LANDING TRAJECTORY RECONSTRUCTION COMPUTER PROGRAM
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## PREFACE

The Lander Trajectory Reconstruction (LTR) program is a computer program developed by the Martin Marietta Corporation to analyze the entry trajectory and atmosphere reconstruction process for the planetary entry of a lander or probe. The program was initially developed in the Viking lander contract, NAS1-9000. In Contract NAS5-11873, the program was modified and expanded to provide for more flexible atmosphere models, a wider variety of measurement types, and an arbitrary entry plane.

The program can be divided into two parts -- the data generator and the reconstructor. The data generator provides the "real" environment in which the lander or probe is presumed to find itself. Thus the data generator integrates the equation of motion from entry to landing using vehicle and atmosphere models that are assumed to model the "real" envi ronnent. These data are then used as inputs to the reconstructor.

The reconstructor reconstructs the entry trajectory and atmosphere using the sensor data generated by the data generator and a Kalman-Schmidt recursive estimation algorithm. The estimation algorithm generates an estimate of the state of the vehicle at each measurement time as well as the statistics associated with the estimate. In addition to the basic state of the vehicle, the state vector may be augmented with a wide variety of vehicle and environmental parameters. These augmented parameters may be treated as either solve-for parameters or consider parameters. The solve-for parameters are estimated along with the basic state variable, whereas the consider parameter uncertainties are used in generating the state and solve-for parameters statistics but are not estimated, i.e., their uncertainties are not improved.

The reconstructor can be operated in either of two modes. Mode A operation is designed for high-Mach number high-altitude regions. The principal atmosphere measurements of temperature and pressure are difficult to obtain accurately in these regions. Consequently vehicle acceleration, based on an atmosphere model and measurements of temperature and pressure, tend to be less accurate than direct accelerometer outputs. Therefore in mode A operation, no a priori atmosphere model is assumed and the vehicle acceleration terms in the equations of motion are obtained directly from accelerometer and, if available, gyro data. In low-Mach number regions, particularly as terminal velocity is approached, accurate temperature and pressure measurements are available and accelerometer
data yield less useful information. Mode B operation is designed for such regions. The vehicle accelerations are based on an a priori atmosphere model and accelerometer data are processed as observables.

The documentation for the LTR program is contained in two volumes: the Analytic/Users' Manual and the Programmers' Manual. Each of these manuals is self contained.

The Analytic/Users' Manual consists of two parts. The first pait provides a unified treatment of the mathematical analysis of the LTR program. The general problem descriptions, formulation, and solution are given in a tutorial manner. This is followed by the detailed analysis of each LTR subroutine. The second part contains the information necessary to operate the program. The input and output quantities are described in detail. Example cases are also given and discussed.
-The Programmers' Manual provides the reader with the information he needs to effectively modify the program. The overall structure of the program and the computational flow and analysis of the individual subroutines are described in this manual.

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PART I
LTR ANALYTICAL MANUAL

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## I. INTRODUCTION

The Londer Trajectory Reconstruction (LTR) program is a planetury entry trajectory and atmosphere reconstruction program. Currently LTR is a preflight mission analysis tool that is used to perform error analyses and simulations of entry trajectory and atmosphere reconstruction processes. The LTR program can provide answers to questions such as How do modeling errors affect our ability to reconstruct the entry trajectory and planetary atmosphere? Is the reconstruction process convergent? What kind of reconstruccion strategy and instrumentation accuracies are required to meet the scientific objectives of the mission?

The LTR program consists of a data generator program and a reconstruction program. The data generator can be run independently as an entry trajectory program, but is used primarily to generate the "actual trajectory and atmosphere" and "actual measurements" for use in the reconstruction program. The reconstruction program is primarily a simulation program that processes these "actual measurements" in an attempt to reconstruct the "actual trajectory and atmosphere." In designing an actual mission, of course, we can never obtain exact values of dynamic and measurement parameters, and our equations of motion always neglect certain dynamical effects and often embody certain simplifications in the interest of computational efficiency. It is important to know the significance of these inherent limitations on our ability to reconstruct the entry trajectory and the planetary atmosphere. This is the basis for the division of the LTR program into two parts. In essence, the mathematical models used in the data generator to compute the "actuals" represent the "real worla," while those models used in the reconstruction program represent the "modeled world."

An independent error analysis mode is currently not available in the LTR reconstruction program. However, all the information that would be generated in an independent error analysis mode is always generated by the LTR reconstruction program. An independent error analysis mode could not be defined for the mode A reconstruction process because "actual" accelerometer measurements are required from the data generator. Although an independent error analysis mode could be defined for the mode B reconstruction process, it would not result in a significant reduction in program operation costs. Furthermore, it is always useful to have simulation information available because of the more difficult problems encountered in designing cunvergent filters for entry missions. The remainder of this chapter will summarize the contents of the remaining chapters in the Analytic section of this manual.

Chapter II presents the dynamic and measurement models that are used in the data generator to compute the "actuals." The equations of motion are written assuming an inverse square gravitational field and Mach number-dependent $\ddagger$ erodynamic coefficients. Provision is available for including the dyamic effects of parachute deployment and release. The atmosphere model is a linear breakpoint model defined by teniperature and molecular weight profiles and surface pressure. A linear breakpoint horizontal wind model is also available. During the terminal descent phase the dynamic model can be replaced with the quasi-static dynamic model. The quasi-static dynamic model improves computational efficiency and avoids certain integrator instabilities. Measurement mels are defined in the data generator for the following measurement types: axial and normal accelerometers, gyro, radar altimeter, stagnation pressure, stagnation temperature, and range and rangerate from three earth-based tracking stations. "Actual" bias and scale factor errors can be incorporated into most of the dynamic and measurement parameters.

Chapter III presents the recursive linear estimation algorithm used in both reconstruction modes. The algorithm is the KalmanSchmidt algorithm with a consider mode. The consider mode permits the uncertainties in certain parameters to be considered in the algorithm without actually attempting to estimate these parameters. This is a device for combating filter divergence. The state transition matrix computational method is also described in Chapter III. Quasi-linear filtering is discussed and the measurement noise models for all measurement types are summarized.

The two reconstruction modes available in the LTR reconstruction program differ primarily in the method employed for modeling aerodynamic forces. Since the mode A reconstruction process, which is described in Chapter IV, uses accelerometer and gyro data to model aerodynamic forces, it requires no model of the planetary atnosphere for its operation. The mode $B$ reconstruction process, which is described in Chapter $V$, uses mathematical models similar to those used in the data generator to model aerodynamic forces and the planetary atmosphere. In either mode provision is available for estimating or considering various dynamic and measurement parameters.

## II. MODELING OF ACTUAL TRAJECTORY, ATMOSPHERE, AND MEASUREMENTS

## A. ENTRY GEOMETRY AND EQUATIONS OF MOTION

Figure II-1 defines the entry geometry modeled in the LTR program. The entry plaite is defined relative to the planetocentric ecliptic coordinate system $X_{\varepsilon} y_{\varepsilon} z_{\varepsilon}$ by the longitude of the ascending node $\Omega_{\varepsilon}$ and the inclination $i_{\varepsilon}$. The reference line from which the downrange angle $\phi i$ measured, is defined in the entry plane by the angle $\phi_{\varepsilon}$. Since only the planar translation and rotational dynamics of the entry vehicle are modeled in LTR, the state of the entry vehicle can be defined by the altitude $h$, velocity $v$, flightpath angle $\gamma$. downrange angle $\phi$, attitude $\theta$, and angular velocity $\omega$. Flightpath angle $\gamma$ is measured from the instantaneous local horizontal. The position of the entry vehicle relative to the planet center is given by

$$
\begin{equation*}
r=h+R_{p} \tag{II-1}
\end{equation*}
$$

The vehicle body axes are denoted by the $x y<$ coordinate system, where $x$ is aligned with the vehicle longitudinal axis, $y$ is normal to the entry plane, and $z$ completes the orthogonal triad. The vehicle attitude angle $\theta$ is measured from the local horizontal (corresponding to $\phi=0$ ) to the $x$ body axis. The vehicle angle of attack $\alpha$ is measured from the relative velocity $v_{r}$ to the $x$ body axis. Velocity $v_{r}$ is the vehicle velocity relative to the planetary atmosphere, and is defined as the vector difference of the inertial velocity $v$ and the atmosphere velocity $v_{a}$, so that

$$
\begin{equation*}
\mathbf{v}_{\mathbf{r}}=\frac{\mathrm{v}-\mathrm{v}_{\mathrm{a}} \cos \gamma}{\cos \varepsilon} \tag{II-2}
\end{equation*}
$$

where $\varepsilon$ is t'le angle between $v$ and $v_{r}$. The angle $\varepsilon$ is given by

$$
\begin{equation*}
\varepsilon=\tan ^{-1}\left[\frac{v_{a} \sin \gamma}{v-v_{a} \cos \gamma}\right] \tag{II-3}
\end{equation*}
$$



Figure II-1 Entry Geometry

If $\vec{\omega}_{p}$ denotes the angular velocity vector of the planet, and if $\vec{e}_{n}$ denotes a unit vector normal to the entry plane, then the atmosphere velocity in the entry plane can be written as

$$
\begin{equation*}
v_{a}=\left(\vec{\omega}_{p} \cdot \vec{e}_{n}\right) r+v_{w} \tag{II-4}
\end{equation*}
$$

where $v_{W}$ is the horizontal wind velocity. The angle of attack can be related to the other angular quantities according to

$$
\begin{equation*}
\alpha=\theta+\phi-\gamma-\varepsilon . \tag{II-5}
\end{equation*}
$$

The entry vehicle geometry is defined in Figure II-2. The probe center of gravity (cg) has location ( $x_{p}, z_{p}$ ). When the parachute is deployed, its centerline is assumed to be aligned with the relative velocity vector so that the force $F_{d}$ exerted by the parachute on the probe is also aligned with the relative velocity vector. The force $F_{d}$ acts at location $\left(X_{d}, 0\right)$ relative to the body axis system.

Axial aerodynamic force $A$, normal aerodynamic force $N$, and aerodynamic (damping) moment $M$ act at the center of pressure and are given by

$$
\begin{align*}
& A=-C_{A} q S  \tag{II-6}\\
& N=-C_{N} q S  \tag{II-7}\\
& M=C_{M_{q}} \omega d^{2} q S / v_{r} \tag{II-8}
\end{align*}
$$

The parachute force is given by

$$
\begin{equation*}
F_{d}=C_{D} q S_{D} \tag{II-9}
\end{equation*}
$$

In these equations $C_{A}, C_{N}$, and $C_{M_{q}}$ are the axial force, normal force, and damping moment coefficients, respectively, and are tabulated functions of angle of attack $\alpha$ and Mach number M. The parachute drag coefficient $C_{D}$ is a tabulated function of $M$ only. The quantities $S$ and $S_{D}$ denote the reference areas of the probe and parachute,


Figure II-2 Entry Vehicle Geometry
respectively; the reference diameter of the probe is denoted by d. Dynamic pressure $q$ is given by

$$
\begin{equation*}
\mathrm{q}=\frac{1}{2} \rho \mathrm{v}_{\mathrm{r}}^{2} \tag{II-10}
\end{equation*}
$$

where $\rho$ is the atmospheric density.
Assuming an inverse square gravitational force in addition to the previously discussed aerodynamic forces and moments, the translational and rotational equations of motion of the entry vehicle can be written as

$$
\begin{align*}
& \mathrm{h}=\mathrm{v} \sin \gamma  \tag{II-11}\\
& \dot{\mathrm{v}}=-\mathrm{g} \sin \gamma+\frac{A}{m} \cos (\alpha+\varepsilon)+\frac{N}{m} \sin (\alpha+\varepsilon)-\frac{F_{d}}{m} \cos \varepsilon  \tag{II-12}\\
& \dot{\gamma}=\left(\frac{v}{r}-\frac{\mathrm{g}}{\mathrm{v}}\right) \cos \gamma+\frac{1}{v}\left[\frac{A}{m} \sin (\alpha+\varepsilon)-\frac{N}{m} \cos (\alpha+\varepsilon)\right. \\
&\left.\quad-\frac{F_{d}}{m} \sin \varepsilon\right]  \tag{II-13}\\
& \dot{\phi}= \frac{v}{r} \cos \gamma  \tag{II-14}\\
& \dot{\theta}= \omega  \tag{II-15}\\
& \dot{\omega}= \frac{1}{I}\left[\left(z_{p}-z_{g}\right) A-\left(x_{p}-x_{g}\right) N+M+z_{g} F_{d} \cos \alpha\right. \\
&\left.-\left(x_{g}-x_{d}\right) F_{d} \sin \alpha\right] \tag{II-16}
\end{align*}
$$

where I denotes the vehicle (pitch) moment of inertia about the center of gravity, and acceleration of gravity $g$ is given by

$$
\begin{equation*}
g=\frac{\mu}{r^{2}} \tag{II-17}
\end{equation*}
$$

where $\mu$ is the gravitational constant. Vehicle mass is denoted by m .

The parachute terms, of course, only appear in the above equations of motion when the parachute is deployed. The parachute can be deployed at a desired altitude, and then released at a lower altitude.

The derivation of the rotational equations of motion assumes that gravity-gradient and attitude control moments are negligible. Out-of-plane rotational dynamics are neglected and are assumed to have negligible coupling with in-plane rotational dynamics.

During certain flight regimes it becomes necessary to modify the equations of motion to avoid excessive computational time. During the terminal velocity regime, for example, it becomes desirable to increase the integration step size, especially when terminal velocities are rather low. Experience has shown, however, that very small step sizes are required during the terminal velocity regime to prevent the onset of an integrator instability. Although the terminal velocity regime is physically characterized by $|\dot{v}| \ll 1$, an unstable $\dot{\mathrm{v}}$ oscillation can occur during this regime if the step size is not chosen small enough. A solution to this problem war obtained by assuming quasi-static motion during the terminal velocity regime. This entails replacing equation (II-12) for $\dot{v}$ with the approximate equation $\dot{v}=0$, and replacing $v$ in the remaining equations with the terminal velocity $\mathrm{v}_{\mathrm{T}}$, which is computed from

$$
\begin{equation*}
v_{T}=\left[\frac{2 m g|\sin \gamma|}{\rho\left(C_{A} S+C_{D} S_{D}\right)}\right]^{\frac{1}{2}} \tag{II-18}
\end{equation*}
$$

This equation was obtained by setting $\dot{v}, \alpha$, and $\varepsilon$ to zero and $v_{r}$ to $v_{T}$ in equation (II-12), and solving for $v_{T}$.

Yet another integrator instability can occur during the maximum dynamic pressure (max q) regime if integration step sizes are not sufficiently small. As the max $q$ regime is entered, rotational oscillations with very high frequencies are induced by the aerodynamic moments acting on the entry vehicle. For the integrator to reproduce these oscillations accurately would require an extremely small step size; too large a step size would drive the integrator unstable. The instability normally becomes apparent in the unstable oscillation of the angle of attack during max $q$. A solution to this problem was devised by approximating the rotational motion during max $q$ so small integration step sizes would not be required. If the actual entry vehicle is aerodynamically stable
during max $q$, it is reasonable to assume that the actual angle of attack oscillations are characterized not only by high frequencies, but very small amplitudes as well. This permits one to set the angle of attack $\alpha$ to zero during max $q$ without significantly disturbing the accuracy of the computed translational motion. Setting $\alpha=0$ in equation (II-5) yields

$$
\begin{equation*}
\theta=\gamma-\phi+\varepsilon \tag{II-19}
\end{equation*}
$$

Differentiating this equation, and assuming $\dot{\varepsilon}$ is negligible, we obtain

$$
\begin{equation*}
\omega=\dot{\gamma}-\dot{\phi} \tag{II-20}
\end{equation*}
$$

Thus, during max $q$ we obtain the rotational state from equations (II-19) and (II-20), instead of by integrating equations (II-15) and (II-16). This approximation is also applied during the initial phase of parachute deployment to avoid integration instabilities in the rotational equations.

## B. PLANETARY ATMOSPHERE MODEL

The planetary atmosphere modeled in LTR assumes only radial variations in all atmospheric parameters; horizontal gradients are neglected. The hydrostatic equation

$$
\begin{equation*}
\frac{d p}{d h}=-\rho g \tag{II-21}
\end{equation*}
$$

and the perfect gas law

$$
\begin{equation*}
\rho=\frac{\mathrm{pM}}{\mathrm{RT}} \tag{II-22}
\end{equation*}
$$

are also assumed to be valid. In these equations $p$ represents ambient pressure; $g$, acceleration of gravity; $\rho$, density; $M$, molecular weight; $T$, ambient tempera亡ure; and $R$, the universal gas constant.

$$
\begin{equation*}
\frac{d p}{d h}=-\frac{p g \mathrm{~g}}{R T} \tag{II-23}
\end{equation*}
$$

Assuming constant $g$, the integral of this equation has the form

$$
\begin{equation*}
p(h)=p\left(h_{k}\right) \exp \left[-\frac{g}{R} \int_{h_{k}}^{h} \frac{M(\zeta)}{T(\zeta)} d \zeta\right] \tag{II-24}
\end{equation*}
$$

This integral is evaluated in LTR by assuming piece-wise linear variations of molecular weight $M$ and temperature $T$ with altitude:

$$
\begin{gather*}
T(h)=T\left(h_{j}\right)+\left[\frac{T\left(h_{j+1}\right)-T\left(h_{j}\right)}{h_{j+1}-h_{j}}\right]\left(h-h_{j}\right) \\
h_{j} \leq h \leq h_{j+1}  \tag{II-25}\\
M(h)=M\left(h_{i}\right)+\left[\frac{M\left(h_{i+1}\right)-M\left(h_{i}\right)}{h_{i+1}-h_{i}}\right]\left(h-h_{i}\right) \\
h_{i} \leq h \leq h_{i+1} \tag{II-26}
\end{gather*}
$$

where the set of altitudes $h_{j}$ define the temperature breakpoints, and the set of altitudes $h_{i}$ define the molecular weight breakpoints. Details of the evaluation of the integral in equation (II-24) are given in the subroutine ATMSET analysis section.

The molecular weight profile defined by equation (II-26) is computed from a set of mole fraction profiles for the component gases present in the planetary atmosphere. The same breakpoints $h_{i}$ are used to define these profiles. Letting $\alpha_{j i}$ denote the mole fraction of the $j$ th gas at altitude $h_{i}$, the molecular weight at $h_{i}$ is given by

$$
\begin{equation*}
M\left(h_{i}\right)=\sum_{j} \alpha_{j i} m_{j} \tag{II-27}
\end{equation*}
$$

where $m_{j}$ is the molecular weight of the $j$ th gas. Up to five gases can be defined in LTR.

A horizontal wind model is also available in LTR. Since a piece-wise linear variation of wind with altitude is assumed, the wind $w$ at altitude $h$ can be written as

$$
\begin{gather*}
w(h)=w\left(h_{n}\right)+\frac{w\left(h_{n+1}\right)-w\left(h_{n}\right)}{h_{n+1}-h_{n}}\left(h-h_{n}\right) \\
h_{n} \leq h \leq h_{n+1} \tag{II-28}
\end{gather*}
$$

where the set of altitudes $h_{n}$ define the horizontal wind breakpoints.

## C. ACCELEROMETER AND GYRO MODELS

Two strapdown accelerometers, or velocity reference units, are modeled in LTR. A third accelerometer is not required because of the planar dynamic model assumed by LTR. The two accelerometers are nominally aligned with the $x$ and $z$ body axes of the entry vehicle and have location ( $x_{m}, z_{m}$ ) relative to the origin of the body axes. Although a number of acclerometer error sources could be modeled, LTR assumes only misalignment, bias, and scale factor errors. The actual output from the accelerometer has quantized form and is not available as a continuous function of time. The derivation of the actual accelerometer output equation will be summarized in the following paragraphs.

The actual nongravitational acceleration at the location of the velocity reference unit (VRU) is given by

$$
\begin{align*}
& a_{x}=a_{x g}-\omega^{2} \bar{x}+\dot{\omega} \bar{z}  \tag{II-29}\\
& a_{z}=a_{z g}-\omega^{2} \bar{z}-\dot{\omega} \bar{x} \tag{II-30}
\end{align*}
$$

where $\bar{x}$ and $\bar{z}$ denote the offset of the VRU relative to the vehicle cg and are given by

$$
\begin{align*}
& \bar{x}=x_{m}-x_{g}  \tag{II-3I}\\
& \bar{z}=z_{m}-z_{g} \tag{II-32}
\end{align*}
$$

and where $a_{x g}$ and $a_{z g}$ denote the $x$ and $z$ components of the actual nongravitational acceleration at the vehicle cg.

Because of accelerometer misalignment errors $\delta_{1}$ and $\delta_{2}$, the actual nongravitational acceleration experience by the VRU is given by

$$
\begin{align*}
& \dot{v}_{x}=a_{x} \cos \delta_{1}-a_{z} \sin \delta_{1}  \tag{II-33}\\
& \dot{v}_{z}=a_{x} \sin \delta_{2}+a_{z} \cos \delta_{2} \tag{II-34}
\end{align*}
$$

The actual output of the VRU is in quantized form and is corrupted by bias errors $C_{b x}$ and $C_{b z}$ and scale factor errors $C_{s x}$ and $C_{s z}$. The equations for the quantized output are given by

$$
\begin{align*}
& v_{x q}\left(t_{k}\right)=Q\left\{C_{s x} \int_{0}^{t_{k}} \dot{v}_{x} d t+C_{b x}\right\}  \tag{II-35}\\
& v_{z q}\left(t_{k}\right)=Q\left\{C_{s z} \int_{0}^{t_{k}} \dot{v}_{z} d t+c_{b z}\right\} \tag{II-36}
\end{align*}
$$

where $Q$ denotes the quantizing operator. If we let I denote the modified greatest integer operator, where $I(x)$ is the integer part of $x$ formed by truncating all digits to the right of the decimal, and $\Delta q$, the quantum level, then

$$
\begin{equation*}
Q()=I\left\{\frac{()}{\Delta q}\right\} \times \Delta q \tag{II-37}
\end{equation*}
$$

Note that $v_{x q}$ and $v_{z q}$ are not true velocities. Rather, they represent the contents of the $x$ and $z$ integrating accelerometer registers at time $t_{k}$. They would be true velocities only if the inertial orientation of the vehicle had remained constant over tie time interval [ $0, t_{k}$ ].

A single strapdown gyro, or attitude reference unit, is modeled in LTR. Three gyros are not required because of the planar dynamic model assumed by LTR. Although a number of gyro error sources could be modeled, LTR assumes only misalignment, bias, and scale factor errors. The actual output from the gyro, like that of the accelerometers, has quantized form and is not available as a continuous function of time. Derivation of the actual gyro output equation is summarized in the following paragraph.

The actual angular velocity of the vehicle and the attitude reference unit ARU in the plane of motion is denoted by $\omega$. Because of the gyro misalignment error $\delta_{3}$, the actual angular velocity experience by the ARII is given by

$$
\begin{equation*}
\dot{A}_{\theta}=\omega \cos \delta_{3} \tag{II-38}
\end{equation*}
$$

The actual output of the ARU is in quantized form and is corrupted by bias error $C_{b \theta}$ and scale factor error $C_{s \theta}$. The equation for the quantized output is given by

$$
\begin{equation*}
A_{\theta q}\left(t_{k}\right)=Q\left\{c_{s \theta} \int_{0}^{t_{k}} \dot{A}_{\theta} d t+c_{b \hat{\theta}} t_{k}\right\} \tag{II-39}
\end{equation*}
$$

## D. PREPRUCESSING OF GYRO AND ACCELEROMETER MEASUREMENTS

The quantized accelerometer and gyro data are not processed directly by the navigation filter, but must first be preprocessed. This preprocessing consists of smoothing the quantized data to generate not only smoothed acceleration and attitude angle, but also angular velocity and angular acceleration. The smoothed angular quantities are particularly important for operation of the mode A reconstruction process. The LTR preprocessor employs a five-point central-point smoother, which simply means that five quantized data points are used to determine smoothed data and their derivatives at the center of the five-point interval. The smoothing of $A_{\theta q}$ data will be discussed in more detail in the following paragraph. The same method is used to smooth $v_{x q}$ and $v_{z q}$.

Suppose we wish to obtain smoothed attitude $\theta_{m}$, angular velocity $\omega_{m}$, and angular acceleration $\dot{\omega}_{m}$ at $t=t_{k}$. Then, as indicated in Figure II-3, smoothing will be performed using all quantized attitude data over the interval $\left[t_{k-2}, t_{k+2}\right]$. We assume $\theta_{m}(t)$ can be expressed as a quadratic function over this interval, so that

$$
\begin{equation*}
\theta_{m}(t)=C_{1}+C_{2}\left(t-t_{k}\right)+C_{3}\left(t-t_{k}\right)^{2} \tag{II-40}
\end{equation*}
$$

The coefficients $C_{1}, C_{2}$, and $C_{3}$ are chosen to obtain a leastsquares fit to the data points $A_{\theta q}\left(t_{k-2}\right)$ through $A_{\theta q}\left(t_{k+2}\right)$. The solution to this problem is given by the following equations:

$$
\left[\begin{array}{l}
C_{1}  \tag{II-41}\\
C_{2} \\
C_{3}
\end{array}\right]=\left(B^{T} B\right)^{-1} B\left[\begin{array}{c}
A_{\theta q}\left(t_{k-2}\right) \\
\vdots \\
A_{\theta q}\left(t_{k+2}\right)
\end{array}\right]
$$

where

$$
B=\left[\begin{array}{ccc}
1 & -2 \Delta & 4 \Delta^{2}  \tag{II-42}\\
1 & -\Delta & 2 \Delta^{2} \\
1 & 0 & 0 \\
1 & \Delta & 2 \Delta^{2} \\
1 & 2 \Delta & 4 \Delta^{2}
\end{array}\right]
$$

and $\Delta=t_{k}-t_{k-1}$. Having determined the coefficients $C_{1}, C_{2}$, and $C_{3}$, we evaluate equation (II-40) at $t=t_{k}$ to obtain:

$$
\begin{equation*}
\theta_{m}\left(t_{k}\right)=C_{1} \tag{II-43}
\end{equation*}
$$

Evaluating the first two derivatives of equation (II-40) at $t=t_{k}$ yieids

$$
\begin{align*}
& \omega_{m}\left(t_{k}\right)=C_{2}  \tag{II-44}\\
& \dot{\omega}_{m}\left(t_{k}\right)=2 C_{3} \tag{II-45}
\end{align*}
$$



Figure II-3 Smoothing of Quantized Data

Smoothed acceleration data $a_{\mathrm{xm}}\left(\mathrm{t}_{\mathrm{k}}\right)$ and $\mathrm{a}_{\mathrm{zm}}\left(\mathrm{t}_{\mathrm{k}}\right)$ are similarly obtained from the quantized accelerometer data. Additional information is available in the subroutine PRPRøS and SMøめT2 documentation.

## E. OTHER ONBOARD MEASUREMENT MODELS

In addition to the gyro and axial and normal accelerometers discussed in the previous section, several other onboard measurement types are modeled in the LTR program. These models are summarized below.

## 1. Stagnation Temperature Measurement

The stagnation temperature measurement is given by

$$
\begin{equation*}
T_{0}=T\left(1+\frac{\gamma-1}{2} M^{2}\right) \tag{II-46}
\end{equation*}
$$

where $T$ is ambient temperature; $\gamma$, ratio of specific heats; and $M$, Mach number. The actual measurement is computed hy multiplying the ideal measurement times a scale factor error, and adding on a bias and noise. The noise error is computed by sampling from a gaussian distribution.

## 2. Stagnation Pressure Measurement

The stagnation pressure measurement is a function of the Mach number regime and is computed using one of the following three equatic $1 \mathrm{~s}:$

M $\geq 3$ :
$\mathrm{p}_{\mathrm{o}}=\frac{1}{2} \mathrm{C}_{\mathrm{p}} \rho \mathrm{v}_{\mathrm{r}}^{2}+\mathrm{p}$
$C_{p}=2-\varepsilon ;$
$1 \leq M<3:$
$p_{o}=\frac{1}{2} C_{p} \rho v_{r}^{2}+p$

$$
\begin{equation*}
\rho_{p}=\frac{p}{8}\left[\left(\frac{\gamma+1}{2} M^{2}\right)^{\gamma / \gamma-1} \times\left(\frac{\gamma+1}{2 \gamma M^{2}-\gamma+1}\right)^{1 / \gamma-1}-1\right] ; \tag{II-50}
\end{equation*}
$$

M < 1 :

$$
\begin{equation*}
p_{0}=p\left(1+\frac{\gamma-1}{2} m^{2}\right)^{\gamma / \gamma-1} \tag{II-51}
\end{equation*}
$$

where $p_{0}$ is stagnation pressure; $p$, ambient pressure; $C_{p}$, coefficient of pressure; $M$, Mach number; $\gamma$, ratio of specific heats; and $\varepsilon$, ratio of densities in front of and behind the shock wave. The ratio $\varepsilon$ is a tabulated function of $v_{r}$. The actual measurement is computed by multiplying the ideal measurement times a scale factor error, and adding on a bias and noise. The noise error is computed by sampling from a gaussian distribution.

## 3. Radar Altimeter Measurement

The onboard radar altimeter measurement is defined as the shortest distance between the spacecraft and the terrain of the planet within the limits of the altimeter sweep angle. Figure II-4 depicts the relevant radar altimeter and terrain height geometry. The spacecraft has altitude $h$ above the mean planet surface. The altimeter has a symmetrical sweep angle of $2 n$. Terrain height $\tau$ above the mean surface is assumed to ba e the form

$$
\begin{equation*}
\tau=C_{1}+C_{2} \sin \left(C_{3} \phi^{\prime}+C_{4}\right)+C_{5} \cos \left(C_{6} \phi^{\prime}+C_{7}\right) \tag{II-52}
\end{equation*}
$$

where constants $C_{1}, C_{2}, \ldots, C_{7}$ are chosen to approximate the terrain height profile of the planet, and $\phi^{\prime}$ represents the difference between the spacecraft downrange angle $\phi$ and angular displacement of the terrain due to planet rotation, and is given by

$$
\begin{equation*}
\phi^{\prime}=\phi-\vec{\omega}_{p} \cdot \vec{e}_{n}\left(t-t_{0}\right) \tag{II-53}
\end{equation*}
$$

In this equation $\vec{\omega}_{p}$ denotes the planet inertial angular velocity $\vec{e}_{n}$ is a unit vector normal to the entry plane (and in the direction of the spacecraft orbit angular momentum). The distance $\tilde{h}$ between the spacecraft and the terrain is given by

$$
\begin{equation*}
\tilde{h}=\left[\left(h+R_{p}\right)^{2}+w^{2}-2 w\left(h+R_{p}\right) \cos (\tilde{\phi}-\phi)\right]^{\frac{1}{2}} \tag{II-54}
\end{equation*}
$$



Figure II-4 Altimeter and Terrain Height Geometry
where $w$ is the distance from the center of the planet to the actual planet surface. Minimization of $\tilde{h}$ is equivalent to minimization of the function

$$
\begin{equation*}
f=w^{2}-2 w\left(h+R_{p}\right) \cos (\tilde{\phi}-\phi) \tag{II-55}
\end{equation*}
$$

The function $f$ is minimized using a direct search over the interval $[\phi-\delta, \phi+\delta]$ with respect to $\phi$. The a.lgle $\delta$ is given by

$$
\begin{equation*}
\delta=\sin ^{-1}\left[\frac{\left(R_{p}+h\right) \sin \eta}{R_{p}}\right]-\eta \tag{II-56}
\end{equation*}
$$

The actual radar altimeter measurement is computed by multiplying the minimum $\tilde{h}$ times a scale factor error and adding on a bias and nois?. The noise error is computed by sampling from a gaussian distribution.

## 4. Angle of Attack

The ratio of measured accelerations $a_{z} / a_{x}$ can be used to define an angle of attack measurement $\tilde{\alpha}$. The ratio of vehicle lift $L$ and drag $D$ can be related to $a_{x}, a_{z}$, and $\tilde{\alpha}$ according to the equation

$$
\begin{equation*}
\frac{L}{D}=\frac{a_{z} \cos \tilde{\alpha}-a_{x} \sin \tilde{\alpha}}{a_{x} \cos \tilde{\alpha}+a_{z} \sin \tilde{\alpha}} \tag{II-57}
\end{equation*}
$$

Solving for $\tilde{\alpha}$, we obtsin

$$
\begin{equation*}
\tan \tilde{\alpha}=\frac{\frac{a_{z}}{a_{x}}-\frac{L}{D}}{1+\left(\frac{a_{z}}{a_{x}}\right)\left(\frac{L}{D}\right)} \tag{II-58}
\end{equation*}
$$

The ratio $L / D$ has the form

$$
\begin{equation*}
\frac{L}{D}=k \tilde{\alpha} \tag{II-59}
\end{equation*}
$$

where $k$ is a tabulated function of Mach number. To compute the angle of attack measurement $\tilde{\alpha}$, equation II-58 is solved iteratively using

$$
\begin{equation*}
\tilde{\alpha}=\frac{\frac{a_{z}}{a_{x}}}{k+1} \tag{II-60}
\end{equation*}
$$

as the initial guess.

## F. EARTH-BASED RANGE AND DOPPLER MEASUREMENT MODELS

The geometry of earth-based tracking is shown in Figure II-5. The trackîng statio is located relative to the geocentric equatorial coordinate system $x_{0} y_{0} z_{0}$ by the latitude $\theta$, longitude $\lambda$, Greenwich hour angle GHA of the vernal equinox, earth radius $R_{0}$, and altitude $h$ above the mean earth sphere. The spacecraft has position $\vec{r}$ and velocity $\vec{r}$ relative to the target planet. These vectors are normally expressed relative to the planetocentric ecliptic coordinate system $x_{\varepsilon} y_{\varepsilon} z_{\varepsilon}$.

The range $\rho$ between the spacecraft and the tracking station is given by

$$
\begin{equation*}
\rho=\left|\vec{r}+\vec{r}_{p}-\vec{r}_{e}-\overrightarrow{\mathbf{r}}_{s}\right| \tag{II-61}
\end{equation*}
$$

where $\vec{r}_{p}$ and $\vec{r}_{e}$ denote the position of the target planet and the earth, respectively, relative to the sun, and $\vec{r}_{s}$ denotes the position of the tracking station relative to the center of the earth. The heliocentric ecliptic components of $\vec{r}_{s}$ are given by

$$
\begin{align*}
& x_{s}=\left(R_{0}+h\right) \cos \theta \cos G  \tag{II-62}\\
& y_{s}=\left(R_{0}+h\right)[\cos \theta \cos \varepsilon \sin G+\sin \theta \sin \varepsilon]  \tag{II-63}\\
& z_{s}=\left(R_{0}+h\right)[-\cos \theta \sin \varepsilon \sin G+\sin \theta \cos \varepsilon] \tag{II-64}
\end{align*}
$$



Figure II-5 Earth-Based Tracking Geometry
where $\varepsilon$ is the obliquity of the ecliptic, and

$$
\begin{equation*}
G=\lambda+\omega_{e}\left(t-t_{0}\right)+G H A\left(t_{0}\right) \tag{II-65}
\end{equation*}
$$

In this last equation, $\omega_{e}$ represents the inertial angular velocity of the earth; $t-t_{0}$, the time interval since epoch $t_{0}$; and GHA ( $t_{0}$ ), the Greenwich hour angle at epoch.

The range rate $\dot{\rho}$ between the spacecraft and the tracking station is given by

$$
\begin{equation*}
\dot{\rho}=\dot{\vec{\rho}} \cdot \vec{e}_{\rho}=\frac{\overrightarrow{\mathrm{p}} \cdot \overrightarrow{\mathrm{p}}}{\rho} \tag{II-66}
\end{equation*}
$$

where $\vec{e}_{\rho}$ is a unit vector directed along the range vector $\vec{\rho}$, and $\vec{\rho}$ is given by

$$
\begin{equation*}
\vec{\rho}=\overrightarrow{\vec{r}}+\overrightarrow{\vec{r}}_{p}-\overrightarrow{\vec{r}}_{e}-\overrightarrow{\vec{r}}_{s} \tag{II-67}
\end{equation*}
$$

where $\vec{r}_{p}$ and $\vec{r}_{s}$ denote the velocity of the target planet and the earth, respectively, relative to the sun, and $\overrightarrow{\mathbf{r}}_{\mathbf{s}}$ denotes the velocity of the tracking station relative to the center of the earth. The heliocentric ecliptic components of $\overrightarrow{\mathbf{r}}_{\mathbf{s}}$ are given by

$$
\begin{align*}
& \dot{x}_{s}=-\omega_{e}\left(R_{0}+h\right) \cos \theta \sin G  \tag{II-68}\\
& \dot{y}_{s}=\omega_{e}\left(R_{0}+h\right) \cos \theta \cos \varepsilon \cos G  \tag{II-69}\\
& \dot{z}_{s}=-\omega_{e}\left(R_{0}+h\right) \cos \theta \sin \varepsilon \cos G \tag{II-70}
\end{align*}
$$

Actual range and doppler (range-rate) measurements are computed in LTR by incorporating the effects of various error sources in the range and doppler measurements computed from the previous equations. Three types of range and doppler error sources are modeled in LTR:
(1) station location errors, (2) instrument bias and noise, and (3) refractivity effects of the planetary atmosphere. Station location errors are modeled as biases in station latitude, longitude, and altitude. Instrumert noise is computed in LTR by sampling from a gaussian distribution.

## III. RECURSIVE STATE ESTIMATION

## A. RECURSIVE ESTIMATION ALGORITHM


#### Abstract

The recursive estimation algorithm refers to the computational procedure that combines the dynamic model and measurement information to generate estimates of the deviation of the basic system state from the nominal and the covariances associated with these estimates. It is also possible to augment the state vector with parameters that are known with some uncertainty. The basic estimation algorithm treats all uncertain parameters as solve-for parameters, i.e., the estimation algorithm generates estimates of these parameters as well as estimates of the basic state. Continued processing of measurements will often reduce state covariances to unrealistically low values, a situation that can induce divergence in the estimation algorithm. One method used to prevent divergence is to incprorate a consider option in the algorithm and divide all uncertain parameters into either solve-for or consider parameters. Consider parameters are not estimated by the algorithm, nor can their covariance be reduced by measurement processing. In essence, by not solving for all parameters in the uncertain parameter set, the algorithm acknowledges that the nominal dynamic and measurement parameter values do not fully describe the real world, and that it is impossible to reduce parameter uncertainties indefinitely.


Thus the basic state vector is augmented with both solve-for parameters and consider parameters. The consider parameters are further categorized into dynamic-consider parameters, measurementconsider parameters, and dynamic/measurement-consider parameters. A dynamic-consider parameter appears in the dynamic equations only, whereas a measurement-consider parameter appears in the measurement equations only. Dynamic/consider parameters appear in both.

Before presenting the estimations algorithm, the dynamic and measurement models will be described. The set of dynamic equations is assumed to have been linearized about a nominal trajectory. The augmented state vector of deviations from nominal may be written in partitioned form as
$x^{A}=\left[\begin{array}{l}x \\ q \\ u \\ v \\ w\end{array}\right]$
where

```
\(x=\) basic state vector,
\(q=\) vector of solve-for parameters,
\(u=\) vector of dynamic-consider parameters,
\(v=\) vector of measurement-consider parameters,
\(\mathbf{w}=\) vector of dynamic/measurement-consider parameters.
```

The linearized dynamic model is assumed to have the form

$$
\begin{equation*}
x_{k+1}^{A}=\phi_{k+1, k}^{A} x_{k}^{A}+q_{N_{k+1, k}}^{A} \tag{III-2}
\end{equation*}
$$

where $\phi_{k+1, k}^{A}$ is the augmented state transition matrix over the interval $\left[t_{k}, t_{k+1}\right]$ and $q_{N_{k, k+1}}^{A}$ represents the effects of dynamic noise over the interval. Since the dynamic noise affects the basic state only,

$$
\mathrm{q}_{\mathrm{N}_{\mathrm{k}+1, \mathrm{k}}}^{\mathrm{A}}=\left[\begin{array}{c}
\mathrm{q}_{\mathrm{N}_{\mathrm{k}+1, \mathrm{k}}}  \tag{III-3}\\
0 \\
0 \\
0 \\
0
\end{array}\right]
$$

The linearized measurement model is assumed to have the form

$$
\begin{equation*}
y_{k}=H_{k}^{A} x_{k}^{A}+\eta_{k} \tag{III-4}
\end{equation*}
$$

where $y_{k}$ represents the deviation of the observation from the nominal observation at $t_{k}, H_{k}^{A}$ relates changes in $X_{k}^{A}$ to changes in the measurement $y_{k}$, and $\eta_{k}$ represents the measurement noise.

Under the usual assumptions of white noise, the dynamic and measurement noise statistics are described by

$$
\begin{aligned}
& E\left[q_{N_{k}}\right]=E\left[\eta_{k}\right]=0 \\
& E\left[q_{N_{j}} q_{N_{k}}^{T}\right]=Q_{N_{k}} \delta_{j k} \\
& E\left[\begin{array}{ll}
\eta_{j} & \eta_{k}^{T}
\end{array}\right]=R_{k} \delta_{j k}
\end{aligned}
$$

The equations constituting the recursive estimation algorithm are of two types -- prediction equations and filtering equation. The prediction equations describe the behavior of the state and covariance matrix as they are propagated forward in time with no measurement processing. The state prediction equation is simply equation III-2 without the dynamic noise term. The filtering equations define the covariance updating procedure whenever a measurement is processed. Details of their derivation may be round in Reference 3.

The covariance of the augmented state is defined as

$$
P_{k}^{A}=E\left[\left(\hat{x}_{k}^{A}-x_{k}^{A}\right)\left(\hat{e}_{k}^{A}-x_{k}^{A}\right)^{T}\right]
$$

where $\hat{X}_{k}^{A}$ is the estimated deviation from nominal and $X_{k}^{A}$ is the actual deviation from nominal. The covariance prediction equation that relates the covariance following the processing of a measurement at $t_{k}, P_{k}^{A+}$, to the covarirnce prior to processing the next measurement at $t_{k+1}=P_{k+1}^{A-}$, is given by

$$
\begin{equation*}
P_{k+1}^{A-}=\Phi_{k+1, k}^{A} P_{k}^{A+}{ }_{\Phi}^{A T+1, k}{ }_{k}^{A T}+Q_{N_{k+1, k}} \tag{III-5}
\end{equation*}
$$

Before presenting the filtering equations, the measurement residual at $t_{k}$ must be defined. The measurement residual, $\varepsilon_{k}$, is the difference between the "actual" measurement $y_{k}^{a}$ and the estimated or expected measurement $y_{k}^{e}, y_{k}^{a}$ is composed of an errorfree component, $y_{k}$, based on the actual state deviation, $x^{A} p l u s$ a random noise component $\nu_{k}$ and a bias component $b$

$$
\begin{equation*}
y_{k}^{a}=y_{k}+v_{k}+t \tag{III-6}
\end{equation*}
$$

The estimated or expected measurement, $y_{k}^{e}$, is composed of an errorfree component $\tilde{y}_{k}$ based on the nominal state, plus a measurement deviation based on the estimated state deviation $\hat{\mathbf{x}}^{\mathrm{A}}$

$$
\begin{equation*}
y_{k}^{e}=\ddot{y}_{k}+H_{k}^{A} \hat{x}_{k}^{A-} \tag{III-7}
\end{equation*}
$$

The measurement residual is then simply

$$
\begin{equation*}
\varepsilon_{k}=y_{k}^{a}-y_{k}^{e} \tag{III-8}
\end{equation*}
$$

The filler equations involve equations fior the measurement residual covariance matrix $J$, the augmented Kalman gain matrix $K^{A}$, the covariance update equation, and the state update equation. The first two equations are

$$
\begin{align*}
& J_{k+1}=H_{k+1}^{A} P_{k+1}^{A-} H_{k+1}^{A} T+R_{k+1}  \tag{III-9}\\
& K_{k+1}^{A}=P_{k+1}^{A-} H^{A T}\left(J_{k+1}\right)^{-1} \tag{III-10}
\end{align*}
$$

Unfortunately there is no compact formulation for the state and covariance update equations in terms of the above matrices. This is true because the consider parameters and their covariances are not updated and thus require special handling. An artifice that may be used is to partition the rows of $K_{k+1}^{A}$ corresponding to the partition of tise augmented state vector in equation III-12:

$$
K_{k+1}^{A}=\left[\begin{array}{l}
K 1_{k+1} \\
K 2_{k+1} \\
K 3_{k+1} \\
K 4_{k+1} \\
K 5_{k+1}
\end{array}\right]
$$

If a modified gain matrix is defined by

$$
K_{k+1}^{A m}=\left[\begin{array}{c}
K 1_{k+1}  \tag{III-12}\\
K 2_{k+1} \\
0 \\
0 \\
0
\end{array}\right]
$$

then the estimated state update equation may be written

$$
\begin{equation*}
\hat{\mathrm{x}}_{k+1}^{\mathrm{A}+}=\hat{\mathrm{x}}_{k+1}^{\mathrm{A}-}+\mathrm{K}_{k+1}^{\mathrm{Am}} \varepsilon_{k+1} \tag{III-13}
\end{equation*}
$$

The covariance update equation may also be written in terms of $\mathrm{K}_{\mathrm{k}+1}^{\mathrm{Am}}$; however, only the partitions of $\mathrm{P}_{\mathrm{k}+1}^{\mathrm{At}}$ on and above the diagonal are valid

$$
\begin{equation*}
P_{k+1}^{A+}=P_{k+1}^{A-}-K_{k+1}^{A m} H_{k+1}^{A} P_{k+1}^{A-} \tag{III-14}
\end{equation*}
$$

To take advantage of the sparceness and symmetry of the above equations, they are computed in partitioned form. The partitioned equations are given in the subroutine FILTER analysis.

## B. MEASUREMENT NOISE MODELS

This section discusses the measurement noise models used to compute the measurement noise covariance matrix $R$ appearing in equation (III-9).

The measurement noise covariance for an accelerometer, stagnation pressure, angle of attack, range, or doppler measurement is assumed to be a constant. For a stagnation pressure measurement $p_{0}, R$ is a two-valued function:

$$
R_{p_{0}}=\left\{\begin{array}{l}
C_{1}, p_{0} \geq 20 \text { millibars }  \tag{III-15}\\
C_{2}, p_{0}<20 \text { millibars }
\end{array}\right.
$$

The measurement noise covariance for a radar altimeter measurement $\tilde{\mathrm{h}}$ can be either set to a constant or computed as a function of the measurement itself. Currently, with the latter option

$$
R=\operatorname{maximum} \begin{cases}{[.005 \tilde{h}, .051]} & , \tilde{h} 26 \\ {[.015 \tilde{h}, .00051]} & , \tilde{h}<6\end{cases}
$$

(III-16)

Although the doppler measurement noise is assumed to be constant, the modeled doppler noise can be adjusted to account for differences between the actual and modeled sample rates using the approximation

$$
\begin{equation*}
\sigma_{\dot{\rho}}=\sigma_{\dot{\rho}} \quad\left(\frac{1}{T_{s}^{\frac{13}{2}}}\right) \tag{III-17}
\end{equation*}
$$

where $\sigma_{\dot{\rho}}$ is the actual or original sample rate (typically $1 \mathrm{~mm} / \mathrm{s}$ for a 1 -minute count time), and $T_{s}$ is the spacing between successive doppler points used in the model. For additional information concerning this approximation, see Reference 4.

## C. COMPUTATION OF ST' ¿ TRANSITION AND OBSERVATION MATRICES

The state transition matrices describe the behavior of a dynamic system in the neighborhood of a nominal trajectory. Before presenting the technique used in the I.TP program for computing state transition matrices, the deviation of the general form of the dynamic system modeled in LTR will be summarized.

The nonlinear equations describing the motion of the lander or probe have the form

$$
\begin{equation*}
\dot{x}^{A}=f\left(X^{A}, t\right) \tag{III-18}
\end{equation*}
$$

where $\mathrm{X}^{\mathrm{A}}$ denotes the augmented state vector. If equation (III-18) is linearized about a nominal trajectory, it takes the form

$$
\begin{equation*}
\dot{x}^{A}=\frac{\partial f}{\partial X^{A}} x^{A} \tag{III-19}
\end{equation*}
$$

where $X^{A}$ represents small deviations from the nominal augmented state $\tilde{X}^{A}$. The partial derivative is evaluated along the nominal trajectory.

The discrete solution of equation (III-19) over the interval [ $t_{k}, t_{k+1}$ ] is given by

$$
\begin{equation*}
x_{k+1}^{A}=\Phi\left(t_{k+1}, t_{k}\right) x_{k}^{A} \tag{III-20}
\end{equation*}
$$

If the augmented state vector is partitioned into the basic state vector, $x$; solve-for parameter vector, $q$; dynamic-consider parameter vector, $u$; measurement parameter vector, $v ;$ and dynamic/ measurement parameter vector, $w$; it is possible to make a corresponding partition of the state transition matrix $\Phi$. Before writing $\Phi$ in partitioned form, it should be observed that all solve-for and consider parameters are assumed to be constant. This means that all partitions of $\Phi$ will be either zero mecrices or identity matrices except for those associated with the basic state vector. Thus the partitioned form of $\Phi$ is

$$
\Phi=\left[\begin{array}{lllll}
\phi & \psi & \theta_{u} & 0 & \theta_{w} \\
0 & I & 0 & 0 & 0 \\
0 & 0 & I & 0 & 0 \\
0 & 0 & 0 & I & 0 \\
0 & 0 & 0 & 0 & I
\end{array}\right]
$$

(III-21)

The specification of the time interval has been dropped in equation (III-21) and will henceforth be assumed to be [ $t_{k}, t_{k+1}$ ] unless shown otherwise.

A numerical differencing technique was chosen for the computation of the partitions of the state transition matrix. This was done because of the resulting ease with which the solve-for/ consider parameter set may be changed or expanded. Before describing the numerical differencing technique, let us adopt the following notation. Express the perturbation in the augmented state at time $t_{k+1}$ due to a perturbation in the state at $t_{k}$ or $x_{k}$ as $x^{A}\left(t_{k+1} ; x_{k}, t_{k}\right)$ and let the $f$ th column of $\Phi$ be designated by $\Phi_{._{j}}$. Now consider the special case of equation (III-20) in which $X_{k}^{A}$ is a vector whose only nonzero element is the jth element:

$$
\begin{equation*}
x_{k}^{A}=\left[0, \cdots, 0, \delta_{j}, 0, \cdots, 0\right]^{T}=d_{j} \tag{III-22}
\end{equation*}
$$

Equation (III-20) becomes

$$
\begin{equation*}
x^{A}\left(t_{k+1} ; d_{j}, t_{k}\right)=\Phi_{\cdot j} \delta_{j} \tag{III-23}
\end{equation*}
$$

from which we obtain the $j$ th column of $\Phi$ as

$$
\Phi_{\cdot j}=\frac{x^{A}\left(t_{k+1} ; d_{j}, t_{k}\right)}{\delta_{j}}
$$

(III-.24)

The numeration of this expression is evaluated by integrating the state equations over the interval $\left[t_{k}, t_{k+1}\right]$ as follows. Let

$$
\begin{array}{rl}
I_{j}=\int_{t_{k}}^{t_{k+1}} & f\left(X^{A}(\tau)+d_{j}, \tau\right) d \tau=X^{A}\left(t_{k+1} ; X_{k}^{A}+d_{j}, t_{k}\right) \\
& -\left(x_{k}^{A}+d_{j}\right) \tag{III-25}
\end{array}
$$

and

$$
\begin{equation*}
I=\int_{t_{k}}^{t_{k+1}} f\left(X^{A}(\tau), \tau\right) d \tau=X^{A}\left(t_{k+1} ; X_{k}^{A}, t_{k}\right)-X_{k}^{A} \tag{III-26}
\end{equation*}
$$

then

$$
\begin{equation*}
x^{A}\left(t_{k+1} ; d_{j}, t_{k}\right)=I_{j}-I+d_{j} \tag{III-27}
\end{equation*}
$$

Thus the state transition matrix is computed by evaluating the integral $I$ once and the integral $I_{j}$ once for each column of $\Phi$.

The computation of partitions of the state transition matrix is controlled by the subroutine STM.

Observation matrices relate the deviations from nominal in the augmented state variable to deviations in observables from their nominal values. The general nonlinear observation equation has the form

$$
\begin{equation*}
Y=Y\left(X^{A} t\right) \tag{III-28}
\end{equation*}
$$

where $Y$ denotes the observable. The linearized versions of equation (III-28) is

$$
\begin{equation*}
y=\frac{\partial Y}{\partial X^{A}} x^{A}=H^{A} x^{A} \tag{III-29}
\end{equation*}
$$

where $y$ and $x^{A}$ represent deviations from the nominal values of $\tilde{\mathrm{Y}}$ and $\tilde{X}^{\mathrm{A}}$.

If we partition the augmented state vector as before, equation (III-29) may be written as

$$
y=[H: M: O: L: G]\left[\begin{array}{c}
x \\
q \\
u \\
v \\
w
\end{array}\right]
$$

(III-30)

The third particion is zero since the dynamic-consider parameters do not affect the observables.

The columns of the augmented observation matrix $H^{A}$ are found by numerical differencing just as with the state transition matrix. However, this time the method is more direct since no integration of state equations is required. If we set $x^{A}=d_{j}$ as before, equation (III-29) may be written

$$
\begin{equation*}
y=Y\left(X^{A}+d_{j}, t\right)-Y\left(X^{A}, t\right)=H^{A} j_{j} \tag{III-31}
\end{equation*}
$$

Thus


The computation of the partitions of the observation matrix aie controlled by the subroutine HMM.

## D. QUASI-LINEAR FILTERING EVENT

The quasi-1inear filtering event option is included in $i_{2}=$ LTR program as an additional means to combat filter divergence. One of the several causes of filter divergence is the failure of the linearization assumption on which the entire estimation process is based. If the vehicle or the environment departs markedly from the current nominal value, the linearization assumptions can become invalid. The quasi-linear filtering event overcomes this difficulty by updating the nominal trajectory to correspond to the present estimate of the state. Specifically, updating the nominal trajectory results ir better computation of the state transition and observation matrix partitions used in the recursive estimation algorithm.

Letting $t_{j}$ denote the time of the quasi-linear filtering event, and using the ( ) ${ }^{-}$and ( ) ${ }^{+}$notations to indicate values immediately before and after the event, respectively, the basic state and solvefor parameter vectors are updated as follows:

$$
\begin{aligned}
& \tilde{x}_{j}^{+}=\tilde{X}_{j}^{-}+\hat{x}_{j}^{-} \\
& \tilde{Q}_{j}^{+}=\tilde{Q}_{j}^{-}+\hat{Q}_{j}^{-} \\
& \hat{x}_{j}^{+}=0 \\
& \hat{q}_{j}^{+}=0
\end{aligned}
$$

where the superscript ~ indicates the nominal value and the superscript ^ indicates an estimated value.

# IV. MODE A STATE ESTIMATION AND ATMOSPHERE RECONSTRUCTION 

## A. MODE A DYNAMIC MODEL

A five-dimensional primary state vector is employed in the mode A reconstruction process. This state vector is defined by

$$
\begin{equation*}
\mathbf{x}=(\mathrm{h}, \mathrm{v}, \gamma, \phi, \mathrm{p})^{\mathrm{T}} \tag{IV-1}
\end{equation*}
$$

where
$h=$ vehicle altitude
$\mathrm{v}=$ vehicle velocity
$\gamma=$ vehicle flightpath angle
$\phi=$ vehicle downrange angle
$\mathrm{p}=$ ambient atmospheric pressure.
The four vehicle state variables are defined in Figure II-1. These four state variables comprise the entire mode B primary state vector. In mode $B$, atmospheric pressure is not treated as a state variable.

