NASA CONTRACTOR
REPORT

NASA CR-61372

A SYSTEM RELIABILITY ANALYSIS FOR STAND-BY SPARES WITH NON-ZERO UNPOWERED FAILURE RATES

By D. S. Taylor, J. E. Weatherbee,
and T. Williams

Computer Sciences Corporation
Field Service Division
Aerospace Systems Center
8300 S. Whitesburg Drive
Huntsville, Alabama 35802

December 1971

Prepared for

NASA-GEORGE C. MARSHALL SPACE FLIGHT CENTER
Marshall Space Flight Center, Alabama 35812

TECHNICAL REPORT STANDARD TITLE PAGE

1. REPORT NO.	2. GOVERNMENT ACCESSION NO.	3. RECIPIENT'S CATALOG NO.
NASA CR-61372		

4. TITLE AND SUBTITLE	5. REPORT DATE
A System Reliability Analysis for Stand-by Spares with Non-Zero Unpowered Failure Rates	December 1971
	6. PERFORMING ORGANIZATION CODE
7. AUTHOR (S) D. S. Taylor, J. E. Weatherbee, and T. Williams	8. PERFORMING ORGANIZATION REPORT #
9. PERFORMING ORGANIZATION NAME AND ADDRESS Computer Sciences Corporation Field Services Division, Aerospace Systems Center 8300 S. Whitesburg Drive Huntsville, Alabama 35802	10. WORK UNIT NO. 11. CONTRACT OR GRANT NO. NAS8-21805
12. SPONSORING AGENCY NAME AND ADDRESS National Aeronautics and Space Administration Washington, D. C. 20546	13. TYPE OF REPORT & PERIOD COVERED Contractor Report 14. SPONSORING AGENCY CODE

15. SUPPLEMENTARY NOTES

16. ABSTRACT

Equations which define the reliability of n-fold parallel systems with stand-by spares, and triply redundant, majority-voting systems with stand-by spares have been derived. The stand-by spares have been assumed to have a non-zero failure rate while in the stand-by mode. A Monte Carlo system simulation has been generated and the results compared to the theoretical reliability predictions. A comparison of these two stand-by configurations is also presented for three (3) through six (6) total units.

17. KEY WORDS	18. DISTRIBUTION STATEMENT
Reliability Probability Stand-by Spares	B. Hodges Computer Systems Division MSFC/Computation Laboratory Unclassified-unlimited

19. SECURITY CLASSIF. (of this report)	20. SECURITY CLASSIF. (of this page)	21. NO. OF PAGES	22. PRICE
Unclassified	Unclassified	26	$ 3.00

MSFC - Form 3292 (May 1969)

TABLE OF CONTENTS

A SYSTEM RELIABILITY ANALYSIS FOR STAND-BY SPARES WITH NON-ZERO UNPOWERED FAILURE RATES

SUMMARY

A theoretical reliability analysis is performed for two different stand-by system configurations. The two system configurations are simulated by utilizing Monte Carlo techniques, and the results agree within one percent of the theoritical reliability predictions. A comparison of the two configurations is presented.

It is concluded that in order for the voting system configuration to be competetive, it must utilize software error detection in the final phases.

SECTION I. INTRODUCTION

The NASA unmanned interplanetary space flights in the 1975 to 1985 time frame will require a spaceborne computer that is highly reliable and which can automatically switch in redundant hardware on a real time basis. The computer division of the Astrionics Laboratory at the Marshall Space Flight Center (MSFC) is developing a condidate advanced aerospace computer called Space Ultrareliable Modular Computer (SUMC).[1] In supporting this effort, several studies have been conducted to determine the best method to configure such a system.

This paper represents the results of one of these studies. The object of this study is to perform a theoretical reliability analysis on two different system configurations and compare the results to a Monte Carlo simulation of the systems.

A. Background

During the past few years several papers have been published on the reliability of parallel redundant and triply redundant majority voting systems. [2,3,4,5, et al.] However, most of these papers are concerned with either system reliability approximations, [3,6,7], powered up redundant units, [2,5] or unpowered stand-by units with a zero stand-by failure rate. [8,9] To the authors' knowledge no one has published any work which will:

1) Exactly predict the system reliability for parallel redundant and triply redundant majority-voting systems, with "n" unpowered stand-by spares which have a non-zero failure rate while in the stand-by mode; and

2) compare the system reliabilities to each other and analyze the results.

The purpose of this paper is to accomplish the above stated objectives. In doing so, two basic stand-by system configurations will be considered

B. SYSTEM CONFIGURATIONS

The first system configuration to be considered will be described as a parallel stand-by system and is illustrated in Figure 1.

A parallel stand-by system has the following characteristics:

1) Only one unit at a time is powered up; and

2) all units possess hardware error detecting capabilities.

