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DIMENSIONAL ANALYSIS CONSIDERATIONS IN THE ENGINE ROTOR FRAGMENT CONTAINMENT/DEFLECTION PROBLEM

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16. Abstract

Dimensional analysis techniques are described and applied to the containment/deflection problem of bursting high-rpm rotating parts of turbojet engines. The use of dimensional analysis to select a feasible set of experiments and to determine the important parameters to be varied is presented. The determination of a "containment coefficient" based on the nondimensionalized parameters is developed for the reduction of experimental data and as an assist to designers of containment/deflection devices.

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FOREWORD

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CONTENTS

SECTION 1

BACKGROUND REMARKS

The Naval Air Propulsion Test Center has been conducting experiments on the interaction between engine rotor fragments and containment/deflection devices. The results of past studies are reported in Refs. 1 through 5 and a report of more recent work is in preparation. Reported in Refs. 6, 7, and 8 are some of the contributions that M.I.T. has made to the study of the fragment containment/deflection problem; these contributions have been in the theoretical-analysis area. The NAPTC experimental work and the M.I.T. theoretical work have been closely complementary and mutually beneficial (see Ref. 9).

While several configurations of various degrees of complexity have been studied at the NAPTC, attention has been directed primarily to the simple (freely supported) ring configuration. In order to lay the groundwork for future experiments, it was decided to proceed with a dimensional analysis of the containment problem. This has been done and the results are currently being used to design experiments and to establish uniform methods for presenting results of pertinent categories of containment studies. In the analysis that follows, the containment device is assumed to consist of a simple ring impacted by bladed-disk fragments from a high-speed rotor.

When the past work of Refs. 1 through 5 is reviewed carefully, it is difficult to perceive how the multitude of data and excellent observations can be used in their present form to aid in the design of fragment-control devices. However, Section 3 of this report indicates how the data may be rearranged, according to the results of the dimensional analysis, as explained in that section, to aid the designer in his choice of design parameters.

SECTION 2

INTRODUCTION

2.1 Problem Statement and Report Scope

The occurrence of failure of high-speed rotating turbojet engine parts has been well documented in the last decade. Uncontained fragments emanating from burst rotors during high rpm service, may travel with high velocity and enter the fuselage of the aircraft and injure passengers or rupture electric, hydraulic, fuel, and control lines of the host aircraft and precipitate a crash. The fragments involved may be simply a blade or two, or they may include large portions of the rotor. It is important, therefore, to provide protection (a) for on-board personnel of aircraft in flight, and (b) for vital components.

Two distinct avenues for providing this protection are evident. First, the structure surrounding the "failure prone" rotor region could be designed to contain (that is, prevent the escape of) the burst-rotor fragments completely. Second, the structure surrounding the rotor could be designed so as to prevent fragment penetration in, and to deflect fragments away from, certain critical regions, and also then to permit fragment escape readily in other "harmless" regions or directions. One or both of these schemes could, in principle, be employed in a given vehicle. In any event, this desired protection is sought for the least weight and/or cost penalty.

The development of information and tools which will permit the engineer to carry out rationally the design of efficient structures to contain or to deflect burst-rotor fragments has been proceeding along both experimental (Refs. 1 through 5) and theoretical (Refs. 6 through 8) lines; these approaches have been closely related to complement and to benefit each other. It is clear that the fragments to be contained or deflected may consist of 1 or 2 blades, or of a portion of a rotor disk with several blades attached, or of multiple complex fragments, etc. Also, the structure used to contain or deflect the subject fragments may consist of one or several layers of materials of different types, and various geometries of both container and fragment may occur. A sequence of fragment-container collisions, fragment-fragment collisions, etc.

may be readily envisioned. It is evident, therefore, that a great many parameters (geometric, material property, and "release condition") are needed to characterize the collision-interaction-response phenomena. Therefore, in view of the vast complexities embedded in this general situation, it is advisable to focus principal experimental and theoretical study on a much simpler subset of this general problem in order to develop an understanding of the primary phenomena involved and to devise adequate prediction methods for these simpler cases. After this has been done, more complicated situations could be studied profitably.

