	13.36	94.23	13.01
34.00	8.38	62.76	13.40	95.78	164.97	13.33	95.94	13.15
36.00	8.84	62.41	13.40	97.38	164.97	13.29	97.58	13.25
38.00	9.30	61.99	13.40	98.91	164.97	13.24	99.16	13.32
40.00	9.76	61.52	13.40	100.39	164.97	13.18	100.68	13.37
42.00	10.21	61.01	13.40	101.83	164.97	13.11	102.15	13.39
44.00	10.66	60.45	13.40	103.23	164.97	13.04	103.59	13.40
46.00	11.11	59.86	13.40	104.60	164.97	12.96	104.99	13.39
48.00	11.56	59.24	13.40	105.93	164.97	12.88	106.36	13.37
50.00	12.01	58.59	13.40	107.25	164.97	12.79	107.70	13.33
52.00	12.45	57.92	13.40	108.53	164.97	12.69	109.02	13.29
54.00	12.86	57.23	13.40	109.80	164.97	12.59	110.31	13.23
56.00	13.32	56.51	13.40	111.05	164.97	12.49	111.59	13.16
58.00	13.75	55.78	13.40	112.29	164.97	12.38	112.85	13.09
60.00	14.18	55.03	13.40	113.51	164.97	12.27	114.09	13.01

Table of normal groundtrack separation, $P_2(\lambda,S)$ in kilometers. Each line of the table gives values of $P_2(\lambda)$ for a fixed value of the groundtrack separation S, in degrees or rev numbers, on the left, and selected positive values of the lunar latitude λ, in degrees.

A(1)= 15.03 180-A(1)= 164.97
A(2)= 13.40 180-A(2)= 166.60

REV	LATITUDE .01	1.00	2.00	3.00	4.00	5.00	6.00	7.00	8.00	9.00	10.00	11.00	12.00	13.00	13.40	14.00	15.00	15.03
1.	7.8	4.1	.3	-3.5	-7.4	-11.3	-15.3	-19.3	-23.4	-27.5	-31.8	-36.2	-40.8	-46.0	-49.4			
2.	15.7	12.0	8.1	4.2	.2	-3.9	-8.1	-12.5	-17.0	-21.6	-26.4	-31.6	-37.2	-44.0	-49.2			
4.	31.4	27.7	23.8	19.7	15.4	10.9	6.2	1.3	-3.9	-9.5	-15.4	-22.0	-29.5	-39.4	-48.4			
6.	47.1	43.5	39.5	35.3	30.8	26.0	20.8	15.3	9.4	3.0	-4.0	-12.0	-21.4	-34.4	-47.1			
8.	62.7	59.2	55.2	50.9	46.3	41.2	35.6	29.6	23.1	15.8	7.7	-1.6	-12.8	-28.8	-45.2			
10.	78.3	74.9	71.0	66.6	61.8	56.5	50.6	44.1	36.9	28.9	19.8	9.2	-3.8	-22.7	-42.8			
12.	93.7	90.5	86.7	82.4	77.4	71.9	65.7	58.8	51.1	42.3	32.2	20.4	5.6	-16.2	-39.9			
14.	109.1	106.1	102.4	98.1	93.1	87.5	81.0	73.7	65.4	56.0	45.0	32.0	15.5	-9.2	-36.5			
16.	124.3	121.6	118.1	113.8	108.9	103.1	96.4	88.8	80.0	69.9	58.1	43.9	25.8	-1.6	-32.5			
18.	139.4	136.9	133.7	129.6	124.6	118.7	111.9	104.0	94.8	84.1	71.5	56.2	36.5	6.3	-28.0			
20.	154.3	152.2	149.2	145.2	140.3	134.5	127.5	119.3	109.7	98.5	85.1	68.8	47.6	14.8	-23.1			
22.	169.1	167.3	164.6	160.8	156.1	150.2	143.2	134.8	124.9	113.1	99.1	81.7	59.1	23.7	-17.6			
24.	183.7	182.3	179.9	176.4	171.7	165.9	158.9	150.3	140.1	128.0	113.2	95.0	71.0	33.1	-11.6			
26.	198.0	197.1	195.0	191.8	187.4	181.7	174.6	166.0	155.5	143.0	127.6	108.5	83.2	42.9	-5.1			
27.	209.4	207.8	206.1	203.1	198.8	193.2	166.2	177.5	166.9	154.1	138.4	118.7	92.4	50.3	.0			
28.	212.1	211.7	210.0	207.1	202.9	197.4	190.4	181.7	171.0	158.1	142.3	122.3	95.8	53.1	1.9			
30.	226.0	226.1	224.9	222.3	218.4	213.0	206.1	197.4	186.6	173.4	157.1	136.4	108.7	63.7	9.4			
32.	239.6	240.2	239.5	237.3	233.8	228.6	221.8	213.2	202.3	188.9	172.1	150.8	121.9	74.8	17.4			
34.	253.0	254.1	253.9	252.2	249.0	244.1	237.5	228.9	216.0	204.4	187.3	165.3	135.4	86.2	25.8			
36.	266.0	267.8	268.1	266.9	264.1	259.5	253.1	244.7	233.8	220.1	202.6	180.1	149.2	98.0	34.7			
38.	278.8	281.2	282.1	281.4	279.0	274.8	268.7	260.4	249.6	235.8	218.1	195.1	163.3	110.2	44.1			
40.	291.2	294.3	295.8	295.7	293.7	289.9	284.1	276.0	265.3	251.5	233.7	210.2	177.6	122.8	53.9			
42.	303.4	307.2	309.3	309.7	308.3	304.9	299.5	291.6	281.1	267.3	249.3	225.5	192.2	135.7	64.2			
44.	315.1	319.7	322.5	323.5	322.6	319.7	314.6	307.1	296.8	283.1	265.0	240.9	207.0	148.9	74.9			
46.	326.6	331.9	335.4	337.0	336.7	334.3	329.7	322.5	312.5	298.9	280.8	256.5	221.9	162.5	86.0			
48.	337.6	343.7	348.0	350.3	350.6	348.7	344.6	337.8	326.0	314.6	296.6	272.1	237.1	176.3	97.6			
50.	348.3	355.2	360.2	363.2	364.2	362.9	359.3	352.9	343.5	330.4	312.4	287.9	252.4	190.5	109.5			
52.	359.6	366.3	372.1	375.9	377.5	376.8	373.7	367.9	358.9	346.0	328.3	303.7	267.9	204.8	121.8			
54.	368.5	377.1	383.7	389.2	390.5	390.5	388.0	382.7	374.1	361.6	344.1	319.5	283.5	219.5	134.5			
56.	378.0	387.5	394.9	400.2	403.3	403.9	402.0	397.3	389.2	377.0	359.8	335.4	299.2	234.4	147.5			
58.	387.1	397.4	405.8	411.9	415.7	417.1	415.8	411.6	404.1	392.4	375.5	351.3	315.0	249.4	160.9			
60.	395.7	407.0	416.2	423.2	427.8	429.9	429.3	425.8	418.8	407.6	391.1	367.1	330.9	264.7	174.6			
ARC ALONG GROUND TRACK BETWEEN EQUATOR AND LATITUDE L (°)																		
ω_1	.04	3.86	7.73	11.64	15.60	19.64	23.77	28.03	32.46	37.10	42.04	47.37	53.30	60.16	63.34	68.89	86.42	89.99
ω_2	.04	4.32	8.66	13.05	17.52	22.09	26.81	31.73	36.91	42.46	49.53	55.42	63.79	76.09	89.99	.00	.00	.00

A(1)= 15.03 180-A(1)= 164.97

A(2)= 13.40 180-A(2)= 166.60

I(SIDE=90)= 6.87 S(SIDE=90)= 27.47 S(P=74 KM)= 7.41

ANGLES AND SIDES OF SHERICAL TRIANGLES BELOW EQUATOR ($\lambda < 0$)

						(INTERSECTION)		(P=74 KM)
S	I	90+E1	A2	90-E2	180-A1	LAT	LONG	LAT
1.00	1.65	171.92	166.60	170.95	15.03	-2.09	-171.19	
2.00	1.70	164.20	166.60	162.26	15.03	-4.05	-162.71	
4.00	1.90	150.85	166.60	146.98	15.03	-7.26	-147.70	
6.00	2.20	140.77	166.60	134.95	15.03	-9.44	-135.74	
8.00	2.55	133.52	166.60	125.76	15.03	-10.84	-126.51	-4.30
10.00	2.94	128.36	166.60	118.66	15.03	-11.73	-119.33	1.23
12.00	3.36	124.68	166.60	113.04	15.03	-12.31	-113.62	4.64
14.00	3.79	122.04	166.60	108.47	15.03	-12.70	-108.95	6.96
16.00	4.24	120.16	166.60	104.69	15.03	-12.96	-105.03	8.61
18.00	4.69	118.82	166.60	101.36	15.03	-13.13	-101.67	9.81
20.00	5.14	117.89	166.60	98.48	15.03	-13.25	-99.72	10.70
22.00	5.60	117.27	166.60	95.92	15.03	-13.33	-96.09	11.38
24.00	6.07	116.89	166.60	93.60	15.03	-13.37	-93.70	11.89
26.00	6.53	116.70	166.60	91.47	15.03	-13.40	-91.51	12.29
27.47	6.87	116.66	165.60	90.06	15.03	-13.40	-90.00	12.52
28.00	6.99	116.67	166.60	89.55	15.03	-13.40	-89.48	12.59
30.00	7.46	116.76	166.60	87.64	15.03	-13.39	-87.57	12.83
32.00	7.92	116.96	166.60	85.88	15.03	-13.36	-85.77	13.01
34.00	8.38	117.24	166.60	84.22	15.03	-13.33	-84.06	13.15
36.00	8.84	117.59	166.60	82.62	15.03	-13.29	-82.42	13.25
38.00	9.30	118.01	166.60	81.39	15.03	-13.24	-80.84	13.32
40.00	9.76	118.48	166.60	79.61	15.03	-13.18	-79.32	13.37
42.00	10.21	118.99	166.60	78.17	15.03	-13.11	-77.85	13.39
44.00	10.66	119.55	166.60	76.77	15.03	-13.04	-76.41	13.40
46.00	11.11	120.14	166.60	75.40	15.03	-12.96	-75.01	13.39
48.00	11.56	120.76	166.60	74.07	15.03	-12.88	-73.64	13.37
50.00	12.01	121.41	166.60	72.75	15.03	-12.79	-72.30	13.33
52.00	12.45	122.08	166.60	71.47	15.03	-12.69	-70.99	13.29
54.00	12.88	122.77	166.60	70.20	15.03	-12.59	-69.69	13.23
56.00	13.32	123.49	166.60	68.95	15.03	-12.49	-68.41	13.16
58.00	13.75	124.22	166.60	67.71	15.03	-12.38	-67.15	13.09
60.00	14.18	124.97	166.60	66.49	15.03	-12.27	-65.91	13.01

TABLE A2 Continued

Table of normal groundtrack separation, $P_2(\lambda,S)$ in kilometers. Each line of the table gives values of $P_2(\lambda)$ for a fixed value of the groundtrack separation S, in degrees or rev numbers, on the left, and selected negative values of the lunar latitude λ, in degrees.

A(1)= 15.03 180-A(1)= 164.97
A(2)= 13.40 180-A(2)= 166.60

REV	LATITUDE -15.03	-15.00	-14.00	-13.40	-13.00	-12.00	-11.00	-10.00	-9.00	-8.00	-7.00	-6.00	-5.00	LATITUDE -4.00	-3.00	-2.00	-1.00	-.01
1.				49.4	49.8	47.8	45.1	42.2	39.1	35.9	32.7	29.3	25.9	22.4	18.8	15.2	11.6	7.9
2.				49.2	51.5	51.1	49.4	47.3	44.8	42.1	39.2	36.2	33.1	29.8	26.4	23.0	19.4	15.8
4.				49.4	54.5	57.3	57.7	57.1	55.9	54.2	52.2	49.9	47.3	44.6	41.6	38.4	35.0	31.5
6.				47.1	57.0	63.0	65.5	66.5	66.5	65.9	64.8	63.3	61.4	59.1	56.5	53.7	50.5	47.2
8.				45.2	59.0	68.3	72.8	75.4	76.8	77.3	77.1	76.4	75.1	73.4	71.3	68.8	66.0	62.8
10.				42.8	60.4	73.0	79.7	83.9	86.7	88.3	89.1	89.2	88.6	87.6	85.9	83.9	81.3	78.3
12.				39.9	61.3	77.2	86.1	92.0	96.1	98.9	100.7	101.7	101.9	101.4	100.3	98.7	96.5	93.8
14.				36.5	61.7	81.0	92.0	99.6	105.1	109.1	112.0	113.8	114.8	115.0	114.5	113.4	111.6	109.2
16.				32.5	61.5	84.2	97.4	106.7	113.7	119.0	122.9	125.6	127.4	128.4	128.5	127.8	126.4	124.4
18.				28.1	60.8	86.8	102.3	113.4	121.9	128.4	133.4	137.1	139.7	141.4	142.2	142.1	141.1	139.5
20.				23.1	59.6	89.0	106.7	119.6	129.5	137.3	143.5	148.2	151.7	154.1	155.6	156.1	155.7	154.4
22.				17.6	57.8	90.6	110.6	125.3	136.8	145.9	153.2	158.9	163.3	166.6	168.7	169.8	169.9	169.1
24.				11.6	55.6	91.8	114.0	130.5	143.5	154.0	162.4	169.2	174.6	178.7	181.5	183.3	184.0	183.7
26.				5.1	52.7	92.3	116.9	135.2	149.8	161.6	171.3	179.2	185.5	190.4	194.1	196.5	197.8	198.0
27.				.0	50.3	92.4	118.7	138.4	154.1	166.9	177.5	186.2	193.2	198.8	203.1	206.1	207.8	209.4
28.				-1.9	49.4	92.4	119.2	139.5	155.6	168.8	179.7	188.7	196.0	201.8	206.3	209.4	211.4	212.1
30.				-9.4	45.5	91.9	121.1	143.2	160.9	175.5	187.6	197.8	206.1	212.8	218.1	222.1	224.7	226.0
32.				-17.3	41.1	90.9	122.4	146.3	165.7	181.7	195.2	206.4	215.8	223.5	229.6	234.3	237.7	239.6
34.				-25.8	36.2	89.3	123.1	149.0	170.0	187.5	202.2	214.6	225.1	233.8	240.8	246.3	250.3	252.9
36.				-34.7	30.8	87.2	123.4	151.2	173.8	192.7	208.8	222.4	233.9	243.6	251.6	257.9	262.7	266.0
38.				-44.1	24.8	84.6	123.1	152.8	177.1	197.5	214.9	229.7	242.4	253.1	262.0	269.2	274.7	278.7
40.				-53.9	18.3	81.4	122.2	153.9	179.8	201.8	220.5	236.6	250.4	262.1	271.9	280.0	286.4	291.2
42.				-64.2	11.4	77.7	120.9	154.4	182.1	205.5	225.6	242.9	257.9	270.7	281.5	290.5	297.8	303.3
44.				-74.9	3.9	73.5	119.0	154.4	183.8	208.7	230.2	248.8	265.0	278.8	290.7	300.6	308.8	315.0
46.				-86.0	-4.1	68.7	116.5	153.9	185.0	211.5	234.4	254.2	271.6	286.6	299.4	310.3	319.3	326.4
48.				-97.5	-12.6	63.4	113.5	152.9	185.6	213.7	238.0	259.2	277.7	293.8	307.7	319.6	329.5	337.5
50.				-109.4	-21.5	57.6	110.0	151.3	185.8	215.3	241.1	263.6	283.3	300.6	315.6	328.5	339.3	349.2
52.				-121.7	-30.9	51.3	105.9	149.1	185.3	216.5	243.6	267.5	288.5	306.9	323.0	336.9	348.7	358.4
54.				-134.4	-40.8	44.4	101.3	146.4	184.4	217.1	245.7	270.9	293.1	312.7	329.9	344.9	357.7	368.3
56.				-147.4	-51.1	37.0	96.1	143.2	182.9	217.1	247.2	273.8	297.3	318.1	336.4	352.4	366.2	377.8
58.				-160.8	-61.9	29.1	90.4	139.4	180.8	216.7	248.2	276.1	300.9	322.9	342.4	359.5	374.3	386.9
60.				-174.5	-73.1	20.7	84.2	135.1	178.2	215.6	248.6	277.9	304.0	327.3	347.9	364.1	382.0	395.5

