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THE APPLICATION OF OPTIMAL CONTROL TECHNIQUES TO ADVANCED MANNED MISSIONS

VOLUME II

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by

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FOREWORD

This report presents in two volumes the results of work performed during the period of May 1970 to February 1972 by Lockheed's Huntsville Research & Engineering Center while under contract to the National Aeronautics and Space Administration for the Aero-Astrodynamics Laboratory of Marshall Space Flight Center (MSFC), Contract NAS8-25578.

The report documents the work performed on the "Application of Optimal Techniques to Advanced Manned Missions," namely the Composite Shuttle Ascent Phase. Mr. J. M. Livingston of NASA-MSFC, Aero-Astrodynamics Laboratory, S&E-AERO-DF, was the MSFC Contracting Officer's Representative. Mr. C. L. Connor was the project engineer at Lockheed. Major contributors were Dr. W. Trautwein, who provided technical assistance, and Mr. A. Hansing, who performed the hybrid programming.

The work reported in this Volume II is included as a detailed source of information to supplement the results of Volume I.

CONTENTS

SUMMARY AND INTRODUCTION

The work reported in this Volume II is included as a detailed source of information to supplement the results of Volume I.

Appendix A describes the hybrid optimization technique in detail. This optimization technique is capable of optimizing an n-dimensional adjustable parameter vector, but a 1-dimensional vector is used as an example to explain the procedure. This allows an easier explanation as opposed to a multi-dimensional case.

Appendix B describes the procedure used to derive the perturbation equations of motion describing the 6-DOF Shuttle Ascent Phase. These equations were programmed on the EAI 8800 analog computer to describe the perturbations of the shuttle vehicle from a nominal zero-lift trajectory due to wind disturbances. Included are the control system equations, trim equations, and wind angle of attack equations.

Appendix C describes the procedure used to derive the equations of motion describing the structural forces at all four attachment points between the orbiter and booster.

Appendix D shows the analog wiring diagrams used in this study. Included are the rotational and translational perturbation EOM, interface loading EOM, control system EOM, wind disturbance generation and performance criteria (J = PC). Manual switches provided capability for the wind disturbances to come from any direction relative to the trajectory plane.

Appendix E includes all raw data and the time-varying coefficients generated by the raw data in SC 4020 plot form. Aerodynamic data were referenced to the center of gravity, X_{cg}, measured from the gimbal point.

Appendix F shows the plots of all state variable responses during shuttle ascent to a 0 deg headwind α_{w_A} and a 90 deg sidewind β_{w_A} for a constant gain controller. These plots were obtained by a digital program using Runge Kutta numerical integration techniques.

Appendix G shows the plots of all state variable responses during shuttle ascent to a 0 deg headwind α_{w_A} and a 90 deg sidewind β_{w_A} for the optimal gain controller obtained from the hybrid studies. The same program used to obtain plots in Appendix F was used to generate these plots.

Appendix A
DETAILED DESCRIPTION OF LOCKHEED'S HYBRID OPTIMIZER

The basic scheme and its operational features were described in generalities in Volume I of this report. The purpose of this section of Volume II is to provide the reader with a more detailed description of the hybrid optimizer and its application to the design of a specific optimal control system (Shuttle Ascent).

A.1 BASIC SCHEME

The basic scheme of the hybrid optimizer program is a direct optimization method, whereby only forward integrations of the dynamic equations are performed. Figure A-1 shows a block diagram of the optimizer being applied to the design of a 3-axis attitude controller for the shuttle launch phase. Complete 6-DOF shuttle dynamics, engine actuator dynamics, controller loops and the performance index penalty function are simulated on the EAI 8800 analog computer. The optimizer is simulated on the EAI 8400 digital computer. Figure A-1 is intended to orient the reader toward a specific application of the optimizer. The specific objective of this block diagram is to compute optimal schedules for the 3-axis controller gains $\left[a_{0_\theta}, a_{1_\theta}, \ldots, a_{1_\phi}\right]^T$ which maximize the orbital insertion payload while desensitizing vehicle performance to the possible occurrence of two disturbance winds.

The operational features of the optimizer will now be described in detail for a general one-dimensional problem. Their application to the specific problem of Fig. A-1 will then be dealt with in a chronological manner.

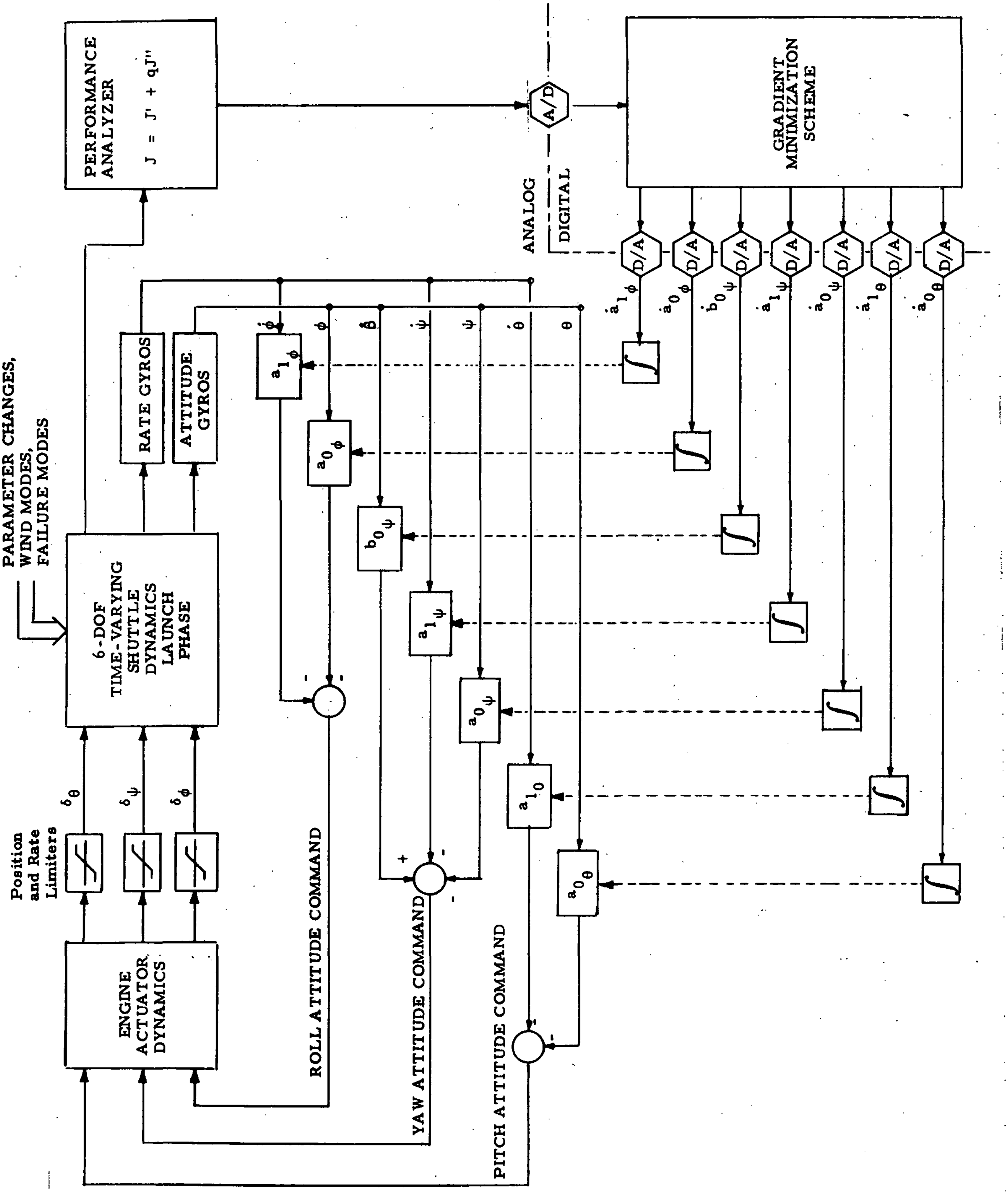


Fig. A-1 - Block Diagram of Hybrid Computer Optimizer as Applied to Optimal Design of a 3-Axis Controller

A.2 STEP-BY-STEP OPTIMIZATION PROCEDURE

Typically, the shuttle dynamics are simulated from 0 to 100 seconds; i.e., from launch through atmospheric ascent. Since shuttle performance is affected most adversely during the region of maximum dynamic pressure, this region is of primary interest. For example, if maximum dynamic pressure occurs at 60 sec, it is only necessary to design the optimal controller during the region from 40 to 80 seconds typically rather than the entire launch period (0 to 100 sec).

The study proceeds in the following steps.

- <u>Step 1</u>

The shuttle is launched (simulated in real time on analog) with all controller gains held constant. A real-time strip chart recorder records desired state variables. At flight time 40 sec, the real-time strip chart is stopped and the optimizer begins its assigned objective; i.e., optimize the controller gains from 40 to 80 sec flight time through a series of fast-time iterations and specified update real-time intervals. Figure A-2 shows this Step 1.

- <u>Step 2</u>

The next step the optimizer performs is to conduct a search of all possible gain slope combinations for all gains to avoid any local minima which may exist. A subroutine entitled "GRID" is called to perform this "grid search." This grid consists of all combinations of all gain slopes defined with finite limits and finess. At each grid point, the vehicle dynamics are integrated for T sec (typically 20 sec) at 1000 times real time from the initial conditions existing at 40 sec for Wind A, and then again for Wind B. For each wind simulation, the performance index is computed. Digital logic maintains in memory the worst performance at each grid point. When the entire grid search is completed, the digital is able to identify the grid search coordinates

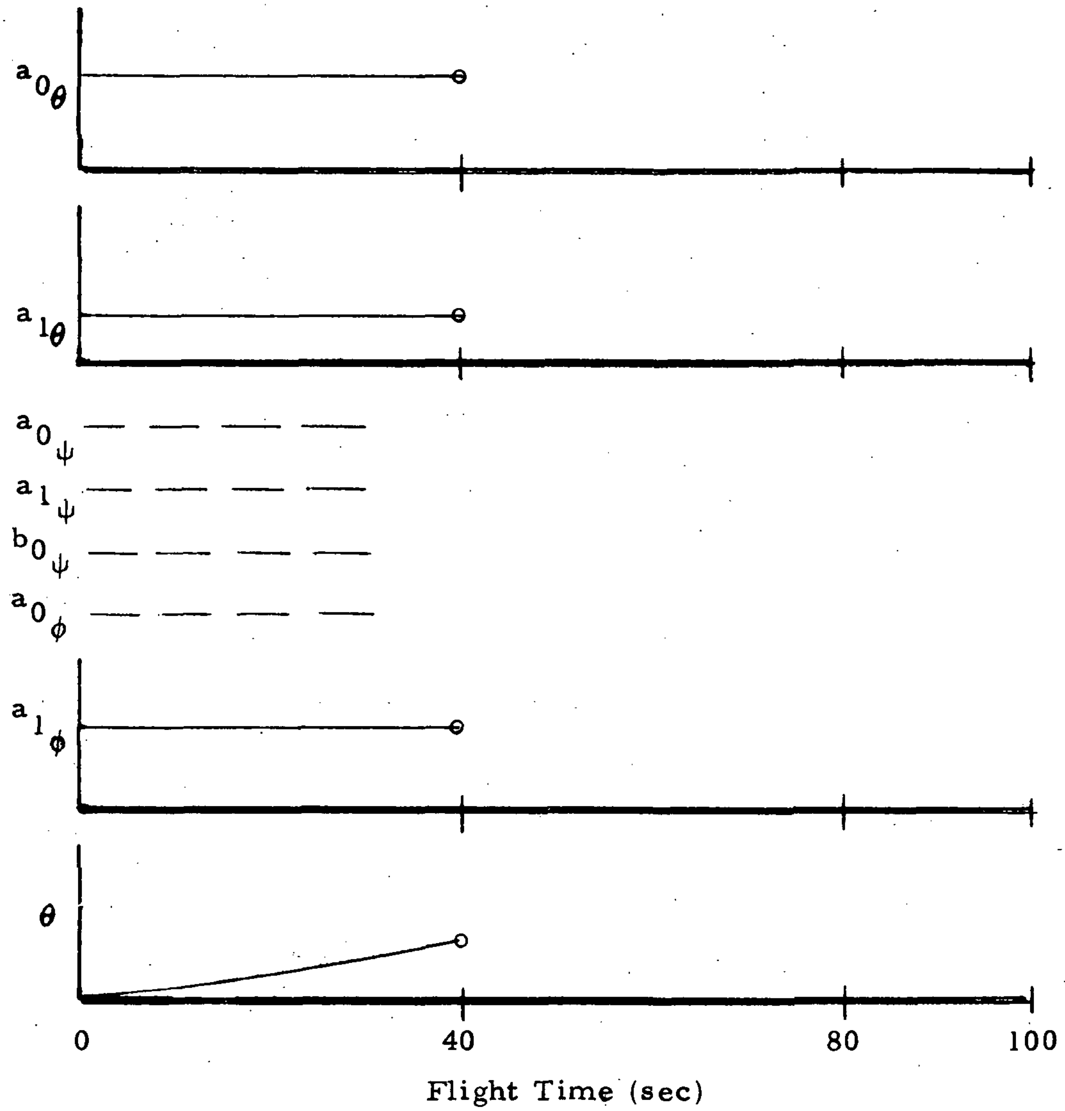


Fig. A-2 - Typical Strip Chart Recordings of the Analog Simulation of 3-Axis Controller for Shuttle Ascent. Shown are typical control gains and a specific state variable integrated on the analog computer from 0 to starting time of optimizer (40 sec).

at which the best performance of all the worst performances occurred. This grid search coordinate is used to initialize the gradient search. Keep in mind, that the optimizer is conducting all grid search and gradient search computations at 1000 times real time. Figures A-3 and A-4 show the grid search operation. The heavy lines of Fig. A-3 indicate the grid point coordinates corresponding to the gain slope combinations which give the best performance for the "look-ahead" interval 40 to 60 sec. The gradient search will improve these slopes. Figure A-4 is included to illustrate a typical performance function J(K) and how the grid search avoids possible local minimum points.

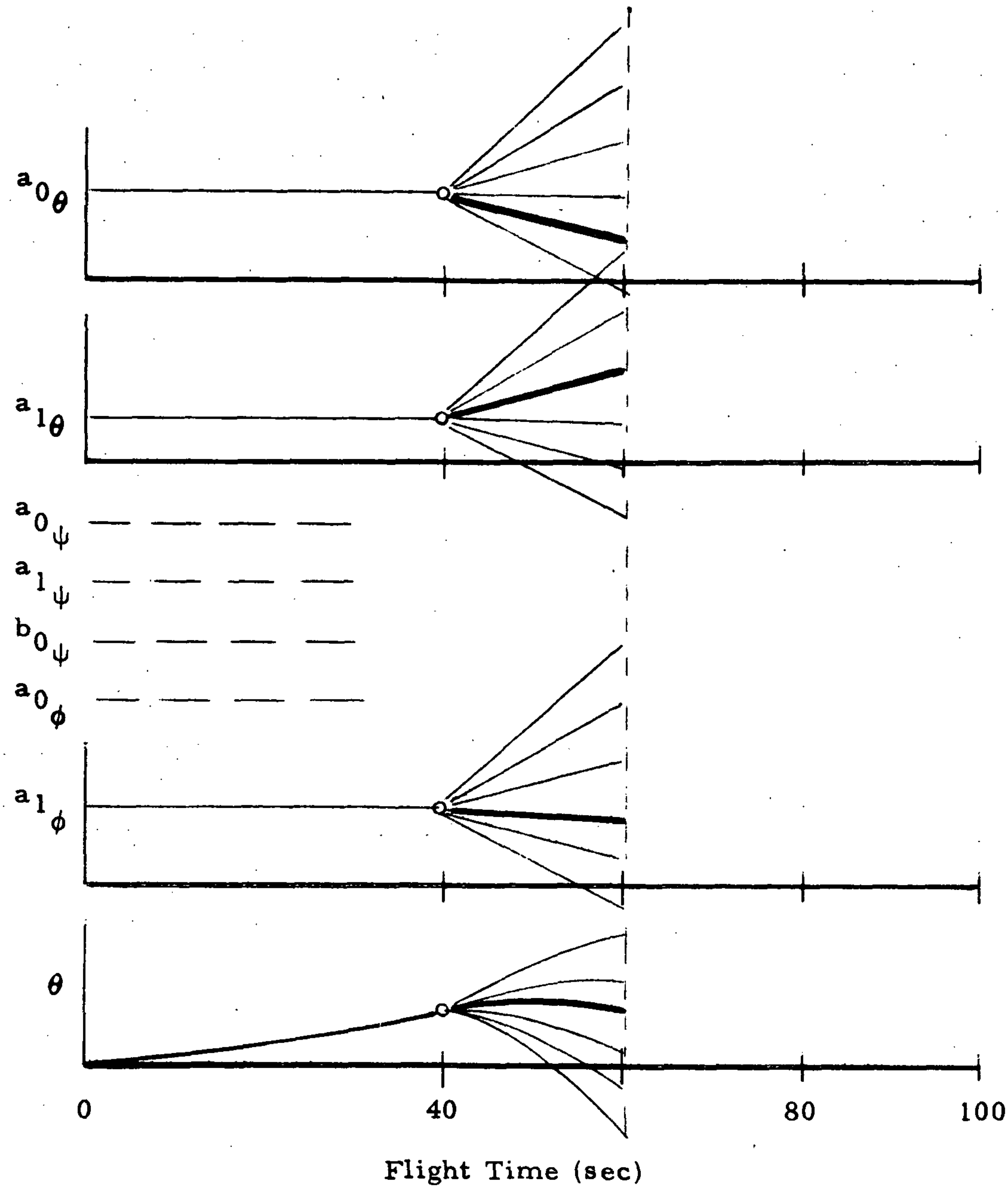
- Step 3

The objective of the gradient search is to refine these grid search coordinates to locate the absolute minimum. The gradient search utilizes this grid point minimum to begin its work. Digital logic has determined which wind case (A or B) caused the worst performance of both winds. The wind causing the worst performance is used to compute the initial gradient computation at $J(K_i)_A$; $\nabla J(K_i)_A$ since α_{w_A} was the worst case. Figure A-5 shows these gradient computations. $\nabla J(K_i)_A$ is computed by a defined perturbation of ΔK which is

$$\nabla J(K_i)_A = \left[J(K_i + \frac{\Delta K}{2})_A - J(K_i - \frac{\Delta K}{2})_A \right] / \Delta K$$

Experience had proven it unnecessary to simulate both winds for the gradient computation since the small perturbations of ΔK had negligible effects on vehicle performance. Techniques developed by Fletcher and Powell compute K_i^* based on the gradient at K_i, $(\nabla J(K_i)_A)$. The vehicle performance is evaluated at K_i^* for Winds A and B, and the worst case is used to compute $\nabla J(K_i^*)_B$ in the same previous manner since α_{w_B} is the worst case for $K^*_{i_B}$. The gradient $\nabla J(K_i^*)_B$ is computed as before

$$\nabla J(K_i^*)_B = \left[J(K_i^* + \frac{\Delta K}{2})_B - J(K_i^* - \frac{\Delta K}{2})_B \right] / \Delta K$$



NOTE: Heavy lines indicate gain slope combination giving best performance of the grid search.

Fig. A-3 - Grid Search Operation at Desired Optimization Starting Time (40 sec). All controller slopes are simulated at 1000 times real time over an integration look-ahead interval of 20 sec. Both winds are simulated for each slope combination. The heavy lines indicate the resulting grid search minimum which the gradient search will improve.

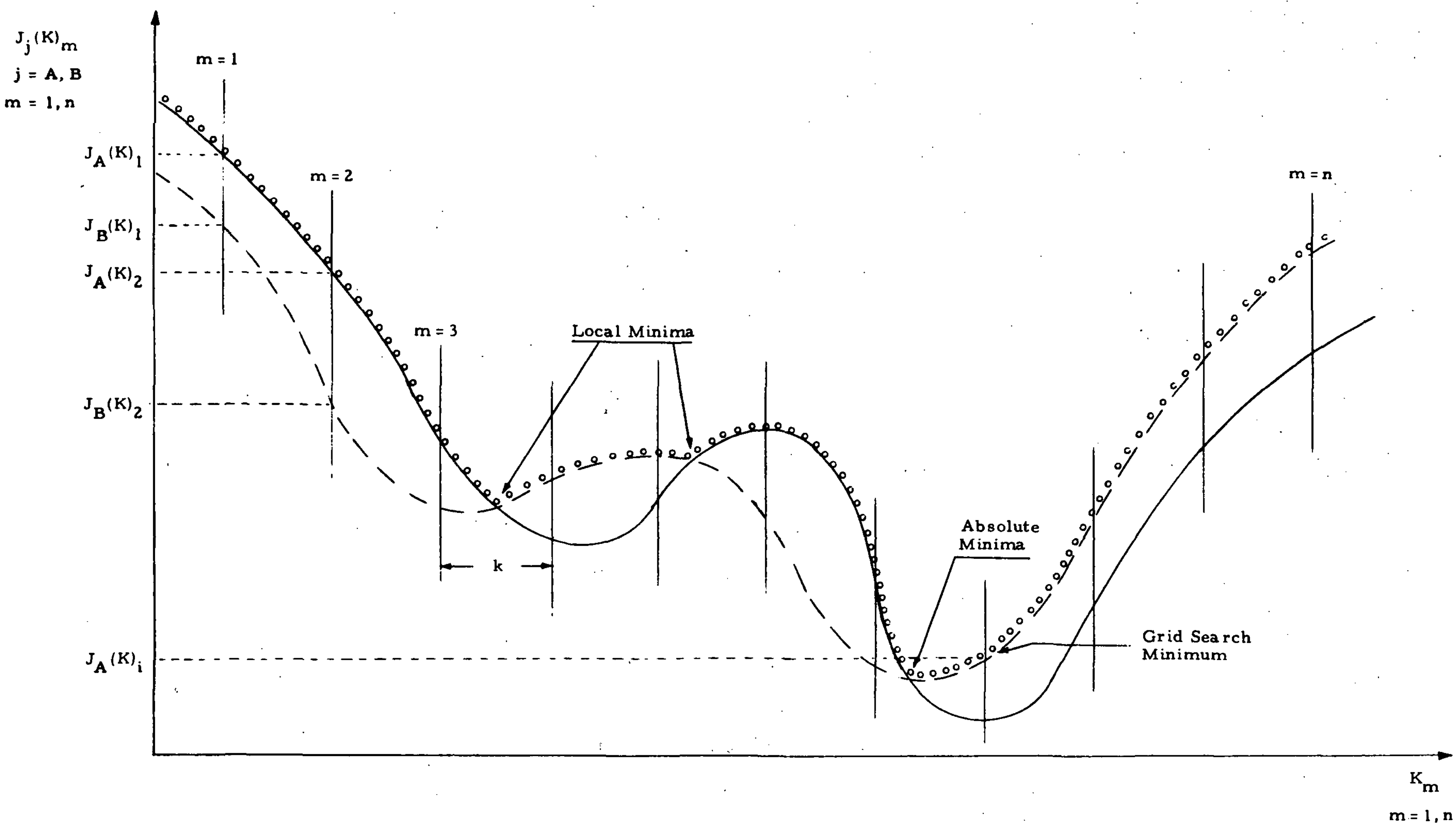


Fig. A-4 - Two-Dimensional View of Grid Search Avoiding Local Minimum of Performance Measure J for Two Adverse Conditions (A and B) for n Increments of the Adjustable Parameter (K). Grid search step represented by adjustable variable (k).

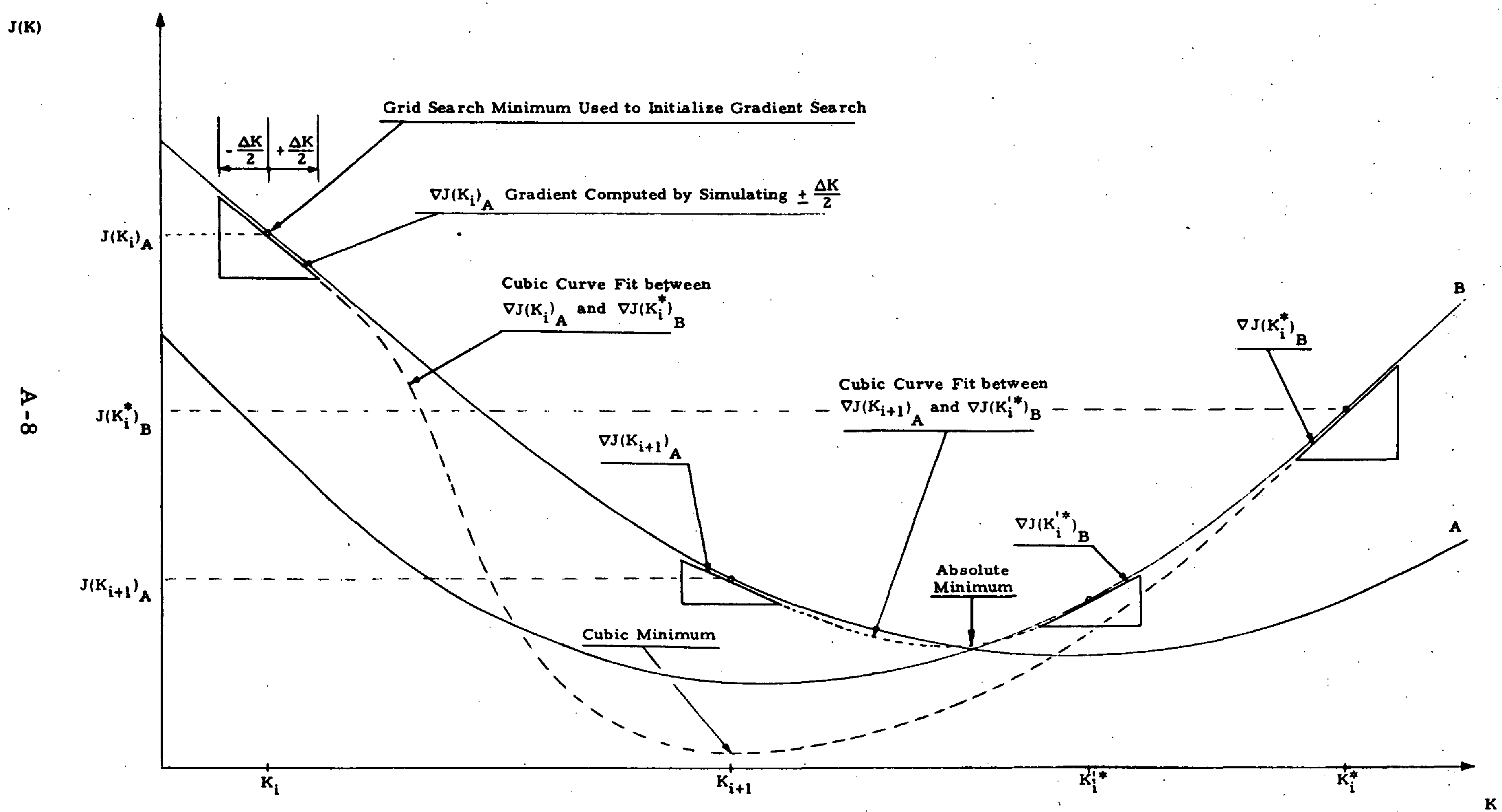


Fig. A-5 - General Operation of a Two-Dimensional View of the Gradient Search Operation Beginning with Grid Search Minimum. This curve J(K) represents an enlarged view of the region near the absolute minimum.

The gradient search continues by fitting a 3rd order polynomial between $\nabla J(K_i)_A$ and $\nabla J(K_i^*)_B$. The minimum of this polynomial is determined by numerical techniques and is shown at K_{i+1}. Performance at K_{i+1} is evaluated for both winds A and B. Since α_{w_A} is the worst case, the gradient $\nabla J(K_{i+1})_A$ is computed at K_{i+1} in the same manner; i.e.,

$$\nabla J(K_{i+1})_A = \left[J\left(K_{i+1} + \frac{\Delta K}{2}\right)_A - J\left(K_{i+1} - \frac{\Delta K}{2}\right)_A \right] / \Delta K$$

Again, a new K_i^* is computed based on $\nabla J(K_{i+1})_A$ and is shown in Fig. A-5 as $K_i'^*$. Performance is again evaluated at $K_i'^*$ for α_{w_A} and α_{w_B}. Since α_{w_B} is the worst case, the gradients at $K_i'^*$ are computed and another 3rd order polynomial is fitted between $\nabla J(K_{i+1})_A$ and $\nabla J(K_i'^*)_B$.

The minimum of this cubic is determined as before to establish a new (K_{i+1}). The gradient search continues in this manner for a specified number of such flip-flop iterations until the absolute minimum $J(K_m)$ is found by the gradient $\nabla J(K_m) \rightarrow 0$. The parameter vector (K_m) which represents the optimal controller gain slopes is transferred to the analog where the vehicle dynamics are integrated in real time from 40 to 45 sec, typically. This response is shown in Fig. A-6. The initial conditions for all state variables are now at 45 sec, and the digital logic returns to the grid search to begin again the optimization cycle. This cycle is continued until the specified flight time of 80 sec, at which time all gain slopes are zeroed and the vehicle dynamics are integrated in real time to completion of trajectory (100 sec). Figure A-7 shows the resulting gain slope schedules and corresponding response of θ. Figure A-8 is included to show a typical operation of the grid search and the speed at which it operates.

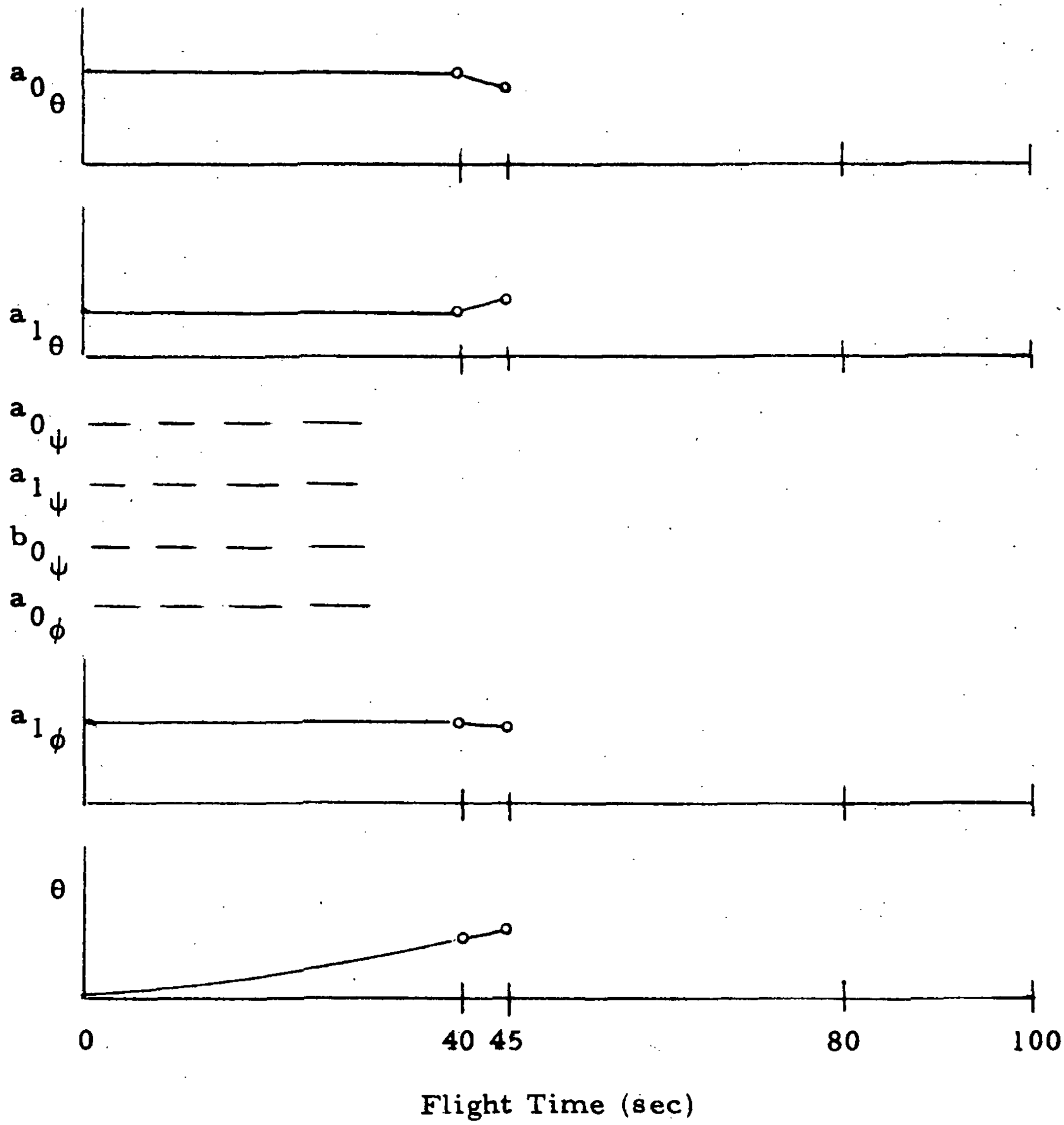


Fig. A-6 - Real Time Update from 40 to 45 sec of Controller Schedules and Shuttle Dynamics Using Optimal Controller Slopes Determined by Optimization Technique. New grid search begins iterations on initial conditions of all states at 45 sec.

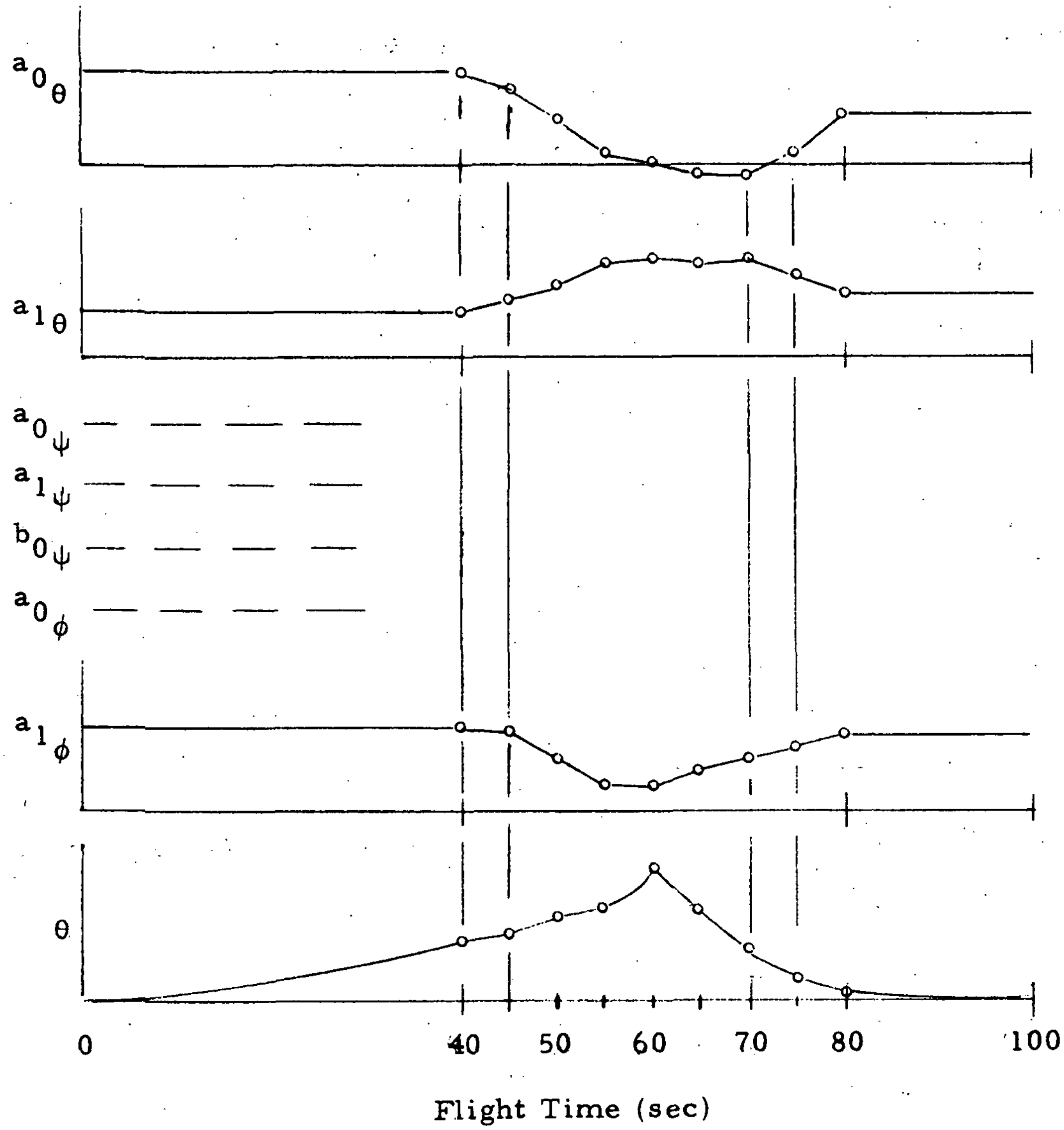


Fig. A-7 - Optimal Controller Schedules and State Variable Response as Obtained from Series of Optimizations and Update Intervals (40 to 45, 45 to 50, ..., 75 to 80 sec). Controller gain slopes are set to zero at optimization stop time (80 sec) and system dynamics integrated to conclusion (100 sec).

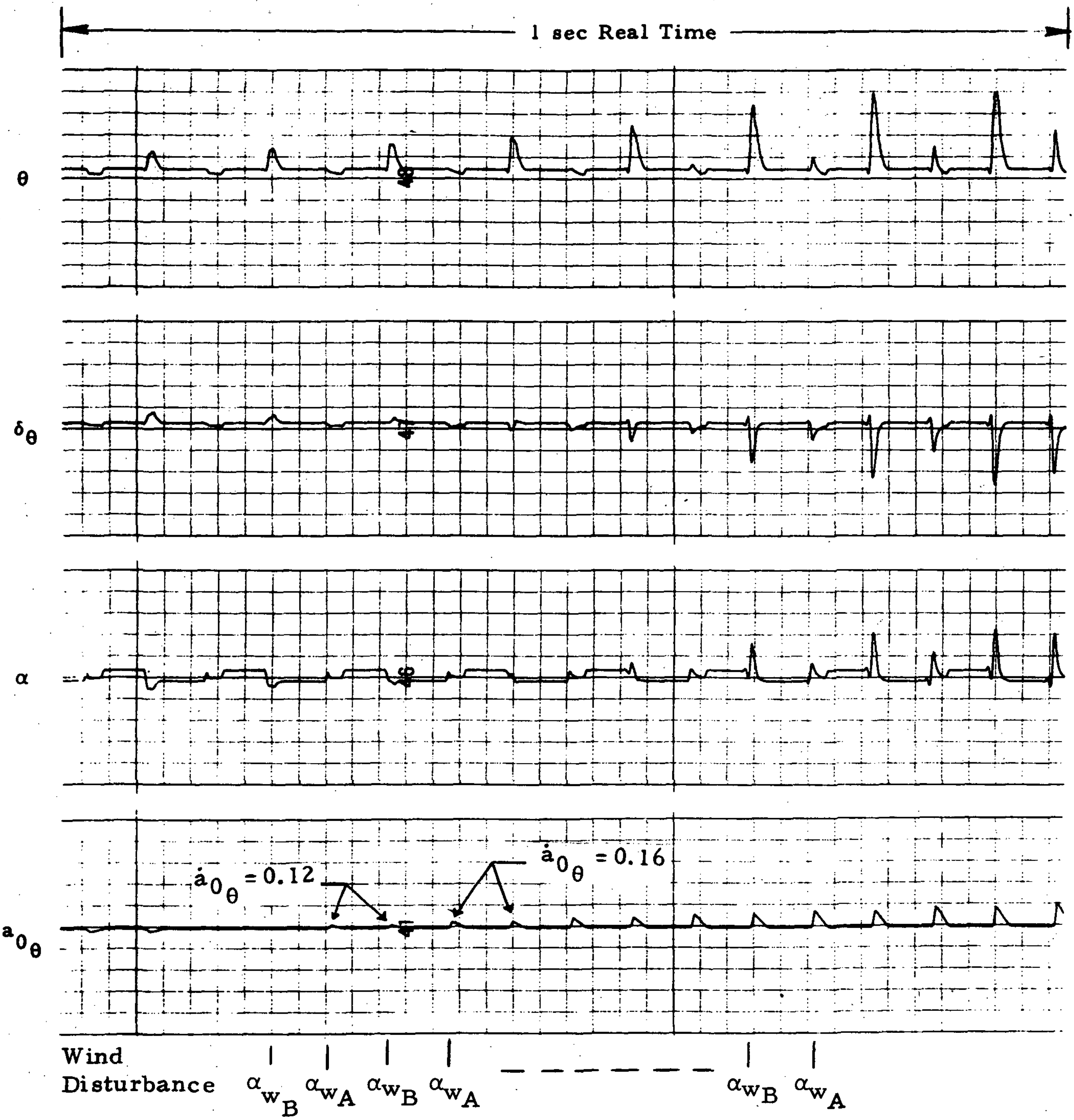
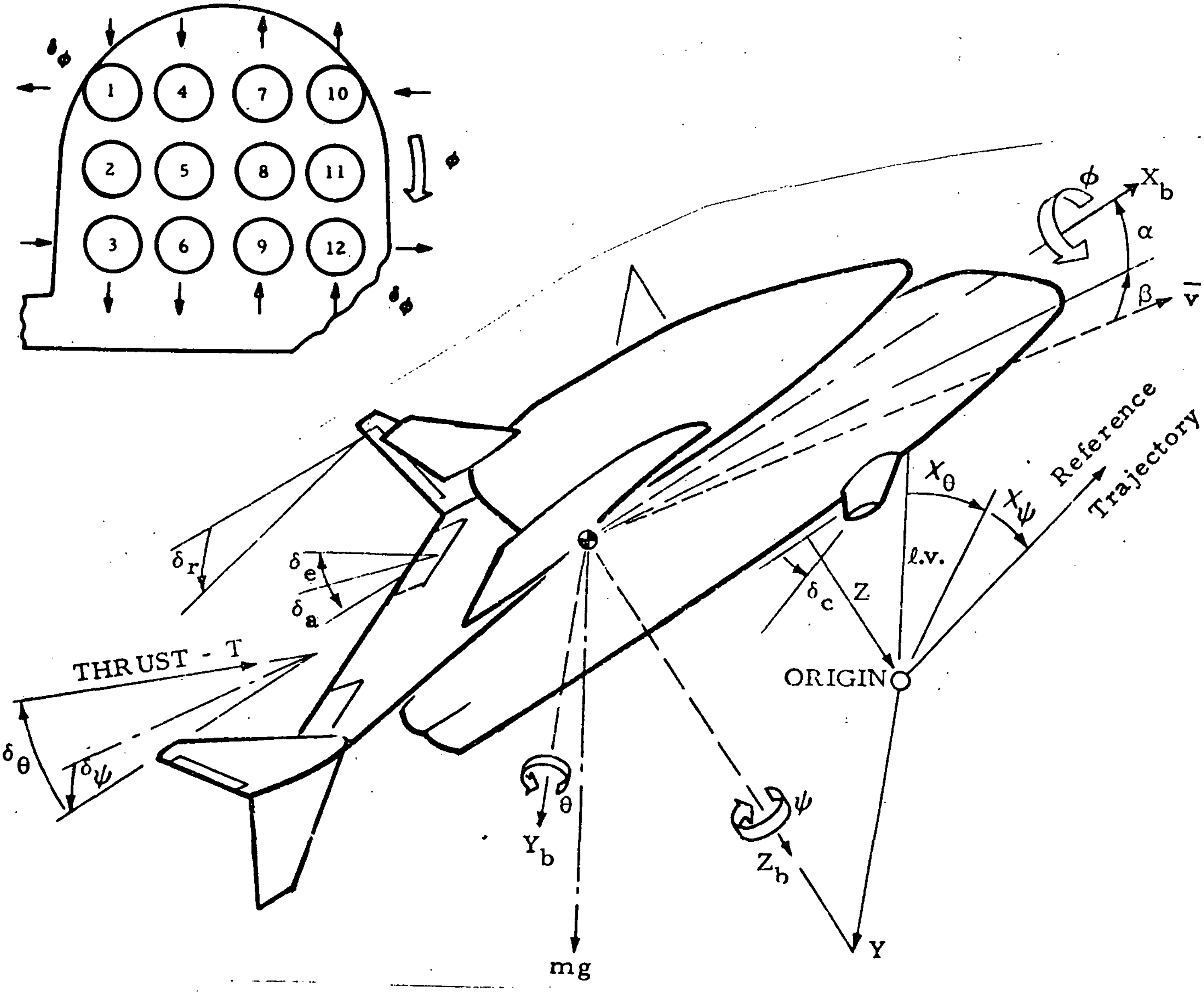


Fig. A-8 - Grid Search Operation Showing Typical Strip Chart Recordings of Several State Variable Responses to Changes in a_{0_θ} for the two Wind Disturbances α_{w_A} and α_{w_B}. Simulations are occurring at 1000 times real time resulting in 17 simulations per sec.

Appendix B

DERIVATION OF THE 6-D PERTURBATION EOM
FOR THE COMPOSITE SHUTTLE LAUNCH PHASE

The equations of motion are derived with respect to the figure below. Engine and surface deflections were defined as positive deflections causing positive attitude changes.



1. Assumptions

a.) I_{xy}, I_{yz} very small

b.) $\frac{I_{xz}}{I_x} >> \frac{I_{xz}}{I_z}$ and $\frac{I_{xz}}{I_y}$

c.) m, I_x, I_y, I_z, I_{xz} are constant 0 - 100 sec.

d.) $\hat{\phi}, \hat{\psi}$ are very small angles

$$\hat{P} = \dot{\hat{\phi}} + \dot{\hat{\theta}} \sin\hat{\phi} \cong \dot{\hat{\phi}}$$

$$\hat{Q} = \dot{\hat{\theta}} \cos\hat{\psi} \cos\hat{\phi} + \dot{\hat{\psi}} \sin\hat{\phi} \cong \dot{\hat{\theta}}$$

$$\hat{R} = \dot{\hat{\psi}} \cos\hat{\phi} - \dot{\hat{\theta}} \cos\hat{\psi} \sin\hat{\phi} \cong \dot{\hat{\psi}}$$

e.) $\cos\hat{\alpha} = 1 \qquad \cos\hat{\beta} = 1$

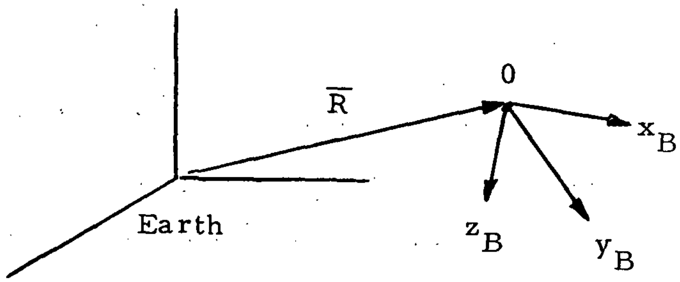
$\sin\hat{\alpha} = \hat{\alpha} \qquad \sin\hat{\beta} = \hat{\beta}$

f.) $\dot{\hat{X}} = \hat{U}$

$\dot{\hat{Y}} = \hat{V} \cong \hat{U}\hat{\beta}$

$\dot{\hat{Z}} = \hat{W} \cong \hat{U}\hat{\alpha}$

g.) Equations of motion are derived with respect to the center of gravity of the composite vehicle; i.e., the equations are body-frame equations.



2. Acceleration

a.) $$\ddot{\vec{R}} = \dot{\vec{U}} + \vec{\omega} \times \vec{U} = \frac{\vec{F}}{m}$$

where

$\ddot{\vec{R}}$ is acceleration of body frame

$\vec{\omega}$ is angular velocity of body frame

$\vec{U}$ is velocity vector of body frame

Since

$$\vec{\omega} = \begin{bmatrix} \hat{P} \\ \hat{Q} \\ \hat{R} \end{bmatrix} \quad \text{and} \quad \vec{U} = \begin{bmatrix} \hat{U} \\ \hat{V} \\ \hat{W} \end{bmatrix}$$

then

b.) $$\vec{\omega} \times \vec{U} = \begin{vmatrix} 1 & 1 & 1 \\ \hat{P} & \hat{Q} & \hat{R} \\ \hat{U} & \hat{V} & \hat{W} \end{vmatrix} = \hat{Q}\hat{W} - \hat{R}\hat{V}] - [\hat{P}\hat{W} - \hat{R}\hat{U} + [\hat{P}\hat{V} - \hat{Q}\hat{U}]$$

Therefore

c.) $$\begin{bmatrix} \dot{\hat{U}} + \hat{Q}\hat{W} - \hat{R}\hat{V} \\ \dot{\hat{V}} + \hat{R}\hat{U} - \hat{P}\hat{W} \\ \dot{\hat{W}} + \hat{P}\hat{V} - \hat{Q}\hat{U} \end{bmatrix} = \begin{bmatrix} \hat{F}_x/m \\ \hat{F}_y/m \\ \hat{F}_z/m \end{bmatrix}$$

where $\left[\hat{F}_x, \hat{F}_y, \hat{F}_z\right]$ are total forces acting on vehicle.

Total forces are

d.) $$\begin{bmatrix} \hat{F}_x \\ \hat{F}_y \\ \hat{F}_z \end{bmatrix} = \begin{bmatrix} \hat{F}_{x\,aero} + \hat{F}_{x\,grav} + \hat{F}_{x\,prop} \\ \hat{F}_{y\,aero} + \hat{F}_{y\,grav} + \hat{F}_{y\,prop} \\ \hat{F}_{z\,aero} + \hat{F}_{z\,grav} + \hat{F}_{z\,prop} \end{bmatrix}$$

which consist of summation of forces due to aerodynamics, gravity and propulsion.

From Eq. (2-c), page B-3, and the assumptions, we have

e.) $$\begin{bmatrix} \ddot{\hat{X}} \\ \ddot{\hat{Y}} \\ \ddot{\hat{Z}} \end{bmatrix} \cong \begin{bmatrix} \hat{U}\hat{\beta}\dot{\hat{\psi}} - \hat{U}\hat{\alpha}\dot{\hat{\theta}} + \hat{F}_x/m \\ \hat{U}\hat{\alpha}\dot{\hat{\phi}} - \hat{U}\dot{\hat{\psi}} + \hat{F}_y/m \\ \hat{U}\dot{\hat{\theta}} - \hat{U}\hat{\beta}\dot{\hat{\phi}} + \hat{F}_z/m \end{bmatrix}$$

Also

f.) $$\begin{bmatrix} \ddot{\hat{\phi}} \\ \ddot{\hat{\theta}} \\ \ddot{\hat{\psi}} \end{bmatrix} \cong \begin{bmatrix} \frac{I_{xz}}{I_x}\ddot{\psi} + \frac{\hat{L}}{I_x} \\ \frac{\hat{M}}{I_y} \\ \frac{\hat{N}}{I_z} \end{bmatrix}$$

3. <u>Separation into Perturbation and Nominal Equations</u>

a.) $$\hat{X} = X_o + x \qquad \dot{\hat{\phi}} \cong \dot{\phi}$$
$$\hat{Y} = Y_o + y \qquad \dot{\hat{\theta}} \cong \dot{\theta}$$
$$\hat{Z} = Z_o + z \qquad \dot{\hat{\psi}} \cong \dot{\psi}$$

i.e., $\dot{\phi}_o$, $\dot{\theta}_o$, $\dot{\psi}_o$ are small

$$\hat{\beta} = \beta \qquad \hat{L} = L_o + L \qquad \hat{F}_x = F_{xo} + F_x$$
$$\hat{\alpha} = \alpha_o + \alpha \qquad \hat{M} = M_o + M \qquad \hat{F}_y = F_{yo} + F_y$$
$$\hat{N} = N_o + N \qquad \hat{F}_z = F_{zo} + F_z$$

$\hat{\delta}_e = \delta_e$; elevon $\qquad \hat{\delta}_\phi = \delta_\phi$

$\hat{\delta}_r = \delta_r$; rudder $\qquad \hat{\delta}_\psi = \delta_\psi$

$\hat{\delta}_a = \delta_a$; aileron $\qquad \hat{\delta}_\theta = \delta_{\theta o} + \delta_\theta$

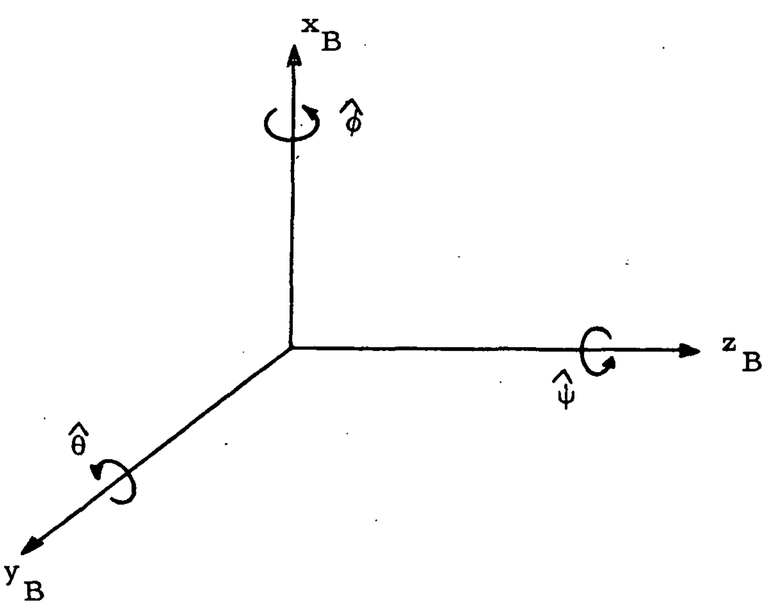
$\hat{U} = U_o + u$

Separate Eq. (2-e) on page B-4 into perturbation and nominal components.

b.) $$\begin{bmatrix} \ddot{X}_o + \ddot{X} \\ \ddot{Y}_o + \ddot{Y} \\ \ddot{Z}_o + \ddot{Z} \end{bmatrix} = \begin{bmatrix} (U_o + u)\beta\dot{\psi} - (U_o + u)(\alpha_o + \alpha)\dot{\theta} + (F_{xo} + F_x)/m \\ (U_o + u)(\alpha_o + \alpha)\dot{\phi} - (U_o + u)\dot{\psi} + (F_{yo} + F_y)/m \\ (U_o + u)\dot{\theta} - (U_o + u)\beta\dot{\phi} + (F_{zo} + F_z)/m \end{bmatrix}$$

4. Gravitational Forces

a.) Assume initial position of shuttle on the launch pad and a 2, 3, 1 rotation of the euler angles ϕ, ψ, θ.



b.) $$\begin{bmatrix} \hat{F}_{xg} \\ \hat{F}_{yg} \\ \hat{F}_{zg} \end{bmatrix} = [\hat{\phi}]_1 [\hat{\psi}]_3 [\hat{\theta}]_2 \begin{bmatrix} -mg \\ 0 \\ 0 \end{bmatrix}$$

Therefore

c.) $$\begin{bmatrix} \hat{F}_{xg} \\ \hat{F}_{yg} \\ \hat{F}_{zg} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & C\hat{\phi} & S\hat{\phi} \\ 0 & -S\hat{\phi} & C\hat{\phi} \end{bmatrix} \begin{bmatrix} C\hat{\psi} & S\hat{\psi} & 0 \\ -S\hat{\psi} & C\hat{\psi} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\hat{\theta} & 0 & -S\hat{\theta} \\ 0 & 1 & 0 \\ S\hat{\theta} & 0 & C\hat{\theta} \end{bmatrix} \begin{bmatrix} -mg \\ 0 \\ 0 \end{bmatrix}$$

Since $\hat{\psi}$ and $\hat{\phi}$ are small, then

d.) $$\begin{bmatrix} \hat{F}_{xg} \\ \hat{F}_{yg} \\ \hat{F}_{zg} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \hat{\phi} \\ 0 & -\hat{\phi} & 1 \end{bmatrix} \begin{bmatrix} 1 & \hat{\psi} & 0 \\ -\hat{\psi} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -mg\cos\hat{\theta} \\ 0 \\ -mg\sin\hat{\theta} \end{bmatrix}$$

which reduces to

e.) $$\begin{bmatrix} \hat{F}_{xg} \\ \hat{F}_{yg} \\ \hat{F}_{zg} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \hat{\phi} \\ 0 & -\hat{\phi} & 1 \end{bmatrix} \begin{bmatrix} -mg\cos\hat{\theta} \\ \hat{\psi}\ mg\cos\hat{\theta} \\ -mg\sin\hat{\theta} \end{bmatrix}$$

$$= \begin{bmatrix} -mg\cos\hat{\theta} \\ \hat{\psi}\ mg\cos\hat{\theta} - \phi mg\sin\hat{\theta} \\ -\hat{\phi}\,\hat{\psi}\ mg\cos\hat{\theta} - mg\sin\hat{\theta} \end{bmatrix}$$

which may be simplified by small angle approximation to

f.)
$$\begin{bmatrix} \hat{F}_{xg} \\ \hat{F}_{yg} \\ \hat{F}_{zg} \end{bmatrix} = \begin{bmatrix} -mg\cos\hat{\theta} \\ \hat{\psi}\, mg\cos\hat{\theta} - \phi mg\sin\hat{\theta} \\ -mg\sin\hat{\theta} \end{bmatrix}$$

Separating into nominal and perturbation components

g.)
$$\begin{bmatrix} F_{xg_o} + F_{xg} \\ F_{yg_o} + F_{yg} \\ F_{zg_o} + F_{zg} \end{bmatrix} = \begin{bmatrix} -mg\cos(\theta_o + \theta) \\ \psi\, mg\cos(\theta_o + \theta) - \phi mg\sin(\theta_o + \theta) \\ -mg\sin(\theta_o + \theta) \end{bmatrix}$$

which is

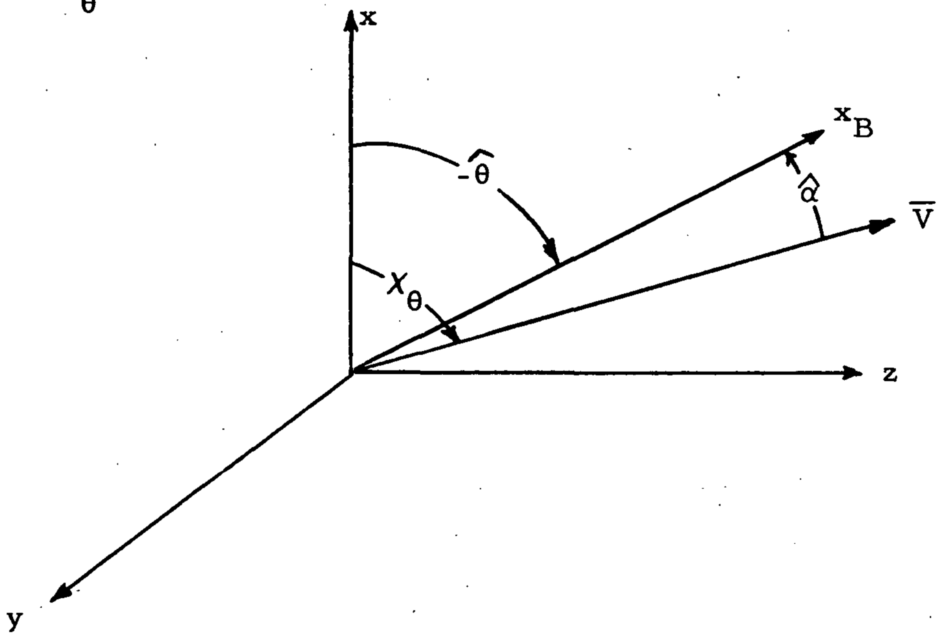
h.)
$$\begin{bmatrix} F_{xg_o} + F_{xg} \\ F_{yg_o} + F_{yg} \\ F_{zg_o} + F_{zg} \end{bmatrix} = \begin{bmatrix} -mg\cos\theta_o\cos\theta + mg\sin\theta_o\sin\theta \\ \psi\, mg\cos\theta_o\cos\theta - \psi\, mg\sin\theta_o\sin\theta - \phi mg\sin\theta_o\cos\theta - \phi mg\cos\theta_o\sin\theta \\ -mg\sin\theta_o\cos\theta - mg\cos\theta_o\sin\theta \end{bmatrix}$$

Separating and simplifying

i.)
$$\begin{bmatrix} F_{xg_o} \\ F_{yg_o} \\ F_{zg_o} \end{bmatrix} = \begin{bmatrix} -mg\cos\theta_o \\ 0 \\ -mg\sin\theta_o \end{bmatrix}$$ Nominal

j.)
$$\begin{bmatrix} F_{xg} \\ F_{yg} \\ F_{zg} \end{bmatrix} = \begin{bmatrix} (mg\sin\theta_o)\theta \\ (mg\cos\theta_o)\psi - (mg\sin\theta_o)\phi \\ -(mg\cos\theta_o)\theta \end{bmatrix}$$ Perturbation

By definition, the shuttle follows a commanded pitch schedule defined with reference to the local vertical. This pitch command is defined by χ_θ.



Therefore,

$$\hat{\theta} = -\chi_\theta + \hat{\alpha}$$

and

$$\theta_o + \theta = -\chi_\theta + \alpha_o + \alpha$$

thus

$$\theta_o = \alpha_o - \chi_\theta$$

$$\theta = \alpha$$

Nominal gravity terms are

k.)
$$\begin{bmatrix} F_{xg_o} \\ F_{yg_o} \\ F_{zg_o} \end{bmatrix} = \begin{bmatrix} -mg\cos(\alpha_o - X_\theta) \\ 0 \\ -mg\sin(\alpha_o - X_\theta) \end{bmatrix}$$

$$= \begin{bmatrix} -mg\cos\alpha_o \cos X_\theta - mg\sin\alpha_o \sin X_\theta \\ 0 \\ -mg\sin\alpha_o \cos X_\theta + mg\cos\alpha_o \sin X_\theta \end{bmatrix}$$

$$= \begin{bmatrix} -mg\cos X_\theta - \alpha_o mg\sin X_\theta \\ 0 \\ -\alpha_o mg\cos X_\theta + mg\sin X_\theta \end{bmatrix}$$

Final Nominal Gravity Equations

Perturbation Gravity Terms are

l.)
$$\begin{bmatrix} F_{xg} \\ F_{yg} \\ F_{zg} \end{bmatrix} = \begin{bmatrix} \theta\, mg\sin(\alpha_o - X_\theta) \\ \psi\, mg\cos(\alpha_o - X_\theta) - \phi mg\sin(\alpha_o - X_\theta) \\ -\theta\, mg\cos(\alpha_o - X_\theta) \end{bmatrix}$$

$$= \begin{bmatrix} \theta\, mg\sin\alpha_o \cos X_\theta - \theta\, mg\cos\alpha_o \sin X_\theta \\ \psi\, mg\cos\alpha_o \cos X_\theta + \psi\, mg\sin\alpha_o \sin X_\theta - \phi\, mg\sin\alpha_o \cos X_\theta + \phi\, mg\cos\alpha_o \sin X_\theta \\ -\theta\, mg\cos\alpha_o \cos X_\theta - \theta\, mg\sin\alpha_o \sin X_\theta \end{bmatrix}$$

m.)
$$\begin{bmatrix} F_{xg} \\ F_{yg} \\ F_{zg} \end{bmatrix} = \begin{bmatrix} \theta\,\alpha_o mg\cos X_\theta - \theta\, mg\sin X_\theta \\ \psi\, mg\cos X_\theta + \psi\,\alpha_o mg\sin X_\theta - \phi\,\alpha_o mg\cos X_\theta + \phi\, mg\sin X_\theta \\ -\theta\, mg\cos X_\theta - \theta\,\alpha_o mg\sin X_\theta \end{bmatrix}$$

which further reduces to

$$\text{n.)} \quad \begin{bmatrix} F_{xg} \\ F_{yg} \\ F_{zg} \end{bmatrix} = \begin{bmatrix} -\theta \, mg \sin X_\theta \\ \psi \, mg \cos X_\theta + \phi \, mg \sin X_\theta \\ -\theta \, mg \cos X_\theta \end{bmatrix}$$

Since $\theta = \alpha$ therefore

$$\text{o.)} \quad \begin{bmatrix} F_{xg} \\ F_{yg} \\ F_{zg} \end{bmatrix} = \begin{bmatrix} -\alpha \, mg \sin X_\theta \\ \psi \, mg \cos X_\theta + \phi \, mg \sin X_\theta \\ -\alpha \, mg \cos X_\theta \end{bmatrix}$$

Final Gravity Perturbation Equations

5. <u>Propulsion Forces</u>

From the figure on page B-1,

$$\text{a.)} \quad \begin{bmatrix} \hat{F}_{xp} \\ \hat{F}_{yp} \\ \hat{F}_{zp} \end{bmatrix} \cong \begin{bmatrix} T_t \\ -T_t \, \delta_\psi \\ T_t \, \delta_\theta \end{bmatrix}$$

where T_t is total thrust

thus

$$\text{b.)} \quad \begin{bmatrix} F_{xp_o} \\ F_{yp_o} \\ F_{zp_o} \end{bmatrix} = \begin{bmatrix} T_{t_o} \\ -T_{t_o} \, \delta_{\psi_o} \\ T_{t_o} \, \delta_{\theta_o} \end{bmatrix}$$

Nominal Thrust Equation

and

c.) $$\begin{bmatrix} F_{xp} \\ F_{yp} \\ F_{zp} \end{bmatrix} = \begin{bmatrix} \Delta T_t \\ -T_{t_o} \delta_\psi - \Delta T_t \delta_{\psi_o} \\ T_{t_o} \delta_\theta + \Delta T_t \delta_{\theta_o} \end{bmatrix}$$

Since ΔT_t is assumed zero; therefore,

d.) $$\begin{bmatrix} F_{xp} \\ F_{yp} \\ F_{zp} \end{bmatrix} = \begin{bmatrix} 0 \\ -T_{t_o} \delta_\psi \\ T_{t_o} \delta_\theta \end{bmatrix}$$ Perturbation Thrust Equations

6. <u>Aerodynamic Forces</u>

a.) $$\begin{bmatrix} \hat{F}_{xa} \\ \hat{F}_{ya} \\ \hat{F}_{za} \end{bmatrix} \begin{bmatrix} -qSC_A - qSC_{A_{\delta_c}} \hat{\delta}_c - qSC_{A_{\delta_e}} \hat{\delta}_e \\ qSC_{y_\beta} \hat{\beta} + qSC_{y_p} \dot{\hat{\phi}} - qSC_{y_r} \dot{\hat{\psi}} \\ -qSC_{L_o} - qSC_{N_\alpha} \hat{\alpha} \end{bmatrix}$$

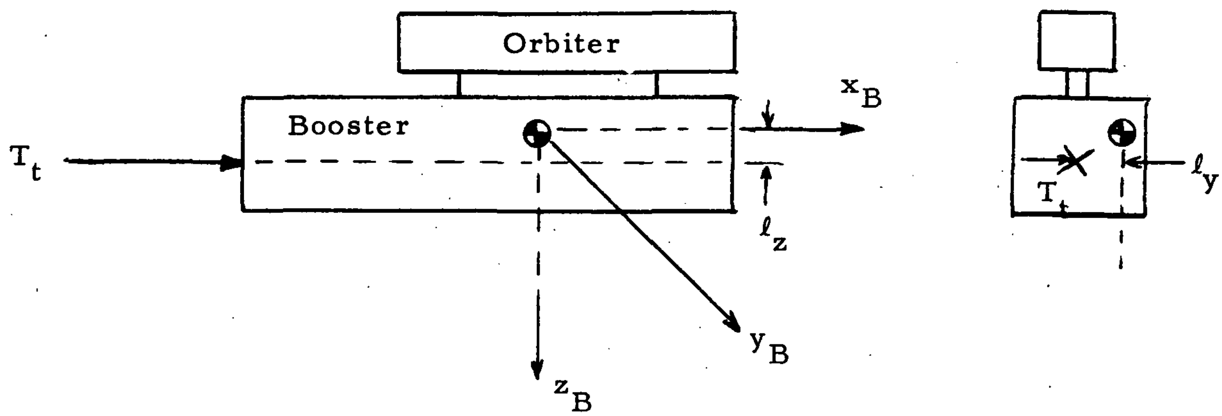
Separating into

b.) $$\begin{bmatrix} F_{xa_o} \\ F_{ya_o} \\ F_{za_o} \end{bmatrix} = \begin{bmatrix} -qSC_A \\ 0 \\ -qSC_{L_o} - qSC_{N_\alpha} \alpha_o \end{bmatrix}$$ Nominal Aerodynamic Forces

and

c.) $$\begin{bmatrix} F_{xa} \\ F_{ya} \\ F_{za} \end{bmatrix} \cong \begin{bmatrix} -qSC_{A_{\delta_c}} \delta_c - qSC_{A_{\delta_e}} \delta_e \\ qSC_{y_\beta} \beta + qSC_{y_p} \dot{\phi} - qSC_{y_r} \dot{\psi} \\ -qSC_{N_\alpha} \alpha \end{bmatrix}$$ Perturbation Aerodynamic Forces

7. Propulsion Moments



a.) $$\begin{bmatrix} \hat{L}_p \\ \hat{M}_p \\ \hat{N}_p \end{bmatrix} = \begin{bmatrix} \hat{F}_{zp} \ell_y - \hat{F}_{yp} \ell_z \\ \hat{F}_{xp} \ell_z - \hat{F}_{zp} \ell_x \\ \hat{F}_{yp} \ell_x - \hat{F}_{xp} \ell_y \end{bmatrix} \begin{bmatrix} \hat{T}_t \hat{\delta}_\theta \ell_y + \hat{T}_t \hat{\delta}_\psi \ell_z \\ \hat{T}_t \ell_z - \hat{T}_t \hat{\delta}_\theta \ell_x \\ -\hat{T}_t \hat{\delta}_\psi \ell_x - \hat{T}_t \ell_y \end{bmatrix}$$

Separating

b.) $$\begin{bmatrix} L_{p_o} \\ M_{p_o} \\ N_{p_o} \end{bmatrix} = \begin{bmatrix} T_{t_o} \delta_{\theta_o} \ell_y \\ T_{t_o} \ell_z - T_{t_o} \delta_{\theta_o} \ell_x \\ -T_{t_o} \ell_y \end{bmatrix}$$ Nominal Propulsion Moments

and

c.) $$\begin{bmatrix} L_p \\ M_p \\ N_p \end{bmatrix} = \begin{bmatrix} T_{t_o} \delta_\theta \ell_y + T_{t_o} \delta_\psi \ell_z \\ -T_{t_o} \delta_\theta \ell_x \\ -T_{t_o} \delta_\psi \ell_x \end{bmatrix}$$ Perturbation Propulsion Moments

8. Aerodynamic Moments

a.) $$\begin{bmatrix} \hat{L}_a \\ \hat{M}_a \\ \hat{N}_a \end{bmatrix} = \begin{bmatrix} q \frac{Sb^2}{2U_o} C_{\ell_p} \dot{\hat{\phi}} + q Sb C_{\ell\beta} \hat{\beta} + q S b C_{\ell_{\delta_a}} \hat{\delta}_a - q Sb C_{\ell_{\delta_r}} \hat{\delta}_r \\ q \frac{S\bar{c}^2}{2U_o} (C_{m_q} + C_{m_{\dot{\alpha}}}) \dot{\hat{\theta}} + q S\bar{c} C_{m_\alpha} \hat{\alpha} + q S\bar{c} C_{m_o} - q S\bar{c} C_{m_{\delta_e}} \hat{\delta}_e \\ q \frac{Sb^2}{2U_o} (C_{n_r} - C_{n_{\dot{\beta}}}) \dot{\hat{\psi}} - q S b C_{N_\beta} \hat{\beta} + q S b C_{n_{\delta_r}} \hat{\delta}_r \end{bmatrix}$$

Separating

b.) $$\begin{bmatrix} L_{a_o} \\ M_{a_o} \\ N_{a_o} \end{bmatrix} = \begin{bmatrix} 0 \\ q S\bar{c} C_{m_\alpha} \alpha_o + q S\bar{c} C_{m_o} \\ 0 \end{bmatrix}$$ Nominal Aerodynamic Moments

and

c.) $$\begin{bmatrix} L_a \\ M_a \\ N_a \end{bmatrix} = \begin{bmatrix} q \frac{Sb^2}{2V_o} C_{\ell_p} \dot{\phi} + q Sb C_{\ell_\beta} \beta + q S b C_{\ell_{\delta_a}} \delta_a - q s b C_{\ell_{\delta_r}} \delta_r \\ q \frac{S\bar{c}^2}{2V_o} (C_{m_q} + C_{m_{\dot{\alpha}}}) \dot{\theta} + q S\bar{c} C_{m_\alpha} \alpha - q S\bar{c} C_{m_{\delta_e}} \delta_e \\ q \frac{Sb^2}{2V_o} (C_{n_r} - C_{n_{\dot{\beta}}}) \dot{\psi} - q S b C_{N_\beta} \beta + q S b C_{N_{\delta_r}} \delta_r \end{bmatrix}$$ Perturbation Aerodynamic Moments

9. Total Nominal Equations (Translation)

a.)
$$\begin{bmatrix} \ddot{X}_o \\ \ddot{Y}_o \\ \ddot{Z}_o \end{bmatrix} = \begin{bmatrix} F_{xo}/m \\ F_{yo}/m \\ F_{zo}/m \end{bmatrix} = \begin{bmatrix} \left[-mg\cos X_\theta - \alpha_o mg \sin X_\theta + T_{t_o} - qSC_A\right]/m \\ \left[-T_{t_o}\delta_{\psi_o}\right]/m \\ \left[-\alpha_o mg\cos X_\theta + mg\sin X_\theta + T_t \delta_{\theta_o} - qSC_{L_o} - qSC_{N_\alpha}\alpha_o\right]/m \end{bmatrix}$$

10. Total Perturbation Equations (Translation)

b.)
$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \begin{bmatrix} -U_o\alpha_o\dot{\theta} + \left[-\alpha\, mg\sin X_\theta - qSC_{A_{\delta_c}}\delta_c - qSC_{A_{\delta_e}}\delta_e\right]/m \\ U_o\alpha_o\dot{\phi} - U_o\dot{\psi} + \left[\psi\, mg\cos X_\theta + \phi\, mg\sin X_\theta - T_{t_o}\delta_\psi + qSC_{y_\beta}\beta + qSC_{y_p}\dot{\phi} - qSC_{y_r}\dot{\psi}\right]/m \\ U_o\dot{\theta} + \left[-\alpha\, mg\cos X_\theta + T_{t_o}\delta_\theta - qSC_{N_\alpha}\alpha\right]/m \end{bmatrix}$$

11. Total Nominal Equations (Rotation)

a.)
$$\begin{bmatrix} \ddot{\phi}_o \\ \ddot{\theta}_o \\ \ddot{\psi}_o \end{bmatrix} = \begin{bmatrix} (T_{t_o}\delta_{\theta_o}\ell_y)/I_x \\ (T_{t_o}\ell_z - T_{t_o}\delta_{\theta_o}\ell_x + qS\bar{c}C_{m_\alpha}\alpha_o + qS\bar{c}C_{m_o})/I_y \\ (-T_{t_o}\ell_y)/I_z \end{bmatrix}$$

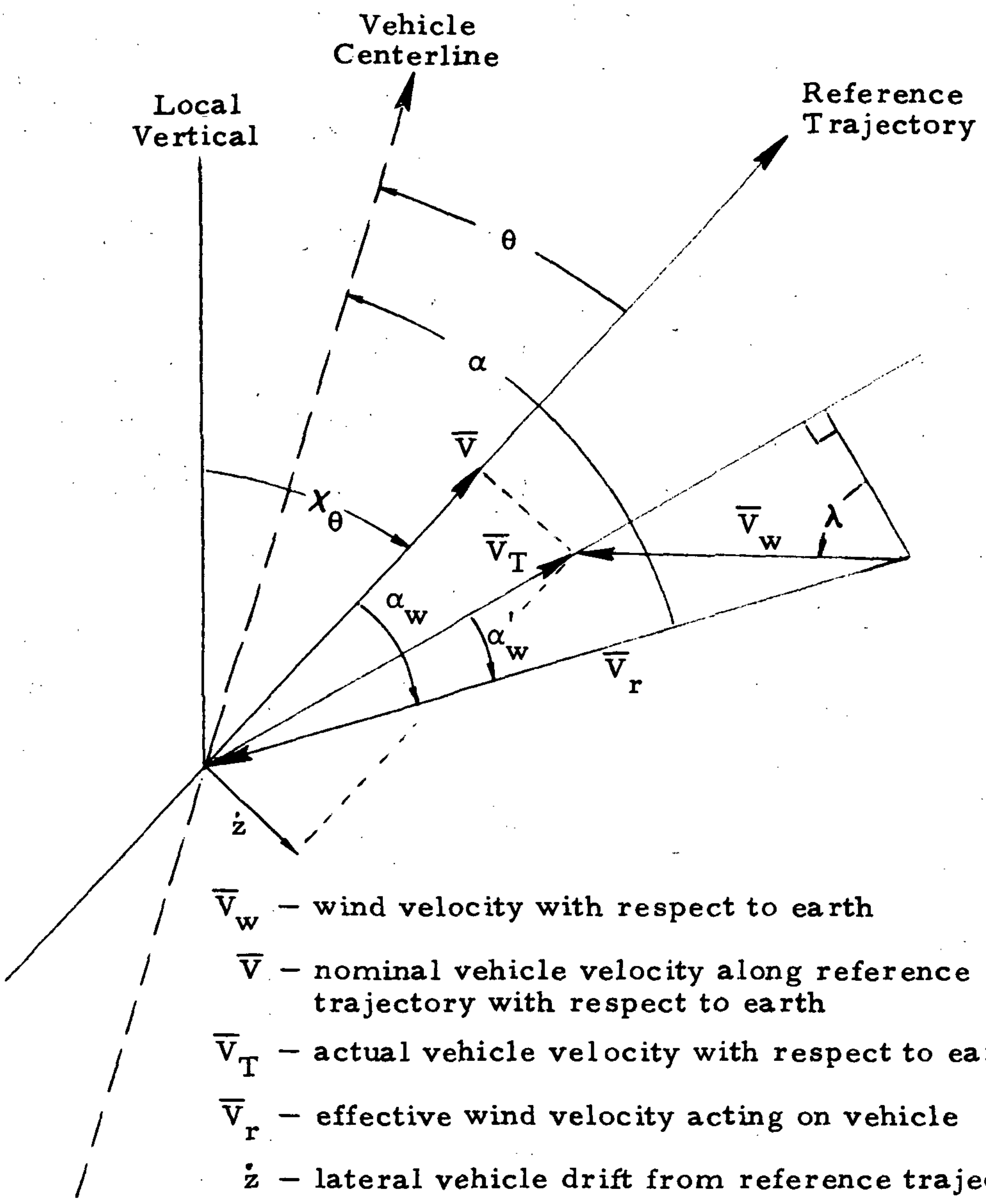
12. Total Perturbation Equations (Rotation)

b.)

$$
\begin{bmatrix} \ddot{\phi} \\ \\ \ddot{\theta} \\ \\ \ddot{\psi} \end{bmatrix}
=
\begin{bmatrix}
\frac{I_{xz}}{I_x}\ddot{\psi} + \Big(T_{t_o}\ell_y\delta_\theta + T_{t_o}\delta_\psi\ell_z + T_{t_o}\ell_\phi\delta_\phi + \frac{qSb^2 C_{\ell_p}}{2U_o}\dot{\phi} \\
\qquad + qSb\,C_{\ell_\beta}\beta + qSb\,C_{\ell_{\delta_a}}\delta_a - qSb\,C_{\ell_{\delta_r}}\delta_r \Big) / I_x \\
\Big(-T_{t_o}\ell_x\delta_\theta + \frac{qS\bar{c}^2}{2U_o}\left(C_{m_q} + C_{m_{\dot{\alpha}}}\right)\dot{\theta} + qS\bar{c}\,C_{m_\alpha}\alpha \\
\qquad - qS\bar{c}\,C_{m_{\delta_e}}\delta_e \Big) / I_y \\
\Big(-T_{t_o}\ell_x\delta_\psi + \frac{qSb^2}{2U_o}\left(C_{n_r} - C_{n_{\dot{\beta}}}\right)\dot{\psi} - qSb\,C_{n_\beta}\beta \\
\qquad + qSb\,C_{n_{\delta_r}}\delta_r \Big) / I_z
\end{bmatrix}
$$

13. Derivation of Angle of Attack Equation

From the following figure



$\overline{V}_w$ — wind velocity with respect to earth

$\overline{V}$ — nominal vehicle velocity along reference trajectory with respect to earth

$\overline{V}_T$ — actual vehicle velocity with respect to earth

$\overline{V}_r$ — effective wind velocity acting on vehicle

$\dot{z}$ — lateral vehicle drift from reference trajectory

χ_θ — commanded pitch angle from local vertical

α_w — wind angle of attack relative to reference trajectory

α'_w — effective wind angle of attack relative to actual vehicle velocity.