The fundamental difference between the mode $A$ and mode $B$ reconstruction processes lies in the manner in which nongravitational forces are modeled. The general translational equations of motion can be written symbolically as

$$
\begin{equation*}
\dot{x}=g(x)+f(x) \tag{IV-2}
\end{equation*}
$$

where in this case $x$ represents the translational state; $g(x)$, the gravitational acceleration acting on the vehicle; and $f(x)$, the nongravitational acceleration. The mathematical form of $g(x)$ is well known and can be used to accurately model gravitational acceleration. This is not the case for the nongravitational acceleration $f(x)$, particularly when $f(x)$ represents an aerodynamic acceleration as is the case for the planetary entry problem. Nevertheless, mode $B$ does use a mathematical model for $f(x)$, which also requires the selection of a mathematical model of the planetary atmosphere. Mode A, however, dispenses entirely with the attempt to mathematically model $f(x)$. Instead, mode A uses acceleration
and gyro data to model $f(x)$. In other words, $f(x)$ is replaced by

$$
f\left(a_{x_{m}}, a_{z_{m}}, \theta_{m}\right)
$$

where $a_{x_{m}}$ and $a_{z_{m}}$ represent (smoothed) axial and normal accelerometer (VRU) data, respectively, and $\theta_{m}$ represents (smoothed)
gyro (ARU) attitude data. Except for a nominal molecular weight profile, mode A requires no model of the planetary atmosphere. The axial acceleration $a_{x}$, however, is essential for mode $A$ operation.

The remainder of this section will treat the mode $A$ dynamic model in more detail. The equations of motion are summarized as

$$
\begin{align*}
& \dot{\mathrm{h}}=\mathrm{v} \sin \gamma \\
& \dot{\mathrm{v}}=-\mathrm{g} \sin \gamma+a_{x_{c}} \cos (\alpha+\varepsilon)+a_{z_{c}} \sin (\alpha+\varepsilon)  \tag{IV-4}\\
& \dot{\gamma}=\left(\frac{v}{r}-\frac{g}{v}\right) \cos \gamma+\frac{1}{v}\left[a_{x_{c}} \sin (\alpha+\varepsilon)-a_{z_{c}} \cos (\alpha+\varepsilon)\right]  \tag{IV-5}\\
& \dot{\phi}=\frac{v}{r} \cos \gamma  \tag{IV-6}\\
& \dot{p}=-g \rho f \tag{IV-7}
\end{align*}
$$

where $a_{x_{c}}$ and $a_{z_{c}}$ represent corrected axial and normal accelerometer data, respectively, and $\alpha+\varepsilon$ can be obtained from equation (II-5)

$$
\begin{equation*}
\alpha+\varepsilon=\theta_{c}+\theta_{0}+\phi-\gamma \tag{IV-8}
\end{equation*}
$$

where attitude $\theta$ has been represented as the sum of the initial attitude $\theta_{0}$ and the change in (corrected) attitude $\theta_{0}$ since $t_{0}$. This permits the mode A filter to treat $\theta_{0}$ as a solve-for or consider parameter. The angle $\varepsilon$ is computed from equations (II-3) and (II-4). A nominal wind profile is assumed by mode A to compute the horizontal wind velocity $v_{w}$. Equation (IV-7) is just the time-differential form of the hydrostatic equation, where $\rho$ represents atmospheric density.

Accelerations $\mathbf{a}_{\mathbf{x}_{\mathbf{c}}}$ and $\mathbf{a}_{\mathbf{z}}$ and attitude $\theta_{c}$ are referred to as corrented quantities since the measured accelerations $a_{x_{m}}$ and $a_{z_{m}}$ and the measured attitude $\theta_{m}$ have been corrected or calibrated for scale factor, bias, and misalignment errors. The equations relating corrected quantities to measured quantities are summarized as

$$
\begin{align*}
& a_{x_{c}}=\frac{1}{\cos \left(\delta_{1_{c}}-\delta_{2}\right)}\left\{\frac{a_{x_{m}}-c_{s x}}{c_{b x}} \cos \delta_{2}\right. \\
& \left.+\frac{a_{z_{m}}-c_{s z}}{c_{b z}} \sin \delta_{1_{c}}\right\}+\omega_{c}^{2} \bar{x}-\dot{\omega}_{c} \bar{z}  \tag{IV-9}\\
& a_{z_{c}}=\frac{1}{\cos \left(\delta_{1_{c}}-\delta_{2}\right)}\left\{-\frac{a_{x_{m}}-c_{s x}}{c_{b x}} \sin \delta_{2}\right. \\
& \left.+\frac{a_{z}-c_{s z}}{c_{b z}} \cos \delta_{1_{c}} \right\rvert\,+\omega_{c}^{2} \bar{z}+\dot{\omega}_{c} \bar{x}  \tag{IV-10}\\
& \theta_{c}=\frac{1}{c_{i s \theta}}\left\{\theta_{m}-c_{d \theta} t\right\} \tag{IV-11}
\end{align*}
$$

where $c_{s x}, c_{s z}$, and $c_{s \theta}$ represent scale factors; $c_{b x}, c_{b z}$ and $c_{b \theta}$, biases; and $c_{d \theta}$, gyro drift error. Equations (II-31) and (II-32) define the accelerometer offsets $\bar{x}$ and $\bar{z}$. Corrected angular velocity $\omega_{c}$ and angular acceleration $\dot{\omega}_{c}$ are given by

$$
\begin{align*}
& \omega_{c}=\frac{1}{c_{s \theta}}\left\{\omega_{m}-c_{d \theta}\right\}  \tag{IV-12}\\
& \dot{\omega}_{c}=\frac{\dot{\omega}_{m}}{c_{s \theta}} \tag{IV-13}
\end{align*}
$$

where $\omega_{m}$ and $\dot{\omega}_{m}$ are measured angular velocity and angular acceleration, respectively. Misalignment angles $\delta_{1_{c}}$ and $\delta_{2_{c}}$ are calibrated using

$$
\begin{align*}
& \delta_{1_{c}}=\delta_{1}-c_{b \delta_{1}}  \tag{IV-14}\\
& \delta_{2}=\delta_{2}-c_{b \delta_{2}} \tag{IV-15}
\end{align*}
$$

where $\delta_{1}$ and $\delta_{2}$ are the nominal misalignment angles, and $c_{b} \delta_{1}$ and $c_{b \delta_{2}}$ are misalignment biases.

Returning to equation (IV-7), it is apparent that a method for obtaining density $\rho$ must be available before this final state equation can be integrated. Unlike mode $B$, an atmospheric model cannot be used for generating $\rho$. Instead, in mode $A$ an approximate relationship between $\rho$ and $a_{x_{c}}$ is used. Comparing equations (IV-4) and (II-12), neglecting temporarily the parachute term in equation (II-12), and using equation (II-6), we obtain

$$
\begin{equation*}
q=-\frac{m}{c_{a} S} a_{x_{c}} \tag{IV-16}
\end{equation*}
$$

The parachute effect can be incorporated approximately by writing

$$
\begin{equation*}
q=-\frac{m}{\left(C_{A} s+C_{D} S_{D}\right)} a_{x_{c}} \tag{IV-17}
\end{equation*}
$$

Having related dynamic pressure $q$ to $a_{z}$, it is a simple matter to relate $\rho$ to ${\underset{X}{c}}$ since

$$
\begin{equation*}
\rho=\frac{2 q}{v_{r}^{2}} \tag{IV-18}
\end{equation*}
$$

where $v_{r}$ is the relative velocity given by equation (II-2). With density $\rho$ available from equations (IV-17) and (IV-18), equation (IV-7) can be integrated to obtain atmospheric pressure p. Atmospheric temperature is then directly available from the equation
of state

$$
\begin{equation*}
T=\frac{p M}{\rho R} \tag{IV-19}
\end{equation*}
$$

where $M$ is the molecular weight and $R$ is the universal gas constant. Molecular weight $M$ is computed from the nominal mole fraction profiles of the component gases in the planetary atmosphere.

If a normal accelerometer is not available, the following substitutions must be made in the previous equations:

(IV-20)

If a gyro is not available, we assume

$$
\begin{align*}
& \omega_{c}=0  \tag{IV-21}\\
& \dot{\omega}_{c}=0
\end{align*}
$$

and delete equation (IV-11).
A quasi-static dynamic model option is also available in mode A. When quasi-static motion is assumed, equation (IV-4) is deleted and velocity is computed from equation (II-18).

## B. MODE A RECURSIVE TRAJECTORY AND ATMOSPHERE RECONSTRUCTION

The equations presented in Section $A$ are used to compute the nominal trajectory and state transition matrices (via numerical differencing) required by the linear recursive estimation process described in Chapter TII. Nominal observations and observation matrices are computed using the equations presented in Chapter II.D and II.E (Part I), with "actual" parameter values replaced by nominal parameter values. Since mode A already employs accelerometer data in its dynamic model, accelerometer data are not treated as a (filtered) measurement in mode $A$ as in mo.? B. Neither are gyro data treated as a (filtered) measurement in mode $A$.

Parameters listed in Table II-1 (Chapter II, Part I) and checked in the mode A column can be augmented to the mode A primary state vector as either solve-for or consider parameters. Note that accelerometer and gyro scale factors, biases, and misalignments can also be treated as augmented parameters, and can thus influence the propagation and update of estimates and covariance matrices.

Estimates of certain parameters that do not appear in the mode A parameter augmentation list can nevertheless be obtained as derived estimates. These estimates are referred to as derived (or secondary) estimates since they are derived from estimates generated by the recursive estimation process, and in no way influence this recursive process. Derived estimates are presently available for atmospheric density and temperature. Tie required equations for both the derived estimates and their variances follow.

The nominal density computed from equations (IV-17) and (IV-18), when combined, yield

$$
\begin{equation*}
\rho=-\frac{2 m a_{x_{C}}}{v_{r}^{2}\left(C_{A} S+C_{D} S_{D}\right)} \tag{IV-22}
\end{equation*}
$$

To obtain a derived estimate of the density deviation from its nominal value, we should, strictly speaking, take the first variation of equation (IV-22) with respect to the primary state variables and all explicit and implicit augmented parameters on which $\rho$ depends through equation (IV-22). Denoting all such parameters as $w$, we would obtain a first variation of equation (IV-22) having the form

$$
\begin{equation*}
\delta \rho=\Gamma_{1}(\delta x, \delta w)^{T} \tag{IV-23}
\end{equation*}
$$

where $\Gamma_{1}$ is the Jacobian matrix

$$
\begin{equation*}
\Gamma_{1}=\left[\frac{\partial \rho}{\partial(x, w)}\right] \tag{J.V-24}
\end{equation*}
$$

Then the derived estimated deviation of density would be given by

$$
\begin{equation*}
\delta \hat{\rho}=\Gamma_{1}(\delta \hat{x}, \delta \hat{W})^{T} \tag{IV-25}
\end{equation*}
$$

where estimates $\delta \hat{\mathbf{x}}$ and $\delta \hat{\omega}$ are available from the recursive estimation process (estimates of any elements of $w$ that are treated as consider parameters are, of course, zero). The variance of the derived estimate $\delta \hat{\rho}$ can be found from

$$
\sigma_{\rho}^{2}=\Gamma_{1}\left[\begin{array}{ll}
P & C_{x W}  \tag{IV-26}\\
C_{X V}^{T} & W
\end{array}\right] \Gamma_{1}^{T}
$$

where $P$ is the primary state sovariance matrix, $W$ the qugmented parameter covariance matrix, and $C_{x w}$ represents the correlation between $x$ and w.

We couid operate on equation (IV-19) in similar fashion to obtain a derived estimate of temperature. Such an estimate would have the form

$$
\begin{equation*}
\delta \hat{T}=\Gamma_{2}(\delta \hat{X}, \delta \hat{W})^{T} \tag{IV-27}
\end{equation*}
$$

where

$$
\begin{equation*}
\Gamma_{2}=\left[\frac{\partial T}{\partial(x, w)}\right] \tag{IV-28}
\end{equation*}
$$

The variance of $\delta \hat{T}$ would be given by

$$
\sigma_{T}^{2}=\Gamma_{2}\left[\begin{array}{ll}
\mathrm{P} & \mathrm{C}_{\mathrm{xW}}  \tag{IV-29}\\
\mathrm{C}_{\mathrm{XW}}^{\mathrm{T}} & \mathrm{~W}
\end{array}\right] \mathrm{\Gamma}_{2}^{T}
$$

Currently, however, derived estimates $\delta \hat{\rho}$ and $\delta \hat{T}$ are computed from considerably simplified expressions. The first variation of $\rho$ is taken only with respect to $\mathbf{v}_{r}$, and then $\delta \hat{v}_{r}$ itself is replaced with $\delta \hat{v}$ to obtain

$$
\begin{equation*}
\delta \hat{\rho}=\frac{2 \rho}{v_{r}} \delta \hat{v} \tag{IV-30}
\end{equation*}
$$

and

$$
\begin{equation*}
\sigma_{\rho}^{2}=\frac{4 \rho^{2}}{v_{r}^{2}} \sigma_{v}^{2} \tag{IV-31}
\end{equation*}
$$

The first variation of $T$ is taken with respect to $p, M$, and $v_{r}$ (with $\delta \hat{v}_{r}$ replaced with $\delta \hat{v}$ ) to obtain

$$
\begin{equation*}
\delta \hat{T}=T\left(\frac{\delta \hat{p}}{\mathbf{P}}+2 \frac{\delta \hat{\mathrm{~V}}}{v_{\mathbf{r}}}+\frac{\delta \hat{M}}{M}\right) \tag{IV-32}
\end{equation*}
$$

and

$$
\begin{equation*}
\sigma_{T}^{2}=\frac{T^{2}}{\mathrm{p}^{2}} \sigma_{\mathrm{p}}^{2}+\frac{4 \mathrm{~T}^{2}}{\mathrm{v}_{\mathrm{r}}^{2}} \sigma_{\sim}^{2}+\frac{\mathrm{T}^{2}}{\mathrm{M}^{2}} \sigma^{2} \tag{IV-33}
\end{equation*}
$$

Eventually equations (IV-30) through (IV-33) should be replaced wi-n equations (IV-25), (IV-26), (IV-27), and (IV-29) to obtain improved derived estimates $\delta \hat{\rho}$ and $\delta \hat{T}$ and more realistic variances of $\sigma_{\rho}^{2}$ and $\sigma_{T}^{2}$.

The entire mode A trajectory and atmosphere reconstruction process is based on the method presented in Reference 1.

## V. MODE B STATE ESTIMATION AND ATMOSPHERE RECONSTRUCTION

A. MODE B DYNAMIC MODEL


#### Abstract

A four-dimensional primary state vector is employed in the mode $B$ reconstruction process. This state vector is defined by $$
\begin{equation*} x=(h, v, \gamma, \phi)^{T} \tag{V-1} \end{equation*}
$$


where

$$
\begin{aligned}
& \mathrm{h}=\text { altitude } \\
& \mathrm{v}=\text { velocity } \\
& \gamma=\text { flightpath angle } \\
& \phi=\text { downrange angle. }
\end{aligned}
$$

These variables are defined in Figure II-1.
The fundamental difference between the mode $A$ and mode $B$ reconstruction processes, which was explained fully in Chapter IV.A, consists in the manner in which the nongravitational forces acting on the entry vehicle are treated. Unlike mode A where all information on the aerodynamic forces and planetary atmosphere are imbedded in the accelerometer and gyro data, mode B assumes a mathematical representation for both aerodynamic forces and the planetary atmosphere.

The remainder of this section will treat the mode $B$ dynamic model in more detail. The equations of motion are summarized below:

$$
\begin{equation*}
\dot{\mathrm{h}}=\mathrm{v} \sin \gamma \tag{V-2}
\end{equation*}
$$

$$
\begin{align*}
& \dot{v}=-g \sin \gamma+\frac{A}{m} \cos (\alpha+\varepsilon)+\frac{N}{m} \sin (\alpha+\varepsilon)-\frac{F_{d}}{m} \cos \varepsilon  \tag{V-3}\\
& \dot{\gamma}=\left(\frac{v}{r}-\frac{g}{v}\right) \cos \gamma+\frac{1}{v}\left[\frac{A}{m} \sin (\alpha+\varepsilon)-\frac{N}{m} \cos (\alpha+\varepsilon)\right. \\
&  \tag{v-4}\\
& \left.-\frac{F_{d}}{m} \sin \varepsilon\right]
\end{align*}
$$

$$
\begin{equation*}
\dot{\phi}=\frac{v}{r} \cos \gamma \tag{V-5}
\end{equation*}
$$

where axial aerodynamic force $A$, normal aerodynamic force $N$, and parachute drag force $F_{d}$ are given by

$$
\begin{align*}
& A=-C_{A} q S  \tag{V-6}\\
& N=-C_{N} q S  \tag{V-7}\\
& F_{d}=C_{D} q S_{D} \tag{V-8}
\end{align*}
$$

where dynamic pressure $q=\frac{1}{2} \rho v_{r}^{2}$. These equations of motion have the same form as the translational equations of motion used by the data generator to compute the "actual" entry trajectory. These latter equations are presented in Chapter II.A. Huwever, mode B uses assumed nominal values of all parameters to integrate these equations, whereas the data generator uses "actual" values. In addition, mode $B$ does not model rotational motion, assumes gyro information is not available, and that the nominal angle of attack $\alpha$ is zero.

Before the aerodynamic forces given by equations (V-6) through ( $V-8$ ) can be evaluated, it is necessary to obtain density $\rho$. Un-• like mode A that extracts density from the axial accelerometer measurement $a_{X_{c}}$, mode $B$ assumes that the planetary atmosphere can be modeled by piece-wise linear temperature and molecular weight profiles. In fact, the mathematical atmosphere model employed by mode $B$ has the same form as the model employed by the data generator to compute the "actual" atmospheric properties. The equations that define such an atmosphere model are presented in Chapter II.B. Of course, mode B uses assumed nominal values of all parameters to define its atmosphere model, whereas the da a generator uses "actual" values to define its atmosphere model.

A quasi-static dynamic model option is also available in mode B. When quasi-static motion is assumed, equation ( $V-3$ ) is deleted and velocity is computed from equation (II-18).

## B. MODE B RECURSIVE TRAJECTORY AND ATMOSPHERE RECONSTRUCTION

The equations presented in Section $A$ (and related equations in Chapter II.A and II.B) are used to compute the nominal trajectory and state transition matrices (via numerical differencing) required by the linear recursive estimation process described in Chapter III. Nominal observations and observation matrices are computed using the equations presented in Chapter II.D and II.E, with "actual" parameter values replaced by nominal parameter values. Unlike mode A, mode B treats accelerometer data as m. ? pments to be used directly in the recursive estimation process 3 uses tie following equations to compute nominal accelerar measurements and accelerometer observation matrices:

$$
\begin{align*}
& a_{x}=\left(\frac{A}{m} \cos \delta_{1}-\frac{N}{m} \sin \delta_{1}\right) c_{s x}+C_{b x}  \tag{v-9}\\
& a_{z}=\left(\frac{A}{m} \sin \delta_{2}+\frac{N}{m} \cos \delta_{2}\right) c_{s z}+c_{b z} \tag{V-10}
\end{align*}
$$

where aerodynamic forces $A$ and $N$ are given by equations ( $V-6$ ) and $(V-7), \delta_{1}$ and $\delta_{2}$ are axial and normal accelerometer misalignment angles, $C_{s x}$ and $C_{s z}$ are scale factors, and $C_{b x}$ and $C_{b z}$ are biases.

Parameters listed in Table II-1 (Chapter II, Part II) and checked in the mode $B$ column can be augmented to the mode $B$ primary state vector as either solve-for or consider parameters. Unlike mode $A$, which can treat only one atmospheric parameter -pressure, in the recursive estimation process, mode $B$ can treat several -- surface pressure, temperature profile parameters, and mole fraction profile parameters. If mode B solves for any of these atmospheric parameters, the final estimates can be used to compute pressure and density as a function of altitude. This could be accomplished by rerunning the data generator program with an atmosphere model defined by these new atmospheric parameter estimates.

The mode $B$ trajectory and atmosphere reconstruction process is an adaptation of the method presented in Reference 2.

## VI. INDIVIDUAL SUBROUTINE ANALYSES

Individual subroutine analyses zre found in Chapter $V$ of the Programmers' Section of the manual.

## VII. REFERENCES

1. F. Hopper. LTR2 Progrom, Philosophy and Implementation. 1643-71-31-V. Martin Marietta Corporation memorandum, April 23, 1971.
2. R. Falce and P. Kusinitz. Mars Entry Trajectory Reconstruction Program, Dynomic and Measurement Equations, April 1, 1970.
3. R. Falce. METR Program Simulation Mode Logic. 1643-71-21-V. Martin Marietta Corporation Memorandum, March 18, 1971.
4. G. Null, H. Gordon, and D. Tito. The Mariner IV Flight Path and Its Determination from Tracking Data. JPi TR 32-1108, August 1, 1967.

PART II
LTR USERS' MANUAL

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## I. INTRODUCTION

The LTR Users' Manual provides the user of the LTR data generator and reconstruction programs with all the information necessary to input these programs and interpret the output.

Chapter II describes the input of the LTR program. This includes a description of the data deck and tape structure, namelist variable definitions, ineasurement and event scheduling, and restrictions on the use of the programs. Chapter III describes the output of the LTR data generator and reconstruction programs. Chapter IV discusses actual sample cases run using the LTR programs. These sample cases are presented primarily to demonstrate the operation and versatility of the LTR programs and to assist the user in the input/output procedure for these programs.

## II. INPUT DESCRIPTION

## A. DATA DECK AND TAPE STRUCTURE

The first card of an LTR data deck must have an integer 1 or 2 in CC 10, followed by another card with Hollerith problem identification information, such as case number, landing date, etc. If the first card has set RUNN $\emptyset=1$, the data generator namelist section ERAN must be input and the data generator and preprocessor will be executed. If the first card has set RUNN $\varnothing=2$, the reconstruction program namelist ERAN must be input and:

1) The data generator must have been executed immediately before, or:
2) The data generator must have v itten logical units 10 and 16 onto magnetic tape during a previous run;
3) The plotting and summary table namelist PLTVAR must be input before the reconstruction and summary modes can be run.

If the reconstruction program is to be executed, a measurement schedule in fixed-field format must follow the PLTVAR section of data. See Section C. 3 for a description of the measurement schedule. If the first card has set $\operatorname{RUNN} \varnothing=3$, the program terminates execution.

## B. DATA GENERATOR INPUT VARIABLE DEFINITIONS

## 1. Namelist Variable Definitions

The namelist variables appearing in the data generator namelist ERAN and read from subroutine SETUP1 are defined below according to several categories. Most of these variables will be preset by the program if they do not appear in the namelist input; these preset values are the quantities enclosed by parentheses in the namelist variable definitions. The required input units are specified in the last column.

## a. Trajectory Variables

| $\mathrm{XN}(1)$ | Initial nomial vehicle altitude h (0.) | km |
| :---: | :---: | :---: |
| XN(2) | Initial nomial vehicle velocity $v$ (0.) | km/s |
| XN(3) | Initial nominal vehicie flight path angle $\gamma$ (0.) | deg |
| $\mathrm{XN}(4)$ | Initial nominal vehicle downrange angle $\phi$ (0.) | deg |
| XN(5) | Initial nominal vehicle attitude angle $\theta$ (0.) | deg |
| XN(6) | Initial nominal vehicle angular velocity $\omega$.(O.) | deg/s |
| XN(7) | Initial value of integral of axial VRU output (O.) | km/s |
| $\mathrm{XN}(8)$ | Initial value of integral of normal VRU output (O.) | km/s |
| XN(9) | Initial value of integral of ARU output (0.) | rad |
|  | Code that defines coordinate system relative to which the entry plane is oriented using the variables ECLINC, ECLØNG, and PHIR (3) <br> $=1$, planetocentric ecliptic <br> 2, planetocentric equatorial <br> 3, subsolar orbital plane | -- |
| ECLINC | Inclination of the entry plane relative to $x y-p l a n e$ of $I C \emptyset \emptyset R$ coordinate system (0.) | deg |
| ECL $\mathrm{N}^{\text {NG }}$ | Longitude of the ascending node of the entry plane relative to ICOjøR coordinate system (0.) | deg |
| PHIR | Angle between the line of nodes and the $\phi$ reference line. Sum of argument of periapsis and initial true anomaly of vehicle (0.) | deg |


| TC | Initial trajectory time (0.) | s |
| :---: | :---: | :---: |
| TF | Final trajectory time | s |
| IYR <br> thru | Initial calendar date corresponding to initial trajectory time TC. | -- |
| SECSI | $\begin{aligned} & \text { IYR = year (integer) } \\ & \text { IMD = month (integer) } \\ & \text { IDAY = day (integer) } \\ & \text { IHR = hour (integer) } \\ & \text { IMIN = minute (integer) } \\ & \text { SECSI = second (floating) } \end{aligned}$ |  |
| DT | Integrator step size (0.1) | s |
| QSALT | Altitude at which the dynamic model is to be replaced with the quasistatic dynamic model (40.) | km |
| QSDT | Integrator step size when quasi-static dynamic model. is used (1.) | S |
| $\emptyset \mathrm{DB}$ | Maximun dynamic pressure permitted for integration of the complete set of equations of motion. Whenever dynamic pressure exceeds $\emptyset \mathrm{DB}$, the motion of the entry vehicle is assumed to be described by the point mass equations of motion. See the last paragraph in Chapter II. A of the Analytice Manual for more details (15. x $10^{5}$ ) | $\mathrm{kg} / \mathrm{km} \cdot \mathrm{s}^{2}$ |
| HD | Parachute deployment altitude (0.) | km |
| HR | Farachute release altitude. HR must be less than HD (0.) | km |
| Planet | Atmosphere Variables |  |
| NTP | Planet code (3) = 2, Mercury <br> 3, Venus <br> 5, Mars <br> 6, Jupiter <br> 7, Saturn <br> 8, Uranus <br> 9, Neptune <br> 10, Pluto | -- |


| RM | Planet radius (6050.) | km |
| :---: | :---: | :---: |
| MU | Planet gravitational constant $\left(3.2486 \times 10^{5}\right)$ | $\mathrm{km}^{3} / \mathrm{s}^{2}$ |
| Gø | Acceleration of gravity at planet surface ( $8.867 \times 10^{-3}$ ) | $\mathrm{km} / \mathrm{s}^{2}$ |
| OMEG | Planet angular velocity ( $2.997 \times 10^{-7}$ ) | $\mathrm{rad} / \mathrm{s}$ |
| ATM $\mathrm{S}_{\text {S (1) }}$ | Surface pressure | $\mathrm{kg} / \mathrm{km}-\mathrm{s}^{2}$ |
| $\begin{aligned} & \text { ATM } \phi \mathrm{S} \text { (18) } \\ & \text { thru } \\ & \text { ATM } \phi \mathrm{S} \text { (33) } \end{aligned}$ | Nominal atmosphere temperature profile. ATM $\varnothing$ S (18) through ATM $\emptyset$ S (25) define the altitude breakpoints in ascending order. ATM $\phi$ S (26) througin ATM $\emptyset$ S (33) define the corresponding temperatures at each of the altitude breakpoints | km ${ }^{\circ} \mathrm{K}$ |
| NTPTS | Number of altitude breakpoints used to define the temperature profile. NTPTS must not exceed 7 (6) | -- |
| XMFH | Altitude breakpoints (in ascending order) for all mole fraction profiles. XMFH (1) must be set equal to ATM A (18) (0., 120., 370., 1000., 0.) | km |
| XMFW | Set of nominal mole fraction profiles for up to five component gases corresponding to the altitude breakpoints appearing in XMFH. Each row of mole fractions corresponds to an altitude breakpoint | -- |
| NMPTS | Number of altitude breakpoints used to define the mole fraction profile in XMFH. NMPTS must not exceed 5 (4) | -- |
| CGMW | Molecular weights of up to five component gases. Order corresponds to order of mole fractions at each altitude breakpoirit (44.011, 28.012, 39,948, 2.016, 0.) | -- |

## AR

Universal gas constant ( $8.31432 \times 10^{-3}$ )
Ratio of specific heats (1.4)
AGAM
Nominal wind profile. WDTBL(1) $=n$, number of altitude -breakpoints WDTBL (2) through WDTBL $(1+\mathrm{n}) \quad \mathrm{km}$ define the sequence of altitude breakpoints in ascending order $\operatorname{WDTBL}(2+\mathrm{n})$ through $\operatorname{WDTBL}(1+2 \mathrm{n}) \quad \mathrm{km} / \mathrm{s}$ define the corresponding sequence of wind magnitudes. Up to 10 altitude breakpoints can be defined (2., $0 .$, 10., 0., 0.)

TH
Nor terrain height profile coeffit $s C_{1}, C_{2}, \ldots, C_{7}$, required to define the profile

$$
\tau(x)=C_{1}+C_{2} \sin \left(C_{3} x+C_{4}\right)
$$ $+C_{5} \sin \left(C_{6} x+C_{7}\right)$.

$C_{1}, C_{2}$, and $C_{5}$ are expressed in units of $\mathrm{km} ; \mathrm{C}_{3}, \mathrm{C}_{4}, \mathrm{C}_{6}$, and $\mathrm{C}_{7}$ are dimensionless

TERHT Logical variable that indicates -whether the data generator is to use the terrain height model defined above: (true)
$=$ true, use terrain height model; false, do not use terrain height model
c. Entry Vehicle Variables

IPHAS Code defining entry phase (1) $=1$, phase prior to parachute deployment
2, parachute phase
3, phase following parachute release

VMASS
Vehicle mass as a function of IPHAS kg (174., 122., 100.)

| VSA | Vehicle reference area as a function of IPHAS $\left(1.474 \times 10^{-6}, 0.292 \times 10^{-6}\right.$, $0.292 \times 10^{-6}$ ) | $\mathrm{km}^{2}$ |
| :---: | :---: | :---: |
| VDIA | Vehicle reference diameter as a function of IPHAS ( $1.37 \times 10^{-3}, 0.61 \times 10^{-3}$, $0.61 \times 10^{-3}$ ) | km |
| VRI | Vehicle rotational inertia as a function of IPHAS ( $1.76 \times 10^{-5}, 0.5 \times 10^{-5}$, $0.5 \times 10^{-5}$ ) | kg-km ${ }^{2}$ |
| XG | Vehicle cg offset along x-axis (0.) | km |
| ZG | Vehicle cg offset along z-axis (0.) | km |
| XD | Parachute bridle apex location along x-axis (-1. x $10^{-3}$ ) | km |
| SDP | Parachute reference area (46. x $10^{-6}$ ) | $\mathrm{km}^{2}$ |
| CDTBL | Parachute drag coefficient table as a function of Mach number. $\operatorname{CDTBL}(1)=n$, number of tabulated $C_{D}$ values; CDTBL(2) through CDTBL (1 + n) define the sequence of tabulated Mach numbers in ascending order; $\operatorname{CDTBL}(2+n)$ through CDTBL $(1+2 n)$ define the corresponding values of drag coefficients. Up to 24 Mach number/drag coefficient pairs can be defined. <br> (5. , $\begin{array}{rll} 0 ., & 0.6,1.4, & 3.2, \\ 0.5, & 0.5, & 0.49, \\ 0.19, & 0.19) \end{array}$ | -- |
| Measurement | Variables |  |
| XM | Velocity reference unit (VRU) location along x-axis (0.) | km |
| ZMM | VRU location along z-axis (0.) | km |
| XSTEP | Quantum level for $x$-axis VRU $\left(1.5 \times 10^{-5}\right)$ | km/s |


| ZSTEP | Quantum level for z-axis VRU ( $1.5 \times 10^{-5}$ ) | km/s |
| :---: | :---: | :---: |
| TSTEP | Quantum level for attitude reference uriit (0.004) | rad |
| DELT | Nominal axial accelerometer, normal accelerometer, and gyro misalignment angles (0., 0., 0.) | rad |
| VXQA | Initial axial accelerometer quantized data for five time points centered about initial time ( 5 * 0.) | km/s |
| VZQA | Initial normal accelerometer quantized data for five time points centered about initial time (5*0.) | km/s |
| THTQA | Initial gyro quantize:, data for five time points celtered about initial time (5*0.) | rad |
| ETA | Radar altimeter sweep half-angle (0.7854) | rad |
| SALT | Array of altitudes above mean earth surface for three tracking stations | km |
| SLAT | Array of latitudes in degrees north for three tracking stations | deg |
| SL@N | Array of longitudes in degrees east for thrce tracking stations | deg |
|  | The following tracking station locations are preset: |  |
|  | SALT SLAT SLQN |  |
|  | 1. Goldstone $1.031 \quad 35.384-116.833$ |  |
|  | 2. Madrid 0540.417 -3.667 |  |
|  | 3. Canberra .05-35.311 149.136 |  |
| NØFRAC | ```Refractivity code (true); nonfunctional = false, refractivity model will be used true, refractivity model will not be used``` | -- |

## e. Other Variables

| ICNTR | The Multiple of DT at which time <br> interval prints are to be made. |
| :--- | :--- |
| If ICNTR $=N$, a print occurs every |  |
| (N * DT) seconds (100) |  |

## 2. Error Definitions

Most of the namelist variables defined in subsection 1 represent nominal values. Actual errors in these variables are specified by inserting the proper $C(j)$ variables in the same namelist. All C(j) variables are defined in Table II-1, along with their required input units. The same table also indicates in whick programs the $C(j)$ variables presently have meaning. The use of the $C(j)$ variab?es in the mode $A$ and mode $B$ reconstruction programs is treated in Section C.2. Room for more than 70 new C(J) variables is still available in the table. All $C(j)$ scale factors are preset to $1 .$, while all $C(j)$ biases are preset to 0 .

As an example of the use of the $C(j)$ variables, suppose one wished to define errors in the initial vehicle state, the scale factor in the aerodynamic coefficient $C_{A}$, and the altimeter bias. The errors in the initial vehicle state are specified as:

$$
\begin{aligned}
C(101) & =10 ., \text { altitude error } \\
C(102) & =.05, \text { velocity error } \\
C(103) & =1.2, \text { flightpath angle error } \\
C(104) & =.5, \text { downrange angle error } \\
C(140) & =-2 ., \text { attitude error } \\
C(106) & =-.03, \text { angular velocity error. }
\end{aligned}
$$

and if the altimeter bias were 0.75 kilometer, we would set

$$
C(72)=.75
$$

Table II-1 $C(j)$ Variables

| j | C(j) | Units | Data Generator | Mode A | Mode B |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Surface pressure $p_{0}$ bias | kg/km-s ${ }^{2}$ | $\checkmark$ |  | $\checkmark$ |
| 2 | Altitude $h_{1}$ bias | km |  | * |  |
| 3 | Temperature $T_{1}$ bias | ${ }^{\circ} \mathrm{K}$ | $\checkmark$ |  | $\checkmark$ |
| 4 | $h_{2}$ | km | $\checkmark$ |  | $\checkmark$ |
| 5 | $\mathrm{T}_{2}$ | ${ }^{\circ} \mathrm{K}$ | $\checkmark$ |  | $\checkmark$ |
| 6 | $h_{3}$ | km | $\checkmark$ |  | $\checkmark$ |
| 7 | $T_{3}$ | ${ }^{\circ} \mathrm{K}$ | $\checkmark$ |  | $\checkmark$ |
| 8 | $\left.h_{4} \quad\right\}$ Temperature | km | $\checkmark$ |  | $\checkmark$ |
| 9 | $T_{4} \quad$ Profile | ${ }^{\circ} \mathrm{K}$ | $\checkmark$ |  | $\checkmark$ |
| 10 | $h_{5}$ | km | $\checkmark$ |  | $\checkmark$ |
| 11 | $T_{5}$ | ${ }^{\circ} \mathrm{K}$ | $\checkmark$ |  | $\checkmark$ |
| 12 | $\mathrm{h}_{6}$ | km | $\checkmark$ |  | $\checkmark$ |
| 13 | $\mathrm{T}_{6}$ | ${ }^{\circ} \mathrm{K}$ | $\checkmark$ |  | $\checkmark$ |
| 14 |  | km | $\checkmark$ |  | $\checkmark$ |
| 15 | $\mathrm{T}_{7} \quad$ | ${ }^{\circ} \mathrm{K}$ | $\checkmark$ |  | $\checkmark$ |
| 16 | Axial aerodynamic coefficient $C_{A}$ bias | -- | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 17 | Normal aerodynamic coefficient $C_{N}$ bias | -- | $\checkmark$ |  | $\checkmark$ |
| 18 | Damping moment aerodynamic coefficient $C_{M_{q}}$ bias | -- | $\checkmark$ |  |  |
| 19 | Center of pressure Xp bias | km | $\checkmark$ |  |  |
| 20 | $C_{A}$ scale factor | -- | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 21 | $C_{N}$ scale factor | -- | $\checkmark$ |  | $\checkmark$ |
| 22 | $C_{M_{q}}$ scale factor | -- | $\checkmark$ |  |  |
| 23 | Xp scale factor | -- | $\checkmark$ |  |  |
| 24 |  |  |  |  |  |
| 25 |  |  |  |  |  |
| 26 | Cg offset in x -direction, Xg | km | $\checkmark$ | $\checkmark$ |  |
| 27 | Cg offset in z -direction, Zg | km | $\checkmark$ | $\checkmark$ |  |
| 28 | VRU offset in $x$-direction, $X_{m}$ | km | $\checkmark$ | $\checkmark$ |  |
| 29 | VRU offset in $z$-direction, $z_{m}$ | km | $\checkmark$ | $\checkmark$ |  |
| 30 | Vehicle mass bias | kg | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 31 | Vehicle rotational inertia bias | $\mathrm{kg}-\mathrm{km}^{2}$ | $\checkmark$ |  |  |

Table II-1 (Cont)


Table [1-1 (Cont)

| j | C(j) | Units | Data Generator | Mode A | Mode 8 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 70 |  |  |  |  |  |
| 71 | Altimeter scale factor | -- | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 72 | Alisimeter bias | km | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 73 |  |  |  |  |  |
| 74 |  |  |  |  |  |
| 75 |  |  |  |  |  |
| 76 |  |  |  |  |  |
| 77 |  |  |  |  |  |
| 78 |  |  |  |  |  |
| 73 |  |  |  |  |  |
| 80 |  |  |  |  |  |
| 81 | Pressure measurement scale factor ( $M \geq 1$ ) | -- | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 82 | Pressure measurement bias ( $M \geq 1$ ) | kg/km-s ${ }^{2}$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 83 | Pressure measurement scale factor $(M<1)$ | -- | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 84 | Pressure measurement bias ( $M<1$ ) | $\mathrm{kg} / \mathrm{km}-\mathrm{s}^{2}$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 85 |  |  |  |  |  |
| 86 |  |  |  |  |  |
| 87 |  |  |  |  |  |
| 88 |  |  |  |  |  |
| 89 |  |  |  |  |  |
| 90 |  |  |  |  |  |
| 91 | Temperature measurement scale factor | -- | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 92 | Temperature measurement bias | ${ }^{\circ} \mathrm{K}$ | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 93 |  |  |  |  |  |
| 94 |  |  |  |  |  |
| 95 |  |  |  |  |  |
| 96 | Parachute $C_{D}$ scale factor | -- | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 97 | Parachute $C_{D}$ bias | -- | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 98 |  |  |  |  |  |
| 99 |  |  |  |  |  |
| 100 |  |  |  |  |  |
| 101 | Initial altitude $h_{0}$ error | km |  |  |  |
| 102 | Initial velocity $v_{0}$ error | km/s | $\checkmark$ |  |  |
| 103 | Initial flightpath angle $\gamma_{0}$ error | deg | $\checkmark$ |  |  |
| 104 | Initial downrange angle $\phi_{0}$ error | deg | $\checkmark$ |  |  |
| 105 |  |  |  |  |  |
| ${ }^{*} \mathrm{C}(105)$ is used as an internal variable for computing sensitivity matrices associated with the 5 th state variable in mode $A$. |  |  |  |  |  |
|  |  |  |  |  |  |

Table II-1 (Cont)

| j | C(j) | Units | Data Generator | Mode A | Mode B |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 106 | Initial angular velocity $\omega_{0}$ error | deg/s | $\checkmark$ |  |  |
| 107 |  |  |  |  |  |
| 108 |  |  |  |  |  |
| 109 |  |  |  |  |  |
| 110 |  |  |  |  |  |
| 111 | Station 1 altitude bias | km | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 112 | Station 1 latitude bias | rad | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 113 | Station 1 longitude bias | rad | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 114 | Station 2 altitude bias | km | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 115 | Station 2 latitude bias | rad | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 116 | Station 2 longitude bias | rad | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 117 | Station 3 altitude bias | km | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 118 | Station 3 latitude bias | rad | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 119 | Station 3 longitude bias | rad | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 120 |  |  |  |  |  |
| 121 |  |  |  |  |  |
| 122 |  |  |  |  |  |
| 123 |  |  |  |  |  |
| 124 | Gyro (ARU) scale factor | -- | $\checkmark$ | $\checkmark$ |  |
| 125 | Gyro (ARU) drift error | $\mathrm{rad} / \mathrm{s}$ | $\checkmark$ | $\checkmark$ |  |
| 126 |  |  |  |  |  |
| 127 |  |  |  |  |  |
| 128 |  |  |  |  |  |
| 129 |  |  |  |  |  |
| 130 |  |  |  |  |  |
| 131 |  |  |  |  |  |
| 132 |  |  |  |  |  |
| 133 |  |  |  |  |  |
| 134 |  |  |  |  |  |
| 135 |  |  |  |  |  |
| 136 |  |  |  |  |  |
| 137 |  |  |  |  |  |
| 138 |  |  |  |  |  |
| 139 |  |  |  |  |  |
| 140 | Initial attitude $\theta_{0}$ error* | deg | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 141 |  |  |  |  |  |
| 142 |  |  |  |  |  |
| 143 |  |  |  |  |  |
| 144 |  |  |  |  |  |
| 145 |  |  |  |  |  |
| $\begin{aligned} & { }^{*} \mathrm{C}(140) \text { may only appear in data generator namelist; index } 140 \text {, however, may appear } \\ & \text { in any parameter augmentation list in modes } A \text { and } B \text {. } \end{aligned}$ |  |  |  |  |  |
|  |  |  |  |  |  |

Table II-1 (Cont)

| j | C(j) | Units | Data Generator | Mode A | Mode B |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 146 |  |  |  |  |  |
| 147 |  |  |  |  |  |
| 148 |  |  |  |  |  |
| 149 |  |  |  |  |  |
| 150 |  |  |  |  |  |
| 151 | Altitude $h_{\alpha_{1}}$ bias | km | See | c(2) |  |
| 152 | Altitude $h_{\alpha_{2}}$ bias Mole Fraction | km | $\checkmark$ |  | $\checkmark$ |
| 153 |  | km | $\checkmark$ |  | $\checkmark$ |
| 154 | Altitude $h_{\alpha_{4}}$ bias | km | $\checkmark$ |  | $\checkmark$ |
| 155 | Altitude $h_{\alpha_{5}}$ bias | km | $\checkmark$ |  | $\checkmark$ |
| 156 | Mole fraction $\alpha(1,1)$ bias | -- | $\checkmark$ |  | $\checkmark$ |
| 157 | $\alpha(2,1)$ | -- | $\checkmark$ |  | $\checkmark$ |
| 158 | $\alpha(3,1) \quad$ at $h_{\alpha_{1}}$ | -- | $\checkmark$ |  | $\checkmark$ |
| 159 | $\alpha(4,1) \quad{ }^{\alpha}$ | -- | $\checkmark$ |  | $\checkmark$ |
| 160 | $\alpha(5,1) \quad\{$ | -- | $\checkmark$ |  | $\checkmark$ |
| 161 | $\alpha(1,2) \quad\{$ | -- | $\checkmark$ |  | $\checkmark$ |
| 162 | $\alpha(2,2)$ | --' | $\checkmark$ |  | $\checkmark$ |
| 163 | $\alpha(3,2) \quad$ at $h_{\alpha}$ | -- | $\checkmark$ |  | $\checkmark$ |
| 164 | $\alpha(4,2) \quad{ }_{2}$ | -- | $\checkmark$ |  | $\checkmark$ |
| 165 | $\alpha(5,2) \quad J$ | -- | $\checkmark$ |  | $\checkmark$ |
| 166 | $\alpha(1,3) \quad$, | -- | $\checkmark$ |  | $\checkmark$ |
| 167 | $\alpha(2,3)$ | -- | $\checkmark$ |  | $\checkmark$ |
| 168 | $\alpha(3,3) \quad$ at $h_{\alpha_{3}}$ | -- | $\checkmark$ |  | $\checkmark$ |
| 169 | $\alpha(4,3) \quad \alpha_{3}$ | -- | $\checkmark$ |  | $\checkmark$ |
| 170 | $\alpha(5,3)$ | -- | $\checkmark$ |  | $\checkmark$ |
| 171 | $\alpha(1,4) \quad\}$ | -- | $\checkmark$ |  | $\checkmark$ |
| 172 | $\alpha(2,4)$ | -- | $\checkmark$ |  | $\checkmark$ |
| 173 | $\alpha(3,4) \quad$ at ${\stackrel{h}{\alpha_{4}}}$ | -- | $\checkmark$ |  | $\checkmark$ |
| 174 | $\alpha(4,4) \quad{ }_{4}$ | -- | $\checkmark$ |  | $\checkmark$ |
| 175 | $\alpha(5,4) \quad\{$ | -- | $\checkmark$ |  | $\checkmark$ |
| 176 | $\alpha(1,5) \quad\{$ | -- | $\checkmark$ |  | $\checkmark$ |
| 177 | $\alpha(2,5)$ | -- | $\checkmark$ |  | $\checkmark$ |
| 178 | $\alpha(3,5) \quad$ at $h_{\alpha_{5}}$ | -- | $\checkmark$ |  | $\checkmark$ |
| 179 | $\alpha(4,5) \quad{ }_{5}$ | -- | $\checkmark$ |  | $\checkmark$ |
| 180 | $\alpha(5,5) \quad \int$ | -- | $\checkmark$ |  | $\checkmark$ |

Tabie II-1 (Concl)

| j | C(j) | Units | Data Generator | Mode A | Mode |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 181 | Altitude $h_{1}$ bias | km | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 182 | Wind $W_{1}$ bias | km/s | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 183 | $h_{2}$ | km | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 184 | $\mathrm{W}_{2}$ | km/s | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 185 | $h_{3}$ | km | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 186 | $w_{3}$ | km/s | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 187 | $h_{4}$ | km | , | $\checkmark$ | $\checkmark$ |
| 188 | $w_{4}$ | km/s | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 189 | $h_{5} \quad$ Wind Profile | km | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 190 | $w_{5}$ | km/s | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 191 | $h_{6}$ | km | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 192 | $w_{6}$ | km/s | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 193 | $h_{7}$ | km | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 194 | $w_{7}$ | km/s | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 195 | $h_{8}$ | km | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 196 | ${ }^{6}$ | km/s | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 197 | $h_{9}$ | km | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 198 | $W_{9}$ | km/s | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 199 | $h_{10}$ | km | $\checkmark$ | $\checkmark$ | $\checkmark$ |
| 200 | $\mathrm{w}_{10} \quad$ | km/s | $\checkmark$ | $\checkmark$ | $\checkmark$ |

Any number of actual errors can be defined in the data generator namelist.

The $C(j)$ variables can also be used to aiter the nominal aerodynamic characteristics of the entry vehicle. Currently all aerodynamic tables are defined in the EL $\emptyset C K$ DATA subroutine. One could, of course, remove the existing aerodynamic tables from BLøCK DATA and replace them with the desired aerodynamic tables. This, however, is a cumbersome task that ia not really required until the aerodynamic characteristics of a articular vehicle have been finalized. For preliminary studies it is far easier to manipulate certain $C(j)$ variables in such a way that the existing BL ${ }^{(1) C K}$ DATA aerodynamic tables approximate the desired aerodynamic tables. For example, the $C_{A}$ table can be modified by using the $C_{A}$ scale factor $C(20)$ and the scale factor bias C(16). Suppose that $C(20)=.9$ and $C(16)=-.1$ would transform the existing $C_{A}$ table to the desired $C_{A}$ table. Then if there were no actual $C_{A}$ errors, one would simply insert $C(20)=.9$ and $C(16)=-.1$ in the data generator (and reconstructor) namelist. If, however, actual errors are defined, say $a+1 \% C_{A}$ scale factor error and a $C_{A}$ bias of .03 , then one would insert

$$
C(20)=.9(1.01)=.909
$$

and

$$
C(16)=-.1+.03=-.07
$$

in the data generator namelist. $C(20)=.9$ and $C(16)=-.1$ would still appear in the reconstructor namelist.

## 3. Restrictions

A successful data generator run depends on selection of proper values for namelist v riables DT, QSALT, QSDT, and ØDB. Improper values can lead to integrator instability in the data generator. Since integrator step size DT is used to integrate both translational. and rotational equations of motion, DT must be chosen small enough to prevent instability or inaccuracies in the integration of the rotational equations, but large enough to avoid exorbitant computational time. High-frequency rotational oscillations, which are likely to occur in the maximum dynamic pressure regime, would require extremely small values of DT . To circumvent this problem, the variable $\emptyset D B$ has been defined. This variable represents the
maximum dynamic pressure permitted for the integration of the complete set of equations of mocion. Whenever dynamic pressure exceeds $\emptyset \mathrm{DB}$, the motion of the entry vehicle is assumed to be described by the point mass equations of motion so the rotational equations of motion need not be integrated. This same approximation is currently employed whenever the parachute is deployed.

Another type of integratcr instability can occur during the terminal velocity regime when $|\dot{v}| \ll 1$. To avoid using very small integration step sizes to prevent this instability, an option for using the quasi-static dynamic model has been developed. When the quasi-static model is used, the $\dot{v}$ equation is not integrated and velocity is computed using equation (II-18). The user sets QSALT to the altitude at which the quasi-static model is to be used, and QSDT to the step-size to be used in the integration of the quasistatic equations of motion. QSDT can be chosen up to 10 times larger than DT, depending, of course, on DT and the particular entry problem. The user should be certain that QSALT is chosen so the quasi-static assumptions are satisfied over the entire altitude range from 0 . to QSALT. The quasi-static assumptions are (1) $|\dot{v}| \ll 1$, and (2) $\gamma \doteq-90^{\circ}$. Since the vehicle motion normally violates the quasi-static assumptions for a few minutes after parachute release, it is recommended that the restriction QSALT < HR < HD be applied.

## C. RECONSTRUCTION PROGRAM INPUT VARIABLE DEFINITIONS

## 1. Namelist Variable Definitions

The namelist variables appearing in the reconstruction program namelist ERAN and read from subroutine SETUP are defined in the following subsections according to several categories. Many of these variables that are identical to those appearing in the data generator namelist are not defined. Refer to Section B. 1 for their definitions. As in Section B.1, preset values of namelist variables are enclosed in parentheses, and required input units are specified in the last column.

## a. Trajectory Variables

| XN(1) thru XN(4) | See Section B. 1 |  |
| :---: | :---: | :---: |
| XN (5) | Initial nominal ambient pressure; required only for mode A (0.) | millibars |
| THETI | Initial nominal vehicle attitude angle $\theta$; required only for mode $A$ (0.) | deg |
| $\begin{aligned} & \mathrm{XO}(1) \\ & \text { thru } \\ & \mathrm{XO}(5) \end{aligned}$ | Initial original nominal vehicle state. $\mathrm{XO}(\mathrm{I})$ corresponds to $\mathrm{XN}(\mathrm{I})$ above for $I=1,2$, ..., 5 | -- |
| ICøضR | See Section B. 1 |  |
| ECLINC | See Section B. 1 |  |
| ECLøNG | See Section B. 1 |  |
| PHIR | See Section B. 1 |  |
| TC | See Section B. 1 |  |
| TF | See Section B. 1 |  |
| IYR <br> thru | See Section B. 1 |  |
| SECSI |  |  |
| $\operatorname{EDN}(1)$ | Initial vehicle altitude estimate $\delta \hat{h}$ (0.) | km |
| EDN (2) | Initial vehicle velocity estimate $\delta \hat{\mathrm{v}}$ (0.) | km/s |
| EDN (3) | Initial vehicle flightpath angle estimate $\delta \hat{\gamma}$ (0.) | rad |
| EDN(4) | Initial vehicle downrange angle estimate $\delta \phi$ (0.) | rad |
| EDN(5) | Initial ambient pressure estimate $\delta \hat{p}$ required only for mode $A$ (0.) | $\mathrm{kg} / \mathrm{km}-\mathrm{s}^{2}$ |


| QEDN | Initial solve-for parameter vector estimate. Order of elements must correspond to order of elements in LISTQ. Units are the same as internal units ( $10 * 0$. ) |  |
| :---: | :---: | :---: |
| DT | Nonfunctional |  |
| QSDT | Integration step size used after time QST; input only if data generator is not run | s |
| SDT | Integration step size used in the data generator; should be input only if data generator is not run | s |
| QST | Time at which dynamic model is changed to quasi-static model. Computed in data generator and transmitted to reconstruction program if these two programs have been run in sequence. Should be input only if data generator is not run | s |
| TD | Time of parachute deployment as determined by the data generator. Should be input only if data generator is not run | s |
| TR | Time of parachute release as determined by data generator. Should be input only if data generator is not run | S |
| TEND | Time of next event. Should be input only if data generator is not run | s |

b. Planet and Atmosphere Variables - All planet and atmosphere variables defined in Section B.1 for the data generator namelist are also defined for the reconstruction program namelist, with the exception that variables NTPTS, ATMøS (1), and ATM ${ }^{(18)}$ (hrough ATMゆS (33) are not used when the reconstruction program is run in mode A. The following variable, which is not defined for the data generator namelist, appears in the reconstruction program namelist:

| GAMTBI. | Table of specific heat ratios as a |
| :--- | :--- |
|  | function of molecular weight. |
|  | GAMTBL(1) $=\mathrm{n}$, number of molecular |
|  | weight breakpoints; GAMTBL(2) througli. |
|  | GAMTBL $(1+2 \mathrm{n})$ define the correspond- |
|  | ing sequence of specific heat ratios. |
|  | Up to four breakpoints can be de- |
|  | fined (2., $0 ., 1000 ., 1.4,1.4)$ |

C. Entry Vehicle Variables - All entry vehicle variables defined in Section B.l for the data generator namelist are also defined for the reconstruction program namelist. The following variable, which is not defined for the data generator namelist, appears in the reconstruction program namelist:

| BKTBL | Table of $k$ (see equation (II-59)) as |
| :--- | :--- |
| a function of Mach number; required |  |
| only if angle of attack measurements |  |
| are scheduled. BKTBL has same struc- |  |
|  | ture as GAMTBL; up to nine breakpoints |
| can be defined (2., 0., 1000., -.922, |  |
|  | -.922 ) |

d. Measurement Variables - Most measurement variables defined in Section B.l for the data generator namelist are also defined for the reconstruction program namelist. Those not defined for the reconstruction program namelist are XSTEr, ZSTEP, TSTEP, VXQA, VZQA, THTQA, and NØFRAC. The following variables, which are not defined for the data generator namelist, appear in the reconstruction program namelist:

| CDEL | ```Logical variable that indicates if misalignment errors are to be treated (true) = true, misalignment errors will be treated false, misalignment errors will not. be treated``` | -- |
| :---: | :---: | :---: |
| NACCEL | Logical variable that indicates if the normal accelerometer is to be deleted (false) <br> $=$ true, normal accelerometer deleted false, normal accelerometer not deleted | -- |

```
NGYR\emptyset Logical variable that indicates if
gyro is to be deleted; applies only
to mode A (false)
= true, gyro deleted
    false, gyro not deleted
```

e. Parameter Augmentation Variables - Parameters appearing in the $C(j)$ table of Section B. 2 can be augmented to the entry vehicle state vector as either solve-for, dynamic-consider, measurementconsider, or dynamic/measurement-consider parameters. This is accomplished by inserting the index $j$ associated with parameter C(j) in one of the parameter lists defined below. Although the order of indices in a given list is arbitrary, once the order has been defined the related covariance matrix partitions (to be defined subsequently) must correspond to this oider.

| NQ | Number of solve-for parameters; must not exceed 10 (0) | -- |
| :---: | :---: | :---: |
| NU | Number of dynamic-consider parameters; must not exceed 20 (0) | -- |
| NV | Number of measurement-consider parameters; must not exceed 20 (0) | -- |
| NW | Number of dynamic/measurementconsider parameters; must not exceed 10 (0) | -- |
| LISTQ | List of augmented solve-for parameters | -- |
| LISTU | List of augmented dynamic-consider parameters | -- |
| LISTV | List of augmented measurementconsider parameters | - |
| LISTW | List of augmented dynamic/measurementconsider parameters | -- |

f. Initial State and Augmented Parameter Covariance Matrices

P State covariance matrix. Structure of -(square) matrix must correspond to the order of state variables $\mathrm{XN}(1), \mathrm{XN}(2)$, $\ldots, \mathrm{XN}(\mathrm{n})$, where $\mathrm{n}=5$ for mode A , and $n=4$ for mode B. Units for $P$ and all remaining covariance variables are appropriate combinations of internal units (km, kg, s, rad). All covariance variables are preset to zero

Q Solve-for parameter covariance matrix. -Structure of (square) matrix must correspond to the order of parameter indices appearing in LISTQ

DU Dynamic-considet parameter covariance -matrix. Since matrix is assumed diagonal, DU is a one-dimensional array of variances whose order must correspond to the order of indices appearing in LISTU

DV Measurement-consider parameter covari- -ance matrix (diagonal). DV is a onedimensional array of variances whose order must correspond to LISTV

DW Dynamic/measurement-consider parameter -covariance matrix (diagonal). DW is a one-dimensional array of variances whose order must correspond to LISTW

CXQ State/solve-for parameter covariance matrix. Dimension $n \times N Q$

CXU State/dynamic-consider parameter covariance matrix. Dimension $n \times N U$

State/measurement-consider parameter covariance matrix. Dimension $n \times N V$

State/dynamic/measurement-consider parameter covariance matrix. Dimension $\mathrm{n} \times \mathrm{NW}$

| CQU | Solve-for parameter/dynamic-consider parameter covariance matrix. Dimension NQ x NU | - |
| :---: | :---: | :---: |
| CQV | Solve-for parameter/measurementconsider parameter covariance matrix. Dimension NQ x NV | -- |
| CQW | Solve-for parameter/dynamic-measurement-consider parameter covariance matrix. Dimension NQ x NW | -- |
| SDMWT | Molecular weight standard deviation used in mode A derived estimation process (this variable will be deleted when the option for augmenting mole fraction parameters in mode $A$ has been developed) | -- |

g. Measurement Noise Statistics

REDRR2 Logical variable used to compute altimeter noise (false) $=$ true, use user-specified measurement noise
false, compute measurement noise in subroutine $\emptyset B S M$

RR Three-dimensional measurement noise variance array: 1st index $I$, indicates measurement type; 2nd index $J$, measurement component; 3 rd index K , regime. Only the accelexometer measurement currently requires more than one component (axial and normal). Only the pressure measurement currently depends on the (Mach number) regime. RR values represent variances whose units are assumed to be internal units. The correspondence between index $I$ and measurement type is indicated:
$I=1$, accelerometer (mode B only)
2, gyro (nonfunctional currently)
3, altimeter
4, stagnation pressure
5, stagnation temperature

```
6, angle of attack (mode A only)
11, doppler, station 1
12, range, station l
13, doppler, station 2
14, range, station 2
15, doppler, station 3
16, range, station 3
```

Two-dimensional actual measurement noise standard deviation array: lst index I, indicates measurement type; 2nd index J, measurement component. Only the accelerometer measurement currently requires more than one component (axial and normal). SD values represent standard deviations whose units are assumed to be internal units. The correspondence between index $I$ and measurement type is identical to that for the preceding RR array
h. Other Variables

| LTR2 | ```Logical mode A variable (true) = true, mode A false, rot mode A``` |
| :---: | :---: |
| LTR1 | ```Logical mode B variable (false) = true, mode B false, not mode B``` |
| ICNTR | Measurement print code. Print will occur after every ICNTR measurements or groups of simultaneous measurements |
| MCNTR | Counter on the TMN and MCDDE event arrays. Whenever MCNTR reaches 250, another batch of 250 events is read from tape 20 into these arrays and MCNTR is reset to 1 . Should be input only if restarting |
| RESTRT | See Section B. 1 |
| NMEAS | Counter on the number of measurements taken up to the current time. Nonfunctional currently |

## 2. Use of $C(j)$ Table in Reconstruction Program

Just as the $C(j)$ variables defined in Table II-1 I can be used to specify actual errors to be incorporated in nominal values of the variables in the data generator, so can the $C(j)$ variables be used in the reconstruction program to change previously defined nominal values to new nominal values. Normally this option is not employed, however, since nominal scale factors are usually set to 1. and nominal biases are usually set to zero, if the $C(j)$ associated with the aerodynamic coefficients are selected in the data generator to alter the nominal aerodynamic tables appearing in BLØCK DATA, the same $C(j)$ variables must be used in the reconstruction program to change the existing aerodynamic tables in BLØCK DATA to the desired nominal values (see Section B. 2 for an example).