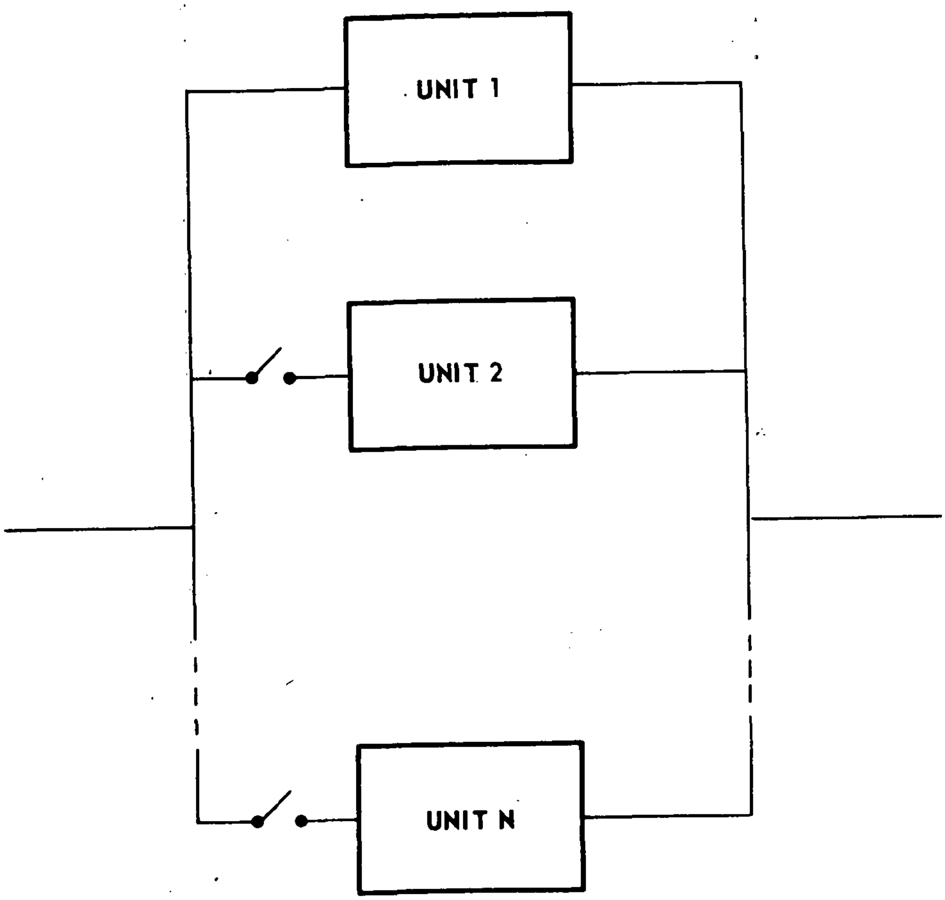

FIGURE 1

PARALLEL STAND-BY CONFIGURATION

The second system configuration under consideration will be described as a triply redundant majority-voting stand-by system and is depicted in Figure 2. The characteristics of this configuration are described below:

1) As long as there are at least three units, three of them will be powered up and the rest will be in an unpowered stand-by mode;

2) when all but two of the units have failed, the voter can than only tell when these final two units are not in agreement. (i.e. it can detect one more failure, but cannot isolate the faulty unit.)

3) Units do not have error detecting hardware, however, software error detection may be introduced whenever the system is down to just two units and one of them fails.

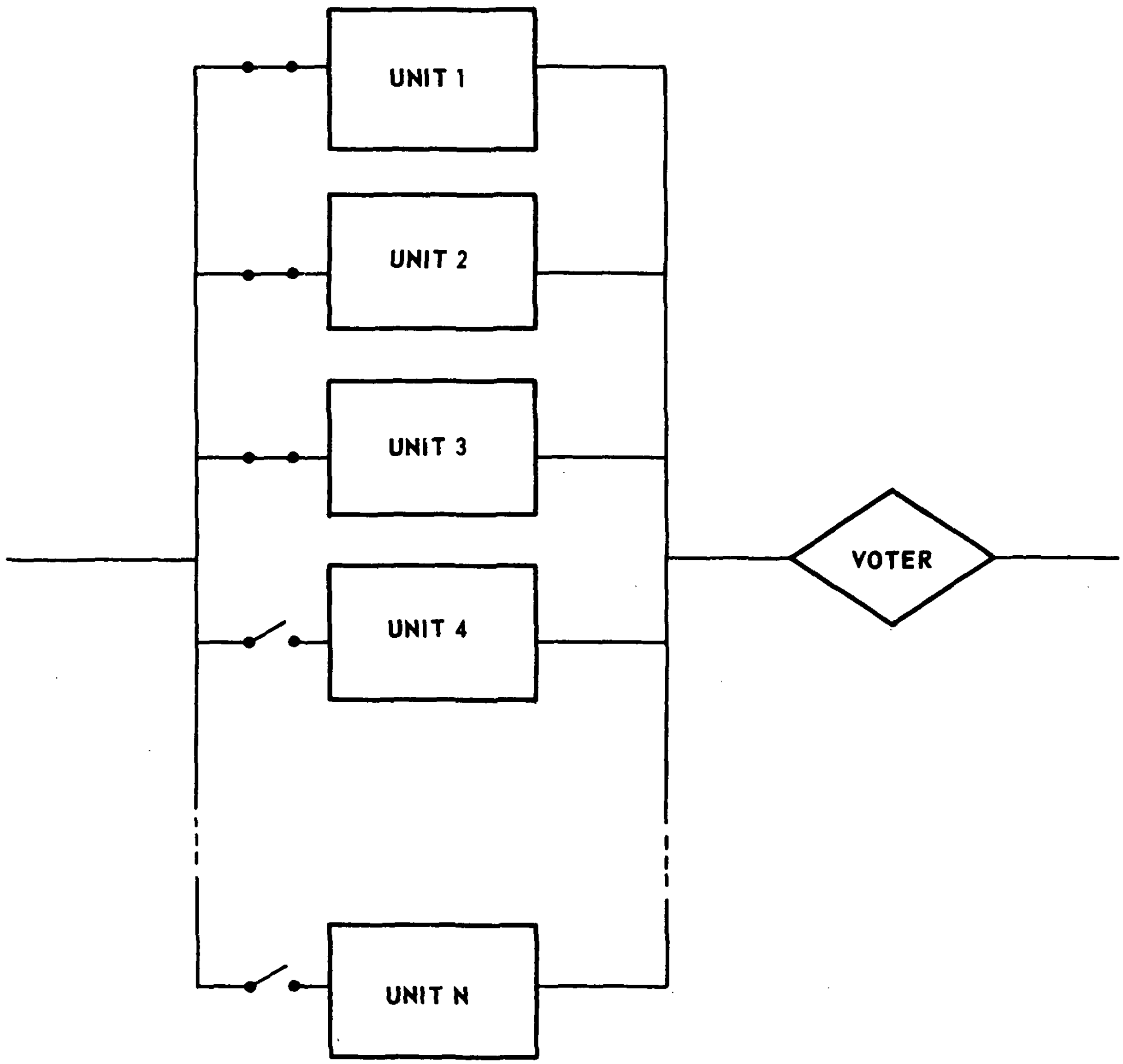

FIGURE 2

TRIPLY REDUNDANT MAJORITY-VOTING STAND-BY CONFIGURATION

SECTION II. THEORETICAL ANALYSIS

A. Theory Of Parallel Stand-By Systems

The reliability of a parallel stand-by system is well defined if the unpowered units cannot fail until they have been powered up. For example, the reliability of such a system N units, of which N-1 are unpowered spares is

$$R = e^{-\lambda t}\left\{\sum_{n=0}^{N} \frac{(\lambda t)^n}{n!}\right\} \tag{1}$$

where

N = total number of units,

λ = powered failure rate, and

t = time.

This simple equation becomes somewhat more complicated if the unpowered stand-by units have a non-zero failure rate ($\lambda' \neq 0$). In order to analyze this system, the reliability will be defined as the sum of the probabilities of the successful paths that the system might follow and still have an operational unit. Utilizing this approach, a system of N units could tolerate no more than N-1 failures, and the reliability equation would consist of N terms of a similar format.

$$R = P_o + P_1 + \ldots + P_{n-1} \tag{2}$$

where

$$P_o = e^{-\lambda t}$$

$$P_1 = \lambda e^{-\lambda t} \int_{t_1 = o}^{t} e^{-\lambda' t_1} \, dt_1 \tag{3}$$

$$P_{n-1} = \left\{\prod_{n=o}^{N-1} (\lambda + n\lambda')\right\} e^{-\lambda t} \int_{t_1=o}^{t} e^{-\lambda' t_1} \int_{t_2=t_1}^{t} e^{-\lambda' t_2} \ldots \int_{t_{n-1}=t_{n-2}}^{t} e^{-\lambda' t_{n-1}} \, dt_1 dt_2 \ldots dt_{n-1}$$

When the integrations in Equation (3) are performed and results substituted into Equation (2), the following expression is obtained:

$$R = e^{-\lambda t}\left\{1 + \sum_{n=1}^{N-1} \frac{(1 - e^{-\lambda' t})^n}{n!\,\lambda'^n} \prod_{i=o}^{n-1} (\lambda + i\lambda')\right\} \qquad (4)$$

where

N = Number of units ≥ 2.