ARC ALONG GROUND TRACK BETWEEN EQUATOR AND LATITUDE L (ω)

ω_1	-90.00	-76.42	-68.89	-63.34	-60.16	-53.30	-47.37	-42.04	-37.10	-32.46	-28.03	-23.77	-19.64	-15.60	-11.64	-7.73	-3.86	-.04
ω_2	.00	.00	.00	-90.00	-76.09	-63.79	-55.42	-48.53	-42.46	-36.91	-31.73	-26.81	-22.09	-17.52	-13.05	-8.66	-4.32	-.04

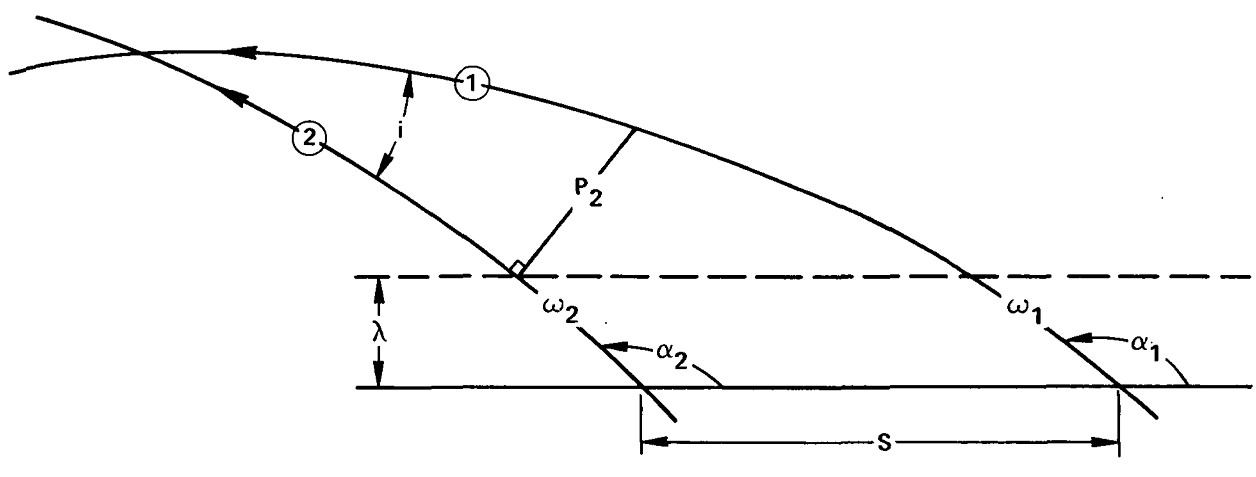


FIGURE 1 - REFERENCE CONFIGURATION FOR INTERSECTION OF TWO GROUND TRACKS

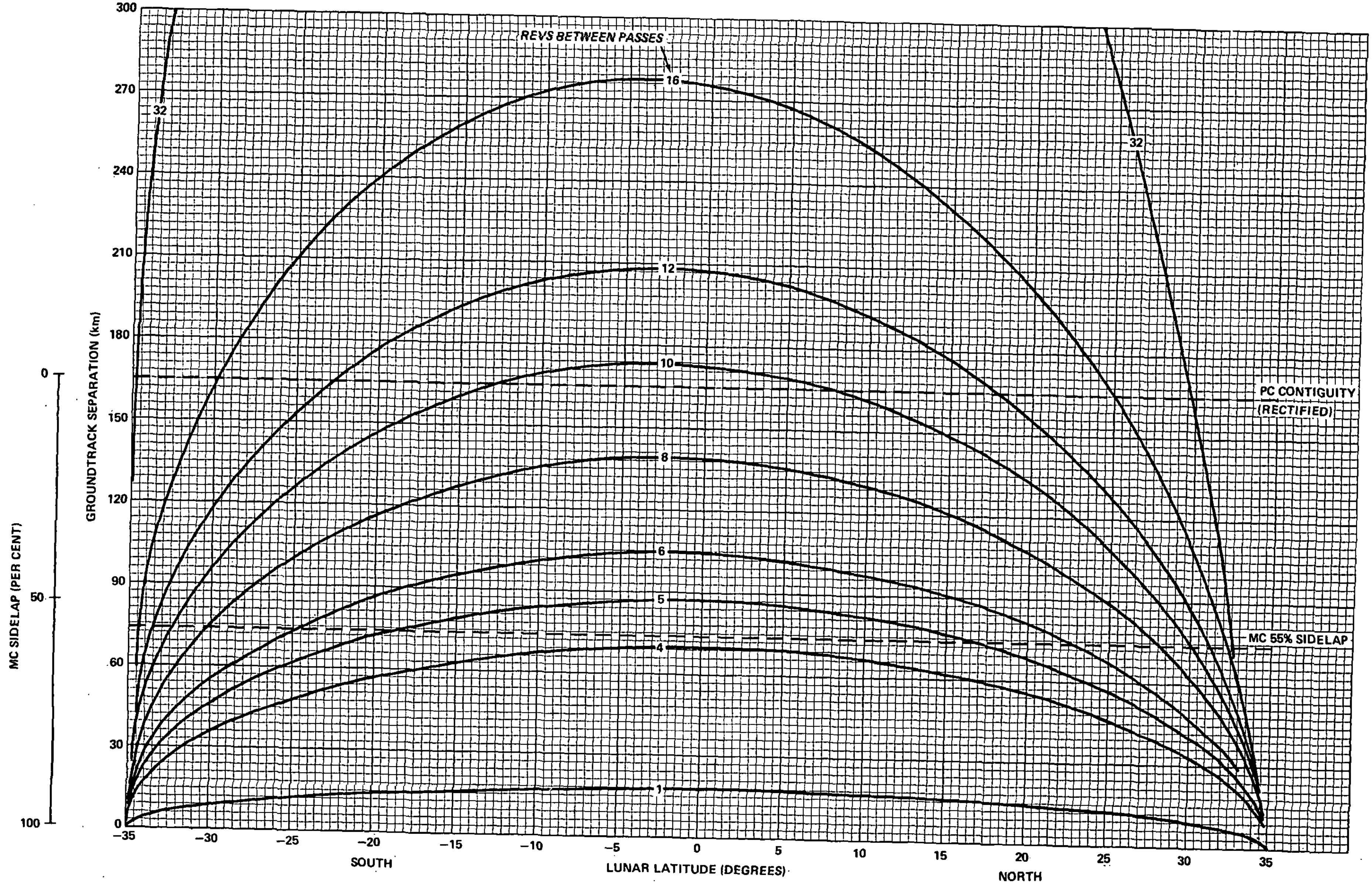


FIGURE 2 - SEPARATION/SIDELAP BETWEEN SUCCESSIVE PHOTO-PASSES AS A FUNCTION OF LUNAR LATITUDE FOR THE INDICATED NUMBER OF REVOLUTIONS BETWEEN PASSES AT AN INCLINATION OF 145°

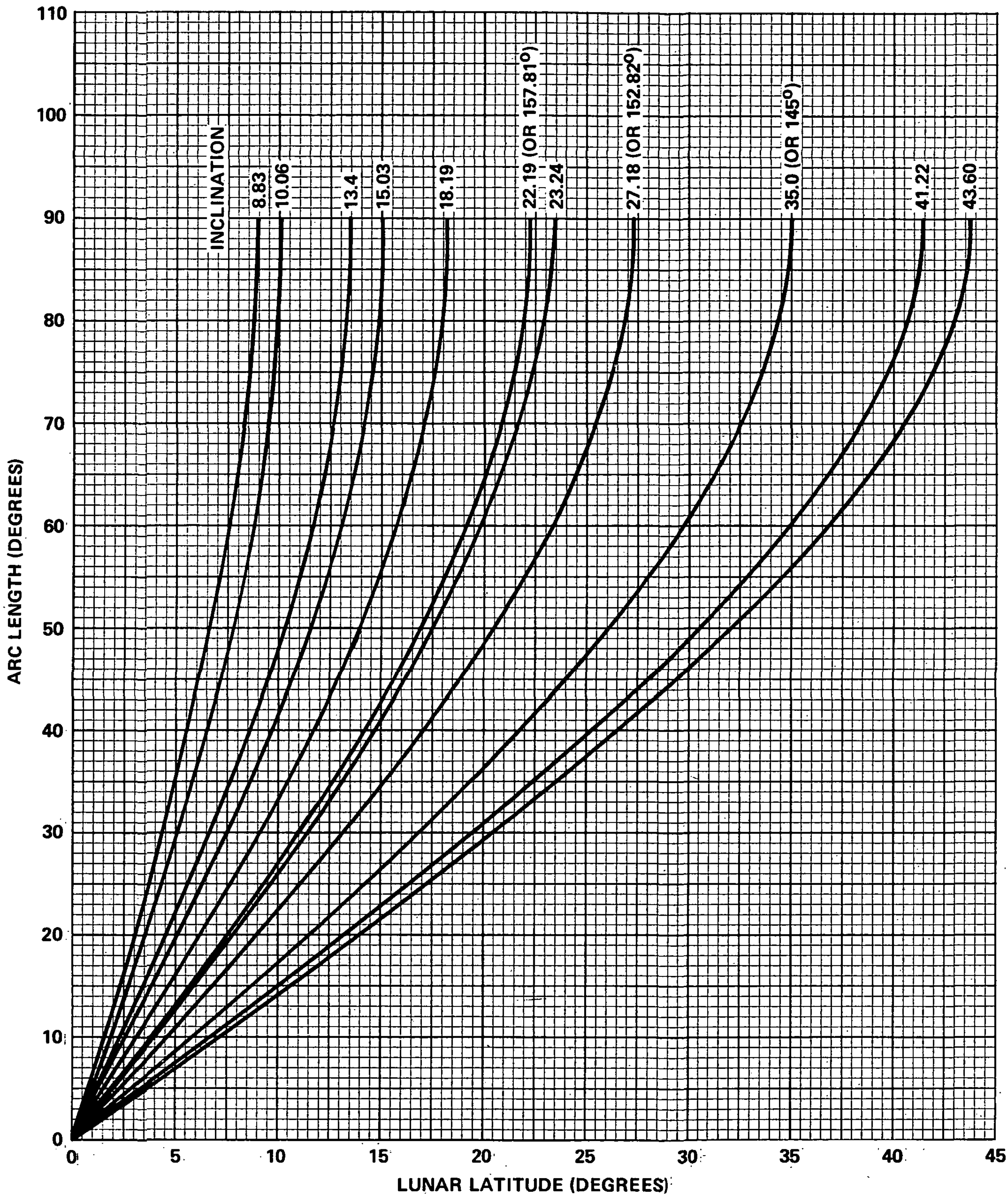


FIGURE 3 - ARC LENGTH ALONG GROUNDTRACK BETWEEN THE LUNAR EQUATOR AND THE INDICATED LATITUDE (NORTH OR SOUTH) FOR GROUNDTRACKS OF VARIOUS INCLINATIONS