We have

$$\alpha_w = \alpha_w' + \tan^{-1}\left(\frac{\dot{z}}{V}\right)$$

$$\cong \alpha_w' + \frac{\dot{z}}{V}$$

Since

$$\lambda \cong \chi_\theta$$

$$V_T \cong V$$

$$|\dot{z}| << V$$

Then

$$\alpha_w' = \tan^{-1}\left(\frac{V_w \cos\lambda}{V_T + V_w \sin\lambda}\right)$$

$$\cong \tan^{-1}\left(\frac{V_w \cos\chi_\theta}{V + V_w \sin\chi_\theta}\right)$$

therefore

$$\alpha_w = \tan^{-1}\left(\frac{V_w \cos\chi_\theta}{V + V_w \sin\chi_\theta}\right) + \frac{\dot{z}}{V}$$

and

$$\alpha = \theta + \alpha_w$$

14. Total Equations in Time-Varying Coefficient Form for Analog Simulation

Translation

$$\ddot{x} = -k_{\dot{\phi}}\dot{\theta} - k_3\alpha + k_{xc}\delta_c + k_{xe}\delta_e$$

$$\ddot{y} = k_{\dot{\phi}}\dot{\phi} + k_{\dot{\psi}}\dot{\psi} + k_1\delta_{\psi} + k_{y\beta}\beta + k_{ya}\delta_a + k_{yr}\delta_r + k_2\psi + k_3\phi$$

$$\ddot{z} = k_{\dot{\theta}}\dot{\theta} - k_1\delta_{\theta} + k_{z\alpha}\alpha + k_{ze}\delta_e + k_{zc}\delta_c$$

Rotations

$$\ddot{\phi} = k_{\phi\psi}\ddot{\psi} + k_{\ell\theta}\delta_\theta + k_{\ell\psi}\delta_\psi + k_{\ell\phi}\delta_\phi + k_{\ell\dot{\phi}}\dot{\phi}$$

$$+ k_{\ell\dot{\psi}}\dot{\psi} + k_{\ell\beta}\beta + k_{\ell a}\delta_a + k_{\ell r}\delta_r$$

$$\ddot{\theta} = k_{m\alpha}\alpha + k_{m\theta}\delta_\theta + k_{m\dot{\theta}}\dot{\theta} + k_{me}\delta_e + k_{mc}\delta_c$$

$$\ddot{\psi} = k_{n\psi}\delta_\psi + k_{n\dot{\phi}}\dot{\phi} + k_{n\dot{\psi}}\dot{\psi} + k_{n\beta}\beta + k_{na}\delta_a$$

$$+ k_{nr}\delta_r$$

Sideslip and Angle of Attack

$$\alpha = \theta + \alpha_w + u_{io}\dot{z}$$

$$\beta = -\psi + \beta_w + u_{io}\dot{y}$$

14. Trim Equations

$$k_{za}\alpha_o - k_1\delta_{\theta_o} + k_3 + k_{n_o} - U_o\dot{\chi}_\theta = 0$$

$$k_{m\alpha}\alpha_o + k_{m_\theta}\delta_{\theta_o} + k_{m_o} - \frac{T \cdot \Delta Z_{cg}}{I_y} = 0$$

These equations were solved simultaneously on an 1108 digital program used to process the time-varying coefficients. Results were outputted in plot form from the SC 4020 Plotter, giving α_o and δ_{θ_o} as functions of flight time. These curves were simulated on diode function generators on the analog.

15. Control Laws

$$\delta_\theta = -H_\theta(s)\left[a_{0\theta}\theta + a_{1\theta}\dot{\theta}\right]$$

$$\delta_\psi = -H_\psi(s)\left[a_{0\psi}\psi + a_{1\psi}\dot{\psi} - b_{0\psi}\beta\right]$$

$$\delta_\phi = -H_\phi(s)\left[a_{0\phi}\phi + a_{1\phi}\dot{\phi}\right]$$

$$\delta_c = -k_c H_c(s)\left[a_{0\theta}\theta + a_1\dot{\theta}\right]$$

$$\delta_e = -k_c H_c(s)\left[a_{0\theta}\theta + a_1\dot{\theta}\right]$$

$$\delta_r = -k_r H_a(s)\left[a_{0\psi}\psi + a_{1\psi}\dot{\psi} - b_{0\psi}\beta\right]$$

$$\delta_a = -k_a H_a(s)\left[a_{0\phi}\phi + a_{1\phi}\dot{\phi}\right]$$

where

$$H_\theta(s) = \frac{15}{s+15}$$

$$H_\psi(s) = \frac{15}{s+15}$$

$$H_\phi(s) = \frac{15}{s+15}$$

$$H_c(s) = \frac{3}{s+3}$$

$$H_e(s) = \frac{3}{s+3}$$

$$H_r(s) = \frac{3}{s+3}$$

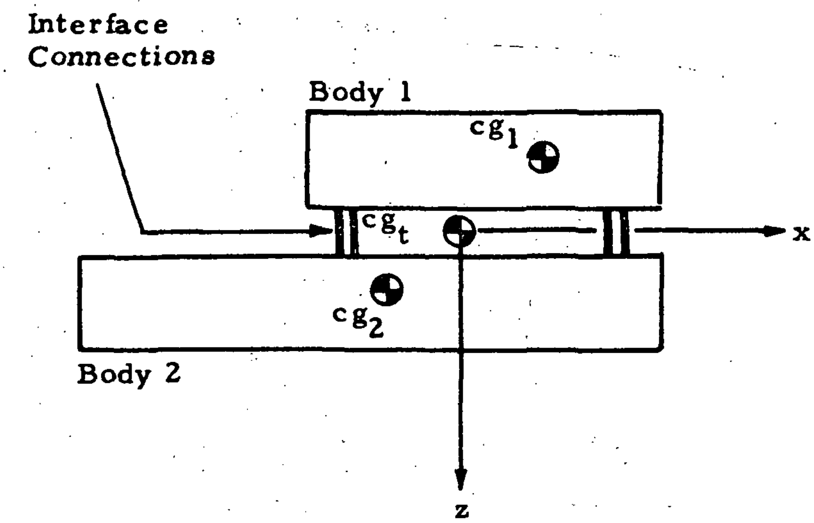
$$H_a(s) = \frac{3}{s+3}$$

Appendix C

DERIVATION OF INTERFACE LOADING EQUATIONS FOR THE COMPOSITE SHUTTLE LAUNCH PHASE

1. Assumptions

Identical to Appendix B.



Body 1 = orbiter

Body 2 = booster

CG_t = total CG of Body 1 + Body 2

2. Total Forces

$$\vec{F}_t = m_t \vec{a} = m_1 \vec{a} + m_2 \vec{a}$$

Total forces on Body 1 are

a. $$\begin{bmatrix} m_1 \dot{\hat{U}} \\ m_1 \dot{\hat{V}} \\ m_1 \dot{\hat{W}} \end{bmatrix} = \begin{bmatrix} (\hat{F}_{x1})_{AERO} + (\hat{F}_{x1})_{CG} + (\hat{F}_{x1})_{INTER} + (\hat{R}\hat{V} - \hat{Q}\hat{W})\, m_1 + (\hat{F}_{x1})_g \\ (\hat{F}_{y1})_{AERO} + (\hat{F}_{y1})_{CG} + (\hat{F}_{y1})_{INTER} + (\hat{P}\hat{W} - \hat{R}\hat{U})\, m_1 + (\hat{F}_{y1})_g \\ (\hat{F}_{z1})_{AERO} + (\hat{F}_{z1})_{CG} + (\hat{F}_{z1})_{INTER} + (\hat{Q}\hat{U} - \hat{P}\hat{V})\, m_1 + (\hat{F}_{z1})_g \end{bmatrix}$$

where

$(\hat{F}_{x1})_{AERO}$ = total aerodynamic forces acting on Body 1 in x-direction

$(\hat{F}_{y1})_{AERO}$ = total aerodynamic forces acting on Body 1 in y-direction

$(\hat{F}_{z1})_{AERO}$ = total aerodynamic forces acting on Body 1 in z-direction

$(\hat{F}_{x1})_{CG}$ = total forces due to offset of CG_1 from CG_t acting on Body 1 in x-direction

$(\hat{F}_{y1})_{CG}$ = total forces due to offset of CG_1 from CG_t acting on Body 1 in y-direction

$(\hat{F}_{z1})_{CG}$ = total forces due to offset of CG_1 from CG_t acting on Body 1 in z-direction

$(\hat{F}_{x1})_{INTER}$ = total interface force from Body 2 acting on Body 1 in x-direction

$(\hat{F}_{y1})_{INTER}$ = total interface force from Body 2 acting on Body 1 in y-direction

$(\hat{F}_{z1})_{INTER}$ = total interface force from Body 2 acting on Body 1 in z-direction

$(\hat{F}_{x1})_{g}$ = total force due to gravity acting on Body 1 in x-direction

$(\hat{F}_{y1})_{g}$ = total force due to gravity acting on Body 1 in y-direction

$(\hat{F}_{z1})_{g}$ = total force due to gravity acting on Body 1 in z-direction

Solve for the interface forces from Eq. (2-a).

b. $$\begin{bmatrix} (\hat{F}_{x1})_{INTER} \\ (\hat{F}_{y1})_{INTER} \\ (\hat{F}_{z1})_{INTER} \end{bmatrix} = \begin{bmatrix} m_1 \left[\dot{\hat{U}} + \hat{Q}\hat{W} - \hat{R}\hat{V}\right] - (\hat{F}_{x1})_{AERO} - (\hat{F}_{x1})_{CG} - (\hat{F}_{x1})_{g} \\ m_1 \left[\dot{\hat{V}} + \hat{R}\hat{U} - \hat{P}\hat{W}\right] - (\hat{F}_{y1})_{AERO} - (\hat{F}_{y1})_{CG} - (\hat{F}_{y1})_{g} \\ m_1 \left[\dot{\hat{W}} + \hat{P}\hat{V} - \hat{Q}\hat{U}\right] - (\hat{F}_{z1})_{AERO} - (\hat{F}_{z1})_{CG} - (\hat{F}_{z1})_{g} \end{bmatrix}$$

where aerodynamic forces are

c. $$\begin{bmatrix} \hat{F}_{x1} \\ \hat{F}_{y1} \\ \hat{F}_{z1} \end{bmatrix} = \begin{bmatrix} -qSC_{A_1} \\ -qSC_{Y_{\beta_1}}\beta \\ -qSC_{L_{o_1}} - qSC_{N_{\alpha_1}}\hat{\alpha} \end{bmatrix}$$

and CG offset forces are

d. $$\begin{bmatrix} \hat{F}_{x1} \\ \hat{F}_{y1} \\ \hat{F}_{z1} \end{bmatrix}_{CG} = -m_1 \begin{bmatrix} \dot{\hat{P}} \\ \dot{\hat{Q}} \\ \dot{\hat{R}} \end{bmatrix} X \begin{bmatrix} \Delta X \\ \Delta Y \\ \Delta Z \end{bmatrix} = \begin{bmatrix} \dot{\hat{R}}\Delta Y - \dot{\hat{Q}}\Delta Z \\ \dot{\hat{P}}\Delta Z - \dot{\hat{R}}\Delta X \\ \dot{\hat{Q}}\Delta X - \dot{\hat{P}}\Delta Y \end{bmatrix} m_1$$

and gravity forces are

e. $$\begin{bmatrix} F_{x1} \\ F_{y1} \\ F_{z1} \end{bmatrix}_g = \begin{bmatrix} -\cos X_\theta - \hat{\alpha}\sin X_\theta \\ \hat{\psi}(\cos X_\theta - \hat{\alpha}\sin X_\theta) + \hat{\phi}(\sin X_\theta - \hat{\alpha}\cos X_\theta) \\ \sin X_\theta - \hat{\alpha}\cos X_\theta \end{bmatrix} m_1 g$$

As before in Appendix A, separate into nominal and perturbation equations with the following assumptions:

$$\ddot{\psi}_o, \dot{\psi}_o = 0 \qquad \dot{Y}_o = 0$$

$$\ddot{\phi}_o, \dot{\phi}_o = 0 \qquad \Delta Y = 0$$

Thus,

f. $$\begin{bmatrix} (F_{x1})_{INTER} \\ (F_{y1})_{INTER} \\ (F_{z1})_{INTER} \end{bmatrix} = \begin{bmatrix} m_1\left[\ddot{X}_o + \dot{\theta}_o \dot{Z}_o\right] - (F_{x1})_{AERO} + \ddot{\theta}_o \Delta Z\, m_1 - (F_{x1})_g \\ -(F_{y1})_{AERO} - (F_{y1})_g \\ m_1\left[\ddot{Z}_o - \dot{\theta}_o \dot{X}_o\right] - (F_{z1})_{AERO} - \ddot{\theta}_o \Delta X\, m_1 - (F_{z1})_g \end{bmatrix}$$

where

$$\dot{X}_o = U_o \cos\alpha_o \qquad \ddot{X}_o = \frac{d}{dt}\dot{X}_o$$

$$\dot{Z}_o = U_o \sin\alpha_o \qquad \ddot{Z}_o = \frac{d}{dt}\dot{Z}_o$$

$$\theta_o = \alpha_o - \chi_\theta \qquad \text{from p. B-8}$$

$$\dot{\theta}_o = \frac{d}{dt}\theta_o \qquad \ddot{\theta}_o = \frac{d}{dt}\dot{\theta}_o$$

and

g. $$\begin{bmatrix} (F_{x1})_{AERO} \\ (F_{y1})_{AERO} \\ (F_{z1})_{AERO} \end{bmatrix} = \begin{bmatrix} -qSC_{A_1} \\ 0 \\ -qSC_{L_{0_1}} - qSC_{N_\alpha}\alpha_o \end{bmatrix}$$

h. $$\begin{bmatrix} (F_{x1})_g \\ (F_{y1})_g \\ (F_{z1})_g \end{bmatrix} = \begin{bmatrix} -\cos\chi_\theta - \alpha_o \sin\chi_\theta \\ 0 \\ \sin\chi_\theta - \alpha_o \cos\chi_\theta \end{bmatrix} m_1 g$$

Therefore, total nominal interface forces can be written as

i. $$\begin{bmatrix} (F_{x1})_{INTER} \\ \\ (F_{y1})_{INTER} \\ \\ (F_{z1})_{INTER} \end{bmatrix} = \begin{bmatrix} m_1 \ddot{X}_o + m_1 \dot{\theta}_o \dot{Z}_o + qSC_{A_1} + \ddot{\theta}_o \Delta Z m_1 \\ + m_1 g \cos X_\theta + m_1 g \alpha_o \sin X_\theta \\ 0 \\ m_1 \ddot{Z}_o - m_1 \dot{\theta}_o \dot{X}_o + qSC_{L_{o1}} + qSC_{N_\alpha} \alpha_o - \ddot{\theta}_o \Delta X m_1 \\ - m_1 g \sin X_\theta + m_1 g \alpha_o \cos X_\theta \end{bmatrix}$$

3. <u>Perturbation Interface Loads</u>

a. $$\begin{bmatrix} (f_{x1})_{INTER} \\ \\ (f_{y1})_{INTER} \\ \\ (f_{z1})_{INTER} \end{bmatrix} = \begin{bmatrix} m_1 \ddot{x} + m_1 \dot{Z}_o \dot{\theta} + m_1 \dot{\theta}_o \dot{z} - (f_{x1})_{AERO} + \ddot{\theta} \Delta Z m_1 \\ - (f_{x1})_g \\ m_1 \ddot{y} + m_1 \dot{X}_o \dot{\psi} - m_1 \dot{Z}_o \dot{\phi} - (f_{y1})_{AERO} - m_1 \Delta Z \ddot{\phi} \\ + m_1 \Delta X \ddot{\psi} - (f_{y1})_g \\ m_1 \ddot{z} - m_1 \dot{X}_o \dot{\theta} - m_1 \dot{\theta}_o \dot{x} - (f_{z1})_{AERO} - m_1 \Delta X \ddot{\theta} \\ - (f_{z1})_g \end{bmatrix}$$

where

b. $$\begin{bmatrix} f_{x1} \\ f_{y1} \\ f_{z1} \end{bmatrix}_{AERO} = \begin{bmatrix} 0 \\ -qSC_{y_\beta} \beta \\ -qSC_{N_\alpha} \alpha \end{bmatrix}$$

c. $$\begin{bmatrix} f_{x1} \\ f_{y1} \\ f_{z1} \end{bmatrix}_g = \begin{bmatrix} -\alpha \sin X_\theta \\ \psi \cos X_\theta + \phi \sin X_\theta \\ -\alpha \cos X_\theta \end{bmatrix} m_1 g$$

Therefore, the total interface loads can be written as

d. $$\begin{bmatrix} (f_{x1})_{INTER} \\ (f_{y1})_{INTER} \\ (f_{z1})_{INTER} \end{bmatrix} = \begin{bmatrix} m_1 \ddot{x} + m_1 Z_o \dot{\theta} + m_1 \dot{\theta}_o \dot{z} + \Delta Z m_1 \ddot{\theta} + m_1 g \cos X_\theta \alpha \\ m_1 \ddot{y} + m_1 \dot{X}_o \dot{\psi} - m_1 \dot{Z}_o \dot{\phi} + qSC_{y_\beta}\beta - m_1 \Delta Z \ddot{\phi} \\ \quad + m_1 \Delta X \ddot{\psi} - m_1 g \cos X_\theta \psi - m_1 g \sin X_\theta \phi \\ m_1 \ddot{z} - m_1 \dot{X}_o \dot{\theta} - m_1 \dot{\theta}_o \dot{x} + qSC_{N_\alpha}\alpha - m_1 \Delta X \ddot{\theta} \\ \quad + m_1 g \cos X_\theta \alpha \end{bmatrix}$$

4. <u>Interface Moments</u>

Since the composite shuttle is considered rigid, therefore

a. $$\begin{bmatrix} \ddot{\hat{\phi}} \\ \ddot{\hat{\theta}} \\ \ddot{\hat{\psi}} \end{bmatrix} = \begin{bmatrix} \frac{I_{xz_1}}{I_{x_1}} \ddot{\hat{\psi}} + \frac{\hat{L}_1}{I_{x_1}} \\ \frac{\hat{M}_1}{I_{y_1}} \\ \frac{\hat{N}_1}{I_{z_1}} \end{bmatrix}$$

Total torques acting on body 1 are equal to torques due to aerodynamics of body 1 and torques from the interface points; therefore

b. $$\begin{bmatrix} \hat{L}_1 \\ \hat{M}_1 \\ \hat{N}_1 \end{bmatrix} = \begin{bmatrix} (\hat{L}_1)_{AERO} + (\hat{L}_1)_{INTER} \\ (\hat{M}_1)_{AERO} + (\hat{M}_1)_{INTER} \\ (\hat{N}_1)_{AERO} + (\hat{N}_1)_{INTER} \end{bmatrix}$$

Thus, substituting Eq. (4-b) into Eq. (4-a), and solving for interface moments, we have

c. $$\begin{bmatrix} (\hat{L}_1)_{INTER} \\ (\hat{M}_1)_{INTER} \\ (\hat{N}_1)_{INTER} \end{bmatrix} = \begin{bmatrix} I_{x1}\ddot{\hat{\phi}} - I_{xz1}\ddot{\hat{\psi}} - (\hat{L}_1)_{AERO} \\ I_{y1}\ddot{\hat{\theta}} - (\hat{M}_1)_{AERO} \\ I_{z1}\ddot{\hat{\psi}} - (\hat{N}_1)_{AERO} \end{bmatrix}$$

Again, separate into nominal and perturbation components

d. $$\begin{bmatrix} (\hat{L}_1)_{INTER} \\ (\hat{M}_1)_{INTER} \\ (\hat{N}_1)_{INTER} \end{bmatrix} = \begin{bmatrix} (L_1)_{INTER} + (\ell_1)_{INTER} \\ (M_1)_{INTER} + (\hat{m}_1)_{INTER} \\ (N_1)_{INTER} + (\hat{n}_1)_{INTER} \end{bmatrix}$$

e. $$\begin{bmatrix} (\hat{L}_1)_{AERO} \\ (\hat{M}_1)_{AERO} \\ (\hat{N}_1)_{AERO} \end{bmatrix} = \begin{bmatrix} (L_1)_{AERO} + (\hat{\ell}_1)_{AERO} \\ (M_1)_{AERO} + (\hat{m}_1)_{AERO} \\ (N_1)_{AERO} + (\hat{n}_1)_{AERO} \end{bmatrix}$$

Therefore, from (4-c)

f. $$\begin{bmatrix} (L_1)_{INTER} \\ (M_1)_{INTER} \\ (N_1)_{INTER} \end{bmatrix} = \begin{bmatrix} I_{x1}\ddot{\phi}_o - I_{xz1}\ddot{\psi}_o - (L_1)_{AERO} \\ I_{y1}\ddot{\theta}_o - (M_1)_{AERO} \\ I_{z1}\ddot{\psi}_o - (N_1)_{AERO} \end{bmatrix} \quad \text{NOMINAL}$$

The nominal aero moments are

g. $$\begin{bmatrix} L_1 \\ M_1 \\ N_1 \end{bmatrix}_{AERO} = \begin{bmatrix} 0 \\ qS\bar{c}C_{m_{\alpha_1}}\alpha_o + qS\bar{c}C_{m_{o_1}} \\ 0 \end{bmatrix}$$

Assume as before $\dot{\phi}_o, \dot{\psi}_o, \beta_o = 0$ and $\theta_o = \alpha_o - X_\theta$. Therefore,

h. $$\begin{bmatrix} (L_1)_{INTER} \\ (M_1)_{INTER} \\ (N_1)_{INTER} \end{bmatrix} = \begin{bmatrix} 0 \\ I_{y1}(\ddot{\alpha}_o - \ddot{X}_\theta) - qS\bar{c}C_{m_{\alpha_1}}\alpha_o - qS\bar{c}C_{m_{o_1}} \\ 0 \end{bmatrix}$$

5. Perturbation Moments

a. $$\begin{bmatrix} (\hat{\ell}_1)_{INTER} \\ (\hat{m}_1)_{INTER} \\ (\hat{n}_1)_{INTER} \end{bmatrix} = \begin{bmatrix} I_{x1}\ddot{\phi} - I_{xz1}\ddot{\psi} - (\hat{\ell}_1)_{AERO} \\ I_{y1}\ddot{\theta} - (\hat{m}_1)_{AERO} \\ I_{z1}\ddot{\psi} - (\hat{n}_1)_{AERO} \end{bmatrix} \qquad \text{PERTURBATION}$$

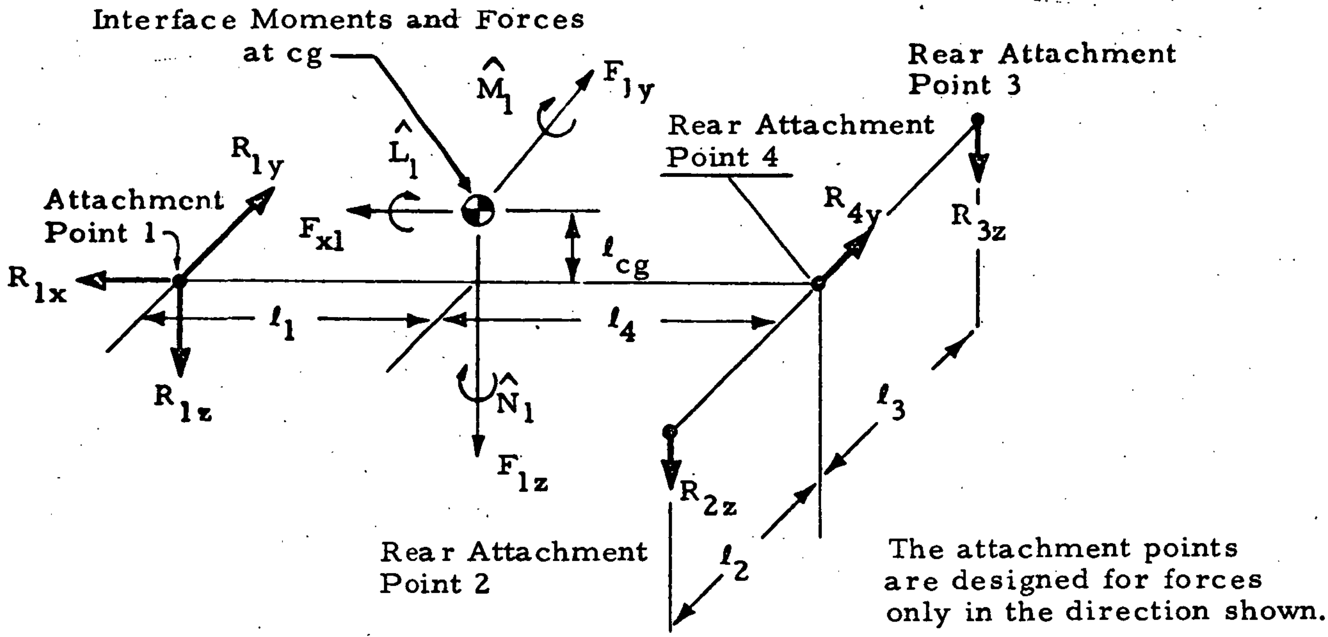
The perturbation aerodynamic moments for body 1 are

b. $$\begin{bmatrix} \hat{\ell}_1 \\ \hat{m}_1 \\ \hat{n}_1 \end{bmatrix}_{AERO} = \begin{bmatrix} 0 \\ qS\bar{c}C_{m_{\alpha_1}}\alpha \\ qSbC_{n_\beta}\beta \end{bmatrix}$$

Therefore, total perturbation moments are

c. $$\begin{bmatrix} \hat{\ell}_1 \\ \hat{m}_1 \\ \hat{n}_1 \end{bmatrix}_{INTER} = \begin{bmatrix} I_{x1}\ddot{\phi} - I_{xz1}\ddot{\psi} \\ I_{y1}\ddot{\theta} - qS\bar{c}C_{m_{\alpha_1}}\alpha \\ I_{z1}\ddot{\psi} - qSbC_{n_\beta}\beta \end{bmatrix} \qquad \text{PERTURBATION}$$

6. Resolution into Each Attachment Point



a.
$$\begin{bmatrix} (\hat{F}_{x1})_{INTER} \\ (\hat{F}_{y1})_{INTER} \\ (\hat{F}_{z1})_{INTER} \end{bmatrix} = \begin{bmatrix} R_{1x} \\ R_{1y} + R_{4y} \\ R_{1z} + R_{2z} + R_{3z} \end{bmatrix}$$

TOTAL FORCES

b.
$$\begin{bmatrix} (\hat{L}_1)_{INTER} \\ (\hat{M}_1)_{INTER} \\ (\hat{N}_1)_{INTER} \end{bmatrix} \quad \begin{bmatrix} -\ell_{CG} R_{1y} - \ell_2 R_{2z} + \ell_3 R_{3z} - \ell_{CG} R_{4y} \\ -\ell_1 R_{1z} + \ell_4 (R_{2z} + R_{3z}) + \ell_{CG} R_{1x} \\ \ell_1 R_{1y} - \ell_4 R_{4y} \end{bmatrix}$$

TOTAL MOMENTS

Eliminating terms and solving for the forces $R_{1x}, \ldots, R_{3z}$ for $\ell_2 = \ell_3$ yields the total forces at each attachment point.

c. $R_{1x} = \hat{F}_{x1}$

$$R_{1z} = -\frac{\hat{M}_1}{\ell_1 + \ell_4} + \frac{\ell_4 \hat{F}_{z1}}{\ell_1 + \ell_4} + \frac{\ell_{CG} R_{1x}}{\ell_1 + \ell_4}$$

$$R_{1y} = \frac{\ell_4 \hat{F}_{y1}}{\ell_1 + \ell_4} + \frac{\hat{N}_1}{\ell_1 + \ell_4}$$

$$R_{4y} = \hat{F}_{y1} - R_{1y}$$

$$R_{2z} = -\frac{\hat{L}_1}{2\ell_2} - \frac{\ell_{CG} \hat{F}_{y1}}{2\ell_2} + \frac{\hat{F}_{z1}}{2} - \frac{R_{1z}}{2}$$

$$R_{3z} = \hat{F}_{z1} - R_{1z} - R_{2z}$$

Since $\hat{F}_{y1}$, $\hat{L}_1$ and $\hat{N}_1$ are nominally zero, the equations may be reduced further. Separating into components and removing $\hat{F}_{y1}$, $\hat{L}_1$ and $\hat{N}_1$ yields

d. $R_{1x} = f_{x1} + F_{x1}$

$$R_{1z} = -\frac{(\hat{m}_1 + M_1)}{\ell_1 + \ell_4} + \frac{\ell_4 (f_{z1} + F_{z1})}{\ell_1 + \ell_4} + \frac{\ell_{CG}(R_{1x})}{\ell_1 + R_4}$$

$$R_{1y} = \frac{\ell_4 f_{y1}}{\ell_1 + \ell_4} + \frac{\hat{n}_1}{\ell_1 + \ell_4}$$

$$R_{4y} = f_{y1} - R_{1y}$$

$$R_{2z} = -\frac{\hat{\ell}_1}{2\ell_2} - \frac{\ell_{CG} f_{y1}}{2\ell_2} + \frac{(f_{z1} + F_{z1})}{2} - \frac{R_{1z}}{2}$$

$$R_{3z} = f_{z1} + F_{z1} - R_{1z} - R_{2z}$$

7. Analog Simulation of Interface Loading Equations

a. Perturbation Forces

$$f_{x1} = a_1\ddot{x} + a_2\dot{\theta} + a_3\dot{z} + a_4\ddot{\theta} - b_7\alpha \qquad \text{(N)}$$

$$f_{y1} = a_1\ddot{y} + b_2\dot{\psi} - a_2\dot{\phi} - a_4\ddot{\phi} + b_5\ddot{\psi} - a_5\psi + b_7\phi \qquad \text{(N)}$$

$$f_{z1} = a_1\ddot{z} - b_2\dot{\theta} - a_3\dot{x} + c_4\alpha - b_5\ddot{\theta} \qquad \text{(N)}$$

where

$$a_1 = m_1 = 377415 \text{ kg}$$

$$a_2 = m_1\dot{Z}_o/57.3 = \text{(curve on page E-30)} \ \frac{\text{kg-m}}{\text{deg-sec}}$$

$$a_3 = m_1\dot{\theta}_o/57.3 = \text{(curve on page E-30) kg/sec}$$

$$a_4 = \Delta Z m_1/57.3 = 47800 \ \frac{\text{kg-m}}{\text{deg}}$$

$$a_5 = m_1\, g \cos X_\theta/57.3 = 49700 \text{ N/deg}$$

$$b_2 = m_1\dot{X}_o/57.3 = \text{(curve on page E-31)} \ \frac{\text{kg-m}}{\text{deg-sec}}$$

$$b_5 = m_1\Delta X/57.3 = 42200 \ \frac{\text{kg-m}}{\text{deg}}$$

$$b_7 = -m_1\, g \sin X_\theta/57.3 = -41400 \text{ N/deg}$$

$$c_4 = \bar{q}\, S\, C_{N_{\alpha_1}} + m_1\, g \cos X_\theta/57.3 = \text{(curve on page E-31) N/deg}$$

b. Perturbation Moments

$$\hat{\ell}_1 = d_1 \ddot{\phi} + d_2 \ddot{\psi} \quad \text{(N-m)}$$

$$\hat{m}_1 = e_1 \ddot{\theta} + e_2 \alpha \quad \text{(N-m)}$$

$$\hat{n}_1 = f_1 \ddot{\psi} \quad \text{(N-m)}$$

where

$$d_1 = I_{x1}/57.3 = 66.3 * 10^3 \ \frac{\text{kg-m}^2}{\text{deg}}$$

$$d_2 = -I_{xz_1}/57.3 = -37.2 * 10^3 \ \frac{\text{kg-m}^2}{\text{deg}}$$

$$e_1 = I_{y1}/57.3 = 1.04 * 10^5 \ \frac{\text{kg-m}^2}{\text{deg}}$$

$$e_2 = -\bar{q} S \bar{c} C_{m_{\alpha_1}} = \text{(curve on page E-32)} \ \frac{\text{kg-m}}{\text{deg-sec}^2}$$

$$f_1 = I_{z1}/57.3 = 1.061 * 10^6 \ \frac{\text{kg-m}^2}{\text{deg}}$$

c. Total Forces

$$R_{1x} = f_{x1} + F_{x1} \quad \text{(N)}$$

$$R_{1z} = g_1(f_{z1} + F_{z1}) + g_2 R_{1x} + g_3(\hat{m}_1 + M_1) \quad \text{(N)}$$

$$R_{1y} = g_4(\hat{n}_1) + g_5(f_{y1}) \quad \text{(N)}$$

$$R_{4y} = f_{y1} - R_{1y} \quad \text{(N)}$$

$$R_{2z} = g_6 \hat{\ell}_1 + g_7 f_{y1} + g_8(f_{z1} + F_{z1}) + g_9 R_{1z} \quad \text{(N)}$$

$$R_{3z} = f_{z1} + F_{z1} - R_{1z} - R_{2z} \quad \text{(N)}$$

where

$$g_1 = \ell_4/(\ell_1 + \ell_4) = 1.0$$

$$g_2 = \ell_{cg}/(\ell_1 + \ell_4) = 0.152$$

$$g_3 = -1.0/(\ell_1 + \ell_4) = -0.056\ m^{-1}$$

$$g_4 = 1.0/(\ell_1 + \ell_4) = +0.056\ m^{-1}$$

$$g_5 = \ell_4/(T_1 + \ell_4) = 1.0$$

$$g_6 = -1.0/2\ell_2 = -0.167\ m^{-1}$$

$$g_7 = -\ell_{cg}/2\ell_2 = -0.45$$

$$g_8 = 1/2 = 0.5$$

$$g_9 = -1/2 = -0.5$$

and

$$\left.\begin{aligned} \ell_1 &= 0\ \text{m} \\ \ell_2 &= 3\ \text{m} \\ \ell_3 &= 3\ \text{m} \\ \ell_4 &= 17.8\ \text{m} \\ \ell_{cg} &= 2.7\ \text{m} \end{aligned}\right\}$$

From figure on page C-10
for MDAC-20 shuttle configuration

F_{x1} = (curve on page E-32)

F_{z1} = (curve on page E-33)

M_1 = (curve on page E-33)

Appendix D

ANALOG WIRING DIAGRAMS

Appendix D

This appendix constitutes the wiring diagrams of all equations, logic, wind disturbances and time varying coefficient generation associated with MDAC-20 Shuttle Configuration. Each page is identified explicitly.

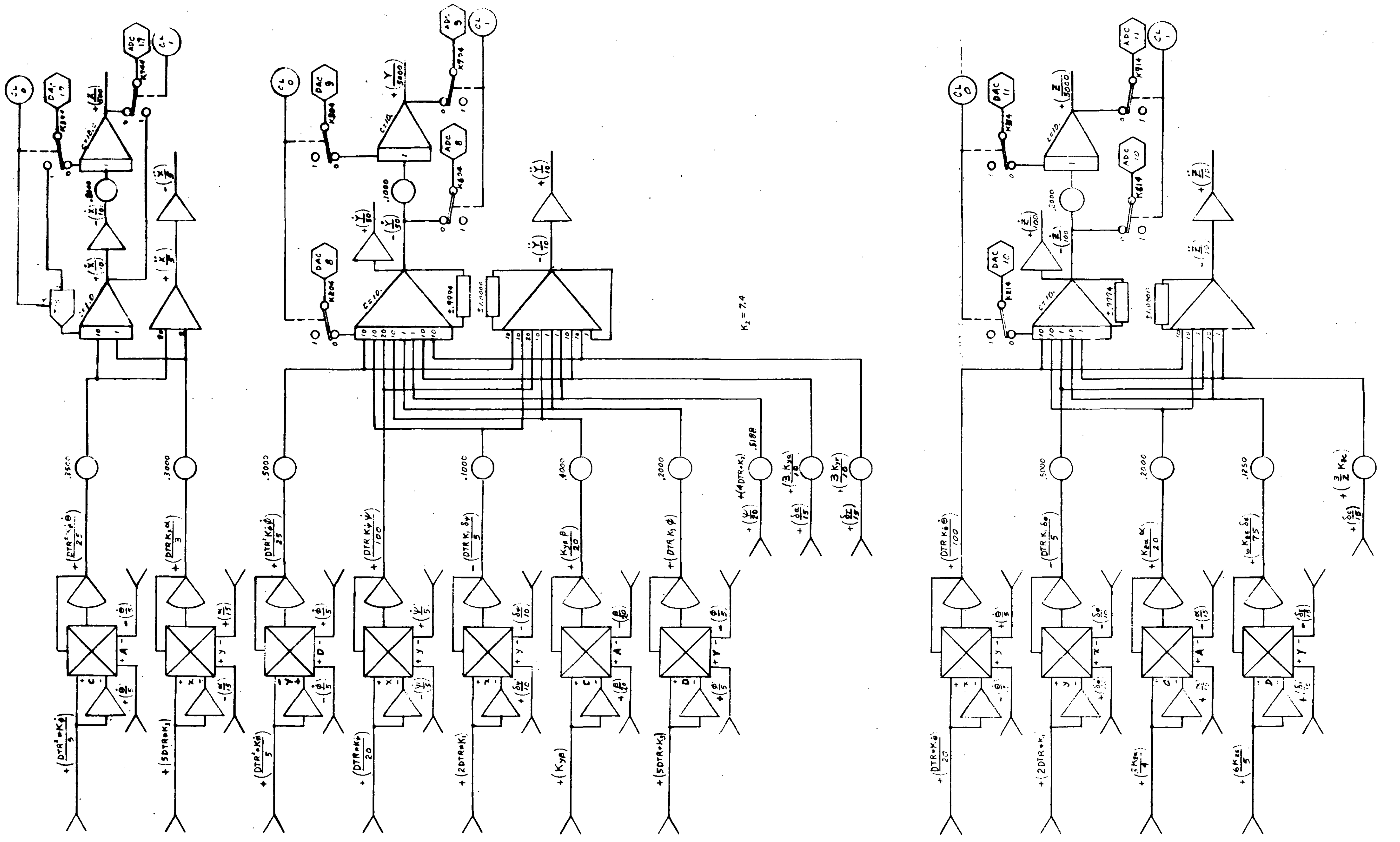


Fig. D-1 - Analog Wiring Diagram for Shuttle Ascent Perturbation Translation Equations

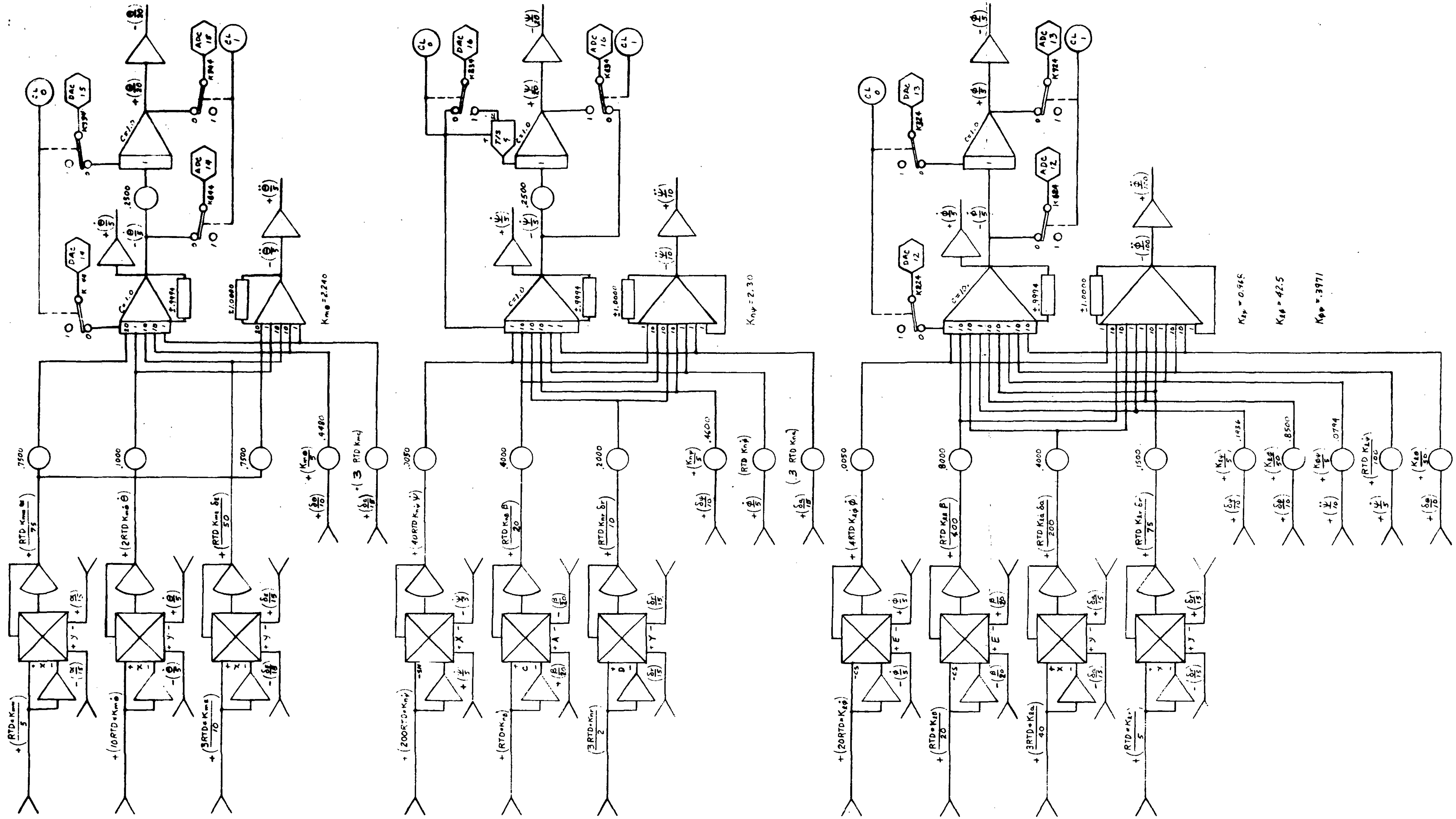


Fig. D-2 - Analog Wiring Diagram for Shuttle Ascent Perturbation Rotation Equations

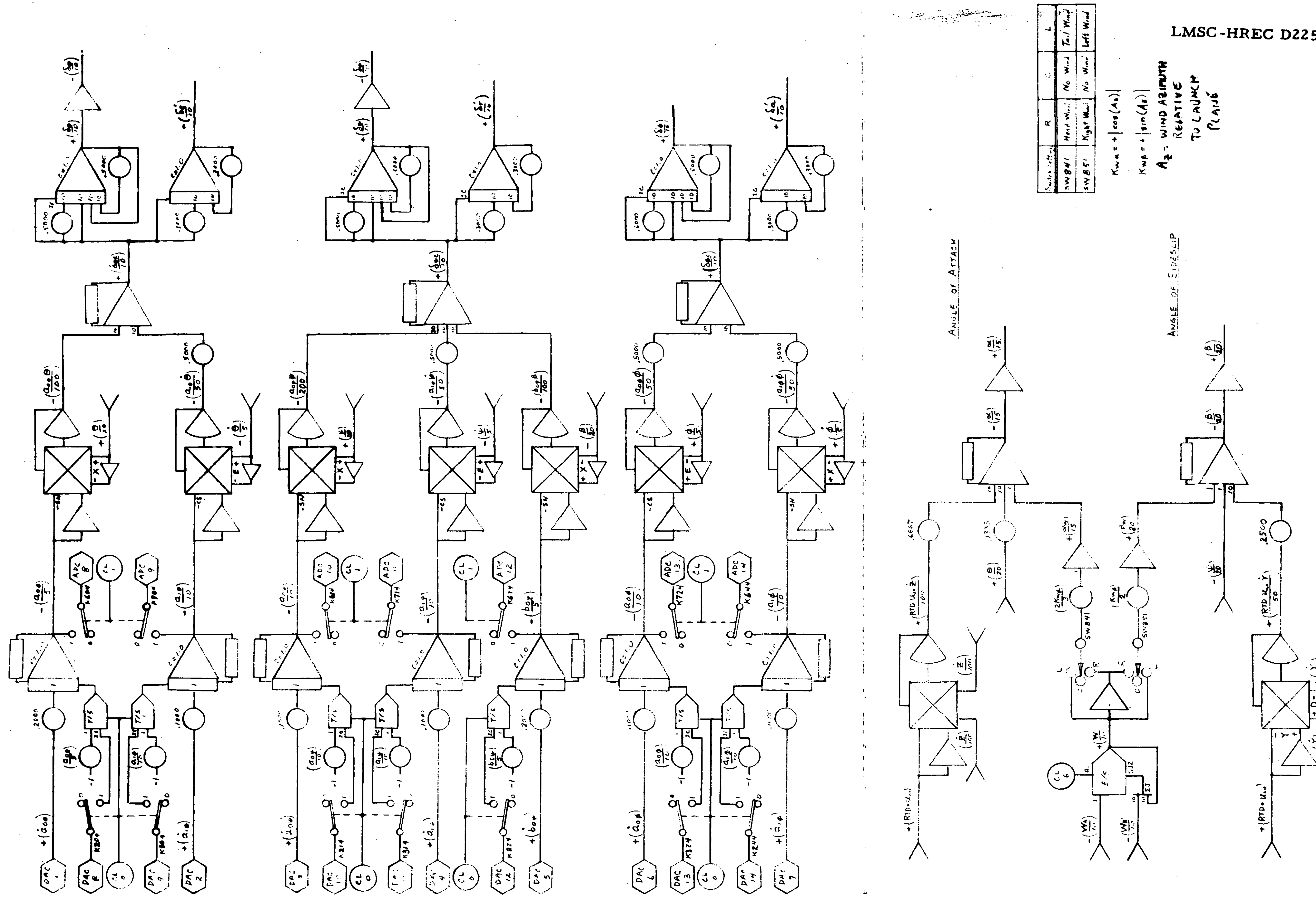


Fig. D-3 - Analog Wiring Diagram for Shuttle Ascent Perturbation Control System Equations

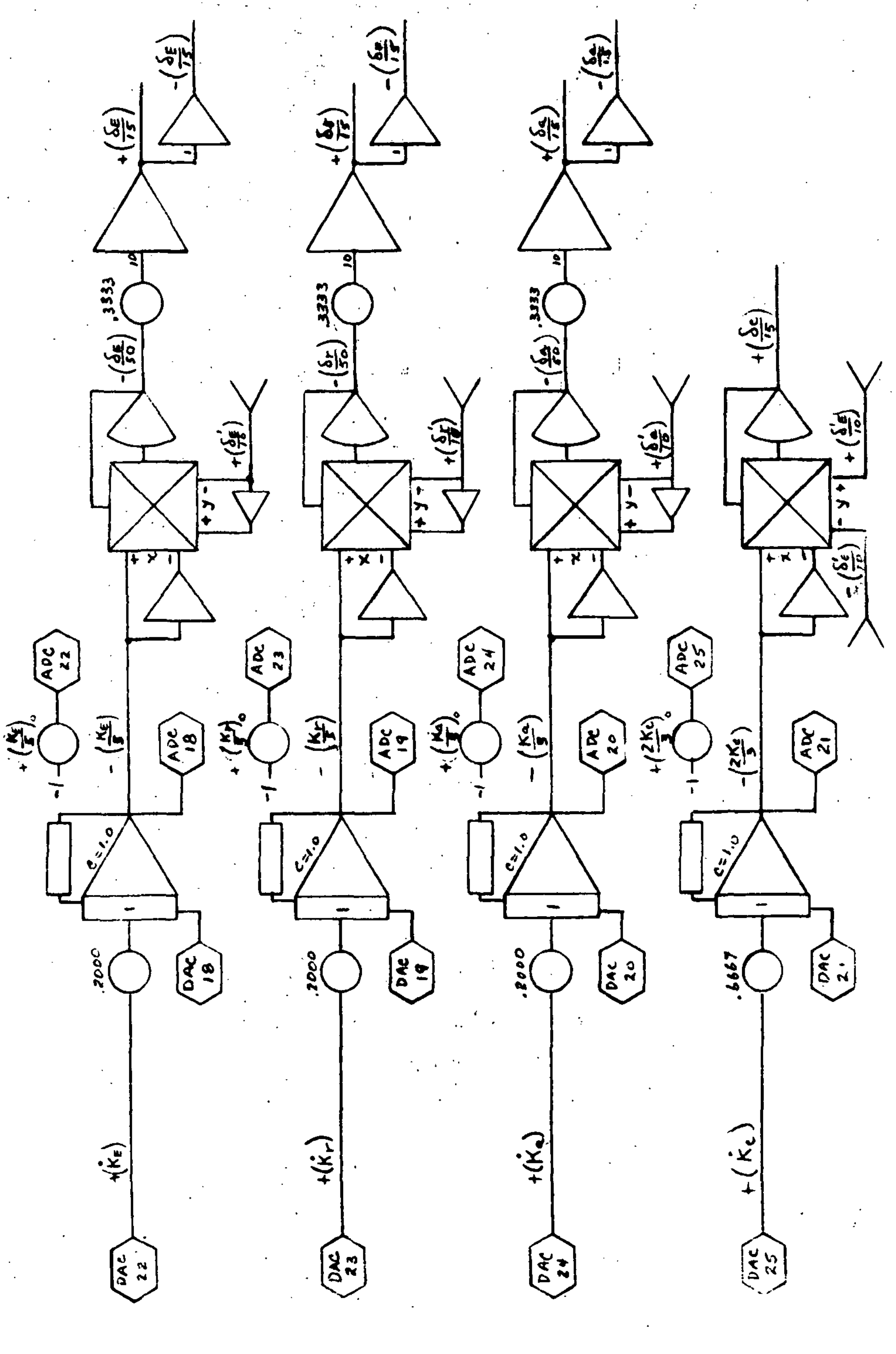


Fig. D-4 - Analog Wiring Diagram for Shuttle Ascent Perturbation Surface Deflection Equations

D-5

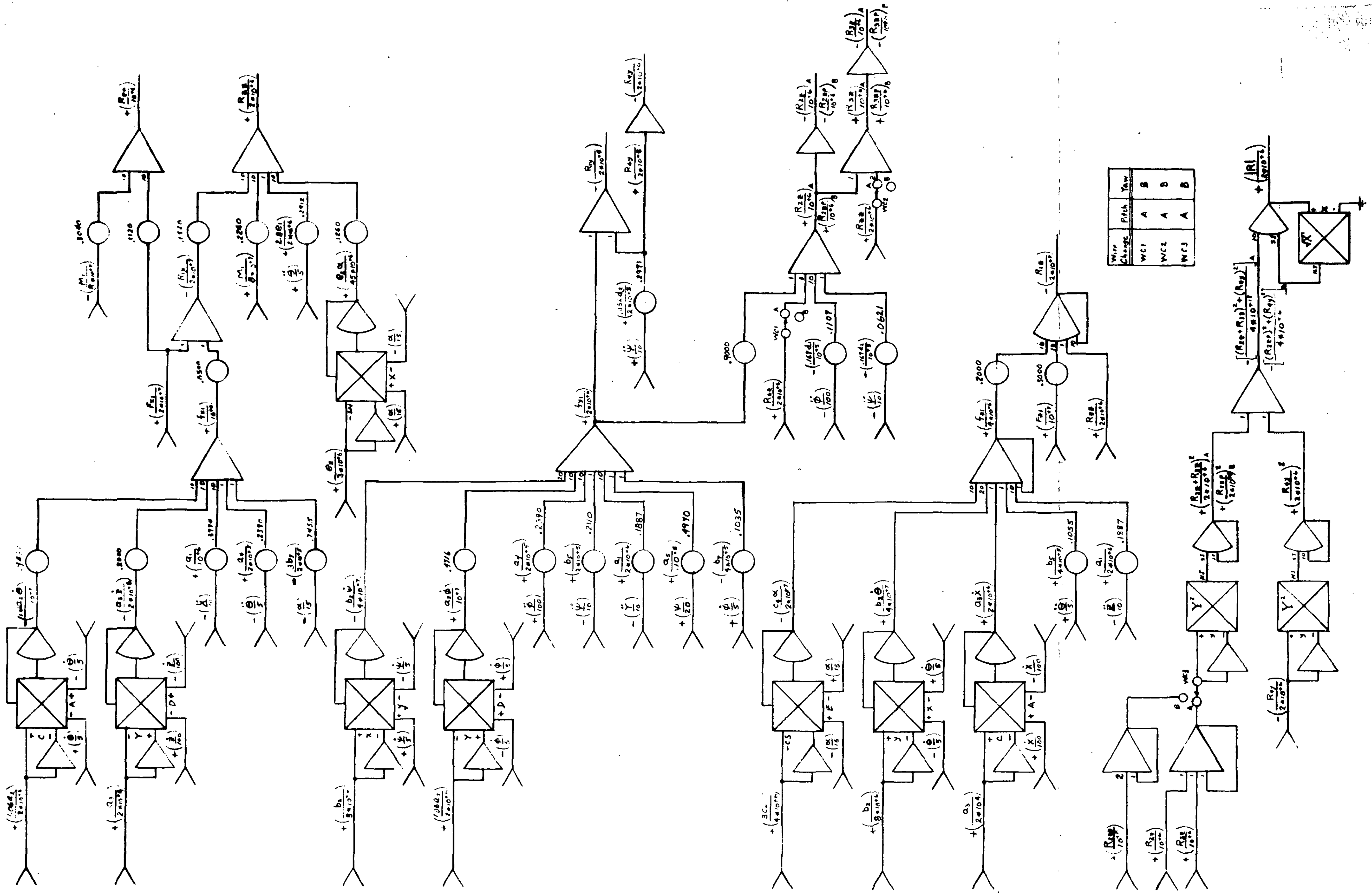


Fig. D-5 - Analog Wiring Diagram for Shuttle Ascent Interface Loading Equations

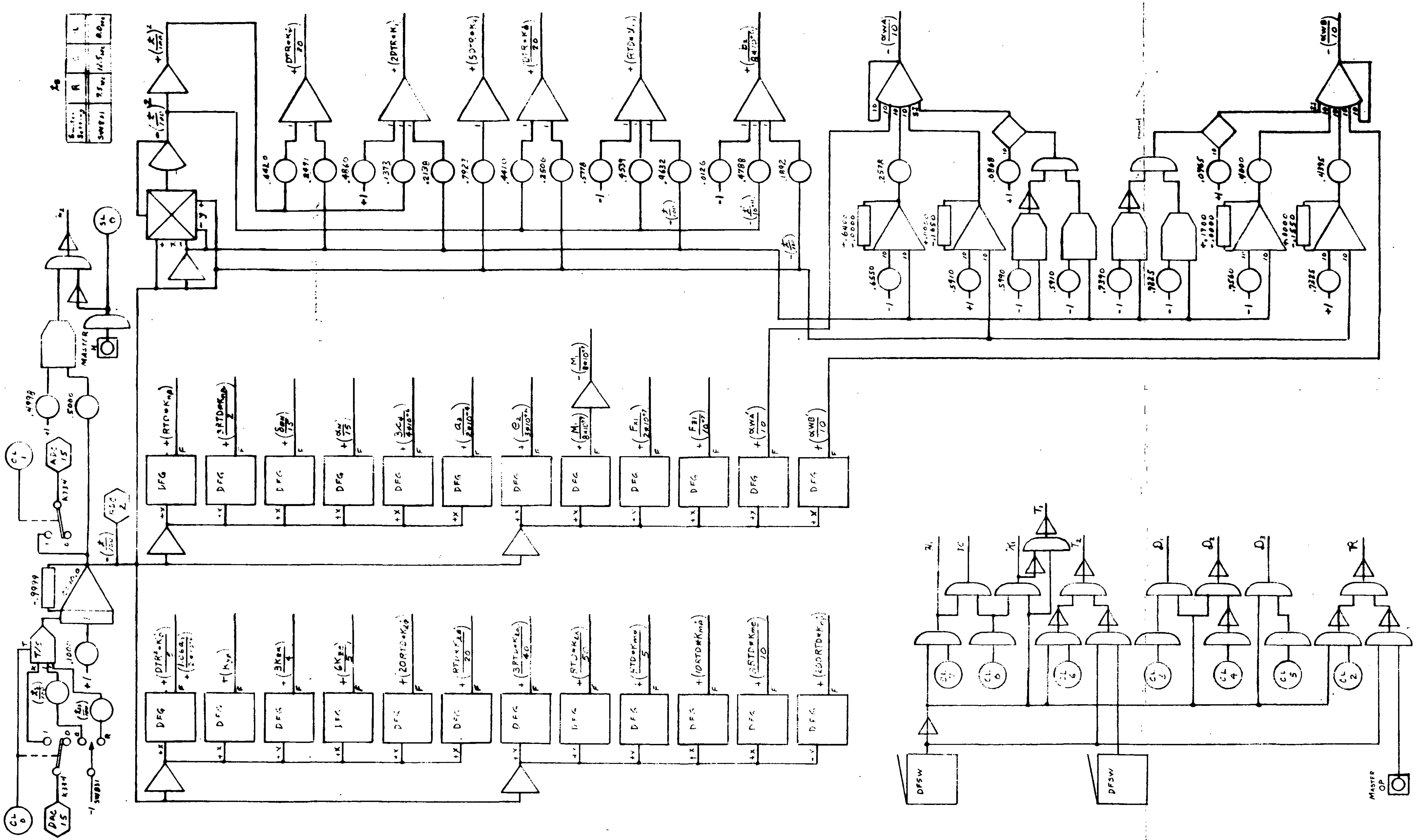


Fig. D-6 - Analog Wiring Diagram for Time Varying Coefficients and Digital/Analog Interface Logic

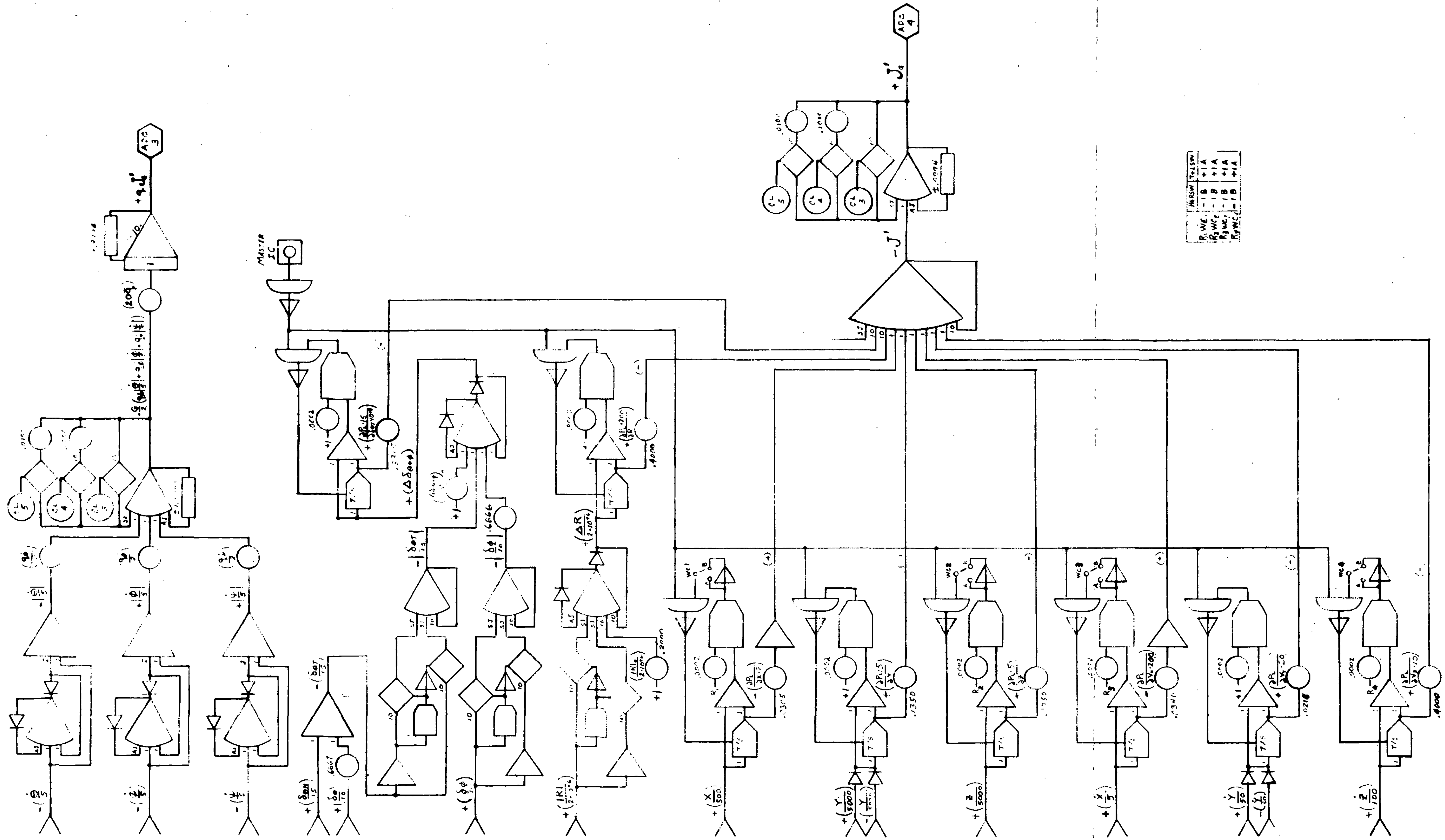


Fig. D-7 - Analog Wiring Diagram for Performance Index Function (J)

Appendix E

SC 4020 DIGITAL PLOTS OF THE MASS, AERODYNAMIC AND TRAJECTORY DATA FOR MDAC-20 SHUTTLE CONFIGURATION AND THE TIME VARYING COEFFICIENTS GENERATED BY THE SHUTTLE DATA REDUCTION PROGRAM

Appendix E

The time varying coefficients for the 6D EOM of Appendix B for MDAC-20 Shuttle Configuration were computed with an IBM 7094 Digital Computer Program for the raw mass, aerodynamic and trajectory data supplied by MSFC. This program is shown in Fig. E-1. Pages E-3 through E-14 show this raw data in SC 4020 plot form. Pages E-15 and E-16 are included for data which are zero or constant. The time varying coefficients are shown on pages E-17 through E-24 in SC 4020 plot form. Some approximations were necessary as dictated by equipment shortages. These approximations as simulated on the analog are straight line segments between the points circled for diode function generation and 2nd order polynomials as indicated. Page E-34 is included to show coefficients which were zero or constant.

