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STUDY OF STABILITY AND CONTROL MOMENT GYRO WOBBLE DAMPING OF FLEXIBLE, SPINNING SPACE STATIONS

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STUDY OF STABILITY AND CONTROL MOMENT GYRO WOBBLE DAMPING OF FLEXIBLE, SPINNING SPACE STATIONS

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Approved: C.J. Witt CNE
C. J. Witt - Section Head
Grumman Guidance and Control
Section

Authors: Howard Berman
Dr. Joel Markowitz
Will Holmer

FOREWORD

This Final Report is submitted to the NASA Manned Spacecraft Center in partial fulfillment of Contract NAS 9-11991. Presented herein are the results of the technical investigations along with conclusions and recommendations for further action in the area of stabilization and control of flexible, spinning space stations.

This report has been segmented into three parts. PART I, which can be considered an Executive Summary, presents an introduction, a concise summary of significant results, and the principal conclusions and recommendations. PART II presents a description of the analyses along with an in-depth discussion of the results. PART III is a User's Guide for the digital computer program that simulates the flexible, spinning space station. The development and submittal of this computer program is a contractual requirement.

The study program began on June 28, 1971. Mr. K. Lindsay of the Guidance and Control Division at NASA/MSC was Contract Technical Monitor. The overall study was directed and coordinated by Mr. H. Berman, Project Engineer. Mr. Berman also led the control analysis activities with aid from Mr. W. Holmer. Dr. J. Markowitz was responsible for the derivation of the dynamic equations of motion and the modal analysis. Dr. F. Austin provided valuable advice in various aspects of the program.

Other contributors to the successful completion of this task were: George Zetkov of the Guidance and Control Section, Arthur Schloth of Loads and Dynamics, and Howard Flicker of Grumman Data Systems.

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PART I

Introduction, Summary, and Conclusions

1.0 Introduction

The open-loop (uncontrolled) stability of the space station and the control moment gyro (CMG) requirements for damping the wobble motion of the space station can be influenced significantly by structural flexibility and vehicle mass distribution. The purpose of this study is to provide an early evaluation of open-loop stability which will allow for sufficient lead time to modify, if necessary, the basic structural design of the space station. If the uncontrolled vehicle is unstable and if changes cannot be made to render the uncontrolled vehicle stable, then CMG reliability would assume vastly greater importance. In addition, early awareness of the CMG system requirements will provide sufficient lead time for hardware development. As a result, the following study objectives are formulated:

- o Determine the effects of structural flexibility on the uncontrolled stability of two dual-spin space station configurations.
- o Investigate CMG wobble damping with an emphasis on control interaction with the structure.
- o Develop and deliver to MSC a digital simulation of the flexible, spinning space station.

The newly developed simulation will be used as an analysis tool to satisfy the first two study objectives; also, it will be available to MSC personnel for future studies.

NASA/MSC has selected for investigation two typical modular space station configurations with extremely different mass distributions. Both configurations consist of a spinning member (a rotor) rotating about a near stationary member (the hub).

In order to accomplish the above outlined objectives for the two general vehicles chosen for investigation, the following study approach is adopted:

- Establish wobble damping requirements, and the level of wobble induced disturbances.
- Formulate the general flexible-body equations of motion and develop the corresponding digital simulation.
- Perform preliminary open-loop stability analyses and CMG wobble damping analyses using digital computer programs developed during previous Grumman studies.
- Use the newly developed simulation to evaluate open-loop stability and analyze sensitivity to variations in the space station structural properties.
- Use the newly developed simulation to evaluate the performance of the CMG wobble damping concept developed during a previous Grumman study.

2.0 Summary of Results

The two space station configurations selected for investigation are referred to as the "T" and "Y." The "T" and "Y" configurations were chosen so that each general class of rotating satellites with extremely different mass distributions is represented; specifically, "T" is a "Min I" vehicle (the inertia about the rotor spin axis is smaller than either transverse inertia), while "Y" is "Max I" (the inertia about the rotor spin axis is greater than either transverse inertia).

The majority of the analyses were performed using two simulations: the Simplified Flexible Body Simulation (developed during a previous study for NASA/MSC) and the General Flexible Body - Stage II simulation (developed during this study). The Simplified Flexible Body simulation idealizes the vehicle as a rigid rotor connected to a rigid hub by a flexible bearing. In the new simulation, the structure is idealized by segmenting the vehicle into several flexible and rigid sections. The sources of structural flexibility considered are:

- The bearing, connecting the rotor to the hub
- The long, thin walled sections that comprise the rotor arms and hub
- The space station-shuttle attachment point.

Much of the previous stability analyses of flexible rotating satellites are based upon many simplifying assumptions, and non-rigorous energy sink methods. Hence, results of the present analyses, using high fidelity structural simulations, enhances the previous knowledge learned from the earlier studies. Using the Simplified Flexible Body Simulation, which physically represents a vehicle with a highly flexible bearing compared to relatively stiff rotor arms and hub, the following uncontrolled stability information is uncovered:

- "Y" is always stable
- "T" is unstable with damping in the bearing outer race
- A stability boundary exists for the "T" configuration with bearing inner race damping.

Using a range of stiffness values for rotor arm and hub flexibility in conjunction with the newly developed simulation, the following occurred:

- o The uncontrolled "Y" configuration is stable even with a flexibly attached shuttle. It should be noted that a "Y" station with a shuttle is a "Min I" vehicle, where stability is always questionable.
- o The uncontrolled "T" configuration is marginally stable. It appears that there is sufficient energy dissipation in the non-rotating portion of the vehicle to over-power the unstable influence of energy dissipation in the rotor. The latter is reaffirmed by demonstrating an instability for an uncontrolled "T" with a rigid hub.

The wobble damping requirements are based upon achieving a 10^{-5}g environment throughout the hub within a specified time from the initiation of the wobble induced disturbance. Although CMG wobble damping is investigated in response to shuttle docking disturbances, CMG requirements will not be based upon the relatively infrequent dockings; rather, successful damping of the wobble induced by typical internal mass motion will determine the CMG requirements.

CMG wobble damping is investigated using a unique CMG configuration that was developed during Grumman's previous study on space-base wobble damping. The CMG control technique employs a single-gimbal CMG with an associated control strategy that forces the wheel spin axis to lag the wobble vector by 90 degrees in inertial space. The interaction of the CMG wobble damping scheme with the flexible space station is evaluated using the General Flexible Body Simulation - Stage II. The results of the investigations are summarized below.

Little modification to the CMG system is required when operating with the "Y" station. A low pass filter with a time constant of 2 seconds is added to decrease the actuator duty cycle. The CMG system required the following modifications in order to perform successfully with the "T" station:

- o A negative limit on gimbal rate to prevent a control instability.
- o A low pass filter with a time constant of 10 seconds is needed to effectively decrease the actuator duty cycle.

- A bias on the gimbal angle command is required in order to minimize the CMG tracking error introduced by the filter.
- The criterion for stopping active CMG damping is raised in order to avoid interacting with the structrual modes at low wobble amplitudes.

In addition to the above noted CMG system modifications required for successful wobble damping of the "T" station, the "T" configuration, as compared with "Y", will in general require a significantly larger CMG and greater torquer motor output power. Specifically, in response to a typical profile of internal mass motion the following requirements are established:

Configuration	Wheel Size Angular Momentum (ft-lb-sec)	Maximum, Average Power Output (watts)
"Y"	400	0.009
"T"	12,000	5.15

The development of the General Flexible Body - Stage II simulation was a contractual requirement. A user's guide for this simulation has been prepared and is presented in Part III of this report.

3.0 Conclusions and Recommendations

There should be no concern over the uncontrolled vehicle stability of a "Y" type space station. The results of all investigations indicate that the vehicle is stable regardless of the amount or location of energy dissipative structural material. In fact, the time duration for wobble convergence will decrease as the amount of structural flexibility is increased; whether the increase in flexibility is in the rotating portion of the vehicle or non-rotating portion, is unimportant. In contrast, the uncontrolled vehicle stability of a "T" type space station is extremely sensitive to the amount and location of energy dissipative material. For example, it is shown that an increase in structural flexibility in the spinning portion has a de-stabilizing influence while an increase in the flexibility of the non-rotating section has a stabilizing effect. Therefore, to enhance the chances for uncontrolled vehicle stability, additional stiffening members should be added to the "T" rotor. It should be noted that the nominal "T" configuration is marginally stable. However, the flexible parameters assumed as nominal represent preliminary estimates. As more structural data is made available, the uncontrolled stability of the "T" configuration should be reaffirmed.

The CMG requirements associated with the "T" configuration are extremely unattractive. Furthermore, to satisfy the wheel size requirement with one CMG would be beyond the "state-of-the-art." In contrast, the CMG requirements associated with the "Y" station can be satisfied without developing additional CMG technology.

Due to the stabilization and control problems associated with the "T" space station, a configuration of this type should be avoided, assuming the option is available. If the option is unrealistic, then early development of a highly reliable CMG system with a very large angular momentum capacity is recommended.

Future Studies

The work accomplished as part of this study solves just a small part of the complex space station control problem. As a result, the following directly related areas of study are recommended for future investigation.

It has previously been established that along with a CMG wobble damping system, the space station will require the following additional spin vector control systems:

- Mass Balancing System
- RCS Attitude Control for Pointing
- Spin Rate Control

Aside from it being important to study the interaction of each control system with the flexible structure and uncover any possible problems, it is equally important to study the integrated spin vector control system and uncover any problems associated with the interaction between control systems.

It is noted that a complex spin vector control system of this type would be advancing present technology and should be studied as early as possible.

In addition to studies in the space station technology area, the simulation that has just been developed as part of this study can be used to investigate the flexible body dynamics of other space vehicles. Specifically, by manipulating the input data, the program can be used to simulate a varity of vehicles with rotating and non-rotating sections, whether it be one rigid body with six degrees of freedom to fifteen flexibly connected bodies with eighty-four degrees of freedom. With minor modifications to the program, the simulation can be made to represent dual spin satellites with an arbitrary number of rotor arms, as well as a high fidelity simulation of a flexible Shuttle Orbiter.

PART II

ANALYSIS

4.0 Physical Vehicle Properties

4.1 Vehicle Configurations

NASA/MSC has selected for investigation two typical modular space station configurations. Both configurations consist of a spinning member (the rotor) rotating about a near stationary member (the hub). The end module of each rotor arm serves as living quarters within which an artificial gravity environment is experienced for the life of the station. Inside the hub, where most of the scientific experiments are performed, a near zero-g environment is provided. The two configurations chosen for study are the "T" and "Y" configurations,and are illustrated in Figures 4.1 and 4.2, respectively. For reference purposes, these two configurations can be found on official MSC drawings SAY44101266, and SAY44101264.

The "T" and "Y" configurations were chosen so that each general class of rotating satellites is represented; specifically, "T" is a "Min I" vehicle (the inertia about the rotor spin axis is smaller than either transverse inertia), while "Y" is "Max I" (the inertia about the rotor spin axis is greater than either transverse inertia). The "Y" configuration may be considered an outgrowth from "T", in that the "T" rotor is comprised of two arms, 180° apart, while the "Y" has a three armed rotor, 120° between arms.

In order to study the dynamics of a flexible space station, the structure is idealized by segmenting the vehicle into several flexible and rigid sections. The sources of structural flexibility considered are:

1. The bearing, connecting the rotor to the hub
2. The long, thin-walled sections that comprise the rotor arms and hub.
3. The shuttle-orbiter attachment point.

The structural properties of each flexible member are detailed in Section 4.3. Section 4.2 presents their mass properties.

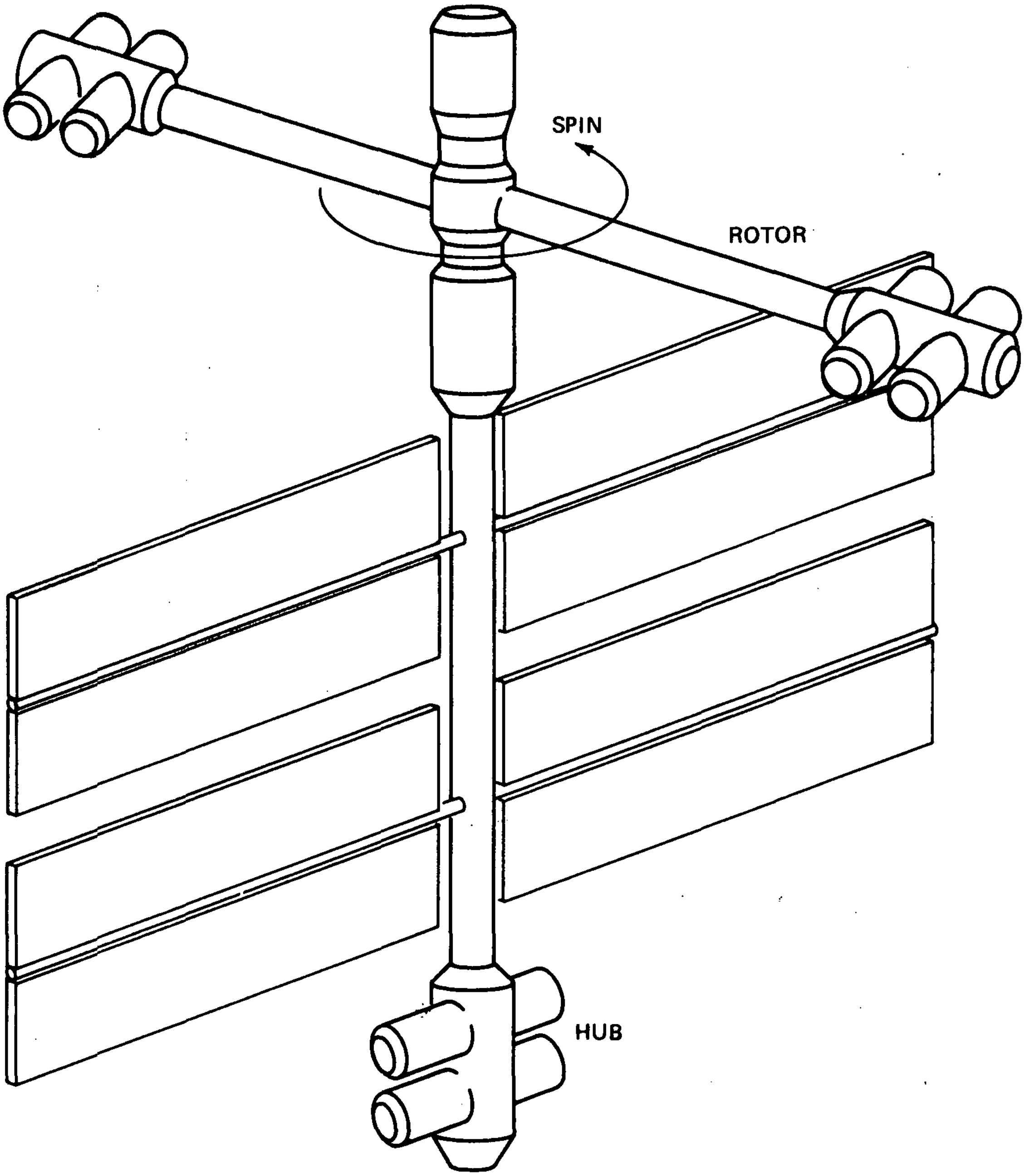


Figure 4-1. T Space Station

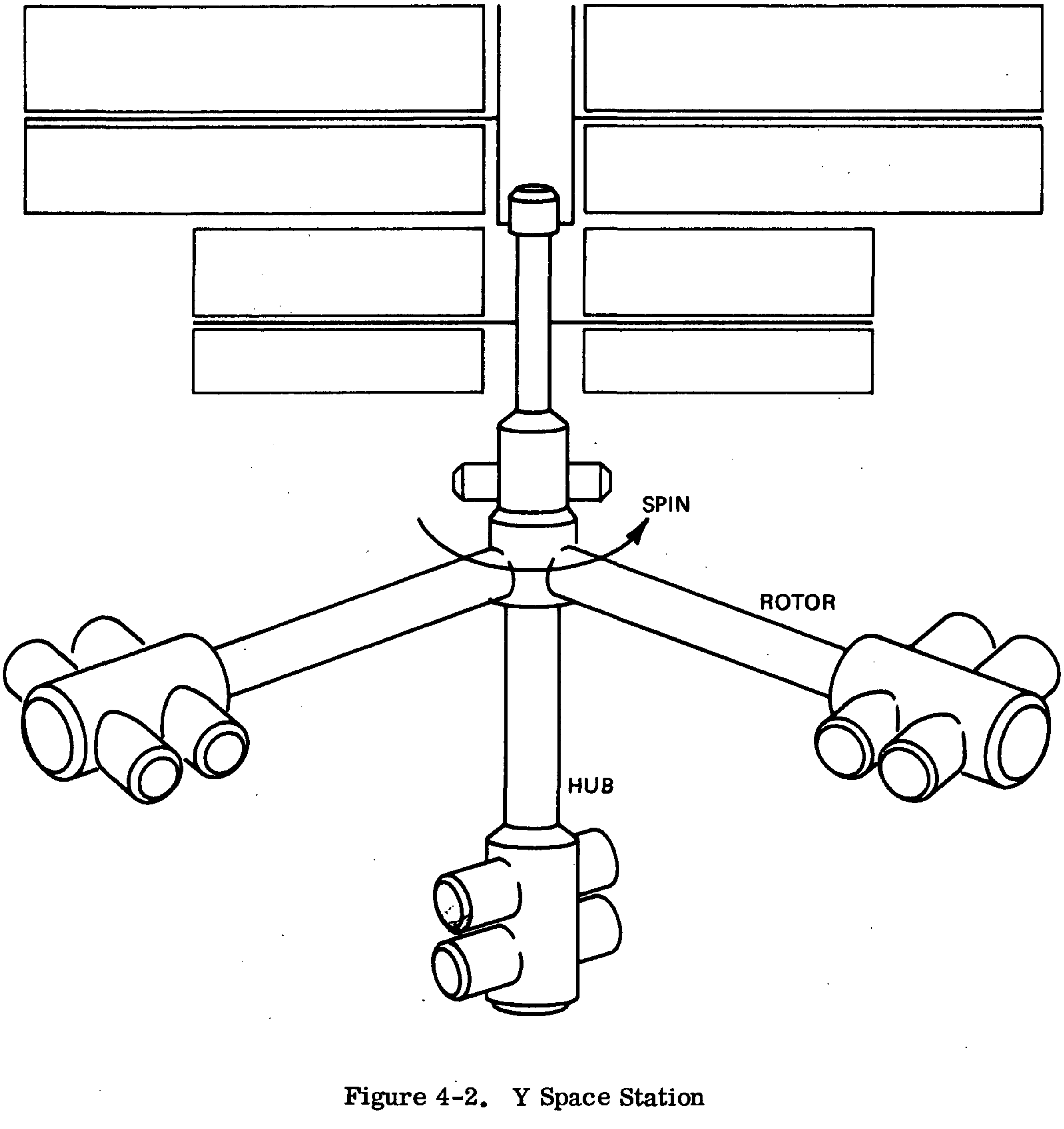


Figure 4-2. Y Space Station

4.2 Mass Properties

Mass property data for the main bodies (rotor and hub) of each configuration was supplied by MSC in Reference 1. A duplicate table, obtained from Reference 1, is shown as Table 4.1 for completeness.

The vehicle idealization used for the dynamic analyses of the flexible space station is a vehicle segmented into several lumped mass sections as shown in Figures 4.3 and 4.4. Specifically, the idealized vehicle is comprised of:

- 3-lumped masses per rotor arm
- 1-rotor ring
- 3-lumped masses for the flexible hub section
- 1-rigid hub section

Mass property data for each lumped mass was developed based upon modular, or building block, data provided in Reference 1. Figures 4.3 and 4.4 contain tabulated mass property data for each lumped mass section. The mass property data shown is in the proper form for direct use in the simulation that was developed during this study.

The analyses that included a shuttle orbiter attached to the station used orbiter mass properties corresponding to a delta wing-25K payload orbiter as obtained from Reference 2, and listed in Table 4.2.

TABLE 4.1 MASS PROPERTY DATA

CONFIGURATION		WEIGHT ($\text{lbs} \times 10^{-3}$)	C.M.-(FT)(1)			INERTIAS ($\text{SLUG-FT}^2 \times 10^{-8}$)(2)		
			X	Y	Z	Ixx	Iyy	Izz
T	Rotor	285	0	0	0	.302	1.127	.889
	Hub	184	0	0	140.7	.637	.661	.025
	Total	469	0	0	55.1	.939	1.788	.914
Y	Rotor	431	0	0	0	.745	.745	1.330
	Hub	156	0	0	- 58.3	.220	.256	.037
	Total	587	0	0	- 15.5	.965	1.001	1.33

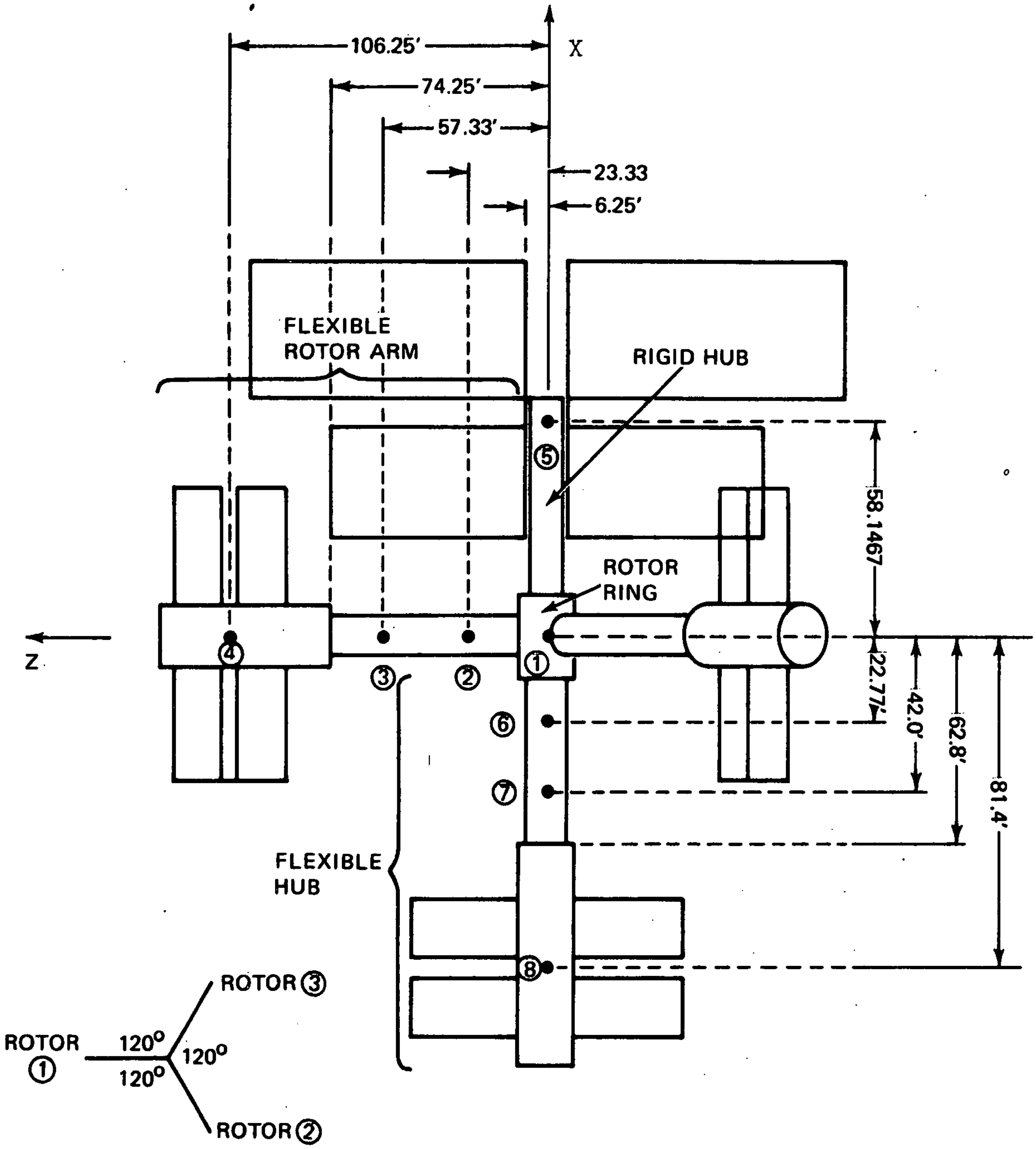
NOTES: 1 The center of the xyz coordinate system is in the rotor plane.

2 Inertias are measured relative to the total CM. Z axis is spin

TABLE 4.2 SHUTTLE ORBITER - DELTA WING - 25 K PAYLOAD MASS PROPERTIES

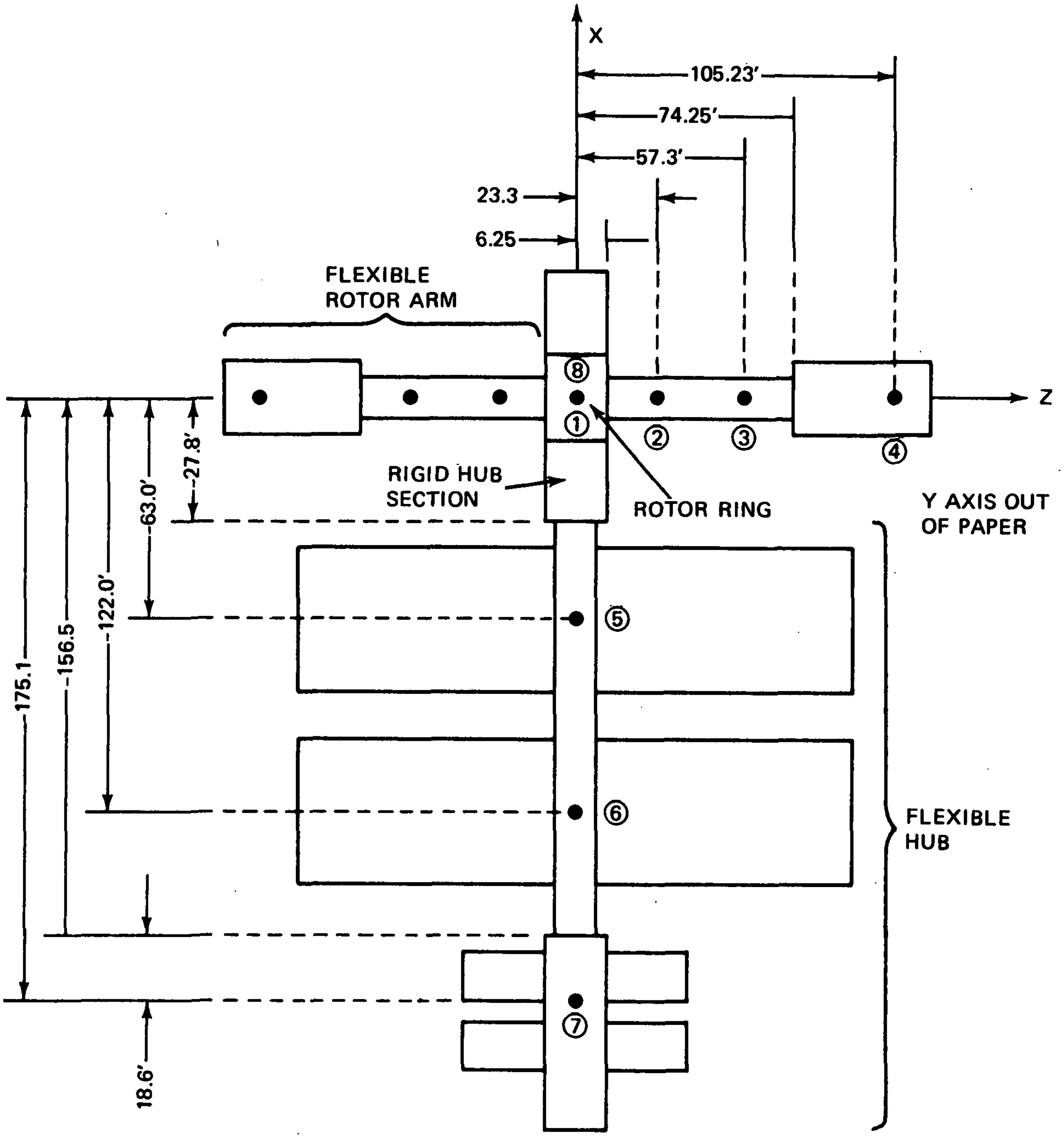
Weight (lb)	X^{sta*} (inches)	Moment of Inertia $\text{slug-ft}^2 \times 10^{-6}$		
		Ixx	Iyy	Izz
316940.	1521.8	2.725	19.883	21.495

*Measured from nose of orbiter



Element Number	Element Type	Ixx (Slug-Ft^2)	Iyy (Slug-Ft^2)	Izz (Slug-Ft^2)	Weight (Lbs)
1	rotor ring	12,692.0	32,781.4	32,781.4	19,000
2	rotor	22,731.1	22,731.3	2,083.	7,250
3	rotor	22,731.3	22,731.3	2,083.	7,250
4	rotor	469,481.85	2,018,651.07	1,642,578.22	123,000
5	rigid hub section	2,007,596.1	2,686,897.86	682,636.59	23,000
6	hub	1,416.43	5,757.926	5,757.926	4,500
7	hub	1,005.43	3,120.47	3,120.47	3,500
8	hub	1,644,085.22	2,034,929.53	485,759.91	125,000

FIGURE 4.3 Y CONFIGURATION IDEALIZATION/MASS PROPERTIES



Element Number	Element Type	Ixx (Slug-Ft^2)	Iyy (Slug-Ft^2)	Izz (Slug-Ft^2)	Weight (Lbs)
1	rotor ring	11,296	15,586	15,586	15,000
2	rotor	22,731.3	22,731.3	2,083	7,250
3	rotor	22,731.3	22,731.3	2,083	7,250
4	rotor	2,034,929.53	485,759.91	1,644,085.22	125,000
5	hub	1,735,329.45	2,012,994.74	292,228.82	23,886.03
6	hub	1,733,430.57	1,829,283.28	98,517.36	17,275.74
7	hub	1,644,085.2	2,034,929.53	485,759.91	125,000.
8	rigid hub section	9,007.36	146,270.93	146,270.93	16,067.25

FIGURE 4.4 T CONFIGURATION IDEALIZATION/MASS PROPERTIES

4.3 Structural Properties

4.3.1 Rotor and Hub Extension Arms

The extension arms for both the rotor and hub are constructed from 7 foot outer diameter cylindrical sections of Aluminum alloy (2219-T87). Moments of inertia (I and J) and elastic moduli (E and G) for these sections were provided by NASA and are listed below:

- $I = 13{,}446 \text{ in}^4$
- $J = 26{,}892 \text{ in}^4$
- $G = 4 \times 10^6$ psi
- $E = 10^7$ psi

Based upon the above polar moment of inertia (J) the computed cross-sectional area is:

- $A = 15.8 \text{ in}^2$

The computed stiffnesses are as follows:

- Bending $EI = 9.34 \times 10^8 \text{ lb-ft}^2$
- Torsion $GJ = 7.47 \times 10^8 \text{ lb-ft}^2$

The above values are considered baseline, lower, stiffness limits. For an upper bound which will also be considered nominal stiffness, the above values were doubled. The rationale for doubling the calculated stiffnesses and using the result for nominal is that stiffening members were excluded from the original calculations.

The above stiffnesses in combination with the geometry of the extension arms are used as input data for the development of a stiffness matrix for both the rotor and hub (refer to Appendix A for the definition of the stiffness matrices). The resulting stiffness matrix is combined with the mode shapes, frequencies, and the percentage of critical damping (as shown in Appendix A), to obtain the damping matrix. One percent structural damping in the extension arms is considered nominal and used for the majority of the analyses.

4.3.2 Rotor-Hub Bearing

The rotor is connected to the hub through a massive bearing that no doubt will be highly flexible. The assumed bearing model is the same as in Reference 3, and is described in section 6.3 of this report.

A reasonable extimate for the elastic characteristics of the bearing can be obtained by assuming a structural natural frequency (f_{NB}) of 0.25 cps. By applying,

$$\frac{K_B}{J_R} = (2\pi f_{NB})^2 \quad ^{(1)} \qquad (4\text{-}1)$$

where K_B = Bearing stiffness

J_R = Maximum transverse inertia of the rotor,

the following estimate for nominal bearing stiffness is obtained,

$$K_B = 2.1156 \times 10^8 \ \frac{\text{ft-lb}}{\text{rad}}$$

Most likely, as part of the bearing fabrication process the rotor arms will either be riveted, bolted, or welded to the bearing outer race. The material at these attachment points will be highly dissapative. Therefore, it is assumed that 10% of critical damping is considered nominal damping for the bearing. By applying,

$$E_B = 2\gamma\sqrt{K_B J_R} \quad ^{(1)} \qquad (4\text{-}2)$$

where E_B = Damping coefficient

γ = Percent critical damping

The following estimate for nominal bearing damping is obtained,

$$E_B = 2.695 \times 10^7 \ \frac{\text{ft-lb}}{\text{rad/sec}}$$

---

[1] The derivation of the elastic parameters for the bearing is based upon a simple model where the hub is fixed and the relative angle between the rotor and hub (θ) is given by,

$$\ddot{\theta} + \frac{E_B}{J_R}\dot{\theta} + \frac{K_B}{J_R}\theta = 0$$

Although the above values are presently thought to be nominal they are subject to change as more structural details become available. It is for this reason that a parametric study was performed to determine the variation in vehicle performance as a function of the bearing parameters (refer to section 8.2).

It is noted that nominal bearing damping, as described above, locates dissipative material in the rotating section of the vehicle. As part of this study there were investigations performed to determine any performance differences associated with locating damping material in the non-rotating section of the bearing (refer to section 8.2).

4.3.3 Shuttle Docking Mechanism

During the analyses that included a Shuttle orbiter attached to the station, a rigid orbiter was assumed to be flexibly attached to the rigid hub section. Nominal values for the elastic coefficients associated with the docking mechanism are obtained in the same manner as the bearing coefficients were obtained. However, the natural frequency for the shuttle connection is assumed to be 0.5 cps, and the structural damping is assumed to be nominally at 1%. The resulting values for the stiffness and damping coefficients are:

$$K_H = 2.119 \times 10^8 \text{ ft-lb/rad}$$

$$E_H = 1.35 \times 10^6 \text{ ft-lb/rad/sec}$$

4.4 Vehicle Spin

A nominal rotor spin rate equal to 2.5 RPM (.262 rad/sec) is used throughout this study, as per an MSC recommendation.

5.0 Wobble Damping Requirements and Disturbances

5.1 Damping Requirements

The desired motion of the space station is pure spin. Any torque free oscillatory motion of the vehicle in inertial space other than pure spin is commonly referred to as WOBBLE. The function of the CMG will be to remove the wobble and in turn restore the vehicle to a pure spin state. Wobble damping requirements consists of an acceptable wobble level along with the time required to damp within the acceptable range. The term wobble level is used synonymously with wobble rate, or the oscillatory component of angular rate viewed on axes perpendicular to the nominal spin axis (i.e. transverse axes).

After reviewing the available control system requirements (as stated in References 4 and 5) and the physiological requirements (established in Reference 6) it is determined that the experimental requirement for a 10^{-5}g environment imposes the most stringent restriction on the wobble level. Table 5.1 contains a summary of the Navigation and Control requirements for an artificial gravity space station.

In order to obtain a specification for the maximum acceptable wobble level (ω_T) based upon the 10^{-5}g requirement, the following inequality is solved:

$$|r\,\dot{\omega}_T| \leq 10^{-5}g \tag{5-1}$$

Where, r = maximum distance from CM of total space station to the farthest point in the hub (feet)

$\dot{\omega}_T$ = transverse angular acceleration (rad/sec)

It should be noted that the $\omega_T^2 r$ contribution to the total linear acceleration is negligible (i.e., $|\dot{\omega}_T| \gg \omega_T^2$) during the periods of interest characterized by low amplitude wobble. Making the appropriate substitutions in (5-1) for $\dot{\omega}_T$ in terms of the wobble rate, the spin rate (P) and the inertia properties, and in turn solving for the wobble amplitude yields,

TABLE 5-1

NAVIGATION AND CONTROL REQUIREMENTS

N and C Requirement	Source	Reference
Maintain Attitude Hold and Rate Stabilization for 30 days: ±0.1 deg, ±.003 deg/sec	MDAC	(4)
Wobble Limit Art-G Experiment: ±0.1 deg/sec	NR	(4)
General Experimental Requirements:	--	(4)
Acceleration Level <.01 g 100% of time <.00001 g 95% of time <.0001 g 5% of time		
Physiological:	TRW	(6)
Ang. Acc. <0.15 deg/sec^2		
Wobble Ang. <15 deg		
Vibration <.005 in. displacement <.01 g pk acceleration		
Routine Operations Attitude Hold: ±0.25° Rate Stability: ±0.05°/sec	--	(5)

$$\omega_r = \begin{Bmatrix} \alpha\,\Omega \text{ for } \alpha > 1 \\ \Omega \text{ for } \alpha \le 1 \end{Bmatrix} = \text{Maximum Acceptable Wobble Level (rad/sec)}$$

where,

$$\Omega = \frac{10^{-5}\ (32.2)}{[\alpha \pm \lambda]\, pr} \left(\frac{\text{rad}}{\text{sec}}\right)$$

Note: 1. Use minus sign for Min I and plus sign for Max I.

2. The x axis is the spin axis.

$$\alpha = \left[\frac{(Ix - Iz)\ Iz}{(Ix - Iy)\ Iy}\right]^{\frac{1}{2}}$$

$$\lambda = \left[\frac{(Ix - Iz)\ (Ix - Iy)}{Iz \quad Iy}\right]^{\frac{1}{2}}$$

The calculated maximum allowable wobble levels corresponding to the space station configurations studied are presented in Table 5.2.

There is presently no firm requirement for the rate of wobble decay. MSC has been using a 60 sec time constant (τ) for CMG sizing purposes (Reference 7), which is considered a reasonable requirement and will, therefore, be adopted for this study with the following slight modification. Since the particular CMG law to be investigated will exhibit a linear decay, rather than exponential, a total decay time requirement of 180 sec is specified, which is equivalent to 3τ.

TABLE 5.2

WOBBLE RATE SPECIFICATION

Configuration	r (feet)	α	λ	Maximum Allowable Wobble ω_r (rad/sec)
"T"	152.9	5.852	.1636	$.824 \times 10^{-5}$
"T" with Shuttle	247.8	1.067	0.800	1.98×10^{-5}
"Y"	84.5	.967	.3526	1.1×10^{-5}
"Y" with Shuttle	151.4	1.030	.623	2.05×10^{-5}

5.2 Disturbances

Deviations from the desired pure spin motion of the space station will be induced by both internal and external disturbances. The three most significant sources of disturbance torques are:

- Internal Mass Motion
- Shuttle Docking
- Environmental

Although momentum exchange devices are applicable for counteracting the oscillatory portion of environmental disturbances, the effect of these disturbances is a precession of the vehicle's momentum vector, and thus is an attitude hold problem rather than a wobble damping problem. As a result, the study of environmental disturbances is considered beyond the scope of these investigations.

The result of a shuttle impact on the space station will be a displacement of the station in addition to a wobble motion about the new displaced position. RCS jets can be used to restore the vehicle to its original position while a CMG system can be used to damp the wobble motion. It should be noted that the use of CMG's for wobble damping decreases the load on the RCS system and will result in less propellant consumption.

Unbalances in the rotor that displace the CMG from the spin axis will result in unacceptable motion of the station which cannot be compensated for by CMG's, thus a mass-balancing system is required. In addition to introducing large cross-products of inertia, which in a sense will create a new spin axis, rotor unbalances will cause the hub to translate in space resulting in large acceleration levels in a preferred near zero-g environment. However, internal mass motion will also induce a wobble motion superimposed on these other effects. It is only the wobble motion that is considered during the course of this study.

Shuttle Docking

The total angular momentum imparted to the station by a planar shuttle docking is given by,

$$\Delta h_{DOCK} = I_s \omega_s + m_s v_s l_s, \quad \text{or} \tag{5-3}$$

$$\Delta h_{DOCK} = I_s \omega_s + M_x [l_D (v_A \cos \theta_M - v_L \sin \theta_M) +$$

$$l_{CM} (v_A \sin \theta_M + v_L \cos \theta_M)] \tag{5-4}$$

Where all variables are defined in Figure 5-1. Applying the maximum docking parameters and docking port contact requirements, as listed in Figure 5-1 (Reference 4: Table 6.1 - Operational Design, and Section 6.2.1.6), results in the following worst case (or 3 sigma) docking disturbance:

"T" - Δh_{DOCK} = 590,163 ft-lb-sec

"Y" - Δh_{DOCK} = 613,760 ft-lb-sec

The corresponding wobble amplitude for the shuttle/station combination will be .0013 $\frac{\text{rad}}{\text{sec}}$, and .00176 $\frac{\text{rad}}{\text{sec}}$ for the "T" and "Y" configurations, respectively. The CMG analyses, presented in Section 9, considers only a nominal (or 1 sigma) docking disturbance.

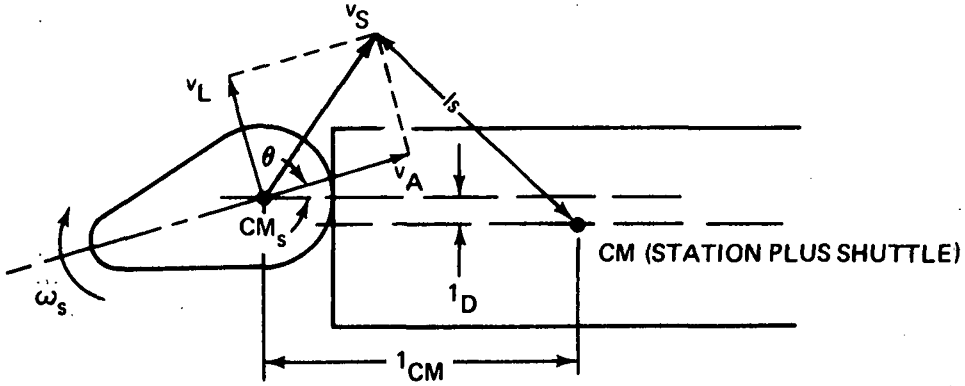
Internal Mass Motion

The wobble amplitude induced by internal mass motion will be a function of the size of the moving mass and its motion profile. MSC has performed a rigid body analysis for CMG sizing purposes using a particular moving mass profile (Reference 7) that induced wobble amplitudes as listed in Table 5-3.

TABLE 5-3

WOBBLE INDUCE BY MASS MOTION

Configuration	Transvers Rate (rad/sec)
"T"	0.4869×10^{-2}
"Y"	0.262×10^{-3}



MAXIMUM DOCKING PARAMETERS (REFERENCE 4):

l_D = 5 INCHES

θ_M = 4 DEG

ω_s = 0.5 DEG/SEC

v_A = 1.0 FPS

v_L = 0.25 FPS

VEHICLE PARAMETERS (SECTION 4)

SHUTTLE INERTIA = I_s = 21.5 X 10^6 SLUG - FT^2

CONFIGURATION	l_{CM} (FEET)	I_{TOTAL} SHUTTLE/STATION (SLUG – FT^2)
T	126.5	4.39 X 10^8
Y	133.99	3.49 X 10^8

1 SIGMA WOBBLE AMPLITUDE

CONFIGURATION	$\frac{(\Delta h_{DOCK})_{1\sigma}}{I_{TOTAL}}$ (RAD/SEC)
T	0.43 X 10^{-3}
Y	0.59 X 10^{-3}

Figure 5-1. Shuttle Docking

These same wobble amplitudes are used as initial conditions for the flexible body analyses performed as part of the present study. The rationale for using identical initial conditions is that the angular momentum transferred to a rigid vehicle by internal mass motion will be approximately equal to the momentum transferred to a flexible vehicle.

6.0 Vehicle Math Model

6.1 Equations of Motion

This section contains the equations of motion of the flexible space station. The analytical formulation of these equations was based upon a model consisting of the following components (See Figure 6-1):

- A rigid section of hub with 6 degrees of freedom (6 d.o.f.)
- A flexible section of hub containing p masses interconnected by massless structure (6p d.o.f.)
- A rigid rotor ring pinned to the rigid section of hub by means of a flexible bearing (3 d.o.f.)
- q flexible rotor arms, each with s masses interconnected by massless structure (6qs d.o.f.)
- A rigid shuttle pinned to the rigid section of hub by means of a flexible docking mechanism (3 d.o.f.)

The equations of motion are presented in Equations 6-1 through 6-8; associated relationships required for the formation of these equations are presented in Equations 6-9 through 6-31. Table 6-1 contains the definitions for all notations. These equations were linearized only with respect to those coordinates which describe the flexible motion of the vehicle; all other nonlinear effects were retained.[1] Mass properties of all bodies were completely general; that is, no assumptions were made regarding symmetries, position of mass centers, etc. Control torques were included on all masses.

Equation 6-43 contains the matrix equations of motion for the two vehicle configurations used in the simulation study[2]; the characteristics of these vehicles are as follows:

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[1] - Linearization of the flexible motion was not total; that is, nonlinear terms were eliminated only if significant simplification resulted.

[2] - Actually, Equation 6-43 can be used for any vehicle, with or without shuttle, for which $p = 3$ and $q = 2$ or 3.

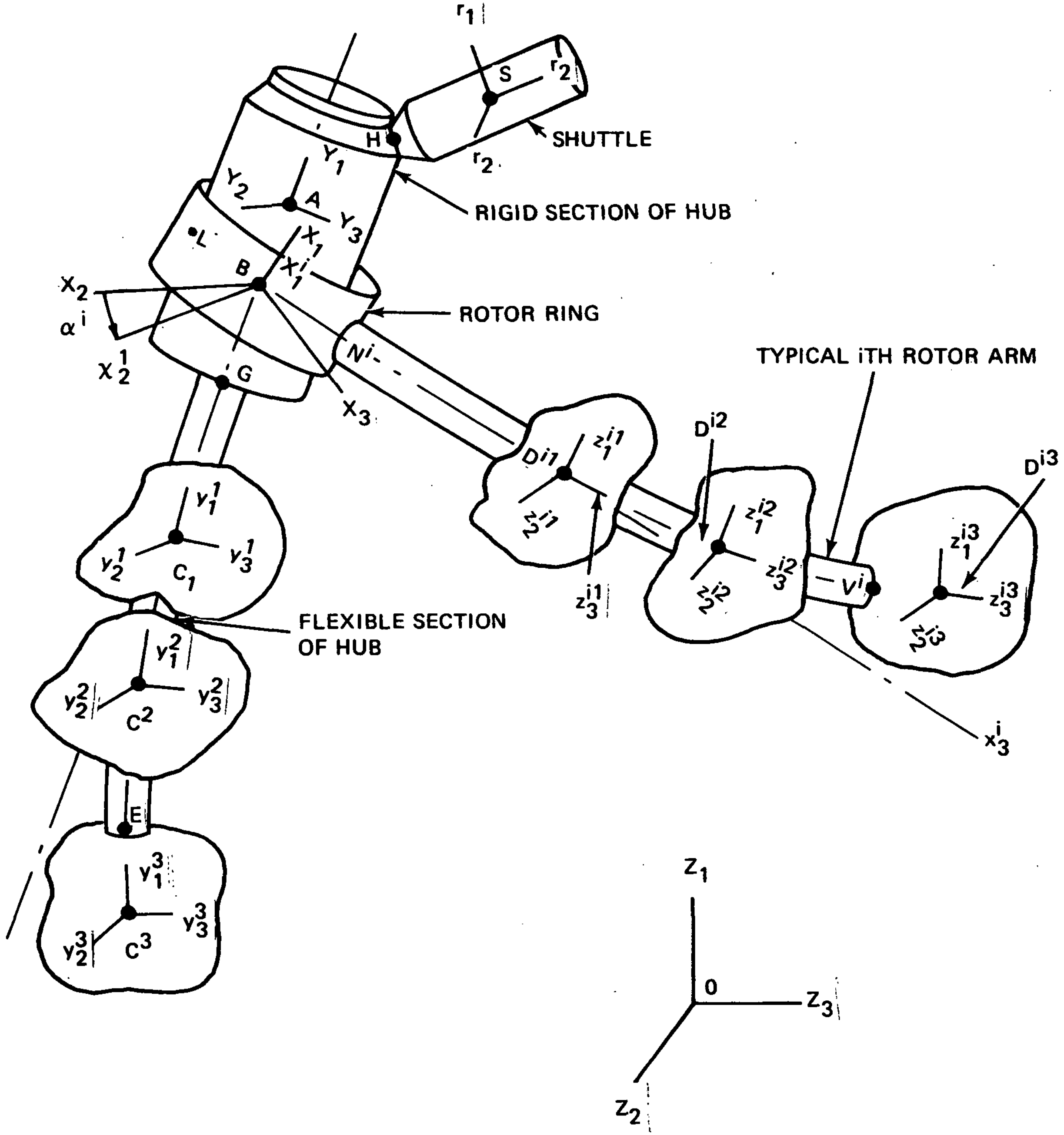

Figure 6-1. Vehicle Idealization

- The flexible section of hub consists of three masses ($p = 3$), the inner two of which are mass points having rotary inertia. The five reference points in the hub, namely the four mass centers and the bearing connection point, are aligned in the undeformed vehicle.
- The two types of symmetric rotors studied are designated as T and Y. The T rotor has two identical arms ($q=2$) 180° apart, and the Y rotor has three identical arms ($q=3$) 120° apart. In both cases, a rotor arm consists of three masses ($s=3$), the inner two of which are mass points having rotary inertia. All mass centers, including that of the rotor ring, lie in a plane; also, the rotor ring mass center coincides with the bearing connection point.
- A docked shuttle may or may not be present; the docking mechanism exhibits elastic and viscous effects (see Equation 6-37)
- The flexible bearing between the hub and the rotor ring exhibits elastic and viscous effects; associated stiffness and damping expressions are given by Equations 6-32 to 6-36. This bearing is discussed in detail in Section 6.3
- The massless structures of the flexible hub and the rotor arms consist of straight beam segments of circular cross-section; the associated stiffness and damping matrices for these structures, discussed in Section 6-2, are integrated into the equations of motion by means of Equations 6-38 to 6-42.

Equations 6-43 contains 84 equations in 84 variables, all of which are needed to describe the motion of the largest simulation vehicle, viz., the vehicle with the Y rotor and the docked shuttle. For computer programming purposes, however, it was decided to preserve the size of the problem when a smaller vehicle was to be studied. This was accomplished by incorporating scalar parameters n_S and n_R into those equations of motion associated with the shuttle and with masses in the third arm of the rotor. For example, when the shuttle is not present ($m_S = 0$, $[I_H]=0$) n_S is set to zero and Euler's equations for the shuttle reduce to $\bar{I}\,\{\ddot{\Psi}\}=0$[1]; then starting from initial conditions $\{\Psi\} = \{\dot{\Psi}\}=0$, the components of $\{\Psi\}$ become meaningless variables with no effect in the analysis. When the shuttle is present, n_S is set to unity and Euler's equations for the shuttle are retained in their usual form. Parameter n_R plays the same role for the third arm of the rotor.

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[1] - Arbitrary scalar $\bar{I}$ is included in order to avoid introducing ill-conditioning effects into the mass matrix.

Although the equations of 6-43 do constitute a legitimate set of equations of motion, they have certain shortcomings for the simulation study. First, they are difficult to use if the motion is constrained in some way, as in the case of a rigid rotor, etc. Second, when used in conjunction with a modal reduction scheme they do not lead to a particularly good reduced set of equations. Both of these difficulties stem from the fact that the forces on the right-hand side of 6-43 do not constitute a set of generalized forces for the problem. Accordingly, it was decided to premultiply both sides of Equation 6-43 by a transformation matrix such that the resulting right-hand side would contain a set of generalized forces[1]. This matrix, denoted as $[Q_{GEN}]$, is presented in Equation (6-44); the associated expressions needed to form $[Q_{GEN}]$ are given by Equation (6-45).

The large size of the problem (84 degrees of freedom) plus the fact that the "λ" and "Q" matrices vary with time made it impractical to use the entire set of equations for the time history study. Accordingly, the modal reduction technique was used to obtain a reduced set of approximate equations in terms of modal coordinates; the degree of approximation depends, of course, on the number of modes used. The essential relationships are presented in Equations (6-46) through (6-49). Equation (6-49) contains two alternate expressions for determining initial values of the modal coordinates. The simpler of the two, (6-49a), was adequate for the needs of the simulation study and was therefore incorporated into the computer program. However, if more complicated initial conditions are to be studied in the future, it would be advisable to use (6-49b).

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[1] - Specifically, the resulting set of generalized forces are associated with the following coordinates: $\{Z_A\}$, $\{\gamma\}$, $\{\Psi\}$, $\{u_C 1\}$, $\{\phi_C 1\}$, ...$\{u_C 3\}$, $\{\phi_C 3\}$, $\{\theta\}$, $\{u_D 11\}$, $\{\phi_D 11\}$, ...$\{u_D 33\}$, $\{\phi_D 33\}$.

TABLE 6-1

NOTATION

Points

A	Mass center of rigid section of hub
B	Fixed point in hub about which rotor rotates
C^j	Mass center of j^{th} mass module in flexible section of hub
D^{ik}	Mass center of k^{th} module in i^{th} rotor arm
E	Connection between flexible section of hub and end module of hub
G	Connection between flexible section of hub and right section of hub
H	Connection between hub and shuttle
L	Mass center of rotor ring
N^i	Intersection of rotor ring and i^{th} rotor arm
O	Origin of inertial frame
S	Mass center of shuttle
V^i	Connection between i^{th} rotor arm and associated end module

Coordinate Frames

$\underset{\sim}{Z}$	Inertial frame fixed at O
$\underset{\sim}{Y}$	Frame fixed in rigid section of hub at A; the Y_1 axis is parallel to the nominal spin axis of hub
$\underset{\sim}{X}$	Frame fixed in rotor ring at B; the X_1 axis is noninally aligned with Y_1.
$\underset{\sim}{x}^i$	Frame fixed in rotor ring at B; x_1^i, coincides with X_1 and z_3^i is nominally aligned with i^{th} rotor arm
$\underset{\sim}{y}^j$	Frame fixed in j^{th} mass module of flexible hub at C^j; this frame is nominally aligned with $\underset{\sim}{Y}$
$\underset{\sim}{z}^{ik}$	Frame fixed in k^{th} mass module of i^{th} rotor arm at D^{ik}, this frame is nominally aligned with x^i
$\underset{\sim}{S}$	Frame fixed in shuttle at S.

Displacements

$\{\gamma\}$	Ordered rotation which comprise transformation from $\underset{\sim}{Z}$ to $\underset{\sim}{Y}$; i) γ_1 about Z_1, ii) γ_2 about "carried" Z_2, iii) γ_3 about "twice carried" Z_3
$\{\theta\}$	Ordered rotations comprising transformation from $\underset{\sim}{Y}$ to $\underset{\sim}{X}$
α^i	Constant rotation about X_1 transforming from $\underset{\sim}{X}$ to $\underset{\sim}{z}^i$
$\{\phi_C{}^j\}$	small elastic rotations comprising transformation from $\underset{\sim}{Y}$ to $\underset{\sim}{y}^j$
$\{\phi_D{}^{ik}\}$	small elastic rotations comprising transformation from $\underset{\sim}{x}^i$ to $\underset{\sim}{z}^{ik}$
$\{\beta\}$	Constant ordered rotations which comprise transformation from $\underset{\sim}{Y}$ to nominal position of $\underset{\sim}{r}$
$\{\Psi\}$	Small elastic rotations comprising transformation from nominal position of $\underset{\sim}{r}$ to instantaneous position of $\underset{\sim}{r}$
$\{u_C{}^j\}$	Small elastic displacements of C^j measured in the $\underset{\sim}{Y}$ frame
$\{u_D{}^{ik}\}$	Small elastic displacements of D^{ik} measured in the x^i frame

Angular Velocities

$\{\omega^Y\}$	Angular velocity of $\underset{\sim}{Y}$ frame projected onto itself
$\{\omega^X\}$	Angular velocity of $\underset{\sim}{X}$ frame projected onto itself
$\{\omega^{x^i}\}$	Angular velocity of $\underset{\sim}{x}^i$ frame projected onto itself
$\{\omega^{y^i}\}$	Angular velocity of $\underset{\sim}{y}^j$ frame projected onto itself
$\{\omega^{z^{ik}}\}$	Angular velocity of $\underset{\sim}{z}^{ik}$ frame projected onto itself
$\{\omega^r\}$	Angular velocity of $\underset{\sim}{r}$ frame projected onto itself

Transformation Matrices

$[a(\gamma)]$, $[a(\theta)]$,...	transformation matrices associated with γ_1, θ_1,...
$[b(\gamma)]$, $[b(\theta)]$,...	transformation matrices associated with γ_2, θ_2,...
$[c(\gamma)]$, $[c(\theta)]$,...	transformation matrices associated with γ_3, θ_3,...
$[\pi(\gamma)]$	transformation matrix going from $\underset{\sim}{Z}$ to $\underset{\sim}{Y}$; $[\pi(\gamma)] = [c(\gamma)]\,[b(\gamma)]\,[a(\gamma)]$

$[\pi(\theta)]$	transformation matrix going from $\underset{\sim}{Y}$ to $\underset{\sim}{X}$
$[\pi(\phi_C j)]$	transformation matrix going from $\underset{\sim}{Y}$ to $\underset{\sim}{y}^j$
$[\pi(\phi_D ik)]$	transformation matrix going from $\underset{\sim}{x}^i$ to $\underset{\sim}{z}^{ik}$
$[\pi(\beta)]$	transformation matrix going from $\underset{\sim}{Y}$ to nominal position of $\underset{\sim}{r}$
$[\pi(\psi)]$	transformation matrix going from nominal position of $\underset{\sim}{r}$ to final $\underset{\sim}{r}$
$[a(\alpha^i)]$	tranformation matrix going from $\underset{\sim}{x}$ to $\underset{\sim}{x}^i$

Physical Properties

m_A	mass of rigid hub section
m_{Ci}	mass of jth module in flexible part of hub
m_L	mass of rotor ring
$m_D ik$	mass of kth module in ith rotor arm
m_S	mass of shuttle
$[I_A]$	inertia matrix of m_A with respect to A (in $\underset{\sim}{Y}$ frame)
$[I_C j]$	inertia matrix of $m_C j$ with respect to C^j (in $\underset{\sim}{y}^j$ frame)
$[I_B]$	inertia matrix of m_L with respect to B (in $\underset{\sim}{X}$ frame)
$[I_D ik]$	inertia matrix of $m_D ik$ with respect to D^{ik} (in $\underset{\sim}{z}^{ik}$ frame)
$[I_H]$	inertia matrix of m_S with respect to H (in $\underset{\sim}{r}$ frame)

Forces and Couples

(a) Structural

$-\{F_C j\}$	structural force acting on $m_C j$ (in $\underset{\sim}{Y}$ frame)
$-\{F_D ik\}$	structural force acting on $m_D ik$ (in $\underset{\sim}{x}^i$ frame)
$-\{F_G\}$	structural force exerted by flexible hub on m_A at point G (in $\underset{\sim}{Y}$ frame)
$-\{M_C j\}$	structural moment acting on $m_C j$ about C^j (in $\underset{\sim}{Y}$ frame)
$-\{M_D ik\}$	structural moment acting on $m_D ik$ about D^{ik} (in $\underset{\sim}{x}^i$ frame)
$-\{M_B\}$	structural moment (exerted by all rotor arms) acting on m_L about point B (in $\underset{\sim}{X}$ frame)
$-\{M_G\}$	structural moment (exerted by flexible hub) acting on m_A about point G (in $\underset{\sim}{Y}$ frame)

(b) Controls

$\{\tau_c j\}$	control moment applied to $m_c j$ (in $\underset{\sim}{y}^j$ frame)
$\{\tau_D ik\}$	control moment applied to $m_D ik$ (in $\underset{\sim}{z}^{ik}$ frame)
$\{\tau_S\}$	control moment applied to m_S (in $\underset{\sim}{r}$ frame)
$\{\tau_A\}$	control moment applied to m_A (in $\underset{\sim}{Y}$ frame)
$\{\tau_L\}$	control moment applied to m_L (in $\underset{\sim}{X}$ frame)

(c) Bearing and Docking

$\{M_{BEAR}\}$	total moment exerted by bearing on rotor ring (in $\underset{\sim}{X}$ frame)
$\{M_{BS}\}$	elastic part of $\{M_{BEAR}\}$
$\{M_{BV}\}$	viscous part of $\{M_{BEAR}\}$
$\{M_{DOCK}\}$	moment exerted by docking mechanism on m_S about point H (in $\underset{\sim}{r}$ frame)

Stiffnesses, Viscosities, etc.

K_{BS1}, K_{BS2}	linear stiffness coefficients of bearing
$[K_{BV}]$	bearing damping matrix
$[K_{BVH}]$	bearing damping matrix when all damping material is fixed to hub
$[K_{BVR}]$	bearing damping matrix when all damping material is fixed to rotor
$[K_{DOCK}]$	stiffness matrix of shuttle docking mechanism
$[C_{DOCK}]$	damping matrix of shuttle docking mechanism
[KTA], [KYA]	stiffness matrices of "T" and "Y" rotors, respectively
[CTA], [CYA]	damping matrices of "T" and "Y" rotors, respectively
[K6HG], [K7H]	hub stiffness matrices
[CHG], [CH]	hub damping matrices

Miscellaneous

$\{X_B\}$, $\{Y_B\}$, etc.	coordinates of any point (here taken to be B) in the $\underset{\sim}{X}$, $\underset{\sim}{Y}$, etc. coordinate frames

$[\Gamma(\lambda)]$	cross-product matrix, formed from the elements of one vector, say $\underset{\sim}{\lambda}$, involved in the cross-product, e.g., if $\underset{\sim}{\lambda} = \begin{Bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{Bmatrix}$ then $[\Gamma(\lambda)] = \begin{bmatrix} 0 & -\lambda_3 & \lambda_2 \\ \lambda_3 & 0 & -\lambda_1 \\ -\lambda_2 & \lambda_1 & 0 \end{bmatrix}$
$[S(\gamma)]$, $[S(\theta)]$	matrices used in conversion between angular velocities $\{\omega^Y\}$, $\{\omega^X\}$ and rates of Euler angles $\{\dot{\gamma}\}$, $\{\dot{\theta}\}$
Ω	nominal spin speed of rotor
$\dot{\delta}$	instantaneous spin speed of rotor minus nominal spin speed of rotor
$\{\nabla\}$	similar to $\{\theta\}$ but with δ replacing θ_1
δ_B	angular misalignment beteen Y_1 and X_1 axes
δ_B^*	value of δ_B below which K_{BS1} applies and above which K_{BS2} applies
η	scalar constant between zero and one; $\eta = 1$ if all bearing damping material is attached to hub $\eta = 0$ if all bearing damping material is attached to rotor ring
n_R	$n_R = 1$ for a three arm rotor; $n_R = 0$ for a two arm rotor
n_S	$n_S = 1$ when a docked shuttle is present; $n_S = 0$ when no shuttle is present
$\bar{m}$	dummy mass value used when $n_R = 0$
$\bar{I}$	dummy inertia value used when $n_R = 0$ or $n_S = 0$
$[\bar{\lambda}]$, $[M]$, $[N(\theta)]$ $[\mu]$, $[J]$, $\{\xi\}$, $\{\Delta\}$ $\{\bar{\sigma}\}$, $\{\rho\}$	miscellaneous combinations of terms appearing in the analysis
$[\lambda]$, $\{\sigma\}$	matrices by means of which the equations of motion are presented
$[Q_{GEN}]$	matrix which premultiplies the equations of motion so as to eliminate forces of constraint
$[A^*]$, $[B^*]$, $\ldots[J^*]$	matrices needed to construct $[Q_{GEN}]$

EQUATIONS OF MOTION

I. Newton's equations for the entire vehicle; all vectors projected onto $\underset{\sim}{Y}$ axes

$$[\lambda_1]\{\ddot{Z}_A\} + [\lambda_2]\{\dot{\omega}^Y\} + \sum_{j=1}^{\ell} [\lambda_3^j]\{\ddot{u}_{cj}\} + [\lambda_4]\{\ddot{\nabla}\} + \sum_{i=1}^{\ell} \sum_{k=1}^{\Delta} [\lambda_5^{ik}]\{\ddot{\mu}_{D^{ik}}\} + [\lambda_6]\{\ddot{\Psi}\} = \{\sigma_1\} \qquad (6\text{-}1)$$

II. Newton's equations for the jth mass of the flexible hub; all vectors projected onto $\underset{\sim}{Y}$ axes

$$[\lambda_7^j]\{\ddot{Z}_A\} + [\lambda_8^j]\{\dot{\omega}^Y\} + [\lambda_9^j]\{\ddot{u}_{cj}\} = \{\sigma_2^j\} \qquad (6\text{-}2)$$

III. Newton's equations for the kth mass of the ith rotor arm; all vectors projected onto $\underset{\sim}{x}^i$ axes

$$[\lambda_{10}^{ik}]\{\ddot{Z}_A\} + [\lambda_{11}^{ik}]\{\dot{\omega}^Y\} + [\lambda_{12}^{ik}]\{\ddot{u}_D^{ik}\} + [\lambda_{13}^{ik}]\{\ddot{\nabla}\} = \{\sigma_3^{ik}\} \qquad (6\text{-}3)$$

IV. Euler's equations for the jth mass of the flexible hub; torques taken about point C^j; all vectors projected onto $\underset{\sim}{y}^i$ axes

$$[\lambda_{14}^j]\{\dot{\omega}^Y\} + [\lambda_{15}^j]\{\ddot{\phi}_{cj}\} = \{\sigma_4^j\} \qquad (6\text{-}4)$$

V. Euler's equations for the kth mass of the ith rotor arm; torques taken about point D^{ik}; all vectors projected onto $\underset{\sim}{z}^{ik}$ axes

$$[\lambda_{27}^{ik}]\{\dot{\omega}^Y\} + [\lambda_{16}^{ik}]\{\ddot{\nabla}\} + [\lambda_{17}^{ik}]\{\ddot{\phi}_D^{ik}\} = \{\sigma_5^{ik}\} \qquad (6\text{-}5)$$

VI. Euler's equations for the shuttle; torques taken about point H; all vectors projected onto $\underset{\sim}{r}$ axes

$$[\lambda_{18}]\{\ddot{Z}_A\} + [\lambda_{19}]\{\dot{\omega}^Y\} + [\lambda_{20}]\{\ddot{\psi}\} = \{\sigma_6\} \tag{6-6}$$

VII. Euler's equations for the rotor ring; torques taken about point B; all vectors projected onto $\underset{\sim}{X}$ axes

$$[\lambda_{21}]\{\ddot{Z}_A\} + [\lambda_{22}]\{\dot{\omega}^Y\} + [\lambda_{23}]\{\ddot{\nabla}\} = \{\sigma_7\} \tag{6-7}$$

VIII. Euler's equations for the rigid hub/shuttle combination; torques taken about point B; all vectors projected onto $\underset{\sim}{Y}$ axes

$$[\lambda_{24}]\{\ddot{Z}_A\} + [\lambda_{25}]\{\dot{\omega}^Y\} + [\lambda_{26}]\{\ddot{\psi}\} = \{\sigma_8\} \tag{6-8}$$

Essential Equations and Definitions

The following set of relationships are required for the calculation of the "λ" and "σ" matrices which constitute the fundamental matrix equations of motion, 6-1 through 6-8.