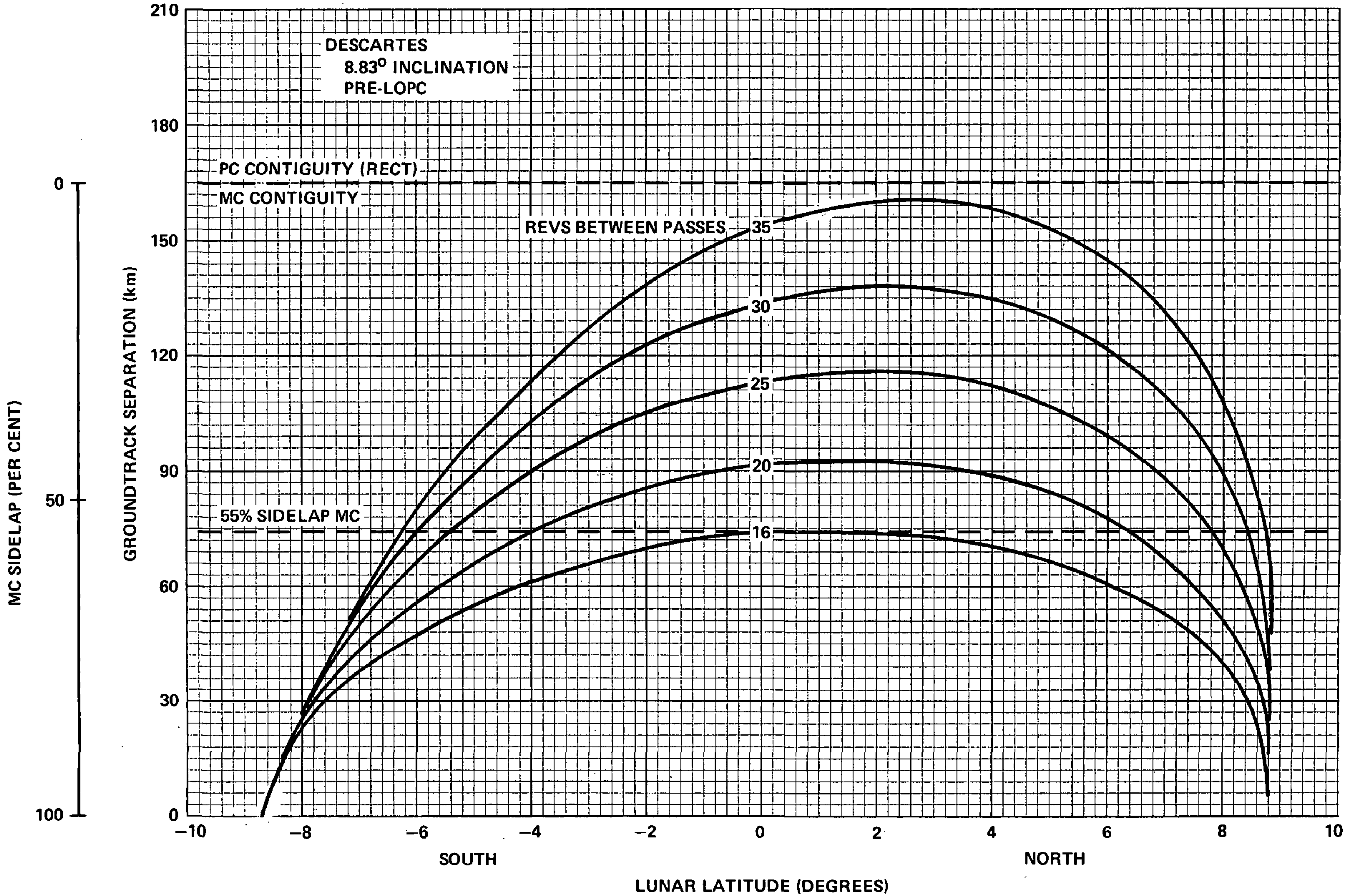


FIGURE 4a - SEPARATION/SIDELAP BETWEEN SUCCESSIVE PHOTO-PASSES AS A FUNCTION OF LUNAR LATITUDE FOR THE INDICATED NUMBER OF REVOLUTIONS BETWEEN PASSES AT AN INCLINATION OF 8.83°

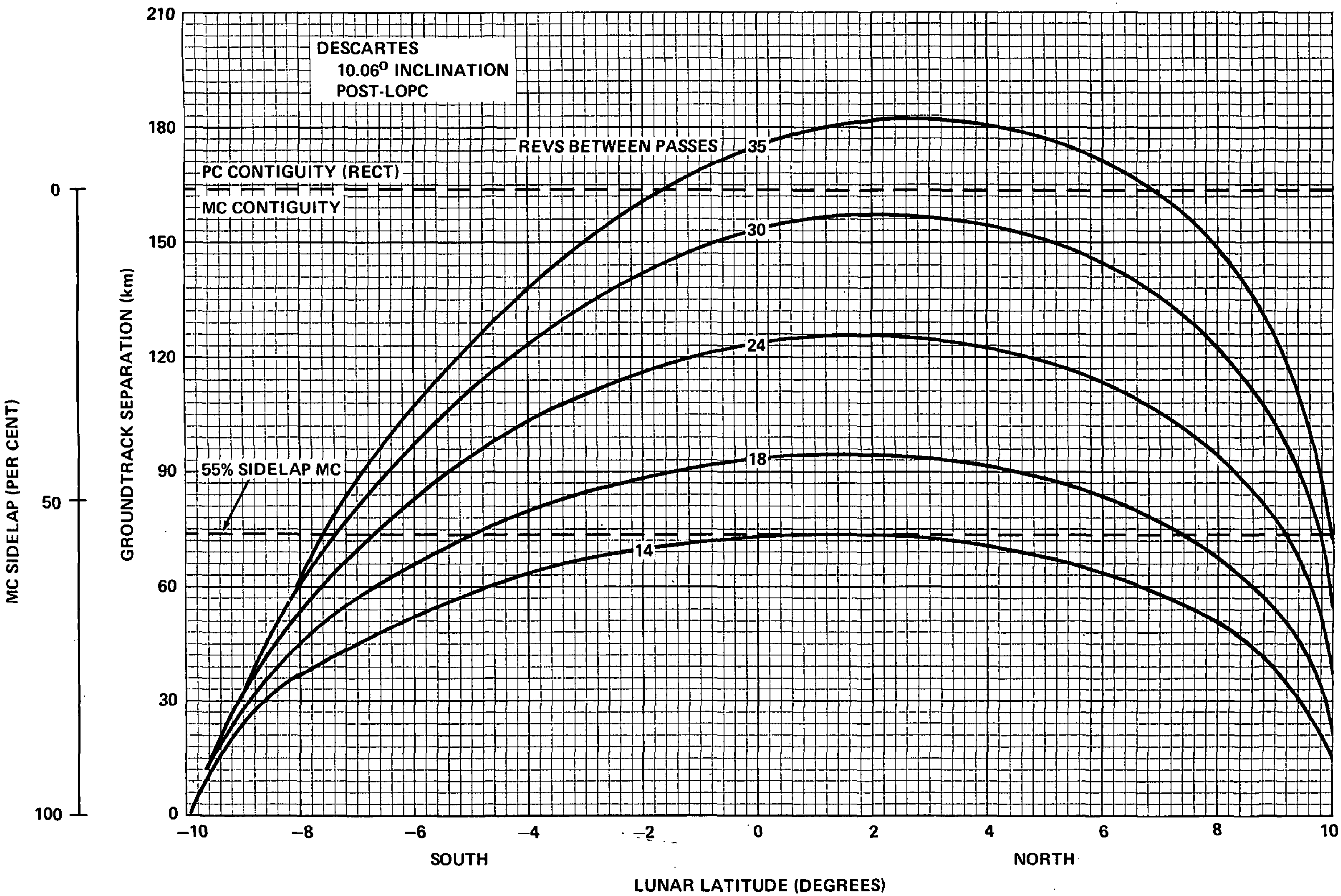


FIGURE 4b - SEPARATION/SIDELAP BETWEEN SUCCESSIVE PHOTO-PASSES AS A FUNCTION OF LUNAR LATITUDE FOR THE INDICATED NUMBER OF REVOLUTIONS BETWEEN PASSES AT AN INCLINATION OF 10.06°

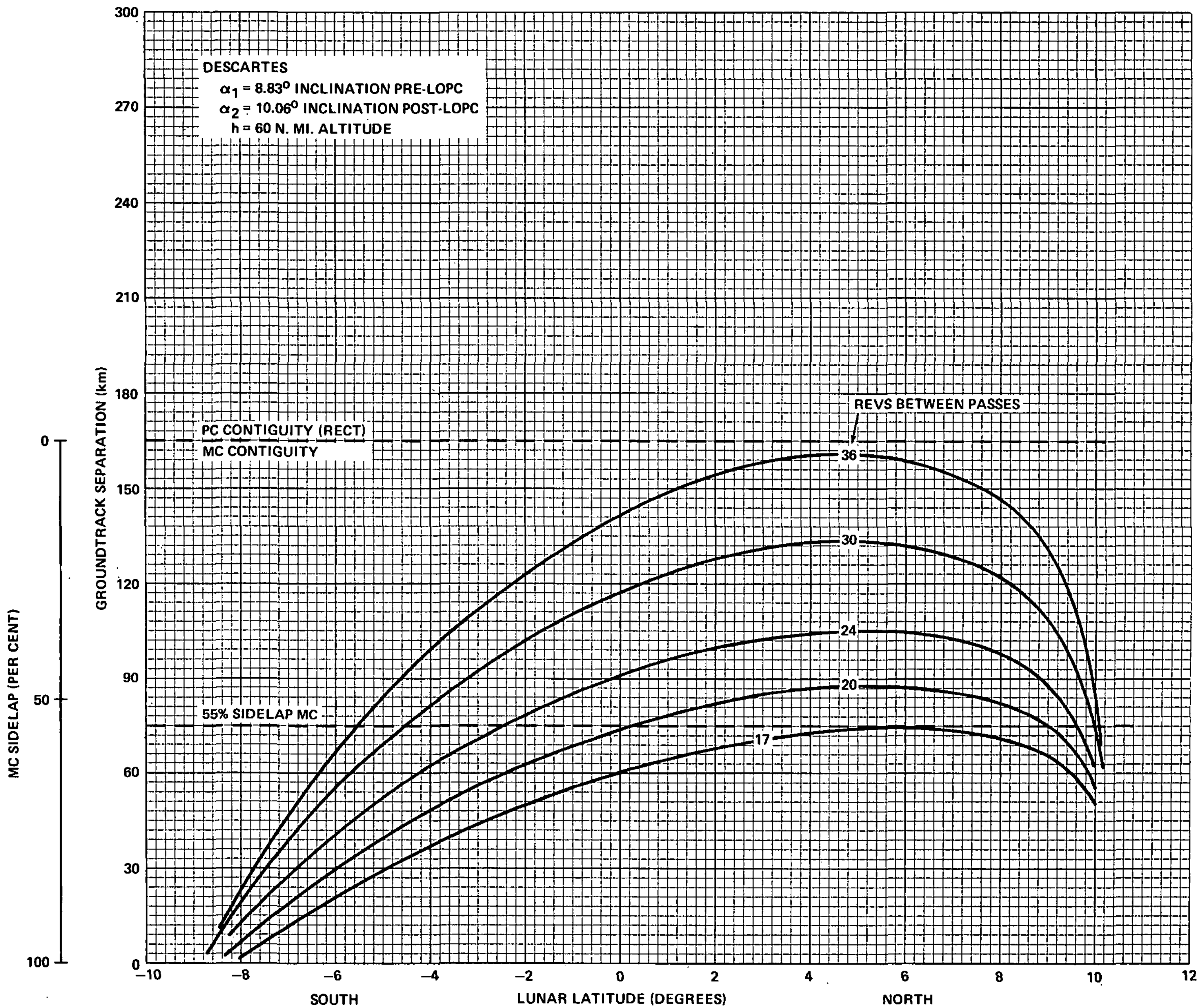


FIGURE 4c - SEPARATION/SIDELAP BETWEEN SUCCESSIVE PHOTO-PASSES PRE- AND POST-LOPC AS A FUNCTION OF LUNAR LATITUDE FOR THE INDICATED NUMBER OF REVOLUTIONS BETWEEN PASSES

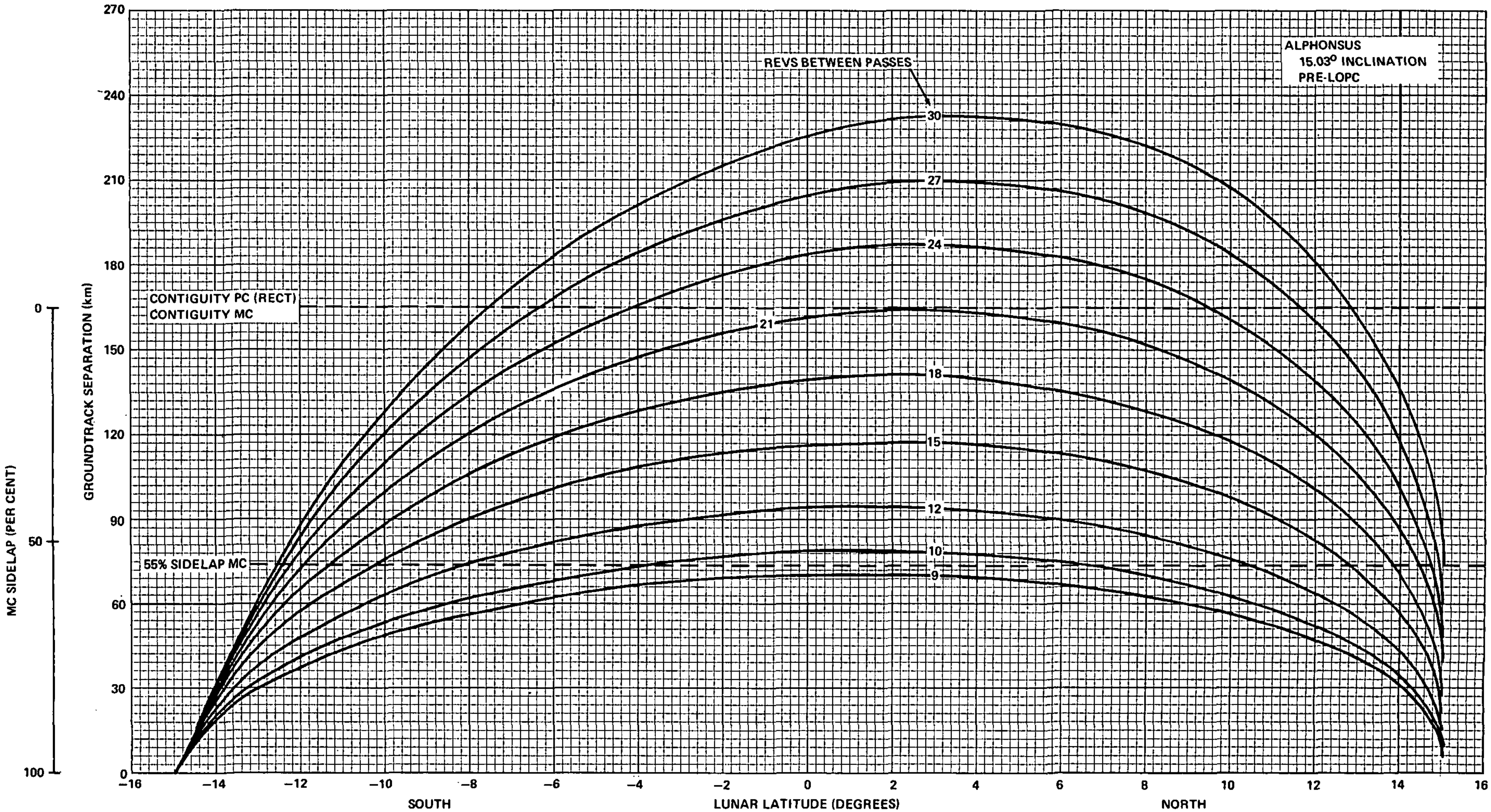


FIGURE 5a · SEPARATION/SIDELAP BETWEEN SUCCESSIVE PHOTO-PASSES AS A FUNCTION OF LUNAR LATITUDE FOR THE INDICATED NUMBER OF REVOLUTIONS BETWEEN PASSES AT AN INCLINATION OF 15.03°