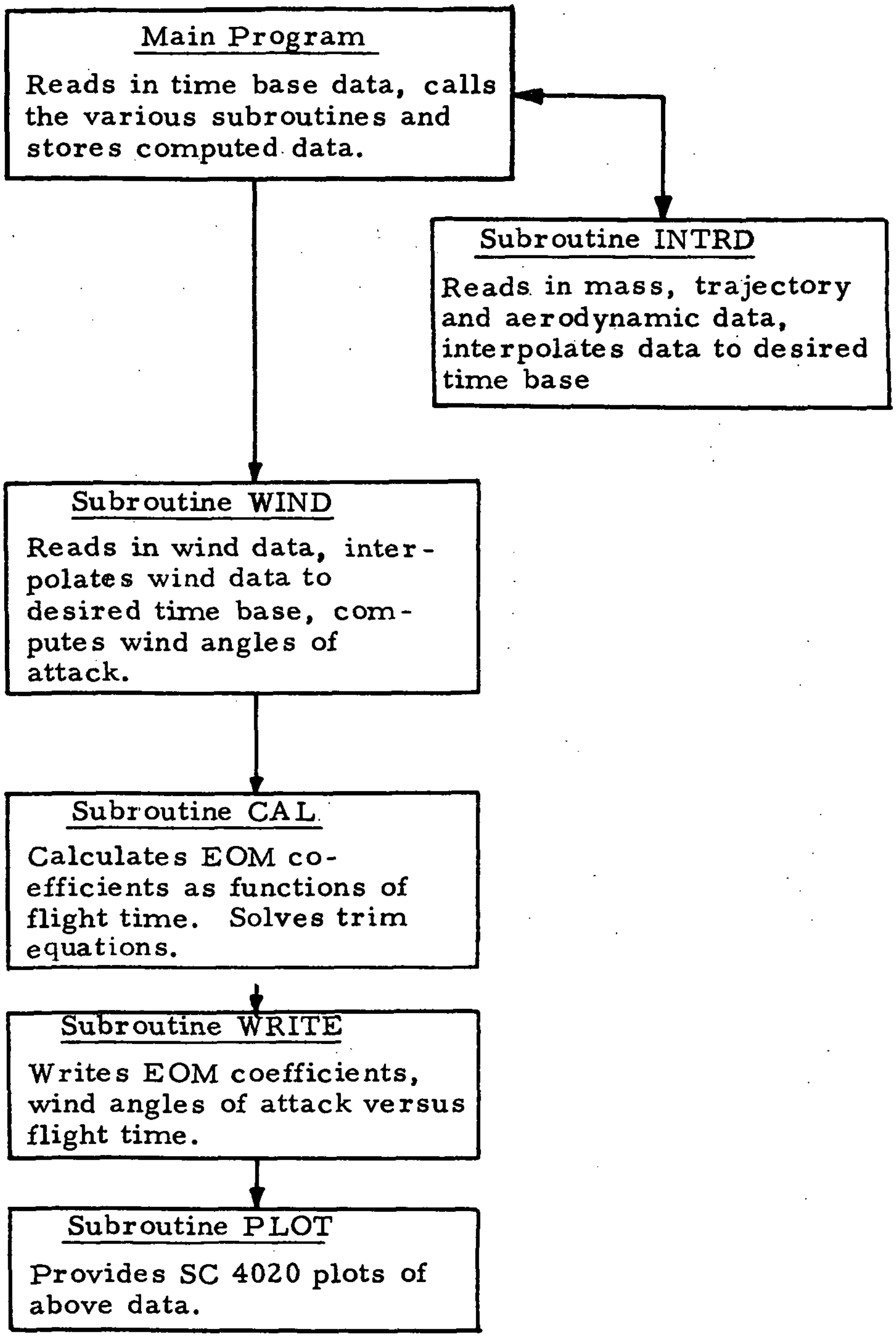
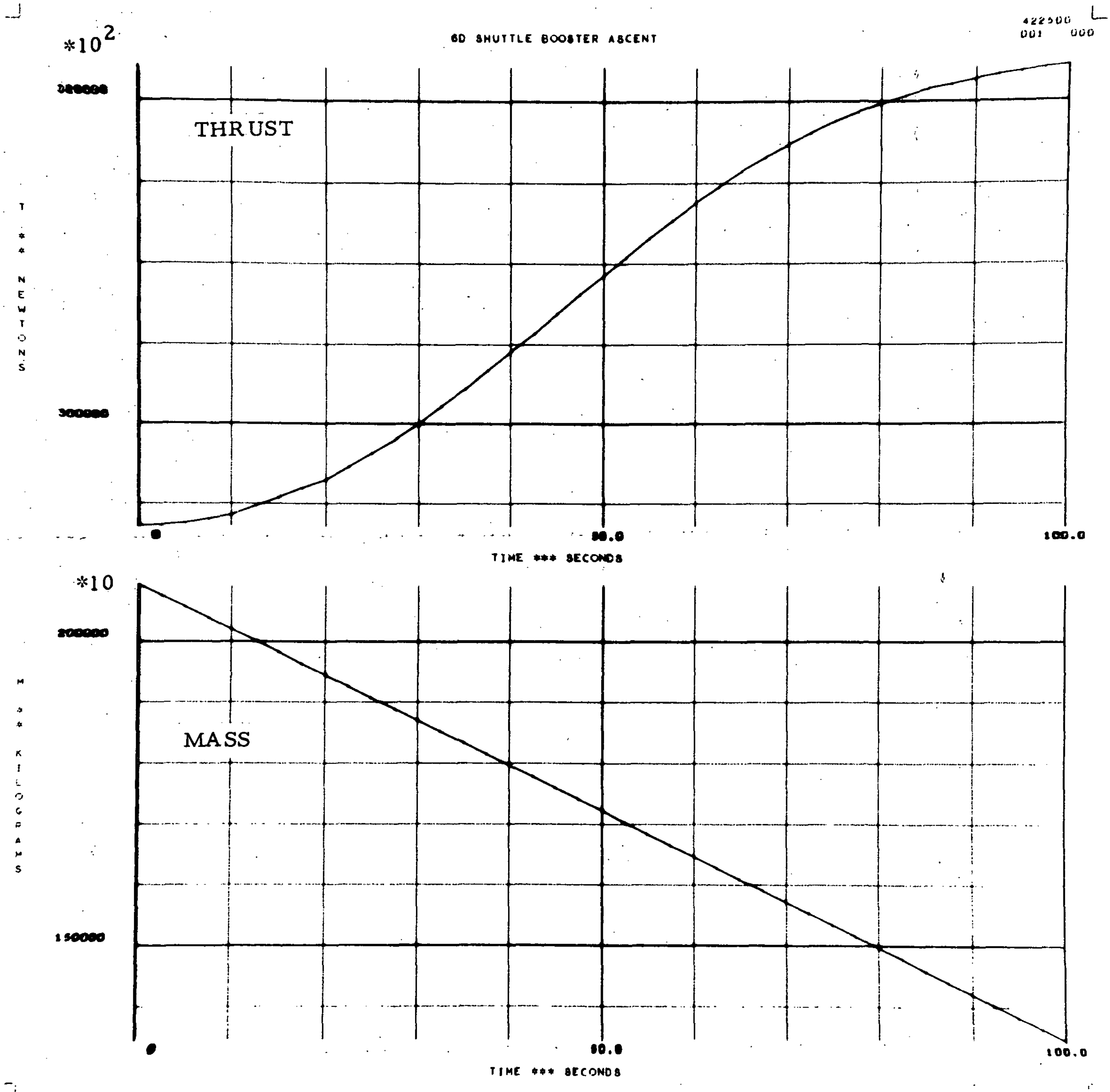
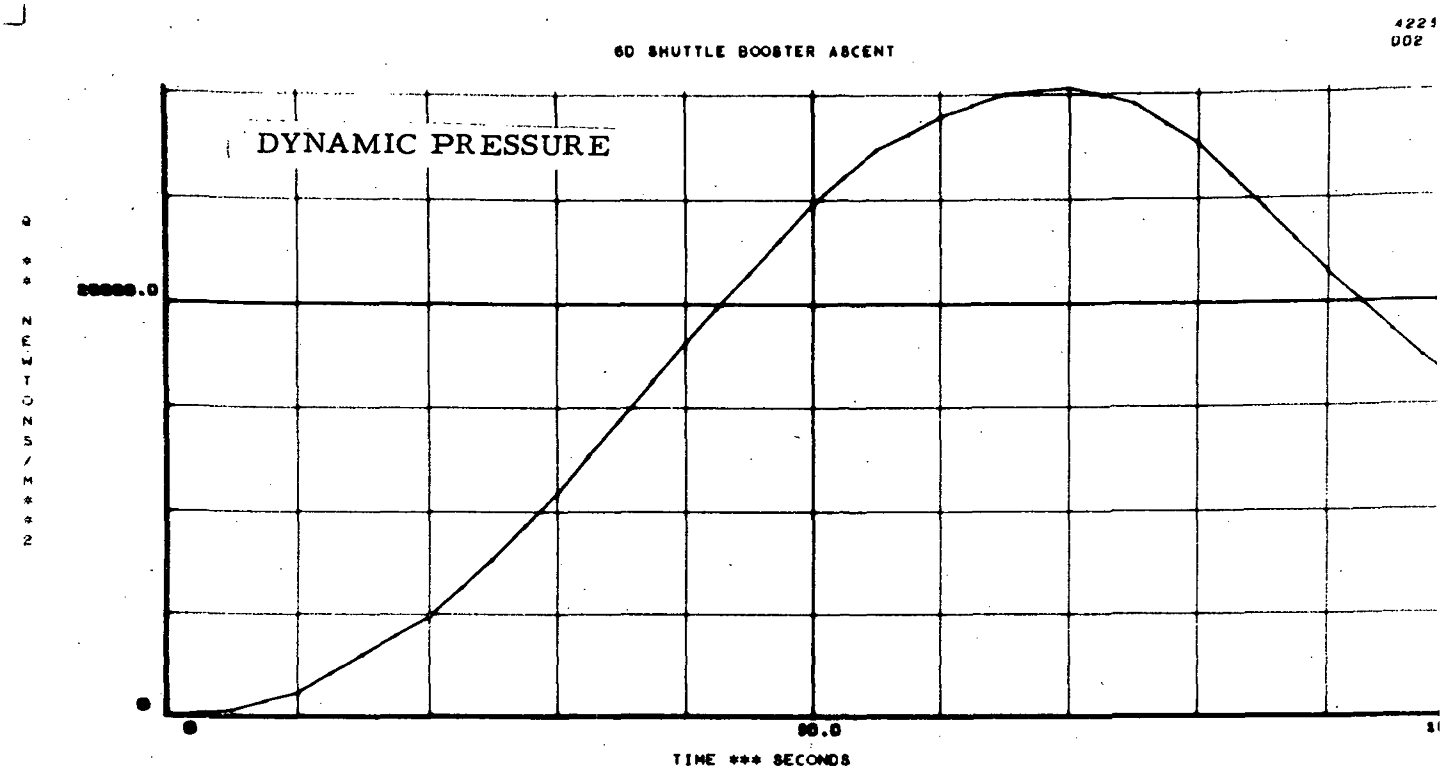
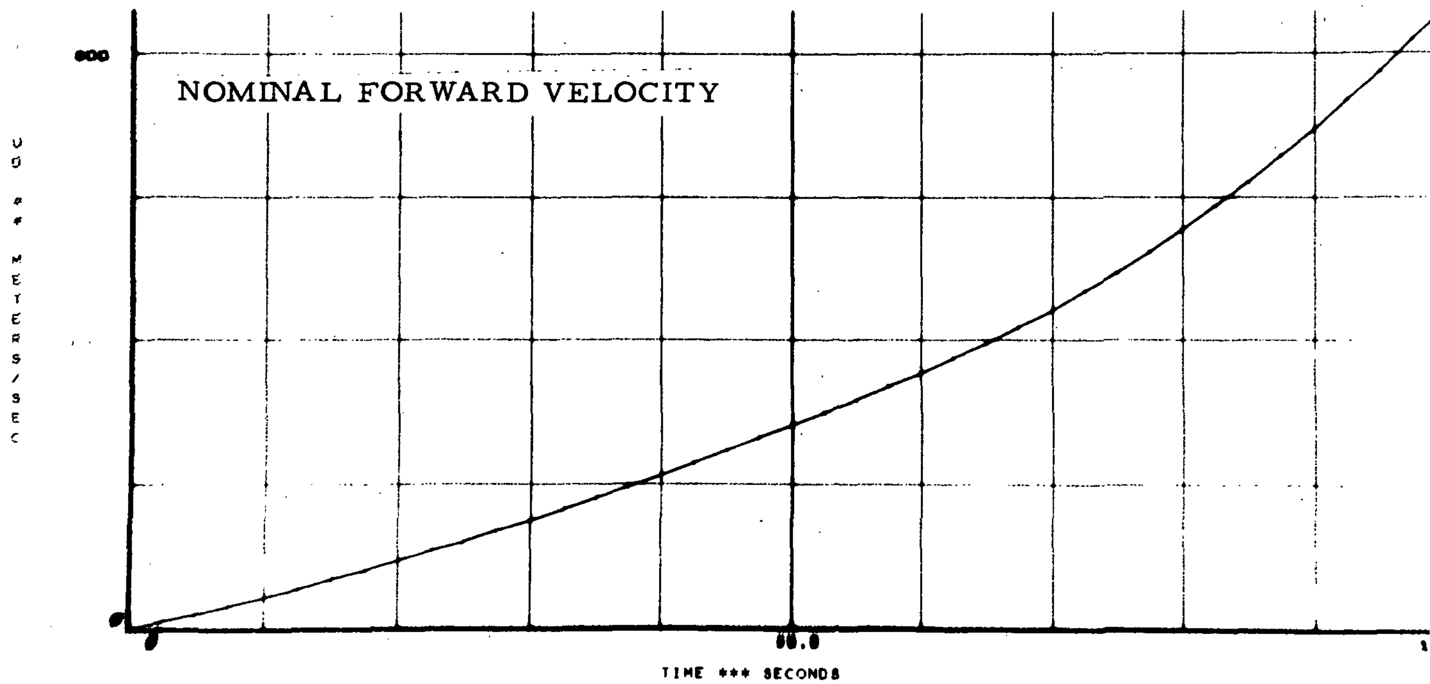
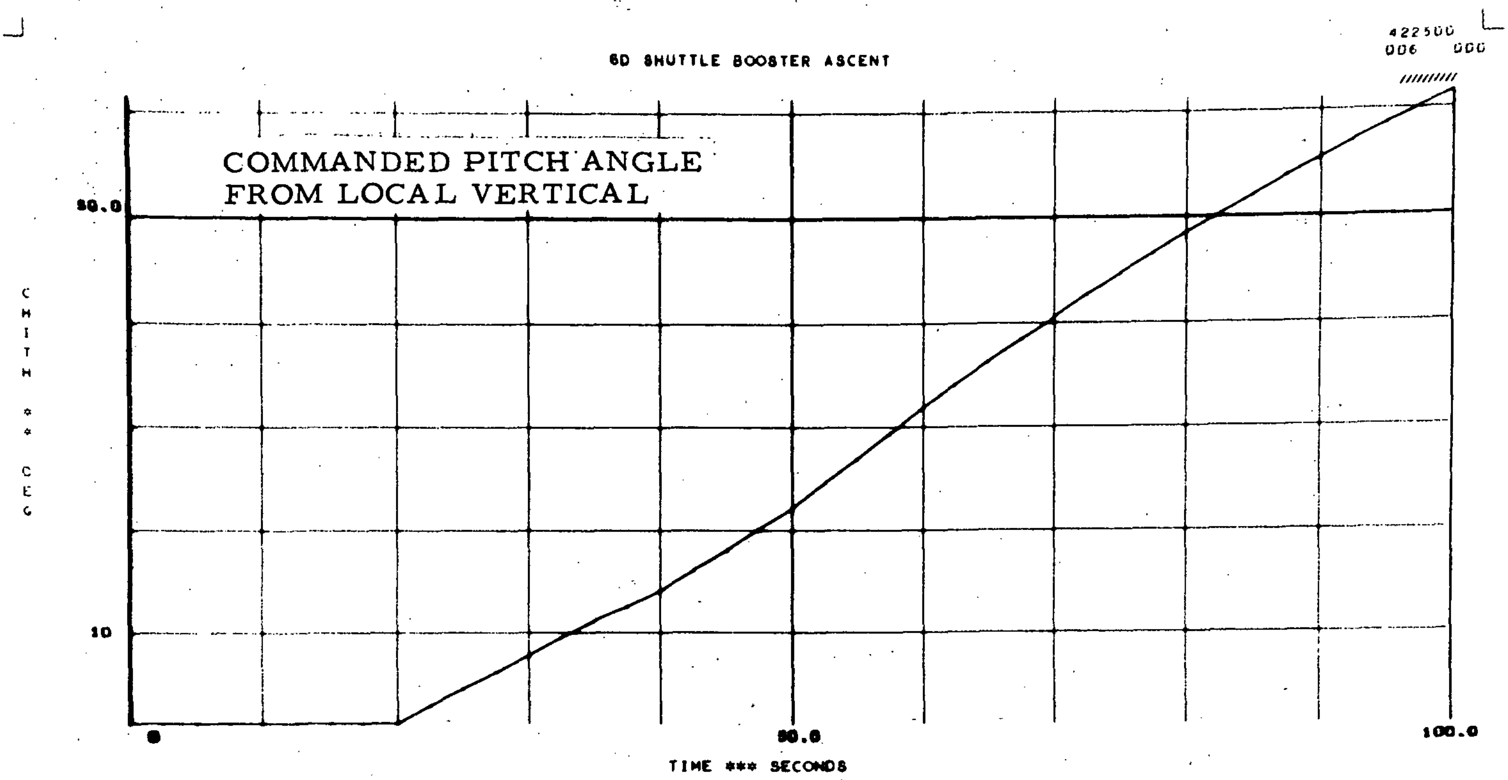


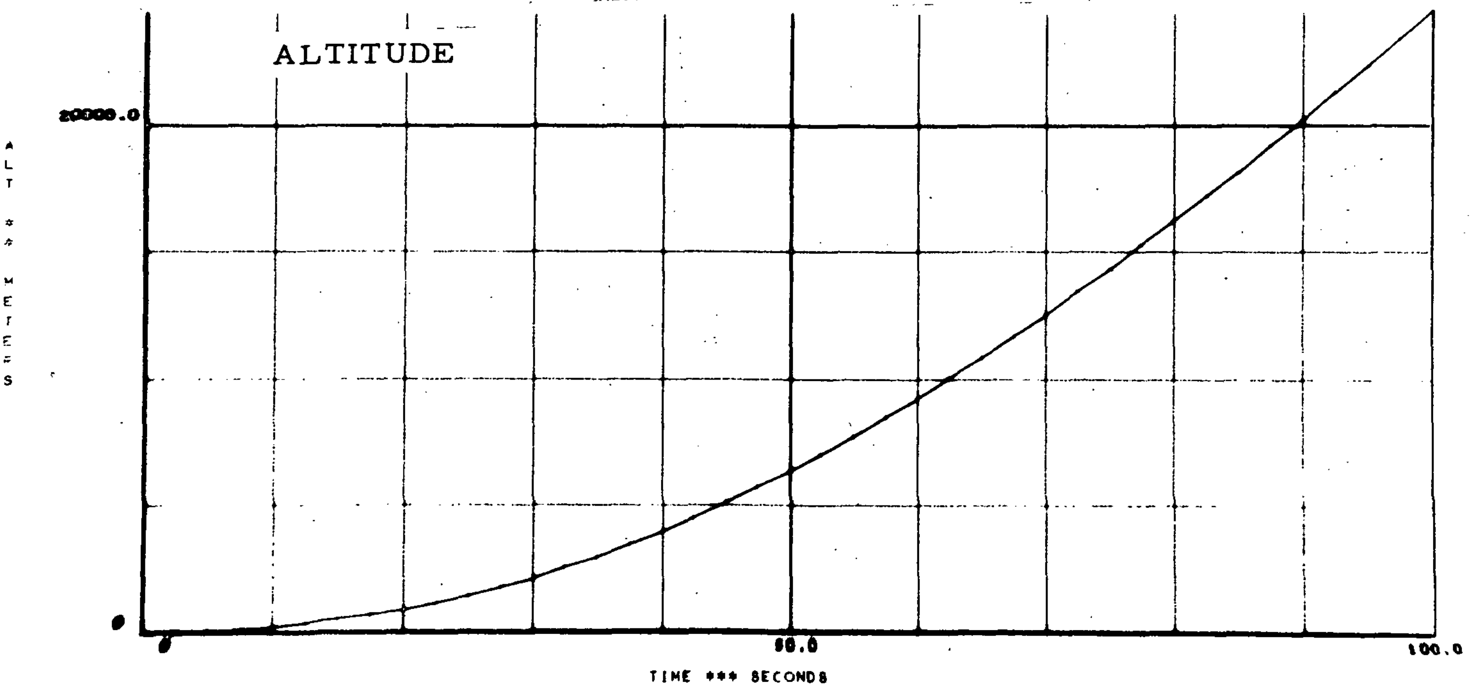
Fig. E-1 - Flow Chart of IBM 7094 Digital Data Program to Calculate and Plot Shuttle EOM Coefficients

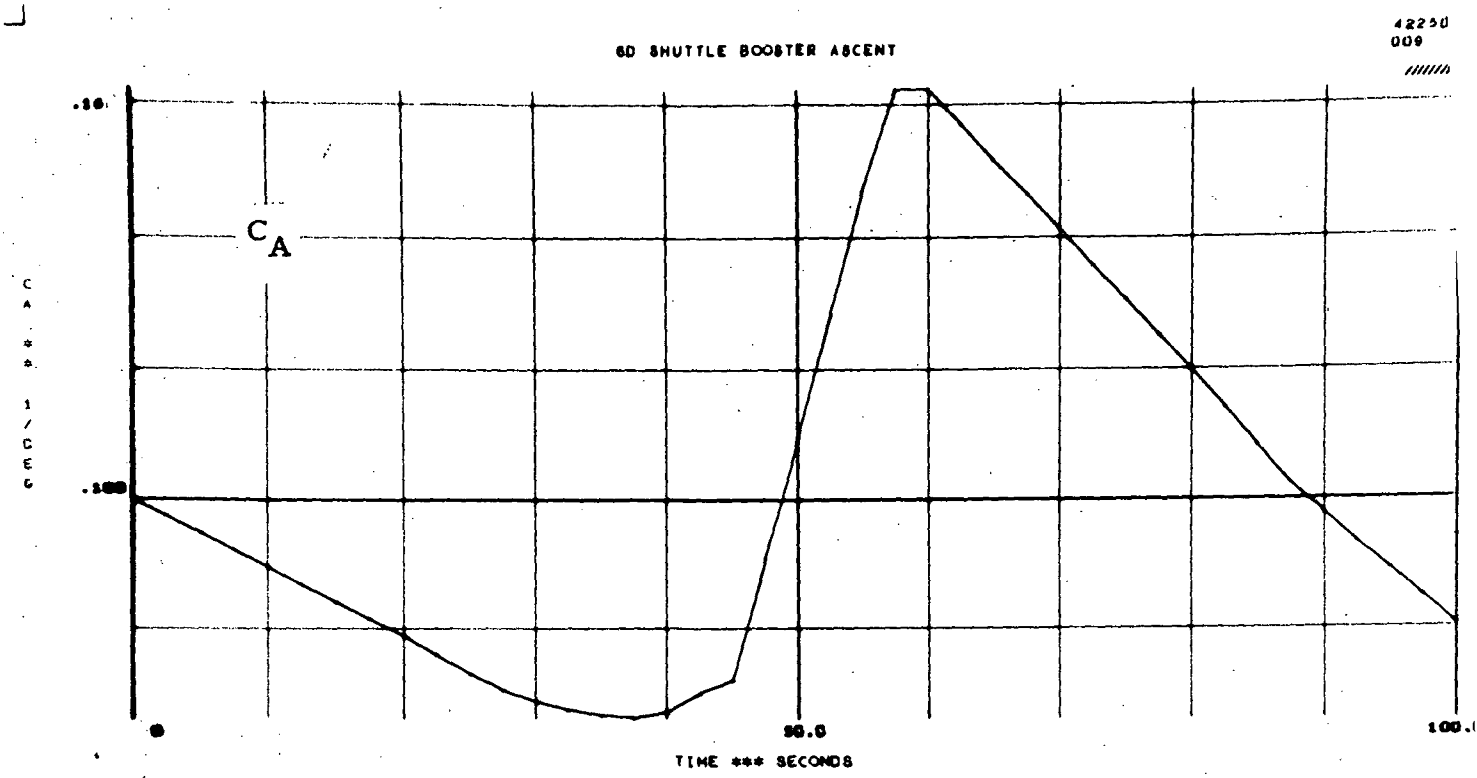

6D SHUTTLE BOOSTER ASCENT
*10^2
THRUST
300000
T ** NEWTONS
TIME *** SECONDS
100.0
*10
MASS
200000
150000
M ** KILOGRAMS
TIME *** SECONDS
100.0

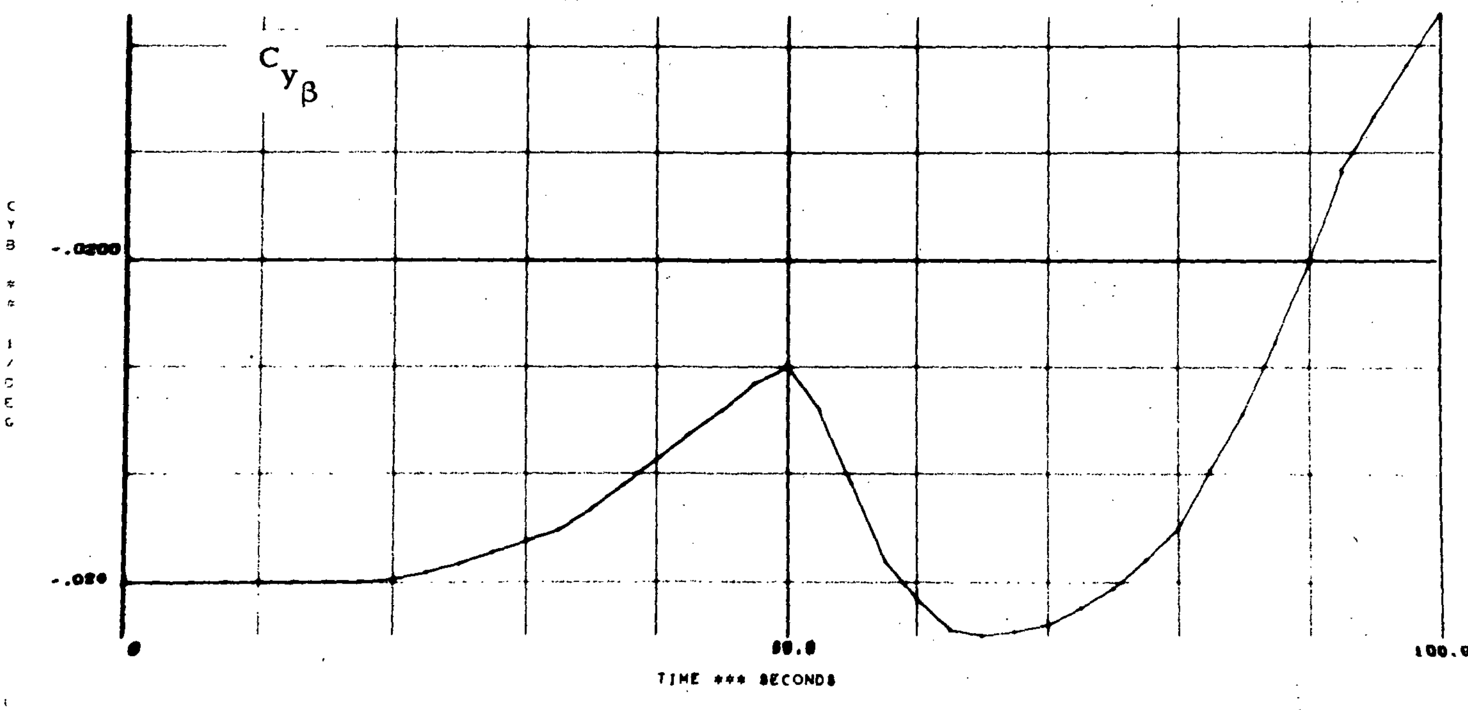

SD SHUTTLE BOOSTER ASCENT
DYNAMIC PRESSURE
Q ** NEWTONS/M**2
TIME *** SECONDS

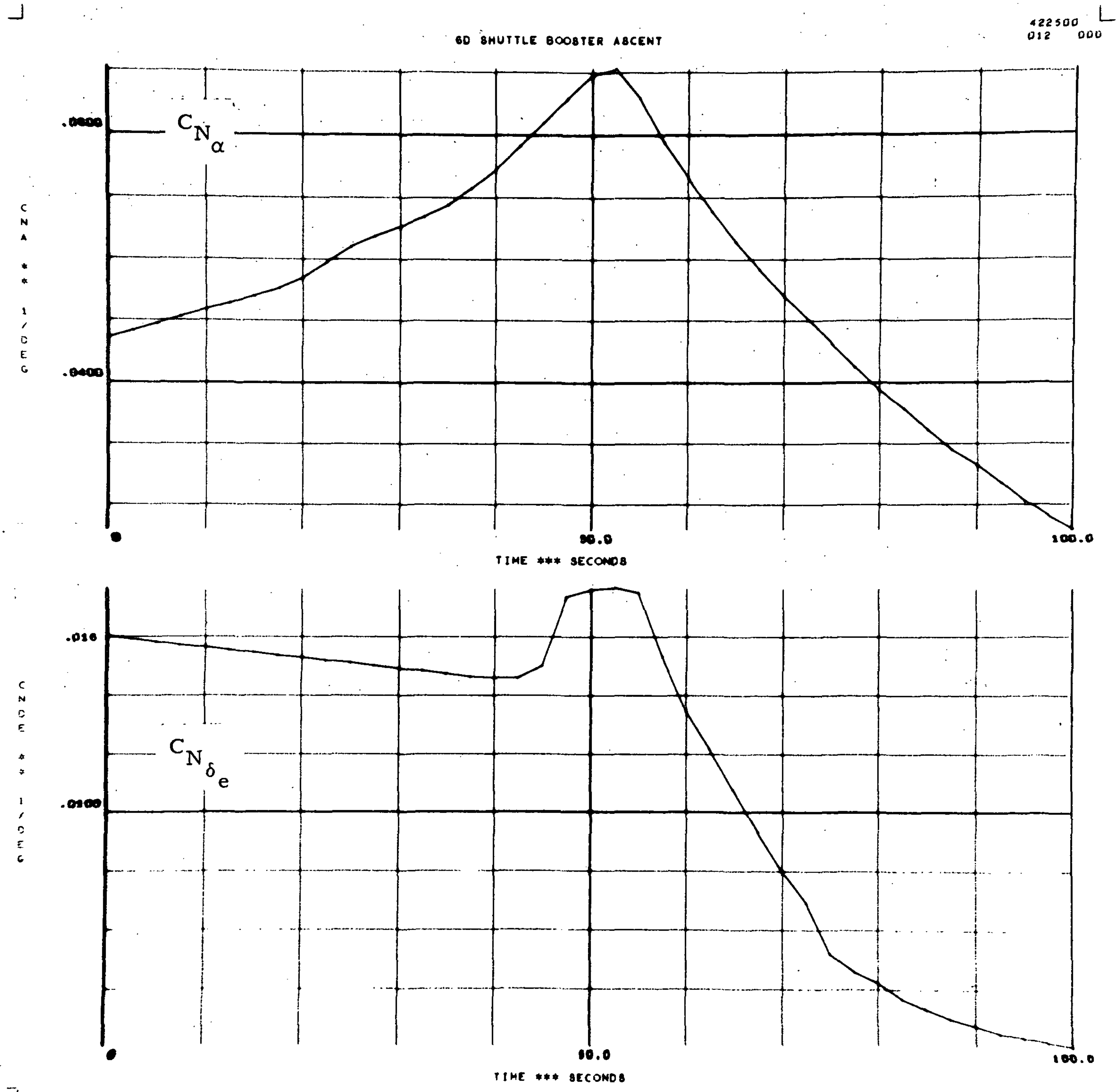

NOMINAL FORWARD VELOCITY
UO ** METERS/SEC
TIME *** SECONDS


422500
006 000
6D SHUTTLE BOOSTER ASCENT
COMMANDED PITCH ANGLE
FROM LOCAL VERTICAL
CHITH ** DEG
10
0
50.0
100.0
TIME *** SECONDS


ALTITUDE
20000.0
ALT ** METERS
0
50.0
100.0
TIME *** SECONDS

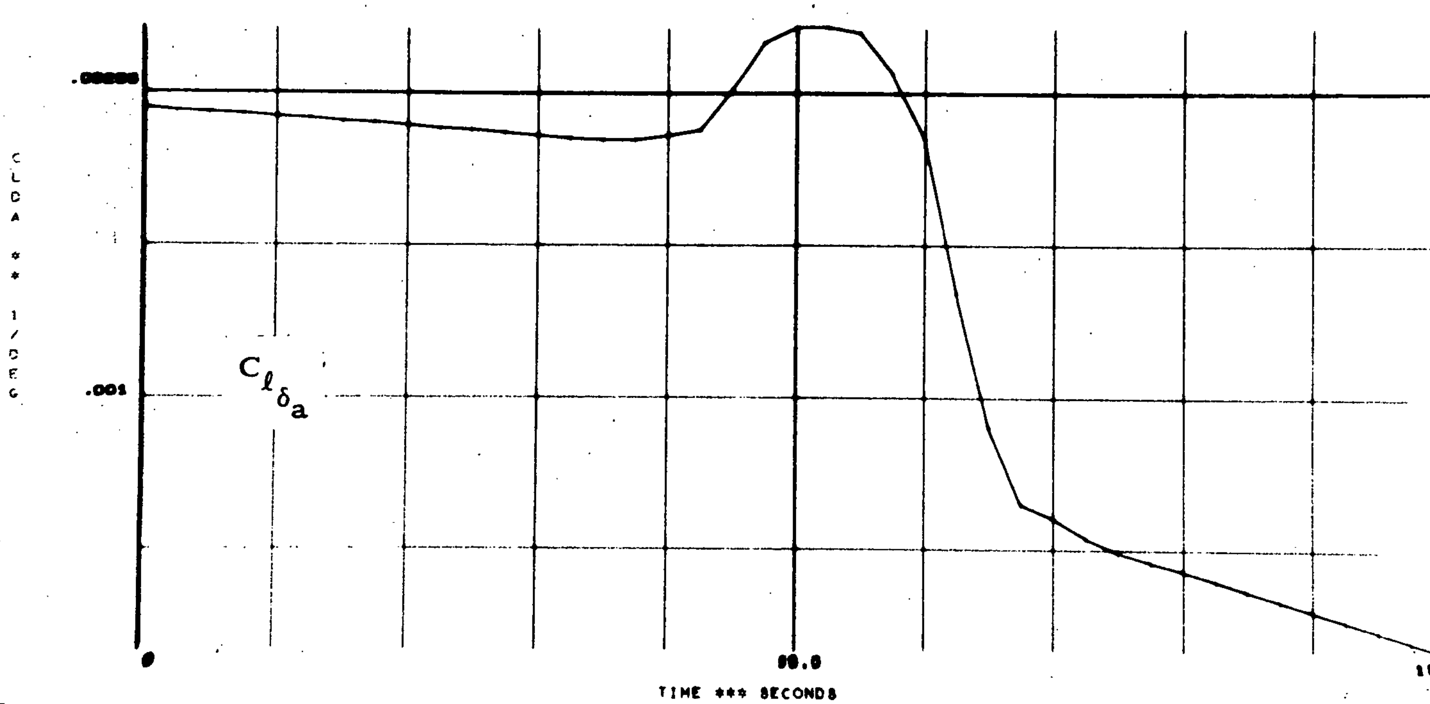

6D SHUTTLE BOOSTER ASCENT
42250
009
C_A
C A ** 1/DEG
.16
.100
0
50.0
100.0
TIME *** SECONDS

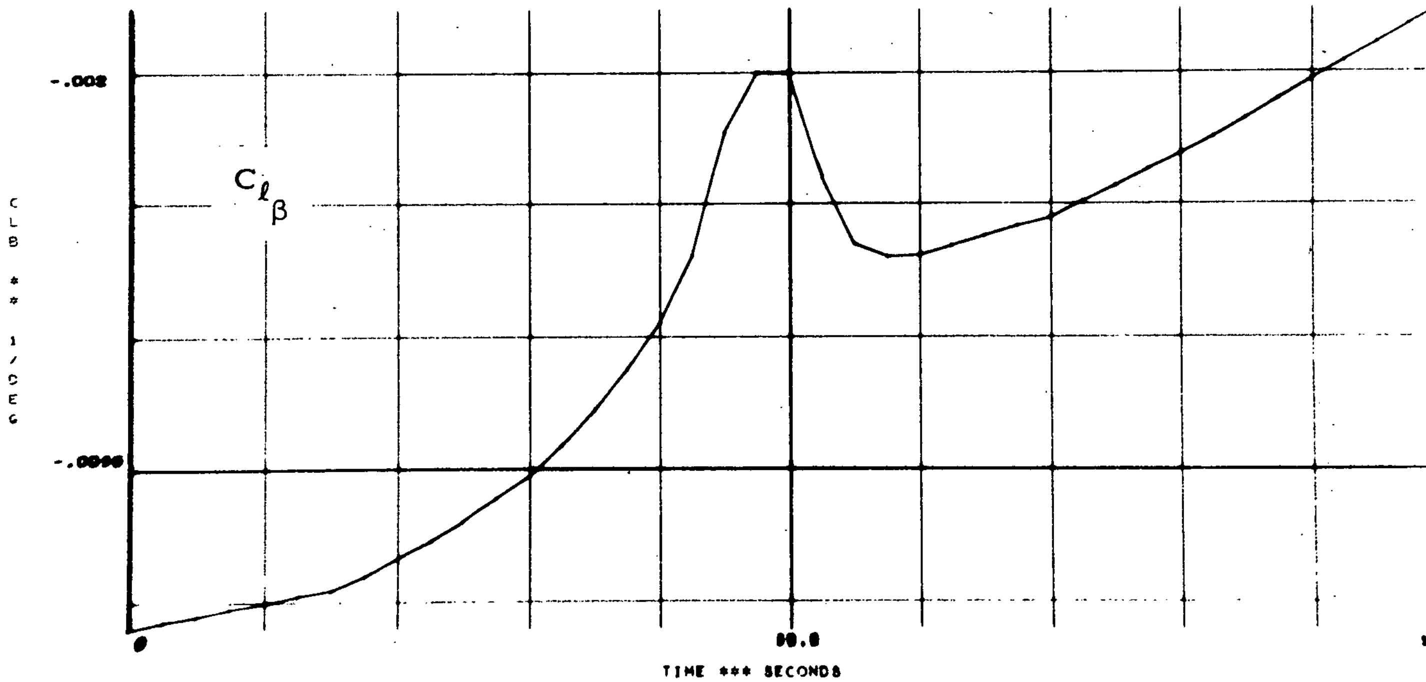

C_{y_β}
C Y B ** 1/DEG
-.0200
-.020
0
50.0
100.0
TIME *** SECONDS

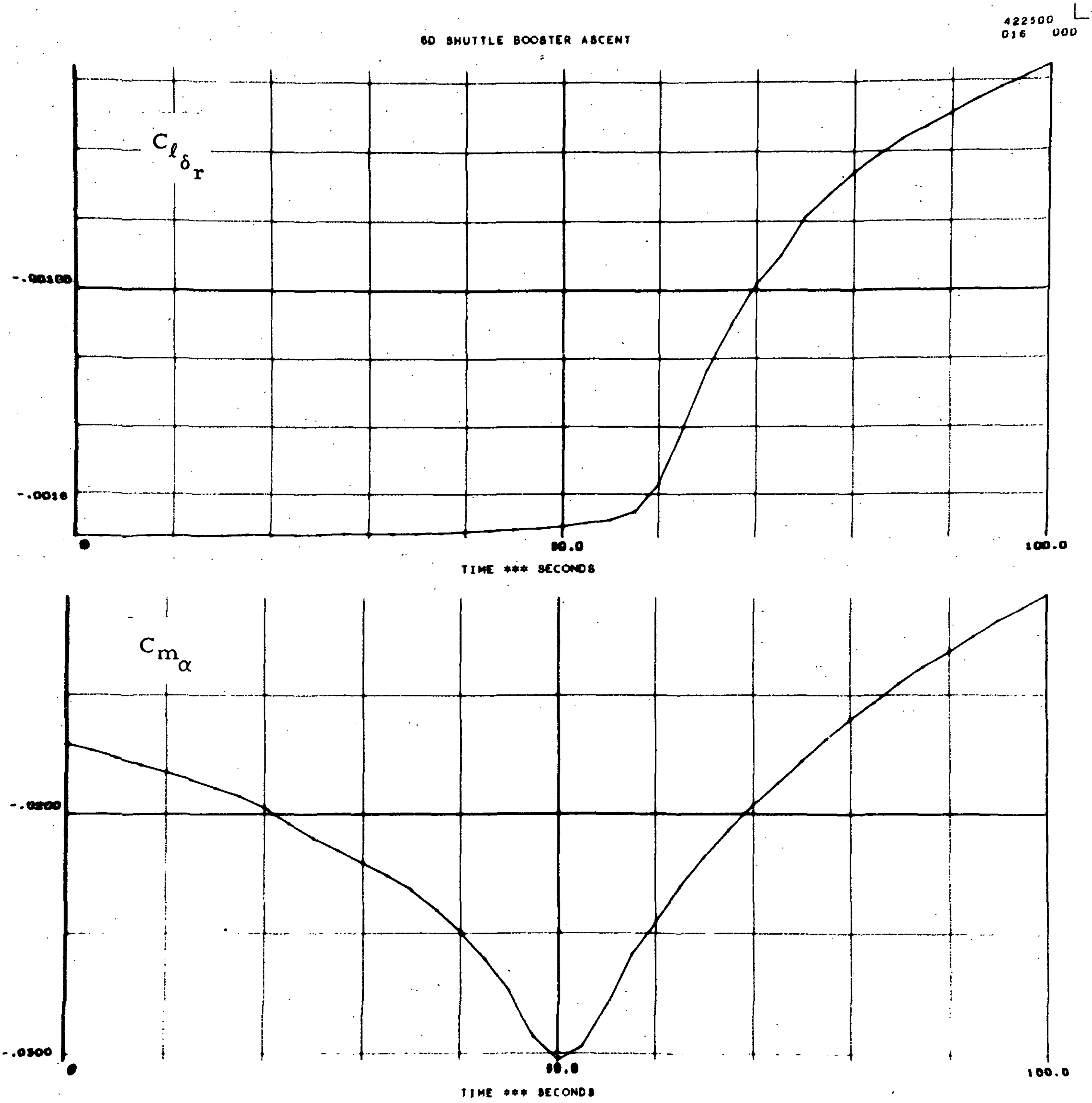

422500
012 000
6D SHUTTLE BOOSTER ASCENT
C_{N_α}
CNA ** 1/DEG
.0600
.0400
0
50.0
100.0
TIME *** SECONDS
$C_{N_{\delta_e}}$
CNDE ** 1/DEG
.016
.0100
0
50.0
100.0
TIME *** SECONDS

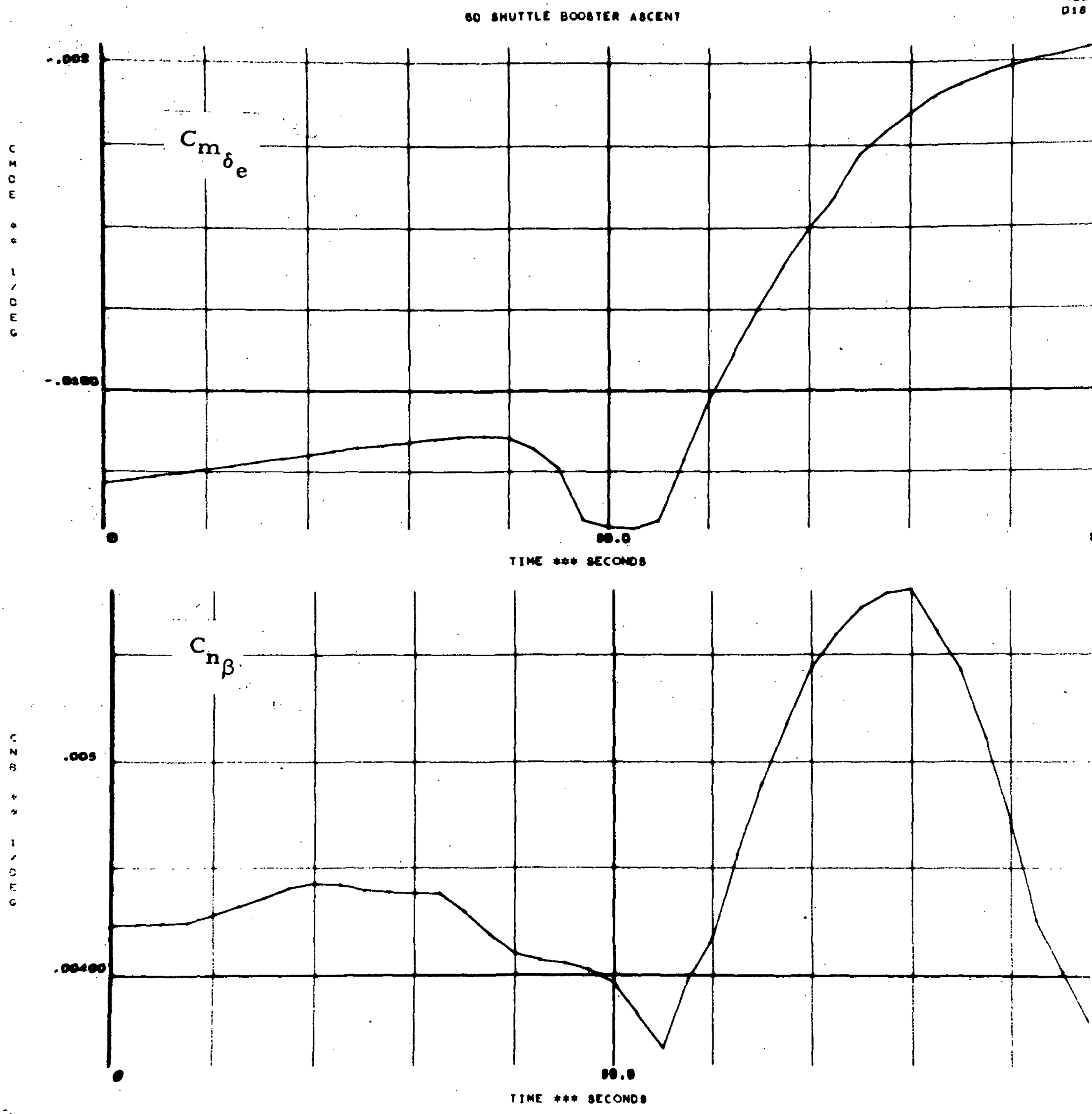
422500
013

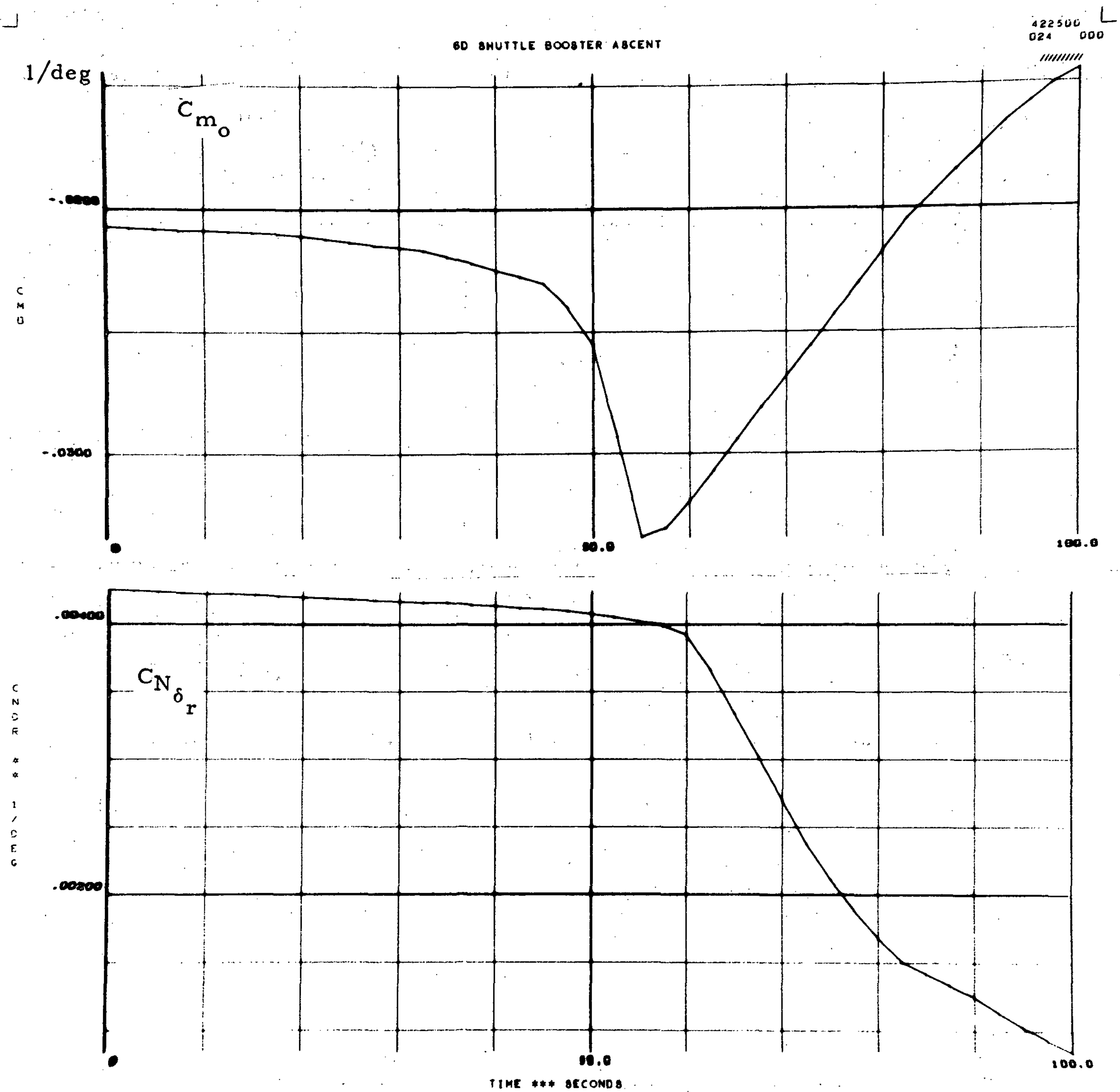
SD SHUTTLE BOOSTER ASCENT

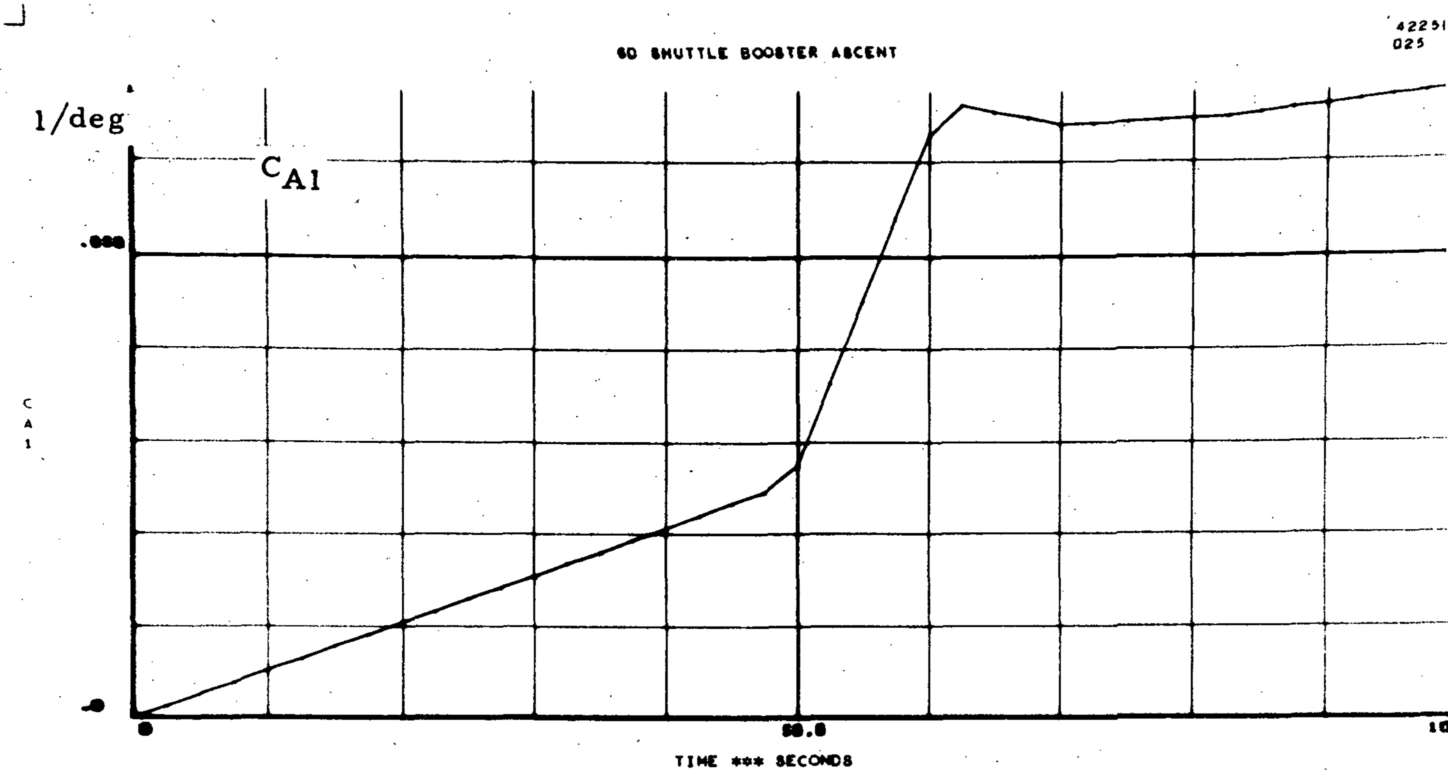




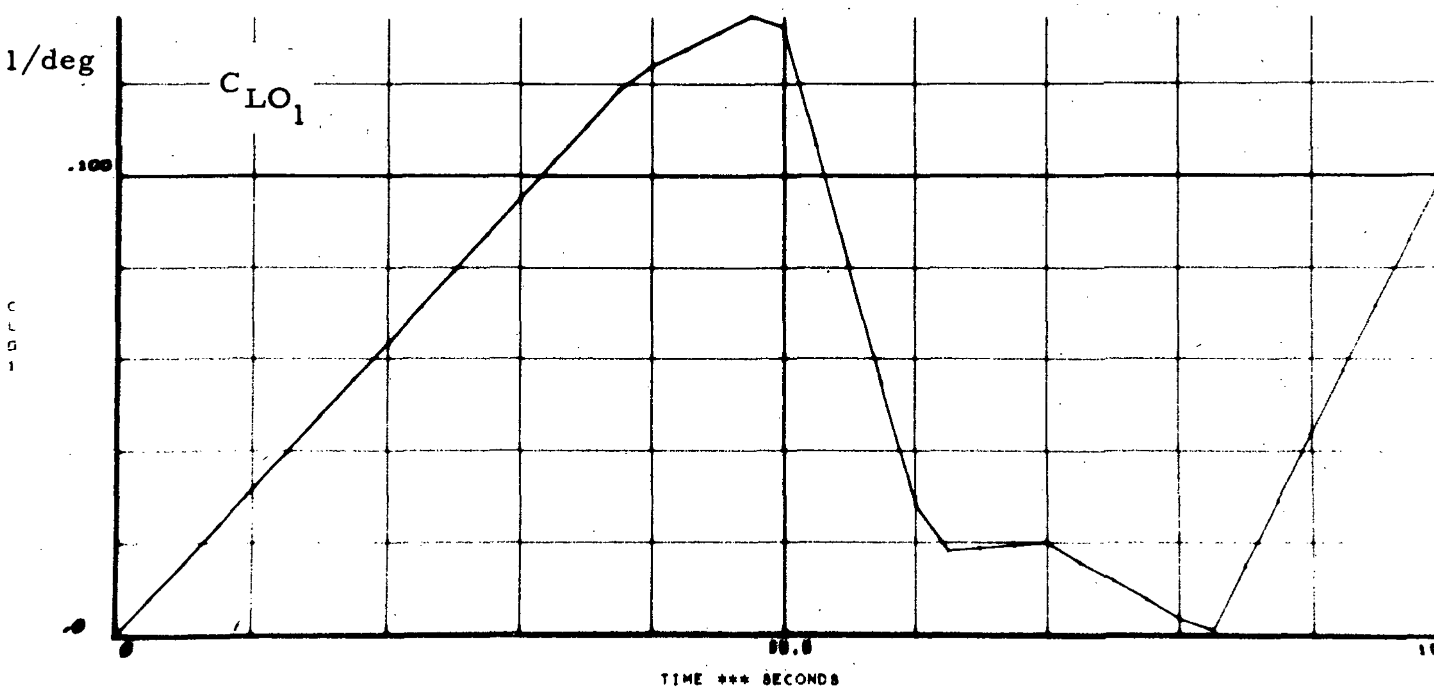

422500
016 000
6D SHUTTLE BOOSTER ASCENT
$C_{\ell_{\delta_r}}$
CLDR ** 1/DEG
-.00100
-.0016
0
50.0
100.0
TIME *** SECONDS
C_{m_α}
CMA ** 1/DEG
-.0200
-.0300
0
50.0
100.0
TIME *** SECONDS


SD SHUTTLE BOOSTER ASCENT
$C_{m_{\delta_e}}$
CMDE ** 1/DEG
TIME *** SECONDS
C_{n_β}
.005
CNB ** 1/DEG
TIME *** SECONDS

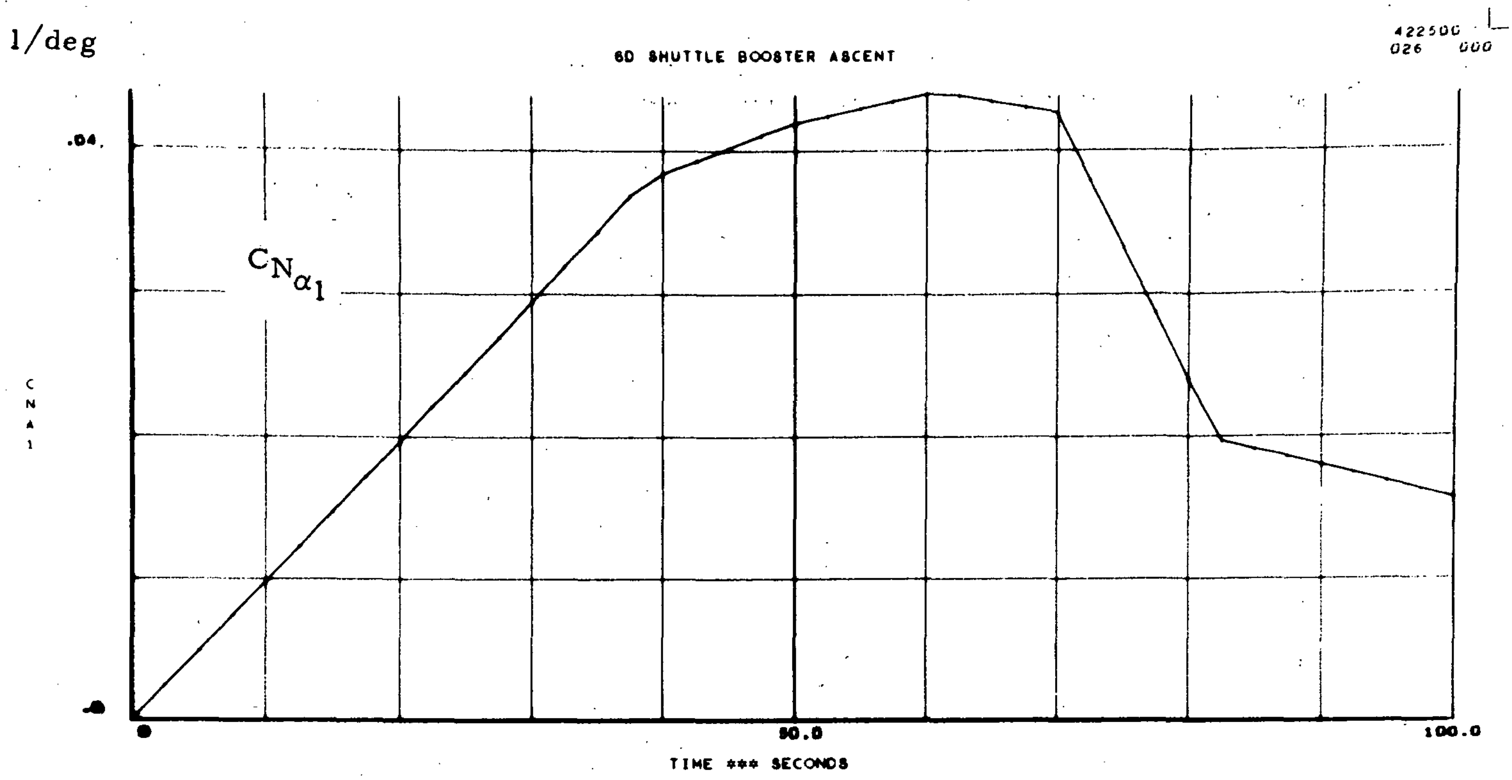

422500
024 000
6D SHUTTLE BOOSTER ASCENT
1/deg
C_{m_o}
-.0300
100.0
$C_{N_{\delta_r}}$
.00400
.00200
100.0
TIME *** SECONDS

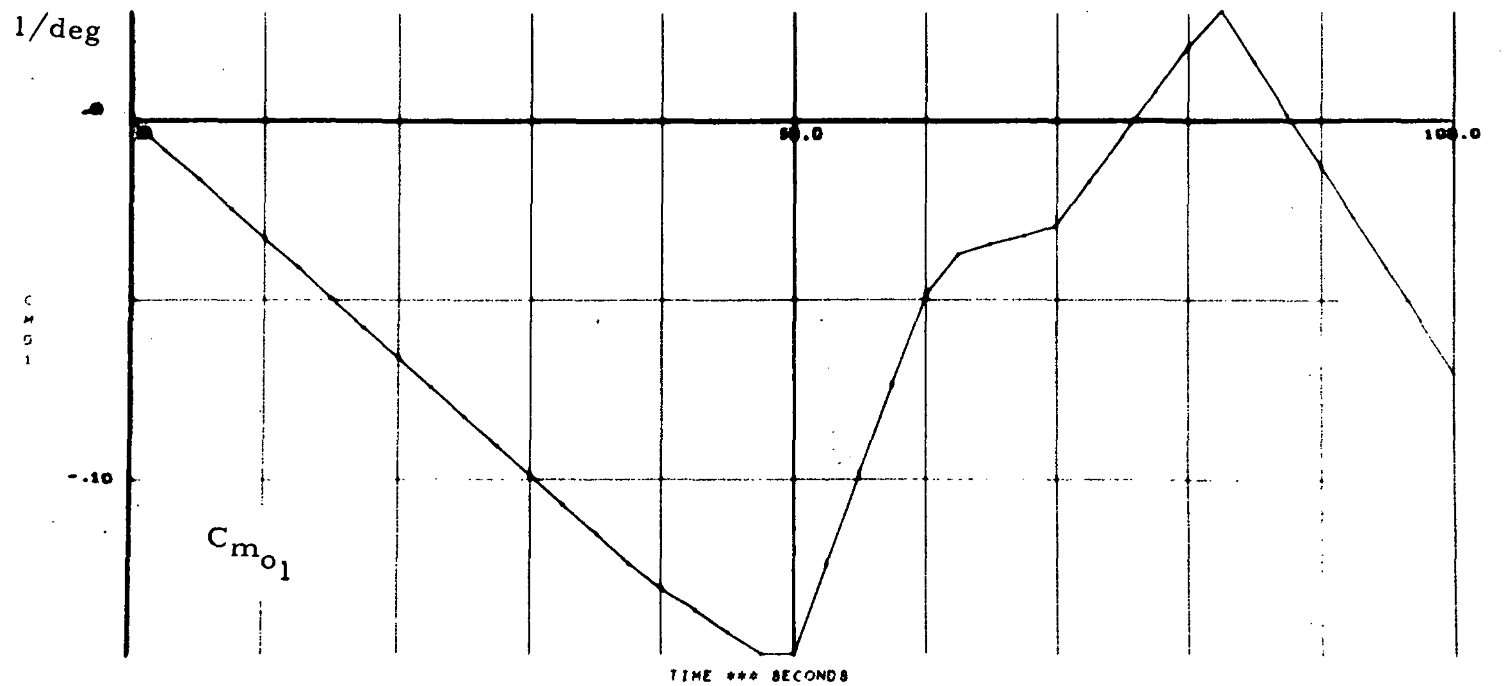


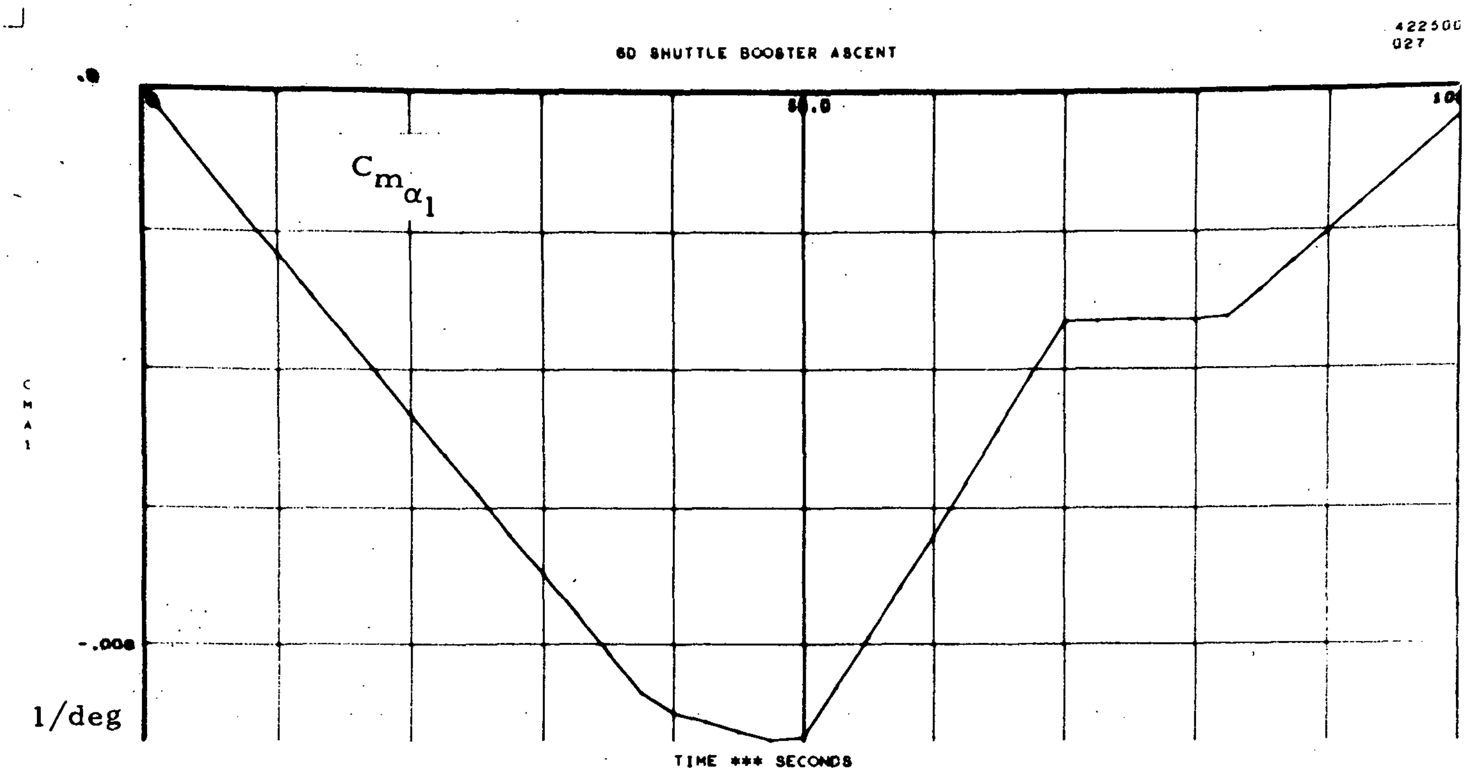
4225
025
6D SHUTTLE BOOSTER ASCENT
1/deg
C_{A1}
.0
C A 1
0
50.0
TIME *** SECONDS



1/deg
C_{LO_1}
.100
.0
C L O 1
0
50.0
TIME *** SECONDS


1/deg
6D SHUTTLE BOOSTER ASCENT
422500
026 000
.04
$C_{N_{\alpha_1}}$
C N A 1
0
50.0
100.0
TIME *** SECONDS


1/deg
0
50.0
100.0
-.10
$C_{m_{o_1}}$
C M O 1
TIME *** SECONDS


4225000
027
6D SHUTTLE BOOSTER ASCENT
$C_{m_{\alpha_1}}$
C M A 1
-.008
1/deg
TIME *** SECONDS

$I_x = .350 * 10^8$ kg-m^2

$I_y = .607 * 10^9$ kg-m^2

$I_z = .596 * 10^9$ kg-m^2

$I_{xz} = .139 * 10^8$ kg-m^2

$x_{cg} = 43.0$ m

$y_{cg} = 0.0$ m

$z_{cg} = -1.7$ m

$C_{a_{\delta_c}} = 0$ 1/deg

$C_{a_{\delta_e}} = 0$ 1/deg

$C_{y_{\delta_a}} = 0$ 1/deg

$C_{y_{\delta_r}} = 0$ 1/deg

$C_{N_{\delta_c}} = 0$ 1/deg

$C_{\ell_p} = -.158 * 10^{-3}$ 1/deg

$C_{\ell_r} = 0$ 1/deg

$C_{\ell_{\dot{\beta}}} = 0$ 1/deg

$C_{m_q} = -.525 * 10^{-2}$ 1/deg

$C_{m_{\dot{\alpha}}} = 0$ 1/deg

$$C_{n_p} = 0 \quad 1/\text{deg}$$

$$C_{n_r} = -.28 * 10^{-3} \quad 1/\text{deg}$$

$$C_{n_{\dot{\beta}}} = 0 \quad 1/\text{deg}$$

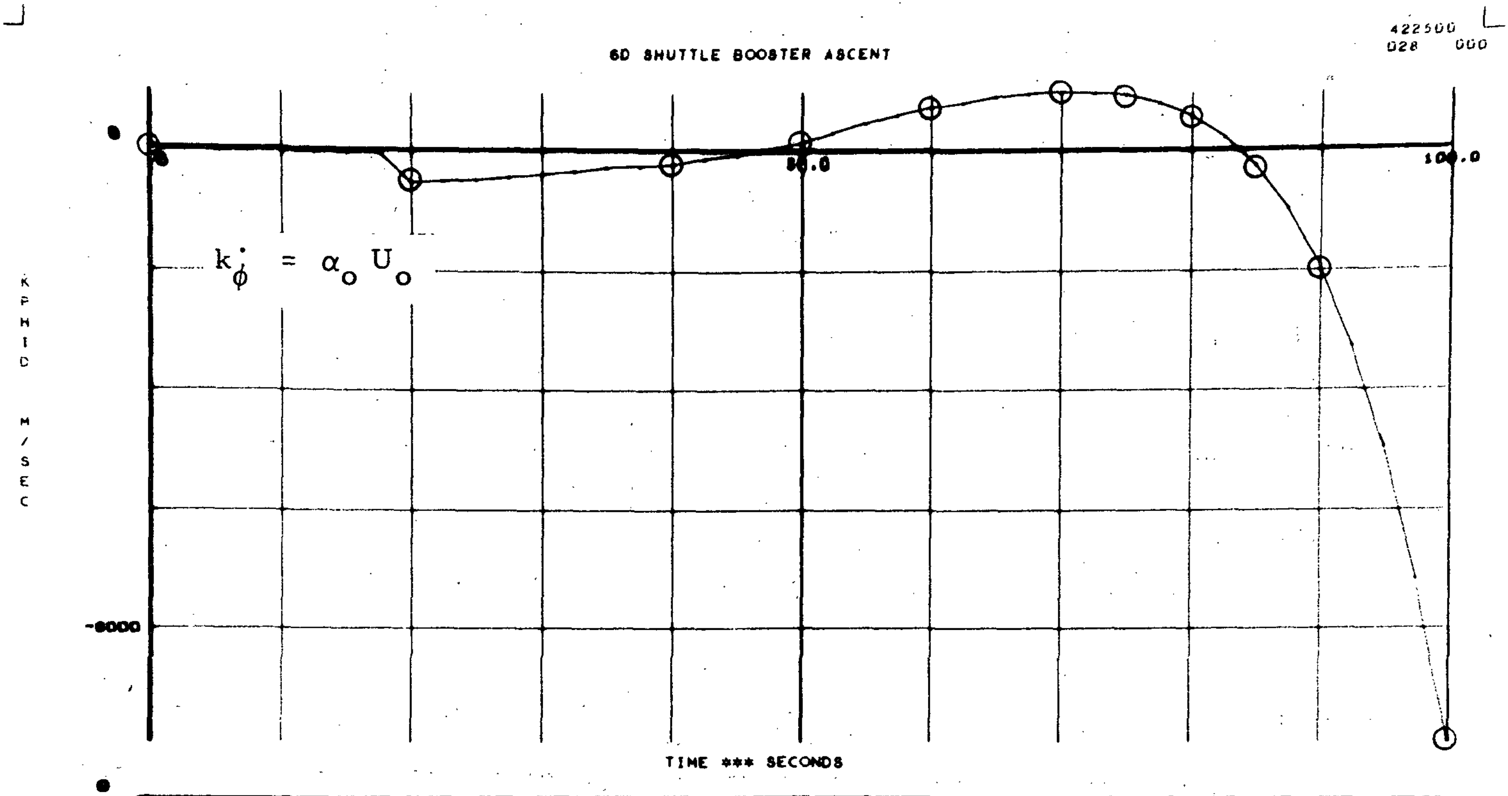
$$C_{n_{\delta_a}} = 0 \quad 1/\text{deg}$$

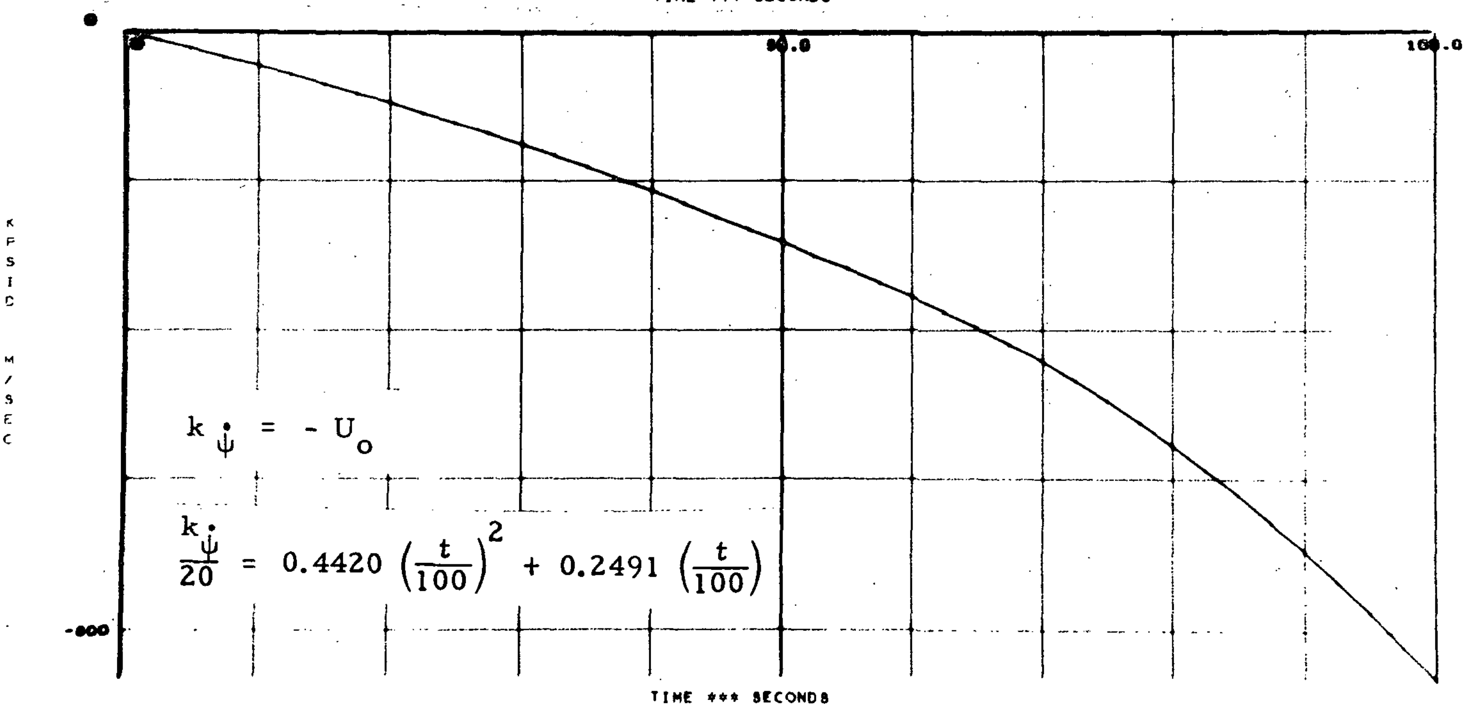
$$M'_{\alpha_1} = .279 * 10^6 \quad \text{N-m/deg}$$

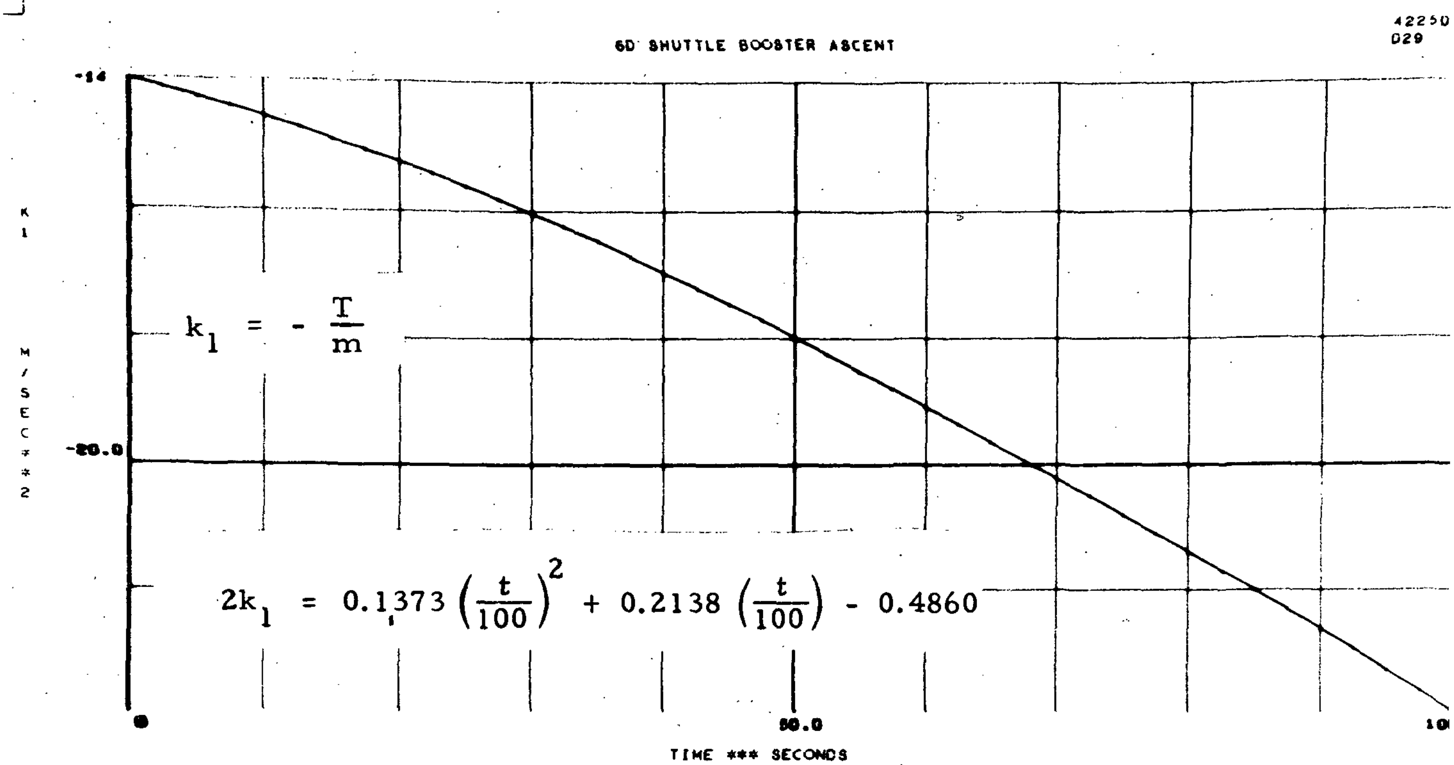
$$M'_{\alpha_2} = .19 * 10^6 \quad \text{N-m/deg}$$