N = 1 implies that there are no spares and the reliability is simply

$$R = e^{-\lambda t}. \qquad (5)$$

If $\lambda' = 0$ in Equation (4), i.e., the unpowered, stand-by spares cannot fail until they become powered up, then L 'Hospital's rule must be employed and Equation (4) reduces Equation (1).

B. Theory of Triply Redundant Majority-Voting Stand-By Systems

Triple redundancy with a voter has been investigated by several authors, [10,11] and even N-fold redundant systems (all units voting and powered up) have been investigated. [12] However, to the authors' knowledge, an analysis of a triply-redundant, majority-voting system with "N" unpowered stand-by units, and which is allowed to degrade to a single operating unit, has not been performed. This configuration, as illustrated in Figure 2, is assumed to operate in the following manner.

As long as two units agree, the voter assumes the third has failed and thus will switch in a previously unpowered spare; providing one is available. (It is assumed that the spares can fail even though they are unpowered.) If no spare is available, the system can still operate as long as the two remaining units agree. However, when they disagree, the voter cannot tell which unit is in error and for that reason the voter is useless from this time on. Since one does not wish a system to fail as long as one good unit remains, software error detection may be introduced at this point to determine the faulty unit. Through the proper use of software error detecting schemes, the final unit could then be utilized until it fails.

The theoretical reliability analysis of a triply redundant majority-voting system with "N" stand-by spares is much more complicated than that of a simple parallel stand-by system. For example, one can consider, as was done in the previous section, that the reliability of a system is the sum of the probabilities of the successful paths which the system might follow and still have an operational unit at time t. Under such an analysis, the reliability of a triply redundant voting system with "N" stand-by spares will generate N + 2 different probablistic terms, with no software error detection, or N + 3 terms if software error detection is utilized. The format of these equations is illustrated in Equations (6) below.

$$P_o(N) = k_o$$

$$P_i(N) = k_i \int_{t_1=o}^{t} e^{-\lambda' t_1} \int_{t_2=t_1}^{t} e^{-\lambda' t_2} \cdots \int_{t_i=t_{i-1}}^{t} e^{-\lambda' t_i} \, dt_1 \, dt_i \cdots dt_i$$

where $t_o = o$ and $i = 1,2,3, \ldots ,N$

$$P_{n+1}(N) = k_{n+1} e^{\lambda t} \int_{t_1=o}^{t} e^{-\lambda' t_1} \cdots \int_{t_n=t_{n-1}}^{t} e^{-\lambda' t_n} \int_{t_{n+1}=t_n}^{t} e^{-\lambda t_{n+1}} \, dt_1 \cdots dt_{n+1} \qquad (6)$$

$$P_{n+2}(N) = 2\lambda k_{n+1} e^{2\lambda t} \int_{t_1=o}^{t} e^{-\lambda' t_1} \int_{t_n=t_{n-1}}^{t} e^{-\lambda' t_n} \int_{t_{n+1}=t_n}^{t} e^{-\lambda t_{n+1}} \int_{t_{n+2}=t_{n+1}}^{t} e^{-\lambda t_{n+2}} dt_1 \ldots dt_{n+2}$$

$$k_o = e^{-3\lambda t}, \quad k_i = k_o \prod_{n=o}^{i-1} (3\lambda + n\lambda), \quad i \geq 1$$

and N = number of stand-by units.

These equations have been writen out in detail for N = 0,1,2,3 in Table 1. Solutions for these equations are then presented in Appendix 1.

TABLE 1

NO SPARES	$P_0(0) = e^{-3\lambda t}$
	$P_1(0) = 3\lambda\, e^{-2\lambda t} \int_{t_1=0}^{t} e^{-\lambda t_1}\, dt_1$
	$^*P_2(0) = 6\lambda^2\, e^{-\lambda t} \int_{t_1=0}^{t} e^{-\lambda t_1} \int_{t_2=t_1}^{t} e^{-\lambda t_2}\, dt_1\, dt_2$
1 SPARE	$P_0(1) = P_0(0)$
	$P_1(1) = 3\lambda\, e^{-3\lambda t} \int_{t_1=0}^{t} e^{-\lambda' t_1}\, dt_1$
	$P_2(1) = (3\lambda + \lambda')\, 3\lambda\, e^{-2\lambda t} \int_{t_1=0}^{t} e^{-\lambda' t_1} \int_{t_2=t_1}^{t} e^{-\lambda t_2}\, dt_1\, dt_2$
	$^*P_2(1) = (3\lambda + \lambda')\, 6\lambda^2\, e^{-\lambda t} \int_{t_1=0}^{t} e^{-\lambda' t_1} \int_{t_2=t_1}^{t} e^{-\lambda t_2} \int_{t_3=t_2}^{t} e^{-\lambda t_3}\, dt_1\, dt_2\, dt_3$
2 SPARES	$P_0(2) = P_0(0)$
	$P_1(2) = P_1(1)$
	$P_2(2) = (9\lambda^2 + 3\lambda\lambda')\, e^{-3\lambda t} \int_{t_1=0}^{t} e^{-\lambda' t_1} \int_{t_2=t_1}^{t} e^{-\lambda' t_2}\, dt_1\, dt_2$
	$P_3(2) = (27\lambda^3 + 27\lambda^2\lambda' + 6\lambda\lambda'^2)\, e^{-2\lambda t} \int_{t_1=0}^{t} e^{-\lambda' t_1} \int_{t_2=t_1}^{t} e^{-\lambda' t_2} \int_{t_3=t_2}^{t} e^{-\lambda t_3}\, dt_1\, dt_2\, dt_3$
	$^*P_4(2) = (54\lambda^4 + 54\lambda^3\lambda' + 12\lambda^2\lambda'^2)\, e^{-\lambda t} \int_{t_1=0}^{t} e^{-\lambda' t_1} \int_{t_2=t_1}^{t} e^{-\lambda' t_2} \int_{t_3=t_2}^{t} e^{-\lambda t_3} \int_{t_4=t_3}^{t} e^{-\lambda t_4}\, dt_1\, dt_2\, dt_3\, dt_4$
3 SPARES	$P_0(3) = P_0(0)$
	$P_1(3) = P_1(1)$
	$P_2(3) = P_2(2)$
	$P_3(3) = (27\lambda^3 + 27\lambda^2\lambda' + 6\lambda\lambda'^2)\, e^{-3\lambda t} \int_{t_1=0}^{t} e^{-\lambda' t_1} \int_{t_2=t_1}^{t} e^{-\lambda' t_2} \int_{t_3=t_2}^{t} e^{-\lambda' t_3}\, dt_1\, dt_2\, dt_3$
	$P_4(3) = (81\lambda^4 + 162\lambda^3\lambda' + 99\lambda^2\lambda'^2 + 18\lambda\lambda'^3)\, e^{-2\lambda t} \int_{t_1=0}^{t} e^{-\lambda' t_1} \int_{t_2=t_1}^{t} e^{-\lambda' t_2} \int_{t_3=t_2}^{t} e^{-\lambda' t_3} \int_{t_4=t_3}^{t} e^{-\lambda t_4}\, dt_1\, dt_2\, dt_3$
	$^*P_5(3) = (162\lambda^5 + 324\lambda^4\lambda' + 198\lambda^3\lambda'^2 + 36\lambda^2\lambda'^3)\, e^{-\lambda t} \int_{t_1=0}^{t} e^{-\lambda' t_1} \int_{t_2=t_1}^{t} e^{-\lambda' t_2} \int_{t_3=t_2}^{t} e^{-\lambda' t_3} \int_{t_4=t_3}^{t} e^{-\lambda t_4} \int_{t_5=t_4}^{t} e^{-\lambda t_5}\, dt_1 dt_2 dt_3 dt_4 dt_5$