In this approach, the use of systematic experiments is extremely valuable. To assist in planning an efficient set of experiments and to devise a rational systematic analysis of the resulting experimental data, one may often employ dimensional analysis to great advantage. Accordingly, this report addresses itself to the application of dimensional analysis to:

(1) a conveniently restricted subset of the general problem of fragment containment/deflection in order that the number of variables involved be small enough to permit the feasible conducting of a set of informative fragment containment experiments and

(2) a broader class of fragment containment/deflection situations for which a great many variables must be taken into account.

In the remainder of this report, various aspects of this dimensional-analysis question are discussed. The balance of Section 2 is devoted to reminding the reader of the use of dimensional analysis in the context of both mathematical models and physical models, with a simple example of each included for illustration. In Section 3, the use of dimensional analysis is illustrated for (1) a restricted subset of the general fragment containment/deflection problem (see Subsection 3.2) and (2) a broader class of such problems (i.e., with many more variables, Subsection 3.3). The application of the dimensional analysis described in Subsection 3.2 in conjunction with an experimental program of feasible limited scope (i.e., a current experimental program in progress at the NAPTC) is discussed in Section 4, including the selection of appropriate experimental conditions and parameters, and recommended means for analyzing the experimental data. Summary observations and comments are given in Section 5.

2.2 The Uses of Dimensional Analysis

There are two ways of studying physical phenomena. In the first method, mathematical models of the event, or series of events, are constructed and the solutions of the equations involved may be used to predict what will happen in similar events.

The alternative method for studying physical phenomena consists of building physical models and testing them. The use of wind tunnels, towing tanks, and spin chambers are common examples of this latter technique.

Before any experimental program can be undertaken, a clear understanding must be obtained as to what parameters should be varied in the experimental studies. The determination of these parameters is usually done in one of the two aforementioned ways.

2.3 Mathematical-Model Example

In the first method, all of the governing equations are written and rendered dimensionless. For example, consider the equation of motion for a simple pendulum (massless string of length ℓ with a concentrated mass M at its end) undergoing small free oscillations:*

$$\ell\ddot{\theta} + g\theta = 0 \tag{1}$$

where θ represents the angular displacement and g is the acceleration of gravity. For convenience, one may rewrite this equation as

$$\ddot{\theta} + (g/\ell)\theta = 0 \tag{2}$$

A nondimensional time, $\bar{t}$, may be introduced such that

$$\bar{t} = \sqrt{g/\ell}\ t \tag{3}$$

Equations 2 and 3 may be combined to obtain the following nondimensional

*$\sin\theta = \theta$ for small oscillations.

equation:

$$\frac{d^2\theta}{d\bar{t}^2} + \theta = 0 \tag{4}$$

A solution to this homogeneous equation for the angular displacement θ is

$$\theta = e^{i\bar{\omega}\bar{t}} \tag{5}$$

The quantity $\bar{\omega}$ is the nondimensional circular frequency. Clearly then

$$\bar{\omega} = 1$$

$$\bar{f} = \frac{1}{2\pi} \tag{6}$$

$$\bar{T} = 2\pi$$

The quantities $\bar{f}$ and $\bar{T}$ are, respectively, the nondimensional frequency and the nondimensional period.

One may combine the results of Eqs. 3 and 6 to produce an expression for the period, T^*, as follows:

$$2\pi = \sqrt{g/\ell}\; T^* \tag{7}$$

Thus, one finds that the only dimensionless parameter associated with the problem is $T^*\sqrt{g/\ell}$ and the following well-known result is obtained:

$$T^* = 2\pi\sqrt{\ell/g} \tag{8}$$

If one chooses to introduce the next order of complication by removing the restriction of small θ, one would find that

$$T^* = 2\pi\sqrt{\ell/g}\; f(\theta) \tag{9}$$

The function of θ, $f(\theta)$, is known to have the properties

$$f(\theta = 0) = 1 \tag{10}$$

$$f(\theta \cong \sin\theta) \cong 1$$

2.4 Physical-Model Example

In the second method, all of the physical properties thought to be involved in the phenomena are written down and the Pi theorem is used to construct a set of nondimensional parameters, or, as they are often called, similarity parameters. Once these parameters have been determined, the experimenter can establish which ones he wishes to vary experimentally in order to study the phenomena at hand.+