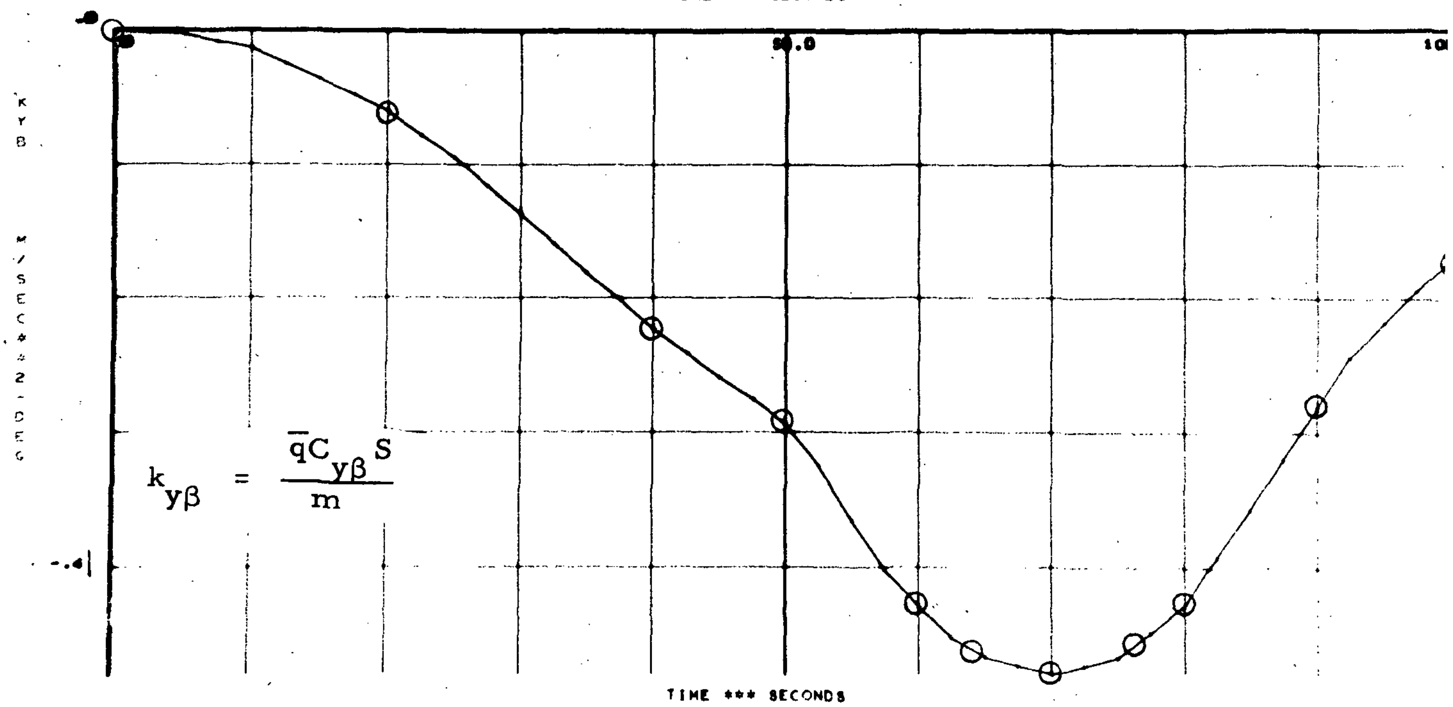
$$M'_{\beta_1} = .116 * 10^8 \quad \text{N-m/deg}$$

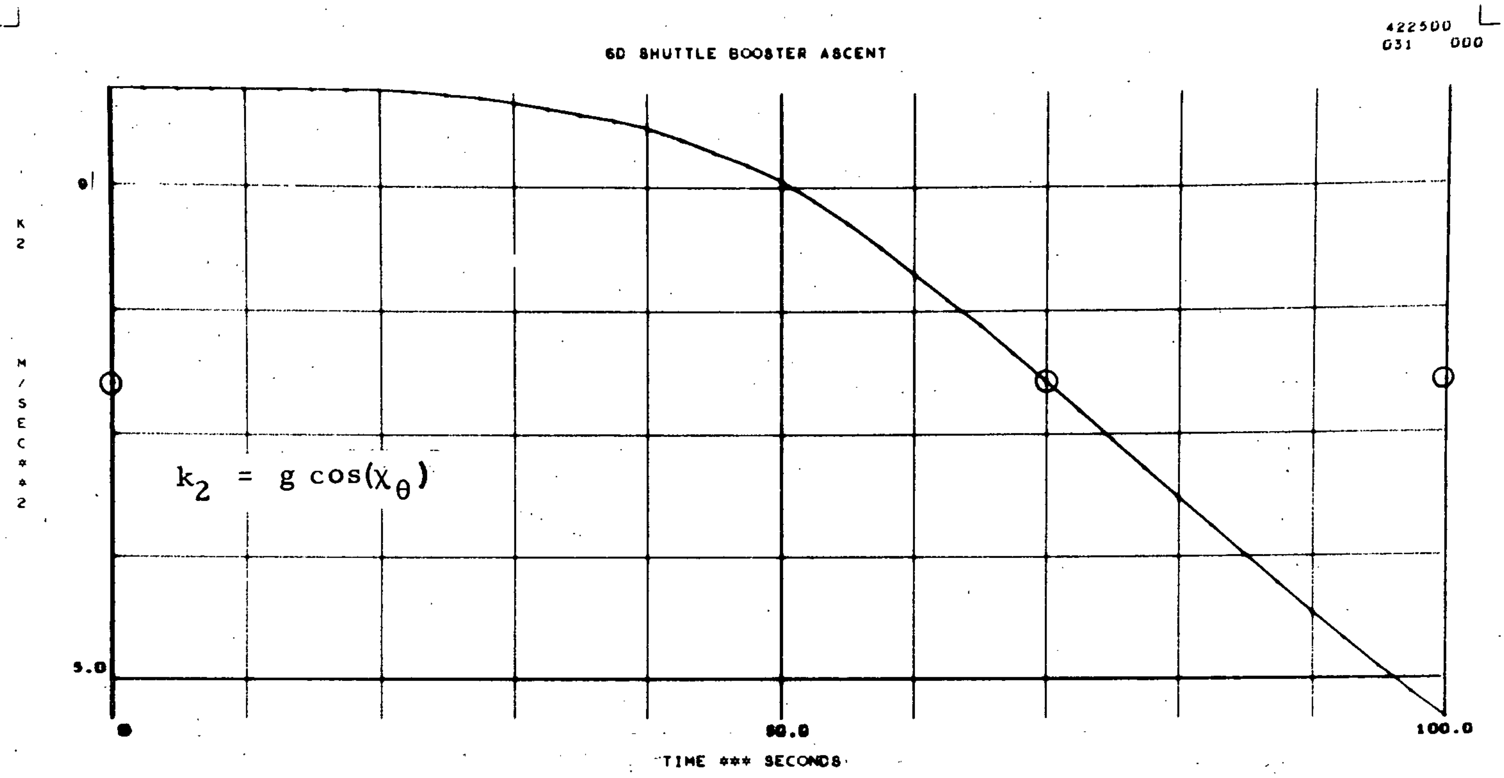
$$M'_{\beta_2} = .332 * 10^8 \quad \text{N-m/deg}$$

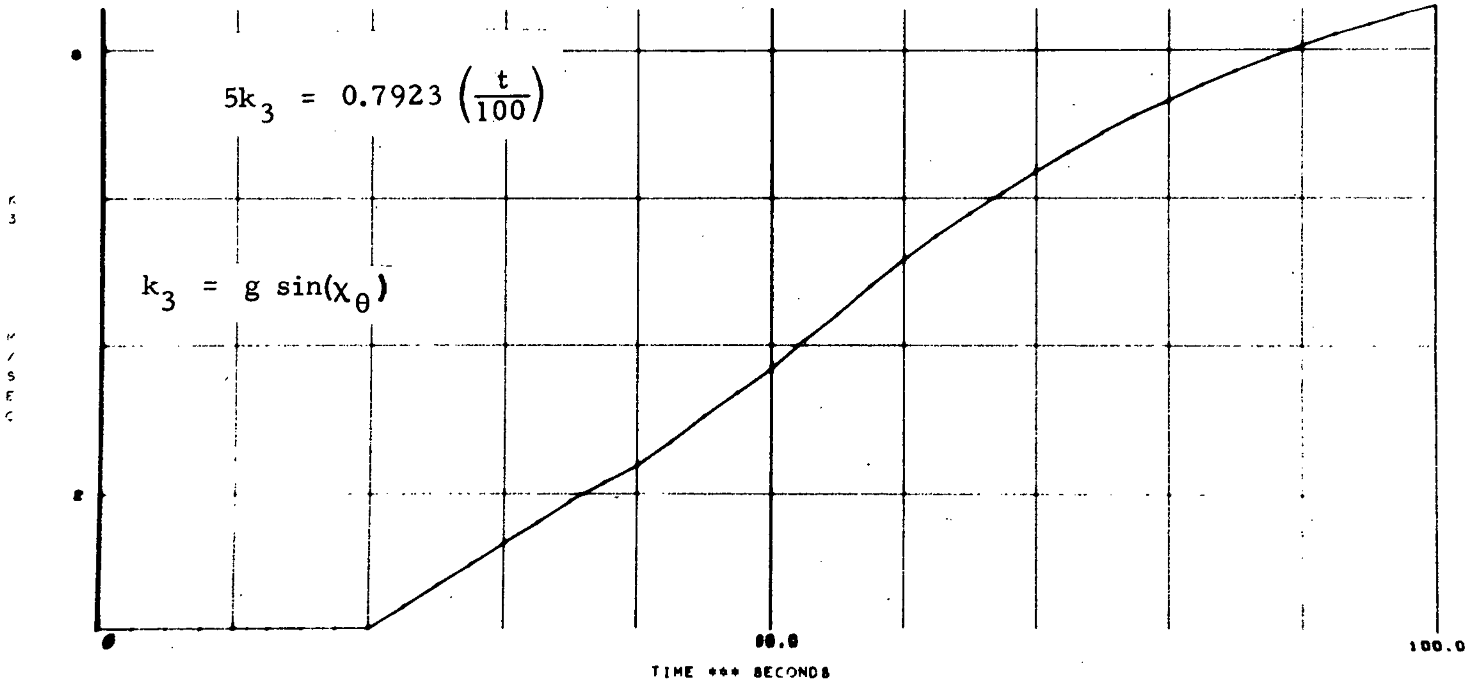

422500
028 000
6D SHUTTLE BOOSTER ASCENT
K P H I D M/SEC
100.0
TIME *** SECONDS
$k_{\dot{\phi}} = \alpha_o U_o$

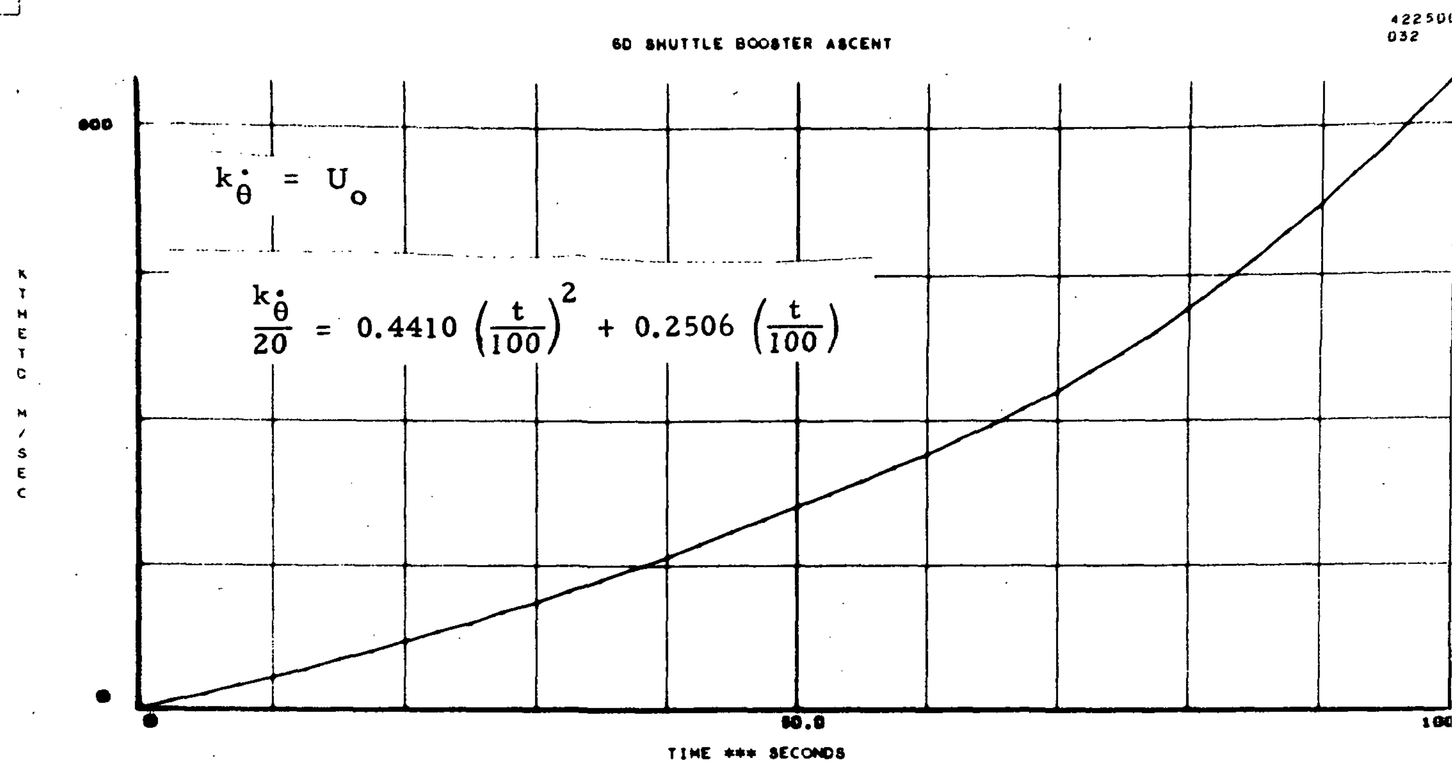

K P S I D M/SEC
100.0
TIME *** SECONDS
$k_{\dot{\psi}} = - U_o$
$\frac{k_{\dot{\psi}}}{20} = 0.4420 \left(\frac{t}{100}\right)^2 + 0.2491 \left(\frac{t}{100}\right)$


42250
029
6D SHUTTLE BOOSTER ASCENT
-14
-20.0
K1 M/SEC**2
$k_1 = -\frac{T}{m}$
$2k_1 = 0.1373 \left(\frac{t}{100}\right)^2 + 0.2138 \left(\frac{t}{100}\right) - 0.4860$
0
50.0
TIME *** SECONDS

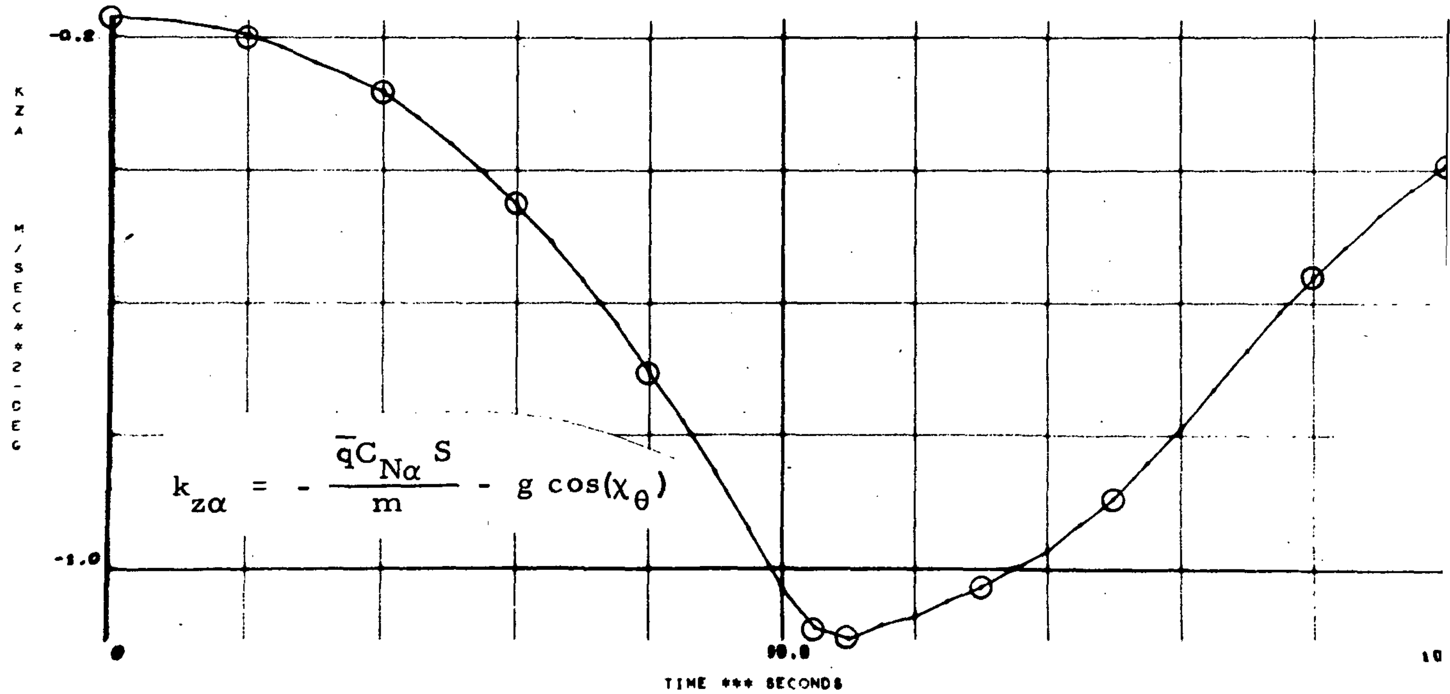

0
50.0
KYB M/SEC**2-DEG
$k_{y\beta} = \frac{\bar{q} C_{y\beta} S}{m}$
-.4
TIME *** SECONDS


422500
031 000
6D SHUTTLE BOOSTER ASCENT
K2
M/SEC**2
$k_2 = g\cos(\chi_\theta)$
5.0
50.0
100.0
TIME *** SECONDS

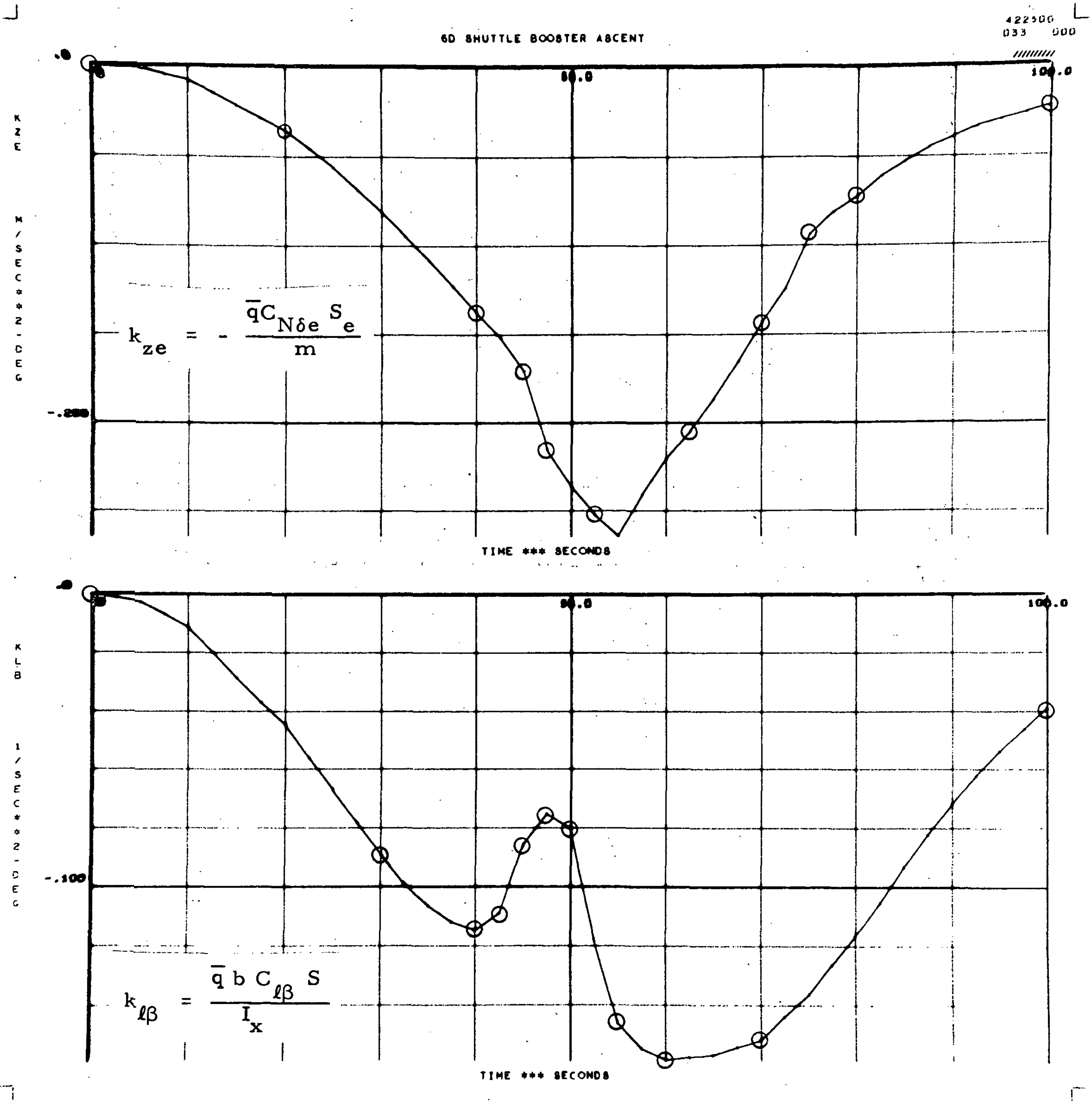

$5k_3 = 0.7923\left(\frac{t}{100}\right)$
$k_3 = g\sin(\chi_\theta)$
K3
M/SEC
50.0
100.0
TIME *** SECONDS

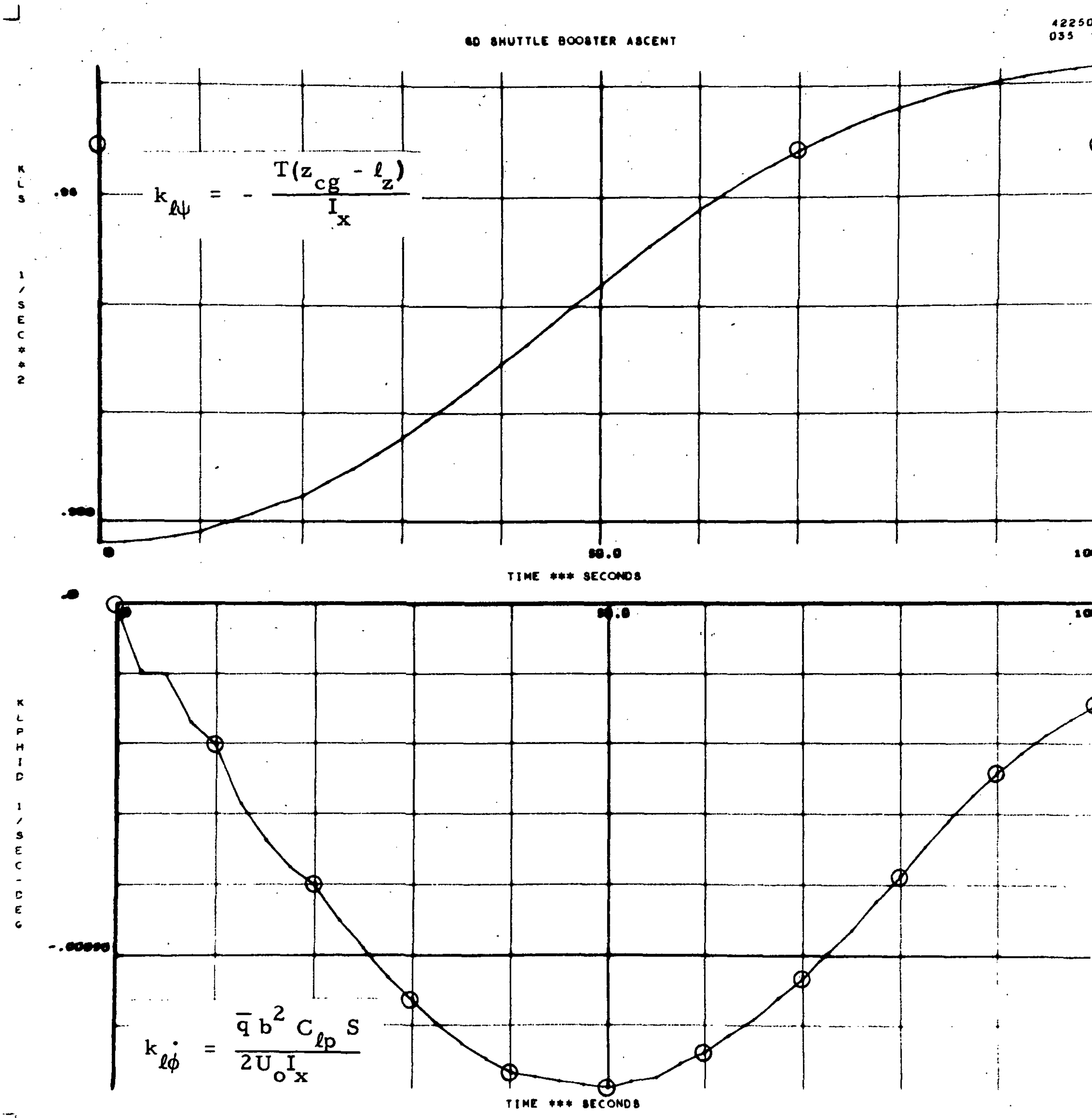


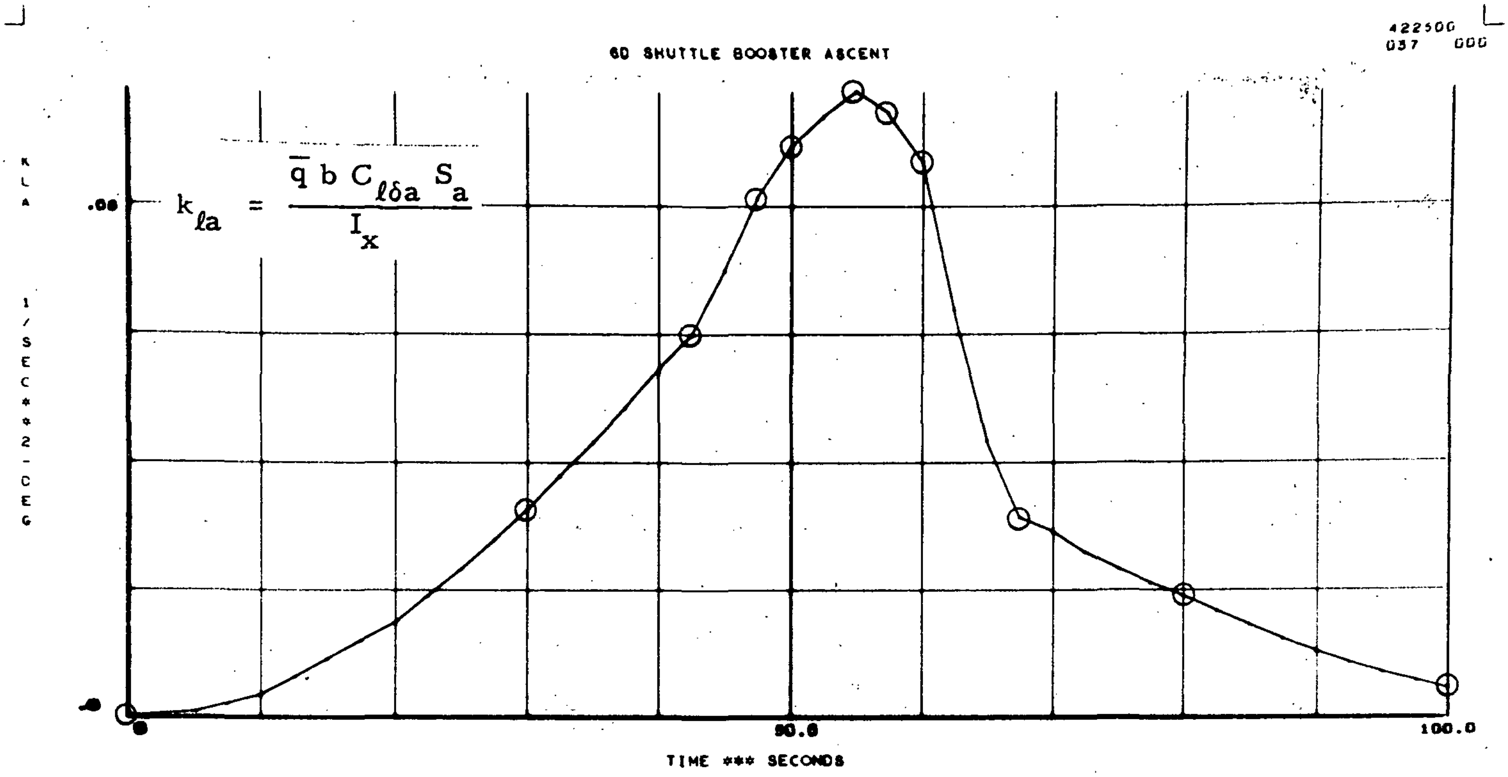
6D SHUTTLE BOOSTER ASCENT
422500
032
800
0
KTHETC M/SEC
$k_{\dot{\theta}} = U_o$
$\frac{k_{\dot{\theta}}}{20} = 0.4410 \left(\frac{t}{100}\right)^2 + 0.2506 \left(\frac{t}{100}\right)$
50.0
TIME *** SECONDS

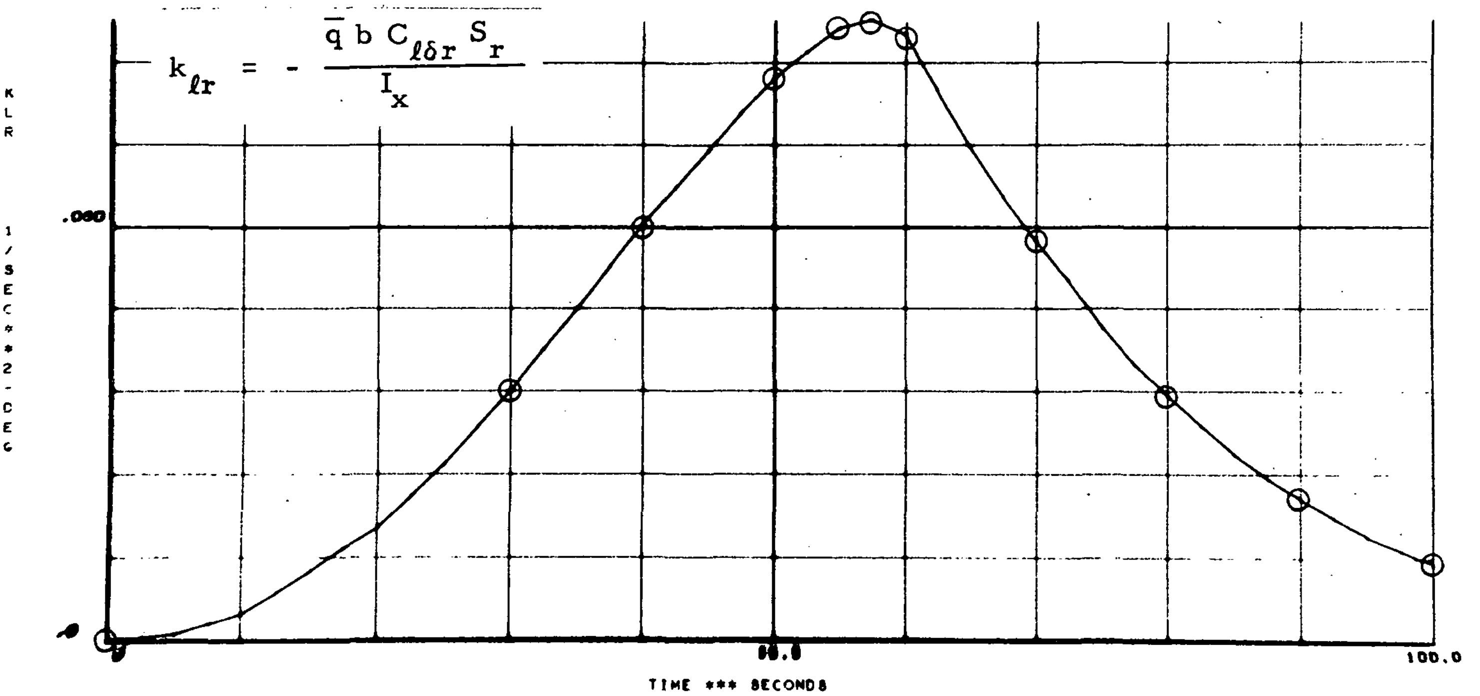


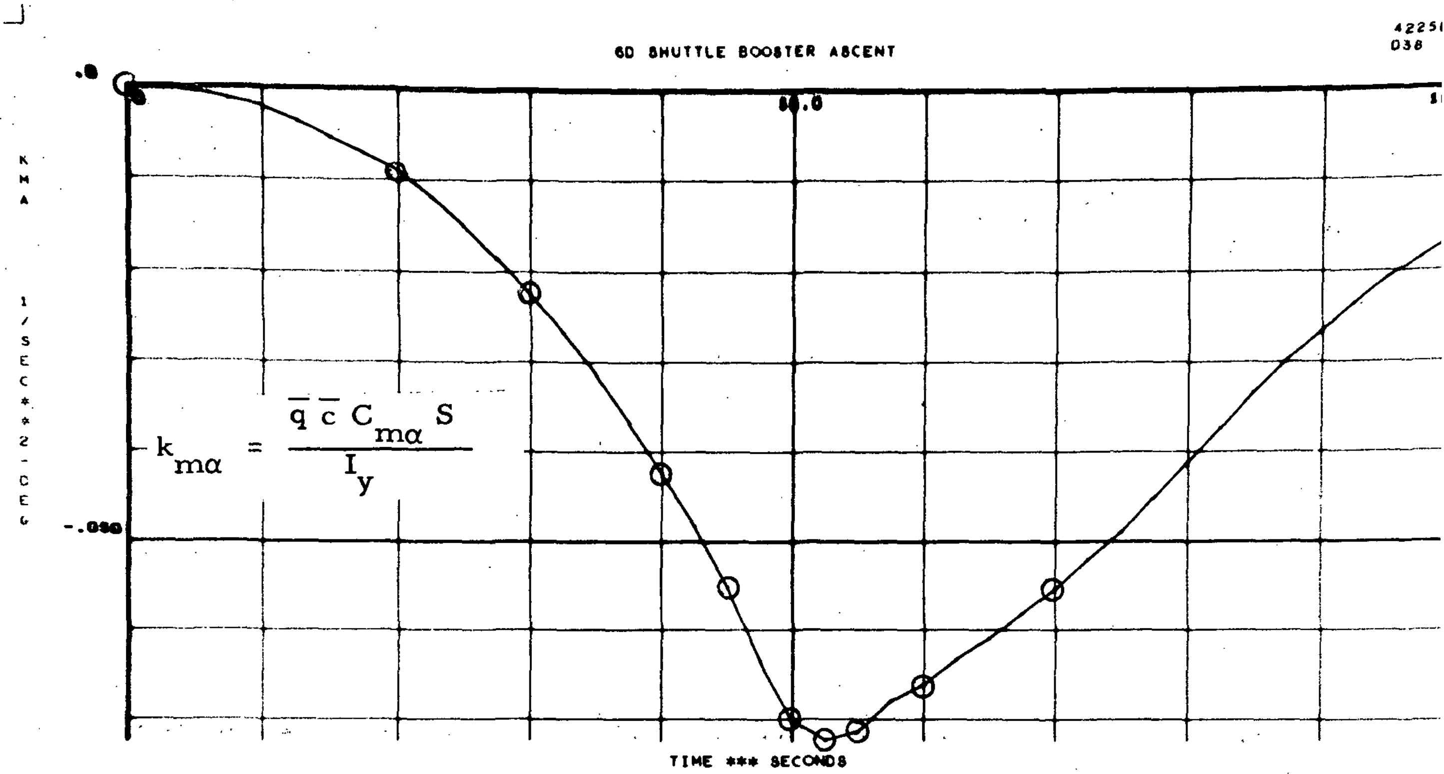
-0.2
-1.0
KZA M/SEC**2-DEG
$k_{z\alpha} = - \frac{\bar{q} C_{N\alpha} S}{m} - g \cos(\chi_\theta)$
50.0
TIME *** SECONDS

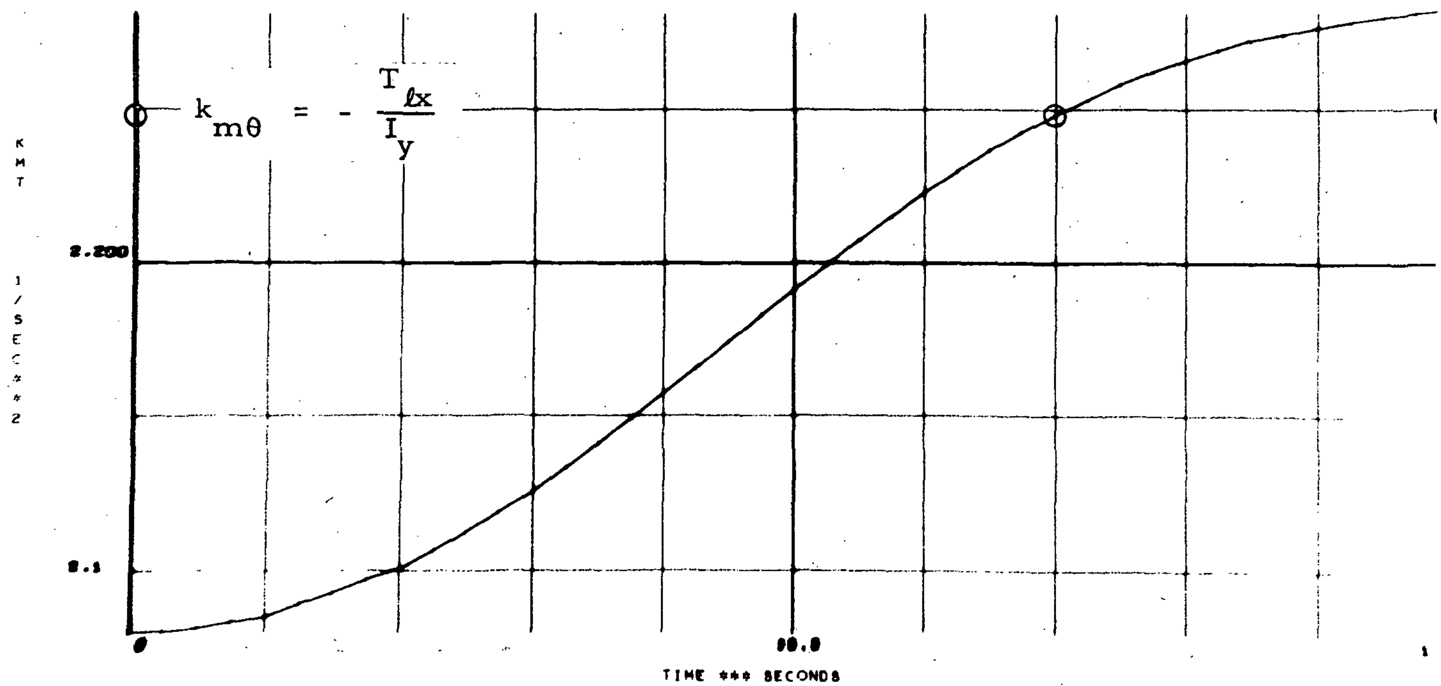

6D SHUTTLE BOOSTER ASCENT
422500
033 000
.0
-.200
50.0
100.0
KZE M/SEC**2-DEG
TIME *** SECONDS
$k_{ze} = -\frac{\bar{q}C_{N\delta e} S_e}{m}$
.0
-.100
50.0
100.0
KLB 1/SEC**2-DEG
TIME *** SECONDS
$k_{\ell\beta} = \frac{\bar{q}\, b\, C_{\ell\beta}\, S}{I_x}$

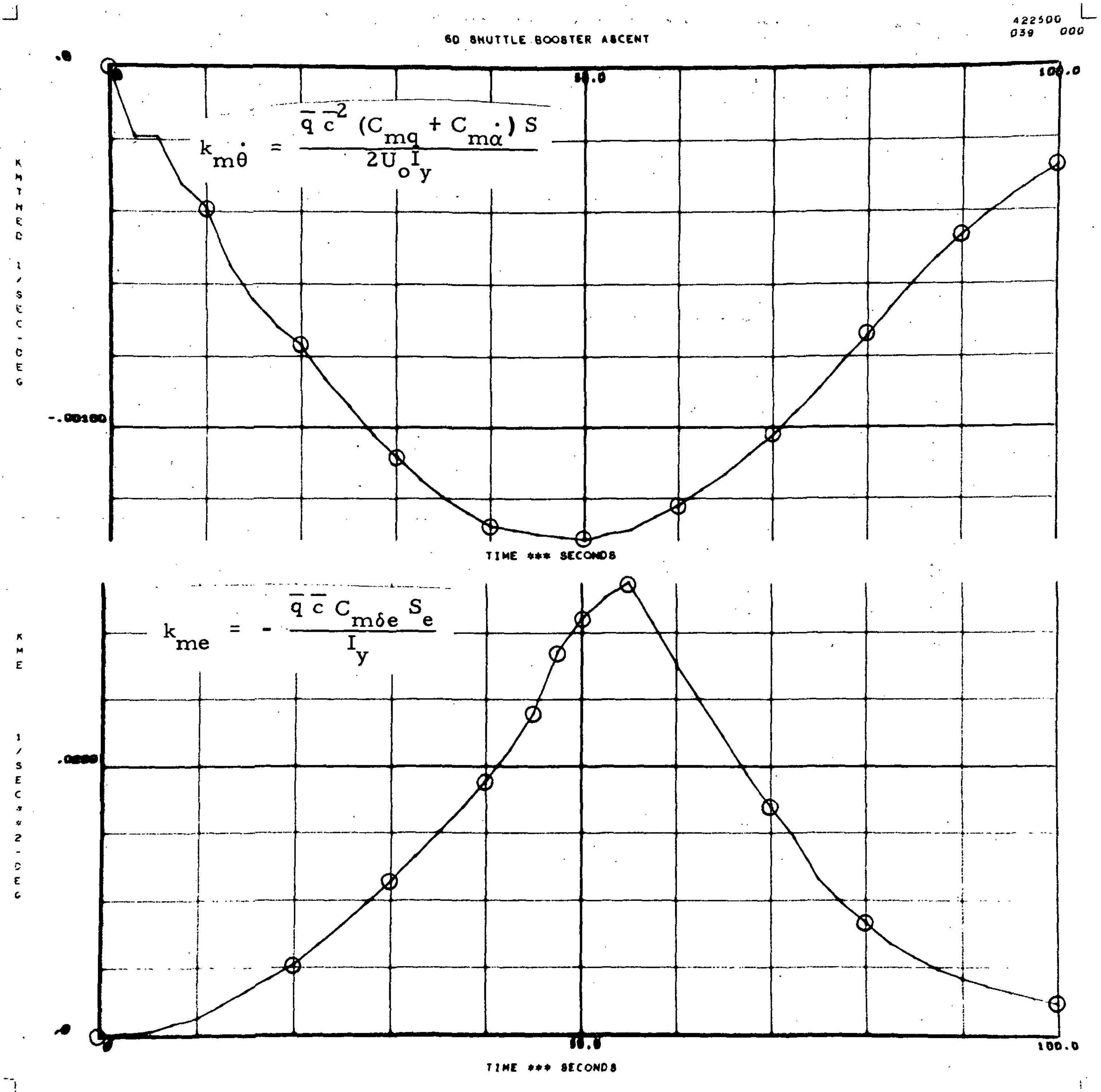

SD SHUTTLE BOOSTER ASCENT
KLS 1/SEC**2
$k_{\ell\psi} = - \frac{T(z_{cg} - \ell_z)}{I_x}$
TIME *** SECONDS
KLPHID 1/SEC-DEG
$k_{\ell\dot{\phi}} = \frac{\bar{q}\, b^2\, C_{\ell p}\, S}{2 U_o I_x}$
TIME *** SECONDS

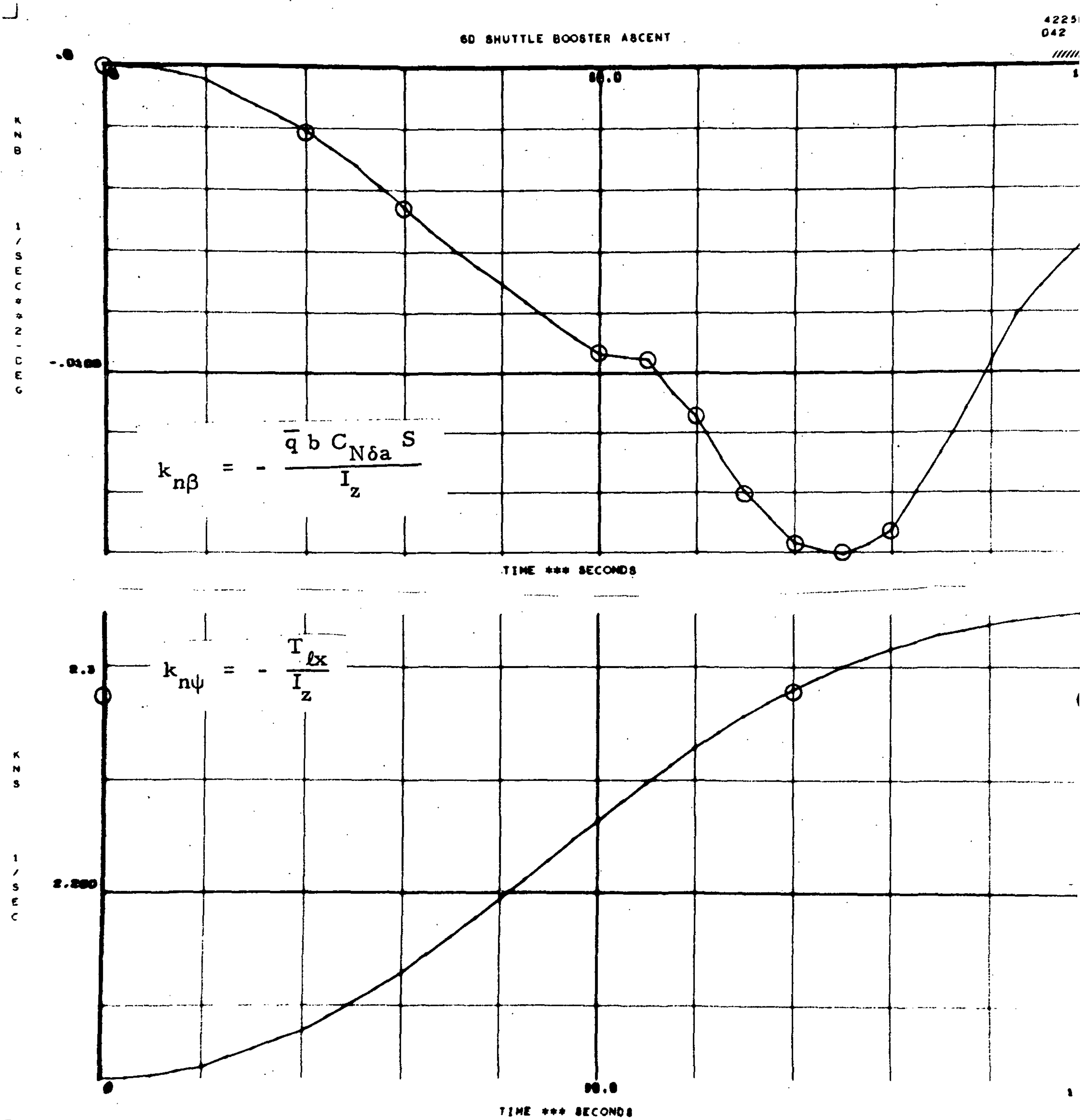

422500
037 000
60 SHUTTLE BOOSTER ASCENT
$k_{\ell a} = \frac{\bar{q}\, b\, C_{\ell \delta a}\, S_a}{I_x}$
KLA 1/SEC**2-DEG
50.0
100.0
TIME *** SECONDS

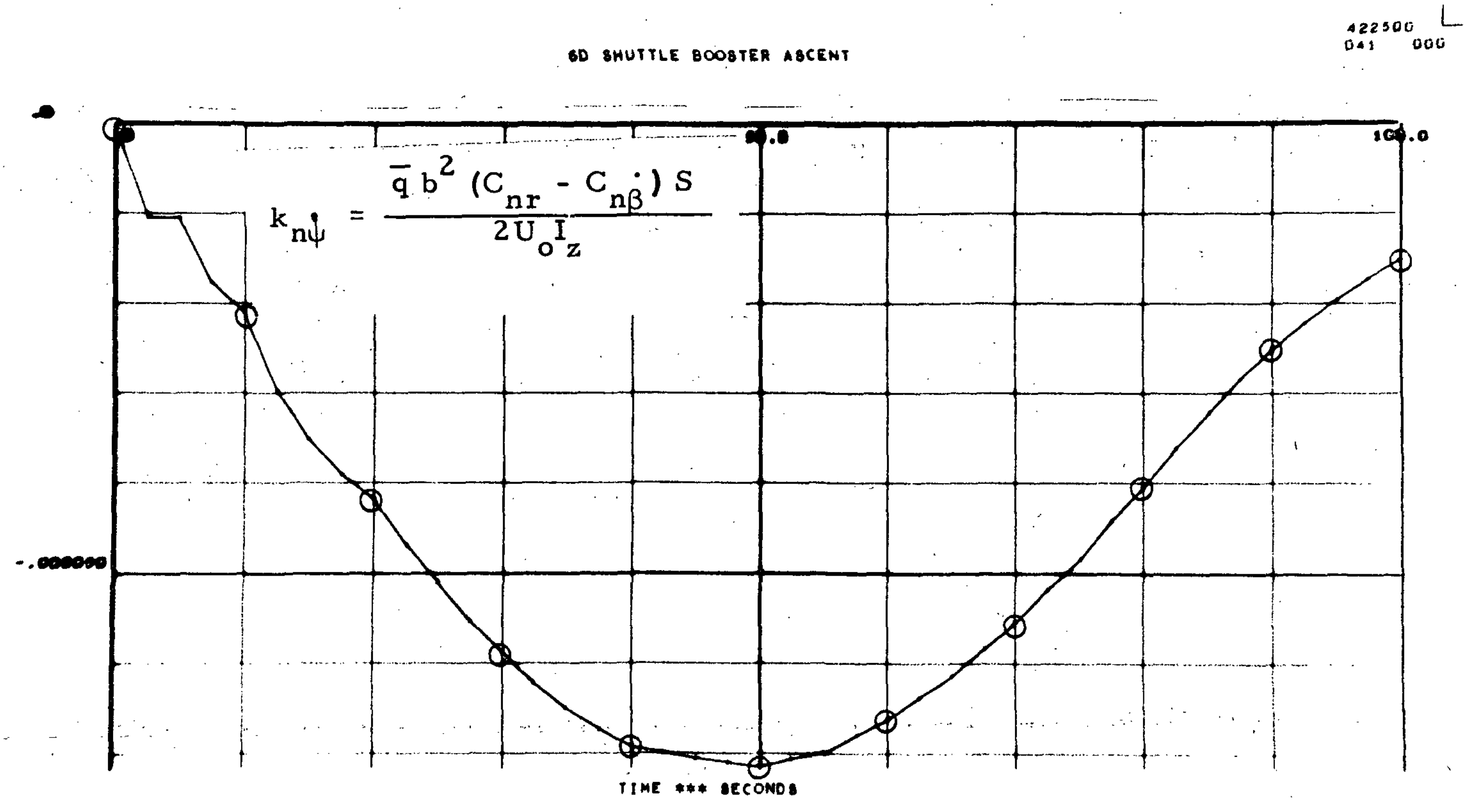

$k_{\ell r} = - \frac{\bar{q}\, b\, C_{\ell \delta r}\, S_r}{I_x}$
KLR 1/SEC**2-DEG
50.0
100.0
TIME *** SECONDS


6D SHUTTLE BOOSTER ABCENT
KMA 1/SEC**2-DEG
TIME *** SECONDS
$k_{m\alpha} = \frac{\bar{q}\,\bar{c}\,C_{m\alpha}\,S}{I_y}$

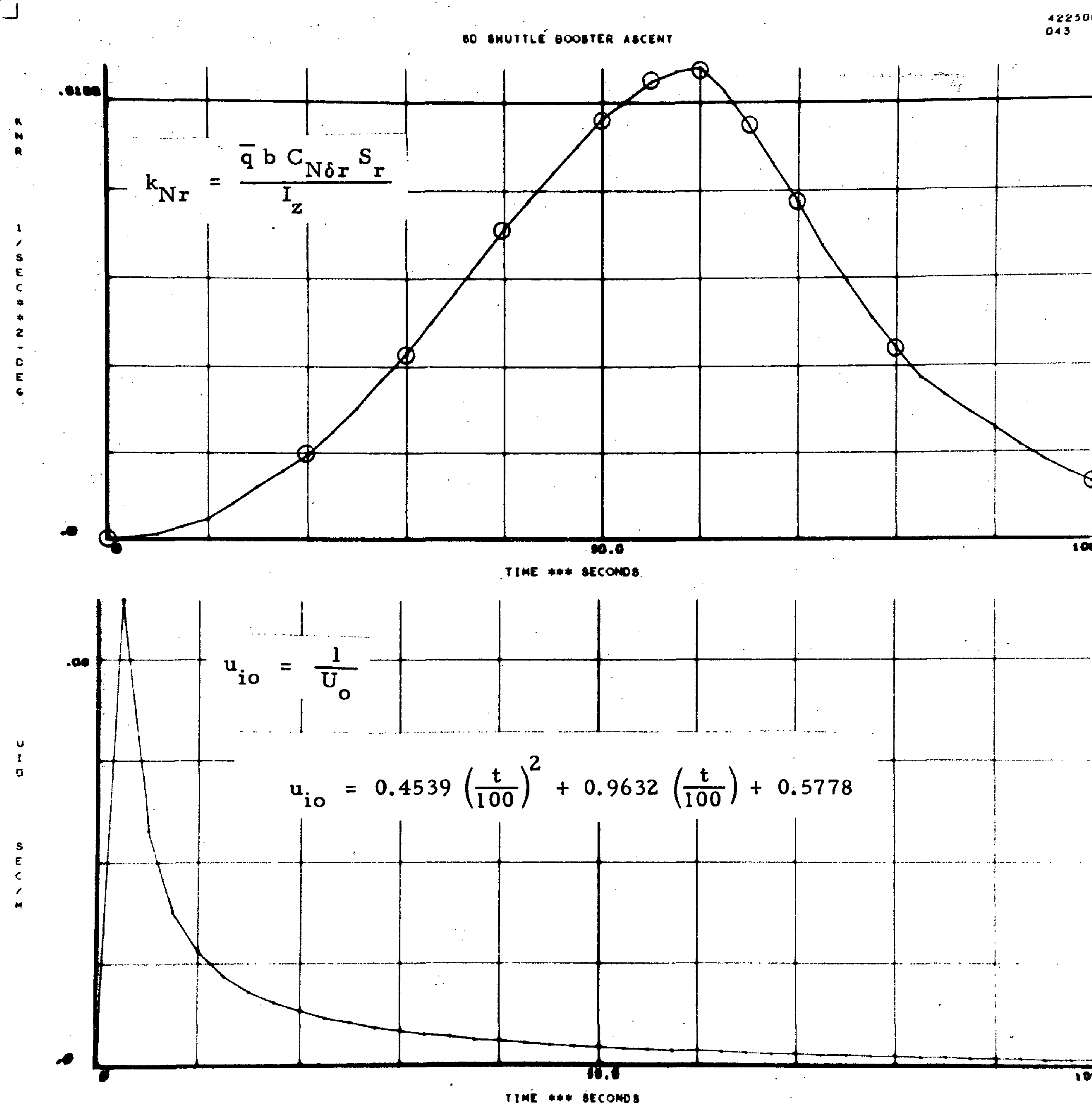

KMT 1/SEC**2
TIME *** SECONDS
$k_{m\theta} = -\frac{T_{\ell x}}{I_y}$

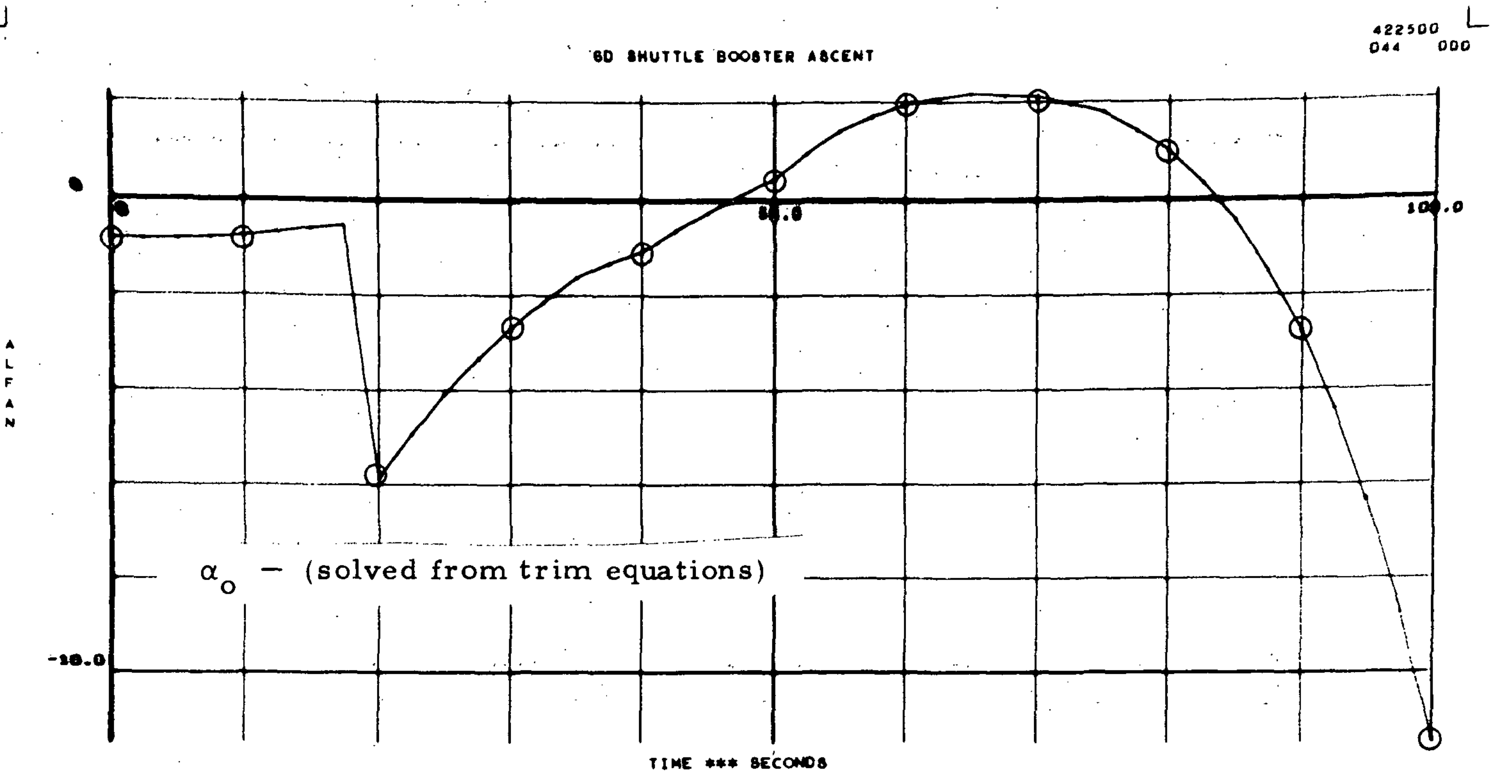

422500
039 000
6D SHUTTLE BOOSTER ASCENT
$k_{m\dot{\theta}} = \frac{\bar{q}\,\bar{c}^2 (C_{mq} + C_{m\dot{\alpha}})\,S}{2U_o I_y}$
KMTHED 1/SEC-DEG
.0
100.0
TIME *** SECONDS
$k_{me} = -\frac{\bar{q}\,\bar{c}\,C_{m\delta e}\,S_e}{I_y}$
KME 1/SEC**2-DEG
100.0
TIME *** SECONDS

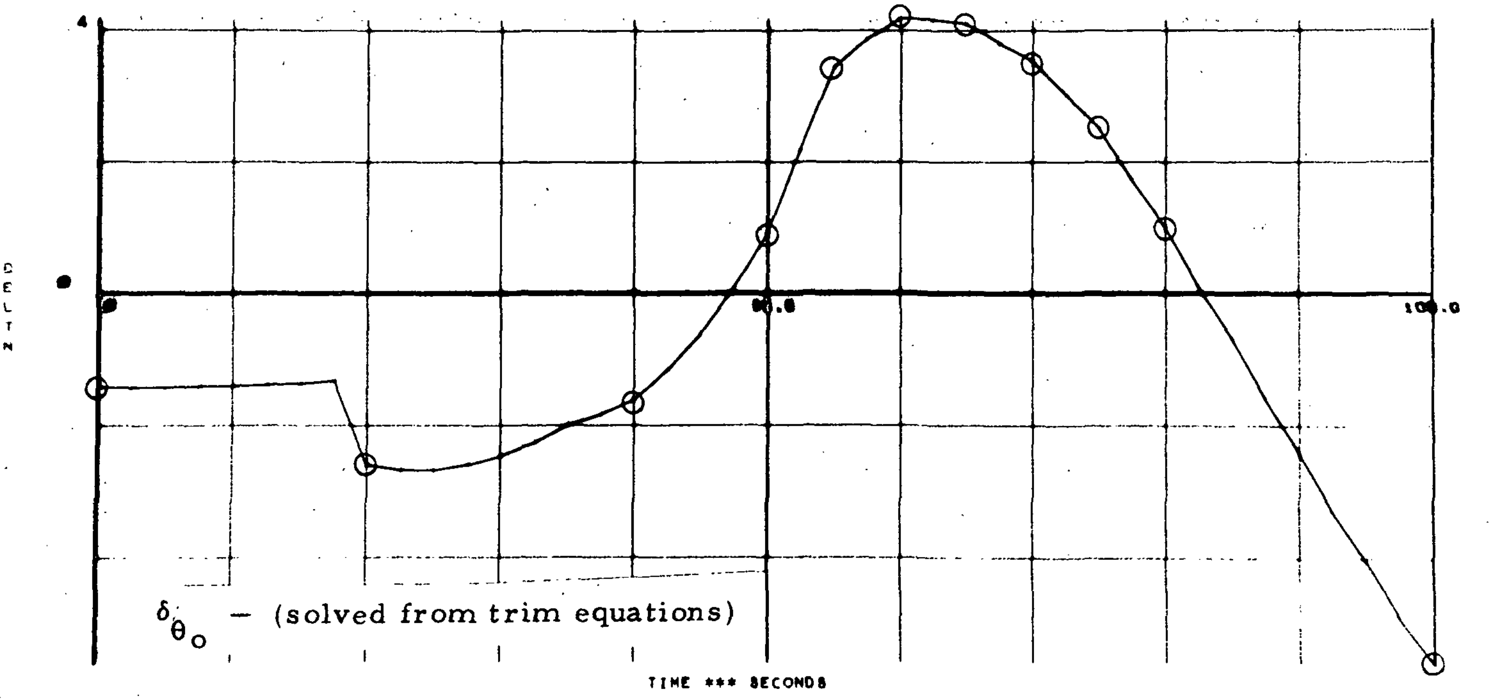

6D SHUTTLE BOOSTER ASCENT
KNB 1/SEC**2-DEG
$k_{n\beta} = - \frac{\bar{q}\, b\, C_{N\delta a}\, S}{I_z}$
TIME *** SECONDS
KNS 1/SEC
$k_{n\psi} = - \frac{T_{\ell x}}{I_z}$
TIME *** SECONDS

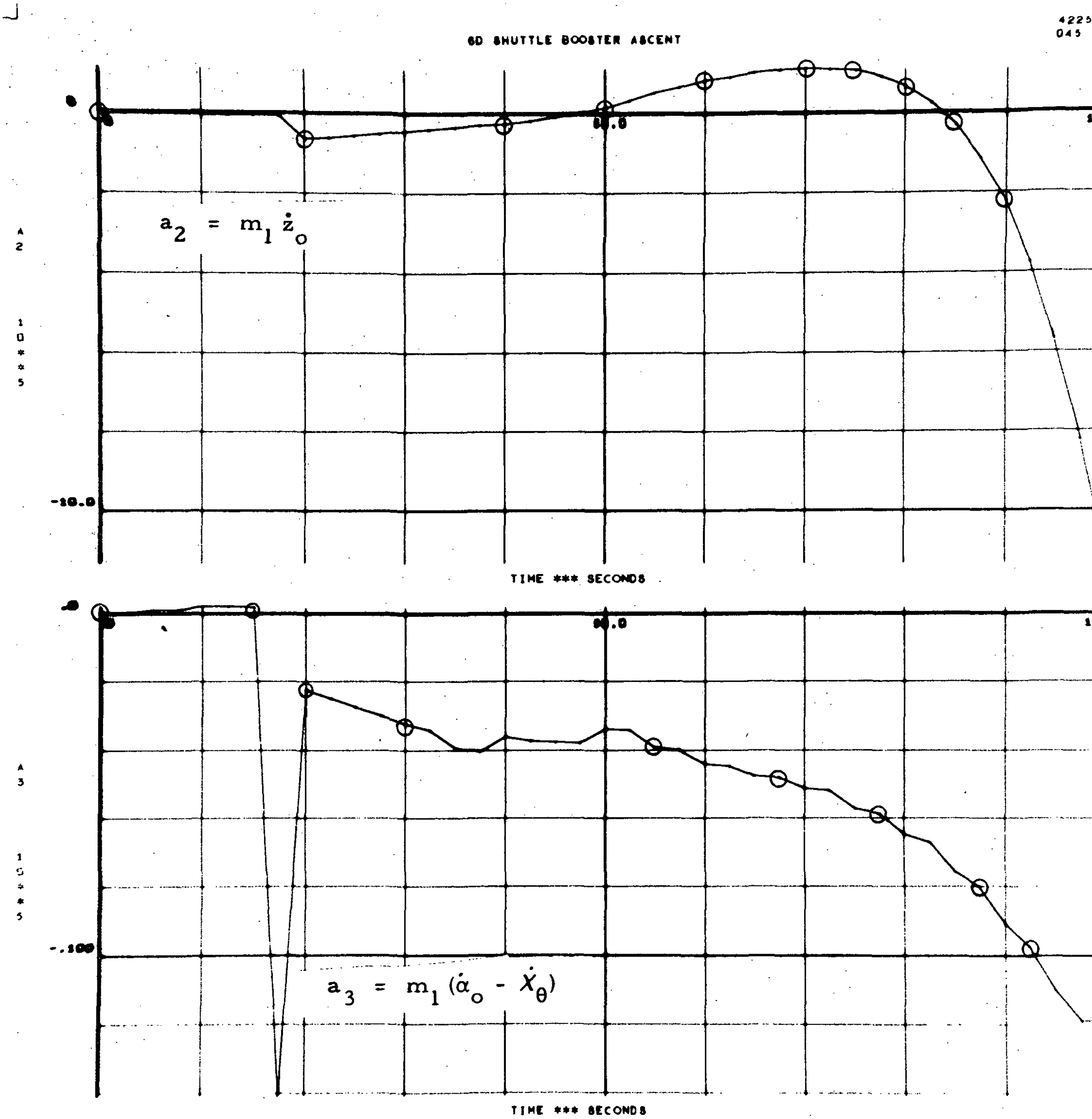


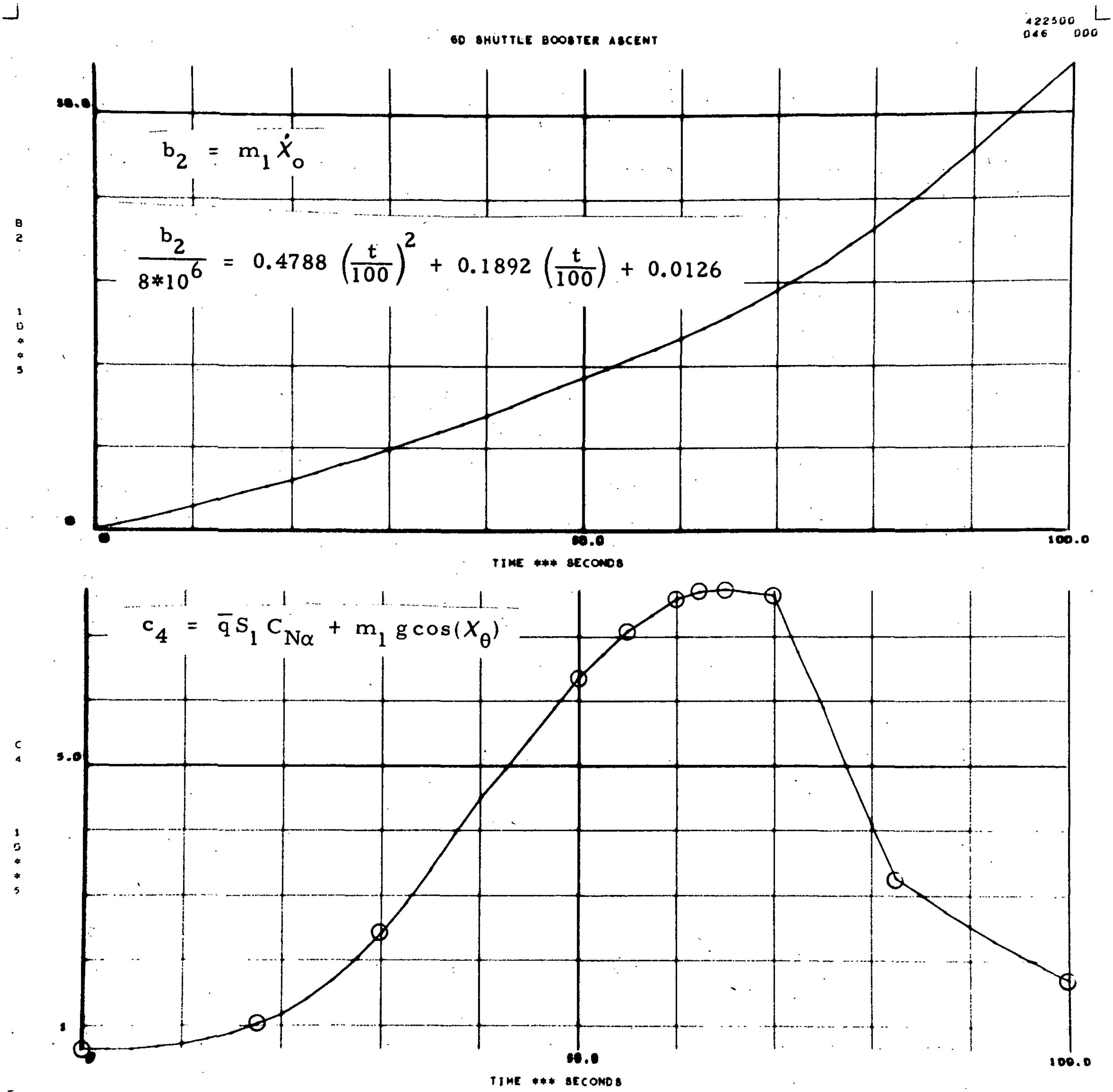
422500
041 000
6D SHUTTLE BOOSTER ASCENT
$k_{n\dot{\psi}} = \frac{\bar{q}\, b^2 (C_{nr} - C_{n\dot{\beta}})\, S}{2 U_o I_z}$
-.000090
KNPSID 1/SEC-DEG
TIME *** SECONDS

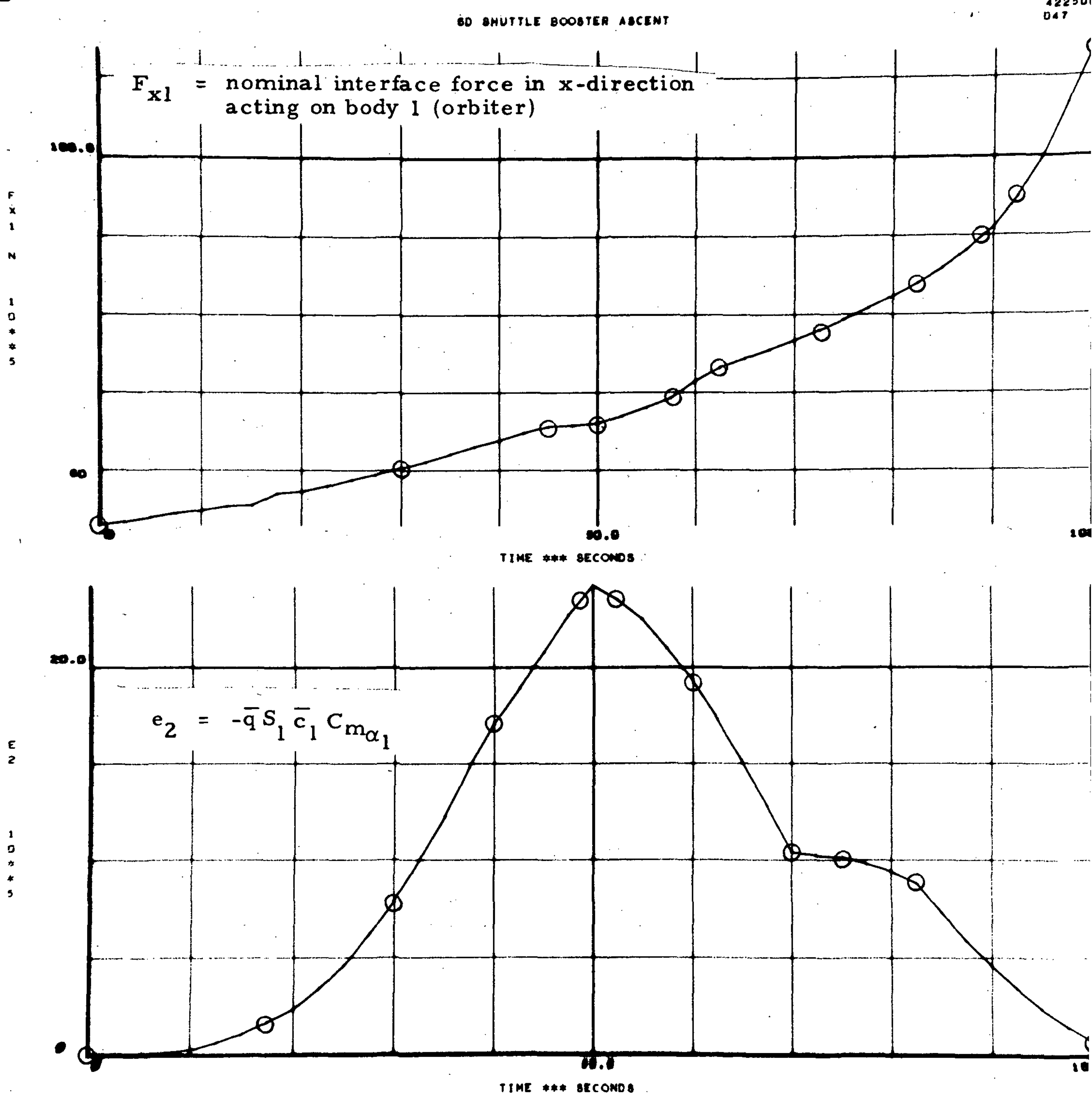

4225
043
6D SHUTTLE BOOSTER ASCENT
K N R   1 / SEC ** 2 - DEG
$k_{Nr} = \frac{\bar{q}\, b\, C_{N\delta r}\, S_r}{I_z}$
TIME *** SECONDS
U I O   SEC / M
$u_{io} = \frac{1}{U_o}$
$u_{io} = 0.4539 \left(\frac{t}{100}\right)^2 + 0.9632 \left(\frac{t}{100}\right) + 0.5778$
TIME *** SECONDS

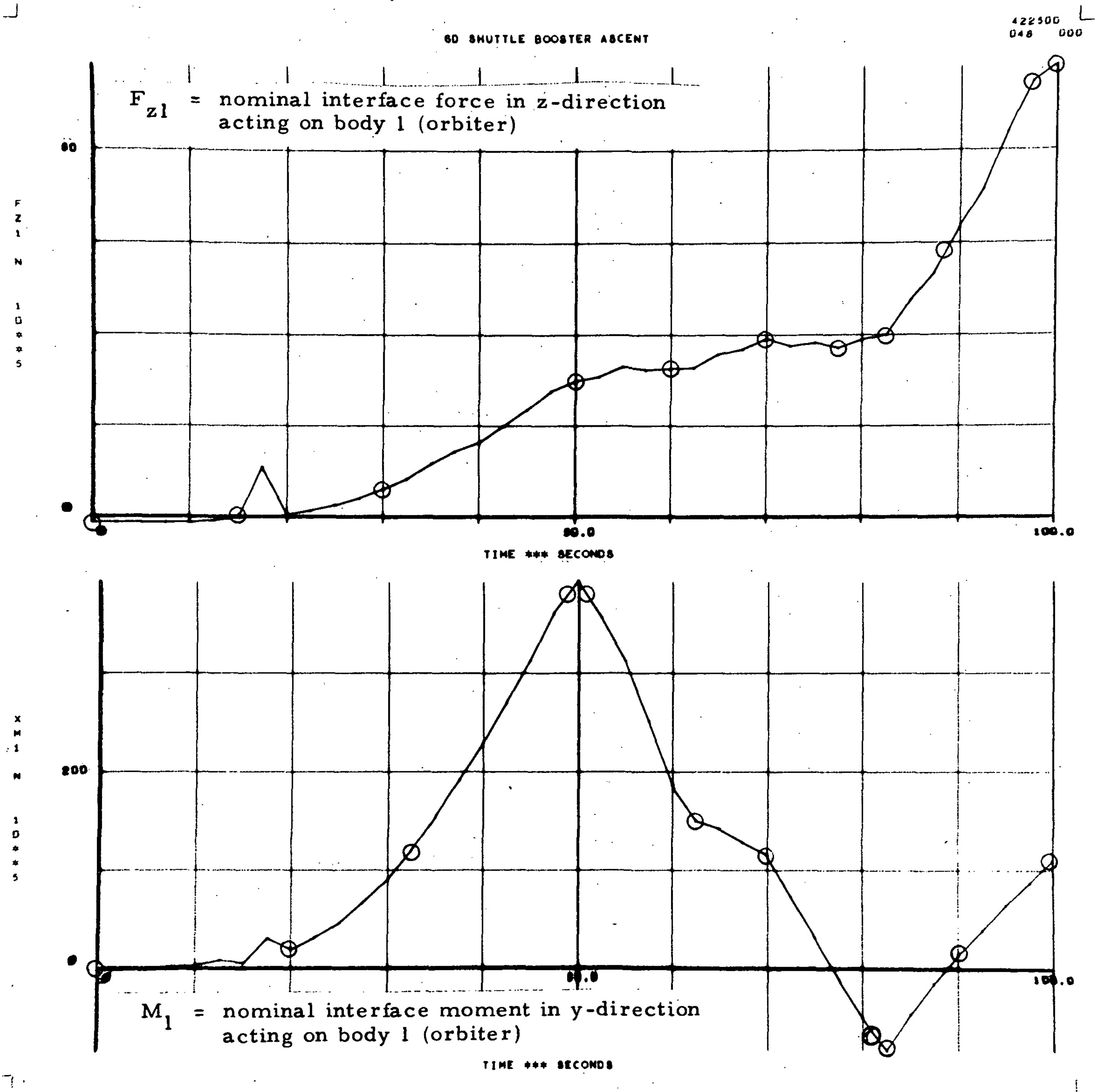

422500
044 000
6D SHUTTLE BOOSTER ASCENT
ALFAN
50.0
100.0
-10.0
α_o — (solved from trim equations)
TIME *** SECONDS


4
DELTN
50.0
100.0
δ_{θ_o} — (solved from trim equations)
TIME *** SECONDS


6D SHUTTLE BOOSTER ASCENT
4225
045
A
2
1
0
*
*
5
-10.0
$a_2 = m_1 \dot{z}_o$
TIME *** SECONDS
A
3
1
0
*
*
5
-.100
$a_3 = m_1 (\dot{\alpha}_o - \dot{X}_\theta)$
TIME *** SECONDS


422500
046 000
6D SHUTTLE BOOSTER ASCENT
$b_2 = m_1 \dot{\chi}_o$
$\frac{b_2}{8*10^6} = 0.4788 \left(\frac{t}{100}\right)^2 + 0.1892 \left(\frac{t}{100}\right) + 0.0126$
B2
10**5
0
100.0
TIME *** SECONDS
$c_4 = \bar{q} S_1 C_{N\alpha} + m_1 g \cos(\chi_\theta)$
C4
10**5
5.0
100.0
TIME *** SECONDS


6D SHUTTLE BOOSTER ASCENT
F_{x1} = nominal interface force in x-direction acting on body 1 (orbiter)
100.0
50
F X 1 N 1 0 * * 5
50.0
TIME *** SECONDS
20.0
$e_2 = -\bar{q} S_1 \bar{c}_1 C_{m_{\alpha_1}}$
E 2 1 0 * * 5
0
TIME *** SECONDS


SD SHUTTLE BOOSTER ASCENT
F_{z1} = nominal interface force in z-direction acting on body 1 (orbiter)
TIME *** SECONDS
100.0
200
M_1 = nominal interface moment in y-direction acting on body 1 (orbiter)
TIME *** SECONDS

$$k_{ya} = \frac{\bar{q} C_{y_{\delta_a}} S_a}{m} = 0 \ \frac{m}{sec^2\text{-deg}}$$

$$k_{yr} = \frac{-\bar{q} C_{y_{\delta_r}} S_r}{m} = 0 \ \frac{m}{sec^2\text{-deg}}$$

$$k_{zc} = \frac{-\bar{q} C_{N_{\delta_c}} S_c}{m} = 0 \ \frac{m}{sec^2\text{-deg}}$$

$$k_{\phi\psi} = \frac{I_{xz}}{I_x} = 0.397142$$

$$k_{\ell\theta} = \frac{-T(y_{cg} - \ell_y)}{I_x} = 0 \ \frac{1}{sec^2}$$

$$k_{\ell\dot{\psi}} = \frac{\bar{q} b^2 (C_{\ell_r} - C_{\ell_\beta}) S}{2 U_o I_x} = 0 \ \frac{1}{sec\text{-deg}}$$

$$k_{mc} = \frac{\bar{q} \bar{c} C_{m_{\delta_c}} S_c}{I_y} = 0 \ \frac{1}{sec^2\text{-deg}}$$

$$k_{na} = \frac{\bar{q} b C_{N_{\delta_a}} S_a}{I_z} = 0 \ \frac{1}{sec^2\text{-deg}}$$

$$k_{n\dot{\phi}} = \frac{\bar{q} b^2 C_{n_p} S}{2 U_o I_z} = 0 \ \frac{1}{sec\text{-deg}}$$

Appendix F

SC 4020 DIGITAL PLOTS OF RUNGE KUTTA INTEGRATED SHUTTLE ASCENT TRAJECTORY SHOWING ALL STATE VARIABLES AND INTERFACE LOADING FOR 0° HEADWIND α_{w_A} AND 90° SIDEWIND β_{w_A} (MDAC CONFIG. 20) FOR CONSTANT GAIN CONTROLLER

Appendix F

A block diagram of the IBM 1108 Digital Program used to generate these curves follows in Fig. F-1. The nomenclature on pages F-3 through F-7 are included to identify the various plots. Response to 0° headwind α_{w_A} for MDAC-20 Configuration is shown in pages F-8 through F-22. Response to 90° sidewind β_{w_A} is shown on pages F-23 through F-39.