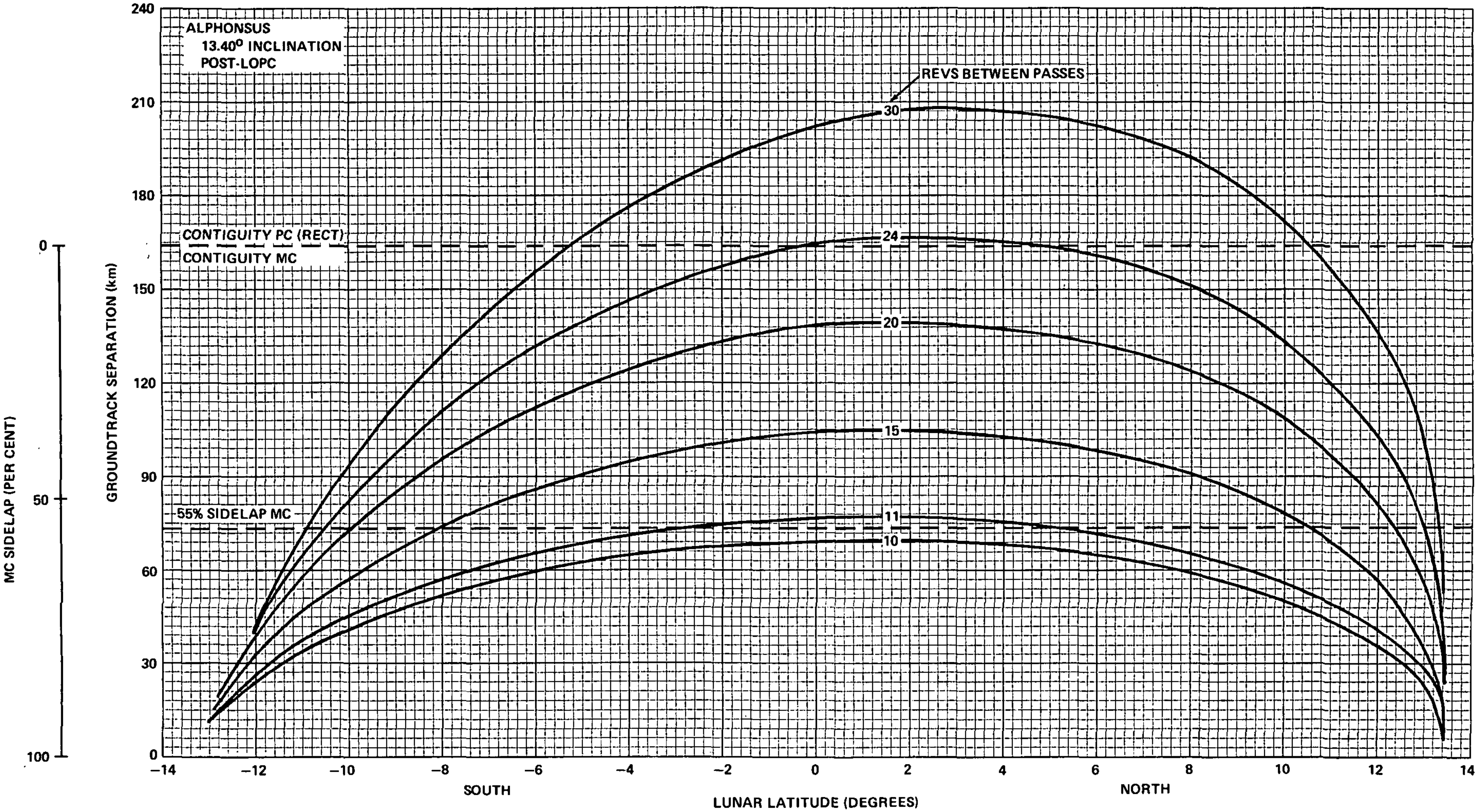


FIGURE 5b - SEPARATION/SIDELAP BETWEEN SUCCESSIVE PHOTO-PASSES AS A FUNCTION OF LUNAR LATITUDE FOR THE INDICATED NUMBER OF REVOLUTIONS BETWEEN PASSES AT AN INCLINATION OF 13.40°

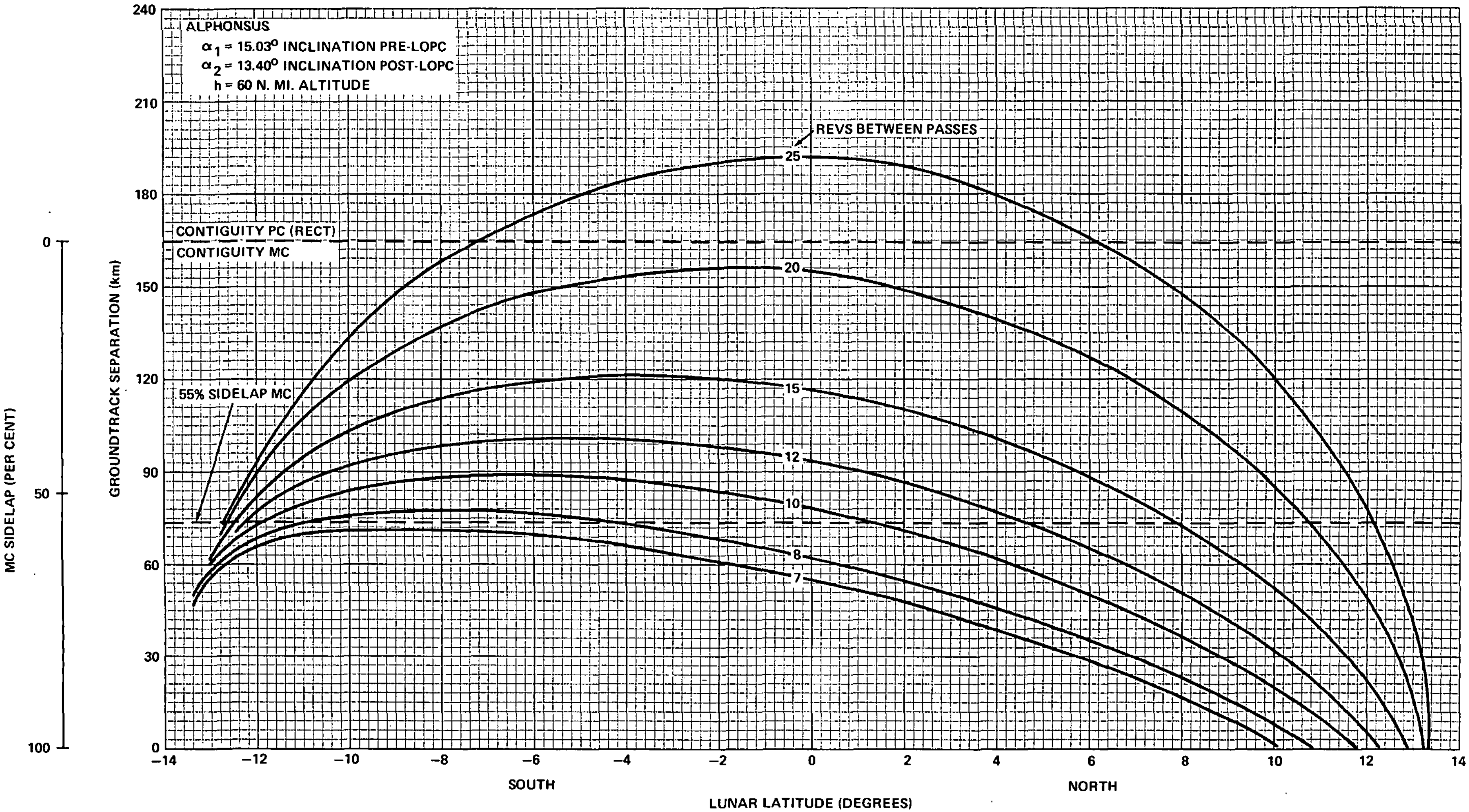


FIGURE 5c - SEPARATION/SIDELAP BETWEEN SUCCESSIVE PHOTO-PASSES PRE- AND POST-LOPC AS A FUNCTION OF LUNAR LATITUDE FOR THE INDICATED NUMBER OF REVOLUTIONS BETWEEN PASSES

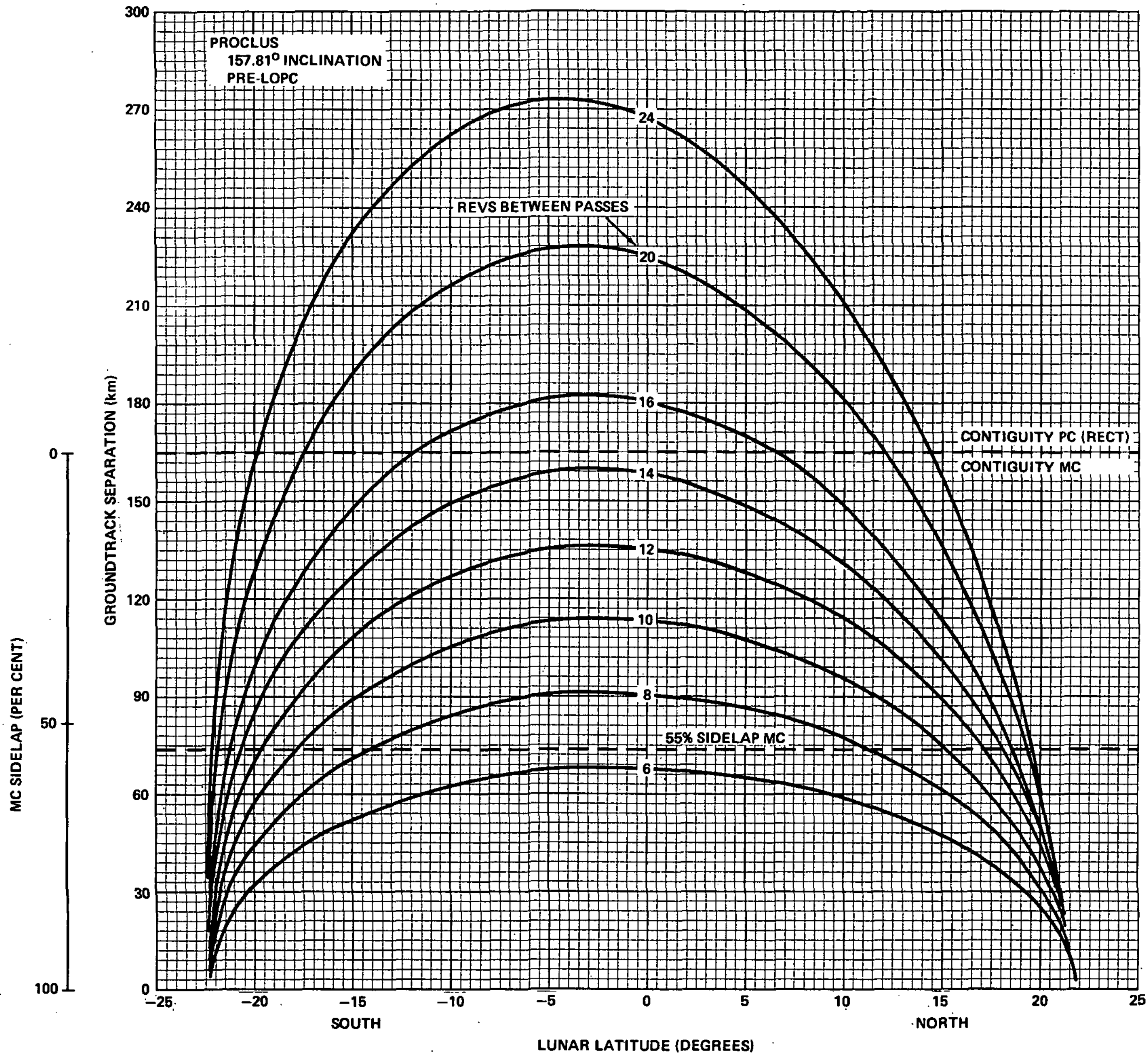


FIGURE 6a - SEPARATION/SIDELAP BETWEEN SUCCESSIVE PHOTO-PASSES AS A FUNCTION OF LUNAR LATITUDE FOR THE INDICATED NUMBER OF REVOLUTIONS BETWEEN PASSES AT AN INCLINATION OF 157.81°