For the case of the simple pendulum, one might be interested in determining the period T^* and one might suppose that the period were influenced by the acceleration of gravity, g, the length of the pendulum, ℓ, and the mass of the pendulum, M. The dimensions of the quantities are listed below. The symbols $[x]$ are to be read: "The dimensions of x are______."

$$\begin{aligned} [T^*] &= T \\ [M] &= FL^{-1}T^2 \\ [\ell] &= L \\ [g] &= LT^{-2} \end{aligned} \tag{11}$$

where T, F, and L denote, respectively, the dimensions: time, force, and length. The only dimensionless quantity that can be formed from this set of variables is $T^* \sqrt{g/\ell}$. This implies that

$$T^* = \text{Const.} \sqrt{\ell/g} \tag{12}$$

Dimensional analysis does not even hint at the fact that the constant is 2π. Experiments (in the absence of a mathematical model) are necessary to evaluate the constant. One may also use dimensional analysis to show that

$$T^* = \text{Const.} \sqrt{\ell/g} \;\; f(\theta) \tag{13}$$

It is clear that in doing experiments with simple pendula, for example, only the ratio of g/ℓ need be varied. One does not have to vary g and then ℓ separately.

2.5 The Focus of This Report

This report is concerned with the use of dimensional analysis in the context of the second method because of the great difficulty inherent in trying to describe the rotor-fragment containment/deflection problem by mathematical means alone.

+See pp 365-367 of Ref. 10.

SECTION 3

DIMENSIONAL ANALYSIS OF SOME CONTAINMENT PROBLEMS

3.1 Introduction

A dimensional analysis of the rotor fragment containment/deflection problem becomes unwieldy and unproductive if one seeks to account for all, or even a large fraction, of the protective-device parameters and fragment parameters which can be varied. To realize the practical benefits of dimensional analysis for this type of problem, therefore, it is necessary to focus attention on a suitably limited sub-group of this general problem; this is discussed in Subsection 3.2. The implications of expanding consideration to include some additional parameter variations in the dimensional analysis, even within this very limited problem sub-group, are discussed in Subsection 3.3.

3.2 Analysis of a Simplified Containment Problem

3.2.1 Problem Definition

In the present discussion, a sub-group of the general containment problem is sought such that one can conduct a feasibly small number of experiments to provide useful data for assisting in the design of a fragment containment structure.

To reduce the number of variables sufficiently to meet this objective, let the containment structure be a uniform homogeneous circular cylindrical shell having: wall thickness h, axial length ℓ, inner-surface radius r, and mass* m; the consideration of more complex containment structures would involve, in general, more variables. In the envisioned experiments, containment rings of only one material would be used; this decision permits one to eliminate containment ring material properties as variables in the present dimensional analysis, and also (as will be seen shortly) reduces the scope of the experiments to a

*One also accounts for the acceleration of gravity, g, and hence ring weight, w = mg, is also a useful quantity.

feasible level.

The second simplification adopted at this point, concerns the type of "attacking" fragments -- a great variety of types, sizes, and attacks (single, multiple, etc.) can occur. To reduce the number of variables requiring consideration, while at the same time utilizing a reasonably plausible (although idealized) fragment attack, let it be assumed that a rotor such as that of a T58 turbine breaks into n equal bladed-disk segments where n may be 2, 3, 4, etc. Also in the experiments, let only one type of rotor be employed; this eliminates fragment material properties as variables in both the dimensional analysis and in the experiments. Accordingly, it is apparent that the properties that most simply characterize a to-be-fragmented spinning rotor include its angular velocity ω, polar mass moment of inertia I, and the number n of equal-sized fragments into which it is prescribed to break.