Coordinate Transformation Matrices

$$[a(\gamma)] \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma_1 & \sin\gamma_1 \\ 0 & -\sin\gamma_1 & \cos\gamma_1 \end{bmatrix} \qquad (6\text{-}9a)$$

$$[b(\gamma)] \quad \begin{bmatrix} \cos\gamma_2 & 0 & -\sin\gamma_2 \\ 0 & 1 & 0 \\ \sin\gamma_2 & 0 & \cos\gamma_2 \end{bmatrix} \qquad (6\text{-}9b)$$

$$[c(\gamma)] \quad \begin{bmatrix} \cos\gamma_3 & \sin\gamma_3 & 0 \\ -\sin\gamma_3 & \cos\gamma_3 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (6\text{-}9c)$$

$$[\pi(\gamma)] = [c(\gamma)]\,[b(\gamma)]\,[a(\gamma)] \qquad (6\text{-}10)$$

Relationships between angular velocities and time rates of Euler angles

$$\{\dot{\gamma}\} = [S(\gamma)]^{-1}\{\omega^{Y}\} \tag{6-11}$$

$$\{\dot{\theta}\} = [S(\theta)]^{-1}\left(\{\omega^{X}\} - [\pi(\gamma)]\{\omega^{Y}\}\right) \tag{6-12}$$

$$\{\omega^{x^{i}}\} = [a(\alpha^{i})]\{\omega^{X}\} \tag{6-13}$$

$$\{\omega^{r}\} = [J]\{\omega^{Y}\} + \{\dot{\psi}\} \tag{6-14}$$

$$\{\omega^{y^{j}}\} = [\pi(\phi_{C}j)]\{\omega^{Y}\} + \{\dot{\phi}_{C}j\} \tag{6-15}$$

$$\{\omega^{z^{ik}}\} = [\pi(\phi_{D}ik)][a(\alpha^{i})]\{\omega^{X}\} + \{\dot{\phi}_{D}ik\} \tag{6-16}$$

$$S(\theta) \begin{bmatrix} \cos\theta_3 \cos\theta_2 & \sin\theta_3 & 0 \\ -\sin\theta_3 \cos\theta_2 & \cos\theta_3 & 0 \\ \sin\theta_2 & 0 & 1 \end{bmatrix} \tag{6-17}$$

$$[S(\theta)]^{-1} = \frac{1}{\cos\theta_2} \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 \\ \sin\theta_3 \cos\theta_2 & \cos\theta_3 \cos\theta_2 & 0 \\ -\cos\theta_3 \sin\theta_2 & \sin\theta_3 \sin\theta_2 & \cos\theta_2 \end{bmatrix} \tag{6-18}$$

Miscellaneous Expressions

$$\{\nabla\} = \begin{Bmatrix} \delta \\ \theta_2 \\ \theta_3 \end{Bmatrix} \tag{6-19}$$

$$\{\dot{\theta}\} = \{\dot{\nabla}\} + \Omega \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} - [\mu]\,[\pi(\theta)]\,\{\mu^{Y}\} \tag{6-20}$$

$$[J] = [\pi(\psi)]\,[\pi(\beta)] \tag{6-21}$$

$$[N(\theta)] = [1] - [S(\theta)]\,[\mu] \tag{6-22}$$

$$[\mu] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{6-23}$$

$$\{\rho\} = [N(\theta)]\,[\dot{\pi}(\theta)]\,\{\mu^{Y}\} + [S(\theta)]\,\{\dot{\theta}\} \tag{6-24}$$

$$[\dot{\pi}(\theta)] = [\Gamma(\,[S(\theta)]\,\{\dot{\theta}\}]\,[\pi(\theta)] \tag{6-25}$$

$$[\dot{S}(\theta)] = \begin{bmatrix} 0 & \dot{\theta}_3 & 0 \\ -\dot{\theta}_3 & 0 & 0 \\ \dot{\theta}_2 & 0 & 0 \end{bmatrix} \tag{6-26}$$

$$\left.\begin{aligned}
[M_1] &= -[\pi(\theta)]^T[r(\chi_L)] \\
[M_2^{ik}] &= \left([a(\alpha^i)][\pi(\theta)]\right)^T \\
[M_3^{ik}] &= -[M_2^{ik}][r(\chi_D{}^{ik} + \mu_D{}^{ik})][a(\alpha^i)] \\
[M_4] &= [J]^T[r(r_H)][J] - [r(Y_H)] \\
[M_5] &= [J]^T[r(r_H)]
\end{aligned}\right\} \text{(6-27}$$

$$\left.\begin{aligned}
\{\zeta_1{}^j\} &= 2[r(\omega^Y)]\{\dot{\mu}_C{}^j\} + [r(\omega^Y)]^2\{Y_C{}^j + \mu_C{}^j\} \\
\{\zeta_2\} &= [r(\omega^Y)]^2\{Y_B\} + [\pi(\theta)]^T[r(\omega^X)]^2\{\chi_L\} \\
\{\zeta_3^{ik}\} &= [r(\omega^Y)]^2\{Y_B\} + [M_2^{ik}](2[r(\omega^{X^i})]\{\dot{\mu}_D{}^{ik}\} + [r(\omega^{X^i})]^2\{\chi_D^i{}^{ik} + \mu_D{}^{ik}\}) \\
\{\zeta_4\} &= [r(\omega^Y)]^2\{Y_H\} - [J]^T([r(\omega^r)]^2\{r_H\} + [r(r_H)][r(\dot{\varphi})][\pi(\beta)]\{\omega^Y\})
\end{aligned}\right\} \text{(6-28)}$$

$$\left.\begin{aligned}
\{\Delta_1\} &= \sum_{j=1}^{\theta}(m_C{}^j\{\zeta_1^j\}) + m_L\{\zeta_2\} + \sum_{i=1}^{4}\sum_{k=1}^{\Delta}(m_D{}^{ik}\{\zeta_3^{ik}\}) + m_S\{\zeta_4\} \\
\{\Delta_2^j\} &= m_C{}^j\{\zeta_1^j\} \\
\{\Delta_3^{ik}\} &= m_D{}^{ik}[a(\alpha^i)][\pi(\theta)]\{\zeta_3^{ik}\} \\
\{\Delta_4^j\} &= -[I_C{}^j][r(\dot{\phi}_C{}^j)]\{\omega^Y\} + [r(\omega^{y^j})][I_C{}^j]\{\omega^{y^j}\} \\
\{\Delta_5^{ik}\} &= -[I_D{}^{ik}][r(\dot{\phi}_D{}^{ik})][a(\alpha^i)]\{\omega^X\} + [r(\omega^{z^{ik}})][I_D{}^{ik}]\{\omega^{z^{ik}}\} \\
\{\Delta_6\} &= -m_S[r(r_H)][J][r(\omega^Y)]^2\{Y_H\} - [I_H][r(\dot{\varphi})][\pi(\beta)]\{\omega^Y\} + [r(\omega^r)][I_H]\{\omega^r\} \\
\{\Delta_7\} &= m_L[r(\chi_L)][\pi(\theta)][r(\omega^Y)]^2\{Y_B\} + [r(\omega^X)][I_B]\{\omega^X\} \\
\{\Delta_8\} &= [r(\omega^Y)][I_A]\{\omega^Y\} + [J]^T([r(\omega^r)][I_H]\{\omega^r\} - [I_H][r(\dot{\varphi})][\pi(\beta)]\{\omega^Y\}) \\
&\quad + m_S[r(Y_H - Y_B)]\{\zeta_4\} - m_S[J]^T[r(r_H)][J][r(\omega^Y)]^2\{Y_H\}
\end{aligned}\right\} \text{(6-2}$$

Tabulated "σ" Vectors

$$\{\bar{\sigma}_1\} = -\{\Delta_1\}$$

$$\{\sigma_1\} = \{\bar{\sigma}_1\} - [\bar{\lambda}_4]\{\rho\}$$

$$\{\sigma_2{}^j\} = -\{\Delta_2^j\} - \{F_C\, j\}$$

$$\{\bar{\sigma}_3^{ik}\} = -\{\Delta_3{}^{ik}\} - \{F_D{}^{ik}\}$$

$$\{\sigma_3{}^{ik}\} = \{\bar{\sigma}_3{}^{ik}\} - [\bar{\lambda}_{13}^{ik}]\{\rho\}$$

$$\{\sigma_4{}^j\} = -\{\Delta_4^j\} - [\pi(\phi_C\, j)]\{M_C\, j\} + \{\tau_C\, j\}$$

$$\{\bar{\sigma}_5{}^{ik}\} = -\{\Delta_5{}^{ik}\} - [\pi(\phi_D\, ik)]\{M_D\, ik\} + \{\tau_D\, ik\}$$

$$\{\sigma_5{}^{ik}\} = \{\bar{\sigma}_5{}^{ik}\} - [\bar{\lambda}_{16}^{ik}]\{\rho\}$$

$$\{\sigma_6\} = -\{\Delta_6\} + \{\tau_S\} + \{M_{DOCK}\} \qquad (6\text{-}30)$$

$$\{\bar{\sigma}_7\} = -\{\Delta_7\} - \{M_B\} + \{\tau_L\} + \{M_{BEAR}\}$$

$$\{\sigma_7\} = \{\bar{\sigma}_7\} - [\bar{\lambda}_{23}]\{\rho\}$$

$$\{\sigma_8\} = -\{\Delta_8\} + \{\tau_A\} + [J]^T\{\tau_S\} - [\pi(\theta)]^T\{M_{BEAR}\} - \{M_G\} - [\Gamma(Y_G - Y_B)]\{F_G\}$$

Tabulated "λ" Matrices

$$[\lambda_1] = \left(m_A + \sum_{j=1}^{\ell} m_{cj} + m_L + \sum_{i=1}^{q}\sum_{k=1}^{\Delta} m_{D}{ik} + m_s\right)[\pi(\gamma)]$$

$$[\bar{\lambda}_2] = m_s[M_4] - \left(m_L + \sum_{i=1}^{q}\sum_{k=1}^{\Delta} m_{D}{ik}\right)[\Gamma(Y_B)] - \sum_{j=1}^{\ell}(m_{cj}[\Gamma(Y_{cj} + \mu_{cj})]$$

$$[\lambda_2] = [\bar{\lambda}_2] + [\bar{\lambda}_4][N(\theta)][\pi(\theta)]$$

$$[\lambda_3^{j}] = m_{cj}[1]$$

$$[\bar{\lambda}_4] = m_L[M_1] + \sum_{i=1}^{q}\sum_{k=1}^{\Delta}\left(m_{D}{ik}[M_3^{ik}]\right)$$

$$[\lambda_4] = [\bar{\lambda}_4][S(\theta)]$$

$$[\lambda_5^{ik}] = m_{D}{ik}[M_2^{ik}]$$

$$[\lambda_6] = m_s[M_5]$$

$$[\lambda_7^{j}] = m_{cj}[\pi(\gamma)]$$

$$[\lambda_8^{j}] = -m_{cj}[\Gamma(Y_{cj} + \mu_{cj})]$$

$$[\lambda_9^{j}] = m_{cj}[1] = [\lambda_3^{j}] \tag{6-31}$$

$$[\lambda_{10}^{ik}] = m_{D}{ik}[a(\alpha^i)][\pi(\theta)][\pi(\gamma)]$$

$$[\bar{\lambda}_{11}^{ik}] = -m_{D}{ik}[a(\alpha^i)][\pi(\theta)][\Gamma(Y_B)]$$

$$[\lambda_{11}^{ik}] = [\bar{\lambda}_{11}^{ik}] + [\bar{\lambda}_{13}^{ik}][N(\theta)][\pi(\theta)]$$

$$[\lambda_{12}^{ik}] = m_{D}{ik}[1]$$

$$[\lambda_{12}^{*3k}] = \eta_R[\lambda_{12}^{3k}] + \bar{m}\,(1-\eta_R)[1]$$

$$[\bar{\lambda}_{13}^{ik}] = -m_{D}{ik}[\Gamma(\chi^i_{D}{ik} + \mu_{D}{ik})][a(\alpha^i)]$$

$$[\lambda_{13}^{ik}] = [\bar{\lambda}_{13}^{ik}][S(\theta)]$$

$$[\lambda_{14}^{j}] = [I_{cj}][\pi(\phi_{cj})]$$

$$[\lambda^{j}] = [I\;j]$$

$$\bar{\lambda}_{16}^{ik}] = [I_p ik][\pi(\phi_p ik)][a(\alpha^i)]$$

$$\lambda_{16}^{ik}] = [\bar{\lambda}_{16}^{ik}][S(\theta)]$$

$$[\lambda_{17}^{ik}] = [I_p ik]$$

$$[\lambda_{17}^{*3k}] = \eta_R[\lambda_{17}^{3k}] + \bar{\bar{I}}(1-\eta_R)[1]$$

$$[\lambda_{18}] = -m_s[r(r_H)][J][\pi(\gamma)]$$

$$[\lambda_{19}] = m_s[r(r_H)][J][r(Y_H)] + [I_H][J]$$

(6-31 Continued)

$$[\lambda_{20}] = [I_H]$$

$$[\lambda_{20}^{*'}] = \eta_s[\lambda_{20}] + \bar{\bar{I}}(1-\eta_s)[1]$$

$$[\lambda_{21}] = m_L[r(X_L)][\pi(\theta)][\pi(\gamma)]$$

$$[\bar{\lambda}_{22}] = -m_L[r(X_L)][\pi(\theta)][r(Y_B)]$$

$$[\lambda_{22}] = [\bar{\lambda}_{22}] + [\bar{\lambda}_{23}][N(\theta)][\pi(\theta)]$$

$$[\bar{\lambda}_{23}] = [I_B]$$

$$[\lambda_{23}] = [\bar{\lambda}_{23}][S(\theta)]$$

$$[\lambda_{24}] = (-m_A[r(Y_B)] + m_s[r(Y_H - Y_B)] - m_s[J]^T[r(r_H)][J])[\pi(\gamma)]$$

$$[\lambda_{25}] = [I_A] + [J]^T[I_H][J] + m_s([r(Y_H - Y_B)][M_4] + [J]^T[r(r_H)][J][r($$

$$[\lambda_{26}] = [J]^T[I_H] + m_s[r(Y_H - Y_B)][M_5]$$

$$[\lambda_{27}^{ik}] = [\bar{\lambda}_{16}^{ik}][N(\theta)][\pi(\theta)]$$

Note: $[\lambda_{12}^{*3k}]$, $[\lambda_{17}^{*3k}]$ and $[\lambda_{20}]$ apply to the vehicle simulation study only.

Expressions for Flexible Bearing and Flexible Docking Mechanism

The following equations for the bearing and docking mechanism were used in the vehicle simulation study:

Bearing Damping

$$[K_{BVH}] = [K_{BVR}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \epsilon_B & 0 \\ 0 & 0 & \epsilon_B \end{bmatrix} \qquad (6\text{-}32a)$$

$$[K_{BV}] = \eta [K_{BVH}] + (1-\eta)[\pi(\theta)][K_{BVR}][\pi(\theta)]^T, \quad 0 \le \eta \le 1 \qquad (6\text{-}32b)$$

$$\{M_{BV}\} = -[K_{BV}]\left(\{\omega^X\} - [\pi(\theta)]\{\omega^Y\}\right) \qquad (6\text{-}33)$$

Bearing Stiffness

$$\{\delta_B\} = \text{arc } \cos\left(\cos\theta_2 \cos\theta_3\right) \qquad (6\text{-}34)$$

$$\{M_{BS}\} = -K_{BS1}\begin{Bmatrix} 0 \\ \theta_2 \\ \theta_3 \end{Bmatrix} \text{ for } \delta_B \le \delta_B^* \qquad (6\text{-}35a)$$

$$\{M_{BS}\} = \left((K_{BS_2} - K_{BS1})\frac{\delta_B^*}{\delta_B} - K_{BS2}\right)\begin{Bmatrix} 0 \\ \theta_2 \\ \theta_3 \end{Bmatrix} \text{ for } \delta_B > \delta_B^* \qquad (6\text{-}35b)$$

Note: δ_B^* should not be less than 10^{-6} rad.

Total Bearing Moment

$$\{M_{BEAR}\} = \{M_{BV}\} + \{M_{BS}\} \qquad (6\text{-}36)$$

Docking Moment

$$\{M_{DOCK}\} = -[K_{DOCK}]\{\psi\} - [C_{DOCK}]\{\dot{\psi}\} \qquad (6\text{-}37)$$

Structural Forces

The following expressions are for the hub and rotor structures used in the simulation study:

Hub

$$\left\{\begin{array}{c} F_{G'} \\ \hdashline F_{HUB} \end{array}\right\} = \left[\begin{array}{c} [K6HG] \\ \hdashline [K7H] \end{array}\right]\{q_{HUB}\} + \left[\begin{array}{c} [CHG] \\ \hdashline [CH] \end{array}\right]\{\dot{q}_{HUB}\} \tag{6-38}$$

where

$$\{F_{G'}\} = \left\{\begin{array}{c} F_G \\ M_G \end{array}\right\}, \quad \{F_{HUB}\} = \left\{\begin{array}{c} F_c^1 \\ M_c^1 \\ F_c^2 \\ M_c^2 \\ F_c^3 \\ M_c^3 \end{array}\right\}, \quad \{q_{HUB}\} = \left\{\begin{array}{c} u_c^1 \\ \phi_c^1 \\ u_c^2 \\ \phi_c^2 \\ u_c^3 \\ \phi_c^3 \end{array}\right\} \tag{6-39}$$

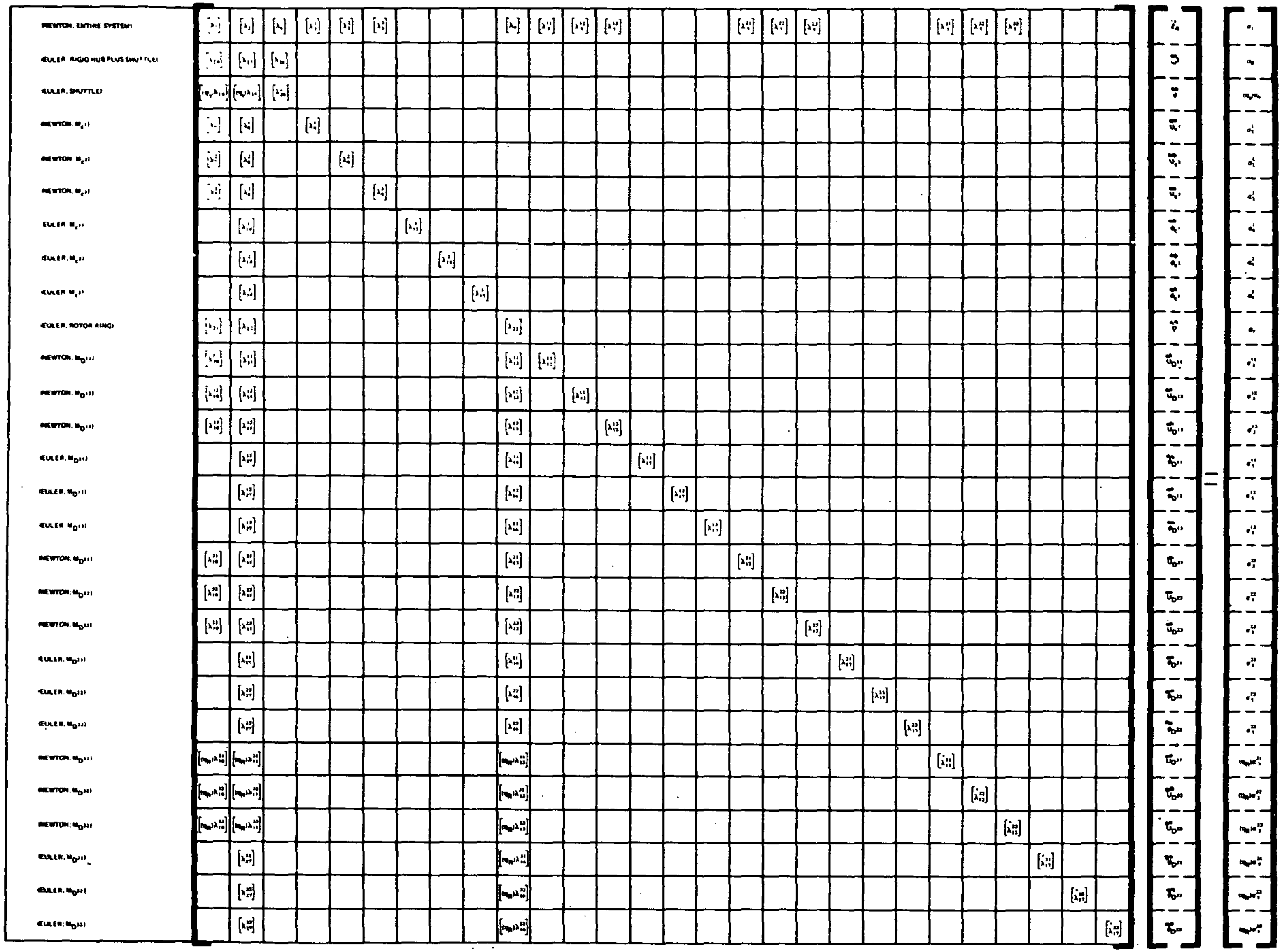
Rotor

$$\{F_{TROT}\} = [KTA]\{q_{TROT}\} + [CTA]\{\dot{q}_{TROT}\} \tag{6-40}$$

$$\{F_{YROT}\} = [KYA]\{q_{YROT}\} + [CYA]\{\dot{q}_{YROT}\} \tag{6-41}$$

where

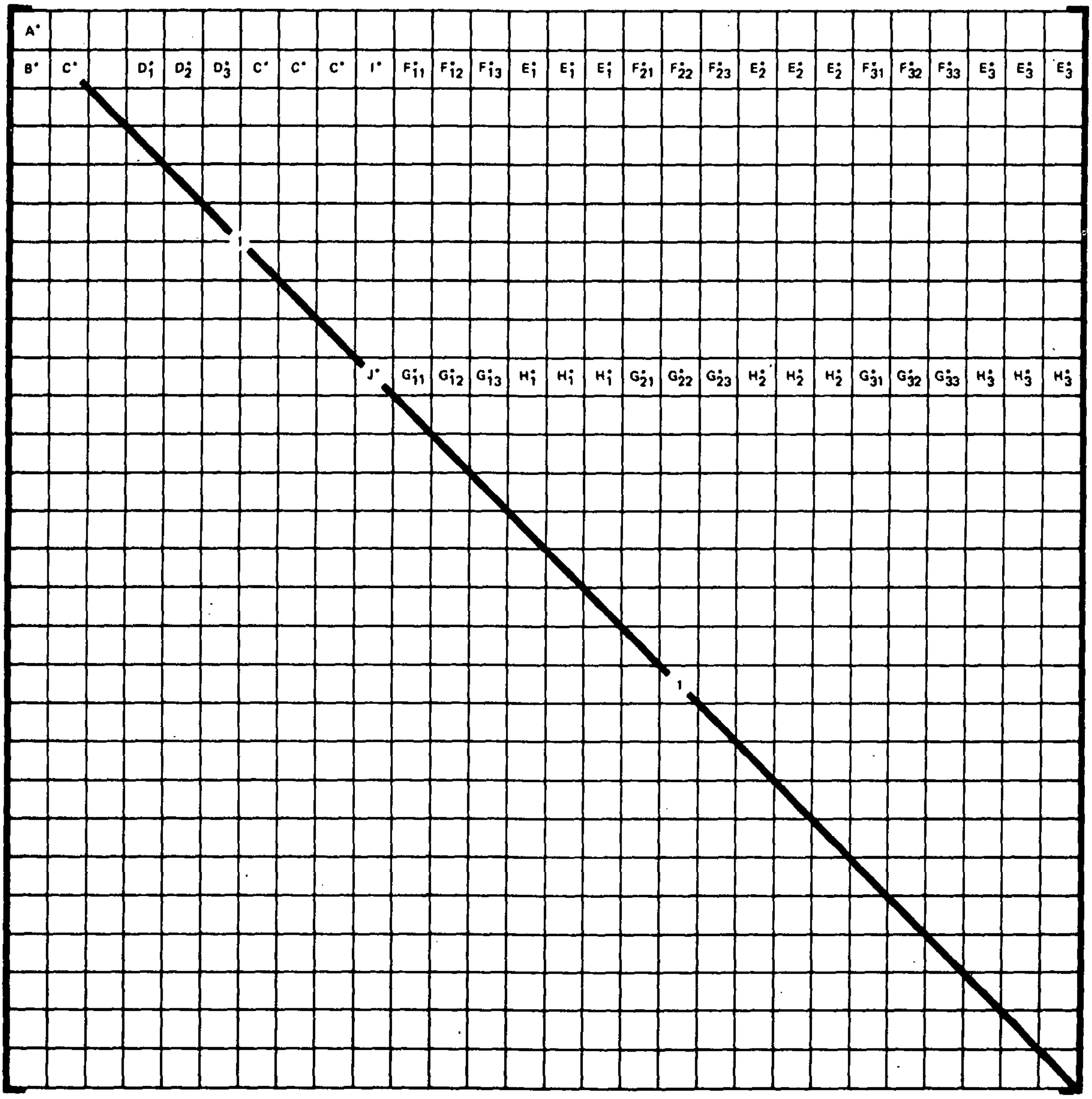
$$\{F_{YROT}\} \left\{\begin{array}{c} \{F_{TROT}\}\left\{\begin{array}{c} M_B \\ F_D^{11} \\ M_D^{11} \\ F_D^{12} \\ M_D^{12} \\ F_D^{13} \\ M_D^{13} \\ F_D^{21} \\ M_D^{21} \\ F_D^{22} \\ M_D^{22} \\ F_D^{23} \\ M_D^{23} \end{array}\right. \\ F_D^{31} \\ M_D^{31} \\ F_D^{32} \\ M_D^{32} \\ F_D^{33} \\ M_D^{33} \end{array}\right\} \qquad \{q_{YROT}\} \left\{\begin{array}{c} \{q_{TROT}\}\left\{\begin{array}{c} \nabla \\ u_D^{11} \\ \phi_D^{11} \\ u_D^{12} \\ \phi_D^{12} \\ u_D^{13} \\ \phi_D^{13} \\ u_D^{21} \\ \phi_D^{21} \\ u_D^{22} \\ \phi_D^{22} \\ u_D^{23} \\ \phi_D^{23} \end{array}\right. \\ u_D^{31} \\ \phi_D^{31} \\ u_D^{32} \\ \phi_D^{32} \\ u_D^{33} \\ \phi_D^{33} \end{array}\right\} \tag{6-42}$$



(6-43)

Matrix Equations of Motion for Simulation Vehicles

$[Q_{GEN}] =$



(6-44)

$[Q_{GEN}]$ MATRIX FOR SIMULATION VEHICLES

Submatrices of $[Q_{GEN}]$

$$[A^*] = [\pi(\gamma)]^T$$

$$[B^*] = [\mathcal{S}(\gamma)]^T[\Gamma(Y_B)]$$

$$[C^*] = [\mathcal{S}(\gamma)]^T$$

$$[D_j^*] = [\mathcal{S}(\gamma)]^T[\Gamma(Y_{cj} + u_{cj} - Y_B)]$$

$$[E_i^*] = [\mathcal{S}(\gamma)]^T[\pi(\theta)]^T[a(\alpha^i)]^T$$

$$[F_{ik}^*] = [E_i^*][\Gamma([a(\alpha^i)][\pi(\theta)]\{Y_B\} + \{\chi_{vik}^i + u_{vik}\})] - [B^*][\pi(\theta)]^T[a(\alpha^i)]$$

$$[G_{ik}^*] = [H_i^*][\Gamma(\chi_{vik}^i + u_{vik})]$$

$$[H_i^*] = [\mathcal{S}(\theta)]^T[a(\alpha^i)]^T$$

$$[I^*] = [\mathcal{S}(\gamma)]^T[\pi(\theta)]^T$$

$$[J^*] = [\mathcal{S}(\theta)]^T \qquad (6\text{-}45)$$

Modal Reduction

Original equations of motion in terms of 84 physical coordinates $\{x\}$:

$$[Q_{GEN}][\lambda]\{\ddot{\chi}\} = [Q_{GEN}]\{\sigma\} \qquad (6\text{-}46)$$

Modal approximation; n modes $[\phi]$; n modal coordinates $\{q\}$:

$$\{x\} = [\phi]\{q\} \tag{6-47}$$

Reduced set of approximate equations in model coordinates:

$$([\phi]^T[Q_{GEN}][\lambda][\phi])\{\ddot{q}\} = [\phi]^T[Q_{GEN}]\{\sigma\} \tag{6-48}$$

Initial conditions in modal coordinates:

$$\{q\} = ([\phi]^T[\phi])^{-1}[\phi]^T\{x\} \tag{6-49a}$$

or

$$\{q\} = ([\phi]^T[Q_{GEN}][\lambda][\phi])^{-1}[\phi]^T[Q_{GEN}][\lambda]\{x\} \tag{6-49b}$$

6.2 Formulation of Structural Characteristics

The details of the procedure by means of which structural modes, frequencies, stiffness matrices and damping matrices were computed are lengthy and are presented in Appendix A. Major features of the formulation are as follows:

- The rigid section of hub was considered to be fixed in space; consequently, cantilever modes were computed for the flexible section of hub.

- Nonrotating modes for the rotors were computed assuming that the rotor ring was pinned to ground (rigid section of hub); rotation of the rotor ring out of the spin plane was resisted by rotory springs having the elastic characteristics of the bearing.

- The stiffening effects due to centrifugal forces in a spinning rotor were accounted for in the analysis by incorporating an incremental beam stiffness matrix; for the modal analysis, the centrifugal forces were represented by a properly oriented set of external loads.

- Due to symmetry of the rotor, some repeated eigenvalues and coupled modes were anticipated. Although mathematically acceptable, this condition could make the mode shapes difficult to interpret. To avoid this problem, rotor motion was divided into its symmetric and antisymmetric parts and two separate eigenvalue problems were solved for each rotor.
- The rotor had one rigid body mode, namely simple rotation in the spin plane. Since the eigenvalue algorithm used could not deal with a zero frequency, it was necessary to eliminate this frequency from the equations. This was accomplished by using a simple frequency shift technique (See Appendix B) rather than the conventional reduction technique.
- With some minor modification, structural damping matrices were based on modal damping.

6.3 Rotor-Hub Bearing Math Model

Calculations for the bearing stiffness and damping parameters are based on an assumed model illustrated in Figure 6-2. In order to assure a tight fit and to compensate for wear, it was assumed that the bearing will be assembled by heating the outer race and permitting it to shrink over the inner bearing structure as it cools. In Section 4.3.2 an approximate analysis was conducted to determine reasonable elastic and damping parameters, K_{BS1} and E_B, for the bearing. It may be assumed that stiffness K_{BS1} is suitable for deflections δ_B below some critical value, δ_B^*. For larger δ_B a section of the elastic dissipative material is assumed to be completely compressed causing bottoming of the outer race against the rotor; hence the structure then has the higher stiffness K_{BS2}. These characteristics may be summarized by the following equations:

$$M_{BS} = K_{BS1}\,\delta_B \quad \text{for } \delta_B \le \delta_B^*$$

$$M_{BS} = [K_{BS2}\,\delta_B - (K_{BS2} - K_{BS1})\,\delta_B^*] \quad \text{for } \delta_B > \delta_B^*$$

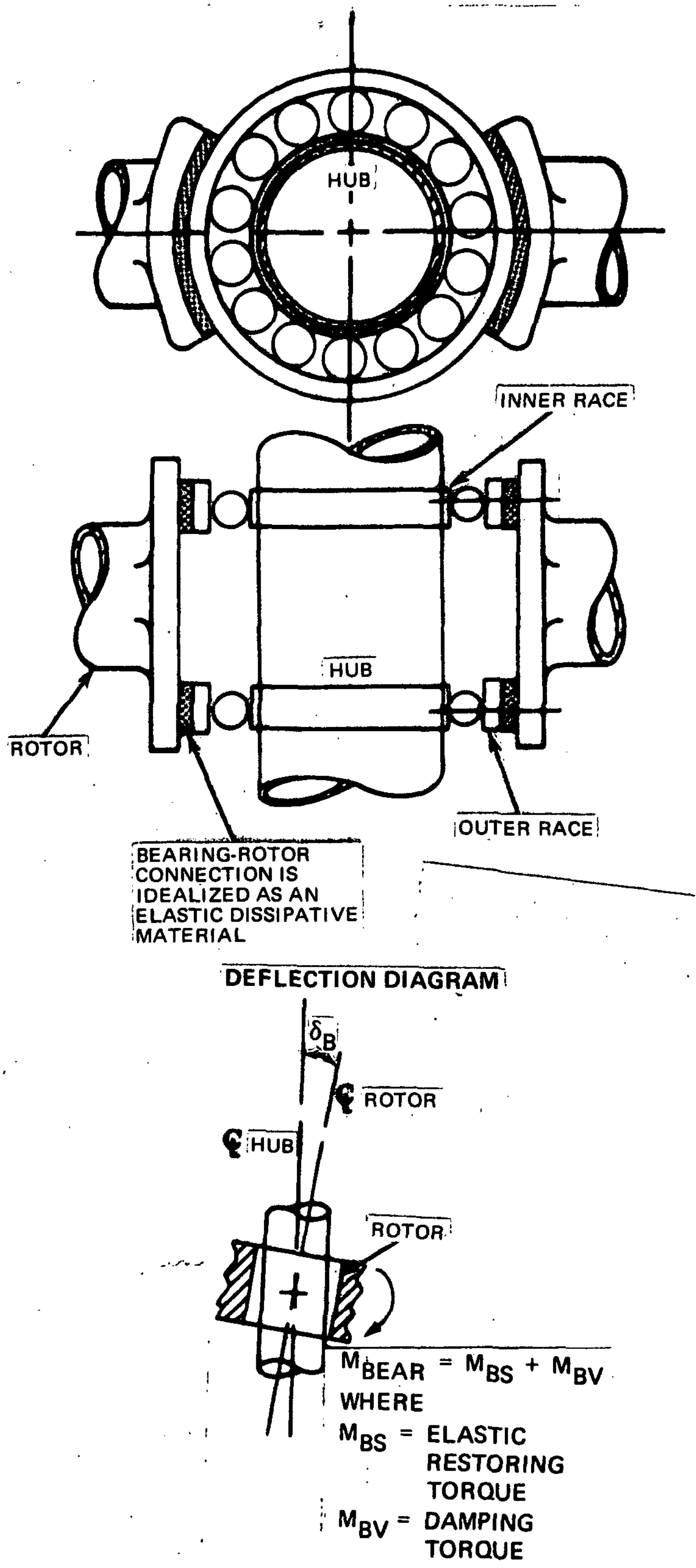


Figure 6-2. Elastic Characteristics of Bearing

By setting $K_{BS1} = 0$, the above equations can also be used to investigate the behavior of a structure with a bearing which is so badly worn that slop occurs.

The damping torque exerted on the rotor by the hub is a function of the spin rates of the viscous materials located in the bearing; also, the damping properties of a bearing with dissipative material which is fixed relative to the hub differ from the damping properties when the dissipative material is fixed relative to the rotor. This point will be explained by using the idealizations illustrated in Figure 6-3.

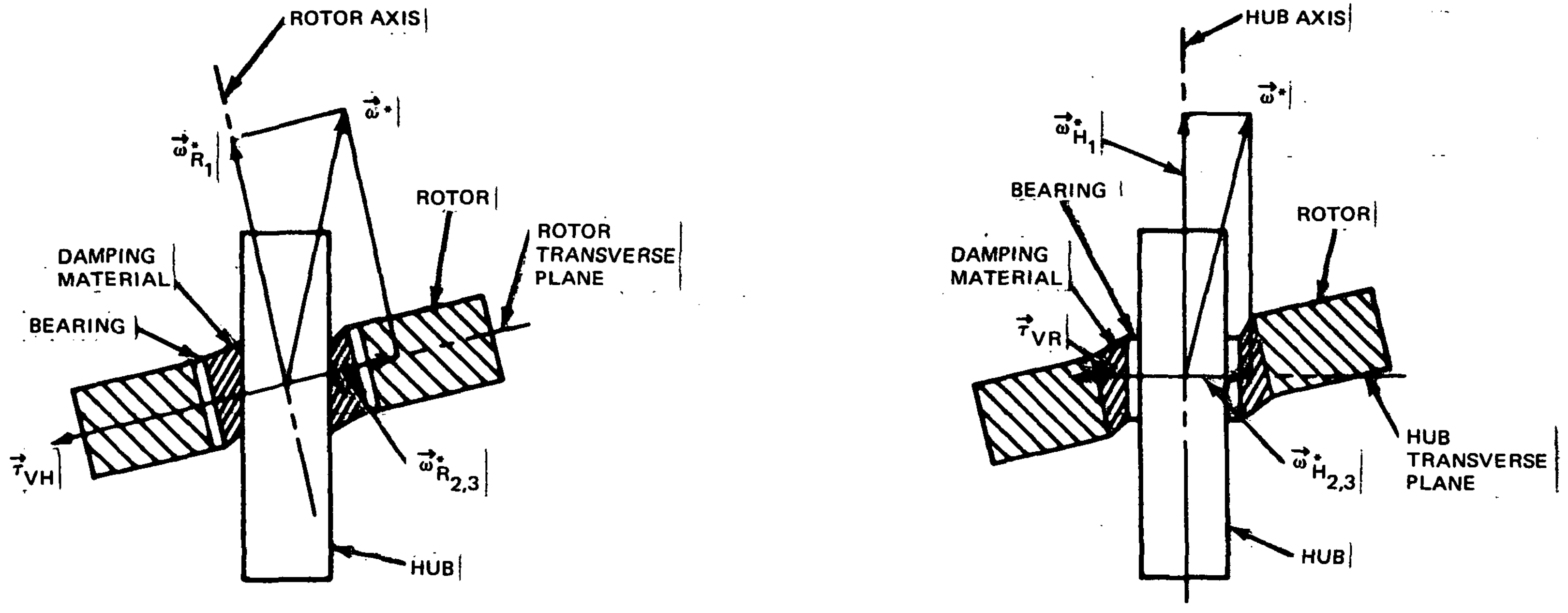
In Figure 6-3a all of the damping material in the bearing is fixed to the hub. A slip ring is assumed to separate the rotor from the non-rotating structure. The vector $\vec{\omega}^*$ denotes the angular velocity of the rotor relative to the hub. If $\vec{\omega}^*$ were directed along the rotor axis ($\vec{\omega}^*_{R_{2,3}} = 0$), the deformed shape of the damping material would not change with time; therefore, no damping would occur. For damping to occur, $\vec{\omega}^*$ must have a nonzero projection on the rotor transverse plane ($\vec{\omega}^*_{R_{2,3}} \neq 0$). Neglecting nonlinear effects, the rate of deformation of the damping material is proportional to $\vec{\omega}^*_{R_{2,3}}$; therefore the damping torque on the rotor is also assumed to be proportional to $\vec{\omega}^*_{R_{2,3}}$ but opposite in direction since it is a restoring torque. Thus, the damping torque on the rotor is given by the following expression when the dissipative material is connected to the hub:

$$\bar{M}_{BVH} = -E_B \vec{\omega}^*_{R_{2,3}}$$

Since E_B is a constant, it may be obtained by testing the nonrotating structure.

Applying the same reasoning the idealization of Figure 6-3b, it can be shown that the damping torque on the rotor would obey the following expression if all of the dissipative material were connected to the rotor:

$$\bar{M}_{BVR} = -E_B \vec{\omega}^*_{H_{2,3}}$$



A. DAMPING MATERIAL CONNECTED TO HUB

B. DAMPING MATERIAL CONNECTED TO ROTOR

Figure 6-3. Idealization Used to Assess Effect of Rotation on Bearing Damping

In practice, it is likely that a combination of the above two effects occurs. This phenomemon was approximated in the equations of motion by assuming that the actual damping torque on the rotor is given by the following weighted combination of the above two effects:

$$\bar{M}_{BV} = \eta\vec{M}_{BVH} + (1-\eta)\vec{M}_{BVR}$$

where η is a scalar parameter between zero and one. If, in particular bearing design, most of the damping material revolves at or near the rotor speed, η would be set close to zero; if the opposite is true, η would be set close to unity. All the essential relations pertaining to the bearing torque are presented in matrix form in equations 6-32 to 6-36.

7.0 Computer Programs

A digital simulation of a flexible, spinning space station, as represented by the equations in Section 6.0, was developed to satisfy the primary objectives of this study. This digital program, referred to as the "General Flexible Body Simulation - Stage II," is described in detail in Part III of this report, where a user's guide is presented. It should be noted that the Stage I simulation, previously developed under contract to MSC (Reference 3), did not include:

- as many flexible body degrees of freedom
- a flexible hub
- a flexible shuttle docking mechanism
- a capability for modal reduction

Furthermore, the fact that the high frequency modes could not be eliminated from the Stage I simulation made it an inefficient analysis tool. Due to the above noted failings of the Stage I simulation the Stage II simulation was developed. However, results from the Stage I simulation were used as a baseline for comparison during Stage II checkout.

The General Flexible Body - Stage II simulation is divided into two main sub-programs:

1. The TIME HISTORY PROGRAM (THP) - which is a numerical integration of the flexible body equations.
2. The MODAL SERVICE PROGRAM (MSP) - which is used to determine the structural matrices (e.g., stiffness matrix, damping matrix, and modal matrix) that is needed for use by the THP.

Concurrent with the development of the Stage II simulation, preliminary analyses were performed with the previously developed Simplified Flexible Body simulation (Reference 3). This digital program simulates a dual spin space station whose rigid rotor is connected to a rigid hub through a flexible bearing.

8.0 Uncontrolled (Open-Loop) Dynamics

8.1 Preliminaries

Prior to proceeding with a discussion of the complex motion of a flexible rotating space station it is advisable to establish a clear understanding concerning the motion of a rigid spinning body. For demonstration purposes, a simple model for the space station is used; specifically, it is assumed that the spin axis (x axis) is a principle axis for both the hub and the rotor and that the hub is symmetrical. As a result, the Euler equations are identical to those for a spinning body with no hub provided I_y and I_z include both the hub and rotor mass and I_x includes only the rotor mass (see Reference 8). It should be noted that both the "T" and "Y" configurations have a near-symmetric hub. The following inertial angular rates projected on coordinates fixed in the rotating body are obtained by solving the uncontrolled linearized Euler equations:

$$\begin{aligned} \omega_x &= p = \text{constant spin} \\ \omega_Y &= \mp \Omega \alpha \sin(\lambda p t + \beta) \\ \omega_z &= \Omega \cos(\lambda p t + \beta) \end{aligned} \tag{8-1}$$

where

$$\begin{aligned} \lambda &\equiv \sqrt{\lambda_Y , \lambda_z} \quad , \quad \alpha \equiv \sqrt{\frac{\lambda_Y}{\lambda_z}} \\ \lambda_Y &\equiv \frac{I_x - I_z}{I_Y} \quad , \quad \lambda_z \equiv \frac{I_x - I_Y}{I_z} \end{aligned} \tag{8-2}$$

and Ω and β are determined by the initial conditions. The upper sign in the expression for ω_y applies for "Max I" vehicles ($I_x > I_y$, $I_x > I_z$), and the lower sign applies for "Min I" vehicles ($I_x < I_y$, $I_x < I_z$). The case where I_x is an intermediate axis of inertia is not considered since configurations with this property have unstable motions when uncontrolled.

The wobble vector (ω_r) is defined as the projection of the station's angular velocity on the transverse plane; i.e.,

$$\{\omega_T\} = \Omega\left[\mp\alpha \sin(\lambda pt)\{j\} + \cos(\lambda pt)\{k\}\right] \tag{8-3}$$

It is noted that the phase angle (β) can be omitted without a significant loss of generality. Examining the above relationship for $\{\omega_T\}$ it is seen that the vector traces an elliptical path as illustrated in Figure 8-1. The frequency of the ellipse, referred to as the wobble frequency (λp), will increase as the separation between the transverse inertias and the spin inertia is increased (see equation 8-2). Furthermore, it is noted that the ellipse degenerates into a circle for symmetric vehicles (i.e., when $\alpha = 1$). Finally, the rotation of the vector is in the same direction as the vehicle spin for a Max I configuration and opposite for Min I.

It is easier to visualize the vehicle motion by observing the wobble rate as seen in despun hub coordinates. The transverse components of the wobble rate as viewed in the hub, is given by

$$\begin{aligned} \omega_{YH} &= (\{\omega_T\}\cdot\{j\})\cos(pt) - (\{\omega_T\}\cdot\{k\})\sin(pt) \\ \omega_{ZH} &= (\{\omega_T\}\cdot\{j\})\sin(pt) + (\{\omega_T\}\cdot\{k\})\cos(pt) \end{aligned} \tag{8-4}$$

Substituting the expression in (8-3) for $\{\omega_T\}$ and rearranging terms results in,

$$\begin{aligned} \omega_{Y_H} &= \frac{\Omega}{2}\{(\alpha \mp 1)\cos[p(1-\lambda)t] + (\alpha \pm 1)\cos[p(1+\lambda)t] \\ \omega_{Z_H} &= \frac{\Omega}{2}\{(\alpha \mp 1)\sin[p(1-\lambda)t] + (\alpha \pm 1)\sin[p(1+\lambda)t] \end{aligned} \tag{8-5}$$

The upper sign in the above expression applies for Max I vehicles and the lower

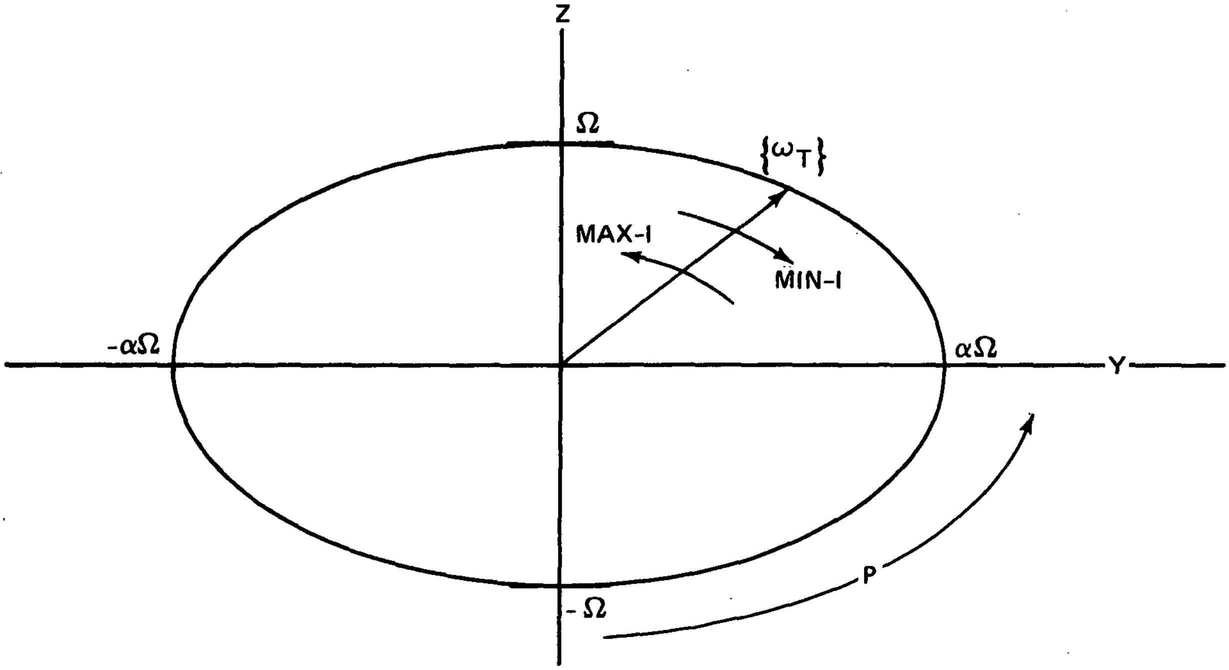


Figure 8-1. Wobble Vector Projected on Rotor Coordinates

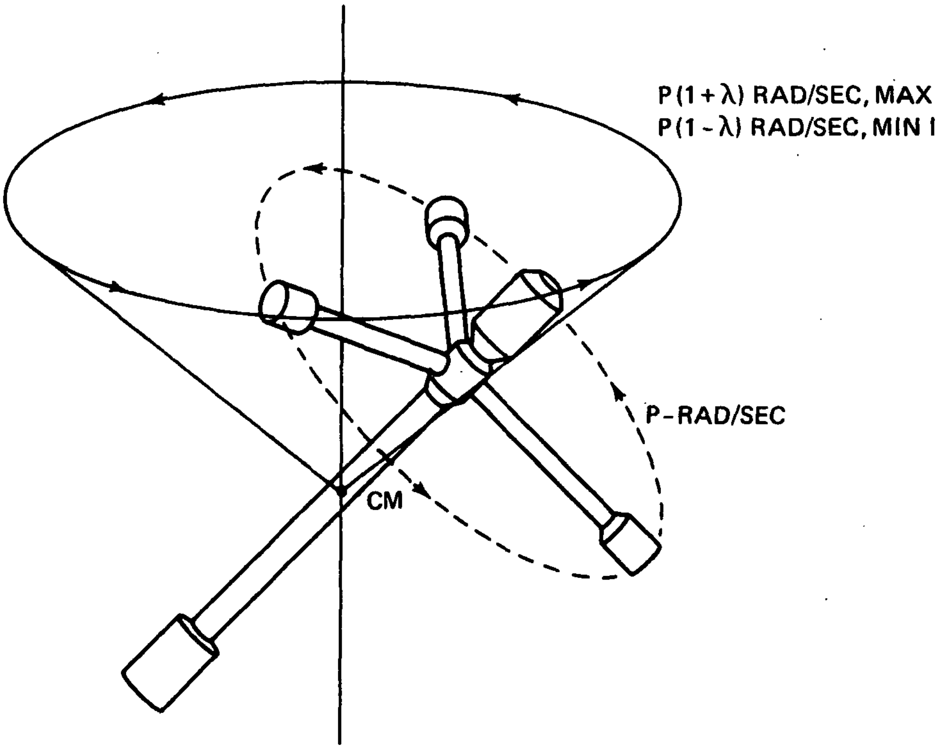


Figure 8-2. Inertial Motion Of A Symmetric Spinning Space Station

sign applies for Min I vehicles. The general expression (8-5) consists of the sum of two sinusoids: one with a frequency equal to the spin plus the wobble frequency, and the other frequency equal to the spin minus the wobble frequency. For most spinning satellites α is close to unity and, in fact, the deviation from unity first becomes appreciable for satellites whose inertia characteristics are close to an "Intermediate Inertia" vehicle. Therefore, the predominate inertial wobble mode for most Max I vehicles is at a frequency equal to spin plus wobble frequency (i.e., $p + \lambda p$). In order to visualize wobble motion let us examine the motion of a symmetric vehicle ($\alpha = 1$) which is characterized by a pure coning motion in inertial space plus the spin motion of the rotor (as shown in Figure 8-2). For an unsymmetric vehicle ($\alpha \neq 1$) small cyclic motions are superimposed on the coning motion shown in Figure 8-2.

When the structure is made flexible and energy dissipation exists then the coning motion, or wobble, will either converge or diverge depending upon the location of the energy absorbing material. The stability of each basic space station was investigated with two sources of structural flexibility:

- Bearing
- Extension Arms (both hub and rotor)

The influence of a flexibly attached shuttle on vehicle stability was also studied.

Section 8.2 presents the results of analyses performed on a vehicle whose only source of flexibility is its bearing. These results allow for the separation of the stability trends due to just a flexible bearing, as well as being representative of total system stability for space stations with extremely stiff extension arms. The results of total system stability investigations is presented in Section 8.4. A description of what are considered the signficant mode shapes is included in Section 8.3. This information will allow the reader to visualize the predominate structural motion of the flexible rotor and flexible hub.

8.2 Stability Sensitivity To Bearing Parameters

The bearing assembly, that connects the rotor to the hub, is idealized as in Reference 3, and is described in Section 6.0 of this report. As previously noted (in Section 6.0), the bearing model permits the inclusion of energy-dissipative material in either the bearing outer race connection to the rotor or the bearing inner race connection to the hub. The nominal values for bearing stiffness and damping have been derived in Section 4.3.2; the effects of variations about these nominal values are investigated.

A characteristic root analysis was performed using the set of unforced Simplified Flexible Body Equations as presented in Reference 3 (Section 4.3.1), and repeated in Figure 8-3 for completeness. The mass properties for each space station, were held constant at their nominal values as presented in Section 4.2. The results of the analysis are as follows:
"Y" Space Station - The "Y" configuration is stable for any stiffness and damping combination which includes damping material located on either the bearing inner race or outer race. The time constant associated with the decay of the wobble mode as a function of bearing stiffness and damping is shown in Figures 8-4 and 8-5 corresponding to outer race damping and inner race damping, respectively. The time constant resulting from using nominal bearing coefficients is 18.6 hr. As seen from Figures 8-4 and 8-5 dissipative material on the inner race has more of an influence on wobble damping then dissipative material on the outer race; that is for the same bearing coefficients the time constant resulting from inner race damping is appreciably less than that of outer race damping.

"T" Space Station - The "T" configuration is unstable for outer race damping for any value of stiffness or damping. The time constant associated with the exponential divergence of the wobble mode is shown in Figure 8-6 as a function of bearing stiffness and damping. The time constant for divergence corresponding to nominal bearing parameters is 5.8 hrs.

The "T" station with inner race damping is either stable or unstable depending upon the amount of damping and corresponding stiffness. A stability boundary is constructed and is shown in Figure 8-7. In general, a low bearing stiffness in combination with a high damping coefficient will improve the possibility of vehicle stability. Figure 8-8 presents the time constant associated with either an exponential decay or divergence as a function of the bearing parameters.

$$I_2\dot{W}_2 = -(I_1 - I_3)pW_3 - \epsilon_B\dot{\theta}_2 - K_B\theta_2 + n_H\epsilon_B p\theta_3$$

$$I_3\dot{W}_3 = (I_1 - I_2)pW_2 - \epsilon_B\dot{\theta}_3 - K_B\theta_3 - n_H\epsilon_B p\theta_2$$

$$\dot{h}_2 = ph_3 - I_1 pW_3$$

$$\dot{h}_3 = -ph_2 + I_1 pW_2$$

$$Q\dot{\theta}_2 = -h_2 + (I_2 + Q)W_2 + Qp\theta_3$$

$$Q\dot{\theta}_3 = -h_3 + (I_3 + Q)W_3 - Qp\theta_2$$

where:

Q = $I_H + M_H a_H^2 + M_R a_R^2$

I_1, I_2, I_3 = moement of inertia of rotor (spin is 1 axis)

I_4 = transverse moment inertia of hub (euqations assume hub symmetry)

M_R, M_H = mass of rotor, hub

a_R, a_H = Distance for combined system CM to CM of rotor, hub

p = spin rate

$\{h\}$ = angular momentum of entire system

$\{0\}$ = ordered rotations from hub to rotor

ω_2, ω_3 = transverse angular velocity components of rotor

K_B = Bearing Stiffness

E_B = Bearing Damping

n_H = $\begin{cases} 0 \text{ - Damping material on bearing outer race} \\ 1 \text{ - Damping material on bearing inner race} \end{cases}$

FIGURE 8-3 SIMPLIFIED FLEXIBLE BODY EQUATIONS

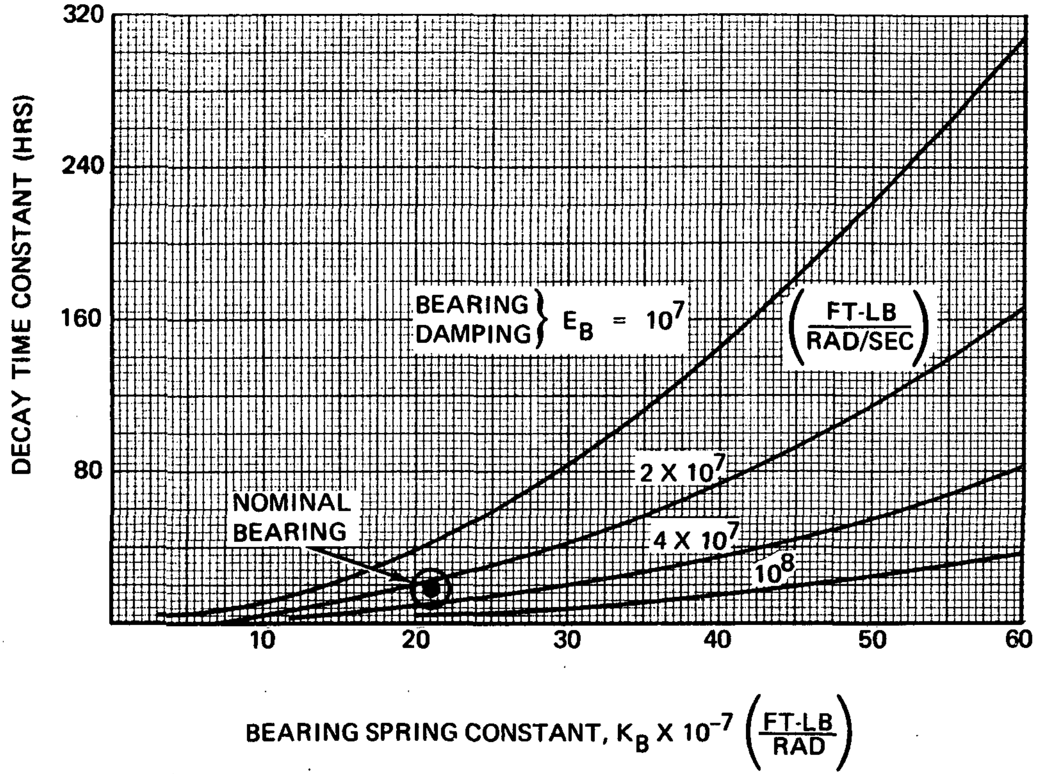


Figure 8-4. Decay Time Constant① for "Y" Outer Race Bearing Damping

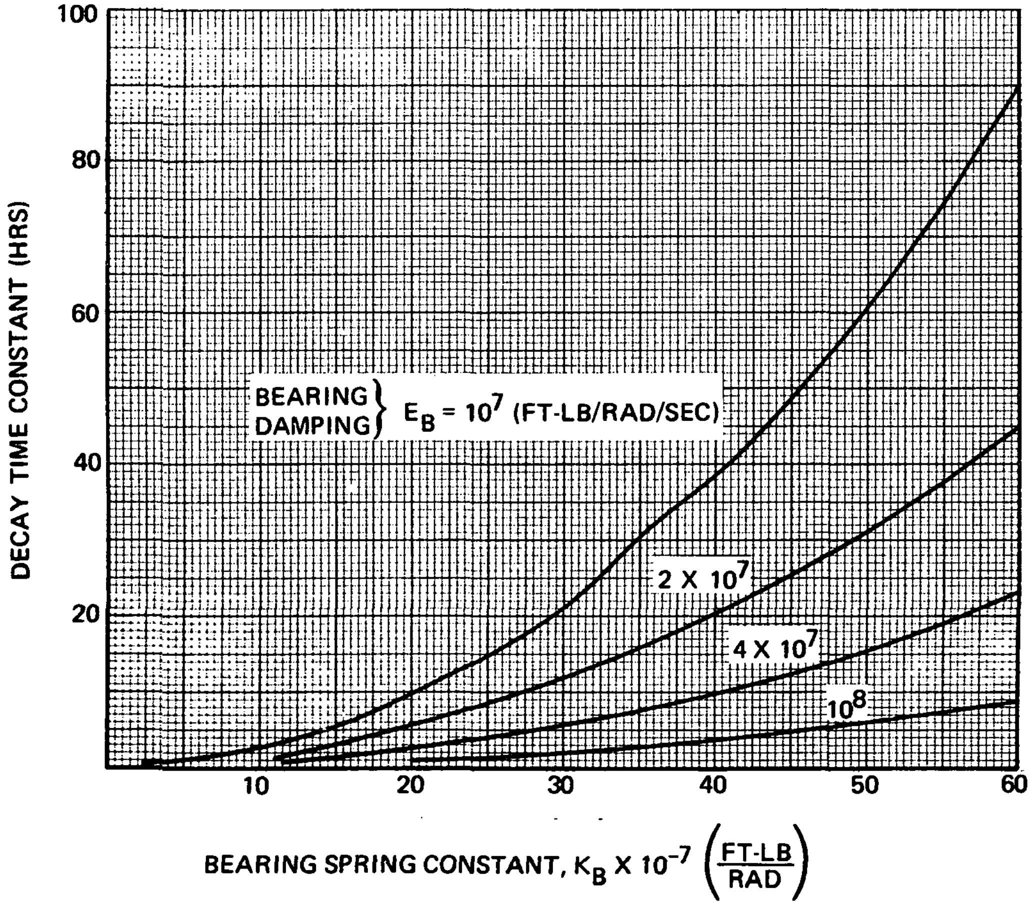


Figure 8-5. Decay Time Constant① For "Y" Inner Race Bearing Damping

① TIME CONSTANT (τ) AS IN, $e^{-t/\tau}$

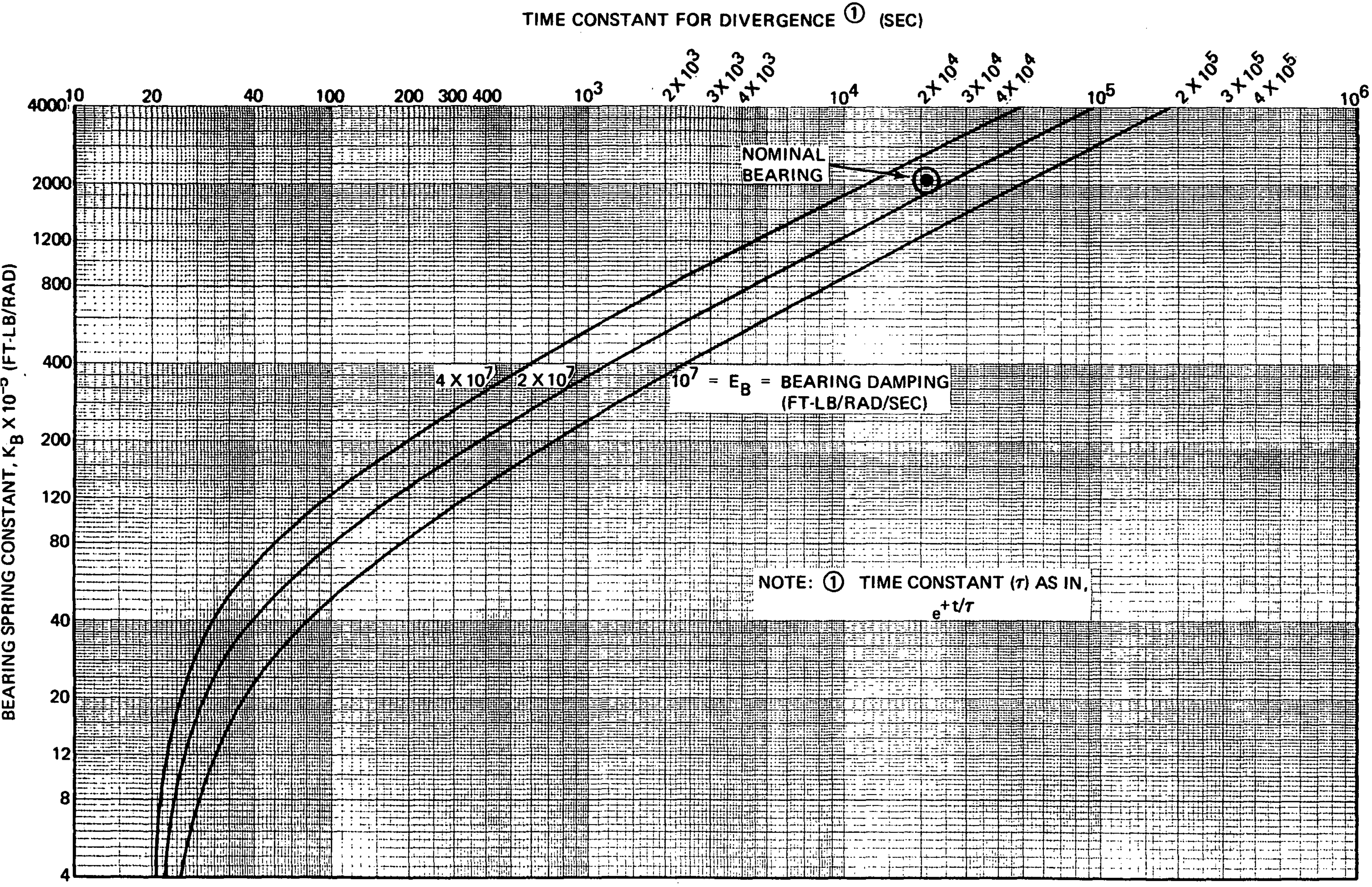


Figure 8-6. Time Constant For Divergence For "T" - Outer Race Bearing Damping

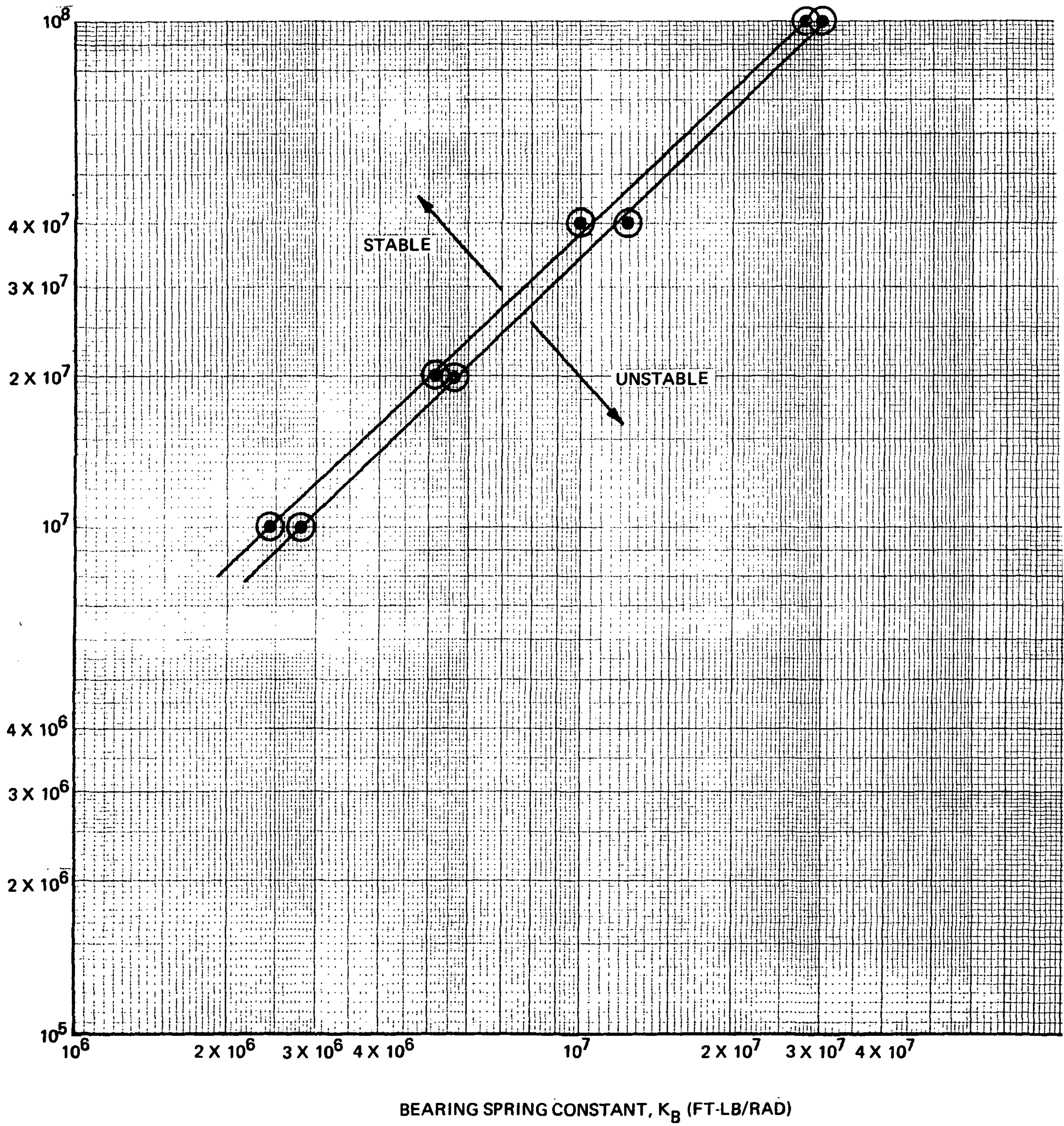


Figure 8-7. Stability Boundary For "T" - Inner Race Bearing Damping

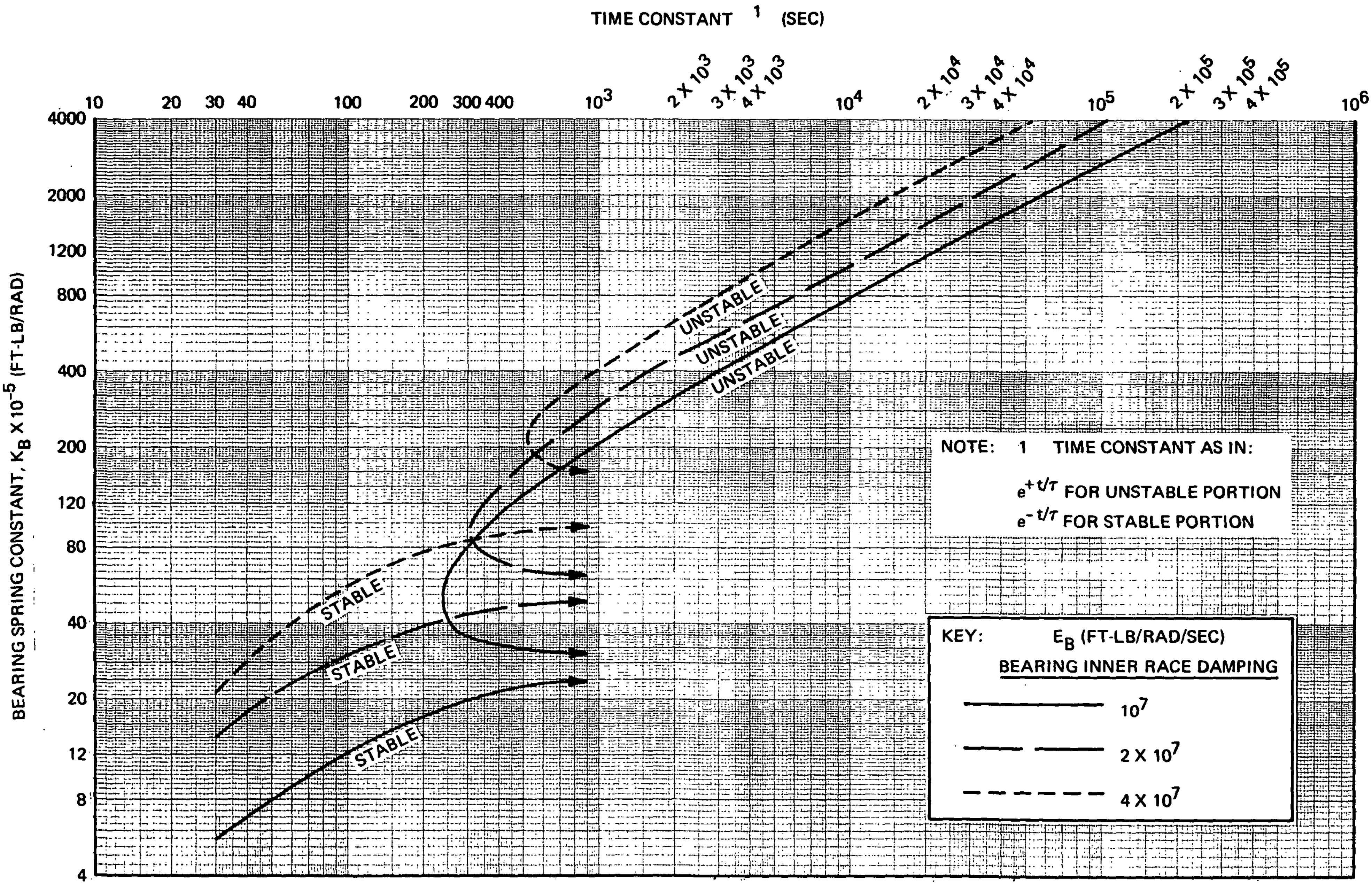


Figure 8-8. Time Constant For "T" - Inner Race Bearing Damping

Until this point in the report the rotational dynamics have been described by words, equations, illustrations and characteristic roots. A useful tool for examining vehicle dynamics is the Time History Simulation and the resulting time history plots. However, before proceeding with analyzing the time history plot resulting from the General Flexible Body simulation (which includes extension arm flexibility), let us first examine the time history plots from a simulation whose only flexible member is the bearing. Figures 8-9 and 8-10 are time histories of the transverse angular rates for the nominal "Y" and "T" configurations, respectively. The "Y" simulation was initialized with a transverse component of angular momentum in the hub while the "T" run was initialized with an angular momentum component in the rotor. The high frequency flexible mode, representing relative motion between the rotor and hub, is seen to damp out completely within the first 20 seconds. The remaining oscillitory motion is wobble. Although unobservable on these plots, the wobble motion of the "Y" station is converging while the wobble of the "T" is diverging. Due to the size of the time constant, convergence or divergence can only be observed by examining the printed data of the wobble magnitude $|\{\omega_T\}|$.