The primary use of the $C(j)$ table in the reconstruction piogram lies in parameter augmentation. The final two columns in Table II-1 indicate which parameters can be augmented to the state vector in each of the two reconstruction modes. Augmentation is accomplished by inserting the index $j$ of the appropriate parameter $C(j)$ in one of the four parameter lists. For example, if the user wished to treat the $C_{A}$ scale factor as a solve-for parameter, the $C_{N}$ scale factor and the vehicle mass bias as dynamic-consider parameters, doppler biases for all three tracking stations as measurementconsider parameters, and the axial accelerometer scale factor as a dynamic/measurement-consider parameter, the following should appear in the namelist:

| LISTQ $=20$, | NQ $=1$ |
| :--- | :--- |
| LISTU $=21,30$, | NU $=2$ |
| LISTV $=67,68,69$, | NV $=3$ |
| LISTW $=51$, | NW $=1$. |

Whether a consicier parameter is to be treated as a dynamic-, measurement-, or dynamic/measurement-consider parameter is a function of the measurement types scheduled and the reconstruction mode. If in doubt, it is always safe to treat the questionable parameter as a dynamic/measurement-consider parameter and insert the associated index $j$ in LISTW. If NACCEL is true (i.e., when the normal accelerometer is deleted), $C(53)$ and $C(54)$ cannot be treated as solve-for parameters although it is still meaningful to treat them as consider parameters. If NGYR $\emptyset$ is true (i.e., when the gyro is deleted in mode A), C(124), $C(125)$, and $C(140)$ can only be treated as consider parameters.

## 3. Measurement/Event Types and Schedules

Measurements and events are input with fixed field formats immediately after the PLTVAR namelist section. Each card contains the following formats and inforanation:

| F10.3 | F10.3 | F10.3 | I10 |
| :--- | ---: | ---: | ---: |
| START | TIMEND | TIMDIF | CøDE |

where

```
START is the time (in seconds) to start a measurement or
            event,
TIMEND is the time (in seconds) to end a measurement or
            event,
TIMDIF is the time (in seconds) between measurements or
            events,
C\emptysetDE is the type of measurement or event to be processed, and
        can take on any of the following values,
        = 1 accelerometer measurement
        = 2 gyro measurement (not functiona1)
        = 3 altimeter measurement
        =4 pressure measurement
        = 5 temperature measurement
        = 6 angle of attack measurement
        = 7 to 10 not used
        = 11 prediction event (not functional)
        = 12 quasi-filtering event
        = 13 print increment set event
        = 14 set internally
        = 15 set internally
```

$=16$ set internally
= 17 print without measurement
= 18 set internally
$=19,20$ not used
$=21$ range-rate measv rement from station 1
$=22$ range measurement from station 1
$=23$ range-rate measurement from station 2
$=24$ range measurement from station 2
$=25$ range-rate measurement from station 3
$=26$ range measuremert from station 3.
The last card of the measurement schedule must have a START value of 100000. to signify the end of measurement input.

## 4. Restrictions

Restrictions on the use of the quasi-static dynamic model and the selection of intecration step sizes and measurement schedules in the LTR reconstruction program are discussed in this section.

The use of the quasi--static dynamic model in the reconstruction program is subject to the same restrictions that apply in the data generator program. In fact, the values selected for DT and QSDT in the data generator must be small enough that the step sizes of $2 * D T$ and $2 *$ QSDT do not lead to integrator instabilities in the reconstruction program because the step sizes used in the reconstriction program must be twice the size of the corresponding step sizes used in the data generator.

The quasi-static dynamic model should be used with care when a wind model has been defined since the quasi-static assurptions are not always satisfied when the entry vehicle encounters winds of sufficient magnitude. This restriction applies to the data generator as well as to the two modes of the reconstruction program.

Since the LTR program performs trajectory reconstruction utilizing data already generated (by the data generator), the user cannot arbitrarily select integration step sizes. Since the present integrator is a two-step Runge-Kutta package, the basic step size in the reconstructor must be an even multiple of the basic step size used in the data generator. In addition, the use of the quasistatic dynamic model introduces more problems:

1) The switch to the quasi-static model in the data generator must occur at a time that corresponds to an even multiple of the basic integration step size so the two-step RungeKutta integrator can be used in the reconstructor;
2) The data generator quasi-static integration step size QSDT must be a multiple of the basic data generator step size DT to insure proper measurement processing in the reconstructor;
3) The switch to the quasi-static model in the data generator must occur at a time that corresponds to an even multiple of the quasi-static model step size to be used by the data generator to prevent improper measurement sequencing in the reconstructor.

The user must also sequence measuremente in the reconstructor with care. If, for example, the user wished to process altimeter measurements from 5 seconds to 100 seconds every 1 second, the reconstructor could not integrate with a step size of 0.75 seconds, either before or after a change to the quasi-static model. The reconstructor step size of 0.75 seconds would require a data generator step size of 0.375 seconds for the same time period and therefore no altimeter data would be available to the reconstructor at 5.000 seconds. The user could choose a reconstructor step size for the quasi-static model of 1.0 second and a basic step size of 0.1 second, thereby requiring step sizes in the data generator of 0.5 second and 0.05 second for the quasi-static and basic models, respectively. This would ensure that all necessary data had been calculated in the data generator. The general rule, then, is that the measurament times must be at even multiples of the data generator quasi-static model step size.

An additional user problem concerns state transition matrices. The assumption of linearity is not valid for all integration step sizes. The user, for example, could not expect linear matrices over an interval of 60 seconds but can assume linearity uver a 1 -second interval. Given the integration step sizes used by the data generator, the measurement sequencing subroutine SCHED will allow an interval between measurements or events of no more than 10 times the step size used at a given time point. The user must therefore determine what step sizes can be used in both the $i: 1 c$ and quasistatic dynamic models that will not violate linea. assumptions.

The user restrictions on integration step siz. are summarized as:

1) The integration step size in the reconstructor must be an even multiple of the scep size used in the data generator, regardless of the dynamic model chosen. The program currently sets the step sizes internally in the reconstructor to twice the step sizes used in the data generator;
2) In the data generator the quasi-static step size QSDT must be a multiple of the basic integration step size DT;
3) Measurement and/or event times (see subroutine SCHED) must be at even multiples of the data generator quasi-static model integration step size QSDT;
4) Integration step sizes must be chosen so the linearity assumption used in the computation of state transition matrices in the reconstructor is not violated.

If the quasi-static dynamic model is not used, the restrictions are fewer:

1) Reconstructor step sizes are still even multiples of data generator step sizes;
2) Measurement/event times must occur at even multiples of the data generator integration step size DT;
3) State transition matrix linearity must still be considered when choosing step sizes.

## III. OUTPUT DESCRIPTION

## A. DATA GENERATOR OUTPUT DESCRIPTION

The initial data generator output consists of the following:

1) Namelist ERAN;
2) Initial actual state vector -- altitude, velocity, flightpath angle, downrange angle, attitude angle, angular velocity, unquantized axial VRU output, unquantized normal VRU output, unquantized ARU output, ambient pressure;
3) Planet and vehicle constants;
4) Initial and final trajectory times in seconds;
5) Actual planet atmosphere model.

At each trajectory printout time, the following output is printed:

1) Trajectory time and integration step size in seconds -entry phase;
2) Actual state vector;
3) Actual state vector derivatives;
4) Actual trajectory, atmosphere, and aerodynamic parameters, i.e.,
a) Vehicle relative velocity,
b) Horizontal wind velocity,
c) Dynamic pressure,
d) Molecular weight of atmosphere,
e) Ambient temperature of atmosphere,
f) Ambient pressure of atmosphere,
g) Density of atmosphere,
h) Angle of attack,
i) Aerodynamic coefficient, $C_{A}$,
j) Aerodynamic coefficient, $C_{N}$,
k) Mach number,
5) Axial aerodynamic force (does not include parachute eifect),
m) Normal aerodynamic force (does not include parachute effect),
n) Center of pressure location along $x$ body axis,
o) Aerodynamic damping moment acceleration (does not include parachute effect),
p) Angle between inertial and relative velocity vectors,
q) Aerodynamic coefficient, $\mathrm{C}_{\mathrm{M}_{\mathrm{q}}}$,
r) Aerodynamic damping moment (does not include parachute effect),
s) Local acceleration of gravity,
t) Total axial aerodynamic acceleration,
u) Total normal aerodynamic acceleration;
6) Actual measurements,
a) Axial accelerometer ( $\mathrm{km} / \mathrm{s}^{2}$ ),
b) Normal accelerometer ( $\mathrm{km} / \mathrm{s}^{2}$ ),
c) Stagnation pressure ( $\mathrm{kg} / \mathrm{km}-\mathrm{s}^{2}$ ),
d) Rate gyro (rad/s),
e) Radar altimeter (km),
f) Stagnation temperature ( ${ }^{\circ} \mathrm{K}$ ),
g) Range from three earth-based tracking stations,
h) Range-rate from three earth-based tracking stations,
i) Refraction ef. cts on range and range-rate measurements (not fun ional currently);
7) Auxiliary trajectory information (computed in subroutine AUXIL),
a) Communication angle,
b) Angle between entry plane and plane of the sky,
c) Latitude/longitude ground trace relative to the planetocentric equatorial, subsolar orbital plane, and planetocentric geographic coordinate systems.
B. RECONSTRUCTION PROGRAM OUTPUT DESCRIPTION

The initial reconstruction program output consists of the following:

1) Namelist ERAN;
2) Array of measurement noise variances for all measurement types;
3) Initial nominal vehicle state vector -- altitude, velocity, flightpath angle, downrange angle;
4) Planet and vehicle constants;
5) Initial and final trajectory times in seconds;
6) Number and list of solve-for parameters (appear only if $N Q \neq 0$ );
7) Number and list of dynamic-consider parameters (appear only if $N U \neq 0$;
8) Number and list of measurement-consider parameters (appear only if $\mathrm{NV} \neq 0$ );
9) Number and list of dynamic/measurement-consider parameters (appear only if $N W \neq 0$ );
10) Primary state covariance matrix -- primary state refers to the unaugmented state used in the recursive estimation process;
ii) Solve-for parameter covariance matrix (appears only if NQ $\neq 0$ ):
11) Vector of dynamic-consider parameter variances (appears only if $\mathrm{NV} \neq 0$ );
12) Vector uf measurement-consider parameter variances (appears only if NV $\neq 0$ );
13) Vector of dynamic/measurement-consider parameter variances (appears only if NW $\neq 0$ );
14) Array of nominal $C_{j}$ 's;
15) Initial original nominal, most recent nominal, and actual vehicle state vectors;
16) Initial most recent nominal and actual atmosphere state vectors -- ambient pressure, density, ambient temperature molecular weight (appear only in mode A);
17) Initial actual and reconstructed VRU and ARU data -attitude angle, angular velocity, axial nongravitational accelcation, normal nongravitational acceleration, normal nongravitatiunal acceleration (appear only in mode A);
18) Entry parameters based on most recent nominal trajectory;
19) Initial estimated and actual vehicle state deviations from most recent nominal and initial vehicle state estimation errors;
20) Initial estimated and actual atmosphere state deviations from mosi recent nominal and initial atmosphere state estimation errors (appear only in mode A);
21) Initial estimated and actual solve-for parameter deviations from most recent nominal and initial solve-for parameter estimation errors;
22) Initial estimated solve-for parameter deviations from original nominal;
23) Additional entry parameters based on most recent nominal trajectory;
24) Initial primary state, solve-for parameter, and consider parameter correlation maïrix partitions. Stariard deviations appear along diagonals of the symmetric partitions and" correlation coefficients comprise the remaining elements;
25) Measurement and event data cards;
26) Measurement schedule;
27) Event schedule;
28) Number of measurements to be processed for each measurement type;
29) Number of events to be executed for each event type.

When measurement information is to be printed, the output summarized below will be available. Items 1 through 20 also appear for a type 17 or when a "print without measurement" event occurs:

1) Measurement type and trajectory time;
2) Message "quasi-static model" if quasi-static dynamic model is being used at current trajectory time;
3) Original nominal, most recent nominal, and actual vehicle state vectors;
4) Most recent nominal and actual atmosphere state vectors (appear only in mode A);
5) Actual and reconstructed VRU and ARU data (appear only in mode A);
6) Entry parameters based on most recent nominal trajectory;
7) Estimated and actual vehicle state deviations from most recent nominal and vehicle state estimation errors immediately before processing the measurement;
8) Estimated and actual atmosphere state deviations from most recent nominal and atmosphere stat estimation errors immediately before processing the measurement (appear only in mode A);
9) Estimated and actual solve-for parameter deviations from most recent nominal and solve-for parameter estimation errors immediately before processing the measurement;
10) Estimated solve-for parameter deviations from original nominal immediately before processing the measurement;
11) Additional entry parameters based on most recent nominal trajectory;
12) State transition matrix for primary state vector;
13) Remaining state transition matrix partitions and lists of all snlve-for and consider parameters. The order of elements in each parameter list corresponds to the order of columns in each state transition matrix partition;
14) Diagonal of dynamic noise covariance matrix;
15) Primary state, solve-for parameter, and consider parameter correlation matrix partitions immediately before processing the measurement. Standard deviations appear along diagonals of the symmetric partitions and correlation coefficients comprise the remaining elements;
16). Measurement noise covariance matrix;
16) Primary state and solve-for parameter gain matrices;
17) Deasity and temperature estimation error standard deviations immediately before prucessing the measurement (appear only in mode A and then only if MACHN $\emptyset$ is true. MACHN $\varnothing$ is an internally set logical that is set true when sufficient aerodynami: decelerations have been attained to make density and temperature estimation feasible);
18) Measurement residual covariance matrix;
19) Nominal measurement;
20) Observation matrix partitions;
21) Estimated and actual measurement deviations from nominal and actual measurement residuals;
22) Estimated and actual vehicle state deviations from most recent nominal and vehicle state estimation errors immediately after processing the measurement;
23) Estimated and actual atmosphere state deviations from most recent nominal and atmosphere state estimation errors immediately after processing the measurement (appear only in mode A);
24) Estimated and actual solve-for parameter deviations from most recent nominal and solve-for parameter estimation errors immediately after processing the measurement;
25) Primary state, solve-for parameters, and consider parameter correlation matrix partitions immediately after processing the measurement. Standard deviations appear along diagonals of the symmetric partitions and correlation coefficients comprise the remaining elements;
26) Measurement noise covariance matrix (redundant; identical to item 17);
27) Primary state and solve-for parameter gain matrices (redundant; identical to item 18);
28) Density and temperature estimation error standard deviations immediately after processing the measurement (appear only in mode $A$ and then only if MACHN $\varnothing$ is true);
29) Actual measurement noise;
30) Actual measurement noise standard deviations.

Quasi event output consists of the following:

1) Original nominal, most recent nominal, and actual vehicle state vectors immediately after quasi event has been executed;
2) Most recent nominal and actual atmcsphere state vectors immediately after quasi event has been executed (appear only in mode A );
3) Estimated and actual vehicle state deviations from most recent nominal and vehicle state estimation errors;
4) Estimated and actual ambient pressure deviation from most recent nominal and ambient pressure estimation error (appear only in mode A);
5) Most recent nominal solve-for-parameters immediately after quasi event has been executed;
6) Estimated and actual solve-for parameter deviations from most recent nominal and solve-for parameter estimation errors immediately after quasi event has been executed;
7) Estimated solve-ior parameter deviations from original nominal.

## IV. SAMPLE CASES

## A. LTR MODE A SAMPLE CASE

The sample case presented here demonstrates the application of the mode A reconstruction process to a Venusian entry problem, and is primarily presented to aid the user in defining the required input data and interpreting the resulting output. Before the reconstruction program can be run, the "actual" trajectory, atmosphere; and measurements used in the reconstruction program must be available from a previous data generator run. For this reason, the input and output for the associated data generator run is presented first.

## 1. Data Generator

a. Input Discussion - The input data for the data generator consist of the following namelist ERAN cards:

```
\(\mathrm{XN}=248 ., 11.08,-38.8,0 .,-38.8,0 .\),
DT=.1, TF=500.,
IYR=1977, IM \(\mathbf{I}=5\), IDAY=16, \(I H R=23, ~ I M I N=54\),
SECSI=41.,
```



```
\(\emptyset D B=15 . E+5\),
```

$\mathrm{G} \emptyset=8.867 \mathrm{E}-3, \mathrm{RM}=6050 ., \emptyset \mathrm{MEG}=2.997 \mathrm{E}-7, \mathrm{MU}=3.2486 \mathrm{E} 5$,
$\operatorname{ATM} \emptyset S(1)=1.104 \mathrm{E} 10$,
ATMøS (18) $=0 ., 60$., 115., 125., 137., 175., 2*0.,
$\operatorname{ATM} \not \mathrm{S}(26)=738 ., 260 ., 170 ., 2 * 210 ., 710$. .
NTPTS $=6$, NMPTS=4,
WDTBL $=2 ., 0 ., 10 ., 0 ., 0 .$,
TERHT=.FALSE., AGAM=1.4,
VMASS $=174 ., 122 ., 100 .$,
VSA $=1.474 \mathrm{E}-6$, .292E-6, .292E-6,
VDIA $=1.37 \mathrm{E}-3, .61 \mathrm{E}-3, .61 \mathrm{E}-3$,
$\mathrm{VRI}=1.7 \mathrm{SE}-5, .5 \mathrm{E}-5, .5 \mathrm{E}-5$,
$X G=0 ., X M=0, Z G=0 ., Z M M=0$,
$X S T E P=1.5 E-5, Z S T E P=1.5 E-5, T S T E P=.004$,
ICNTR=20,
RESTRT=.FALSE.,

```
C(101)=-10.,.025, .4, .5, C(140)=1.5,
C(16)=-.1, C(20)=1.034, 2.1, 4., .23,
C(51)=1.00066, C(53)=1.00066,
C(64)=.25,C(67)=1.E-6, C(71)=1.001, C(81)=1.01,
C(83)=1.01, C(91)=.99, C(96)=.97,
C(111)=1.2E-3, -1.3E-7, 5.E-7,
C(152)=6., C(156)=.03, -.03, C(161)=.01, -.01,.
```

The first group of cards defines the nominal entry conditions and certain integration variables. The initial nominal entry state is specified by the XN vector and the variables ICめØR through ECLØNG. These latter variables define the orientation of the entry plane, while XN defines the vehicle state in that plane. An integration step size of.$l$ second will be used in generation of the "actual" trajectory. Point-mass motion will be assumed whenever dynamic pressure $q$ exceeds 15 millibars, as is indicated by $\emptyset D B$. This is necessary to maintain integrator stability through the max q regime.

Planetary physical characteristics are specified by the next group of cards, including the planetary atmosphere model. This planetary atmosphere model is defined by the surface pressure ATM $\varnothing$ S(1), a seq̧uence of six temperature breakpoints defined by the ATM $\emptyset(18)$ vector, and the six corresponding temperatures defined by the ATMDS(26) vector. Mole fraction profiles are also required to complete definition of the atmosphere model. Since the desired mole fraction profiles are preset by the program, they need not appear in the above namelist.

The third group of cards specifies vehicle and certain instrumentation characteristics. The mass (VMASS), reference area (VSA), etc are given as three vectors, which correspond to the three available phases of entry in LTR--aeroshell, parachute, and terminal (with parachute released). However, since variables $H D$ and $H R$ do not appear in the above namelist, the entire sample case deals with only the aeroshell phase.

The final group of cards defines the "actual" dynamic and measurement errors and other differences between the "actual" and nominal models. Elements $C(101)$ through $C(104)$ define initial errors in the vehicle translational state and $C(140)$ defines an initial vehicle attitude error. Use of the aerodynamic coefficient C(J)s to alter preset nominal aerodynamic coefficient tables, as well as to define "actual" errors in the vehicle aerodynamic coefficients, has been explained in Chapter II of this section. In this partinular sample case $C(16), C(22)$, and $C(23)$ are used only to alter the
preset nominal coefficient tables; $C(20)$ is used only to specify an "actual" error; while $C(21)$ performs both functions. To convert the preset $C_{N}$ table to the desired table requires that the preset table be multiplied by a factor of 2. We also desire to introduce a $5 \%$ "actual" $\mathrm{C}_{\mathrm{N}}$ "cale factor error into this table. Thus we set

$$
C(21)=2(1.05)=2.1
$$

The remaining $C(J)$ elements are used to specify "actual" errors in certain sensors and in the nominal mole fraction profiles.
b. Output Discussion - Selected pages from the output of the data generator portion of this sample case appear in section $D$, where it is referred to as case A-1. The selected pages show the "actual" state and state derivatives, various vehicle and atmosphere parameters corresponding to this state, and "actual" values of all measurement types available in the LTR program at selected trajectory times. A trajectory time of 24 . seconds corresponds to mex q. Since dynamic pressure obviously exceeds $\emptyset \mathrm{DB}$, the pointmass ijnamic model was used to generate the information shown at 24. seconds. This also explains why ALPHA and the normal acceleration are zero at this point. The output for this sample case was generated at the CDC 6400/6500 computer at the Martin Marietta Corporation.

## 2. Reconstruction Program

a. Input Discussion - The input data for the reconstruction program consist of a narelist and a measurement/event schedule. The namelist, which is also entitled ERAN, consists of the following cards:

```
XN=248., 11.08, -38.8, 0., 5.4E-9,
X }\varnothing=248., 11.08, -38.8, 0., 5.4E-9
THETI=-38.8,TF=500.,
IYR=1977, IM }\emptyset=5, IDAY=16, IHR=23, IMIN=54, SECSI=41.,
IC\emptyset\emptysetR=3, PHIR=0., ECLINC=140.61, ECL\emptysetNG=68.2,
G\emptyset=8.86 7E-3, RM=6050. , \emptysetMEG=2.997E-7, MU=3.2486E-5,
WDTBL=2., 0., 10., 0., 0.,
TERHT=.FALSE.,
VMASS=174., 122., 100.,
VSA=1.474E-6,.292E-6,.292E-6,
VDIA=1.37E-3,.61E-3,.61E-3,
```

$\mathrm{VRI}=1.76 \mathrm{E}-5, .5 \mathrm{E}-5, .5 \mathrm{E}-5$, $\mathrm{XG}=0 ., \mathrm{XM}=0 ., \mathrm{ZG}=0 ., \mathrm{ZMM}=0 .$, ICNTR=1, RESTRT=.FALSE., NGYR $\varnothing=$. TRUE. , LTR2=.TRUE.,
$C(16)=-.1, C(21)=2 ., 4 ., .23$,

| $\mathrm{P}=200 .$, | $0 .$, | $0 .$, | $0 .$, | $0 .$, |
| ---: | :--- | :--- | :--- | :--- |
| $0 .$, | $2.5 \mathrm{E}-4$, | $0 .$, | $0 .$, | $0 .$, |
| $0 .$, | $0 .$, | $1.22 \mathrm{E}-3$, | $0 .$, | $0 .$, |
| $0 .$, | $0 .$, | $0 .$, | $3.234 \mathrm{E}-4$, | $0 .$, |
| $0 .$, | $0 .$, | $0 .$, | $0 .$, | $25 . \mathrm{E}-10$, |

$\mathrm{NV}=9, \operatorname{LISTV}=67,64,111,112,113,81,83,91,71$,
$\mathrm{DV}=2 . \mathrm{E}-12, .08, .15 \mathrm{E}-5, .19 \mathrm{E}-13, .33 \mathrm{E}-13,1 . \mathrm{E}-4,1 . \mathrm{E}-4,1 . \mathrm{E}-4,1 . \mathrm{E}-6$,
$\mathrm{NW}=5, \mathrm{LISTW}=140,20,96,51,53$,
$\mathrm{DW}=.25 \mathrm{E}-2,2.78 \mathrm{E}-4, .006, .1 \mathrm{E}-6, .1 \mathrm{E}-6$,
SDMNT=3.

REDRR2=.TRUE.,
$\operatorname{RR}(11,1,1)=1 . E-12, .001$,
$\operatorname{RR}(3,1,1)=.01,1 . E+8,1 ., \operatorname{RR}(4,1,2)=1.3 E+10$,
$\operatorname{SD}(11,1)=1 . E-7, .02$,
$S D(3,1)=.05,1 . E+5, .5,$.
The first group of cards defines the initial nominal primary state used in the mode $A$ reconstruction process. The first four elements of the XN vector define the vehicle translational state and are identical to the first four elements of the $X N$ vector appearing in the data generator namelist. The fifth element of the primary state and the fifth element of XN is the ambient pressure. The variable THETI defines the initial nominal vehicle attitude.

Planetary physical characteristics are specified by the next group of cards. Note that an atmosphere model does not appear since the mode A reconstruction process does not employ such a model.

The third group of cards is essentially the same as the third group appearing in the data generator namelist except for the addition of NGYR $\emptyset$ and LTR2. Setting NGYR $\emptyset$ true indicates that gyro measurements will not be processed. Setting LTR2 true indicates that the mode A reconstruction process will be used.

The four C(J) elements appear next and are used solely to alter the preset nominal aerodynamic coefficient tables.

The remaining cards define the statistics of the error sources acknowledged in the design of the filter. The P-array is the initial covariance matrix for the primiry state XN. The filter considers nine measurement-consider parameters and five dynamic/meas-urement-consider parameters. These parameters are defined in LISTV and LISTW, respectively, and their variances are given in the $D V$ and $D W$ vectors, respectively. The assumed molecular weight standard deviation is given by SDMWT. Measurement noise variances assumed by the filter are defined by the RR variables while the "actual" measurement noise standard deviations are defined by the SD variables.

The measurement/event schedule cards used in this sample case are listed.

| 1800. | 2000. | 20. | 3 |
| ---: | ---: | ---: | ---: |
| 60. | 2000. | 50. | 4 |
| 80. | 2000. | 100. | 5 |
| 1. | 2000. | 30. | 21 |
| 10. | 100. | 30. | 22 |
| 150. | 2000. | 200. | 22 |
| 200. | 200. | 10. | 12 |
| 350. | 350. | 10. | 12 |
| 700. | 700. | 10. | 12 |
| 1200. | 1200. | 10. | 12 |
| 1400. | 1400. | 10. | 12 |
| 100000. |  |  |  |

Not all these measurements and events will be processed in the sample case since $T F$ was set to 500 . in the previous namelist.
b. Output Discussion - Selected pages from the reconstruction program portion or this sample case appear in section $D$, where: it is referred to as case A-2. The measurement output for a range measurement at 10. seconds and a doppler measuremen: at 121. seconds is shown. The range measurement at 10 . secronds reduced altitude errors from 8.718 to 8.171 kilometers, flightpath angle errors from -.943 to $-.515^{\circ}$ and downrange angle errors from -.38 to $.090^{\circ}$. The velocity error increases slightly. Reconstructed ARU data are zero, and will remain zero since NGYR $\varnothing$ was set true in the namelist. Reconstructed VRU data are zero since sufficient axial aerodynamic deceleration has not yet developed. According to the data generator output at 10. seconds, the axial aerodynamic deceleration is only on the order of $10^{-7}$. However, a reconstructed axial
acceleration appears in the doppler measurement at 121. seconds. The reconstructed normal acceleration is still zero, and will remain so since the integrated normal acceleration never exceed the normal accelerometer quantum level ZSTEP. The output for this sample case was generated on the CDC 6400/6500 computer at the Martin Marietta Corporation.

## B. LTR MODE B SAMPLE CASE

The sample case presented here demonstrates the application of the mode $B$ reconstruction process to a Venusian entry problem, and is presented primarily to aid the user in defining required input data and interpreting the resulting output. As in Section A, the input and output of the associated data generator run is presented first.

## 1. Data Generator

a. Input Discussion - The input data for the data generator consists of the following namelist ERAN cards.

```
XN=248., 11.06, -74., 0., -65., 0.,
DT=.1, QSDT=.5, QSALT=55., TF=500.,
IYR=1G77, IM }\emptyset=5, IDAY=16, IHR=23, IMIN=54, SECSI=41.,
IC\emptyset\emptysetR=3, PHIR=-14.624, ECL\emptysetNG=137.82,
ECLINC=89.36,
\emptysetD=15.E+5,
G\emptyset=8.867E-3, RM=6050., \emptysetMEG=2.997E-7, MU=3.2486E+5,
ATM\phiS(1)=1.104E+10,
ATM\phiS(18)=0., 60., 115., 125., 137., 175., 2*0.,
ATM\emptysetS(26)=738., 260., 170., 2*210., 710.,
NTPTS:=6, NMPTS=4,
AGAM=1.4,
WDTBL=2., 0., 10., 0., 0.,
TERHT=.FALSE.,
VMASS =22.6, VDIA =.448E-3, VSA=.158E-6,
VRI=1.085E-6,
XG=0., XM=0., ZG=0., ZMM=0.,
XSTEP=1.5E-5, ZSTEP=1.5E-5, TSTEP=.004,
ICNTR=20,
RESTRT=.FALSE.,
```

$$
\begin{aligned}
& C(101)=-10 ., .025, .4, .5, C(140)=1 ., \\
& C(16)=-.1, C(20)=1.034,2.1,4 ., .23, \\
& C(67)=1 . E-4, C(81)=1.01, C(83)=1.01, C(91)=.99, \\
& C(111)=1.2 E-3,-1.3 E-7,5 . E-7, \\
& C(1)=5 . E 8, C(3)=5 ., 2.5,-4 ., 3.5,2 .,-4 .,-7 ., 6 .,-4 ., 8 ., 6 ., \\
& C(152)=6 ., 8 ., C(156)=.03,-.03, C(161)=.01,-.01, \\
& C(166)=.01, .015,-.015,-.01, .
\end{aligned}
$$

Since these data are very similar to the data presented for the mode A sample case (data generator) in Section $A$, only the differences will be explained here.

The first difference concerns the appeararice of the variables QSDT and QSALT in the above namelist. These iables indicate that the quasi-static dynamic model will be used when the vehicle descends to an altitude of 55. kilome ers. The integration step size will be increased from .1 second to .5 second. The second difference concerns the physical characteristics of the entry vehicle. The mode A sample case involves an entry vehicle representative of the Planetary Explorer main probe, while the mode B sample case involves an entry vehicle representative of the Planetary Explorer miniprobe. This accounts for the different values used for the variables VMASS through VRI in each case.
b. Output Discussion - Selected pages from the output of the data generator portion of this sample case appear in section $D$, where it is referred to as case $B-1$. The data at the trajectory time of 40. seconds were generated with the standard dynamic model, although the point-mass assumption was employed since the dynamic pressure exceeded the input value of $\emptyset \mathrm{DB}$. This is still true at the trajectory time of 228. seconds. In addition, the data at this latter time were generated using the quasi-static dynamic model since the vehicle altitude is less than the input value of QSALT. The fact that the derivative of the velocity is zero is a consequence of using the quasi-static dynamic mode1. The output for this sample case was generated in the CDC 6400/6500 computer at the Martin Marietta Corporation.

## 2. Reconstruction Program

a. Input Discussion - The input data for the reconstruction program consist of a namelist and a measurement/event schedule. The namelist, which is also entitled ERAN, consists of the following cards.

```
XN=248., 11.06, -74., 0.,
X }\emptyset=248., 11.06., -74., 0.
```

```
IYR-1977, IM }\varnothing=5, IDAY=16, IHR=23, IMIN=54, SECSI=41.,
IC\emptyset\emptysetR=3, PHIR=-14.624, ECL\emptysetNG=137.82, ECLINC=89.36,
TF=500.,
G\emptyset=8.867E-3, RM=6050., \emptysetMEG=2.997E-7,
MU=3.2486E+5,
ATM\phiS (1)=1.104E+10,
ATM\emptysetS (18)=0., 60., 115., 125., 137., 175., 2*0.,
ATM\emptysetS (26)=738., 260., 170., 2*210., 710.,
NTPTS=6, NMPTS=4,
AGAM=1.4,
TERHT=.FALSE.,
WDTBL=2., 0., 10., 0., 0.,
VMASS=22.6, VDIA =.448E-3, VSA=.158E-6,
VRI=1.085E-6,
XG=0., XM=0., ZG=0., ZMM=0.,
ICNTR=1,
RESTRT=.FALSE.,
NACCEL=.TRUE., NGYR\emptyset=.TRUE.,
LTR1=.TRUE.,
C(16)=.1, C(21)=2.. 4., .23,
\begin{tabular}{rlll}
\(\mathrm{P}=200\), & \(0 .\), & \(0 .\), & \(0 .\), \\
\(0 .\), & \(2.5 \mathrm{E}-4\), & \(0 .\), & \(0 .\), \\
\(0 .\), & \(0 .\), & \(1.22 \mathrm{E}-3\), & \(0 .\), \\
\(0 .\), & \(0 .\), & \(0 .\), & \(3.234 \mathrm{E}-4\),
\end{tabular}
\(\mathrm{NQ}=4, \operatorname{LIST} \mathrm{Q}=3,5,7,9\),
\begin{tabular}{crlr}
\(Q=25 .\), & \(0 .\), & \(0 .\), & \(0 .\), \\
\(0 .\), & \(10 .\), & \(0 .\), & \(0 .\), \\
\(0 .\), & \(0 .\), & 1.5, & \(0 .\), \\
\(0 .\), & \(0 .\), & \(0 .\), & \(40 .\),
\end{tabular}
\(\mathrm{NU}=2\), LISTU=20, 140, \(D U=2.78 \mathrm{E}-4, .25 \mathrm{E}-2\),
\(\mathrm{NV}=7\), LISTV=67, 111, 112, 113, 81, 83, 91,
\(\mathrm{DV}=12 . \mathrm{E}-8, .15 \mathrm{E}-5, .19 \mathrm{E}-13, .33 \mathrm{E}-13,1 . \mathrm{E}-4,1 . \mathrm{E}-4,1 . \mathrm{E}-4\),
NW=8, LISTW=4, 6, 8, 152, 153, 156, 161, 1, \(D W=4 ., 8 ., 14 ., 25 ., 50 ., 6 . E-4,1 . E-4,2 . E+9\), SDMWT=3.,
```

$\operatorname{RR}(4,1,1)=1 . \operatorname{E8}, \operatorname{RR}(4,1,2)=1.3 E 10, \operatorname{SD}(4,1)=1 . \operatorname{E5}$, $\operatorname{RR}(5,1,1)=1 ., \operatorname{SD}(5,1)=.5$,
$\operatorname{RR}(11,1,1)=1 . E-8, \operatorname{SD}(11,1)=1 . E-5,$.
Since these data are very similar to the data presented for the mode A sample case (recinstruction program) in part A, only the differences will be discus: $\mathbf{i}$ here.

The mode B primary state vector consists of only four components. This explains why the XN vectors in the two cases have different dimensions. Since the mode $B$ reconstruction process, unlike mode A, requires an atmosphere model, the pertinent ATM $\phi$ S variables must appear in the above namelist.

Both NACCEL and NGYR $\emptyset$ are set true to remove both the normal accelerometer and the gyro from the reconstruction process. Setting LTR1 true indicates that the mode $B$ reconstruction process will be employed.

The $4 \times 4$ P-array defines the initial covariance matrix corresponding to the primary mode $B$ four-dimensional state vector. The mode $B$ filter in this sample case also solves for the temperatures at the first four temperature breakpoints. The second, third, and fourth temperature breakpoints are treated as consider parameters by the filter. Certain of the component mole fraction profile parameters are also considered by the filter.

The measurement/event schedule cards used in this sample case are listed.

| 30. | 2000. | 100. | 4 |
| ---: | ---: | ---: | ---: |
| 40. | 2000. | 150. | 5 |
| 10. | 2000. | 60. | 21 |
| 60. | 60. | 10. | 12 |
| 200. | 200. | 10. | 12 |
| 600. | 600. | 10. | 12 |
| 1500. | 1500. | 10. | 12 |
| 100000. |  |  |  |

Not all these meas rrements and events will be processed in the sample case since TF was set to 500 . in the previous namelist.
b. Output Discussion - Selected pages from the reconstruction portion of this sample case appear in section $D$ where they are referred to as case B-2. The output for a temperature measurement at 40 . seconds, a quasi-event at 200 . seconds, and a pressure measurement at 230. seconds is shown. The temperature measurement at 40. seconds reduces the velocity and downrange estimation errors, although the altitude and fligitpath angle errors have increased. The temperature estimation errors at the temperature breakpoints of 0 . and 60 . kilometers have been reduced, while those at the higher temperature breakpoints have not beeñ significantly affected. This is to be expected since the vehicle is at an altitude of 66.6 kilometers when this temperature measurement was made. The nominal trajectory is updated at the quasi-event at 200. seconds, but only the altitude and downrange angle components of the nominal trajectory have been improved as a result. Examining all estimation errors at this quasi-event shows that all initial errors, except for the temperature error at the third temperature breakpoint, have been reduced at 200 . seconds. The pressure measurement at 230. seconds, which is the first measurement following the previous quasi-event, does not have much of an effect on the temperature solve-for parameters, although altitude and velocity estimation errors are reduced. Note that all state and solve-for parameter estimation errors at this point easily fall within the $\pm 3 \sigma$ range predicted by the filter. For example, compare the altitude error of -.751 kilometer with the predicted 1-a standard deviation of .823 kilometer, and the surface temperature error of $-4.55^{\circ} \mathrm{K}$ with the predicted $1-\sigma$ standard deviation of $4.97^{\circ} \mathrm{F}$. This indicates that the filter is convergent at this point in the reconstruction process. The output for this sample case was generated on the SDC 6400/6500 computer at the Martin Marietta Corporation.

## C. QUASI-STATIC DYNAMIC MODEL SAMPLE CÅSE

The results of a study performed to establish the validity of the quasi-static dynamic model in the terminal descent phase of a Vernusian entry mission are presented here. The assumptions and equations defining the quasi-static dynamic model are given in Chapter II of the Analytic Section of this manual. The LTR data generator program was used to compute the true vehicle velocity. The quasi-static velocity, of course, was computed from the analytic terminal velocity solution.

Vertical motion ( $\gamma=-90^{\circ}$ ) was assumed for this study. The initial vehicle velocity was $863 \mathrm{~m} / \mathrm{s}$ at an $85-\mathrm{ki}$ lometer altitude. A ballistic coefficient of $30.01 \times 10^{6} \mathrm{~kg} / \mathrm{km}^{2}$ was assumed until the parachute was deployed at the $50-\mathrm{kilometer}$ altitude, after which the ballistic coefficient was changed to $25.539 \times 10^{6} \mathrm{~kg} / \mathrm{km}^{2}$.

The Venusian atmosphere model used in this study was based on the GSFC No. 3609 Venusian model. In LTR, atmosphere models are approximated with a surface pressure and linear temperature and molecular weight breakpoint models. The validity of the hydrostatic equation and the perfect gas law is also assumed. The temperature profile used in the study is defined.

| Altitude (km) | Temperature $\left({ }^{\circ} \mathrm{K}\right)$ |
| :---: | :---: |
|  |  |
| 13.5 | 738. |
| 42. | 640. |
| 60. | 387. |
| 115. | 170. |
| 125. | 210. |

A constant molecular weight of 43.2 over the altitude range under consideration was assumed. Surface pressure was set to $1.104 \times 10^{5}$ millibars.

The results of the study are summarized. True velocity (computed by LTR) and quasi-static velocity (computed analytically) are shown as functions of altitude

| Altitude (km) | Quasi-Static <br> Velocity (m/s) | True Velocity (m/s) |
| :---: | :---: | :---: |
|  | 190. | 406. |
| 80. | 80.3 | 83.4 |
| 70. | 32.7 | 32.9 |
| 50. | 15.55 | 15.87 |
| 40. | 9.14 | 9.15 |
| 30. | 6.06 | 5.86 |
| 20. | 4.30 | 4.30 |
| 10. | 3.17 | 3.21 |

The initial disagreement at the 80 -kilometer altitude is due to the fact that the initial velocity at the $85-\mathrm{kilometer}$ altitude was chosen to be $863 \mathrm{~m} / \mathrm{s}$, which is much greater than the terminal velocity at that altitude. However, after 70 kilometers, the agreement between the quasi-static and true velocities is quite good.
D. SELECTED PAGES FROM LTR SAMPLE CASES

Case A-1: LTR Mode A Data Generator Sample Case

```
ACTUAL STATE VFCTOR AT 10.00 SFCONDS INTFGPATION STEP SIZE = . 10 PHASE= 1
    H = 1.F921663714170E+02 KM K
```

$V=1.1155334181572 E+01$ KM/SEC V7 $=-2.7665354886475 \mathrm{E}-$-19 KM/SEG

GAMMA $=-3.7934412160168 E+01$ OEGREES OMEGA $=-4.941527481226$ 0E-04 OEGREES/SEC THTQ $=-1.1314188220029 E-030 E G R E E S / S E C$

```
state dfoivatives
\[
\begin{aligned}
H \text { DOT } & =-6.8594570405358 E+00 \\
\text { OHI DOT } & =9.1064099393858 E-02 \\
D V X & =-1.8835435736657 E-07 \\
\eta O R F S ? & =1.3102150226326 E-02
\end{aligned}
\]
```

GAMYA DOT $=4.7043150714520 E-02$ $\begin{aligned} \text { OUEGA DOT } & =-2.2992247512274 E-04 \\ \text { DTHT } & =-6.6245924625211 E-06\end{aligned}$ DTHT $=-8.6245924625211 E-06$

RELATIVE VELDCTTY
WIND VELORITY
DYNAYIC PPESSURE
MCLECULAD WFIGHT TFMPERATURF
AYTAL FORCF
MOPMAL FORCE
CENTER OF PDESSURE
MCYENT ACCELERATION EPSILOH

$V$ DOT $=5.1631404479111 \mathrm{E}-03$ DOT $=-4.941527483 .7280 \mathrm{E}-04$
DVZ $=-1.2877259846811 \mathrm{E}-09$

```
measupeyent values
\begin{tabular}{rrr} 
AFCFLFRGMFTERS & \(=\) & \(-1.8836436 \mathrm{~F}-07\) \\
PRESSURE & \(=\) & \(-1.2877260 \mathrm{E}-09\) \\
& \(2.0413347 E+01\)
\end{tabular}
\begin{tabular}{llll} 
PRESSURE & \(=\) & \(3.2236860003164 E-07\) & MILLIBARS \\
OENSITY & \(=2.2745279735908 E-01\) & KG/KMF*3 \\
ALPHA & \(=\) & \(1.93070145157740 E+00\) & DEGREES \\
CA & \(=1.5705846403379 E+00\) & UNIT FREE \\
CN & \(=-1.0737077230424 E-02\) & UNIT FREE \\
CMO & \(-5.5411155496466 E-01\) & UNIT FREE \\
MOMENT & \(=1.6776432892351 E-17\) & KG-KH/SEC**2 \\
GRAYITY & \(=8.39894755704900 E-03\) & KH/SEGFE2 \\
AXIAL ACCEL & \(=-1.8836435736657 E-07\) & KM/SECF*2 \\
NORMAL ACCEL & \(=-1.2877259846811 E-09\) & KH/SECF*2
\end{tabular}
```


RANGE DATE $=\quad$ P.30803786E+01 2.25059150E+01 2.27705017E+01
RFFRARTIVITY Valufs
OELTA PANGE = C.
OELTA P-RATF = 0 .
0. 0 .
$\begin{array}{ll}0 . & 0 . \\ 0 . & 0 .\end{array}$
0.

```
COMMUNICATIDN ANGLE IS \(4.61810635 E+01\)
ANGLF RETWEEN ENTRY PLANE ANJ SKY IS 6.55079225E+01
```

peffrencf plane latttuje
LONG ITUDE

OATA GENERATOR PROBLEM PRELIMINARY MAIN PROBE-MORE A-VENUS-P AS IN 2-3M


## state derivatives

$$
\begin{aligned}
\text { H DOT } & =-2.8007667569590 E+00 \\
\text { PHI DOT } & =3.4261833305652 E-32 \\
\text { DUX } & =-2.3138423473662 E+00 \\
\text { DPRFSZ } & =3.9462553662688 E+05
\end{aligned}
$$

GAMMA DOT $=-5.6237637716223 E-02$ OMEGA DOT $=2.4700173026663 \mathrm{E}-01$ DTHT $=-1.5795145133445 E-03$
relative velocity
WIND VELOCITY
OYNAMIG PRESSURE
HOLECULAR WEIGH
TEMPERATURE
AXIAL FORCE
AXIAL FORCE
NORMAL FORCE
CENTER JF PRESSURE
MEMENT ACCELERATION
EPSTLON

| $4.6137801719457 E+00$ | KM/SEC |
| :---: | :---: |
| 0. | KM/SEC |
| 1.7341350370286E+03 | MILLIBARS |
| 4.3167746647707E+01 | KG-MOL |
| 2.2881825483449E+02 | DEGREES K |
| 1.3574360391822E+01 | UNIT FREE |
| -4, $0260856844173 \mathrm{E}+02$ | KG-KM/SEC**Z |
| 0. | KG-KM/SECFF2 |
| -3.1825100000000E-04 | KM |
| 4.36054:1001409E-10 | KM/SEC** |

PRESSURE $=7.1805405223274 \mathrm{E}+00$
OENSITY $=1.6292927013319 \mathrm{E}+07$
ALPKA $=0$.

MILLIBARS KG/KMFF 3 gegrees UNIT FREE
UNIT FREE UNIT FREE KM/SEG**2 KHMSECF* KM/SEC**2


[^0]| RFFEPENTE PLANE LATITUDF | LONGITUOE |  |
| :---: | :---: | :---: |
| QGANFTO-EQUATORIAL | $-3.79103376 E+00$ | $3.95641068 E+01$ |


reference plane latitude LONGITUDEPLANETO-EQUATORIAL - $3.79090863 E+00 \quad 3.95639611 E+01$

| ACTUAL STATE VECTOR AT 200.00 | seconos | INTEGRATION STEP SIZE = | -1.0 PHASE= | 1 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $H=5.6551268231716 E+01$ | KM | $v=5.4740069601004 \mathrm{E}-02$ | KY/SEC | GAMMA $=-9.1420096769027 E+01$ | OEGREES |
| PHI $=2.4981797959445 E+00$ | DFGREES | THETA $=-9.2498338384194 \mathrm{E}+01$ | DEGREES | OMEGA $=6.4126499096196 \mathrm{E}-05$ | DEGREES/SEC |
| $V \mathrm{X}=-1.2630869707362 \mathrm{E}+01$ | KM/SE | $\mathrm{VZ}=-6.5174$ ? $\mathrm{E}^{\text {r }} \mathrm{z} 27280 \mathrm{E}-06 \mathrm{~K}$ | KM/SEC | THTQ $=-5.5358643325049 \mathrm{E}+01$ DEGR | REES/SEC |

## STATE NERIVATIVES

```
H OOT = -5.4723255658393E-02
PHI OOT =-4.2728663648431E-05
    TVX = - &.8949674167073E-03
DPRES2 = - 4.0360239323714E+05
```

$\begin{aligned} V \text { OOT } & =-1.881838697628 \mathrm{CE}-04 \\ \text { ETA OOT } & =6.4126499096196 E-05\end{aligned}$
OOT $=6.4126499096196 E-05$
DVZ $=2.9180580642419 E-11$

GAMMA DOT $=-4.8697732113918 E-03$ OYEGA OOT $=-1.1567453640475 E-05$ OTHT $=1.1192185470058 \mathrm{E}-06$

```
RELATIVE VFLOCITY = 5.47232566585594E-02 KM/SEC
WINO VELORITY = 0.+723256658594E-02 KM/SEC
DYNAMTG PQESSUPE = 1.2576269020245E+01 MILLIRARS
HOLECULAD WFIGHT
TEMPERATURE
ACH NUMPFE
AXIAL FORCE
AKJAL FORCE = -1.5485943305071F+80 KG-KM/SEC**2
NORMAL FOPCF = 5.0774210317810E-09 KG-KM/SEC**2
NORNAL FOPPFRESSURE = 5.07742101017810E-09 KG
```



```
    EPSILON
```

| ESSURE | 4.6813409283365E+02 | MILLIBARS |
| :---: | :---: | :---: |
| DENSIT | 8.4659865292781E+08 | KG/KM** |
| ALPHA | -4.3861.363128632E-06 | OEGREES |
| CA | $8.2879807570890 \mathrm{E}-01$ | UNIT FREE |
| CN | 2.7174042276947E-19 | UNIT FREE |
| CMQ | -7.9999824554547E-02 | UNIT FRE |
| MOMENT | -5.7380200732552E-12 | KG-KM/SEC** |
| gravity | 8,7117263439054E-03 | KM/SEG**2 |
| AXIAL ACCEL $=$ | -8.8999674167074E-03 | KM/SEG**2 |
| NORMAL ACCEL | 2.9180580642419E-1 | KM/SEC**2 |

measurement valufs

| AGCELEROMETERS | $=$ | $-8.8999674 \mathrm{E}-03$ |
| ---: | ---: | ---: |
| PRFSSURE | $2.9180581 \mathrm{E}-11$ |  |
|  | $=$ | $4.3574276 \mathrm{E}+07$ |

RATE GYRO $=$
ALTIMETER TEMPERATURE =
1.11.92185E-06
$5.6607819 E+01$
2.8630200E+02

| ISN TPAKING FOR | STATION 1 | STATION ? | STATION 3 |  |
| :---: | :---: | :---: | :---: | :---: |
| RANGE = | 7.07732751E+07 | 7.07753967E+07 | 7.07676430E+07 | KM |
| RANGE RATE = | 1.31707447E+01 | 1.259384305+01 | $1.23663416 \mathrm{E}+01$ | KM/SEC |
| REFRACTIVITY VALUES |  |  |  |  |
| BELTA PANGE = | 0. | 0. | 0. | KM |
| DELTA マ-RATE = | 0. | 0. | 0. | KM/SEC |

communicatton angle is
$4.715542545+01$

ANGLE RETHEEN ENTPY PLANE ANT SKY IS $6.550704 \mathrm{~g} 6 \mathrm{E}+01$
peferfnce plane latituoe longitluoe
PLANFTU-ERUATORIAL -3.79024791E+00 3.95632264E+01

Case A-2: LTR Mode A Reconstruction Program Sample Case

| AT TRAJECTORY TIME 10 OBSEGPRORLEM PRELIMINARY MAIN PRORE-MOUE A-VENUS-P AS IN 2-3M |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| TRAJECTORY |  |  |  |  |
|  | ORIGINAL NOMINAL | MOST REGENT NOMINAL | ACTUAL | UNITS |
| H | $1.7875831848536 \mathrm{E}+02$ | 1.7875831848536E+02 | 1.692166391417CE+02 | KM |
| $v$ | 1.113163 n848014E+01 | $1.1131635848014 \mathrm{E}+01$ | $1.1156334181572 \mathrm{E}+01$ | KM/SEC |
| gamma | -3.8339720316418E+01 | -3.8339720316418E+0i | $-3.7934412160 \leq 68 \mathrm{E}+01$ | degrees |
| PHI | ?.9431193263621E-G1 | 7.9431193263621E-01 | 1.3017819706084E+00 | DEGREES |
| ATMOSPHERE |  |  |  |  |
|  |  | most recent hominal | actual | UNITS |
| pressure |  | 6. 2740266824954 E -08 | 3. $2236860003154 \mathrm{E}-07$ | MILLIBRS |
| OENSITY |  | 1.0000000000000E-02 | 2.2745279735908 E -11 | KG/KM ${ }^{\text {+ }}$ |
| TEMP |  | $3.0000000000000 E+02$ | $6.3390314660135 \mathrm{E}+02$ | DEGREE K |
| MOL. WT |  | 3.5892540874251E+01 | $3.7186747277807 \mathrm{E}+01$ |  |
| VRU-ARU DATA |  |  |  |  |
|  | ACTUAL | RECONSTRUCTED | UNITS |  |
| THETA | -3.7301131418811E+01 | 0. | Degrees |  |
| OMEGA | -4.9415274812280E-04 | 0. | DEGREES/SEC |  |
| AxC | $-1.8836435736657 E-07$ | 0. | KM/SEC**2 |  |
| A 26 | -1.2877259346811E-09 | 0. | KM/SEC**2 |  |

ENTRY PARAMETERS GASED ON MOST RECENT NOMINAL

HIND VELOCITY $=0$. KM/SEC
RELATIVE VELOGITY $=1.1132716112061 E+01 K M / S E C$
EPSILON $=4.4172658172132 E-03$ DEGREES
$\begin{array}{lll}\text { ALPH } & =4.4172658172132 E-03 \text { DEGREES } \\ & =0 . & \text { DEGREES }\end{array}$

DYNAMIC PRESSURE MACH NUMBER
CA
GRAVITY
$1.3448418402448 \mathrm{E}+01$ HILL IBAPS 1.000000000000 OE+01 UNIT FREE 7.7116235865525E-01 UNIT FREE
 DEvIATIONS FROM MOST RECENT NOMINAL

TRAMECTORY

## Estimated

$\begin{array}{ll} & \text { ESTIMATED } \\ H & -8.2363247454919 E-01 \\ V & 1.3922044387907 E-03\end{array}$
GZMAA PHI
$1.3922044387917 \mathrm{E}-03$
$-5.3796774737613 \mathrm{E}-01$ $-5.3796774737613 \mathrm{E}-01$
$1.2723889611897 \mathrm{E}-01$
actual
$-9.5416793436580 E+00$ $2.4703333557966 \mathrm{E}-02$ $4.053081562497 \mathrm{EE}-01$ 5.0747003797218E-01
(ERROR EST-ACT) 8.7180468691087E +00 $-2.3311129119175 \mathrm{E}-02$ -3.8023114185321E-01

# UNITS 

<M KM/SES DEGREES
OEGREES
(ERROR EST- AGT) $-2.5893970029033 \mathrm{E}-07$ 0.