The constant gain controller consisted of $a_{0\theta} = 1.5$ and $a_{1\theta} = 0.5$ sec.

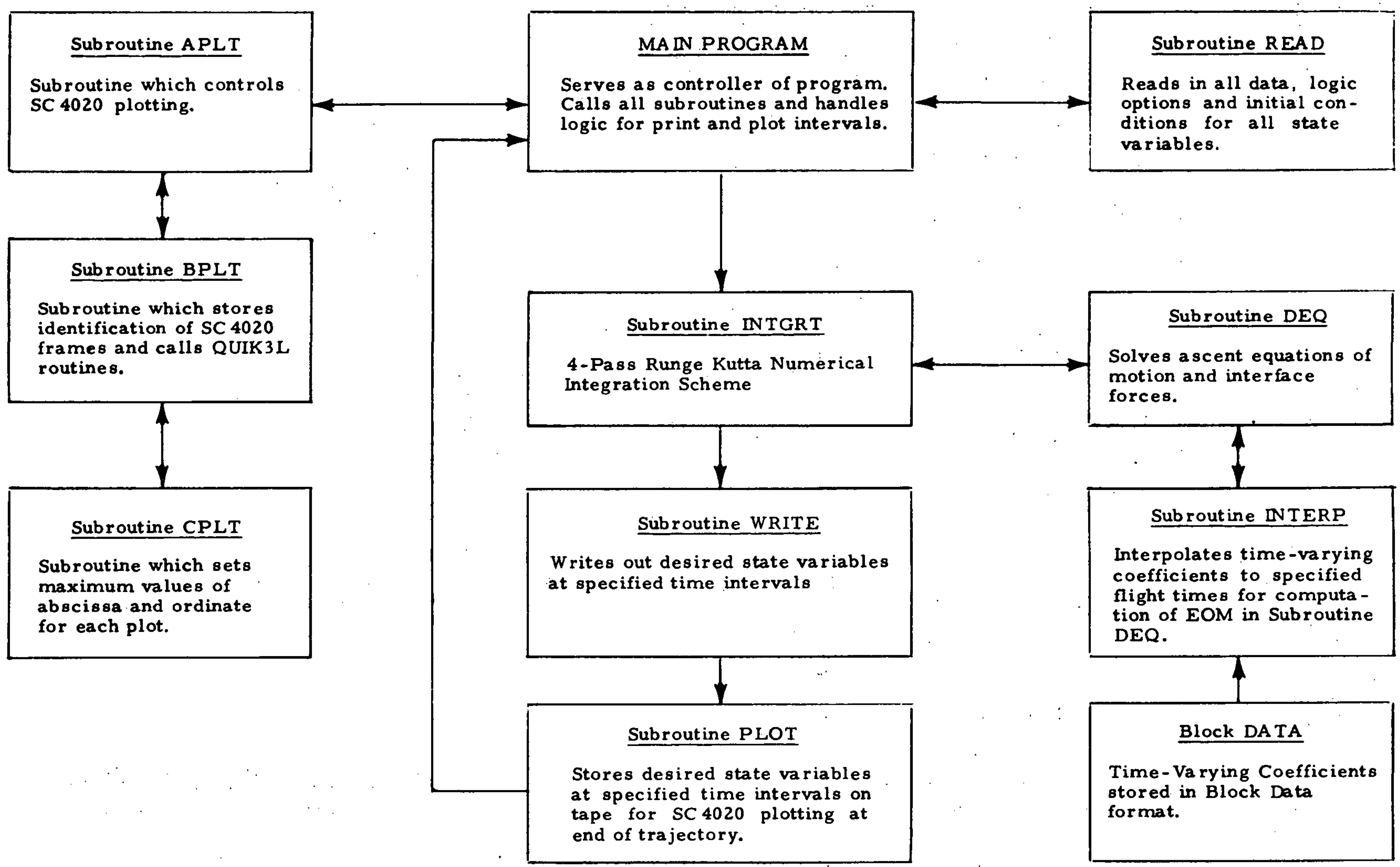


Fig. F-1 - Block Diagram of IBM 1108 Digital Program Used to Integrate Shuttle Ascent EOM and Solve Interface Loading Equations (MDAC - Config. 20)

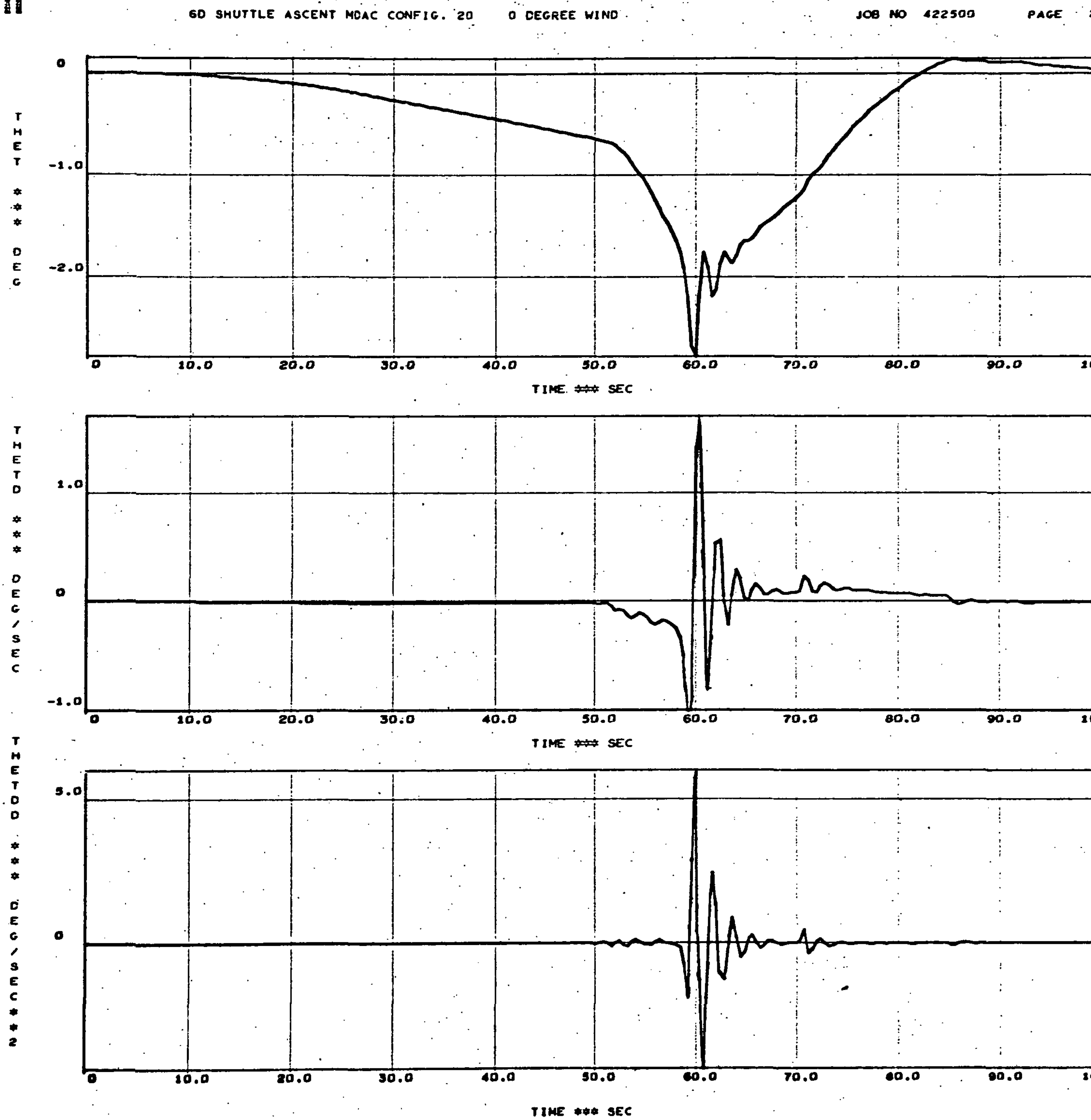
Fortran Term		Definition
ALFAW	α_ω	pitch plane component of wind disturbance, all cases included in digital plots are α_{w_A}.
ALFA	α	perturbation angle of attack
ALFAT	α_T	total angle of attack
BETAW	β_ω	yaw plane component of wind disturbance
BETA	β	sideslip angle
BETAT	β_T	total sideslip angle
DELP	δ_p	thrust vector roll gimbal angle
DELT	δ_θ	perturbation thrust vector pitch gimbal angle
DELTT	$\delta_{\theta T}$	total thrust vector pitch gimbal angle
DELTTT	$\delta_{\theta TT}$	total thrust vector gimbal angle of pitch and roll
DELS	δ_ψ	thrust vector yaw gimbal angle
DELA	δ_a	aileron surface deflection angle
DELR	δ_r	rudder surface deflection angle
DELE	δ_e	elevon surface deflection angle
DELC	δ_c	canard surface deflection angle
JPP	J''	stability integral component of J function
JP	J'	payload penalty component of J function
J	J	total penalty function
ETA10	η	component of J function representing longitudinal drift rate $\eta_{10} = \frac{\partial P}{\partial \dot{x}} * \dot{x}$

Fortran Term		Definition
ETA11	η_{11}	component of J function representing lateral drift rate $$\eta_{11} = \frac{\partial P}{\partial \dot{y}} * \dot{y}$$
ETA12	η_{12}	component of J function representing lateral drift rate $$\eta_{12} = \frac{\partial P}{\partial \dot{z}} * \dot{z}$$
ETA13	η_{13}	component of J function representing longitudinal drift $$\eta_{13} = \frac{\partial P}{\partial x} * x$$
ETA14	η_{14}	component of J function representing lateral drift $$\eta_{14} = \frac{\partial P}{\partial y} * y$$
ETA15	η_{15}	component of J function representing lateral drift $$\eta_{15} = \frac{\partial P}{\partial z} * z$$
ETA8	η_8	component of J function representing interface loading $$\eta_8 = \frac{\partial P}{\partial R} * \Delta R; \quad \Delta R > 0$$ $$\eta_8 = 0 ; \quad \Delta R \leq 5 * 10^5 \text{ N}$$
ETA9	η_9	component of J function representing gimbal deflection $$\eta_9 = \frac{\partial P}{\partial \delta_{\theta_{TT}}} * \Delta\delta_{\theta\phi} ; \quad \Delta\delta_{\theta\phi} > 7$$ $$\eta_9 = 0 ; \quad \Delta\delta_{\theta\phi} \leq 7$$
FXP1	$f_{x_{p_1}}$	interface perturbation force in x direction on body 1 (orbiter

Fortran Term		Definition
FYP1	$f_{y_{p_1}}$	interface perturbation force in y direction on body 1 (orbiter) due to wind disturbance
FZP1	$f_{z_{p_1}}$	interface perturbation force in z direction on body 1 (orbiter) due to wind disturbance
XLP1	$\hat{\ell}_{p_1}$	interface perturbation moment vector in x direction on body 1 due to wind disturbance
XMP1	$\hat{m}_{p_1}$	interface perturbation moment vector in y direction on body 1 due to wind disturbance
XNP1	$\hat{n}_{p_1}$	interface perturbation moment vector in z direction on body 1 due to wind disturbance
R1X	R_{1x}	total force in x direction on attachment point 1
R1Y	R_{1y}	total force in y direction on attachment point 1
R1Z	R_{1z}	total force in z direction on attachment point 1
R4Y	R_{4y}	total force in y direction on attachment point 4
R2Z	R_{2z}	total force in z direction on attachment point 2
R3Z	R_{3z}	total force in z direction on attachment point 3
R	R	total force affecting structure $R = \sqrt{(R_{2z} + R_{3z})^2 + R_{4y}{}^2}$
DR	ΔR	arbitrary limit of R for optimizer to maintain $\Delta R = R - 5 * 10^5$
DDTP	$\Delta\delta_{\theta\phi}$	arbitrary limit of $\delta_{\theta TT}$ for optimizer to maintain $\Delta\delta_{\theta\phi} = \delta_{\theta TT} - 7^o$

Fortran Term		Definition
BMX1	M_{B_1}	total bending moment at station 1 used in early studies
BMX2	M_{B_2}	total bending moment at station 2 used in early studies
AOT	$a_{o\theta}$	pitch attitude position feedback controller gain
A1T	$a_{1\theta}$	pitch attitude rate feedback controller gain
KC	k_c	canard blending ratio feedback controller gain
KE	k_e	elevon blending ratio feedback controller gain
AOS	$a_{o\psi}$	yaw attitude position feedback controller gain
A1S	$a_{1\psi}$	yaw attitude rate feedback controller gain
BOS	$b_{o\psi}$	sideslip position feedback controller gain
KR	k_r	rudder blending ratio feedback controller gain
AOP	$a_{o\phi}$	roll attitude position feedback controller gain
A1P	$a_{1\phi}$	roll attitude error feedback controller gain
KA	k_a	aileron blending ratio feedback controller gain
R1XP	R_{1xp}	perturbation force in x direction on attachment point 1
R1ZP	R_{1zp}	perturbation force in z direction on attachment point 1
R1YP	R_{1yp}	perturbation force in y direction on attachment point 1
R4YP	R_{4yp}	perturbation force in y direction on attachment point 4

Fortran Term		Definition
R2ZP	R_{2zp}	perturbation force in z direction on attachment point 2
R3ZP	R_{3zp}	perturbation force in z direction on attachment point 3
RRP	RR_p	total perturbation force affecting structure $RR_p = \sqrt{(R_{2zp} + R_{3zp})^2 + (R_{4yp})^2}$
R1XO	R_{1x_o}	nominal force in x direction at attachment point 1
R1ZO	R_{1z_o}	nominal force in z direction at attachment point 1
R1YO	R_{1y_o}	nominal force in y direction at attachment point 1
R4YO	R_{4y_o}	nominal force in y direction at attachment point 4
RRO	RR_o	total nominal force affecting structure $RR_o = \sqrt{(R_{2z_o} + R_{3z_o})^2 + (R_{4y_o})^2}$


6D SHUTTLE ASCENT MDAC CONFIG. 20 0 DEGREE WIND
JOB NO 422500
PAGE
THET *** DEG
0
-1.0
-2.0
0
10.0
20.0
30.0
40.0
50.0
60.0
70.0
80.0
90.0
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-1.0
TIME *** SEC
THETDD *** DEG/SEC**2
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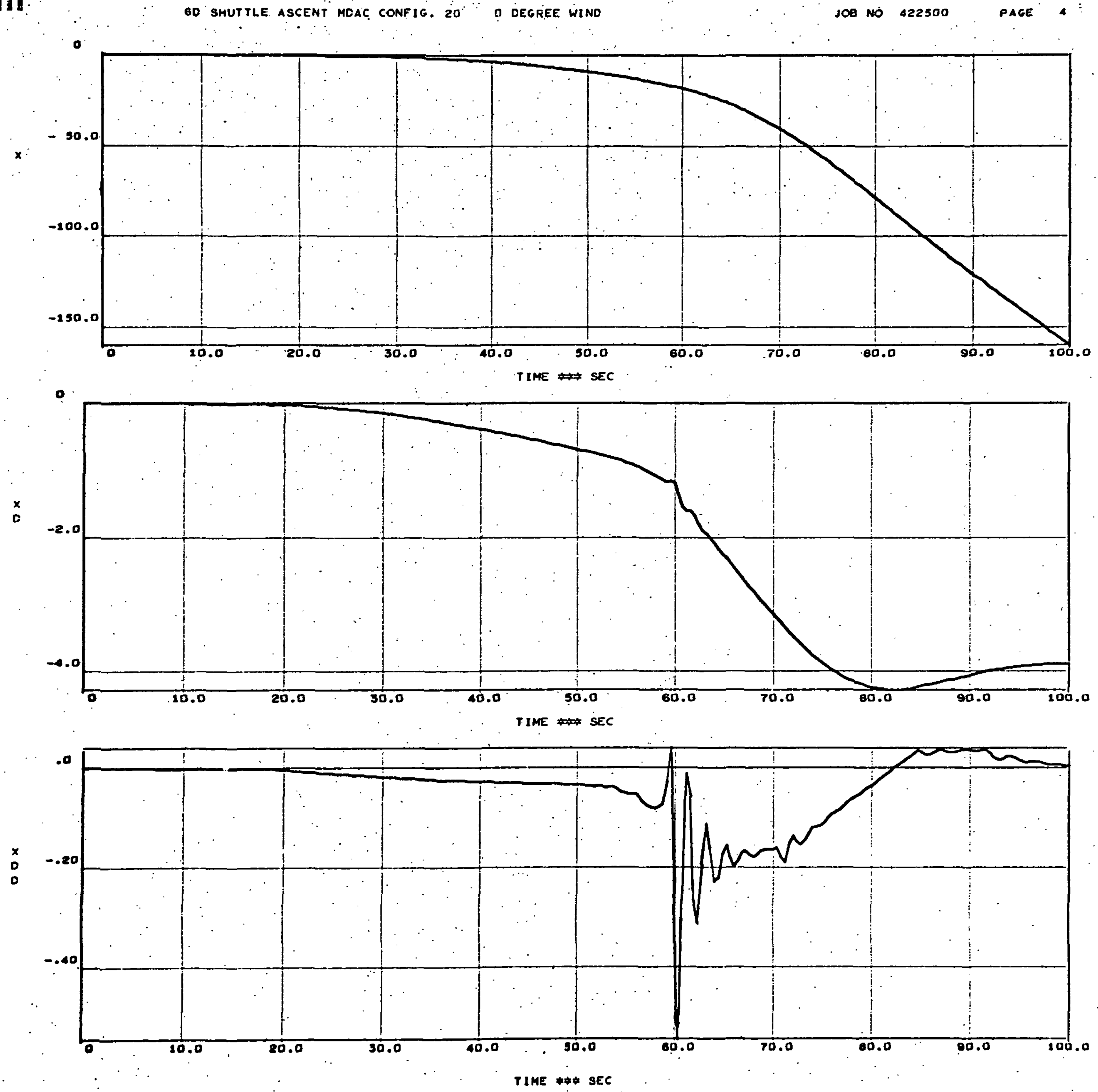
TIME *** SEC



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50.0
60.0
70.0
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90.0
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D
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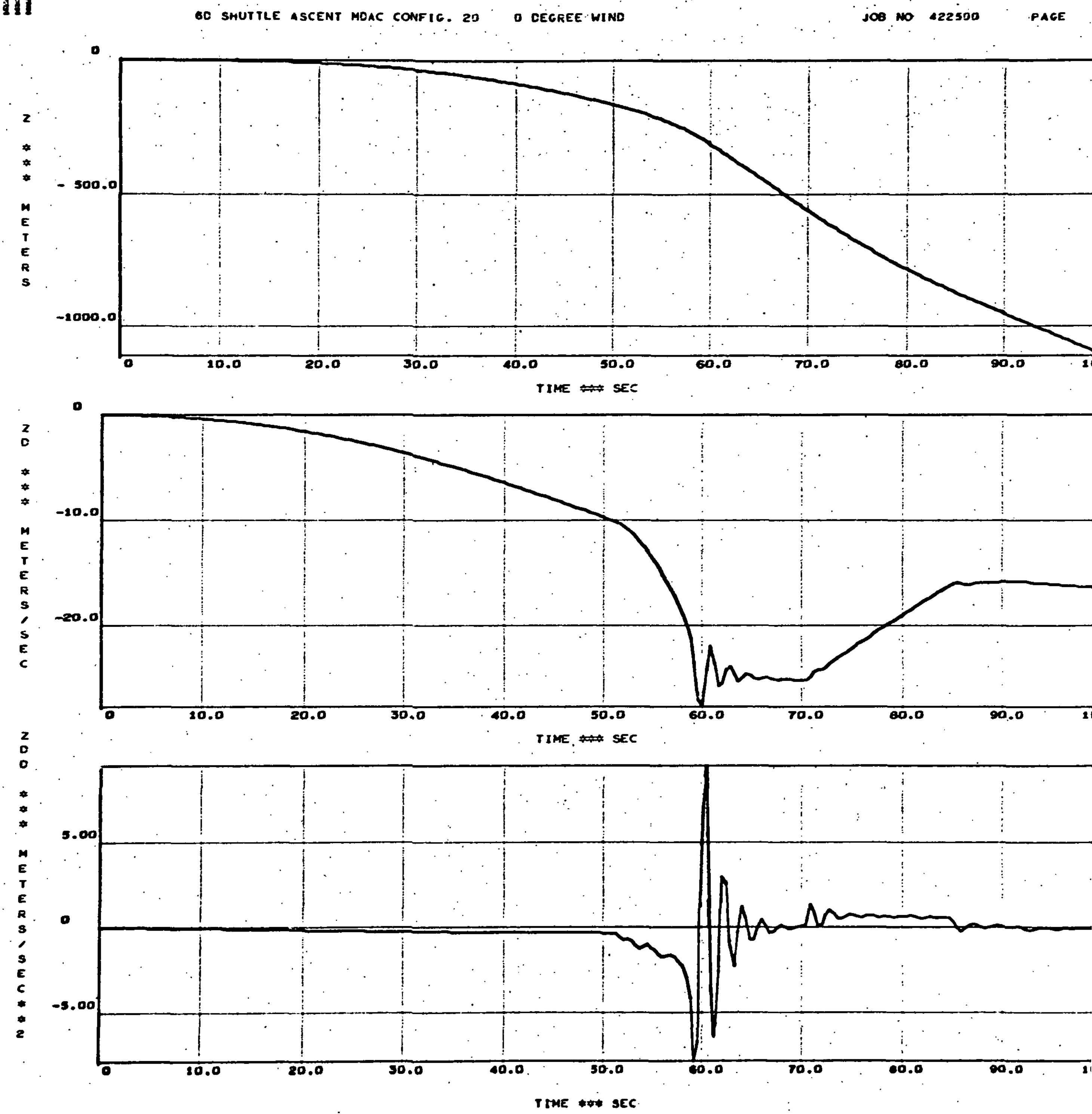
TIME *** SEC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TIME *** SEC
ZD *** METERS/SEC
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-10.0
-20.0
0 10.0 20.0 30.0 40.0 50.0 60.0 70.0 80.0 90.0
TIME *** SEC
ZDD *** METERS/SEC**2
5.00
0
-5.00
0 10.0 20.0 30.0 40.0 50.0 60.0 70.0 80.0 90.0
TIME *** SEC

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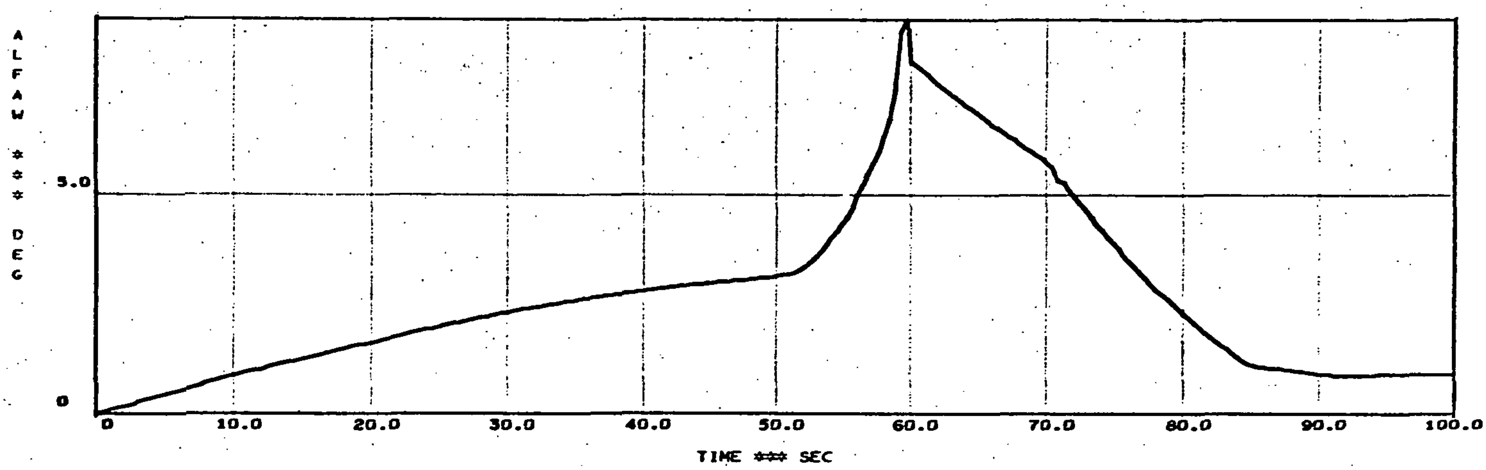

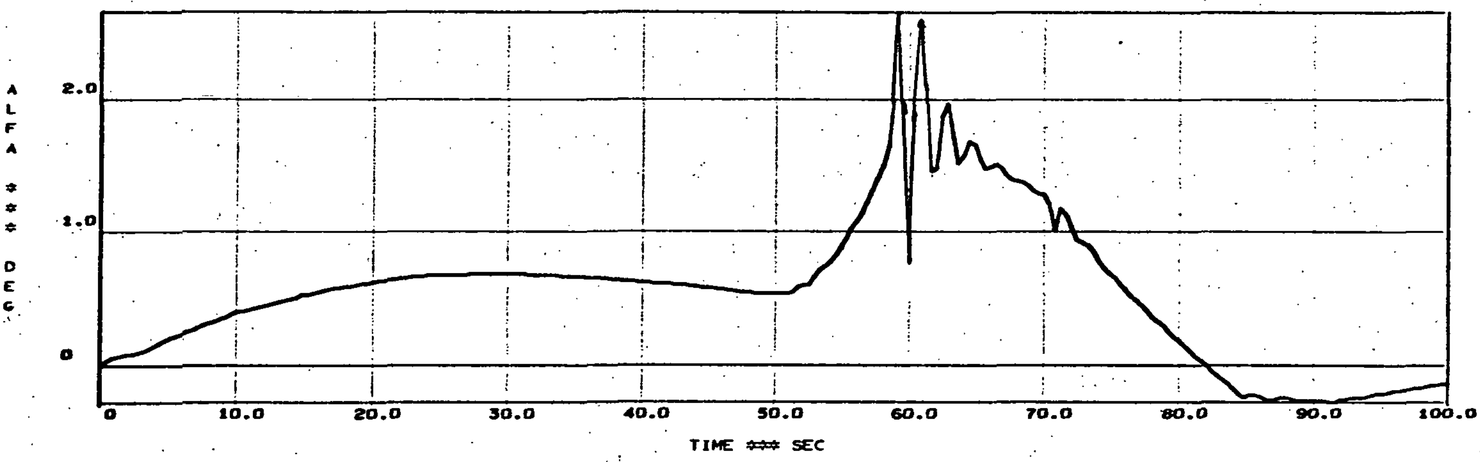

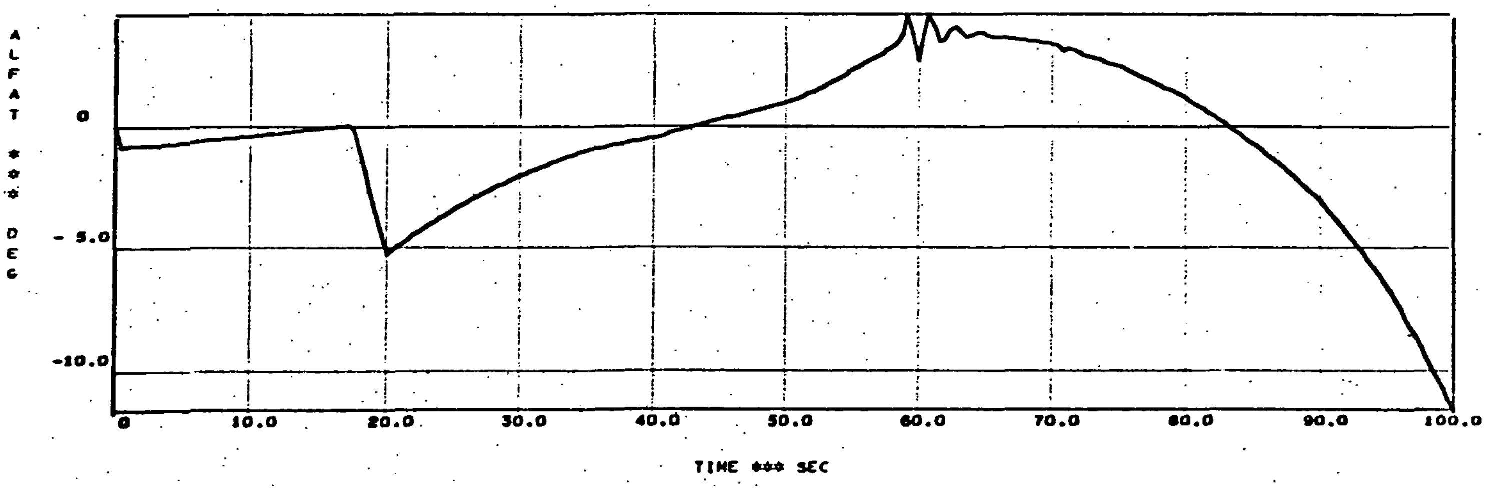

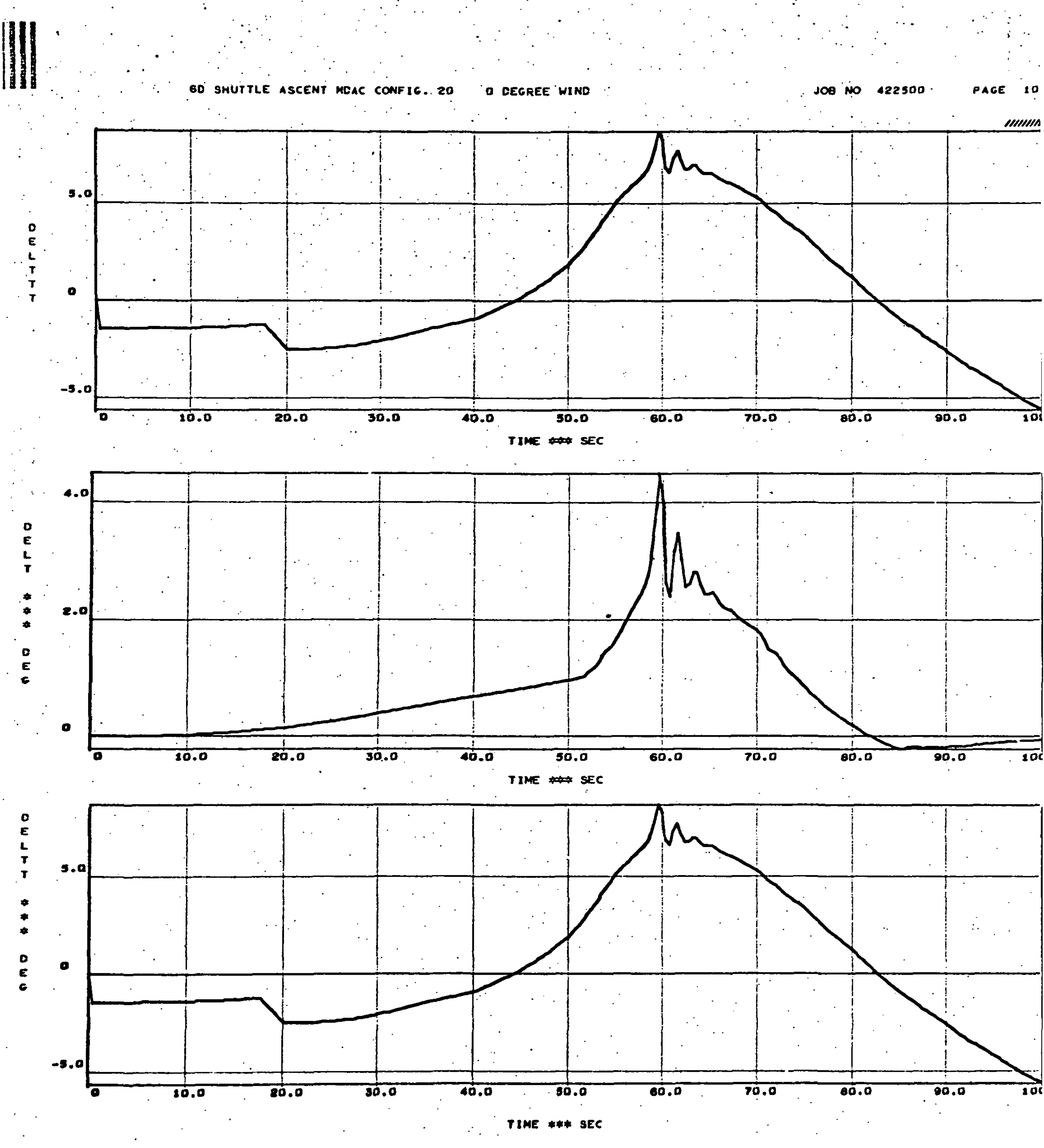
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20.0
30.0
40.0
50.0
60.0
70.0
80.0
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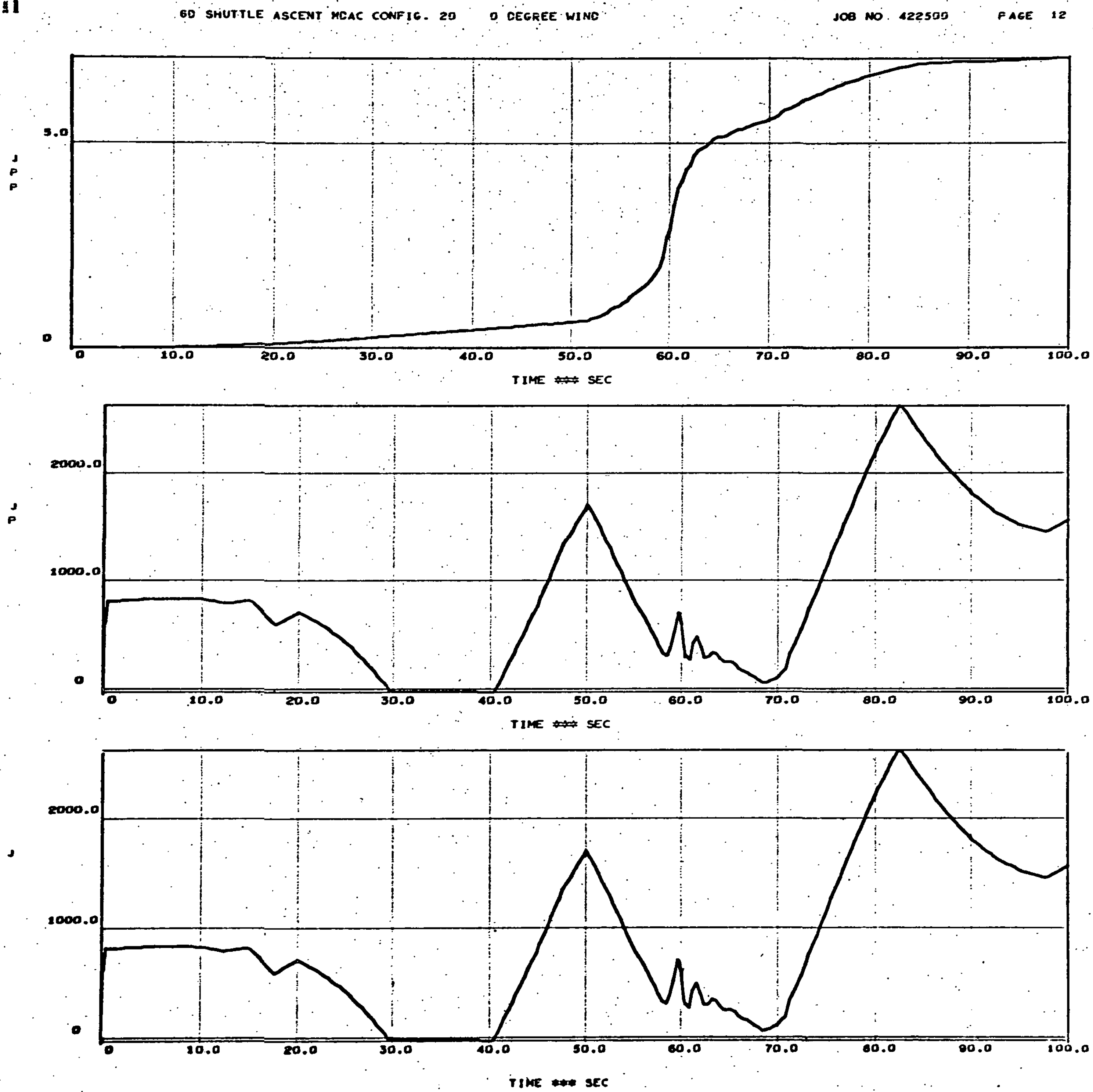
TIME *** SEC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TIME *** SEC
2000.0
1000.0
0
JP
0 10.0 20.0 30.0 40.0 50.0 60.0 70.0 80.0 90.0 100.0
TIME *** SEC
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0
J
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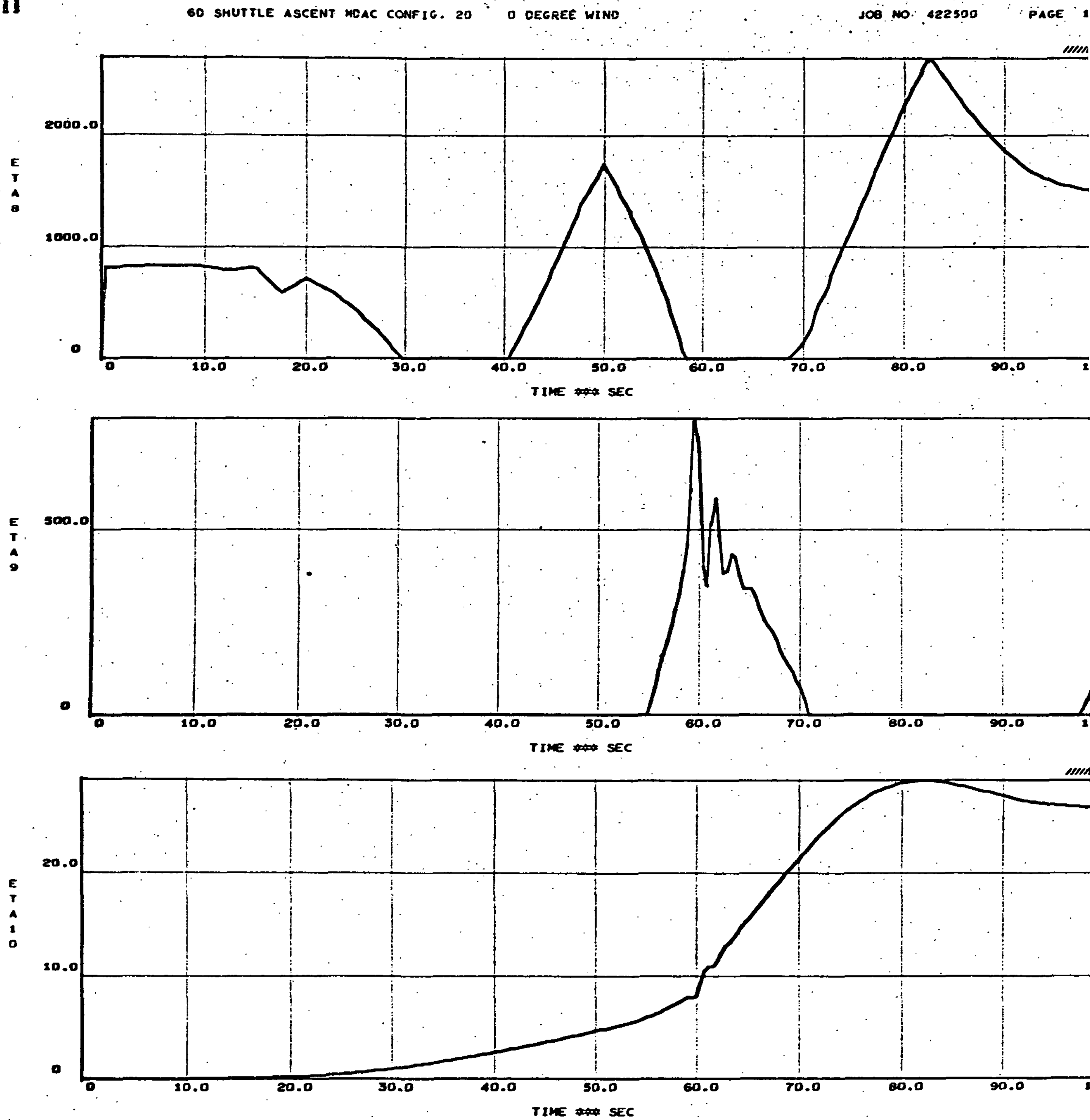
TIME *** SEC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TIME *** SEC
500.0
0
ETA9
0 10.0 20.0 30.0 40.0 50.0 60.0 70.0 80.0 90.0
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10.0
0
ETA10
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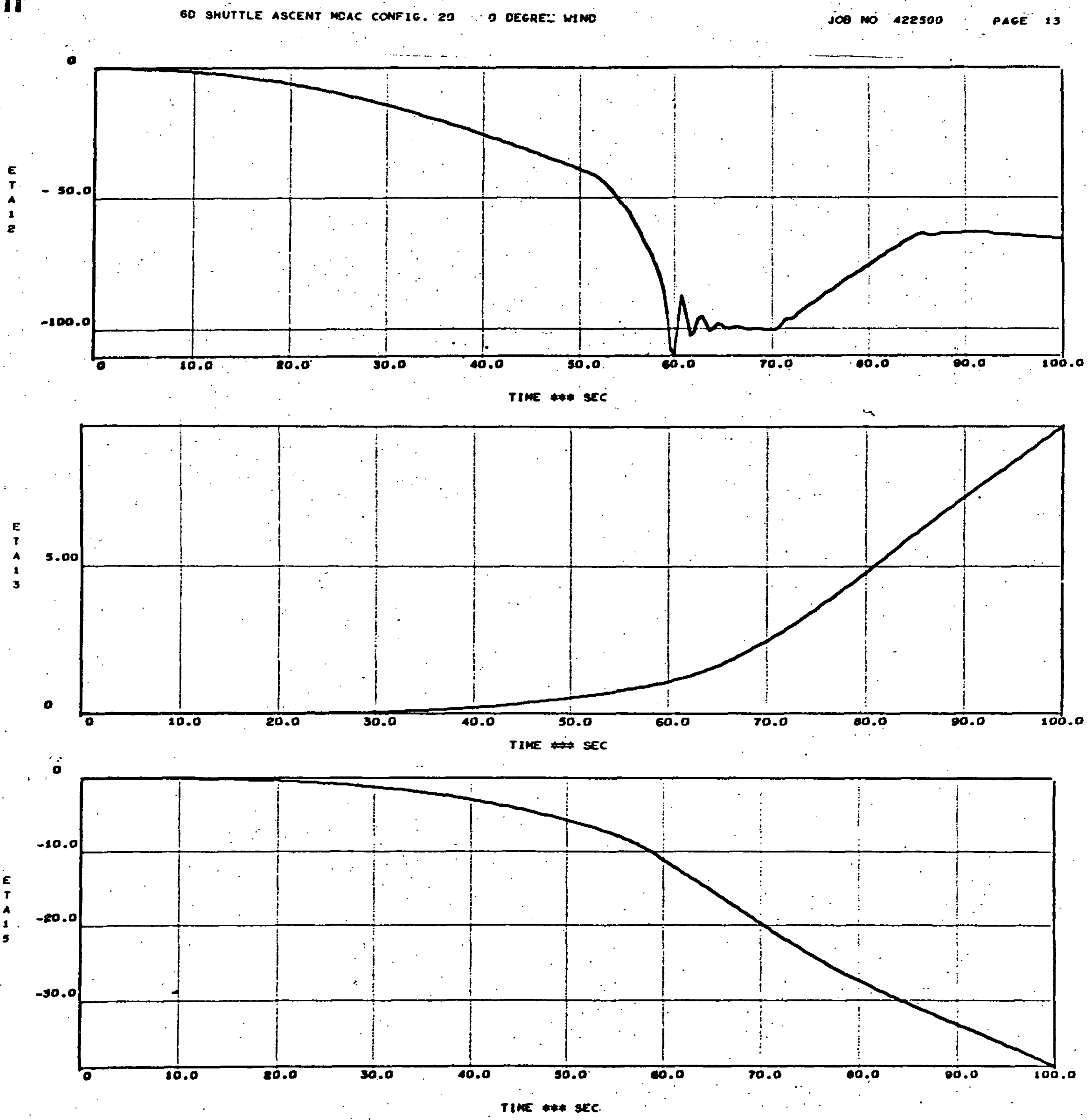
TIME *** SEC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JOB NO 422500
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-100.0
ETA13
5.00
0
ETA15
0
-10.0
-20.0
-30.0
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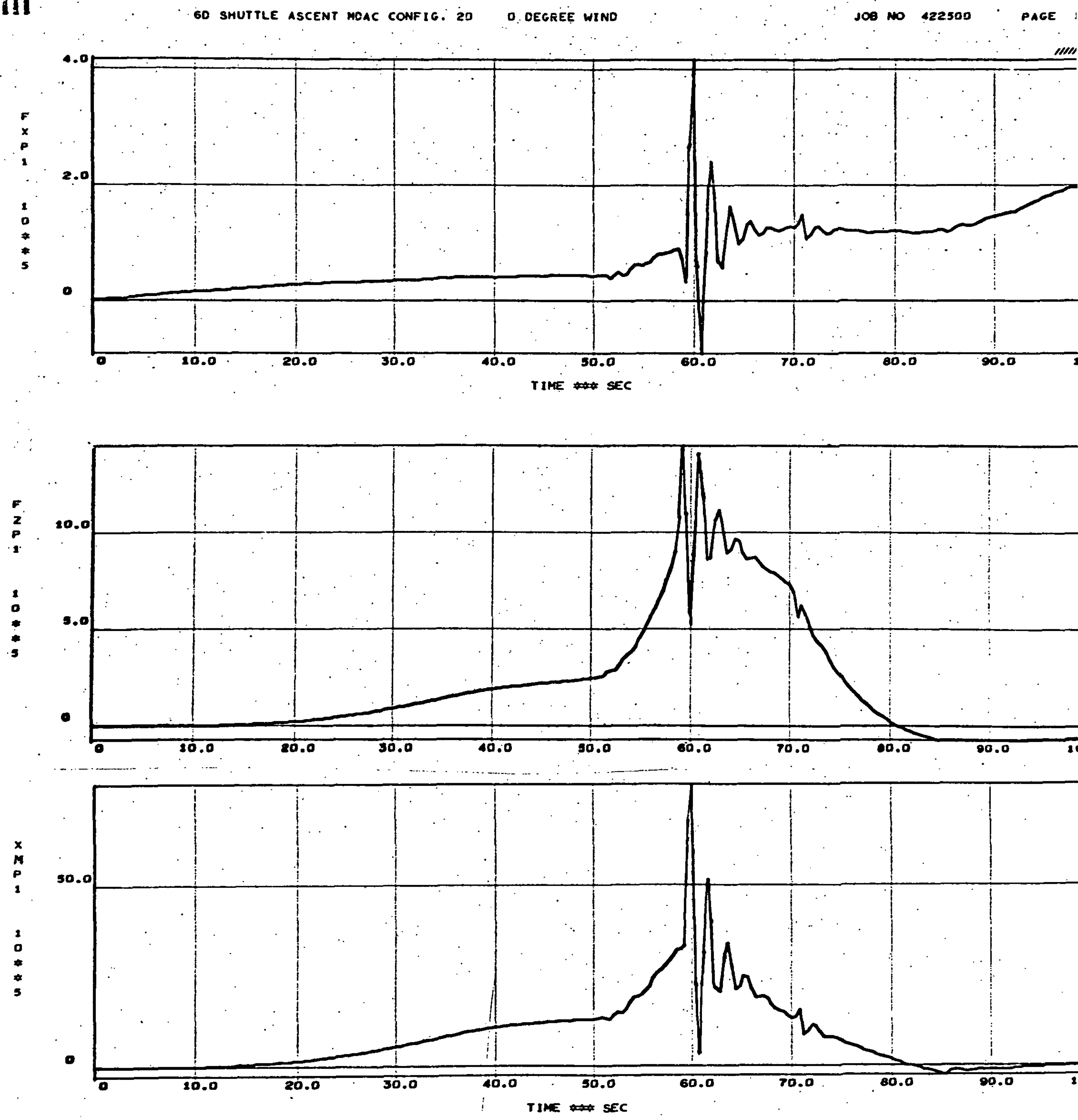
TIME *** SEC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JOB NO 422500 PAGE
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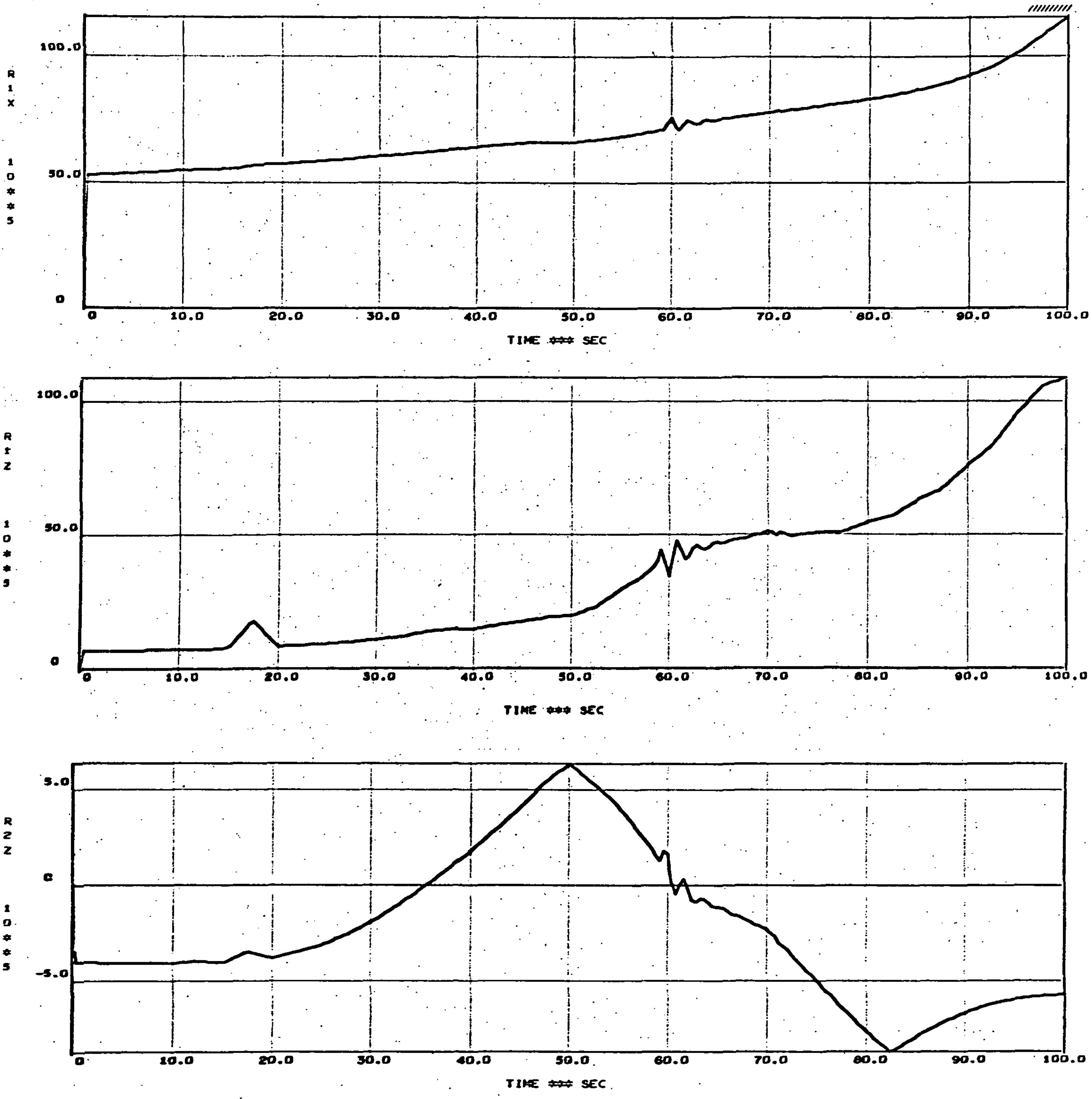
2.0
0
0 10.0 20.0 30.0 40.0 50.0 60.0 70.0 80.0 90.0
TIME *** SEC
FZP1 10**5
10.0
5.0
0
0 10.0 20.0 30.0 40.0 50.0 60.0 70.0 80.0 90.0
XMP1 10**5
50.0
0
0 10.0 20.0 30.0 40.0 50.0 60.0 70.0 80.0 90.0
TIME *** SEC

6D SHUTTLE ASCENT MDAC CONFIG. 20   0 DEGREE WIND   JOB NO 422500   PAGE 18



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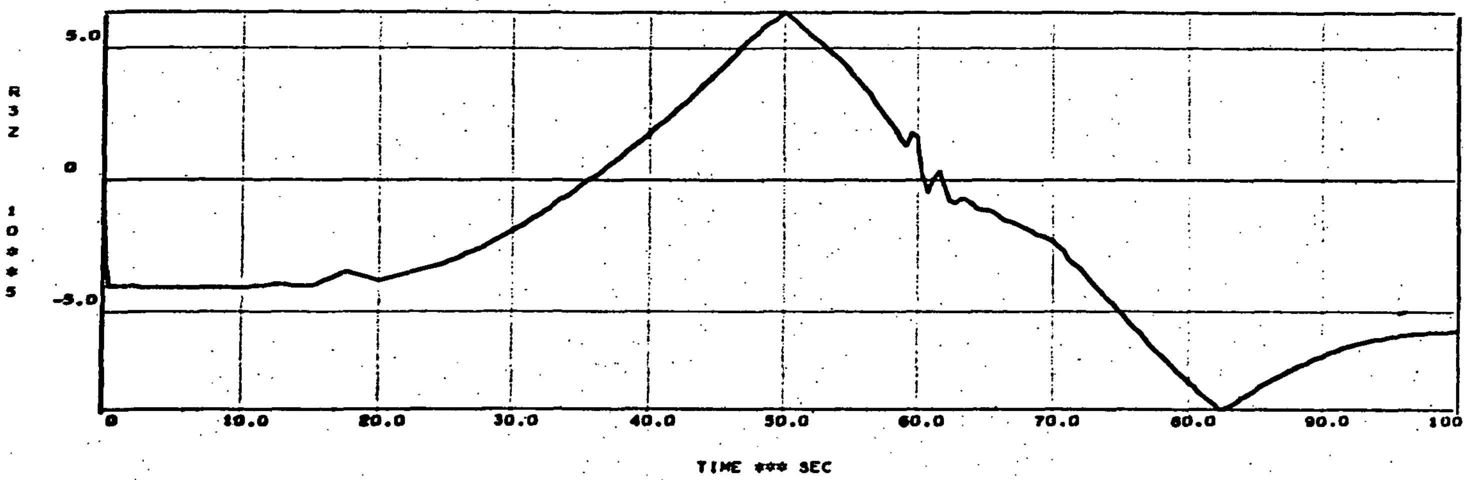
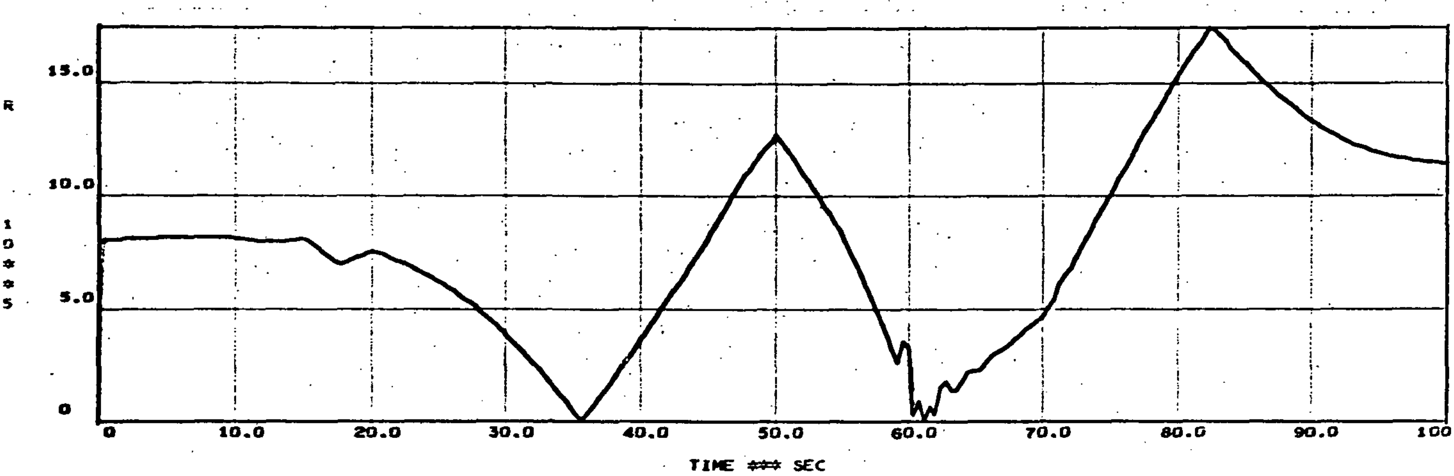
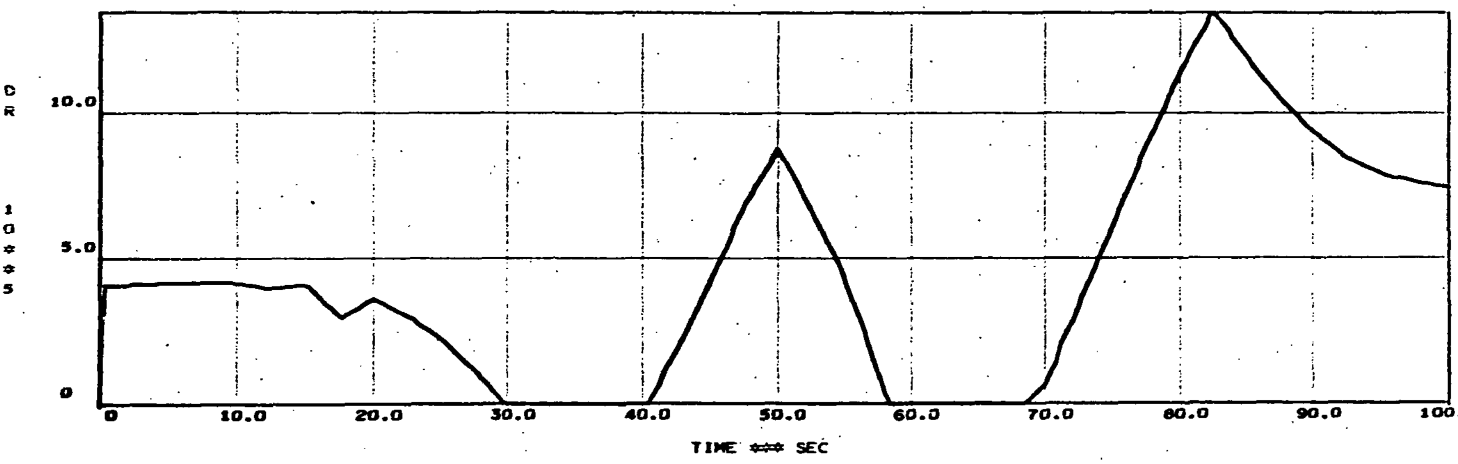
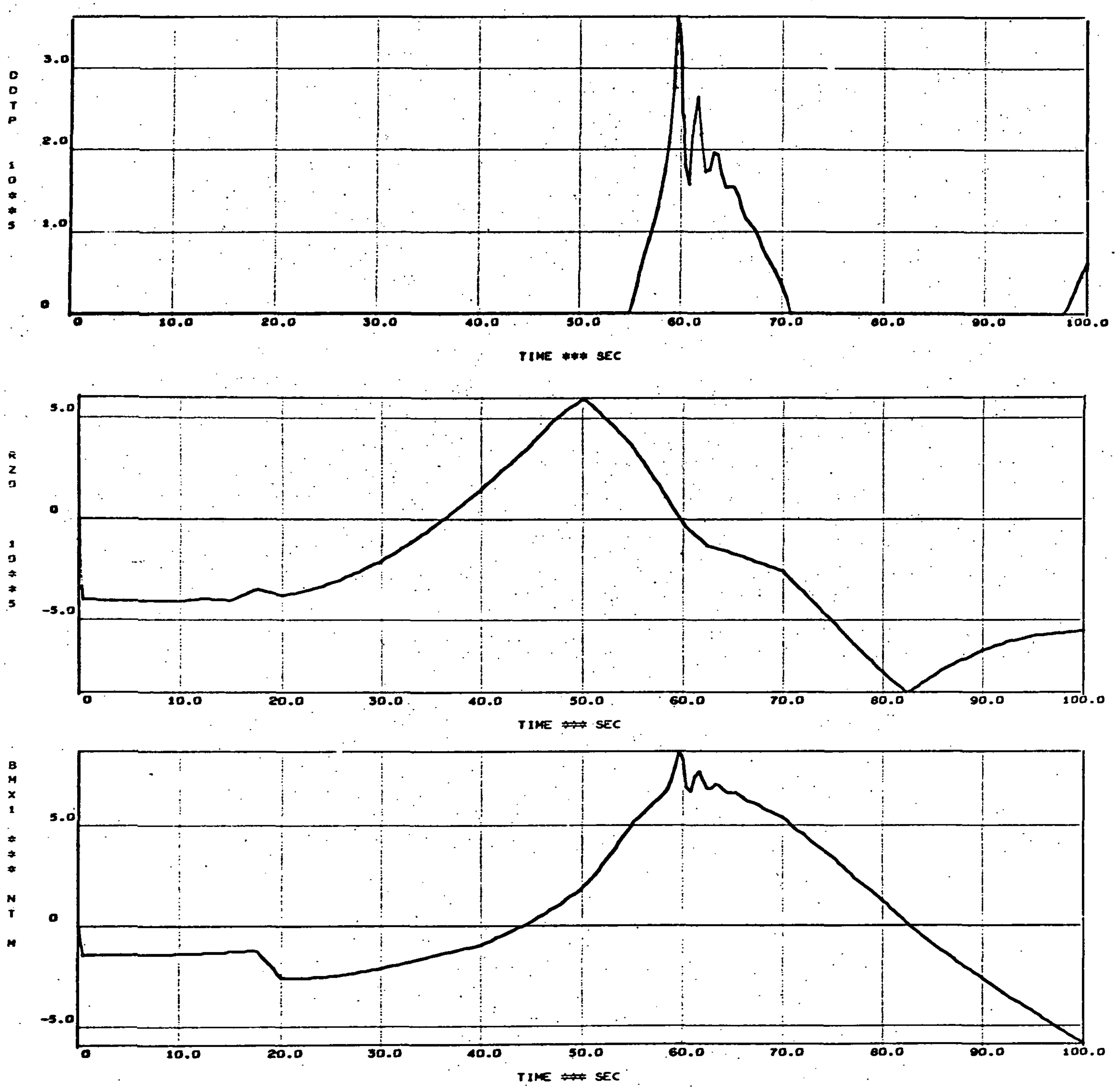
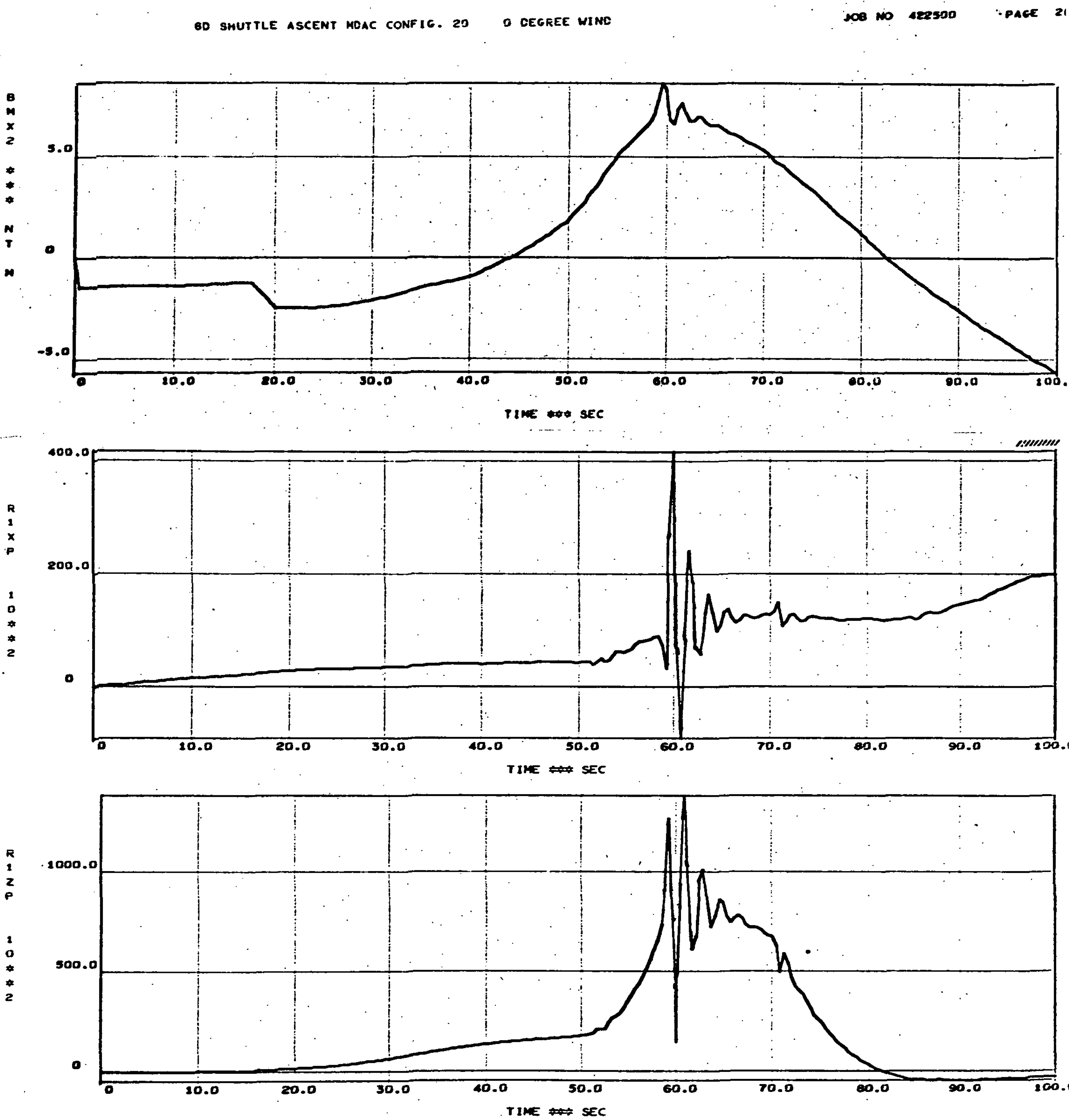