It is interesting to note that since α for "Y" is closer to one, a predominant frequency equal to spin plus wobble frequency $(p + \lambda p)$ can be observed when viewing hub transverse rates. In contrast, the "T" station is close to being an "Intermediate Inertia" configuration (i.e., $\alpha = 5.85$) and as a result the hub rates exhibit two frequency components $(p \pm \lambda p)$ of relatively equal weights. These time history plots will be used as a basis of comparison when examining the time histories of vehicles with additional flexible members (refer to Section 8.4).

It should be noted that when a shuttle is rigidly attached to the station, the "Y" configuration as well as "T" are "Min I". As a result, both stations when combined with the shuttle are unstable for the condition of nominal bearing flexibility provided there is no other source of energy dissipation.

FIGURE 8.9 "Y" WITH BEARING FLEXIBILITY

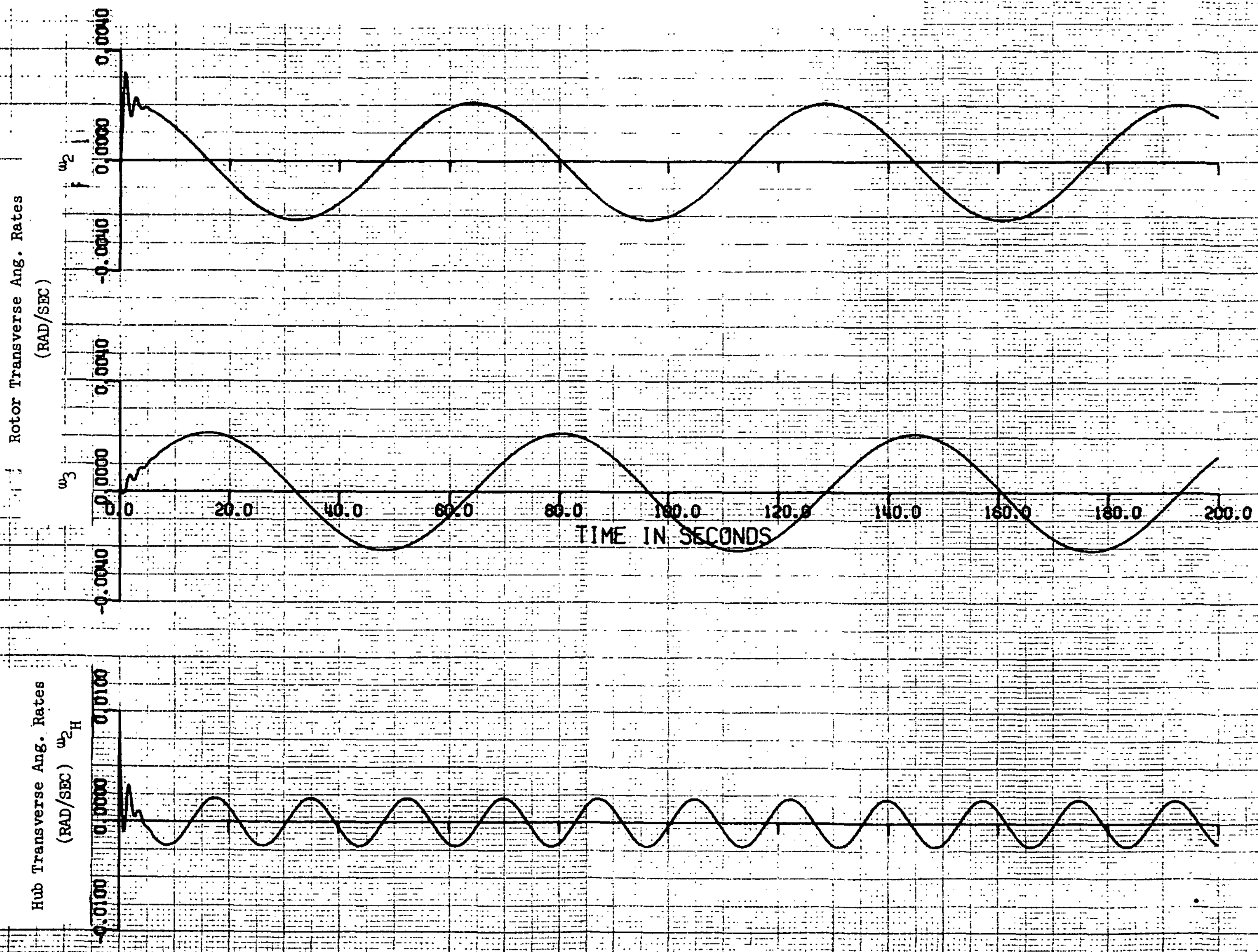
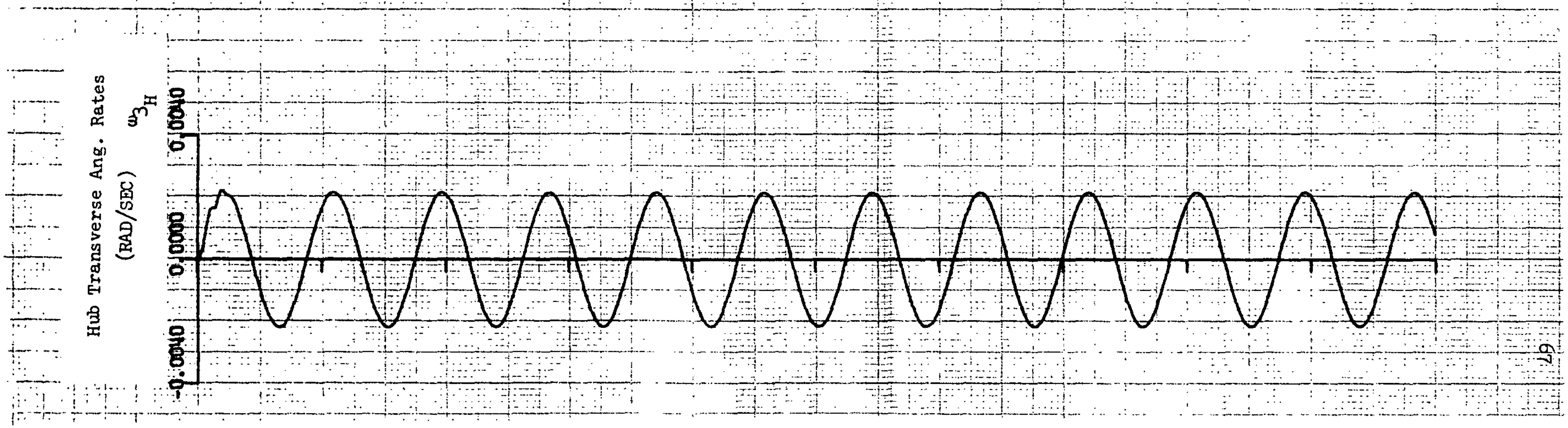


FIGURE 8.9 (Con't)



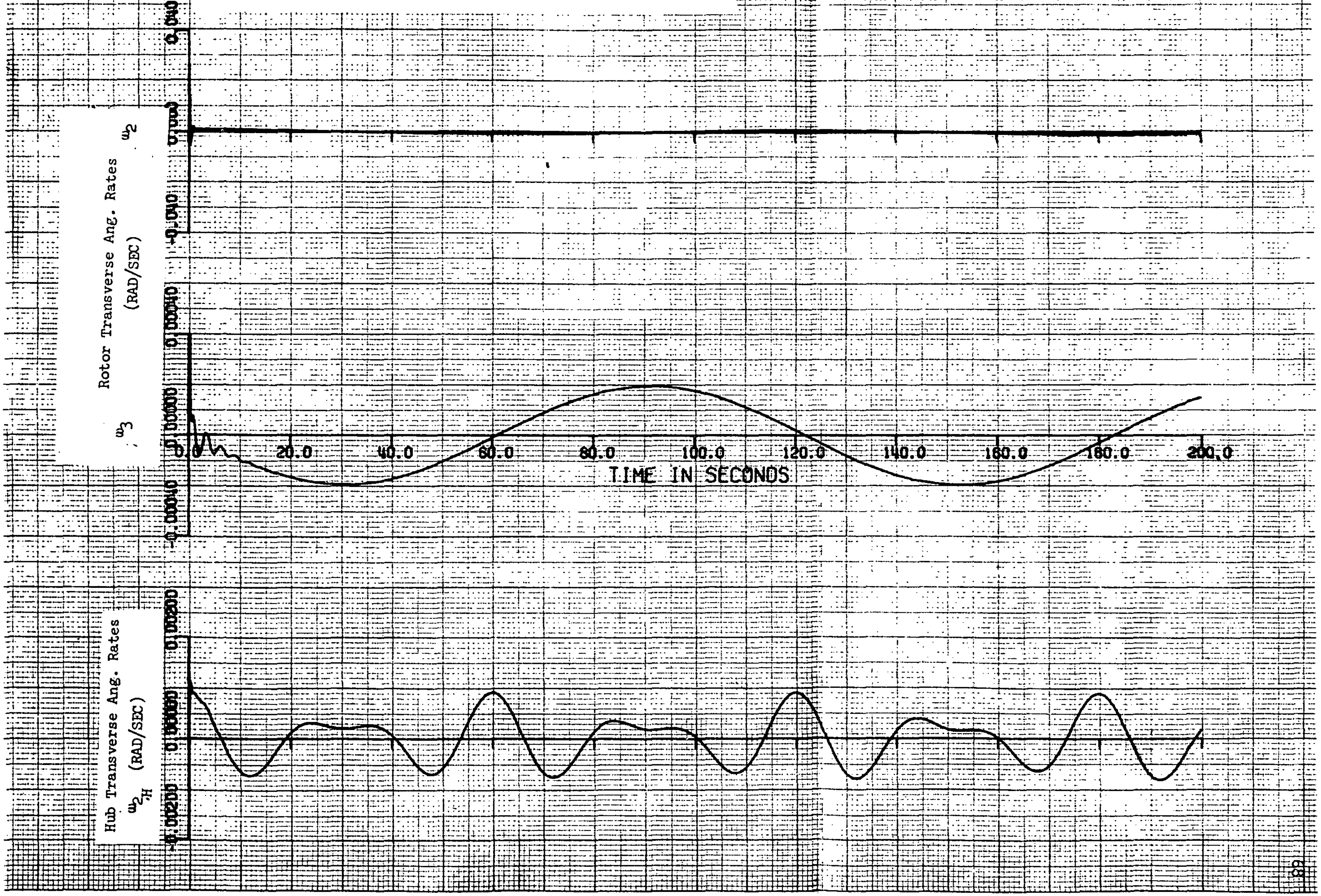
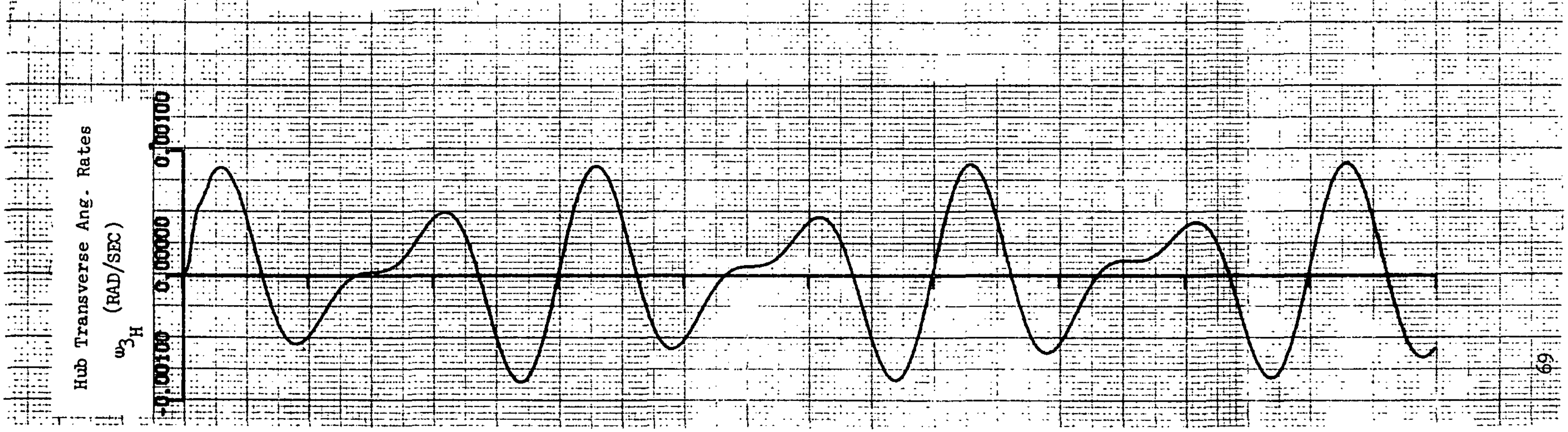


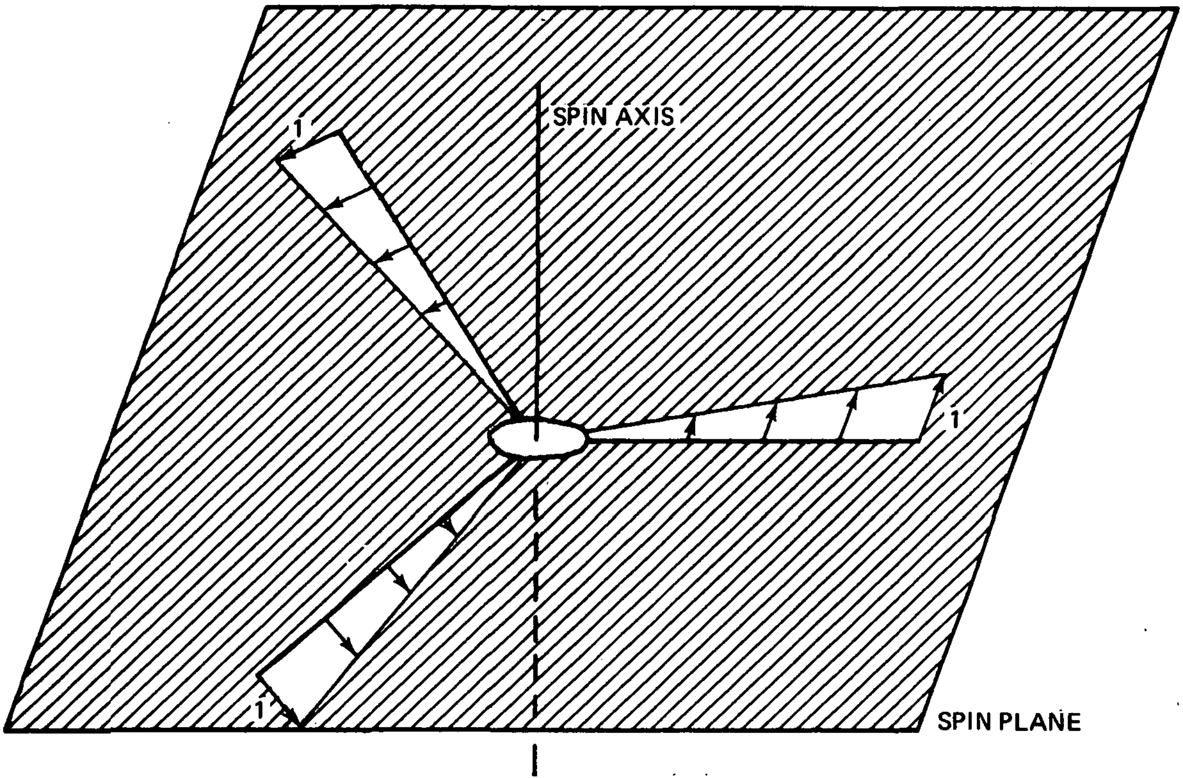
FIGURE 8.10 "T" WITH BEARING FLEXIBILITY

FIGURE 8.10 (Con't)

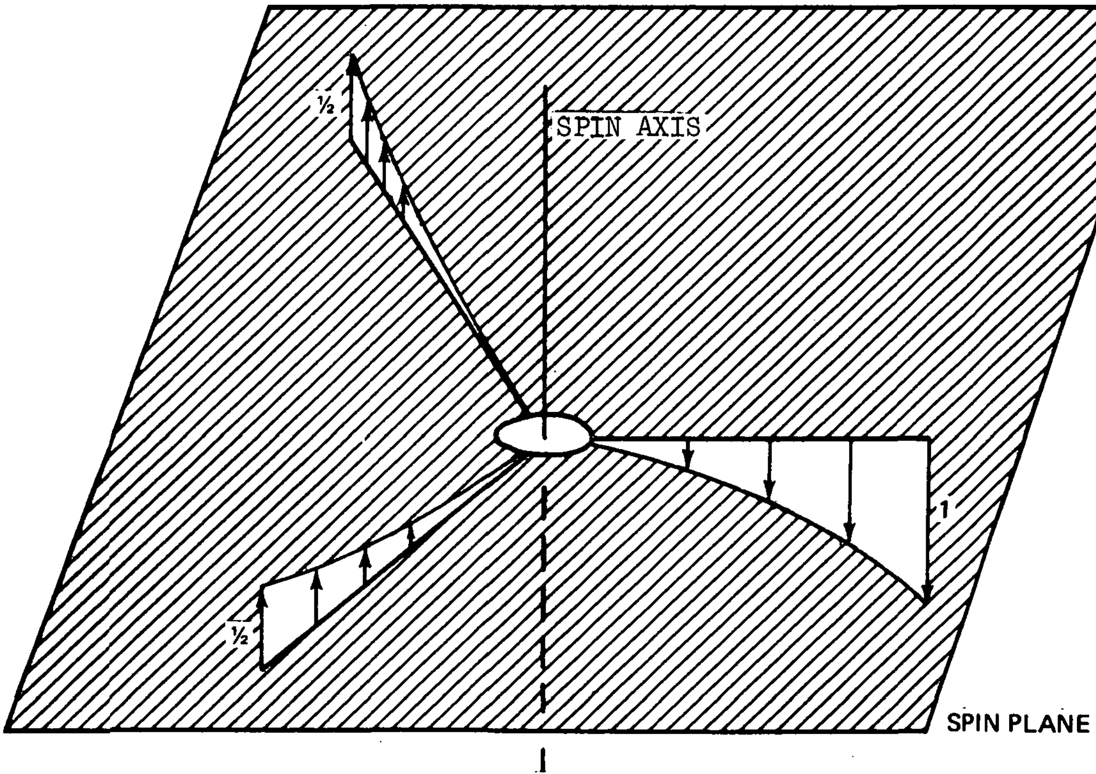


8.3 Significant Mode Shapes

Figures 8-9 through 8-15 show the rotor and hub modes used for the majority of simulation studies. The listed modal frequencies correspond to the "Y" space station with baseline properties. Six rotor modes and two hub modes were used for each vehicle. For the rotor, the predominant motions are i) in plane bending, ii) out-of-plane bending, and iii) torsion. Torsion in the T-rotor occurs independently of bending and is easy to visualize (Figures 8-14a and b); for the Y-rotor, however, some torsion accompanies out-of-plane bending and is difficult to show; e.g. there is some torsion in arms two and three in Figure 8-9(b). Predominant hub motions are bending in each of the two transverse planes. As was expected, axial extension of the beams occurs at very high frequency and is not present in any of the modes shown.

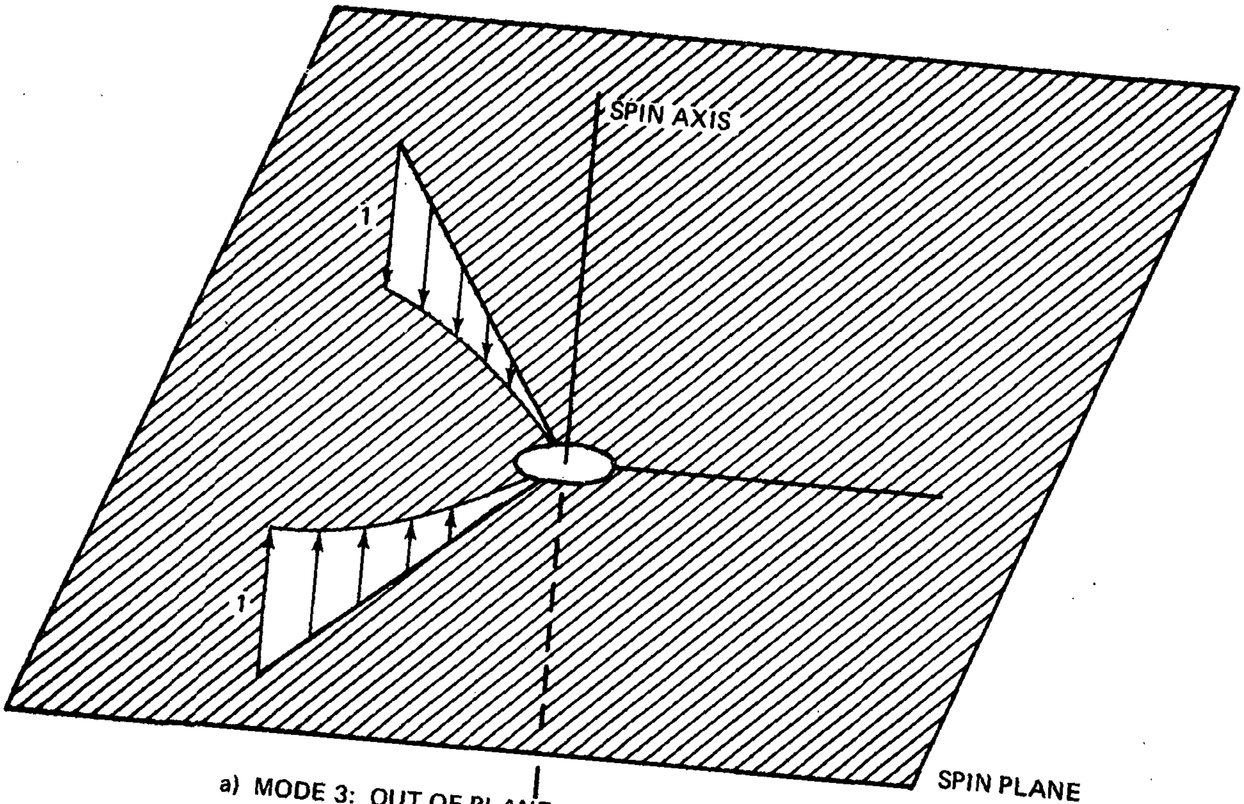


a) MODE 1: RIGID BODY MODE, IN PLANE (f = 0)

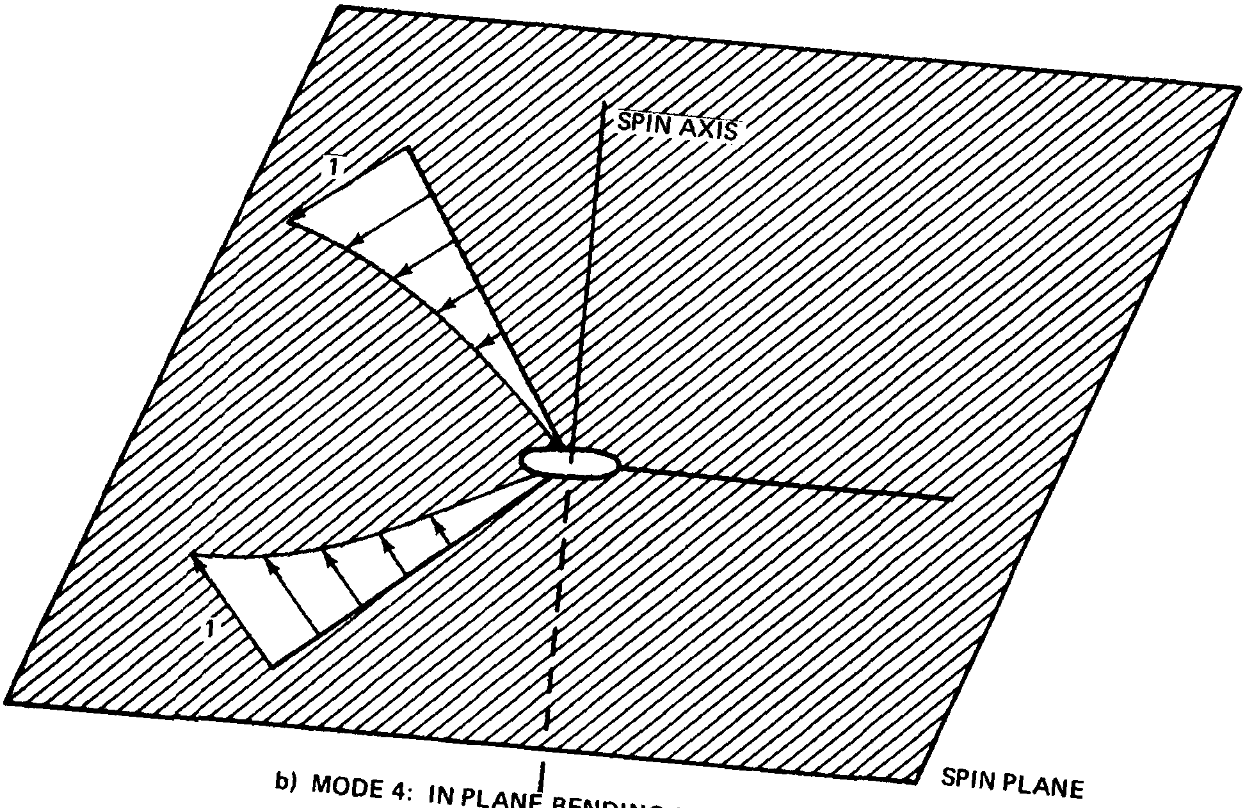


b) MODE 2: OUT OF PLANE BENDING (f = 0.12881 CPS)

Figure 8-9. Y Rotor Modes

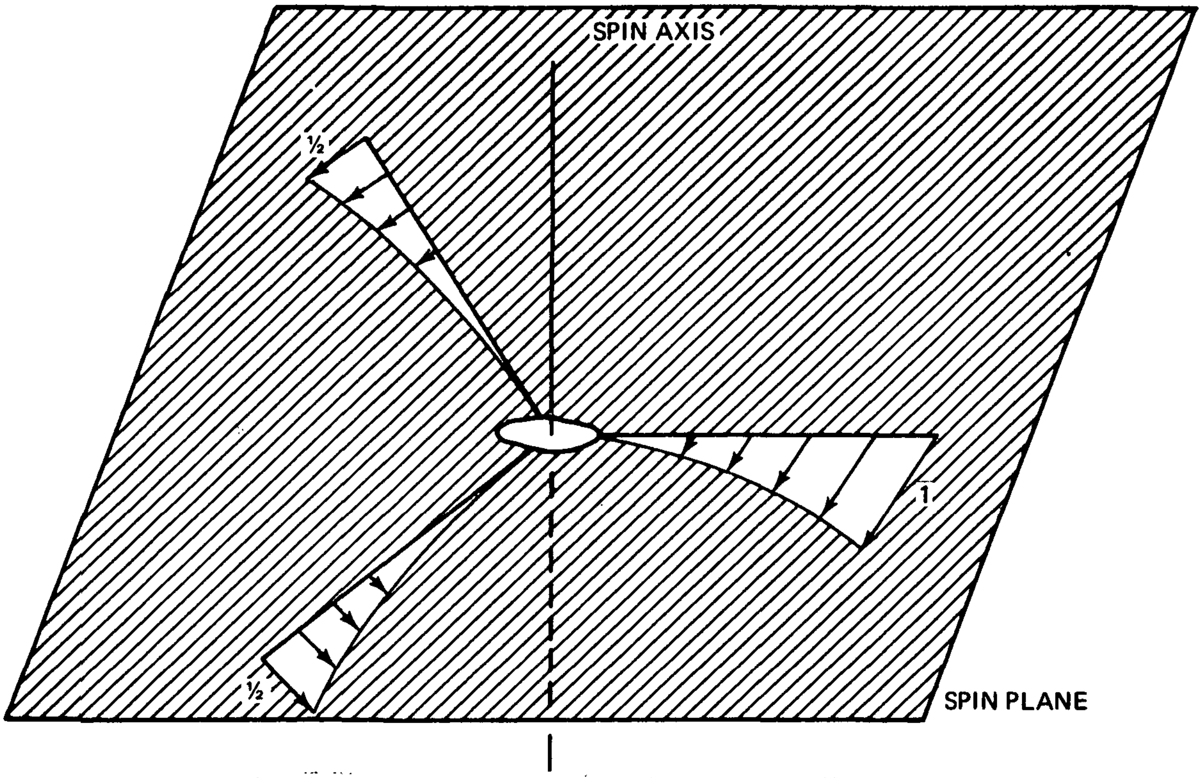


a) MODE 3: OUT OF PLANE BENDING (f = 0.12885 CPS)

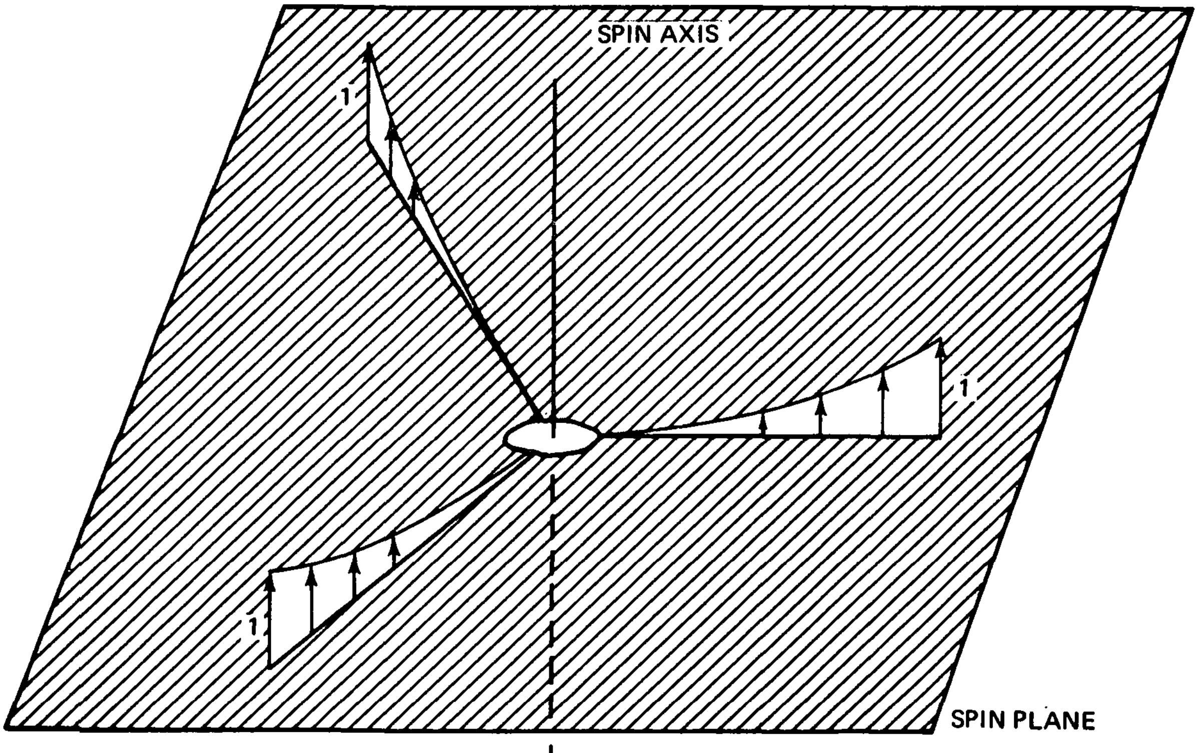


b) MODE 4: IN PLANE BENDING (f = 0.14138 CPS)

Figure 8-10. Y Rotor Modes (Cont.)

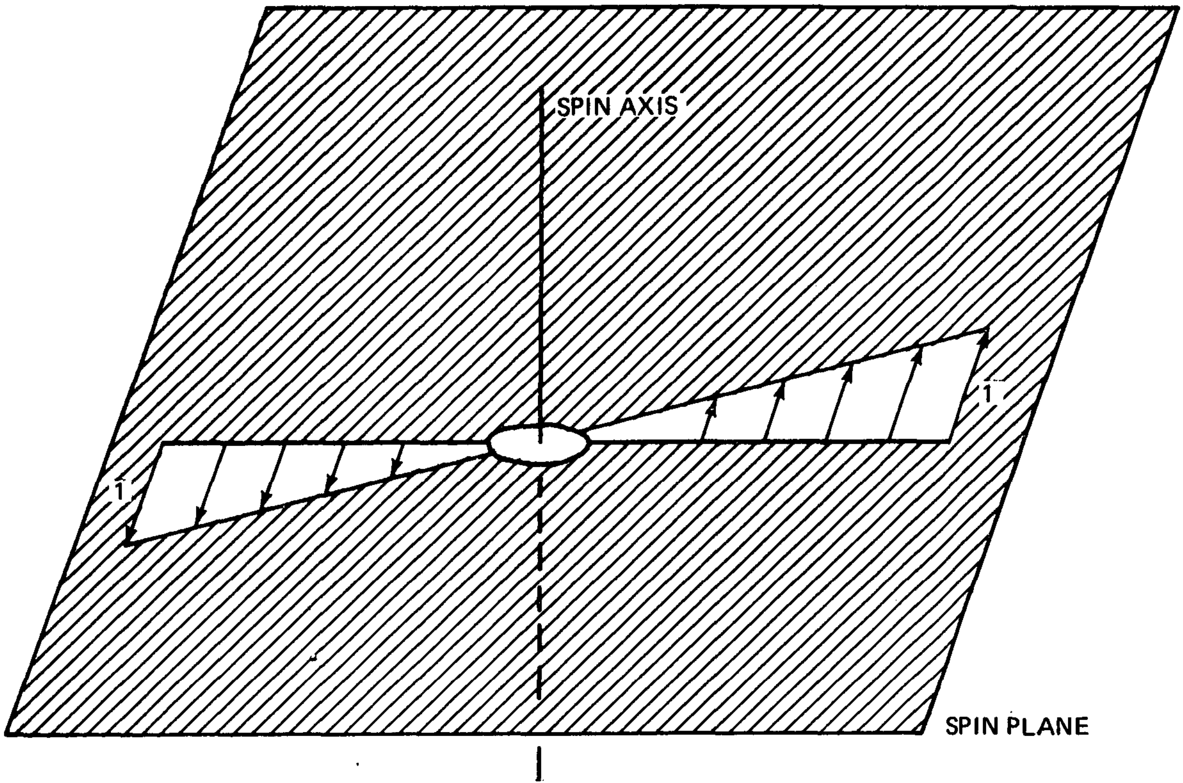


a) MODE 5: IN PLANE BENDING (f = 0.14155 CPS)

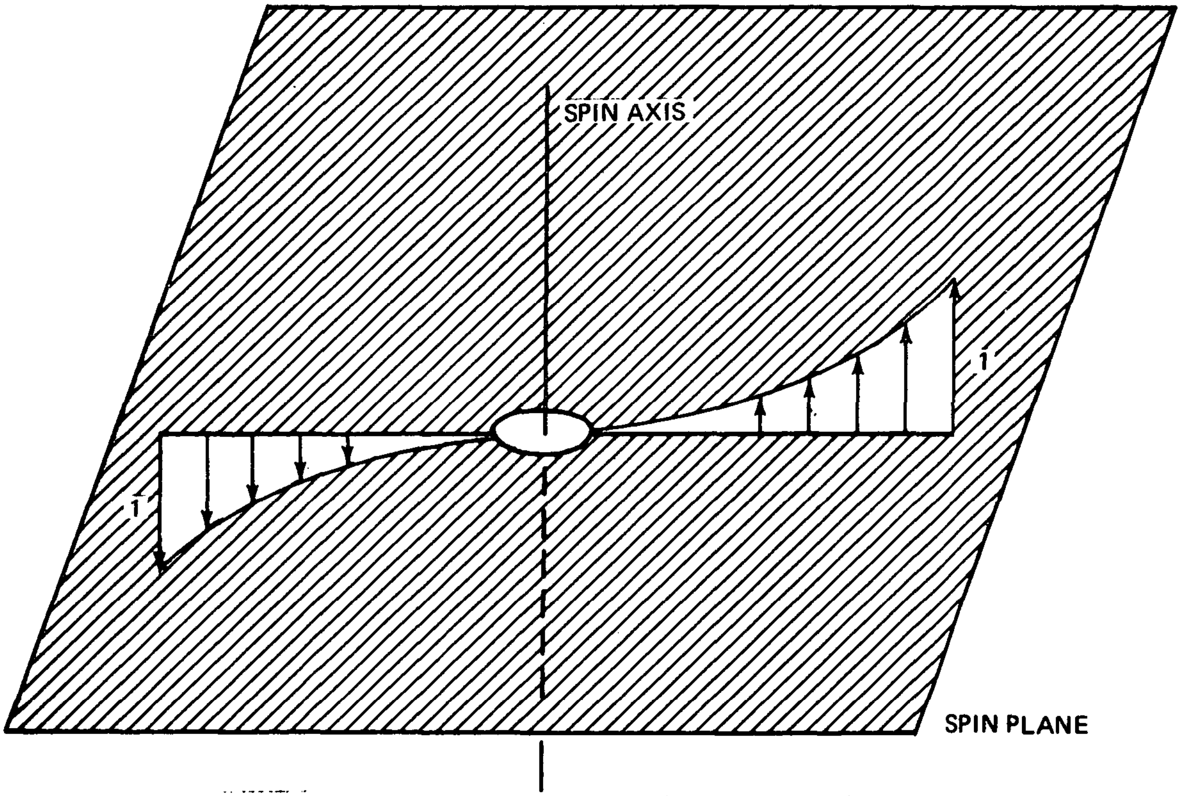


b) MODE 6: OUT OF PLANE BENDING (f = 0.14196 CPS)

Figure 8-11. Y Rotor Modes (Cont.)

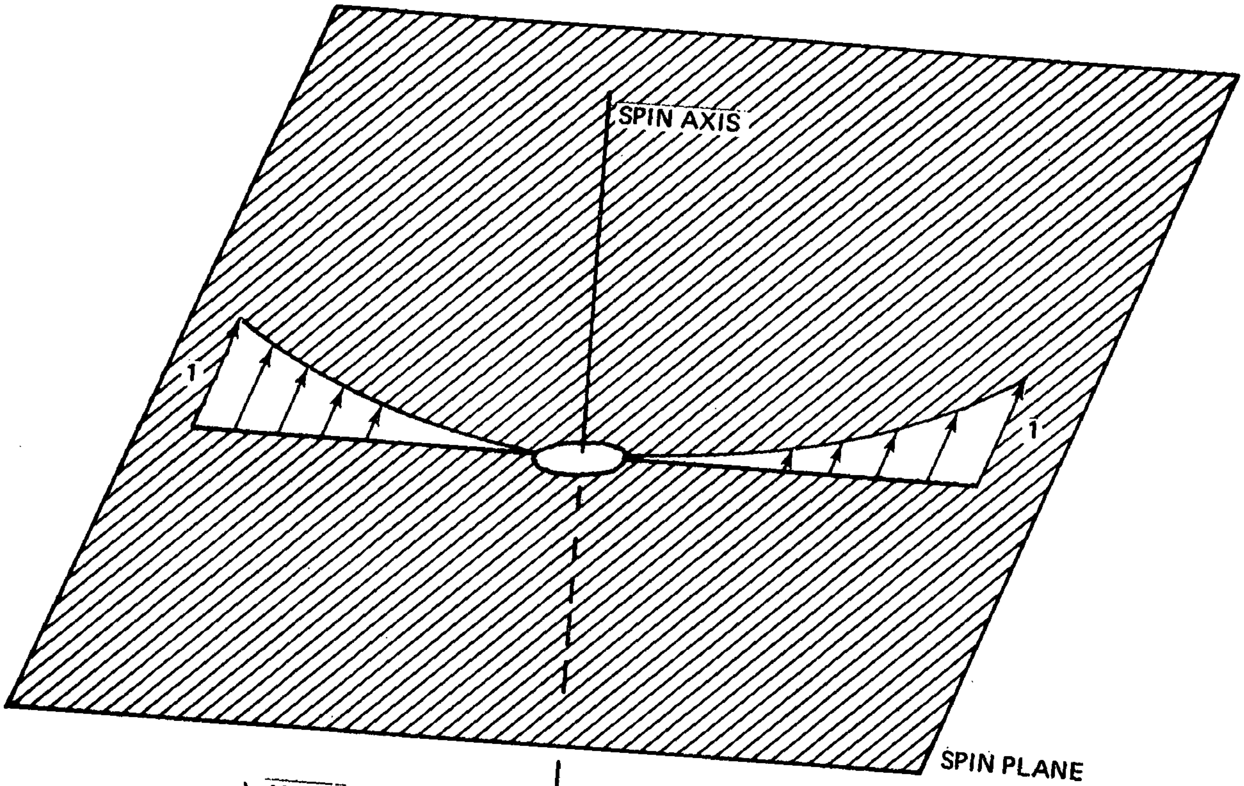


a) MODE 1: RIGID BODY MODE, IN PLANE (f = 0)

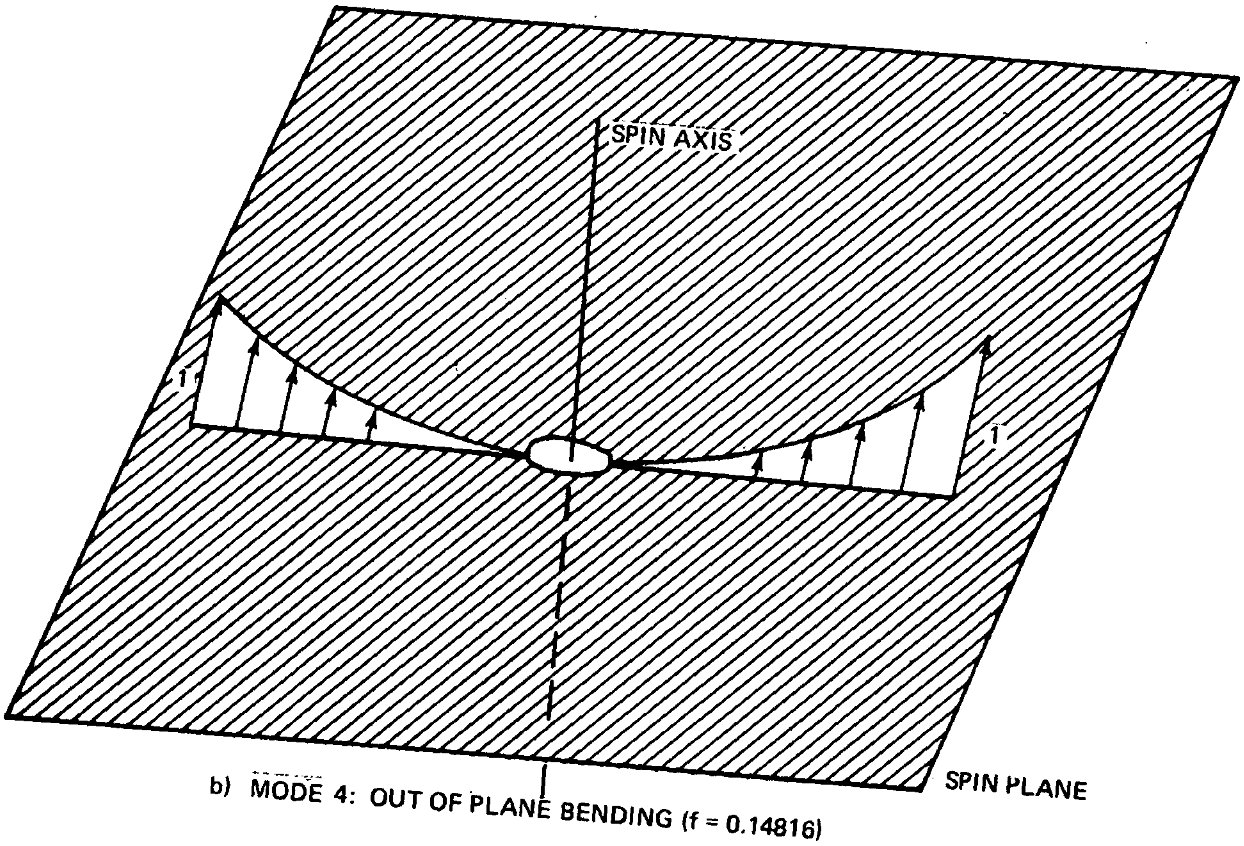


b) MODE 2: OUT OF PLANE BENDING (f = 0.13007 CPS)

Figure 8-12. T Rotor Modes

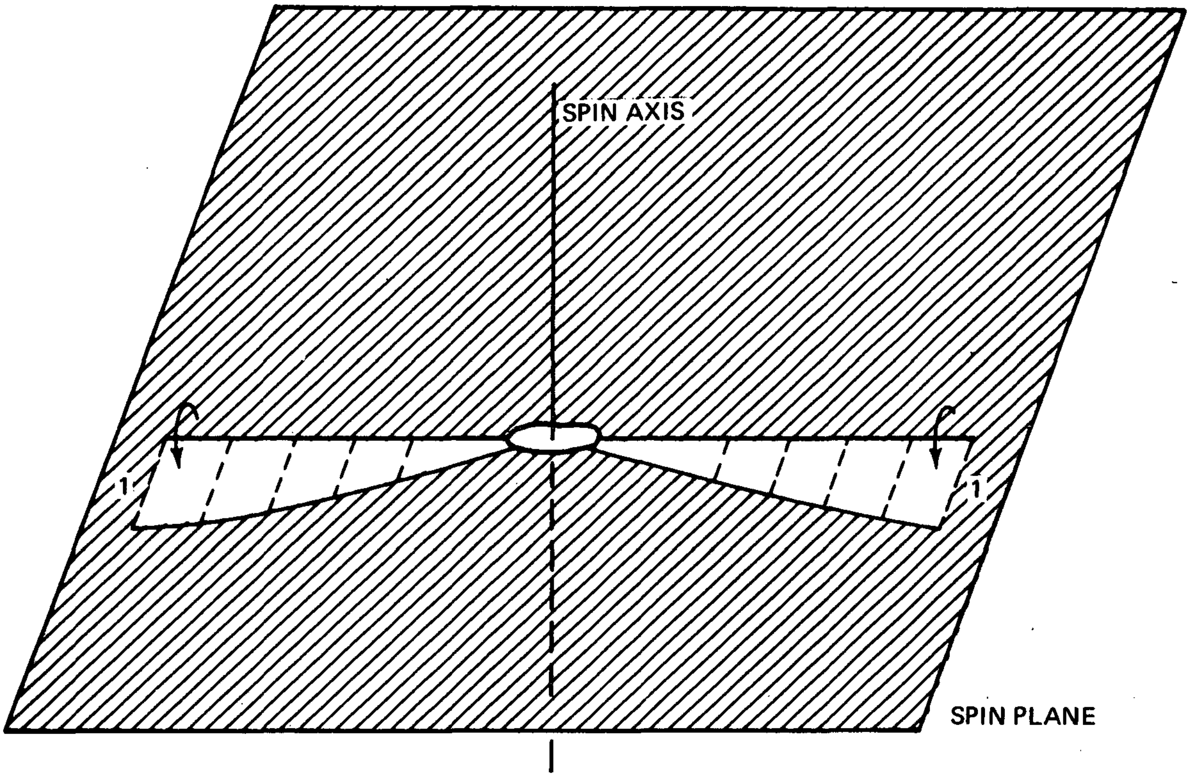


a) MODE 3: IN PLANE BENDING (f = 0.13719 CPS)

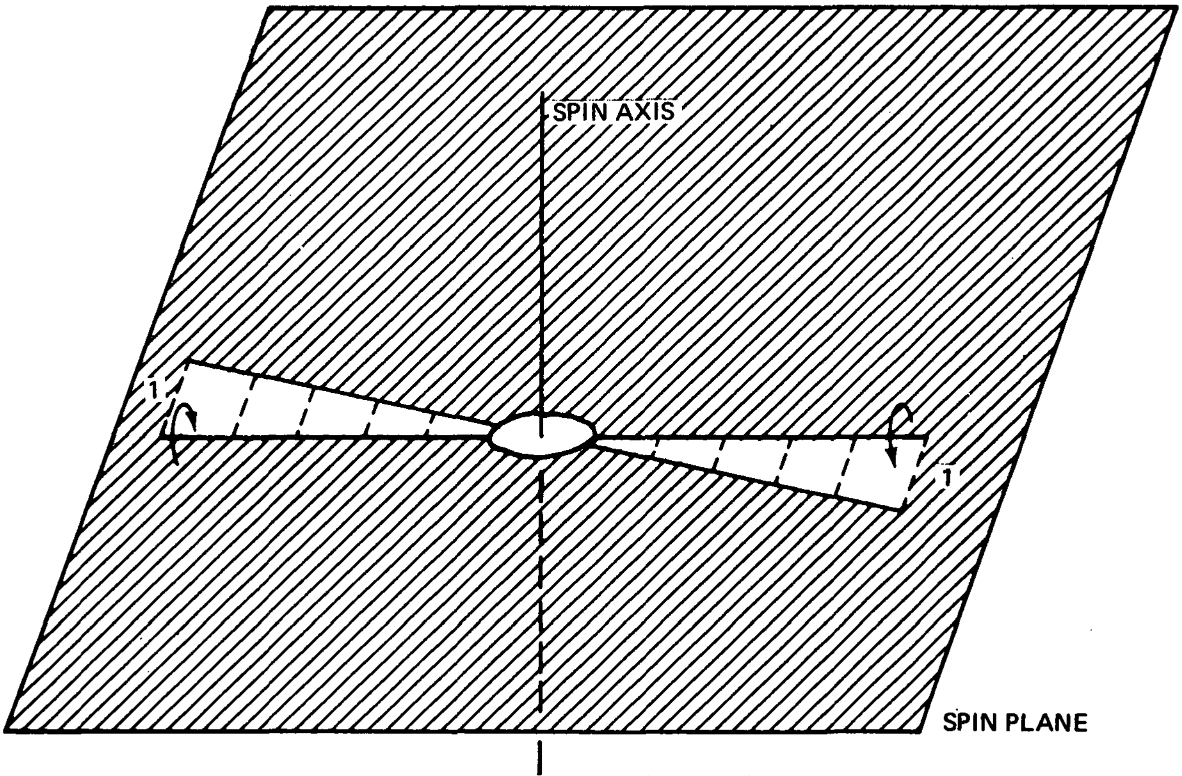


b) MODE 4: OUT OF PLANE BENDING (f = 0.14816)

Figure 8-13. T Rotor Modes (Cont.)



MODE 5: TORSION (f = 0.39320 CPS)



MODE 6: TORSION (f = 0.41336 CPS)

Figure 8-14. T Rotor Modes (Cont.)

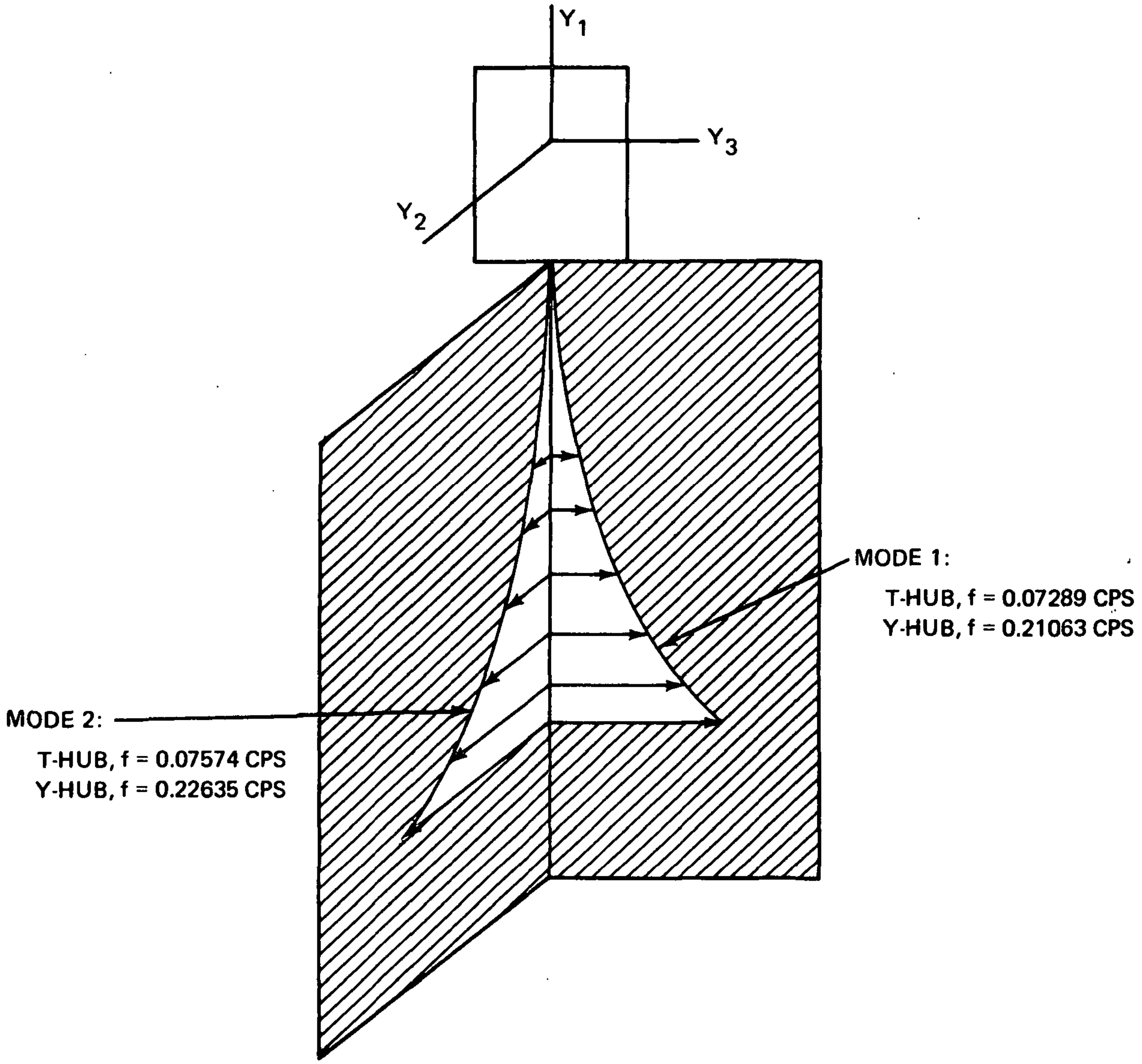


Figure 8-15. Hub Modes (T And Y)

8.4 General Flexible Body Dynamics

The flexible body dynamics of both the "T" and "Y" configurations were investigated using the General Flexible Body Simulation - Stage II. This simulation, specifically developed for this study program, is described briefly in Section 7.0, and is presented in more detail in Part III of this report in the form of a User's Guide. The simulated equations of motion are presented in Section 6.0. These equations have been formulated to include the effects of the following flexible structural members: rotor arms, hub, bearing, and shuttle docking mechanism.

An exact solution to the equations of motions is obtained when all the flexible modes are numerically integrated. Unfortunately, such a task would require an unacceptable amount of digital computer time. A good compromise is to find the minimum number of low frequency flexible modes that can represent the vehicle motion with a negligible error. It should be noted that the low frequency modes are considered most important since they generally exhibit larger flexible motions, they don't damp as fast, and they are most troublesome from a control point of view. The approach used to determine the minimum number of significant modes was to systematically perform simulation runs with fewer and fewer modes until significant changes in the dynamics were observed. It was determined that at least 6 rotor modes and 2 hub modes are required for an accurate representation of both "T" and "Y" dynamics. Section 8.3, contains a detailed description of the significant mode shapes.

Stability is determined by observing either the convergence or divergence of the magnitude of the transverse angular rate as viewed in the rotor ring $|\omega_T|$ (i.e., simulation variable WTMM). Prior to examining $|\omega_T|$ some time should be allowed to elapse until the significant flexible motion has damped.