UNITS MILLIBRS KG/KMF: 3 DEGREE K
 $\begin{array}{ll}\text { TEMP } & 0 . \\ 0 .\end{array}$

SOLVE FOR PARAMETERS PAR
0.

ACTUAL
2.5952833320669E-07 0.5
0.
0.
0.
0.

ESTIMATED DEVIATIONS FROM ORIGINAL NOMINAL OF SOLVE FORPARAMETERS 0 .

ENTRY PARAMETERS FASED ON MOST RECENT NOMINAL

| WIND VELOCITY | $=$ | 0. | KM/SEC |
| :---: | :---: | :---: | :---: |
| DYNAMIC PRESSURE | = | i. $3448418402448 \mathrm{E}+01$ | MILLIBARS |
| NOLECUL AR WEIGHT | = | 3.5892540874251E+01 | KG-MOL |
| TEMPERA TURE | = | $3.000000000000 \mathrm{E}+02$ | DEGREES K |
| MACH NUMBER | = | $1.000000000000 E+01$ | UNIT FREE |
| AXIAL FORCE | = | 0. | KG-KM/SEC**2 |
| NORMAL FORCE | = | \%. | KG-KM/SEC**2 |
| CENTER OF PRESSURE | = | 0. | KM |

## DENSITY ALPHA <br> CA CN CN CHO MOMENT GRAYITY GRAVITY EPSILON

$\qquad$
-YNAMTC PRESSUR OLECULAR WEIGHT
MACH NUMBER AXIAL FORCE CENTER OF PRESSURE

### 1.0000000000000E-02

 0.0. 
1. $3732349902621 E-03$
4.4172656172132E-03

KG/KMFE3 DEGREES UNIT FREE UNIT FREE UNIT FREE KG=KH/SEC**2 KH/SECFH2 DEGREES


| state po | MA FRIX |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.42098 05E+01 | 3.6926249E-03 | 9.5871129t-02 | 8. 3040918E-02 | -2.8741853E-01 |  |  |  |
| 3.6926249E-03 | 1.5294533E-02 | $2.5925316 \mathrm{E}-01$ | -1.3844639E-01 | -1.7068817E-01 |  |  |  |
| 9.5871129E-02 | 2.5925316E-01 | 9.2540509E-01 | 9.2059198E-01 | -8.9651606E-01 |  |  |  |
| 8.984091 RE-02 | -1.3844639E-01 | 9.2059198E-01 | 9.3976710E-01 | -3.4887805E-01 |  |  |  |
| -2.8741853E-01 | -1.7068817F-1 | -8.9651806E-01 | -8.4887805E-01 | 1.2835235E-09 |  |  |  |
| CXV CORR | Matrix |  |  |  |  |  |  |
| $3.1655320 E-06$ | 0 . | 1.5794790E-07 | -8.0744827E-08 | -3.3795047E-08 | 0. | 0. | 0. |
| 0. |  |  |  |  |  |  |  |
| -4.9709295E-05 | 0. | -2.4303030E-07 | 1.2679602E-07 | 5.3069372E-08 | B. | 0 。 | 0. |
| 0. |  |  |  |  |  |  |  |
| 3.1746450E- 05 | 0. | 1.5840260E-06 | -8.0977277E-07 | -3.3892337E-07 | 0. | 0. | 0. |
| 0. |  |  |  |  |  |  |  |
| -7.39.38472F-05 | 0. | -3.6892460E-07 | 1.8859860E-07 | 7.8936310E-08 | 0. | 0. | 0. |
| $-2.9239134 E-15$ | 0. | -1.4619143E-06 | $7.4734784 \mathrm{E}-87$ | $3.1279596 E-0.7$ | 3. | $0 \cdot$ | 0. |
| $\mathrm{n}_{0}$ |  |  |  |  |  |  |  |
| CXW GORR | MATRIX |  |  |  |  |  |  |
| 0. | $n$. | 0. | 0. | 0. |  |  |  |
| 0. | 0. | 0. | 0. | 0. |  |  |  |
| 0. | 0. | 0. | 0. | 0. |  |  |  |
| 0. | 0. | 0. | 0. | 0. |  |  |  |
| 0. | 0. | 0. | 0. | 0. |  |  |  |

$\underset{R}{\text { attual }}$ tYnamig notse

## UNMODELED DYNAMIC NOISE COVA?IANCE MATRIX

D. D. 0 .

## -

 0.
## GAIN MATRICES

$-1.789389^{\mathrm{KLE}}-02$
$-3.5534010 \mathrm{E}-02$
$2.4414950 \mathrm{E}-04$
$2.6997264 E-04$ $-1.7185631 \mathrm{E}-06$

## RESIDUAL UNCERTAINITY MATRIX

3.5900373E+03 MATRIX


## HESUREMENT CONSIDER PARAMETERS

L
HaTPIX
$9.9999773 E-31 \quad 1.2536515 E-01$
$-1.3007120 E+03$
5． $0503118 \mathrm{E}+03$
0.

0 ．

MFASUREMENTS

Estimated
T．07705957360C1E＋07

AC TUAL
7．0770626328030E＋07

## actual

$-9.5416793436580 E+00$ $2.4703333557965 \mathrm{E}-02$ 4．05305：5624970E－01 5．0747003797218E－01

## （ERROR EST－ACT）

 8． $1706362272036 E+00$ 2．4398186560341E－02 -5.15 •3210068233E－01 9．29；－${ }^{-1429790 E-02}$RESIJUALS $-3.0592028617859 \mathrm{E}+01$


SOLVE FOR DARAMETERS
C．

ETCTE MATRIX
$\begin{array}{rrrr}1.4158288 F-01 & -5.9003944 F-03 & 3.3976 n 38 E-01 & 9.9958568 E-01 \\ -5.9003944 E-03 & 1.5145594 E-02 & 9.1956973 E-01 & -5.9158265 E-03 \\ 3.887503=5-01 & 0.1856773 \mathrm{E}-01 & 3.9204907 E-01 & 3.8889563 E-01\end{array}$
3．067503Eース1 ．1月673E－01 3．9204907E－01
9．9958558E－01－5．9168265E－03
3．3899563E－01
CXV COPR MATRIX
2．5747212E－06 3．5721727E－34 －6．124．1594F－06 6．E359494E－14 0．1AB？719E－G5－1．0092132E－02－8．9393446E－07解．5？259755－06－2．8247391E－02 5． 0743351 F－05 6．3449981E－03 CXW CO？P MATRIX

| 0. | 0. | 0. |
| :--- | :--- | :--- |
| $0_{0}$ | $0_{0}$ | 0. |
| $0_{0}$ | $0_{0}$ | $0_{0}$ |
| $0_{0}$ | $0_{0}$ | 0. |
| $0_{0}$ | 0 | 0. |

0. 
1. 

R
MATRIX

## $1.0000000 \mathrm{E}-03$

UNMGOELED QYNAMIf NOISE g GUARIANCE MATRIX
UNM
0.
0.
0.
0.

GAIN MATRIEES
41 TRIX
$-1.7893898 \mathrm{E}-02$
$-3.5534010 \mathrm{E}-05$
2. $4414950 \mathrm{E}-04$
2. 699 ?264E-04
$2.698631 \mathrm{E}-06$
-1.718561

RESIDUAL UNCERTAINITY MATRIX
$3.59083782+03$
ACTUAL MEASUREMENT NOISE
2.0606645999998E-02
measurevent covariange matrix
2.000000000000GE-02

|  | ORIGINAL NCMINAL | MOST RECENT NOMINAL | ACTUAL | UNITS |
| :---: | :---: | :---: | :---: | :---: |
| H | $7.3000690547543 E+01$ | 7. $3000690547543 E+01$ | 6.1662346266752E+01 |  |
| $\checkmark$ | 4.9961921135524E-02 | 4.9961921135524E-0゙2 | 7.9081973389546E-02 | KM/SEC |
| g amma | -0.1554833498257E+01 | -9.1554833498257E + 01 | -9.0920537591225E+01 | OEGREES |
| PHI | 1.9720070532197E+30 | 1.9720070532197E+00 | 2.4991777056531E+00 | DEGREES |
| ATMESPHFRE |  |  |  |  |
|  |  | MOST RECENT NOMINAL | actual | UNITS |
| Pfessidef |  | 2.6532843168221E*02 | 1.9467594165644E+02 | HILLIBfs |
| OENSITY |  | 1.0470131331500E+09 | 3.9326328524518E+00 | KG/K14** 3 |
| TEMP |  | 1.3211517800296E+02 | $2=5727979701804 \mathrm{E}+02$ | OEGREE K |
| MCL. WT |  | 4.3183123008242E+01 | 4.3211917003504E+01 |  |
| VRU-ARU Data |  |  |  |  |
|  | ACTUAL | REGONSTRUGTED | UNITS |  |
|  | $-9.2434940732901 E+71$ |  |  |  |
| OMEGA | -1.1433582106660E-J2 | 0. | DEGREES/SEC |  |
| AXC | -9.2178174750768E-13 | -9.22500000035 E0E-03 | KM/SEC**2 |  |
| C.2s | -1.1625497438391E-08 | 0. | KM/SEC**2 |  |



DEVIATIONS FROM MOST RECENT NOMINAL
TRAJECTORY

|  | estimateo |
| :---: | :---: |
| H | -5.048998425239RE+00 |
| $v$ | 3.1563133075245E-02 |
| gamma | -7.2164712276593E400 |


| AGTUAL | (ERROR EST-ACT) |
| :---: | ---: |
| $-1.1338344280792 E+01$ | $6.2893458555519 E+00$ |
| $2.9120052254022 E-02$ | $2.4430808212228 E-03$ |
| $6.3424590703198 E-01$ | $-7.8507172346913 E+00$ |
| $5.2717065243338 E-01$ | 7.24758618848 E2E-02 |

KNITS
KM
KM/SEC
DEGREES
DEGREES

ATMOSPHERF
fstimaten
PRESSURE-9.0684408565250E +11
DFNSITY -1.3233757170274E+19
TFMP 1.2200235772552E+02
solve for faphmf.ters
0.

ACTUAL
$-7.1652490025768 \mathrm{E}+01$
$-6.5374985290477 \mathrm{E}+08$
$1.2516451901508 \mathrm{E}+02$
$1.2516451901508 \mathrm{E}+02$
0.

TYNAMIC PRESSURE

GRA:ITY
$=\quad 2.6465644962714 E-01$ UNIT FREE
$=9.3394537345388 E-01$ UNIT FREE
$=8.6649811672799 E-03 K M / S E C * * 2$

ESTIMATEO DFYTATIONS FROM ORIGINAL NOMINAL OF SOLVE FORPARAMETERS 0.

ENTRY PARAMETfRS BASED ON MOST RECENT NOMINAL

```
WINO VELOTITY
OYNAMIT PRESSURE MCLFTULAR WFIGH TEMP FRATURE MACH NUMBER AXIAL FORCE NORMAL FORCE CFNTER OF PRESSURE
```

KH/SET:
$1.0470131331500 \mathrm{E}+09$
KG/KM** 3

| hate transition phi | MATRIX PARTI MATRIX |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| 1.0000014E+00 | -9.9964088E-01 | 1.7101101E-03 | -2.0501803 0 -06 | 0. |
| -2.826.3199F-06 | $1.00000095+00$ | $1.2360266 \mathrm{E}-05$ | 2.3304725E-04 | 0. |
| -1.5362928E-06 | 6.6601822E-03 | $1.0099667 \mathrm{E}+00$ | -1.6858250E-01 | 0. |
| $3.6121098 \mathrm{E}-11$ | -4.3771470E-06 | A. $2375263 \mathrm{E}-06$ | 9. 3999929E-01 | 0. |
| -1.3431218E+02 | -8.8805322F+06 | -3.6582419E+04 |  | 1.0012767E+00 |


DIAGONAL OF OYNAMIC NOISE MATRIX
0.
state
pf

| state <br> pF | MA TRIX |
| :---: | :---: |
| 8. $4029630 E+00$ | -5.4846817E-31 |
| -5.4846817E-01 | 1.0325922E-03 |
| 1.6377953E-01 | -6.2217679\%-01 |
| 9.9864434E-01 | -5.5563250E-01 |
| -3.7̈r8907E-02 | -1.20022775-01 |
| CXV CORR | MATRIX |
| -9.3243304E-05 | 9.4898714E-33 |
| 0. |  |
| 2.4553557E-04 | -1.104?227E-02 |
| 0. |  |
| -1.1935480F-04 | 1.0821265E-03 |
| 0. |  |
| -1.05064255-104 | -3.9482699E-02 |
| 1. |  |
| 6. $3905508 \mathrm{E}-05$ | 3.3865545E-03 |
| 0. |  |
| CXW CORR | MATRIX |
| -2.0615520E-01 | -3.E784709E-01 |
| 6.0120694E-01 | -5.2965872E-02 |
| -9.9572003E-01 | 1.1921598E-02 |
| -2.2296605E-01 | -3.5721444E-01 |
| 1.1280193E-01 | -5.1782654E-01 |


| 1.6377353E-01 | 9.9864434E-01 | -3.7789077E-02 |
| :---: | :---: | :---: |
| -6.2217679E-01 | -5.5563250E-01 | -1.2002273E-01 |
| 1.4526209E+61 | 1.3067182E-01 | -8.6785530E-02 |
| 1.8067182E-01 | 9.0223853E-02 | -3.9786560E-02 |
| -8.6785530E-02 | -3.9786560E-02 | 2. 70118 A8E+00 |
| 5.4107560E-06 | -9.2469046E-05 | 3.2389631E-05 |
| -1.1622519E-05 | 1.3037097E-05 | -3.4367343E-05 |
| -6.9768243E-06 | 4. $2615543 \mathrm{E}-06$ | 5.8624460E-06 |
| -2.2457125E-05 | 2.2517578E-05 | -1.2615641E-64 |
| 1.3553299E-05 | -8.637487JE-06 | 7.7743196E-06 |
| -2.5091499E-06 | 9. $2675990 \mathrm{E}-02$ | -4.5043896E-08 |
| 3.4688924E-06 | 4.1169786E-02 | 6.2273055E-08 |
| -1.4595007E-07 | -5.3297492E-03 | -2.6200745E-09 |
| -2.5014297E-06 | 9. $2430545 \mathrm{E}-02$ | -4.4905365E-03 |
| -1.7408803E-06 | -3.7198216E-02 | -3.1252031E-08 |

actual nynamic noise

## UNWODELED DYNAMIC NOTSE COVAPIANCE MATRIX

RICES
${ }^{k} 1$
MATRIX
$1.3691434 \mathrm{E}+02$
$-6.3757771 F-02$
3. $6919767 E+01$
3. $6919767 E+01$ $2.95927845-02$
$-4.1004767 E+05$

RESINUAL UNCERTAINITY MATRIX
4.6330578E-n5 MATRIX

STANDARD DEVIATIONS ON DENSITY AND TEMPERATURE DENSITY 4. $3294405840480 E+07$


## MEASUREMFNT CONSIDER PARAMETERS



MEASUREMENTS
estimated
1.3185377760577E+01

AGTUAL
1.3187763104700E+01

RESIDUALS
-2.3853441234110E-03

DEVIATIONS FROM MOST RECENT NOMINAL TRAJECTORY

| ESTIMATED |  |
| :--- | ---: |
| $H$ | $-4.7226491505354 E+00$ |
| $V$ | $3.1363341968245 \mathrm{E}-02$ |
| GAMMA | $-2.1706412392143 \mathrm{E}+0 \mathrm{D}$ |
| FHI | $6.0369095455678 \mathrm{E}-01$ |

ATMOSPHERE

$$
\begin{aligned}
& \text { ESTIMATED } \\
& \text { PRESSUPE-9.0782219046090E } 01
\end{aligned}
$$

$$
\text { DENSITY }-1.3149984990818 E+09
$$

$$
\begin{array}{ll}
\text { TEMP } & 1.2089682637227 E+02
\end{array}
$$

solve for parameters
0.

ACTUAL
$-1.1338344280792 E+01$ 2.9120052254022E-02 2. $9120052254022 \mathrm{E}-02$ $6.3717065243338 \mathrm{E}-01$

## ACTUAL

$-7.1652490025768 \mathrm{E}+01$ $-6.5374985290477 E+08$ 1. $2516461901508 \mathrm{E}+02$
0.
(ERROR EST-ACT) $6.6156951302563 \mathrm{E}+00$ 2. $2432897142235 \mathrm{E}-03$ $2.8048871462463 \mathrm{E}+0 \mathrm{C}$
$7.5520312123396 \mathrm{E}-02$
(ERROR EST- ACT) 1. $9129729020312 \mathrm{E}+0$ $-6.6124864617701 \mathrm{E}+0$ $4.2677926428050 \mathrm{E}+\mathrm{CO}$
UNITS
KH/SEC DEGREES
DEGREES
UNITS MIELIBRS KG/KM**3 OEGREE K

| $\begin{aligned} & \text { STATE } \\ & P P \end{aligned}$ | MATRIX |  |
| :---: | :---: | :---: |
| 8. $3506410 \mathrm{C}+00$ | -5.8869172E-01 | 6.3844001E-01 |
| -5.8869172E-01 | R.5890231E-04 | -9.9800187E-01 |
| 6.3844001E-01 | -9.9800187E-01 | $1.2076083 \mathrm{E}+00$ |
| 9.9877640E-01 | -5.8704716E-01 | 6.3597231E-01 |
| -2.652216se-012 | -2.1478128E-01 | 2.0243263E-01 |
| CXV CORR | MatRIX |  |
| -1.0566536E-04 | 9.5143594E-03 | 5.2339935F-06 |
| 0. |  |  |
| 3.6564838E-04 | -1.3067064E-02 | -1.2718969E-05 |
| 0. |  |  |
| -2.7013629E-03 | 9.2777199E-03 | -1.0644697E-04 |
| 0. |  |  |
| -1.1963605E-04 | -3.9853753E-02 | -2.2398913E-05 |
| 0. |  |  |
| 7.5281122E-05 | 3.4375565c-03 | 1.3823279E-05 |
| 0. |  |  |


| $9.9877340 \mathrm{E}-01$ | $-2.6522168 \mathrm{E}-02$ |  |
| ---: | ---: | ---: |
| $-5.9704716 \mathrm{E}-01$ | $-2.1478123 \mathrm{E}-01$ |  |
| $6.3697231 \mathrm{E}-01$ | $2.0243263 \mathrm{E}-01$ |  |
| $8.3474546 \mathrm{E}-02$ | $-2.6793127 \mathrm{E}-02$ |  |
| $-2.6793127 \mathrm{E}-02$ | $2.6865741 \mathrm{E}+00$ |  |
| $-9.3524403 \mathrm{E}-06$ | $3.2441203 \mathrm{E}-05$ | 0. |
| $1.5955795 \mathrm{E}-05$ | $-4.0416250 \mathrm{E}-05$ | 0. |
| $4.6172820 \mathrm{E}-05$ | $5.4334922 \mathrm{E}-05$ | 0. |
| $2.2651073 \mathrm{E}-05$ | $-1.2738785 \mathrm{E}-04$ | 0. |
| $-8.6401323 E-06$ | $7.9575252 \mathrm{E}-06$ | 0. |


| $2.0307130 \mathrm{E}-01$ | $1.5005630 \mathrm{E}-02$ |
| ---: | ---: |
| $1.3325769 \mathrm{E}-02$ | $-5.770917 \mathrm{BE}-01$ |
| $1.2511905 \mathrm{E}-03$ | $5.5069290 \mathrm{E}-01$ |
| $2.0323068 \mathrm{E}-01$ | $1.4961700 \mathrm{E}-02$ |
| $-7.0292928 \mathrm{E}-01$ | $2.7733798 \mathrm{E}-01$ |


| -9.6031236E-02 | -3.6118506E-01 | -2.5497173E-06 | 9.3496352E-02 | -4.5754204E-08 |
| :---: | :---: | :---: | :---: | :---: |
| 5.9568919E-02 | -5.7154332E-02 | 4.3123162E-06 | 4.806861 $\mathrm{aE}-02$ | 7.7414078E-08 |
| -6.4204936E-02 | 2.6242000E-02 | -4.3051413E-06 | -3.8484363E-32 | -7.7285275E-08 |
| -9.59540.4.5F-02 | -3.6147302E-01 | -2.5499563E-06 | 9.3481737E-02 | -4.5776448E-08 |
| 9.6127395E-13 | -5.1962260E-01 | -1.7281359E-06 | -3.7623862E-02 | -3.1023245E-08 |
| $\underset{R}{\operatorname{ACTUAL}}$ | NAMIG NOISE MATPIX |  |  |  |

## Case B-1: LTR Mode B Data Generator Sample Case




Case B-2: LTR Mode B Reconstruction Program Sample Case

ENTRY PARAMETERS BASEO ON MOST REGENT NOMINAL
WIND VELOCITY $=0 . \quad$ KM/SEC
RELATIVE VELOCITY $=1.2658218348745 E-01 K M / S E C$
$\begin{array}{ll}\text { EPSILON } \\ \text { ALPH } & =-5.1418159731060 E-02 \text { DEGREES } \\ \text { DEGREES }\end{array}$
beviations from most recent nominal
TRAJEGTORY

| ESTIMATE0 |  |
| :--- | :--- |
| H | $1.0384352704990 E-01$ |
| V | $1.2605552265850 \mathrm{E}-03$ |
| GAMMA | $2.8455291020724 \mathrm{E}-02$ |
| PHI | $7.1750476365649 \mathrm{E}-02$ |
| SORVE FOR PARAMETERS |  |
|  | $-4.5375564512887 E-03$ |
|  | $-3.3011947905498 \mathrm{E}=03$ |
|  | $7.1375020134447 \mathrm{E}-05$ |
|  | $-1.3252864935987 \mathrm{E}-06$ |

ACTUAL 9.4197431116481E-01 -3.3161228775906E-03 43.3715223349145E-01
$5.0000000000000 E+00$ $-4.0000000000000 \mathrm{E}+00$ $2.0000000000000 E+00$

| OYNAMIC PRESSURE | $=1.6559191150278 E+$ O1MILLIBARS |
| :--- | :--- |
| MACH NUMBER | $=4.8522446962874 E-01$ UNIT FREE |
| CA | $=9.4974284655508 E-01$ UNIT FREE |
| GRAUITY | $=8.6858208398923 E-03 K M / S E C * F 2$ |

## MACH NUMBER

 GRAVITY$=1.6559191150278 \mathrm{E}+01 \mathrm{MILLIBARS}$
4.8522446962874E-01 UNIT FREE
$=\quad 9.4974284655508 E-01$ UNIT FREE
$9.4974284655508 E-01$ UNIT FREE
$8.6858208398923 E-03 K M / S E G *-2$

ESTIMATED DEVIATIONS FROM ORIGINAL NOMINAL OF SOLVE FORPARAMETERS
0.
0.
0.
0.

ENTRY PARAMETERS BASED ON MOST RECENT NOMINAL

RELATIVE VEL OCITY KING VELOCITY MOLECG AR MEIGHT HoLECULAR NEIGHT MACH NUMBER AXIAL FORCE NORHAL FORGE CENTER OF PRESSURE

## 1.

 ©. 1.6559191150270E+01 $4.2868539999999 E+01$ $4.2868539999999 E+01$$2.5075307570466 E+02$ $2.5075307570466 \mathrm{E}+02$
$4.8522446962874 \mathrm{E}-01$ $-2.4848617876750 \mathrm{E}-01$ 0.
-1.5061784311685E-04

KHISEC KH/SEC
KG-MOL
KG-MOL
OEGREES K
KG-KM/SEC**2
KG-KH/SEC**2 KM

## PRESSURE

 DENSITYALPHA
CA
CN
CMa
MOMENT gravity EPSILON
(ERROR EST-ACT)
-8.3813078411491E-01 4.5766781041756E-03 3.6560752451217E-0
-4.1090007765473E-01

> KNITS KM KN/SEC OEGREES OEGREES

## -5.0045375564513E+00 <br> 3. $9966988052094 E+00$ 1.9999206249799E+0

KH KH/SEG DEGREES

| PHI | MATRIX |  |  |
| :---: | :---: | :---: | :---: |
| 9.9913567E-01 | -8.9993435E-01 | 1.7898026E-02 | 0. |
| $1.6835441 \mathrm{E}-03$ | 8.1850107E-01 | -1.0318177E-03 | 0. |
| 4.1405765E-05 | 5.2729421E-02 | 9.3489874E-01 | 0. |
| 1.5969620E-08 | 1.7551806E-05 | $1.9999049 \mathrm{E}-05$ |  |


| SOLVE FOR PARAI | $S=35$ | 9 |  |
| :---: | :---: | :---: | :---: |
| PSI | MATRIX |  |  |
| 2.4732436E-05 | 4.8384831E-05 | -1.3491545E-06 | $0 \cdot$ |
| -4.8000347E-05 | -9.3748332E-05 | 2.6157182E-06 | 0. |
| -1.1222942E-06 | -2.1969336E-06 | 6.1207046E-08 | 0 。 |
| -4.6746358E-10 | -9.1454587E-10 | 2.5500852E-11 | 0. |


| DYNAMIC CONSIOER PARAMETERS $=$ | 20 | 140 |
| :---: | :---: | :---: |
| THU | MATRIX |  |
| $5.7327025 E-03$ | $-5.9841157 E-04$ |  |
| $-1.1127897 E-02$ | $8.9685259 E-05$ |  |
| $-2.6009058 E-04$ | $-7.6937024 E-02$ |  |
| $-1.0835255 E-07$ | $-8.0476345 E-07$ |  |


oIAGONAL OF DYNAMIC NOISE MATRIX
0 .
0.
0.
0.


| 0. | $6.2681937 E-02$ | 0 |
| :--- | :--- | :--- |
| 0. | $2.7378387 E-01$ | 0 |
| 0. | $1.5922693 E-02$ | 0 |
| 0. | $5.2769429 E-03$ | 0 |



## GAIN MATRICES

7.9383341E-02
$7.9383341 E-02$
$2.4932652 E-04$
$2.4932652 E-04$
$-1.0950281 E-03$
$-1.1950281 E-03$
$-5.2515406 E-06$
-5. $2515406 \mathrm{K2}$
-6.2684259E-02
4. $3774940 \mathrm{E}-01$
6.6556582E-03
3.0426702E-0

RESIOUAL UNCERTAINITY MATRIX
2.8613001EF01


| -1.9127504E-05 | -3.8918073E-09 | 1.9910819E-09 | 8. $3590739 \mathrm{E}-10$ | 0. | 2.8344664E-01 | -1.9041106E-01 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.8660839E+04 | 3.7968567E-08 | $-1.9425044 E-08$ | -8. $1551328 \mathrm{E}-09$ | 0. | 1.6432531E-02 | 2.7445563E-02 |  |
| $\begin{aligned} & -8.3512026 E-04 \\ & \text { CXH CORR } \end{aligned}$ | $\begin{aligned} & -1.6991905 E-07 \\ & \text { MATRIX } \end{aligned}$ | 8.6932037E-08 | 3.6496305E-08 | 0. | 5.2920020E-03 | 8.3695743E-04 |  |
| 3.2730389 E -01 | -2.2154490E-02 | -4.8439307E-08 | 1.0469356E-09 | 4.2379475E-10 | -5.2182862E-01 | -1.2.977071E-01 | 2.1536195E-35 |
| 2.9056477E-01 | 2.2231015E-02 | -3.0367218E-07 | -6.8773171E-06 | -2.8977 150E-09 | -3.4384192E-01 | -1.5979468E-01 | 3.8471126 E |
| 3. 704710 HED 02 | 6.5464265E-03 | 2.327 ¢120E-07 | 1.3249595E- 87 | B. $0548012 \mathrm{E}-09$ | $1.8392130 \mathrm{E}-03$ | -2.0353175E-03 | -1.7674756E-07 |
| 7. 258488 JE-03 | -1.775867 0E-04 | -6.5354146E-07 | -5.1601228E-07 | -3. $3320353 \mathrm{E}-08$ | -1.7798353E-03 | -1.5013132E-04 | 1. $2606.547 \mathrm{E}-07$ |
| $\begin{aligned} & \text { SOLVE } F \\ & 00 \end{aligned}$ | OR MATRIX |  |  |  |  |  |  |
| 4.9923851E+00 | 4.0169585E-02 | 1.6600546E-03 | 3.1965218E-09 |  |  | + |  |
| 4.0169585E-02 | 2.5360854E+00 | -2.2705229E-02 | -1.2492021E-08 |  |  |  |  |
| 1.6600546E-03 | -2.2705229E-02 | $1.2241751 E+00$ | -7.6779222E-10 |  |  |  |  |
| 3.1965218E-09 | -1.2492021E-06 | -7.6779222E-10 | 6.3245553E+00 |  |  |  |  |
| CQU CORR | Matrix |  |  |  |  |  |  |
| -2.4925371E-03 | 1.5176504E-03 |  |  |  |  |  |  |
| $4.687281 .0 E-12$ | -4.3398876E-02 |  |  |  |  |  |  |
| 1.61793 04E-03 | -1.5952005E-03 |  |  |  |  |  |  |
| $\begin{aligned} & -1.3251628 E \sim 08 \\ & \text { COV CORR } \end{aligned}$ | $2.4812081 E-06$ <br> MatRIX |  |  |  |  |  |  |
| 2.49968 03E-06 | 5.0860135E-10 | -2.5020480E-10 | -1.0924067E-10 | 0. | 7.9836892E-05 | 3.2967051E-0? |  |
| 1.2538060E-05 | 2.55107596-09 | -1.3051523E-09 | -5.4793648E-10 | 0. | -8.6863085E-03 | -4.5320153E-01 |  |
| 1.3531684E= 07 | 2.7532453E-11 | -1.4085838E-11 | -5.9135971E-12 | 0. | -3.0091566E-04 | -1.8564627E-02 |  |
| $\begin{aligned} & 2.6627566 \mathrm{E}-09 \\ & \text { COW CORR } \end{aligned}$ | 5.4178193E-13 MatRIX | -2.7718027E-13 | -1.1636740E-13 | 0. | 8.7457333E-09 | -1.2631498E-08 |  |
| 1.0173148E-02 | 6.1102242E-03 | -6.1904747E-09 | -6. $8090182 \mathrm{E}-11$ | 3.9586511E-11 | $1.4544808 \mathrm{E}-02$ | 2.7980813E-03 | -5.1632517E-07 |
| -2.6371887E-01 | -6.3571715E-02 | 1.0633720E-08 | 7.8981516E-09 | 5. $0650597 \mathrm{E}-10$ | -1.9401289E-01 | -3.7539096E-02 | 6.8818275E-06 |
| -1.06902.65E-02 | -3.4268251E-03 | 1.0403181E-09 | 2.6819530E-10 | 1.2304020E-11 | -7.9954952E-03 | -1.5452299E-03 | 2.8102434E-07 |
| 6.6819499E-09 | -2.0234321E-09 | 2.4806202E-12 | 1.7238999E-12 | 1. $0917191 \mathrm{E}-13$ | -1.2256836E-08 | -2.1115579E-09 | 5.3141838E-13 |
|  | NAMIC NOISE MATRIX |  |  |  |  |  |  |

UNHODELED OYNAMIC NOISE COVARIANGE MATRIX
0.
0.
0.
0.

GAIN MATRICES
7.9383341E-02
$2.4932652 \mathrm{E}-04$
-1. $0950281 E-03$
-5, 2515406E-06
K2
$595-02$
matrix
$-6.2684259 \mathrm{E}-02$
4. $3774940 \mathrm{E}-01$
. 655658 2E-0
3. 0426702E-08

## RESIOUAL UNCERTAINITY MATRIX

$1.8613001 \mathrm{E}+01$
AGTUAL MEASUREMENT NOISE
7. $0620339999994 \mathrm{E}=01$

MEASUREMENT COYARIANGE MATRIX
5. $0000000000000 \mathrm{E}-01$



ENTRY PARAMETERS BASED ON MOST RECENT NOMINAL

| relitive velogity | 4.7276103513401E-02 | KM/SEC | PRESSURE | $\pm$ | 8.7973770811783E+02 | MILLIBARS |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| WTND VELOCITY | 0. | KH/SEC | DENSITY | = | 1.4316554063095E+09 | KG/KM ${ }^{\text {+F }} 3$ |
| OTNAMIS PRESSURE | $1.5998963651897 E+01$ | hillitiars | ALPHA | $=$ | 0. | DEGREES |
| molegul ar height | $4.2888539999999 E+01$ | KG-HOL | CA | = | 7.7962128296895E-01 | UNIT FREE |
| TEMPERATURE | $3.1697808454498 E+02$ | DEGREES K | CN | = | 0. | UNIT FREE |
| NACM NUMBER | 1.6118339613134E-01 | UNIT FREE | cma | = | 0. | UNIT FREE |
| AXEAL FORCE | -1.97 17549456176E-01 | KG-KM/SEC*+2 | MOMENT | = | 0 | KG-KM/SECFF\% |
| NORHAL FORGE | 0. | KG-KM/SEC** 2 | Gravity | = | 8.7228717129088E-03 | KM/SEC*F2 |
| Center of pressure | -1.394881i237912E-04 | KM | EPSILON | I | -1.3822765148786E-01 | DEGREES |



| SOLVE FOR PARAME | $\underset{\text { MATRIX }}{\text { TERS }}=3$ | 7 | 9 |
| :---: | :---: | :---: | :---: |
| 9.2647650E-05 | 6.3978079E-05 | 0. | 0. |
| 0. | 0. | 0. | 0. |
| 8.9537541E-07 | 6.1329513E-07 | 0. | 0. |
| -3.2100993E-11 | -2.2179182E-11 | 0 。 | 0. |


| OYNAMI: CONSIDER PARAMETERS $=$ | 20140 |
| :---: | :---: |
| THU | MATRIX |
| $2.5258 O B B E-02$ | $4.4230524 E-04$ |
| 0. | 0. |
| $2.4431996 E-04$ | $-1.6351681 E-01$ |
| $-8.7624918 E-09$ | $-6.5259512 E-07$ |



DIAGONAL OF OVNANIC NOISE MATRIX
0.

0 .
0.

STATE
MATRIX
. 27443APE-01 2.1011491E-01
$\begin{array}{ll}2.1011491 \mathrm{E}-01 & 7.4465541 \mathrm{E}-04 \\ 1.3216089 \mathrm{E}-01 & 5.2761690 \mathrm{E}-01\end{array}$
$-9.6466669 \mathrm{E}-02-1.2389246 \mathrm{E}-01$
CXO CORR MATRIX
1.9974101E-01
$-9.2152917 E-03$ ©.3373301E-02
$-9.2152917 E-03 \quad 4.0366289 E-02$
$\begin{array}{rr}-2.4288006 E-02 & 5.9804023 E-02 \\ 4.1167191 E-03 & -3.0477655 E-03\end{array}$
4. 1167191E-03 -3.0477655E-03
CXU CORR MATRIX

CXU CORR MATRIX

1. $0422991 \mathrm{E}-01 \quad-1.3204176 \mathrm{E}-01$
$-2.9155121 \mathrm{E}-04 \quad-5.2684189 \mathrm{E}-01$
$\begin{array}{rrr}-3.7620075 \mathrm{E}-02 & -9.1067104 \mathrm{E}-02\end{array}$ CXV CORR MATRIX
1.3216089E-01 -9.6466669E-02
$5.2761690 \mathrm{E}-01$-1.2389246E-01 $2.7835248 \mathrm{E}+00 \quad 9.0928365 \mathrm{E}-02$ 9.0928365E-02 9.3319703E-01
$1.8766235 \mathrm{E}-03 \quad-7.2027546 \mathrm{E}-08$ $-2.2955273 \mathrm{E}-03$ $3.7949632 \mathrm{E}-04$ .9751997E-04 7. $3298085 \mathrm{E}-08$ 4. $8359249 E-08$ 8802681E-07

| $1.5116182 E-05$ | $6.9334783 E-06$ |
| :--- | :--- |
| $5.8978652 E-05$ | $2.6947627 E-05$ |
| $5.5106246 E-0 B$ | $2.5488575 E-08$ | $5.5106246 \mathrm{E}-0 \mathrm{~B}$ 4.1448364E-06

6.9334783E-06 $2.6947627 E-05$
$2.5488575 \mathrm{E}-08$ 1.7767715E-06 1.7767715E-06

```
5.3006556E-02 -5.4599940E-02 \(1.8484716 \mathrm{E}-01-1.2435163 \mathrm{E}-01\) 2.5939312E-04 -1.2405979E-04
```

$\begin{array}{ll}-1.4610396 E-C 1 & -2.9626268 E-05 \\ -5.6992 B 46 E-01 & -1.1559243 E-04\end{array}$ $-5.3287327 E-04 \quad-1.0800308 E-07$
$-3.9914475 \mathrm{E}-02-8.1229459 \mathrm{E}-06$
$-6.5367176 E-04-1.6481471 \mathrm{E}-04$ 6.0035842E-09
-14 $1.9568113 E-14$
$-7.0878113 E-19$

CXH CORR MATRIX

$2.8461625 E-08$
$9.6799629 E-0$
9．6799629E－0
$6.6623892 E-1$
$-6.2341214 E-07$
2．0920956E－08
－． 2306952 E －0
5．8565417E－11
$-5.0769895 E-07$
$\begin{array}{ll}1.3369915 E-09 & -8.1569003 E-01 \\ 5.3680492 E-09 & -1.2375571 E-01\end{array}$
3． $8357578 \mathrm{E}-12$
－3．2923859E－0B
$-1.2375571 E-01$
$-8.4927924 E-05$
$-8.4927924 E-05$
$5.8148126 E-02$
－8．8703579E－02 －2．3938529E－02 $-1.2791774 \mathrm{E}-05$ $2.8159005 \mathrm{E}-02$
4.375592 3E－05 1．0932359E－п5 $1.7033356 \mathrm{E}-0$ －8．3327448E－09

## SOLVE FOR

| 00 | MATRIX |  |
| :---: | :---: | ---: |
| $4.9684828 \mathrm{E}+00$ | $1.7175284 \mathrm{E}-02$ | $8.9853774 \mathrm{E}-04$ |
| $1.71752844 \mathrm{E}-02$ | $2.3828783 \mathrm{E}+00$ | $-2.99012570 \mathrm{E}-02$ |
| $8.9853774 \mathrm{E}-04$ | $-2.9012570 \mathrm{E}-02$ | $1.2239861 \mathrm{E}+00$ |
| $2.5324868 \mathrm{E}-09$ | $-3.1337431 \mathrm{E}-08$ | $-2.7444515 \mathrm{E}-10$ |

2．5324868E－09 $-3.1337431 \mathrm{E}-00$ $-3.1337431 E-00$
$-2.7444515 E-10$ 6． $3245553 \mathrm{E}+00$

1．4179132E－07 1．4328014E－07 $-1.9001483 E-06 \quad-8.4149919 E-07$ $-3.7461367 E-07-1.7002759 E-07$ 1． $3271169 \mathrm{E}-12$
3．072592ロー12
－3．9102993E－09 $6.4844249 \mathrm{E}-00$ $2.5165534 E-12$
$-3.3682128 E-09$
$1.1362169 E-08$
$-4.8440190 \mathrm{E}-11$

| $2.2319276 \mathrm{E}-02$ | $3.4241990 \mathrm{E}-02$ |  |
| ---: | ---: | ---: |
| $-2.2908800 \mathrm{E}-02$ | $-3.6715894 \mathrm{E}-01$ |  |
| $-1.0107860 \mathrm{E}-03$ | $-1.5028904 \mathrm{E}-02$ |  |
| $-2.4887853 \mathrm{E}-08$ | $4.3873903 \mathrm{E}-08$ |  |
|  |  |  |
| $3.1781108 \mathrm{E}-02$ | $6.6502089 \mathrm{E}-03$ | $-3.7647792 \mathrm{E}-07$ |
| $-5.2784497 \mathrm{E}-03$ | $7.8426687 \mathrm{E}-03$ | $3.7883604 \mathrm{E}-07$ |
| $1.0180349 \mathrm{E}-04$ | $-6.0050524 \mathrm{E}-04$ | $-1.9761125 \mathrm{E}-07$ |
| $4.847027 \mathrm{EE}-08$ | $2.4921007 \mathrm{E}-08$ | $2.3115018 \mathrm{E}-13$ |

NHODELED DYNAMIG mínISE COVARIANGE MATRIX
B．

Kit
MATRIX
1．840749GEー08
$-9.520135 \mathrm{E}-11$
$-3.311923 \mathrm{E}-09$
－6． $1961091 \mathrm{E}=12$
1.892433 K2 -08

1． $2398394 \mathrm{E}-08$ 1．0313545E－0 9 -1.57357 0．OE -14

## RESIDUAL UNCERTAINITY MATRIX

1．9433686E－13

| $\begin{array}{r} \text { OASERVATIOI } \\ H \\ .1 .2573654 E+07 \end{array}$ | N MATRIX PART MATRIX $6.8842557 E+07$ | IONS $-7.8192698 E+03$ |
| :---: | :---: | :---: |
|  | Parameters MATRIX <br> 5.:192561E+05 | 0. |
| $\begin{gathered} \text { DYNAMIC-MEI } \\ \text { G } \\ 4.0611272 F+06 \end{gathered}$ | a Surement consi MATRIX <br> 0 . | der Parameters <br> 0. |
| MeASUREMENT | IT CONSIOER PARA MATRIX | METE RS |
| 0. | 0 - | 0 - |

0. 

.
0.
0.
0.
$-3.2483136 E+08-1.0995444 E+08$
8.1145005E-03
$8.9584086 E+07$ 0.

MEASUREMENTS

```
ESTIMATED
8. \(9584085553111 E+07\)
```

RESIDUAL
$-1.5470925164847 E+06$

DEVIATIONS FROM MOST RECENT NOMINAL

TRAJEGTORY

| H | $2.84780999957196 \mathrm{E}-02$ |
| :--- | ---: |
| $\mathbf{V}$ | $-1.4728529611394 \mathrm{E}-04$ |
| GAMMA | $-2.9357513181557 \mathrm{E}-01$ |
| PHI | $-7.2651845172863 \mathrm{E}-04$ |

SOLVE FOR PARAMETERS
$2.9277689147708 \mathrm{E}=02$
$1.9181463333361 \mathrm{E}-02$
$1.5956007602780 \mathrm{E}-03$
-2.4344584322059E-D8

## STATE

| STATE PP | MAT |
| :---: | :---: |
| 8. $2345519 \mathrm{E}-01$ | 3.2232781E-01 |
| $3.22827^{81 E-01}$ | 6.1512324E-04 |
| 1.7028882E-01 | 4.5469693E-01 |
| -9.6715564E-02 | -1.51495565-01 |
| CXO CORR | MA IRIX |
| 1.9908191E-01 | 8.1538237E-02 |
| 3.0031614E-04 | 6.4533205E-02 |
| -2.0177E68E-02 | 6.9948490E-02 |
| 4.1545639E-03 | -2.9976777E-03 |
| cxu COR | MATRIX |
| $3011937 E-01$ | -1.6226194E-01 |
| 5. $2869625 \mathrm{E}-01$ | -4.3298236E-01 |
| -8.1584683E-02 | -9.5388425E-01 |
| 1048798F-02 | -9.0401426E-02 |
| CXV CORR | MATRIX |
|  |  |

7.7970115340827E-01
$-4.9495749933293 E-04$
$7.3925024253185 \mathrm{E}-04$
$4.5262891154025 \mathrm{E}-01$ 4.5262891154025E-01
(ERROR EST-ACT)
-7.5122305345107E-0 $3.4767220321898 \mathrm{E}-04$ $-2.9431438205810 \mathrm{E}-01$ $-4.5335542999198 \mathrm{E}-01$

## $-4.5523870818135 E+00$ <br> 2.1527477007121E+00 -2. $0445610369656 E+00$ 6. $9999983023128 \mathrm{E}+00$

KM
KM/SEC begrees DEGREES
1.7028882E-01 4.5469693E-01 $2.6548491 E+00$ 9.4636726E-02
$1.5196694 E-03$
$-2.4453737 E-04$ $-2.4453737 E-04$ $7.7256924 \mathrm{E}-04$ $1.0057698 \mathrm{E}-03$
-9. 6715564E-02 -1.5149556E-01 9. $4636726 E-02$
9. $3319474 \mathrm{E}-01$
$-7.1295572 E-08$
-9. $6216500 \mathrm{E}-08$ 4. $7247093 \mathrm{E}-08$ 5.8800393E-07

| -4.5620510E-01 | -9.2530373E-05 | 4.7211715E-05 | 2.1557647E-05 | 0. | 2.8118772E-51 | -1.2221903E-01 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1.0738873E-01 | 2.1779088E-05 | -1.1112328E-05 | -5.0832360E-06 | 0. | $2.6788210 \mathrm{E}-02$ | 1.2948247E-02 |  |
| -3.9154585E-02 | -7.9688360E-06 | 4.0662050E-06 | 1.7407999E-06 | 0. | -3.6151603E-02 | 6.1222892E-02 |  |
| CXh CORR | R Matrix |  |  |  |  |  |  |
| 3.7914087E-01 | -3.3197361E-03 | 3.3516262E-08 | 2.5635841E-08 | 1.6479983E-09 | -8.3263188E-01 | -8.5670630E-02 | $4.6026375 \mathrm{E}-05$ |
| 2.9948347E701 | -4.5401148E-03 | 8.3142299E-08 | 6.7697149E-08 | 4.3900285E-09 | -5.9874259E-02 | -5.2952920E-02 | -1.0175979E-06 |
| 2.45298 98E-01 | -2.8135075E-03 | -1.5651452E-08 | -1.4690342E-08 | -9.6971012E-10 | $4.1448793 \mathrm{E}-12$ | -1.1085064E-02 | -6.5641016E-06 |
| -5.6039519E-02 | 5.8109855E-03 | -6.2352436E-07 | -5.0780406E-07 | -3.2930796E-08 | $5.8440711 \mathrm{E}-02$ | 2.8081126E-02 | -5.4672708E-08 |
| SOLVE | OR MATRIX |  |  |  |  |  |  |
| 4.9677824E+00 | 1.679693 0E-02 | 8.3628967E-04 | 2.7170372E-09 |  |  |  |  |
| 1.6796930E-02 | $2.3822513 E+00$ | -2.9105631E-02 | -3.1094029E-08 |  |  |  |  |
| F. $3628967 E 04$ | -2.9105631E-02 | $1.2239777 \mathrm{E}+00$ | -2.3370431E-10 |  |  |  |  |
| 2.7170372E-09 | -3.1094029E-08 | -2.3370431E-10 | 6. $3245553 \mathrm{E}+00$ |  |  |  |  |
| CQU CORR | R MATRIX |  |  |  |  |  |  |
| -3.8892932E-02 | 1.9263609E-02 |  |  |  |  |  |  |
| 4.7789288E-02 | -6.6698887E-02 |  |  |  |  |  |  |
| 1.81964 07E-03 | -7.3746549E-04 |  |  |  |  |  |  |
| $\begin{aligned} & 4.4740032 E-08 \\ & \text { COV CORR } \end{aligned}$ | $\begin{aligned} & -4.5146398 E-08 \\ & \text { MATRIX } \end{aligned}$ |  |  |  |  |  |  |
| -7.2161645E-03 | -1.4447605E-06 | 7.3713350E-07 | 4.1564089E-07 | 0. | 2.0909213E-02 | $3.3549796 \mathrm{E}-02$ |  |
| 1.0473318E-02 | 2.1309857E-06 | -1.0873086E-06 | -4.6964415E-07 | 0. | -2.4845582E-02 | -3.6820785E-01 |  |
| 2.346063 0E-03 | 4.7612501E-07 | -2.4293360E-07 | -1.0978823E-07 | 0. | -1.3233809E-03 | -1.5183185E-02 |  |
| $\begin{aligned} & -2.5847720 E-08 \\ & \text { COW GORR } \end{aligned}$ | $-5.2613442 E-12$ | 2.68377 06E-12 | 1.1492437E-12 | 0. | -2.3964846E-08 | 4.4329148E-08 |  |
| 3. $022785 \mathrm{EE-02}$ | 7.5252174E-03 | -3.0729676E-09 | -2.5824783E-09 | -1.6789839E-10 | 2.9571788E-02 | 7.2412216E-03 | -2.7734694E-08 |
| -5.4932258E-02 | -8.0014191E-02 | 6.6006044E-08 | 1.2439292E-00 | 4. $4456746 \mathrm{E}-10$ | -8.3043737E-03 | 8.6509036E-03 | 8.5820026E-07 |
| 5.80.43906E-03 | -3.4198536E-03 | $1.9748498 \mathrm{E}-09$ | 1.2546554E-10 | -4.6980 034E-12 | -3.8787898E-04 | -4.6998738E-04 | -1.2001795E-07 |
| -3.1527718E-08 | 3.1371118E-09 | 2.5160062E-12 | 1.7352360E-12 | 1.0975337E-13 | $4.9916173 \mathrm{E}-08$ | 2.4535612E-08 | 2.0351951E-15 |
|  | NAMIC NOISE MATRIX |  |  |  |  |  |  |

## UNMODELEO DYNAMIC NOISE GOVARIANGE MATRIX

 0.GAIN MATRICES
K1
1.8407496E-08
-9.5201350E-11
3. $3119239 \mathrm{E}-09$
$1.092433^{K 2}$
MATRIX

1. $8924330 \mathrm{E}-08$
$1.2398394 \mathrm{E}-08$
$1.0313545 \mathrm{E}-09$
-1. $5735700 \mathrm{E}-14$
MATRIX

## RESIDUAL UNCERTAINITY HATRIX $1.9433886 \mathrm{E}+13$ MATRIX

AGTUAL MEASUREMENT NOISE
$8.5621099999883 E+03$
measurenent covariance matrix
1.0000000000000E+05

LTR PROGRAMMERS' MANUAL

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## I. INTRODUCTION

This Lander Trajectory Reconstruction Programmers' Manual is intended to supply the reader with sufficient information about the lander trajectory reconstruction (LTR) program to enable him to modify it for his own uses. Both the overall structure of the program and the computational flow of individual subroutines are presented in this manual.

Chapters II and III describe the overall flow and operation of the program and present the graphical structure of the various segments.

Chapter IV of this volume contains the definitions of common variables. The variables are defined in the order of appearance within the common block, and the common blocks are presented alphabetically.

Chapter $V$ documents each of the subroutines in detail. The purpose of the subroutine is given, and subroutines required by the subroutine are listed. Arguments to the subroutine and local variables of interesi are defined, and usage of common variables is noted. The mathematical analysis and a flow chart are provided whenever nacessary for a thorough understanding of the subroutine.

## II. SUMMARY OF LTR OPERATION

The LTR program is segmented by an overlay structure to conserve core storage. There are five overlay sections, which are discussed in the following sections.

## A. MAIN

The executive subroutine MAIN controls overall program execution and calls the remaining sections according to values from input data. The rest of the subroutines in this overlay section are calied from several of the remaining overlay sections and appear in the main overlay to conserve core.

## B. DATGEN

The data generator overlay section generates the actual trajectory, the actual measurements, the quantized ARU-VRU data, and the actual atmosphere experienced by the vehicle as it descends. Several input/output files are written for use by subsequent overlay sections in reconstructing the atmosphere and trajectory.

## C. PRPRDS

The preprocessor overlay section operates on the quantized ARU-VRU data generated by DATGEN and filters them using a leastsquares curve-fitting technique. File 16 is written with the smoothed data for use by the reconstructor.
D. LTRCON

The reconstructor overlay section utilizes the $I / \emptyset$ files written by DATGEN and PRPR $\varnothing$ S to reconstruct the atmosphere and descent trajectory. It is presumed that files 10 and 16 have been previously written on magnetic tape or are generated immediately prior to the start of reconstruction. Derived measurements are used to drive the dynamic equations, and a Kalman-Schmidt filter is employed to refine the estimated errors vetween "actual" and "assumed" parameters. Up to 15 data files may be written for plotting purposes, according to data supplied from NAMELIST input.
E. SUMMRY

The summary overlay section prints the summary tables from $I / \emptyset$ files written by LTRC $\varnothing \mathrm{N}$ and calls the user-written plot package according to information supplied by LTRC $\emptyset$ N. A sample plot package is described in Appendix A.

## III. LTR Program Structure

A. LTR SUBROUTINE CLASSIFICATION

The subroutines chat make up the LTR program are listed according to category ia Table III-1. Table III-2 lists the subroutines again with a briaf summary of their purposes: The individual subroutines are documented in detail in alphabetical order in Chapter V.

## B. LTR SUBROUTINE HIERARCHY

As described in Chapter II, the LTR program is composed of three distinct parts. Each part is governed by an executive routine that is called from the main routine according to the value of RUNNØ. The calling hierarchy of the LTR program is given in Figure III-1. Each of the three parts is broken down separately in Figures III-2 thru III-4. Subroutines in parentheses are called by the preceding subroutine. An asterisk indicates an expansion elsewhere in the hierarchy charts. BLDCK DATA in Figure III-1 is adjoined to LTR by dotted lines, indicating that data initialized there are available to all subroutines.