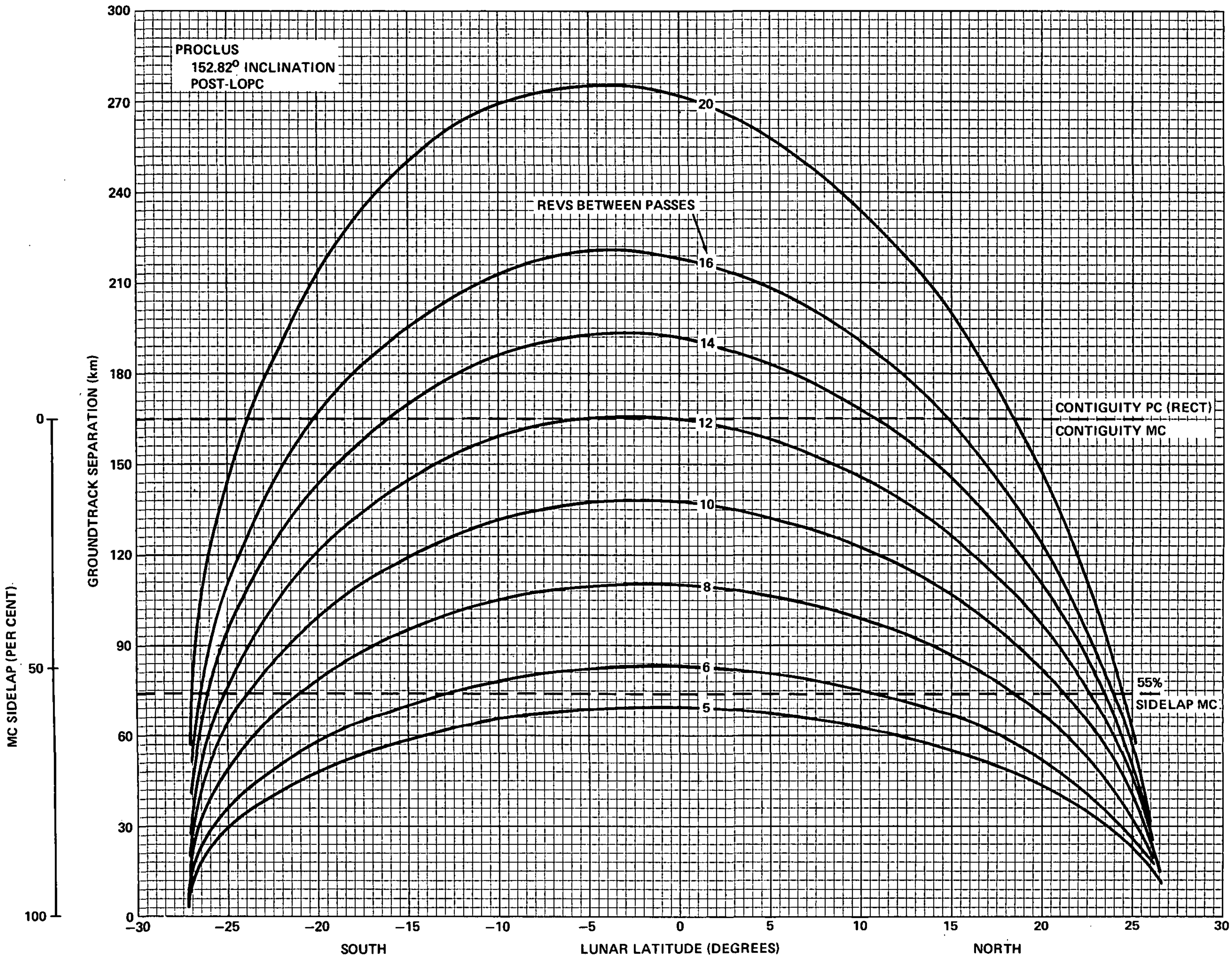


FIGURE 6b - SEPARATION/SIDELAP BETWEEN SUCCESSIVE PHOTO-PASSES AS A FUNCTION OF LUNAR LATITUDE FOR THE INDICATED NUMBER OF REVOLUTIONS BETWEEN PASSES AT AN INCLINATION OF 152.82°

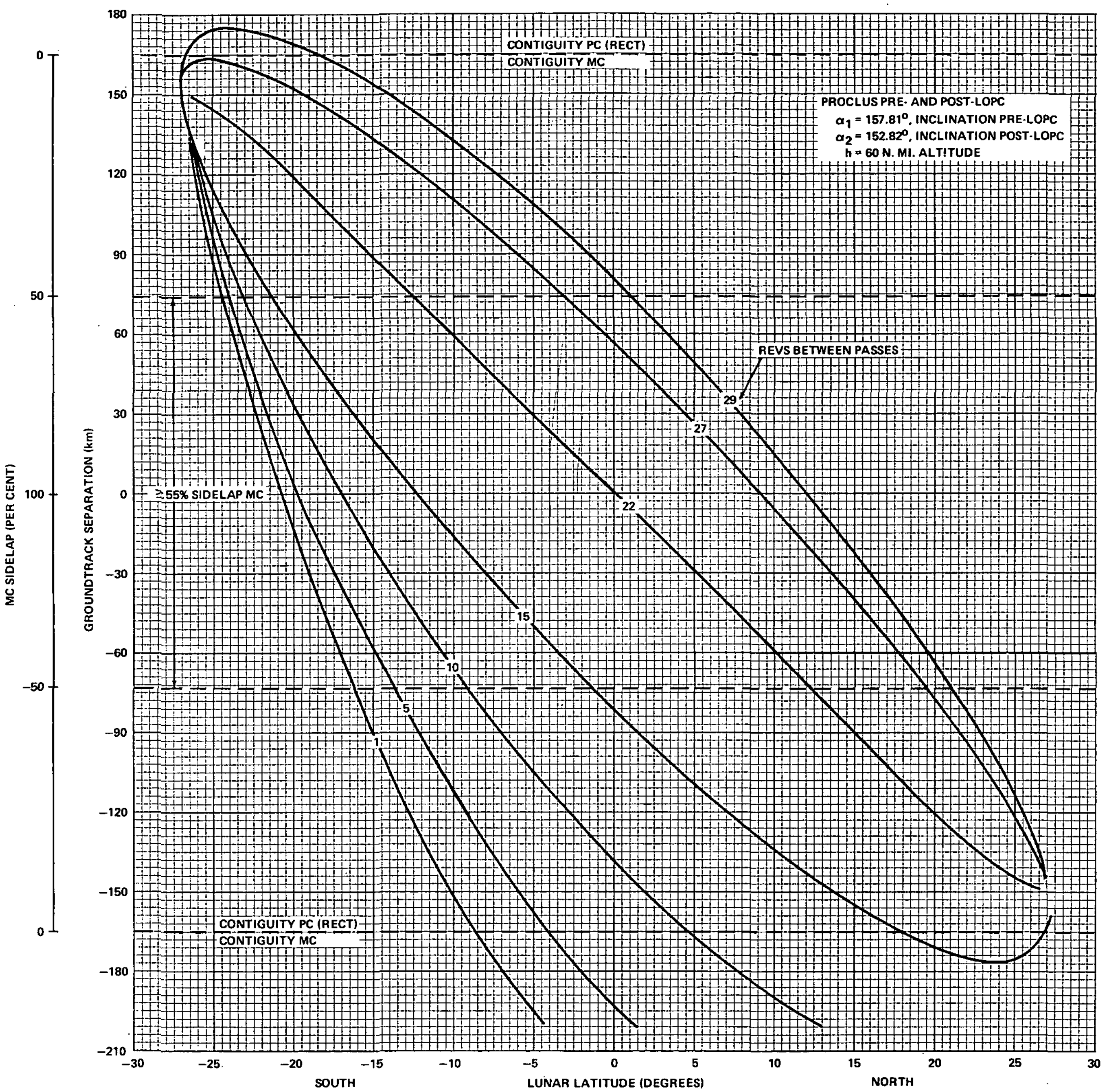


FIGURE 6c - SEPARATION/SIDELAP BETWEEN SUCCESSIVE PHOTO-PASSES PRE- AND POST-LOPC AS A FUNCTION OF LUNAR LATITUDE FOR THE INDICATED NUMBER OF REVOLUTIONS BETWEEN PASSES

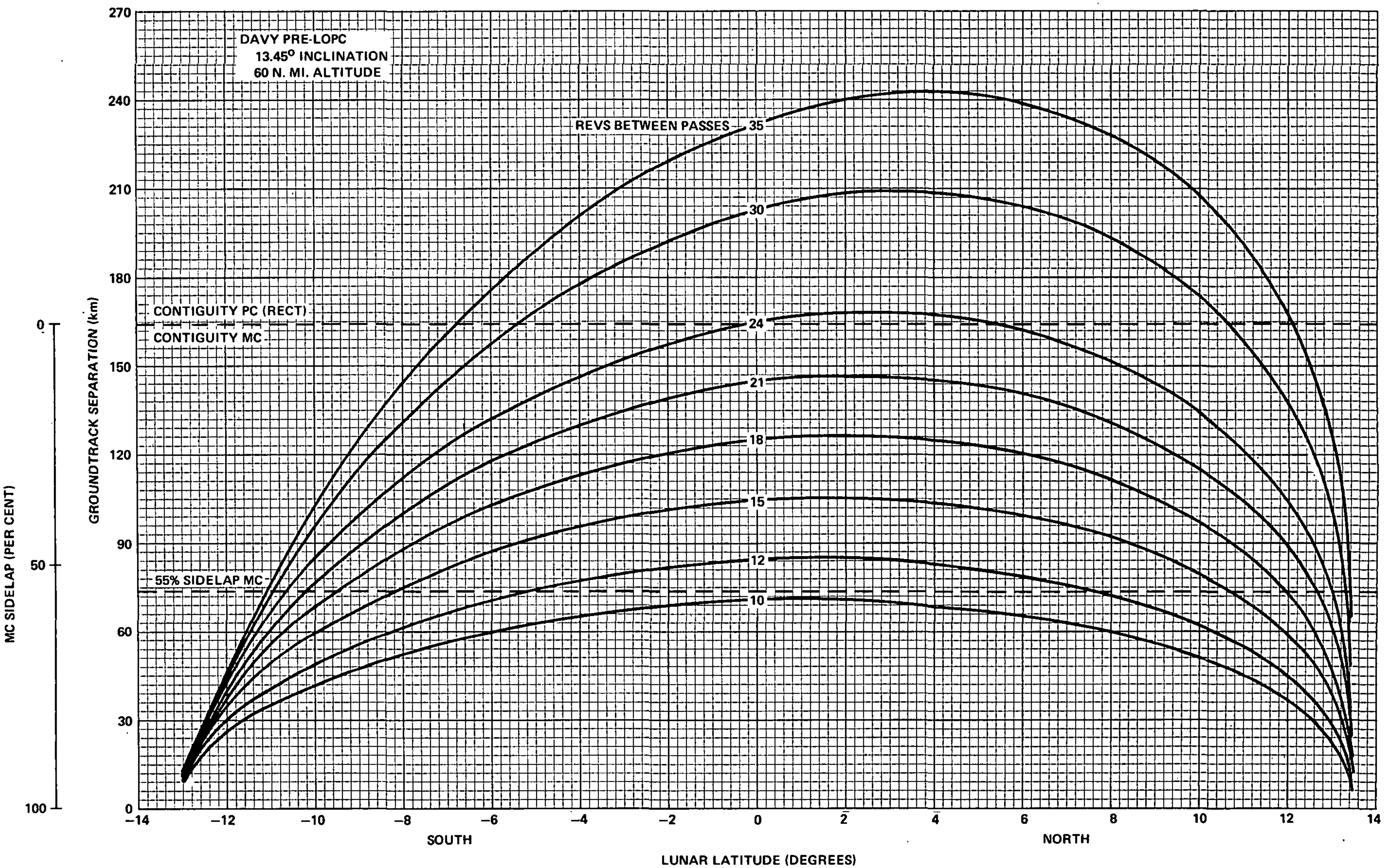


FIGURE 7a - SEPARATION/SIDELAP BETWEEN SUCCESSIVE PHOTO-PASSES AS A FUNCTION OF LUNAR LATITUDE FOR THE INDICATED NUMBER OF REVOLUTIONS BETWEEN PASSES AT AN INCLINATION OF 13.45°

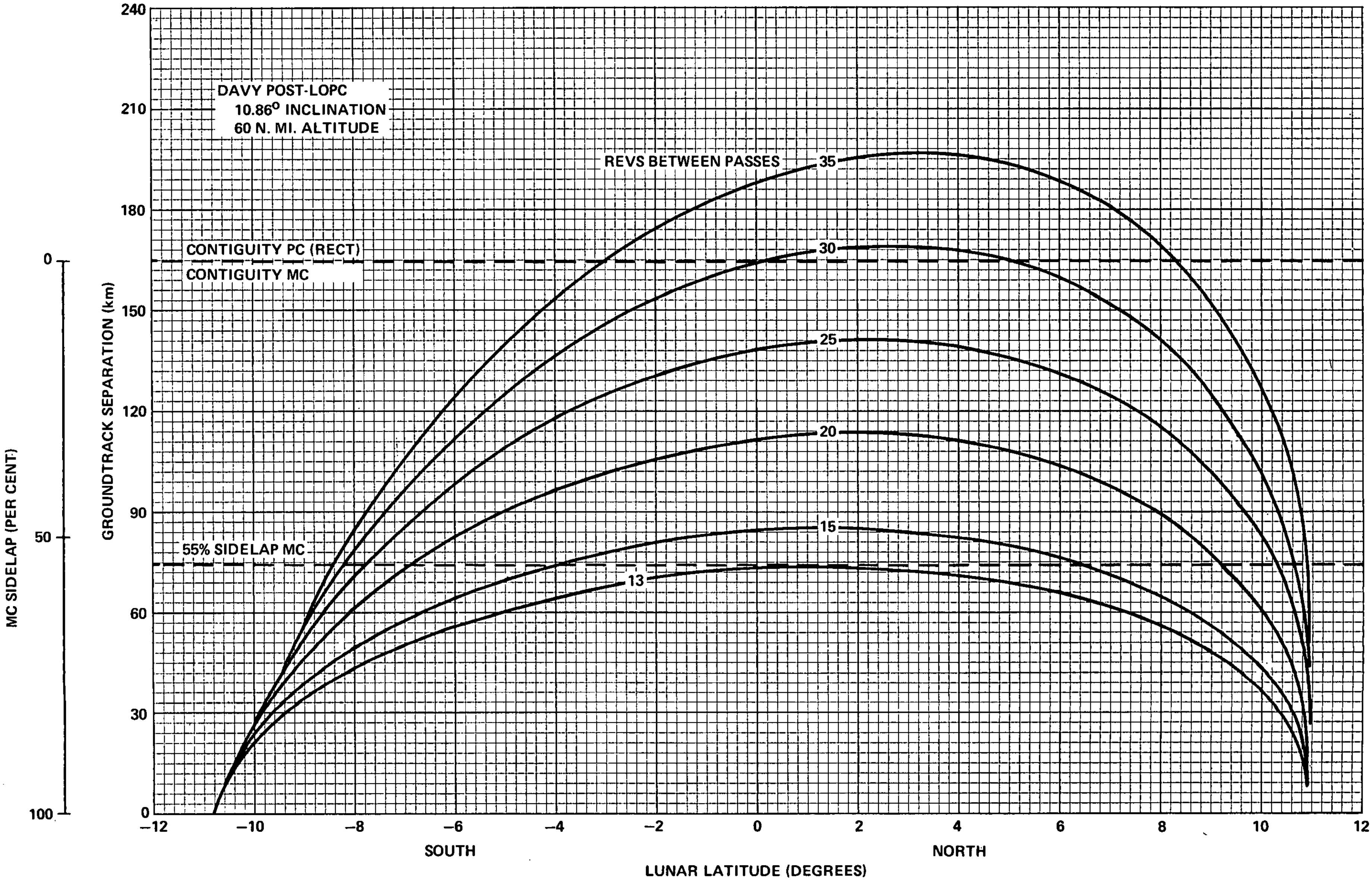


FIGURE 7b - SEPARATION/SIDELAP BETWEEN SUCCESSIVE PHOTO-PASSES AS A FUNCTION OF LUNAR LATITUDE FOR THE INDICATED NUMBER OF REVOLUTIONS BETWEEN PASSES AT AN INCLINATION OF 10.86°