The quantities which may be treated as variables in seeking to determine the "containment threshold" of this simple containment vessel, which is subjected to idealized fragment attack, are summarized in the following, together with their dimensions in terms of F, L, and T, for force, length, and time, respectively:

Quantity	Symbol	Dimensions
Containment Vessel		
Wall Thickness	h	L
Axial Length	ℓ	L
Inner-Surface Radius	r	L
Mass	m	$FL^{-1}T^2$
Acceleration of Gravity (near-earth value)	g	LT^{-2}
Spinning Rotor		
Angular Velocity	ω	T^{-1}
Polar Mass Moment of Inertia	I	FLT^2
Number of Equal-Sized Fragments	n	1

3.2.2 Dimensional Analysis

According to Buckingham's Pi theorem (Ref. 11, pp 18-20), there are at most 5 dimensionless parameters that cannot be expressed as products or powers of products of the other dimensionless parameters (or products of different powers of these dimensionless parameters). A convenient set is:

$$h/r,\ \ell/r,\ \frac{mgr}{\frac{1}{2} I\omega^2},\ g(r\omega^2),\ n \qquad (14)$$

Hence, one can describe the containment threshold* for the containment ring by

$$\frac{wr}{\frac{1}{2} I\omega^2} = f(h/r,\ \ell/r,\ g/(r\omega^2),\ n) \qquad (15)$$

where w denotes the sea-level weight (w = mg) of the containment ring and $(I\omega^2)/2$ is recognized as the kinetic energy of the entire rotor at the instant of bursting. In principle, this containment threshold function can be determined from either experiment or theoretical analysis provided that all necessary phenomena are taken into account properly in the theory. In the present discussion, however, attention is restricted to utilizing experiments.

Before discussing specific experiments and their use in determining the containment threshold, it is useful to note that rotor failures will typically occur at high rotational velocities (of roughly the order of 2000 to 800 radians/sec); with r of the order of 1 to 5 feet, respectively, the parameter $g/(r\omega^2)$ (or $mg/mr\omega^2$) will be both small and will have a small range of values for all altitudes, failure rotational speeds, and radii. Hence, unless the coefficients of the series expansion of the threshold function f in powers of $g/(r\omega^2)$ are enormous (which is not expected to be the case from physical considerations), the parameter $g/(r\omega^2)$ may be assumed to have negligible significance and hence is dropped from further consideration. Incidentally, the ratio

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*That is, the "dividing line" between fragment containment and noncontainment.

$g/(r\omega^2)$ or $mg/(mr\omega^2)$ may be interpreted as the ratio of the weight to the centrifugal force acting on a particle of mass m at radius r.

3.2.3 Application of Experiments

In view of the preceding discussion, the containment threshold for the simple containment vessel under discussion may be expressed from Eq. 15 as a function of 3 instead of 4 parameters:

$$\frac{wr}{\frac{1}{2} I\omega^2} = f(h/r,\ \ell/r,\ n) \tag{16}$$

Ideally, experiments to determine the containment threshold could proceed as follows. One could conduct all tests by using an "identical" rotor, and rotor failure could be designed to occur at a given ω; all tests, therefore would occur at fixed rotor kinetic energy $(I\omega^2)/2$. By fixing r, ℓ, and n, successive tests could be carried out with containment vessels of various wall thicknesses h. However, since discretely different wall thickness values are used, one would observe, typically, complete containment of the rotor fragments for the large h values, and containment vessel fracturing with attendant actual or imminent fragment escape would be noted when a sufficiently small h were used. For each test one can compute the dimensionless ratio $(2wr)/(I\omega^2)$ which is f itself. Hence when the vessel contains the fragments, call

$$f \equiv C_c = [(2wr)/(I\omega^2)]_c \tag{17}$$

and when the vessel fails, call

$$f \equiv C_f = [(2wr)/(I\omega^2)]_f \tag{18}$$

Thus, one could conveniently display the experimental results as a function of h/r and ℓ/r for a fixed number n of rotor fragments, for example, as follows:

Values of the Containment Function $f \equiv (2wr)/(I\omega^2) \equiv C_c$ or C_f			
n = 3 fragments			
h/r \ ℓ/r	1/20	1/10	1/5
1/5	C_c =	C_c =	C_c =
1/10	C_f =	C_f =	C_c =
1/20	C_f =	C_f =	C_f =
1/30	C_f =	C_f =	C_f =

Such a chart may be prepared for each value of n. Each experiment will provide a value for either C_c (containment) or C_f (failure), and tabular entries may be made as illustrated. Such a tabulation provides the analyst with an indication of the uncertainty bounds between containment and noncontainment (or failure), which will depend upon the wealth or scarcity of test data points near the failure threshold. Additional tests could be conducted, if desired, to increase the resolution of the containment threshold -- with attendant additional expense.