Table III-1 LTR Subroutines
I. Executive Subroutines

1. LTRTWø
2. LTRCON
3. DATGEN
4. SUMMRY
II. Integrator Subroutines
5. ATMSET
6. DERIVE
7. DERIV1
8. NTM
9. RKUTDG
10. NTM2
11. ATM@SP
12. RKUT3
13. DERIV 3
14. RKUTL3
III. Matrix Manipulation Subroutines
15. $A D D$
16. CDPY
17. CØRMAT
18. CØRR
19. CORRD
20. DMULTT
21. DTAB
22. INVPD2
23. INVPSD
24. MULT
25. MULTD
26. MULTT
27. SUB
28. SYMTRZ
29. TMULT
30. TMULTT
31. TAB
32. MATRIX
IV. Measurement Subroutines
33. $\emptyset B S M 1$
34. MEAZUR
35. SENS $\emptyset R$
36. $\emptyset B S M$
37. ATTACK
V. Range/Doppler Measurement Subroutines
38. AECEQ
39. GHA
40. ECLIP
41. PLANE
42. ELCAR
43. SUBSØL
44. EPHEM
45. TIME
46. EQUATR
47. GEDG

## Table III-1 (Conl)

VI. Filter Subroutines

1. DYNQIZ
2. FILTER
3. HMM
4. NøRMNZ
5. JACOBN
6. RNUM
7. STM
VII. Input/Output Subroutines
8. MATgUT
9. TIMEX
10. PRINT1
11. SETUP1
12. PRPRQS
13. SMDDT2
14. CONVRT
15. NE:TAA
16. NEXTIM
17. ØUTPHI
18. QUTPP
19. PRINT
20. PSTøRE
21. READAC
22. RSTART
23. SCHED
24. SETICN
25. SETPLT
26. SETUP
27. PLDTS

VIII. Data Initialization Subroutines
28. BLKDAT
29. BEGIN

## Table III-2 Purpose of Subroutines

SUBROUTINE PURPOSE
AECEQ Computes transformation from ecliptic to geocentric equatorial

ATMSET
(ATMDAT)
ADD
ATTACK

ATMDSP

BLKDAT
BEGIN

CgPY

CøRMAT

CøRR

CØRRD


DTAB

Initializes and computes atmospheric parameters

Adds matrix $X$ and matrix $Y$
Computes angle of attack measurement

Calculates atmospheric parameters in mode A

Initializes common variables
Resets common variables for reconstructor

Copies matrix/vector $X$ into matrix/vector $Y$

Converts covariance matrix into correlation matrix

Computes correlations for offdiagonal block of partitioned covariance matrix

Computes correlations for diagonal block of partitioned covariance matrix

Converts state vectors to output units

Multiplies a diagonal matrix $X$ times the transpose of matrix $Y$

Performs table lookup with two independent variables

Table III-2 (Cont)

SUBROUTINE

| DATGEN | Drives data generator to provide input to reconstructor |
| :---: | :---: |
| DERIV1 | Computes derivatives for integration by the data generator |
| DERIV3 | Computes derivatives for integration by the mode B reconstructor |
| DERIVE | Computes derivatives for integration by the mode A reconstructor |
| DYN@IZ | Computes dynamic noise covariance matrix |
| ECLIP | Computes planetocentric ecliptic state of spacecraft for DSN tracking |
| EPHEM | Computes heliocentric ecliptic coordinates of the planets |
| EQUATR | Computes transformation from geoequatorial to planetoequatorial |
| FILTER | Computes estimates and covariance matrices |
| GEDG | Computes transformation from planetoequatorial to planetogeographical |
| GHA | Computes Greenwich hour angle of the vernal equinox |
| HMM | Computes observation matrix $H$ for Kalman filter equations |
| INVPD2 | Inverts positive definite matrix $X$ ( NxN ) |
| INVPSD | Inverts positive definite $2 \times 2$ matrix X |

## Table III-2 (Cont)

SUBROUTINE PURPOSE

| JACDBN | Computes sensitivity matrices by numerici ${ }^{1}$ differencing |
| :---: | :---: |
| LTRTWø | Controls , srall program for data generation and trajectory reconstruction |
| LTRCON | Drives reconstructor portion of LTR |
| matøut | Prints matrix X |
| MULT | Multiplies matrix $X$ times matrix $Y$ |
| MULTD | Multiplies matrix $X$ times diagonal matrix $Y$ |
| MULTT | ```Multiplies matrix X times matrix Y transposed``` |
| MATRIX | Performs matrix alegbra through use of multiple entry points |
| MEAZUR | Processes measurements |
| NEXTAA | Takes accelerometer data from preprocessor tape |
| NEXTIM | Selects measurement or other event |
| N@RMNZ | Computes normally distributed noise |
| NTM | Propagates most recent nominal traiectory |
| NTM2 | Propagates original nominal trajectory |
| ØBSM1 | Contains measurement equations for the data generator |

Table III-2 (Cont)

| SUBROUTINE | PURPOSE |
| :---: | :---: |
| $\emptyset B S M$ | Contains measurement equations for the reconstructor |
| QUר゙¢I | Prints intermediate variables and phi matrix |
| QUTPP | Prints correlation, gain, and dynamic noise matrices |
| PLANE | Cormputes entry plane orientation to reference plane |
| PRINT1 | Prints trajectory and measurements from the data generator |
| PRPRDS <br> (SMøøT2) | Preprocesses accelerometer and gyro measurements for the reconstructor |
| PRINT | Prints trajectory and measurements from the reconstructor |
| PSTøRE | Stores data for plot package |
| PLøTS | Drives system plot package |
| RKUTDG | Integrator for data generator |
| QEADAC | Reads "actual" measurements created by data generator |
| RSTART | Punches restart cards for reconstructor |
| RKUT3 | Integrator for mode A reconstructor |
| RKUTL3 | Integrator for mode B reconstructor |
| RNUM | Generates measurement noise on "actual" measurements |
| SUBSØL | Computes transformation from planetocentric ecliptic to subsolar orbital plane |

## Table III-2 (Concl)

SUBROUTINE
SENS@R

SETUP1
SCHED

SETICN
SETPLT
SETUP
STM
SUMMRY
TMULT

TMULTT

TAB

TIMEX
TIME

PURPOSE
Quantizes accelerometer and gyro measurements

Reads input data for data generator
Reads and sequences measurements and events

Sets iteration counters for printout Reads plot control variables

Reads input data for reconstructor
Calculates state transition matrices
Controls summary print and plotting
Multiplies matrix $X$ transposed times matrix $Y$

Multiplies matrix $X$ transposed times matrix $Y$ transposed

Performs table lookup with one independent variable

Calculates central processor time
Converts Julian date to/from calendar date, epoch 1900


Figure III-1 LTR Executive Flow Diagram


Figure III-2 DATGEN Executive Flow Diagram


Figure III-3 LTRCØN Executive Flow Diagram



Figure III-3 (Concl)

## Iv. COMMON VARIABLE DEFINITIONS

## A. COMMON VARIABLES BY BLOCKS

In this section common blocks are listed in alphabetical order. Variables within tnese blocks are defined in the order they appear within the block.
/ACCEL/

ACCLX Actual acceleration along the X-axis (written in DATGEN)

ACCLZ Actual acceleration along the Z-axis (written in DATGEN)
/ACT/

RHøA Actual density (read in READAC)
TRMPA Actual stagnation temperature (read in READAC)
ACCLXC Actual acceleration along the X -axis (read in READAC)

ACCLZC Actual acceleration along the Z-axis (read in READAC)

MWTA Actual molecular weight (read in READAC)

GENDAT Flag to determine if data generator has been run before reconstructor (set .TRUE. in SETUP1)

RESTRT Flag to determine if data generator an? reconstructor are being restarted (read in SETUP1, SETUP)

MWTM Nominal molecular weight (input in SETUP)
THETI Initial attitude angle (input in SETUP)
/ATMS/

BKTBL(20) Breakpoints of ratios of lift to drag versus Mach number used to calculate angle of attack measurement

GAMTBL(20) Breakpoints of ratios of specific heats versus molecular weight used to calculate Mach number and speed of sound

## /AX/

AXC Reconstructed acceleration along $X$-axis as experienced by VRU

AZC Reconstructed acceleration along Z-axis as experienced by VRU

THTC Change in inertial pitch attitude since TZER
ØMGC Reconstructed angular velocity as experienced by ARU

ALPHA Reconstructed angle of attack

## /BM/

EDNBM(30) Estimated deviations from most recent nominal trajectory before a measurement

QEDNBM(30) Estimated deviations in solve-for parameters from most recent nominal before a measurement

| P(36) | State covariance matrix |
| :---: | :---: |
| Q(100) | Solve-for parameter covariance matrix |
| DU(20) | Dynamic-consider parameter covariance matrix |
| DV(20) | Measurement-consider parameter covariance matrix |
| DW(10) | ```Dynamic/measurement-consider parameter covariance matrix``` |
| $\operatorname{CXQ}$ (60) | Covariance matrix relating state parameters to solve-for parameters |
| CXU (120) | Covariance matrix relating state parameters to dynamic-consider parameters |
| CXV(120) | Covariance matrix relating state parameters to measurement-consider parameters |
| $\operatorname{CXW}$ (60) | Covariance matrix relating state parameters to dynamic/measurement-consider parameters |
| CQU(200) | Covariance matrix relating solve-for parameters to dynamic-consider parameters |
| CQV(200) | Covariance matrix relating solve-for parameters to measurement-consider parameters |
| CQW(100) | Covariance matrix relating solve-for parameters to dynamic/measurement-consider parameters |
| SP(36) | $P$ matrix saved before a new measurement, event, etc |
| SQ(100) | $Q$ matrix saved before a new measurcment, event, etc |
| SDU(20) | DU matrix saved before a new measurement, event, etc |
| SDV(20) | DV matrix saved before a new measurement, event, etc |


| SDW (10) | DW matrix saved before a new measurement, event, etc |
| :---: | :---: |
| SCXQ (60) | CXQ matrix saved before a new measurement, event, etc |
| S CXU (120) | CXU matrix saved before a new measurement, event, etc |
| SCXV (120) | CXV matrix saved before a new measurement, event, etc |
| SCXW (60) | CXW matrix saved before a new measurement, event, etc |
| SCQU (200) | CQU matrix saved before a new measurement, event, etc |
| SCQV(200) | CQV matrix saved before a new measurement, event, etc |
| SCQW(.200) | CQW matrix saved before a new measurement, event, etc |
| CXQC (60) | Correlation matrix of CXQ matrix |
| CXUC(120) | Correlation matrix of CXU matrix |
| CxvC (120) | Correlation matrix of CXV matrix |
| CXWC (60) | Correlation matrix of CXW matrix |
| CQUC (200) | Correlation matrix of CQU matrix |
| CQVC( 200 ) | Correlation matrix of CQV matrix |
| CQWC (100) | Correlation matrix of CQW matrix |
| PHI (36) | State transition matrix |
| PSI (60) | Sensitivity matrix relating state parameters to solve-for parameters |
| THU(120) | Sensitivity mairix relating state parameters to dynamic-consider parameters |
| THW (60) | Sensitivity matrix relating state parameters to dynamic/measurement-consider parameters |

HM(24) Partition of observation matcix relating observables to state

MM(40) Partition of observation matrix relating observables to solve-for parameters

LM(80) Partition of observation matrix relating observables to measurement-consider parameters

GM(40) Partition of observation matrix relating observables to dynamic/measurement-consider parameters

JM(16) Kalman filter J matrix
W1(24)
W2 (40)
W3(80)
W4 (80)
W5 (40)
K1(24) Kalman gain matrix for state parameters
K2(40) Kalman gain matrix for solve-for parameters
WøRK(400) Working matrices for filter equations
$W(120)$
$R(16) \quad$ Measurement noise matrix
DYN(36) Dynamic noise matrix
PP(36) Correlation matrix of $P$ matrix
QQ(100) Correlation matrix of $Q$ matrix
JIN(16) J inverse of Kalman filter equations
SQDU(20) Standard deviations of dynamic-consider parameters

SQDV(20) Standard deviations of measurement-consider parameters

SQDW Standard deviations of dynamic/measurement-consider parameters

PPC(36) PP matrix converted to output units
QEDN(10) Estimated deviations of solve-for parameters from nominal values

QEDNBC(10) QEDN matrix before a quasi-event
/DET/

CDELT1 Cocines of calibrated misalignments of the VRU
CDELT3 and ARU after biasing

SDELT1 Sines of calibrated misalignments of the VRU and SDELT2
SDELT3 ARU after biasing

SUBDL1 Intermediate term used to calculate axial and normal acceleration

## /DøPLER/

TZER $\varnothing$ Trajectory time TC at start of data generator and reconstructor

DATEJ Julian date, epoch 1900, corresponding to TZER $\varnothing$ (calculated in SETUP1, SETUP)

SALT(3) Station location altitudes for DSN tracking in kilometers

SLAT(3) Station location latitudes for DSN tracking in radians

SLめN(3) Station location longitudes for DSN tracking in radians

| RANGE (3) | Actual range measurement (km) |
| :---: | :---: |
| RANGER(3) | Actual range-rate measurements (km/s) |
| ØMEGAE | Angular velocity of earth (rad/s) |
| $\emptyset$ BLIC | Obliquity of the ecliptic (radians) |
| REARTH | Radius of the earth (kilometers) |
| GHATØ | Greenwich hour angle of the vernal equinox at TZERD |
| SCPEC(6) | Spacecraft planetocentric ecliptic coordinates based on ECLめNG(1), ECLINC(1), PHIR(1) |
| PHIR(3) | Reference angle phi for ecliptic, planetoequatorial, and subsolar orbital planes, respectively |
| ECLøNG(3) | Reference longitude for ecliptic, planetoequatorial, and subsolar orbital planes, respectively |
| ECLINC(3) | Reference inclination for ecliptic, planetoequatorial, and subsolar orbital planes, respectively |
| DELRR(3) | Range perturbations due to refractivity |
| DELRRR(3) | Range-rate perturbations due to refractivity |
| RØTNØ | The target planet angular velocity component normal to the entry plane |
| NTP | Integer number of the target planet (see EPHEM for range of values) |
|  | /DERIV/ |
| VA | Velocity of atmosphere at vehicle position |
| SGAM | Sine of vehicle flightpath angle |
| CGAM | Cosine of vehicle flightpath angle |
| V | Velocity of vehicle |



| GEND | Integer to indicate number of gyro elements |
| :--- | :--- |
| IAA | Indicates which accelerometer data partition to <br> use to calculate state derivatives |
| ICNTR | Indicates number of increments between print points |
| IEND | Indicates end of accelerometer data partitions <br> for a given interval |
| IGYRø | Indicates which gyro data partition to use to <br> calculate state derivatives |
| INDEP(15) | Indicators of independent variables for plot <br> package |
| IPRINT | Print increment counter used with ICNTR to control <br> print points |
| IX | Not used <br> LASTIM |
| Not used |  |
| LICNTR(15) | Array of values for ICNTR |
| LISTS(6) | Not used |


| LISTQ(10) | List of solve-for parameters |
| :---: | :---: |
| LISTU (20) | List of dynamic-consider parameters |
| LISTV(20) | List of measurement-consider parameters |
| LISTW (10) | List of dynamic/measurement-consider parameters |
| M | Parameter used to quantize VRU and ARU data |
| MCNTR | Indicates which measurement or event is currently being processed |
| MCøDE (250) | Array of values of MCNTR |
| M ${ }^{\text {d }}$ DE | Not used |
| N | Parameter used to quantize VRU and ARU data |
| NE | Number of state parameters in LISTS |
| NICNTR | Indicates LICNTR value of interest |
| NM | Number of observables in a measurement |
| NMEAS | Not used |
| NMPTS | Number of breakpoints of altitude versus molecular weight |
| NPRED | Not used |
| NQS | Set to NQ and used to set up plot package |
| NQUASI | Not used |
| NTPTS | Number of breakpoints of altitude versus ambient temperature |
| NVAR(15) | Array of number of dependent variables for plot package |
| NS | Number of state parameters in LISTS |
| NQ | Number of solve-for parameters in LISTQ |

Number of dynamic-consider parameters in LISTU
NV Number of measurement-consider parameters in LISTV
NW Number of dynamic/measurement-consider parameters in LISTW

PRøB(40) Array of Hollerith data for problem identification
RUNNФ Indicates which part of LTR is being executed (data generator, reconstructor, etc)

SUM Not used
SUMFAR Not used
SIZEP Not used
TYPE $\quad$ Current value of MCøDE used to process event or measurement

CDEL Logical to reduce time needed to compute state derivatives when $C(55), C(56), C(63)$ are not perturbed

HITGND Logical set to . TRUE. When vehicle impacts the planet

LASTYM Logical used to quantize VRU and ARU data
LINEAR(15) Array of logicals to set linear scales for plot package

LøG(15) Array of logicals to set log scales for plot package

LTRI Logical to control mode B logic
LTR2 Logical to control mode A logic
MACHN $\varnothing$ Logical to control updating of Mach number for LTR2 mode of reconstructor

```
PARACH Not used
PLØTL(15) Array of logicals used to control storage of plot data (subroutine PSTØRE)
REDRR1 Not used
```

REDRR2 Logical to control calculation of measurement

SUMTB(15) Array of logicals to control print of summary tables (subroutine SUMMRY)

TERHT Logical used to control terrain height modeling
UPDATE Not used

## /øBSERV/

$\operatorname{ACC}(3,3) \quad$ Not used

ACCDT Not used

ACCT Not used

AQUANT Not used

BF(16,4) Bias factors used to perturb actual measurements read in READAC

BTBL(50) Not used
DELT(18) Misalignment angles for gyro and accelerometer measurements

EPSM(50) Table of shock wave density ratios versus velocity to calculate stagnation pressure measurement

ETA Altimeter beam angle used in altimeter measurements

GQUANT Not used
GYRøDT Not used
$\operatorname{RR}(16,4,3)$ Array of measurement noise, dimensioned on measurement type, measurement component, and noise option
$S D(16,4) \quad$ Array of measurement noise standard deviations, dimensioned by measurement type and measurement component

SF(16,4) Scale factors used to perturb actua? measurements read in READAC


PREDIC Not used
PREDND(50) Not used
STC Not used
XNPM(30) Not used
XNPMS(30) Not used

AEEDEN Actual error in estimated deviation from most recent nominal value of density

AEESLV(10) Actual error in estimated deviations from most recent nominal solve-for values after a measurement

AEESTT(6) Actual errors in estimated deviations from most recent nominal trajectory state

AEETMP $\quad$ Actual error in estimated deviation from most recent nominal value of ambient temperature

ALPHAA Actual angle of attack in degrees
DENS Estimated deviation from most recent nominal value of density

EDNC(6) Estimated deviations from most recent nominal trajectory state

ØMGCC Reconstructed angular velocity converted to degrees
PPD(6) Di.agonal elements of PP matrix after a measurement
PPDBM(6) Diagonal elements of PP matrix before a measurement

PPXD Actual dynamic pressure in willibars
QQD(10) Diagonal elements of $Q Q$ matrix after a measuremeni
QQDBM(10) Diagonal elements of $Q Q$ matrix before a measurement

RESI(4) Measurement residuals
SODENS Standard deviation in density after a measurement
SDMWT2 Not used
SDTEMP Standard deviation in ambient temperature after a measurement

| THETRC | Reconstructed angle THETA |
| :---: | :---: |
| XNAC(6) | Actual state in output units |
| $\mathrm{XNC}(6)$ | Most recent nominal state in output units |
| SDENBM | Standard deviation in density before a measurement |
| STEMBM | Standard deviation in ambient temperature before a measurement |
| TEMEDN | Estimated deviation in ambient temperature after a measurement |
| DENSBM | Estimated deviation in density before a measurement |
| TEMDBM | Estimated deviation in ambient temperature before a measurement |

EDNBQC(30) Converted estimated deviations from most recent nominal trajectory before a quasi-event

EDNBMC(30) EDNBM converted to output units before a quasievent

XNBQC(30) Most recent nominal trajectory (converted) before a quasi-event

CARCØR(6) Cartesian coordinates of heliocentric ecliptic position and velocity of a specified planet

CøNEL(7) Conic elements of heliocentric ecliptic orbit of a specified planet and gravitational constant of the planet

XSTEP Quantizing factor for axial acceleration
ZSTEP Quantizing factor for normal acceleration
TSTEP Quantizing factor for raice gyro attitude

VXQA(9) Axial acceleration values before smoothing
VZQA(9) Normal acceleration values before smoothing
THTQA(9) Rate gyro values before smoothing
CAN $(3,3) \quad B$ transposed times $B$
$D(3,3) \quad$ Inverse of CAN matrix
$E(3,9) \quad$ Pseudoinverse of $B$
$B(9,3) \quad$ Least-squares filter matrix used to smooth accelerometer and gyro data

A1(3) Quadratic coefficients used by reconstructor for smoothed axial acceleration values

A2(3) Quadratic coefficients used by reconstructor for smoothed normal acceleration values

A3(3) Quadratic coeffisients used by reconstructor for smoothed gyro values
$A A(3,3,50)$ Values of $A 1, A 2, A 3$ stored by SMDØT2 and read by NEXTAA for each integration interval

VXQ Latest axial acceleration stored in VXQA array for curve fitting by least-squares filter

VZQ Latest normal acceleration stored in VZQA array for curve fitting by least-squares filter

THTQ Latest rate gyro value stored in THTQA array for curve fitting by least-squares filter integrators divided into four par*.
A. CA table of ALPHA versus Mach au su:
B. CN table of ALPHA versus Mach numiver
C. CMQ table of ALPHA versus Mach «umber
D. CP table of ALPHA versus Mach number

AF Axial force calculated from surface area and dynamic pressure

AGAM Ratio of specific heats used to calculate Mach number and speed of sound

ALPH Computed angle of attack assuming an atmosphere stationary with respect to the rotating planet

AM Moment acceleration computed by data generator
AR Universal gas constant
ATMøSS (33,5) Five tables of breakpoints of molecular weights and temperatures versus altitude

ATM $\phi$ (33) ATMDSS table chosen according to NATM $\phi$ S
AX Axial aerodynamic acceleration
AY Normal aerodynamic acceleration
C(200) Biases and scale factors used to calculate "real world" model in the data generator and to calculate sensitivity matrices and state deviations in the reconstructor (for a breakdown of the elements of the $C$ array, see input description of the Users' Manual)

CACT(200) Data generator values of the $C$ array used by the reconstructor to compute actual deviations from most recent nominal values of solve-for parameters

CBQ(30) Scale factors and biases before a quasi-event used to compute estimated deviations from most recent nominal values of solve-for parameters

CDTBL(50) Parachute drag coefficient table
CMQ Moment coefficient computed from AR $\varnothing$ TBL tables
CN Normal force coefficient computed from ARøTBL tables
Cø(200) Original reconstructor $C$ array values used to calculate estimated deviations from original nominal values of solve-for parameters

DIA Vehicle base diameter
DP Dynamic pressure
DT Integration step size (seconds) for data generator and reconstructor

DXN(30) First derivatives of state parameters, actual VRUARU data, and ambient pressure deivative
$\operatorname{EDN}(30) \quad$ Estimated deviations from most recent nominal trajectory after a measurement

EDNBQ(30) EDN array before a quasi-event
EPS Epsilon, the angle between the inertial velocity and relative velocity vectors

FD Parachute drag force
GA Local gravitational acceleration
Gø Gravitational acceleration at zero altitude
MACH Mach number

| MASS | Mass of the vehicle |
| :---: | :---: |
| MASSA | Perturbed vehicle mass |
| MEAS (4) | Reconstructed measurements calculated in $\emptyset B S M$ to drive the filter equations |
| MEZACT(4) | Measurements calculated by the data generator and f:rturbed by noise, scale, and bias factors |
| MEZEST(10) | Estimated measurements calculated from MEAS array for filter equations |
| MEZN $\emptyset \mathrm{Z}(16,4)$ | Measurement noise components used to calculate MEZACT array |
| MU | Gravitational constant of the target planet |
| MWT | Actual molecular weight used to calculate Mach number |
| ФMEG | Rotational rate of the target planet |
| PRES | Ambient pressure state variable |
| RAD | Conversion factor from radians to degrees |
| RH $\varnothing$ | Atmospheric density |
| RI | Rotational inertia of the vehicle |
| RM | Radius of the target planet |
| SA | Reference surface area of the vehicle |
| SDP | Parachute reference area |
| SS | Speed of sound |
| TAPETM | Trajectory time stored on unit 10 for processing groups of events and measurements |
| TC | Current trajectory time (seconds) |
| TDIFF | Difference between current trajectory time and time of next measurement or event |


| TEMP | Ambient temperature |
| :---: | :---: |
| TEND | Trajectory time of the next event or measurement |
| TF | Final trajectory time |
| $T: \mathbb{N}(250)$ | Array of measurement and event times |
| VR | Relative velocity of the vehicle |
| VW | Actual wind velocity for data generator |
| WDTBL(50) | Table of breakpoints of wind velocity versus altitude |
| XD | Location of parachute bridle apex relative to origin of vehicle body axes |
| XG | Axial distance to center of gravity |
| XM | Axial distance to accelerometer location |
| XN(30) | Most recent nominal trajectory state |
| XNA(30) | Actual trajectory state (read in READAC) |
| XNAS ( 30 ) | Not used |
| XNBQ ( 30 ) | Most recent nominal trajectory state before a quasi-event |
| XNS (30) | Not used |
| X $\emptyset$ ( 30) | Original nominal trajectory state |
| XøS (30) | Not used |
| XP | Axial location of center of pressure |
| YG | Not used |
| YM | Not used |
| ZM | Moment force calculated from surface area, dytiamic pressure, and relative velocity |

Normal force calculated from surface area and dynamic pressure

Normal distance to center of gravity
ZMM
Normal distance to accelerometer location

RMACHB
Mach number at the beginning of an integration interval for calculation of sensitivity matrices


| XMFW $(5,5)$ | Mole fractions of component gases |
| :---: | :---: |
| CGMW (5) | Molecular weights of component gases |
| VMASS (3) | Vehicle mass before parachute deployment, after deployment, and after release |
| VSA(3) | Vehicle reference surface area before parachute deployment, after deployment, and after release |
| VDIA(3) | Vehicle base diameter before parachute deployment, after deployment, and after release |
| VRI (3) | Vehicle rotational inertia before parachute deploymeit, after deployment, and after release |
| HD | Altitude at which to deploy parachute |
| HR | Altitude at which to release parachute |
| TD | Value of TC at which parachute was deployed |
| TR | Value of TC at which parachute was released |
| TH(7) | Terrain height mode1 for altimeter measurements |
| $\emptyset \mathrm{DB}$ | Bound on dynamic pressure to control calculation of vehicle attitude and angle of attack |
| CAC | Coefficient of axial force (CA) perturbed by bias and scale factors |
| CDC | Parachute drag coefficient (CD) perturbed by bias and scale factors |

V. INDIVIDUAL SUBROUTINE DOCUMENTATION

PRECEDING PAGE BLANK NOT FITMED ADD-A

SUBROUTINE ADD
PURPOSE : TO ADO TWO REGTANGULAR MATRICES' AND STORE IN A THIRD MATRIX

EMTRY PARAMETERS
NCX
nUmber of columns of $x, y$, and $z$ matrices
NRX NUMBER OF ROWS OF $X, Y$, ANJ $Z$ MATRICES
$x$ INPUT MATRIX
$\gamma$ INPUT MATRIX
$Z$ CUTPUT MATRIX (SJM OF $X$ AND $Y$ )

LOCAL SYMBOLS
I
I NOEX
N
nUmber of elements of $x, r$, and $z$ matrices


## AECEQ Analysis

Subroutine AECEQ computes the coordinate transformation from geocentric ecliptic to geocentric equatorial coordinates. If A denotes the coordinate transformation matrix, then

$$
\vec{x}_{\text {equatorial }}=\mathrm{A} \overrightarrow{\mathrm{x}}_{\text {ecliptic }}
$$

and

$$
A=\left[\begin{array}{ccc}
1 & 0 & 0 \\
0 & \cos \varepsilon & -\sin \varepsilon \\
0 & \sin \varepsilon & \cos \varepsilon
\end{array}\right]
$$

where obliquity of the ecliptic is given (in degrees) by $\varepsilon=23^{\circ} .452294-0^{\circ} .0035626 D-0^{\circ} .000000113 D^{2}+0.0000000103 D^{3}$
and

$$
D=J u l i a n \text { date }\left(\text { epoch 1900) } * 10^{-4}\right.
$$



## ATMØSP Analysis

Subroutine ATMøSP computes Mach number and atmospheric density and temperature for the mode $A$ reconstruction process. The required equations are derived in Chapter IV.

Dynamic pressure q can be related to the calibrated axial accelerometer measurement $a_{x_{s}}$ according to

$$
\begin{equation*}
q=-\frac{\left(m+C_{30}\right) a_{x_{c}}}{\left(C_{20} \cdot C_{A}+C_{16}\right)+\left(C_{96} \cdot C_{D}+C_{97}\right)} \tag{1}
\end{equation*}
$$

so that density can be immediately obtained from

$$
\begin{equation*}
\rho=\frac{2 q}{v_{r}^{2}} \tag{2}
\end{equation*}
$$

These equations correspond to equations (IV-17) and (IV-18), respectively, but with relevant scale factors and biases incorporated. These scale factors and biases are defined as follows:

```
\(C_{16}=\) axial aerodynamic coefficient \(C_{A}\) bias
\(C_{20}=C_{A}\) scale factor
\(\mathrm{C}_{30}=\) mass m bias
\(C_{96}=\) parachute drag coefficient \(C_{D}\) scale factor
\(C_{97}=C_{D}\) bias.
```

Mach number $M$ is computed from the equation

$$
\begin{equation*}
M=\left[\frac{2 q}{\gamma p}\right]^{\frac{1}{2}} \tag{3}
\end{equation*}
$$

where $\gamma$ is the ratio of specific heats and $p$ is the ambient pressure, which has been obtained by integrating the hydrostatic equation in subroutine DERIVE.

## ATM@SP-2

Temperature is obtained from the equation of state

$$
\begin{equation*}
T=\frac{p M}{\rho R} \tag{4}
\end{equation*}
$$

where $M$ denotes molecular weight and $R$ denotes the universal gas constant.

ATMDSP Flow Chart




| USFJ/COMMN-*- | AGAM MHT TPT | APO NMPTS GGMW | AR NTPTS XMFH | 60 PRES XMFW | MOL TEMP | $\begin{aligned} & \text { MPT } \\ & \text { TMP } \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SETTCOMMUN-a- | $\begin{aligned} & \text { MPT } \\ & \text { MDL } \end{aligned}$ | MWT | PRPS | RHO | S5 | TEMP |
| FCT CALLED-*- | F1 | F2 |  |  |  |  |
| FCT DFND --* | F1 | F2 |  |  |  |  |
| ENTRY PNT --- | ATMDAT | ATMSET |  |  |  |  |

## ATMSET Analysis

ATMSET determines the temprature, molecular weight, pressure, density, and speed of sounc of the atmosphere as a function of height above tine mean surface, $h$. The atmosphere is modeled by assuming piece-wise linear representation for the temperature and molecular weight versus height. The remaining atmospheric parameters are then found from the hydrostatic equations and the perfect gas law.

The temperature $T$ at height $h$ between the $j: d d j+1$ temperature breakpoints is given by

$$
\begin{equation*}
T(h)=T s_{j} h+T b_{j} \tag{1}
\end{equation*}
$$

The molecular weight $M$ at height $h$ between the $i$ and i+1 molecular weight breakpoints is given by

$$
\begin{equation*}
M(h)=M s_{i} h+M b_{i} \tag{2}
\end{equation*}
$$

The hydrostatic equation

$$
\begin{equation*}
\frac{d P}{d h}=-\rho g \tag{3}
\end{equation*}
$$

where $g=$ acceleration due to gravity, and the perfect gas law,

$$
\begin{equation*}
\rho(h)=\frac{P(h)}{R} \cdot \frac{M(h)}{T(h)} \tag{4}
\end{equation*}
$$

where $R=$ gas constant, may be integrated from the atmosphere breakpoint (temperature or molecular weight) immediately below the height $h$ to give the pressure $P(h)$

$$
\begin{equation*}
P(h)=P\left(h_{k}\right) \operatorname{EXP}\left[-\frac{g}{R} \int_{h_{k}}^{h} \frac{M(\zeta)}{T(\zeta)} d \zeta\right] \tag{5}
\end{equation*}
$$

$\int_{h_{k}}^{h} \frac{M(\zeta)}{T(\zeta)} d \tau=\left\{\begin{array}{l}\frac{1}{T b_{j}}\left(h-h_{k}\right)\left\{M b_{i}+\frac{1}{2} M s_{i}\left(h-h_{k}\right)\right\}, T s_{j}=0 \\ \frac{M s_{j}}{T s_{j}}\left(h-h_{k}\right)+\frac{M b_{i} \cdot T s_{j}-T b_{j} \cdot M s_{i}}{T s_{j}} \ell n \frac{T b_{j}+T s_{j} h}{T b_{j}+T s_{j} h_{k}}, T s_{j} \neq 0\end{array}\right.$
where

$$
\begin{aligned}
i= & \text { the index of the molecular weight breakpoint immediately } \\
& \text { below } h, \\
j= & \text { the index of the temperature breakpoint immediately } \\
& \text { below } h .
\end{aligned}
$$

For a given surface pressure $P(h o)$, the pressure $P(h)$ may be found by repeated application of the above expression

$$
\begin{equation*}
P(h)=P(h o)\left\{\frac{P\left(h_{k}\right)}{P(h o)}\right\} \operatorname{EXP}\left[-\frac{g}{R} \int_{h_{k}}^{h} \frac{M(\zeta)}{T(\zeta)} d \zeta\right] \tag{7}
\end{equation*}
$$

The density is then found from equation (4) and the speed of sound at height $h$ is given by

$$
\begin{equation*}
s s(h)=\gamma_{s}\left[\frac{R T(h)}{M(h)}\right]^{\frac{1}{2}} \tag{8}
\end{equation*}
$$

where $\gamma_{s}=$ ratio of specific heats.
The subroutine has two entry points -- ATMST and ATMDAT. Entry ATMSET computes and stores the ratios $P\left(h_{\ell}\right) / P(h o)$ for each atmosphere breakpoint: . Entry ATMDAT computes the temperature, molemolecular weight, pressure, density, and speed of sound at height $h$.

The flow of the ATMSET subroutine is illustrated.



## ATTACK Analysis

Subroutine ATTACK computes the actual angle of attack measurement, which is currently defined only for the mode A reconstruction process. The ratio of calibrated accelerations $a_{z_{c}} / a_{x_{c}}$ is used to define the angle of attack measurement $\tilde{\alpha}$. The vehicle lift/ drag ratio can be related to $a_{x_{c}}, a_{z_{c}}$, and $\tilde{\alpha}$ as follows:

$$
\begin{equation*}
\frac{L}{D}=\frac{a_{z_{c}} \cos \tilde{\alpha}-a_{x_{c}} \sin \tilde{\alpha}}{a_{x_{c}} \cos \tilde{\alpha}+a_{z_{c}} \sin \tilde{\alpha}} \tag{1}
\end{equation*}
$$

Furthermore, $\frac{L}{D}$ has the form

$$
\begin{equation*}
\frac{\mathrm{L}}{\mathrm{D}}=\mathrm{k} \tilde{\alpha} \tag{2}
\end{equation*}
$$

where $k$ is a tabulated function of Mach number. Eliminating $\frac{L}{D}$ from equations (1) and (2) yields

$$
\begin{equation*}
\tan \tilde{\alpha}=\frac{\zeta-k \tilde{\alpha}}{1+\zeta k \tilde{\alpha}} \tag{3}
\end{equation*}
$$

where

$$
\zeta=a_{z} / a_{x_{c}}
$$

Equation (3) is solved iteratively for $\tilde{\alpha}$ using a standard Newton iteration technique. Rewriting equation (3) as

$$
\begin{equation*}
F=(1+\zeta k \tilde{\alpha}) \tan \tilde{\alpha}+k \tilde{\alpha}-\zeta=0 \tag{4}
\end{equation*}
$$

the iteration process is defined by

$$
\begin{equation*}
\tilde{\alpha}_{i+1}=\tilde{\alpha}_{i}-\left(\frac{F}{\partial F / \partial \tilde{\alpha}}\right)_{i} \tag{5}
\end{equation*}
$$

where

$$
\begin{align*}
& \frac{\partial F}{\partial \tilde{\alpha}}=k+1+\tilde{\alpha}\left[-2 \zeta k+\tilde{\alpha}\left(1-\frac{4}{3} k \zeta \tilde{\alpha}\right)\right]  \tag{6}\\
& \tilde{\alpha}_{0}=\frac{\zeta}{k+1} \tag{7}
\end{align*}
$$

which is an approximate solution of equation (3) for small $\tilde{\alpha}$ and $\zeta$.


## AUXIL Analysis

Subroutine AUXIL computes the following auxiliary information:

1. Latitude and longitude ground trace relative to three coordinate systems:
a. Planetocentric equatorial,
b. Subsolar orbital-plane,
c. Planetocentric geographical;
2. Communication angle;
3. Angle between the entry plane and the plane of the sky.

Given the spacecraft position components ( $x, y, z$ ) relative to an arbitrary orthogonal coordinate system, the latitude and longitude are given by the following equations:
a. Latitude (relative to $x y-p l a n e$ )

$$
\theta=\tan ^{-1}\left(\frac{z}{\sqrt{x^{2}+y^{2}}}\right)
$$

b. longitude (relative to $x$-axis)

$$
\lambda=\tan ^{-1}(y / x)
$$

The communication angle $\psi$ is defined as the angle between the spacecraft and earth position vectors relative to the center of the target planet. Thus

$$
\psi=\cos ^{-1}\left[\frac{\overrightarrow{\mathbf{r}} \cdot\left(\vec{r}_{e}-\vec{r}_{p}\right)}{|\vec{r}| \cdot\left|\vec{r}_{e}-\vec{r}_{p}\right|}\right] \quad, \quad 0 \leq \psi \leq \pi
$$

where $\vec{r}$ is the spacecraft position relative to the target planet, and $\vec{r}_{e}$ and $\vec{r}_{p}$ are the position vectors of the earth and the target planet, respectively, relative to the sun.

The angle $\eta$ between the entry plane and the plane of the sky is defined as the angle between the normals of each plane. The unit vector $\vec{e}_{n}$ normal to the entry plane is given by

$$
\vec{e}_{\mathbf{n}}=\left[\begin{array}{ccc}
\sin i_{\varepsilon} & \sin \Omega_{\varepsilon} \\
-\sin i_{\varepsilon} & \cos \Omega_{\varepsilon} \\
\cos i_{\varepsilon} & &
\end{array}\right]
$$

relative to the planetocentric ecliptic system, where inclination $i_{\varepsilon}$ and longitude of the ascending node $\Omega_{\varepsilon}$ define the orientation of the entry plane relative to the same system (see subroutine ECLIP). The plane of the sky is defined as the plane perpendicular to the range vector $\vec{\rho}$ from the earth to the spacecraft. The unit vector normal to this plane is

$$
\vec{e}_{\rho}=\frac{\vec{\rho}}{\rho}
$$

Then

$$
\begin{gathered}
n=\cos ^{-1}\left(\vec{e}_{\rho} \cdot \vec{e}_{n}\right) \\
0 \leq n \leq \pi
\end{gathered}
$$



## BEGIN Analysis

BEGIN resets common variables prior to trajectory reconstruction, which may have been changed by the data generator. BEGIN is called by subroutine SETUP prior to reading input data for the reconstructor.


## BLøCK DATA-1

BLØCK DATA Analysis
Common variables are preset by data statements for use in the data generator (DATGEN). The variables are reinitialized in subroutine BEGIN for use in the reconstructor (LTRC $\emptyset \mathrm{N}$ ). For a general descrintion of storage in ARøTBL, see subroutine DTAB.

# PURPOSE : CONVERTS A VECTOR OF INTERNAL VALUES ANJ STORES <br> INTS AN DUTPUT VECTOR 

## entry parameters

A VECTOR OF INTERNAL PROGRAM VALUES
B OUTPUT VEGTOR OF CONVERTED VALUES
N LOGIC VARIABLE TO CONTROL CONVERSION

COMRONS : TRAJ
local symbols none
USEJ/COMMN--- RA)

## SUBROUTINE COPY

PURPOSE : S'T ONE MATRIX EQUAL TO ANOTHER

## ENTRY PARAMETERS

NCZ NUMBFR OF COLUMNS IN 2 NATRIX
NRZ NUMBER OF ROHS IN $Z$ MATRIX
$w \quad$ MATRIX TO PE COPIED
2 MATRIX WHICH IS SET EQUAL TO $W$ MATRIX

LOGAL SYMBOLS
I Ivoex
:! penDuct of NRZ AND NCZ

```
SUBROUTINE CORMAT
PURPOSE & COMPUTE A MATRIX OF CORRELATION COEFFICIENTS FROM
a Covariance matrix
ENTRY PARAMETERS
    A COVARIANCE MATRIX (N X N)
    B MATRIX WHOSE DIAGONAL ELEMENTS ARE THE SQUARE ROOTS
        OF THE CORRESPONOING ELEMENTS OF A AND WHOSE OFF-
        jiagonal elements are the gjrrelation goeffigients
        OF THE CORRESPONIING ELEMENTS OF A
    N
    DIMENSION OF A AND B (N X N)
LOCAL SrMBOLS
    I INDEX
    II index of diagonal element of I-th row
    \ INDEX
    JJ INDEX OF OIAGONAL ELEMEMT OF J-TH COLUNN
    K index of the IJ-Th ElEment
```

SUBROUTINE CORR

PURPOSE $:$ COMPUTE CORRELATIOM COEFFICIENTS FOR OFF-DIAGONAL BLOCK OF A PARTITIONED COVARIANCE MATRIX

```
ENTRY PARAMETERS
    A DIAGONAL BLOCK OF COVARIANCE MATRIX WHERE ROWS
        GORRESPOND TO THE ROWS OF E
    B JIAGONAL RLOCK OF COVARIANCE MATRIX WHOSE COLUMNS
        CORRESPONO TO THE COLUMNS OF C
    C DFF-DIAGONAL BLOGK OF COVARIANGE MATRIX
    D MATRIX WHOSE ELEMENTS ARE THE CORRELATION COEFFICIENTS
        DF THE CORRESPONDING ELEMENTS OF C
    N1 NUMBER OF ROWS OF C
    N2 NUMAER OF COLUMNS DF C
SURROUTINES CALLED: COPY
LOGAL SYMBOLS
    I INDEX
    J INDEX
    N NUMPER OF ELEMENTS IN C
    X SGUARE ROOT OF DIAGONAL ELEMENT OF COVARIANCE MATRIX
```

CORR Analysis
CORR computes the correlation coffficient corresponding to elements of an off-diagonal block of a partitioned covariance matrix.

Let the covariance matrix be partitioned as

$$
\mathbf{P}=\left[\begin{array}{ccccc}
\cdots & \cdot & \cdot & \cdot & \cdot \\
\cdot & \mathbf{A} & \cdot & \mathbf{C} & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot \\
\cdot & \cdot & \cdot & \mathbf{B} & \cdot \\
\cdot & \cdot & \cdot & \cdot & \cdot
\end{array}\right]
$$

where $A$ and $B$ are diagonal blocks and $C$ is an off-diagonal block having rows and columns in common with $A$ and $B$ respectively. The matrix whose elements are the correlation coefficient of the corresponding elements of $C$ is given by

$$
D_{i j}=\frac{C_{i j}}{\sqrt{A_{i i} B_{j j}}}
$$

## SUBROUTINE CORRD

## PURPOSE : COMPUTE THE CORRELATION COEFFIEIENTS FOR THE OFFJIAGONAL BLOCK OF A PARTITIONED GOVARIANCE MATRIX

## ENTRY PARA METERS

A DIAGONAL RLOCK OF COVARIANCE MATRIX WHOSE ROHS CORRESPONO TO THE ROWS DF :

8 ELEMENTS OF DIAGONAL BLOCK OF CO/ARIANGE MATRIX WHOSE COLUMNS GORRESPOND TO THE COLUMNS OF $C$

C OFF-DIAGONAL BLOCK OF COVARIANCE MATRIX
D MATRIX WHOSE ELEMENTS ARE THE CORRELATION COEFFICIENTS DF THE CORRESPONDING ELEMENTS OF C

N1 NUMBER OF ROWS OF C
N2 NUMBER OF COLUMNS OF $C$

```
SURROUTINES CALLEJ: COPY
```

LOCAL SYMBOLS
I INDEX
J INDEX
N NUMBER OF ELEMENTS OF $L$
$X$ SQUARE ROOT OF DIAGONAL ELEMENT OF COVARIANCE MATRIX

CORRD Analysis
CORRD computes the correlation coefficients corresponding to elements of an off-diagonal block of a partitioned covariance matrix when the diagonal block having columns corresponding to the offdiagonal block is diagonal.

Let the covariance matrix be parti oned as

$$
P=\left[\begin{array}{cccc}
\cdot & \cdot & \cdot & \cdot \\
\cdot & A & \cdot & C \\
\cdot & \cdot & \cdot \\
\cdots & \cdot & \cdot & \cdot \\
\cdots & \cdot & \cdot
\end{array}\right], B=\operatorname{diag}\left(b_{i j} \cdots b_{n_{2}}\right)
$$

where A and B are diagonal blocks and C is an off-diagonal block having rows and columns in common with $A$ and $B$ respectively. The matrix where elements are the correlation coefficients of the corresponding elements of $C$ is given by

$$
D_{i j}=\frac{c_{i j}}{\sqrt{A_{i i} b_{j}}}
$$

| PUPPOSE : EXECU | EXECUTIVE CONTROL FOR DATA GENERATOR |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| SUEROUTINES CALLED: | altfile SETUP1 | OERIV1 | OBSM1 | PRINT1 | RKUTOG |
| Sommons : accel | INTCOM | TRAJ | QMPT I | L0GMOJ | PHASE |
|  |  |  |  |  |  |
| NC | ItERATIVE COUNTER FOR PRINTOUT |  |  |  |  |
| UPDAIT | dummy call argument |  |  |  |  |
| USED/COMMN--- $\begin{array}{r}\text { DT } \\ \text { TC }\end{array}$ | $\underset{\text { TF }}{\substack{\text { ITG }}}$ |  |  | QSALT | QSot |
| $\begin{array}{cc} \text { WRITTEN } & --- \\ \text { AGOLX } \\ \text { TG } \end{array}$ | $\begin{aligned} & \text { ACCL } \\ & \text { TEMP } \end{aligned}$ |  | M | PRES | RHO |
| SET/COMMON--- TC | то | TR |  | G QST | OT |
| LOADED --- NG |  |  |  |  |  |

## DATGEN Analysis

Subroutine DATGEN is the executive subroutine for the LTR data generator and controls the entire computational flow for actual trajectory propagation, actual atmosphere parameter computation, actual measurement computation, and printout.

DATGEN Flow Chart





DERIV1-B


## DERIV1 Analysis

Subroutine DERIV1 is the dynamic model subroutine used in the generation of the actual trajectory, actual VRU and ARU outputs, and actual atmospheric parameters. Subroutine DERIV1 computes derivatives of the variables $h, v, \gamma, \phi, \theta, \omega, v_{x}, v_{z}, A_{\theta}$, and $p$ for use in the integration subroutine RKUTDG.

Certain preliminary calculations are required before the required derivatives can be evaluated. First, the local acceleration of gravity is computed from

$$
\begin{equation*}
g=\frac{\mu}{r^{2}} \tag{1}
\end{equation*}
$$

where $\mu$ is the planet gravitational constant and $r$ is the radial distan e from the planet center. Atmosphere velocity $v_{a}$, vehicle relative velocity $v_{r}$, and the angle $\varepsilon$ between the insrtial velocity $v$ and the relative velocity are computed from the following relations:

$$
\begin{align*}
& v_{a}=r \omega_{n}+v_{w}  \tag{2}\\
& v_{r}=\frac{v-v_{a} \cos \gamma}{\cos \varepsilon}  \tag{3}\\
& \varepsilon=\tan ^{-1}\left[\frac{v_{a} \sin \gamma}{v-v_{a} \cos \gamma}\right] \tag{4}
\end{align*}
$$

where $\omega_{n}$ denotes the component of the planet rotational velocity in the entry plane. Argle of attack is given by

$$
\begin{equation*}
\alpha=\theta+\phi-\gamma-\varepsilon \tag{5}
\end{equation*}
$$

Axial, normal, and parachute drag forces are given, respectively, by

$$
\begin{align*}
& A=-C_{A} q S  \tag{6}\\
& N=-C_{N} q S  \tag{7}\\
& F_{d}=C_{D} q S_{D} \tag{8}
\end{align*}
$$

The aerodynamic damping moment is computed from

$$
\begin{equation*}
M=C_{M_{q}} \omega d^{2} q S / v_{r} \tag{9}
\end{equation*}
$$

The equations of motion which constitute the dynamic model used to compute the actual entry trajectory are summarized below:

$$
\begin{align*}
\dot{h} & =v \sin \gamma \\
\dot{v} & =-g \sin \gamma+\frac{A}{m} \cos (\alpha+\varepsilon)+\frac{N}{m} \sin (\alpha+\varepsilon)-\frac{F_{d}}{m} \cos \varepsilon(11) \\
\dot{\gamma} & =\left(\frac{v}{r}-\frac{g}{v}\right) \cos \gamma+\frac{1}{v}\left[\frac{A}{m} \sin (\alpha+\varepsilon)-\frac{N}{m} \cos (\alpha+\varepsilon)-\frac{F_{d}}{m} \cos \varepsilon\right] \\
\dot{\phi} & =\frac{v}{r} \cos \gamma \\
\dot{\theta} & =\omega  \tag{13}\\
\dot{\omega} & =\frac{1}{I}\left[\left(z_{p}-z_{g}\right) A-\left(x_{p}-x_{g}\right) N+M+z_{g} F_{d} \cos \alpha\right.  \tag{14}\\
& \left.-\left(x_{g}-x_{d}\right) F_{d} \sin \alpha\right]
\end{align*}
$$

The parachute terms, of course, appear only when the parachute is deployed (IPHAS = 2).

The actual nongravitational acceleration experienced by the VRU is given by

$$
\begin{align*}
& \dot{v}_{x}=a_{x} \cos \delta_{1}-a_{z} \sin \delta_{1}  \tag{16}\\
& \dot{v}_{z}=a_{x} \sin \delta_{2}+a_{z} \cos \delta_{2} \tag{17}
\end{align*}
$$

The actual angular velocity experienced by the ARU is given by

$$
\begin{equation*}
\dot{A}_{\theta}=\omega \cos \delta_{3} \tag{18}
\end{equation*}
$$

The rate of change of ambient pressure is computed from
$\dot{\mathrm{p}}=-\rho \mathrm{g} \dot{\mathrm{h}}$
which is just the time-differential form of the hydrostatic equation.

If the quasi-static dynamic model is to be used, equation (11) is replaced with

$$
\begin{equation*}
\dot{\mathrm{v}}=0 \tag{20}
\end{equation*}
$$

and $v$ is computed from the terniinal velocity solution

$$
\begin{equation*}
v=\left[\frac{2 m g|\sin \gamma|}{\rho\left(C_{A} S+C_{D} S_{D}\right)}\right] \tag{21}
\end{equation*}
$$

The logical variable C $\emptyset \mathrm{ND}$ is set to true if either dynamic pressure exceeds $\emptyset D B$ or if the parachute is deployed. Whenever C $\varnothing$ ND is true, the angle of attack and the rotational state are computed as follows:

$$
\begin{align*}
& \alpha=0 \\
& \theta=\gamma-\phi+\varepsilon  \tag{22}\\
& \omega=\dot{\gamma}-\dot{\phi} \tag{23}
\end{align*}
$$

DERIV1 Flow Chart





| purpose : compute moje a vehicle state derivatives |  |  |
| :---: | :---: | :---: |
| - NTRY PARAMETERS trajfictory time (not currently useos |  |  |
|  | UPDAIT | LCGICAL TO CONTROL UPDATING OF VEhICLE STATE VEGTOP |
|  | XNE | vemigle state vector at time t |
| SURROUTINES CALLED: |  | atmdat diab |
| COMMONS : TRAJ |  | GY INTCOM DOPLER LOGMOD PHASE |
| LOCAL SYMBOLS ${ }_{\text {LERO }}$ AEROOYNAMIC FORCE COFFFICIENTS |  |  |
| Alf |  | absolute value of angle of attack |
| CAE |  | COSINE OF ALPH PLUS EPS |
| casa |  | CA TIMES SA |
| cGAm |  | cosine of gam |
| DEPS |  | intermediate varianle to compute alph |
| FE |  | perturbed value of vehitle down range angle |
| GAM |  | perturbed value of vehicle flight path angle |
| H |  | perturbed value df vehitale altitude |
| Parde |  | Intermediate varianle to compute alph |
| Radius |  | distance from center of planet to vehicle |
| SADP |  | vehicle reference area times dynamic fressure |
| SAE |  | SINE OF ALPH PLUS EPS |
| SGAM |  | SINE OF GAM |
| $v$ |  | perturpeo value of vehicle velocity |
| VA |  | atmosphere velocity |


| USEJ/COMMN--* | AF | Ary | ALPH | AX | AY | BGY |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | C | CA | CMA | CN | DIA | DP |
|  | DT | DXN | EPS | GA | GB | GD |
|  | GS | GYRO | IGYRO | MASS | MASSA | MU |
|  | RH: | RI | RM | ROTNO | SA | SS |
|  | TC | VR | VW | WDTBL | XG | XP |
|  | 26 | ZM | ZN | IPHAS | QSMCHG |  |
| SET/COMMON--* | AF | ALPH | AM | AK | AY |  |
|  | CH2 | CN | DP | OXN | EPS | GA |
|  | IGYRO | MACH | MASSA | VR | VW | WDTBL |
|  |  |  | ZN |  |  |  |
| FCT CALLEJ-o- | TA" | HINJV |  |  |  |  |
| FCT DFND | F |  |  |  |  |  |

## DERIV3 Analysis

Surbourine DERIV3 is the iilter dynamic model subroutine employed in the mode $B$ reconstruction process. The primary purpose of DERIV3 is to evaluate the derivatives of the state variables $h, v$, $\gamma$, and $\phi$ for use in the integration subroutine RKUTL3 in the computation of both the nominal trajectory and the state transition matrix partitions. State transition matrices are computed by perturbing the relevant $C_{j}^{\prime} s$ that appear in the DERIV3 equations.

Certain preliminary calculations are required before the derivatives of the state variables can be evaluated. The local acceleration of gravity is computed from

$$
\begin{equation*}
g=\frac{\mu}{r^{2}} \tag{1}
\end{equation*}
$$

where $\mu$ is the planet gravitational constant and $r$ is the radial distance from the planet center. Atmosphere velocity $v_{a}$, vehicle relative velocity $v_{r}$, and the angle $\varepsilon$ between the inertial velocity $v$ and the relative velocity are computed from the following relations:

$$
\begin{align*}
v_{a} & =r \omega_{n}+v_{w}  \tag{2}\\
v_{r} & =\frac{v-v_{a} \cos \gamma}{\cos \varepsilon}  \tag{3}\\
\varepsilon & =\tan ^{-1}\left[\frac{v_{a} \sin \gamma}{v-v_{a} \cos \gamma}\right] \tag{4}
\end{align*}
$$

where $\omega_{n}$ denotes the component of the planet rotational velocity in the entry plane.

Angle of attack $\alpha$ is given by

$$
\begin{equation*}
\alpha=\theta+\phi-\gamma-\varepsilon \tag{5}
\end{equation*}
$$

However, attitude angle $\theta$ is not available in the mode $B$ reconstruction process since gyro measurements are not permitted in this mode. Thus, in mode $B \alpha$ is nominally set to zero. It is
still necessary, however, to compute the perturbations in $\alpha$ resulting from perturbations in the state variables and other parameters in order to compute valid state transition matrix partitions. The perturbation $\delta \alpha$ is given by

$$
\begin{equation*}
\delta \alpha=\delta \theta+\delta \phi-\delta \gamma-\delta \varepsilon \tag{6}
\end{equation*}
$$

where $\delta \theta=C_{140}, \delta \phi=C_{104}, \delta \gamma=C_{103}$, and

$$
\begin{align*}
\delta \varepsilon=\frac{\sin ^{2} \varepsilon}{v_{a}^{2} \sin ^{2} \gamma}[-v & \omega_{n} \sin \gamma \cdot \delta h+\left(v \cos \gamma-v_{a}\right) v_{a} \delta \gamma \\
& \left.-v_{a} \sin \gamma \cdot \delta \vee\right] \tag{7}
\end{align*}
$$

In this latter equation, which was derived by differentiating equation (4), $\delta h=C_{101}$ and $\delta v=C_{102}$.