6D SHUTTLE ASCENT MDAC CONFIG. 20   0 DEGREE WIND   JOB NO 422500   PAGE 20

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JOB NO 422500
PAGE
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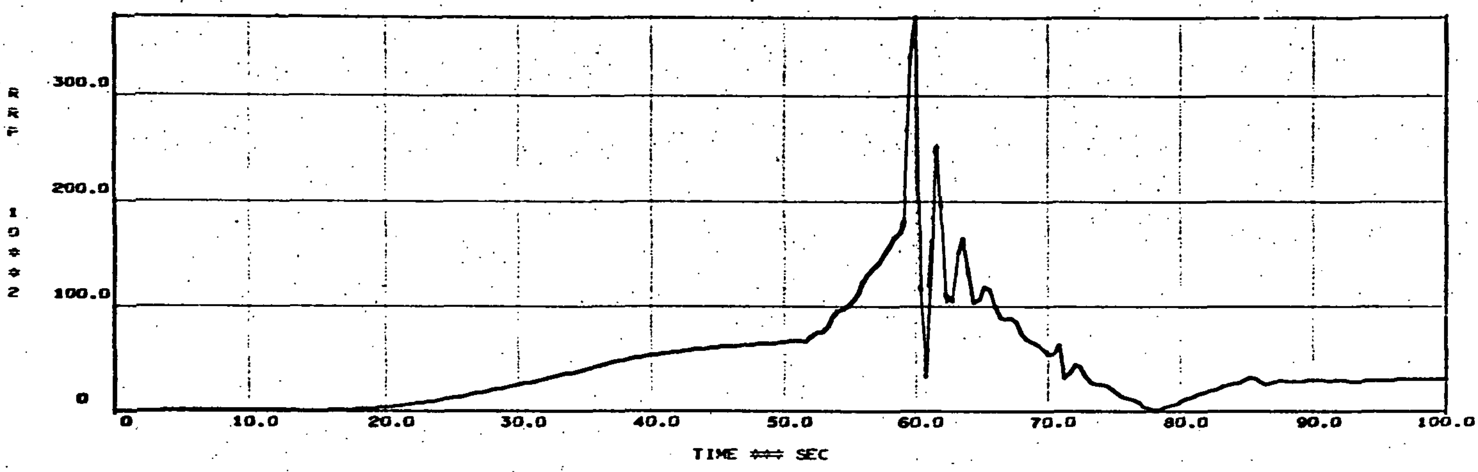
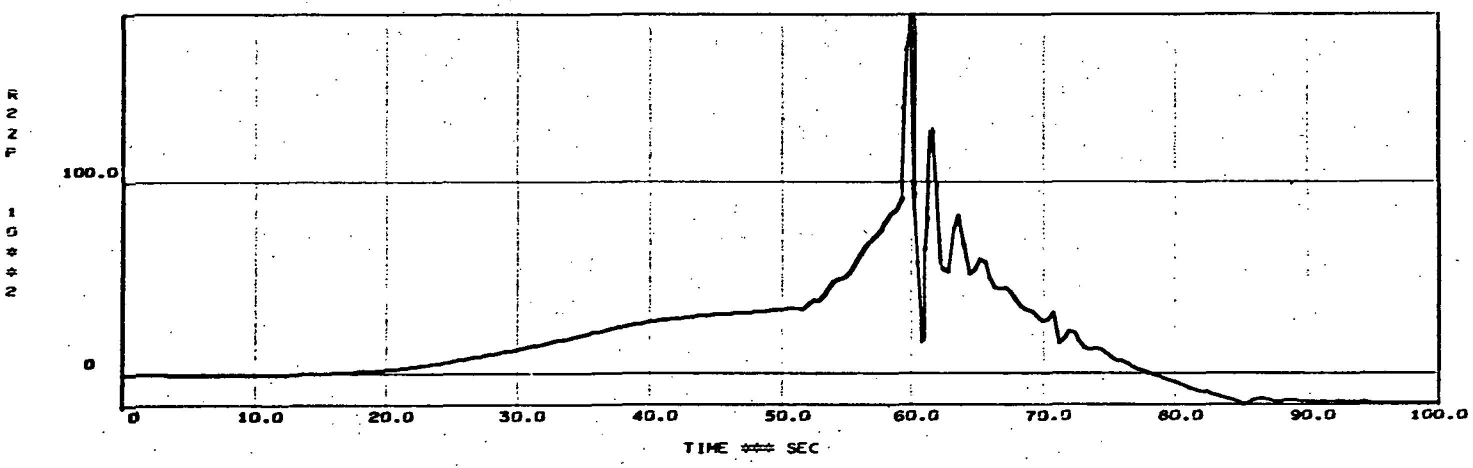
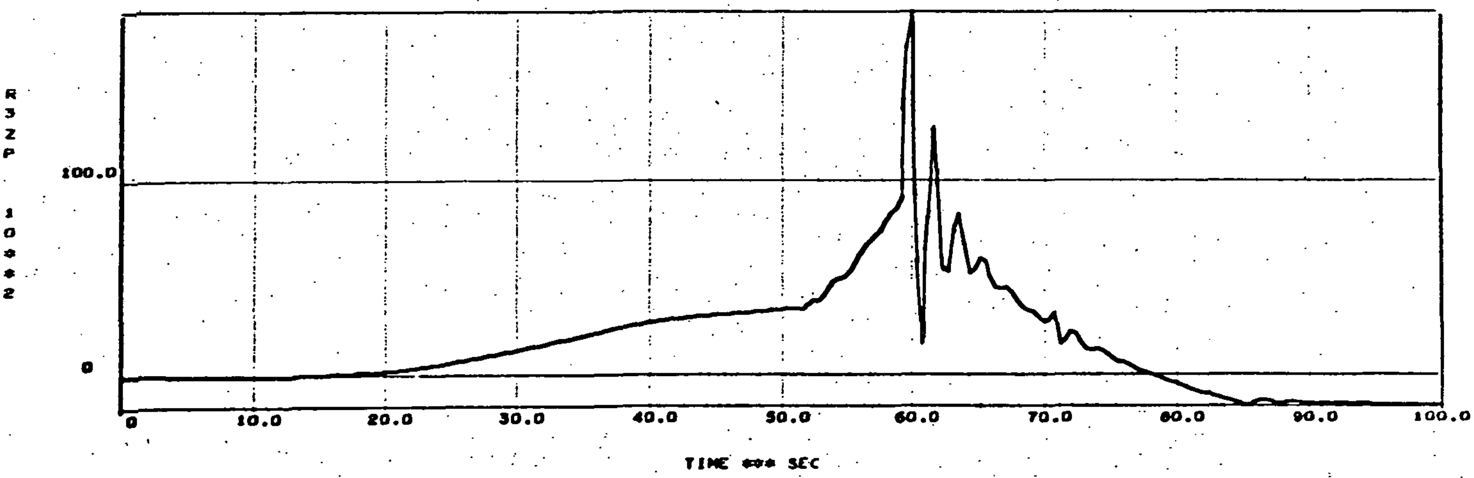
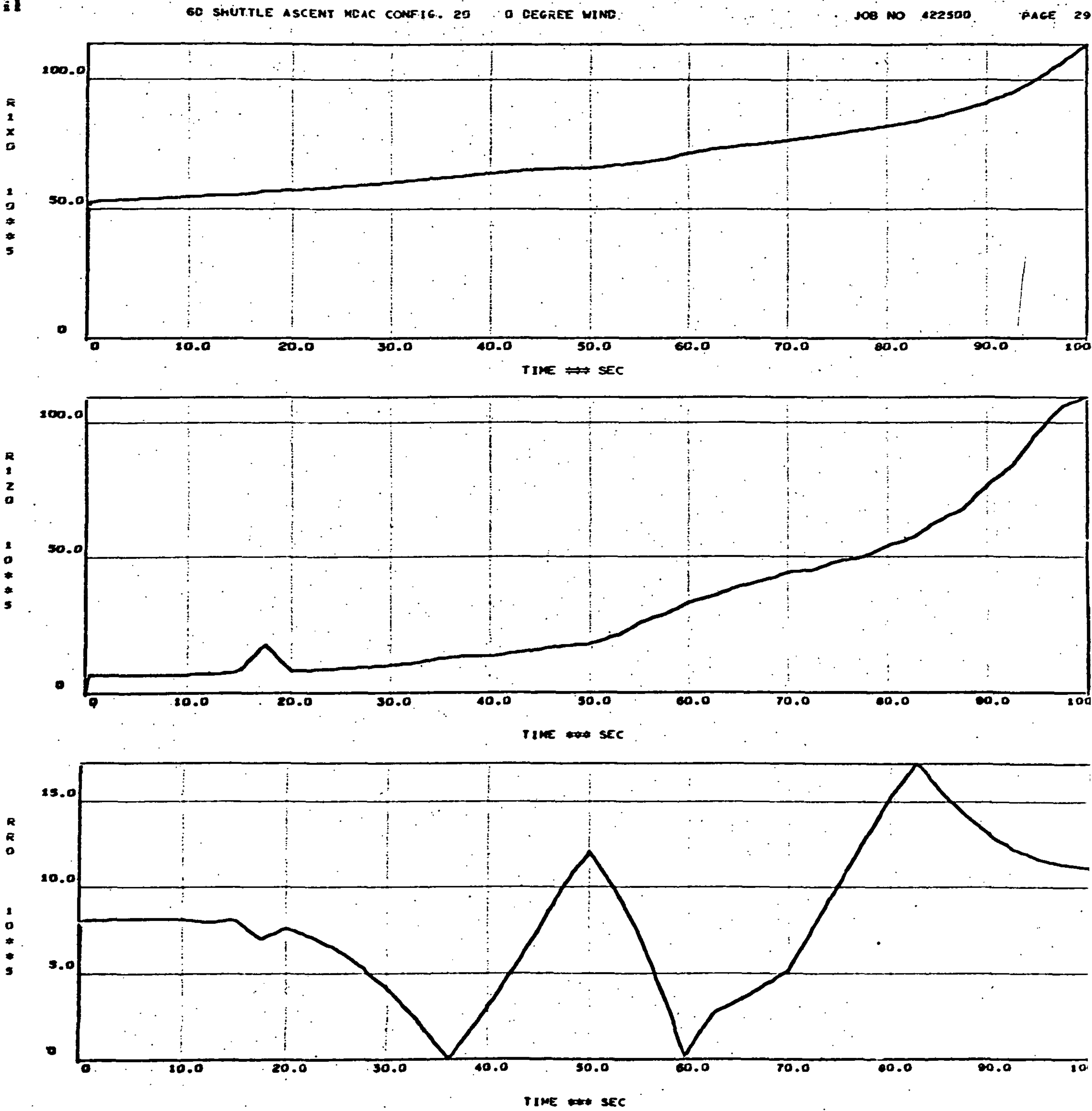
5.0
0
-5.0
0 10.0 20.0 30.0 40.0 50.0 60.0 70.0 80.0 90.0 100.
TIME *** SEC
R1XP 10**2
400.0
200.0
0
0 10.0 20.0 30.0 40.0 50.0 60.0 70.0 80.0 90.0 100.
TIME *** SEC
R1ZP 10**2
1000.0
500.0
0
0 10.0 20.0 30.0 40.0 50.0 60.0 70.0 80.0 90.0 100.
TIME *** SEC

6D SHUTTLE ASCENT MDAC CONFIG. 29 0 DEGREE WIND JOB NO 422500 PAGE 28





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JOB NO 422500
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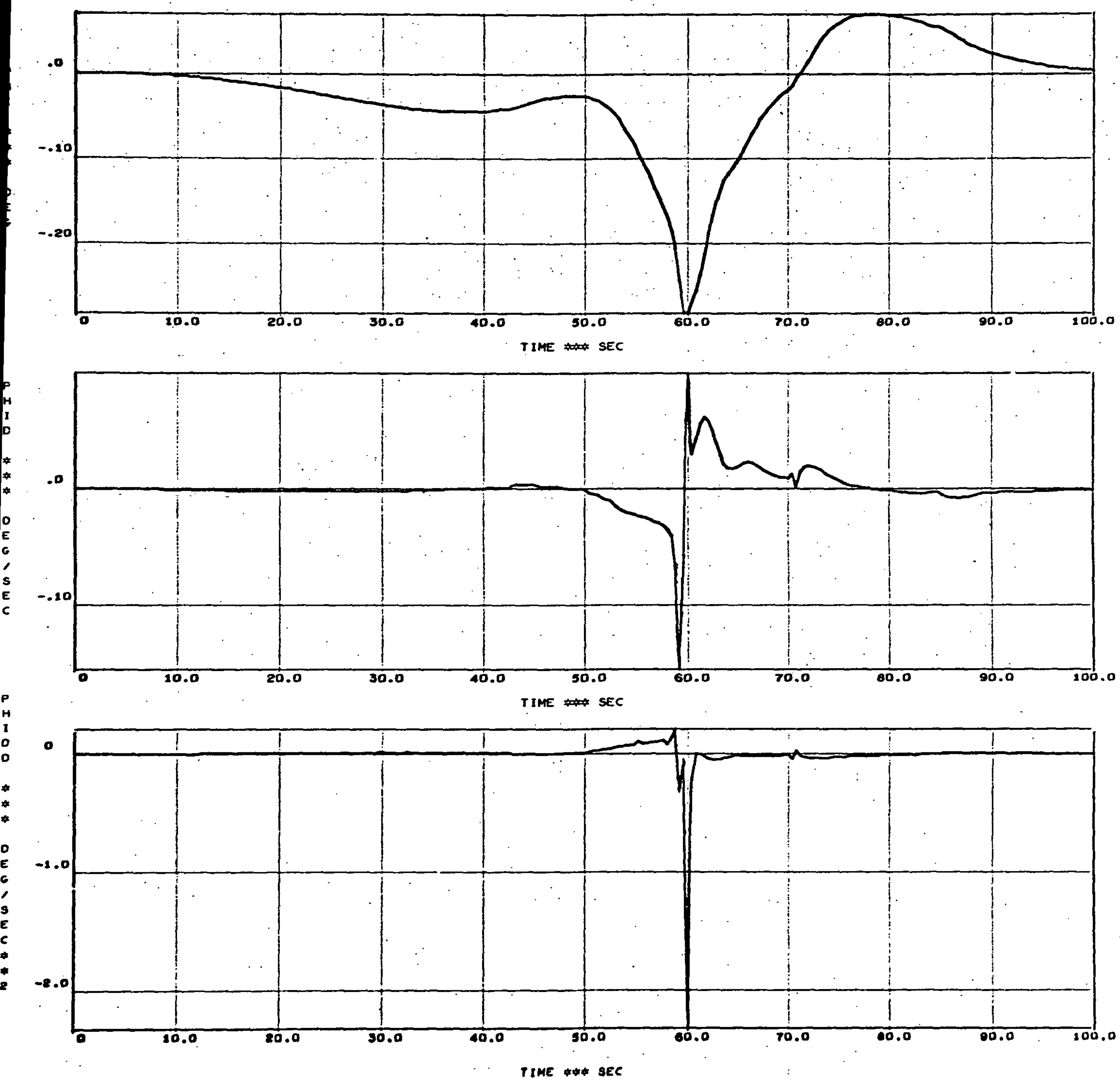
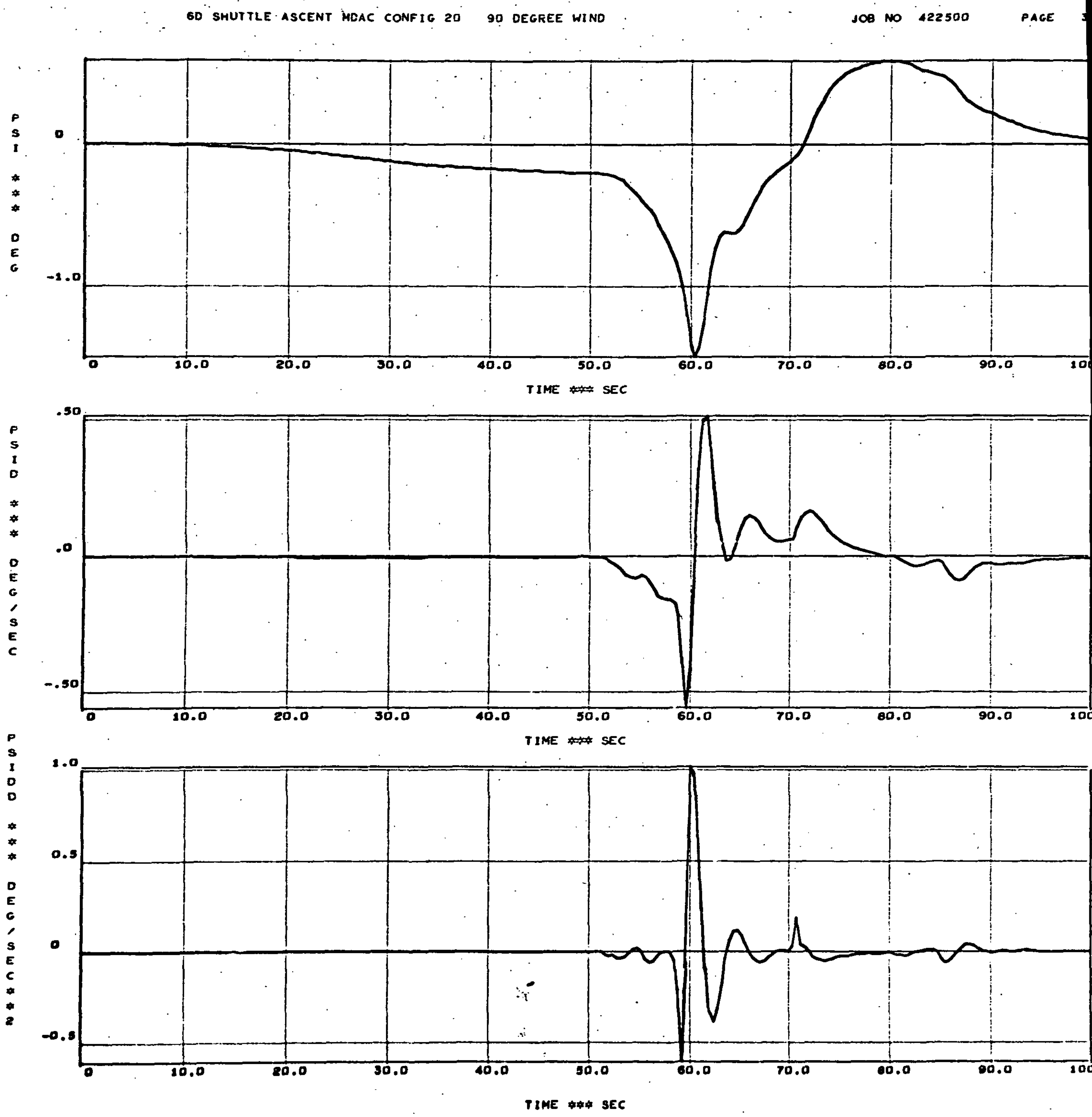
1 0 * * 5
100.0
50.0
0
0
10.0
20.0
30.0
40.0
50.0
60.0
70.0
80.0
90.0
100
TIME *** SEC
R1ZO
1 0 * * 5
100.0
50.0
0
TIME *** SEC
RRO
1 0 * * 5
15.0
10.0
5.0
0
TIME *** SEC

60 SHUTTLE ASCENT MDAC CONFIG 29   90 DEGREE WIND   JOB NO 422500   PAGE 1

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JOB NO 422500
PAGE
PSI *** DEG
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-1.0
0
10.0
20.0
30.0
40.0
50.0
60.0
70.0
80.0
90.0
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.50
.0
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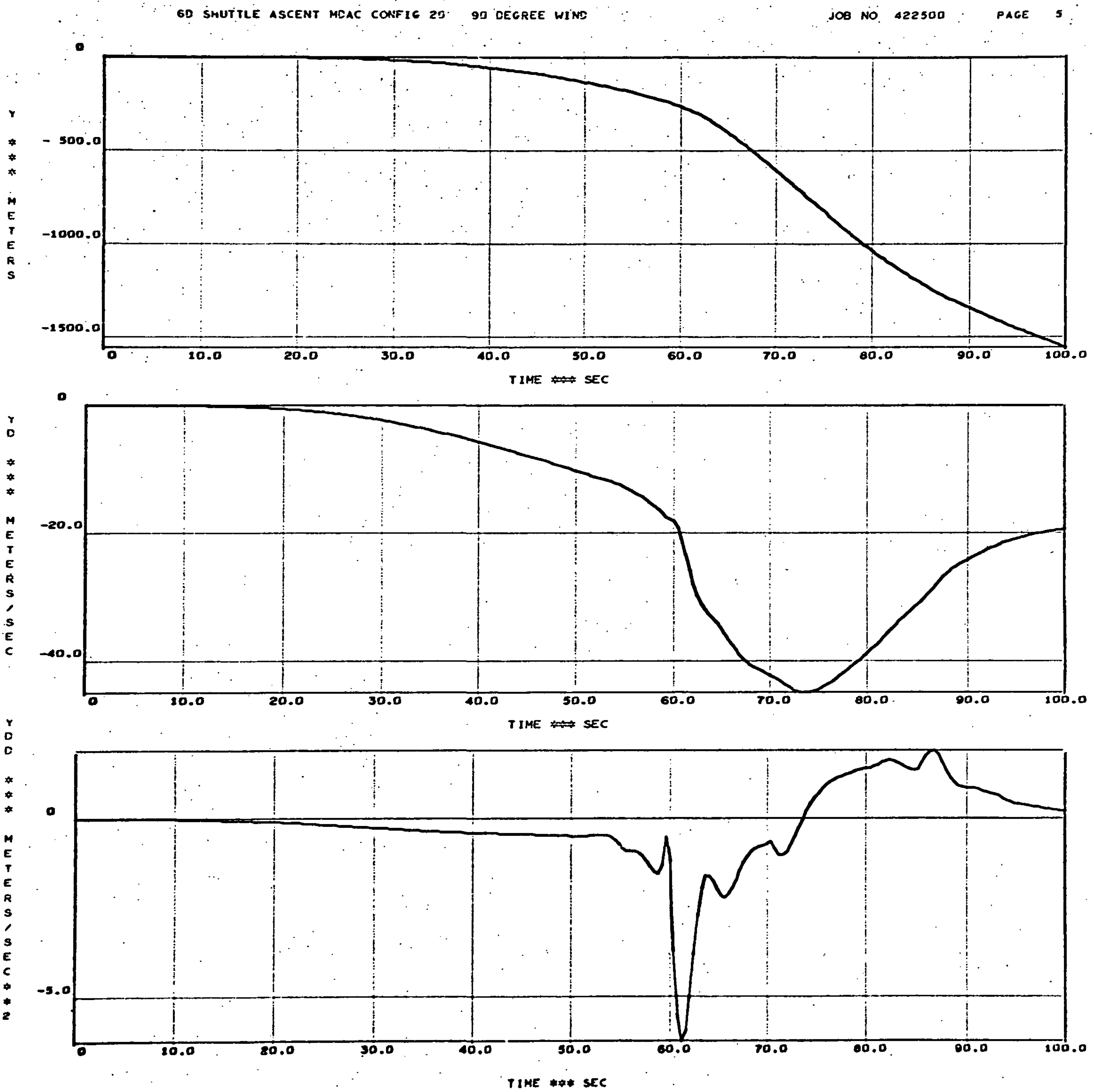
0
-0.5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JOB NO 422500
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-1000.0
-1500.0
0 10.0 20.0 30.0 40.0 50.0 60.0 70.0 80.0 90.0 100.0
TIME *** SEC
YD *** METERS/SEC
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-20.0
-40.0
0 10.0 20.0 30.0 40.0 50.0 60.0 70.0 80.0 90.0 100.0
TIME *** SEC
YDD *** METERS/SEC**2
0
-5.0
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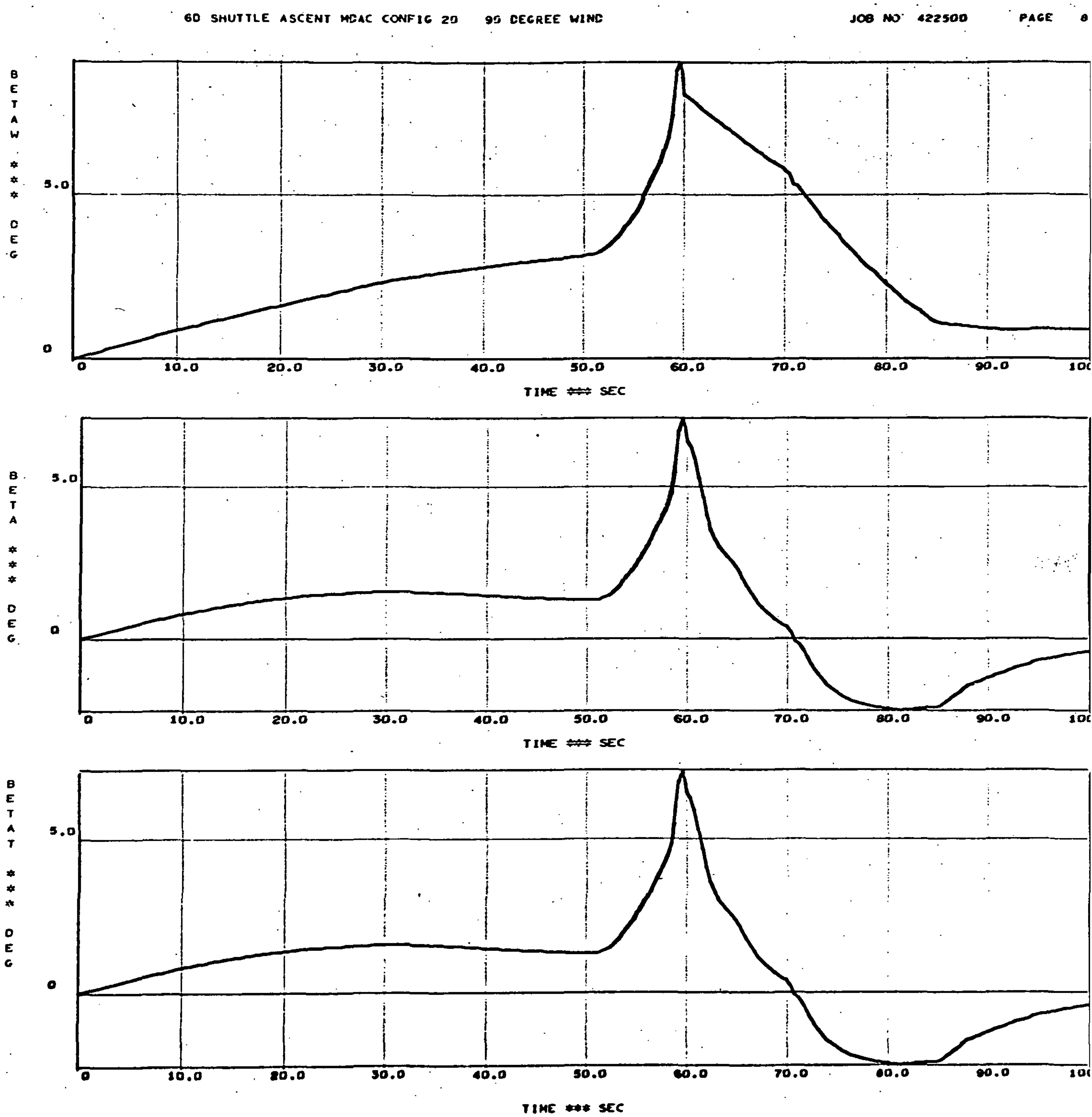
TIME *** SEC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JOB NO 422500
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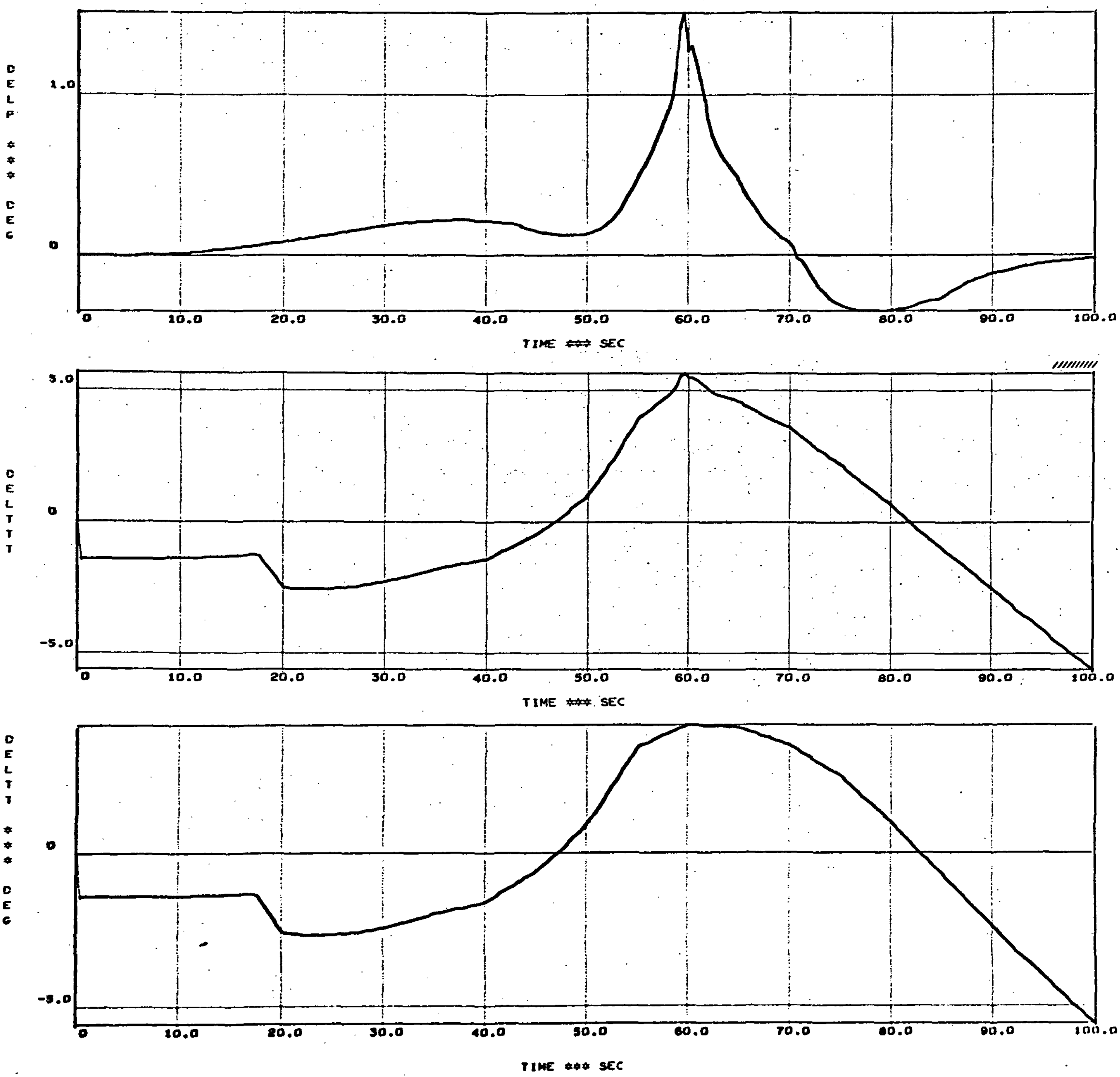
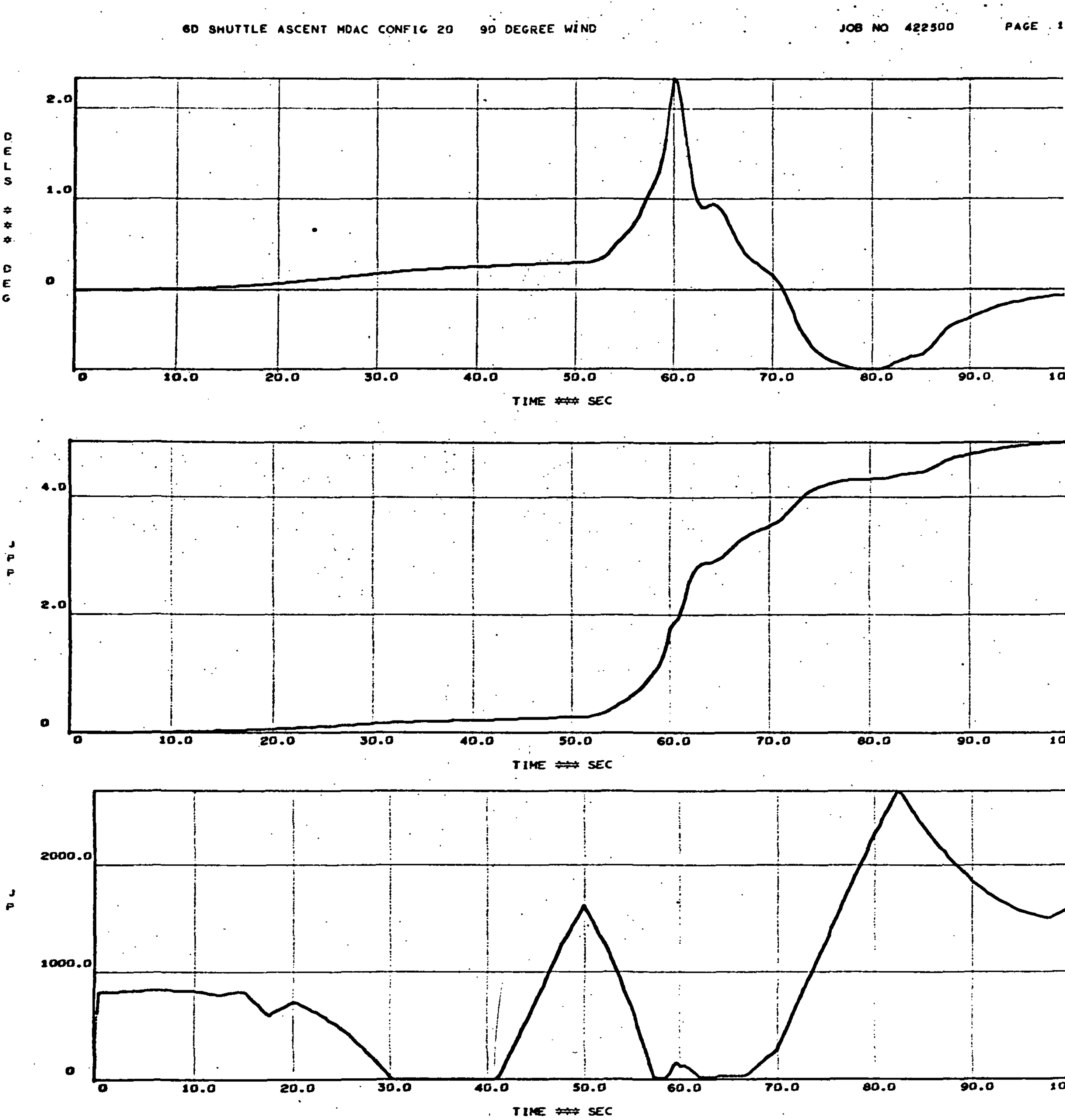
0
0
10.0
20.0
30.0
40.0
50.0
60.0
70.0
80.0
90.0
TIME *** SEC
BETA *** DEG
5.0
0
TIME *** SEC
BETAT *** DEG
5.0
0
TIME *** SEC

6D SHUTTLE ASCENT MDAC CONFIG 25 90 DEGREE WIND JOB NO 422500 PAGE 9

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JOB NO 422500
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DELS *** DEG
2.0
1.0
0
0
10.0
20.0
30.0
40.0
50.0
60.0
70.0
80.0
90.0
TIME *** SEC
JPP
4.0
2.0
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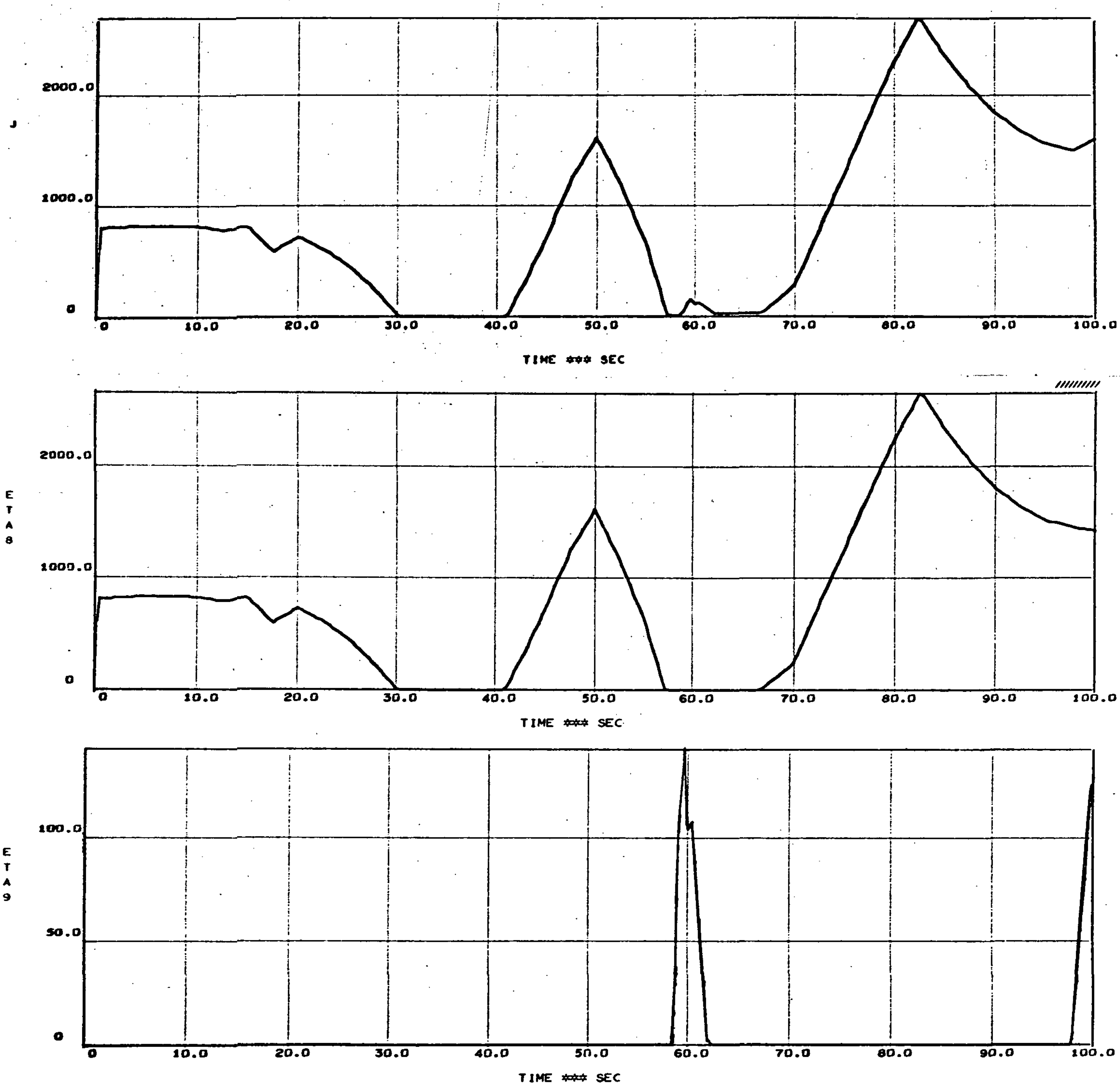
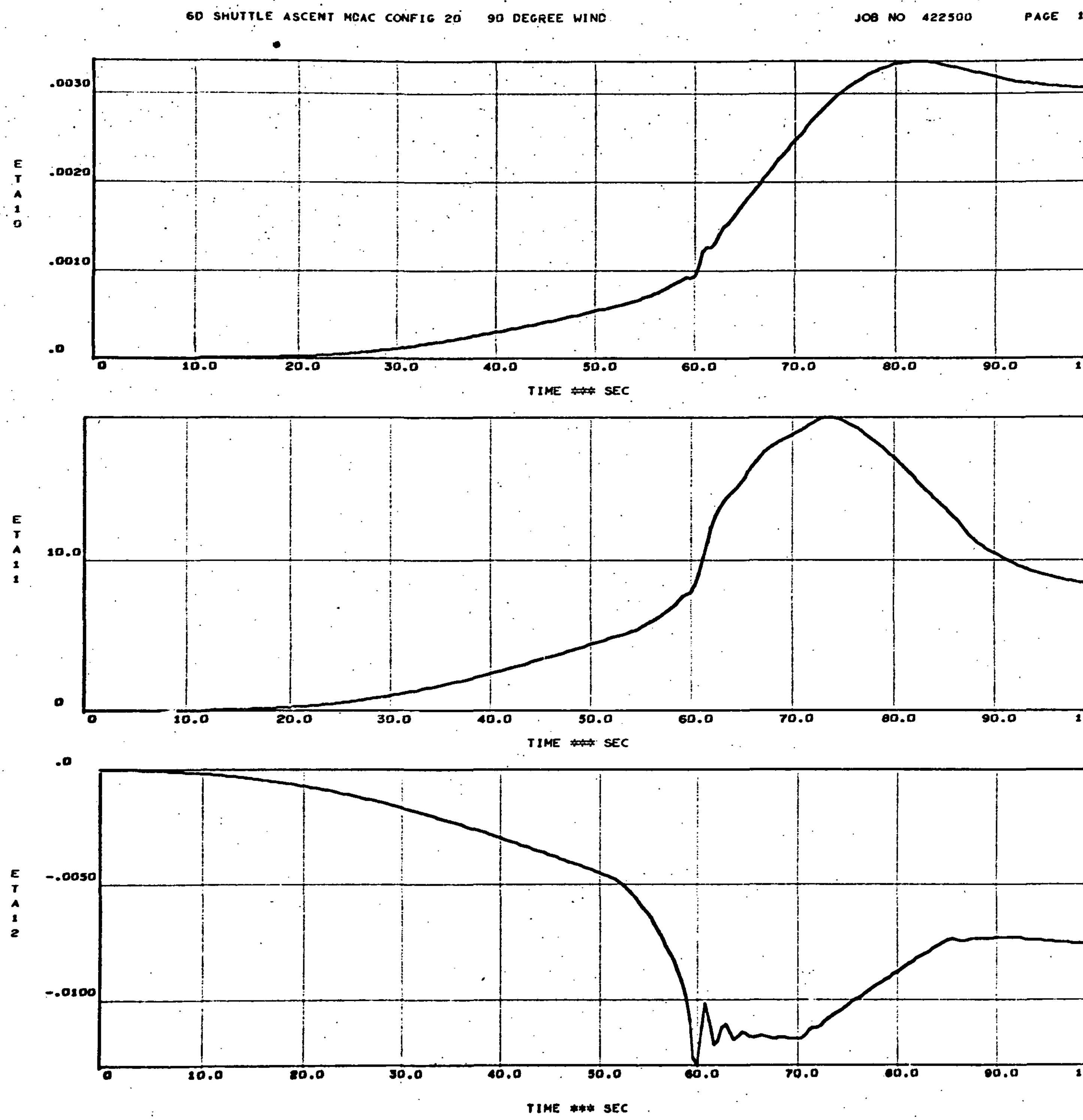
TIME *** SEC

6D SHUTTLE ASCENT MDAC CONFIG 20   90 DEGREE WIND   JOB NO 422500   PAGE 12

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JOB NO 422500
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0
10.0
20.0
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40.0
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60.0
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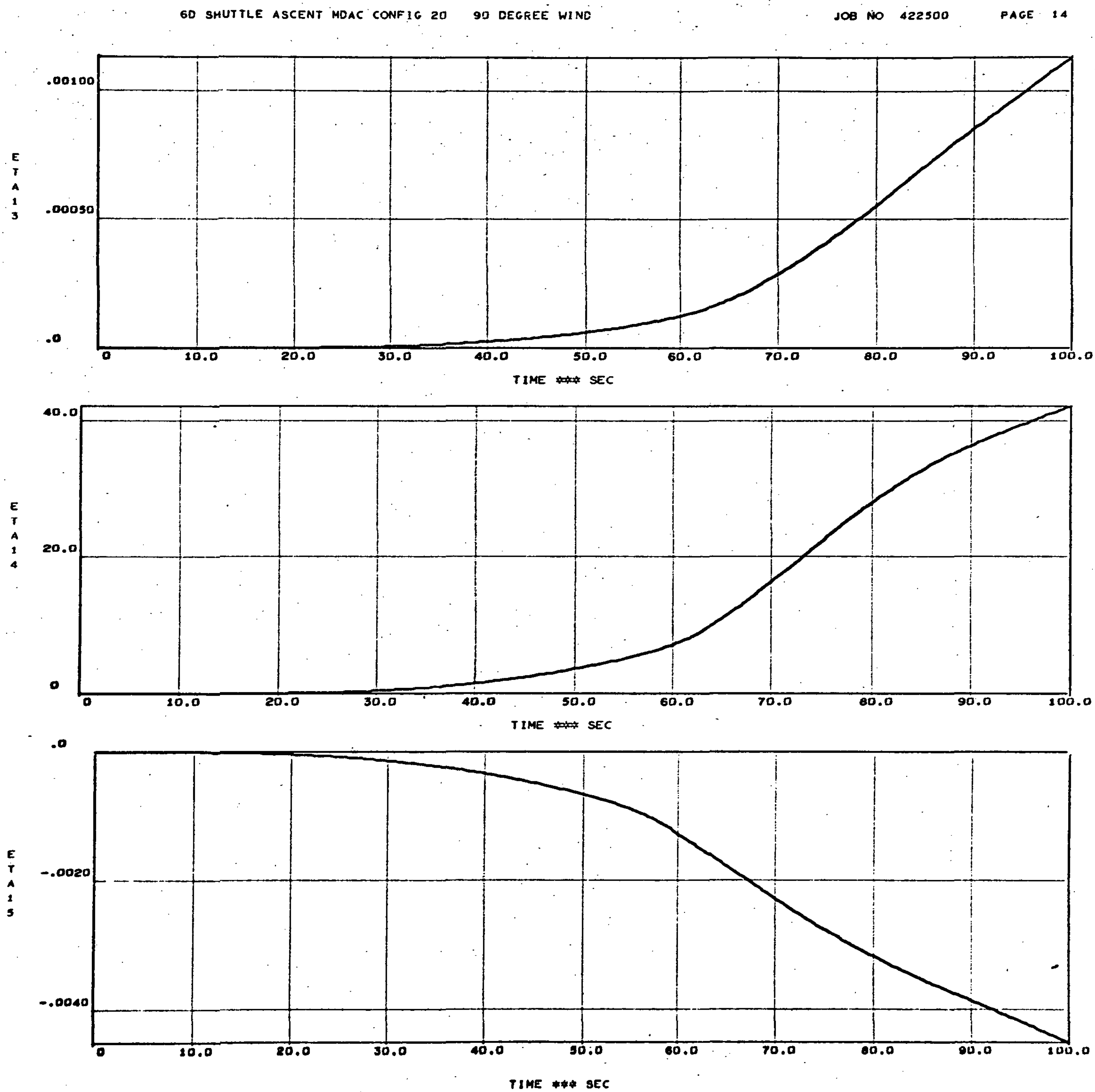
TIME *** SEC
ETA12
.0
-.0050
-.0100
TIME *** SEC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JOB NO 422500
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ETA15
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-.0020
-.0040
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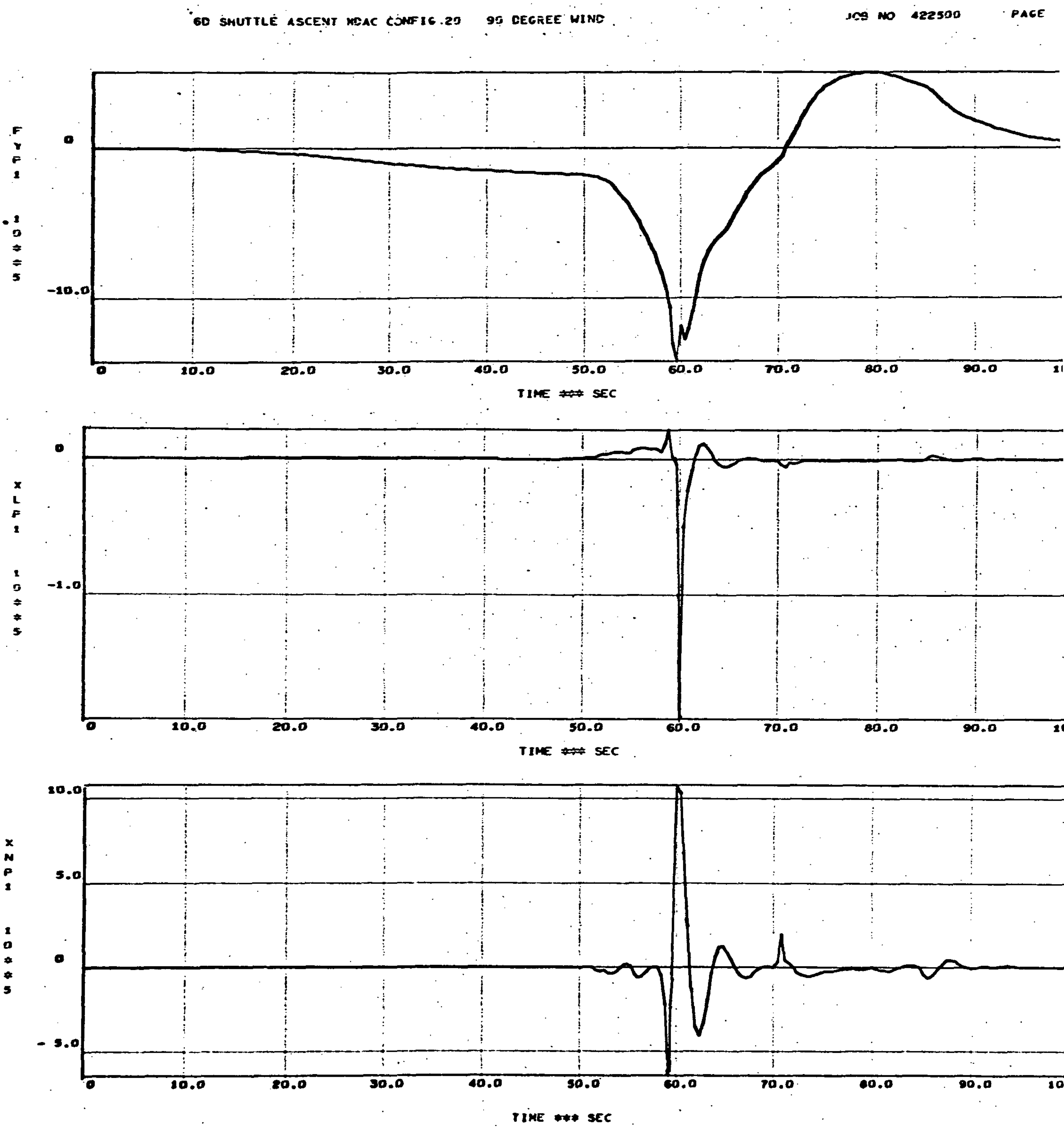
TIME *** SEC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10**5
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-1.0
0 10.0 20.0 30.0 40.0 50.0 60.0 70.0 80.0 90.0
TIME *** SEC
XNP1
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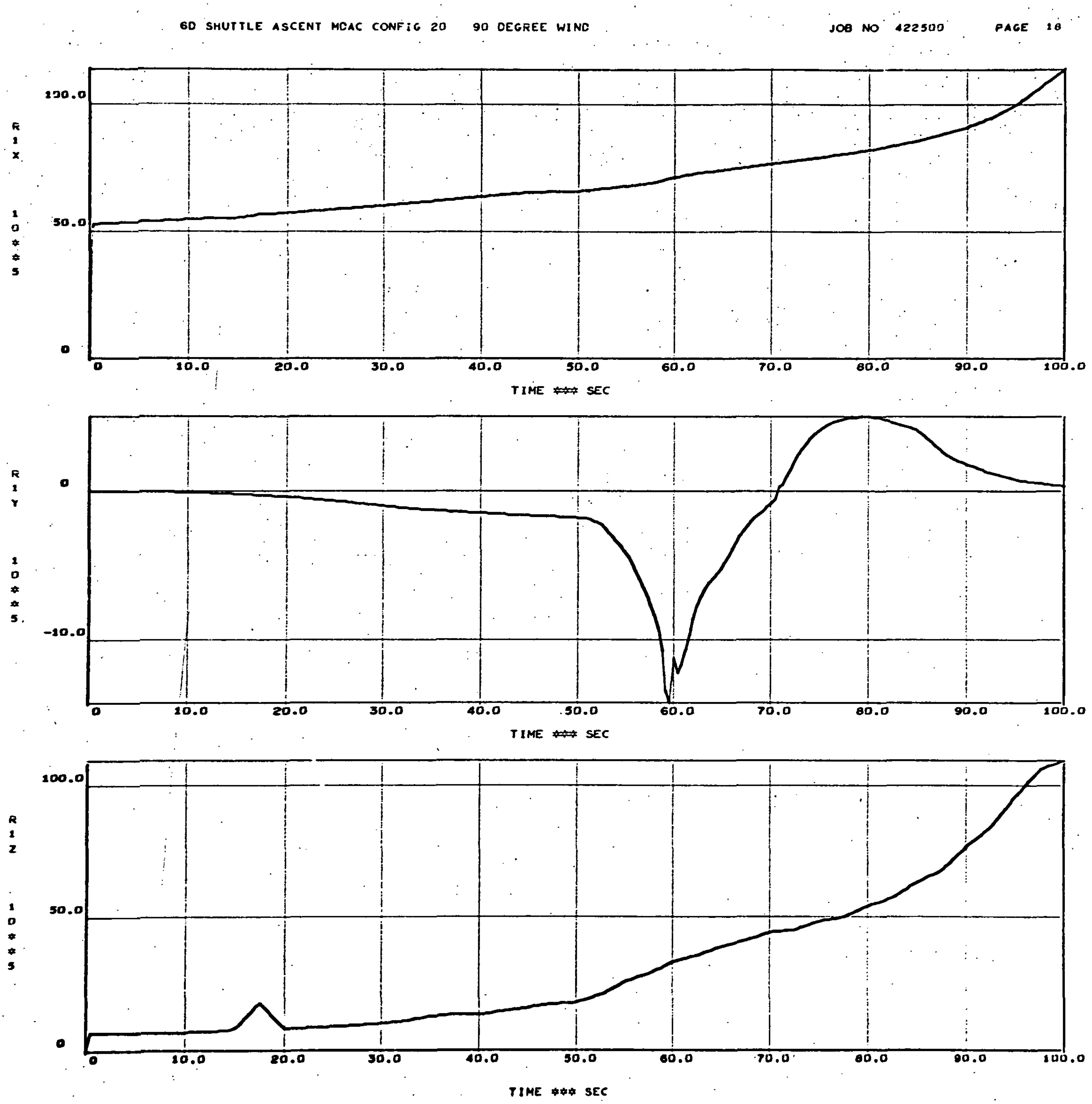
TIME *** SEC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JOB NO 422500
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R1Y 10**5
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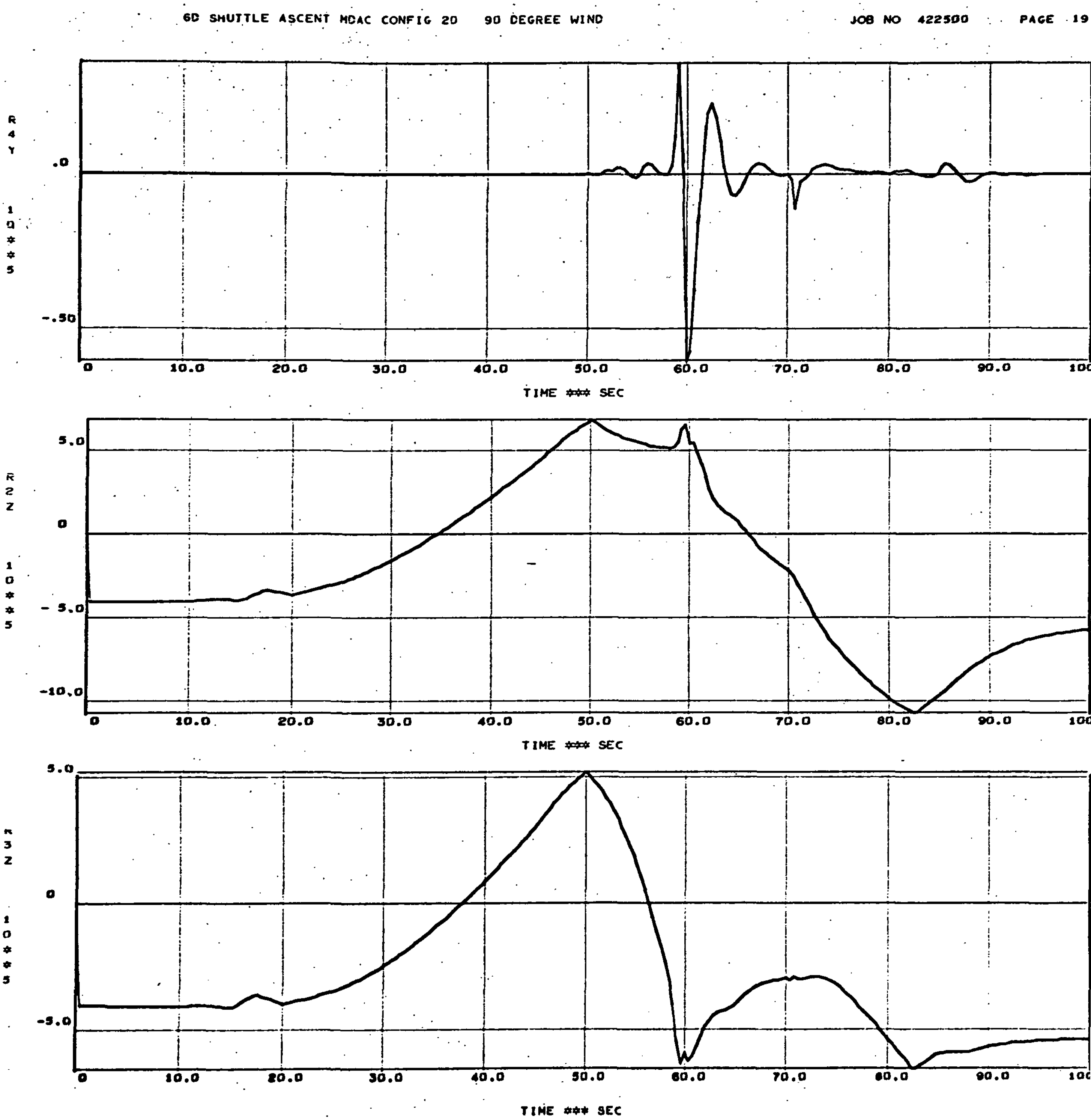
TIME *** SEC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10**5
-.50
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80.0
90.0
TIME *** SEC
R2Z
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TIME *** SEC
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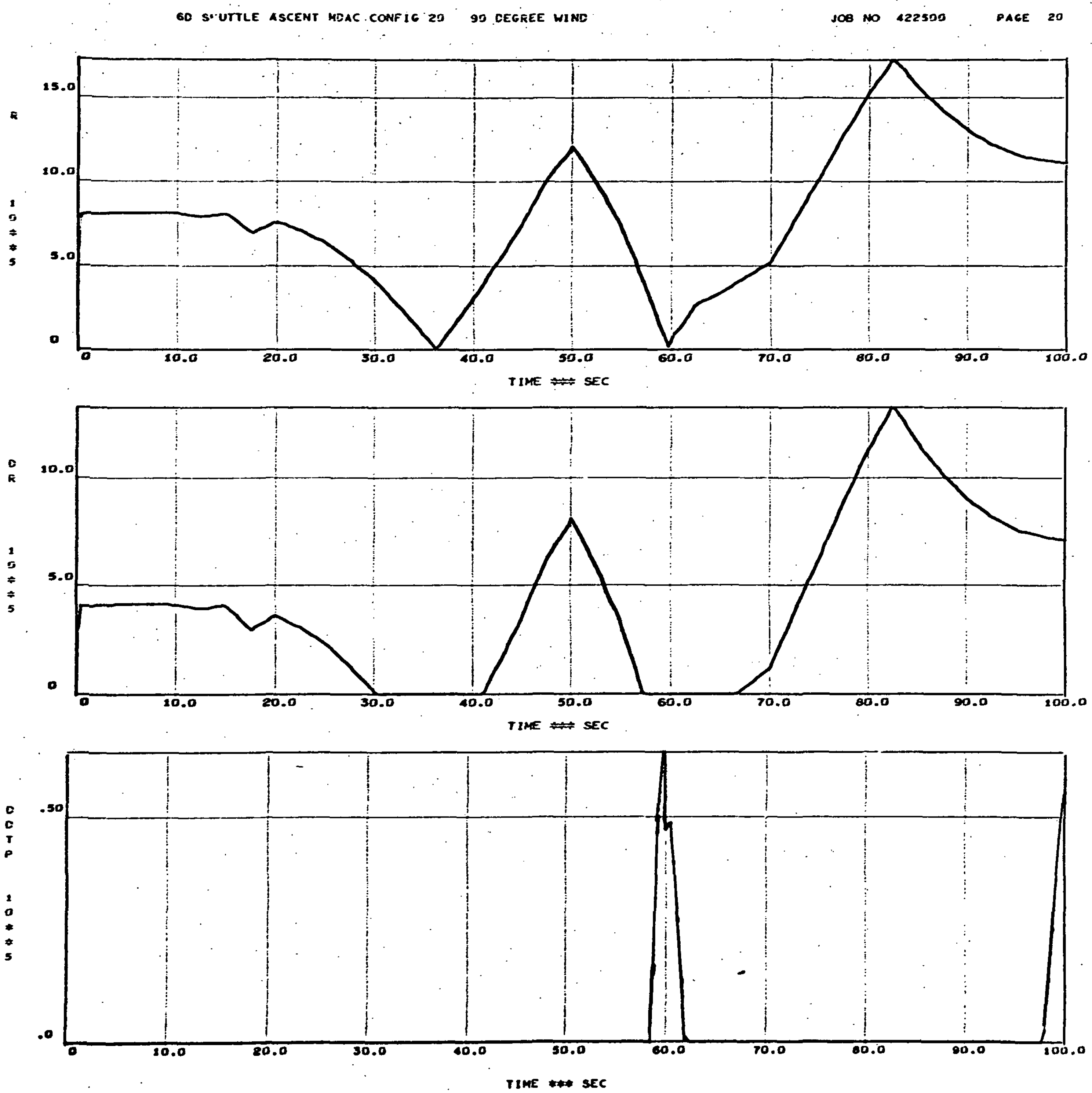
TIME *** SEC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20.0
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40.0
50.0
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70.0
80.0
90.0
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TIME *** SEC
DR
10**5
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CCTP
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.50
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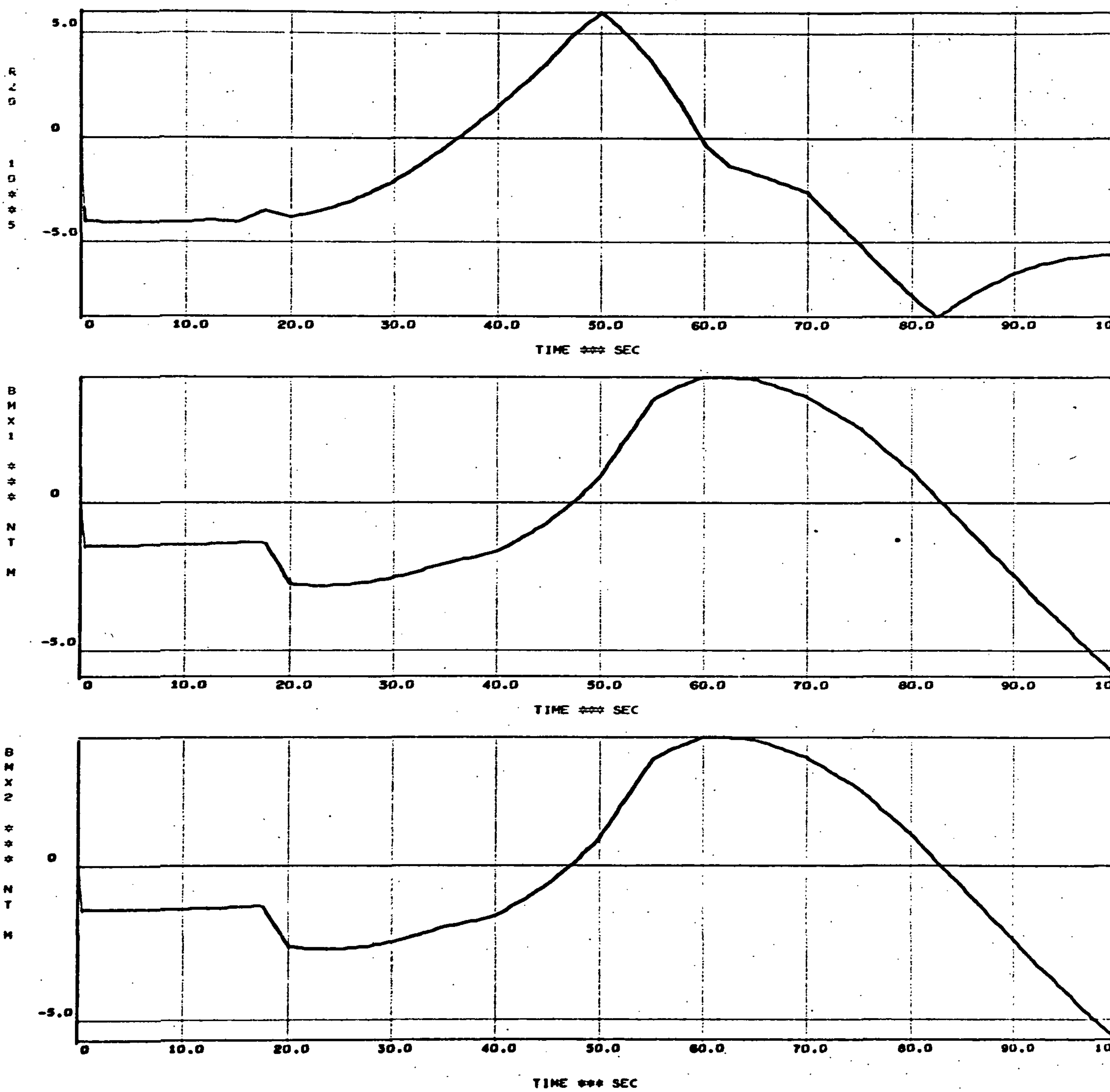
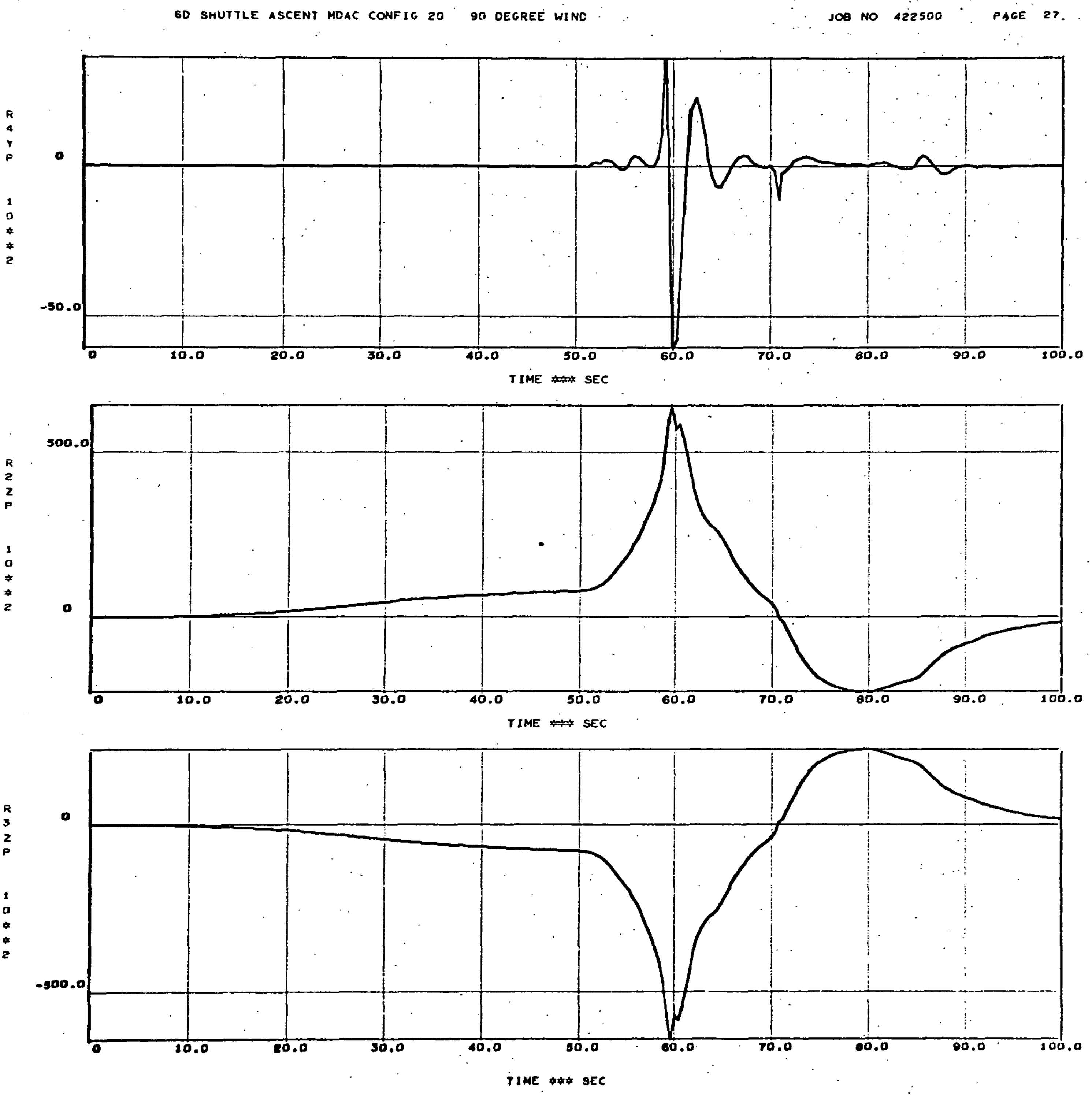
TIME *** SEC