* THIS TERM IS ADDED TO THE SYSTEM RELIABILITY IF AND ONLY IF SOFTWARE ERROR DETECTION IS INTRODUCED AFTER THE N+2nd FAILURE.

SECTION III. SYSTEM SIMULATION

As can be seen in Appendix 1, the equations necessary to calculate the reliability of a triply-redundant system become increasingly complex as the number of unpowered stand-by units increases. In order to verify the results obtained from these equations, a Monte Carlo simulation model was developed which provides estimates of system reliability for "M-redundant" majority-voting stand-by systems (see Figure 3). Note that M-redundant systems include both the parallel redundant system (see Figure 1) and the triply-redundant system (see Figure 2).

The model simulates the life of a system a specified number of times and uses the results from these simulations to calculate estimates of system reliability. Both powered and unpowered failures are assumed to be exponentially distributed with mean failure rates λ and λ', respectively. Exponential distributions with means provided by input data are sampled during execution of the model to find the failure times of the various units. When a powered unit fails, it is replaced by a spare as long as one is available.

The simulated time at which the number of operable units falls below a preset minimum is recorded as the duration of a system's life. System reliability, for a particular time t, is obtained by dividing the number of systems lasting longer than t by the total number of systems simulated.

Naturally, the accuracy of these reliability estimates is a function of the number of times the system life is simulated (i.e, the sample size used). Model runs were made with sample sizes of 1000, 5000, and 10000. Table 2 shows typical results from the three model runs when compared to the results obtained from the equations in Appendix 1. These results are for a triply-redundant system with 2 spares and failure rates $\lambda = .75$ and $\lambda' = .01$. Simulator results using a sample size of 10,000 were generally within 1% of the results from the equations in Appendix 1. This close agreement is sufficient to verify the equations.

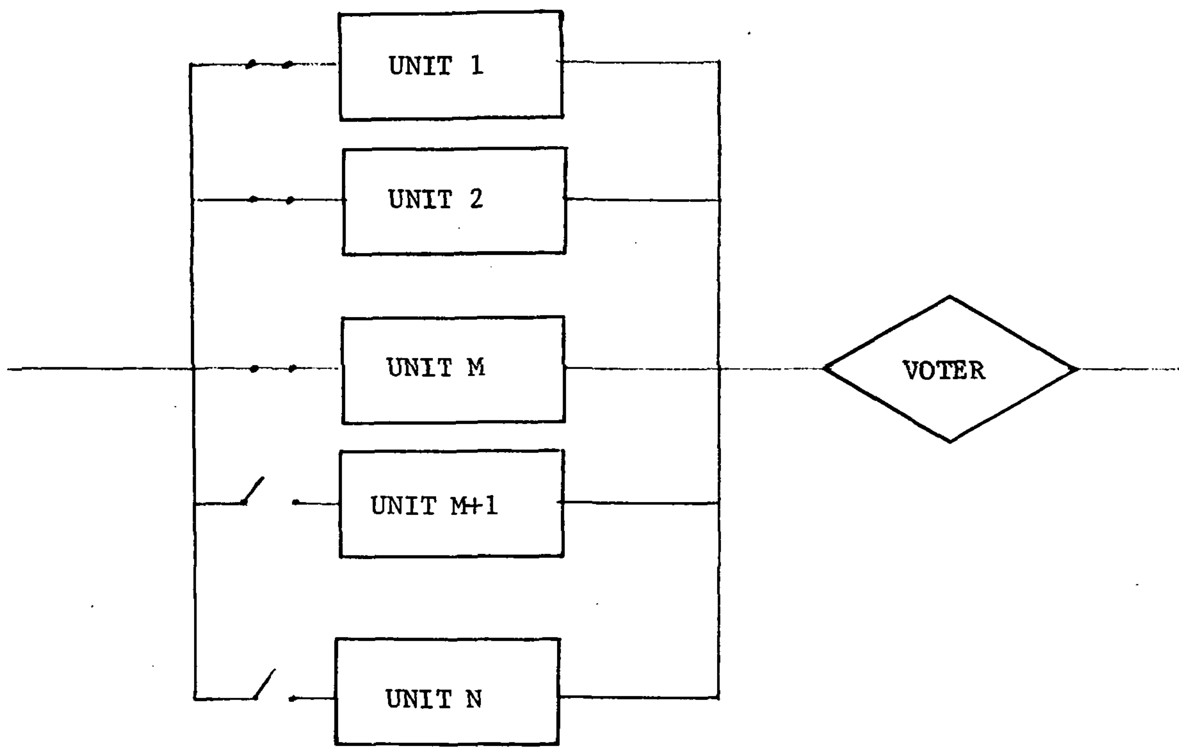

FIGURE 3: M-REDUNDANT SYSTEMS WITH SPARES

TABLE 2: COMPARISON OF SIMULATOR RESULTS WITH ACTUAL RELIABILITIES

TIME	SIMULATOR ESTIMATES			ACTUAL RELIABILITY
	SAMPLE SIZE			
	1000	5000	10000	
1	.9750	.9738	.9754	.9746
2	.8150	.7758	.7855	.7842
3	.5390	.5502	.5061	.5049
4	.3060	.2818	.2817	.2816
5	.1610	.1426	.1462	.1449
6	.0740	.0770	.0744	.0715
7	.0340	.0404	.0364	.0345
8	.0120	.0176	.0164	.0165

SECTION IV. COMPARISON OF THE TWO STAND-BY CONFIGURATIONS

The reliability of the two stand-by configurations depicted in Figures 1 and 2 and described mathematically by Equations (3) and (6) respectively, depends upon four independent variables, i.e., λ, λ', N and t. In order to compare these configurations fairly, the following assumptions have been made.