Axial, normal, and parachute drag aerodynamic forces are given, respectively, by

$$
\begin{align*}
& A=-C_{A} q S  \tag{8}\\
& N=-C_{N} q S \tag{9}
\end{align*}
$$

$$
\begin{equation*}
F_{d}=C_{D} q S_{D} \tag{10}
\end{equation*}
$$

The equations of motion that constitute the mode B filter dynamic model are summarized as

$$
\begin{align*}
& \dot{\mathrm{h}}=\mathrm{v} \sin \gamma  \tag{11}\\
& \dot{\mathrm{v}}=-\mathrm{g} \sin \gamma+\frac{A}{m} \cos (\alpha+\varepsilon)+\frac{N}{m} \sin (\alpha+\varepsilon)-\frac{F_{d}}{m} \cos \varepsilon  \tag{12}\\
& \dot{\gamma}=\left(\frac{v}{r}-\frac{g}{v}\right) \cos \gamma+\frac{1}{v}\left[\frac{A}{m} \sin (\alpha+\varepsilon)-\frac{N}{m} \cos (\alpha+\varepsilon)\right. \\
& \left.-\frac{F_{d}}{m} \sin \varepsilon\right] \tag{13}
\end{align*}
$$

$$
\begin{equation*}
\phi=\frac{\mathbf{V}}{\mathbf{r}} \cos \tag{14}
\end{equation*}
$$

The parachute terms, of course, appear only when the parachute is deployed (IPHAS = 2)

If the quasistatic dynamic model is to be used: equation (12) is replaced with

$$
\begin{equation*}
\dot{\mathbf{v}}=0 \tag{15}
\end{equation*}
$$

and $v$ is computed from the terminal velocity solution

$$
\begin{equation*}
v=\left[\frac{2\left(m+C_{30}\right) g|\sin \gamma|}{D\left(C_{A} S+C_{D} S_{D}\right)}\right]^{\frac{1}{2}}+C_{102} \tag{16}
\end{equation*}
$$

## DERIV3 Flow Chart






| USEOICOMMN--* | AA <br> CDELTI <br> ERS <br> MWTM <br> RM <br> TC <br> XM | ALPH <br> CDELT2 <br> FE <br> NACCEL <br> ROTNO <br> THETI <br> $2 G$ | AXC <br> CGAM <br> GA <br> NGYRO <br> SDEL T1 <br> THTC <br> 7.MM | AZC <br> DELT <br> GAM <br> OMGC <br> SDELT2 <br> V <br> IPHAS | C <br> DT <br> IAA <br> PSIN <br> SGAM <br> VA <br> QSMCHG | CDEL <br> DXN <br> MU <br> RHO <br> SUBDL 1 <br> XG |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SETYCOMMON-m- | $\begin{aligned} & \text { ALDH } \\ & \text { CDFLT } 3 \\ & \text { GAY } \\ & \text { SJELT2 } \\ & \text { VA } \end{aligned}$ | AXC <br> CGAM <br> IAA <br> SJELT3 <br> VR | AZC <br> DKN <br> MWT <br> SGAM | C EPS OMGC SUBDL 1 | ```CDELT1 FE PRES THTC``` | ```CDELT2 GA SJELT1 V``` |

```
FCT CALLED--- WINDV
```

FCT DFND --a F

## DERIVE Analysis

Subroutine LERIVE is the filter dynamic model subroutine employed in the mode A reconstruction process. The primary purpose of DERIVE is to evaluate the derivatives of the state variables $h, v, \gamma, \phi$, and $p$ for use in the integration subroutine RKUT3 in computation of the nominal trajectory and ambient pressure and the state transition matrix partitions. State transition matrices are computed by perturbing the relevant $C_{j}^{\prime} s$ that appear in the DERIVE equations.
Certain preliminary calculations are required before the derivatives of the state variables can be evaluated. The first computation concerns the calibration of cg and VRU offsets and VRU and ARU misalignments using the following equations:

$$
\begin{align*}
& \bar{x}_{c}=\left(x_{\text {II }}+c_{28}\right)-\left(x_{g}+c_{26}\right)  \tag{1}\\
& \bar{z}_{c}=\left(z_{m}+c_{29}\right)-\left(z_{g}+c_{27}\right)  \tag{2}\\
& \delta_{1_{c}}=\delta_{1}-c_{55}  \tag{3}\\
& \delta_{2_{c}}=\delta_{2}-c_{56}  \tag{4}\\
& \delta_{3_{c}}=\delta_{3}-c_{63} \tag{5}
\end{align*}
$$

where $x_{m}$ and $z_{m}$ define the nominal VRU location; $x_{g}$ and $z_{g}$, the nominal cg location; $\delta_{1}$ and $\delta_{2}$, the nominal VRU misalignment angles; $\delta_{3}$, the nominal ARU misalignment; and $\mathrm{C}_{26}, \mathrm{C}_{27}, \mathrm{C}_{28}, \mathrm{C}_{29}, \mathrm{C}_{55}, \mathrm{C}_{56}$, and $\mathrm{C}_{63}$, biases in all these quantities.

The measured VRU and ARU data are obtained from the AA(I, J, K) array, which contains the coefficients $a_{i j}$ generated by the preprocessor smoothing process. Index I specifies the sensor type:

```
I = 1 : axial VRU;
I = 2 : normal VRU;
I = 3 : ARU.
```

Index $J$ refers to the coefficient in the quadratic function that is fitted in a least-squares sense to five quantized data points (see Section II.D). Index $K$ is the time index in the $A A(I . J, K$ ) array. The measured axial and normal VRU data are obtained from

$$
\begin{align*}
& \mathbf{a}_{\mathbf{x}_{\mathrm{m}}}=\mathrm{a}_{12}  \tag{6}\\
& \mathbf{a}_{\mathrm{z}_{\mathrm{m}}}={ }^{R_{2}} 22 \tag{7}
\end{align*}
$$

The measured ARU data are obtained from

$$
\begin{align*}
& \theta_{m}=a_{31}  \tag{8}\\
& \omega_{m}=a_{32}  \tag{9}\\
& \dot{\omega}_{m}=2 a_{33} \tag{10}
\end{align*}
$$

If normal VRU data are not available, $a_{z_{m}}$ and misalignment $\delta_{2}$ are set to zero. In this situation it is no longer meaningful to treat the normal VRU scale factor $C_{53}$ as a solve-for or consider parameters. For this reason $C_{53}$ has a fixed value of 1. and cannot be perturbed. However, the normal VRU bias $C_{54}$ can still be treated as a consider parameter representing the anticipated, but not measured, normal accelerations. Since these normal accelerations are not constant, $C_{54}$ cannot be treated as a solve-for parameter when ncrmal VRU data are not available. If ARU data are absent, $\omega_{m}$, $\dot{\omega}_{m}$, and misalignment $\delta_{3}$ are set to zero and the nominal angle of attack $\alpha$ is assumed to be zero.

The measured VRU and ARU data are calibrated for scale factor, bias, and misalignment errors using the following equations:

$$
\begin{align*}
& a_{x_{c}}=\frac{1}{\cos \left(\delta_{1_{c}}-\delta_{2_{c}}\right)}\left[\frac{a_{x_{m}}-c_{52}}{c_{51}} \cos \delta_{2}+\frac{a_{z_{m}}-c_{54}}{c_{53}} \sin \delta_{1_{c}}\right] \\
& \\
& \quad+\omega_{c}^{2} \bar{x}_{c}-\dot{\omega}_{c} \bar{z}_{c} \\
& a_{z_{c}}= \\
& \quad+\frac{1}{\cos \left(\delta_{1_{c}}-\delta_{2}{ }_{c} \bar{z}_{c}+\dot{\omega}_{c} \bar{x}_{c}\right.}\left[-\frac{a_{x_{m}}-c_{52}}{c_{51}} \sin \delta_{2_{c}}+\frac{a_{z_{m}}-c_{54}}{c_{53}} \cos \delta_{1_{c}}\right]  \tag{13}\\
& \theta_{c}=  \tag{14}\\
& \frac{1}{C_{124}}\left[\theta_{m}-c_{125}\left(t-t_{0}\right)\right]  \tag{15}\\
& \omega_{c}=\frac{1}{c_{124}}\left(\omega_{m}-c_{125}\right) \\
& \dot{\omega}_{c}=\frac{\dot{\omega}_{m}}{c_{124}}
\end{align*}
$$

The local acceleration of gravity is computed from

$$
\begin{equation*}
g=\frac{\mu}{r} \tag{16}
\end{equation*}
$$

where $\mu$ is the planet gravitational constant and $r$ is the radial distance from the planet center. Atmosphere velocity $v_{a}$, vehicle relative velocity $\mathrm{v}_{\mathrm{r}}$, and the angle $\varepsilon$ between the inertial velocity v and the relative velocity are computed from the following relations:

$$
\begin{equation*}
v_{a}=r \omega_{n}+v_{w} \tag{17}
\end{equation*}
$$

$$
\begin{align*}
v_{r} & =\frac{v-v_{a} \cos \gamma}{\cos \varepsilon}  \tag{18}\\
\varepsilon & =\tan ^{-1}\left[\frac{v_{a} \sin \gamma}{v-v_{a} \cos \gamma}\right] \tag{19}
\end{align*}
$$

where $\omega_{n}$ denotes the component of the planet rotational velocity in the entry plane.

Angle of attack $\alpha$ is given by

$$
\begin{equation*}
\alpha=\theta_{c}+\theta_{0}+\phi-\gamma-\varepsilon+C_{140} \tag{20}
\end{equation*}
$$

where $\theta_{0}$ is the initial attitude angle and the calibrated attitude measurement $\theta_{c}$ represents the change in attitude since initial time $t_{0}$. Parameter $C_{140}$ represents the initial attitude error. When nominal $\alpha$ is chosen to be zero, as it is when ARU data are not available or when the parachute is deployed, perturbations in $\alpha$ resulting from perturbations in the state variables and other parameters are computed from

$$
\begin{equation*}
\delta \alpha=\delta\left(\theta_{c}+\theta_{0}\right)+\delta \phi-\delta \gamma-\delta \varepsilon \tag{21}
\end{equation*}
$$

where $\delta\left(\theta_{c}+\theta_{0}\right)=C_{140}, \delta \phi=C_{104}, \delta \gamma=C_{103}$, and

$$
\begin{gather*}
\delta \varepsilon=\frac{\sin ^{2} \varepsilon}{v_{a}^{2} \sin ^{2} \gamma}\left[-v \omega_{n} \sin \gamma \cdot \delta h+\left(v \cos \gamma-v_{a}\right) v_{a} \delta \gamma\right. \\
\left.-v_{a} \sin \gamma \cdot \delta v\right] \quad . \tag{22}
\end{gather*}
$$

In this latter equation, which was derived by differentiating equation (19), $\delta h=C_{101}$ and $\delta v=C_{102}$.

Subroutine ATMøSP is not called until significant axial aerodynamic deceleration has developed. Currently, ATM $\varnothing$ SP is called when

$$
a_{x_{c}} \leq-0.5 \times 10^{-3} \mathrm{~km} / \mathrm{s}^{2}
$$

in order to compute Mach number and atmospheric density and temperature.

The equations of motion that constitute the mode A filter dynamic model are summarized as

$$
\begin{align*}
& \dot{i}=v \sin \gamma  \tag{23}\\
& \dot{\mathrm{v}}=-\mathrm{g} \sin \gamma+a_{x_{c}} \cos (+\varepsilon)+a_{z_{c}} \sin (\alpha+\varepsilon)  \tag{24}\\
& \dot{\gamma}=\left(\frac{v}{r}-\frac{g}{v}\right) \cos \gamma+\frac{1}{v}\left[a_{x_{c}} \sin (\alpha+\varepsilon)-a_{z_{c}} \cos (\alpha+\varepsilon)\right]  \tag{25}\\
& \dot{\phi}=\frac{v}{r} \cos \gamma  \tag{26}\\
& \dot{p}=-g \rho \dot{h} \tag{27}
\end{align*}
$$

If the quasistatic dynamic model is to be used, equation (24) is replaced with

$$
\begin{equation*}
\dot{\mathrm{v}}=0 \tag{28}
\end{equation*}
$$

and $v$ is computed from the terminal velocity solution

$$
\begin{equation*}
v=\left[\frac{2\left(m+C_{30}\right) g|\sin \gamma|}{\rho\left(C_{A} S+C_{D} S_{D}\right)}\right]^{\frac{1}{2}}+C_{102} \tag{29}
\end{equation*}
$$

DERIVE Flow Chart




> Use TAB to interpolate on XMT to compute the molecular weight

Compute the radial distance of the vehicle from the planet center and the local acceleration of gravity

Use WINDV :o compute the local
horizontal wind velocity $v_{w}$ horizontal wind velocity $v_{w}$



```
SURROUTINE DMULTT
PURPOSE : TO MULTIPLY A DIAGONAL MATRIX BY THE TRANSPOSE OF A
    RECTANGULAR MATRIX ANO STORE INTO A RECTANGULAR MATRIX
ENTRY PARAMETERS
    NGY NUMAER OF COLUMNS OF }Y\mathrm{ MATRIX
    ND NUMBER OF DIAGONAL ELEMENTS OF }x\mathrm{ mATRIX
    NRY NUMBER OF ROWS OF }Y\mathrm{ MATRIX AND NUMBER OF COLUMNS
    OF Z MATRTX
    X gIAGONAL INPUT MATRIX
    r rectangular input matrix
    z RECTANGULAR OUTPUT MATRIX (X TIMES Y TRANSPOSED)
lOCAL SYMBOLS
    I
    INDEX
    J INDEX
    P I-TH DIAGONAL ELEmENT OF X MATRIX
```


## SUBROUTINE DTAB

PURPOSE : PERFORMS LINEARLY INTERPOLATED DOURLE TARLE LOOKUP

## EHTRY PARAMETERS

| - | A | OUTPUT VECTOR OF INTERPOLATED VALUES (JEPENJENT VARIABLES) |
| :---: | :---: | :---: |
|  | $L T$ | LENGTH OF EACH PARTITION OF TABLE WHENEVER NT.GT. 1 |
|  | NT | NUMBER OF ELEMENTS OF A TO RE CALCULATED |
|  | TABLE | INPUT TABLE Of BREAK POINTS AND COEFFICIENTS |
|  | X | FIRST INOEPENDENT VARIABLE |
|  | V | SECOND INDEPENDENT VARIARLE |
| LOCAL | SYMBOLS |  |
|  | COORL | POINTERS TO RREAKPOINTS NEAREST TO $x$ AND $r$, USEJ TO FINJ N |
|  | FRAC | PER CENT OIfFERENCES BASED ON $x$ AND $Y$, RESPECTIVELY. FRAC(1) IS USED TO FIND Wi,W2 AND FRAC(2) TO FIND A(I) |
|  | I | INJEX |
|  | $J$ | DO LOOP INITIALIZER. |
|  | $K$ | DO LOOP TERMINATOR |
|  | $L$ | INDEX |
|  | $N$ | INDEX TO FIND W1 AND H2 |
|  | N1 | INTEGER VALUE OF TAALE (1) |
|  | POINT | LOCAL VALUES OF $x$ and $Y$ |
|  | W1 | LOWER BOUND OF A(I) |
|  | H2 | UPPER BOUNJ OF A(I) |

DTAB Analysis

TABLE is a partitioned matrix, each submatrix containing:

1) N1 - The number of values in the $X$ table;
2) N2 - The number of values in the $Y$ table;
3) The N1 values of the $X$ table;
4) The N2 values of the $Y$ table;
5) The first $N 1$ values of $X$ versus $Y$;
6) The second N1 values of $X$ versus $Y$;

- 
- 

7) The last (= N2) N1 values of $X$ versus $Y$.

Thus, each partition contains $N 1 x \mathrm{~N} 2+\mathrm{N} 1+\mathrm{N} 2+2$ elements. If X1, X2 from 3) above are the bounds of $X$ so X1.LE.X.LE.X2 and Y., Y2 from 4) above are the bounds of $Y$ so Y1.LE.Y.LE.Y2, then W1 represents $A(I)$ only if $Y=Y 1$ and $W 2$ represents $A(I)$ only if $Y=Y 2$. That is, W1 and W2 are lower and upper bounds of $A(I)$, which are computed according to standard single table lookup schemes. $A(I)$ is then computed by

$$
A(I)=F R A C(2) *(W 2-W 1)+W 1
$$

as in the standard formulae.

SUBROUTINE DYNOIZ
PURPOSE : CURRENTLY SETS DYNAMIC NOISE MATRIX TO ZERO

| COMMONS : COVARP | INTCOM |
| :---: | :--- |
| LOCAL SYMBOLS |  |
| I | INJEX |
| NN | NUMRER OF STATE VARI ABLES SQUARED |

USED/COMMN--- NS
SET/COMMON-- OYN

## SUBROUTINE ECLIP


USEJ/COMMN-- EGLINC ECLDNG PHIR PM

## ECLIP Analysis

Subroutine ECLIP transforms the standard LTR spacecraft state variables $h, v, \gamma, \phi+\phi_{\varepsilon}, \Omega_{\varepsilon}$, and $i_{\varepsilon}$ to planetocentric ecliptic Cartesian components $r_{x}, r_{y}, r_{z}, v_{x}, v_{y}$, and $v_{z}$. In the figure below $x_{\varepsilon} y_{\varepsilon} z_{\varepsilon}$ denotes the planetocentric ecliptic coordinate system and $R_{p}$ denotes the planet radius.


The transformation equations are summarized as:

$$
\begin{aligned}
& r_{x}=r\left(\cos \theta \cos \Omega_{\varepsilon}-\sin \theta \cos i_{\varepsilon} \sin \Omega_{\varepsilon}\right) \\
& r_{y}=r\left(\cos \theta \sin \Omega_{\varepsilon}+\sin \theta \cos i_{\varepsilon} \cos \Omega_{\varepsilon}\right) \\
& r_{z}=r \sin \theta \sin i_{\varepsilon} \\
& v_{x}=v\left(\cos \psi \cos \Omega_{\varepsilon}-\sin \psi \cos i_{\varepsilon} \sin \Omega_{\varepsilon}\right) \\
& v_{y}=v\left(\cos \psi \sin \Omega_{\varepsilon}+\sin \psi \cos i_{\varepsilon} \cos \Omega_{\varepsilon}\right) \\
& v_{z}=v \sin \theta \sin i_{\varepsilon}
\end{aligned}
$$

where

$$
\begin{aligned}
& r=R_{p}+h \\
& \theta=\phi+\phi_{\varepsilon}, \\
& \psi=\phi+\phi_{E}-\gamma+\frac{\pi}{2} .
\end{aligned}
$$

surroutine elcar
PURPOSE: TRANSFORMATION OF CONIC ELENENTS TO CARTESIAN COOROINATES

| ENTRY | PARAMETERS <br> A | SEMIMAJOR AXIS |
| :---: | :---: | :---: |
|  | E | Eccentricity |
|  | GM | gravitational constant jf gentral gooy |
|  | R | position vector in peference srstem |
|  | -RM | POSITION MAGNITUDE |
|  | TA | true anomaly |
|  | tFP | TIME FROM PERIAPSIS |
|  | $v$ | velogity vector in reference system |
|  | vir | velocity magnituje |
|  | W | ARGUMENT OF PERIAPSTS |
|  | XI | INCL INATION IN REFERENGE SYSTEM |
|  | XN | Longitude of ascenoing node |
| LOCAL | SYMBOLS |  |
|  | AUXF | ECCENTRIC ANOMALY (HyPERPOLIC CASE) |
|  | AVA | MEAN anomaly (elliptic case) |
|  | CI | COSINE OF Inclination |
|  | CK | velocity factor usej to calculate final velocity vector |
|  | CN | Cosine of longitude of ascending node |
|  | COSEA | cosine of eccentric anomaly (elliptic case) |
|  | CT | COSINE OF TRUE anomaly |
|  | CH | cosine of sum of argument of periapsis and true anomaly, also cosine of argument of periapsis |
|  | JIV | Intermediate variable used to calculate tfp |
|  | EA | ECCENTRIC anomaly (ELLIPTIC CASE) |
|  | P | SEMI-LATUS RECTUM |


| RAD | CONVERSION FAGTOR FROM JEGREES TO RADIANS |
| :--- | :--- |
| SI | SINE OF INCLINATION |
| SINEA | SINE OF EGCENTRIC ANOMALY |
| SINHF | HYPERROLIG SINE OF AUXF |
| SN | SINE OF LONGITUDE OF ASCENOING NODE |
| ST | SINE OF TRUE ANOMALY |
| SW | SINE OF THE SUM OF ARGUMENT OF PERIAPSIS ANJ TRUE |
|  | ANOMALY, ALSO SINE OF ARGUMENT OF PERIAPSTS |
| TANG | INTERMEDIATE VARIABLE USED TO CALCULATE SINHF |

## ELCAR Analysis

ELCAR transforms the standard con'*: elements of a massless point referenced to a gravitational body :o Cartesian position and velocity components with respect to that body.

Let the gravitational constant of the body be denoted $\mu$ and the given conic elements ( $a, e, i, \omega, \Omega, f$ ). The semilatus rectum $p$ is

$$
\begin{equation*}
p=a\left(1-e^{2}\right) \tag{1}
\end{equation*}
$$

Then the magnitude of the radius vector is given by

$$
\begin{equation*}
r=\frac{p}{1+e \cos f} \tag{2}
\end{equation*}
$$

The unit vector in the direction of the position vector is

$$
\begin{align*}
& u_{x}=\cos (\omega+f) \cos \Omega-\cos i \sin (\omega+f) \sin \Omega \\
& u_{y}=\cos (\omega+f) \sin \Omega+\cos i \sin (\omega+f) \cos \Omega  \tag{3}\\
& u_{z}=\sin (\omega+f) \sin i . \tag{3}
\end{align*}
$$

The position vector $\vec{r}$ is therefore

$$
\begin{equation*}
\overrightarrow{\mathbf{r}}=\mathbf{r} \hat{u} . \tag{4}
\end{equation*}
$$

The velocity vector $\vec{v}$ is given by

$$
\begin{align*}
& v_{x}=\sqrt{\mu / p} \quad[(e+\cos f)(-\sin \omega \cos \Omega-\cos i \sin \Omega \cos \omega) \\
&-\sin f(\cos \omega \cos \Omega-\cos i \sin \Omega \sin \omega)] \\
& v_{y}=\sqrt{\mu / p} \quad[(e+\cos f)(-\sin \omega \sin \Omega+\cos i \cos \Omega \cos \omega) \\
&-\sin f(\cos \omega \sin \Omega+\cos i \cos \Omega \sin \omega)] \\
& v_{z}=\sqrt{\mu / p}[(e+\cos f) \sin i \cos \omega-\sin f \sin i \sin \omega] \tag{5}
\end{align*}
$$

The conic time from periapsis $t_{p}$ is computed from different formulae, depending on the sign of the semimajor axis. For a>0
(elliptical motion)

$$
\begin{align*}
& t_{p}=\sqrt{a^{3} / \mu}(E-e \sin E) \\
& \cos E=\frac{e+\cos f}{1+e \cos f} \quad \sin E=\frac{\sqrt{1-e^{2}} \sin f}{1+e \cos f} . \tag{6}
\end{align*}
$$

For $a<0$ (hyperbolic motion), the time from periapsis is
$t_{p}=\sqrt{a^{3} / \mu}(e \sinh H-H)$
$\tanh \frac{H}{2}=\sqrt{\frac{e-1}{e+1}} \tan \frac{f}{2}$.

| PURPOSE: | $\begin{array}{ll}\text { SE: } & \text { COMP } \\ \text { COMPO }\end{array}$ | PUTE HEL TOCENTRIC ECLIPTIC POSITION AND VELOCITY ONENTS OF AN ARBITRARY PLANET |
| :---: | :---: | :---: |
| ENTRY PARAMETERS DJ |  | JULIAN DATE, EPOCH 1900, JAMUARY 0. |
| NP P |  | planet code number |
| SUPROUTINES CALLEJ: |  | : ElCAR |
| commons: | ns: State |  |
| LOCAL SY | SYMBOLS |  |
|  | AU | CONVERSION FACTOR FROM A. U. TO KILOMETERS |
|  | CAPOM | longituje of the ascenjing noje |
|  | CD | JJLIAN DATE IN UNITS OF 10,000 EPHEMERIS DAYS |
|  | CDC | co cubed |
|  | cos | cJ Squarej |
|  | costa | COSTAE OF TRUE ANOMALY |
|  | EA | ECCENTRIC ANOMALY |
|  | ECAN | INTERMEDIATE VARIABLE USED IN ITERATIVE SOLUTION OF KEPLER EQUATION |
|  | ECC | ECGENTRICITY |
|  | I | INDEX |
|  | IJ | I NDEX |
|  | IJKL | Index |
|  | ITEAP | mean anomaly dividej oy 360 Degrees |
|  | OMEGA | ARGUMENT OF PERIAPSIS |
|  | OMEGAT | LONGITUJE of pertapsis |
|  | PI | COMSTANT $=3.141592653589793$ |
|  | PMU | ARRAY OF GRAVITATIONAL CONSTANTS |
|  | RAJ | CONVERSIOM FACTOR FROM zadians to oegrees |
|  | FM | planet hel iocentric postition magnitude |



## EPHEM Analysis

Subroutine EPHEM computes the heliocentric ecliptic position and velocity components of an arbitrary planet at a given Julian date. The elements are referred to the mean equinox and ecliptic of date except for Pluto. The time interval from the epoch is denoted by T when measured in Julian centuries of 36,525 ephemeris days, by $D=3.6525 \mathrm{~T}$ when measured in units of 10,000 ephemeris days. Times are measured with respect to the epoch 1900 January 0.5 E.T. = J.D. 2415020.0. Angular relations are expressed in radians.

The first step in this process consists of computing the six mean orbital elements of the planet using standard ephemeris polynomials. The six orbital elements are semimajor axis a, eccentricity e, inclination $i$, longitude of the ascending node $\Omega_{3}$ argument of periapsis $\omega$, and mean anomaly M. Kepler's equation

$$
M=E-e \sin E
$$

is then solved iteratively to determine the eccentric anomaly E . Subsequent computations are basic conic manipulations:

$$
\begin{aligned}
& \mathrm{p}=\mathrm{a}\left(1-\mathrm{e}^{2}\right) \\
& \mathrm{r}=\mathrm{a}(1-\mathrm{e} \cos \mathrm{E}) \\
& \mathrm{v}=\sqrt{\mu\left(\frac{2}{r}-\frac{1}{a}\right)} \\
& \cos f=\frac{p-r}{e r} \quad \sin f=\sqrt{1-\cos ^{2} \mathrm{f}} \cdot \operatorname{sgn}(\sin E) \\
& \cos \gamma=\frac{\sqrt{\mu p}}{r v} \\
& \omega=\tilde{\omega}-\Omega .
\end{aligned}
$$

The heliocentric ecliptic position and velocity components of the planet are then

$$
\begin{aligned}
& \overrightarrow{\mathbf{r}}=r_{x} \hat{\mathbf{i}}+r_{y} \hat{\mathbf{j}}+r_{z} \hat{\mathbf{k}} \\
& r_{x}=r \cos (\omega+f) \cos \Omega-r \sin (\omega+f) \sin \Omega \cos i \\
& r_{y}=r \cos (\omega+f) \sin \Omega+r \sin (\omega+f) \cos \Omega \cos i \\
& r_{z}=r \sin (\omega+f) \sin i \\
& \vec{v}=\frac{v}{r}[(\hat{w} \mathbf{x} \vec{r}) \cos \gamma+\vec{r} \sin \gamma]
\end{aligned}
$$

where $\hat{\mathbf{w}}=(\sin i \sin \Omega) \hat{i}-(\sin i \cos \Omega) j+(\cos i) \hat{k} \quad$.

EPHEM-2

## Planetary Ephemerides*

## Mean Elements of Mercury

$i=0.1222233228+3.24776685 \times 10^{-5} \mathrm{~T}-3.19770295 \times 10^{-7} \mathrm{~T}^{2}$
$\Omega=0.8228518595+2.068578774 \times 10^{-2} \mathrm{~T}+3.034933644 \times 10^{-6} \mathrm{~T}^{2}$
$\tilde{\omega}=1.3246996178+2.714840259 \times 10^{-2} \mathrm{~T}+5.143873156 \times 10^{-6} \mathrm{~T}^{2}$
$\mathrm{e}=0.20561421+0.00002046 \mathrm{~T}-0.000000030 \mathrm{~T}^{2}$
$M=1.785111955+7.142471 .000 \times 10^{-2} \mathrm{~d}+8.72664626 \times 10^{-9} \mathrm{D}^{2}$ $a=0.3870986$ A.U. $=57,909,370 \mathrm{~km}$.

Mean Elements of Venus
$i=0.0592300268+1.755510339 \times 10^{-5} \mathrm{~T}-1.696847884 \times 10^{-8} \mathrm{~T}^{2}$
$\Omega=1.3226043500+1.570534527 \times 10^{-2} \mathrm{~T}+7.155849933 \times 10^{-6} \mathrm{~T}^{2}$
$\tilde{\omega}=2.2717874591+2.457486613 \times 10^{-2} \mathrm{~T}+1.704120089 \times 10^{-5} \mathrm{~T}^{2}$
$e=0.00682069-0.00004774 T+0.000000091 T^{2}$
$M=3.710626172+2.796244623 \times 10^{-2} d+1.682497399 \times 10^{-6} \mathrm{D}^{2}$ $\mathrm{a}=0.7233316$ A.U. $=108,209,322 \mathrm{~km}$.

Mean Elements of Earth (Barycenter)
$i=0$
$\Omega=0$
$\tilde{\omega}=1.7666368138+3.000526417 \times 10^{-2} \mathrm{~T}+7.902463002 \times 10^{-6} \mathrm{~T}^{2}$
$+5.817764173 \times 10^{-8} \mathrm{~T}^{3}$
$e=0.01675104-0.00004180 T-0.000000126 T^{2}$
$M=6.256583781+1.720196977 \times 10^{-2} \mathrm{~d}-1.954768762 \times 10^{-7} \mathrm{D}^{2}$
$-1.22173048 \times 10^{-9} \mathrm{D}^{3}$
$a=1.0000003$ A.U. $=149,598,530 \mathrm{~km}$.

[^1]EPHEM-3

## Mean Elements of Mars

$$
\begin{aligned}
\mathrm{i}= & 0.0322944089-1.178097245 \times 10^{-5} \mathrm{~T}+2.201054112 \times 10^{-7} \mathrm{~T}^{2} \\
\Omega= & 0.8514840375+1.345634309 \times 10^{-2} \mathrm{~T}-2.424068106 \times 10^{-8} \mathrm{~T}^{2} \\
& -9.308422677 \times 10^{-8} \mathrm{~T}^{3} \\
\tilde{\omega}= & 5.8332085089+3.212729365 \times 10^{-2} \mathrm{~T}+2.266503959 \times 10^{-6} \mathrm{~T}^{2} \\
& -2.084698829 \times 10^{-8} \mathrm{~T}^{3} \\
\mathrm{e}= & 0.09331290+0.000092064 \mathrm{~T}-0.000000077 \mathrm{~T}^{2} \\
\mathrm{M}= & 5.576840523+9.145887726 \times 10^{-3} \mathrm{~d}+2.365444735 \times 10^{-7} \mathrm{D}^{2} \\
& +4.363323130 \times 10^{-10} \mathrm{D}^{3} \\
\mathrm{a}= & 1.5236915 \text { A.U. }=227,941,963 \mathrm{~km} .
\end{aligned}
$$

Mean Elements of Jupiter

$$
\begin{aligned}
& i=0.0228410270-9.696273622 \times 10^{-5} \mathrm{~T} \\
& \Omega=1.7355180770+1.764479392 \times 10^{-2} \mathrm{~T} \\
& \tilde{\omega}=0.2218561704+2.812302353 \times 10^{-2} \mathrm{~T} \\
& \mathrm{e}=0.0483376+0.00016302 \mathrm{~T} \\
& M=3.93135411+1.450191928 \times 10^{-3} \mathrm{~d} \\
& \mathrm{a}=5.202803 \mathrm{~A} \cdot \mathrm{U} \cdot=778,331,525 \mathrm{~km} .
\end{aligned}
$$

## Means Elements of Saturn

$$
\begin{aligned}
& i=0.0435037861-7.757018898 \times 10^{-8} \mathrm{~T} \\
& \Omega=1.9684445802+1.523977870 \times 10^{-2} \mathrm{~T} \\
& \tilde{\omega}=1.5897996653+3.419861162 \times 10^{-2} \mathrm{~T} \\
& e=0.0558900-0.00034705 \mathrm{~T} \\
& M=3.0426210430+5.837120844 \times 10^{-4} \mathrm{~d} \\
& a=9.538843 \mathrm{~A} \cdot \mathrm{U} .=1,426,996,160 \mathrm{~km} .
\end{aligned}
$$

Mean Elements of Uranus
$i=0.0134865470+0.9696273622 \times 10^{-5} \mathrm{~T}$
$\Omega=1.2826407705+8.912087493 \times 10^{-3} \mathrm{~T}$
$\tilde{\omega}=2.9502426085+2.834608631 \times 10^{-2} \mathrm{~T}$
$e=0.0470463+0.00027204 \mathrm{~T}$
$M=1.2843599198+2.046548840 \times 10^{-4} \mathrm{~d}$
$a=(19.182281-0.00057008 \mathrm{~T})$ A.U. $=(2,869,640,310-85271 \mathrm{~T}) \mathrm{km}$.

## Mean Elements of Neptune

$i=0.0310537707-1.599885148 \times 10^{-4} \mathrm{~T}$
$\Omega=2.2810642235+1.923032859 \times 10^{-2} \mathrm{~T}$
$\tilde{\omega}=0.7638202701+1.532704516 \times 10^{-2} \mathrm{~T}$
$e=0.00852849+0.00007701 \mathrm{~T}$
$\mathrm{M}=0.7204851506+1.033089473 \times 10^{-4} \mathrm{~d}$
$\mathrm{a}=(30.057053+0.001210166 \mathrm{~T}) \mathrm{A} . \mathrm{U} .=(4,496,490,000+181039 \mathrm{~T}) \mathrm{km}$.
Mean Elements of Pluto
$i=0.2996706970859694$
$\Omega=1.1914337550102258$
$\tilde{\omega}=3.909919302791948$
$e=0.2488033053623924$
$\mathrm{M}=3.993890007+0.6962635708298997 \times 10^{-4} \mathrm{~d}$
$a=39.37364135300176$ A.U. $=5,890,213,786.146730 \mathrm{~km}$.

| SUAROUTINE EQUATP |  |
| :---: | :---: |
| PURPOSE: COMPUTE | E COORDINATE TRANSFORMATION MATRIX FROM TRIC EQUATORIAL TO PLANETOCENTRIC EQUATORIAL |
| ENTRY PARAMETERS AGCAC COOROINATE TRANSFORMATION FROM GEOCENTRI EQUATORIAL TO PLANETOCENTRIC EQUATORIAL |  |
| 0 | JULIAN DATE, EPOCH 1900 |
| NP | TARGET Planet cone |
| SUBROUTINES CALLED: | EPHEM EXIT |
| COMRONS 8 STATE |  |
| LOGAL SYMBOLS <br> AHCGC <br> COORDINATE TRANSFORMATION FROM GEOGENTRI EQUATORIAL TO GEOCENTRIC ECLIPTIC |  |
| Csoecl | COSINE OF DECL |
| CSEOBL | COSINE OF EORL |
| CSINM | COSINE OF INM |
| CSNDM | COSINE OF NODEM |
| CSRASC | COSINE OF RASC |
| DECL | declination of target planet pole |
| DGTR | CONVERSION FACTOR FROM DEGREES TO RADIANS |
| ECEQ | COORJINATE TRANSFORMATION FROM GEOCENTRIC EGLIPTIC TO PLANETOCEMTRIC EQUATORIAL |
| ED | JULIAN DATE, EPOCH 4713 B.C. |
| EOBL | OBLIQUITY OF THE ECLIPTIC |
| I | Index |
| $J$ | INDEX |
| K | INDEX |
| NORM | UNIT VEGTOR NORMAL TO TARGET PLANET ORBITAL |
| PGAR | CROSS PROJUCT OF POLE ANJ NORM |
| PMAS | MAGNITUDE OF PBAR |


| Pole | Unit vector aligned with target planet polap axis |
| :---: | :---: |
| polmag | magnitude of pole |
| QRARP | Cross product of pole and prar |
| amag | magnitude of abarp |
| PASC | RIGHt ascenston of target planet pole |
| SNJECL | SINE OF Jecl |
| SNEOBL | SINE OF EOBL |
| SNINM | SINE OF INCLINATION INM |
| SNNJM | SINE OF NOJE NDM |
| SNRASC | SINE OF RASC |
| $T$ | JULIAN DATE, EPOCH 1900, DIVIDED ©Y 36525 |
| TPRIM | besselian date |
| MN--- INY | nodem |

## EQUATR Analysis

Subroutine EQUATR computes the coordinate transformation matrix A from geocentric equatorial to planetocentric equatorial coordinates. Matrix A is computed from

$$
\begin{equation*}
A=A_{1} A_{2} \tag{1}
\end{equation*}
$$

where $A_{2}$ is the coordinate transformation matrix from geocentric equatorial to geocentric ecliptic. coordinates and is given by

$$
A_{2}=\left[\begin{array}{ccc}
1 & 0 & 0  \tag{2}\\
0 & \cos \varepsilon & \sin \varepsilon \\
0 & -\sin \varepsilon & \cos \varepsilon
\end{array}\right]
$$

where $\varepsilon$ is the obliquity of the ecliptic. Matrix $A_{1}$ is the coordinate transformation matrix from geocentric ecliptic to planetocentric equatorial coordinates. The derivation of $A_{1}$ is summarized below.

The coordinate transformation $A_{1}$ is defined by

$$
A_{1}=\left[\begin{array}{l:l:l}
\hat{X} & \hat{Y} & \hat{Z} \tag{3}
\end{array}\right]^{T}
$$

where $\hat{X}, \hat{Y}$, and $\hat{Z}$ are unit vectors aligned with the planetocentric equatorial coordinate axes and referenced to the geocentric ecliptic coordinate system. Unit vector $\hat{Z}$ is aligned with the planet pole. Unit vector $X$ lies along the intersection of the planet equatorial and orbital planes and points at the planet vernal equinox. Unit vector $\hat{\mathrm{Y}}$ completes the orthogonal triad and is given by

$$
\begin{equation*}
\hat{Y}=\hat{Z} \times \hat{X} . \tag{4}
\end{equation*}
$$

It remains to obtain expressions for $\hat{X}$ and $\hat{Z}$. Let $\hat{N}$ denote the unit vector normal to the planet orbital plane, and let $\hat{P}$ denote the unit vector aligned with the planet polar axis. Then

$$
\begin{equation*}
\hat{Z}=\hat{\mathrm{P}} \tag{5}
\end{equation*}
$$

and

$$
\begin{equation*}
\hat{X}=\frac{\hat{P} \times \hat{N}}{|\hat{P} \times \hat{N}|} \tag{6}
\end{equation*}
$$

The unit vector $\hat{N}$, referred to the ecliptic coordinate system, is given by

$$
\hat{\mathrm{N}}=\left[\begin{array}{lll}
\sin & i & \sin \Omega  \tag{7}\\
-\sin & i & \cos \Omega \\
\cos & i
\end{array}\right]
$$

where $i$ and $\Omega$ are the inclination and longitude of the ascending node, respectively, of the planet orbital plane. The unit vector $\hat{P}$, referred to the ecliptic system, is given by

$$
\hat{\mathrm{P}}=\left[\begin{array}{c}
\cos \alpha \cos \delta  \tag{8}\\
\cos \varepsilon \sin \alpha \cos \delta+\sin \varepsilon \sin \delta \\
-\sin \varepsilon \sin \alpha \cos \delta+\cos \varepsilon \sin \delta
\end{array}\right]
$$

where $\alpha$ and $\delta$ are the right ascension and declination, respectively, of the planet polar axis relative to the geocentric equatorial coordinate system, and $\varepsilon$ is the obliquity of the ecliptic. Expressions for $\alpha$ and $\delta$ for each planet were obtained from JPL TR 32-1306, Constants and Related Information for Astrodynamic Calculacions, 1968, by Melbourne, et al.

The use of subroutine EQUATR is restricted to planets other than the earth and moon.

EQUATR Flow Chart



FILTER Analysis
The augmented state deviation vector is defined as

$$
x^{A}=\left[\begin{array}{c}
x  \tag{1}\\
q \\
u \\
v \\
w
\end{array}\right]
$$

where

$$
\begin{aligned}
& \mathbf{x}=\text { basic staie vector, } \\
& \mathbf{q}=\text { vector of solve-for parameters, } \\
& \mathbf{u}=\text { vector of dynamic consider parameters, } \\
& \mathbf{v}=\text { vector of measurement consider parameters, } \\
& \mathbf{w}=\text { vector of dynamic/measurement consider parameters. }
\end{aligned}
$$

The dynamic model for the linearized equations has the form

$$
\begin{equation*}
x_{k+1}^{A}=\Phi_{k+1, k}^{A} x_{k+1}^{A}+Q_{N_{k+1, k}}^{A} \tag{2}
\end{equation*}
$$

where the augmented state transition matrix, $\Phi_{k+1, k}^{A}$, may be partitioned as

$$
\Phi^{A}=\left[\begin{array}{lllll}
\phi & \psi & \theta_{\mathbf{u}} & \theta_{\mathbf{v}} & \theta_{W}  \tag{3}\\
0 & \mathrm{I} & 0 & 0 & 0 \\
0 & 0 & \mathrm{I} & 0 & 0 \\
0 & 0 & 0 & \mathrm{I} & 0 \\
0 & 0 & 0 & 0 & \mathrm{I}
\end{array}\right]
$$

Henceforth the state transition matrix partitions will be written without stating the associated time interval, which will always be assumed to be [ $t_{k}, t_{k+1}$ ]. The augmented dynamic noise vector $Q_{N_{k+1, k}}^{A}$

$$
\mathrm{Q}_{\mathrm{N}_{k+1, h}}^{A}=\left[\begin{array}{l}
\mathrm{Q}_{\mathrm{N}}  \tag{4}\\
0 \\
0 \\
0 \\
0
\end{array}\right]
$$

where the associated time interval is again understood to be $\left[t_{k}, L_{k+1}\right.$ ].

The measurement deviation vector is related to the augmented state deviations through the relation

$$
y_{k}=H_{k}^{A} x_{k}^{A}+\eta_{k}
$$

where the augmented measurement matrix may be partitioned as

$$
\mathrm{H}_{\mathrm{k}}^{\mathrm{A}}=\left[\begin{array}{lllll}
\mathrm{H} & \mathrm{M} & 0 & \mathrm{~L} & \mathrm{G} \tag{5}
\end{array}\right]
$$

and $\eta_{k}$ is the measurement noise.
The augmented state covariance matrix may be written in partitioned form as

$$
P^{A}=\left[\begin{array}{lllll}
P & C_{x q} & C_{x u} & C_{x v} & C_{x w}  \tag{6}\\
C_{x q}^{T} & Q & C_{q u} & C_{q v} & C_{q w} \\
C_{x u}^{T} & C_{q u}^{T} & D_{u} & 0 & 0 \\
C_{x v}^{T} & C_{q v}^{T} & 0 & D_{v} & 0 \\
C_{x w}^{T} & C_{q w}^{T} & 0 & 0 & D_{w}
\end{array}\right] .
$$

Prediction and filtering equations for the partitions appearing in the previous equations will be written below. Equations need not be written for the consider parameter covariances $D_{u}, D_{v}$, and $D_{w}$ since they remain constant. A minus supers ript on a covariance partition indicates its value immediately prior to processing a
measurement; a plus superscript indicates its value immediately after processing a measurement. To improve numerical accuracy and to avoid nonpositive definite covariance matrices, $P$ and $Q$ are symmetrized after the computations.

At, entry point FILM, the following computations are made. First, the measurement residual covariance matrix

$$
\begin{align*}
J_{k+1}= & H_{k+1}^{A} P_{k+1}^{A-} H_{k+1}^{A T}+R_{k+1} \\
= & H_{k+1}\left\{P_{k+1}^{-} H_{k+1}^{T}+C_{x q_{k+1}}^{-} M_{k+1}^{T}+C_{x v_{k+1}}^{L_{k+1}}+C_{x w_{k+1}}^{-} G_{k+1}^{T}\right\} \\
& +M_{k+1}\left\{C_{x q_{k+1}}^{-T} H_{k+1}^{T}+Q_{k+1}^{-} M_{k+1}^{T}+C_{q v_{k+1}}^{-} L_{k+1}^{T}+C_{q w_{k+1}}^{-} G_{k+1}^{T}\right\} \\
& +L_{k+1}\left\{C_{x v_{k+1}}^{-T} H_{k+1}^{T}+C_{q v_{k+1}}^{-T} M_{k+1}^{T}+D_{v} L_{k+1}^{T}\right\} \\
& +G_{k+1}\left\{C_{x w}^{-T} H_{k+1}^{T}+C_{q w_{k+1}}^{-T} M_{k+1}^{T}+D_{w} G_{k+1}^{T}\right\}+R_{k+1} \tag{7}
\end{align*}
$$

The Kalman gain matrix

$$
\begin{gather*}
K_{k+1}^{A}=P_{k+1}^{A-} H_{k+1}^{A T}\left(J_{k+1}\right)^{-1}=\left[\begin{array}{l}
K_{k+1} \\
K 1_{k+1} \\
K 3_{k+1} \\
K 4_{k+1} \\
K 5_{k+1}
\end{array}\right]  \tag{8}\\
K_{k+1}^{A M}=\left[\begin{array}{c}
K 1_{k+1} \\
K 2_{k+1} \\
0 \\
0 \\
0
\end{array}\right] \tag{BA}
\end{gather*}
$$

## FILTER-4

Only the $K 1_{k+1}$ and $K 2_{k+1}$ partitions are used,
$K 1_{k+1}=\left\{P_{k+1}^{-} H_{k+1}^{T}+C_{x q_{k+1}}^{-} M_{k+1}^{T}+C_{x v_{k+1}}^{-} L_{k+1}^{T}+C_{x w_{k+1}}^{-} G_{k+1}^{T}\right\}\left(J_{k+1}\right)^{-1}$
$K 2_{k+1}=\left\{C_{x q_{k+1}}^{-T} H_{k+1}^{T}+Q_{k+1}^{-} M_{k+1}^{T}+C_{q v_{k+1}}^{-} L_{k+1}^{T}+C_{q_{k+1}}^{-} G_{k+1}^{T}\right\}\left(J_{k+1}\right)^{-1}$
The partitions of the covariance update equation

$$
\begin{equation*}
\mathrm{P}_{\mathrm{k}+1}^{\mathrm{A}+}=\mathrm{P}_{\mathrm{k}+1}^{\mathrm{A}-}-\mathrm{K}_{k+1}^{\mathrm{A}} \mathrm{H}_{k+1}^{\mathrm{A}} \mathrm{P}_{\mathrm{k}+1}^{\mathrm{A}-} \tag{11}
\end{equation*}
$$

are given by

$$
\begin{equation*}
P_{k+1}^{+}=P_{k+1}^{-}-K 1_{k+1}\left\{H_{k+1} P_{k+1}^{-}+M_{k+1} C_{x q_{k+1}}^{-T}+L_{k+1} C_{x v_{k+1}}^{-T}+G_{k+1} C_{x w_{k+1}}^{-T}\right\} \tag{12}
\end{equation*}
$$

$$
\begin{align*}
& C_{x q_{k+1}^{+}}^{+}=C_{x q_{k+1}}^{-}-K_{k+1} \Delta_{k+1}  \tag{13}\\
& C_{x u_{k+1}}^{+}=C_{x u_{k+1}}^{-}-K 1_{k+1} \Gamma_{k+1}  \tag{14}\\
& C_{x v_{k+1}}^{+}=C_{x v_{k+1}}^{-}-K 1_{k+1} \Omega_{k+1}  \tag{15}\\
& C_{x w_{k+1}^{+}}^{+}=C_{x w_{k+1}}^{-}-K 1_{k+1} \Lambda_{k+1}  \tag{16}\\
& Q_{k+1}^{+}=Q_{k+1}^{-}-K 2_{k+1} \Delta_{k+1}  \tag{17}\\
& C_{q u_{k+1}^{+}}^{+}=C_{q u_{k+1}}^{-}-K 2_{k+1} \Gamma_{k+1}  \tag{18}\\
& C_{q v_{k+1}}^{+}=C_{q v_{k+1}}^{-}-K 2_{k+1} \Omega_{k+1}  \tag{19}\\
& C_{q w_{k+1}^{+}}^{+}=C_{q w_{k+1}}^{-}-K 2_{k+1} \Lambda_{k+1} \tag{20}
\end{align*}
$$

where

$$
\begin{align*}
& \Delta_{k+1}=H_{k+1} C_{x q_{k+1}}^{-}+M_{k+1} Q_{k+1}^{-}+L_{k+1} C_{q v_{k+1}}^{-T}+G_{k+1} C_{q w_{k+1}}^{-T}  \tag{21}\\
& \Gamma_{k+1}=H_{k+1} C_{x u_{k+1}}^{-}+M_{k+1} C_{q u_{k+1}}^{-}  \tag{22}\\
& \Omega_{k+1}=H_{k+1} C_{x v_{k+1}}^{-}+M_{k+1} C_{q v_{k+1}}^{-}+L_{k+1} D_{v}  \tag{23}\\
& \Lambda_{k+1}=H_{k+1} C_{x w_{k+1}}^{-}+M_{k+1} C_{q w_{k+1}}^{-}+G_{k+1} D_{w} \tag{24}
\end{align*}
$$

The remaining partitions, $D_{u}, D_{v}, D_{w}$, are not updated since they are associated with consider parameters.

At entry point PREM, the following computations are made. First the partitions of the covariance prediction equation

$$
\begin{equation*}
\mathrm{P}_{k+1}^{\mathrm{A}-}=\Phi_{k+1, h}^{A} \quad \mathrm{P}_{k}^{\mathrm{A}+} \Phi_{k+1, h}^{\mathrm{AT}} \tag{25}
\end{equation*}
$$

are given by

$$
\begin{align*}
& P_{k+1}^{-}=\left\{\phi P_{k}^{+}+\psi C_{x q_{k}}^{+T}+\theta_{u} C_{x u_{k}}^{+T}+\theta_{w} C_{x w_{k}}^{+T}\right\} \phi^{T} \\
& +C_{x q_{k+1}}^{-} \psi^{T}+C_{x u_{k+1}}^{-} \theta_{u}^{T}+C_{x w_{k+1}}^{-} \theta_{w}^{T}  \tag{26}\\
& C_{x q_{k+1}}^{-}=\phi C_{x q_{k}}^{+}+\psi Q_{k}^{+}+\theta_{u} C_{q u_{k}}^{+T}+\theta_{w} C_{q W_{k}}^{+T}  \tag{27}\\
& C_{x u_{k+1}}^{-}=\phi C_{x u_{k}}^{+}+\psi C_{q u_{k}}^{+}+\theta_{u} D_{u}  \tag{28}\\
& C_{x v_{k+1}}^{-}=\phi C_{x v_{k}}^{+}+\psi C_{q v_{k}}^{+}  \tag{29}\\
& C_{x W_{k+1}}^{-}=\phi C_{x W_{k}}^{+}+\psi C_{{ }_{q W_{k}}}^{+}+\theta_{w} D_{w} \tag{30}
\end{align*}
$$

Since all solve-for and consider parameter deviations are assumed to be constant between measurements, the following relations are used

$$
\begin{align*}
& Q_{k+1}^{-}=Q_{k}^{+}  \tag{31}\\
& \mathrm{C}_{\mathrm{qu}{ }_{k+1}}^{-}=\mathrm{c}_{\mathrm{qu}{ }_{k}}^{+}  \tag{32}\\
& \mathrm{C}_{\mathrm{qv}_{\mathrm{k}+1}}^{-}=\mathrm{C}_{\mathrm{qv}_{\mathrm{k}}}^{+}  \tag{33}\\
& \mathrm{c}_{\mathrm{q} \mathrm{w}_{\mathrm{k}+1}}^{-}=\mathrm{c}_{\mathrm{qw}_{\mathrm{k}}}^{+} \tag{34}
\end{align*}
$$

Again since the solve-for and consider parameter deviations are constant between measurements, only the basic state partition of the estimated state prediction or propagation equation is required

$$
\begin{equation*}
\hat{x}_{k+1}^{-}=\phi \hat{x}_{k}^{+}+\psi \hat{q}_{k}^{+} \tag{35}
\end{equation*}
$$

At entry point SIMM, the following computations are made. First the measurement residual is computed as

$$
\varepsilon_{k+1}=y_{k+1}^{a}-\left\{\tilde{y}_{k+1}+H_{k+1} \hat{\mathrm{x}}_{k+1}^{-}+\mathrm{M}_{k+1} \hat{\mathrm{q}}_{\mathrm{k}+1}^{-}\right\}
$$

Then the partition of the estimated state update equation

$$
\tilde{x}_{k+1}^{A+}=\hat{x}_{k+1}^{A-}+\hat{k}_{k+1}^{A m} \varepsilon_{k+1}
$$

where

$$
\mathrm{K}_{\mathrm{k}+1}^{\mathrm{Am}}=\left[\begin{array}{c}
\mathrm{K} 1 \\
\mathrm{~K} 2 \\
0 \\
0 \\
0
\end{array}\right]
$$

is given by

$$
\begin{aligned}
& \hat{\mathrm{x}}_{\mathrm{k}+1}^{+}=\hat{\mathrm{x}}_{\mathrm{k}+1}^{-}+\mathrm{K} 1_{\mathrm{k}+1} \varepsilon_{\mathrm{k}+1} \\
& \hat{\mathrm{q}}_{\mathrm{k}+1}^{+}=\hat{\mathrm{q}}_{\mathrm{k}+1}^{-}+\mathrm{K} 2_{\mathrm{k}+1} \varepsilon_{\mathrm{k}+1}
\end{aligned}
$$

At entry point QUASI, the computations associated with a quasilinear filtering event are made

$$
\begin{aligned}
& \tilde{\mathbf{x}}^{+}=\tilde{\mathrm{x}}^{-}+\hat{\mathrm{x}}^{-} \\
& \tilde{\mathrm{q}}^{+}=\tilde{\mathrm{q}}^{-}+\hat{\mathrm{q}}^{-} \\
& \hat{\mathbf{x}}^{+}=0 \\
& \hat{\mathrm{q}}^{+}=0
\end{aligned}
$$

where the superscript ~ indicates the nominal value of the state or solve-for parameter. The + superscript indicates the value after the quasi-linear filtering event, whereas the - superscript indicates the value before.

The flow of the FILTER subroutine is illustrated.


FILTER FIow Chart

```
SUBROUTINE GEOG
PUPPOSE: COMPUTE THE CD-ORDINATE TRANSFJRMATION FROM PLANETOCENTRIC
        equarorial plane to planetocentric geographical plane
ENTRY PARAMETERS:
    NP TARGET PLANET CODE
    D JULIAN JATE, EPOCH JANUARY 0, 1900
    EQGF CO-ORDINATE TRANSFORMATION FROM PLANETOCENTRIC
    EQUATORIAL TO PLANETOCENTRIC GEOGRAPHICAL
LOCAL srMBOL S:
    ED JULIAN DATE, EPOCH 4713 Bs i.
    gGTR GONVERTS DEGREES TO RAJIANS
    veha hour angle of the vernal equinox
```


## GEDG Analysis

Subroutine GE $\phi$ G computes the coordinate transformation from planetocentric equatorial to planetocentric geographical coordinate for an arbitrary planet. The geographical coordinate system is defined so the 2 -axis is aligned with the planet spin vector and the $x$-"axis lies in the plane of the planet prime meridian. The prime meridian is oriented relative to the planet vernal equinox $T$ by the hour angle of the vernal equinox $V$. In the figure shown below the xyz axes define the planetocentric geographical system, the $x_{e q} y_{e q}{ }^{r}{ }_{e q}$ axes define the planetocentric equatorial system.


The expressions used to evaluate $V$ for each planet were obtained from JPL TR32-1306, Constants and Related Information for Astrodynamic Calculations, 1968, by Melbourne et al.

The coordinate transformation matrix is given by

$$
A=\left[\begin{array}{ccc}
\cos \mathrm{V} & \sin \mathrm{~V} & 0 \\
-\sin \mathrm{V} & \cos \mathrm{~V} & 0 \\
0 & 0 & 1
\end{array}\right]
$$

Thus

$$
\vec{x}_{\text {geographical }}=A \vec{x}_{\text {equatorial }}
$$

## GEDG Flow Chart



```
SUBROUTINE GHA
purpose: gomputes greenwich hour angle of the vernal equinox
ENTRY PARAMETERS:
    OATEJ JULIAN DATE, EPOCH AT JANUARY O, 1900
    gh GreENHIGH hour angle of the vernal equinox in radians
LOGAL SYMBOLS:
    EOMEG EARTH ROTATION RATE IN JEGREES/DAY
    REFJD JULIAN DATE OF 1950 JANUARY 1, EPOCH AT 4713 B.C.
    TSTAR JULIAN DATE, EPOCH AT 1950 JANUARY 1
    ID INTEGER PART OF TSTAR
    D ID CONVERTED TO REAL
    TFRAC FRACTIONAL PART OF TSTAR
```


## GHA-1

GHA Analysis
Subroutine GHA computes the Greewich hour angle GHA of the vernal equinox at a given Julian date (JD), epoch 1900 January $0^{\text {d }} 12^{\mathrm{h}}$, using

GHA $=100.0755426+0.985647346 \mathrm{~d}+2.9015 \times 10^{-13} \mathrm{~d}^{2}+\omega t$
where
$\omega=$ Earth's rotation rate (deg/day)
$\mathrm{d}=$ integer part of $\mathrm{T}^{*}$
$t=$ fractional part of $T^{*}$
and
$\mathbf{T}^{*}=$ Julian date, epoch 1950 January $1^{d} 0^{\text {h }}$.