3.2.4 Comments

In the present simplified rotor-fragment containment problem under discussion which involves the prescribed fixing of a number of parameters (discussed subsequently) while permitting only a few parameters to vary, it is seen that the containment threshold function $f \equiv (2wr)/I\omega^2$ is a function of only three variables or parameters: h/r, ℓ/r, and n. Thus, for a rotor of given size, a containment threshold test program might well consist of fixing r while using perhaps 4 values of n, 3 values of ℓ/r, and 4 values of h/r; this would result in 4x3x4 = 48 experiments. As noted later in Section 4, a test program of this nature, but involving rotors of two different sizes (to explore rotor size [scale] effects for two common rotors) and hence, tentatively, 96 experiments is in progress at the NAPTC. If the initial series of tests demonstrates that f has a weak or negligible dependence upon, for example, the number of fragments n (or perhaps ℓ/r), the number of experiments will be reduced.

Cost factors motivate an experimental program which seeks not to vary many parameters. The reason for eliminating the variation of some parameters, such as material properties, from an experimental program and the associated dimensional analysis is due to the fact that the experimental costs rise rapidly as the number of independent parameters is increased. The reason for this is presented below.

Suppose, for example, that one must explore the effects of p variables and that k experiments are needed for each of the p variables in order to determine the containment threshold function f. In this case, one would require k^p experiments. The "required number of experiments" could readily become very large unless suitable constraints on the number of parameters permitted to vary are introduced.

Since the cost of an experimental program of this type is essentially proportional to the number of tests conducted, it is important to utilize dimensional analysis to aid in minimizing the number of variables which must be included in the experimental program.

Finally, it should be noted that the designer might design his containment vessel to provide a certain C_c value (determined from test data which includes the proper physical parameters: free ring, restrained ring, type of restraint, ring material, rotor material, etc. -- all of these and more could be important). The particular value of C_c that he elects to use may depend upon his assessment of how much deformation and/or damage of the appropriate containment vessel will be acceptable to the cognizant certifying agency. Therefore, for each entry of C_c or C_f in the tabulated test results, postmortem photographs (one or more) and/or supplementary descriptions of the postmortem condition of the fragment containment vessel should be provided to aid the designer in his design decisions.

Until other than "free"-ring data are available, the initial design may be based on these data rather than on restrained-ring data which may be more appropriate when available.

It is appropriate to mention at this juncture that the containment threshold is not a sharply-defined surface. Experimental data will yield scattered results and thus an appropriate concept to introduce at this point is that of the probability of successful containment or deflection at a given level of the containment coefficient for a given material and geometry.

For example, if out of ten nominally identical experiments, seven are successful (contain or deflect as wanted) and three are unsuccessful (fail to contain or deflect, as required) then the probability of containment/deflection may be said to be approximately 70%. Data are currently being reduced and presented in this manner.

3.3 Analysis of the Simplified Containment Problem with Additional Variables

If the simplified combination of containment vessel geometry and rotor fragment type discussed in Subsection 3.2 is retained but one now considers exploring the effects of using different isotropic materials for the rotor, and/or containment vessel, the following additional variables should be taken into account for each: (1) elastic modulus E, (2) Poisson's ratio ν, (3) yield stress σ_o, (4) ultimate stress σ_u, and perhaps (5) ultimate strain ε_u. Additional material property parameters would need to be taken into account if anisotropic materials were employed in one or both parts (fragments and ring) of this system. Thus, the "new" set of variables and associated dimensions are:

Quantity	Symbol	Dimensions
Containment Vessel		
Wall Thickness	h	L
Axial Length	ℓ	L
Inner-Surface Radius	r	L
Mass	m	$FL^{-1}T^2$
Acceleration of Gravity (near-earth value)	g	LT^{-2}
*Elastic Modulus	E	FL^{-2}
*Poisson's Ratio	ν	1
*Yield Stress	σ_o	FL^{-2}
*Ultimate Stress	σ_u	FL^{-2}
*Ultimate Strain	ε_u	1
Spinning Rotor		
Angular velocity	ω	T^{-1}
Polar Mass Moment of Inertia	I	FLT^2
Number of Equal-Sized Fragments	n	1
*Elastic Modulus of Fragment	E_f	FL^{-2}
*Poisson's Ratio of Fragment	ν_f	1
*Yield Stress of Fragment	σ_{of}	FL^{-2}
*Ultimate Stress of Fragment	σ_{uf}	FL^{-2}
*Ultimate Strain of Fragment	ε_{uf}	1

*"New variables"

If one considers a rotor of given geometry and material properties and the effects of varying only the geometric parameters and material properties of the containment vessel, it is seen that there are 13 variables of which 3 are dimensionless. Hence, one may choose the following 10 dimensionless variables to characterize this situation:

$$h/r,\ \ell/r,\ (2wr)/(I\omega^2),\ g/(r\omega^2),\ n,\ \sigma_o/E,\ \sigma_u/E,\ \varepsilon_u,\ \nu,\ w/(Eh\ell) \qquad (19)$$

Thus, the containment threshold may be expressed by (dropping $g/(r\omega^2)$ and for similar reasons $w/(Eh\ell)$):

$$(2wr)/(I\omega^2) = f(h/r,\ \ell/r,\ n,\ \sigma_o/E,\ \sigma_u/E,\ \varepsilon_u,\ \nu) \tag{20}$$

If a complete matrix of experiments were to be conducted, it is evident that a great many experiments would be required to define the containment threshold. However, in carrying out such experiments in a well-selected sequence, it is usually possible to identify very early that f depends only very weakly upon several or perhaps even most of these parameters. In this case, the scope of the necessary experiments can be radically reduced; experiments must be conducted and planned judiciously.

If on the other hand, one seeks similarly to determine the effects of varying the material properties of both the containment vessel and the attacking fragments, it is seen that there are 18 variables of which 5 are dimensionless. Hence, one may choose the following 15 dimensionless variables to characterize this situation:

$$h/r,\ \ell/r,\ (2wr)/(I\omega^2),\ g/(r\omega^2),\ n,\ \sigma_o/E,\ \sigma_u/E,\ \varepsilon_u, \tag{21}$$

$$\nu,\ w/(Eh\ell),\ E_f/E,\ \nu_f,\ \sigma_{of}/E_f,\ \sigma_{uf}/E_f,\ \varepsilon_{uf}$$

Accordingly, the containment threshold may be expressed (again dropping $g/(r\omega^2)$ and $w/(Eh\ell)$:

$$(2wr)/(I\omega^2) = f(h/r,\ \ell/r,\ n,\ \sigma_o/E,\ \sigma_u/E,\ \varepsilon_u,\ \nu,\ E_f/E,\ \nu_f,\ \sigma_{of}/E_f,\ \sigma_{uf}/E_f,\ \varepsilon_{uf}) \tag{22}$$

Dimensional analysis serves to identify the number of truly independent dimensionless parameters contained in a set of quantities which the analyst conceives of as possibly being important. It is the role of experiments and/or further analysis to determine whether or not those quantities do in fact importantly affect the behavior in question (that is, the containment threshold in the present case).

3.4 Comments Concerning More General Containment Situations

In Subsections 3.2 and 3.3, a set of useful but highly restricted combinations of containment vessel parameters and fragment parameters was considered. There are, of course, many other plausible combinations which are worthy of study -- not only for fragment containment but also for fragment deflection. The value of conducting carefully chosen experiments to obtain definitive data for design cannot be overestimated because of the myriad of complexities involved in this type of physical situation. Clearly, however, the amount of testing could easily become prohibitively large and expensive while producing few results of design importance unless the experiments are chosen very carefully and conducted in a well-chosen sequence.