An appreciation of flexible body dynamics is obtained by observing time history traces of several variables that represent the vehicle motion at different points located throughout the space station. Specifically, the time history plots that are included herein are of the following variables:

- ω^X - angular rate of rotor ring
- ω^Y - angular rate of rigid section of hub
- ω^{y^3} - angular rate of the hub end mass
- $\omega^{z^{i3}}$ - angular rates of the end mass of the ith rotor arm
- $\dot{\mu}^{c^3}$ - linear velocity of the hub end mass relative to the rigid hub section
- $\dot{\mu}^{D^{i3}}$ - linear velocity of the end mass of the ith rotor arm relative to the rotor ring

The stability of both the "T" and "Y" configurations were investigated using stiffness values for the extension arms that ranged from the baseline numbers as presented in Section 4.3.1 to twice the baseline stiffness which will be referred to as nominal, specifically,

RANGE OF EXTENSION ARM STIFFNESS		
	Baseline, Lower	Nominal, Upper
• Bending EI	9.34×10^8 lb-ft^2	18.68×10^8 lb-ft^2
• Torsion GJ	7.47×10^8 lb-ft^2	14.94×10^8 lb-ft^2

During the stability analysis all other physical vehicle properties were maintained at their nominal values, as presented in Section 4.0. The results of the uncontrolled stability analysis using the General Flexible Body simulation are as follows:

"Y" - Space Station - The initial conditions for all investigations are equivalent to that experienced as a result of internal mass motion (refer to Section 5.2). The resulting time histories shown in Figures 8-16 and 8-17 correspond to the baseline and nominal stiffness conditions, respectively. Since the resolution of the time history plots is not sufficient, the stability was determined by viewing the printed output data. All stiffness conditions investigated resulted in convergence of the "Y" station wobble, or absolute stability. In fact, the addition of dissipative material to the

FIGURE 8.16 "Y" - BASELINE STRUCTURE, UNCONTROLLED

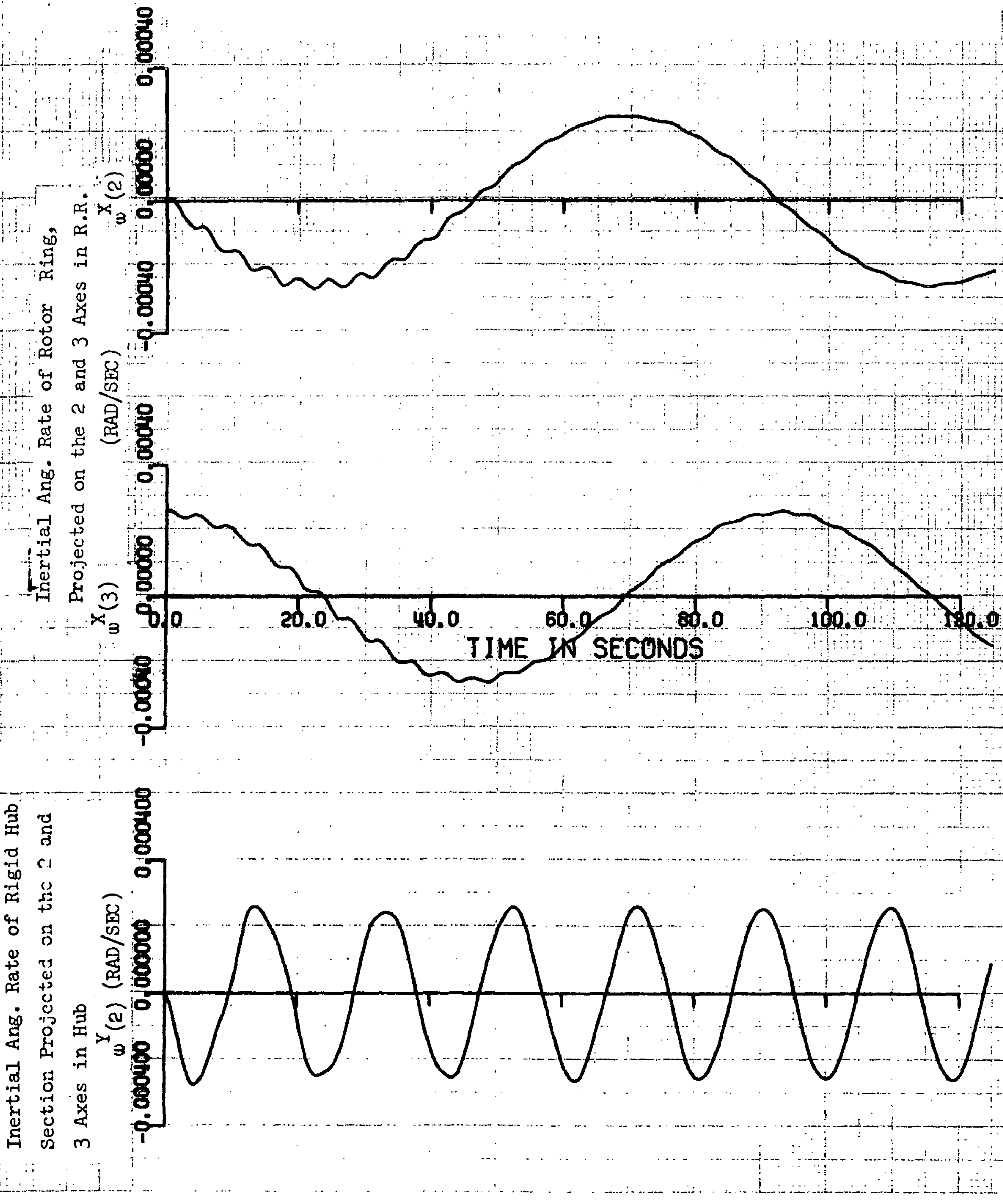
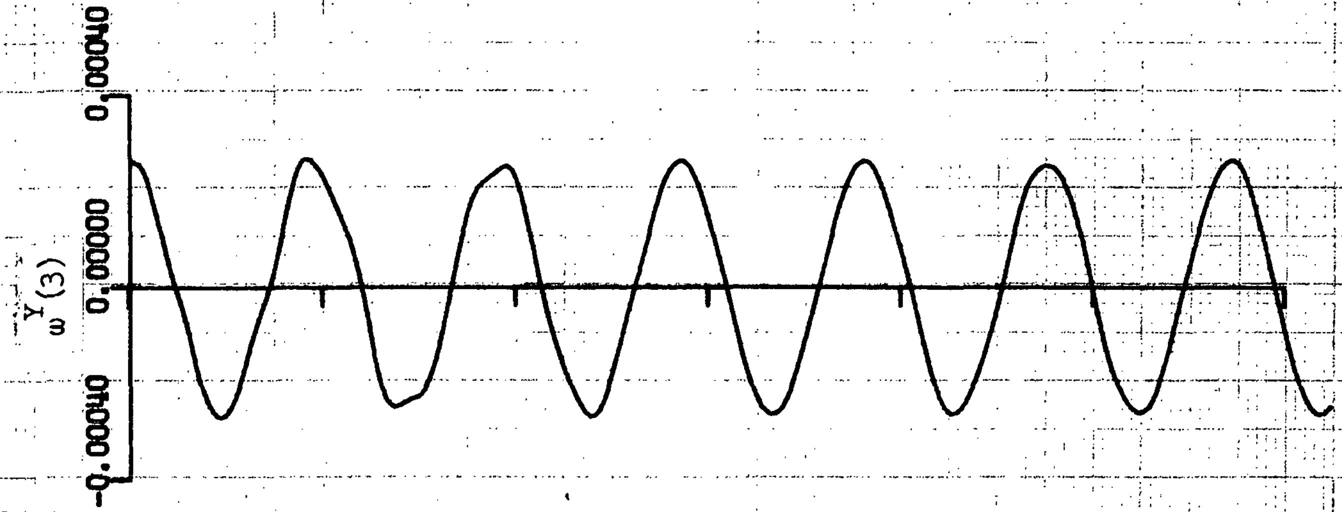
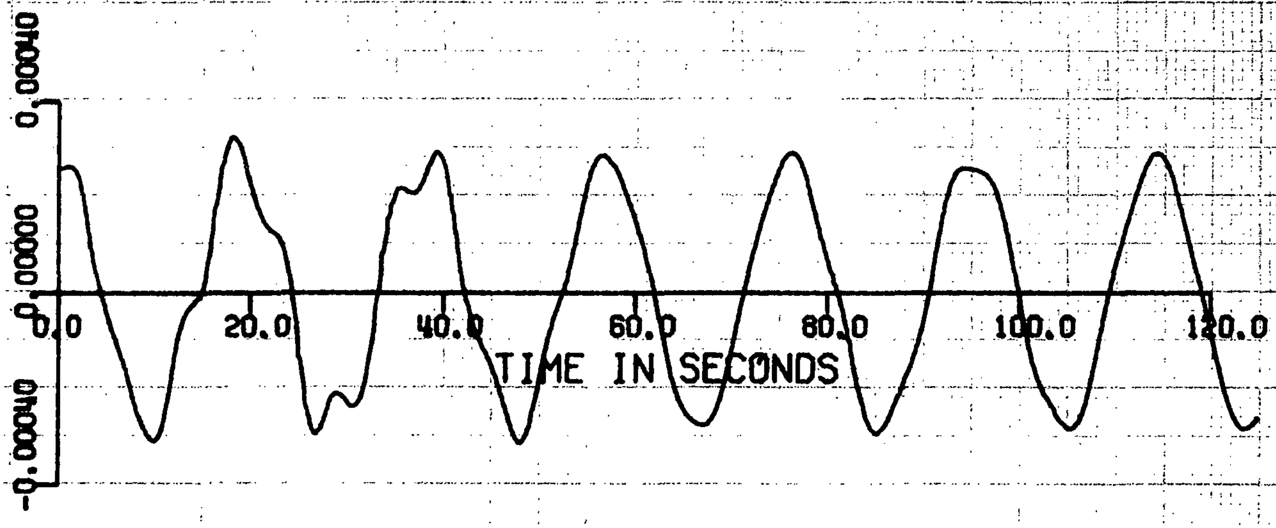
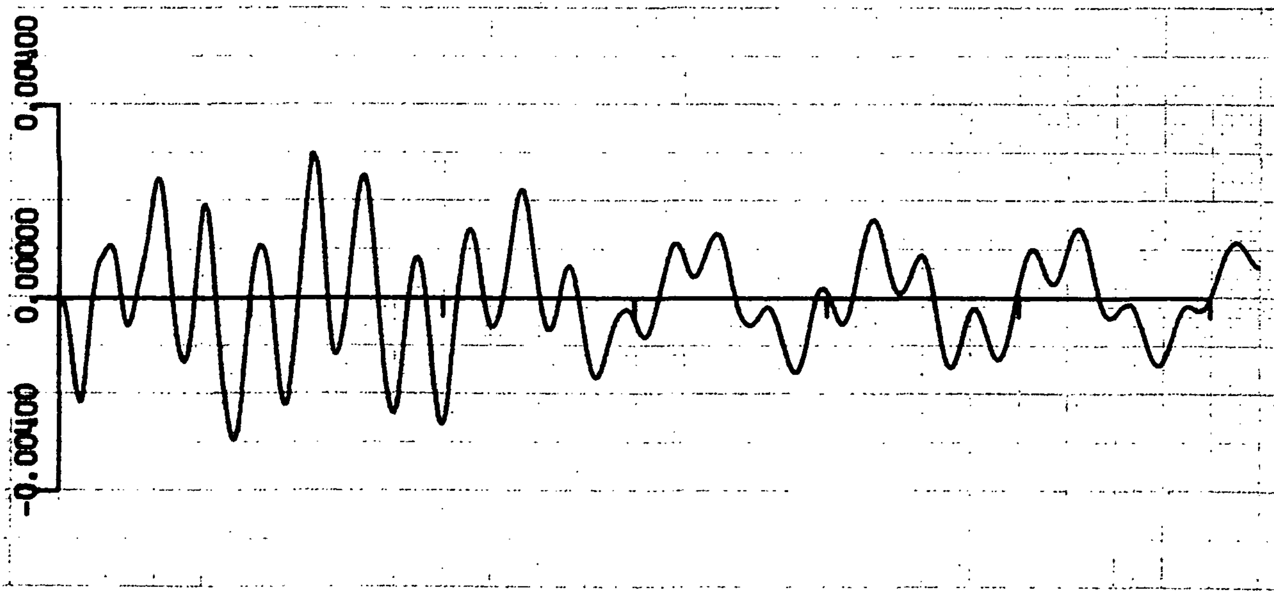


FIGURE 8.16 (Con't)





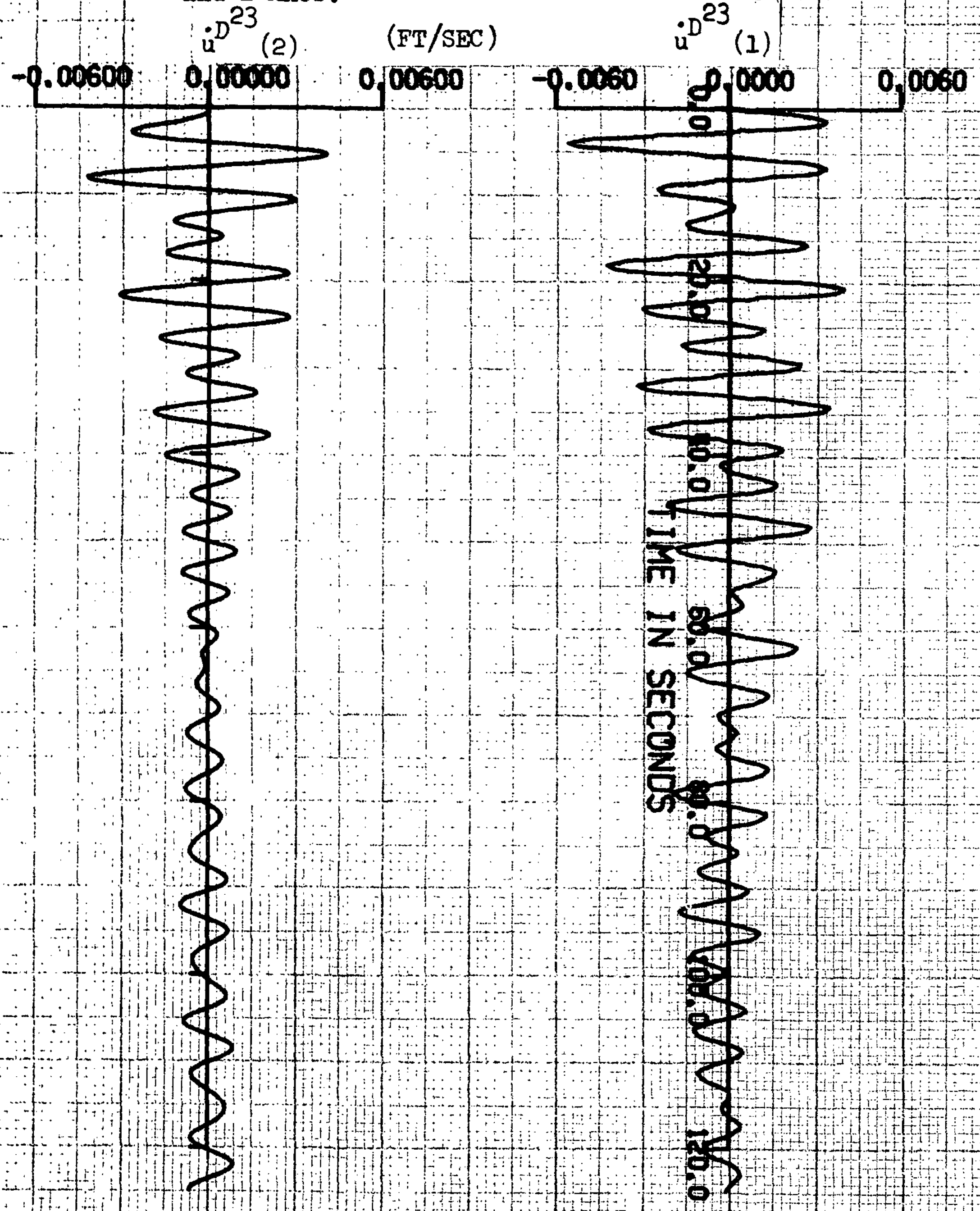
Inertial Ang. Rate of Third Hub Mass Projected on the Mass' 3 Axis. $\omega^{y^{c^3}}(3)$ (RAD/SEC)



Linear Velocity of the Third Hub Mass Relative to the Rigid Hub Section, Projected on the Mass' 2 Axis. $\dot{u}^{c^3}(2)$ (FT/SEC)

FIGURE 8.16 (Con't)

Linear Velocity of the Third Rotor Mass in the Second Arm, Relative to the Rotor Ring, Projected on the Mass' 1 and 2 Axes.



Linear Velocity of Third Rotor Mass in the First Arm, Relative to the Rotor Ring, Projected on the Mass' 1 Axis.

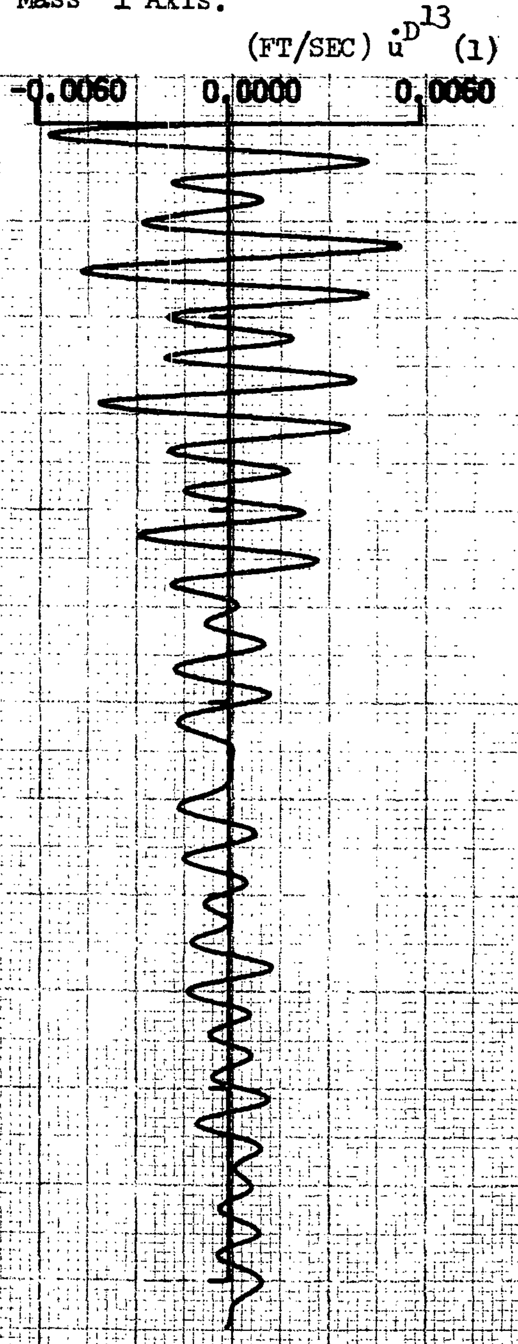
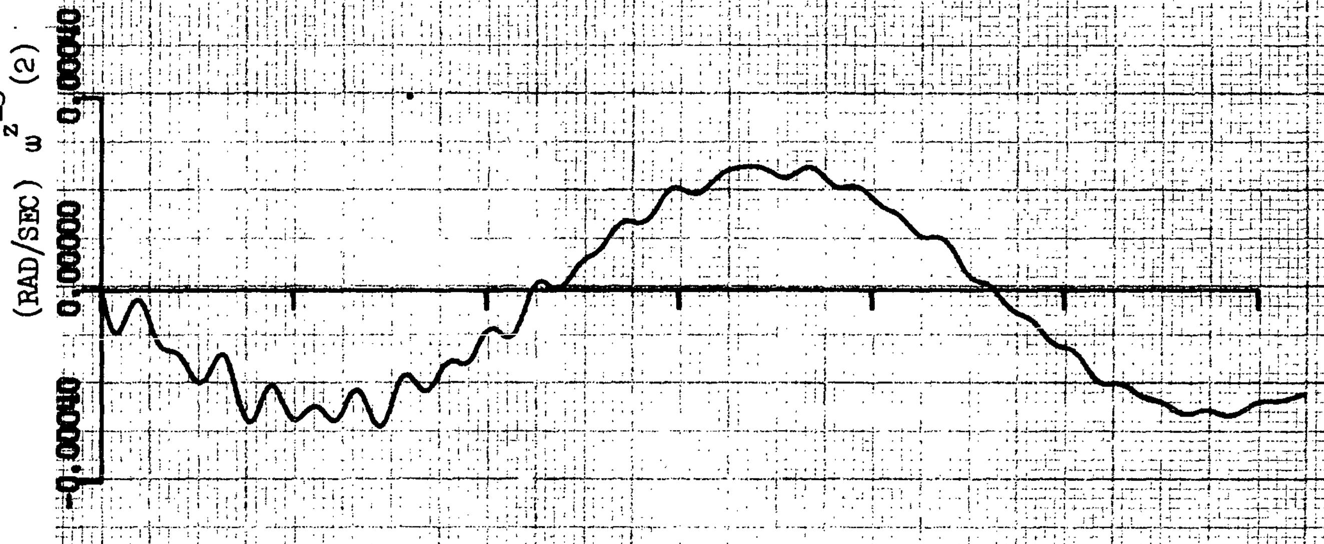
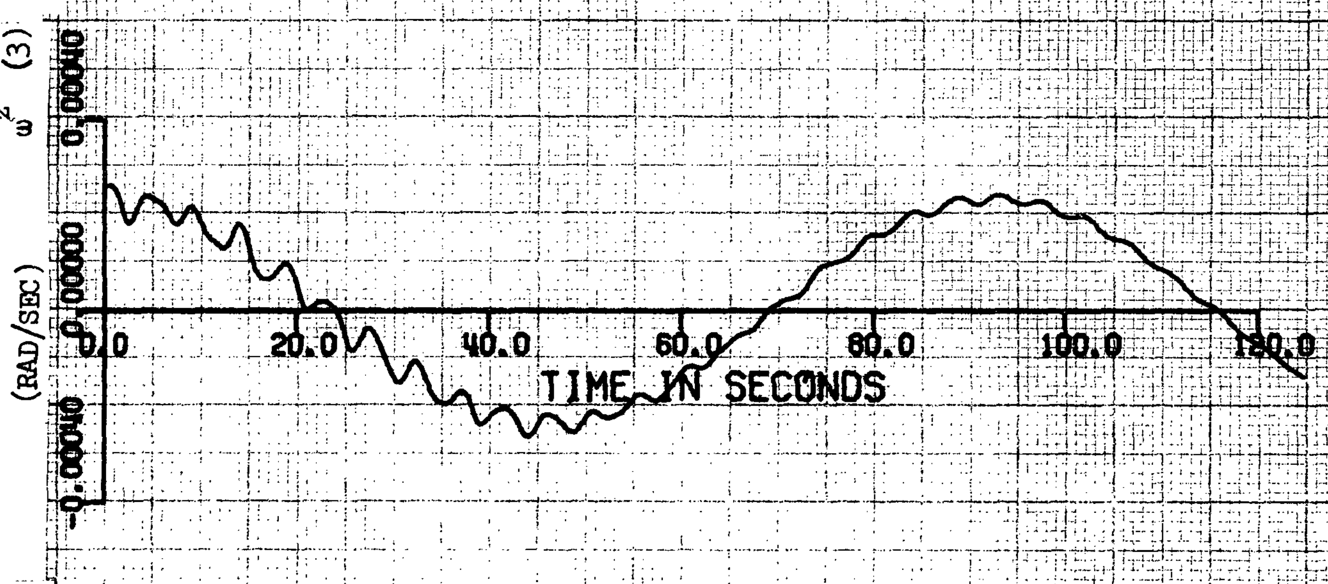


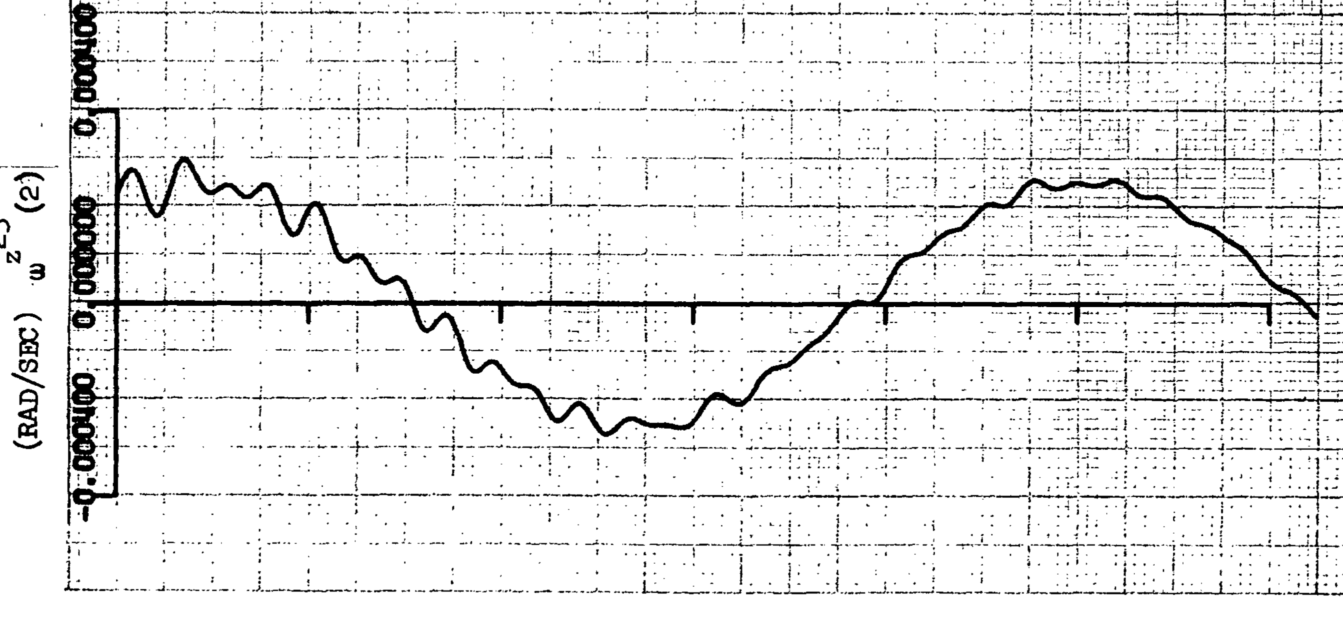
FIGURE 8.16 (Con't)



Inertial Angular Rate of the Third Rotor Mass in the First Arm, Projected on the Mass' 2 Axis.



Inertial Angular Rate of the Third Rotor Mass in the First Arm, Projected on the Mass' 3 Axis



Inertial Angular Rate of the Third Rotor Mass in the Second Arm, Projected on the Mass' 2 Axis

FIGURE 8.17 . "Y" - NOMINAL STRUCTURE, UNCONTROLLED

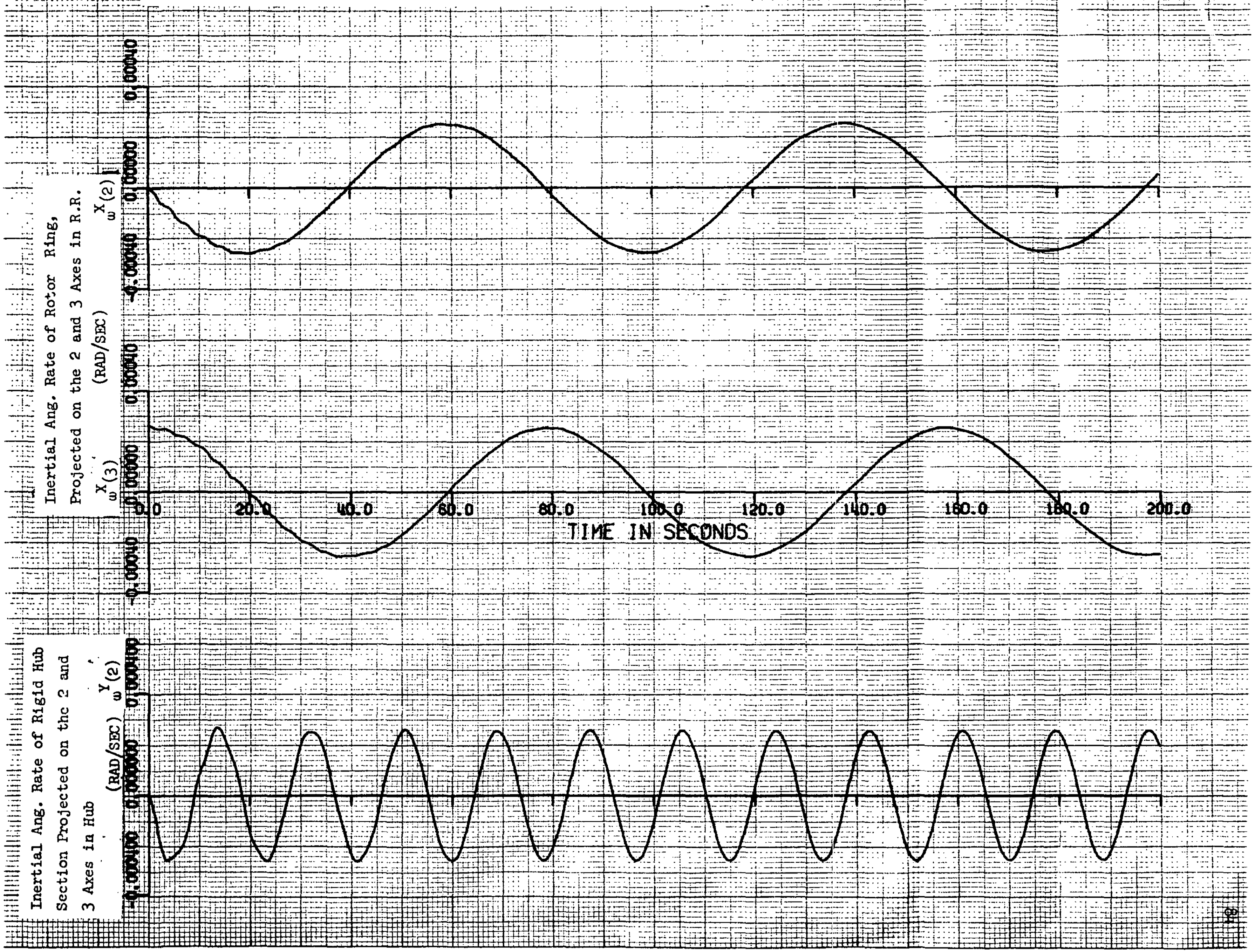


FIGURE 8.17 (Con't)

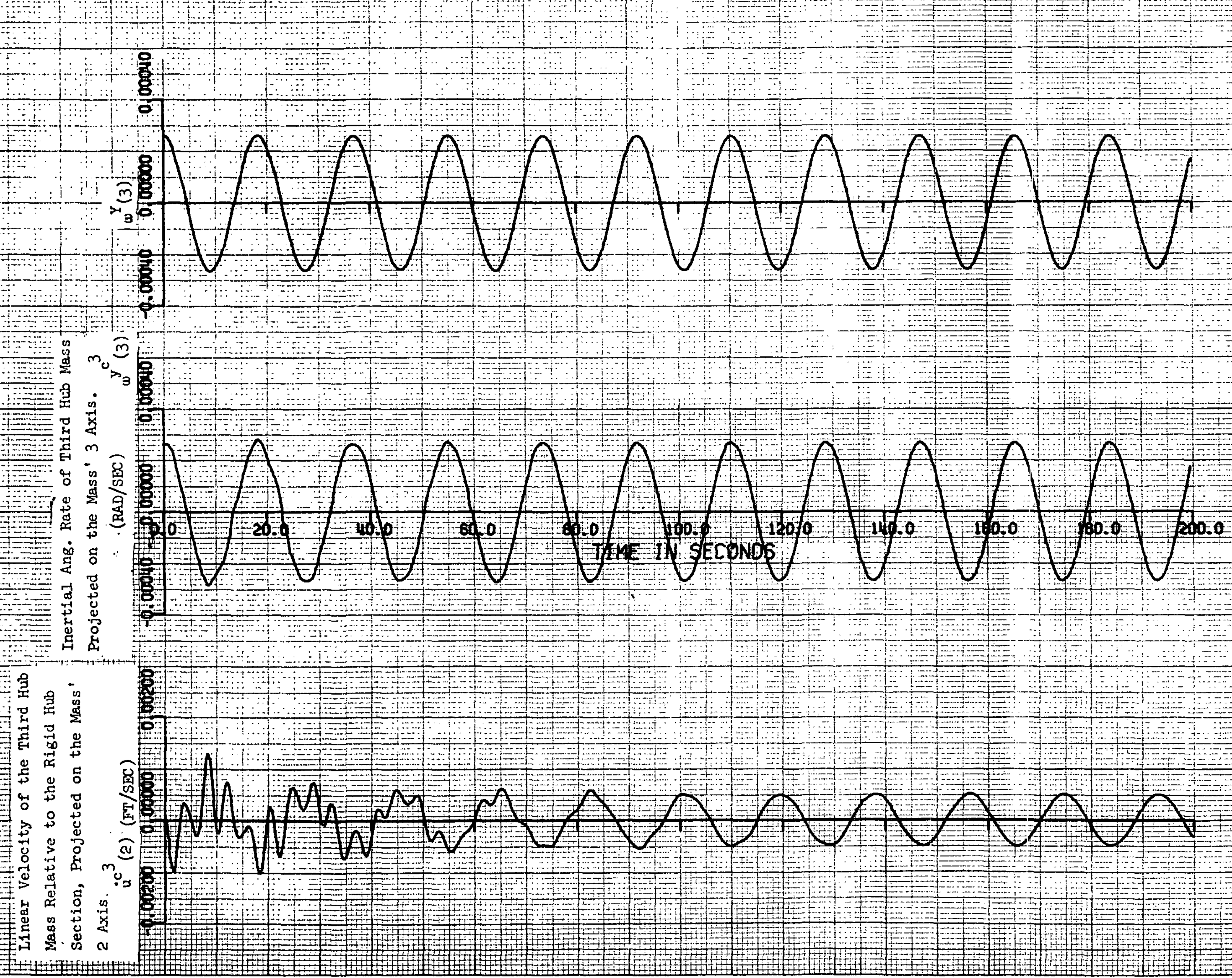
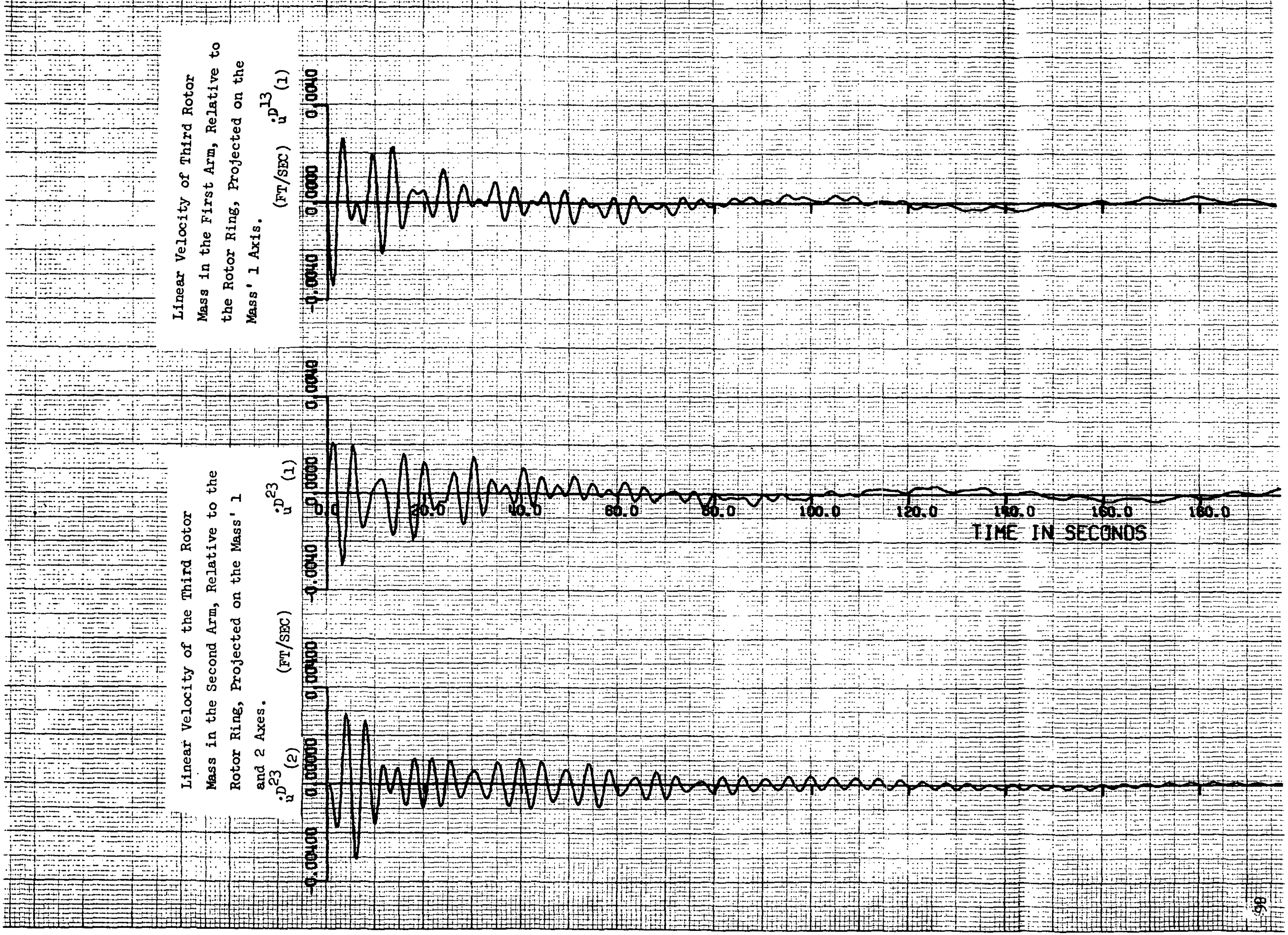


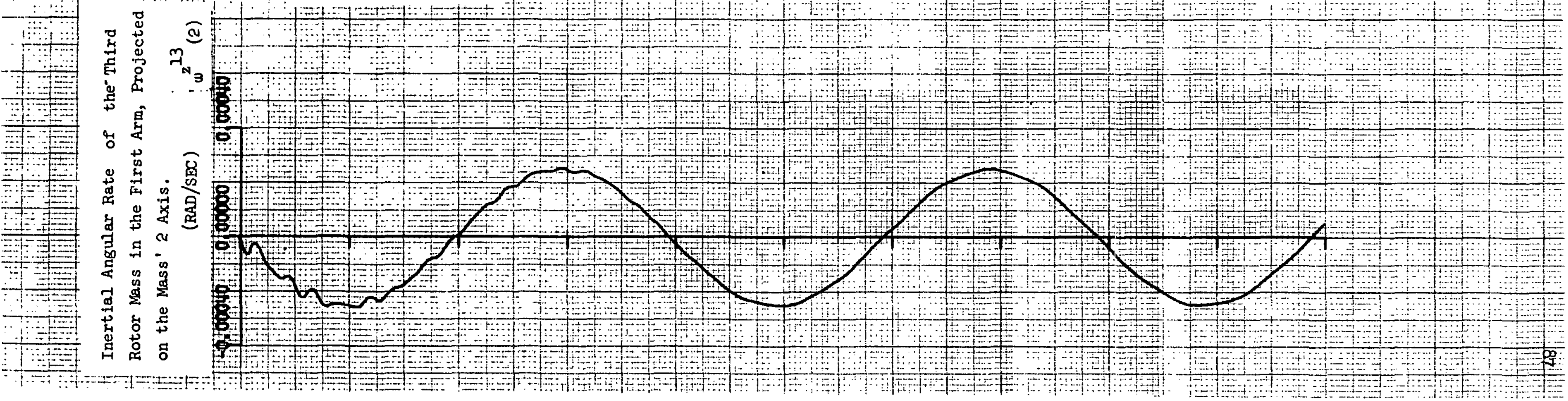
FIGURE 8.17 (Con't)



Linear Velocity of Third Rotor Mass in the First Arm, Relative to the Rotor Ring, Projected on the Mass' 1 Axis.

Linear Velocity of the Third Rotor Mass in the Second Arm, Relative to the Rotor Ring, Projected on the Mass' 1 and 2 Axes.

FIGURE 8.17 (Con't)



extension arms, both rotor and hub, increased the wobble damping. For example, the simulation condition with nominal stiffness experienced wobble decay with an estimated 2 hr. time constant in contrast to the 18.6 hr decay-time constant associated with perfectly stiff extension arms (as noted in Section 8.2, the results corresponding to bearing flexibility).

Comparing these time histories with those resulting from just a flexible bearing (Figure 8-9) it is seen that flexible motion has increased significantly as indicated by the high frequency oscillation, superimposed on the wobble motion. It should be noted that although a major portion of the flexible motion is damped during the course of the simulation run, there is a significant amount of flexible motion remaining which is being forced by the wobble motion (e.g., refer to the relative motion time histories of $\dot{\mu}^C$ and $\dot{\mu}^D$)

It is interesting to note that as the extension arm stiffness is altered the rigid body wobble characteristics change significantly. That is, as the extension arms were made stiffer it was observed that the wobble frequency increased. Specifically, the wobble period for different arm stiffness conditions are:

$$\text{"Y" Wobble Period} = \begin{cases} 93 \text{ sec, Baseline Stiffness} \\ 79 \text{ sec, Nominal Stiffness (2 x Base)} \\ 65 \text{ sec, Infinite Stiffness} \end{cases}$$

In other words, the effective transverse inertia of the total vehicle is lowered as the structure is made stiffer. A similar phenomenon was observed during studies when just the bearing stiffness was varied, and also during investigations of the flexible "T" configuration.

"Y" With Shuttle - Figure 8-18 are time histories resulting from the simulation of a "Y" space station with a shuttle flexibly attached to the rigid hub section. The initial conditions correspond to a nominal (or 1 sigma) docking disturbance as derived in Section 5.1. All of the physical properties are nominal (i.e., the arm stiffness is twice the baseline value) as presented in Section 4.0. As noted by the direction of rotation of the wobble vector, the addition of a shuttle transforms the "Y" configuration, which was

FIGURE 8.18 "Y"/SHUTTLE - NOMINAL, UNCONTROLLED

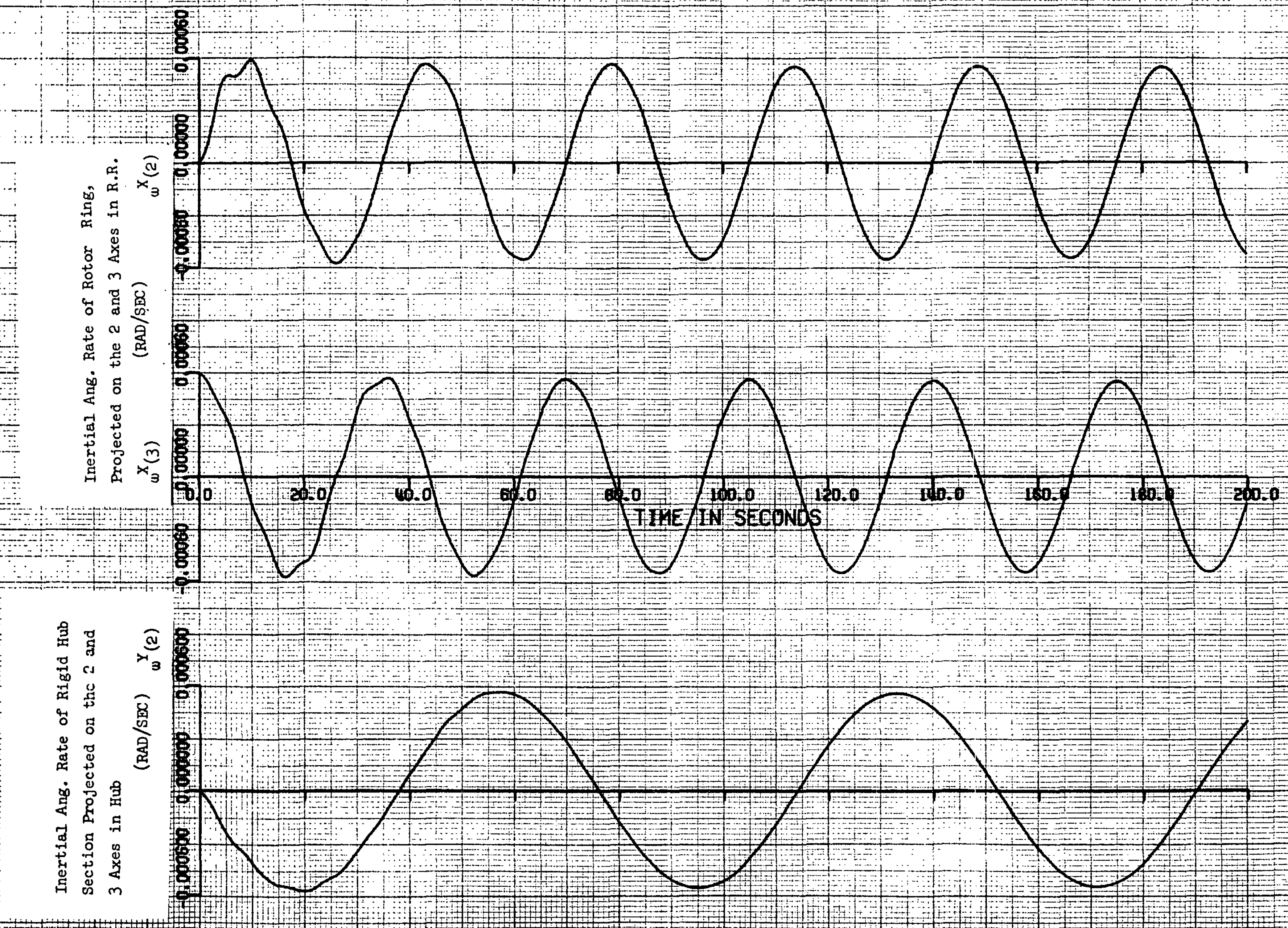
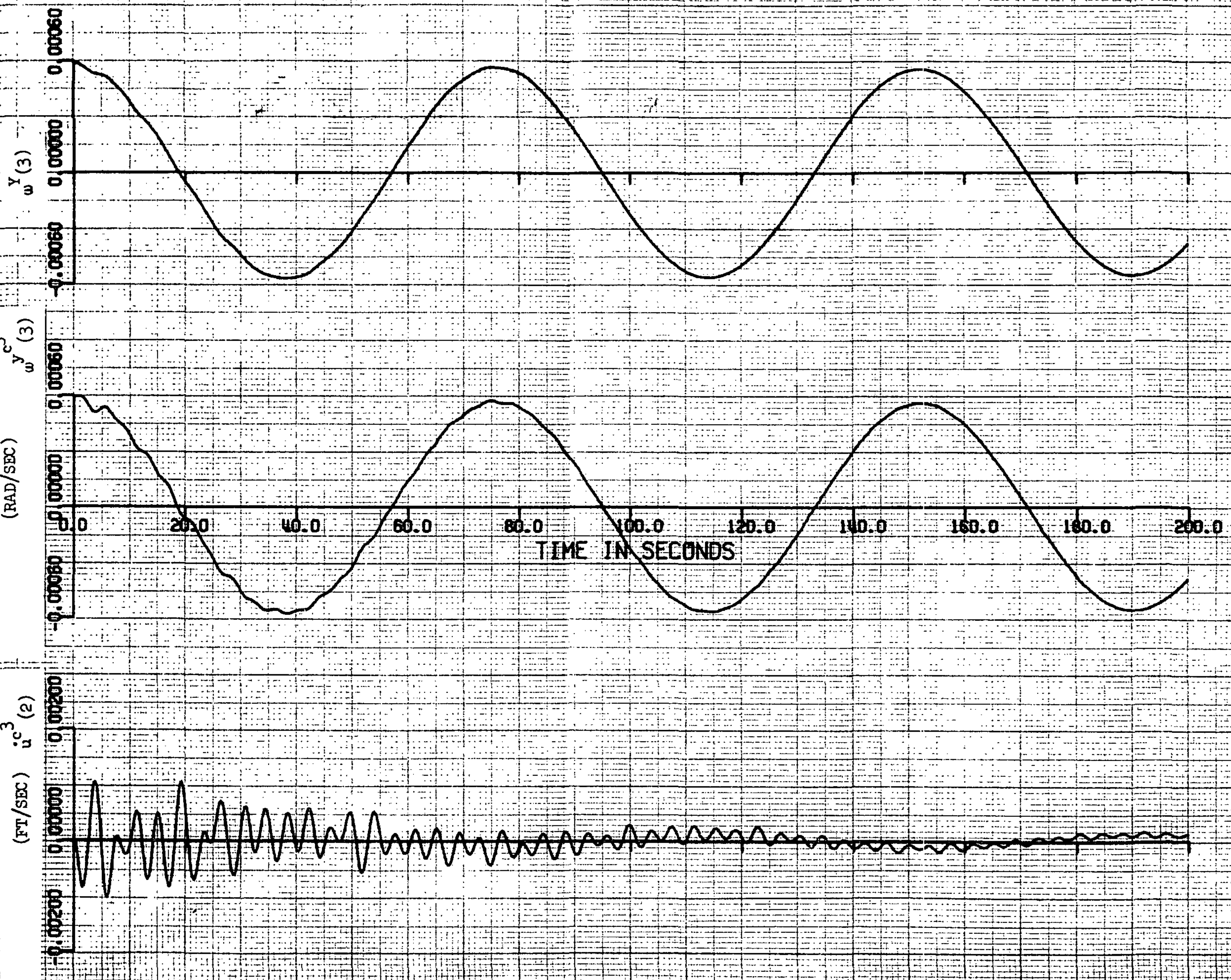
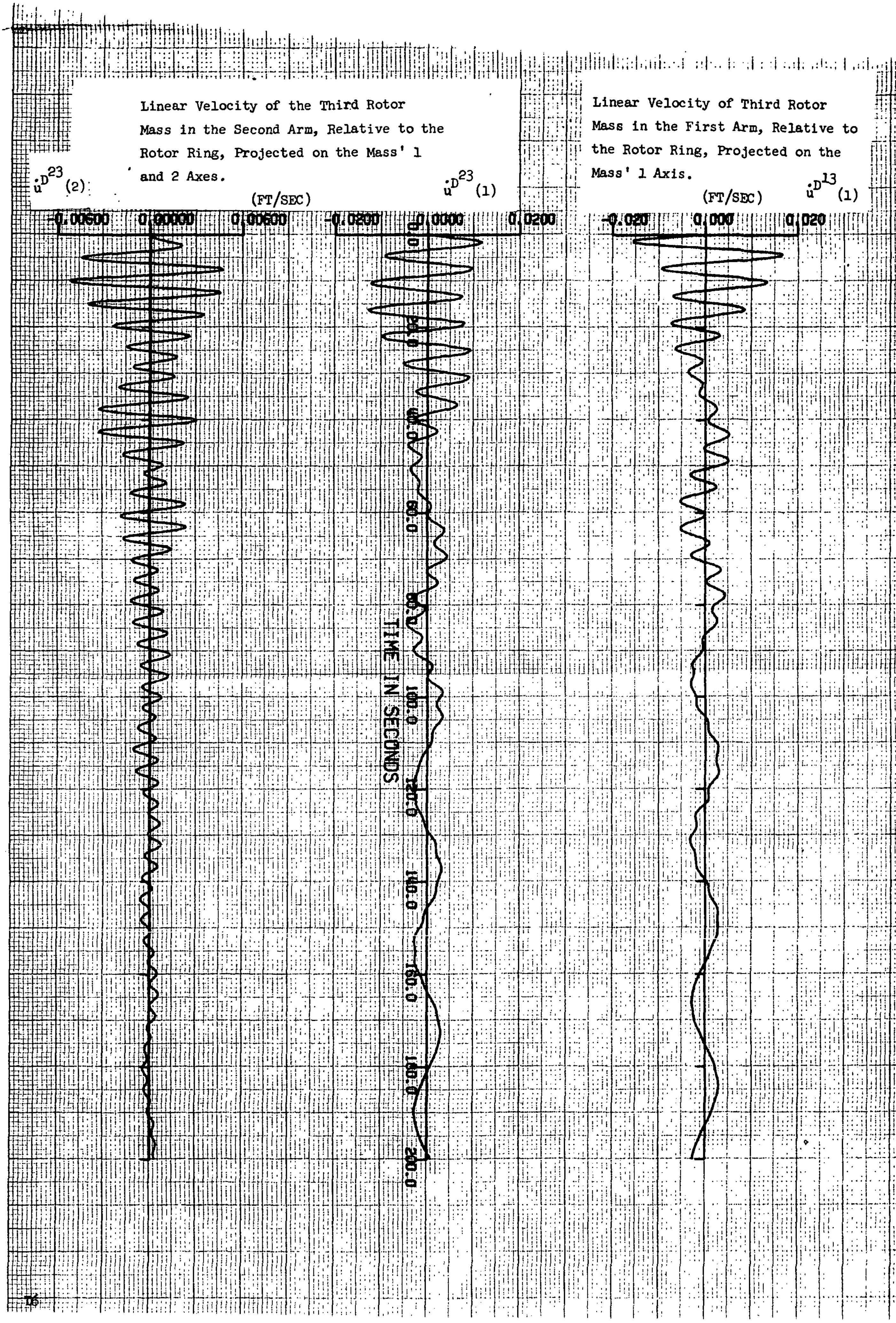


FIGURE 8.18 (Con't)



Inertial Ang. Rate of Third Hub Mass
Projected on the Mass' 3 Axis
(RAD/SEC)

Linear Velocity of the Third Hub
Mass Relative to the Rigid Hub
Section, Projected on the Mass'
2 Axis.
(FT/SEC)



Linear Velocity of the Third Rotor Mass in the Second Arm, Relative to the Rotor Ring, Projected on the Mass' 1 and 2 Axes.

Linear Velocity of Third Rotor Mass in the First Arm, Relative to the Rotor Ring, Projected on the Mass' 1 Axis.

FIGURE 8.18 (Con't)

Inertial Angular Rate of the Third Rotor Mass in the First Arm, Projected on the Mass' 2 Axis.

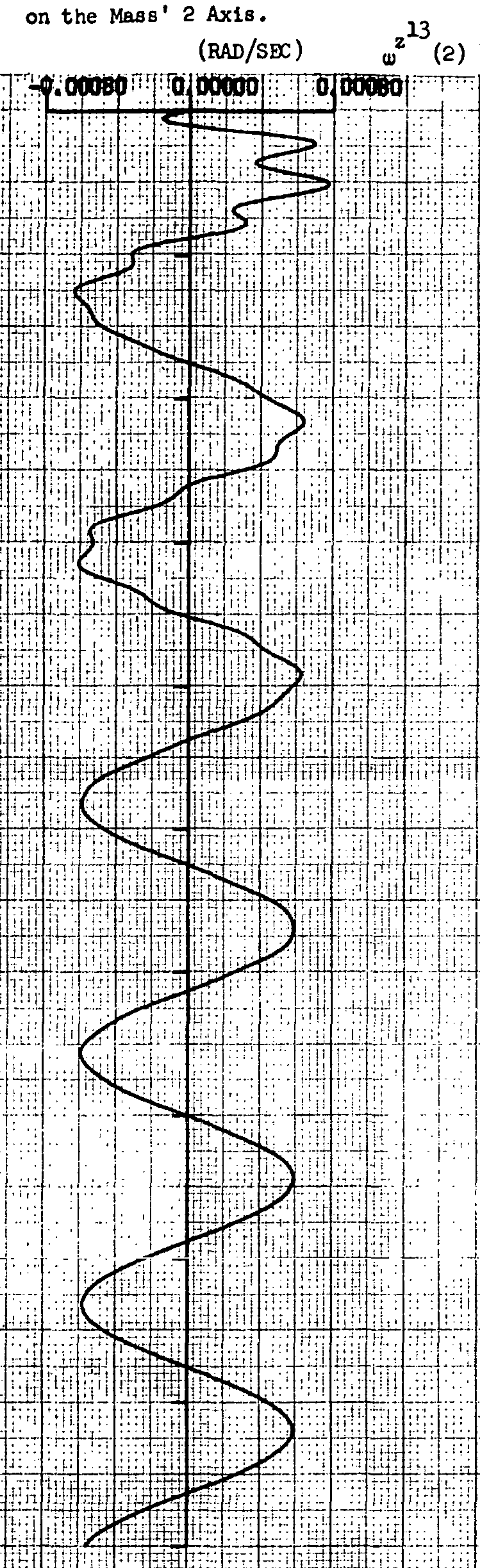


FIGURE 8.18 (Con't)

originally "Max I", into a "Min I" configuration. Although this configuration was unstable with just bearing flexibility, the additional energy dissipative material that now is present in the hub and in the shuttle docking mechanism has stabilized the uncontrolled "Y"/Shuttle configuration. The estimated decay time constant, as obtained from printed simulation output data, is 0.73 hrs. This particular vehicle configuration is one of the few conditions investigated where the actual decay is observable on the time history traces.

T - Space Station - The time history traces shown in Figures 8-19 and 8-20 are the result of simulated "T" configuration tests with baseline and nominal stiffness conditions, respectively. The initial conditions represent a disturbance resulting from internal mass motion (refer to Section 5.2). Although all the structural stiffness conditions investigated resulted in a seemingly stable wobble mode, the stability is extremely marginal. That is, the decay time constant is of the order of several hours. In addition, as the wobble mode approaches neutral stability the accuracy to which a time history program can detect absolute stability is questionable. If a more accurate determination of stability is required then a characteristic root analysis should be performed, which would require the application of Floquet Theory to accomodate the time varying coefficients (refer to Reference 9).

A simulated condition of a "T" station with infinitly stiff hub in combination with nominal flexible rotor arms and bearing was investigated. The resulting time histories are shown in Figure 8-21. The wobble mode for this condition is definitely unstable. Therefore, energy dissipative material added to the hub has a stabilizing influence as indicated by the previously described results where marginal stability was observed.

When viewing the time history traces it is observed that the amplitude of the relative flexible motions are generally higher for the "T" configuration as opposed to "Y" configuration results with identical stiffness characteristics. These larger flexible motions are attribued to the mass distribution associated with the "T" configuration.

As experienced during "Y" investigations, the frequency of the rigid body wobble mode varied with changes in structural stiffness. Consistent with the "Y" results, an increase in arm stiffness caused a decrease in the effective total

FIGURE 8.19 "T" - BASELINE, UNCONTROLLED

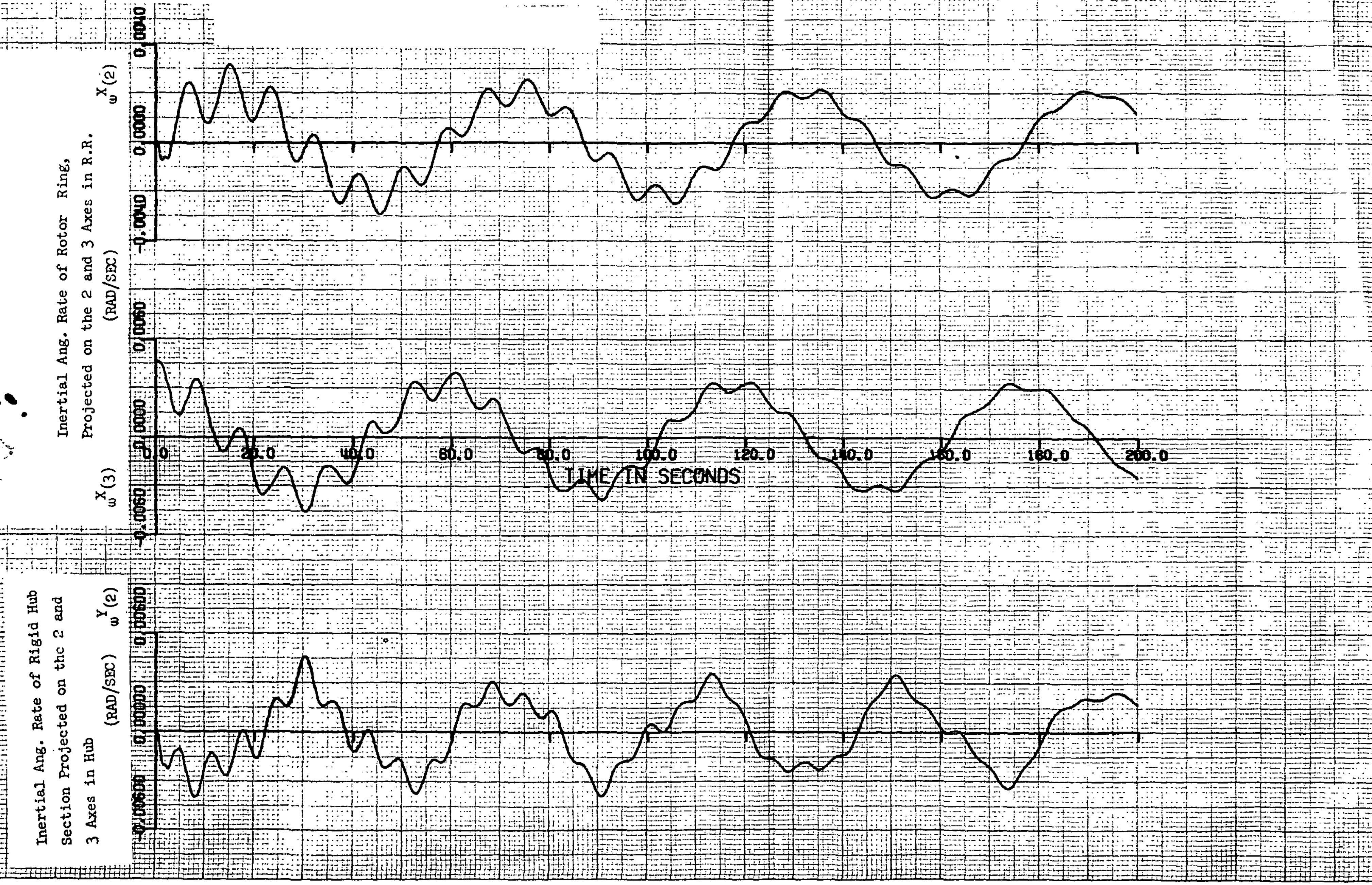
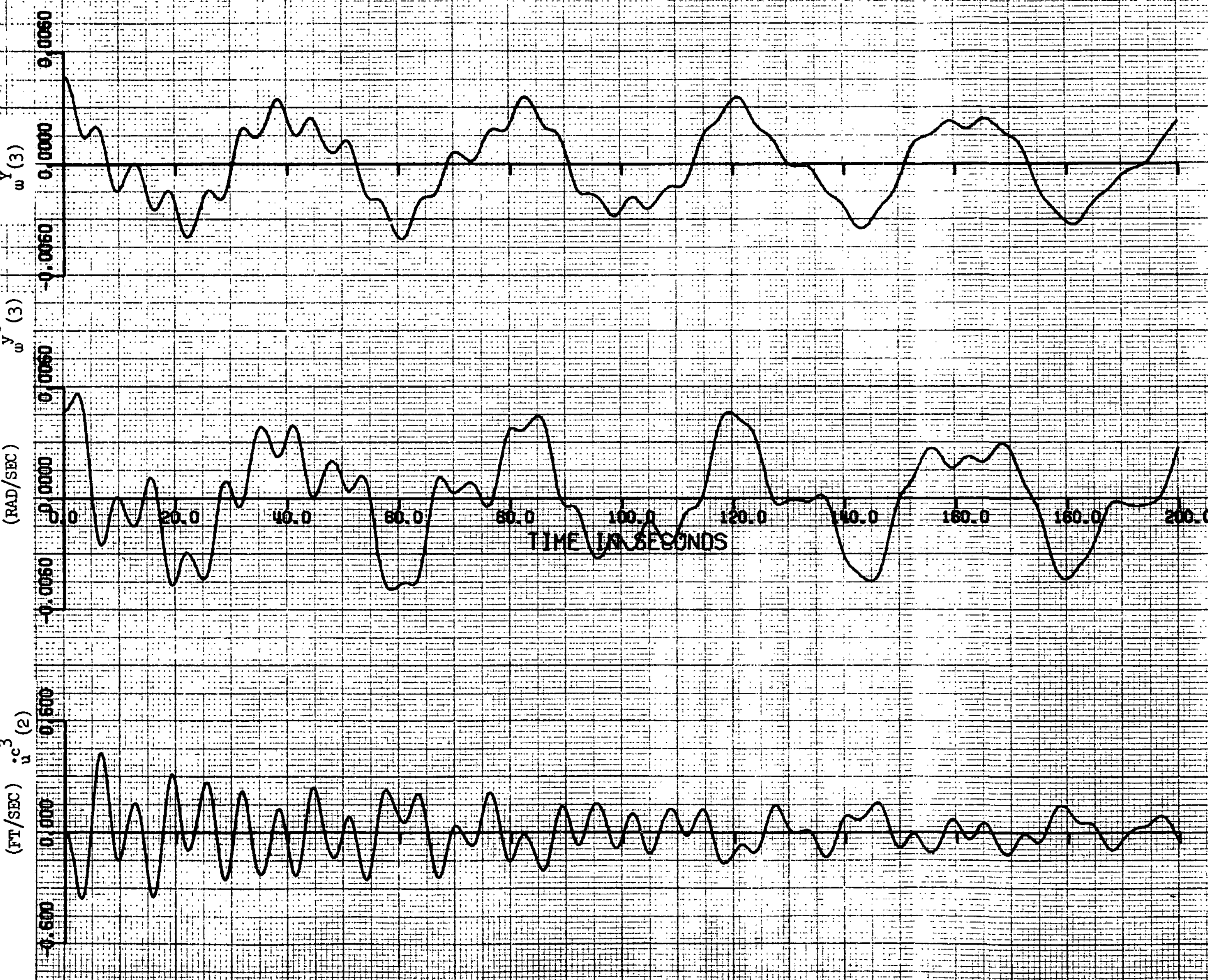


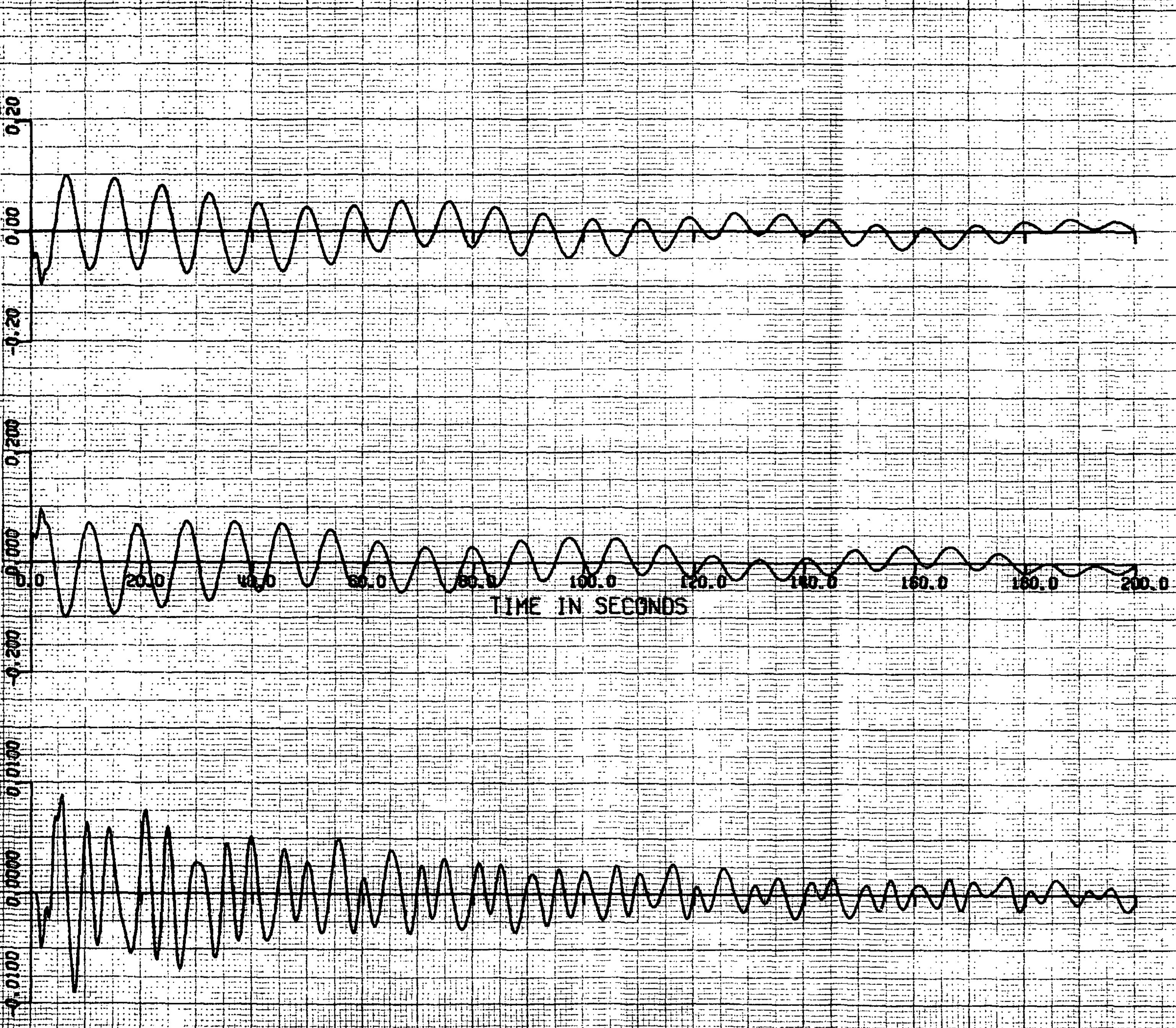
FIGURE 8.19 (Con't)



Inertial Ang. Rate of Third Hub Mass
Projected on the Mass' 3 Axis.