The Julian dates relative to epochs 1900 and 1950 are related as follows:

$$
T^{*}=J D+2415020.0-2433282.5
$$

where
$2415020.0=1900$ January $0^{\text {d }} 12^{\text {h }}$ referenced to 4713 BC January $0^{d} 12^{\text {h }}$ and
$2433282.5=1950$ January $1^{d} o^{h}$ referenced to 4713 BC January $0^{d} 12^{h}$

```
SUPROUTINE HMM
PURPOSE : CONTROLS COMPUTATION OF OBSERVATION MATRIX PARTITIONS
SURROUTINES CALLED: JACOON
COMMONS : TRAJ COVARP INTCOM
LOCAL SYMBOLS
    ORSM EXTERNAL VARIAMLE NAME USEJ QY SACORN FOR COMPUTATION OF MEASUREMENT VALUES
\begin{tabular}{lllllll} 
USED/COMMN-- & C & DU & DV & DW & GM & HM \\
& LISTQ & LISTS & LISTV & LISTW & LM & MM \\
& NM & NQ & NS & NU & NH & P \\
& C & & & & &
\end{tabular}
```


## HMM Analysis

Subroutine $H M M$ is an executive routine that controls the computtation of the partitions of the observation matrix. The matrix partitions are all computed by numerical differencing, which is carried out by calling JACOBN. The indices of the variables to be perturbed to compute columns of the observation matrix are stored in LIS'S, LISTQ, LISTV, and LISTW for the H, M, L, and G partitions, respectively. The size of the perturbations are governed by the variance of the parameters that are stored in arrays $P, Q, D V$ and $D W$. The unperturbed measurement values are stored in the MEAS array.

The linearized measurement equation in partitionsd form is given by

$$
y=\left[\begin{array}{l:l:l:l:l}
H & M & 0 & L & G
\end{array}\right]\left[\begin{array}{l}
x \\
q \\
u \\
v \\
w
\end{array}\right]
$$

SUBROUTINE INVPD?
PURPOSE : INVERTS A POSITIUE DEFINTTE SYMMETRIC MATRIX


INVPD2 Analysis
This subroutine inverts a positive definite symmetric matrix by a modified Cholesky method. Let $S$ be the positive definite matrix to be inverted. The method proceeds by determining matrices $L$ and $D$ so $L$ is lower triangular with $1 s$ on the diagonal, $D$ is diagonal, and

$$
S=L D L^{T}
$$

$L$ and $D$ may be found recursively from the relations

$$
\begin{gathered}
d_{j}=s_{j j}-\sum_{k=1}^{j-1} d_{k} \ell_{j k}^{2} \\
\ell_{i j}=\frac{\left\{s_{i j}=\sum_{k=1}^{j-1} d_{k} \ell_{i k} \ell_{j k}\right\}}{d_{j}}, \quad i>j
\end{gathered}
$$

The inverse of $S$ is then given by

$$
S^{-1}=\left(L^{T}\right)^{-1} \cdot D^{-1} L^{-1}
$$



```
SURROUTINE INVPSO
PURPOSE: TO INVERT A IX1 OR 2X2 MATRIX
ENTRY PARAMETERS
    N SIZE OF X AND Y MATRICES
    X HATRIX TO BE INVERTED
    Y INVERSE MATRIX (OUTPUTI
I.OCAL SYMBOLS
    I INOEX
    RECDET REGIPROCAL OF THE JETERMINANT OF }
```

```
SUBROUTINE JACOBN
PURPOSE : COMPUTE THE JACORIAN MATRIX OF A VECTOR FUNCTION WITH
    RESPECT TO A SPECIFIC SUBSET OF PARAMETERS BY
    NUMERICAL DIFFERENGING
ENTRY PARAMETERS
    C VFCTOR OF PAFAMETERS
    GOVAR COVARIANCE MATRIX CONTAINING THE VARIANCE
        of the parameters
    FCT EXTERNAL FUNCTION USED TO COMPUTE VALUES
        OF THE VECTOR FUNCTION
    ZACOBN THE JACOBIAN MATRIX
    LIST LIST OF INOICATORS OF THE SUBSET OF PARAMETERS
        TO HE USED
        DIMENSION OF COVAR
        NUMBER Of PARAMETERS IN THE SUBSET
        dIMENSION OF THE VECTOR FUNCTION
        7ADD NOMINAL value of the vegtor function
SUBROUTINES CALLED: FCT (EXTERNAL SUPPLIED AS ENTRY PARAMETER)
logal gYmbols
    CSAVE TEMPORARY STORAGE FOR UNPERTURBED VALUE OF PARAMETER
    OIFF PERTURBATION APPLIED TO PARAMETER
    E CONSTANT USED IN COMPUTING THE SIZE OF the
        PERTURBATION INDEX
    II INDEX OF the I-th diagonal element of covar
    l InOEX of the parameter meing perturbej
    ZADDP PERTURBED VALUE OF THE VECTOR FUNCTION
    ZP DUMMY PARAMETER
LOADEJ --- E
```


## JACøBN Analysis

JACøBN computes the Jacobian matrix of an $N Z$ dimension vector with respect to the subset of parameters in the C Array whose indicates are in LIST. The computation is carried out by numerical differencing. The vector function is evaluated by calling FCT. The unperturbed values of the function are stored in ZADD. The parameters are perturbed by an amount depending on this variance. The variances are stored in the array CøVAR.

| Purpose : ExEcu | EXECUTIVE CONTROL FOR REGONSTRUCTOR |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{gathered} \text { SUBROUTINES CALLEO: } \\ \text { NTM } \\ \text { OUASI } \end{gathered}$ | COPY NTM2 READAC | DYNOIZ PDUMP RESTRT | measur PREDIC SETICN | $\begin{aligned} & \text { NEXTAA } \\ & \text { PREM } \\ & \text { SETUP } \end{aligned}$ | NEXTIM PRINT STM |
| COMMONS : SMO | GY | TRAJ | SUMRY |  |  |
| LOCAL SYMBOLS INDEX |  |  |  |  |  |
| $J$ | Index |  |  |  |  |
| z | most receht nominal state at start and end OF CURRENT INTEGRATION INTERVAL |  |  |  |  |
| 2 ADO | change in 2 vector over the interval |  |  |  |  |
| $\begin{aligned} \text { USED/COMMN--- AA } \\ \text { TENJ } \end{aligned}$ | IEN | LTR | LTR2 | TC | toiff |
| SET/COMMON--- AA | TC | TIN |  |  |  |

## LTRCON Analysis

Subroutine LTRCøN is the executive subroutine for the LTR reconstruction program and controls the entire computational flow fior trajectory propagation, state transition matrix computation, measurement processing, event execution, and printout.

LTRCON Flow Chart




```
SUBROUTINE MAIN
PURPOSE : CONTROLS OVERALL PROGRAM FLOW FOR DATA GENERATION,
        PREPROCESSING, TRAJECTORY RECONSTRUCTON, AND
        SUMMARY OUTPUT
\begin{tabular}{|c|c|c|c|c|}
\hline SURROUTINES CAILEJ: & EXIt & TIMEX & & \\
\hline COMmons : A: REDY & DOPLER & INTCOM & LOGEOM & \\
\hline USFO/COMMN--- LTR1 & LTR2 & runno & & \\
\hline SET/COMMSN---GENJAT & NTP & OMEGAE & REARTH & RESTRT \\
\hline
\end{tabular}
```

MAIN Analysis
RUNN $\varnothing$ controls program flow. If RUNN $\emptyset=1$, the data generator and preprocessor are executed and control goes to statement 10, where another value of RUNN $\varnothing$ is read. If RUNN $\varnothing=2$, the main LTR program is called to reconstruct the lander trajectory and print the summary output, which may include a plotting package. Control then passes to statement 10. If RUNN $\varnothing=3$, the program exits to the system.

SUBROUTINE MATOUT
PURPOSE : MATRIX PRINTOUT HITH HOLLERITH NAME

| ENTRY PARAMETERS |  |
| :---: | :--- |
| NAME | HOLLERITH NAME OF $X$ MATRIX |
| NC | NUMBER OF COLUMNS OF $X$ MATRIX |
| NR | NUMBER OF ROWS OF $X$ MATRIX |
| $X$ | MATRIX TO RE PRINTED OL: |

LOCAL SYMBOLS
I
INDEX
$N$ TOTAL NUMPER OF ELEMENTS U. $x$
NEND LOCATION IN $x$ OF THE END OF THE I-TH ROW
NSTART LOCATION IN $X$ OF THE START OF THE I-TH ROW

WRITTEN -- NAME $X$

MATøUT-1

## MATQuT Analysis

The matrix $X$ is written out by rows with up to 8 values per line and can be a column vector or a rectangular matrix. Each row of $X$ starts a new line, and a return is generated when NEND 2 N .


SUPROUTINE MEATUR

| PURPOSE : PRICES | PRICESSES MEASUREMENTS THROUGHT THE FILTER EQUATIONS |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| SURROUTINES CALLED: | ATtACK | FILM | HMM | 09SM | SIMM |
| COMMONS 1 AX | INTCOM | TRAJ | SMO |  |  |
| LOCAL SYMBOLS <br> TMLAST | TRAJECT | TIME | LAST | REMEN |  |
| XX | DUMMY C | ARGUP |  |  |  |


| USED/COMMN--- | ALPHA THN | IPRINT TYPE | $\begin{aligned} & \text { MCNTR } \\ & \text { AA } \end{aligned}$ | MEZACT | NMEAS |
| :---: | :---: | :---: | :---: | :---: | :---: |
| SET/COMMON-- - | IPRINT | mezact | NMEAS |  |  |
| LOABED --- | TMLASt |  |  |  |  |

## MEAZUR Analysis

Subroutine MEAZUR is the executive measurement processing subroutine. It controls the computation of all quantities required to generate a new estimate of the state and the associated error covariance matrix partitions. Subroutine MEAZUR also computes the actual angle of attack measurement for mode $A$ and the actual accelerometer measurements for mode B. Unlike other measurement types available in the LTR program, the actual values of these two measurement types cannot be computed in the data generator since they are computed from information generated in the preprocessor, which is always run after the data generator has been run.

subrolitine mult
PURPOSE : TO NULTIPLY ONE RECTANGULAR MATRIX BY ANOTHER AND STORE INTO A THIRS RECTANGULAR MATRIX

ENTRY PARAMETERS
NCX NUMBER OF COLUMNS OF $X$ NIATRIX AND NUMRER OF RONS OF Y MATRIX

NCY NUMBFR OF COLUMNS OF $Y$ MATRIX AND $z$ MATRIX
NRX NUMRFR OF ROWS OF $X$ MATRIX AND 7 MATRIX
$x$ INPUT MATKIX
$\checkmark$ INPUT MATRIX
7 PROJUCT OF $x$ ANJ $Y$ MATRICES (OUTPUT)

LOCAL SYMBOLS
I
index
J INDEX
$K$ INDEX
SUM DOT PRODUCT OF I-TH ROM OF $X$ AND J-TH COLUMN OF $Y$

SURROUTINE MULTD
PURPOSE : TO MULTIPLY A RECTANGULAR MATRIX TIMES A DIAGONAL MATRIX

ENTRY PARAMETERS
NCX NUMBER OF COLUMNS OF $X$ AND NUMBER OF COLUMNS OF $Z$
ND NUMBER OF DTAGONAL ELEMENTS DF $Y$
NRX NUMAER DF ROUS OF $x$ AND NUMRER OF ROWS OF $z$
$x$ RECTANGULAR INPUT MATRIX
$Y$ DIAGONAL INPUT MATRIX
$Z$ RECTANGULAR OUTPUT MATRIX

LOCAL SYMRBLS
INDEX
J INJEX
$P \quad J-T H$ OIAGONAL ELEMENT OF $Y$

```
SUBROUTINE MULTT
PURPOSE : TO MULTIPLY ONE RECTANGULAR MATRIX AY THE TRANSPOSE
        OF ANOTHFR MATRIX AND STORE INTO A THIRD MATRIX
ENTRY PARAMETERS
    NGX NUMBER OF COLUMNS OF }x\mathrm{ and }Y\mathrm{ matrices
    NRX NUMBER OF ROHS OF }X\mathrm{ and }z\mathrm{ mATRICES
    NRY NUMBER OF RONS OF Y AND NUMBER OF COLUMNS DF Z
    x rectangular input matrix
    r RECTANGULAP INPUT MATRIX (TO BE TRANSPOSED)
    z OUTPUT MATRIX ( }x\mathrm{ TIMES }\gamma\mathrm{ TRANSPOSED)
LOCAL SYMBOLS
    I INDEX
    J INDEX
    K INDEX
    SUM DOT PRODUCT OF I-TH column of x and J-TH COLUMN OF Y
```

| PURPOSE : READS | SMOOTHED GYRO AND AGCELEROMETER DATA ntegration to next event |
| :---: | :---: |
| Subroutines calleds | altfile |
| COMMENS : SMO | TRAJ |
| LOCAL SYMBOLS | I NDEX |
| $J$ | index |
| NALT | DUMMY CALL ARGUMENT |
| NG | Cal Culated number of regords to be read |
| time | time corresponjing to each recoro |
| USEB/COMMN--- DT | IEND TC TDIFF |
| REA $\quad-\infty$ an | time |
| SETYCOMMON--- TAA | IEND TENO |

## NEXTAA Analysis

Subroutine NEXTAA reads from file 15 the smoothed accelerometer and gyro coefficients (as 3etermined by subroutine PREPR $\phi S$ ) required to cover the time interval to the next event.

The first time NEXTAA is callen, TEND is zero, which causes the coefficients for time zero to be read. Thereafter, the coefficients for the begiming $G$ the interval are obtained from the last point of the previous interval.

Subroutine NEXTAA aisn detemines the nunber NG of records to be read to cover the in erval Erom TC through TEND. However, if an end-of-file is cacountered white chese coeficients are being read, then the number IEND of coefficient records read is adjusted and TEND is reset to correspond to the last record read.


SUBROUTINE NEXTIM


## NEXTIM-1

## NEXTIM Analysis

Subroutine NEXTIM computes the next event time and the time difference between the current time and the next event time. The logic proceeds as follows:
a. If the event schedule buffer has been used up, as determined by MCNTR $=250$, then another 250 elements of the schedule is read from file 20 and MCNTR is set to zero.
b. Current time TC is updated, schedule index MCNTR is incremented, and the time TEND and type TYPE of the next event are taken from the schedule.
c. If the current time $T C$ is equal to the time QST to change to the quasi-static dynamic model, then QSMCHG is set to true and the integration step size is changed to the quasi-static integration step size.
d. The entry phase IPHAS is determined by comparing current time TC to the time of parachute deployment TD and the time of parachute deployment TD and the time of parachute release TR. The vehicle parameters MASS, RI, SA, and DIA are then selected for this phase.

```
SUBROUTINE NORMNZ
PURPOSE & COMPUTES RANDOM VARIARLES FROM A DISTRIQUTION WITH
    ZERO MEAN AND STANDARD DEVIATION ONE
ENTRY PARAMETERS
    SIGMA
OUTPUT RANDOM VARIARLE
LDCAL SYMBOLS
        A
        SUM OF THE vALUES OF RR
        N INTEGER PORTION OF SS MULTIPLIED BY 1.E-7
    NX CONTROLS START OF RANCOM SELECTION
    RR OIFFERENEE OF SS AND N
    SS INTERMEDIATE SUM OF SS, WH, YY, AND ZZ
    WH SEED VALUE FOR BUILDING SS
    YY SEEJ UALUE FOR BUILJING SS
    ZZ SEEO VALUE FOR BUILDING SS
LOADES --* NK
```


## NØRMNZ Analysis

N $\varnothing$ RMNZ builds a random number from a distribution with a mean of zero and standard deviation of one. From preset seed values ( $\mathrm{NX}=0$ ) or from values stored in a previous call ( $\mathrm{NX}=1$ ), the variables WW, YY, $Z Z$ are always positive and, when summed with SS, yield a number $X$ such that $X$ is in the open interval between $1 . E+7$ and 1.E +8 , with occasional (i.e., greater than 3 sigma) values outside this interval. RR is then found as
$\mathrm{RR}=\mathrm{X}$ modulo (integer portion of X )
that RR is normally distributed over ( 0,1 ). Finally,

$$
\text { SIGMA }=\left(\sum_{i=1}^{12} \begin{array}{c}
\text { RR }
\end{array}\right)-6
$$

NTM-A


NTM-1

## NTM Analysis

Subroutine NTM controls the propagation of the most recent nominal state vector over the time interval [TC, TEND] for both mode A and mode B.


SUBROUTINE NTMZ
PURPOSE : CONTROLS INTEGRATION OF ORIGINAL NOMINAL STATE VECTOR FROY TIME TC TO TIME TEND

EnTRY firameters
$X$ ORIGINAL NOMINAL STATE AT TIME TEND

SUBROUTINES CALLED: ATMSET RKUTL3 RKUT 3 DERIVE DERIV3
COMAONS : LOGCOM TRAJ LOGMOD
LOGAL SYMBOLS XADD DIFFERENCE IN STATE VECTOR OVER THE INTERVAL

KXX DUMMY CALL ARGUMENT

USEO/COMMN-- LTR1 QSMCHG TC

## NTM2 Analysis

Subroutine NTM2 controls the propagation of the original nominal state vector over the time interval [TC, TEND] for both mode A and mode B. A flow chart for NTM2 is not $p \times$ esented since it would be quite similar to the NTM flow chart. Sf NTM Analysis for more details.


| HESP | hel iocentric ecliptic state of target planet |
| :---: | :---: |
| HEST | geocentric ecliptic state of osn station |
| highet | dounrange angle at end of oirect search MINIMIZATION PROCESS |
| icode | CURRENT MEASUREMENT TYPE REING PROCESSEJ |
| IJ | Index of rangegrange-rate measurement |
| ITEST | Intermediate integer to find gin station numper |
| minalt | minimum distance betheen vehicle and planet terrain |
| SING | SINE OF GANG |
| SINLAT | SINE OF ALAT |
| SINOB | SINE OF OBLIC |
| STEP | ANGULAR STEPSIZE EMPLOYED IN DIRECT SEARCH MINIMIZATION PROCESS |
| TM | NUMBER Of SECONDS PER day |
| UPDAIT | dummy call argument |
| Wh | oistance from planet center to planet terrain |
| XGA | PERTURBED AXIAL DISTANGE TO CENTER OF GRAVITY |
| XMA | perturbed axial distance to accelerometer location |
| $z$ | FUNCTION actually minimized in radar altimeter direct search minimization process |
| 26A | PERTURRED NORMAL distange to center of gravity |
| zma | PERTURBED NORMAL DISTANGE TO ACCELEROMETER LOCATION |


| USEO/COMMN-®- | ACC <br> AqUANT DP <br> GYRODT <br> MENTR <br> RANGER <br> RH <br> SCPEC <br> TRERO <br> 26 | ACCET <br> BTBL <br> DXN <br> LTR1 <br> MGODE <br> REARTH <br> RMACHB <br> SLAT <br> VR <br> 2HM | ACCT <br> C <br> EPSM <br> LTR2 <br> OBLIC <br> REJRR1 <br> ROTNO <br> SL ON <br> XG <br> 2N | AF <br> CARCOR <br> ETA <br> MACH <br> OHEGAE <br> REJRR2 <br> RR <br> TC <br> XH | AGAM <br> DATEJ <br> GHATO <br> MASS <br> PRES <br> RHO <br> SA <br> TEMP <br> XN | ALPH <br> DELT <br> GQUANT <br> MASSA <br> RANGE <br> RI <br> SALT <br> TERHT <br> XP |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SETYCOMMON--- | ACC <br> RANGE | ACCT RANGER | MACH | MASSA | NM | $\mathbf{R}$ |
| FCT CALLED-- <br> FCT DFND --- | $\begin{aligned} & F F \\ & F F \end{aligned}$ | TAB |  |  |  |  |

## ØBSM Analysis

Subroutine $\emptyset \mathrm{BSM}$ has three functions:
a. Compute nominal measurement for each measurement type.
b. Compute perturbed measurement for use in the numerical differencing computation of observation matrix partitions.
c. Compute measurement noise covariance matrix for each measurement type.

The computation of nominal measurements in $\emptyset B S M$ is very similar to the computation of actual measurements in $\emptyset \mathrm{BSM1}$. The equations used to compute nominal radar altimeter, stagnation pressure, stagnation temperature, range, and range-rate measurements have the same form as those used to compute the corresponding actual measurements in $\emptyset$ BSMl and will not be discussed further (see subroutine $\emptyset$ BSM1 for details).

The $C_{j}$ in subroutine $\emptyset B S M 1$ represent actual errors. In $\emptyset B S M$, however, the $C_{j}$ represent both nominal and perturbed values of the errors. The $C_{j}$ are perturbed only when $\emptyset$ BSM is being used in the numerical differencing computation of observation matrix partitions.

If a measurement is being processed, $\emptyset \mathrm{BSM}$ also computes the measurement noise covariance matrix. The equations used to compute the measurement noise covariance matrix for each measurement type are summarized in section 3.2 of the Aralytic Manual.

Accelerometer and angle of attack measurements require further discussion since their treatment in 0 BSM differs from their treatment in $\emptyset B S M 1$. Accelerometer measurements are used in the filter observation model only for the mode $B$ reconstruction process. In mude A accelerometer measurements are treated as part of the dynamic model and all computations relating to mode $A$ accelerometer measurements are performed in subroutine DERIVE; none are performed in $\emptyset B S M$. The following equations are used in $\emptyset B S M$ to compute the accelerometer measurements for mode $B$ :

$$
\begin{align*}
& a_{x}=\left[\frac{A}{\left(m+c_{30}\right)} \cos \left(\delta_{1}+c_{55}\right)-\frac{N}{\left(m+c_{30}\right)} \sin \left(\delta_{1}+c_{55}\right)\right] c_{51} \\
& \quad+c_{52} \\
& a_{z}= {\left[\frac{A}{\left(m+c_{30}\right)} \sin \left(\delta_{2}+c_{56}\right)+\frac{N}{\left(m+c_{30}\right)} \cos \left(\delta_{2}+c_{56}\right)\right] c_{53} } \\
& \quad+c_{54} \tag{2}
\end{align*}
$$

where A and N are the axial and normal aerodynamic forces (including effect of parachute), $\delta_{1}$ and $\delta_{2}$ are misalignment angles, and $m$ is vehicle mass. Bias terms $C_{30}, C_{52}, C_{54}, C_{55}$, and $C_{56}$ are readily identifiable, as are scale factors $C_{51}$ and $C_{53^{\circ}}$

The angle of attack measurement, which is currently defined only for mode $A$, is defined as the angle of attack $\alpha$ computed in subroutine DERIVE.

Prior to computing any measurement, $\emptyset$ BSM calls the relevant dynamic model subroutines (DERIVE, if mode A; DERIV3 and ATMSET, if mode $B$ ) to ensure that all dynamic quantities have the proper values at the time of the measurement, since many of these quantities are required in the computation of measurements.

| PURPOSE |  | COMPUTE MEASUREMENTS FOR DATA GENERATOR |
| :---: | :---: | :---: |
| COMMONS | S : TRAJ | STATE DOPLER OBSERV LOGCOM |
| L.OCAL SY | SYMBOLS ACRAME | ACTUAL RANGE AND RANGE-RATE VECTORS |
|  | AL | DISTANGE FIROM EARTH CENTER TO DSN STATION |
|  | AL AT | LATITUDE OF DSN STATION |
|  | AL ON | LONGITUDE OF DSN STATION |
|  | ANG | DOWNRANGE ANGLE AT REGINNING OF JIRECT SEARCH MINIMIZATION PROCESS |
|  | ARG1 | AXIAL NON-GRAVITATIONAL ACCELERATION AT VEHICLE CENTER OF GRAVITY |
|  | ARG2 | NORMAL NON-GRAVITATIONAL ACCELERATION AT VEHICLE CENTER OF GRAVITY |
|  | $\cos 6$ | COSINE OF GANG |
|  | costat | COSINE OF ALAT |
|  | COSOB | COSINE OF OBLIC |
|  | DELTA | half the angular distance (relative to planet center) COVEREJ IN THE DIRECT SEARCH MINIMIZATION PROCESS |
|  | DJUL | JULIAN DATE AT TIME TC |
|  | GANG | LONGITUDE OF DSN STATION AT TIME TC |
|  | HESE | HELIOCFNTRIC ECLIPTIC STATE OF EARTH |
|  | HESP | HELIOCENTRIC ECLIPTIC STATE OF TARGET PLANEY |
|  | HEST | GEOCENTRIC ECLIPTIC STATE OF ISN STATION |
|  | HI GHPT | DOWnRANGE aNGLE AT END JF DIRECT SEARCH MINIMIZATION PROCESS |
|  | I | INJEX |
|  | IJ | INDEX OF RANGE, RANGE-RATE MEASUREMENT |
|  | MINALT | MINIMUM distance betheen vehicle and planet terrain |
|  | SING | SINE OF GANG |


| SINLAT |  | SINE of alat |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| SINOB |  | SINE OF OBLIC |  |  |  |  |
| STEP |  | ANGULAR STEPSIZE EMPLOYED IN DIRECT SEARCH MINIMIZATION PROCESS |  |  |  |  |
| TM |  | NUMBER OF SECONDS PER DAY |  |  |  |  |
| W - distance from planet genter to planet terrain |  |  |  |  |  |  |
| z |  | function actually minimized in radar altitude oIrect seareh minimization process |  |  |  |  |
| USED/COMMN--- | AGAM <br> CJELTZ <br> ETA <br> RANGE <br> SAL T <br> TC <br> XM | AX <br> CJELTS GHATO RANGER SCPEC TEMP XN | AY <br> DATEJ MACH REARTH SDELTI TERHT ZG | ```C DP OBLIC RHO SDELT2 TZERO ZHN``` | CARCOR <br> DXN <br> omegae <br> RM <br> SLAT <br> VR | CJELTi <br> EPSM PRES ROTNO SL DN XG |
| SETICOMMON--- M | meass | range | RANGER |  |  |  |
| FCT Callej--- ${ }^{\text {f }}$ | F | tab |  |  |  |  |
| FCT DFND --- F |  |  |  |  |  |  |

## ØBSM1 Analysis

Subroutine $\emptyset B S M 1$ computes the actual measurements for most weasurement types available in the LTR program and incorporates the effects of all error sources except noise into these measurements. Those measurements not computed in $\emptyset$ BSM1 are the quantized VRU and ARU measurements, which are computed in subroutine SENS $\emptyset R$. The equations used to compute the actual measurements in $\emptyset$ BSMI are summarized below.

If the terrain height model is not used, the radar altimeter measurement is given by

$$
\begin{equation*}
\tilde{h}=c_{71} h+c_{72} \tag{1}
\end{equation*}
$$

where $h$ is the vehicle altitude, $C_{71}$ is the altimeter scale factor, and $C_{72}$ is the altimeter bias. If the terrain height model is used, the radar altimeter measurement is defined as the shortest distance between the vehicle and the planet terrain within the altimeter sweep angle $2 \eta$. The altimeter measurement is computed from

$$
\begin{equation*}
\tilde{h}=C_{71}\left[\left(h+R_{p}^{2}\right)+\tilde{f}\right]^{\frac{1}{2}}+C_{72} \tag{2}
\end{equation*}
$$

where $\tilde{f}$ is the minimum value of

$$
\begin{equation*}
f=W^{2}-2 W\left(h+R_{p}\right) \cos (\tilde{\phi}-\phi) \tag{3}
\end{equation*}
$$

with respect to $\tilde{\phi}$, and is found using a direct search technique. For more details see section 2.4 of the Analytic Manual.

Unquantized accelerometer (VRU) and rate gyro (ARU) measurements, which are currently not used in the LTR reconstruction program, are given by

$$
\begin{align*}
& \dot{v}_{x}=a_{x} \cos \delta_{1}-a_{z} \sin \delta_{1}  \tag{4}\\
& \dot{v}_{z}=a_{x} \sin \delta_{2}+a_{z} \cos \delta_{2} \tag{5}
\end{align*}
$$

and

$$
\begin{equation*}
\dot{A}=\omega \cos \delta_{3} \tag{6}
\end{equation*}
$$

where $\delta_{1}, \delta_{2}$, and $\delta_{3}$ are the axial accelerometer, normal accelerometer, and rate gyro misalignment angles, respectively, $\omega$ is the vehicle angular velocity, and $a_{x}$ and $a_{z}$ are the axial and normal nongravitational accelerations at the VRU location. These latter accelerations are computed from

$$
\begin{align*}
& a_{x}=a_{x g}-\omega^{2} \bar{x}+\dot{\omega} \bar{z}  \tag{7}\\
& a_{z}=a_{z g}-w^{2} \bar{z}-\dot{\omega} \bar{x} \tag{8}
\end{align*}
$$

where $a_{x g}$ and $a_{z g}$ are the axial and normal nongravitational acceleration at the vehicle cg location, and $\bar{x}$ and $\bar{z}$ denote the offset of the VRU relative to the cg. Scale factor and bias errors for these unquantized measurements are currentiy undefined.

The stagnation pressure measurement $p_{0}$ is a function of Mach number regime. If Mach number $M \geq 3$, then

$$
\begin{equation*}
p_{0}=C_{81}\left[\frac{1}{2} C_{p} \rho v_{r}^{2}+p\right]+C_{82} \tag{9}
\end{equation*}
$$

where $\rho$ is the density, $p$ is the ambient pressure, and the coefficient of pressure $C_{p}$ is given by

$$
\begin{equation*}
C_{p}=2-\varepsilon \tag{10}
\end{equation*}
$$

where $\varepsilon$ is the ratio of densities in front of an behind the shock wave. Scale factor $C_{81}$ and bias $C_{82}$ are the error terms used in the supersonic regimes. If $1 \leq M<3$, then $p_{0}$ is again given by equation (9), but $C_{p}$ is now given by

$$
\begin{equation*}
C_{p}=\frac{p}{8}\left[\left(\frac{\gamma+1}{2} M^{2}\right)^{\frac{\gamma}{\gamma-1}} \cdot\left(\frac{\gamma+1}{2 \gamma M^{2}-\gamma+1}\right)^{\frac{1}{\gamma-1}}-1\right] \tag{11}
\end{equation*}
$$

where $\gamma$ is the ratio of specific heats. If $M<1$, then

$$
\begin{equation*}
p_{0}=c_{83}\left[p\left(1+\frac{\gamma-1}{2} M^{2}\right)^{\frac{\gamma}{\gamma-1}}\right]+c_{84} \tag{12}
\end{equation*}
$$

where $C_{83}$ and $C_{84}$ are the subsonic scale factor and bias errors, respectively.

The stagnation temperature measurement is computed from

$$
\begin{equation*}
T_{0}=C_{91}\left[T\left(1+\frac{\gamma-1}{2} M^{2}\right)\right]+C_{92} \tag{13}
\end{equation*}
$$

where $T$ is the ambient temperature and $C_{91}$ and $C_{92}$ are the scale factor and bias errors, respectively.

Range and range-rate measurements are given by

$$
\begin{equation*}
\rho=|\stackrel{\rightharpoonup}{\rho}| \tag{14}
\end{equation*}
$$

and

$$
\begin{equation*}
\dot{\rho}=\frac{\dot{\vec{\rho}} \cdot \stackrel{\rightharpoonup}{\rho}}{\rho} \tag{15}
\end{equation*}
$$

respectively, where

$$
\begin{align*}
& \stackrel{\rightharpoonup}{\rho}=\stackrel{\rightharpoonup}{r}+\stackrel{\rightharpoonup}{r}_{p}-\stackrel{\rightharpoonup}{r}_{\ell}-\stackrel{\rightharpoonup}{r}_{s}  \tag{16}\\
& \stackrel{\rightharpoonup}{\rho}=\stackrel{\ddot{r}}{\mathbf{r}}+\stackrel{\ddot{\dot{r}}}{p}-\stackrel{\ddot{\dot{r}}}{\ell}-\stackrel{\rightharpoonup}{r}_{s} \tag{17}
\end{align*}
$$

$(\stackrel{\rightharpoonup}{r}, \stackrel{\rightharpoonup}{r})=$ vehicle state relative to target planet $\left(\vec{r}_{p}, \stackrel{\rightharpoonup}{r}_{p}\right)=$ target planet state relative to Sun $\left(\stackrel{\rightharpoonup}{r}_{\ell}, \stackrel{\stackrel{\rightharpoonup}{r}}{ }_{\ell}\right)=$ Earth state relative to Sun $\left(\stackrel{\rightharpoonup}{r}_{s}, \stackrel{\rightharpoonup}{r}_{s}\right)=$ tracking station state relative to Earth.

All vectors are assumed to be referred to an ecliptic coordinate system. The geocentric ecliptic coordinates of the i-th tracking station state are given by

$$
\begin{align*}
& x_{s}=\left(R_{0}+h_{i}^{\prime}\right) \cos \theta_{i}^{\prime} \cos G_{i}^{\prime}  \tag{18}\\
& y_{s}=\left(R_{0}+h_{i}^{\prime}\right)\left[\cos \theta_{i}^{\prime} \cos \varepsilon \sin G_{i}^{\prime}+\sin \theta_{i}^{\prime} \sin \varepsilon\right]  \tag{19}\\
& z_{s}=\left(R_{0}+h_{i}^{\prime}\right)\left[-\cos \theta_{i}^{\prime} \sin \varepsilon \sin G_{i}^{\prime}+\sin \theta_{i}^{\prime} \cos \varepsilon\right]  \tag{20}\\
& \dot{x}_{s}=-\omega_{\ell}\left(R_{0}+h_{i}^{\prime}\right) \cos \theta_{i}^{\prime} \sin G_{i}^{\prime} \tag{21}
\end{align*}
$$

$$
\begin{align*}
& \dot{y}_{s}=\omega_{l}\left(R_{0}+h_{i}^{\prime}\right) \cos \theta_{i}^{\prime} \cos \varepsilon \cos G_{i}^{\prime}  \tag{22}\\
& z_{s}=-\omega_{\ell}\left(R_{0}+h_{i}^{\prime}\right) \cos \theta_{i}^{\prime} \sin \varepsilon \cos G_{i}^{\prime} \tag{23}
\end{align*}
$$

where

$$
\begin{align*}
& h_{i}^{\prime}=h_{i}+c_{108+3 i}  \tag{24}\\
& \theta_{i}^{\prime}=\theta_{i}+c_{109+3 i}  \tag{25}\\
& G_{i}^{-}=\lambda_{i}+\omega_{\ell}\left(t-t_{0}\right)+\operatorname{GHA}\left(t_{0}\right)  \tag{26}\\
& \lambda_{i}^{-}=\lambda_{i}+c_{110+3 i} \tag{27}
\end{align*}
$$

In these equations $R_{0}$ denotes the Earth radius; $h_{i}$, the altitude of the i-th station; $\theta_{i}$, the latitude; $\lambda_{i}$, the longitude; $\omega_{l}$, the Earth's angular velocity; $t$, the current time; $t_{0}$, the initial time; GHA ( $t_{0}$ ), the initial Greenwich hour angle of the vernal equinax: $\varepsilon$, the obliquity of the ecliptic; and $C_{108+3 i}, C_{109+3 i}$, and $C_{110+3 i}$, station location errors. A range bias $C_{63+1}$ is added to the range computed using equation (14), and a range-rate bias $C_{66+i}$ is added to the range-rate computed using equation (15).

OBSM1 F10w Chart




OUTPHI Analysis
The following entry parameters are printed, based on the most recent nominel trajectory:

1) Relative velocity ( $\mathrm{km} / \mathrm{s}$ );
2) Stagnation (atmospheric) pressure (millibars);
3) Wind velocity ( $\mathrm{km} / \mathrm{s}$ ) ;
4) Atmospheric density ( $\mathrm{kg} / \mathrm{km**3}$ ) ;
5) Dynamic pressure (millibars);
6) Angle of attack (degrees);
7) Molecular weight (kg-mol);
8) Coefficient of axial force (unit free);
9) Atmospheric temperature (degrees K );
10) Coeffllcient of normal force (unit free);
11) MACH number (unit free);
12) Coefficient of dynamic moment (unit free);
13) Axial force ( $\mathrm{kg}-\mathrm{km} / \mathrm{s} * * 2$ ) ;
14) Moment (kg-km/s**2);
15) Normal force ( $\mathrm{kg}-\mathrm{km} / \mathrm{s} * * 2$ );
16) Gravitational acceleration ( $\mathrm{km} / \mathrm{s} * * 2$ );
17) Centex of pressure (km);
18) Angle between inertial velocity and relacive velocity (degrees).

If time is other than $T_{0}$, the following matrix partitions are printed:

1) State transition matrix $\Phi$;
2) Solve-for parameter matrix $\Psi$;
3) Dynamic-consider parameter matrix;
4) Dynamic/measurement-consider parameter matrix;
5) Diagonal of the dynamic noise matrix.



|  |  |
| :--- | :--- |
| SINI | SINF OF REFERENCE INGLINATION |
| SINP | SINE OF REFERENCE PLANE LATITUDE |
| SINPHI | SINE OF CALCULATED REFERENCE PLANE LATITUDE |
| SUMEN | INTERMEDIATE SUM |
| SUMER | INTERMEJIATE SUM |
| TEMPOR | TEMPORARY TRANSFORMATION ATRIX |


| USED/COMMN-- ECLINC | ECLONG | OMEG | PHIR |
| :--- | :--- | :--- | :--- |
| SETYCOMMON--- ECLING | ECLONG | PHIR | ROTNO |

## PLANE Analysis

Given the orientation of the entry plane and the $\phi$ reference line relative to 1 of 3 coordinate systems, subroutine PLANE computes the orientation of the entry plane and the $\phi$ reference line relative to the remaining two coordinate systems. The orientation of the entry plane is defined by the inclination $i$ and the longitude of the ascending node $\Omega$, and $t$ location of the $\phi$ reference line in the entry plane is defined $b$. $\phi_{\text {ref }}$ (see subroutine ECLIP).
These quantities are computed relative to the following three coordinate systems: (1) planetocentric ecliptic, (2) planetocentric equatorial, and (3) subsolar orbital plane.

Given $i, \Omega$, and $\phi_{\text {ref }}$ relative to one of the three coordinate systems, the unit vector $\vec{e}_{n}$ normal to the entry plane and the unit vector $\vec{e}_{r}$ aligned with the $\phi$ reference line can be computed from

$$
\begin{aligned}
& \vec{e}_{n}=\left[\begin{array}{cc}
\operatorname{sir} & i \\
\sin \Omega \\
-\sin & i \\
\cos i
\end{array}\right] \\
& \vec{e}_{r}=\left[\begin{array}{l}
\cos \phi_{\text {ref }} \cos \Omega-\sin \phi_{r e f} \cos i \cos \Omega \\
\cos \phi_{\text {ref }} \sin \Omega+\sin \phi_{\text {ref }} \cos i \cos \Omega \\
\sin \phi_{\text {ref }} \sin i
\end{array}\right]
\end{aligned}
$$

The coordinate transformations from the given coordinate system to the remaining two coordinate systems are then computed, and $\vec{e}_{n}$ and $\vec{e}_{r}$ are transformed to these systems.

Denoting the components $\approx=$ the transformed $\vec{e}_{n}$ and $\vec{e}_{r}$ as

$$
\begin{aligned}
& \vec{e}_{n}=\left(e_{n_{x}}, e_{n_{y}}, e_{n_{z}}\right) \\
& \vec{e}_{r}=\left(e_{r_{x}}, e_{r_{y}}, e_{r_{z}}\right),
\end{aligned}
$$

the angles $i^{\prime}, \Omega^{\prime}$, and $\phi_{\text {ref }}^{\prime}$ defining the entry plane and $\phi$ reference line relative to the new coordinate system can be computed as follows:

$$
\begin{aligned}
\Omega^{\prime} & =\tan ^{-1}\left(\frac{e_{n_{x}}}{-e_{n_{y}}}\right) \\
i^{\prime} & =\cos ^{-1}\left(e_{n_{z}}\right) \\
\phi_{\text {ref }}^{\prime} & =\tan ^{-1}\left(\frac{\sin \phi_{\text {ref }}^{\prime}}{\cos \phi_{\text {ref }}^{\prime}}\right)
\end{aligned}
$$

where

$$
\begin{aligned}
& \sin \phi_{\text {ref }}^{\prime}=\frac{\stackrel{e}{r}_{z}}{\sin i^{\prime}} \\
& \cos \phi_{\text {ref }}^{\prime}=\left[\frac{\vec{e}_{z} \times \vec{e}_{n}}{\left|\vec{e}_{z} \times \vec{e}_{n}\right|}\right] \cdot e_{r}
\end{aligned}
$$

and $\vec{e}_{z}$ is a unit vector aligned with the z-axis of the new coordinate system.

Subroutine PLANE also computes the component of the planet inertial angular velocity normal to the entry plane. Letting $\omega_{p}$ denoce the inertial angular velocity of the planet and $\omega_{n}$, the component normal to the entry plane, we can compute $\omega_{n}$ as follows:

$$
\omega_{n}=\omega_{p} \vec{e}_{\omega} \cdot \vec{e}_{n}
$$

where $\vec{e}_{\omega}=(0,0,1)$ is a unit vector aligned with the planet spin axis and $\vec{e}_{n}$ is a unit vector normal to the entry plane. Both unit vectors are referred to the planetocentric equatorial coordinate system.

## SUBROUTINE PLOTS




USED/COMMN-- XMAT
LOADED -- DEVAR XLABEL YLABFL

PLøTS Analysis
Subroutine PLøTS functions as an executive program to plot data of interest. For a complete description of the DD280 plotter, see Appendix B.


| L 3 | hollerith array of a tmosphere parameters |
| :---: | :---: |
| L4 | HOLLERITH ARRAY OF DUTPUT UNITS |
| 15 | HOLLERITH ARRAY OF DUTPUT UNITS |
| OXACT | agtual deviations from most recent nominal TRAJEGTORY AFTER A DUASI EVENT |
| TMPACT | agtual deviation in temperaturf from most recent value |
| UL AREL | HOLLERITH ARRAY OF DUTPUT UNITS |
| XOC | original nominal trajectory values |
| XRECEN | actual deviations from most recent nominal trajectory hefore or after a measurement |
| XxX | DUMMY ARGUMENT |


| USED COMMN--* | ACCLXC <br> ALPHA <br> CO <br> EDNC <br> LTR1 <br> MEZEST <br> NY <br> PPD <br> QEDNRM <br> TEMDBM <br> THTC | ACCL $2 C$ <br> AXC <br> DENS <br> NGYRO <br> LTR2 <br> MHT <br> NW <br> PPDBM <br> QQ <br> TEMEDM <br> tYpe | AEEJEN <br> AZC <br> DENSBM <br> ICNTR <br> MACHND <br> MHTA <br> OMGC <br> PPXD <br> RAJ <br> TEMP <br> VR | AEESTT <br> C <br> DP <br> IPRINT <br> MCNTR <br> NiM <br> OMGCC <br> PRSOAT <br> RHO <br> TEMPA <br> XNAC | AEETMP <br> CACT <br> EDNBMC <br> QSMCHG <br> mCODE <br> NQ <br> PARACH <br> QEDN <br> RHOA <br> THETI <br> XNBQC | ALPH <br> CBA <br> EONBAC <br> LISTQ <br> MEZACT <br> NS <br> PP <br> QEDNBC <br> SJMHT <br> THETRC <br> XNC |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| MRITTEN -mis | AEFgua <br> BESTAT <br> DEVQ <br> GA <br> MACH <br> QEJNBM <br> SDTEMP <br> XHAC | aeeque <br> CA <br> DPP <br> LABEL <br> MEAS <br> OXACT <br> STEHBM <br> XNC | aEesty <br> CACTUL <br> EDNBMC <br> LISTA <br> mezagt <br> RESI <br> TC <br> XOC | AESOLB <br> CORG IN <br> EDNC <br> LI <br> mezest <br> SD <br> UL ABEL <br> XRECEN | AEESTT <br> CQ <br> EPS <br> $L 3$ <br> MEZNOZ <br> SDDENS <br> VR | AL PHO <br> DEV QEDN $L 5$ PROB SDENBM VW |
| SETYCOMMON-E- | aEEDEN <br> DENSBM <br> QaDBM <br> STEMBM | AEESLY <br> OMGCC <br> RESI <br> TEMDBH | AEESTT <br> PPD <br> SODENS <br> TEMEDN | AEETMP PPDBM SDENBM THETRC | AL PHAA PPXD SDMWT2 | DENS <br> QOD <br> SDTEMP |
| loages -os | Lanel <br> ULABEL | 11 | 12 | 13 | 14 | 1.5 |



## PRINT1 Analysis

The problem identification is printed. If the parachute has been deployed, a message is printed. The current time, actual state vectors, and state derivatives are printed in appropriate output units. The following atmospheric and acceleration terms are printed:

| 1) | Relative velocity ( $\mathrm{km} / \mathrm{s}$ ) ; | 11) | Mach number (unit free) ; |
| :---: | :---: | :---: | :---: |
| 2) | Stagnation pressure (millibars); | 12) | Coefficient of moment (unit free); |
| 3) | Wind velocity ( $\mathrm{km} / \mathrm{s}$ ) ; | 13) | Axial force ( $\mathrm{kg}-\mathrm{km} / \mathrm{s}^{2}$ ) ; |
| 4) | Atmospheric density ( $\mathrm{kg} / \mathrm{km}^{3}$ ) | 14) | Moment ( $\mathrm{kg} / \mathrm{km} / \mathrm{s}^{2}$ ) ; |
| 5) | Dynamic pressure (millibars | 15) | Normal force ( $\mathrm{kg}-\mathrm{km} / \mathrm{s}^{2}$ ) ; |
| 6) | Angle of attack (degrees); | 16) | Gravitational acceleration (km/s ${ }^{2}$ ); |
| 7) | Molecular weight (kg-mol) ; | 17) | Center of pressure (km) ; |
| 8) | Coefficient of axial force (unit free); | 18) | Axial acceleration ( $\mathrm{km} / \mathrm{s}^{2}$ ); |
| 9) | Stagnation temperature (degrees K ); | 19) | Moment acceleration (km/s ${ }^{2}$ ) ; |
|  |  | 20) | Normal acceleration ( $\mathrm{km} / \mathrm{s}^{2}$ ); |
| 10) | Coefficient of normal force (unit free); | 21) | Angle between $V$ and $V_{R}$ (degrees). |

Measurement values that do not affect the dynamic equations are also printed.


## PRPRQS Analysis

Subroutine PRPRøS is the executive preprocessor subroutine and controls computation of the coefficients used to smooth quantized VRU and ARU data. The operation of PRPRøS is more easily described by including a description of the operation of SM $\varnothing \emptyset T 2$.

As quantized VRU and ARU data are input into PRPR $\varnothing \mathrm{S}$, the quantized data arrays are shifted up and the new data are inserted in the bottom of each array (in SMめDI2) so the arrays hold exactly the five most recent data points. The coefficients of the smoothing quadratic for each data array are determined as follows:

$$
\left[\begin{array}{l}
c_{1}  \tag{1}\\
c_{2} \\
c_{3}
\end{array}\right]=E \cdot\left[\begin{array}{c}
q_{k-2} \\
\vdots \\
q_{k+2}
\end{array}\right]
$$

where

$$
\begin{equation*}
E=\left(B^{T}\right)^{-1} B \tag{2}
\end{equation*}
$$

$$
\mathbf{B}=\left[\begin{array}{ccc}
1 & -2 \Delta & 4 \Delta^{2}  \tag{3}\\
1 & -\Delta & 2 \Delta^{2} \\
1 & 0 & 0 \\
1 & \Delta & 2 \Delta^{2} \\
1 & 2 \Delta & 4 \Delta^{2}
\end{array}\right]
$$

$$
\begin{equation*}
\Delta=t_{k}-t_{k-1} \tag{4}
\end{equation*}
$$

and the $C_{j}$ are the desired coefficients and $q_{k-2}, \ldots, q_{k+2}$ represent a set of five evenly spaced quantized data points over the time interval $\left[t_{k-2}, t_{k+2}\right.$ ]. The matrix $E$ is computed only twiceat the initial time, and when the dynamic model is changed to the quasistatic dynamic model.

An exception to this scheme occurs when PRPRØS is first called. In this case the coefficients are not determined until three data points are available. The two preceding data points are assumed to be zero by the five-point smoother.

Another exception occurs at the end of the process. After all quantized data have been input, the coefficients for the last two time points must still be computed. This is accomplished by calling SM $\emptyset \mathrm{T} 2$ twice in succession without reading any more quantized data. This is equivalent to assuming that the final two quantized data points are equal to the last quantized data point actually read.



[^2]SURROUTINE READAG


READAC Analysis
READAC perturbs the actual measurement data with noise, scale, and bias factors and passes the perturbed measurements to the reconstructor for processing. If several measurements are taken at the same time, unit 10 is not reinterrogated. PARACH and HITGND are set to .TRUE. whenever actual altitude reaches the appropriate values. Subroutine RNUM is called to calculate the random noise MEZNøZ.


## RKUTDG Analysis

Subroutine RKUTDG is the integration subroutine employed in the LTR data generator program. The algorithm employed is a modified Runge-Kutta method, although the classical fourth-order RungeKutta is used to start the integration process.

The system of equations to be integrated has the form

$$
\dot{x}=f(x, t)
$$

where $x$ is the n-dimensional state vector and $t$ is the time. The classical fourth-order Runge-Kutta algorithm is summarized as:

$$
\begin{aligned}
& k_{1}=h f\left(x_{k}, t_{k}\right) \\
& k_{2}=h f\left(x_{k}+\frac{1}{2} k_{1}, t_{k}+\frac{h}{2}\right) \\
& k_{3}=h f\left(x_{k}+\frac{1}{2} h_{2}, t_{k}+\frac{h}{2}\right) \\
& k_{4}=h f\left(x_{k}+k_{3}, t_{k}+h\right) \\
& x_{k}^{*}=x_{k} \\
& x_{k+1}=x_{k}+\frac{1}{6}\left(k_{1}+2 k_{2}+2 k_{3}+k_{4}\right)
\end{aligned}
$$

where $h$ is the step size, $X_{k}$ is the state at the beginning of the interval, and $x_{k+1}$ is the state at the end of the interval. The state $x_{k}^{*}$ is required by the modified Runge-Kutta algorithm, which is summarized as:

$$
\begin{aligned}
& \ell_{1}=k_{1} \\
& k_{1}=h_{1} f\left(x_{k+1}, t_{k+1}\right) \\
& k_{2}=3.6 k_{1}-4.2\left(x_{k+1}-x_{k}^{*}\right)+1.6 \ell_{1} \\
& k_{3}=h f\left(x_{k+1}+\frac{3}{4} k_{1}+\frac{3}{4} k_{2}, t_{k+1}+\frac{h}{2}\right)
\end{aligned}
$$

$$
\begin{aligned}
& k_{4}=h f\left(x_{k+1}-k_{2}+2 k_{3}, t_{k+1}+h\right) \\
& x_{k+1}^{*}=x_{k+1} \\
& x_{k+2}=x_{k+1}+\frac{1}{6}\left(k_{1}+4 k_{3}+k_{4}\right) .
\end{aligned}
$$

The advantage of using the modified Runge-Kutta algorithm lies in the fact that the state derivatives need be evaluated only three times, and not four times as is required in the classical RungeKutta algorithm.

Another function of RKUTDG is to determine , hen the vehicle hits the planet surface. The first component of the state $x$, which is the vehicle altitude, is compared with the terrain height. If the two are equal, RKUTDG sets the logical variable HITGND to true, sets TEND to the current time, and returns.

[^3]```
SUBROUTINE RKUTL3
PURPOSE : INTEGRATOR FOR MODE B RECONSTRUCTOR
ENTRY PARAMETERS
    TEND FINAL TIME OF INTEGRATION
    tstart starting tIme of Integration
    UPDAIT LOGICAL VARIABLE TO CONTROL UPDATING OF STATE
        vECTOR WHEN STATE DERIVATIVES ARE COMPUTED
    X STATE VECTOR BEING INTEFRATED
    XADD CHANGE IN STATE VECTOR OVER THE INTERVAL
SURROUTINES CALLEJ: DERIVZ
COMWONS : TRA.4 LOGGOM INTCOM
logal symbols
    H INTEGRATION STEPSIZE
    KK INTERMEDIATE WORKING ARRAY
    Ki INTERMEOIATE WORKINE ARRAY
    k2 INTERMEDIATE WORKING ARRAY
    k3 INTERMEDIATE HORKING ARRAY
    K4 INTERMEDIATE WORKING ARRAY
    l INDEX ON NUMBER OF INTEGRATION STEPS REQUIRED TO
    INTEGRATE THROUGH THE TIME INTERVAL
    L1 INTERMEDIAYE HORKING ARRAY
    M NUMPER GF INTEGRATION STEPS REQUIRED TO INTEGRATE
    OVEN. THE ENTIRE INTERVAL
    ST TOTAL INTEGRATION INTERYAL
    T GURRENT TIME OF INTEGRATION
    W INTERMEOIATE HORKING ARRAY
    hi INTERMEDIATE varIAble
    XC INTERMEJIATE WORKING ARRAY
USEDSCOMMN--- DT
    DXN NE
```


## RKUTL3 Analysis

Subroutine RKUTL 3 is the integration subroutine employed in the mode $B$ reconstruction program, and employs the same Runge-Kutta algorithm that is used in subroutine RKUT3. The derivatives required by RKUTL3 are computed in subroutine DERIV3 (see subroutine RKUT3 for more details).

```
SUBROUTINE RKUT3
PURPOSE : INTEGRATOR FOR MODE A RECONSTRUCTOR
ENTRY PARAMETERS
    TEND
    TSTART STARTING TIME OF INTEGRATION
    UPDAIT LOGICAL rO CONTROL UPDATING OF STATE VECTOR
                                    WHEN STATE DERIVATIVES ARE COMPUTED
    X STATE UEGTOR (OF SIZE NE) BEING INTEGRATED
    kadd change in state vector over the interval
Suproutines callej: jerive
COMMONS : TRAJ LOGCOM INTGOM
LOCAL SYMBOLS
    H
    INTEGRATION STEPSIZE
    KK INTERMEDIATE HORKING ARRAY
    Ki INTERMEOIATE WORKING ARRAY
    K2 INTERMEDIATE HORKING ARZAY
    k3 INTERMEJIATE MORKING ARRAY
    K4 INTERMEDIATE HORKING ARRAY
    l INDEX ON NUMPER OF STEPS REQUIRED TO INTEGRATE
    THROUGH THE TIME INTERVAL
    L1 INTERMEDIATE WORKINS ARZAY
    M NUMRER OF INTEGRATION STEPS REQUIRED TO INTEGRATE
        OVER THE ENTIRE INTERVAL
    ST TOTAL INTEGRATION INTERVAL
    T CURRENT TIME OF INTEGRATION
    H INTERMEDTATE WORKING ARRAY
    WI INTERMEDIATE VARIABLE
    XC TNTERMEJIATE WORKING ARRAY
USED/COMMN--- DT
DXN
NE
```


## RKUT3 Analysis

Subroutine RKUT3 is the integration subroutine employed in the mode A reconstruction program. The Runge-Kutta algorithm is the same as that employed in subroutine RKUTDG, except that the classical Runge-Kutta algorithm is used initially whenever RKUT3 is called. This procedure is required since RKUT3 is used to integrate more than one trajectory (original nominal, most recent nominal, and perturbed trajectories) and the local variables that contain information from the last integration may not correspond to the desired trajeetory. Because the total interval TEND-TSTART may not be an exact multiple of the step size DT, DT is always adjusted so an exact multiple is attained.

Subroutine RKUT3 also computes the variable XADD, which is used in the computation of the state transition matrix and is defined as

$$
X A D D=x_{k+1}-x_{k}
$$

where $x_{k}$ and $x_{k+1}$ are the states at the beginning and end, respectively, of the integration interval.

Subroutine RKUT3 does not have the hit-ground test appearing in RKUTDG since impact occurs when the actual trajectory, not the nominal trajectory, impacts the planet surface.