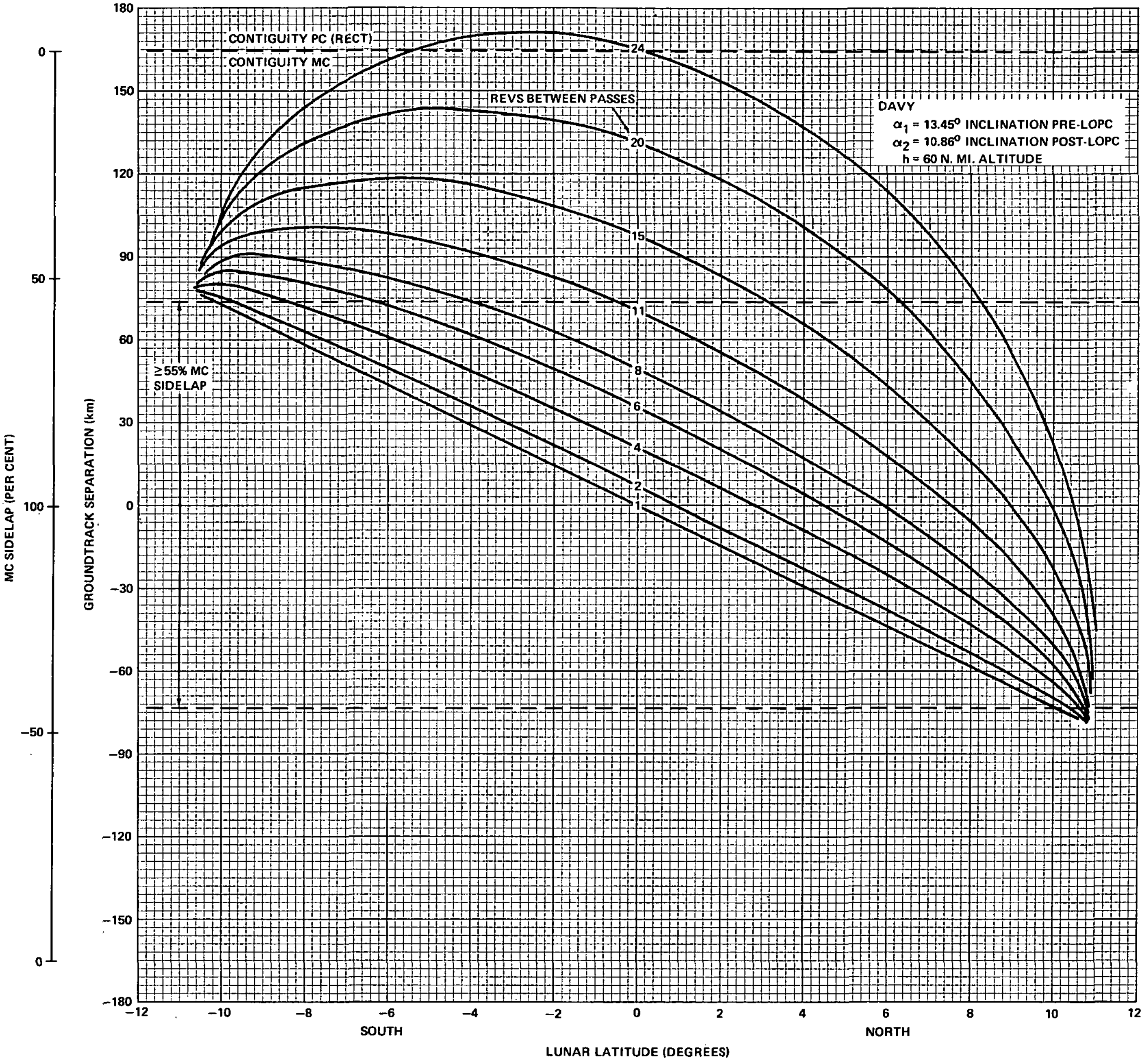


FIGURE 7c - SEPARATION/SIDELAP BETWEEN SUCCESSIVE PHOTO-PASSES PRE- AND POST-LOPC AS A FUNCTION OF LUNAR LATITUDE FOR THE INDICATED NUMBER OF REVOLUTIONS BETWEEN PASSES

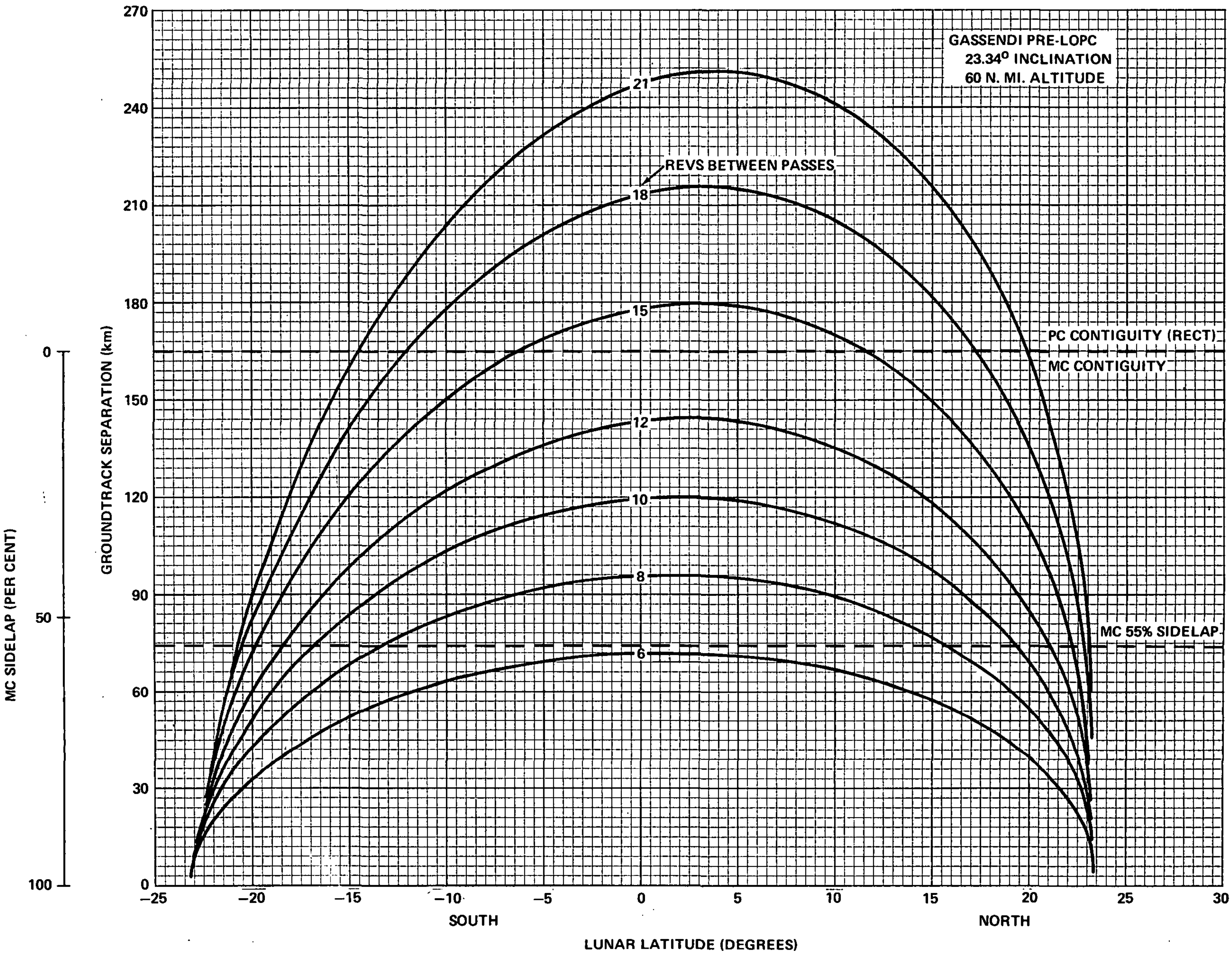


FIGURE 8a - SEPARTION/SIDELAP BETWEEN SUCCESSIVE PHOTO-PASSES AS A FUNCTION OF LUNAR LATITUDE FOR THE INDICATED NUMBER OF REVOLUTIONS BETWEEN PASSES AT AN INCLINATION OF 23.34°

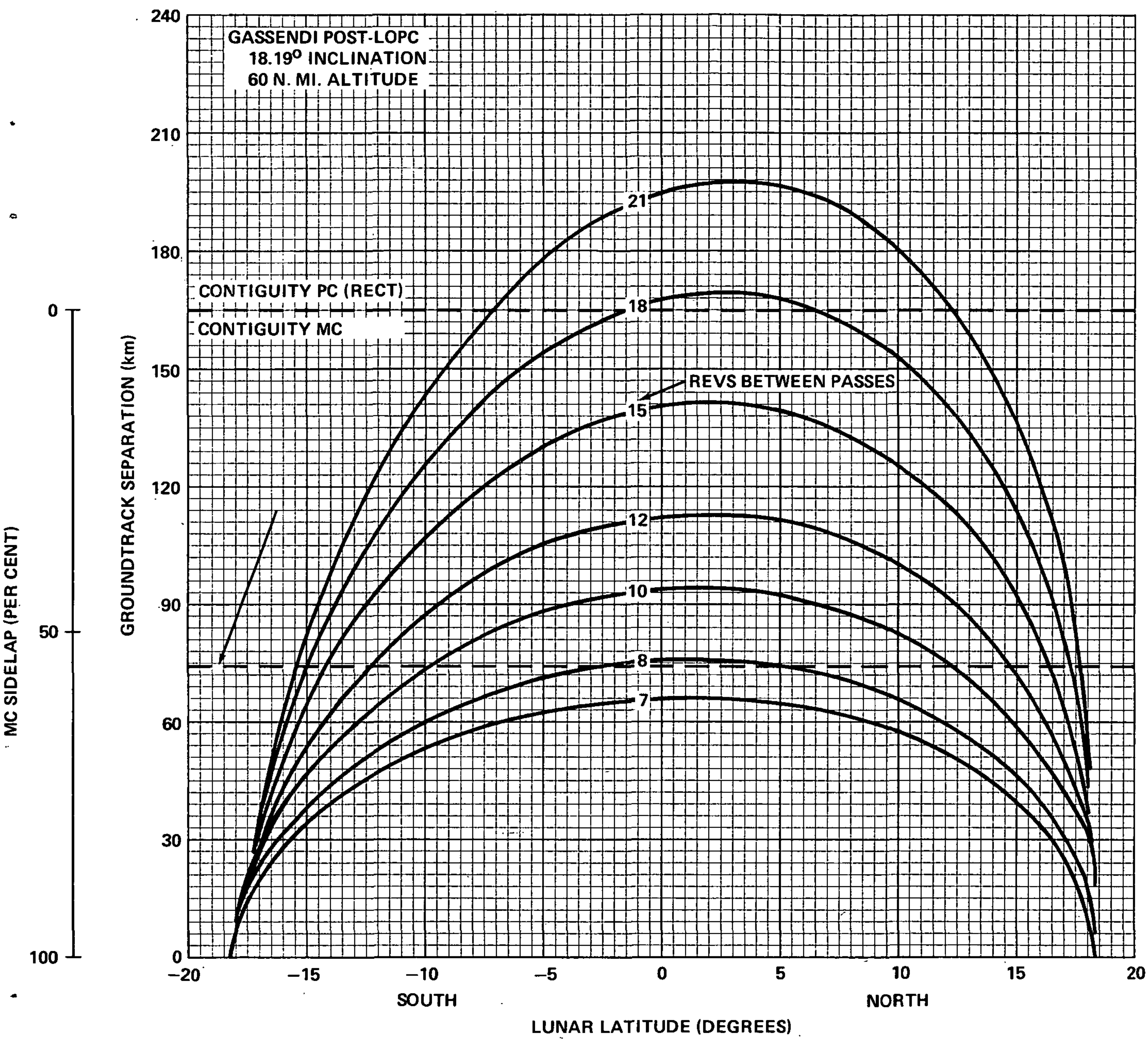


FIGURE 8b - SEPARATION/SIDELAP BETWEEN SUCCESSIVE PHOTO-PASSES AS A FUNCTION OF LUNAR LATITUDE FOR THE INDICATED NUMBER OF REVOLUTIONS BETWEEN PASSES AT AN INCLINATION OF 18.19°