(1) The parallel configuration units must contain more hardware (due to error checking capabilities) than the voting units, thus $\lambda p > \lambda v$, and for the purpose of comparison it will be assumed that $\lambda v = .75 \lambda p$.

(2) It has been assumed that the reliability of the voter and switching devices is sufficiently close to 1.0 that they can be ignored in this initial comparison.

With these assumptions in mind, a comparison between the two configurations is shown in Figures 4 through 7. These curves are presented in such a way that the units of time are the same as the units of $1/\lambda$.

SECTION V. CONCLUSIONS

Equations (3) and (6) enable one to calculate the reliability of two distinct stand-by configurations. These equations have been derived subject to the following assumptions:

(1) The reliability of the switching has been ignored, and

(2) If and when a unit fails, its failure can be detected, and a spare unit (if available) can be successfully switched into operation.

The primary conclusions that one can draw from Figures (4) through (7) are that:

(1) The system reliability will increase as the number of stand-by spares increases. (Actual amount of increase is optimistic because of the assumptions mentioned above;

(2) The voting system is not as sensitive to the unpowered failure rate (λ') of the stand-by units as the parallel system;

(3) The voting system is <u>not</u> competitive unless it utilizes some form of software error detection in the final phases;

(4) If the number of units are the same and $\lambda >> \lambda'$, the parallel system yields the higher reliability, while if $\lambda \approx \lambda'$, the voting system yields the higher reliability; and

(5) In order to obtain extremely high system reliabilities, the mission time (t) must be near the mean time between failure ($1/\lambda$) of the individual units.

Finally, it is obvious that the system reliability is very dependent upon the powered and unpowered failure rates, λ and λ' respectively. Thus, to obtain realistic numbers for estimates of system reliability, an effort needs to be made to obtain realistic values for these important parameters.

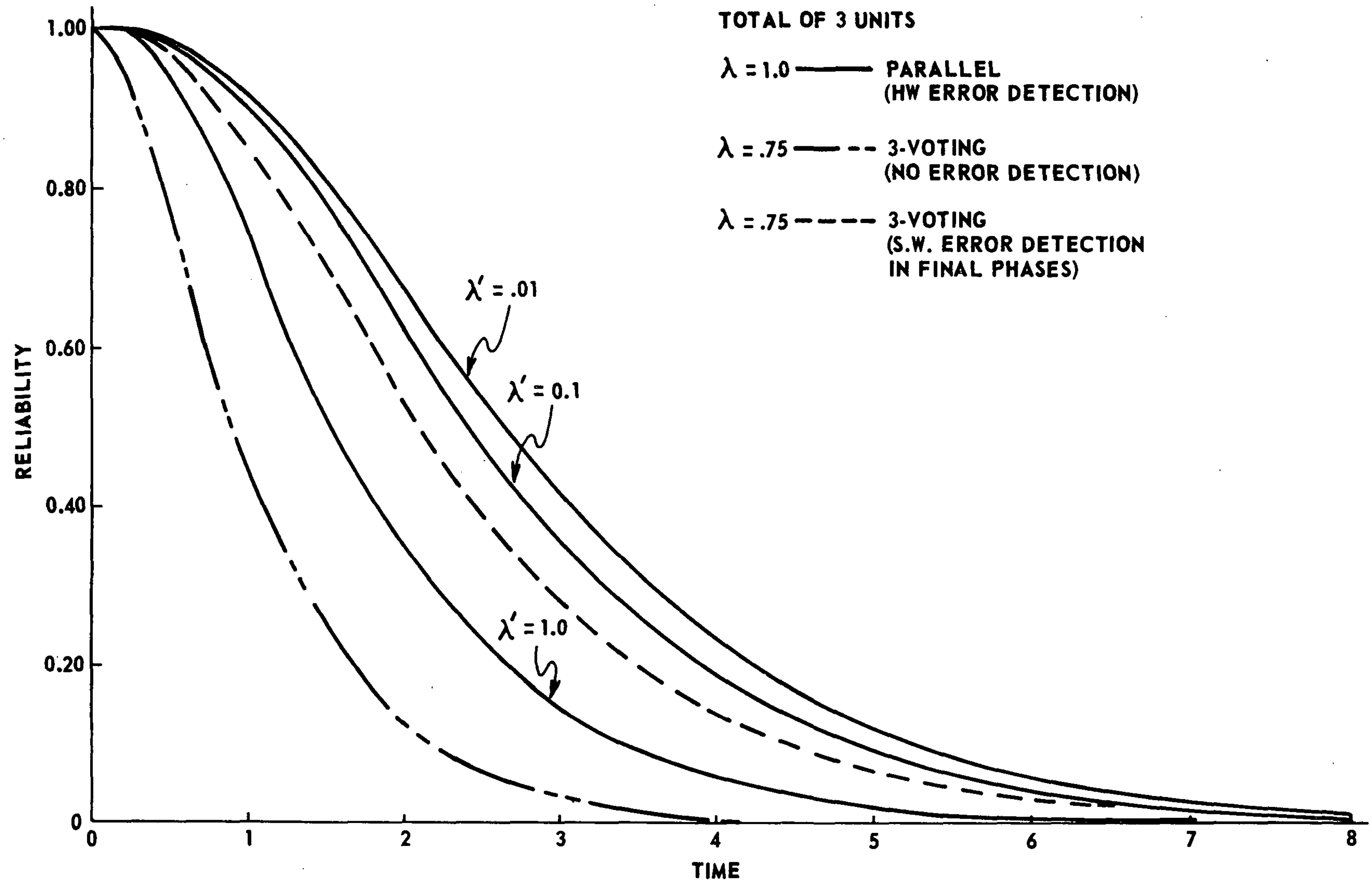
TOTAL OF 3 UNITS
λ = 1.0 PARALLEL (HW ERROR DETECTION)
λ = .75 3-VOTING (NO ERROR DETECTION)
λ = .75 3-VOTING (S.W. ERROR DETECTION IN FINAL PHASES)
λ′ = .01
λ′ = 0.1
λ′ = 1.0
RELIABILITY
1.00
0.80
0.60
0.40
0.20
0
TIME
0
1
2
3
4
5
6
7
8