Linear Velocity of the Third Hub
Mass Relative to the Rigid Hub
Section, Projected on the Mass'
2 Axis.

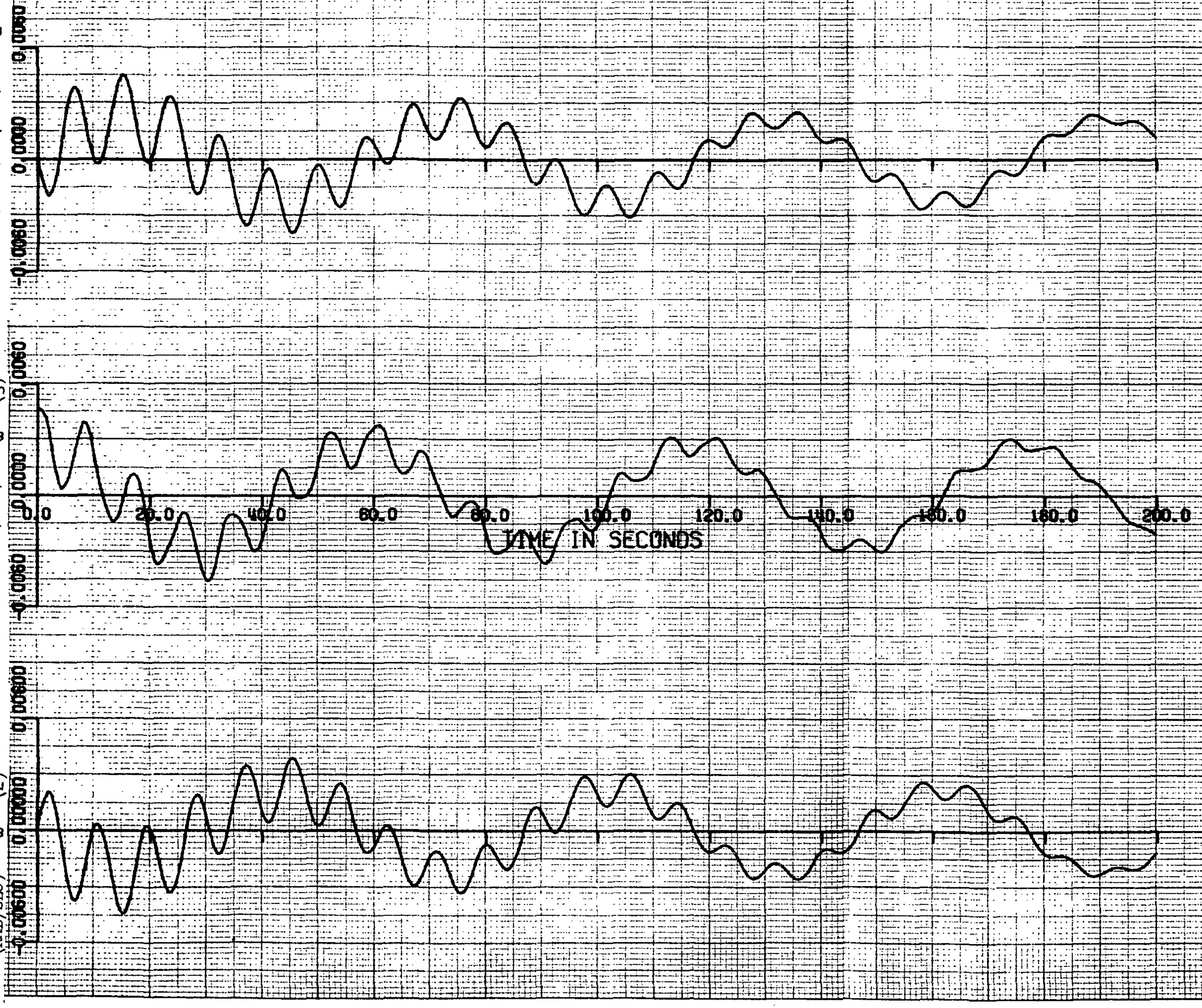
FIGURE 8.19 (Con't)



Linear Velocity of Third Rotor Mass in the First Arm, Relative to the Rotor Ring, Projected on the Mass' 1 Axis.

Linear Velocity of the Third Rotor Mass in the Second Arm, Relative to the Rotor Ring, Projected on the Mass' 1 and 2 Axes.

FIGURE 8.19 (Con't)



Inertial Angular Rate of the Third Rotor Mass in the First Arm, Projected on the Mass' 2 Axis. $\omega_z^{13}(2)$ (RAD/SEC)

Inertial Angular Rate of the Third Rotor Mass in the First Arm, Projected on the Mass' 3 Axis $\omega_z^{13}(3)$ (RAD/SEC)

nertial Angular Rate of the Third otor Mass in the Second Arm, Projected n the Mass' 2 Axis $\omega_z^{23}(2)$ (RAD/SEC)

FIGURE 8.20 "T" - NOMINAL, UNCONTROLLED

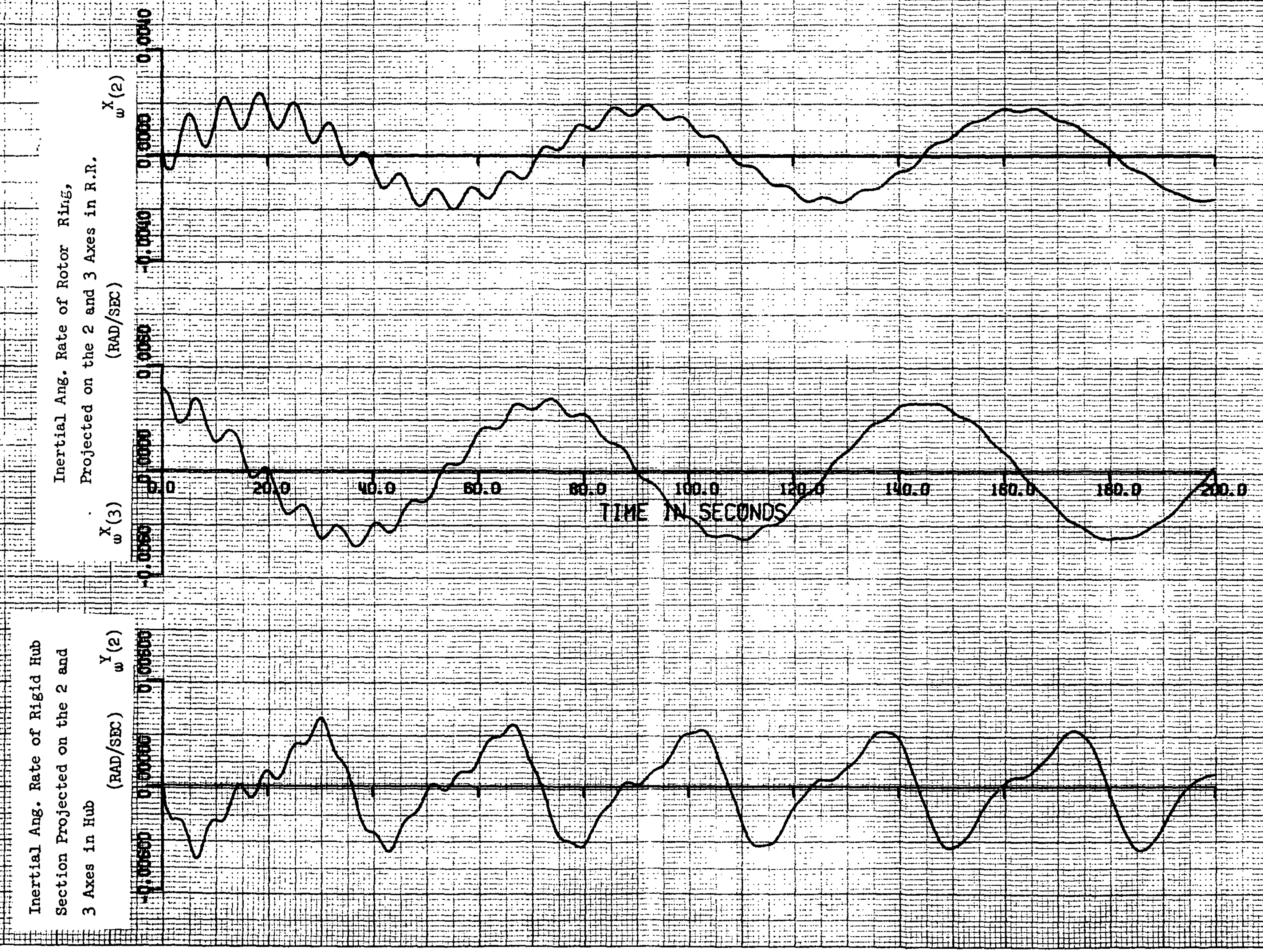
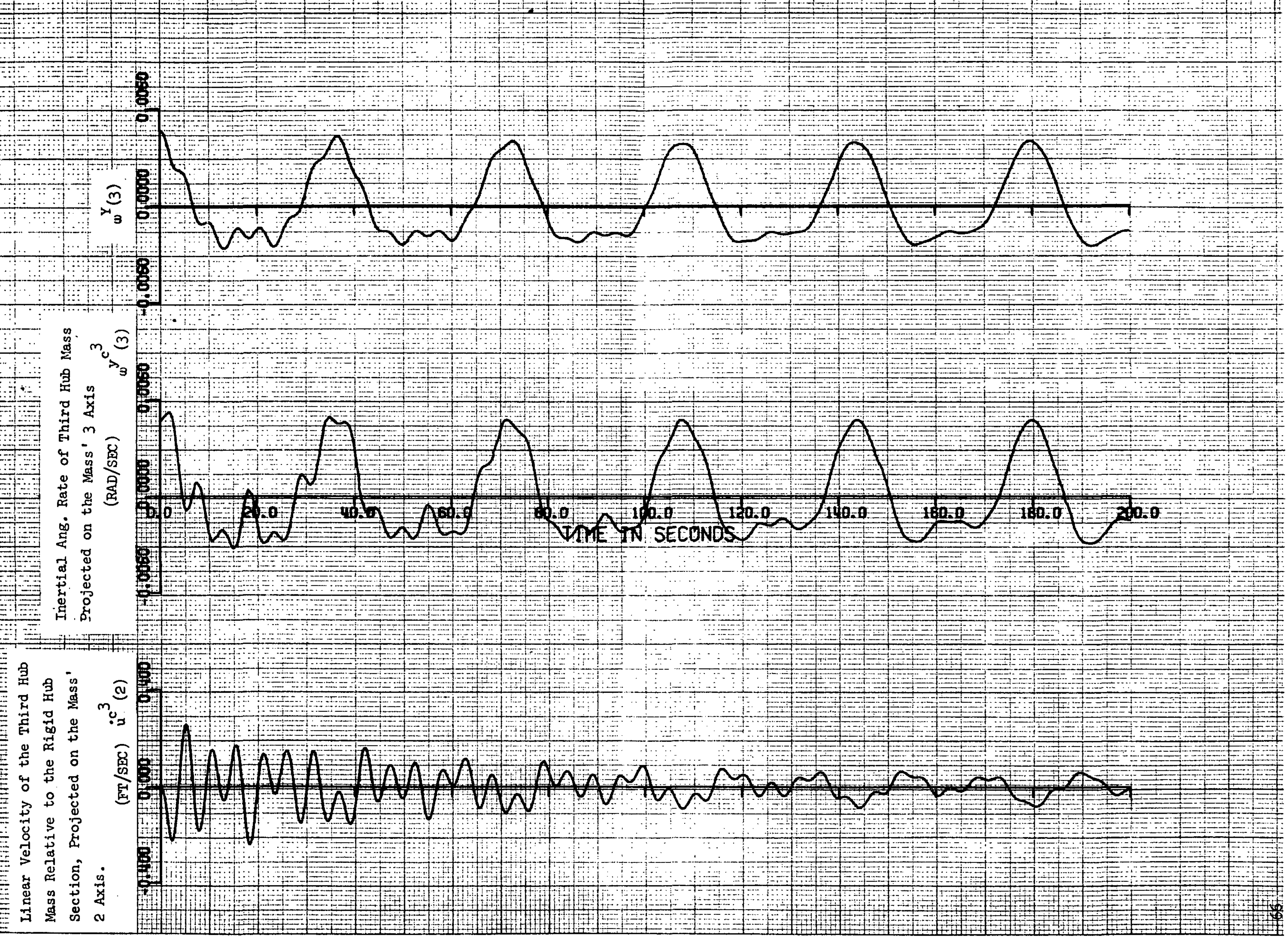


FIGURE 8.20 (Con't)

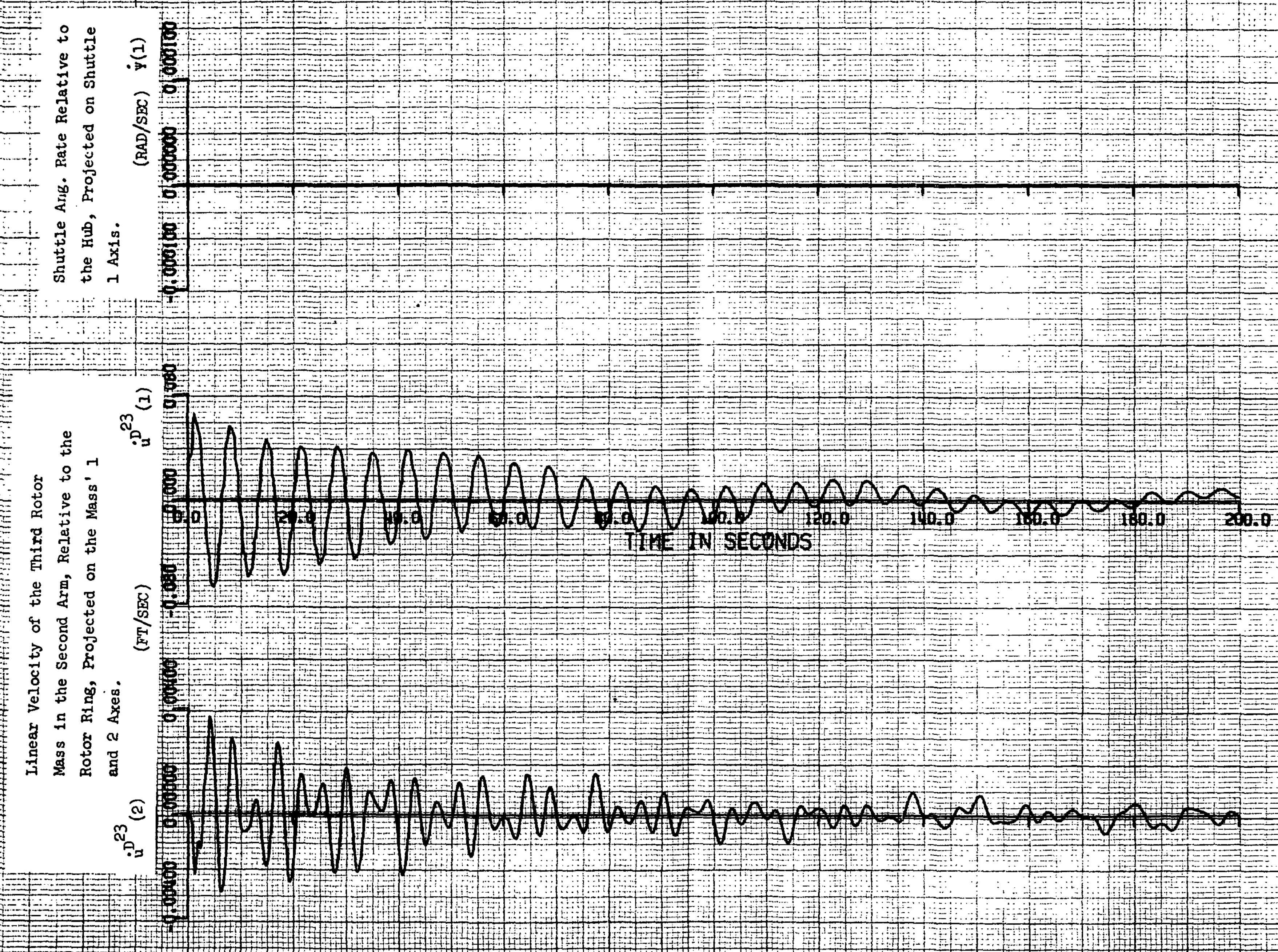


$\omega^{Y}(3)$

Inertial Ang. Rate of Third Hub Mass Projected on the Mass' 3 Axis (RAD/SEC) $\omega^{y}{}_{c^3}(3)$

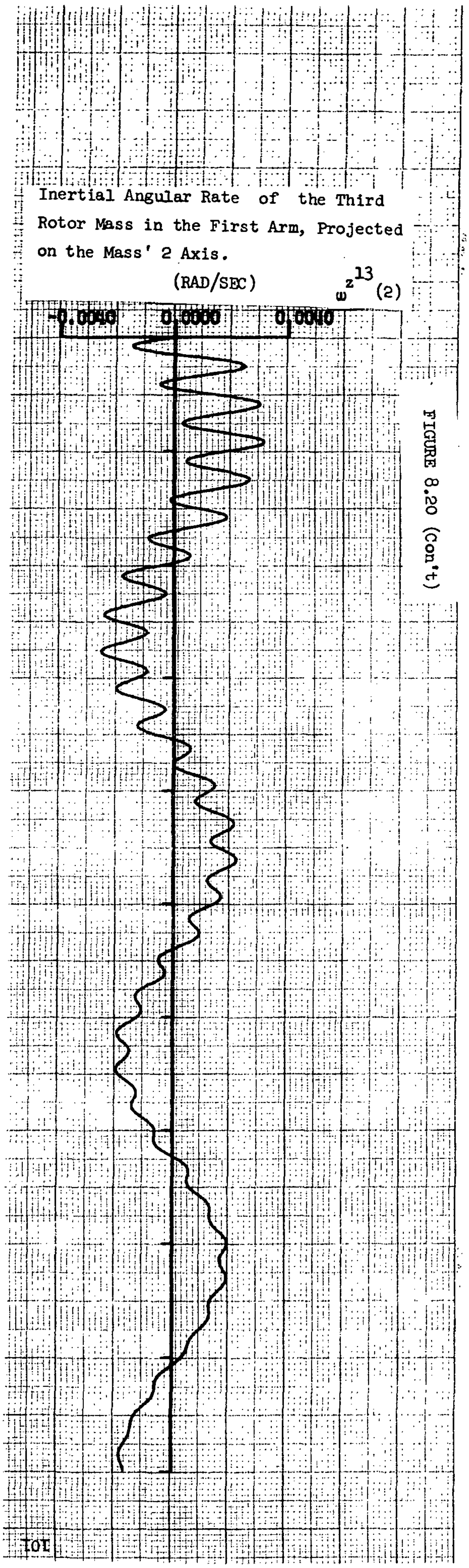
Linear Velocity of the Third Hub Mass Relative to the Rigid Hub Section, Projected on the Mass' 2 Axis. (FT/SEC) $\dot{u}_{c^3}(2)$

FIGURE 8.20 (Con't)



Shuttle Ang. Rate Relative to the Hub, Projected on Shuttle 1 Axis.

Linear Velocity of the Third Rotor Mass in the Second Arm, Relative to the Rotor Ring, Projected on the Mass' 1 and 2 Axes.



Inertial Angular Rate of the Third Rotor Mass in the First Arm, Projected on the Mass' 2 Axis.

FIGURE 8.20 (Con't)

FIGURE 8.21 "T" WITH RIGID HUB - NOMINAL ROTOR, UNCONTROLLED

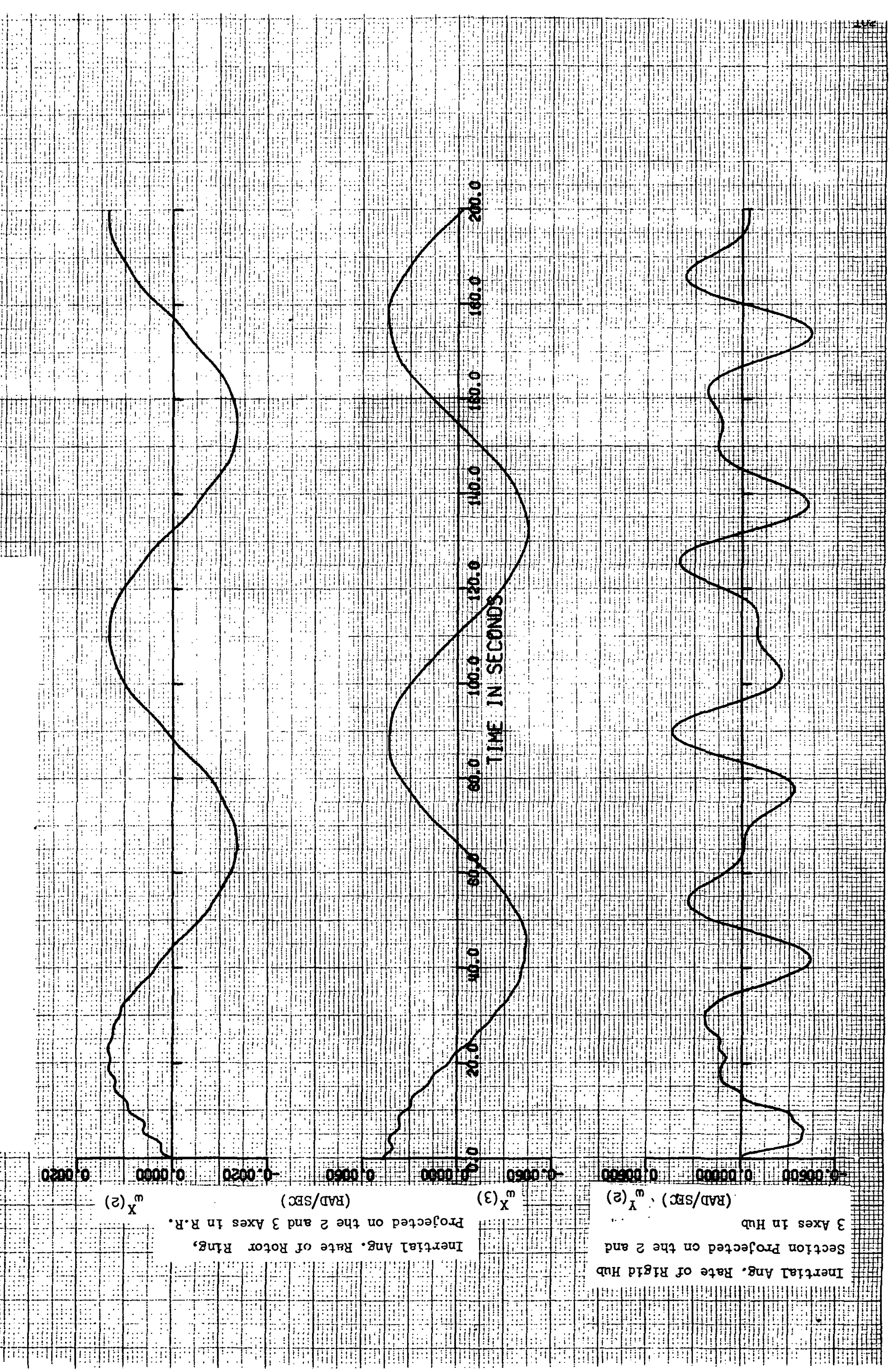
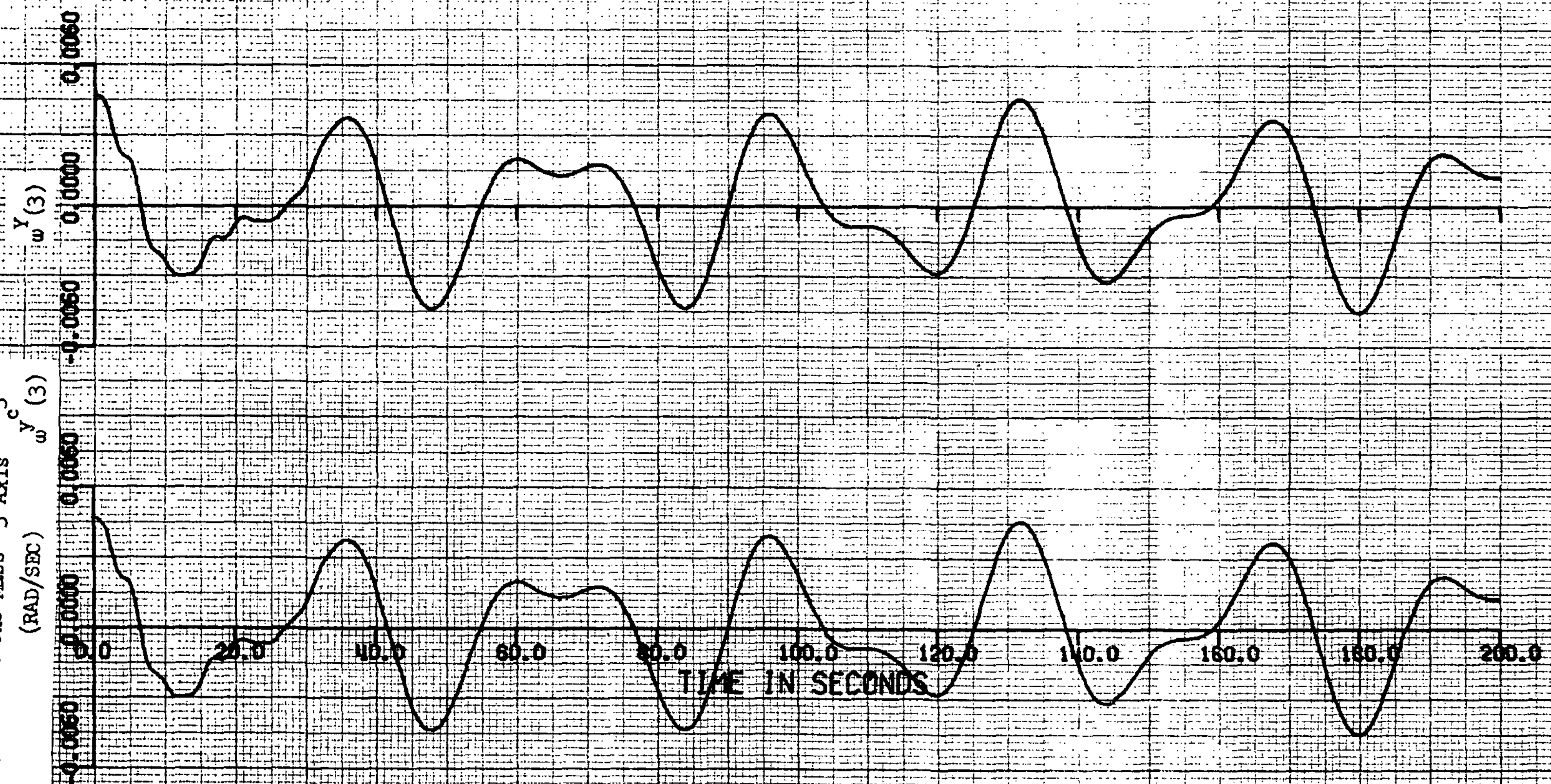
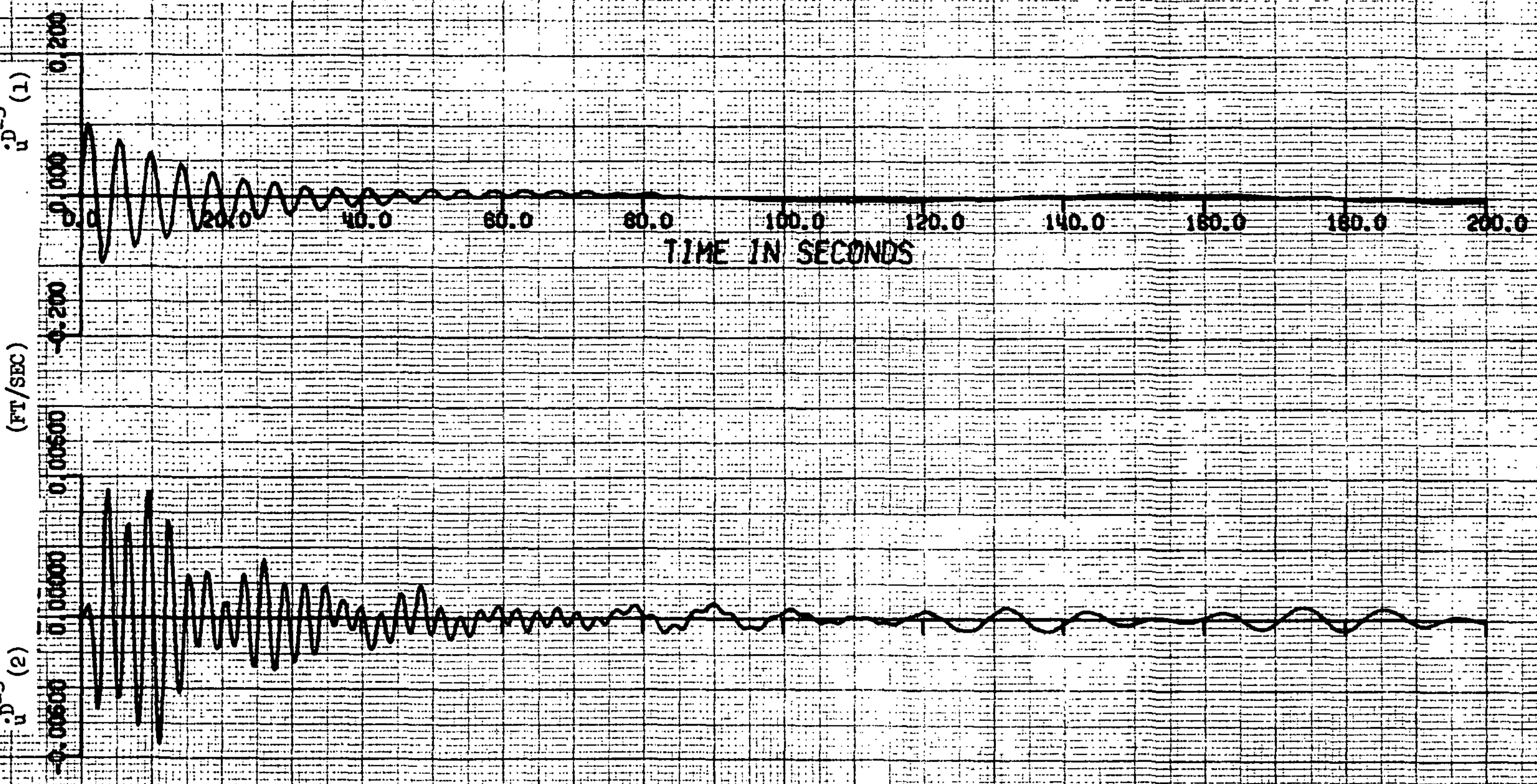


FIGURE 8.21 (Con't)



Inertial Ang. Rate of Third Hub Mass
Projected on the Mass' 3 Axis
(RAD/SEC)

FIGURE 8.21 (Con't)



Linear Velocity of the Third Rotor Mass in the Second Arm, Relative to the Rotor Ring, Projected on the Mass' 1 and 2 Axes.

Inertial Angular Rate of the Third
Rotor Mass in the First Arm, Projected
on the Mass' 2 Axis.

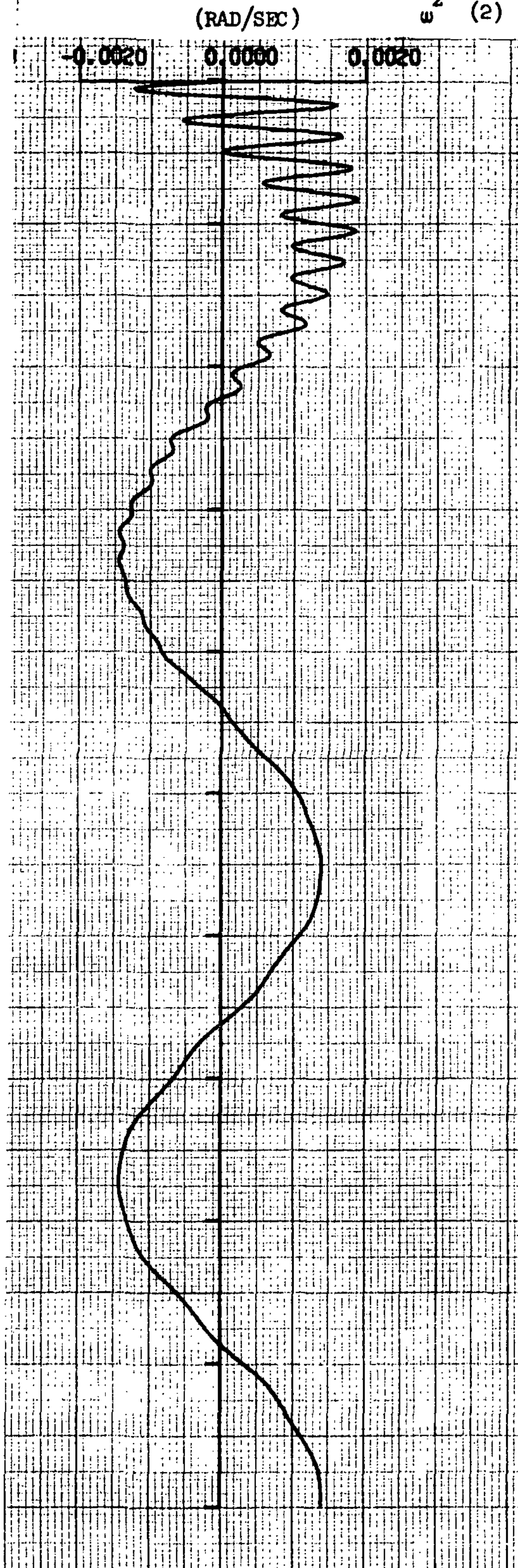


FIGURE 8.21 (Con't)

vehicle transverse inertia. However, in the case of the "T" configuration a decrease in transverse inertia results in a decrease in wobble frequency. Specifically, the wobble period for different arm stiffness conditions are:

$$\text{T Wobble Period} = \begin{cases} 59 \text{ sec, Baseline Stiffness} \\ 73 \text{ sec, Nominal Stiffness} \\ 91 \text{ sec, Infinite Stiffness} \end{cases}$$

<u>"T" With Shuttle</u> - The time histories resulting from the simulation of the "T" space station with a flexibly attached shuttle are shown in Figure 8-22. All of the physical properties are nominal as presented in Section 4.0. The initial conditions correspond to a 1 sigma docking disturbance (refer to Section 5.1). The wobble mode of this configuration is also marginally stable with a decay time constant of the order of a few hours. Again, there is sufficient energy dissipation in the shuttle docking mechanism and in the despun hub to stabilize this uncontrolled "Min I" configuration.

An additional time history trace was added to this set of traces in order to indicate the extent of the relative motion between the shuttle and the station. Examination of ψ indicates damped relative motion with a residual wobble forced vibration.

It is interesting to note that this configuration is no longer close to being an "Intermediate Inertia" vehicle. As a result, a predominate frequency equal to spin minus wobble frequency ($p - \lambda p$) is associated with the wobble motion as viewed in the hub.

FIGURE 8.22 "T"/SHUTTLE - NOMINAL, UNCONTROLLED

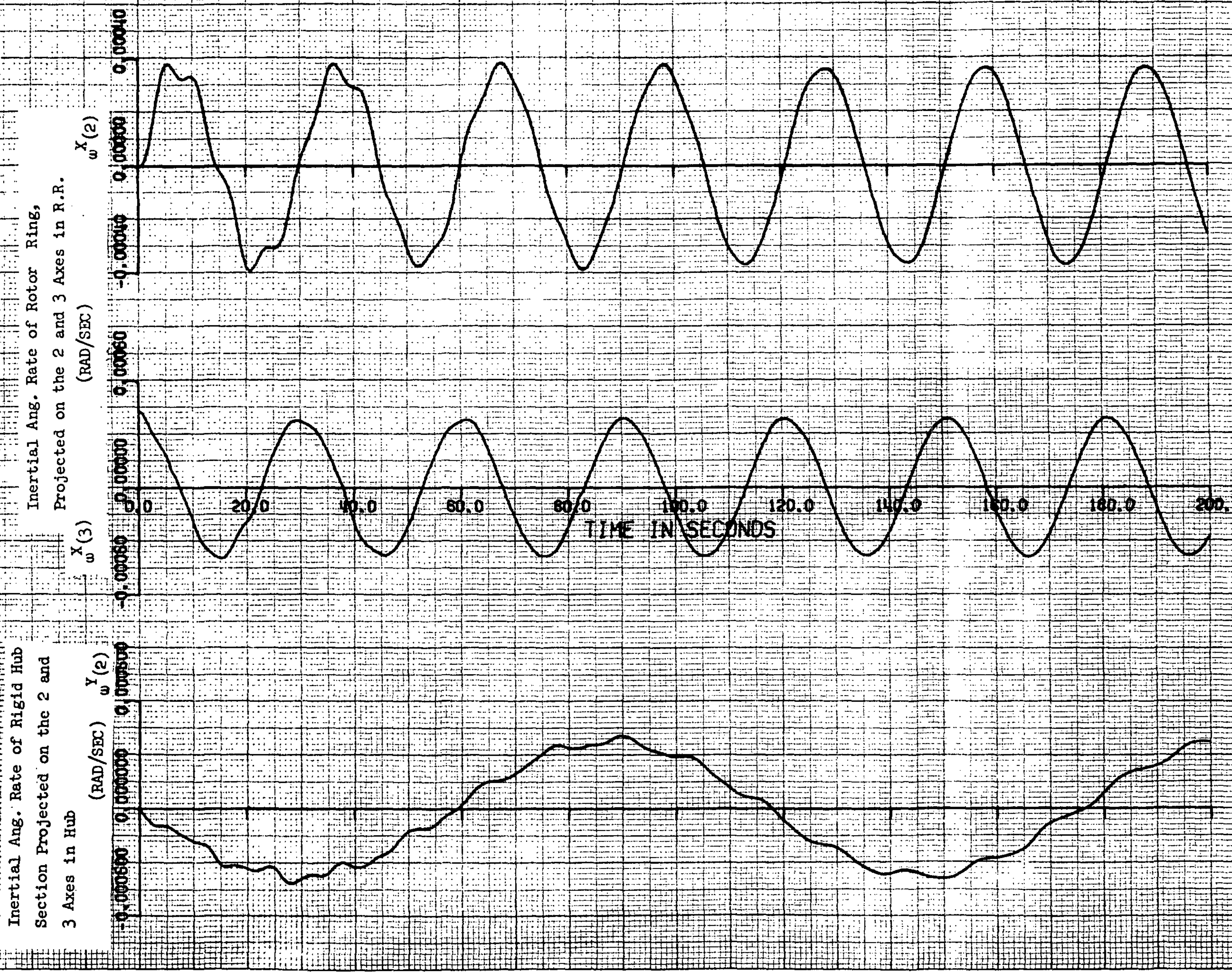


FIGURE 8.22 (Con't)

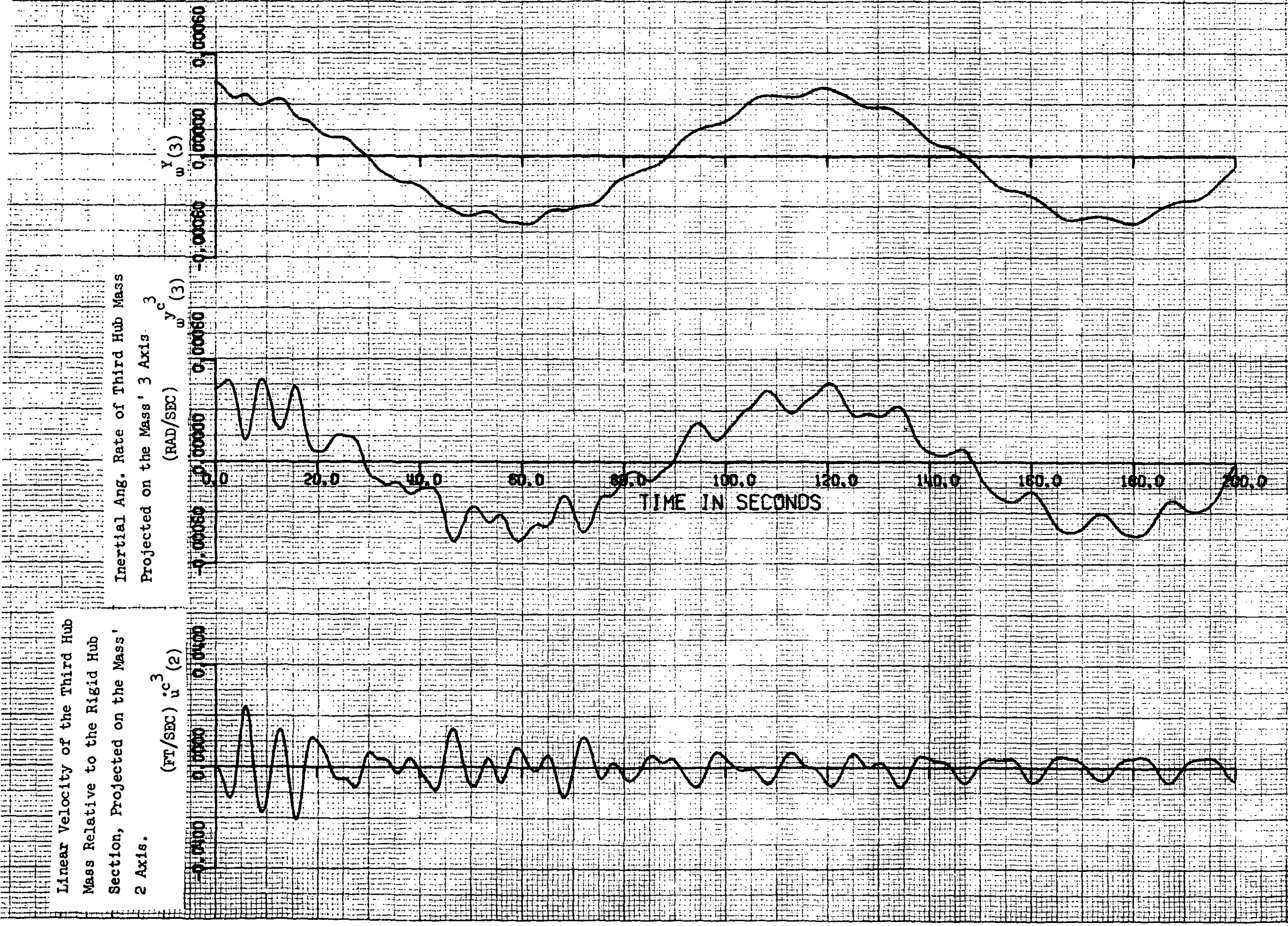
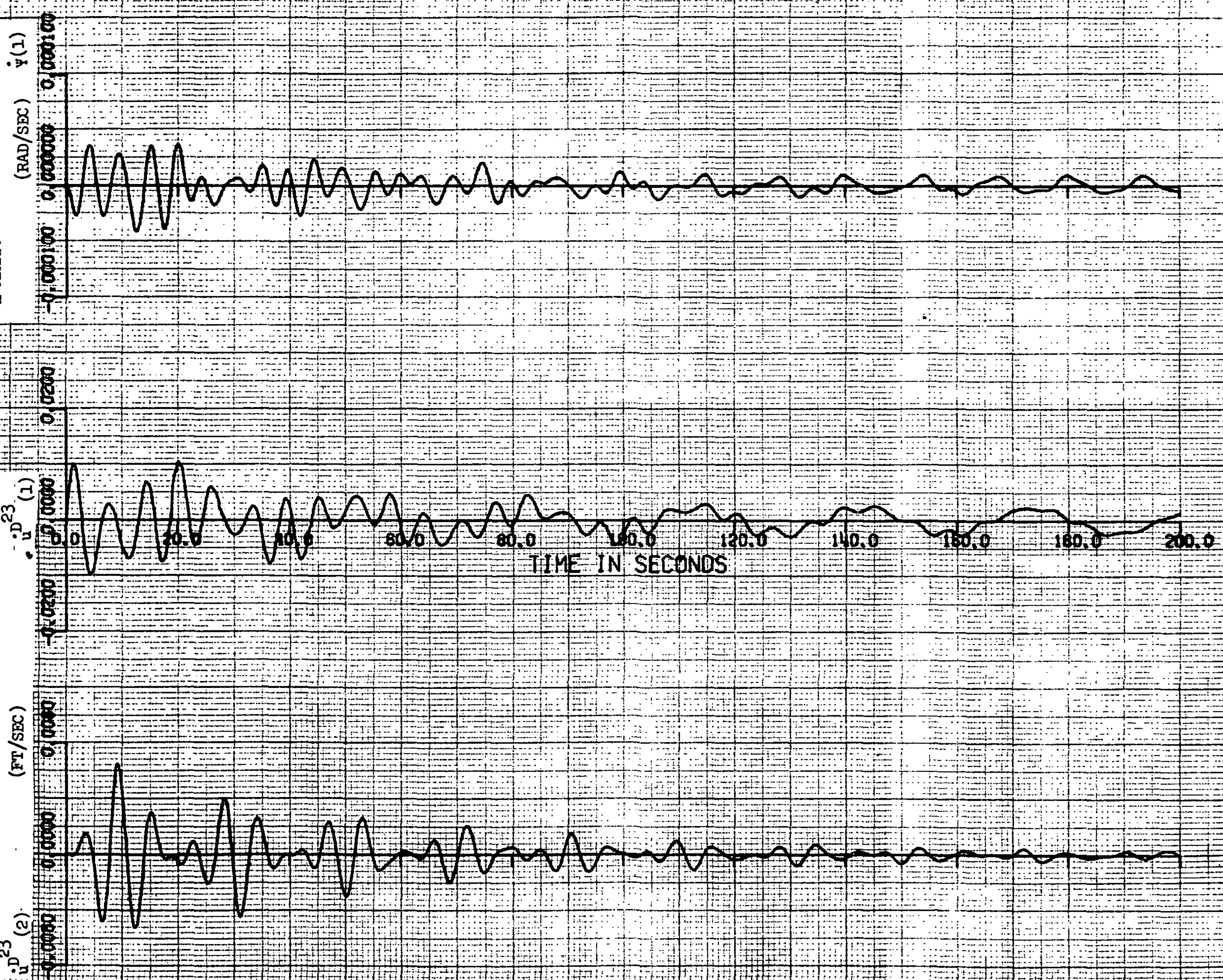


FIGURE 8.22 (Con't)



Shuttle Ang. Rate Relative to the Hub, Projected on Shuttle 1 Axis.

Linear Velocity of the Third Rotor Mass in the Second Arm, Relative to the Rotor Ring, Projected on the Mass' 1 and 2 Axes.

Inertial Angular Rate of the Third Rotor Mass in the First Arm, Projected on the Mass' 2 Axis.

(RAD/SEC) $\omega_z^{13}(2)$

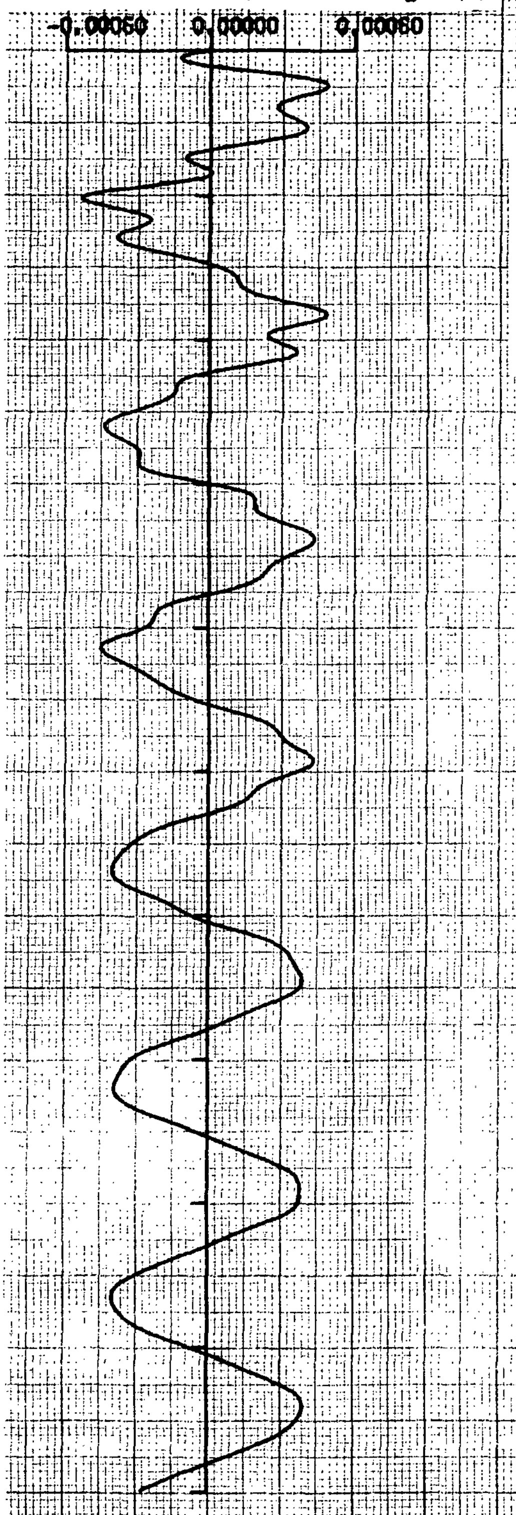


FIGURE 8.22 (Con't)

9.0 CMG Wobble Damping

9.1 Preliminaries

The formulation of the preferred CMG wobble damping technique evolved from a series of analyses beginning with the previous study Grumman performed for NASA/MSC concerning CMG applications to the Space Base (Reference 3). After investigating many different CMG configurations and associated control laws, a unique wobble damping concept was developed. The CMG control technique employs a single-gimbal CMG with the gimbal axis aligned to the vehicle spin axis. The associated control strategy, which is illustrated in Figure 9-1, is to drive the gimbal so that the CMG spin axis (i.e., the CMG momentum vector, h) lags the wobble vector $\{\omega_T\}$ by 90 degrees in inertial space. This strategy is called the 90 degree H-Lag Law. The above control technique was chosen because it accomplishes the following desriable control characteristics:

- The transverse component of the control torque is always in opposition to the wobble vector.
- The total torque impulse (or angular momentum) applied in opposition to the wobble, per wobble period, is near the maximum deliverable as limited by the wheel size, h.

The latter characteristics can be demonstrated as follows:

The control torque $\{T_c\}$ is given by,

$$\{T_c\} = - \{\dot{h}\} = - \{\omega_{CMG}\} \times \{h\} \tag{9-1}$$

where

$\{\dot{h}\}$ = the inertial time derivative of the CMG angular momentum vector.

$\{\omega_{CMG}\}$ = total inertial angular rate of the CMG $= \{\omega\} + \{\omega_G\}$

$\{\omega\}$ = total vehicle angular rate $= P \{i\} + \{\omega_T\}$

$\{\omega_G\}$ = angular rate of the CMG gimbal relative to the vehicle

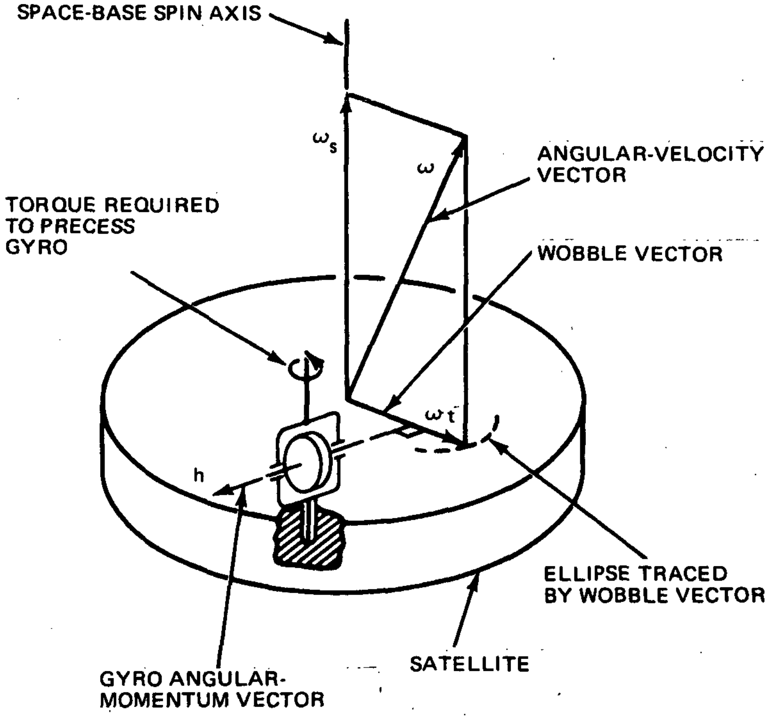


Figure 9-1. CMG 90-deg H-Lag Law

and the transverse torque component $\{T_{C_T}\}$ is,

$$\{T_{C_T}\} = -(P + \omega_G)\{i\} \times \{h\} \tag{9-2}$$

If $\{h\}$ lags $\{\omega_T\}$ by 90 degrees then it is easily seen that the above torque component will always oppose $\{\omega_T\}$, provided $\omega_G > -P$. Furthermore, in order to track the wobble vector and maintain the proper 90 degree phase relationship, the gimbal rate, ω_G, is constrained to equal the rotation rate of the wobble vector, which will always be greater than minus spin. The wobble vector rotation rate $\dot{\Phi}$ can be derived by defining Φ as,

$$\tan \Phi = \frac{\omega_Y}{\omega_Z} \tag{9-3}$$

Using equation (8-1) for the wobble components ω_Y and ω_Z, differentiating the result and making the appropriate trigonometric substitutions results in,

$$\dot{\Phi} = \pm \lambda P \frac{2\alpha}{(\alpha^2 + 1) + (1 - \alpha^2)\cos 2\lambda P t} \tag{9-4}$$

Where α and λ are defined in equation (8-2), and the plus and minus signs refer to "Max I" and "Min I" vehicles, respectively. For symmetric vehicles, where $\alpha = 1$, the rotation rate of the wobble vector and in turn ω_G are both equal to the wobble frequency, λP. The total torque impulse, I_T, applied in opposition to the wobble, per inertial wobble period, is given by,

$$I_T \equiv \int_0^{\frac{1}{(P \pm \lambda P)}} |T_{C_T}| d\tau = (P + \omega_G) h \left(\frac{1}{P \pm \lambda P}\right) \tag{9-4}$$

For a symmetric vehicle $\omega_G = \pm \lambda P$, so that

$$I_T = h, \tag{9-5}$$

which is the maximum deliverable angular momentum.

A previous analysis performed at Grumman (Reference 10) proved, using an energy sink method, that the 90 degree-H-Lag Law is time optimum for a symmetric vehicle and is an averaging approach to the time optimal law for non-symmetric vehicles. In fact, for most vehicles, symmetric or unsymmetric, the damping time penality associated with 90 degree-H-Lag Law is negligible. Furthermore, to mechanize the true optimal law would not be practical (as noted in Reference 10).

In order to stop the application of control torques, the CMG gimbal is driven so that the CMG momentum vector is stationary in inertial space. That is, when the CMG is located in the rotating portion of the vehicle it is driven at a rate equal to minus the spin rate. The stop criterion is based upon whether the wobble amplitude is within the Target Ellipse, or not. The semi-major and semi-minor axes of the Target Ellipse are defined as $\alpha \Omega_r$ and Ω_r, respectively; for $\alpha < 1$, the definitions are reversed.

Furthermore,

$$\Omega_r = \begin{cases} \dfrac{\omega_r}{\alpha}, & \text{for} \quad \alpha \geq 1 \\ \omega_r, & \text{for} \quad \alpha < 1, \end{cases} \qquad (9\text{-}6)$$

Where the values for the maximum allowable wobble levels (ω_r), corresponding to both "T" and "Y" configurations, are given in Table 5.2.

The basic CMG wobble damping concept, as presented in the above paragraphs evolved from rigid body analyses. Applying this basic concept to the flexible space station resulted in interactions with the structure which necessitated a few modifications. The results of the flexible body analyses are presented in Section 9.2.

Before proceeding to the flexible body analysis, an estimate for the required CMG wheel size (h) is obtained. The wobble damping task is specified by stipulating that the amplitude of a component of wobble rate (Ω) is to be reduced within a specified time (t_f) to a value below a specified threshold, $\Omega(t_f)$. Based upon a rigid body analysis, Reference 10 shows that the 90 degree-H-Lag-Law control concept will require a wheel size that is given by,

$$h = \frac{\Omega(0) - \Omega(t_f)}{Kt_f} \tag{9-7}$$

where

$$K = \frac{2I_x \lambda P \bar{G}(\alpha)}{\pi I_z \left| I_x - I_z \right|}$$

$$\bar{G}(\alpha) \equiv \begin{cases} F(k);\ 1 \leq \alpha \leq \infty,\ k = \frac{1}{\alpha}\left(\alpha^2 - 1\right)^{\frac{1}{2}} \\ \alpha F(k^1);\ 0 \leq \alpha \leq 1,\ k^1 = \left(1 - \alpha^2\right)^{\frac{1}{2}} \end{cases}$$

$F(k) \equiv$ the complete elliptic integral of the first kind.

All other variables are defined as in Section 8.1.

Applying the above relationships to the nominal "T" and "Y" configurations (as defined in Section 4.0) in combination with the wobble damping requirements as specified in Section 5.1 results in the following specifications for the CMG wheel size.

Configuration		Required Wheel Size h (ft-lb-sec)
"Y"	- in response to internal mass motion[1]	385
"T"	- in response to internal mass motion[1]	9,670
"Y"/Shuttle	- in response to one sigma docking[2]	11,170
"T"/Shuttle	- in response to one sigma docking[2]	17,870

---

[1] Mass motion is defined in Reference 7. Initial disturbance is presented in Section 5.2.

[2] Docking disturbance is derived in Section 5.2.

9.2 Wobble Damping of the Flexible Space Station

The CMG wobble damping concept, previously described, was applied to the General Flexible Body Simulation - Stage II. A functional block diagram of the math model for the torquer motor and gimbal dynamics along with the associated control logic is shown in Figure 9-2. The natural frequency and damping for the torquer-gimbal combination used for all studies were 3 rad/sec and 0.8, respectively. The components of the applied control torque are derived in the simulation as follows:

$$\begin{aligned} T_{c_1} &= h(\omega_3 \sin\beta + \omega_2 \cos\beta) \\ T_{c_2} &= -h(\omega_1 + \dot{\beta}) \cos\beta \\ T_{c_3} &= -h(\omega_1 + \dot{\beta}) \sin\beta \end{aligned} \qquad (9\text{-}8)$$

where

$\omega_1, \omega_2, \omega_3$ - are vehicle angular rates of the body that the CMG is within

$\beta, \dot{\beta}$ - CMG gimbal angle, rate. Where β is measured positively from axis 2, and $0 \le \beta \le 2\pi$.

Nominal values for all the physical vehicle properties (refer to Section 4.0) were used during the flexible body analyses with CMG control. The initial disturbance, for CMG sizing purposes, is the result of a particular mass motion profile corresponding to a 2,000 lb mass moving within rotor arm #1. The initial transverse angular rate resulting from this disturbance was derived in Reference 7 and presented in Section 5.2 for completeness.