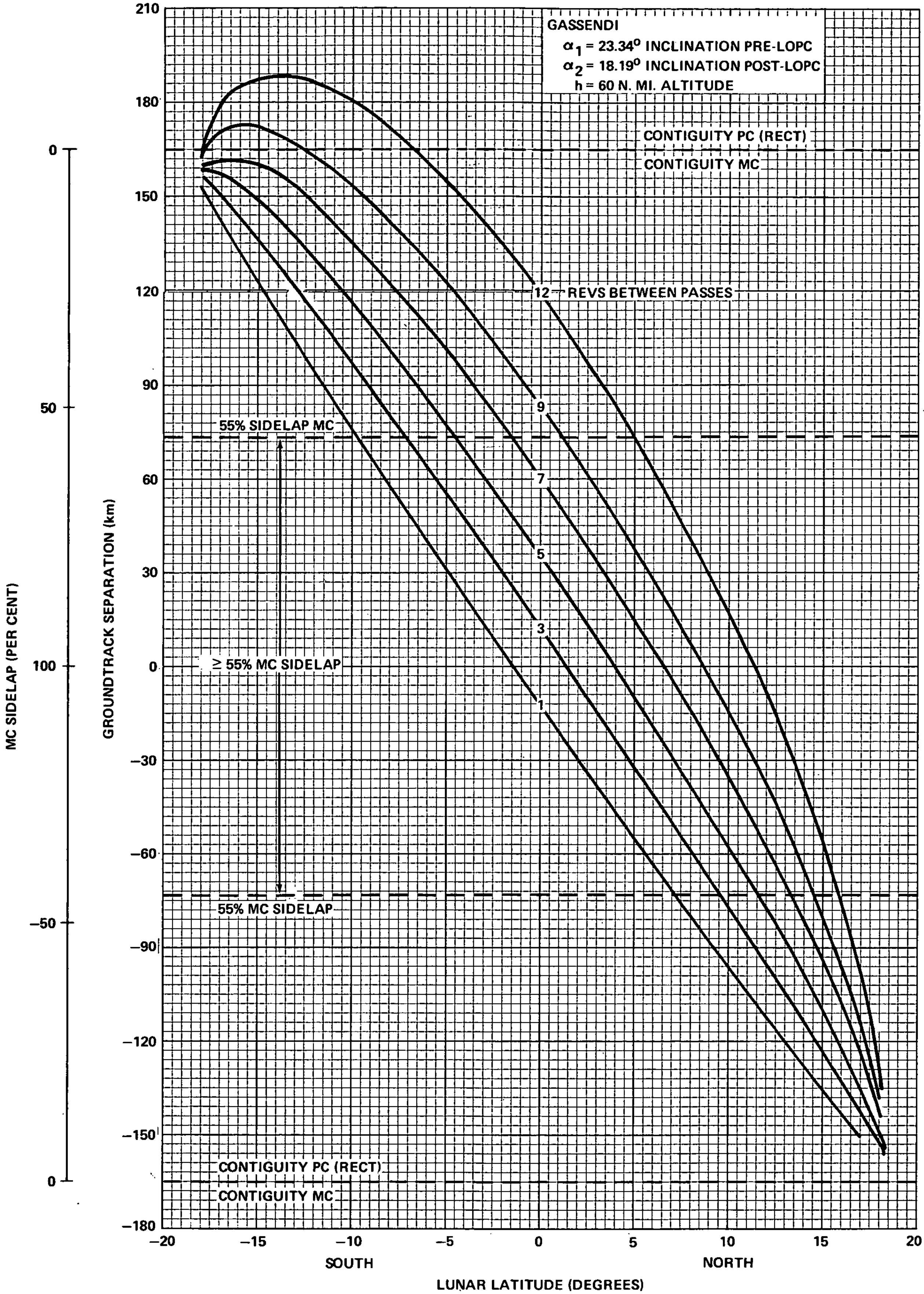


FIGURE 8c - SEPARATION/SIDELAP BETWEEN SUCCESSIVE PHOTO-PASSES PRE- AND POST-LOPC AS A FUNCTION OF LUNAR LATITUDE FOR THE INDICATED NUMBER OF REVOLUTIONS BETWEEN PASSES

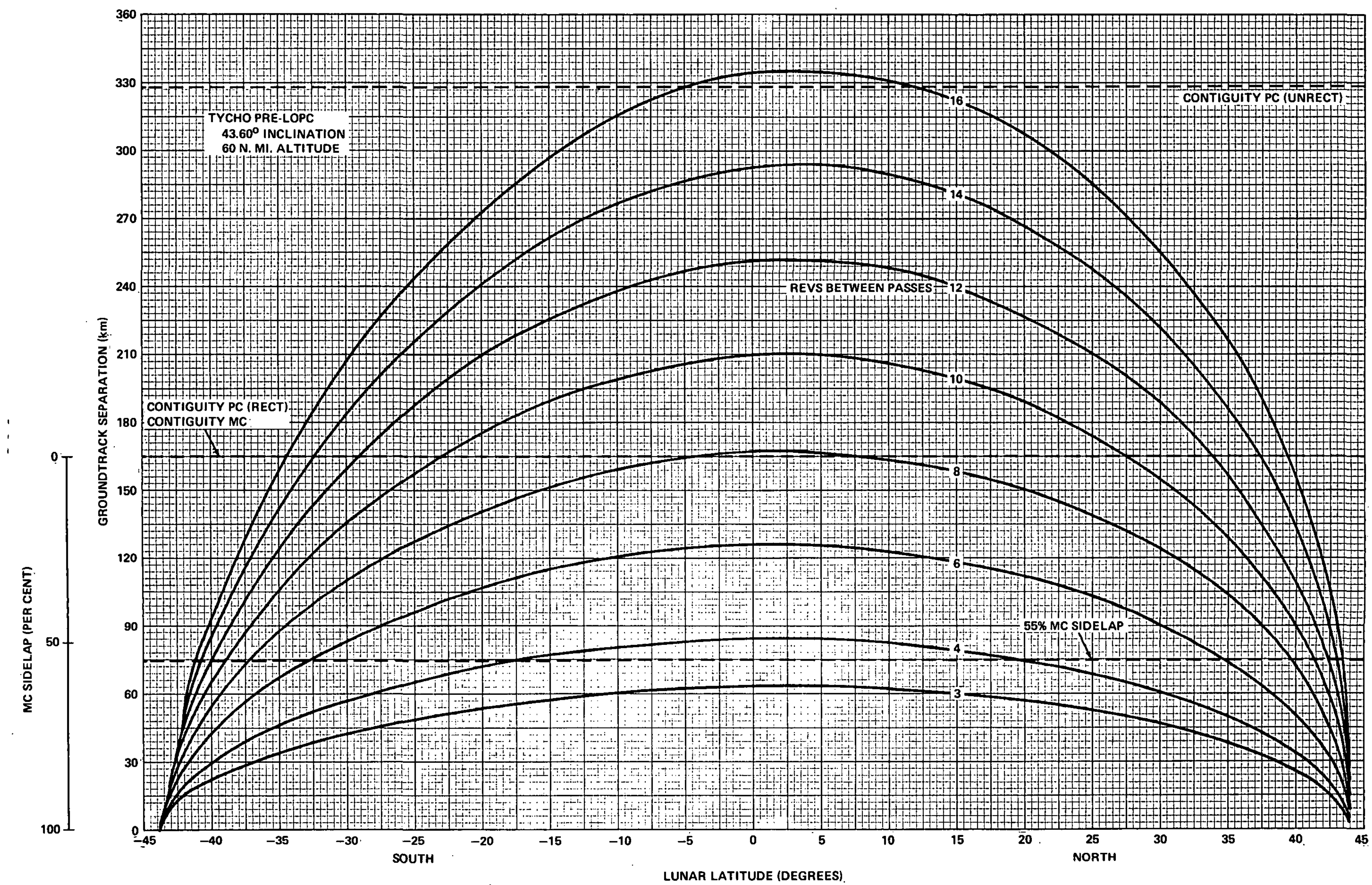


FIGURE 9a - SEPARATION/SIDELAP BETWEEN SUCCESSIVE PHOTO-PASSES AS A FUNCTION OF LUNAR LATITUDE FOR THE INDICATED NUMBER OF REVOLUTIONS BETWEEN PASSES AT AN INCLINATION OF 43.60°

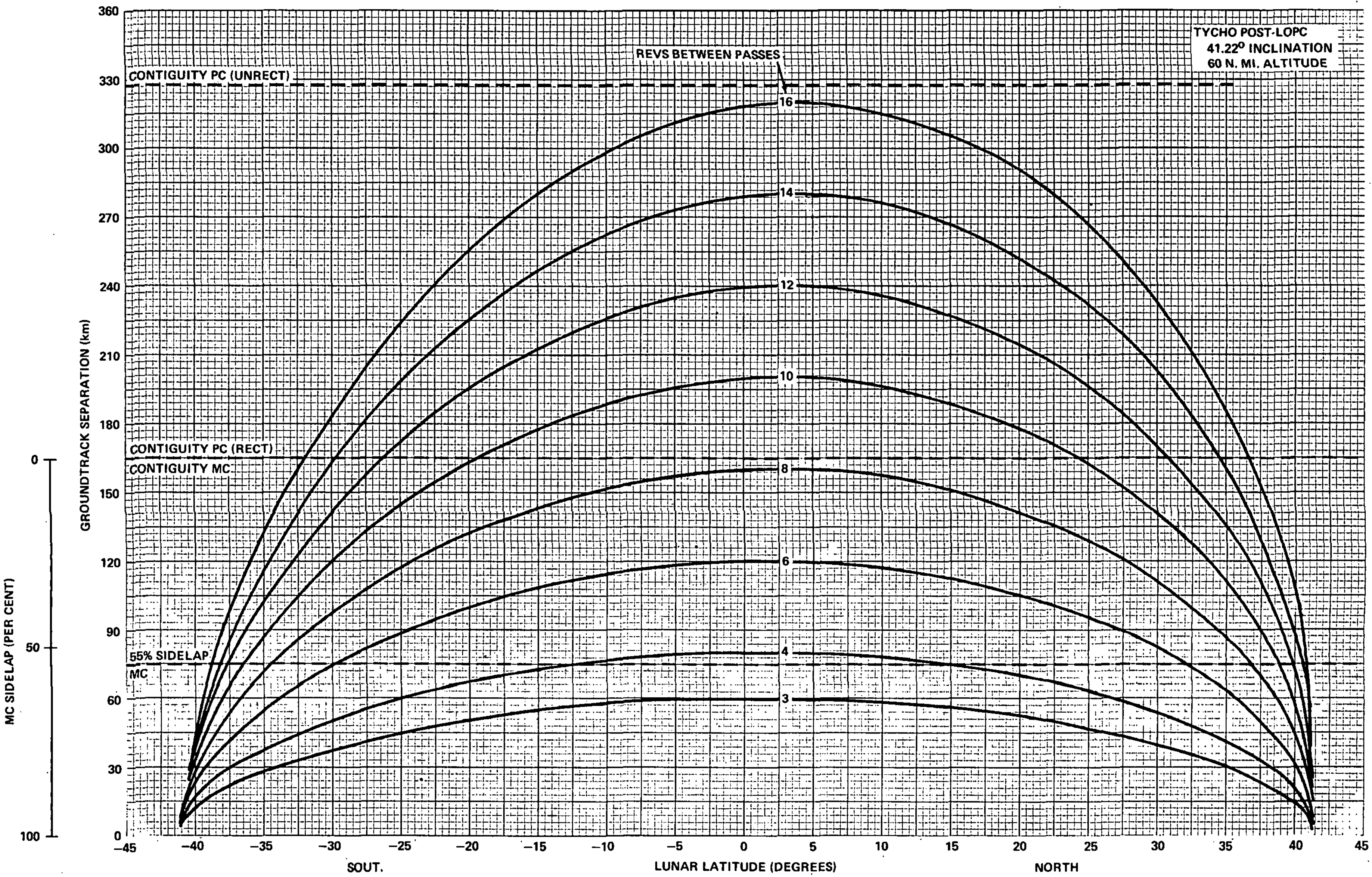


FIGURE 9b - SEPARATION/SIDELAP BETWEEN SUCCESSIVE PHOTO-PASSES AS A FUNCTION OF LUNAR LATITUDE FOR THE INDICATED NUMBER OF REVOLUTIONS BETWEEN PASSES AT AN INCLINATION OF 41.22°

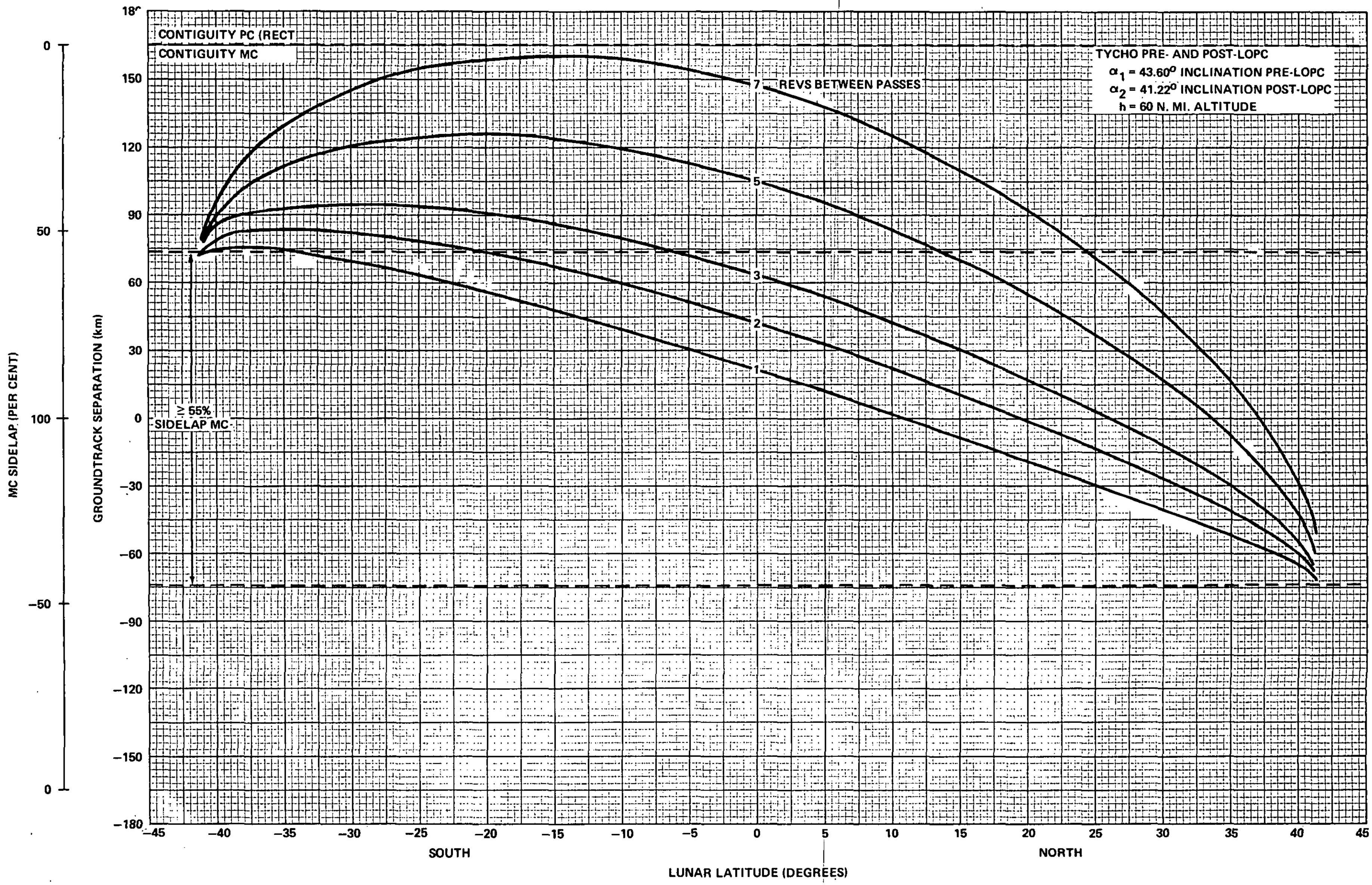


FIGURE 9c - SEPARATION/SIDELAP BETWEEN SUCCESSIVE PHOTO-PASSES PRE- AND POST-LOPC AS A FUNCTION OF LUNAR LATITUDE FOR THE INDICATED NUMBER OF REVOLUTIONS BETWEEN PASSES