FIGURE 4

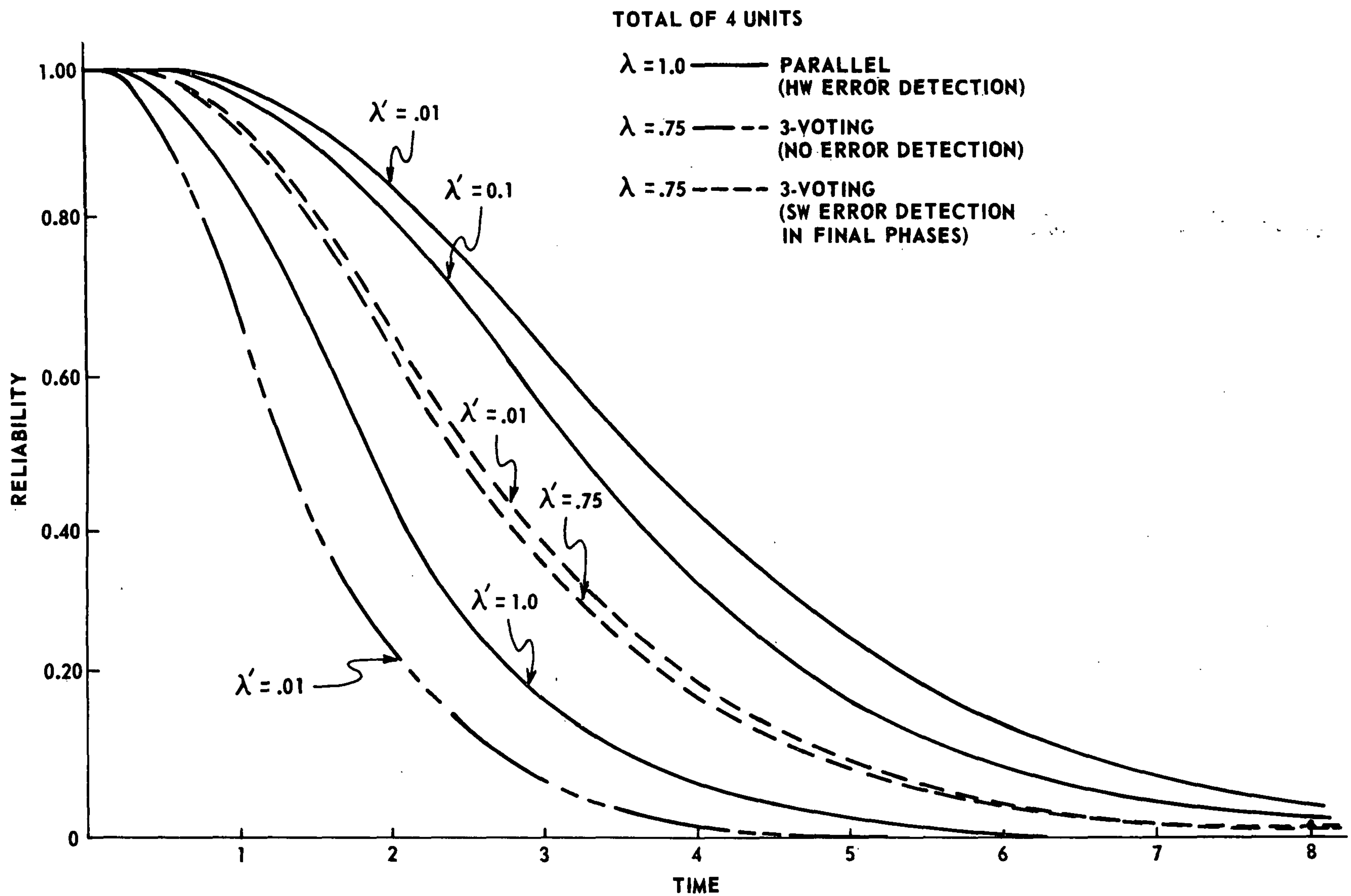
TOTAL OF 4 UNITS
λ = 1.0 PARALLEL (HW ERROR DETECTION)
λ = .75 3-VOTING (NO ERROR DETECTION)
λ = .75 3-VOTING (SW ERROR DETECTION IN FINAL PHASES)
λ' = .01
λ' = 0.1
λ' = .01
λ' = .75
λ' = 1.0
λ' = .01
RELIABILITY
1.00
0.80
0.60
0.40
0.20
0
TIME
1
2
3
4
5
6
7
8