```
SUBROUTINE RNUM
PURPOSE : CALGULATES RANDOMLY SAMPLED MEASUREMENT NOISE FOR
    A GIVEN MEASUREMENT TYPE
ENTRY PARAMETERS
    ICODE MEASUREMENT TYPE
    NCOMP NUMBER OF COMPONENTS TO BE STORED
SUBROUTINES GALLED: NORMNZ
COMHONS : TRAS OBSERV
LOCAL SYMBOLS
    J
        INDEX
        NOISE NORMALLY DISTRIBUTED NUMBER OF MEAN ZERO AND
        STANDARD DEVIATION UNITY
USEJ/COMMN--* SJ
SETYCOMMON--- HEZNOZ
```

```
SURROUTINE RSTART
PURPISE : PROUIDE RESTART CAPARILITY RY PUNCHING MATRICES OF INTEREST
SUBROUTINES CALLED: COPY
COMMONS: COVARP TRAJ INTGOM
```

```
LOCAL SYMBOLS
    I
    INDEX
    WQQ NQ SQUARES
    nou na times nu
    NQV NQ TIMES NV
    NQN NQ TIMES NH
    NSS NS SQUARED
    NXQ NS TIMES NQ
    NXU NS TIMES NU
    NXV NS TIMES NV
    NXH NS TIMES NH
    XNAC GOPY OF XNA, ACTUAL STATE VEGTOR
    XNG COPY OF XN, MOST RECENT NOMINAL STATE VECTOR
    XOC COPY OF XO, ORIGINAL NOMINAL STATE VECTOR
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline \multirow[t]{4}{*}{USED/COMMN---} & cais & cav & CQW & Cxa & Cxu & CxV \\
\hline & CXH & Ju & JV & 31 & NQ & NS \\
\hline & Nu & NV & NW & P & Q & QEDN \\
\hline & \multicolumn{6}{|l|}{Rat} \\
\hline \multirow[t]{3}{*}{HRITTEN} & \multirow[t]{2}{*}{EDV} & TC & xnac & XNC & xoc & P \\
\hline & & CXO & cxu & cxv & CXW & CQU \\
\hline & cav & COW & DU & DV & DH & QEDN \\
\hline
\end{tabular}
```



| USEB/COMMN--- | $\begin{aligned} & \mathbf{0 T} \\ & \mathbf{T O} \end{aligned}$ | $\begin{aligned} & \text { MCOJE } \\ & T R \end{aligned}$ | $\begin{aligned} & \text { TC } \\ & \text { QST } \end{aligned}$ | TF | THN |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| READ | coje | timend | mbode | START | TIMDIF | TMN |
| WRITTEN --- | $\begin{aligned} & \text { COJE } \\ & \text { TMN } \end{aligned}$ | TIMENJ total | mcode | nevent | START | TIMDIF |
| SET/COMMON--- | LIENTR | mgode | NPRED | PREONO | TMN |  |


#### Abstract

SCHED Analysis SCHED reads and sequences measurements and other events for use by the reconstructor. START, TIMEND, TIMDIF, and CØDE(N) are read and written for identification purposes. START, TIMEND, and TIMDIF are converted to integer numbers of integration steps DT and stored. The process is repeated until a START value of 100000 . or some other hard-wired value is read. All values read are then separated into groups according to $\mathrm{C} \emptyset \mathrm{DE}(\mathrm{N})$ and written with identifiers. Groups of 250 measurements and events are then ordered on time, with TMN and MCøDE used as storage, TMN and MCøDE are written on unit 20 for processing by subroutine NEXIIM. The process continues until the final measurement time exceed TF. Whenever a start time exceeds an end time for an event type (start times are incremented), the start time is set to $1 . E+10$. The last group of times and codes is then written, urit 20 is rewound, and the total number of events and total num' sr of each type of event are printed with identifiers.





## SENSOR Analysis

Subroutine SENSOR computes the quantized output of an accelerometer or gyro sensor. The quantized output of a sensor is found by first modifying the integral of the actual sensor input by appropriate scale factor, $C_{s}$, and bias, $C_{b}$, terms. This is then divided by the quantization step size, $\Delta$. The greatest integer contained in this number is the sensor output count. The quantized output is then the output count times the quantization step size.

Let the operation of finding the largest integer contained in a number be designated by enclosing the number in brackets, \{ \}. The quantized accelerometer outputs are given by

$$
\begin{aligned}
& v_{x q}=\left\{\frac{c_{s x} v_{x}+c_{b x}}{\Delta_{x}}\right\} \Delta_{x} \\
& v_{z q}=\left\{\frac{c_{s z} v_{z}+c_{b z}}{\Delta_{z}}\right\}_{z}
\end{aligned}
$$

where $V_{X}$ and $V_{z}$ are the integrals of the actual accelerations experienced by the $x$ and $z$ accelerometers, respectively.

The quantized output of the gyro is found similarly except that the bias term is also integrated,

$$
A_{\theta q}=\left\{\frac{c_{s \theta} \dot{A}_{\theta}+c_{b \theta} t}{\Delta_{\theta}}\right\} \Delta_{\theta}
$$

where $A_{\theta}$ is the integral of the actual angular rate experience by the gyro and $t$ is the time since the instrument was last initialized.

## SUAROUTINE SETPLT



| USED/CDMMN--- | NQS |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| REAJ | INJEP SUMTE | $L$ INEAR | LOG | NV AR | PL OTL | PLotvar |
| SET/COMMON--- | INDEP | $L$ INEAR | LOG | NVAR | PLOTL | Sumte |

```
SUBROUTINE SETICN
PURPOSE : INITIALIZES PRINT INCREMENT COUNTERS AT AN ITERATION
    COUNTER SET EVFNT
COMMONS : INTCOM
LOCAL SYMBOLS : NONE
USFD/COMMN--- LICNTR NICNTR
SET/COMMON--- ICNTR IPRINT NICNTR
```


## SETICN Analysis

IPRINT is reset to 0 for later incrementing and usage. NICNTR is the counter for the $N$-th iteration counter set event, incremented by 1. ICNTR is the $N$-th value of LICNTR, a vector of print increments that allows the user to change print increments for denser print at critical trajectory intervals. SETICN is called whenever TYPE $=13$ in LTRC $\neq$. TYPE is set in subroutine NEXTIM. IPRINT, the counter for groups of measurements, is updated in subroutine MEASUR.


| USEO/COMMN--* | APS | AROTRL | C | OATEJ | DEL T | ECLINC |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | EGL ONG | GENDAT | LISTU | LISTW | LTR1 | LTR2 |
|  | MODE | NACSEL | NE | NGYRO | NQ | NS |
|  | NU | NV | NW | PHIR | RAD | RESTRT |
|  | RR | SLAT | SLON | TC | THET I | XN |
|  | XNAS | $\times 0$ |  |  |  |  |
| REAO | AA | ACCDT | ACCL XE | ACCLZC | AGAM | aquant |
|  | AR | ATMOSS | BKTBL | BTBL | C | CACT |
|  | CDEL | COTBL | CQU | CQV | CQW | CXO |
|  | CXU | CXV | CXW | DELT | DIA | DT |
|  | DU | DV | OW | DYN | ECLINC | ECL ONG |
|  | EJN | ERAN | ETA | GAMTBL | GO | GQUANT |
|  | GYRODT | ICNTR | ICOOR | IDAY | IHR | III |
|  | IMIN | IMO | IYR | LISTQ | LISTS | LISTU |
|  | LISTV | LISTW | LTR1 | LTR2 | MASS | MCNTR |
|  | MOJE | MSATS | MU | MHTA | MWTM | NACCEL |
|  | Nathos | NE | NGYRO | NMEAS | NMPTS | NQ |
|  | NS | NTP | NTPTS | NU | NV | NW |
|  | OMEG | P | PHIR | PLOT | PROR | PRSDAT |
|  | 0 | QEJN | REJRRI | REJRR2 | RESTRT | RHOA |
|  | RI | RM | RR | SA | SALT | SD |
|  | SOMWT | SDP | SECSI | SLAT | SLION |  |
|  | TAFETM | TAPSAV | TC | TEMPA | TEND | TERHT |
|  | TF | THETI | TIME | WSTBL | $X 3$ | XG |
|  | XH | XN | XNA | $\times 0$ | YG | YM |
|  | ZG | ZMM |  |  |  |  |
| WRITTEN | C | DIA | I | J | LISTQ | LISTU |
|  | LLISTV | LISTW | L1 | 12 | MASS | MnAme |
|  | MU | NQ | NU |  |  |  |
|  | NV | NW | OMEG | PROB | RAD | RI |
|  | R.1 | RR | SA | TC | TF | ULAB |
|  | ULABEL | XG | XM | XN | 26 | 7.MM |
| SET/COMHON--- | ACC | ALPHA | AROTBL | AXC | AZC | BF |
|  | CDELT1 | CDEL 72 | CDELT3 | CQU | CQUC | Cav |
|  | cave | COW | CQWC | CXa | CXQC | CXU |
|  | CXUC | CXV | CXVC | CXH | CXWC | EU |
|  | DV | OH | DYN | ECLINC | ECL ONG | EDN |
|  | GM | HM | ICNTR | IEND | IPRINT | IX |
|  | IEND | IPRINT | IX | LASTIM |  | LTR2 |
|  | K5 | LISTQ | LISTS | LISTSM | LISTU | LISTV |
|  | LISTH | LM | LTR2 | MCNTR | MM | NACCEL |
|  | NE | NGYRO | NICNTR | MMEAS | NQS | NQUASI |
|  | NS | OBL IC | OMGC | P | PHI | PHIR |
|  | op | PPC | PSI | Q | QEJN | QEJNBC |
|  | 20 | R | RESTRT | ScQu | scav | SCOW |
|  | SCXQ | SCXU | SCXV | SEXH | So | SDELT! |
|  | SOELT2 | SDELT3 | SDU | SOV | SDW | SF |
|  | STZEP | SLAT | SLION | SP | SQ | SQDU |
|  | SQDV | SQDW | SURDL 1 | SU MF AR | THETI | THTC |
|  | THU | THW | TYPE | TZERO | W | WORK |
|  | H1 | H2 | W3 | W4 | W5 | XN |
|  | XHAS | $\times 0$ |  |  |  |  |
| LOABED --- | ICOMM | L1 | $\bigcirc .2$ | miname | UL AB | UL ABEL |

## SETUP Analysis

Subroutine SETUP reads and initializes the data necessary for the reconstruction program. Subroutine BEGIN is called to reset data changed by the data generator. Print counters, logic variables, and dynamic equation parameters are initialized. Scale factors are set to une and standard deviations and bias factors are set to zero. Subroutine $N \emptyset R M N Z$ is called to seed the random noise generator. If logic variable GENDAT is . FALSE., the data generator was not run (i.e., the actual trajectory resides on previously generated data tapes), and the ARDTBL array must be converted. A series of data cards containing Hollerith information is read and printed. If the first character was a $C$, the card is presumed to be a comment card. Successive cards are read until the array PR $\mathrm{D}_{\mathrm{B}}$ contains the problem identification.

The matrices associated with the Kalman filter equations are set to zero, and the namelist section ERAN is read and written. The basis integration step size DT is set to twice the step used in the computation of the actual trajectory in the data generator, and the vehicle physical properties are chosen according to IPHAS. The number of state parameters NS is set according to mode $A$ or mode B, the LISTS array is initialized, and subroutine SETPLT is called to read the plot package variables. SETUP then checks the deletion of accelerometer or gyro data for the mode A dynamic equations. If such data are deleted, C(54) or C(140) must appear as a consider parameter if either appears at all.

Subroutine TIME is called to calculate the Julian date at TZER $\varnothing$, the earth's obliquity is computed, and GHA is called to find the Greenwich hour angle at TZERØ. Subroutine PLANE is called to calculate the orientation of the entry plane to the three reference coordinate systems. If GENDAT is . FALSE., the DSN station locations are converted to radians.

The initial trajectory conditions, vehicle characteristics, planetary values, and problem identification are printed. The lists of augmentation parameters and associated covariance matrices are printed. The nominal values of the $C$ array are printed, the trajectory state is stored in the XNS and X $\varnothing S$ arrays, and the actual atmosphere variables at TZER $\emptyset$ are read from unit 10. If the trajectory is not being restarted, the most recent nominal trajectory is also the original nominal trajectory and $X N$ is stored in Xø. Covariance and correlation matrices are stored in saving matrices, and initial accelerometer values are read from unit 16. Subroutine PRINT is called to print the trajectory and atmosphere values at TZER $\varnothing$, and subroutine SCHED is called to read and sequence measurement and event information. Control then returns to LTRCøN.

## SUBROUTZNE SETUPI

PURPOSE : INITIALI'E ANJ REAS JATA FOR THE JATA GENERATOR


| USED/COMMN--- | AP) | AROTPL |  | DATEJ | JEL T | ECL ONG |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| E.CLINC | GHATO | ICOOR | IPHAS | MASS | MOL | MPT |
| NE | NMPTS | NTPTS | PHIR | RAD | RESTRT | SLAT |
| SLUN | TMD | TPT | WDTBL | XG | XM | XN |
| 7 G | 24M |  |  |  |  |  |
| READ --. | AR | C | DELT | DT | ERAN | ETA |
|  | 60 | ICNTR | LTR1 | LTR2 | NATMOS | PROB |
|  | RESTRT | TC | TDIF | TEND | TERHT |  |
|  | TSTEP | XG | XM | XN | XSTEP | 2G |
|  | 244 | ZSTFP |  |  |  |  |
| WRITTEN --- | ACSLX | ACCLZ | APP | C | OIA | MASS |
|  | MEASS | MOL | MPT | MU | MWT | OMEG |
|  | PR? 5 | PROA | RAT | RHO | RI |  |
|  | SA | TC | TEMP | TF | TMP | TPT |
|  | XG | XM | XN | 2G | ZMM |  |
| SET/COMMON--- | AP? | AROTBL | C | CJEL T1 | c Jeltz | CDELT3 |
| DELT | DTA | IPHAS | MASS | MOL | MPT |  |
| RI | SA | SDELT1 | SDEL T2 | SDEL T3 | TMP | TPT |
| TZERO | WDTBL | XG | XM | XN | 76 | 7 MM |

SETUP1 Analysis
SETUP1 is called from DATGEN to initialize and read data via NAMELIST for the data generator. Elements of the ARฤTBL array are converted to radians and the variable GENDAT is set . TRUE. so that the reconstructor (see subroutine SETUP) will not convert AR $\emptyset$ TBL elements. Problem identification and namelist ERAN are read and subroutine TIME is called to calculate the Julian date, epoch 1900, from the input calendar date. The obliquity of the ecliptic is calculated and trajectory time TC is stored as TZER $\varnothing$. Subroutine GHA is called to compute the Greenwich hour angle at TZER $\varnothing$. Since one set has been read into the first elements of PHIR, ECLØNG, and ECLINC regardless of the value of ICøøR, subroutine PLANE computes the orientation of the remaining reference planes. DSN tracking station latitudes and longitudes are converted to radians, and the desired target planet atmosphere is stored according to NATM $\varnothing$ S. ARU-VRU misalignment errors are added to nominal location values. If RESTRT is false, state parameter values are perturbed with nominal errors read from input. Input data are converted to internal units and the atmosphere and vehicle characteristics are written, together with the perturbed state parameters and problem identification. Subroutines ATMSET, DERIV1, SENS $\emptyset$ R, and $\emptyset$ BSM1 are called to initialize the trajectory integration at TZER $\emptyset$ and PRINT1 is called to print the data generator output at TZER $\varnothing$. Control then returns to DATGEN for the data generator execution.


SMøФT2 Analysis
Subroutine SM $\varnothing \emptyset_{\mathrm{T}}$ \% computes the smoothing quadratic coefficients used to smooth quantized VRU and ARU serasor data. See subroutine PRPR $\emptyset$ S for more details.

## SMDDT2 Flow Chart



```
SUBROUTINE STM
PURPOSE : CONTROLS THE CALGULATION OF THE AUGMENTED STATE TRANSITION MATRIX PARTITIONS
ENTRY PARAMETERS
        ZADD
        NOMINAL CHANGE IN THE STATE VECTOR OVER THE
                INTERVAL OF INTEREST
SURROUTINES CALLED:
JACOBN
COMHONS : TRAJ
    COVARP INTCOM
LOCAL SYMBOLS
        I
    INDEX
    NTM EXIERNAL VARIABLE FOR INTEGRATION OF STATE EQUATION
    N1 NUMBER OF BASIC STATE VARIABLES SQUARED
    N2 NUMBER OF EASIC STATE VARIABLES PLUS 1
USED/COMMN--- NS
PHI
SET/COMMON-®- PHI
```


## STM Analysis

STM is an executive routine that controls the calculation of the partitions of the augmented state transition matrix.

The augmented state vector, $\bar{X}$, may be partitioned into the basic state vector, $\bar{x}$; solve-for parameter, $\bar{q} ;$ dynamic consider parameter, $\bar{u}$; measurement consider parameters, $\overline{\mathbf{v}}$; and dynamic/measurement consider parameters, $\overline{\mathrm{w}}$. When the state transition matrix is partitioned to correspond with the augmented state vector partitions, the state equation may be written
$\bar{X}\left(t_{F}\right)=\Phi_{t_{F}, t_{o}} \bar{X}\left(t_{0}\right)=\left[\begin{array}{ccccc}\phi_{t_{F},}, t_{0} & \psi_{t_{F}, t_{0}} & \theta_{u_{t_{F}, t}} & 0 & \theta_{w_{t_{F}}, t} \\ 0 & I & 0 & 0 & 0 \\ 0 & 0 & I & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I\end{array}\right]\left[\begin{array}{l}\bar{x}\left(t_{0}\right) \\ \bar{q}\left(t_{0}\right) \\ \bar{u}\left(t_{0}\right) \\ \bar{v}\left(t_{0}\right) \\ \bar{w}\left(t_{0}\right)\end{array}\right]$
The partitions $\phi, \psi, \theta_{u}$, and $\theta_{w}$ are computed by numerical differencing, i.e., the value of the $j$-th element of $\bar{X}\left(t_{0}\right)$ is perturbed by an amount $\delta j$, and the resulting change in $\bar{X}\left(t_{F}^{0}\right)$ is found by integrating the equations of motion. The $j-t h$ column of $\Phi$ is then given by $\Delta \overline{\mathrm{X}}\left(\mathrm{t}_{\mathrm{F}}\right) / \delta \mathrm{j}$.

The actual computation of the partitions of $\Phi$ are obtained by calling JACOBN once for each partition. The elements of $\bar{X}\left(t_{0}\right)$ to be perturbed are indicated by indices stored in the arrays LISTS, LISTQ, LISTU, and LISTW. The magnitude of the perturbation $\delta j$ is determined from the variance of the parameter, $\sigma_{x_{j}}^{2}$. These variances are stored in the covariance matrices $P, Q, D_{u}$ and $D_{w}$.

## SURROUTINE SUQ

PUPPOSE : TO SUBTRACT ONE RECTANGULAR MATRIX FROM ANOTHER AND STORE INTO A THIRD PECTANGULAR MATRIX

## ENTRY PARAMETERS

NCX
NUMBER DF COLUMNS OF $K$, $Y$, AND $Z$ MATRICES
NRX NUMBER OF POWS OF $X, Y$, AND $z$ MATRICES
$X$ MATRIX TO SURTRACT FROM $Y$
$Y$ MATRIX TO BE SUBTRACTED FROM
$Z$ OUTPUT MATRIX ( $Y$ - $X$ )

LOCAL SYMBOLS

| $I$ | INDEX |
| :--- | :--- |
| $N$ | TOTAL NUMRER OF ELEMEATS OF $X, Y$, ANJ $Z$ MATRICES |

```
SUBROUTINE SUBSOL
PURPOSE: COMPUTES THE CO-ORDINATE TRANSFJRMATION FROM PLANETOCENTRIC
    ECLIOTIC PLANE TO SUB-SOLAR PLANET-ORBITAL PLANE
SURROUTINES CALLED: EPHEM
ENTRY PARAMETERS:
    NP TARGET PLANET COJE
    D JULIAN DATE, EPOCH JANUARY O, 1900
    EQSS CO-ORDINATE TRANSOFRMATION FROM PLANETOCENTRIC
        ECLIPTIC PLANE TO SUB-SOLAR PLANET-ORBITAL PLANE
COMMONSR . . STATE
LOCAL SYMBOLS:
        CROSS PRODUCT OF PLANET POSITION AND VELOCITY vECTORS,
        OR UNIT VECTOR ALLIGNED WITH Z-AXIS OF SUR-SOLAR
        pLANET-DRBITAL PLANE
    C1 MAGNITUDE OF EZS
    C2 magmitude of planet position vegtor
    EXS UNIT VECTOR ALLIGNED WITH X-AXIS OF SUB-SOLAR
    PL ANET-ORBITAL PLANE
    EYS UNIT VECTOR ALLIGNED WITH Y-AXIS OF SUR-SOLAR
        PLANET-ORBITAL PLANE
    XP PLANET POSITION AND VELOCITY VECTORS
USED/COMMON: CARCOR
```

SUBSØL Analysis
Subroutine SUBS $\emptyset$ L computes the transformation from planetocentric ecliptic coordinates to subsolar planet orbital plane coordinates for an arbitrary planet. The subsolar planet orbital flane coordinate system is defined as the planetocentric system whose x -axis points directly at the sun, whose $z$-axis is normal to the planet's orbital plane, and whose y-axis is normal to the xz-plane and lies in the planet's orbital plane. In the figure below $\vec{r}$ and $\vec{v}$ denote the position and velocity vectors; respectively, of the planet relative to the sun. Unit vectors $\vec{e}_{x}, \vec{e}_{y}$, and $\vec{e}_{z}$ are aligned with the axes of the subsolar planet orbital plane system.


These unit vectors are defined as

$$
\begin{aligned}
& \vec{e}_{x_{s}}=-\frac{\vec{r}}{r} \\
& \vec{e}_{y_{s}}=\vec{e}_{z_{s}} \times \vec{e}_{x_{s}} \\
& \vec{e}_{z_{s}}=\frac{\vec{r} \times \vec{v}}{|\vec{r} \times \vec{v}|}
\end{aligned}
$$

If these unit vectors are referred to the ecliptic coordinate system, the coordinate transformation A from planetocentric ecliptic to subsolar planet orbital plane coordinates is given by


## Thus

$$
\vec{x}_{\text {subsolar }}=\mathrm{A} \overrightarrow{\mathrm{x}}_{\text {ecliptic }}
$$

SUBSØL Flow Chart



## SUMMRY Analysis

If SUMTB(I) is . TRUE., the I-th summary table is written (containing problems identification, title, and label information) and plot values are stored on unit 10. Subroutine PL $\emptyset T S$ is called to plot using the system plot package.

## SUBROUTIAE SYMTR?

```
PURPCSE : TO DFIERMINE THE SYMMETRIC COMPJNENTS OF A SQUARE MATRIX by taking one half the sum of the matrix and its transpose
```

ENTRY PARAMETERS
N TIMENSTON DF $X$
$x$ SQUARE matrix which is replaced by its symmetric COMPONENT

| LOCAL SYMROLS |  |
| :---: | :---: |
| I | INDEX |
| $J$ | INDEX |

```
FUNCTION TAB
PURPOSE : TO PERFORM A LINEARLY INTERPOLATED TABLE LOOKUP
ENTRY PARAMETERS
    TABLE SINGLY OIMENSIONED ARRAY HHOSE FIRST ENTRY INDIGATES
        THE NUMQER N OF PREAK POINTS. THE N BREAK POINTS
        of the independent variable are next ano the remaining
                                N values are the break points of the dependent variable
    x
        value of the independent variarle
lOGAL SYMBOLS
    K INDEX
    l index
```

TAB Analysis
The index $K$ is set to TABLE(1)+1, and $X$ is tested against TABLE(L), $\mathrm{L}=3$, K. If $\mathrm{X} \leq \operatorname{TABLE}(\mathrm{L})$,

$$
M=K+L-1
$$

and
$\operatorname{TAB}=\frac{(\mathrm{X}-\operatorname{TABLE}(\mathrm{L}-1))}{(\operatorname{TABLE}(\mathrm{L})-\operatorname{TABLE}(\mathrm{L}-1))} *(\operatorname{TABLE}(\mathrm{M})-\operatorname{TABLE}(\mathrm{M}-1)+\operatorname{TABLE}(\mathrm{M}-1))$
where TABLE is a singly dimensioned array whose first entry indicates the number $N$ of break points. The $N$ break points of the independent variable are next, and the remaining N values are the break points of the dependent variable.

```
SUBROUTINE TIME
PURPOS'E: TRANSFORM CALENDAR DATE TO/FROM JULIAN DATE, EPOCH 1900
entry Pirameters
```



IYR GALENDAR YEAR

## MO CALENJAR MONTH

rday calendar day
IHR HOUR OF THE DAY
MIN MINUTE OF THE HOUR
SEC FRACTIONAL SEGONDS
ICODE OPERATIONAL MODE
=1, JULIAN JATE IS INPUT, CALENDAR DATE•IS OUTPUT $=0$, CALENDAR DATE IS INPUT, JULIAN DATE IS OUTPUT
LOCAL SYMBOLS
IA
IB
IP NUMBER OF MONTH (BASED ON MARCH AS NUMBER ZERO)
IQ NUMBER OF YEARS
IR NUMBER OF CEntURIES DIVIDED by 4
IS NUMRER OF YEARS SINGE LAST 400 YEAR SECTION GEGAN
It NUMBER OF LEAP YEARS IN PRESENT CENTURY
IU NUMBER OF YEARS SINGE LASt LEAP YEAR
IV NUMPER OF JAYS IN LAST PEAR
IX INTERMEDIATE VARIABLE
J intermediate variable
JD NUMBER OF DAYS IN JULIAN DATE
$P$ JULIAN DATE
R FRACTIONAL PORTION OF JAY IN JULTAN JATE

```
SUBROUTINE TIMEX
purpose : to p{int time elapsed since last call
ENTRY PARAMETERS
    Name a hollerith name of a subroutine
SUBROUTINES CALLED: CPHMS XRCL
LOCAL SYMBOLS
        N LOGIC VARIARLE SET TO +1 OR -1
        T ElAPSED TIME IN SECONDS (T2-T1)
        T1 PreviOúS tImE IN SECONDS
        tz GURRENT TIME IN SECONDS
HRITTEN --- NAME T
logdes -- N
```

TIMEX Analysis
XFCL and CPWMS are Martin Marietta/CDC system routines that, together, return real clock time in seconds. If $\mathbf{N}<0$, elapsed time and a Hollerith subroutine name are printed. T 1 is set to T 2 and N to -N prior to return.

```
SURROUTINE TMULT
PURPOSE : TO MULTIPLY THE TRANSPOSE OF A REGTANGULAR MATRIX BY
        ANOTHER RECTANGULAR MATRIX AND STORE IN A THIRD MATRIX
EMTRY PARAMETERS
    NCX NUNBER OF COLUNNS OF }X\mathrm{ AND NUMBER OF ROWS OF }
    NGY NUMBER OF COLUMNS OF \ AND }2\mathrm{ MATRICES
    NRX NUMBER OF ROWS OF }x\mathrm{ AND }Y\mathrm{ MATRICES
    X INPUT RECTANGULAR MATRIX (TO BE TRANSPOSEDS
    Y INPUT RECTANGULAR MATRIX
    Z OUTPUT MATRIX ( X TRANSPJSED TIMES Y %
LOCAL SYMBOLS
    I INDEX
    J INDEX
    K INDEX
    SUM PRODUCT OF I-TH COLUMN OF }X\mathrm{ AND J-TH COLUMN OF Y
```

```
SUBROUTINE TMULTT
PURPOSE : TO MULTIPLY THE TRANS゙SOSES OF PWO RECTANGULAR MATRICES
    ANJ STORE INTO A THIRO RECTANGULAR MATRIX
```



```
LOCAL SYMBOLS
    I INDEX
    J INDEX
    K INDEX
    SUM DOT PROUCT OF I-TH COLUMN OF }X\mathrm{ ANO J-TH ROW OF Y
```



## 1. INTRODUCTION

This document has been written to provide the user with software information that he might require in using the CDC 280 for producing microfilm or hardcopy.

These routincs are a part of the CDC 6000 MACE Operating System and utilize the CDC 280 as an on-line peripheral device.
2. THE CDC 6000 OPERATION SYSTEM INTERFACE
2.1 Interfacing the 280 Display and Recorder with the MACE Operation System requires the addition of two special filc names, Film Plot (FILMPL) and Film Print (FILMPR). Film Plot filcs (FILMPL) and other I/O files, such as OUTPUT, that are used by the program must be declared on the program name card for the particular job. The Film Print file (FILMPR) can be declared in the same manner, or it can be established via a control card. Data sent to either of these files during job execution is written on the system disk in the same manner as print data.

Each file being filmed can be controlled by Output control point commands. The commands are: END, REPEAT, and SUPPRESS.

280 CONSOLE OPERATION
When the RUN mode is selected on the console, output data directed to the 280 is transferred to both the console CRT and the recorder CRT. (The RUN mode should not be confused with the RUN control card. By use of the 280 console keyboard, the operator has the options to stop the 280 and monitor each page of output. The operator accomplishes this by selecting the STEP mode on the console and stepping through successive pages of output display until he desires to return to the normal RUN mode. The operator can change modes by pressing keys in the following manner:

| $\frac{\text { TYPE KEY }}{\text { R }} \quad$ | $\frac{\text { ACTION }}{\text { Return to RUN mode }}$ |
| :---: | :--- |
| $S$ | Change to STEP mode |
| $G$ | Go to next page if in STEP mode |

Jobs can automatically be put in the STEP mode by keying in (1. ON SW1.) at the 6000 console. This STEP mode of operation can then be removed by keying in (1. OFF SW1.).
2.2 Examples of Job Generation Film Print and Film Plot Files D205,3,500,50000.

## Charge.

IDENTIFY (FILMPL,3) (See 2.3 for an explanation on IDENTIFY (FILMPR,3) use of IDENTIFY)

Run.
EOR
PROGRAM TEST (FILMPR, FILMPL,TAPE5 = FILMPR,TAPE6 = FILMPL)


DIMENSION XLABEL(3), YLABEL(3), TITLE (3)
DATA ( $(\operatorname{XLABEL}(I), I=1,3)=10 \mathrm{H}$ XA,10HXIS -- LIN,10HEAR)
DATA ( (XLABEL(I), $I=1,3)=10 \mathrm{H}$ YA,10HXIS -- LIN,10HEAR)
DATA ((TITLE $(I), I=1.3)=10 H L I N E A R ~ T E S, 10 H T$ PLOT,10H DEJ)
C Initialize Linear Graph Plot (straight line)
CALL BPLT
CALL SPLT ( )
$\mathrm{X}=0$.
DO $1 I=1,11$
$\mathbf{Y}=\mathbf{X}$

C Plot a Point in the Linear Graph
CALL FPLT $(X, Y)$
$1 \mathrm{X}=\mathrm{X}+100000.0$

## A-6

M-69-27

C Close Out or Terminate Graph
CALL EPLT
-
-

ETC.
2.3 The IDENTIFY card is the method by which the device and type of microfilm and/or hardcopy is specified for nonstandard options.

Standard: FILMPR(BCD) TO MICROFILM $=$ (no IDENTIFY card required) FILMPL(BINARY) TO HARDCOPY = (no IDENTIFY card required)

Nonstandard: IDENTIFY (FILE1,FORM)
FILE1 = FILMPL(BINARY FILE)
$=$ FILMPR (BCD FILE)
FORM $=1$ (HARDCOPY)
= 2 (MICROFIIM)
$=3$ (BOTH)
Example: Binary and BCD files to both hardcopy and mjerofilm.

## New Method

IDEVTIFY (FILMPL, 3)
IDENTIFY (TILMPR,3)
3. PLOTTING

### 3.1 DD202

These FORTRAN subroutines have been rewritten to produce a file (FILMPL) for plot ting on the CDC 280. The only change required in a user's program is to identify the file FILMPL in the PROGRAM statement in place of TAPE44. DD202 has five entries: BPLT, SPLT, FPITT, EPLT, REVPL.

### 3.1.1 BPLT

The function of BPLT is to provide initialization and need be called only at the start of the job.

CALL BPLT(A,B)
CALL BPLT (2HNB, 2HLC) wnere
2HNB indicates no background grid lines.
2HLC indicates a larger character size.
3.1.2 SPLT

The function of SPLT is to provide frame identification for the data that are to be plotted This frame identification is repeated for as many frames as are necessary to plot the data. The frame identification consists of a title, symbolic names of the dependent and independent variables, the scales of the dependent variables, and the scale of the independent variable properly incremented on all frames produced. Thus, SPLT should be called only whenever it is desired to change any of the frame identification variables. The frame identification information is supplied by ?

CALL SPLT (XO,XS, XN, TITLE, O., T, YMIN1, YMAXI, $\left.\mathrm{YNL}, \ldots . . . . \mathrm{YMIN}_{i}, \mathrm{YMAX}_{i}, \mathrm{YN}_{i}\right)$
where

XO is the value of the independent variable at which plotting is to begin.

XS is the scale of the independent variable. Since each frame of a plot is divided into 10 major divisions, $10 * X S=$ total range of the independent variable over one frame. If the value of the independent variable exceeds the value of $10 * X S$, plotting is continued onto a new frame with the frame identification repeated as already noted.

[^4]```
The number of remaining arguments for SPLT
depends on the number of dependent vari-
ables to be plotted. A maximum of \(i=10\)
dependent variables is allowed and three
arguments are required for each additional
variable in the same order as YMIN1, YMAXI,
YNl. The scale of each dependent variable
is computed as
\(\frac{\mathrm{YMAX}_{i}-\mathrm{YMIN}_{\hat{i}}}{10}\) per major grid division;
however if more than three dependent vari-
ables are requested, all N variables will
be plotted at the
\(\frac{\operatorname{YMAX}_{i}=\text { YMIN }_{i}}{10}\) scale but the BCD name of
the first variable is the only one that
will appear as part of the frame identi-
fication.
If \(\mathrm{YMAX}_{i}=\mathrm{YMIN}_{i}\), the action taken is
identical to that described under the XS
argument discussion.
```


### 3.1.3 FPLT

The third entry to be called is FPLT. FPLT must be called once for each point (or set of points) to be plotted.

FPLT is used by a:
CALL FPLT (X,Y1...... $Y_{i}$ )
where:
$X$ is the value of the independent variable.
Y1 is the value of the first dependent variable at point $X$. Again, there may be a maximum of $i=10$ dependent variable values. The number of dependent

# variable values specified in FPLT must agree with the number specified by the SPLT arguments. The FPLT arguments must have floating point values. 

### 3.1.4 EPLT

The function of EPLT is to terminate the plot information. Thus, it must be called before tise user program terminates to insure that all plotting information is put on the file. It must also be called before a new SPLT is called to insure that all of the previous frame identifications is processed befure the new frame identification specs are input through SPLT. EPLT is called by

CALL EPLT (0)

### 3.1.5 REVPL

The function of REVPL is to provide an option for switching from vector to print plotting or vice versa. Each time the REVPL entry is called, the mode of plotting is reversed. The applications for this option might be in plotting discrete functions to eliminate a vector between points of discontinuity.

CALL REVPL

## 3.1 .6 FRAMECT

The function of FRAMECT is to place on the dayfile the number of frames tinat have been advanced. FRAMECT is automatically called by EPLT.

CALL FRAMECT ( $\mathrm{N}, \mathrm{I}$ )
where:
$N$ is the number of frames
$I=0$ - no dayfile message
1 - a dayfile message.

### 3.2 LRL-KAFB Package

> 3.2.1 Most of this report was taken from "CRT Plotting Routines in Use at LRL-Livermore" written by Judith D. Ford and Marilyn J. Welsh (UCRL-14427-T), and modified by Lt. Peter R. Keller of KAFB.
> This report describes a system of plotting routines. These FORTRAN routines provide a flexible package for point, line, and character plotting via a CDC 280 display device.
> This report gives detailed descriptions of the 280 routines, including purpose, operation, usage, and examples. The routines are separated into the following classes:

1. Mapping routines.

These routines set up scale factors for converting the user's coordinates to the 280 raster point coordinates (raster point defined later). These routines may also draw scales with grid lines or short marks along the axes.
2. Arrow, line, and point plotting routines.

These routines provide the facility for plotting various types of curves.
3. Character plotting routines.

These routines provide the facility for plotting alphanumeric information.
4. Absolute plotting routines.

These routines position the beam independent of the scaling defined by the mapping routines.
5. Utility routines.

These routines give the facilities for fraung and initializing the plot pack. age.
6. Internal routines.

Internal routines perform various functions necessary to the operation of the system, and the user is normally not aware of their existence.

The CDC 280 plane is defined to be a (1024 by 1024) square of addressable points on the face of a cathode ray tube (CRT). These points are called raster points. Information is displayed by unblanking the CRT beam. The beam may be moved to a new position without unblanking (i.e., without plotting a line). Points may only be positioned at a raster point. Lines may only be drawn between two raster points (i.e., the beam unblanked between these two raster points may or may not intersect other raster points).

In the following description of the 280 routines, it is assumed that all arguments are given in the same mode as the dummy arguments, using the standard FORTRAN conventions for the names of integer and floating point variables. The dummy arguments spelled -DUM- are not used by the routine. These arguments are reserved in some cases for future options.

For the purposes of these routines this 280 plane is regarded as having the usual $X, Y$ cartesian coordinates, both of which range from 0 . to 1. with the origin at the lower left corner. If no mapping routine is called all coordinates for the plotting routines are assumed to be between 0 , and 1.
3.2.2 This group of routines makes it unnecessary for the user to scale his own numbers for plotting on the 280 . This is accomplished by establishing a mapping from the user's coordinate plane onto some portion of the 280 plane. This, by the way, allows more than one graph to be plotted on a frame.

CALL MAP (XMIN, XMAX, YMLN, YMAX, XML, XMA, YMI, YMA)
XMIN, XMAX, YMIN, YMAX are the user's maximum and minimum cartesian coordinates.

XMI, XMA, YMI, YMA are the maximum and minimum coordinates of the 280 plane desired to be used.

This description encompasses a group of twelve routines, each of which establishes a mapping from the rectangle in the user's plane with corners (XMIN, YMIN), (XMAX, YMAX) onto the rectangle in the 280 plane with corners (XMI, YMI), (XMA, YMA). Unless reset, this mapping applies to all subsequent plotting, except the absolute plotting routines.

Linear mappings are established by -MAP-, -MAPG-, and -MAPS-.

MAP establishes a mapping only.
MAPG plots a grid with scale numbers.
MAPS plots a rectangle with scale numbers and short marks along the axes.

The suffixes -LL-, -SL-, and -LS- may be used with any of -MAP-, -MAPG-, or -MAPS- to modify the mapping as follows:

LL establishes a log-log mapping.
SL establishes a semi-log mapping with the X-axis linear.

LS establishes a semi-log mapping with the Y -axis linear.

The cycles are determined automatically.

Examples:
CALL MAP (0., 1., 0., 1., 0., 1., 0., 1.) sets up a linear-linear mapping,

CALL MAPSLL (1., 10., 1., 100000., .1, .999, .1, .999) sets up a 1 cycle by 5 cycle scale, and

CALL MAPGSL ( $-100 ., 10 ., 1 ., 100 ., .1, .5$, .1, .999) sets up a linear by 2 cycle grid.

The mapping function is initially set
XMIN - YMIN $=\mathrm{XMI}=\mathrm{YMI}=0$. and $\mathrm{XMAX}=\mathrm{YMAX}=$ $\mathrm{YMA}=1$.

The scale numbers will overplot the grid lines if XMI or YMI is less than .078125 for linear scaling or .043 for logarithmic scaling.

Plotting routines specifying point(s) out of the defined user domain are handled in two ways.

1. If the scaled coordinate is within the 280 range then the routine is executed at the scaled coordinate.
2. If the scaled coordinate is outside of the 280 range then this coordinate is projected on the nearest extreme edge and the routine executes there.

An error message is printed whenever a mapping routine is called with

XMIN $\geq$ XMAX, YMIN $\geq$ YMAX, XMI $\geq X M A$, YMI $\geq$ YMA
or a log mapping is called with a nonpositive argument.

CALL MAPP (RMAX, XMI, XMA, YMI)
RMAX is the maximum radius for the user's polar coordinates.

XMI, XMA, YMI are the same as in -MAP- above.
-MAPP- establishes a mapping from the circle of radius RMAX in the user's polar coordinate plane into the square in the 280 plane with corners (XMI, YMI), (XMA, YMA I where YMA $=\mathrm{YMI}+(\mathrm{XMA}-\mathrm{XMI})$.

Vertical and horizontal reference axes will be plotted, with scale numbers along the zero-degree axis, and with the origin at the center of the square. All ( $\mathrm{X}, \mathrm{Y}$ ) pairs given in later plotting routines will be interpreted as polar coordinates ( $R, \theta$ ) until another mapping routine is called.

CALL MAPX (XMIN, XMAX, YMIN, YMAX, XMI, XMA, YMI, YMA, I)
I is an integer $1 \leq I \leq 13$.
The remaining arguments are the same as in -MAPabove.
-MAPX- allows the mapping to be specified at execution time, according to the value of I. A call to -MAPX- is equivalent to a call to one of the above mapping routines, with the integers 1-13 corresponding to these routines in the following order:

MAP, MAPSL, MAPLS, MAPLL, MAPG, MAPGSL, MAPGLS, MAPGLL, MAFS, MAPSSL, MAPSLS, MAPSLL, MAPP.

When $I=13$ the arguments in MAPX correspond to MAPP as follows

CALL MAPX (DUM, RMAX, DUM, DUM, XMI, XMA, YMI, DUM, 13).

## ARROW, LINE AND POINT PLOTTING ROUTINES

These routines may be used to display and/or photograph data in graphic form. The user's $(X, Y)$ coordinates in these plotting routines are scaled by the scale factors set up by a mapping routine. If no mapping routine has been called, these coordinates are assumed to be in the range 0 . to 1 .

CALL ARROW (X1, Y1, X2, Y2, Z)
( $\mathrm{X} 1, \mathrm{Y} 1$ ) and $\mathrm{X} 2, \mathrm{Y} 2$ ) are coordinates of two points.
$Z$ is a floating roint number 1 .
-ARRGW- sweeps a line from (X1, Y1) to (X2, Y2) and dravs an arrowhead at (X2, Y2). The arrowhead measures $Z$ raster points in length. $Z=10$ is a normal size arrowhead. The intensity is set by -LINEOPT-. The final beam position is (X2, Y2).

CALL LINE (X1, Y1, X2, Y2)
(X1, Y1) and (X2, Y2) are the coordinates of two points. -LINE- will sweep a line from (X1, Y1) to (X2, Y2) with intensity set by -LINEOPT-.

CALL LINEOPT (DUM, INTEN)
DUM is a dummy argument.
INTEN is the intensity at which all arrows, lines, points, and vectors will be plotted.

0 low intensity (fine line).
1 high intensity (heavy line).
-LINEOPT- is initially set to low inteissity.
The mapping routines reset -LINEOPT- from within.

CALL LINEP (X1, Y1, X2, Y2, K)
(X1, Y1) and (X2, Y2) are the coordinates of two points.
$K$ is an integer.
-LINEP- plots a line consisting of every Kth raster point between (X1, Y1) and (X2, Y2). Intensity is set by -LINEOPT-.

CALL LINES ( $\mathrm{X}, \mathrm{Y}, \mathrm{N}$ )
$X$ and $Y$ are the names (first word addresses) of arrays of the $X$ and $Y$ coordinates of points.

N is the number of points.
-LINES- connects the N points given by the arrays $X$ and $Y$ with line segments. The final beam position is ( $\mathrm{X}(\mathrm{N}), \mathrm{Y}(\mathrm{N})$ ). The lines are swept with intensity as set by -LINEOPT-.

CALL POINT ( $\mathrm{X}, \mathrm{Y}$ )
$X$ and $Y$ are the coordinates of a point.

- POINT- will plot a point at (X, Y) with intensity set by -LINEOPT-.

CALL POINTS ( $\mathrm{X}, \mathrm{Y}, \mathrm{N}$ )
$X$ and $Y$ are the names (first word addresses) of arrays of the $X$ and $Y$ coordinates of points.

N is the number of points.
-POINTS- plots the $N$ points given by the arrays $X$ and $Y$. The intensity is set by -LINEOPT-.

## CALL SETB!:AM (X, Y)

$X$ is the abscissa at which the beam is to be positioned.

Y is the ordinate at which the beam is to be positioned.
-SETBEAM- causes the beam to be posi.tioned at ( $\mathrm{X}, \mathrm{Y}$ ) without unblanking.

CALL VECTOR (X2, Y2)
(X2, Y2) is the coordinate of a point.
-VECTOR- sweeps a line from the current beam position to ( $\mathrm{X} 2, \mathrm{Y} 2$ ) with intensity set by -LINEOPT-.
3.2.3 Character Plotting Routines

This group of routines allows the plotting of alphanumeric information, either to label the various curves, lines, etc., produced by the point and line plotting routines, or as a more versatile alternative to ar off-line printer (this is distinct from the -FILMPR- option. -FILMPR- merely simulates the printer). This versatility derives from:

1) The capability of positioning a line of alphanumeric information anywhere on the current frame (vs the top-to-bottom progression of a page printer).
2) The two orientations, two intensities and four character sizes that are available, and
3) The expanded character set, which includes many non-key punchable characters (not immediately available).

## CALL CHAROPT (DUM, DUM, ISIZE, IOR, DUM)

```
ISIZE = 0 miniature
```

            1 small
            2 medium
            3 large
    IOR $=0$ horizontal $\left(0^{\circ}\right)$
1 vertical ( $90^{\circ}$ )
-CHAROPT- specifies the size (ISIZE) and orienta-
tion (IOR) of all characters to be plotted. The
option is changed by a second call to -CHAROPT-.

The maximum string length and line limits for the variou's sizes are:

|  | Symbols/ <br> Line | Lines/ <br> Frame |
| :--- | :---: | :---: |
| Miniature | 128 |  |
| Small | 86 |  |
| Medium | 64 | 43 |
| Large | 43 | 32 |
|  |  | 22 |

In the character plotting routines, the 280 plane is considered to be a grid of rectangles, each containing one character of the chosen size. The number and dimensions of these rectangles depend on the character size and orientation. Characters are drawn within the rectangle. The rectangle is positioned such that the current beam position is in the center of the rectangle. After the character has been drawn the beam is positioned in the center of the next rectangle.

CALL NUMBER (X, F)
$X$ is a variable (fixed or floating).
F is any allowable FORTRAN format 10 characters.
-NUMBER- converts the variable X under the given format, determines the field width and plots the resulting characters as -SYMBOL- would.

Example:
$\mathrm{X}=1 . \mathrm{E} 5$
CALL NUMBIER (X, 5HE10.2)
-
-
would plot
bbbl. 00E05
and
$I=42$
CALL NUMBER (I, $9 \mathrm{H} 4 \mathrm{HIN}=$, I3)
would flot
INb $=\mathrm{b} 42$
CALL SYMBOL (A)
or
CALL SYMBOL (MH...\$.)
A is te first word of $B C D$ data. The end of string is derignated by $\$$.

MH... $\$$. is a Hollerith text of $M$ characters. The last two characters must be $\$$. , which designates the end of string.
-SYMBOL- encodes BCD data into the 280 character set and plots it starting at the current beam position with options as given by -CHAROPT-.

If $\$$. does not appear at the end of string -SYMBOLattempts to plot words up to the field length.

### 3.2.4 Absolute Plotting Routines

These routines position the beam independently of the defined mapping function. The arguments range from 0. to 1 . Out of range points are projected on the nearest extreme edge of the plotting area.

## CALL ABSBEAM (X, Y)

$X, Y$ are coordinates of a point.
-ABSBEAM- causes the beam to be positioned at (X, Y) without unblanking.

CALL ABSLINE (X1, Y1, X2, Y2)
(X1, Y1), (X2, Y2) are coordinates of two points.
-ABSLINE- draws a line from (X1, Y1) to (X2, Y2).
CALL ABSPT (X, Y)
$X, Y$ are coordinates of a point.
-ABSPT- plots a point at (X, Y).
CALL ABSVECT (X, Y)
$\mathrm{X}, \mathrm{Y}$ are coordinates of a point.
-ABSVECT- draws a vector from the last beam position to ( $\mathrm{X}, \mathrm{Y}$ ).
3.2.5 Utilicy Routines

CALL INIT280
-INIT280- initializes the 280 routines and must be called before any of the plotting routines.

CALL FRAME
-FRAME advances the microfilm to the next blank f:ame after emptying the buffer.
-FRAME $\boldsymbol{7}^{\text {should be the last foutine called in }}$ order to empty the buffer.

```
3.2.6 Internal Routines
These routines are essential to the plotting rou-
tines; but are not called directly by the user,
only by other routines in the system.
-GRID80- is called by the mapping routines which
draw scale marks or grid lines and label them with
scale number.
-GTRF-, -GEQF-, -EQLF-, -SEQF-, -SMLF-, -UNQF-, and
-ZGTRF- are functions which are used in -GRID80-.
Each has two arguments and returns a value of l if
the first argument stands in the indicated relation
to the second, a value of 0 otherwise.
```


## FUNCTION

| GTRF | greater than |
| :--- | :--- |
| GEQF | greater than or equal to |
| EQLF | equal to |
| SEQF | less than or equal to |
| SMLF | less than |
| UNQF | not equal to |

These functions call -ZGTRF- to establish the value.
-TEST- is called by the mapping routines to establish legal arguments.
-ADJOST- is called by some of the plotting routines to convert nonlinear arguments to linear before scaling.
-LENGTH- is called by number to count the number of characters to be plotted.
-STREND- is called by symbol to test for end of string symbol.
-PSCALE- is called by the mapping routines to establish the scaling.

- PLO'Q- is called by the plotting routines and forms the 280 instructions.


### 3.3 SC4020 Conversion

3.3.1 SC4020 Binary PLOT Files may be converted to a CDC 280 FIIMPL File by calling the FORTRAN Subroutine SCDD. The calling sequence is:

CALL $\operatorname{SCDD}(I, J, K)$
where $I=$ Number of files to be converted.
$J=$ Tape number of SC4020 FILE
$K=0$, debug printout is inhibited
1, debug printout is not inhibited

### 3.3.2 Example:

PROGRAM TEST (OUTPUT, TAPE45, FILMPL)

CALL ENDPLOT
ENDFILE 45
REWIND 45
CALL $\operatorname{SCDD}(1,45,0)$
3.3.3 SC4020 Bisary Plot Tapes produced on the IBM 360 or 7094 must be processed by program DD219. This program will read tapes written in 36 -bit increments.

## 4. PRINTING

### 4.1 CDC 6000 Print Files

4.1.1 Print files may be recorded by the CDC 280 with the following format:

Up to 128 characters per line are accepted.
The first character of each line is interpreted as the vertical spacing control and is replaced with a space code. The control characters are:
$0=12$ (BCD) Double Sp:
$1=01$ (BCD) Eject
$t=60$ (BCD) Suppress Sp
Any other character carses single spacing.
Vertical spacing control is accomplished before the line is filmed (preprint spacing).

A maximum of 64 lines per frame is admissible. More than 64 lines force an automatic frame advance.
*40 FR will produce 40 blank frames of microfilm for spacing purposes.

### 4.2 Non-CDC 6000 Print Files

4.2.1 Print Files may be created on other computers for recording on the CDC 280.

Tapes must be written in the following manner:
Unlabeled 7 track tape (BCD)
Single blocked records of 130 characters
(Last 2 characters blank).
Blocked records:
Maximum size is 1820 characters.

```
4.3 Forms Flash
    A Forms Flash may be programed for use as an outline for
    each frame of CDC 280 recording of Print Data.
    Those desiring the use of a Forms Flash should design a
    Forms Flash on a grid layout with the following specifi-
    cations.
    Grid size allows }128\mathrm{ characters per line and 64
    lines per page for data.
    Symbol sizes may be:
        128 characters per line
        86 characters per line
        64 characters per line
        4 3 \text { characters per line}
    Symbols may be oriented horizontally (left to right)
    or oriented 90* councer clockwise.
    The Grid Layout should be submitted to Dept. }6643\mathrm{ for
    programing and implementation.
4.4 Special Capabilities
    Jobs requiring Secret or Confidential output on micro-
film or hardcopy may be obtained as follows:
FILMPR - If this file is utilized to generate secret or confidential output, a forms flash must be used to label the microfilm or hardcopy as secret or confidential. This forms flash is generated by initializing FILMPR with one of the following \(B C D\) records:
```


## Col 1

```
FORMSFLASH6A3 For a secret file.
FORMSFLASH6A4 For a confidential file.
```

Fxample to initialize FILMPR with secret forms f.lash:

```
Job Card
CHARGE.
RFQUEST TAPEl,HY.
COPYCR (INPUT, FILMPR, 1)
RUN(S)
LGD
...
78
F\emptysetRMSFLA SH6A3
789
...
```


## 5. ILLUSTRATIONS

Figure A-1 was generated by the following sequence of instructions:

```
CALL INIT280
    CALL MAPG(-1000., 1000., 50., 100., .1, 1., .1, 1.)
    CALL ABSLINE (0., 0., 0., 1.)
    CALL ABSVECT (1., 1.)
    CALL ABSVECT (1., 0.)
    CALL ABSVECT (0., 0.)
    CALL ABSBEAM (. \(4, .05\) )
    CALL CHAROPT ( \(0,0,0,0,0\) )
    CALL SYMBOL (8HX-AXIS\$.)
    CALL ABSBEAM (.05, .4)
    CALL CHAROPT ( \(0,0,0,1,0\) )
    CALL SYMBOL (8HY-AXIS\$.)
    CALL CHAROPT ( \(0,0,0,0,0\) )
    CALL AsSbEAM (.45, .02)
    CALL SYMBOL (11HFIGURE 1.\$.)
    CALL FRAME
```

    Figure A-2 was generated by the following sequence of instruc-
    tions:
    ```
CALL MAPGSL (-1., 1., 1., 100000., .1, .5, .1, 1.)
CALL MAPGLL (1., 10., 100., 1000., .6, 1., .1, .5)
CALL MAPS (-10., 10, 6., 7., .6, 1., .6, 1.)
CALL CHAROPT (0, 0, 0, 0, 0)
CALL ABSBEAM (.45, .001)
CALL SYMBOL (11HFIGURE 2.$.)
CALL FRAME
```

```
Figure A-3 was generated by the following sequence of instruc-
tions:
    DIMENSION X(100), Y(100)
    DO 1. I=1, 100
    X(I)=1
    1 Y(I)=7.2*1
    CALL MAPP (100., 0., .5, 0.)
    DO 2 I=1, 100
2 CALL POINT (X(I), Y(I))
    CALL MAPP (100., .5, 1., .5)
    DO 3 I=1, 98, 4
    3 CALL ARROW (X(I), Y(I), X(I+2), Y(I+2), 8.)
    CALL CHAROPT (0, 0, 0, 0, 0)
    CALL ABSBEAM (.45, .001)
CALL SYMBOL (IlHFIGURE 3.$.)
CALL FRAME
```




FIGURE 2.



[^0]:    AVGLE RFTWFEM ENTRY PLANE AND SKY IS 6.55075855E+B1

[^1]:    *Space Research Conic Program, Phase III, JPL, May 1969 (Planetary Constants).

[^2]:    PSTøRE Analysis
    PSTøRE stores trajectory parameters, estimates, and deviations from nominal values. If $N Q=0$, information relating to solvefor parameters is not calculated or stored. Information is stored if the appropriate value of PLøTL is .TRUE.

[^3]:    $\overline{\text { Reference A. S. Chai: A Modified Runge-Kutta Method, Simulation, }}$ May 1968.

[^4]:    In this case, plotting may not be resumed on a previous frame, i.e., subsequent values of the independent variable nay not be less than that value which caused a new frame to be produced. If the user attempts to do this or if the user inadvertently supplies an XS of 0.0 , an error message is printed. The job is not terminated, but no more plotting will be done.

    XN is the name of the independent variable in Hollerith and many consist of one to six alphanumeric characters.

    TITLE is a 60 contiguous character Hollerith array which will be printed below each frame generated.
    O. is self evident, and at present is a dummy argument.

    T is a flag specifying whether point plots ( $\mathrm{T}=0$ ) or vector plots ( $\mathrm{T}=1$ or $\mathrm{T}=1$.) are desired. If the plots are Secret or Confidential, the Secret or Confidential label is generated $b_{j}$ setting the last four characters of the sixty-character TITLE parameter of SPLT to either "SECR." or "CONF".

    YMIN1 is the minimum grid value for the first dependent variable.

    YMAXI is the maximum grid value of the first dependent variable.
    (If the dependent variable goes olit of the interval (YMIN-YMAX) no plotting is done off the grid, but is resumed normally at the point where $Y_{i}$ reenters the interval)

    YN1 is the name of the first dependent variable in Hollerith and may consist of one to six alphanumeric characters.