If, for example, a different type of fragment attack were considered to be both likely and critically severe (such as, for example, the loss of a single blade followed by blade impact upon the containment vessel, rebound, collision with one or more of the blades remaining on the spinning rotor, further fragment production, etc.), a similar test program could be conducted to obtain definitive containment data. In this case also, to carry out a test program of feasible scope, it would be necessary to limit the number of parameters which is permitted to be varied in the test program. Perhaps a test program of scope similar to that discussed in Subsection 3.2 would be desirable, and could be modified according to pertinent prior test experience.

Other carefully circumscribed sub-groups of the multitude of possible rotor fragment containment/deflection conditions could be assessed selectively and then explored in more or less depth as the results warrant. In each case, however, dimensional analysis can be used to advantage to assist in minimizing the test program and in presenting the results in a concise and useful manner.

SECTION 4

APPLICATION TO A TEST PROGRAM OF LIMITED AND FEASIBLE SCOPE

4.1 Objectives and Scope of the Experiments

The goal of the experimental program described herein is to obtain typical values of the containment coefficient to aid the designer-analyst.

Consistent with dimensional-analysis considerations, the Naval Air Propulsion Test Center has proposed (Ref. 12) that a limited number of carefully-planned experiments be conducted. There will be, tentatively, 96 experiments in all; as the program proceeds, the developed information may permit some reduction of the number of tests. A brief description follows.

Two turbine-rotor sizes (approximately 35.6 cm (14" for T58) and 78.7 cm (31" for J65) in diameter, but not exactly similar geometrically) will be used to generate bladed-disk fragments. There will be "containment rings", then, of two basic radii. The clearance between the blade tip and the containment vessel in each instance will be small.

For each of the two sizes under consideration, experiments will be conducted using containment rings having four values of the ring thickness. There will also be three values of the axial length of the rings. In each experiment, a selected number n of equal-sized bladed-disk fragments will be utilized (n will be 2, 3, 4, or 6).

The total number of experiments is thus seen to be

$$2 \times 4 \times 3 \times 4 = 96 \qquad (23)$$

The non-dimensional parameters that will be varied in the experimental program are:

$$h/r,\ \ell/r,\ n,\ \text{and}\ 2wr/(I\omega^2) \qquad (24)$$

These experiments will be used to evaluate C_c, C_f, etc., and to identify the associated "damage". These results will be reported in a future document.

4.2 Extension of the Test Results

Values of the containment coefficient will necessarily be determined for a limited range of the non-dimensional parameters. In some actual designs, the appropriate parameters (h/r, ℓ/r, etc.) will lie outside of the range considered. Values of the containment coefficient will then have to be obtained by formal-mathematical extrapolation procedures. The usual cautions associated with extrapolation will certainly apply. Also, of course, mathematical interpolation of the data can be utilized.

SECTION 5

CONCLUDING REMARKS

The concepts of dimensional analysis have been reviewed and their relationship to experimental programs pointed out. A dimensional analysis for simplified versions of the containment problem has been set down in detail and suggestions for "reducing" experimental data have been made. An extension of the analysis which incorporates the effects of material properties has been described.

A conclusion of this report is that the systematic experimental study of the effects of material properties on the phenomena associated with containment would require an enormously-expensive experimental program. This conclusion may be reached by studying the entries in the list of dimensionless parameters and by reviewing the limited number of experiments already available.

It is possible to predict the large-deflection, elastic-plastic, transient responses of simple containment rings by methods outlined in Ref. 8, for example. However, the forcing function (the time and space description of the forces which the fragments exert on the ring) must be known. Since these forcing functions are not presently known, a second conclusion of the report must be that as long as the details of the forcing functions remain unknown, there will be an ongoing need for an experimental program to determine the nature of the containment surface.

The dimensional analysis discussed in this report can serve effectively as a guide for future experimental programs of the type outlined in this report.

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