FIGURE 5

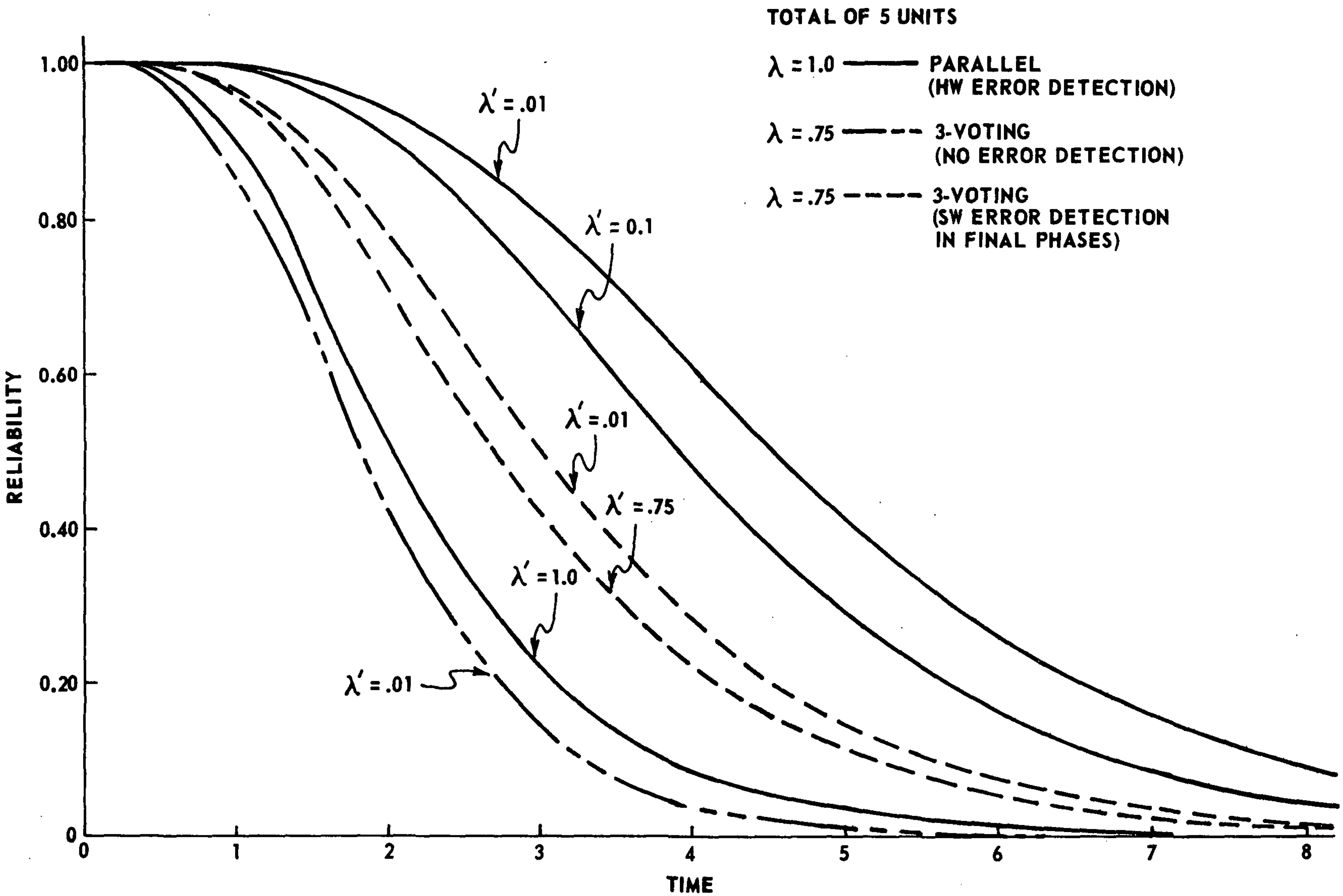
TOTAL OF 5 UNITS
λ = 1.0 PARALLEL (HW ERROR DETECTION)
λ = .75 3-VOTING (NO ERROR DETECTION)
λ = .75 3-VOTING (SW ERROR DETECTION IN FINAL PHASES)
λ' = .01
λ' = 0.1
λ' = .01
λ' = .75
λ' = 1.0
λ' = .01
RELIABILITY
1.00
0.80
0.60
0.40
0.20
0
TIME
0
1
2
3
4
5
6
7
8

FIGURE 6

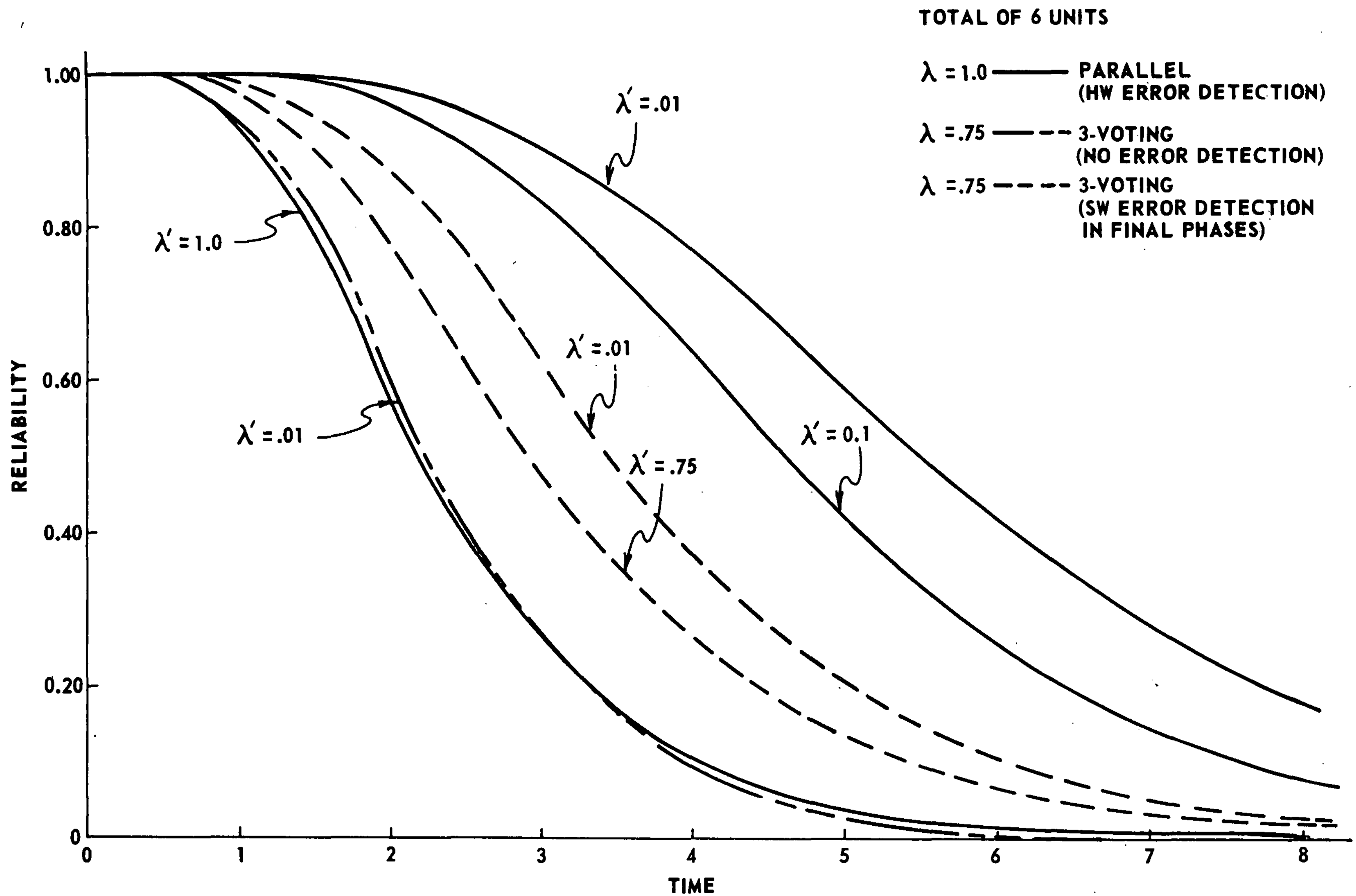
TOTAL OF 6 UNITS
λ = 1.0 PARALLEL (HW ERROR DETECTION)
λ = .75 3-VOTING (NO ERROR DETECTION)
λ = .75 3-VOTING (SW ERROR DETECTION IN FINAL PHASES)
λ′ = .01
λ′ = 1.0
λ′ = .01
λ′ = .01
λ′ = .75
λ′ = 0.1
RELIABILITY
1.00
0.80
0.60
0.40
0.20
0
TIME
0
1
2
3
4
5
6
7
8