6D SHUTTLE ASCENT MDAC CONFIG 20 90 DEGREE WIND JOB NO. 422500 PAGE 2

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30.0
40.0
50.0
60.0
70.0
80.0
90.0
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TIME *** SEC
R3ZP 10**2
0
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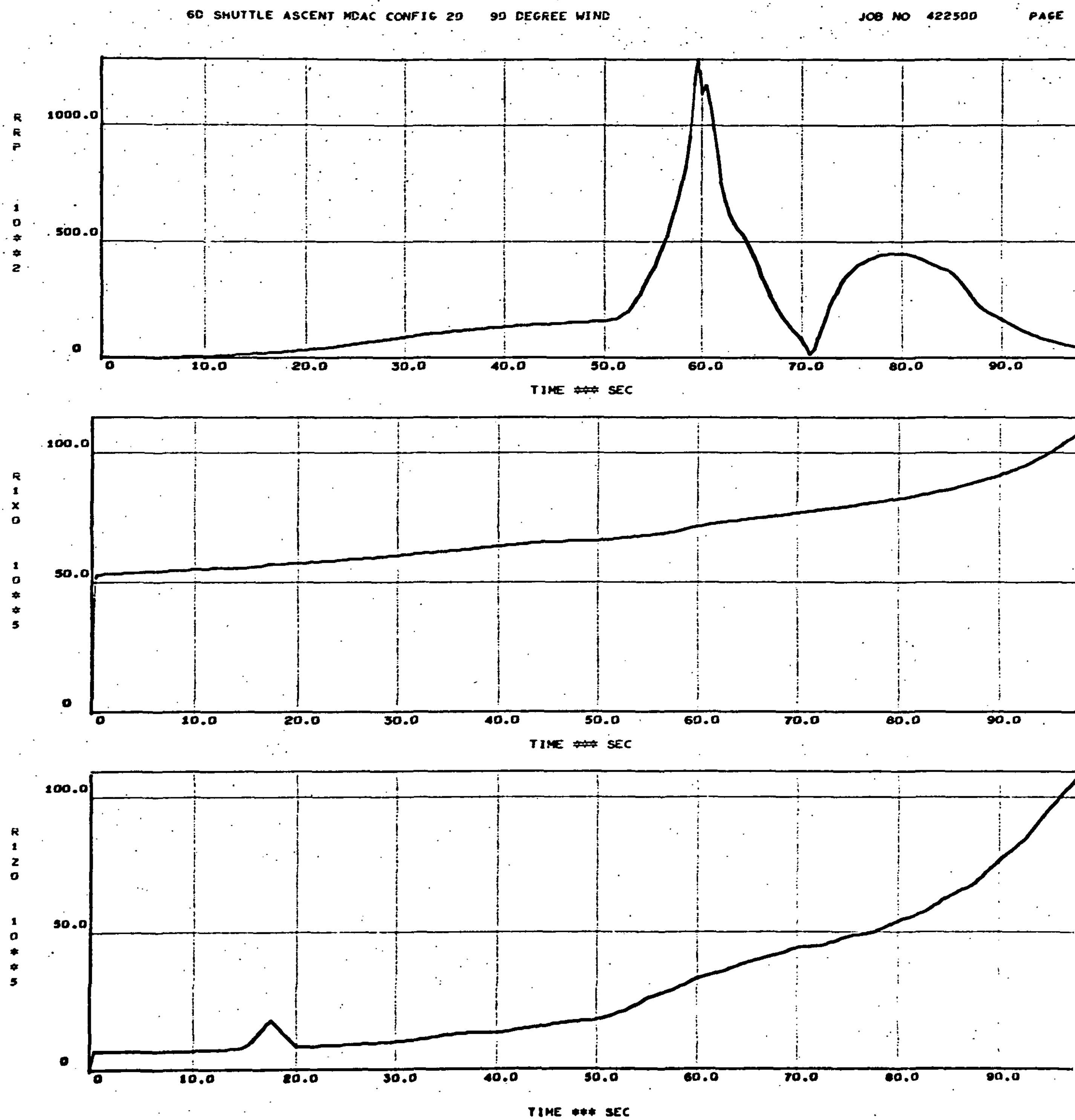
TIME *** SEC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10.0
20.0
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70.0
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TIME *** SEC
R1ZO 10**5
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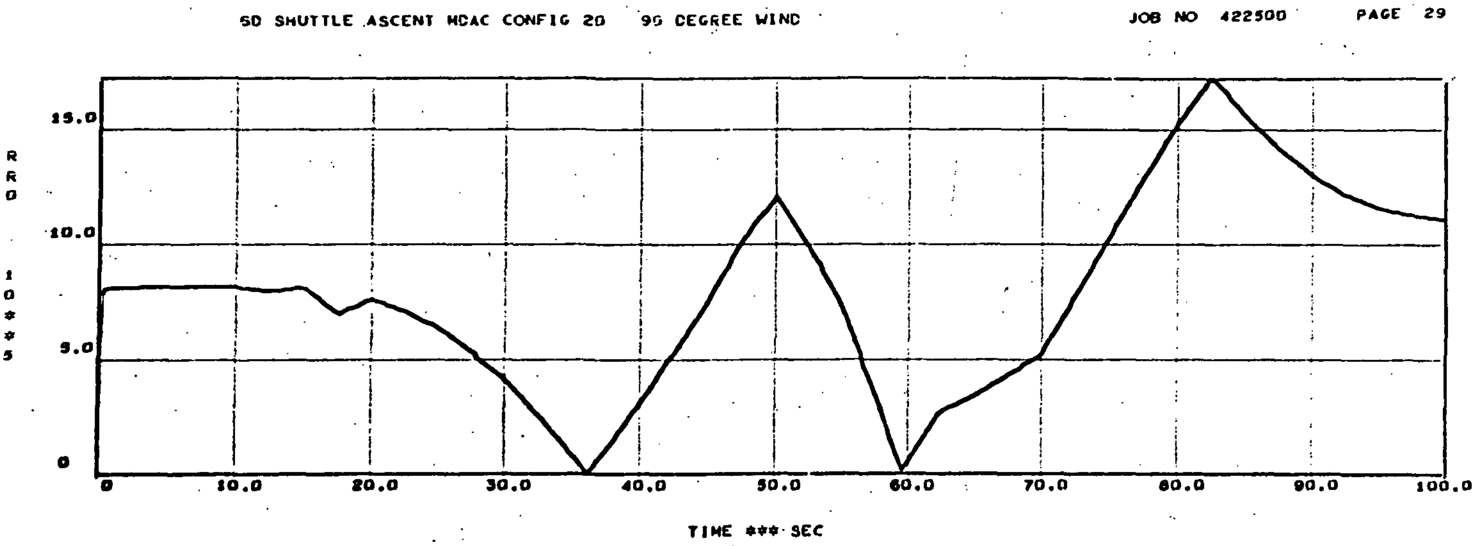
TIME *** SEC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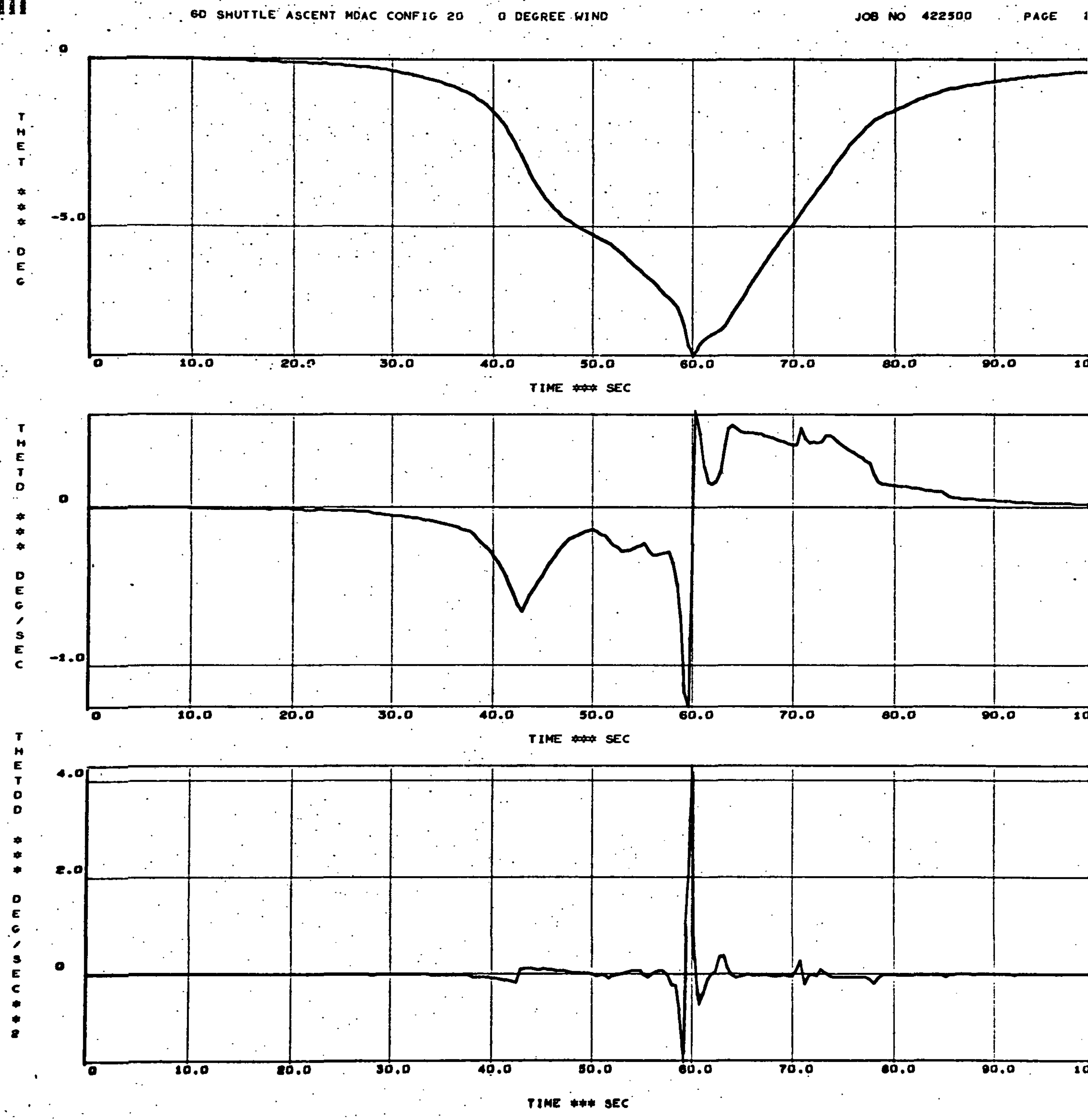
5.0
0
10.0
20.0
30.0
40.0
50.0
60.0
70.0
80.0
90.0
100.0
TIME *** SEC

Appendix G

SC 4020 PLOTS OF RUNGE KUTTA INTEGRATED SHUTTLE ASCENT TRAJECTORY SHOWING ALL STATE VARIABLES AND INTERFACE LOADING FOR 0° HEADWIND α_{wA} AND 90° SIDEWIND β_{wA} (MDAC-CONFIGURATION 20) FOR OPTIMAL CONTROLLER

Appendix G

Response to headwind α_{w_A} for optimal pitch controller is shown on pages G-2 through G-17. Response to sidewind β_{w_A} for optimal yaw/roll controller is shown on pages G-18 through G-36.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JOB NO 422500
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-5.0
0
10.0
30.0
40.0
50.0
60.0
70.0
80.0
90.0
TIME *** SEC
THETD *** DEG/SEC
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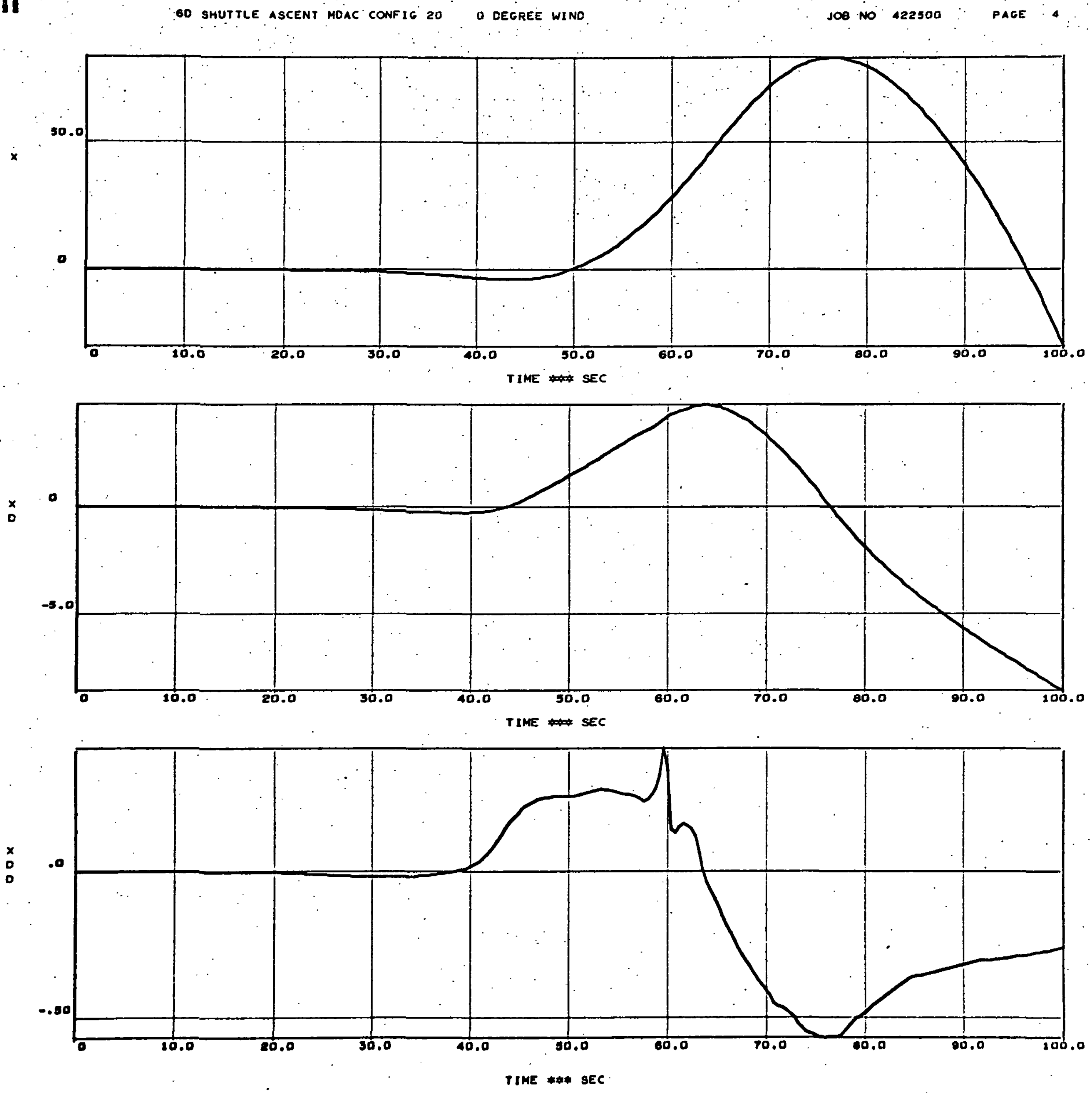
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100.0
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D
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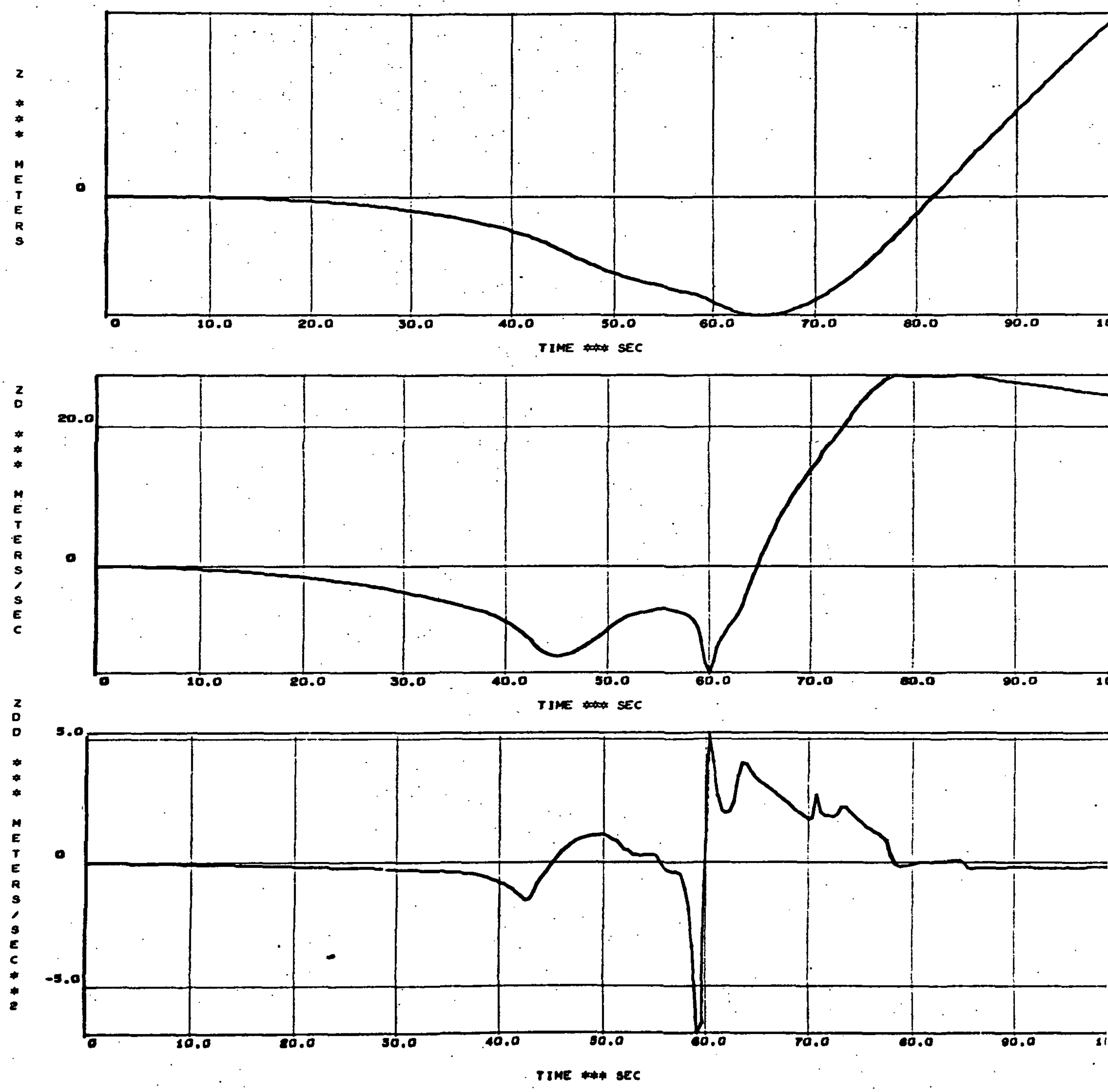
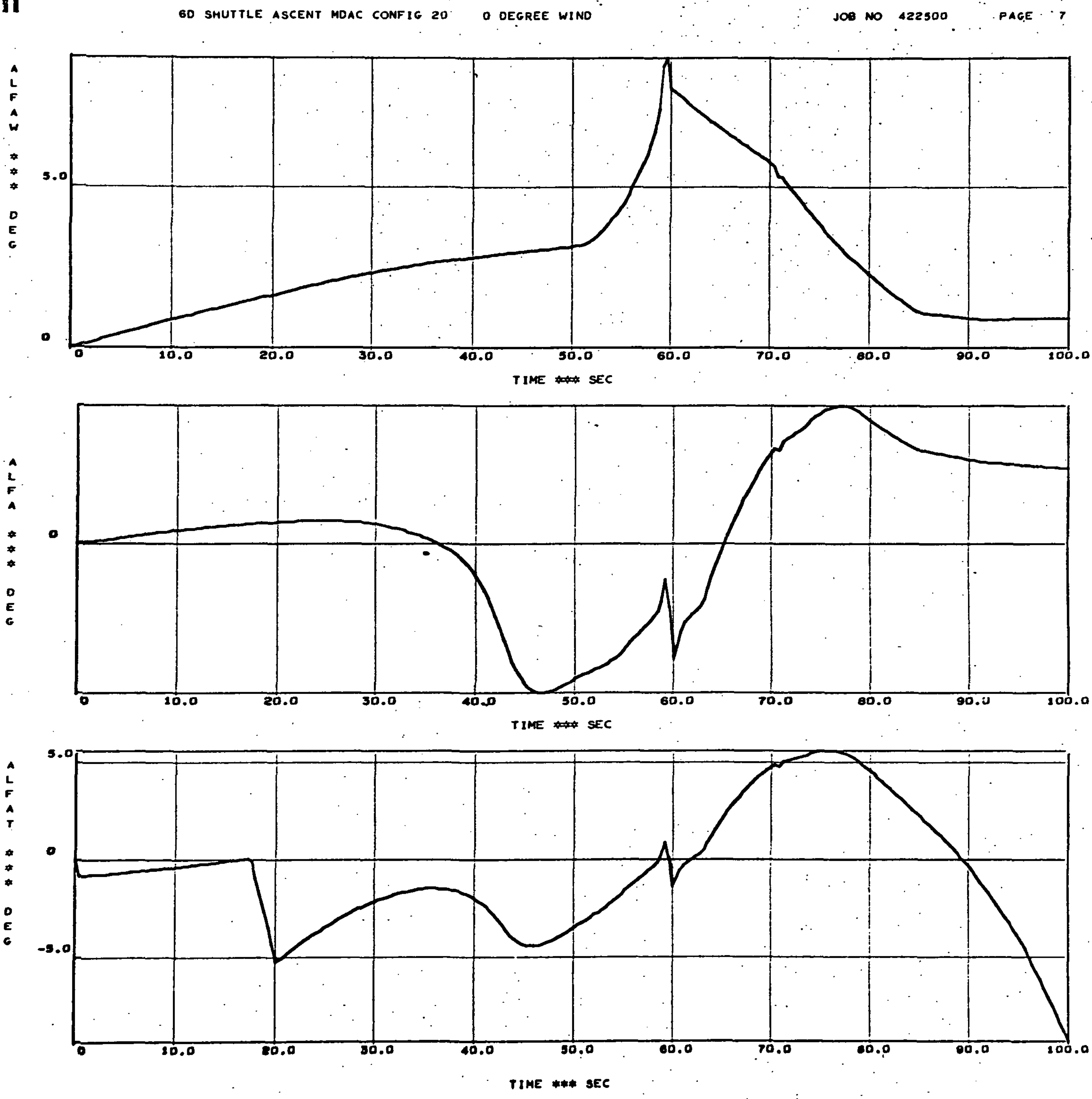
TIME *** SEC

6D SHUTTLE ASCENT MDAC CONFIG 20   0 DEGREE WIND   JOB NO 422500   PAGE





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TIME *** SEC
ALFA *** DEG
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0 10.0 20.0 30.0 40.0 50.0 60.0 70.0 80.0 90.0 100.0
TIME *** SEC
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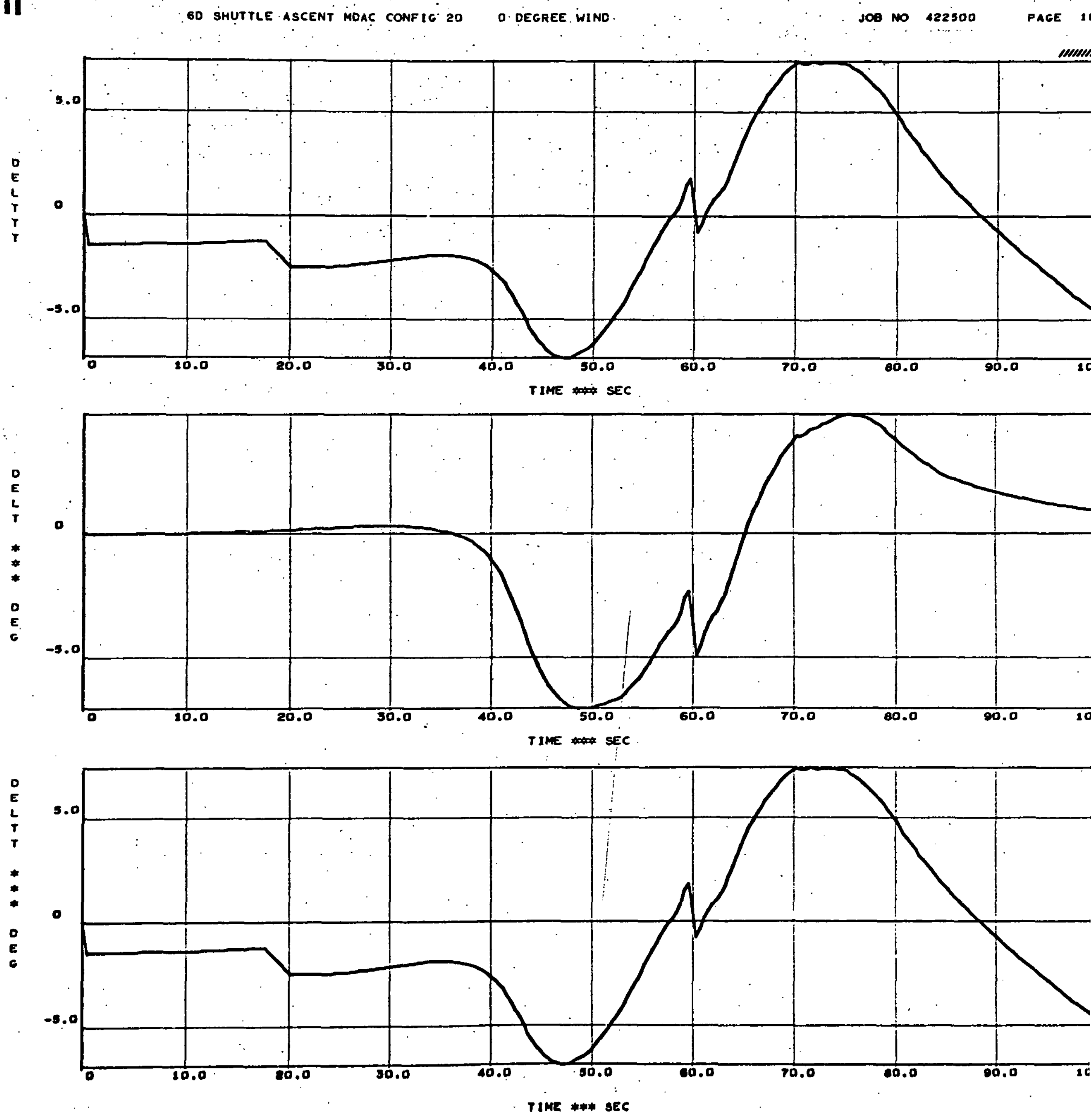
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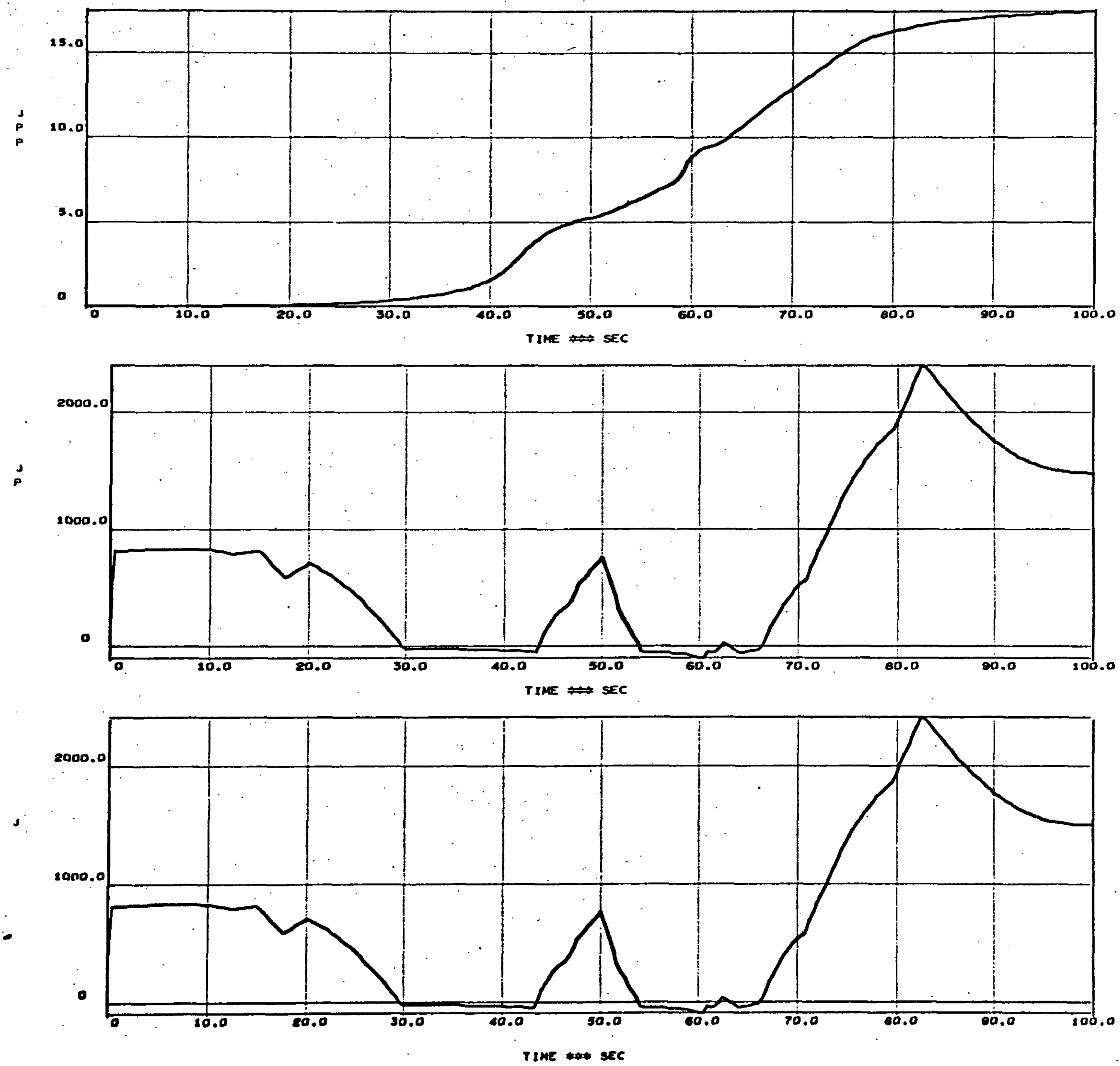
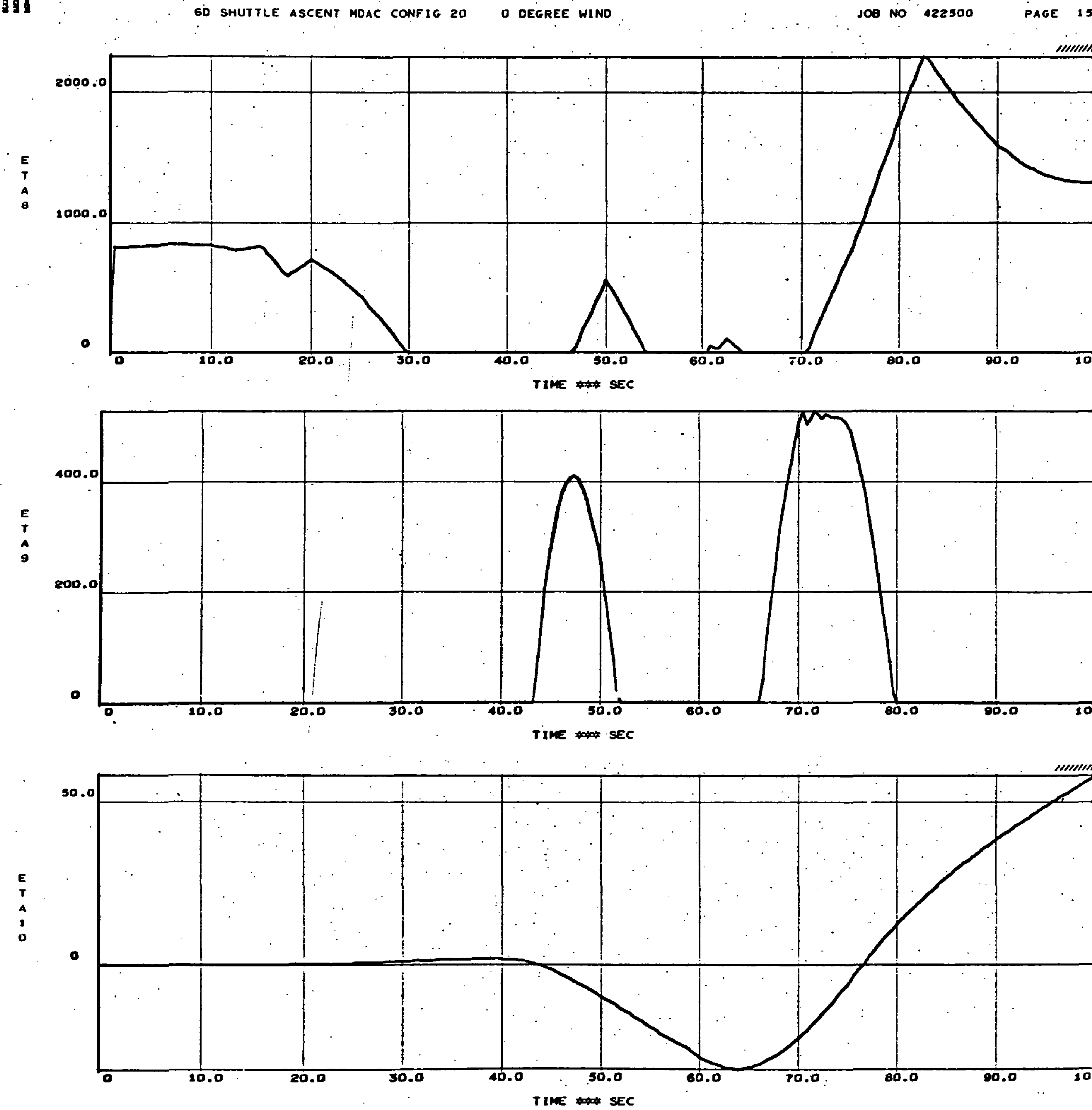
TIME *** SEC

60 SHUTTLE ASCENT MDAC CONFIG 20 0 DEGREE WIND JOB NO 422500 PAGE 12

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ETA10
50.0
0
10.0
20.0
30.0
40.0
50.0
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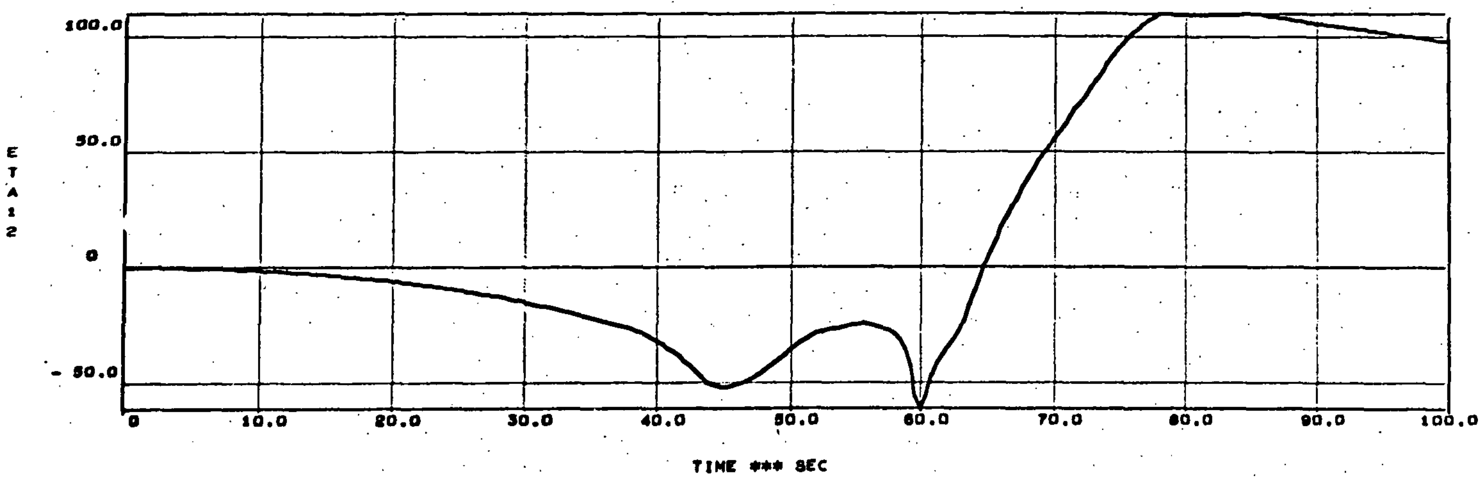
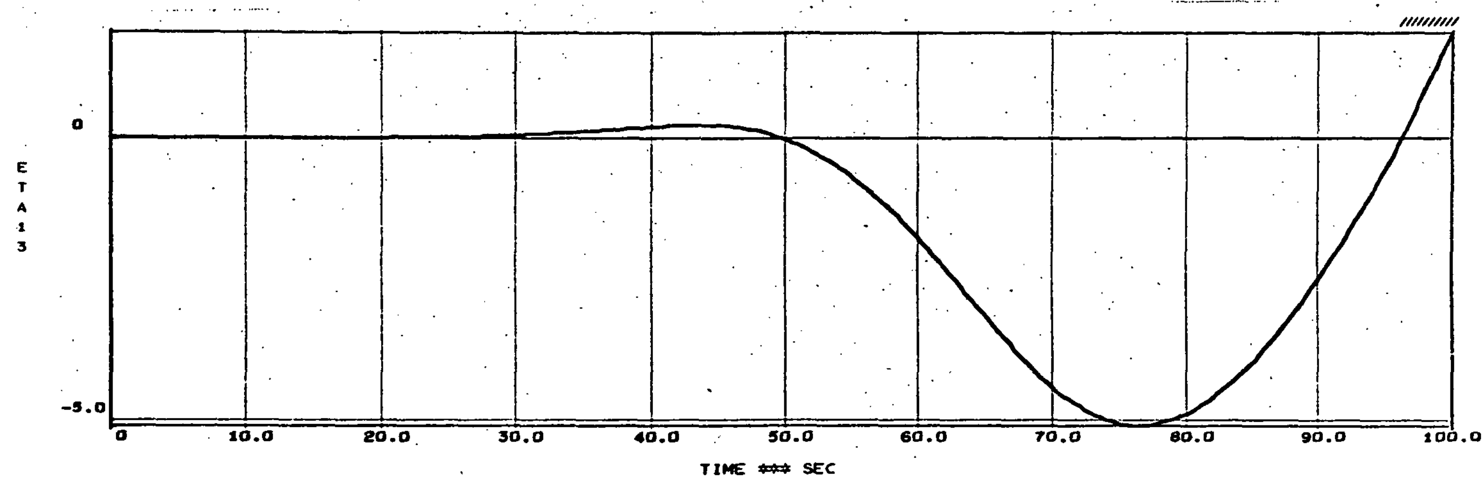
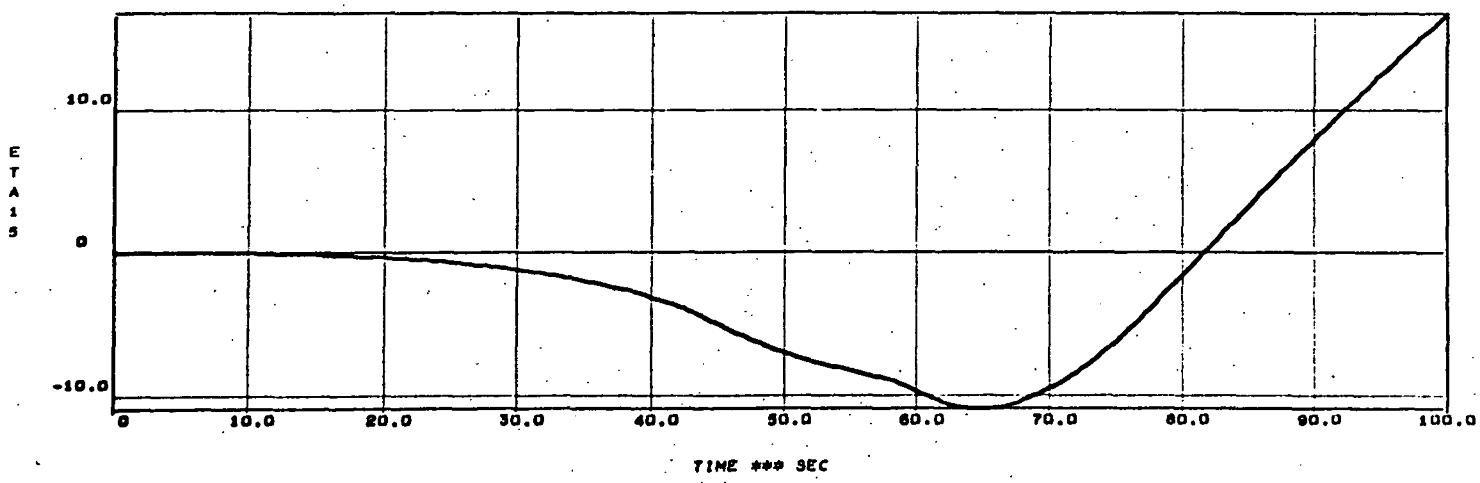
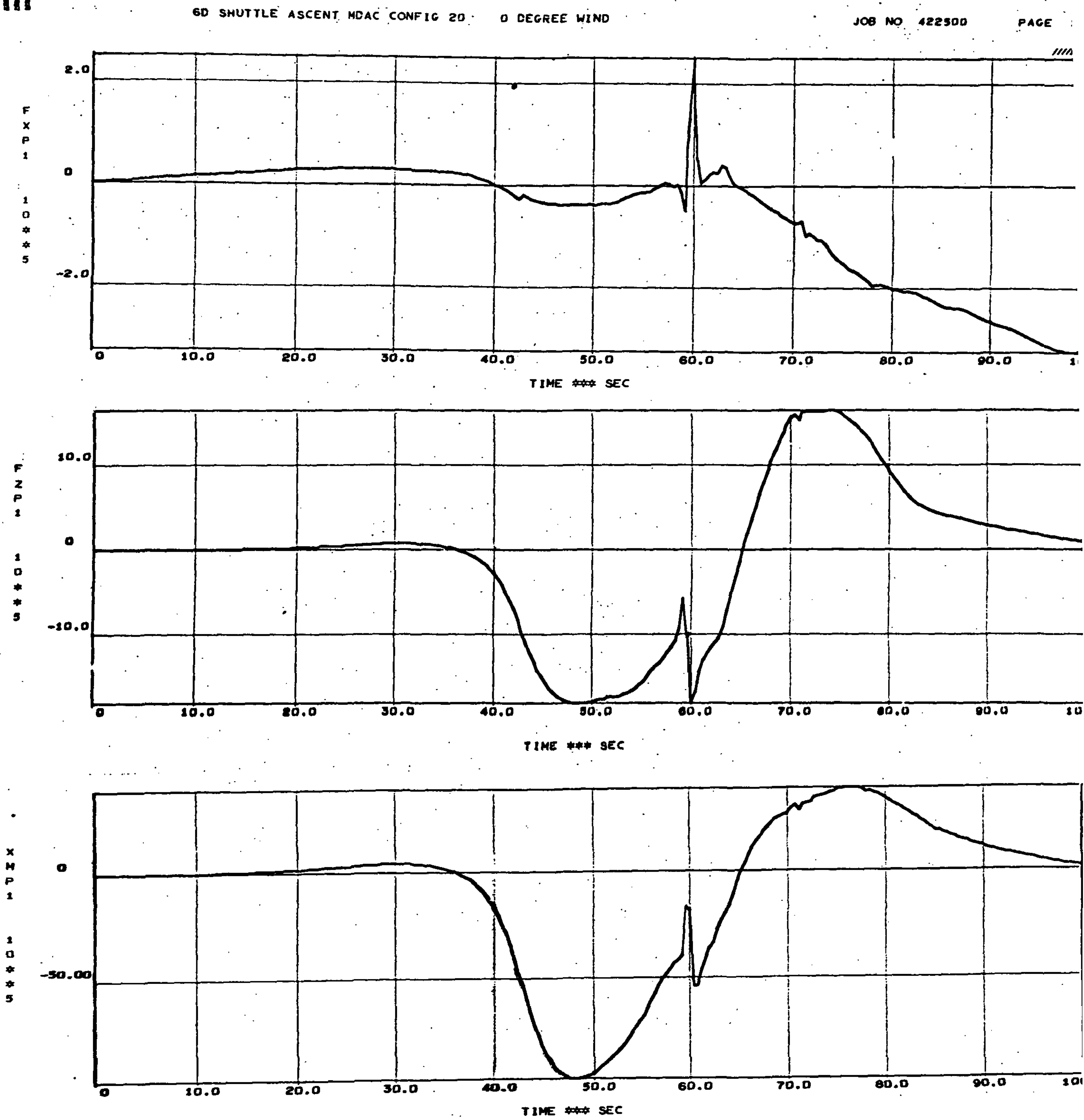
80.0
90.0
TIME *** SEC

6D SHUTTLE ASCENT MCAC CONFIG 20   0 DEGREE WIND   JOB NO 422500   PAGE 13





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TIME *** SEC
F Z P 1 10**5
10.0
0
-10.0
TIME *** SEC
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30.0
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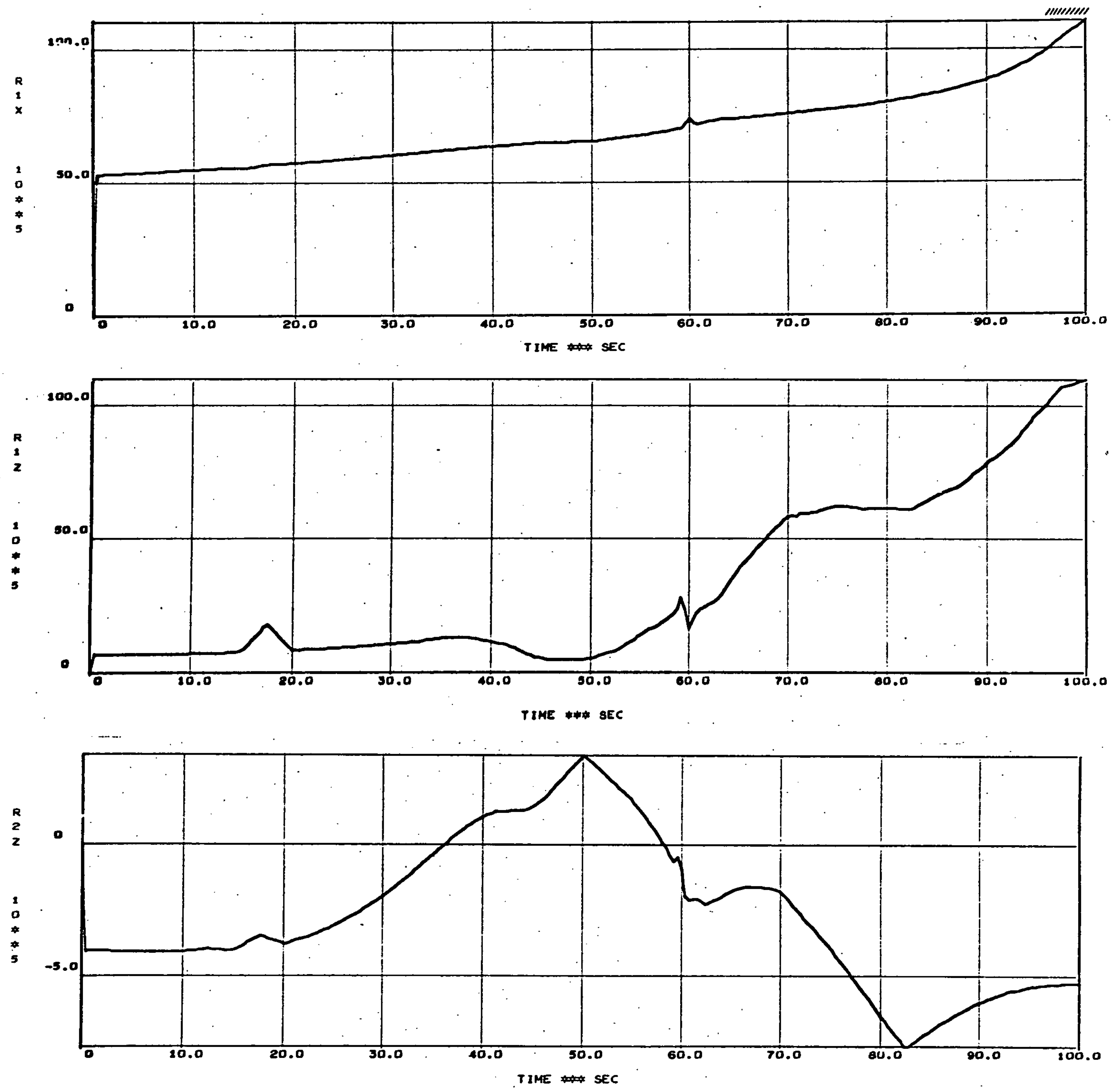
6D SHUTTLE ASCENT MDAC CONFIG 20   0 DEGREE WIND   JOB NO 422500   PAGE 18



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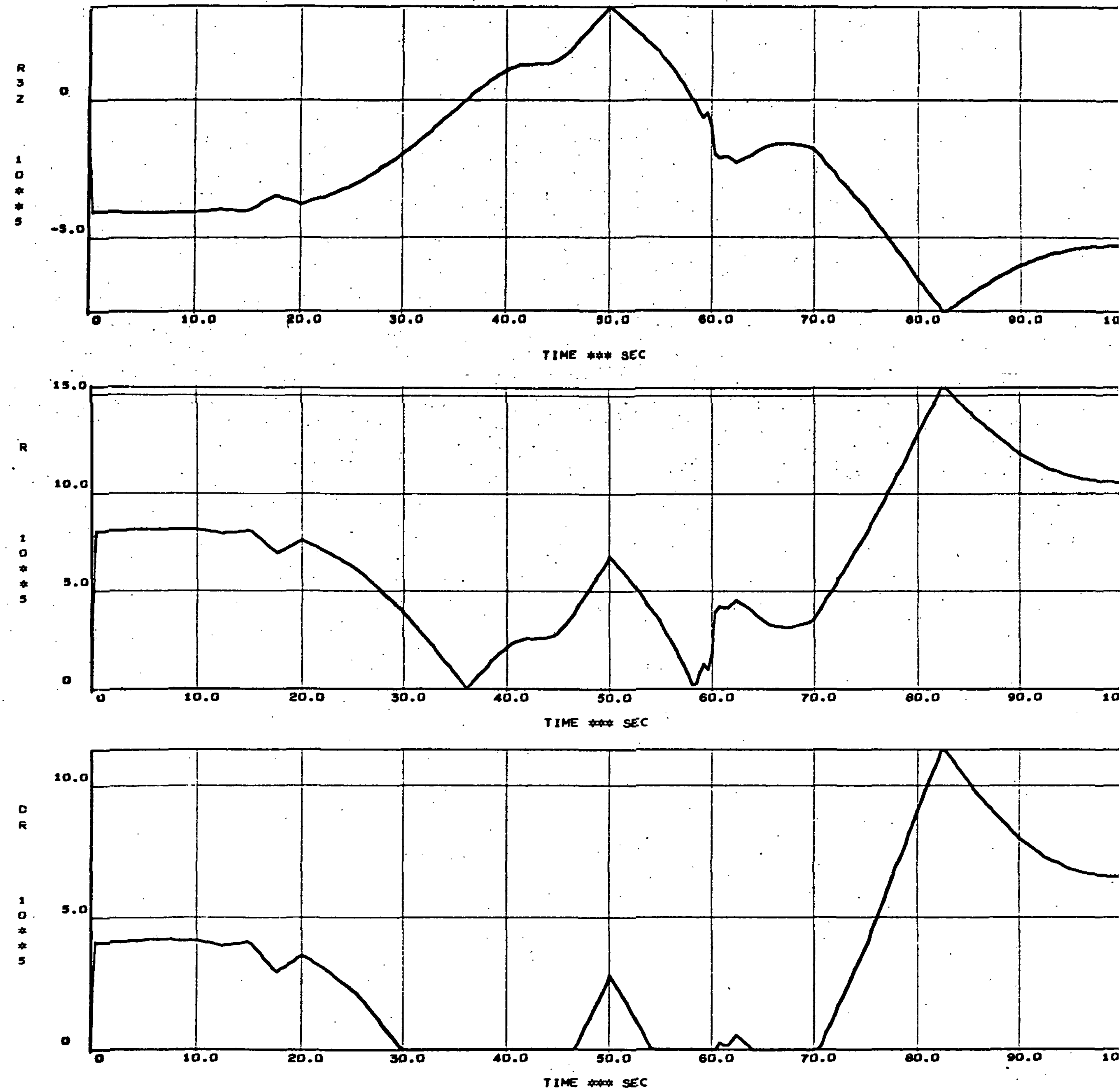

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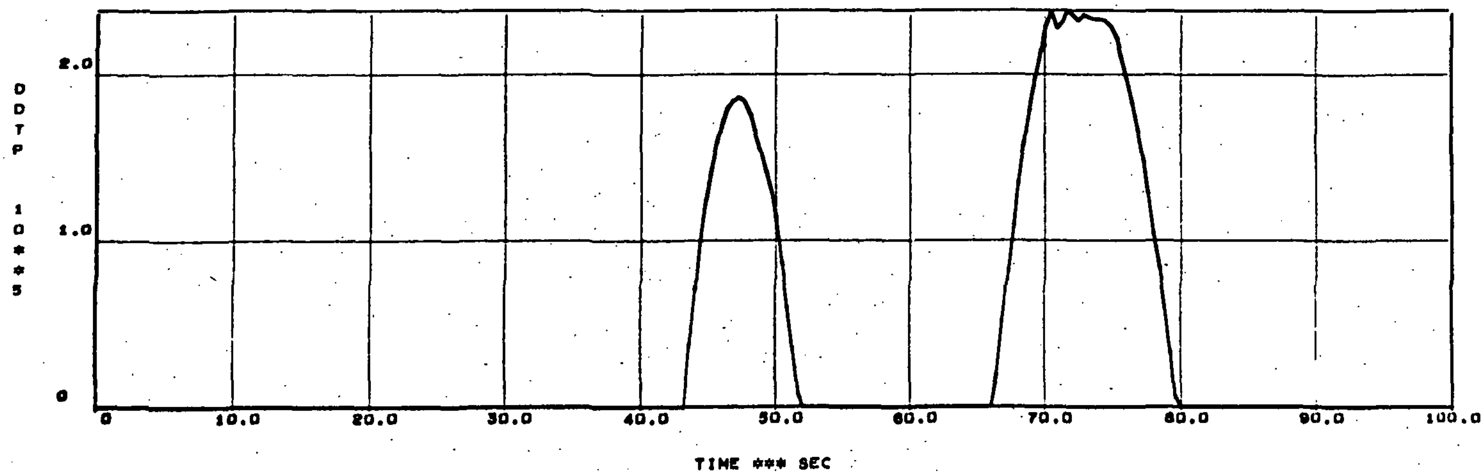

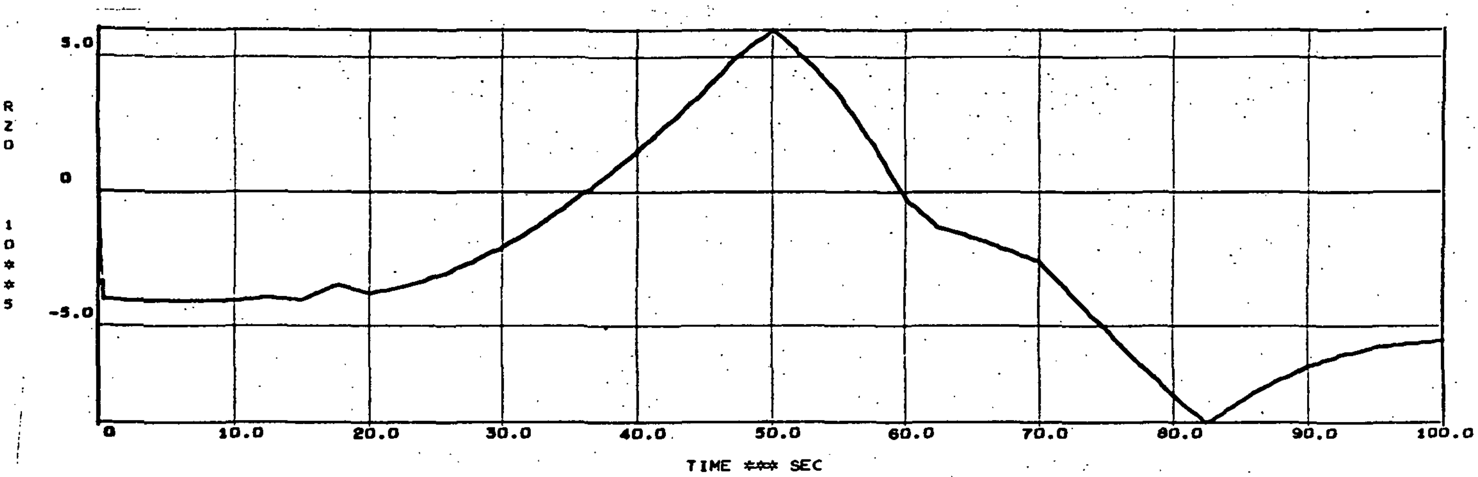

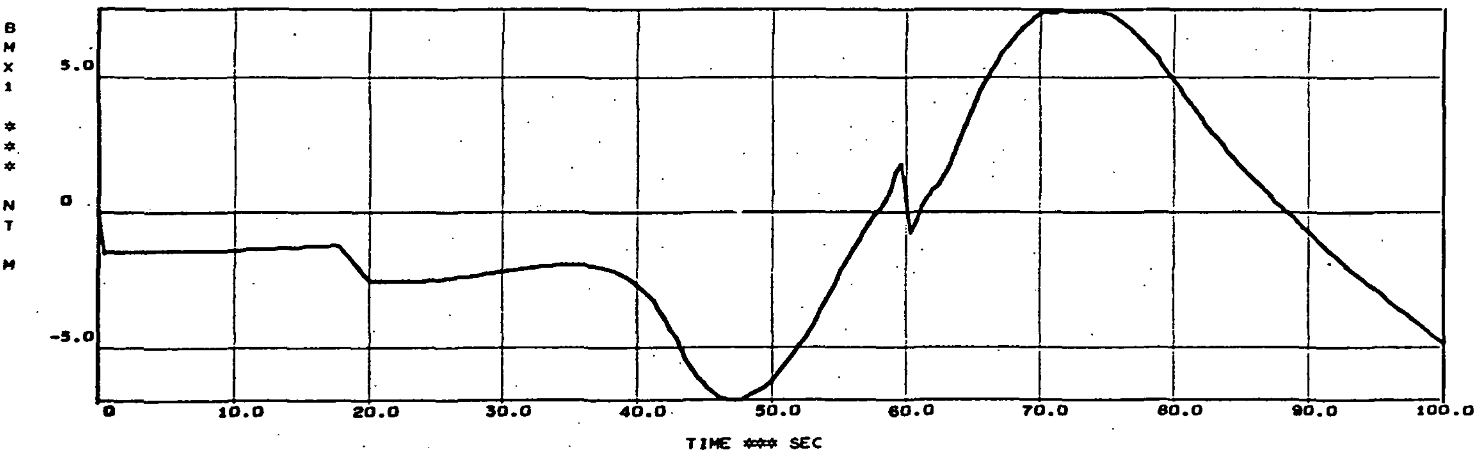

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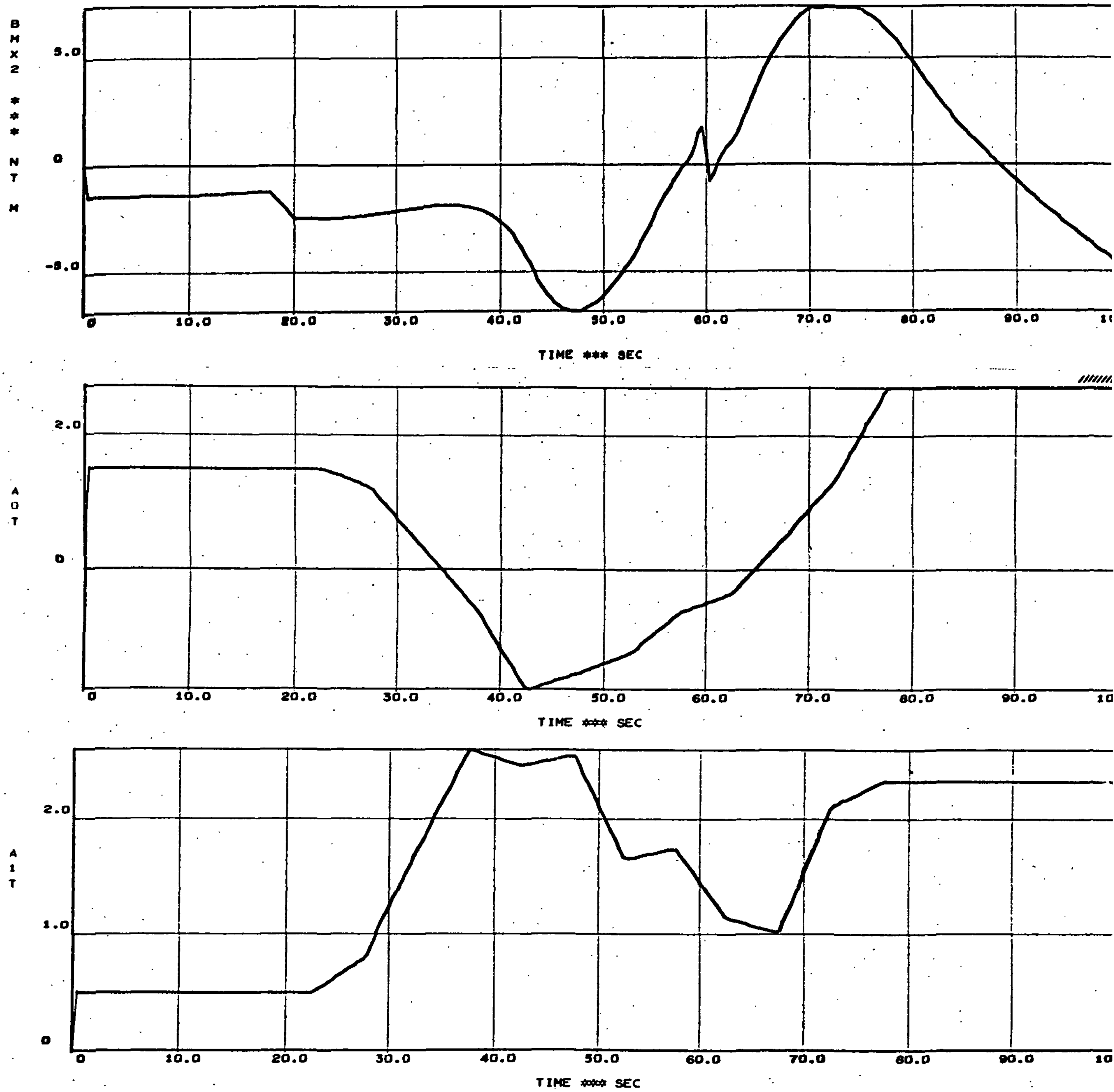
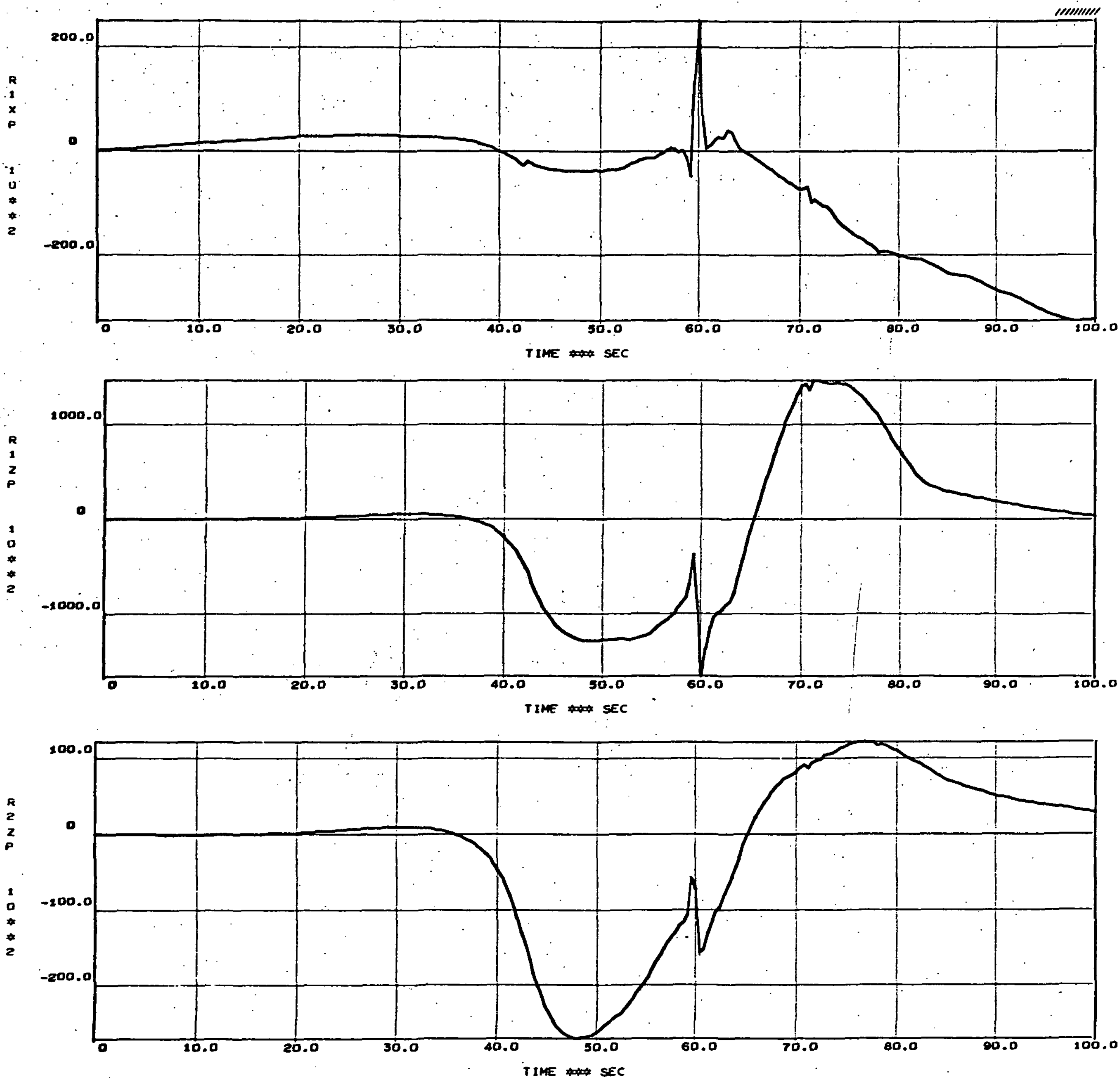
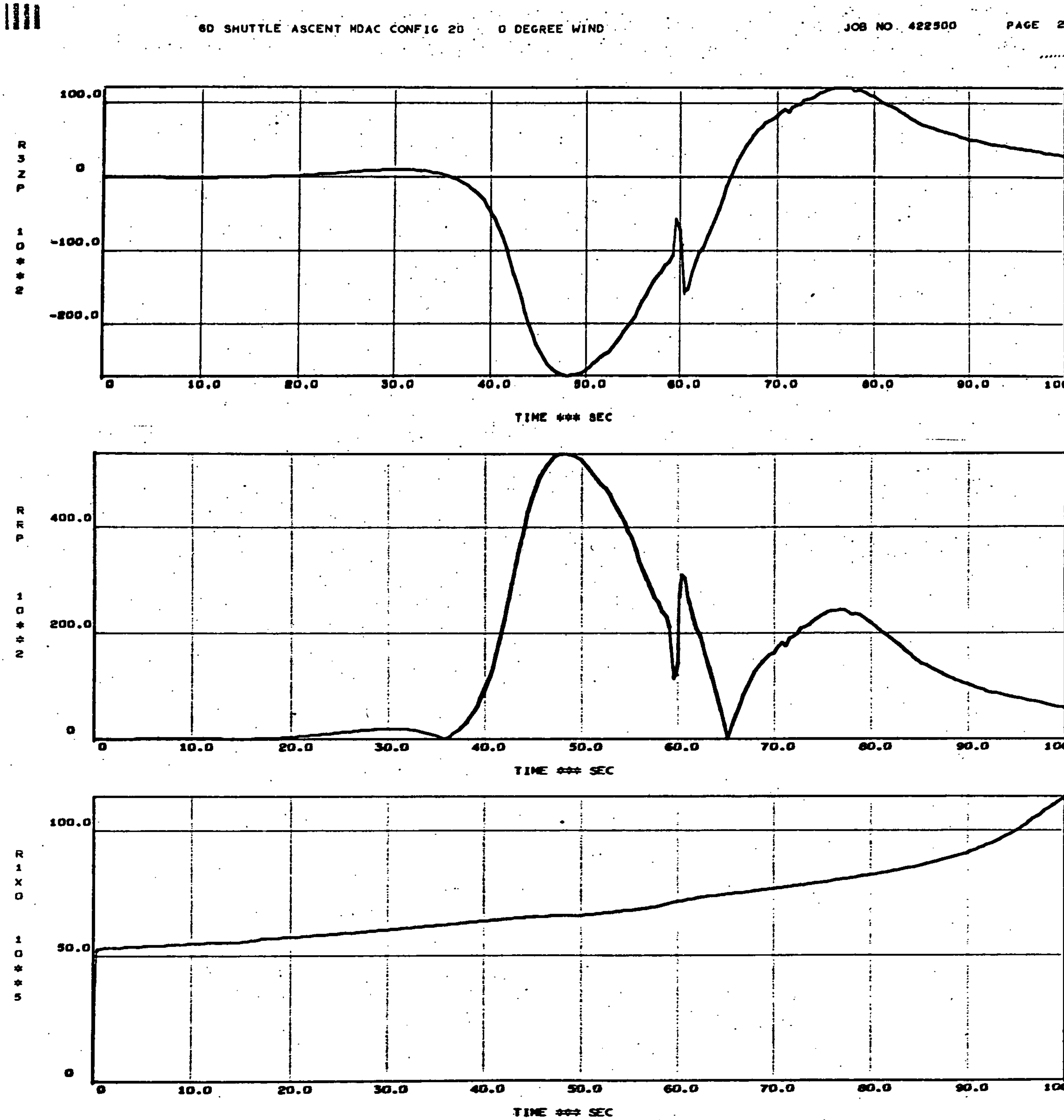


6D SHUTTLE ASCENT MDAC CONFIG 20 0 DEGREE WIND JOB NO 422500 PAGE 26

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TIME *** SEC
RRP 10**2
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200.0
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TIME *** SEC
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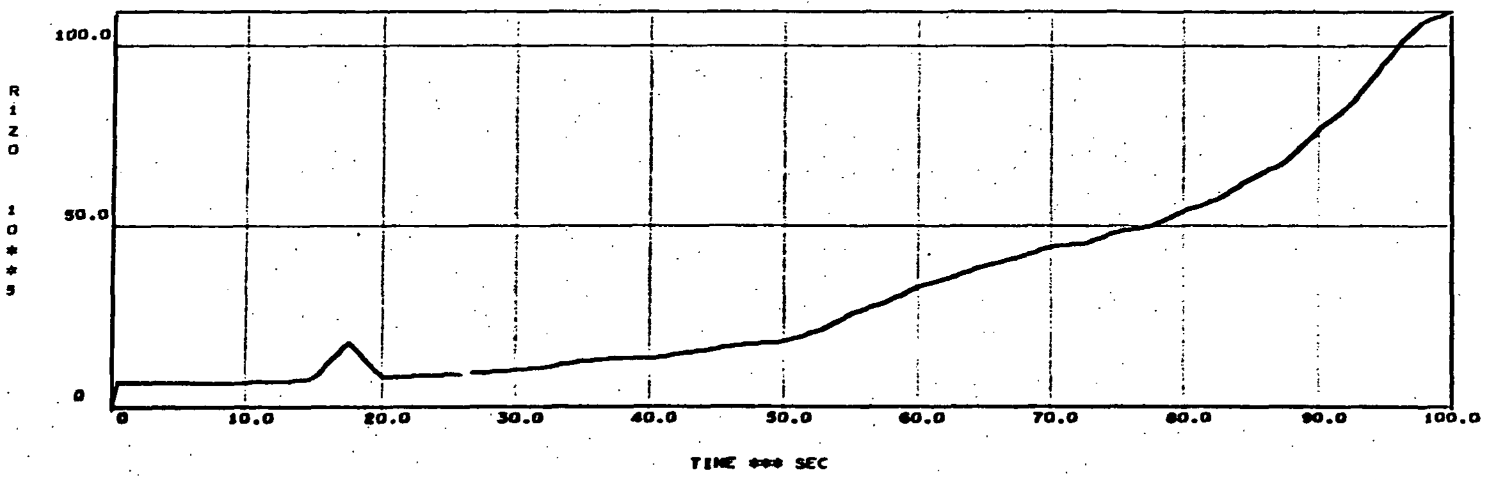
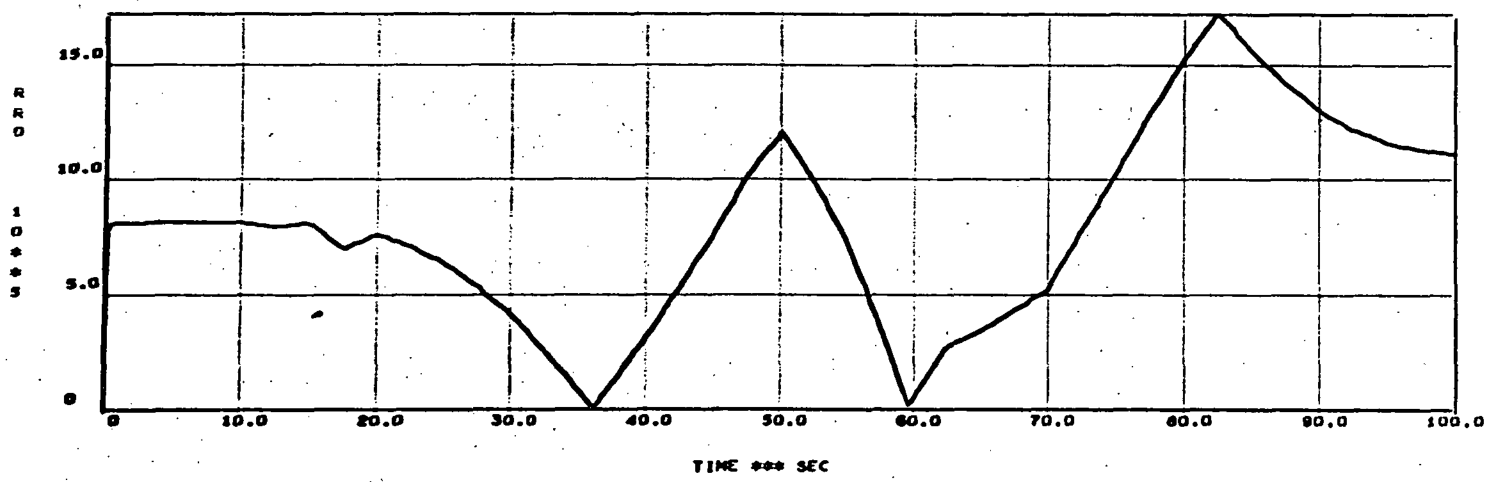
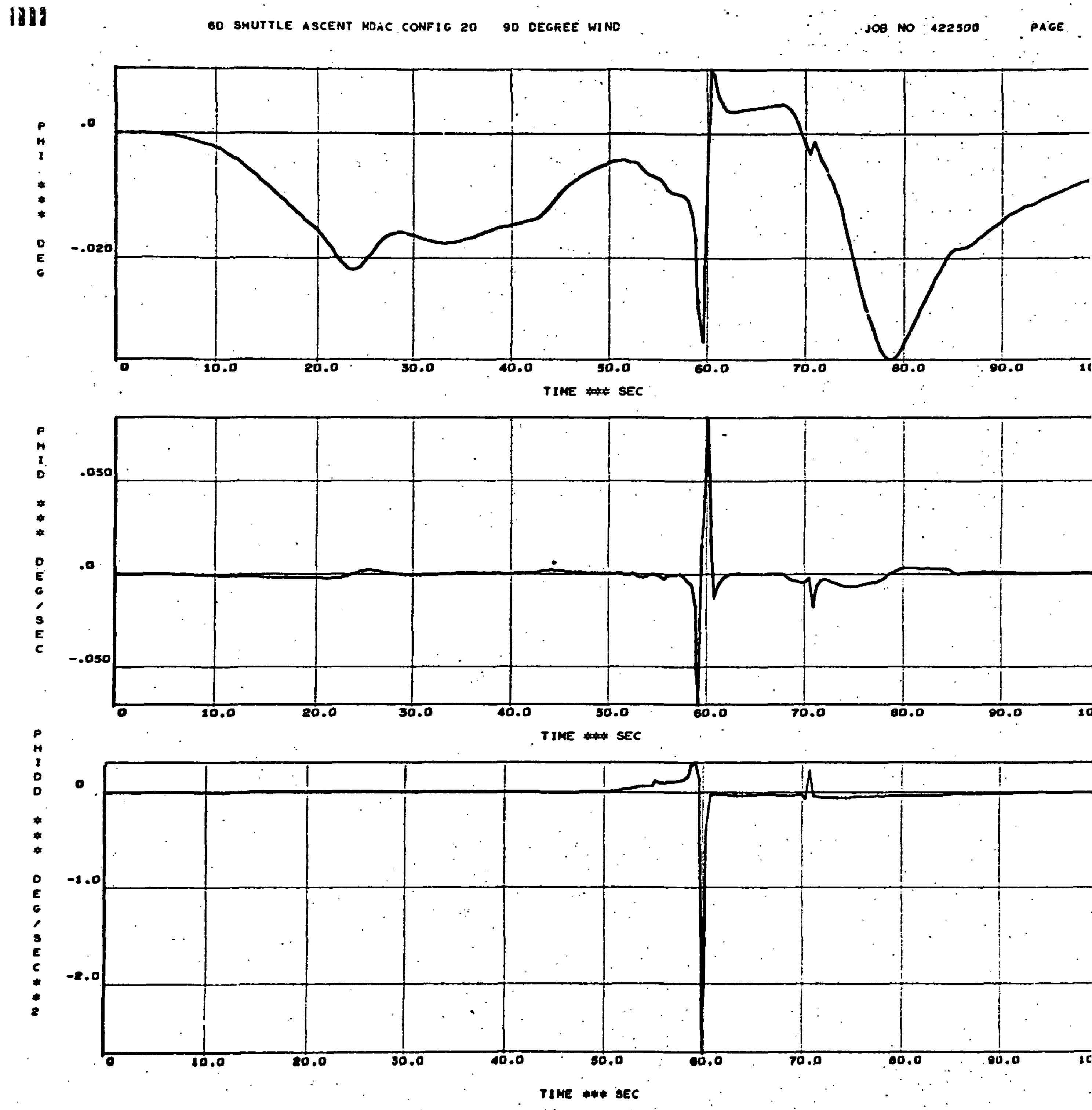
TIME *** SEC