Applying the basic CMG wobble damping concept to the "Y" space station resulted in successful wobble damping without modification to the control system. A low pass filter, with a two second time constant, was added to the system so as to avoid unnecessary interaction with the structure and in turn decrease the actuator duty cycle. Figure 9-3 are the resulting time history traces with a 400 ft-lb-sec CMG placed in the "Y" rotor. It is noted that all motion

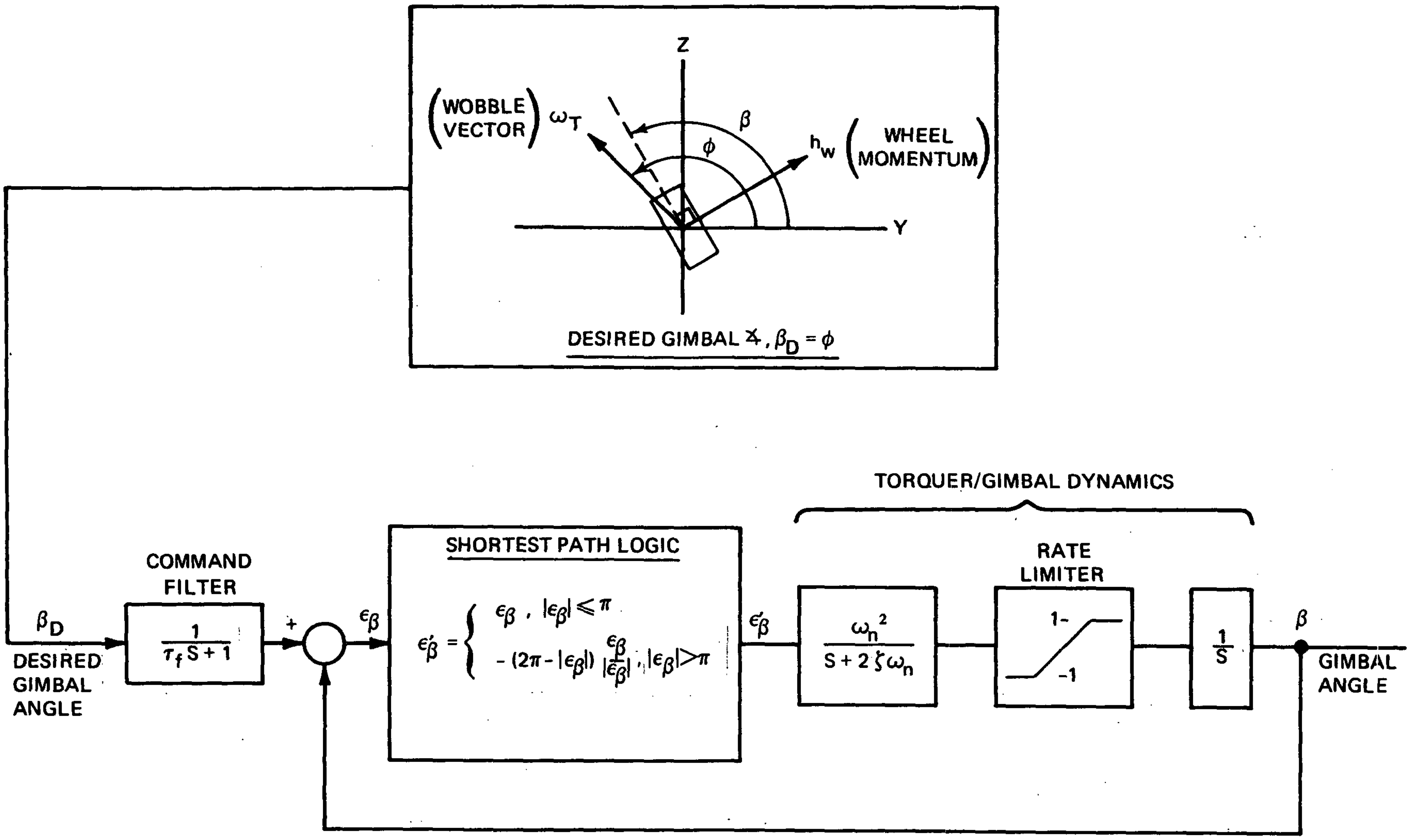


Figure 9-2. CMG Wobble Damping Logic And Simulated Dynamics

FIGURE 9.3 "Y" - NOMINAL, CMG CONTROL
(H = 400 FT-LB-SEC)

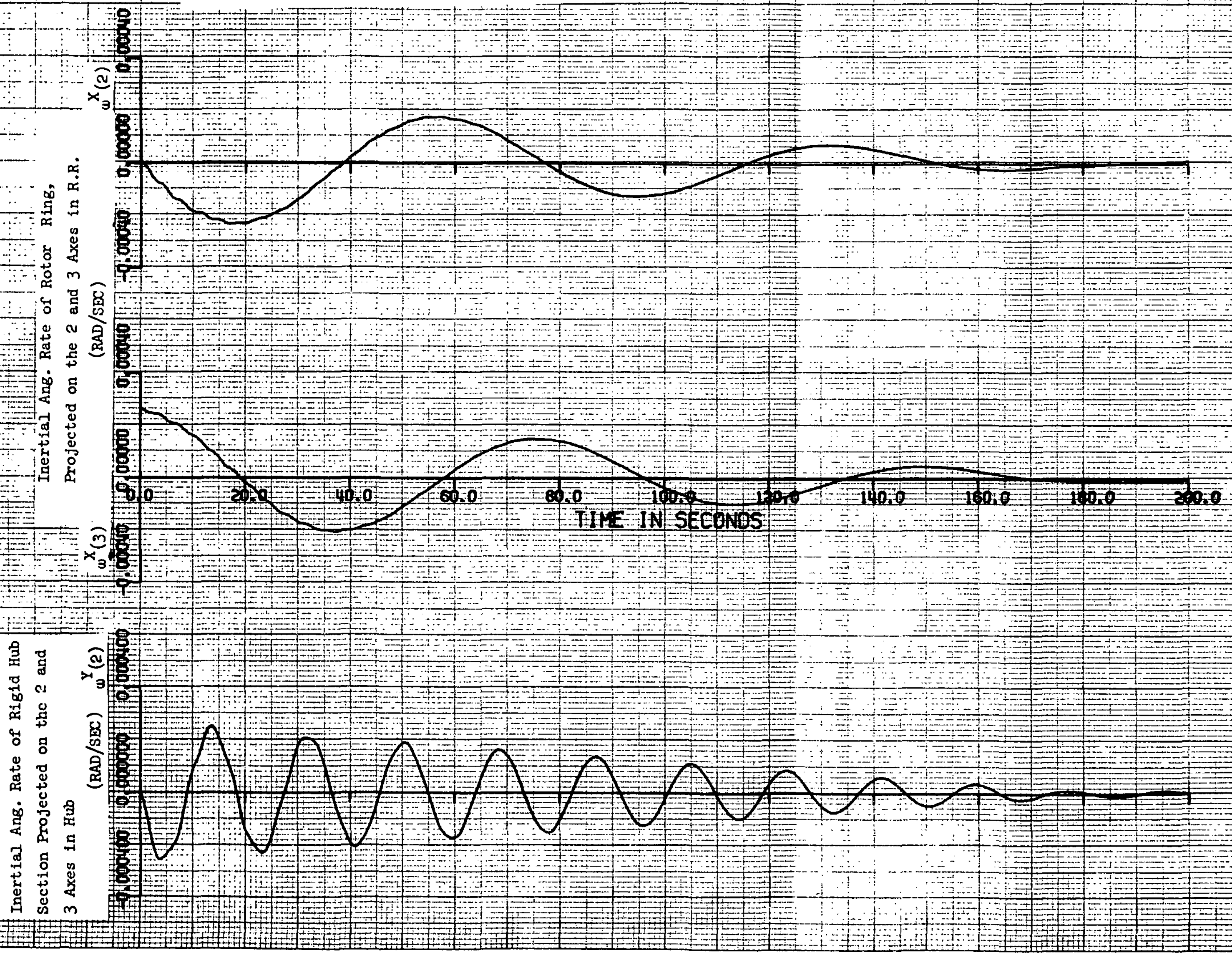


FIGURE 9.3 (Con't)

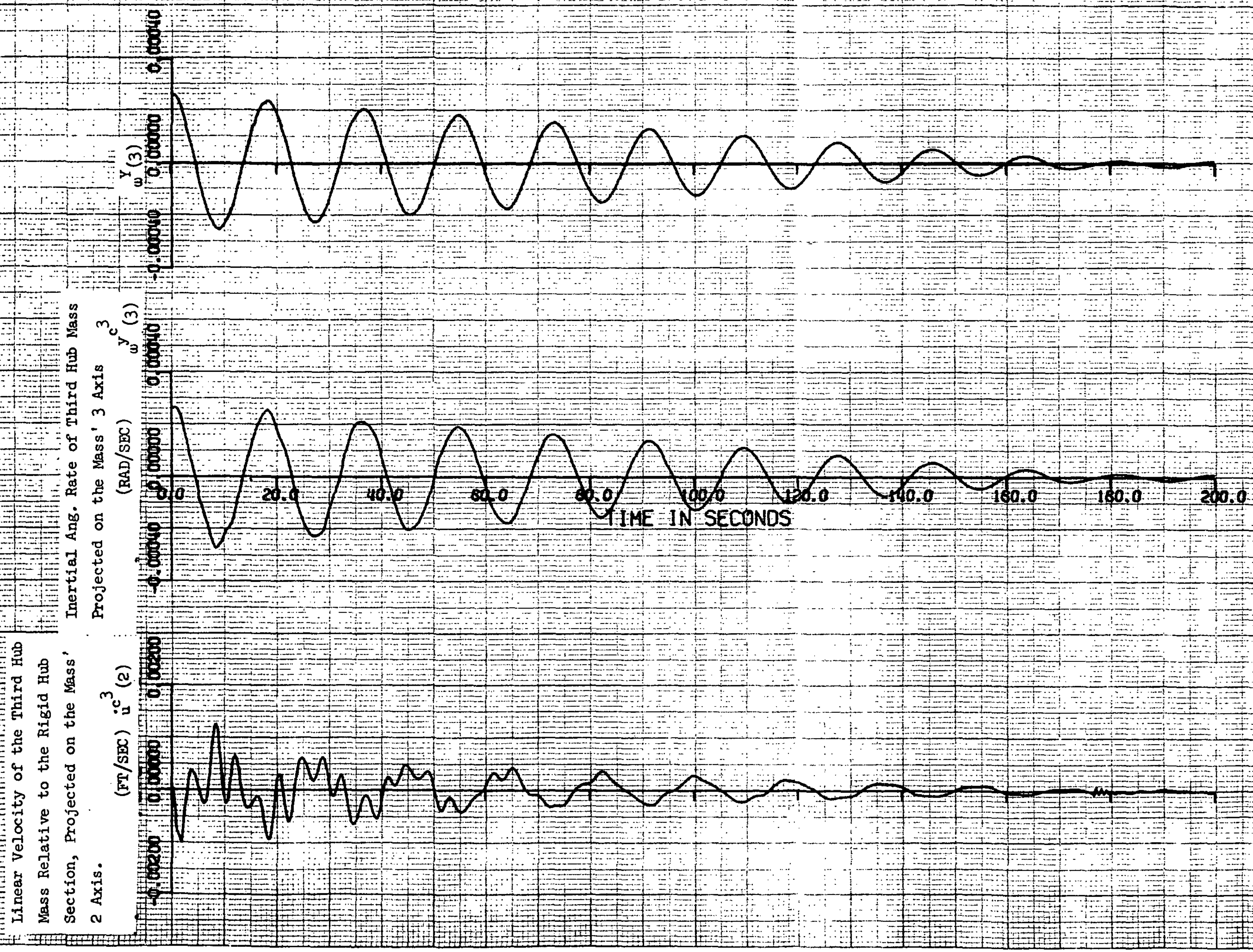
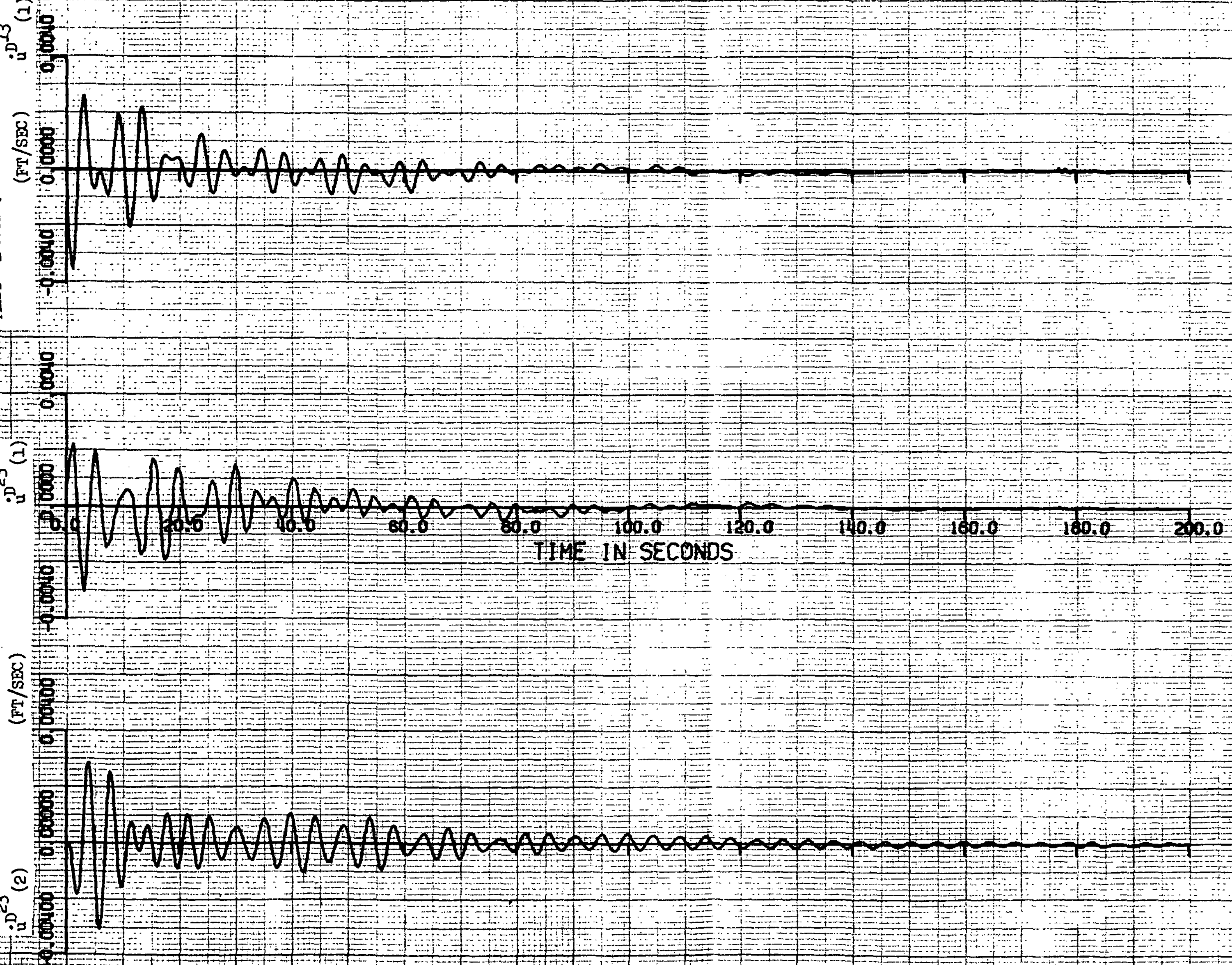


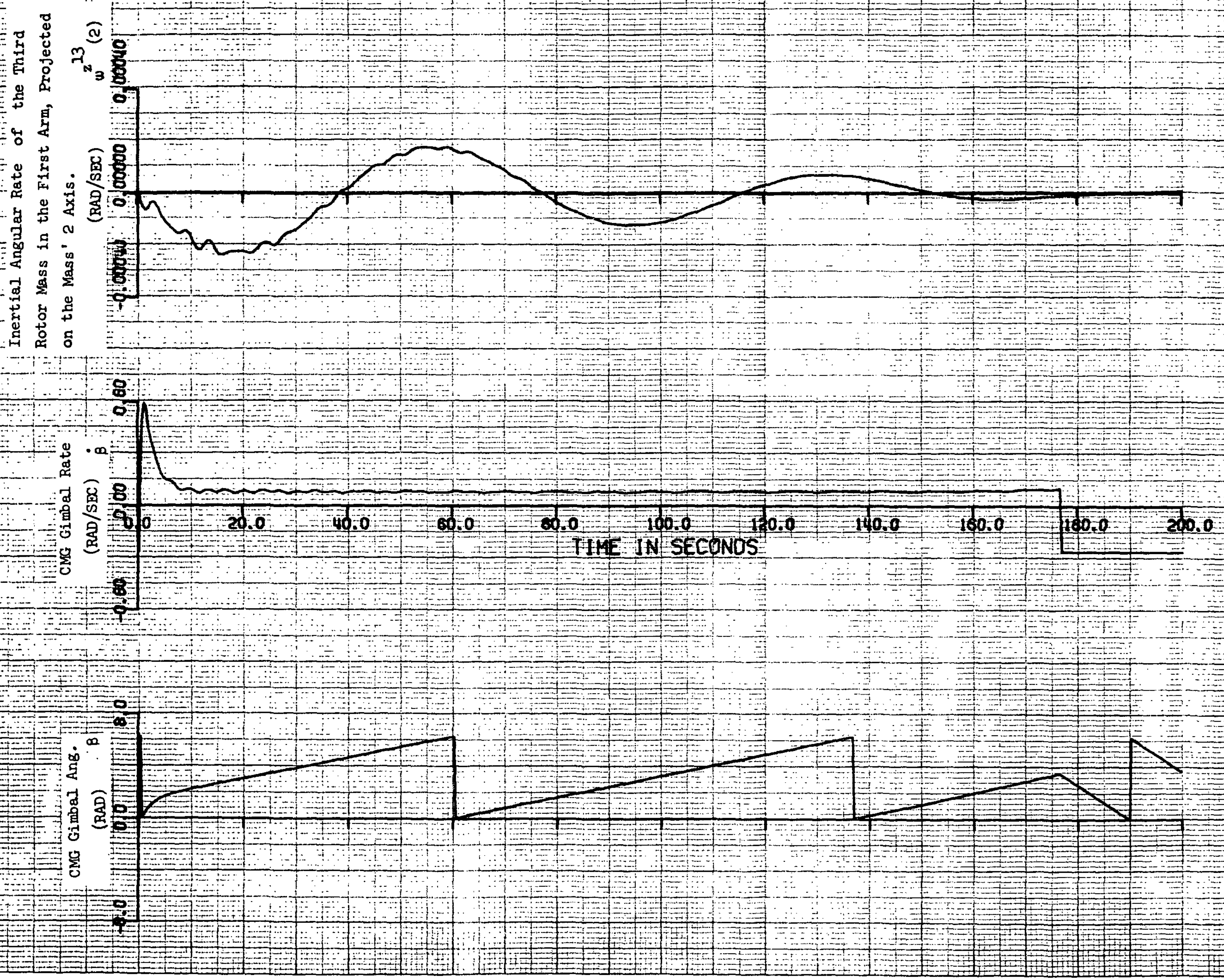
FIGURE 9.3 (Con't)



Linear Velocity of Third Rotor Mass in the First Arm, Relative to the Rotor Ring, Projected on the Mass' 1 Axis.

Linear Velocity of the Third Rotor Mass in the Second Arm, Relative to the Rotor Ring, Projected on the Mass' 1 and 2 Axes.

FIGURE 9.3 (Con't)



is damped including the relative flexible motion as exemplified by the traces of relative linear velocities ($\dot{u}^D$ and $\dot{u}^C$). Referring to the time history for gimbal rate ($\dot{\beta}$) it is seen that the wobble damping task is satisfied within the required time (180 sec). For example, when the desired wobble level is reached, the CMG is driven at minus spin rate. Although the gimbal is 90 degrees out of phase at the start of the simulation, the CMG gimbal is at its desired position within 10 seconds and maintains track until the wobble is damped within the desired level.

Applying the same CMG system to a "Y" station with the Shuttle attached, and disturbed by a 1 sigma docking, results in the time histories shown in Figure 9-4. Although the gimbal does lock-on and track the wobble vector, the amount of damping during the run duration is slight due to the small CMG wheel size (400 ft-lb-sec). It is estimated that it would take 5,000 seconds to completely damp the wobble induced by the docking using a 400 ft-lb-sec CMG.

It should be noted that it is considered unreasonable to size the CMG to accommodate the relatively infrequent docking disturbances. If faster damping is required in response to a docking, then the reaction jet system can be used.

Unlike the immediate successful results experienced with the "Y" station, the application of the basic wobble damping concept to the "T" station rotor required modifications for proper operation. The first difficulty encountered was a high frequency limit cycle resulting from the gimbal rate being less than minus spin rate. It is noted that a rigid "T" station requires a gimbal rate close to negative spin and with the addition of small flexible motions there are periods of a commanded gimbal rate below negative spin. The result of this phenomenon is a reversal of the direction of rotation of the wheel angular momentum vector in inertial space; in turn, control torques are aiding the wobble rather than opposing it. It should be noted that for any rigid body vehicle this anomaly would not occur. In order to fix this problem a negative rate limit equal to minus the spin rate is incorporated.

FIGURE 9.4 "Y"/SHUTTLE - NOMINAL, CMG CONTROL
(H = 400 FT-LB-SEC)

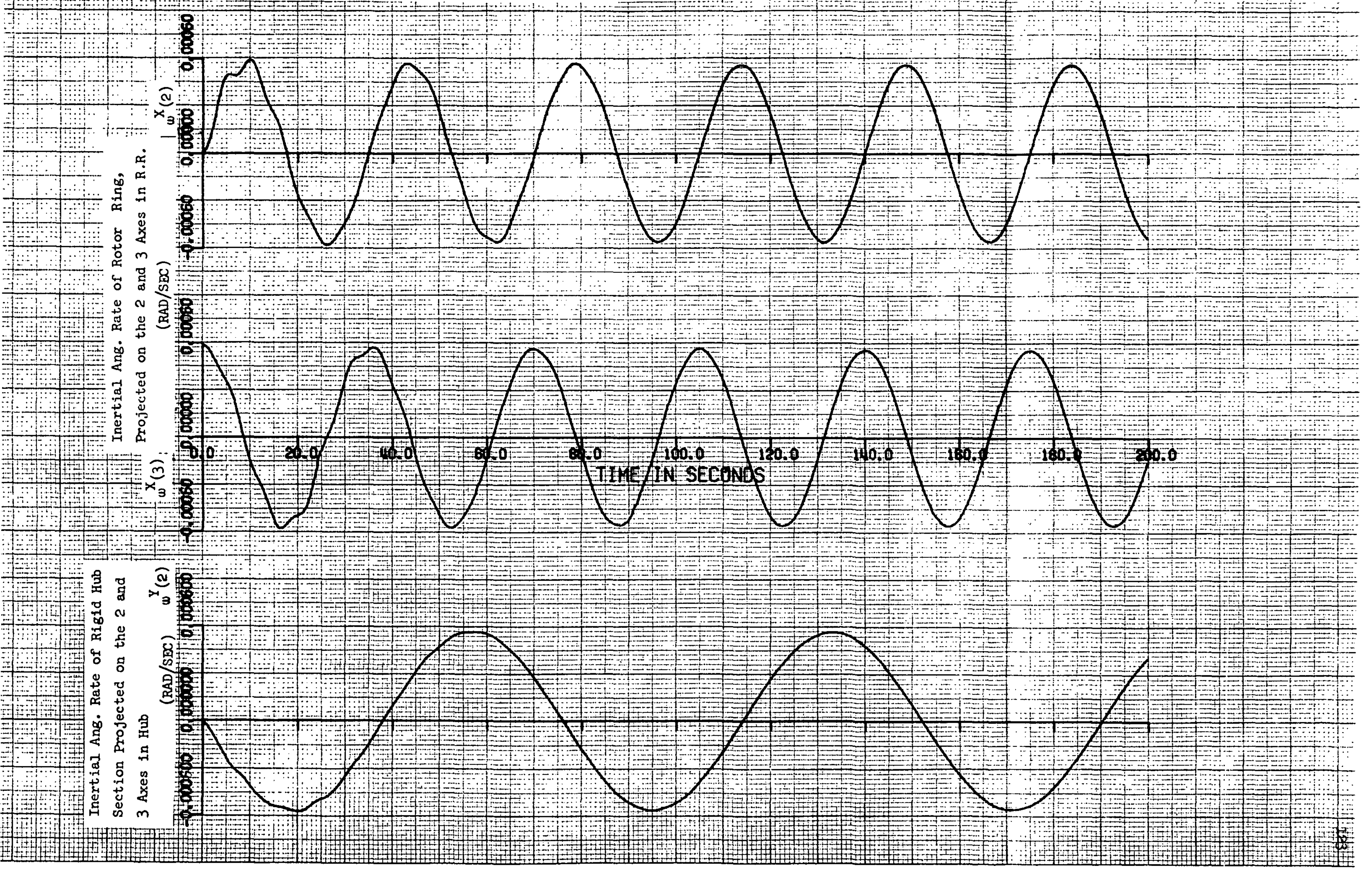


FIGURE 9.4 (Con't)

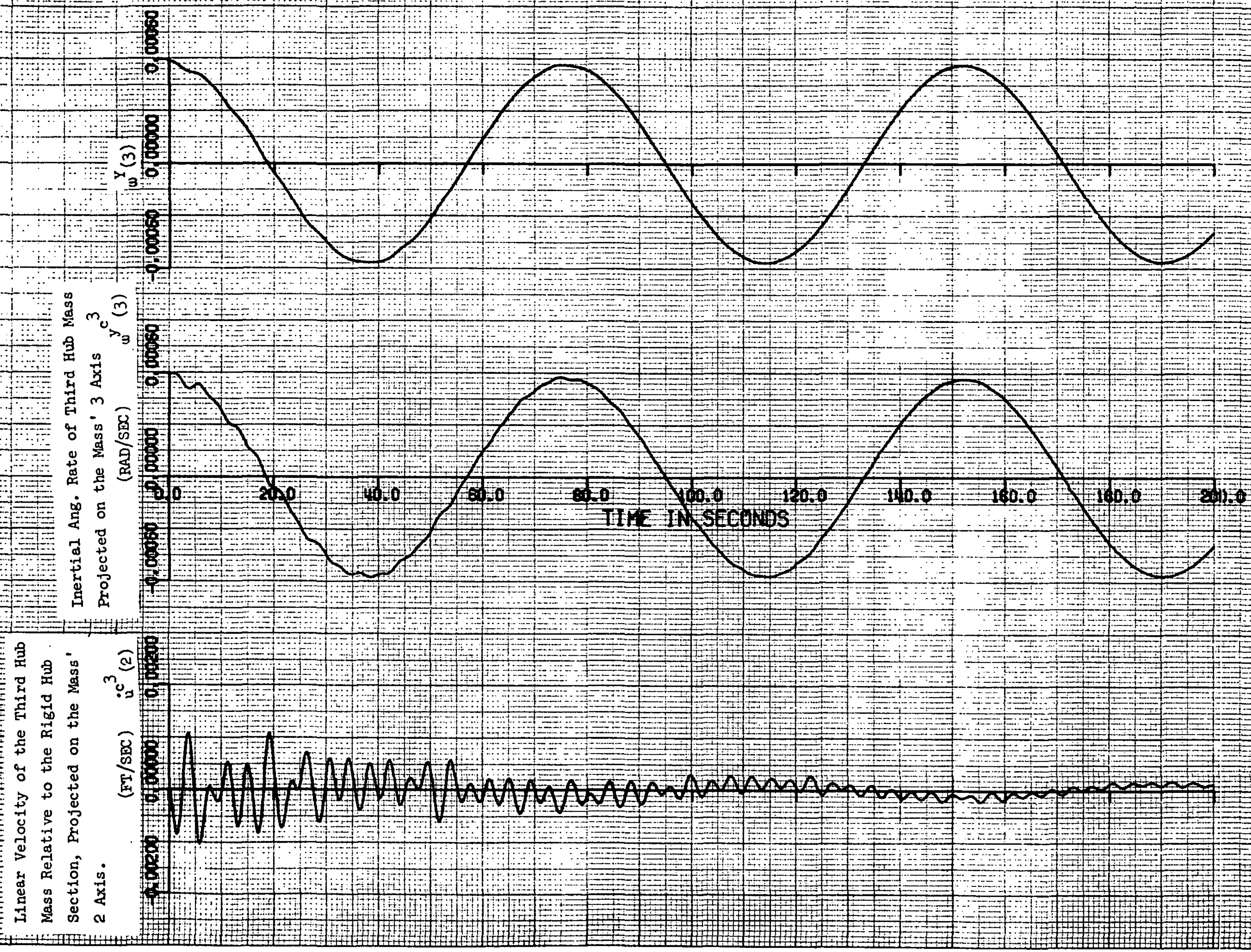
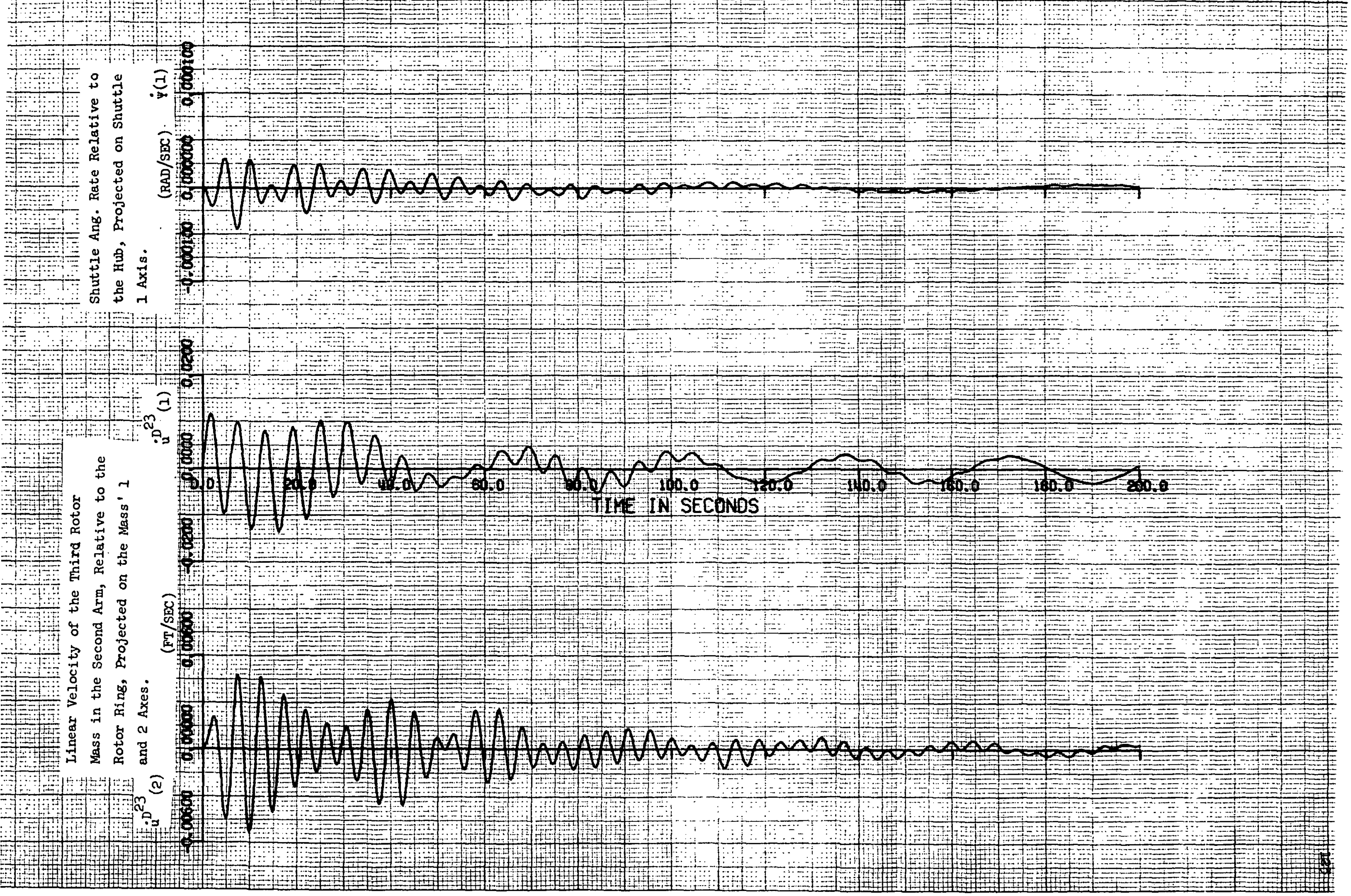


FIGURE 9.4 (Con't)



Shuttle Ang. Rate Relative to the Hub, Projected on Shuttle 1 Axis.

Linear Velocity of the Third Rotor Mass in the Second Arm, Relative to the Rotor Ring, Projected on the Mass' 1 and 2 Axes.

FIGURE 9.4 (Con't)

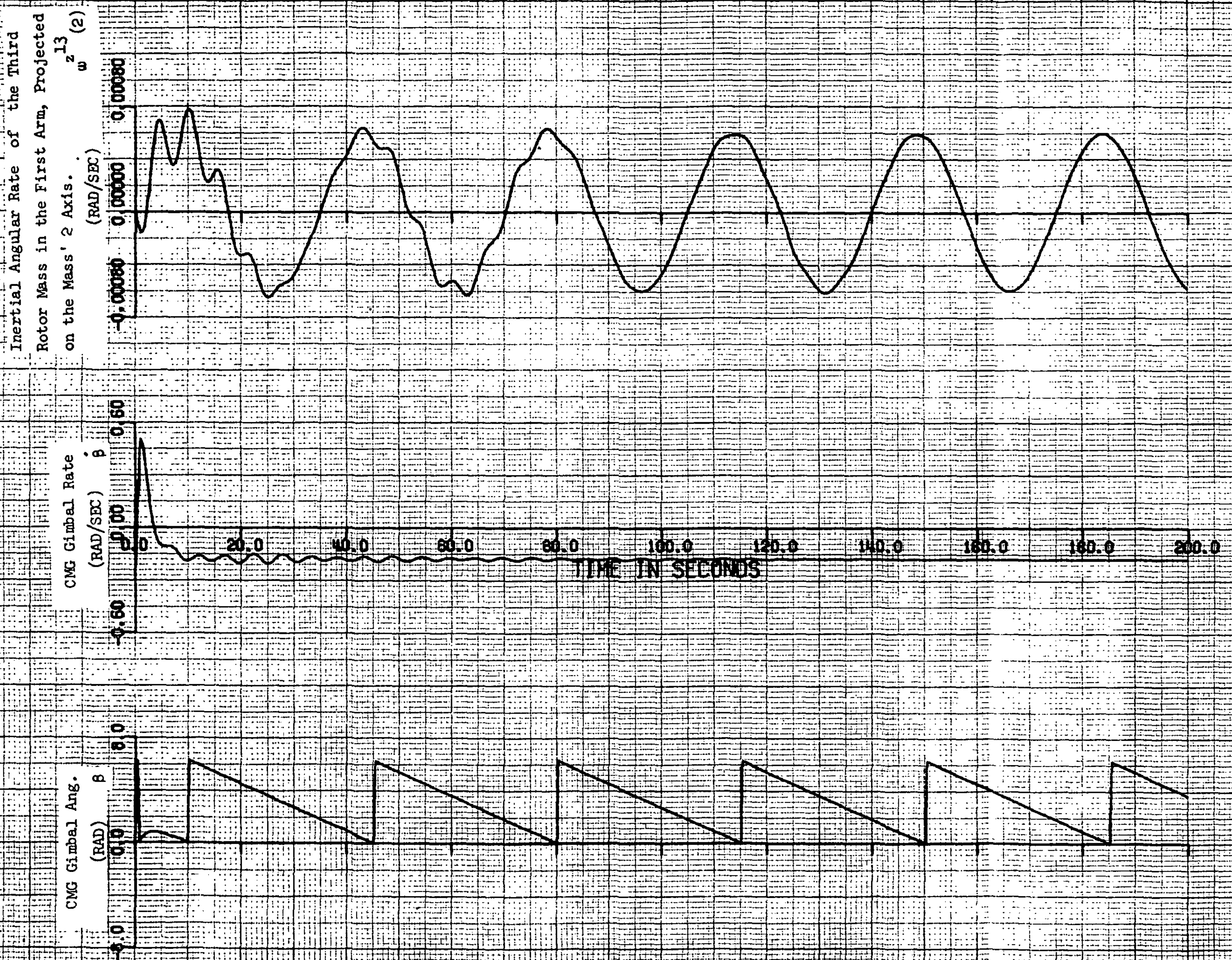


Figure 9-5 contains "T" configuration time history traces resulting from the application of the basic CMG system with the additional negative rate limit, a 10,000 ft-lb-sec CMG, and a low pass filter with a 2 second time constant. Two undersirable effects are noted:

1. The wobble damping task is not accomplished within the required 180 seconds.
2. There is a significant amount of control interaction with the structure as indicated by the activity on the $\dot{\beta}$ trace.

The solution to the first problem is to increase the wheel size. A 12,000 ft-lb-sec CMG was shown to be sufficient. Although there was no structural instability observed as a result of item 2, the possibility for an instability does exist as shown in Reference 3. In addition, high frequency oscillation of the CMG should be avoided from a wear point of view. In order to avoid control/structural interaction, the filter bandwidth, decreased. However, as the bandwidth of the filter is decreased, the tracking error is increased, which results in an inefficiency. As a result, the recommended final modifications for the "T" configuration CMG system are as follows:

- increase the filter time constant to 10 seconds.
- add a gimbal angle command bias to compensate for the additional tracking error.

A good approximation for the tracking error and in turn the gimbal command bias is, $(\lambda P)\tau_f$, the wobble frequency times the filter time constant. The time history traces resulting from the application of the CMG system with all of the above mentioned modifications are shown in Figure 9-6. As indicated by these time histories, the wobble damping task is successfully accomplished. It should be noted that the criterion for stopping active CMG damping was rasied slightly in order to avoid interaction with the structural modes at low wobble amplitudes.

Figure 9-7 contains time history traces resulting from the application of the final CMG system, with all required modifications, to a "T" station with an attached shuttle. The initial disturbance is equivalent to a 1 sigma docking. Although wobble damping is

FIGURE 9.5 "T" - NOMINAL, CMG CONTROL
(H = 10,000 FT-LB-SEC)

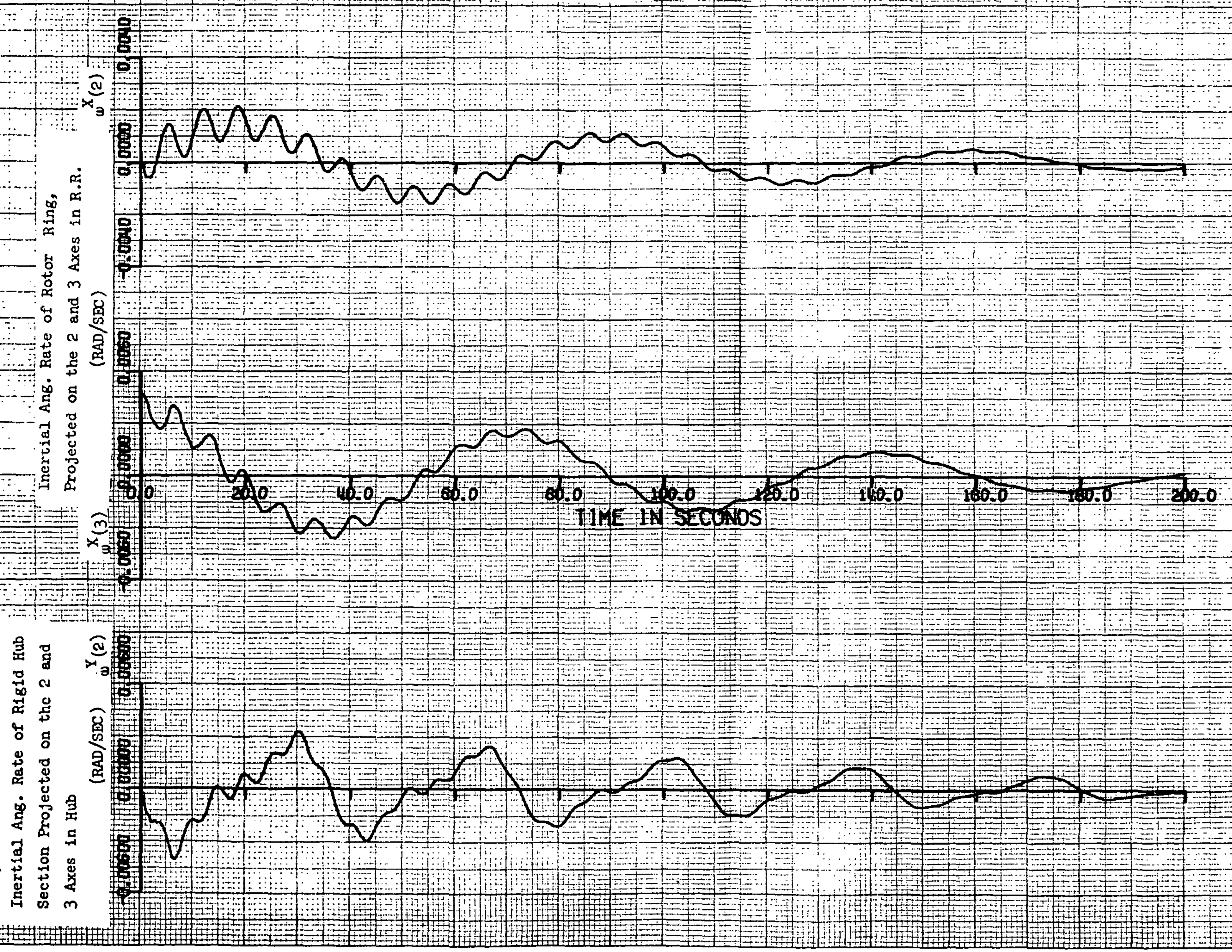
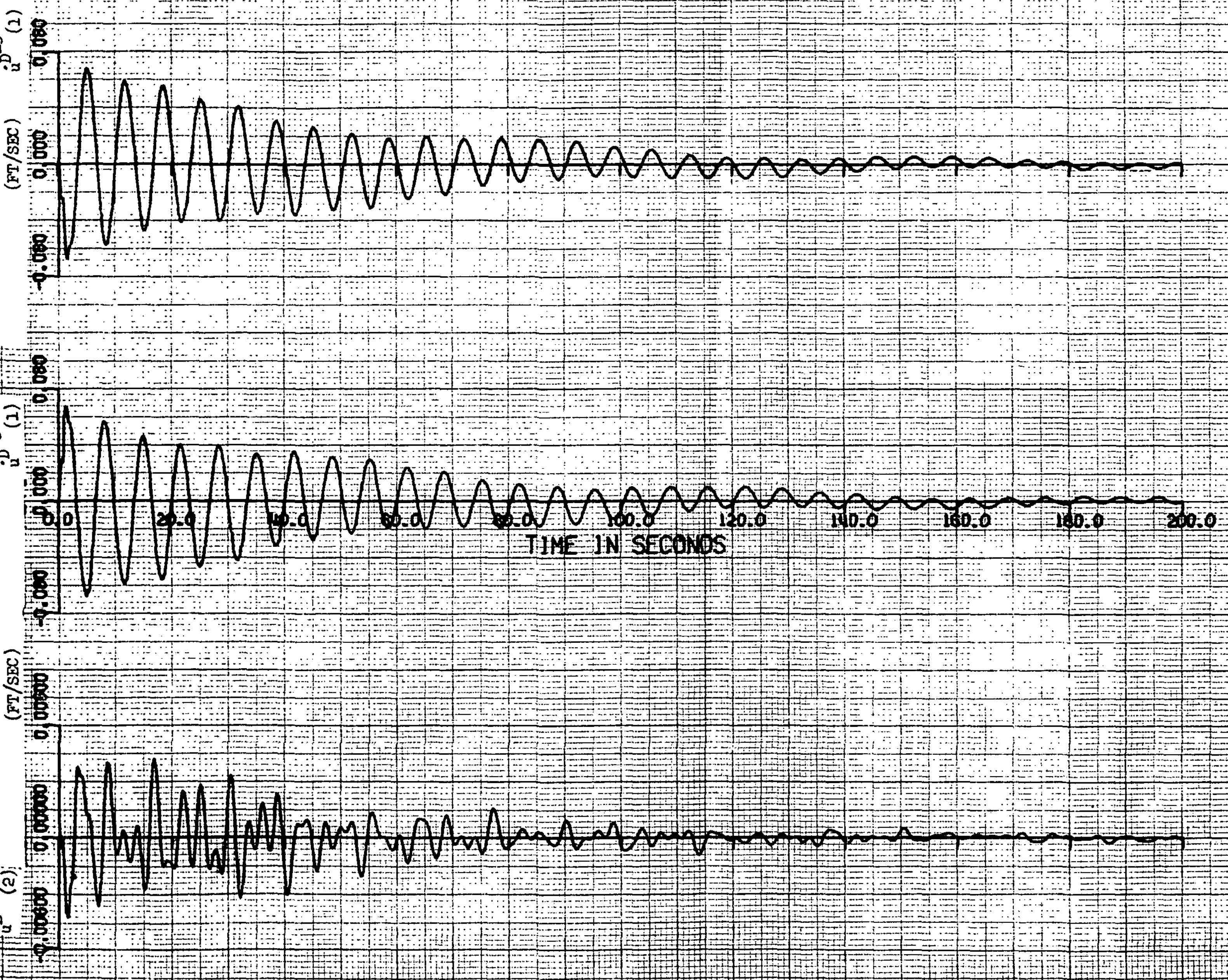


FIGURE 9.5 (Con't)

Inertial Ang. Rate of Third Hub Mass Projected on the Mass' 3 Axis

Linear Velocity of the Third Hub Mass Relative to the Rigid Hub Section, Projected on the Mass' 2 Axis.

FIGURE 9.5 (Con't)



Linear Velocity of Third Rotor Mass in the First Arm, Relative to the Rotor Ring, Projected on the Mass' 1 Axis.

Linear Velocity of the Third Rotor Mass in the Second Arm, Relative to the Rotor Ring, Projected on the Mass' 1 and 2 Axes.

FIGURE 9.5 (Con't)

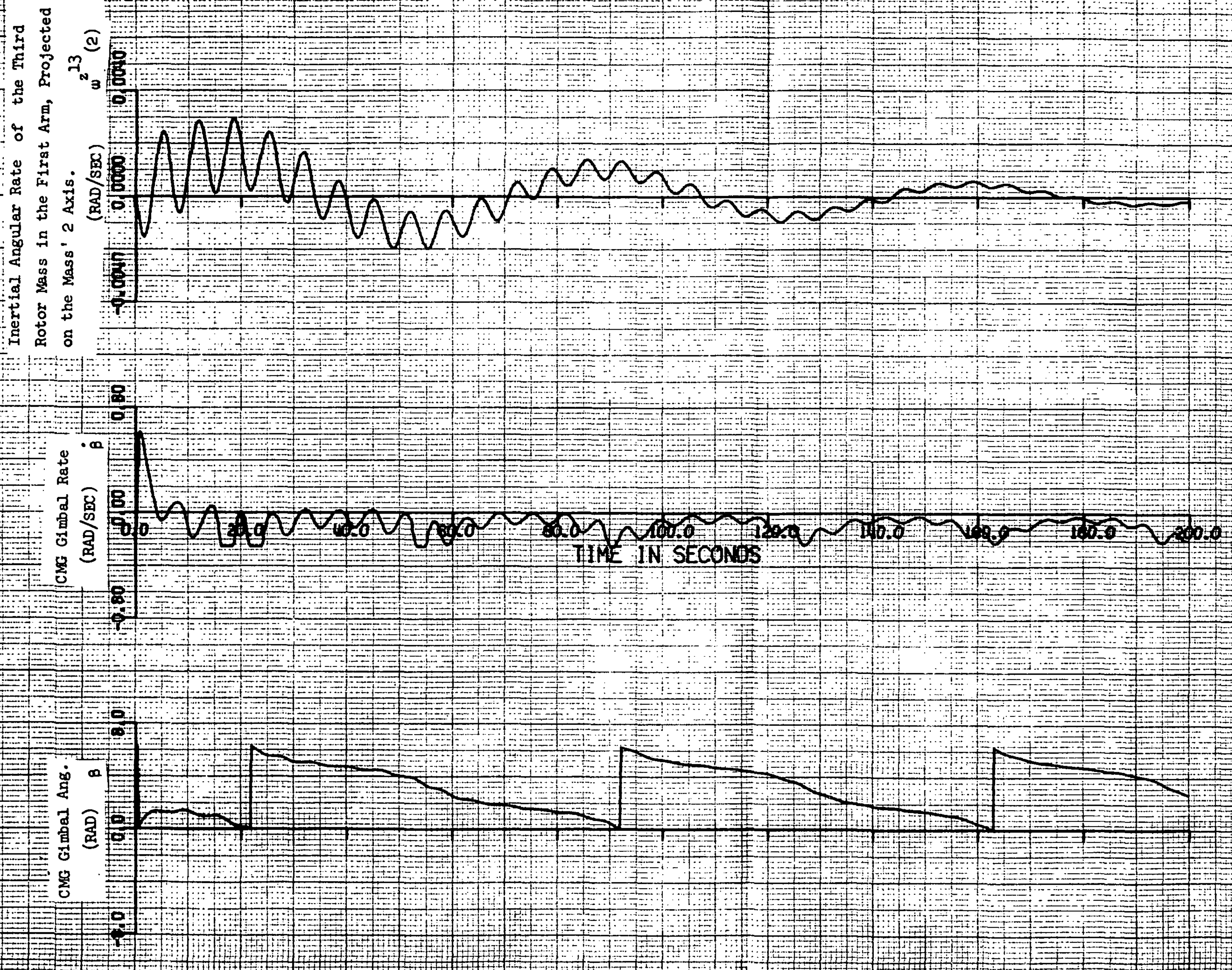
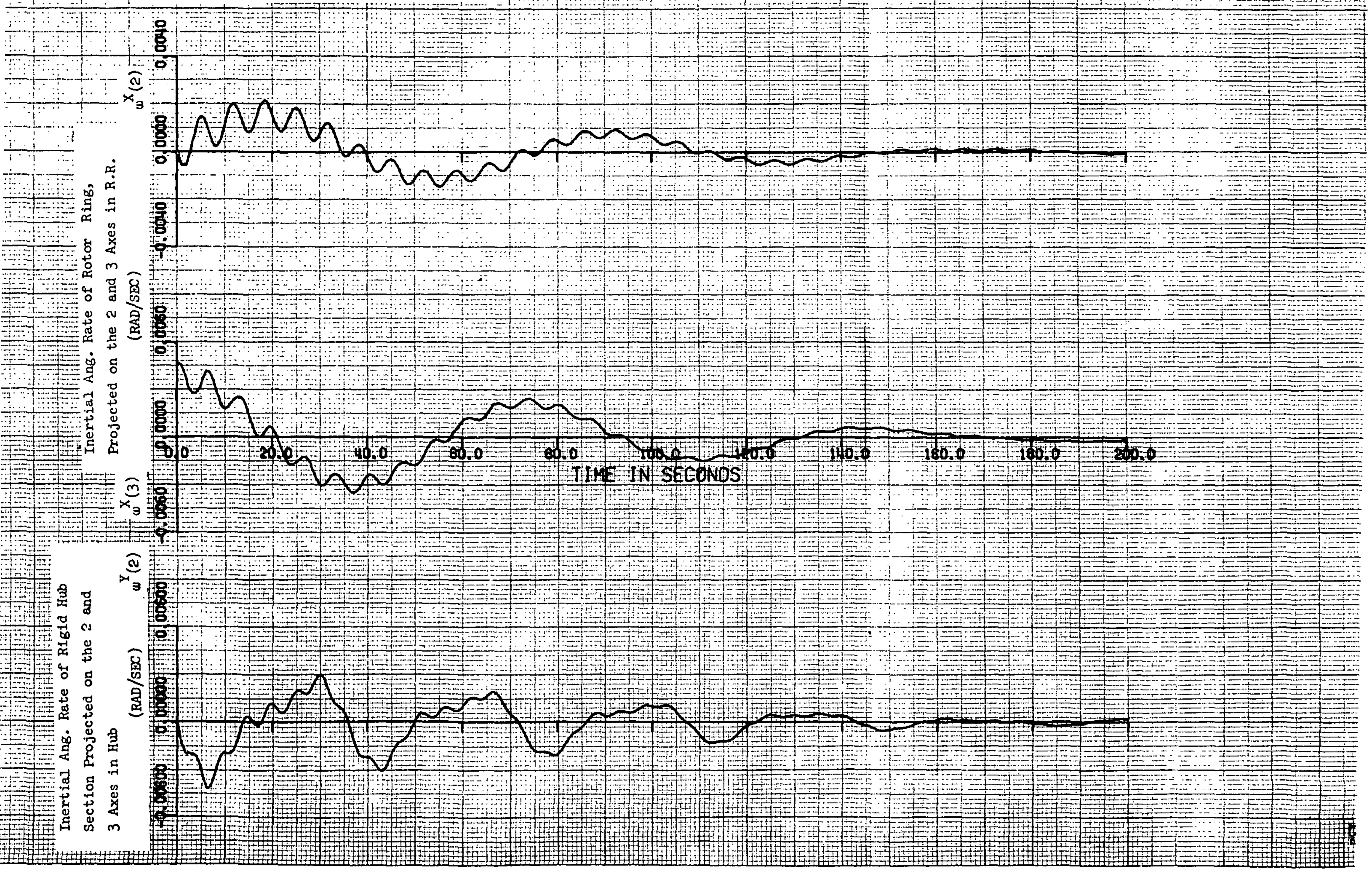


FIGURE 9.6 "T" - NOMINAL, CMG CONTROL
(H = 12,000 FT-LB-SEC, τ_f = 10 SEC,
GIMB ANG BIAS)



Linear Velocity of the Third Hub Mass Relative to the Rigid Hub Section, Projected on the Mass' 2 Axis.

Inertial Ang. Rate of Third Hub Mass Projected on the Mass' 3 Axis.

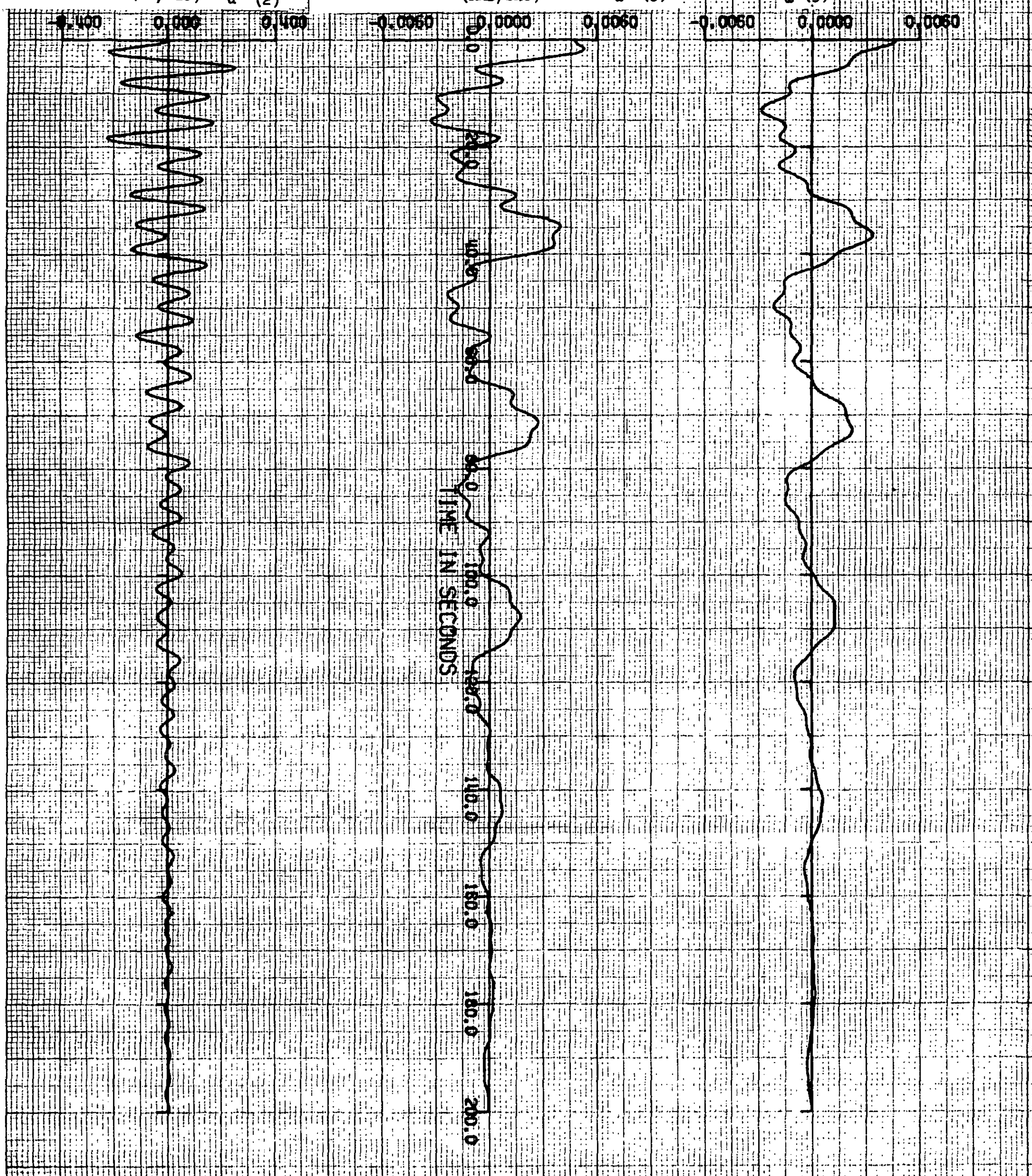
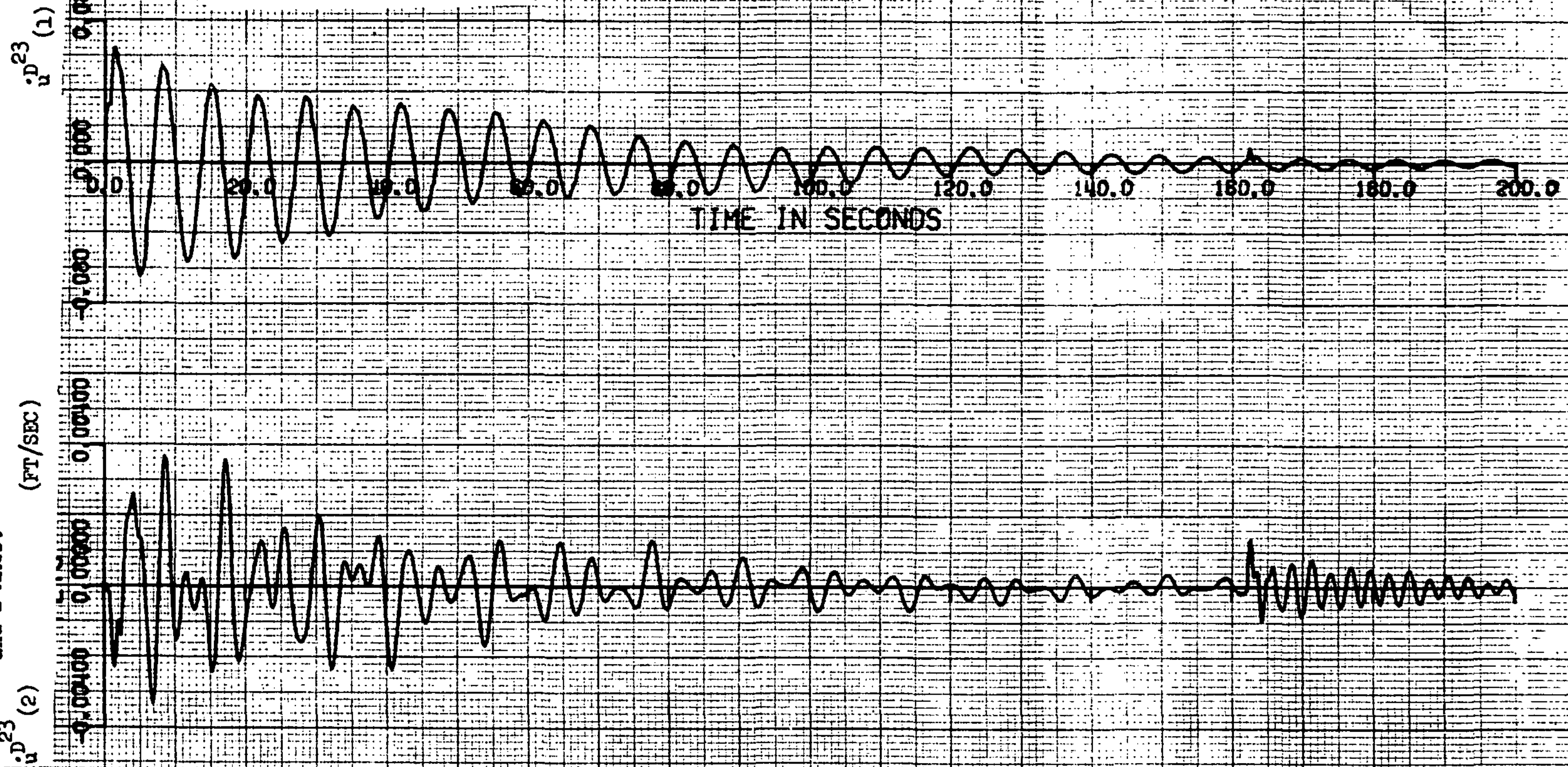


FIGURE 9.6 (Con't)

FIGURE 9.6 (Con't)



Linear Velocity of the Third Rotor Mass in the Second Arm, Relative to the Rotor Ring, Projected on the Mass' 1 and 2 Axes.