FIGURE 7

APPENDIX 1

The results of integrating the equations in Table 1 are presented below.

$$P_0(0) = P_0(1) = P_0(2) = P_0(3) = e^{-3\lambda t}$$

$$P_1(0) = 3e^{-3\lambda t}\,(e^{\lambda t} - 1)$$

$$P_2(0) = 3e^{-3\lambda t}\,(1 - e^{\lambda t})^2$$

$$P_1(1) = P_1(2) = P_1(3) = \frac{3\lambda}{\lambda'}\,e^{-3\lambda t}\,(1 - e^{-\lambda' t})$$

$$P_2(1) = 3(3\lambda + \lambda')\,e^{-3\lambda t}\left\{\frac{e^{\lambda t} - e^{-\lambda' t}}{\lambda + \lambda'} - \frac{1 - e^{-\lambda' t}}{\lambda'}\right\}$$

$$P_3(1) = 3(3\lambda + \lambda')\,e^{-3\lambda t}\left\{\frac{e^{2\lambda t} - e^{-\lambda' t}}{2\lambda + \lambda'} + \frac{1 - e^{-\lambda' t}}{\lambda'} - \frac{2e^{\lambda t} - 2e^{-\lambda' t}}{\lambda + \lambda'}\right\}$$

$$P_2(2) = P_2(3) = \frac{3\lambda(3\lambda + \lambda')}{2\lambda'^2}\,e^{-3\lambda t}\,(1 - e^{-\lambda' t})^2$$

$$P_3(2) = 3(3\lambda + \lambda')(3\lambda + 2\lambda')e^{-3\lambda t}\left\{\frac{e^{\lambda t} - e^{-2\lambda' t}}{(\lambda + \lambda')(\lambda + 2\lambda')} - \frac{e^{-\lambda' t} - e^{-2\lambda' t}}{\lambda'(\lambda + \lambda)} - \frac{(1 - e^{-\lambda' t})^2}{2\lambda'^2}\right\}$$

$$P_4(2) = 3(3\lambda + \lambda')(3\lambda + 2\lambda')e^{-3\lambda t}\left\{\frac{e^{2\lambda t} - e^{-2\lambda' t}}{(2\lambda + \lambda')(2\lambda + 2\lambda')} - \frac{e^{-\lambda' t} - e^{-2\lambda' t}}{(2\lambda + \lambda')\lambda'} + \frac{(1 - e^{-\lambda' t})^2}{2\lambda'^2} - \frac{2e^{-\lambda t} - 2e^{-2\lambda' t}}{(\lambda + \lambda')(\lambda + 2\lambda')} + \frac{2e^{-\lambda' t} - 2e^{-2\lambda' t}}{(\lambda + \lambda')\,\lambda'}\right\}$$

$$P_3(3) = \frac{3\lambda\,(3\lambda + \lambda')(3\lambda + 2\lambda')}{6\lambda'^3}\left\{1 - e^{-\lambda' t}\right\}^3$$

$$P_4(3) = 3(3\lambda + \lambda')(3\lambda + 2\lambda')(3\lambda + 3\lambda')\, e^{-3\lambda t} \left\{ \frac{e^{\lambda t} - e^{-3\lambda' t}}{(\lambda + \lambda')(\lambda + 2\lambda')(\lambda + 3\lambda')} \right.$$

$$\left. - \frac{e^{-2\lambda' t} - e^{-3\lambda' t}}{\lambda'(\lambda + \lambda')(\lambda + 2\lambda')} - \frac{(1 - e^{-\lambda' t})(e^{-\lambda' t} - e^{-2\lambda' t})}{2\lambda'^2(\lambda + \lambda')} - \frac{(1 - e^{-\lambda' t})^3}{6\lambda'^3} \right\}$$

$$P_5(3) = 3(3\lambda + \lambda')(3\lambda + 2\lambda')(3\lambda + 3\lambda')\, e^{-3\lambda t} \left\{ \frac{e^{2\lambda t} - e^{-3\lambda t}}{(2\lambda + \lambda')(2\lambda + 2\lambda')(2\lambda + 3\lambda')} \right.$$

$$- \frac{e^{-2\lambda' t} - e^{-3\lambda' t}}{(2\lambda + \lambda')(2\lambda + 2\lambda')\lambda'} - \frac{e^{-\lambda' t}(1 - e^{-\lambda' t})^2}{2\lambda'^2 (2\lambda + \lambda')} + \frac{(1 - e^{-\lambda' t})^3}{6\lambda'^3}$$

$$\left. - \frac{2e^{-\lambda t} - 2e^{-3\lambda' t}}{(\lambda + \lambda')(\lambda + 2\lambda')(\lambda + 3\lambda')} + \frac{2e^{-2\lambda' t} - 2e^{-3\lambda' t}}{(\lambda + \lambda')(\lambda + 2\lambda')\lambda'} + \frac{e^{-\lambda' t}(1 - e^{-\lambda' t})^2}{\lambda'^2(\lambda + \lambda')} \right\}$$

REFERENCES

1. Sperry Rand, "MSFC Advanced Aerospace Computer," Report No. SP-232-0384, July 6, 1970.

2. P. Beckmann, "Reliability of Loaded Parallel-Redundant Systems," IEEE Annual Symposium on Reliability, January 10-12, 1967, p. 54-58.

3. D. K. Rubin, "The Approximate Reliability of Triply Redundant Majority-Voted Systems," IEEE Computer Conference, 1st Annual, September 6-8, 1967, p. 46-49.

4. D. E. Anderson, "Reliability of a Special Class of Redundant Systems," IEEE Transactions on Reliability, Volume R-18, Number 1, February, 1969, p. 21-28.

5. N. Deo, "Generalized Parallel Redundancy in Digital Computers," IEEE Transactions on Computers, June, 1968, p. 600.

6. R. B. Coffelt, "Automated System Reliability Prediction," Proceedings of 1967 Annual Symposium on Reliability, Washington, D. C., January, 1967.

7. P. A. Jensen, and M. Bellmore, "An Algorithm to Determine the Reliability of a Complex System," IEEE Transactions on Reliability, Volume R-18, Number 4, November, 1969.

8. I. Bazovsky, Reliability Theory and Practice, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1961.

9. ARINC Research Corporation, Reliability Engineering, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1964.

10. R. H. Wilcox and W. C. Mann, Redundancy Techniques for Computing Systems, Spartan, Washington, D. C., 1962.

11. W. H. Pierce and P. A. Jensen, Failure-Tolerant Computer Design, Academic Press, New York, 1965.

12. N. Deo, "Redundancy Optimization for Space Computers" NASA, Space Program Summary, Volume 4, Number 37-43, February, 1967.

MSFC—RSA, Ala