60 SHUTTLE ASCENT MDAC CONFIG 29   0 DEGREE WIND   JOB NO 422500   PAGE 28



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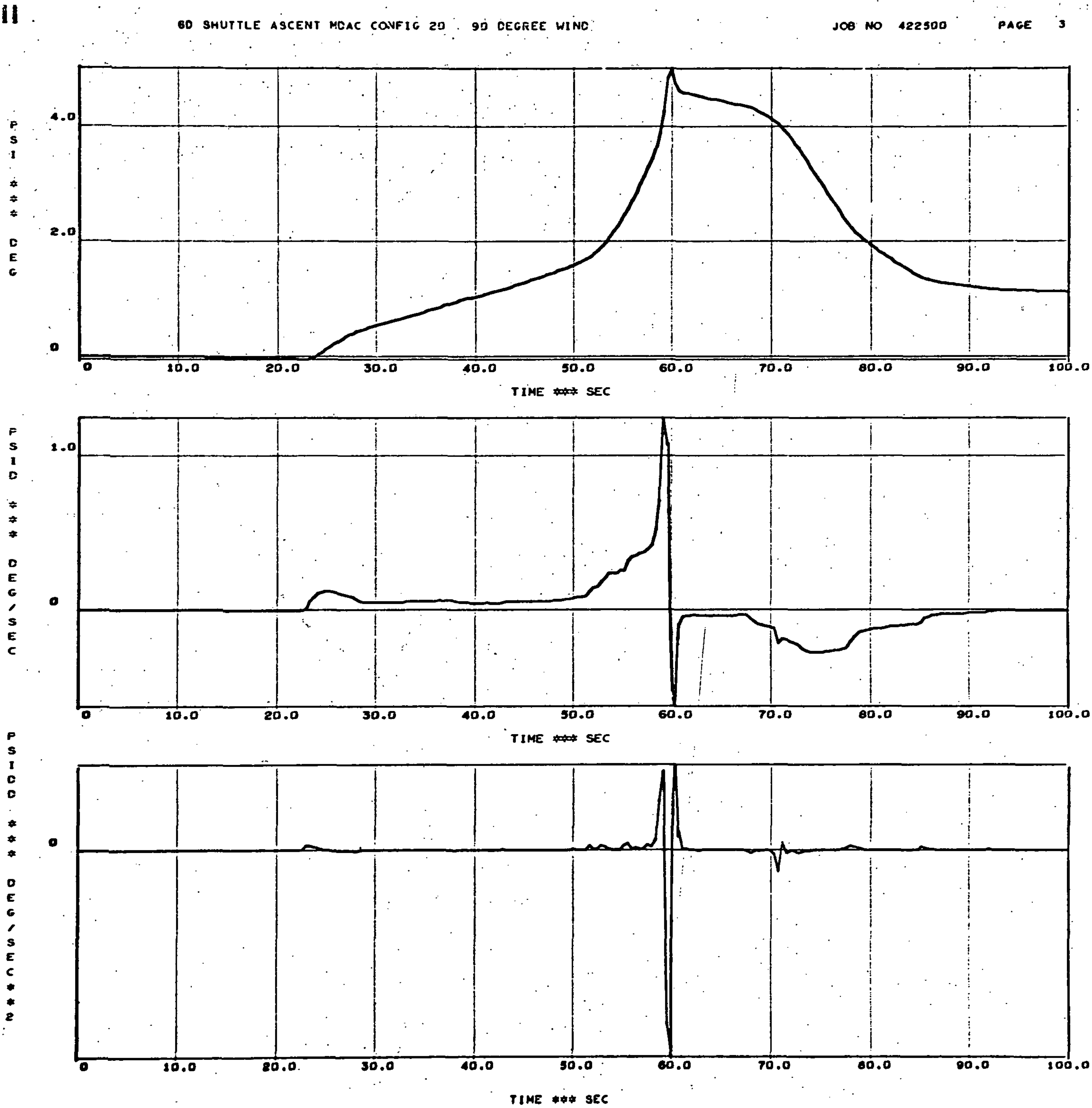

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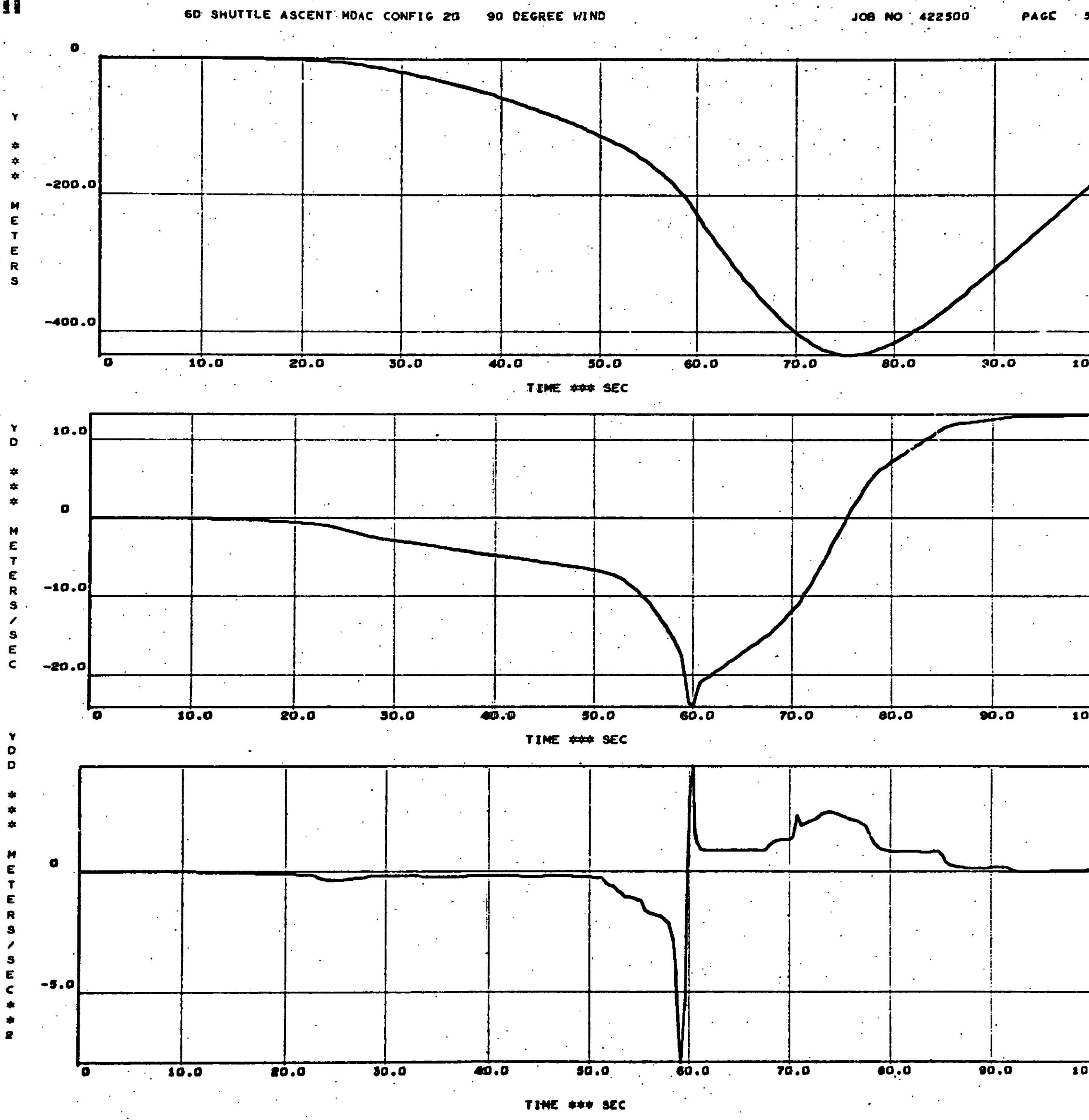
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Y D *** METERS/SEC
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-10.0
-20.0
Y D D *** METERS/SEC**2
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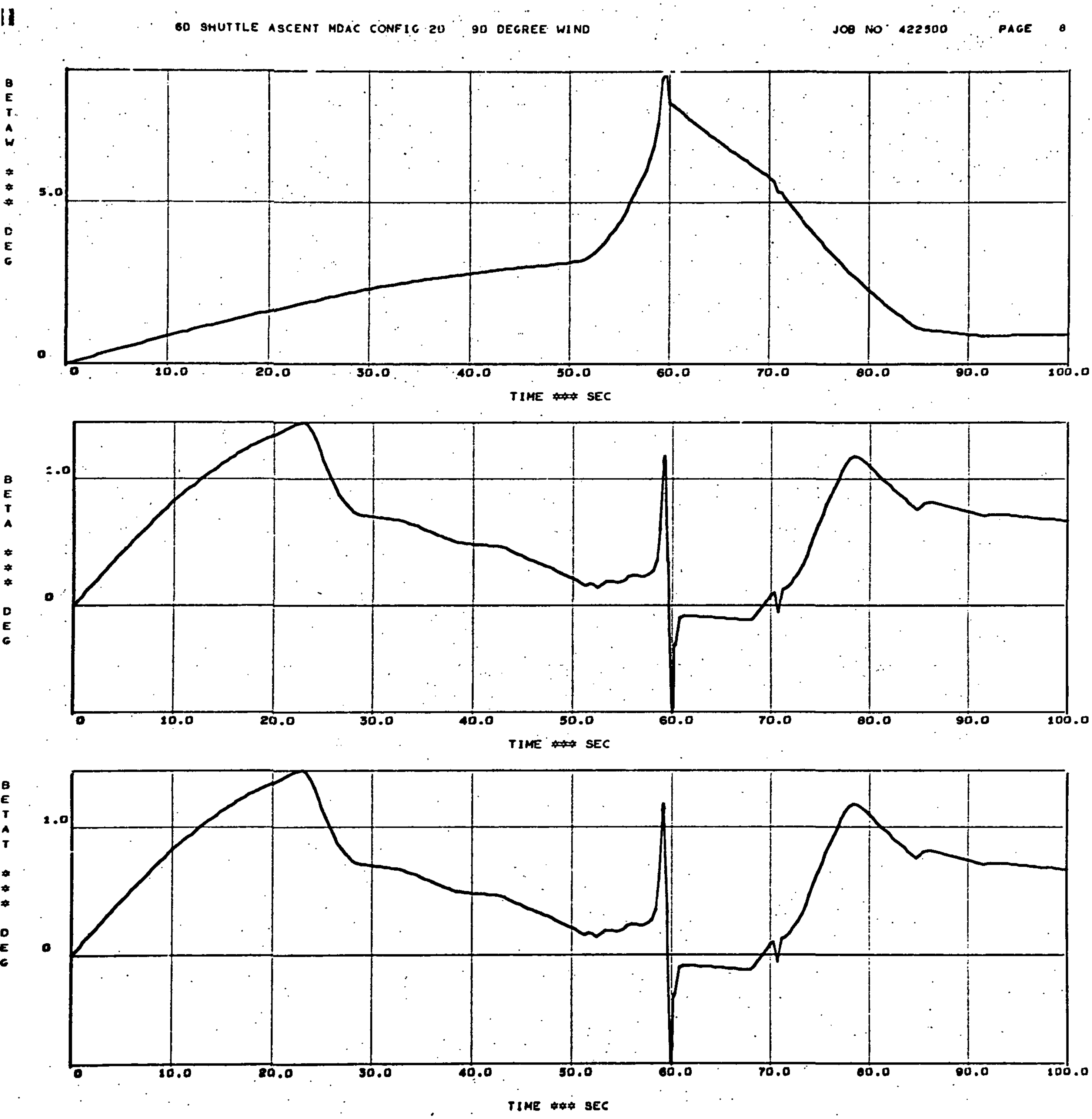
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TIME *** SEC
BETA *** DEG
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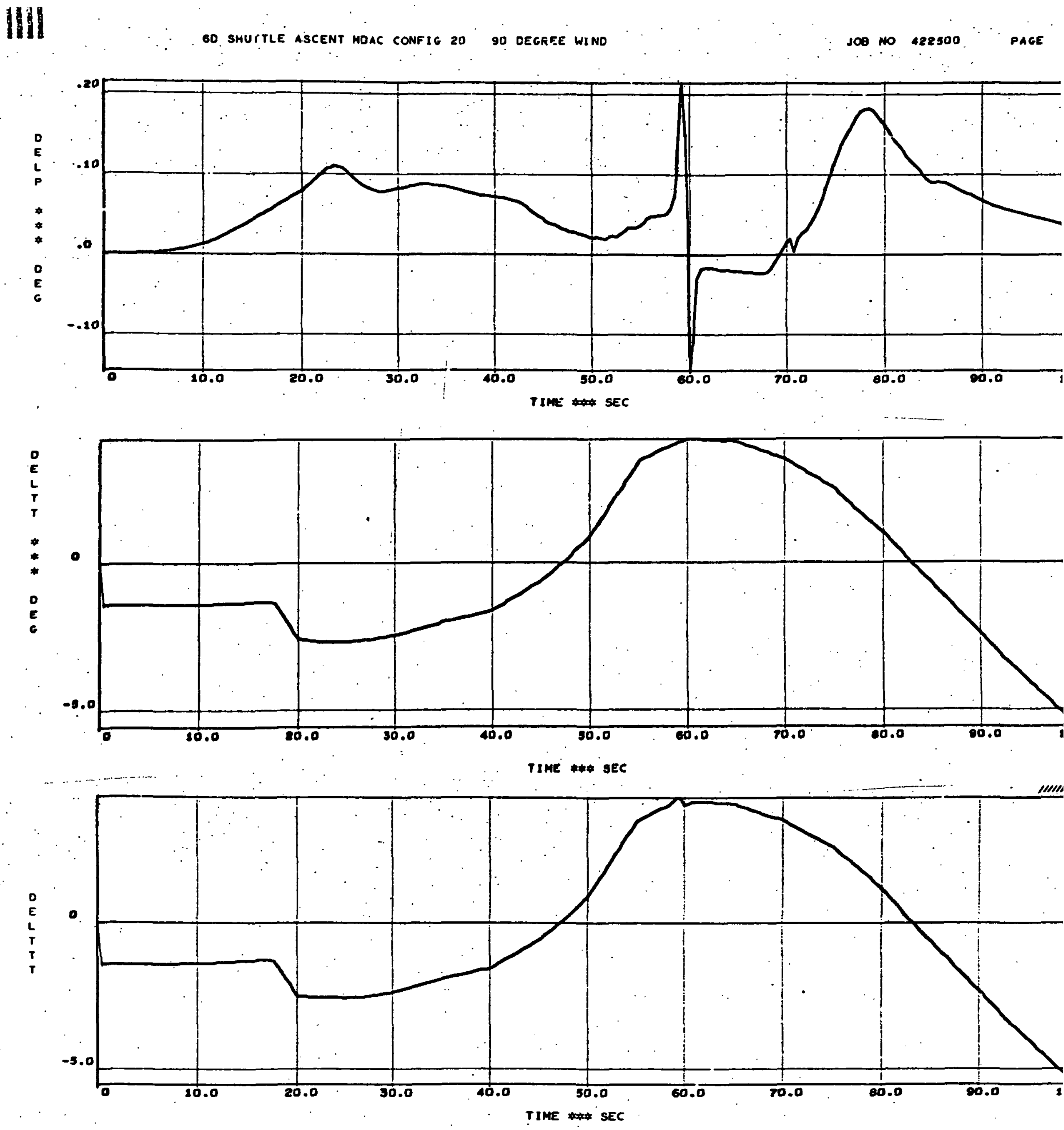
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TIME *** SEC
DELTT *** DEG
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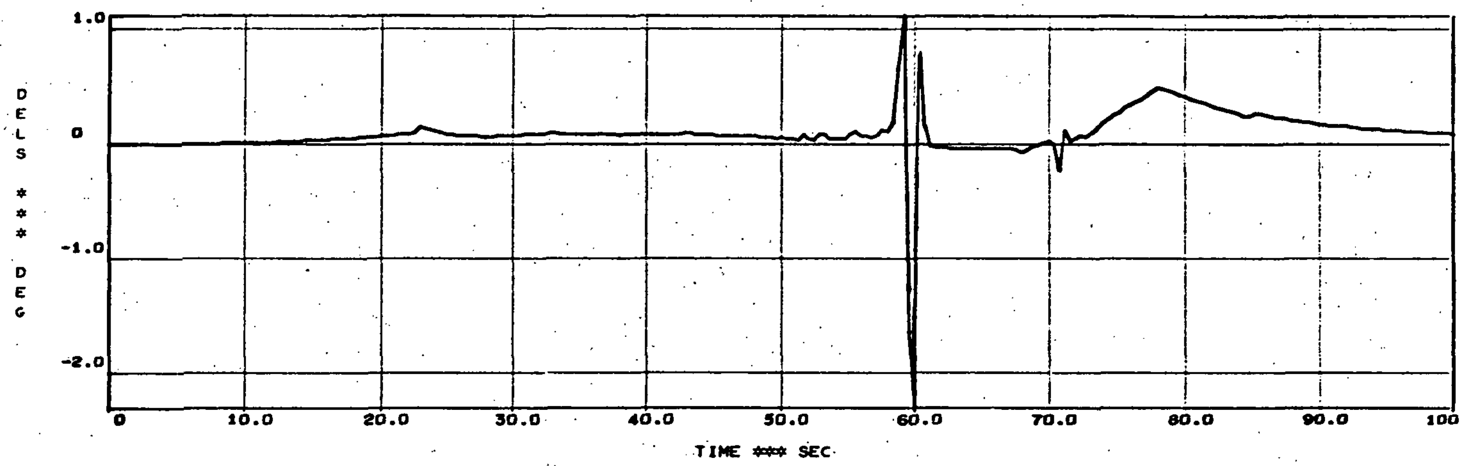
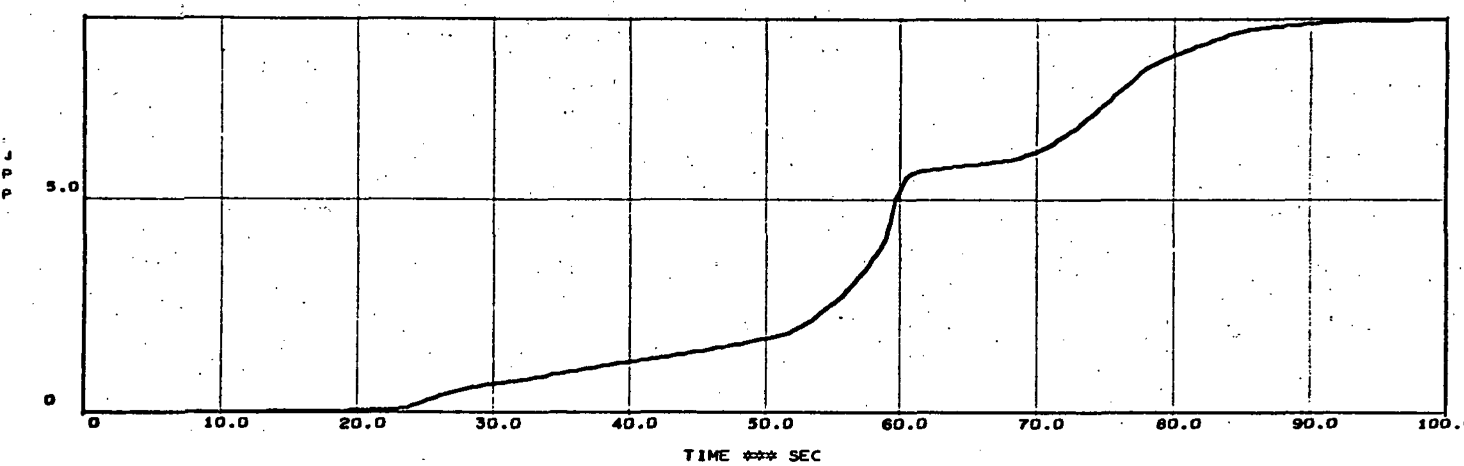
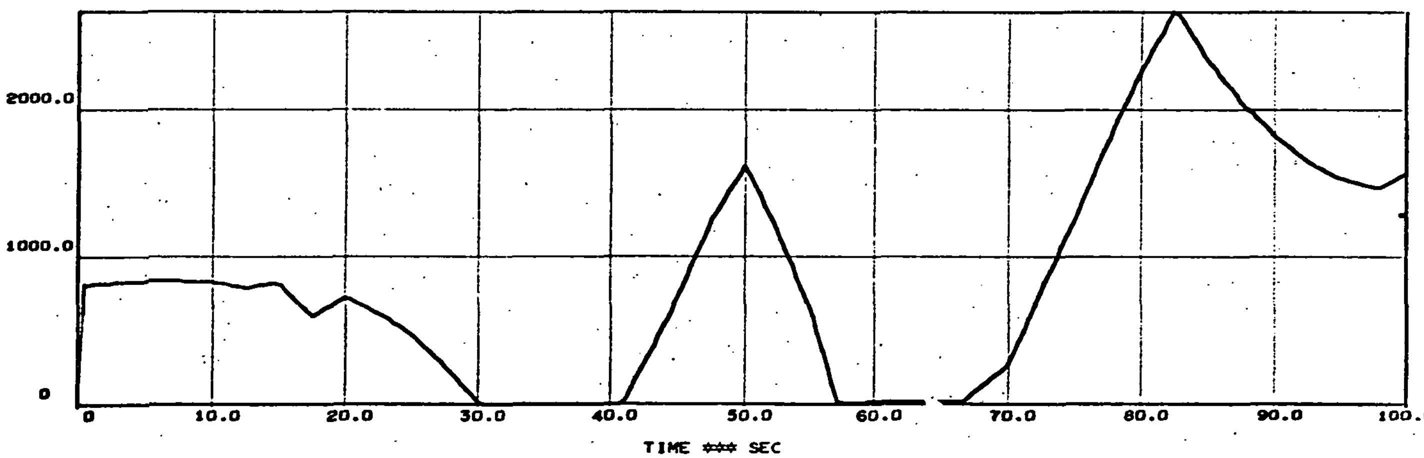
6D SHUTTLE ASCENT MDAC CONFIG 20 90 DEGREE WIND

JOB NO 422500 PAGE 12







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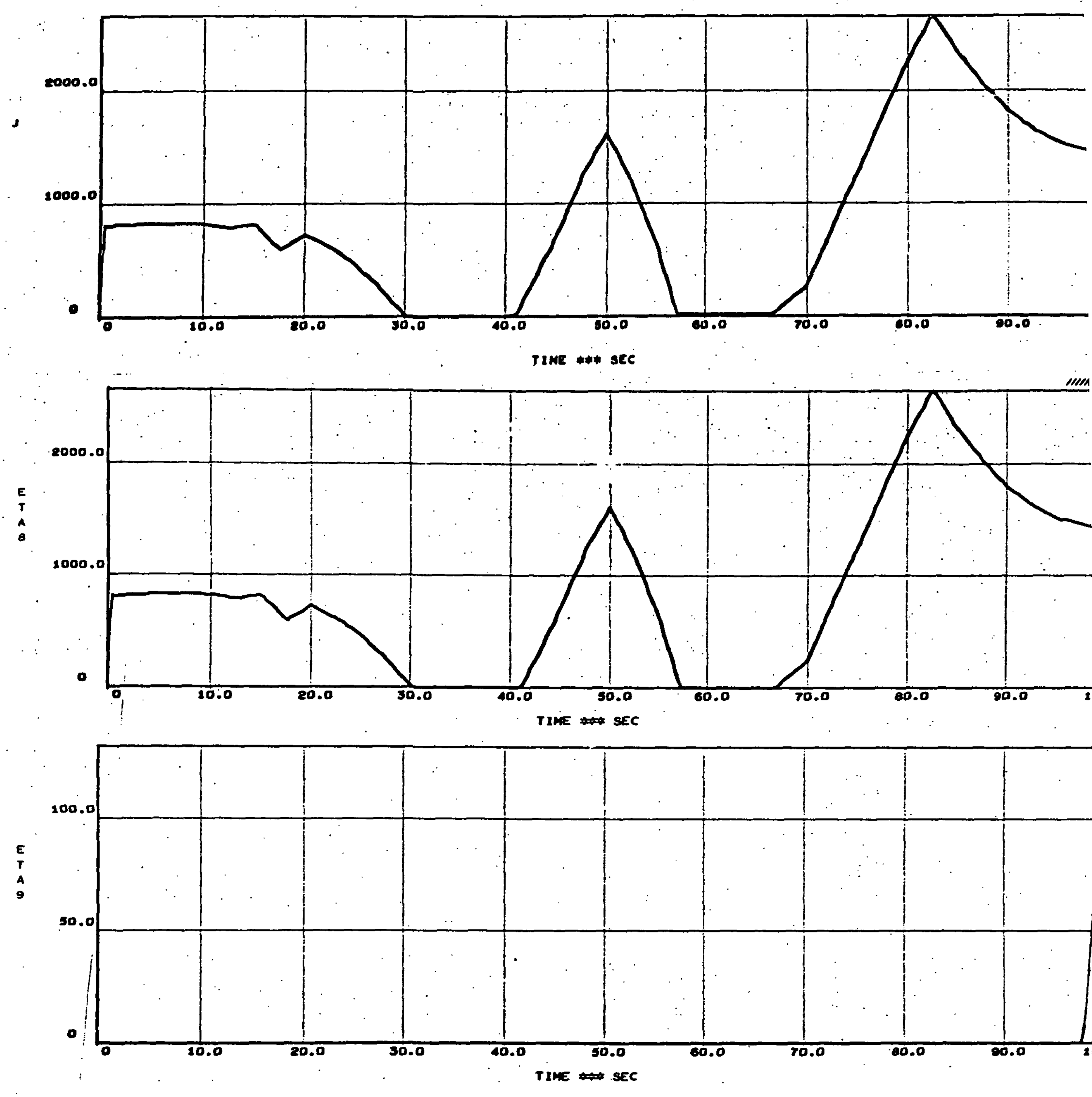

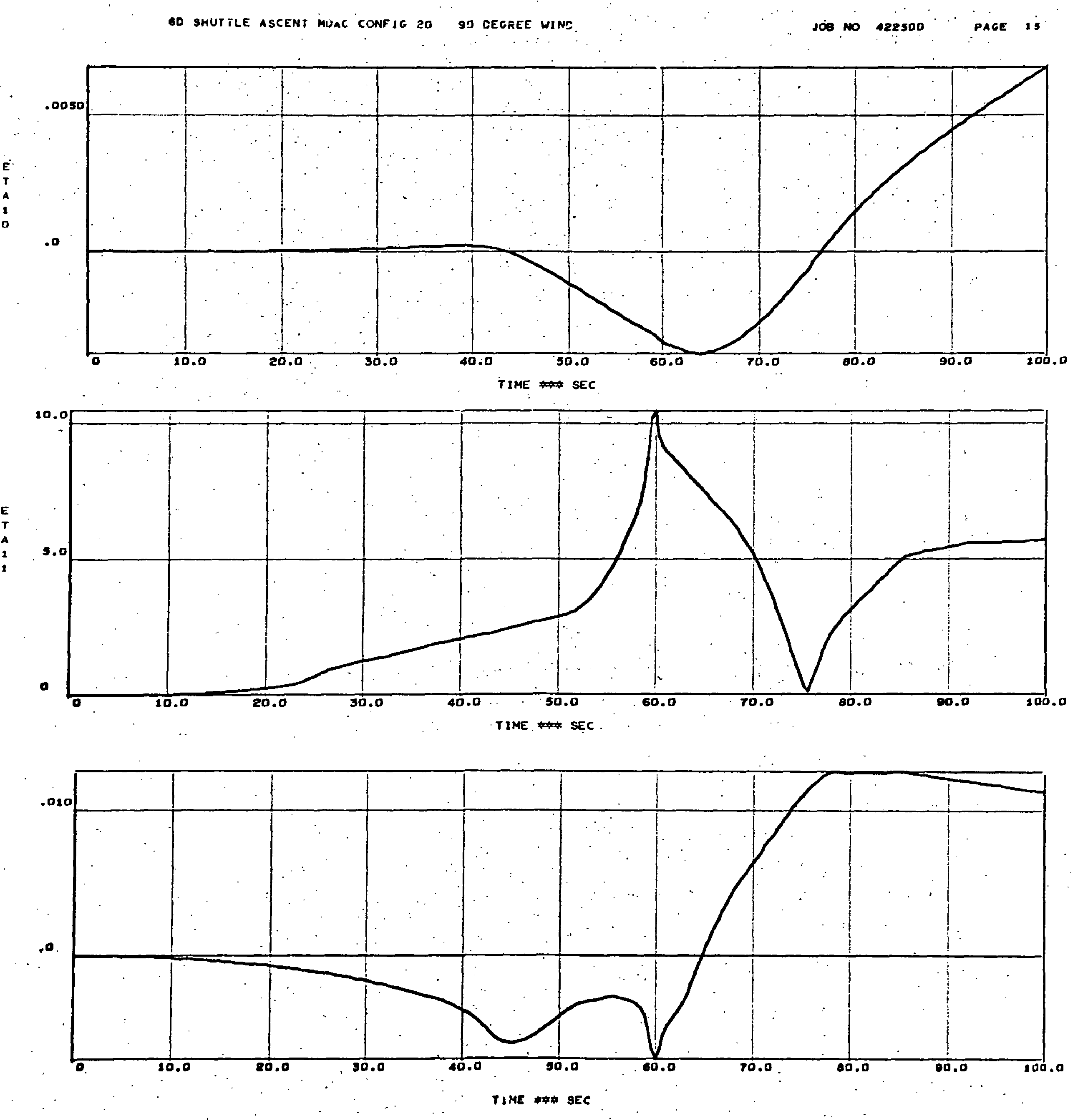
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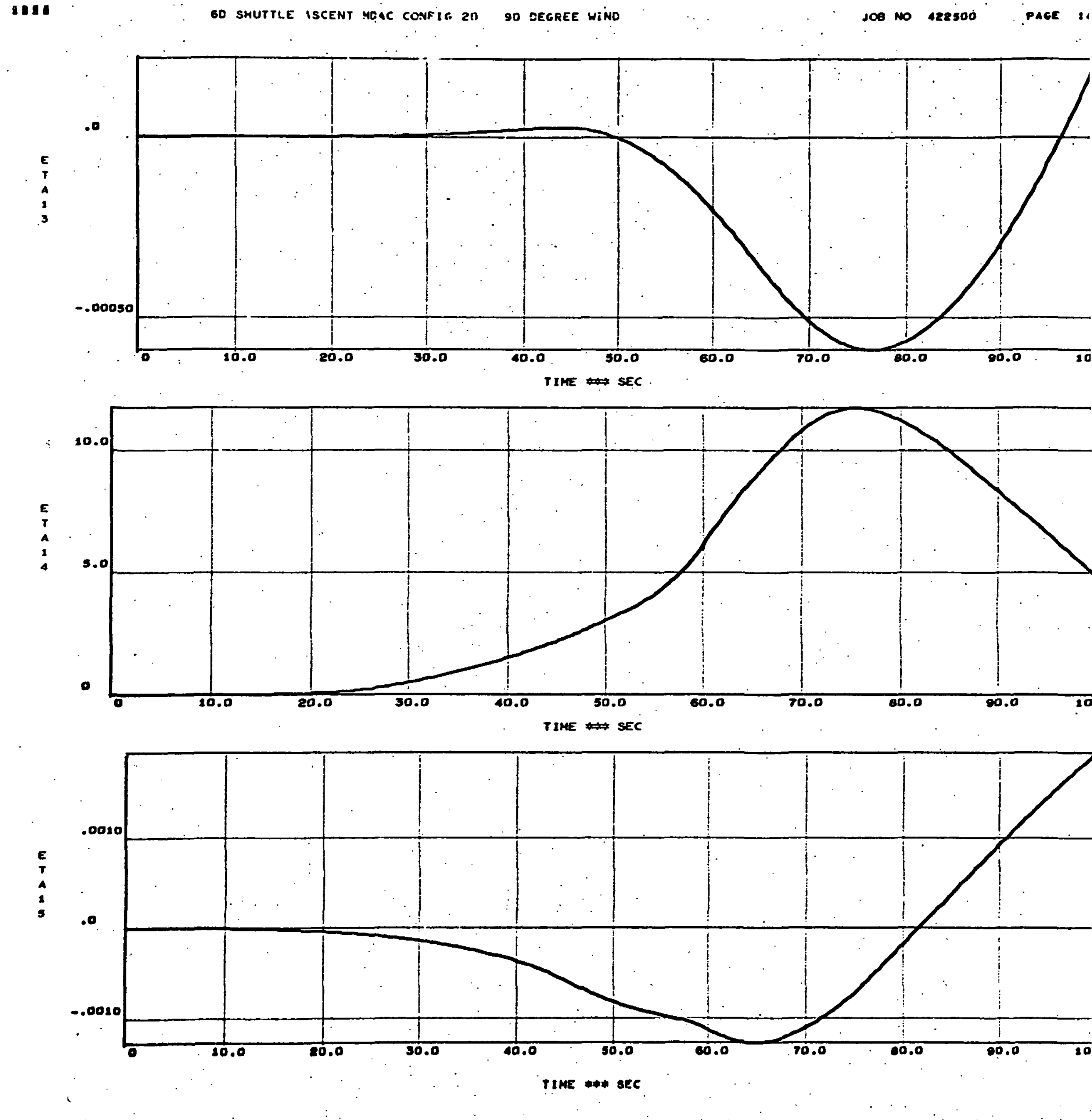
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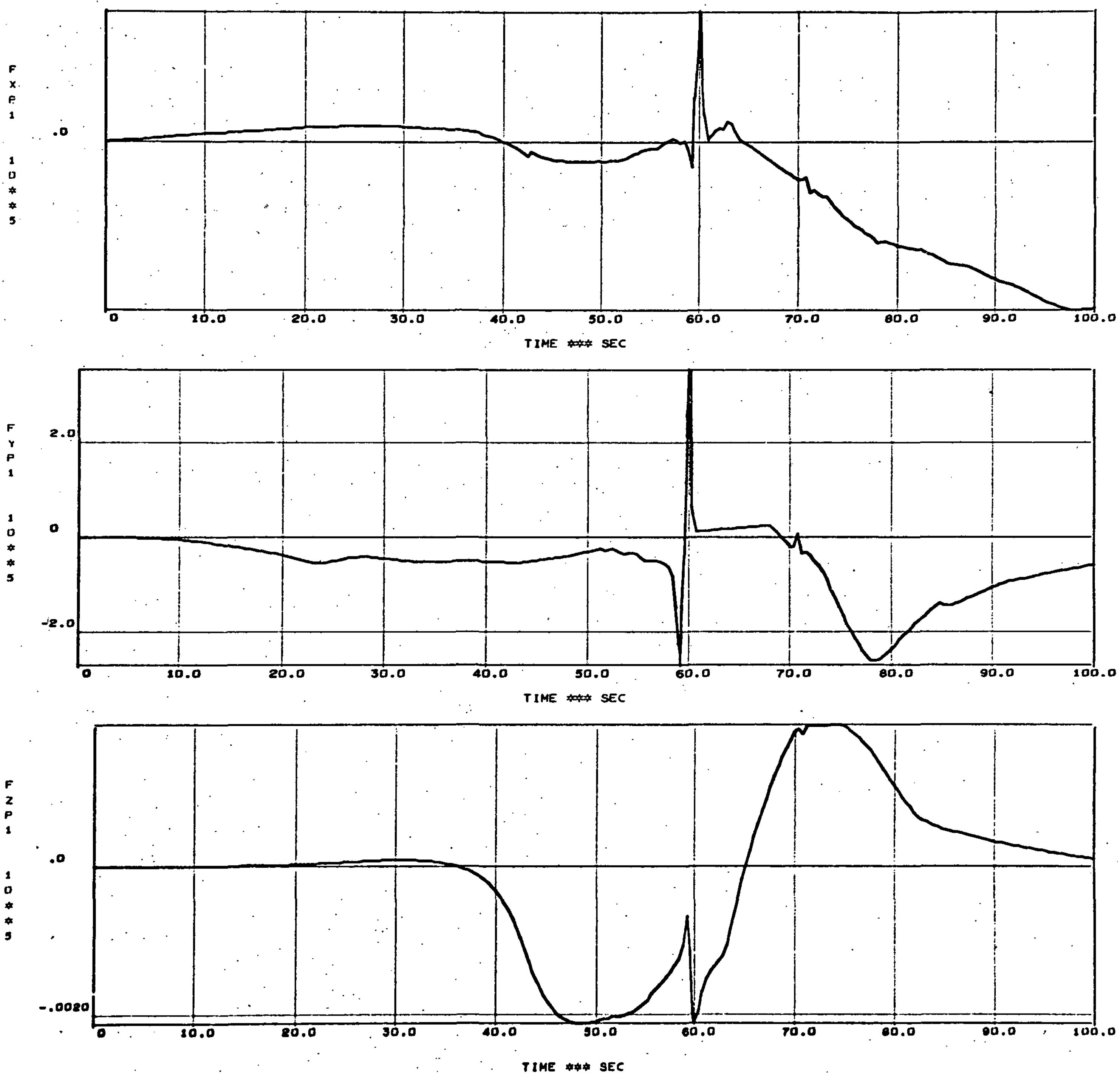
6D SHUTTLE ASCENT MDAC CONFIG 20 90 DEGREE WIND JOB NO 422500 PAGE 16



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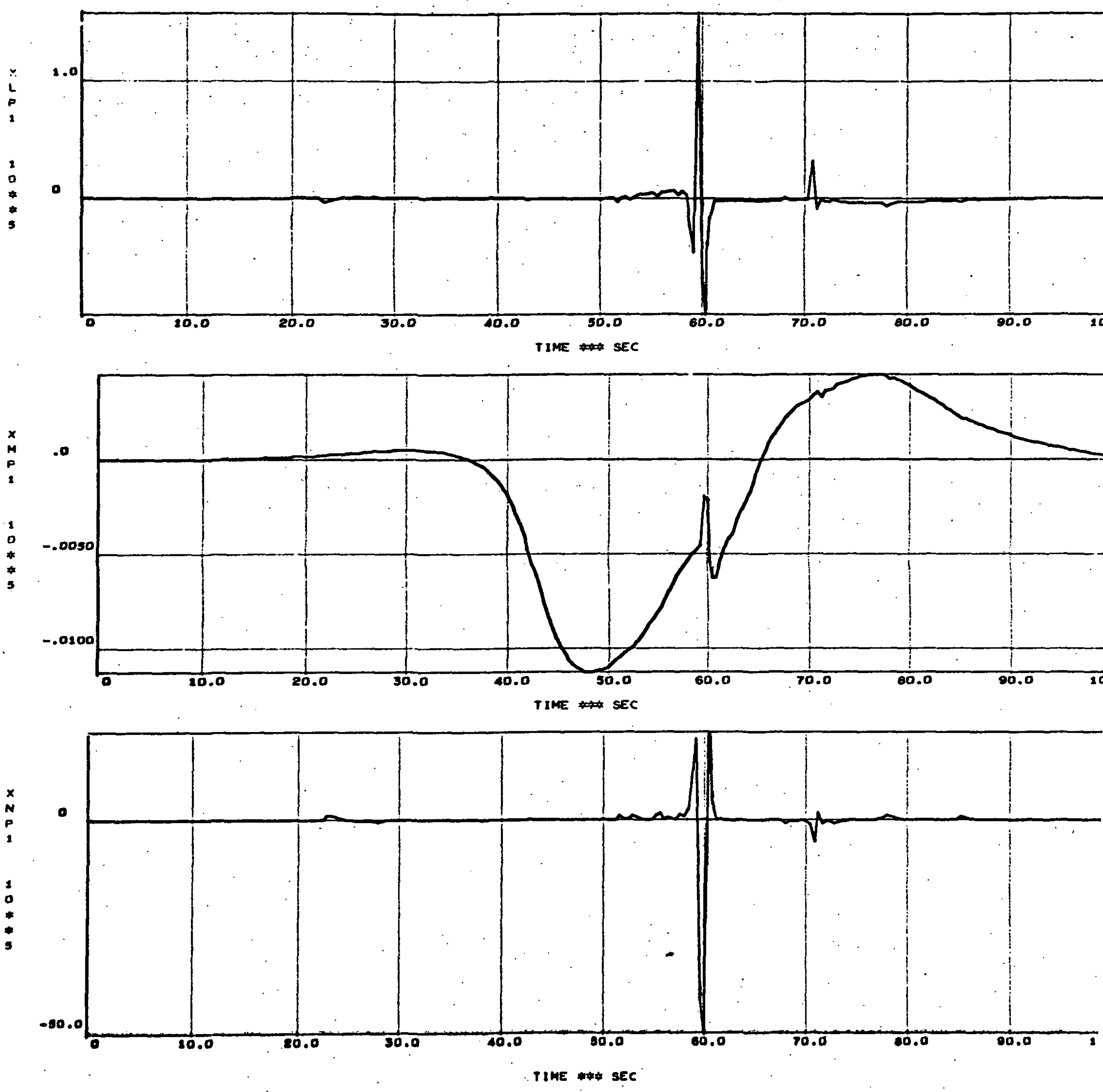
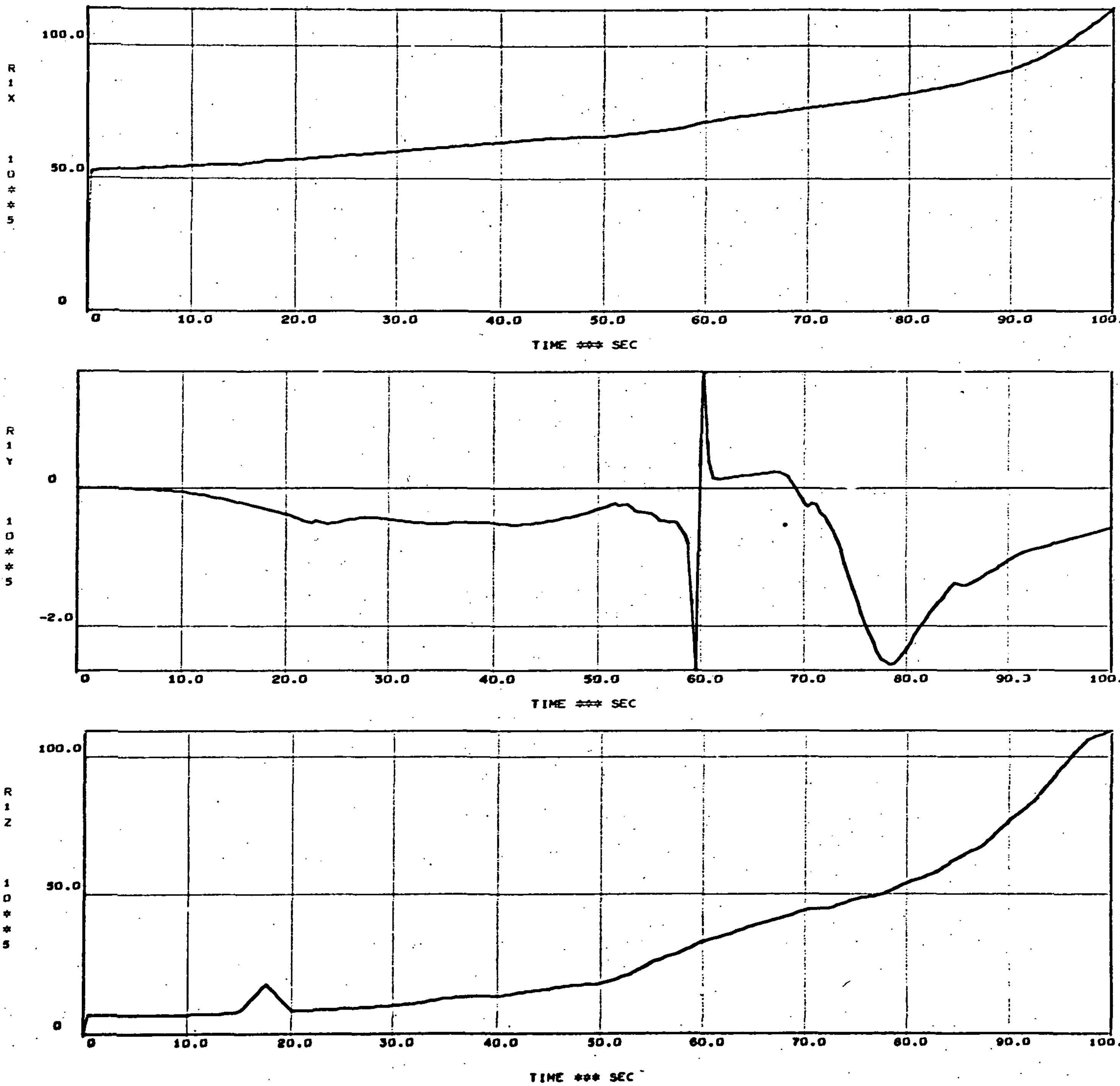
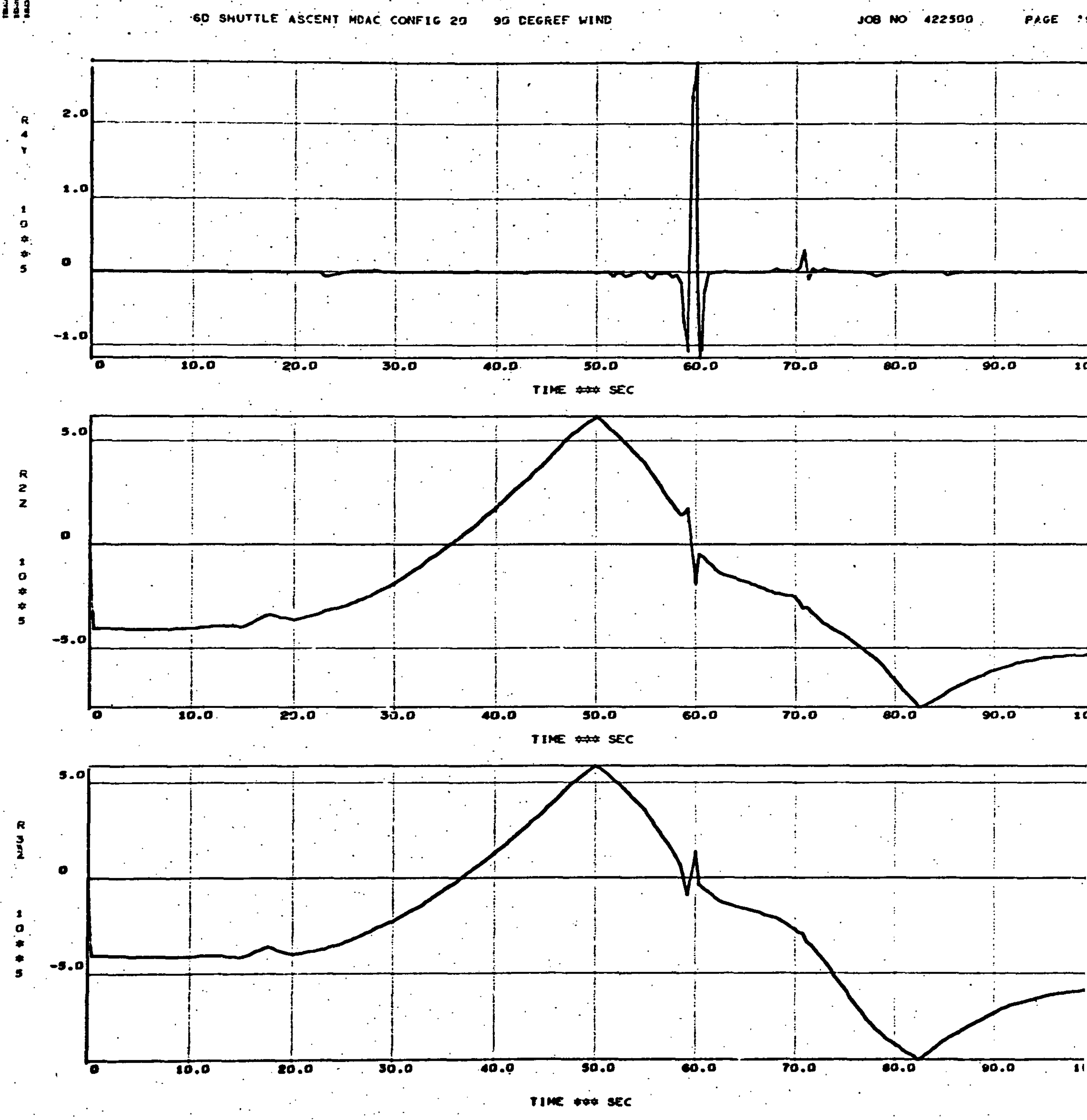


6D SHUTTLE ASCENT MDAC CONFIG 20 90 DEGREE WIND JOB NO 422500 PAGE 18

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30.0
40.0
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70.0
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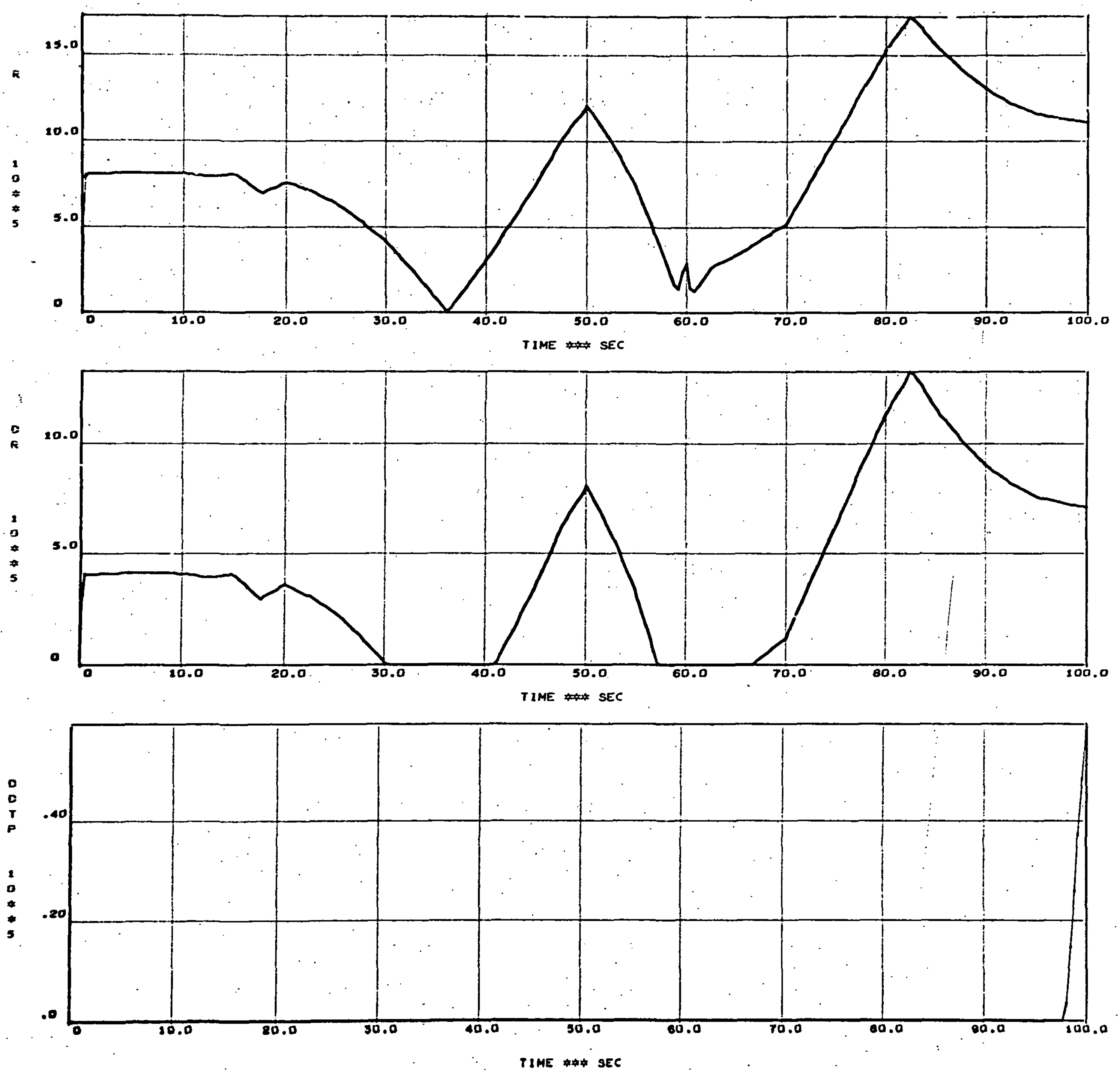
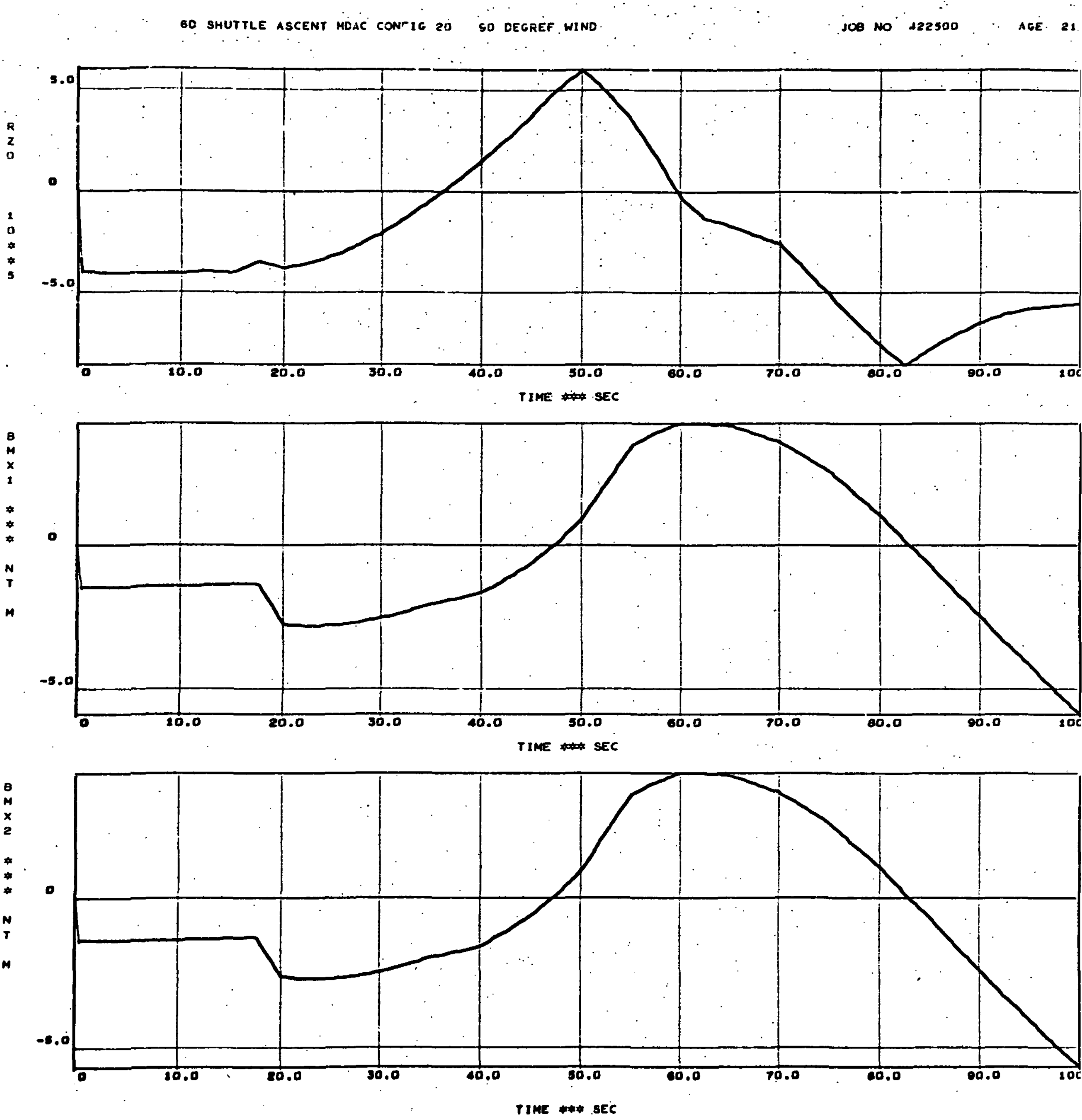
TIME *** SEC

GO SHUTTLE ASCENT MDAC CONFIG 2C 90 DEGREE WIND JOB NO 422500 PAGE 20

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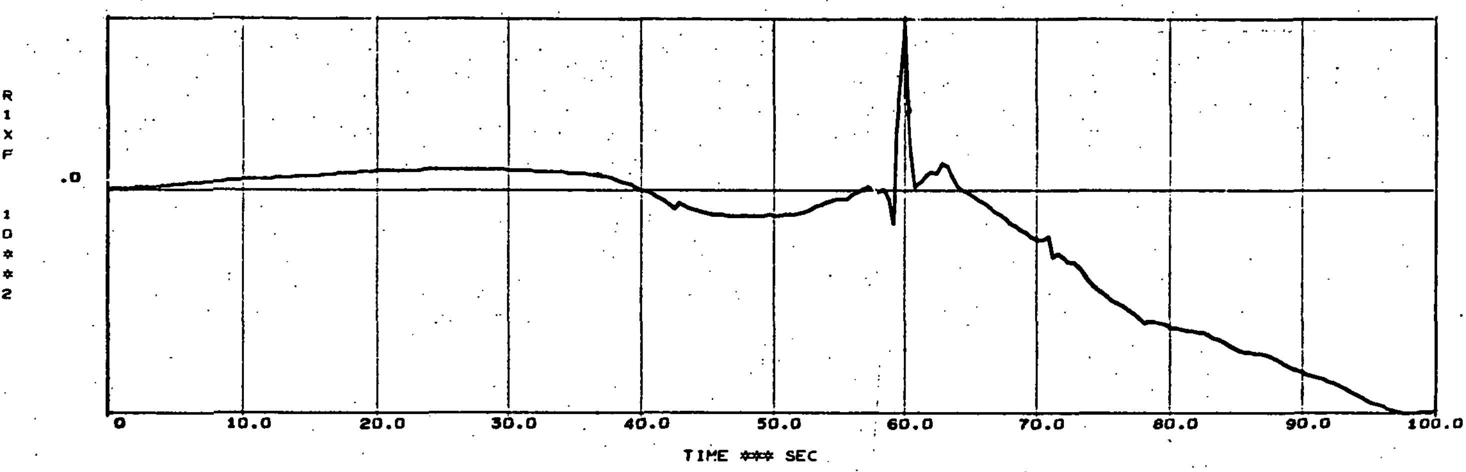
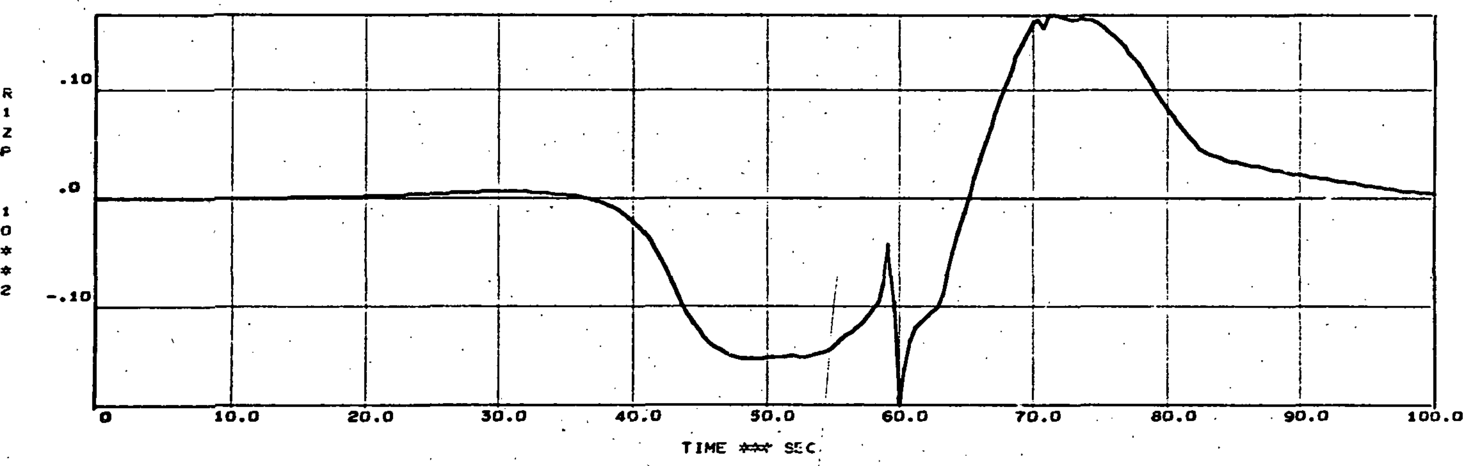
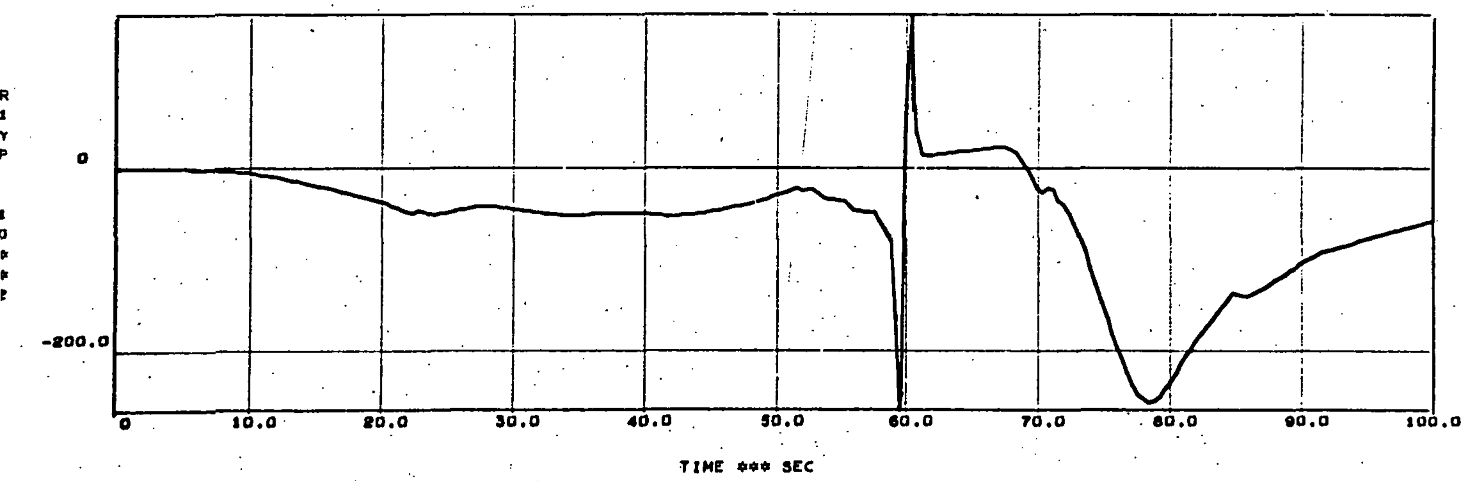
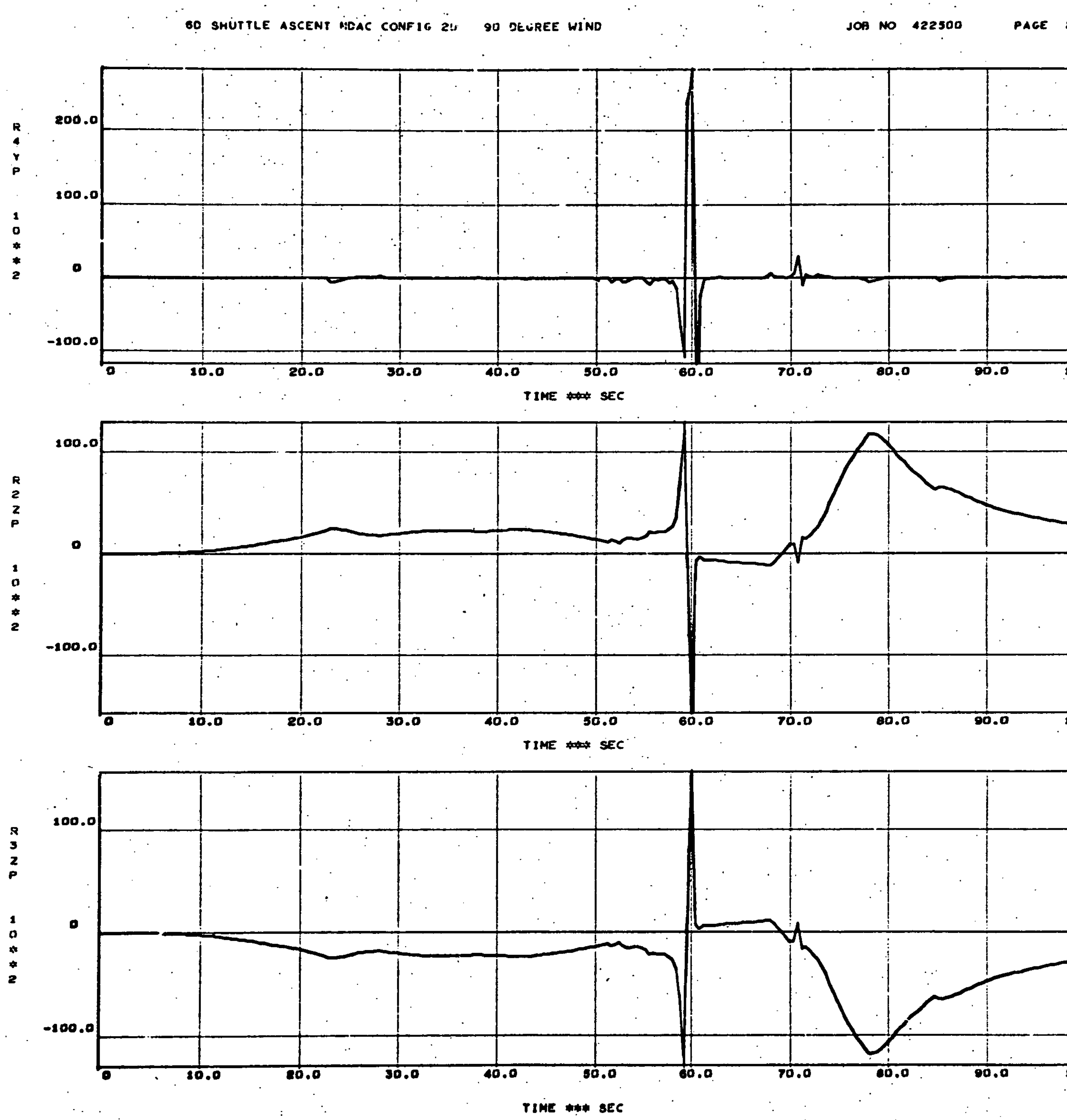
60 SHUTTLE ASCENT MDAC CONFIG 20 30 DEGREE WIND JOB NO 422500 PAGE 26









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TIME *** SEC
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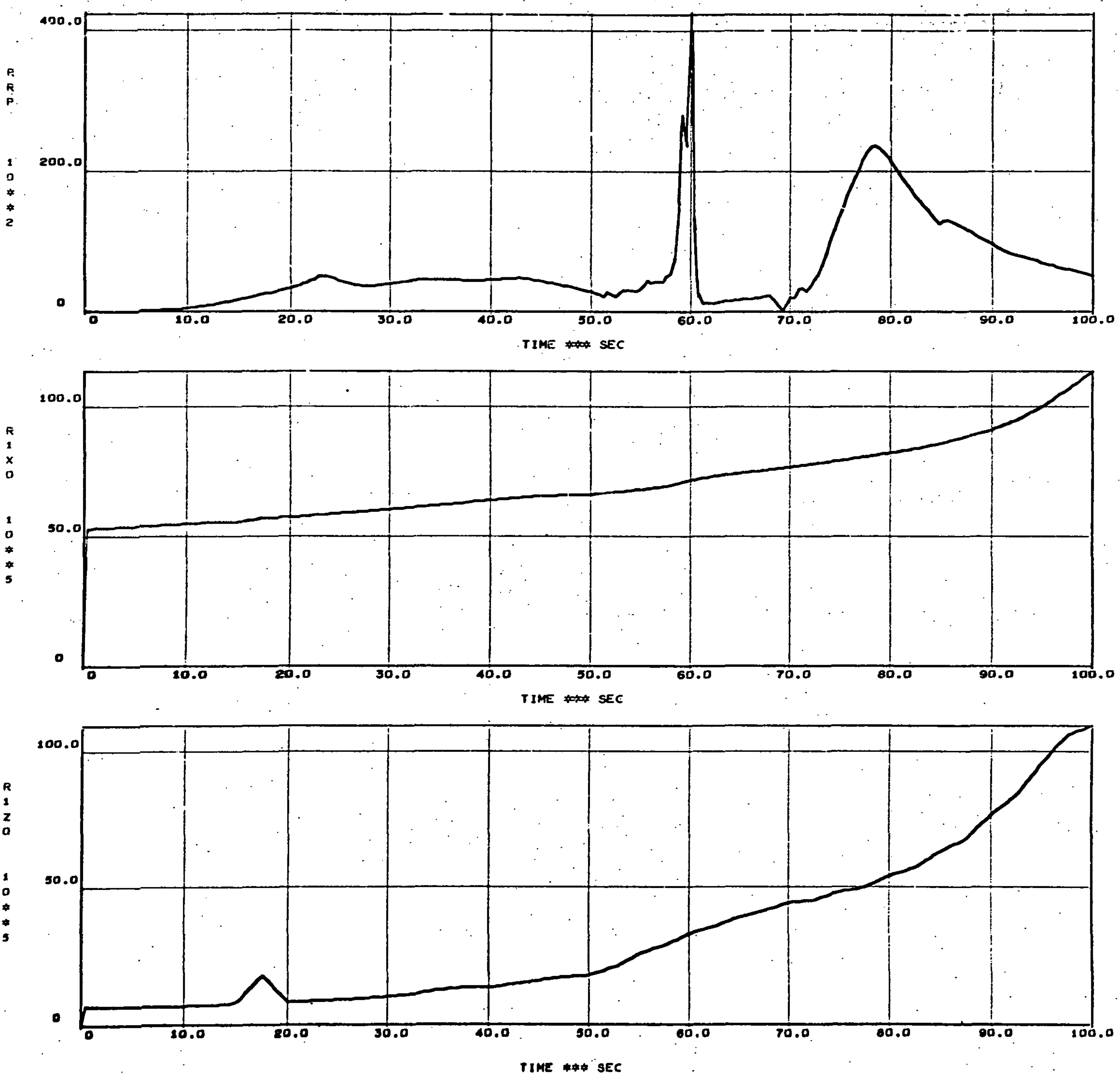
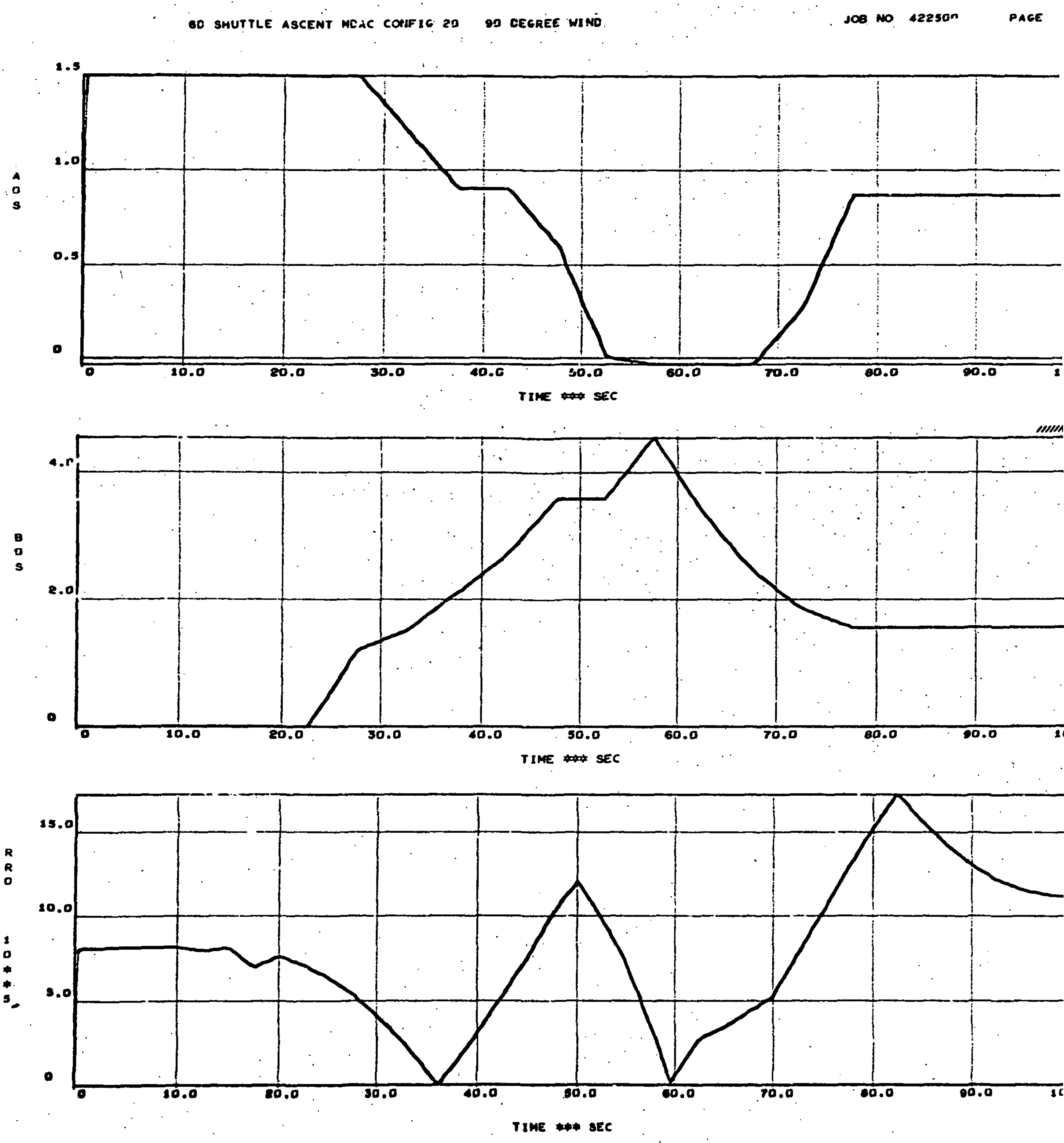
TIME *** SEC

6D SHUTTLE ASCENT MDAC CONFIG 20 90 DEGREE WIND JOB NO 423500 PAGE 28

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TIME *** SEC
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BOS
TIME *** SEC
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RRO
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TIME *** SEC