FIGURE 9.6 (Con't)

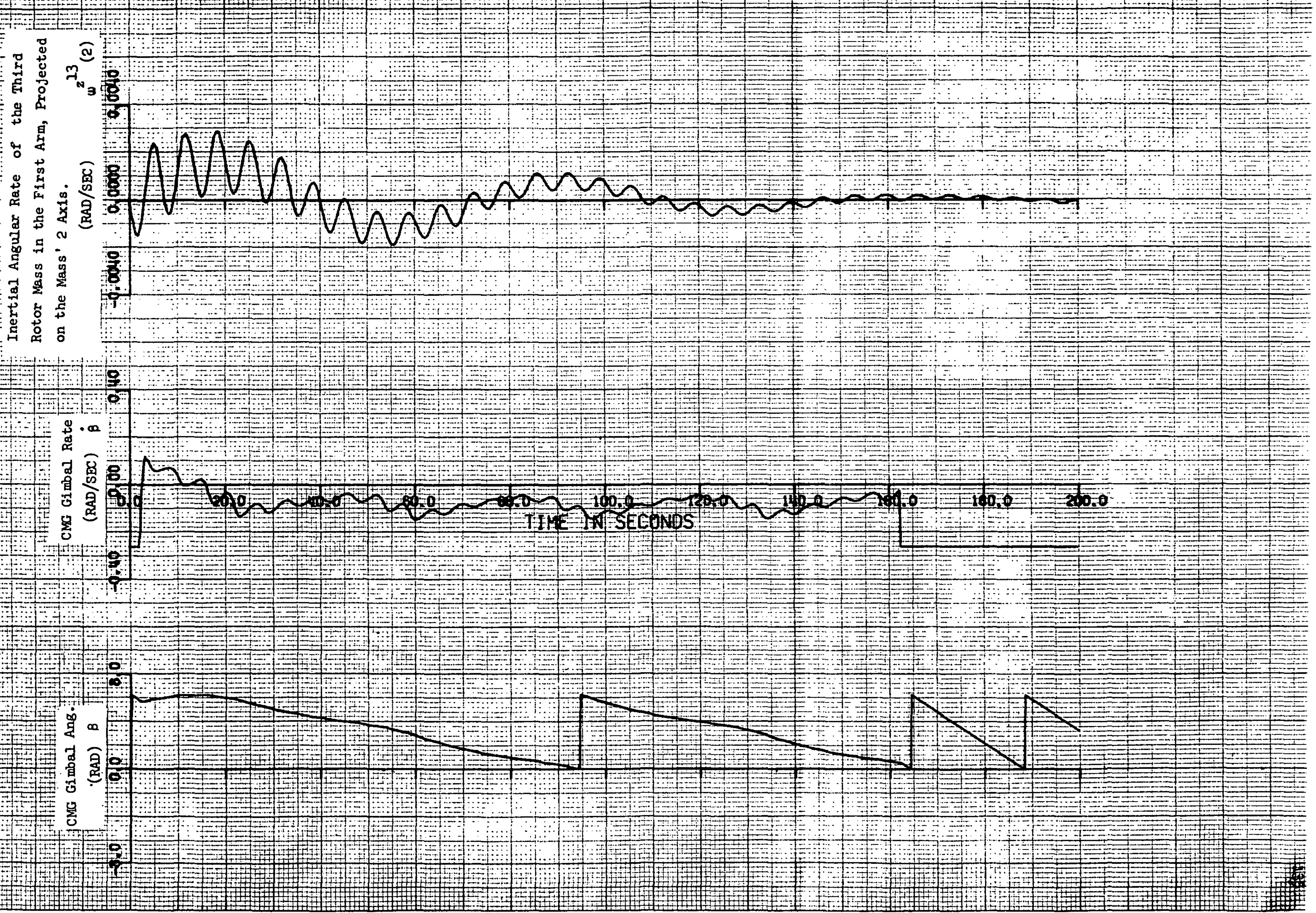


FIGURE 9.7 "T"/SHUTTLE - NOMINAL, CMG CONTROL
(H = 12,000 FT-LB-SEC, τ_f = 10 SEC,
GIMB ANG BIAS)

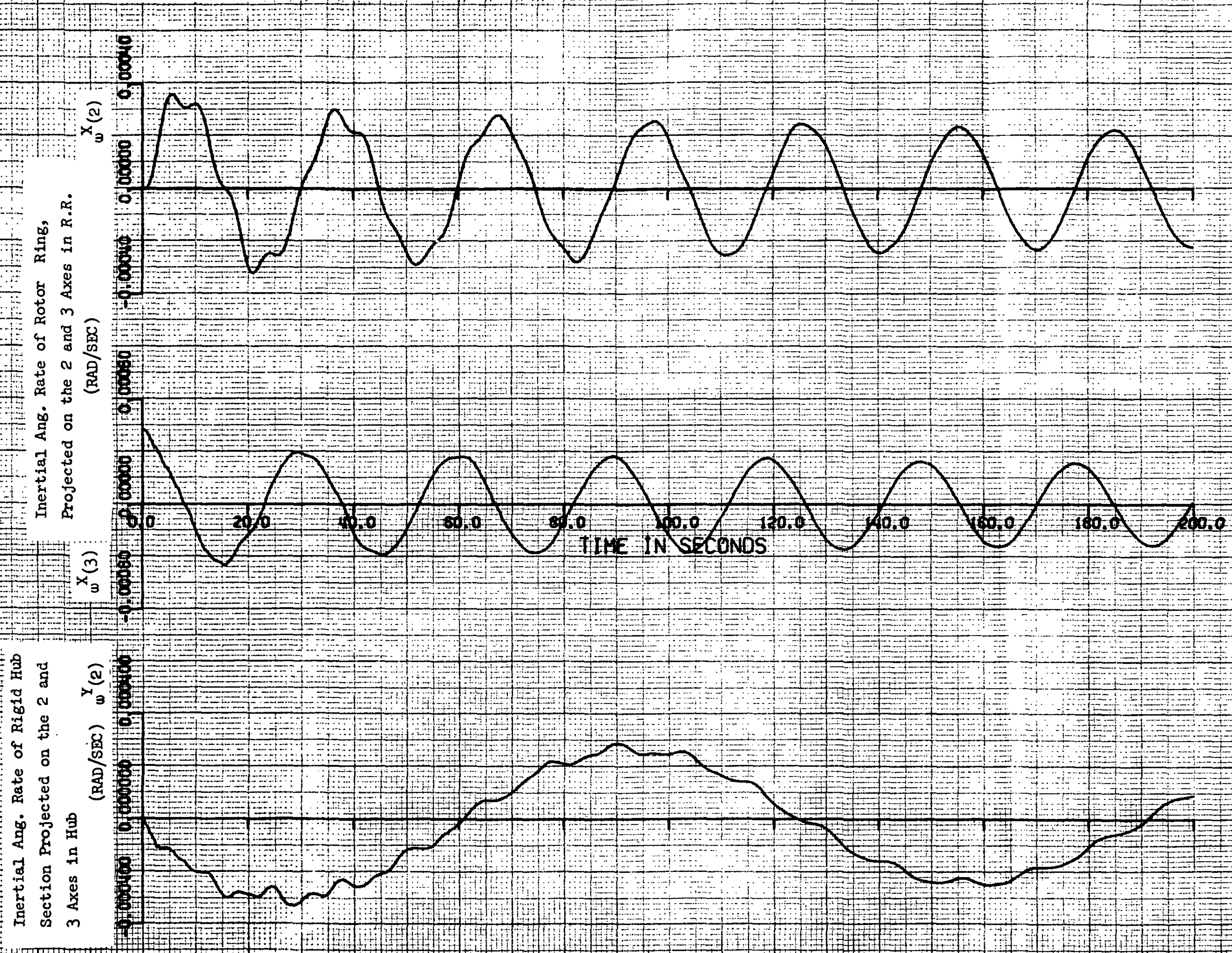


FIGURE 9.7 (Con't)

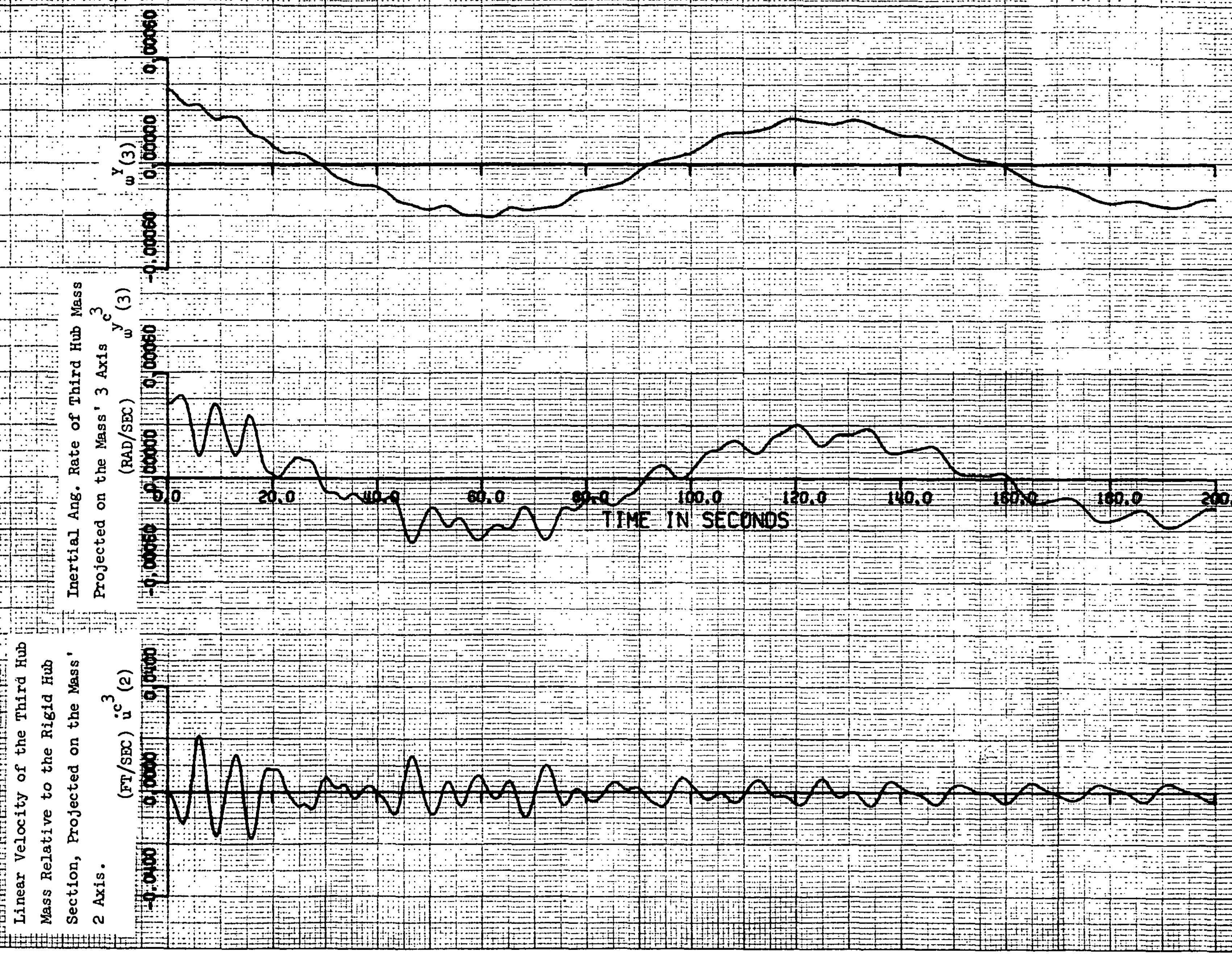
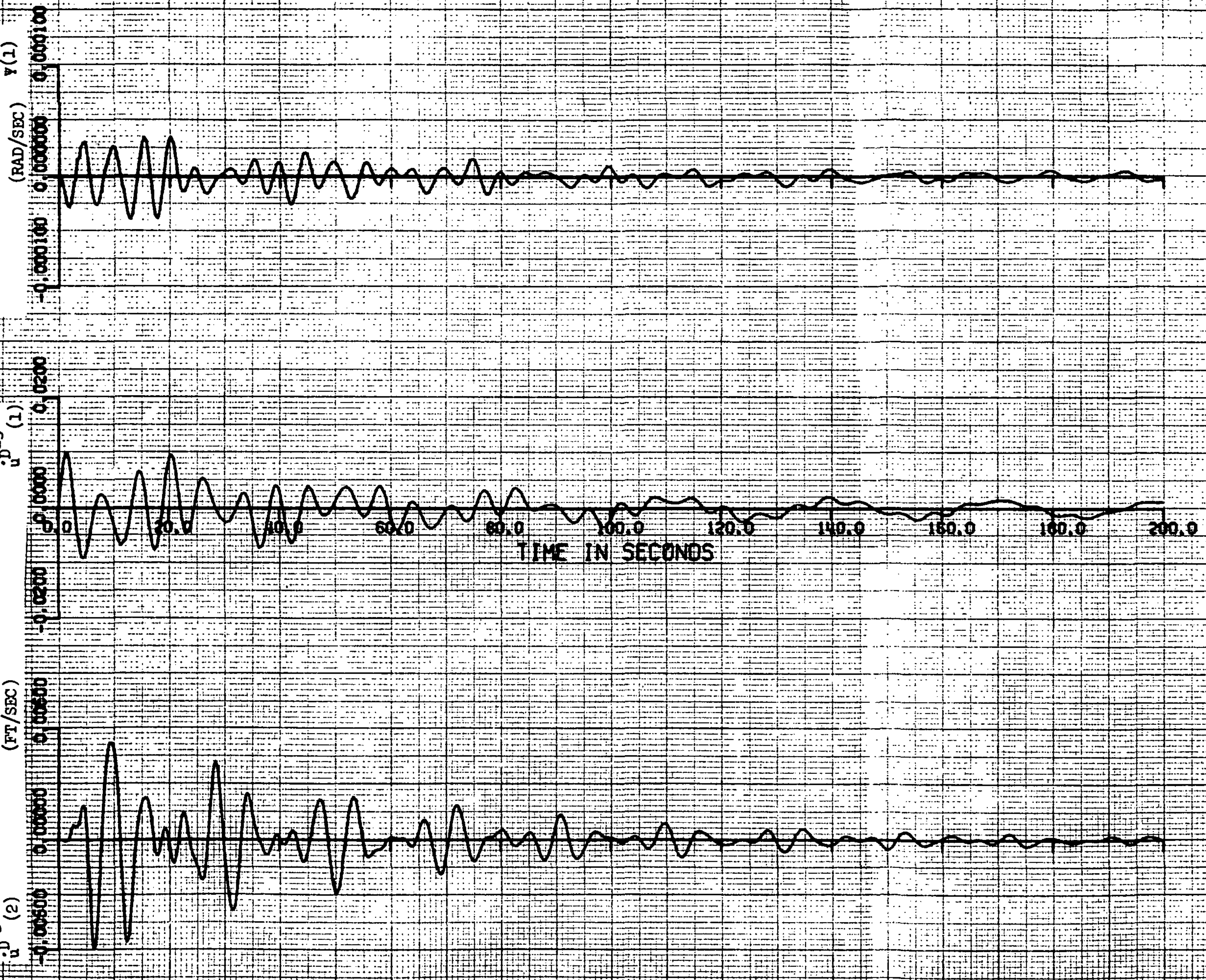


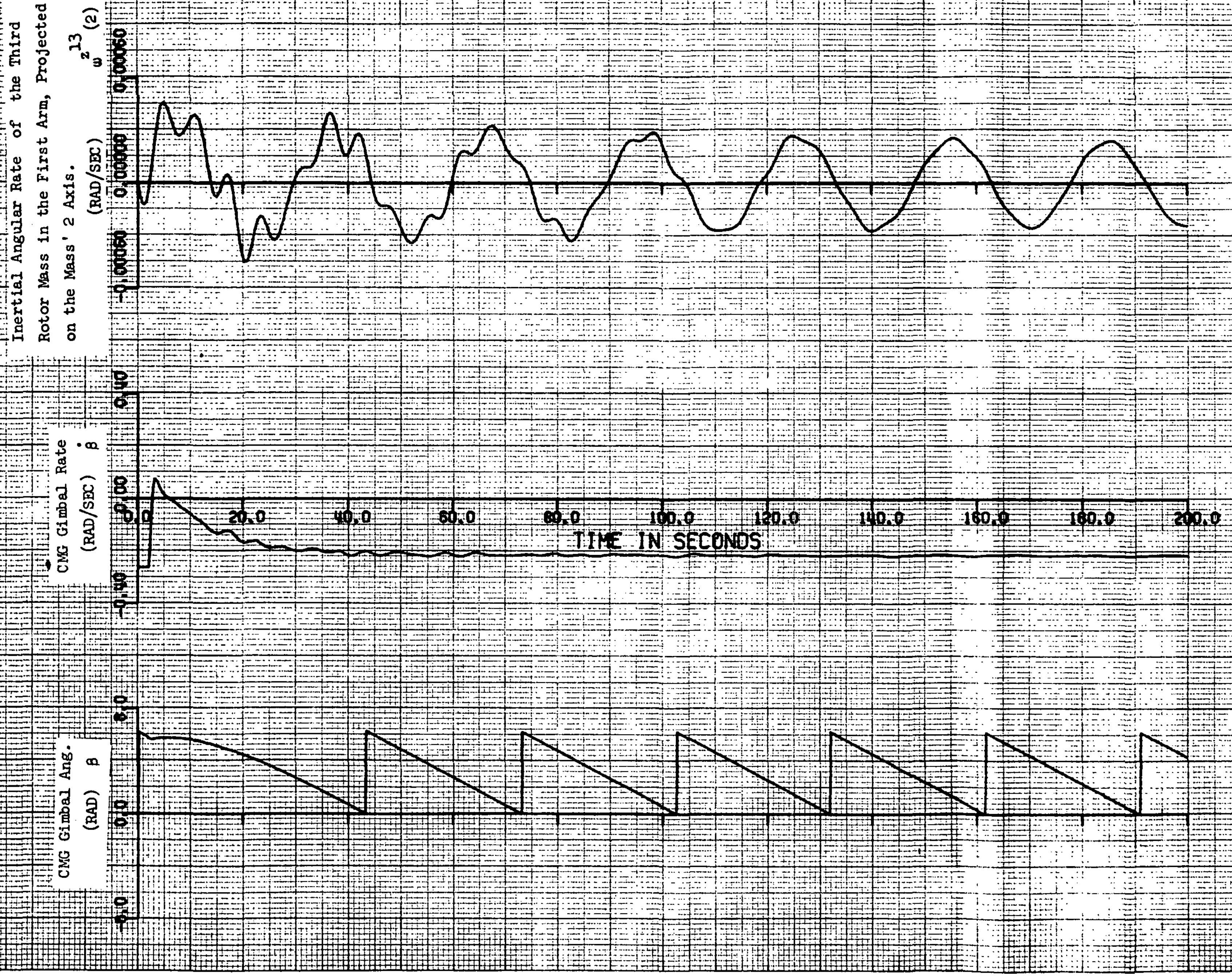
FIGURE 9.7 (Con't)



Shuttle Ang. Rate Relative to the Hub, Projected on Shuttle 1 Axis.

Linear Velocity of the Third Rotor Mass in the Second Arm, Relative to the Rotor Ring, Projected on the Mass' 1 and 2 Axes.

FIGURE 9.7 (Con't)



observable, it noted that the wheel size is not sufficient to complest the damping task within the specified time. However, as previously stated, CMG sizing will not be based upon infrequent docking disturbances.

9.3 CMG Requirements

Summarizing the results of the flexible body simulation studies, the following table lists the CMG system requirements.

TABLE 9-1

CMG System Requirements

				Torquer/Gimbal		
Configuration	Wheel Size (ft-lb-sec)	Filter, Nat. Freq, (rad/sec)	Command Bias (rad)	Nat. Freq. (rad/sec)	Damp Const.	Max. Average Output Power (watts)
"Y"	400	0.5	--	3	0.8	.009
"T"	12,000	0.1	-0.97	3	0.8	5.15

The output power, P_{out}, is derived by using,

$$P_{out} = T_{c_1} \omega_G \tag{9-9}$$

Applying $T_{c_1} = |\omega_T| h$, along with equation (9-4) for the gimbal rate, and averaging over the period of the cyclic portion of ω_G, results in in

$$(P_{out})_{AV} = |\omega_T| h \; (\lambda P) \frac{2\alpha}{(\alpha^2 + 1)} \tag{9-10}$$

As noted from equation (9-10), the average output power will vary with the wobble rate magnitude $|\omega_T|$. The maximum average power is defined as the average output power required at the beginning of the wobble damping task. Using actual values for α and wobble frequency (λP), as experienced during the flexible body simulations, estimates for required output power are obtained and are listed in Table 9-1.

It is interesting to note that if the CMG system were placed in the hub, rather than in the rotor, the average output power is given by,

$$(P_{out})_{AV}^{hub} = |\omega_T| h \, [P \pm \lambda P \frac{2\alpha}{(\alpha^2 + 1)}] \qquad (9\text{-}11)$$

Where the plus sign is used with "Max I" vehicles, and the minus sign with "Min I" vehicles. Comparing the above equation with equation (9-10) it is seen that:

- o The output power requirement for "Max I" vehicles will always be greater for hub mounted CMG's as compared with rotor mounted CMG's.
- o The output power requirement for "Min I" vehicles will be greater for hub mounted CMG's only if the average gimbal rate is less than 1/2 the spin rate.

Corresponding to the nominal "Y" and "T" configurations, the required output power for a hub mounted CMG would be 0.047 watts and 15.25 watts, respectively. Referring to Table 9-1, it is seen that a rotor mounted CMG requires less output power for both the "T" and "Y" configurations.

PART III

USER MANUAL FOR - SPACE STATION GENERAL FLEXIBLE BODY SIMULATION

STAGE II

10.0 Introduction

The digital computer simulation described herein was developed as partial fulfillment of the requirements of Contract NAS 9-11991. The digital program is a special purpose simulation of the flexible body dynamics of the "T" and "Y" space station configurations. The version of the program submitted to MSC, includes the simulation of a Control Moment Gyroscope along with the corresponding control logic used for wobble damping.

Although the digital program is presently assembled to simulate two particular space station configurations, by manipulating the input data, the program can be used to simulate a varity of vehicles with rotating and non-rotating sections, whether it be one rigid body with six degrees of freedom to fifteen flexibly connected bodies with eighty four degrees of freedom. With minor modifications to the program, the simulation can be made to represent dual spin satellites with an arbitrary number of rotor arms (presently limited to three), as well as a high fidelity simulation of a flexible Shuttle Orbiter.

The present version of the program is set up to accept external torques to be applied to any of the fifteen bodies. Thus, torque applying actuators can be placed within any of the bodies. A minor modification to the program is required to allow external forces to be applied to any of the individual bodies.

The digital program is divided into two main sub-programs:

1. The TIME HISTORY PROGRAM (THP) is a numerical integration of the flexible body equations of motion presented in Section 6.0. Wobble damping control is added in the form of a subroutine called CMG (refer to Section 9.2 for the model description). A modal reduction technique is used to decrease the number of coordinates (or integrations) required to describe the vehicle motion.

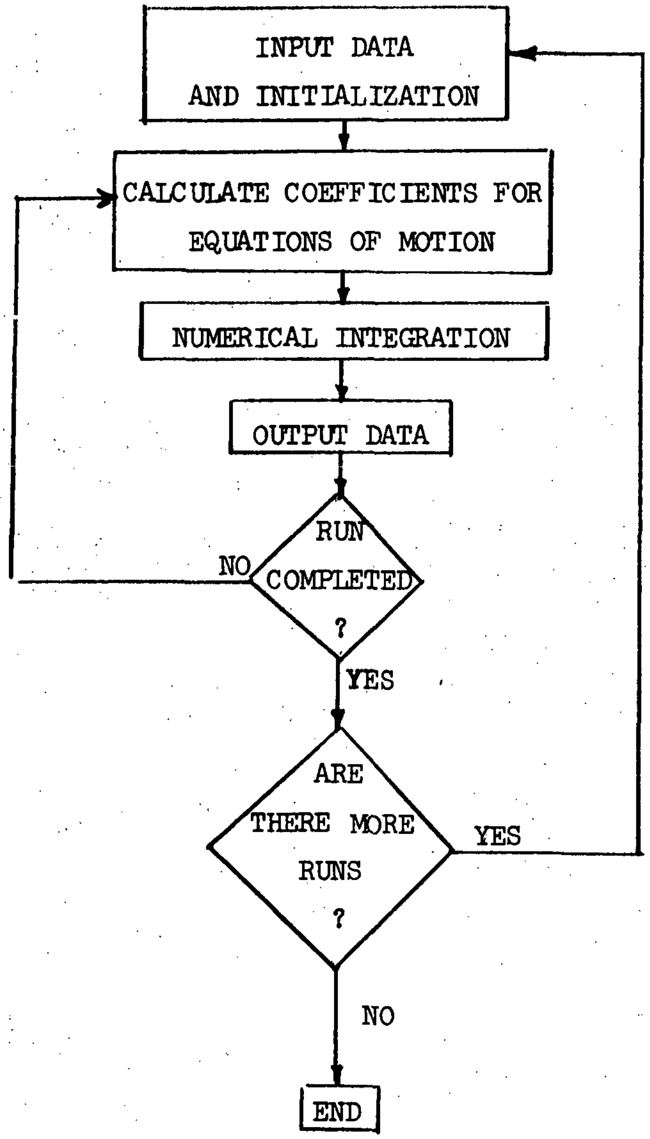
2. The MODAL SERVICE PROGRAM (MSP) is used to determine the mode shapes, frequencies, stiffness matrices, and damping matrices corresponding to the flexible rotor and the section of flexible hub. This data is required for use by the THP. Output from the MSP is in the form of punched cards, in the proper format, for direct input to the THP.

The programming language used was FORTRAN IV. Although the program was developed using the IBM 360/75, particular care was taken to insure compatibility with the UNIVAC 1108 compiler.

A description of the THP and MSP are found in Sections 10.1 and 10.2, respectively.

10.1 TIME HISTORY PROGRAM (THP)

The THP accomplishes the numerical integration of the flexible body equations presented in Section 6.0. The program is structured as follows:



The basic program functions, as pictured above, is accomplished by several subroutines which are described in Section 10.1.2.

The program user can observe at a time increment of his choice, the displacements, rates, and accelerations of each of the 84 coordinates that describe the motion of the vehicle. The number of variables that are actually numerically integrated is equal to the number of flexible modes, chosen by the user, plus eight rigid body modes. The numerical integration technique, known as "Modified Adams," is a corrector-predictor technique that uses a fixed integration interval (DT). A good rule of thumb for chosing a value for DT is that it should be less than (1/20th) the period corresponding to the highest frequency.

The operating procedures for the THP are presented in Section 10.1.3.

10.1.1 Variable Definitions

Table 10.1 contains the definitions of all the program variables used in the Time History Program. The notation used in the equations of motion are used as definitions for program variables that are directly correlated with the equations of motion (refer to Table for definitions).

TABLE 10.1

Definitions of Program Variables

NOTE: The superscripts included in the table below are defined as:

j - lumped hub mass number

k - lumped rotor mass number

i - rotor arm number

Symbol	Definition
AALPHA (I, J, i)	$A(\alpha^i)$
ALPHA (i)	α^i
ANTTWO	Logical variable to initialize problem to second antisymmetric mode of Y-rotor
B	CMG gimbal angle
BDE	Desired gimbal angle
BDED, BDEDD	Gimbal rate, gimbal acceleration
BDEF	Filtered desired gimbal angle
BDR	Limited gimbal rate
BETA (I)	β
BIAS	CMG commanded angle bias, filter compensation.
DD(I)	Input data array
DEL x (I), DEL x (I, i), DEL x (I, k, i)	Δ_x, Δ_x^i, Δ_x^{ik}
DEL (I), DELD (I)	∇, $\dot{\nabla}$
DELTB	δ^*

Symbol	Definition
DT	Iteration time
DTPLOT	Number of seconds per inch of calcomp plot. Also used as logical variable to determine if plotting is desired.
DTPRNT	Print time interval
CH (I, J)	CH
CHG (I, J)	CHG
CONTR	Logical variable to define CMG control or not, CONTR = 0, uncontrolled.
CR (I, J)	CY or CT
ETA	η
FC (I, j)	$F_c^{\,j}$
FD (I, k, i)	$F_D^{\,ik}$
FG (I)	F_G
GAM (I), GAMD (I)	γ , $\dot{\gamma}$
GPSID (I, J)	$\Gamma(\dot{\Phi})$
GYB (I, J)	$\Gamma(Y_B)$
GXL (I, J)	$\Gamma(X_L)$
GYBM (I, J)	(Total rotor mass) $\bullet$ $\Gamma(Y_B)$
GYGB (I, J)	$\Gamma(Y_G - Y_B)$
GYH (I, J)	$\Gamma(Y_H)$
GWR (I,J)	$\Gamma(\omega^r)$
GWX (I, J)	$\Gamma(\omega^X)$
GWXRA (I, J, i)	$\Gamma(\omega^{x^i})$
GWY (I, J)	$\Gamma(\omega^Y)$

Symbol	Definition
GWYC (I, J, i)	$\Gamma({}_{\omega}y^{i})$
GWYS (I, J)	$\Gamma^{2}({}_{\omega}Y)$
H	CMG angular momentum
HCMASS(j)	$m_c{}^j$
HMASS	m_A
IA (I, J)	I_A
IB (I, J)	I_B
IC (I, J, j)	$I_c{}^j$
ID (I, J, k, i)	$I_D{}^{ik}$
IDN	Identification number for data input
IH (I, J)	I_H
IOIN	Data input device indicator
INOUT	Data output device indicator
JT (I, J)	$\pi(\Psi) \bullet \pi(\beta)$
KBS1, KBS2	K_{BS1}, K_{BS2}
KBVH (I,J)	KBVH
KBVR (I, J)	KBVR
KH (I, J)	K7H
KHG (I, J)	K6HG
KHS (I, J)	K_{DOCK}
KHV (I, J)	C_{DOCK}
KR (I, J)	KYA or KTA
LAM (I, J)	Equations of motion mass matrix (not reduced)
LMxx (I, J), LMxx (I, J, i), LMxx (I, J, k, i)	λ_{xx} , $\lambda^{i}{}_{xx}$, $\lambda^{ik}{}_{xx}$
LMxB (I, J) LMxxB (I, J, k, i)	$\bar{\lambda}_{xx}$, $\bar{\lambda}^{ik}{}_{xx}$

Symbol	Definition
M	Number of physical degrees of freedom
Mx (I, J) Mx (I, J, i) Mx (I, J, k, i)	M_x , M_x^i , M_x^{ik}
MAST	Total vehicle mass
MASS (I, J)	Reduced mass matrix; also used to store inverted mass matrix.
MB (I)	M_B
MBEAR (I)	M_{BEAR}
MC (I, j)	M_c^j
MD (I, k, i)	M_D^{ik}
MDOCK (I)	M_{DOCK}
MG (I)	M_G
N	Number of modal coordinates
NHUB	Number of hub modes
NPOINT	Number of plot points per variable
NROT	Number of rotor modes
NRUN	Run number
NS	Logical variable to determine shuttle attached. NS = 0, shuttle not attached.
NSYM	Number of rotor symmetric modes
NTHET (I, J)	$N(\theta)$
P	Number of rotor arms
PASS	Logical variable defining first pass or not. PASS = 0, first pass; = 1, all other passes.
PBLOCK	Number of plot blocks; three variables plotted per block.
PIBETA (I, J)	$\pi(\beta)$
PIGAM (I, J)	$\pi(\gamma)$
PIPHIC (I, J, j)	$\pi(\Phi_c^j)$

Symbol	Definition
PIPHID (I, J, k, i)	$\pi(\Phi_D ik)$
PIPSI (I, J)	$\pi(\Psi)$
PITHET (I, J)	$\pi(\theta)$
PHIC (I, j)	$\Phi_C j$
PHICD (I, j)	$\dot{\Phi}_C j$
PHID (I, k, i)	$\Phi_D ik$
PHIDD (I, k, i)	$\dot{\Phi}_D ik$
PSI (I)	Ψ
PSID (I)	$\dot{\Psi}$
PHIMR (I, J)	ΦYRF or ΦTRF (rotor mode shapes)
PHIMH (I, J)	ΦHF (hub mode shapes)
RDMASS (k, i)	$m_D ik$
RH (I)	r_H
RMASS	Total rotor mass
RRMASS	Rotor ring mass
RTS (I)	Right side of equations of motion (not reduced)
SGAM (I, J)	$S(\gamma)$
SIx (I),	$\xi_x \quad , \quad \xi_x^j \quad , \quad \xi_x^{ik}$
SIx (I, j),	
SIx (I, k, i)	
SIGx (I)	$\sigma, \quad \sigma^j \quad , \quad \sigma^{ik}$
SIGx (I, j)	
SIGx (I, k, i)	
SMASS	m_S
SPIN	Nominal rotor spin rate
STHE (I, J)	$S(\theta)$
STHET (I, J)	$S(\theta)^{-1}$

Symbol	Definition
START	CMG start criterion
STOP	CMG stop criterion
SUM1 (I, k, i)	$[x_D^{ik} + u_D^{ik}]$
T	Simulation time
TAUA (I)	τ_A
TAUC (I, j)	τ_c^j
TAUD (I, k, i)	τ_D^{ik}
TAUF	CMG filter time constant
TAUL (I)	τ_L
TAUS (I)	τ_S
TC (I)	Control torques applied to vehicle from CMG
TEND	Specified duration of simulation test run (seconds)
THET (I)	θ
THETD (I)	$\dot{\theta}$
RHO (I)	ρ
UC (I, j)	u_c^j
UCD (I, j)	$\dot{u}_c^j$
UD (I, k, i)	u_D^{ik}
UDD (I, k, i)	$\dot{u}_D^{ik}$
WAIT	Option to delay CMG control until gimbal error is within WAIT radians
WNG	CMG natural frequency
WOKx (I, J)	Work matrices used for intermediate storage
WTMM	Maximum magnitude of the rotor ring transverse rate over one print cycle
WR (I)	ω^r
WXR (I)	ω^X
WXRA (I, i)	ω^{X^i}
WXY (I)	$\omega^{X/Y}$

Symbol	Definition
WYC (I, j)	$\omega^{y^{j}}$
WYH (I)	ω^{Y}
WZ (I, k, i)	$\omega^{z^{ik}}$
XC (I), XCD (I)	Used as work vectors. Primarily used for storing relative rates and displacements in FLX15
XD (I, k, i)	$x_D{}^{ik}$
XN (I, i)	$x_N{}^{i}$
XL (I)	X_L
XLL (I)	Used as work vector
XVEC (I), XVECD (I), XVECDD (I)	Modal coordinate vector; displacements, rates, and acclerations
YB (I)	Y_B
YC (I, j)	$Y_c{}^{j}$
YE3 (I)	y_E^3
YG (I)	Y_G
YH (I)	Y_H
ZA (I)	Z_A
ZAD (I)	$\dot{Z}_A$
ZETG	CMG damping
ZV3 (I, i)	z_v^{i3}

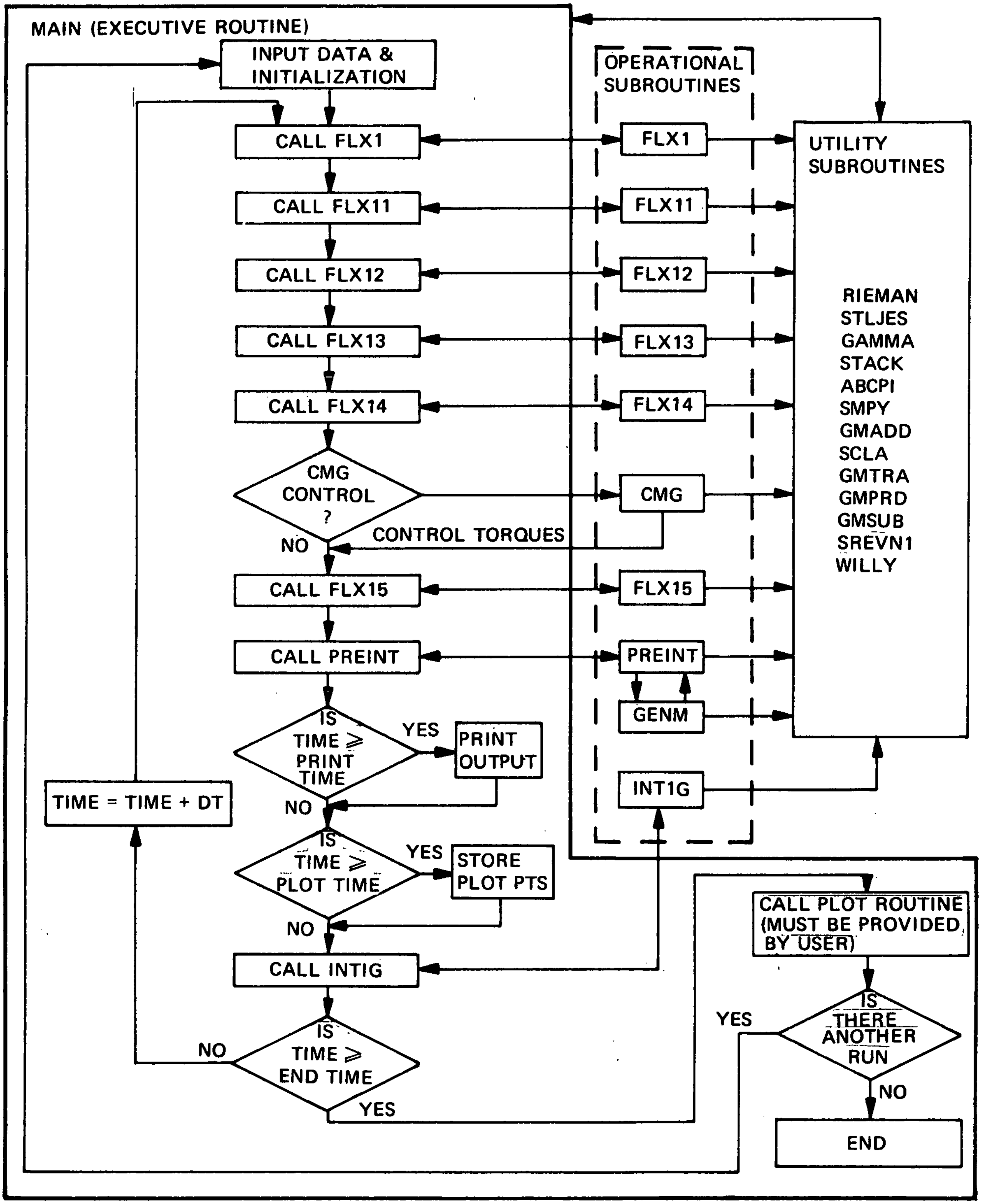


Figure 10-1. Time History Program Flow Diagram

10.1.2 Program Description (with listing)

A flow diagram involving the THP subroutines is shown in Figure 10.1. There are three general subroutine categories; Executive, Operational, and Utility. The MAIN, or Exectuive, routine controls the sequencing of the operational subroutines, the timing of the iteration process, and the means for inputting and outputting data. The formulation of the equations of motion, and the control system equations, are accomplished within the Operational subroutines. Communication between the MAIN routine and the Operational subroutines is achieved via Common blocks. The Utility subroutines are used by both the MAIN routine and the Operational subroutines to perform special purpose mathematical operations such as: matrix inversion, matrix manipulations (e.g., addition, subtraction, multiplication), vector cross products, coordinate transformations, and numerical integration.

10.1.21 MAIN (Executive) Routine

The fortran listing of the routine is shown in Figure 10.2. The program variables are defined in Table 10.1.

The first function of the Main routine is to accept all input data and assign the values to the proper variables. Data format and the required order for the input data is discussed in the Operating Instructions section (Section 10.1.3). Prior to entering the main iteration loop, any variables that require initialization that weren't initialized by input data, are now initialized. During each iteration cycle, the Main routine calls, in sequence, the operational subroutines.

Prior to the integration step (CALL INTIG) tests are made concerning data output. Due to the large number of variables to be observed it was considered convenient to use NAMELIST blocks for printing. A number of the namelists are used for trouble shooting only (e.g., TEST1, TEST2, TEST3). The values of the variables that comprise the main namelist, STATE, are printed every DTPRNT seconds. If a shuttle is attached to the station, or if CMG wobble damping is activated, then the variables in namelists SHUT, and/or CMGPR are also printed every DTPRNT seconds. The important program constants are found in namelist TESTC which is printed at time zero only.

Following the execution of the print instructions, the variables to be plotted are stored on disk to be accessed at the end of the run. It should be noted that the plotting routine provided, PLO3AX, is compatible with the Calcomp system at Grumman and most likely will have to be converted for use at the MSC facility.

If the problem time (T) is less than the specified run duration time (TEND), then the state variables will be integrated via (CALL INTIG) and time will be updated (T = T + DT).

10.1.2.2 Operational Subroutines

FLX1, FLX11, FLX12, FLX13, FLX14, FLX15

The coefficients for the flexible body equations are formulated within these subroutines. The next few paragraphs summarize the sequence of operations performed within each FLXxx subroutine. Reference will be made to the actual variables associated with the equations of motion as defined in Section 6.0.

FLX1 (refer to Figure 10.3 for listing)-coordinate transformations are formulated ($A(\gamma)$, $\pi(\beta)$, $\pi(\gamma)$, $\pi(\theta)$, $\pi(\Psi)$, $\pi(\Phi_C j)$, $\pi(\Phi_D ik)$, J). Inertial angular rates are calculated based upon the vehicle state. The vector variable ρ is determined.

FLX11 (Figure 10.4) - Calculate the M matrices and start the formulation of the λ_k matrices.

FLX12 (Figure 10.5) - Complete the formulation of all the λ_k matrices.

FLX13 (Figure 10.6) - Start setting up the right side of the equations of motion by computing Δ_k.

FLX14 (Figure 10.7) - Continue formulating the right side of the equations of motion by computing Δ_k.

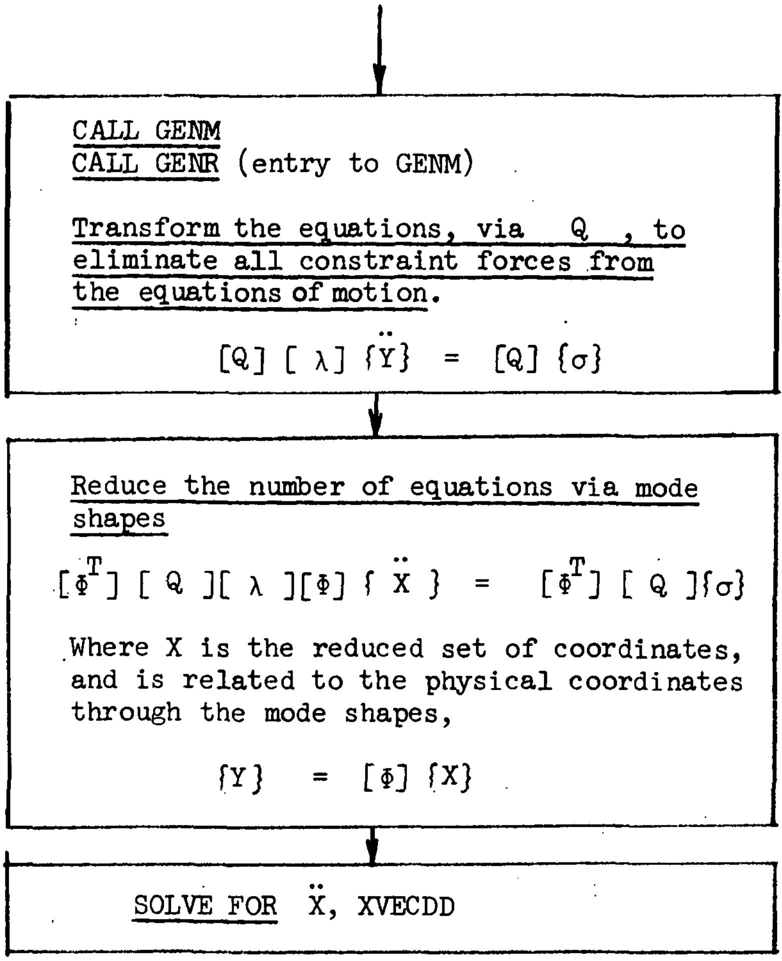
FLX15 (Figure 10.8) - Complete the formulation of the right side of the equations of motion. All elastic forces and moments are computed by multiplying the stiffness and damping matrices by the appropriate relative displacements and rates. Finally, σ_k is computed.

PREINT (Figure 10.9) - The purpose of this subroutine is to solve for the second derivates of the modal coordinates which are subsequently integrated in subroutine INTIG. The process is outlined below:

Set up total mass matrix λ, and right side σ.

$$[\lambda] \{\ddot{Y}\} = \{\sigma\}$$

Where Y is the physical coordinates, or state vector.



CALL GENM
CALL GENR (entry to GENM)

Transform the equations, via Q, to eliminate all constraint forces from the equations of motion.

$$[Q]\,[\lambda]\,\{\ddot{Y}\} = [Q]\,\{\sigma\}$$

Reduce the number of equations via mode shapes

$$[\Phi^T]\,[Q]\,[\lambda]\,[\Phi]\,\{\ddot{X}\} = [\Phi^T]\,[Q]\,\{\sigma\}$$

Where X is the reduced set of coordinates, and is related to the physical coordinates through the mode shapes,

$$\{Y\} = [\Phi]\,\{X\}$$

SOLVE FOR $\ddot{X}$, XVECDD

INTIG (Figure 10.10) - Perform a double integration, via CALL RIEMAN, to obtain the displacements and rates in modal coordinates. Transform modal displacements and rates to physical displacements and rates by,

$$\{\dot{Y}\} = [\Phi]\,\{\dot{X}\}, \qquad \{Y\} = [\Phi]\,\{X\} \tag{10-1}$$

The modal displacements and rates are initialized within INTIG by applying,

$$\{X(0)\} = (\Phi^T\Phi)^{-1}\,\Phi^T\{Y(0)\} \tag{10-2}$$

Finally, Euler rates that aren't included in the state vector (e.g. $\dot{\gamma}$, and $\dot{\theta}_1$) are integrated to obtain the corresponding Euler angles.

GENM (Figure 10.11) - Eliminates all constraint forces from the equations of motion.

CMG (Figure 10.12) - Develops control torques, TC, for wobble damping. The simulated CMG configuration and associated control logic is presented in Section 9.2.

The user has the option to place the CMG in any of the bodies that form the flexible vehicle. This is accomplished by the first and last three executable statements in the subroutine. The first three statements equate the angular rates of the body to the rates used within the control equations. The last three statements equate the external torques applied to the particular body to the derived control torques.

Control variables that require numerical integration is accomplished via a direct call to STLJES from CMG.

It is noted that an additional common block, called CONT, is used to communicate between MAIN and CMG.

10.1.2.3 Utility Subroutines

RIEMAN (Figure 10.13) - A Modified-Adams numerical integration, for a second order differential equation, using a fixed integration interval. Basically, the integrated variable at $t + \Delta t$ is approximated by the first three terms of a Taylor series expansion about Δt. That is,

$$f(t + \Delta t) = f(t) + \dot{f}(t)\ \Delta t + \ddot{f}(t)\ \frac{\Delta t^2}{2} \qquad (10\text{-}3)$$

The integration scheme is a corrector-predictor technique where the predicted value (i.e., calculated at t for $t + \Delta t$) is corrected during the next iteration by applying current rates and accelerations to the above expansion.

STLJES (Figure 10.14) - A special version of RIEMAN for the purpose of integrating a first order differential equation.

GAMMA (Figure 10.15) - The GAMMA matrix, $\Gamma(a)$, is formed to facilitate the vector cross product operation. Specifically,

$$\{a\} \times \{b\} = \Gamma(a)\{b\} \qquad (10\text{-}4)$$

STACK (Figure 10.16) - Places small matrices at specified locations within a large matrix.

ABCPI (Figure 10.17) - Formulates the coordinate transformation $(\pi(\theta))$ for three ordered rotations $(\theta_1, \theta_2, \theta_3)$, and/or the coordinate transformation for a single rotation $A(\theta_1)$.

SMPY (Figure 10.18) - Multiplies a scalar by a matrix. (Part of IBM Sientific Subroutine Package).

GMADD (Figure 10.19) - Matrix Addition (IBM Scientific Subroutine Package).

SCLA (Figure 10.20) - Set each element of a matrix equal to a given scalar. (IBM Scientific Subroutine Package).

GMTRA (Figure 10.21) - Matrix transpose. (Part of IBM Scientific Subroutine Package)

GMPRD (Figure 10.22) - Multiplies two matrices (Part of IBM Scientific Subroutine Package)

GMSUB (Figure 10.23) - Matrix subtraction. (Part of IBM Scientific Subroutine Package).

SREVN1 (Figure 10.24) - Inverts a real, single precision matrix. The method used is Gauss-Jordan elimination with partial pivoting.

WILLY (Figure 10.25) - Prints a matrix in an easily readable format.

FIGURE 10.2 MAIN

```
20.1 (AUG 71)                                      OS/360  FORTRAN H

   COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                     SOURCE,BCD,NOLIST,DECK,LOAD,MAP,NOEDIT,ID,NOXREF
0002              IMPLICIT REAL (A-Z)
0003              COMMON/TIME/T,DT,NSYM,ANTTWC
0004              COMMON/CONT/
                 *TC(3),TAUF,START,STOP,H,WNG,ZETG,BDR,B,FE,BDE,BDEF,BDEDD,BDED,
                 *BIAS,WAIT
0005              COMMON/FLX/
                 *AALPHA(3,3,3),ALPHA(3),BETA(3),DEL1(3),DEL2(3,3),DEL3(3,3,3),
                 *DEL4(3,3),DEL5(3,3,3),DEL6(3),DEL7(3),DEL8(3),DEL8,
                 *CR(57,57),CH(18,18),CHG(6,18),DEL(3),DELD(3),
                 *EIH(3),EIR(3),EH,ER,ETA,FC(3,3),FD(3,3,3),FG(3),GAM(3),GAMD(3),GH,
                 *GJH(3),GJR(3),GPSID(3,3),GR,GYGB(3,3),GXL(3,3),DD(500),
                 *GPH(3,3),GWR(3,3),GWX(3,3),GWXRA(3,3,3),GWY(3,3),GWYC(3,3,3),
                 *GWYS(3,3),GYB(3,3),GYBM(3,3),GYH(3,3),HCMASS(3),HI,HJ,HMASS
0006              COMMON/FLX/
                 *IA(3,3),IB(3,3),IC(3,3,3),ID(3,3,3,3),IH(3,3),JT(3,3),KBS1,KBS2,
                 *KBVH(3,3),KBVR(3,3),KHS(3,3),KHV(3,3),LAM(84,84),LM1(3,3),
                 *LM2(3,3),LM3(3,3,3),LM4(3,3),LM5(3,3,3,3),LM6(3,3),LM7(3,3,3),
                 *LM8(3,3,3),LM9(3,3,3),LM10(3,3,3,3),LM11(3,3,3,3),LM12(3,3,3,3),
                 *LM13(3,3,3,3),LM14(3,3,3),LM15(3,3,3),LM16(3,3,3,3),LM17(3,3,3,3),
                 *LM18(3,3),LM19(3,3),LM20(3,3),LM21(3,3),LM22(3,3),LM23(3,3),
                 *LM24(3,3),LM25(3,3),LM26(3,3),LM4B(3,3),LM13B(3,3,3,3),KHG(6,18),
                 *LM16B(3,3,3,3),KR(57,57),KH(18,18)
0007              COMMON/FLX/
                 *M,M1(3,3),M2(3,3,3),M3(3,3,3,3),M4(3,3),M5(3,3),MAST,MASS(84,84),
                 *MB(3),MBEAR(3),MC(3,3),MD(3,3,3),MDOCK(3),MG(3),NROT,NHUB,
                 *N,NS,P,PASS,PIBETA(3,3),PIGAM(3,3),PIPHIC(3,3,3),PIPHID(3,3,3,3),
                 *PIPSI(3,3),PITHET(3,3),PHIC(3,3),PHICD(3,3),PHID(3,3,3),
                 *PHIDD(3,3,3),PSI(3),PSID(3),PHIMR(57,57),PHIMH(18,18),NTHET(3,3)
0008              COMMON/FLX/
                 *RDMASS(3,3),RH(3),RI,RJ,RMASS,RRMASS,RTS(84),SI1(3,3),SI2(3),
                 *SI3(3,3,3),SI4(3),SIG1(3),SIG2(3,3),SIG3(3,3,3),SIG4(3,3),
                 *SIG5(3,3,3),SIG6(3),SIG7(3),SIG8(3),SGAM(3,3),SMASS,STHET(3,3),
                 *SUM1(3,3,3),TAUA(3),TAUC(3,3),TAUD(3,3,3),TAUL(3),TAUS(3),
                 *THET(3),THETD(3),RHO(3),STHE(3,3),SPIN
0009              COMMON/FLX/
                 *UC(3,3),UCD(3,3),UD(3,3,3),UDD(3,3,3),WOK1(3,3),WOK2(84,84),
                 *WOK3(84),WOK4(3,3),WOK5(3,3),WCK5(3,3),WR(3),WXR(3),WXRA(3,3),
                 *WXY(3),WYC(3,3),WYH(3),WZ(3,3,3),XC(84),XCD(84),XD(3,3,3),XN(3,3),
                 *XL(3),XLL(84),XVEC(84),XVECD(84),XVECDD(84),YC(3,3),YB(3),YE3(3),
                 *YG(3),YH(3),ZA(3),ZAD(3),ZV3(3,3)
0010              NAMELIST/TIM/T
0011              NAMELIST/STATE/ZAD,ZA,WYH,GAM,UCD,UC,PHICD,PHIC,DELD,DEL,WXR,
                 *UDD,UD,PHIDD,PHID,WZ,XVEC,XVECD,XVECDD,RTS,WTMM
0012              NAMELIST/SHUT/PSID,PSI,WR,JT,MDOCK,PIPSI,TAUS
0013              NAMELIST/TESTC/ALPHA,BETA,ETA,HMASS,HCMASS,IA,IB,IC,ID,IH,
                 *KBS1,KBS2,KBVH,KBVR,KHS,KHV,M,MAST,NROT,NHUB,N,NS,P,PASS,
                 *GXL,GPH,GYB,GYBM,GYH,PIBETA,RDMASS,RH,RMASS,RRMASS,SMASS,SPIN,
                 *XN,XL,YB,YE3,YG,YH,ZV3,CR,CH,CHG,KHG,KR,KH,PHIMR,PHIMH
0014              NAMELIST/TEST1/DEL1,DEL2,DEL3,DEL4,DEL5,DEL6,DEL7,DEL8,FC,FD,FG,
                 *GAMD,GYGB,GWXRA,GWY,GWYS,LM1,LM2,LM3,LM4,LM5,
                 *LM6,LM7,LM8,LM9,LM10,LM11,LM12,LM13,LM14,LM15,LM16,LM17,LM18,LM19,
                 *LM20,LM21,LM22,LM23,LM24,LM25,LM26,LM4B,LM13B,LM16B
0015              NAMELIST/TEST2/M1,M2,M3,M4,M5,MB,MBEAR,MC,MD,MG,PIGAM,
                 *PIPHIC,PIPHID,PITHET,NTHET,SI1,SI2,SI3,SI4,SIG1,SIG2,SIG3,
                 *SIG4,SIG5,SIG6,SIG7,SIG8,SGAM,STHET,TAUA,TAUC,TAUD,TAUL,THET,
                 *THETD,RHO,STHE,WXRA,WXY,WYC,WZ,XD,YC
```

```
      NAMELIST/TEST3/RTS,XVEC,XVECD,XVECDD
      NAMELIST/CMGPR/WXR,FE,RDF,RDEF,R,RDEDD,RDED,RDR,FC
      INTEGER II,JJ,KK,LL,NN
      INTEGER IOUT,IOIN,IDN,NRUN
      INTEGER NROT,NHUB,N,M,NFCINT,NSYM
      INTEGER NROW(5),CCL
      COMMON/BUF/BUFFER(512)
      DIMENSION VALU(5)
      DIMENSION QNAM(11)
      IOIN=5
      IOUT=6
      PAUSED=0.0
      DO 1 II=1,500
    1 DD(II)=0.0
  121 READ(IOIN,22)NRUN,QNAM
   22 FORMAT(I3,11A4)
      WRITE(IOUT,23)NRUN,QNAM
   23 FORMAT(1H1,'RUN NUMBER=',I3,5X,11A4/)
      IF(NRUN.GT.0)GO TO 25
      IF(PAUSED.EQ.0.)GO TO 24
      CALL EPLOT
   24 CALL EXIT
   25 READ(IOIN,26)IDN,VAL,QNAM
   26 FORMAT(I5,E20.5,11A4)
      IF(IDN.LE.0.)GO TO 27
      DD(IDN)=VAL
      WRITE(IOUT,29)IDN,VAL,QNAM
   29 FORMAT(5X,'DD(',I5,')=',E20.5,5X,11A4)
      GO TO 25
   27 DTPLOT=DD(1)
      IF(DTPLOT.LE.0.0.OR.PAUSED .NE. 0.0)GO TO 75
      CALL PLOTS(BUFFER,512)
      PAUSED=1.0
   75 CONTINUE
C INPUT DATA ARRAY
C    ITERATION TIME
      DT=DD(2)
C    RUN TIME
      TEND=DD(3)
C    PRINT INTERVAL
      DTPRNT=DD(4)
C  MASS PROPERTY DATA
C     RIGID HUB MASS
      HMASS=DD(7)/32.174
C     SHUTTLE MASS - SET TO ZERO FOR NO SHUTTLE
      SMASS=DD(8)/32.174
C  SHUTTLE ATTACHED NS=1.,NOT ATTACHED NS=0.
      NS=DD(5)
C  NUMBER OF ROTOR ARMS
      R=DD(6)
C    ROTOR RING MASS
      RRMASS=DD(9)/32.174
      DO 10 II=1,3
C    MASS OF LUMPED HUB MASS
      HCMASS(II)=DD(9+II)/32.174
C    MASS OF LUMPED ROTOR MASS-EACH ARM IS SAME
      RDMASS(II,1)=DD(12+II)/32.174
      RDMASS(II,2)=RDMASS(II,1)
```

GAC PLOT ROUTINES.
REPLACE BY EQUIVALENT
MSC PLOT ROUTINES

```
      IF(P.EQ.2.)GO TO 8
      RDMASS(II,3)=RDMASS(II,2)
      GO TO 10
    8 RDMASS(II,3)=0.0
   10 CONTINUE
      DO 15 II=1,3
      DO 15 JJ=1,3
C     RIGID HUB INERTIA
      NN=15+JJ+3*(II-1)
      IA(JJ,II)=DD(NN)
C     SHUTTLE MASS - INPUT ZERO FOR NO SHUTTLE
      IH(JJ,II)=DD(NN+9)
C     ROTOR RING INERTIA
      IB(JJ,II)=DD(NN+18)
      DO 15 KK=1,3
C     INERTIA OF LUMPED HUB MASS - II REPRESENTS EACH LUMPED MASS
      LL=42+KK+3*(JJ-1)+9*(II-1)
      IC(KK,JJ,II)=DD(LL)
C     INERTIA OF LUMPED ROTOR MASS-ALL ARMS IDENTICAL
      ID(KK,JJ,II,1)=DD(LL+27)
      ID(KK,JJ,II,2)=ID(KK,JJ,II,1)
      IF(P.EQ.2.)GO TO 12
      ID(KK,JJ,II,3)=ID(KK,JJ,II,2)
      GO TO 15
   12 ID(KK,JJ,II,3)=0.0
   15 CONTINUE
C   GEOMETRY
      DO 30 II=1,3
C     LOCATE SHUTTLE
      YH(II)=DD(96+II)
      RH(II)=DD(99+II)
C     LOCATE ROTOR RING PIVOT
      YB(II)=DD(102+II)
C     LOCATE FLEX HUB
      YG(II)=DD(105+II)
      YF3(II)=DD(108+II)
C     LOCATE MASS CENTER OF ROTOR RING
      XL(II)=DD(111+II)
C     LOCATE FLEX ARM - ALL ARMS IDENTICAL
      XN(II,1)=DD(114+II)
      XN(II,2)=XN(II,1)
      XN(II,3)=XN(II,2)
      ZV3(II,1)=DD(117+II)
      ZV3(II,2)=ZV3(II,1)
      ZV3(II,3)=ZV3(II,2)
      DO 30 JJ=1,3
      NN=120+JJ+3*(II-1)
C LOCATE HUB LUMPED MASS
      YC(JJ,II)=DD(NN+90)
C     LOCATE ROTOR LUMPED MASS
      XD(JJ,II,1)=DD(NN)
      XD(JJ,II,2)=DD(NN)
   30 XD(JJ,II,3)=DD(NN)
C     ARM ROTATION
      ALPHA(1)=0.
      ALPHA(2)=DD(130)
      ALPHA(3)=DD(131)
C     SHUTTLE ROTATION
```

```
      DO 40 II=1,3
   40 BETA(II)=DD(131+II)
C STIFFNESS AND VISCOSITIES
C   BEARING
      KBS1=DD(135)
      KBS2=DD(136)
C     DISCONTINUITY DEFLECTION
      DELTB=DD(137)
C   AMOUNT OF DAMPING ON INNER RACE (ZERO TO ONE)
      ETA=DD(138)
      DO 50 II=1,3
      DO 50 JJ=1,3
C    OUTER RACE DAMPING
      NN=JJ+3*(II-1)+138
      KBVR(JJ,II)=DD(NN)
C    INNER RACE DAMPING
      KBVH(JJ,II)=DD(NN+9)
C   SHUTTLE PIVOT STIFFNESS
      KHS(JJ,II)=DD(NN+18)
C   SHUTTLE PIVOT DAMPING
   50 KHV(JJ,II)=DD(NN+27)
C   ROTOR ARMS - ALL IDENTICAL
C    E
      ER=DD(175)
C    I
      RI=DD(176)
C    G
      GR=DD(177)
C    J
      RJ=DD(178)
C   HUB SECTIONS
C    E
      EH=DD(179)
C    I
      HI=DD(180)
C    G
      GH=DD(181)
C    J
      HJ=DD(182)
C    EQUATING EACH SECTION
      EIR(1)=ER*RI
      GJR(1)=GR*RJ
      EIH(1)=EH*HI
      GJH(1)=GH*HJ
      DO 60 II=2,3,1
      EIR(II)=EIR(1)
      GJR(II)=GJR(1)
      EIH(II)=EIH(1)
   60 GJH(II)=GJH(1)
C  NOMINAL SPIN RATE
      SPIN=DD(200)
      IF(NRUN.EQ.1)GO TO 315
      IF(NPCT.EQ.DD(210))GO TO 315
C  ZERO PHIMP FOR NEW MODAL DATA
      DO 310 II=1,57
      DO 310 JJ=1,57
  310 PHIMP(JJ,II)=0.0
  315 CONTINUE
```

```
C   NUMBER OF ROTOR MODES
       NROT=DD(210)
C    NUMBER OF HUB MODES
       NHUB=DD(211)
C   NUMBER OF SYM. ROTOR MODES
       NSYM=DD(212)
C   ANTTWO=1. TO INITIALIAZE THE ROTOR 2ND ANTI-SYM MODE
       ANTTWO=DD(213)
C    NUMBER OF PLOT BLOCKS
       PBLOCK=DD(214)
C   DATA FOR CMG CONTROL
C    FOR CMG CONTROL SET CONT .GT. 0.0
       CONTR=DD(220)
C    STOP DAMPING FOR WOBBLE .LT. STOP
       STOP=DD(221)
C    START DAMPING FOR WOBBLE .GT. START
       START=DD(222)
C    TIME CONSTANT OF GIMBAL ANGLE COMMAND LOW PASS FILTER,NO FILTER IF
       TAUF=DD(223)
C    ANGULAR MOMENTUM OF WHEEL
       H=DD(224)
C    NATURAL FREQ.  OF TORQUER MOTOR
       WNG=DD(225)
C    DAMPING OF TORQUER MOTOR
       ZETG=DD(226)
C   BIAS=+OR- WOBBLE FREQ.   MINUS FOR MINI
C   NOTE.. SET BIAS=0. FOR NO GIMBAL ANGLE BIAS
       BIAS=DD(227)
C   WAIT OPTION FOR ACTIVE CONTROL
C   SET WAIT=MIN. ERROR(IN RAD)BEFORE ACTIVE CONTR IS ALLOWED
       WAIT=DD(228)
C   INITIALIZE STATE VECTOR
       DO 70 II=1,3
       ZA(II)=DD(300+II)
       PSI(II)=DD(306+II)
       DO 70 JJ=1,3
       NN=9+JJ+3*(II-1)
       UC(JJ,II)=DD(NN+300)
       PHIC(JJ,II)=DD(NN+309)
       DO 70 KK=1,3
       LL=30+KK+3*(JJ-1)+18*(II-1)
       UD(KK,JJ,II)=DD(LL+300)
    70 PHID(KK,JJ,II)=DD(LL+309)
C   INITIALIZE DERIVATIVE OF STATE
       DO 80 II=1,3
       ZAD(II)=DD(400+II)
       WYH(II)=DD(403+II)
       PSID(II)=DD(406+II)
       WXY(II)=DD(427+II)
       DO 80 JJ=1,3
       NN=9+JJ+3*(II-1)
       UCD(JJ,II)=DD(NN+400)
       PHICD(JJ,II)=DD(NN+409)
       DO 80 KK=1,3
       LL=30+KK+3*(JJ-1)+18*(II-1)
       UDD(KK,JJ,II)=DD(LL+400)
    80 PHIDD(KK,JJ,II)=DD(LL+409)
C   INITIALIZE EULER ANGLES
```

FIGURE 10.2 (Cont'd)

```
      DO 90 II=1,3
      GAM(II)=DD(484+II)
   90 THET(II)=DD(487+II)
C
C  ZERO MODAL,STIFFNESS AND DAMPING MATRICES
      IF(NRUN.GT.1)GO TO 205
      DO 201 JJ=1,57
      DO 201 II=1,57
      PHIMR(II,JJ)=0.0
      KR(II,JJ)=0.0
  201 CR(II,JJ)=0.0
      DO 204 JJ=1,18
      DO 203 II=1,18
      PHIMH(II,JJ)=0.0
      KH(II,JJ)=0.0
  203 CH(II,JJ)=0.0
      DO 204 II=1,6
      KHG(II,JJ)=0.0
  204 CHG(II,JJ)=0.0
  205 CONTINUE
C INPUT MODAL,STIFFNESS AND DAMPING MATRICES
  206 READ(IPIN,207)COL,(NROW(JJ),VALU(JJ),JJ=1,5),IDN
  207 FORMAT(I3,5(I3,F12.5),I2)
      IF(IDN.EQ.0)GO TO 300
      II=1
      IF(IDN.GT.1)GO TO 208
      LL=19
      IF(P.EQ.3.)LL=28
      IF(COL.GT.LL)GO TO 209
      IF(COL.GT.NSYM)GO TO 206
      GO TO 208
  209 COL=COL-LL+NSYM
      IF(COL.GT.NROT)GO TO 206
  208 IF(NROW(II).EQ.0)GO TO 206
      GO TO (210,220,230,240,250,260,270,280),IDN
C                 IDN              INPUT DATA TO-
C                  1                 PHIMR
C                  2                 PHIMH
C                  3                KTA OR KYA
C                  4                CT OR CY
C                  5                  KH
C                  6                  CH
C                  7                  KHG
C                  8                  CHG
C READ IN MODAL DATA - ROTOR
  210 PHIMR(NROW(II),COL)=VALU(II)
      GO TO 290
C  HUB MODES
  220 PHIMH(NROW(II),COL)=VALU(II)
      GO TO 290
C INPUT ROTOR STIFFNESS AND DAMPING MATRICES
C     STIFFNESS
  230 KR(NROW(II),COL)=VALU(II)
      GO TO 290
C    DAMPING
  240 CR(NROW(II),COL)=VALU(II)
      GO TO 290
C INPUT HUB STIFFNESS AND DAMPING
```

FIGURE 10.2 (Cont'd)

```
C     STIFFNESS
  250 KH(NROW(II),COL)=VALU(II)
      GO TO 290
C     DAMPING
  260 CH(NROW(II),COL)=VALU(II)
      GO TO 290
C     STIFFNESS FOR FORCES AT POINT G
  270 KHG(NROW(II),COL)=VALU(II)
      GO TO 290
C     DAMPING FOR FORCES AT POINT G
  280 CHG(NROW(II),COL)=VALU(II)
  290 II=II+1
      IF(II.LT.6)GO TO 208
      GO TO 206
  300 CONTINUE
C INITIALIZATION BLOCK
C TIME
      T=0.
      PASS=0.
      WTMM=0.0
C PRINT AND PLOT INTERVALS
      IF(DTPLOT.GT.0.)DTPL=TEND/850.
      TPRINT=0.0
      TPLOT=0.0
      NPOINT=0
C   GIMBAL ANGLE AND GIMBAL RATE
      B=0.0
      BDP=0.0
C     EXTERNAL TORQUES
      DO 460 II=1,3
      TAUL(II)=0.0
      TAUA(II)=0.0
      TAUS(II)=0.0
      DO 460 JJ=1,3
      TAUC(JJ,II)=0.0
      DO 460 KK=1,3
  460 TAUD(KK,JJ,II)=0.0
C
C MAIN ITERATION LOOP
  150 CALL FLX1
      CALL FLX11
      CALL FLX12
      CALL FLX13
      CALL FLX14
      IF(CONTR.EQ.0.)GO TO 151
      CALL CMG
  151 CONTINUE
      CALL FLX15
C   CALCULATE WOBBLE MAGNITUDE
      WTM=SQRT(WXR(2)**2+WXR(3)**2)
C     MAX WOBBLE OVER PRINT CYCLE
      WTMM=AMAX1(WTM,WTMM)
      IF(T.LT.TPRINT)GO TO 152
      WRITE(IOUT,TIM)
  152 CALL PREINT
      IF(T.LT.TPRINT)GO TO 155
      TPRINT=TPRINT+DTPRNT
      WRITE(IOUT,STATE)
```

```
      WTMM=0.0
      IF(CONTR.EQ.0.)GO TO 156
      WRITE(IOUT,CMGPR)
  156 CONTINUE
      IF(T.GT.-.03)GO TO 153
      CALL WILLY('  LAM   ',LAM,84,84,1,84,1,84)
C     CHECK OUTPUT
      WRITE(IOUT,TEST1)
      WRITE(IOUT,TEST2)
      WRITE(IOUT,TEST3)
  153 IF(PASS.GT.0.)GO TO 154
      WRITE(IOUT,TESTC)
  154 IF(NS.EQ.0.)GO TO 155
      WRITE(IOUT,SHUT)
  155 CONTINUE
      IF(DTPLOT.LE.0.)GO TO 125
      IF(PASS.GT.0.)GO TO 122
      WRITE(1)NRUN,PBLOCK,TEND,DTPLOT
  122 CONTINUE
      IF(T.LT.TPLOT)GO TO 125
      TPLOT=TPLOT+DTPL
      IF(NPOINT.GE.850)GO TO 125
      NPOINT=NPOINT+1
      WRITE(1)T,WXR(2),WXR(3),WYH(2)
      WRITE(1)WYH(3),WYC(3,3),UCD(2,3)
      WRITE(1)PSID(1),UDD(1,3,2),UDD(2,3,2)
      WRITE(1)WZ(2,3,1),BDR,B
  125 CONTINUE
      IF(T+DT .LT. TEND)GO TO 120
      IF(DTPLOT .LE.0.)GO TO 121
      END FILE 1
      CALL PLO3AX
      GO TO 121
  120 CONTINUE
      CALL INTIG
      T=T+DT
      PASS=1.
      GO TO 150
      END
```