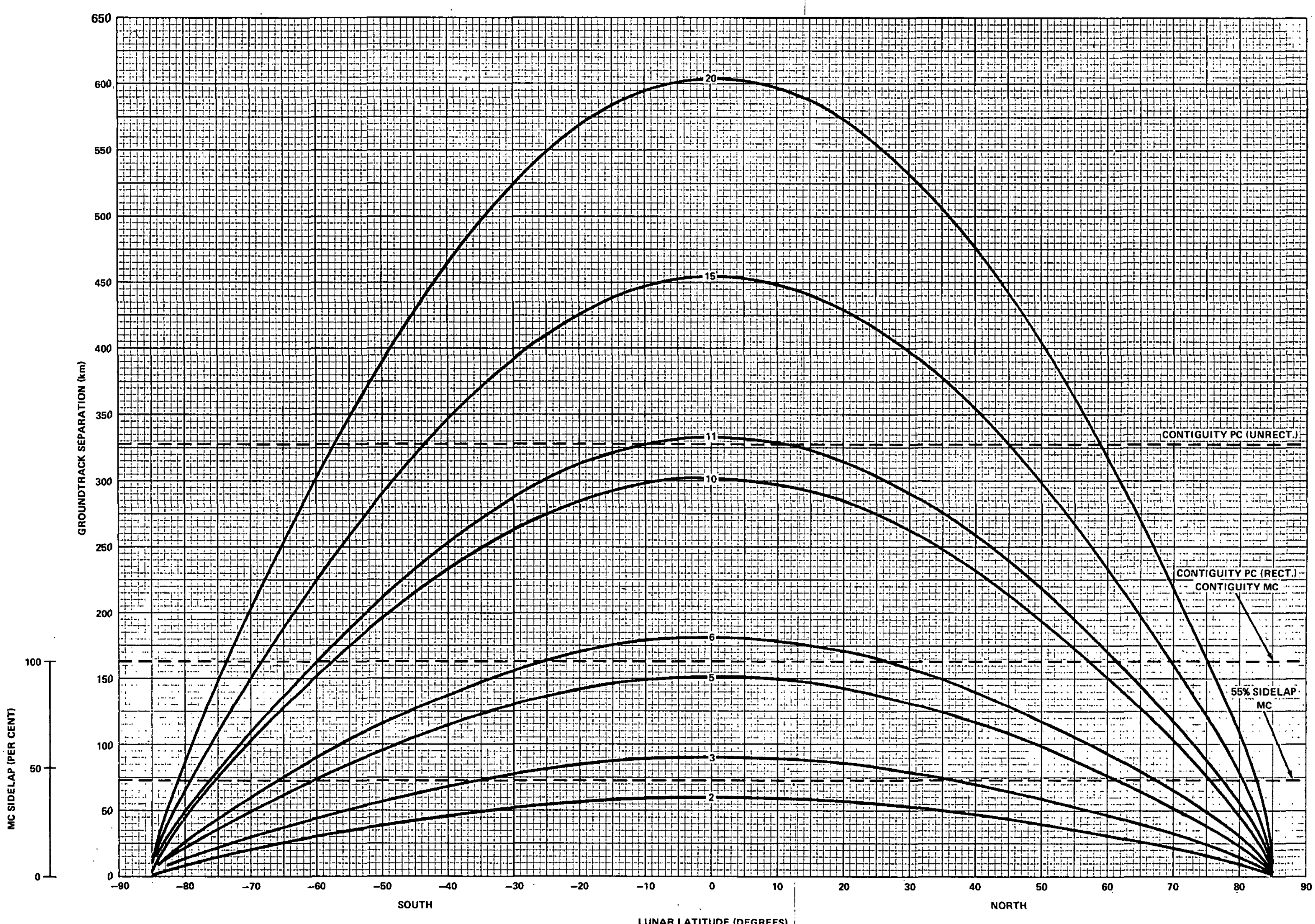


FIGURE 10 - SEPARATION/SIDELAP BETWEEN SUCCESSIVE PHOTOGRAPHIC PASSES AS A FUNCTION OF LUNAR LATITUDE FOR THE INDICATED NUMBER OF REVOLUTIONS BETWEEN PASSES AT AN AZIMUTH AT THE NODE OF 85° (OR 95°)

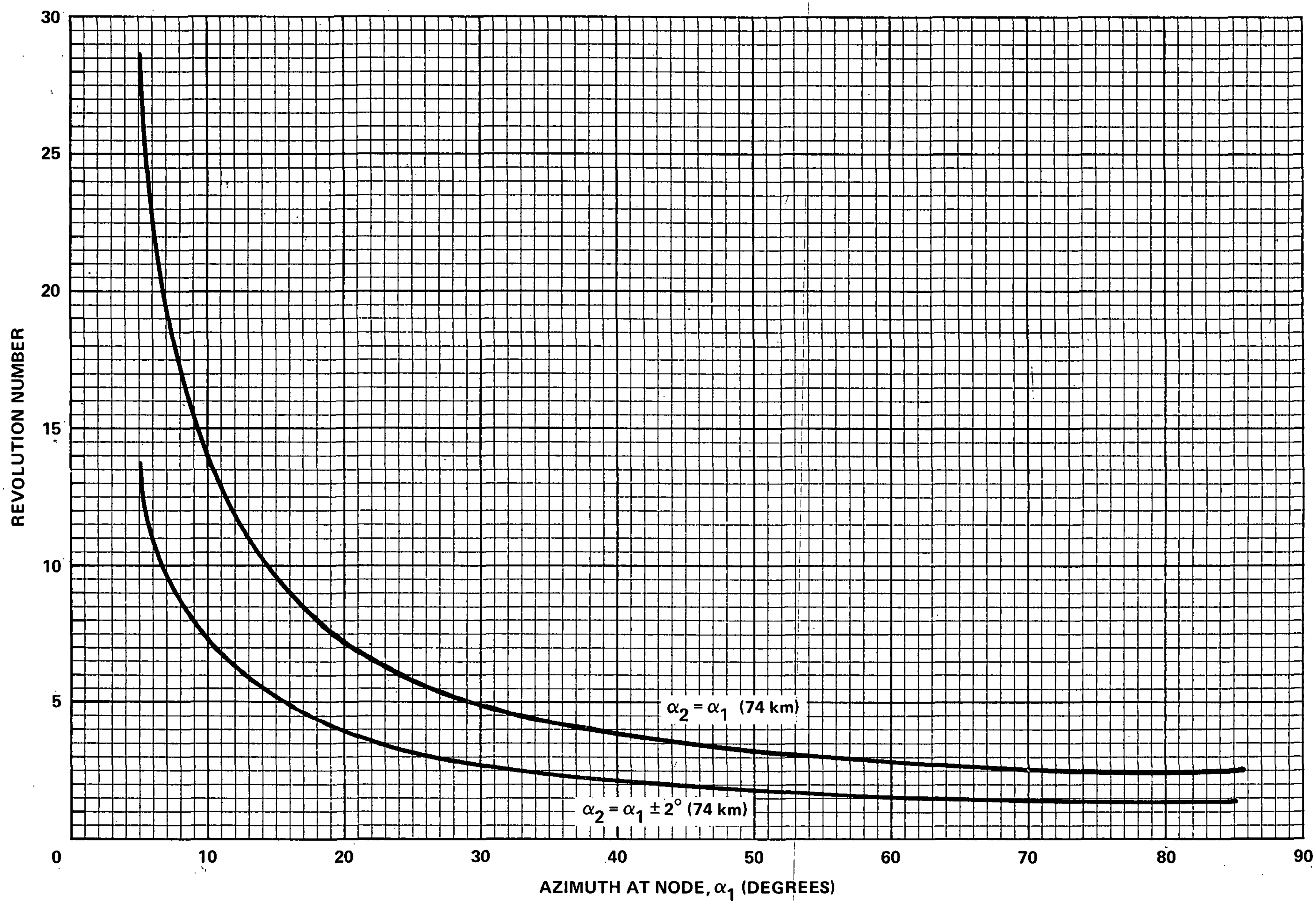


FIGURE 11 - NUMBER OF REVOLUTIONS WHICH CAN ELAPSE BETWEEN SUCCESSIVE MC PHOTO-PASSES AND ATTAIN SIDELAP OF ≥55% AS A FUNCTION OF THE AZIMUTH AT THE NODE, α_1

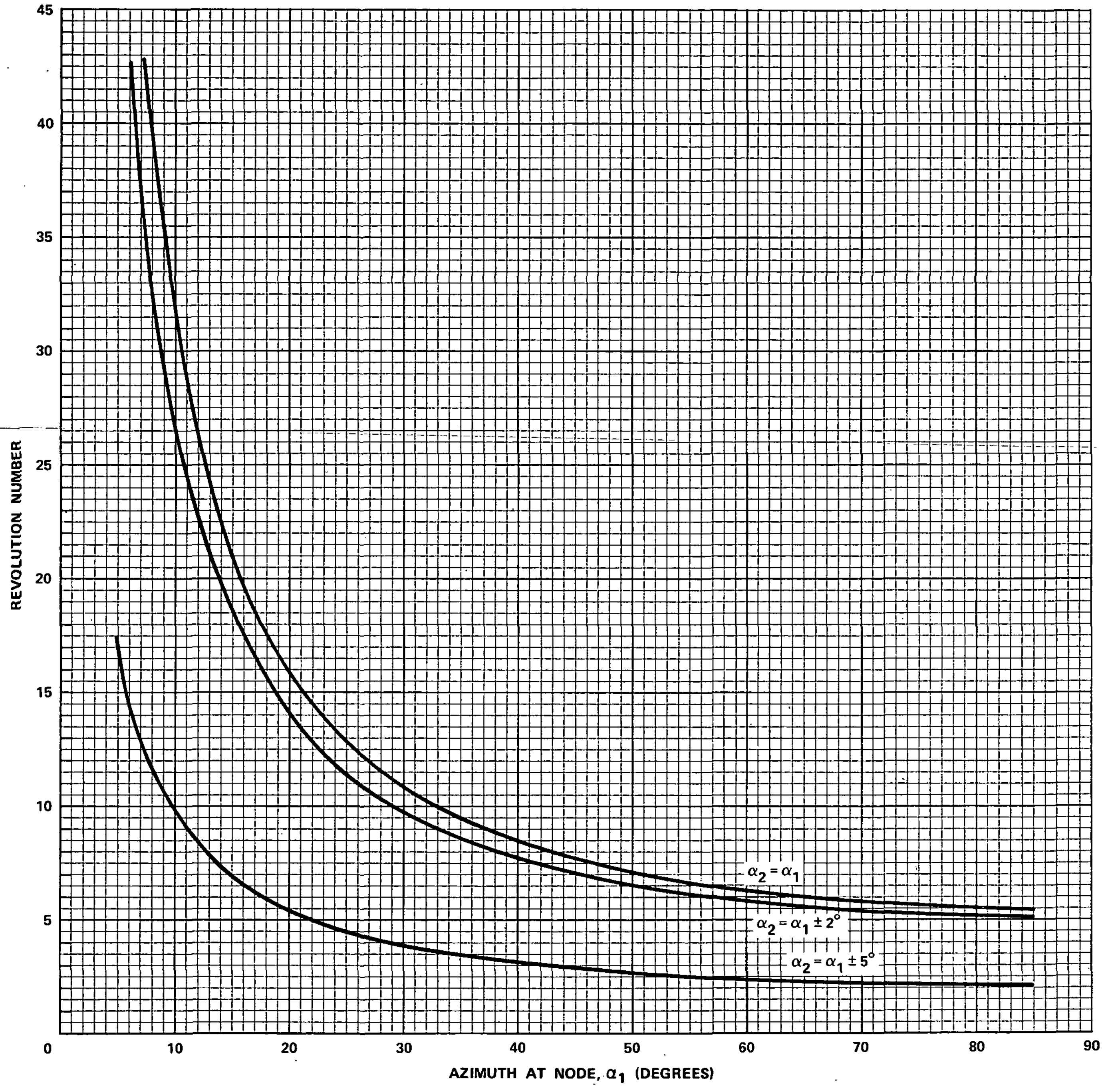


FIGURE 12 - NUMBER OF REVOLUTIONS WHICH CAN ELAPSE BETWEEN SUCESSIVE PHOTO-PASSES AND STILL RETAIN MC CONTIGUITY AND/OR RECTIFIED PC CONTIGUITY AS A FUNCTION OF THE AZIMUTH AT THE NODE