FIGURE 10.3 FLX1

```
20.1 (AUG 71)                                      OS/360  FORTRAN H

  COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                        SOURCE,BCD,NOLIST,DECK,LOAD,MAP,NOEDIT,ID,NOXREF
0002              SUBROUTINE FLX1
0003              IMPLICIT REAL (A-Z)
0004              COMMON/FLX/
                 *AALPHA(3,3,3),ALPHA(3),BETA(3),DEL1(3),DEL2(3,3),DEL3(3,3,3),
                 *DEL4(3,3),DEL5(3,3,3),DEL6(3),DEL7(3),DEL8(3),DELTB,
                 *CP(57,57),CH(18,18),CHG(6,18),DEL(3),DELD(3),
                 *FIH(3),FIP(3),EH,EP,ETA,FC(3,3),FD(3,3,3),FG(3),GAM(3),GAMD(3),GH,
                 *GJH(3),GJP(3),GPSID(3,3),GR,GYGB(3,3),GXL(3,3),DD(500),
                 *GPH(3,3),GWP(3,3),GWX(3,3),GWXRA(3,3,3),GWY(3,3),GWYC(3,3,3),
                 *GWYS(3,3),GYB(3,3),GYBM(3,3),GYH(3,3),HCMASS(3),HI,HJ,HMASS
0005              COMMON/FLX/
                 *IA(3,3),IB(3,3),IC(3,3,3),ID(3,3,3,3),IH(3,3),JT(3,3),KBS1,KBS2,
                 *KBVH(3,3),KBVP(3,3),KHS(3,3),KHV(3,3),LAM(84,84),LM1(3,3),
                 *LM2(3,3),LM3(3,3,3),LM4(3,3),LM5(3,3,3,3),LM6(3,3),LM7(3,3,3),
                 *LM8(3,3,3),LM9(3,3,3),LM10(3,3,3,3),LM11(3,3,3,3),LM12(3,3,3,3),
                 *LM13(3,3,3,3),LM14(3,3,3),LM15(3,3,3),LM16(3,3,3,3),LM17(3,3,3,3),
                 *LM18(3,3),LM19(3,3),LM20(3,3),LM21(3,3),LM22(3,3),LM23(3,3),
                 *LM24(3,3),LM25(3,3),LM26(3,3),LM4B(3,3),LM13B(3,3,3,3),KHG(6,18),
                 *LM16B(3,3,3,3),KP(57,57),KH(18,18)
0006              COMMON/FLX/
                 *M,M1(3,3),M2(3,3,3),M3(3,3,3,3),M4(3,3),M5(3,3),MAST,MASS(84,84),
                 *MB(3),MBEAR(3),MC(3,3),MD(3,3,3),MDOCK(3),MG(3),NROT,NHUB,
                 *N,NS,P,PASS,PIBETA(3,3),PIGAM(3,3),PIPHIC(3,3,3),PIPHID(3,3,3,3),
                 *PIPSI(3,3),PITHET(3,3),PHIC(3,3),PHICD(3,3),PHID(3,3,3),
                 *PHIDD(3,3,3),PSI(3),PSID(3),PHIMR(57,57),PHIMH(18,18),NTHET(3,3)
0007              COMMON/FLX/
                 *PDMASS(3,3),PH(3),RI,RJ,RMASS,RRMASS,RTS(84),SI1(3,3),SI2(3),
                 *SI3(3,3,3),SI4(3),SIG1(3),SIG2(3,3),SIG3(3,3,3),SIG4(3,3),
                 *SIG5(3,3,3),SIG6(3),SIG7(3),SIG8(3),SGAM(3,3),SMASS,STHET(3,3),
                 *SUM1(3,3,3),TAUA(3),TAUC(3,3),TAUD(3,3,3),TAUL(3),TAUS(3),
                 *THET(3),THETD(3),RHC(3),STHE(3,3),SPIN
0008              COMMON/FLX/
                 *UC(3,3),UCD(3,3),UD(3,3,3),UDD(3,3,3),WOK1(3,3),WOK2(84,84),
                 *WOK3(84),WOK4(3,3),WOK5(3,3),WOK5(3,3),WR(3),WXR(3),WXRA(3,3),
                 *WXY(3),WYC(3,3),WYH(3),WZ(3,3,3),XC(84),XCD(84),XD(3,3,3),XN(3,3),
                 *XL(3),XLL(84),XVEC(84),XVECD(84),XVECDD(84),YC(3,3),YB(3),YE3(3),
                 *YG(3),YH(3),ZA(3),ZAD(3),ZV3(3,3)
0009              INTEGER II,JJ,KK,LL,NN
0010              DIMENSION DUMMY(3,3)
            C SET UP TRANSFORMATION MATRICES
            C     TRANSFORMATION A OF ALPHA-STEP1
0011              IF(PASS.GT.0.)GO TO 10
0013              DO 2 II=1,3
0014              DO 1 JJ=1,3
0015            1 AALPHA(JJ,II,1)=0.0
0016            2 AALPHA(II,II,1)=1.0
0017              KK=P
0018              DO 5 II=2,KK,1
0019            5 CALL ARCPI(AALPHA(1,1,II),DUMMY,ALPHA(II),0)
            C ADJUST AALPHA FOR NUMERICAL INACCURACIES
0020              IF(P.EQ.3.)GO TO 6
0022              AALPHA(2,2,2)=-1.0
0023              AALPHA(3,2,2)=0.0
0024              AALPHA(2,3,2)=0.0
0025              AALPHA(3,3,2)=-1.0
0026              GO TO 7
```

FIGURE 10.3 (Cont'd)

```
    6 AALPHA(2,2,2)=-.50
      AALPHA(3,2,2)=-.8660254
      AALPHA(2,3,2)=+.8660254
      AALPHA(3,3,2)=-.50
      AALPHA(2,2,3)=-.50
      AALPHA(3,2,3)=+.8660254
      AALPHA(2,3,3)=-.8660254
      AALPHA(3,3,3)=-.50
    7 CONTINUE
C     TRANSFORMATION PI OF BETA - STEPS 2 AND 3
      CALL ABCPI(DUMMY,PIBETA,BETA,1)
   10 CONTINUE
C     TRANSFORMATIONS PI OF GAMMA,THETA,PSI,PHI OF C AND D.-STEPS 5,6
      CALL ABCPI(DUMMY,PIGAM,GAM,1)
      CALL ABCPI(DUMMY,PITHET,THET,1)
      CALL ABCPI(DUMMY,PIPSI,PSI,1)
      DO 20 II=1,3
      CALL ABCPI(DUMMY,PIPHIC(1,1,II),PHIC(1,II),1)
      DO 20 JJ=1,3
   20 CALL ABCPI(DUMMY,PIPHID(1,1,JJ,II),PHID(1,JJ,II),1)
C     TRANSFORMATION J -FROM HUB TO SHUTTLE - STEP 7
      CALL GMPRD(PIPSI,PIBETA,JT,3,3,3)
C
C  SET UP FOR EULER RATE CALCULATIONS - STEP 8
      S2=SIN(GAM(2))
      S3=SIN(GAM(3))
      C2=COS(GAM(2))
      C3=COS(GAM(3))
      SGAM(1,1)=C3/C2
      SGAM(2,1)=S3
      SGAM(3,1)=-C3*S2/C2
      SGAM(1,2)=-S3/C2
      SGAM(2,2)=C3
      SGAM(3,2)=S3*S2/C2
      SGAM(1,3)=0.
      SGAM(2,3)=0.
      SGAM(3,3)=1.
      S2=SIN(THET(2))
      S3=SIN(THET(3))
      C2=COS(THET(2))
      C3=COS(THET(3))
      STHET(1,1)=C3/C2
      STHET(2,1)=S3
      STHET(3,1)=-C3*S2/C2
      STHET(1,2)=-S3/C2
      STHET(2,2)=C3
      STHET(3,2)=S3*S2/C2
      STHET(1,3)=0.
      STHET(2,3)=0.
      STHET(3,3)=1.
C COMPUTE STHET INVERSE
      STHE(1,1)=C3*C2
      STHE(2,1)=-S3*C2
      STHE(3,1)=S2
      STHE(1,2)=S3
      STHE(2,2)=C3
      STHE(3,2)=0.
      STHE(1,3)=0.
```

```
      STHE(2,3)=0.
      STHE(3,3)=1.
C   CALCULATE NTHET
      NTHET(1,1)=1.-STHE(1,1)
      NTHET(2,1)=-STHE(2,1)
      NTHET(3,1)=-STHE(3,1)
      NTHET(1,2)=0.0
      NTHET(2,2)=1.0
      NTHET(3,2)=0.0
      NTHET(1,3)=0.0
      NTHET(2,3)=0.0
      NTHET(3,3)=1.0
C
C COMPUTE INERTIAL ANGULAR RATES-STEP 9 ,WXR,WXY-ROTOR RING
      CALL GMPRD(PITHET,WYH,WOK1,3,3,1)
      IF(PASS.EQ.0.)GO TO 30
      CALL GMPRD(STHE,THETD,WXY,3,3,1)
   30 CALL GMADD(WXY,WOK1,WXR,3,1)
C
      KK=P
C    WR - SHUTTLE RATES
   35 IF(NS.EQ.0.)GO TO 40
      CALL GMPRD(JT,WYH,WOK1,3,3,1)
      CALL GMADD(WOK1,PSID,WR,3,1)
   40 DO 50 II=1,3
C   WYC - RATES OF HUB LUMPED MASSES
      CALL GMPRD(PIPHIC(1,1,II),WYH,WOK1,3,3,1)
      CALL GMADD(WOK1,PHICD(1,II),WYC(1,II),3,1)
C   WXRA - RATES OF ROTOR ARMS
      IF(II.GT.KK)GO TO 50
      CALL GMPRD(AALPHA(1,1,II),WXR,WXRA(1,II),3,3,1)
   50 CONTINUE
C   WZ - RATES OF ROTOR LUMPED MASSES
      DO 55 JJ=1,KK
      DO 55 II=1,3
      CALL GMPRD(PIPHID(1,1,II,JJ),WXRA(1,JJ),WOK1,3,3,1)
      CALL GMADD(WOK1,PHIDD(1,II,JJ),WZ(1,II,JJ),3,1)
   55 CONTINUE
C
C   EULER RATES FIRST PASS ONLY
      CALL GMPRD(PITHET,WYH,WOK1,3,3,1)
      IF(PASS.GT.0.)GO TO 70
C     GAMMA-DOT
      CALL GMPRD(SGAM,WYH,GAMD,3,3,1)
C     THETA-DOT
      CALL GMPRD(STHET,WXY,THETD,3,3,1)
C   CALCULATE INITIAL VALUES FOR DEL AND DELD
      DEL(1)=0
      DEL(2)=THET(2)
      DEL(3)=THET(3)
      DELD(1)=THETD(1)-SPIN+WOK1(1,1)
      DELD(2)=THETD(2)
      DELD(3)=THETD(3)
   70 CONTINUE
C   CALCULATE RHO
      CALL GAMMA(WOK4,WXY)
      CALL GMPRD(WOK4,WOK1,WOK5,3,3,1)
      CALL GMPRD(NTHET,WOK5,WOK1,3,3,1)
```

```
0127          RHO(1)=THETD(3)*THETD(2)-WCK1(1,1)
0128          RHO(2)=-THETD(3)*THETD(1)-WCK1(2,1)
0129          RHO(3)=THETD(2)*THETD(1)-WCK1(3,1)
      C
0130          RETURN
0131          END
```

FIGURE 10.4 FLX11

```
20.1 (AUG 71)                                     OS/360  FORTRAN H

     COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                        SOURCE,BCD,NOLIST,DECK,LOAD,MAP,NOEDIT,ID,NOXREF
0002                SUBROUTINE FLX11
0003                IMPLICIT REAL (A-Z)
0004                COMMON/FLX/
                   *AALPHA(3,3,3),ALPHA(3),BETA(3),DEL1(3),DEL2(3,3),DEL3(3,3,3),
                   *DEL4(3,3),DEL5(3,3,3),DEL6(3),DEL7(3),DEL8(3),DELTB,
                   *CP(57,57),CH(18,18),CHG(6,18),DEL(3),DELD(3),
                   *FIH(3),FIP(3),EH,ER,ETA,FC(3,3),FD(3,3,3),FG(3),GAM(3),GAMD(3),GH,
                   *GJH(3),GJR(3),GPSID(3,3),GR,GYGB(3,3),GXL(3,3),DD(500),
                   *GPH(3,3),GWR(3,3),GWX(3,3),GWXRA(3,3,3),GWY(3,3),GWYC(3,3,3),
                   *GWYS(3,3),GYB(3,3),GYBM(3,3),GYH(3,3),HCMASS(3),HI,HJ,HMASS
0005                COMMON/FLX/
                   *IA(3,3),IB(3,3),IC(3,3,3),ID(3,3,3,3),IH(3,3),JT(3,3),KBS1,KBS2,
                   *KBVH(3,3),KBVP(3,3),KHS(3,3),KHV(3,3),LAM(84,84),LM1(3,3),
                   *LM2(3,3),LM3(3,3,3),LM4(3,3),LM5(3,3,3,3),LM6(3,3),LM7(3,3,3),
                   *LM8(3,3,3),LM9(3,3,3),LM10(3,3,3,3),LM11(3,3,3,3),LM12(3,3,3,3),
                   *LM13(3,3,3,3),LM14(3,3,3),LM15(3,3,3),LM16(3,3,3,3),LM17(3,3,3,3),
                   *LM18(3,3),LM19(3,3),LM20(3,3),LM21(3,3),LM22(3,3),LM23(3,3),
                   *LM24(3,3),LM25(3,3),LM26(3,3),LM4B(3,3),LM13B(3,3,3,3),KHG(6,18),
                   *LM16B(3,3,3,3),KP(57,57),KH(18,18)
0006                COMMON/FLX/
                   *M,M1(3,3),M2(3,3,3),M3(3,3,3,3),M4(3,3),M5(3,3),MAST,MASS(84,84),
                   *MB(3),MBEAR(3),MC(3,3),MD(3,3,3),MDOCK(3),MG(3),NROT,NHUB,
                   *N,NS,P,PASS,PIBETA(3,3),PIGAM(3,3),PIPHIC(3,3,3),PIPHID(3,3,3,3),
                   *PIPSI(3,3),PITHET(3,3),PHIC(3,3),PHICD(3,3),PHID(3,3,3),
                   *PHIDD(3,3,3),PSI(3),PSID(3),PHIMR(57,57),PHIMH(18,18),NTHET(3,3)
0007                COMMON/FLX/
                   *RDMASS(3,3),RH(3),RI,RJ,RMASS,RRMASS,RTS(84),SI1(3,3),SI2(3),
                   *SI3(3,3,3),SI4(3),SIG1(3),SIG2(3,3),SIG3(3,3,3),SIG4(3,3),
                   *SIG5(3,3,3),SIG6(3),SIG7(3),SIG8(3),SGAM(3,3),SMASS,STHET(3,3),
                   *SUM1(3,3,3),TAUA(3),TAUC(3,3),TAUD(3,3,3),TAJL(3),TAUS(3),
                   *THET(3),THETD(3),PHC(3),STHE(3,3),SPIN
0008                COMMON/FLX/
                   *UC(3,3),UCD(3,3),UD(3,3,3),UDD(3,3,3),WOK1(3,3),WOK2(84,84),
                   *WOK3(84),WOK4(3,3),WOK5(3,3),WOK5(3,3),WR(3),WXR(3),WXRA(3,3),
                   *WXY(3),WYC(3,3),WYH(3),WZ(3,3,3),XC(84),XCD(84),XD(3,3,3),XN(3,3),
                   *XL(3),XLL(84),XVEC(84),XVECD(84),XVECDD(84),YC(3,3),YB(3),YE3(3),
                   *YG(3),YH(3),ZA(3),ZAD(3),ZV3(3,3)
0009                INTEGER II,JJ,KK,LL,NN
0010                DIMENSION W7(3,3)
             C  COMPUTE THE M MATRICES THAT WILL BE USED AS PART OF THE MASS MATRIX
             C      FORMULATION - STEP 11       M1
0011                IF(PASS.GT.0.)GO TO 10
0013                CALL GAMMA(GXL,XL)
0014             10 CALL GMTRA(PITHET,WOK1,3,3)
0015                CALL GMPRD(WOK1,GXL,M1,3,3,3)
0016                CALL SMPY(M1,-1.,M1,3,3)
             C  M4,M5
0017                IF(PASS.GT.0.)GO TO 20
0019                CALL GAMMA(GPH,PH)
0020                CALL GAMMA(GYH,YH)
0021             20 CALL GMTRA(JT,WOK1,3,3)
0022                CALL GMPRD(WOK1,GPH,M5,3,3,3)
0023                CALL GMPRD(M5,JT,WOK1,3,3,3)
0024                CALL GMSUB(WOK1,GYH,M4,3,3)
             C  M2,M3
0025                KK=P
```

FIGURE 10.4 (Cont'd)

```
      DO 30 II=1,KK
      CALL GMPRD(AALPHA(1,1,II),PITHET,WOK1,3,3,3)
      CALL GMTRA(WOK1,M2(1,1,II),3,3)
      DO 30 JJ=1,3
      CALL GMADD(XD(1,JJ,II),UD(1,JJ,II),SUM1(1,JJ,II),3,1)
      CALL GAMMA(WOK1,SUM1(1,JJ,II))
      CALL GMPRD(WOK1,AALPHA(1,1,II),WOK4,3,3,3)
      CALL GMPRD(M2(1,1,II),WOK4,WOK3,3,3,3)
      CALL SMPY(WOK3,-1.,M3(1,1,JJ,II),3,3)
   30 CONTINUE
C
C
C  CALCULATE THE LAMDA MATRICES
C     COMBINE SOME MASSES
      IF(PASS.GT.0.)GO TO 60
      RMASS=0.
      DO 40 II=1,KK
      DO 40 JJ=1,3
   40 RMASS=RDMASS(JJ,II)+RMASS
      RMASS=RMASS+RRMASS
      MAST=0.
      DO 45 II=1,3
   45 MAST=MAST+HCMASS(II)
      MAST=MAST+RMASS+HMASS+SMASS
C
C SOME CONSTANT LAMDA MATRICES FOLLOW -LM3,LM9,LM15,LM17,LM20,LM12
      DO 55 II=1,3
      DO 55 JJ=1,3
      DO 50 KK=1,3
      LM3(KK,JJ,II)=0.0
      LM9(KK,JJ,II)=0.0
      LM15(KK,JJ,II)=IC(KK,JJ,II)
      DO 50 NN=1,3
      LM12(NN,KK,JJ,II)=0.0
   50 LM17(KK,JJ,II,NN)=ID(KK,JJ,II,NN)
      DO 51 KK=1,3
   51 LM12(KK,KK,JJ,II)=RDMASS(JJ,II)
      LM20(JJ,II)=IH(JJ,II)
      LM3(JJ,JJ,II)=HCMASS(II)
   55 LM9(JJ,JJ,II)=HCMASS(II)
   60 CONTINUE
      RETURN
      END
```

FIGURE 10.5 FLX12

```
20.1 (AUG 71)                              OS/360   FORTRAN H

   COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                      SOURCE,BCD,NOLIST,DECK,LOAD,MAP,NOEDIT,ID,NOXREF
0002             SUBROUTINE FLX12
0003             IMPLICIT REAL (A-Z)
0004             COMMON/FLX/
                *AALPHA(3,3,3),ALPHA(3),BETA(3),DEL1(3),DEL2(3,3),DEL3(3,3,3),
                *DEL4(3,3),DEL5(3,3,3),DEL6(3),DEL7(3),DEL8(3),DELTB,
                *CR(57,57),CH(18,18),CHG(6,18),DEL(3),DELD(3),
                *EIH(3),EIR(3),EH,ER,ETA,FC(3,3),FD(3,3,3),FG(3),GAM(3),GAMD(3),GH
                *GJH(3),GJR(3),GPSID(3,3),GR,GYGB(3,3),GXL(3,3),DD(500),
                *GRH(3,3),GWR(3,3),GWX(3,3),GWXRA(3,3,3),GWY(3,3),GWYC(3,3,3),
                *GWYS(3,3),GYB(3,3),GYBN(3,3),GYH(3,3),HCMASS(3),HI,HJ,HMASS
0005             COMMON/FLX/
                *IA(3,3),IB(3,3),IC(3,3,3),ID(3,3,3,3),IH(3,3),JT(3,3),KBS1,KBS2,
                *KBVH(3,3),KBVR(3,3),KHS(3,3),KHV(3,3),LAM(84,84),LM1(3,3),
                *LM2(3,3),LM3(3,3,3),LM4(3,3),LM5(3,3,3,3),LM6(3,3),LM7(3,3,3),
                *LM8(3,3,3),LM9(3,3,3),LM10(3,3,3,3),LM11(3,3,3,3),LM12(3,3,3,3),
                *LM13(3,3,3,3),LM14(3,3,3),LM15(3,3,3),LM16(3,3,3,3),LM17(3,3,3,3)
                *LM18(3,3),LM19(3,3),LM20(3,3),LM21(3,3),LM22(3,3),LM23(3,3),
                *LM24(3,3),LM25(3,3),LM26(3,3),LM4B(3,3),LM13B(3,3,3,3),KHG(6,18),
                *LM16B(3,3,3,3),KR(57,57),KH(18,18)
0006             COMMON/FLX/
                *M,M1(3,3),M2(3,3,3),M3(3,3,3,3),M4(3,3),M5(3,3),MAST,MASS(84,84),
                *MB(3),MBEAR(3),MC(3,3),MD(3,3,3),MDOCK(3),MG(3),NROT,NHUB,
                *N,NS,P,PASS,PIBETA(3,3),PIGAM(3,3),PIPHIC(3,3,3),PIPHID(3,3,3,3),
                *PIPSI(3,3),PITHET(3,3),PHIC(3,3),PHICD(3,3),PHID(3,3,3),
                *PHIDD(3,3,3),PSI(3),PSID(3),PHIMR(57,57),PHIMH(18,18),NTHET(3,3)
0007             COMMON/FLX/
                *RDMASS(3,3),RH(3),RI,RJ,RMASS,RRMASS,RTS(84),SI1(3,3),SI2(3),
                *SI3(3,3,3),SI4(3),SIG1(3),SIG2(3,3),SIG3(3,3,3),SIG4(3,3),
                *SIG5(3,3,3),SIG6(3),SIG7(3),SIG8(3),SGAM(3,3),SMASS,STHET(3,3),
                *SUM1(3,3,3),TAUA(3),TAUC(3,3),TAUD(3,3,3),TAUL(3),TAUS(3),
                *THET(3),THETD(3),RHC(3),STHE(3,3),SPIN
0008             COMMON/FLX/
                *UC(3,3),UCD(3,3),UD(3,3,3),UDD(3,3,3),WOK1(3,3),WOK2(84,84),
                *WOK3(84),WOK4(3,3),WOK5(3,3),WOK6(3,3),WR(3),WXR(3),WXRA(3,3),
                *WXY(3),WYC(3,3),WYH(3),WZ(3,3,3),XC(84),XCD(84),XD(3,3,3),XN(3,3),
                *XL(3),XLL(84),XVEC(84),XVECD(84),XVECDD(84),YC(3,3),YB(3),YE3(3),
                *YG(3),YH(3),ZA(3),ZAD(3),ZV3(3,3)
0009             INTEGER II,JJ,KK,LL,NN
0010             DIMENSION W7(3,3)
        C
0011             KK=P
        C  LM1
0012             CALL  SMPY(PIGAM,MAST,LM1,3,3)
        C  LM8,PARTIAL LM2
0013             CALL  SCLA(LM2,0.,3,3)
0014             DO 70 II=1,3
0015             CALL  GMADD(YC(1,II),UC(1,II),WOK1,3,1)
0016             CALL  GAMMA(WOK4,WOK1)
0017             CALL  SMPY(WOK4,-HCMASS(II),LM8(1,1,II),3,3)
0018          70 CALL  GMADD(LM8(1,1,II),LM2,LM2,3,3)
0019             IF(NS.EQ.0.)GO TO 80
0021             CALL  SMPY(M4,SMASS,WOK1,3,3)
0022             CALL  GMADD(WOK1,LM2,LM2,3,3)
0023          80 IF(PASS.GT.0.)GO TO 90
0025             CALL  GAMMA(GYB,YB)
0026             CALL  SMPY(GYB,RMASS,GYBN,3,3)
```

FIGURE 10.5 (Cont'd)

```
   90 CALL GMSUB(LM2,GYBM,LM2,3,3)
C  LM4,LM5,COMPLETE LM2
      CALL SMPY(M1,PRMASS,LM4,3,3)
      DO 100 II=1,KK
      DO 100 JJ=1,3
      CALL SMPY(M3(1,1,JJ,II),RDMASS(JJ,II),WOK1,3,3)
      CALL GMADD(LM4,WOK1,LM4,3,3)
  100 CALL SMPY(M2(1,1,II),RDMASS(JJ,II),LM5(1,1,JJ,II),3,3)
      CALL GMPRD(LM4,NTHET,WOK4,3,3,3)
      CALL GMPRD(WOK4,PITHET,WOK1,3,3,3)
      CALL GMADD(WOK1,LM2,LM2,3,3)
      CALL SMPY(LM4,1.,LM4B,3,3)
      CALL GMPRD(LM4B,STHE,LM4,3,3,3)
C  LM6
      CALL SMPY(M5,SMASS,LM6,3,3)
C  LM7
      DO 110 II=1,3
  110 CALL SMPY(PIGAM,HCMASS(II),LM7(1,1,II),3,3)
C  LM10,LM11,LM12,LM13
      CALL GMPRD(PITHET,PIGAM,WOK1,3,3,3)
      CALL GMPRD(PITHET,GYB,WOK4,3,3,3)
      DO 120 II=1,KK
      CALL GMPRD(AALPHA(1,1,II),WOK1,WOK5,3,3,3)
      CALL GMPRD(AALPHA(1,1,II),WOK4,WOK6,3,3,3)
      CALL GMPRD(AALPHA(1,1,II),PITHET,W7,3,3,3)
      DO 120 JJ=1,3
      CALL SMPY(WOK5,RDMASS(JJ,II),LM10(1,1,JJ,II),3,3)
      CALL SMPY(W7,RDMASS(JJ,II),WOK3,3,3)
      CALL GMPRD(WOK3,M3(1,1,JJ,II),LM13B(1,1,JJ,II),3,3,3)
      CALL SMPY(WOK6,-RDMASS(JJ,II),LM11(1,1,JJ,II),3,3)
      CALL GMPRD(LM13B(1,1,JJ,II),NTHET,WOK3,3,3,3)
      CALL GMPRD(WOK3,PITHET,WOK2,3,3,3)
      CALL GMADD(WOK2,LM11(1,1,JJ,II),LM11(1,1,JJ,II),3,3)
  120 CALL GMPRD(LM13B(1,1,JJ,II),STHE,LM13(1,1,JJ,II),3,3,3)
C  LM14
      DO 130 II=1,3
  130 CALL GMPRD(IC(1,1,II),PIPHIC(1,1,II),LM14(1,1,II),3,3,3)
C  LM15
      DO 140 II=1,KK
      DO 140 JJ=1,3
      CALL GMPRD(PIPHID(1,1,JJ,II),AALPHA(1,1,II),WOK1,3,3,3)
      CALL GMPRD(ID(1,1,JJ,II),WOK1,LM16B(1,1,JJ,II),3,3,3)
  140 CALL GMPRD(LM16B(1,1,JJ,II),STHE,LM15(1,1,JJ,II),3,3,3)
C  LM18,LM19
      IF(NS.EQ.0.)GO TO 150
      CALL GMPRD(GPH,JT,WOK1,3,3,3)
      CALL GMPRD(WOK1,PIGAM,WOK4,3,3,3)
      CALL SMPY(WOK4,-SMASS,LM18,3,3)
      CALL GMPRD(WOK1,GYH,WOK4,3,3,3)
      CALL SMPY(WOK4,SMASS,LM19,3,3)
      CALL GMPRD(IH,JT,WOK1,3,3,3)
      CALL GMADD(LM19,WOK1,LM19,3,3)
C  LM21,LM22,LM23
  150 CALL GMPRD(GXL,PITHET,WOK1,3,3,3)
      CALL GMPRD(WOK1,PIGAM,WOK4,3,3,3)
      CALL SMPY(WOK4,PRMASS,LM21,3,3)
      CALL GMPRD(WOK1,GYB,WOK4,3,3,3)
      CALL SMPY(WOK4,-PRMASS,LM22,3,3)
```

```
      CALL GMPRD(IB,NTHET,WOK4,3,3,3)
      CALL GMPRD(WOK4,FITHET,WOK1,3,3,3)
      CALL GMADD(WOK1,LM22,LM22,3,3)
      CALL GMPRD(IB,STHE,LM23,3,3,3)
C   LM24,LM25,LM26
      CALL SMPY(GYB,-HMASS,LM24,3,3)
      CALL SMPY(LM24,1.,WOK1,3,3)
      CALL SMPY(IA,1.,LM25,3,3)
      CALL SCLA(LM26,0.,3,3)
      IF(NS.EQ.0.)GO TO 160
      CALL GMSUB(YH,YB,WOK1,3,1)
      CALL GAMMA(WOK4,WOK1)
      CALL SMPY(WOK4,SMASS,WOK1,3,3)
      CALL GMADD(WOK1,LM24,LM24,3,3)
      CALL GMPRD(WOK1,N5,LM26,3,3,3)
      CALL GMPRD(WOK1,N4,WOK4,3,3,3)
      CALL GMADD(LM25,WOK4,LM25,3,3)
      CALL GMTRA(JT,WOK1,3,3)
      CALL GMPRD(WOK1,IH,WOK4,3,3,3)
      CALL GMADD(WOK4,LM26,LM26,3,3)
      CALL GMPRD(WOK4,JT,WOK5,3,3,3)
      CALL GMADD(LM25,WOK5,LM25,3,3)
      CALL GMPRD(WOK1,GRH,WOK4,3,3,3)
      CALL GMPRD(WOK4,JT,WOK1,3,3,3)
      CALL SMPY(WOK1,SMASS,WOK4,3,3)
      CALL GMSUB(LM24,WOK4,WOK1,3,3)
      CALL GMPRD(WOK4,GYH,WOK5,3,3,3)
      CALL GMADD(LM25,WOK5,LM25,3,3)
  160 CONTINUE
      CALL GMPRD(WOK1,FIGAM,LM24,3,3,3)
      RETURN
      END
```

FIGURE 10.6 FLX13

```
20.1 (AUG 71)                                    OS/360  FORTRAN H

     COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                          SOURCE,BCD,NOLIST,DECK,LOAD,MAP,NOEDIT,ID,NOXREF
 0002                     SUBROUTINE FLX13
 0003                     IMPLICIT REAL (A-Z)
 0004                     COMMON/FLX/
                         *AALPHA(3,3,3),ALPHA(3),BETA(3),DEL1 (3),DEL2(3,3),DEL3(3,3,3),
                         *DEL4(3,3),DEL5(3,3,3),DEL6(3),DEL7(3),DEL8(3),DELTB,
                         *CP(57,57),CH(18,18),CHG(6,18),DEL(3),DELD(3),
                         *EIH(3),EIR(3),EH,ER,ETA,FC(3,3),FD(3,3,3),FG(3),GAM(3),GAMD(3),G
                         *GJH(3),GJR(3),GPSID(3,3),GR,GYGB(3,3),GXL(3,3),DD(500),
                         *GRH(3,3),GWP(3,3),GWX(3,3),GWXRA(3,3,3),GWY(3,3),GWYC(3,3,3),
                         *GWYS(3,3),GYB(3,3),GYBN(3,3),GYH(3,3),HCMASS(3),HI,HJ,HMASS
 0005                     COMMON/FLX/
                         *IA(3,3),IB(3,3),IC(3,3,3),ID(3,3,3,3),IH(3,3),JT(3,3),KBS1,KBS2,
                         *KBVH(3,3),KBVR(3,3),KHS(3,3),KHV(3,3),LAM(84,84),LM1(3,3),
                         *LM2(3,3),LM3(3,3,3),LM4(3,3),LM5(3,3,3,3),LM6(3,3),LM7(3,3,3),
                         *LM8(3,3,3),LM9(3,3,3),LM10(3,3,3,3),LM11(3,3,3,3),LM12(3,3,3,3),
                         *LM13(3,3,3,3),LM14(3,3,3),LM15(3,3,3),LM16(3,3,3,3),LM17(3,3,3,3
                         *LM18(3,3),LM19(3,3),LM20(3,3),LM21(3,3),LM22(3,3),LM23(3,3),
                         *LM24(3,3),LM25(3,3),LM26(3,3),LM4B(3,3),LM13B(3,3,3,3),KHG(6,18),
                         *LM16B(3,3,3,3),KR(57,57),KH(18,18)
 0006                     COMMON/FLX/
                         *M,M1(3,3),M2(3,3,3),M3(3,3,3,3),M4(3,3),M5(3,3),MAST,MASS(84,84),
                         *MB(3),MBEAR(3),MC(3,3),MD(3,3,3),MDOCK(3),MG(3),NROT,NHUB,
                         *N,NS,P,PASS,PIBETA(3,3),PIGAM(3,3),PIPHIC(3,3,3),PIPHID(3,3,3,3),
                         *PIPSI(3,3),PITHET(3,3),PHIC(3,3),PHICD(3,3),PHID(3,3,3),
                         *PHIDD(3,3,3),PSI(3),PSID(3),PHIMR(57,57),PHIMH(18,18),NTHET(3,3)
 0007                     COMMON/FLX/
                         *RDMASS(3,3),RH(3),RI,RJ,RMASS,RRMASS,RTS(84),SI1(3,3),SI2(3),
                         *SI3(3,3,3),SI4(3),SIG1(3),SIG2(3,3),SIG3(3,3,3),SIG4(3,3),
                         *SIG5(3,3,3),SIG6(3),SIG7(3),SIG8(3),SGAM(3,3),SMASS,STHET(3,3),
                         *SUM1(3,3,3),TAUA(3),TAUC(3,3),TAUD(3,3,3),TAJL(3),TAUS(3),
                         *THET(3),THETD(3),RHO(3),STHE(3,3),SPIN
 0008                     COMMON/FLX/
                         *UC(3,3),UCD(3,3),UD(3,3,3),UDD(3,3,3),WOK1(3,3),WOK2(84,84),
                         *WOK3(84),WOK4(3,3),WOK5(3,3),WOK5(3,3),WR(3),WXR(3),WXRA(3,3),
                         *WXY(3),WYC(3,3),WYH(3),WZ(3,3,3),XC(84),XCD(84),XD(3,3,3),XN(3,3
                         *XL(3),XLL(84),XVEC(84),XVECD(84),XVECDD(84),YC(3,3),YB(3),YE3(3),
                         *YG(3),YH(3),ZA(3),ZAD(3),ZV3(3,3)
 0009                     INTEGER II,JJ,KK,LL,NN
                C   SET UP THE RIGHT SIDE OF THE EOM
                C     COMPUTE LOWER CASE PSI -STEP13
                C   SI1
 0010                     CALL GAMMA(GWY,WYH)
 0011                     CALL GMPRD(GWY,GWY,GWYS,3,3,3)
 0012                     DO 10 II=1,3
 0013                     CALL GMADD(YC(1,II),UC(1,II),WOK1,3,1)
 0014                     CALL GMPRD(GWYS,WOK1,WOK4,3,3,1)
 0015                     CALL GMPRD(GWY,UCD(1,II),WOK1,3,3,1)
 0016                     CALL SMPY(WOK1,2.,SI1(1,II),3,1)
 0017                  10 CALL GMADD(SI1(1,II),WOK4,SI1(1,II),3,1)
                C   SI2
 0018                     CALL GAMMA(GWX,WXR)
 0019                     CALL GMPRD(GWX,GWX,WOK1,3,3,3)
 0020                     CALL GMPRD(WOK1,XL,WOK4,3,3,1)
 0021                     CALL GMTRA(PITHET,WOK1,3,3)
 0022                     CALL GMPRD(WOK1,WOK4,SI2,3,3,1)
 0023                     CALL GMPRD(GWYS,YB,WOK1,3,3,1)
```

FIGURE 10.6 (Cont'd)

```
      CALL GMADD(SI2,WOK1,SI2,3,1)
C   SI3
      KK=P
      DO 20 II=1,KK
      CALL GAMMA(GWXRA(1,1,II),WXRA(1,II))
      CALL GMPRD(GWXRA(1,1,II),GWXRA(1,1,II),WOK4,3,3,3)
      DO 20 JJ=1,3
      CALL GMADD(XD(1,JJ,II),UD(1,JJ,II),WOK5,3,1)
      CALL GMPRD(WOK4,WOK5,WOK3,3,3,1)
      CALL GMPRD(GWXRA(1,1,II),UDD(1,JJ,II),WOK6,3,3,1)
      CALL SMPY(WOK6,2.,WOK5,3,1)
      CALL GMADD(WOK3,WOK5,WOK6,3,1)
      CALL GMPRD(W2(1,1,II),WOK6,WOK5,3,3,1)
   20 CALL GMADD(WOK1,WOK5,SI3(1,JJ,II),3,1)
C   SI4
      CALL SCLA(SI4,0.,3,1)
      IF(NS.EQ.0.)GO TO 30
      CALL GMPRD(GWYS,YH,SI4,3,3,1)
      CALL GAMMA(GPSID,PSID)
      CALL GMPRD(GPSID,FIBETA,WOK1,3,3,3)
      CALL GMPRD(WOK1,WYH,WOK4,3,3,1)
      CALL GMPRD(GRH,WOK4,WOK1,3,3,1)
      CALL GAMMA(GWR,WR)
      CALL GMPRD(GWR,GWR,WOK4,3,3,3)
      CALL GMPRD(WOK4,RH,WOK5,3,3,1)
      CALL GMADD(WOK5,WOK1,WOK4,3,1)
      CALL GMTRA(JT,WOK1,3,3)
      CALL GMPRD(WOK1,WOK4,WOK5,3,3,1)
      CALL GMSUB(SI4,WOK5,SI4,3,1)
   30 CONTINUE
      RETURN
      END
```

FIGURE 10.7 FLX14

```
20.1 (AUG 71)                                    OS/360   FORTRAN H

   COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                      SOURCE,BCD,NOLIST,DECK,LOAD,MAP,NOEDIT,ID,NOXREF
 0002                SUBROUTINE FLX14
 0003                IMPLICIT REAL (A-Z)
 0004                COMMON/FLX/
                    *AALPHA(3,3,3),ALPHA(3),BETA(3),DEL1(3),DEL2(3,3),DEL3(3,3,3),
                    *DEL4(3,3),DEL5(3,3,3),DEL6(3),DEL7(3),DEL8(3),DELTB,
                    *CR(57,57),CH(18,18),CHG(6,18),DEL(3),DELD(3),
                    *FIH(3),FIR(3),FH,FP,ETA,FC(3,3),FD(3,3,3),FG(3),GAM(3),GAMD(3),G
                    *GJH(3),GJR(3),GPSID(3,3),GR,GYGB(3,3),GXL(3,3),DD(500),
                    *GPH(3,3),GWR(3,3),GWX(3,3),GWXFA(3,3,3),GWY(3,3),GWYC(3,3,3),
                    *GWYS(3,3),GYB(3,3),GYBM(3,3),GYH(3,3),HCMASS(3),HI,HJ,HMASS
 0005                COMMON/FLX/
                    *IA(3,3),IB(3,3),IC(3,3,3),ID(3,3,3,3),IH(3,3),JT(3,3),KBS1,KBS2,
                    *KBVH(3,3),KBVR(3,3),KHS(3,3),KHV(3,3),LAM(84,84),LM1(3,3),
                    *LM2(3,3),LM3(3,3,3),LM4(3,3),LM5(3,3,3,3),LM6(3,3),LM7(3,3,3),
                    *LM8(3,3,3),LM9(3,3,3),LM10(3,3,3,3),LM11(3,3,3,3),LM12(3,3,3,3),
                    *LM13(3,3,3,3),LM14(3,3,3),LM15(3,3,3),LM16(3,3,3,3),LM17(3,3,3,3
                    *LM18(3,3),LM19(3,3),LM20(3,3),LM21(3,3),LM22(3,3),LM23(3,3),
                    *LM24(3,3),LM25(3,3),LM26(3,3),LM4B(3,3),LM13B(3,3,3,3),KHG(6,18)
                    *LM16B(3,3,3,3),KR(57,57),KH(18,18)
 0006                COMMON/FLX/
                    *M,M1(3,3),M2(3,3,3),M3(3,3,3,3),M4(3,3),M5(3,3),MAST,MASS(84,84),
                    *MB(3),MBEAR(3),MC(3,3),MD(3,3,3),MDOCK(3),MG(3),NROT,NHUB,
                    *N,NS,P,PASS,PIBETA(3,3),PIGAM(3,3),PIPHIC(3,3,3),PIPHID(3,3,3,3),
                    *PIPSI(3,3),PITHET(3,3),PHIC(3,3),PHICD(3,3),PHID(3,3,3),
                    *PHIDD(3,3,3),PSI(3),PSID(3),PHIMR(57,57),PHIMH(18,18),NTHET(3,3)
 0007                COMMON/FLX/
                    *RDMASS(3,3),RH(3),RI,RJ,RMASS,RRMASS,RTS(84),SI1(3,3),SI2(3),
                    *SI3(3,3,3),SI4(3),SIG1(3),SIG2(3,3),SIG3(3,3,3),SIG4(3,3),
                    *SIG5(3,3,3),SIG6(3),SIG7(3),SIG8(3),SGAM(3,3),SMASS,STHET(3,3),
                    *SUM1(3,3,3),TAUA(3),TAUC(3,3),TAUD(3,3,3),TAUL(3),TAUS(3),
                    *THET(3),THETD(3),PHC(3),STHE(3,3),SPIN
 0008                COMMON/FLX/
                    *UC(3,3),UCD(3,3),UD(3,3,3),UDD(3,3,3),WOK1(3,3),WOK2(84,84),
                    *WOK3(84),WOK4(3,3),WOK5(3,3),WOK6(3,3),WR(3),WXR(3),WXRA(3,3),
                    *WXY(3),WYC(3,3),WYH(3),WZ(3,3,3),XC(84),XCD(84),XD(3,3,3),KN(3,3
                    *XL(3),XLL(84),XVEC(84),XVECD(84),XVECDD(84),YC(3,3),YB(3),YE3(3),
                    *YG(3),YH(3),ZA(3),ZAD(3),ZV3(3,3)
 0009                INTEGER II,JJ,KK,LL,NN
           C  CONTINUE TO SET UP RIGHT SIDE OF EOM
           C COMPUTE DELTA-STEP 14
           C  DEL1,DEL2,DEL3
 0010                KK=P
 0011                CALL SCLA(DEL1,0.,3,1)
 0012                CALL SCLA(WOK1,0.,3,3)
 0013                DO 40 JJ=1,KK
 0014                CALL GMPRD(AALPHA(1,1,JJ),PITHET,WOK4,3,3,3)
 0015                DO 40 II=1,3
 0016                CALL SMPY(SI3(1,II,JJ),RDMASS(II,JJ),WOK5,3,1)
 0017                CALL GMPRD(WOK4,WOK5,DEL3(1,II,JJ),3,3,1)
 0018             40 CALL GMADD(DEL1,WOK5,DEL1,3,1)
 0019                DO 41 JJ=1,3
 0020                CALL SMPY(SI1(1,JJ),HCMASS(JJ),DEL2(1,JJ),3,1)
 0021             41 CALL GMADD(WOK1,DEL2(1,JJ),WOK1,3,1)
 0022                CALL GMADD(DEL1,WOK1,DEL1,3,1)
 0023                CALL SMPY(SI2,RRMASS,WOK1,3,1)
 0024                CALL GMADD(DEL1,WOK1,DEL1,3,1)
```

```
      IF(NS.EQ.0.)GO TO 50
      CALL SMPY(SI4,SMASS,WOK1,3,1)
      CALL GMADD(DEL1,WOK1,DEL1,3,1)
   50 CONTINUE
C   DEL4,DEL5
      DO 60 JJ=1,3
      CALL GMPRD(IC(1,1,JJ),WYC(1,JJ),WOK1,3,3,1)
      CALL GAMMA(WOK4,WYC(1,JJ))
      CALL GMPRD(WOK4,WOK1,DEL4(1,JJ),3,3,1)
      CALL GAMMA(WOK4,PHICD(1,JJ))
      CALL GMPRD(WOK4,WYH,WOK1,3,3,1)
      CALL GMPRD(IC(1,1,JJ),WOK1,WOK4,3,3,1)
      CALL GMSUB(DEL4(1,JJ),WOK4,DEL4(1,JJ),3,1)
      DO 60 II=1,KK
      CALL GAMMA(WOK1,WZ(1,JJ,II))
      CALL GMPRD(WOK1,ID(1,1,JJ,II),WOK4,3,3,3)
      CALL GMPRD(WOK4,WZ(1,JJ,II),DEL5(1,JJ,II),3,3,1)
      CALL GAMMA(WOK1,PHIDD(1,JJ,II))
      CALL GMPRD(WOK1,AALPHA(1,1,II),WOK4,3,3,3)
      CALL GMPRD(WOK4,WXB,WOK1,3,3,1)
      CALL GMPRD(ID(1,1,JJ,II),WOK1,WOK4,3,3,1)
   60 CALL GMSUB(DEL5(1,JJ,II),WOK4,DEL5(1,JJ,II),3,1)
C   DEL6
      IF(NS.EQ.0.)GO TO 70
      CALL GMPRD(IH,WB,WOK1,3,3,1)
      CALL GMPRD(GWP,WOK1,DEL6,3,3,1)
      CALL GMPRD(IH,GPSID,WOK1,3,3,3)
      CALL GMPRD(WOK1,PIBETA,WOK4,3,3,3)
      CALL GMPRD(WOK4,WYH,WOK1,3,3,1)
      CALL GMSUB(DEL6,WOK1,DEL6,3,1)
      CALL GMPRD(GPH,JT,WOK1,3,3,3)
      CALL GMPRD(WOK1,GWYS,WOK4,3,3,3)
      CALL GMPRD(WOK4,YH,WOK1,3,3,1)
      CALL SMPY(WOK1,SMASS,WOK4,3,1)
      CALL GMSUB(DEL6,WOK4,DEL6,3,1)
C DEL7
   70 CALL GMPRD(GWX,IF,WOK1,3,3,3)
      CALL GMPRD(WOK1,WXB,DEL7,3,3,1)
      CALL GMPRD(GXL,PITHET,WOK1,3,3,3)
      CALL GMPRD(WOK1,GWYS,WOK4,3,3,3)
      CALL GMPRD(WOK4,YB,WOK1,3,3,1)
      CALL SMPY(WOK1,PFMASS,WOK4,3,1)
      CALL GMADD(DEL7,WOK4,DEL7,3,1)
C DEL8
      CALL GMPRD(GWY,IA,WOK1,3,3,3)
      CALL GMPRD(WOK1,WYH,DEL8,3,3,1)
      IF(NS.EQ.0.)GO TO 80
      CALL GMPRD(GWP,IH,WOK1,3,3,3)
      CALL GMPRD(WOK1,WB,WOK4,3,3,1)
      CALL GMPRD(IH,GPSID,WOK1,3,3,3)
      CALL GMPRD(WOK1,PIBETA,WOK5,3,3,3)
      CALL GMPRD(WOK5,WYH,WOK1,3,3,1)
      CALL GMSUB(WOK4,WOK1,WOK5,3,1)
      CALL GMTRA(JT,WOK1,3,3)
      CALL GMPRD(WOK1,WOK5,WOK4,3,3,1)
      CALL GMADD(DEL8,WOK4,DEL8,3,1)
      CALL GMSUB(YH,YB,WOK5,3,1)
      CALL GAMMA(WOK4,WOK5)
```

FIGURE 10.7 (Cont'd)

```
      CALL GMPRD(WOK4,SI4,WOK5,3,3,1)
      CALL GMPRD(WOK1,GRH,WOK4,3,3,3)
      CALL GMPRD(WOK4,JT,WOK1,3,3,3)
      CALL GMPRD(WOK1,GWYS,WOK4,3,3,3)
      CALL GMPRD(WOK4,YH,WOK1,3,3,1)
      CALL GMSUB(WOK5,WOK1,WOK4,3,1)
      CALL SMPY(WOK4,SMASS,WOK1,3,1)
      CALL GMADD(DEL8,WOK1,DEL8,3,1)
   80 CONTINUE
      RETURN
      END
```

FIGURE 10.8 FLX15

```
20.1 (AUG 71)                                OS/360  FORTRAN H

     COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                        SOURCE,BCD,NOLIST,DECK,LOAD,MAP,NOEDIT,ID,NOXREF
0002              SUBROUTINE FLX15
0003              IMPLICIT REAL (A-Z)
0004              COMMON/FLX/
                 *AALPHA(3,3,3),ALPHA(3),BETA(3),DEL1(3),DEL2(3,3),DEL3(3,3,3),
                 *DEL4(3,3),DEL5(3,3,3),DEL6(3),DEL7(3),DEL8(3),DELTB,
                 *CR(57,57),CH(18,18),CHG(6,18),DEL(3),DELD(3),
                 *EIH(3),EIR(3),EH,ER,ETA,FC(3,3),FD(3,3,3),FG(3),GAM(3),GAMD(3),G
                 *GJH(3),GJR(3),GPSID(3,3),GR,GYGB(3,3),GXL(3,3),DD(500),
                 *GPH(3,3),GWR(3,3),GWX(3,3),GWXRA(3,3,3),GWY(3,3),GWYC(3,3,3),
                 *GWYS(3,3),GYB(3,3),GYBM(3,3),GYH(3,3),HCMASS(3),HI,HJ,HMASS
0005              COMMON/FLX/
                 *IA(3,3),IB(3,3),IC(3,3,3),ID(3,3,3,3),IH(3,3),JT(3,3),KBS1,KBS2,
                 *KBVH(3,3),KBVR(3,3),KHS(3,3),KHV(3,3),LAM(84,84),LM1(3,3),
                 *LM2(3,3),LM3(3,3,3),LM4(3,3),LM5(3,3,3,3),LM5(3,3),LM7(3,3,3),
                 *LM8(3,3,3),LM9(3,3,3),LM10(3,3,3,3),LM11(3,3,3,3),LM12(3,3,3,3),
                 *LM13(3,3,3,3),LM14(3,3,3),LM15(3,3,3),LM16(3,3,3,3),LM17(3,3,3,3
                 *LM18(3,3),LM19(3,3),LM20(3,3),LM21(3,3),LM22(3,3),LM23(3,3),
                 *LM24(3,3),LM25(3,3),LM26(3,3),LM48(3,3),LM13B(3,3,3,3),KHG(6,18),
                 *LM16B(3,3,3,3),KR(57,57),KH(18,18)
0006              COMMON/FLX/
                 *M,M1(3,3),M2(3,3,3),M3(3,3,3,3),M4(3,3),M5(3,3),MAST,MASS(84,84),
                 *MR(3),MBEAR(3),MC(3,3),MD(3,3,3),MDOCK(3),MG(3),NROT,NHUB,
                 *N,NS,P,PASS,PIBETA(3,3),PIGAM(3,3),PIPHIC(3,3,3),PIPHID(3,3,3,3),
                 *PIPSI(3,3),PITHET(3,3),PHIC(3,3),PHICD(3,3),PHID(3,3,3),
                 *PHIDD(3,3,3),PSI(3),PSID(3),PHIMR(57,57),PHIMH(18,18),NTHET(3,3)
0007              COMMON/FLX/
                 *RDMASS(3,3),RH(3),RI,RJ,RMASS,RRMASS,RTS(84),SI1(3,3),SI2(3),
                 *SI3(3,3,3),SI4(3),SIG1(3),SIG2(3,3),SIG3(3,3,3),SIG4(3,3),
                 *SIG5(3,3,3),SIG6(3),SIG7(3),SIG8(3),SGAM(3,3),SMASS,STHET(3,3),
                 *SUM1(3,3,3),TAUA(3),TAUC(3,3),TAUD(3,3,3),TAUL(3),TAUS(3),
                 *THET(3),THETD(3),RHO(3),STHE(3,3),SPIN
0008              COMMON/FLX/
                 *UC(3,3),UCD(3,3),UD(3,3,3),UDD(3,3,3),WOK1(3,3),WOK2(84,84),
                 *WOK3(84),WOK4(3,3),WOK5(3,3),WOK6(3,3),WR(3),WXR(3),WXRA(3,3),
                 *WXY(3),WYC(3,3),WYH(3),WZ(3,3,3),XC(84),XCD(84),KD(3,3,3),XN(3,3)
                 *XL(3),XLL(84),XVEC(84),XVECD(84),XVECDD(84),YC(3,3),YB(3),YE3(3),
                 *YG(3),YH(3),ZA(3),ZAD(3),ZV3(3,3)
0009              DIMENSION MBS(3),MBV(3),KBV(3,3)
0010              INTEGER II,JJ,KK,LL,NN
            C COMPLETE THE RIGHT SID OF EQN
            C CALCULATE ELASTIC MOMENT AT DOCKING MECHANISM
            C   MDOCK
0011              IF(NS.EQ.0.)GO TO 10
0013              CALL GMPRD(KHS,PSI,WOK1,3,3,1)
0014              CALL GMPRD(KHV,PSID,WOK4,3,3,1)
0015              CALL GMADD(WOK1,WOK4,WOK1,3,1)
0016              CALL SMPY(WOK1,-1.,MDOCK,3,1)
0017           10 CONTINUE
            C CALCULATE BEARING TORQUES
            C    MBS-SPRING MOMENT
0018              DLTB=ARCOS(COS(THET(2))*COS(THET(3)))
0019              MBS(1)=0.0
            C    SPRING DISCONTINUITY
0020              IF(DLTB.GT.DELTB)GO TO 20
0022              KBS=-KBS1
0023              GO TO 30
```

FIGURE 10.8 (Cont'd)

```
   20 KBS=(KBS2-KBS1)*DELTB/DLTE-KBS2
   30 MBS(2)=KBS*THET(2)
      MBS(3)=KBS*THET(3)
C      DAMPING MOMENT-MBV
C       DAMPING MATRIX -KBV
        CALL GMTRA(PITHET,WOK1,3,3)
        CALL GMPRD(KBVR,WOK1,WOK4,3,3,3)
        CALL GMPRD(PITHET,WOK4,WOK1,3,3,3)
        ETA1=1.-ETA
        CALL SMPY(WOK1,ETA1,WOK4,3,3)
        CALL SMPY(KBVH,ETA,WOK1,3,3)
        CALL GMADD(WOK1,WOK4,KBV,3,3)
        CALL GMPRD(PITHET,WYH,WOK1,3,3,1)
        CALL GMSUB(WOK1,WXP,WOK1,3,1)
        CALL GMPRD(KBV,WOK1,MBV,3,3,1)
C      TOTAL BEARING MOMENT
        CALL GMADD(MBS,MBV,MBEAR,3,1)
C
C     CALCULATE ELASTIC FORCES AND MOMENTS DUE TO ROTOR ARM BENDING
C       SET UP TEMPORY ROTOR STATE IN XC AND DERIV. IN XCD
        DO 40 II=1,3
        XC(II)=DEL(II)
   40 XCD(II)=DELD(II)
        LL=P
        DO 50 KK=1,LL
        DO 50 JJ=1,3
        DO 50 II=1,3
        NN=3+6*(JJ-1)+18*(KK-1)
        XC(II+NN)=UD(II,JJ,KK)
        XC(II+NN+3)=PHID(II,JJ,KK)
        XCD(II+NN)=UDD(II,JJ,KK)
   50 XCD(II+NN+3)=PHIDD(II,JJ,KK)
        NN=39
        IF(P.EQ.3.)NN=57
        DO 55 II=1,NN
        WOK3(II)=0.0
        DO 55 KK=1,NN
   55 WOK3(II)=WOK3(II)+KR(II,KK)*XC(KK)+CR(II,KK)*XCD(KK)
   60 DO 62 II=1,3
   62 MP(II)=WOK3(II)
        DO 65 KK=1,LL
        DO 65 JJ=1,3
        DO 65 II=1,3
        NN=3+6*(JJ-1)+18*(KK-1)
        FD(II,JJ,KK)=WOK3(II+NN)
   65 MD(II,JJ,KK)=WOK3(II+NN+3)
C     CALCULATE FORCES AND MOMENTS DUE TO HUB BENDING
C      SET UP TEMP HUB STATE IN XC AND ITS DERIV IN XCD
        DO 70 JJ=1,3
        DO 70 II=1,3
        NN=6*(JJ-1)
        XC(II+NN)=UC(II,JJ)
        XCD(II+NN)=UCD(II,JJ)
        XC(II+NN+3)=PHIC(II,JJ)
   70 XCD(II+NN+3)=PHICD(II,JJ)
        CALL GMPRD(KH,XC,WOK2,18,18,1)
        CALL GMPRD(CH,XCD,WOK3,18,18,1)
        CALL GMADD(WOK3,WOK2,WOK3,18,1)
```

```
      DO 75 JJ=1,3
      DO 75 II=1,3
      NN=6*(JJ-1)
      FC(II,JJ)=WOK3(II+NN)
   75 MC(II,JJ)=WOK3(II+NN+3)
      CALL GMPRD(KHG,XC,WOK2,6,18,1)
      CALL GMPRD(CHG,XCD,WOK3,6,18,1)
      CALL GMADD(WOK3,WOK2,WOK3,6,1)
      DO 80 II=1,3
      FG(II)=WOK3(II)
   80 MG(II)=WOK3(II+3)
C
C CALCULATE THE SIGMA'S
C    SIG1
      CALL GMPRD(LM4B,RHC,WOK1,3,3,1)
      CALL SMPY(DEL1,-1.,SIG1,3,1)
      CALL GMSUB(SIG1,WOK1,SIG1,3,1)
C    SIG2
      KK=P
      DO 90 JJ=1,3
      CALL SMPY(DEL2(1,JJ),-1.,WOK1,3,1)
      CALL GMSUB(WOK1,FC(1,JJ),SIG2(1,JJ),3,1)
C    SIG4
      CALL GMSUB(TAUC(1,JJ),DEL4(1,JJ),WOK1,3,1)
      CALL GMPRD(PIPHIC(1,1,JJ),MC(1,JJ),WOK4,3,3,1)
      CALL GMSUB(WOK1,WOK4,SIG4(1,JJ),3,1)
C   SIG3
      DO 90 II=1,3
      IF(JJ.GT.KK)GO TO 90
      CALL SMPY(DEL3(1,II,JJ),-1.,WOK1,3,1)
      CALL GMSUB(WOK1,FD(1,II,JJ),WOK1,3,1)
      CALL GMPRD(LM13B(1,1,II,JJ),RHC,WOK4,3,3,1)
      CALL GMSUB(WOK1,WOK4,SIG3(1,II,JJ),3,1)
C   SIG5
      CALL GMSUB(TAUD(1,II,JJ),DEL5(1,II,JJ),WOK1,3,1)
      CALL GMPRD(PIPHID(1,1,II,JJ),MD(1,II,JJ),WOK4,3,3,1)
      CALL GMSUB(WOK1,WOK4,WOK1,3,1)
      CALL GMPRD(LM16B(1,1,II,JJ),RHC,WOK4,3,3,1)
      CALL GMSUB(WOK1,WOK4,SIG5(1,II,JJ),3,1)
   90 CONTINUE
C   SIG6
      IF(NS.EQ.0.)GO TO 95
      CALL GMADD(TAUS,MDOCK,SIG6,3,1)
      CALL GMSUB(SIG6,DEL6,SIG6,3,1)
      GO TO 100
   95 IF(PASS.GT.0.)GO TO 100
      DO 97 II=1,3
   97 SIG6(II)=0.0
C   SIG7
  100 CALL GMADD(TALL,MBEAR,SIG7,3,1)
      CALL GMSUB(SIG7,MB,SIG7,3,1)
      CALL GMSUB(SIG7,DEL7,SIG7,3,1)
      CALL GMPRD(IB,RHC,WOK1,3,3,1)
      CALL GMSUB(SIG7,WOK1,SIG7,3,1)
C   SIG8
      IF(PASS.GT.0.)GO TO 200
      CALL GMSUB(YG,YB,WOK1,3,1)
      CALL GAMMA(GYGB,WOK1)
```

FIGURE 10.8 (Cont'd)

```
200 CALL GMPRD(GYGR,FG,WOK1,3,3,1)
    CALL GMSUB(TAUA,WOK1,SIG8,3,1)
    CALL GMSUB(SIG8,DEL8,SIG8,3,1)
    CALL GMSUB(SIG8,MG,SIG8,3,1)
    CALL GMTRA(FITHET,WOK1,3,3)
    CALL GMPRD(WOK1,MBEAR,WOK4,3,3,1)
    CALL GMSUB(SIG8,WOK4,SIG8,3,1)
    IF(NS.EQ.0.)GO TO 210
    CALL GMTRA(JT,WOK1,3,3)
    CALL GMPRD(WOK1,TAUS,WOK4,3,3,1)
    CALL GMADD(SIG8,WOK4,SIG8,3,1)
210 CONTINUE
    RETURN
    END
```

FIGURE 10.9 PREINT

```
20.1 (AUG 71)                                OS/360  FORTRAN H

   COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                      SOURCE,BCD,NOLIST,DECK,LOAD,MAP,NOEDIT,ID,NOXREF
0002              SUBROUTINE PREINT
0003              IMPLICIT REAL (A-Z)
0004              COMMON/FLX/
                *AALPHA(3,3,3),ALPHA(3),BETA(3),DEL1(3),DEL2(3,3),DEL3(3,3,3),
                *DEL4(3,3),DEL5(3,3,3),DEL6(3),DEL7(3),DEL8(3),DELTB,
                *CR(57,57),CH(18,18),CHG(6,18),DEL(3),DELD(3),
                *EIH(3),EIR(3),EH,ER,ETA,FC(3,3),FD(3,3,3),FG(3),GAM(3),GAMD(3),GH
                *GJH(3),GJR(3),GPSID(3,3),GR,GYGR(3,3),GXL(3,3),DD(500),
                *GRH(3,3),GWR(3,3),GWX(3,3),GWXRA(3,3,3),GWY(3,3),GWYC(3,3,3),
                *GWYS(3,3),GYR(3,3),GYRM(3,3),GYH(3,3),HCMASS(3),HI,HJ,HMASS
0005              COMMON/FLX/
                *IA(3,3),IB(3,3),IC(3,3,3),ID(3,3,3,3),IH(3,3),JT(3,3),KBS1,KBS2,
                *KBVH(3,3),KBVR(3,3),KHS(3,3),KHV(3,3),LAM(84,84),LM1(3,3),
                *LM2(3,3),LM3(3,3,3),LM4(3,3),LM5(3,3,3,3),LM6(3,3),LM7(3,3,3),
                *LM8(3,3,3),LM9(3,3,3),LM10(3,3,3,3),LM11(3,3,3,3),LM12(3,3,3,3),
                *LM13(3,3,3,3),LM14(3,3,3),LM15(3,3,3),LM16(3,3,3,3),LM17(3,3,3,3)
                *LM18(3,3),LM19(3,3),LM20(3,3),LM21(3,3),LM22(3,3),LM23(3,3),
                *LM24(3,3),LM25(3,3),LM26(3,3),LM4B(3,3),LM13B(3,3,3,3),KHG(6,18),
                *LM16B(3,3,3,3),KR(57,57),KH(18,18)
0006              COMMON/FLX/
                *M,M1(3,3),M2(3,3,3),M3(3,3,3,3),M4(3,3),M5(3,3),MAST,MASS(84,84),
                *MB(3),MBEAR(3),MC(3,3),MD(3,3,3),MDOCK(3),MG(3),NROT,NHUB,
                *N,NS,P,PASS,PIBETA(3,3),PIGAM(3,3),PIPHIC(3,3,3),PIPHID(3,3,3,3),
                *PIPSI(3,3),PITHET(3,3),PHIC(3,3),PHICD(3,3),PHID(3,3,3),
                *PHIDD(3,3,3),PSI(3),PSID(3),PHIMR(57,57),PHIMH(18,18),NTHET(3,3)
0007              COMMON/FLX/
                *RDMASS(3,3),RH(3),RI,RJ,RMASS,RRMASS,RTS(84),SI1(3,3),SI2(3),
                *SI3(3,3,3),SI4(3),SIG1(3),SIG2(3,3),SIG3(3,3,3),SIG4(3,3),
                *SIG5(3,3,3),SIG6(3),SIG7(3),SIG8(3),SGAM(3,3),SMASS,STHET(3,3),
                *SUM1(3,3,3),TAUA(3),TAUC(3,3),TAUD(3,3,3),TAUL(3),TAUS(3),
                *THET(3),THETD(3),PHC(3),STHE(3,3),SPIN
0008              COMMON/FLX/
                *UC(3,3),UCD(3,3),UD(3,3,3),UDD(3,3,3),WOK1(3,3),WOK2(84,84),
                *WOK3(84),WOK4(3,3),WOK5(3,3),WOK6(3,3),WR(3),WXR(3),WXRA(3,3),
                *WXY(3),WYC(3,3),WYH(3),WZ(3,3,3),XC(84),XCD(84),XD(3,3,3),XN(3,3)
                *XL(3),XLL(84),XVEC(84),XVECD(84),XVECDD(84),YC(3,3),YB(3),YE3(3),
                *YG(3),YH(3),ZA(3),ZAD(3),ZV3(3,3)
0009              INTEGER II,JJ,KK,LL,NN
0010              INTEGER IJ,N,M,MN
0011              INTEGER NROT,NHUB
0012              DIMENSION LM27(3,3,3,3)
            C FORM THE COMPLETE MASS MATRIX  LAM(84,84)
            C
            C MODIFY MASS TERMS FOR DELETIONS
            C     NS=0. SHUTTLE NOT ATTACHED,NS=1. SHUTTLE ATTACHED
0013              IF(NS.EQ.1.)GO TO 10
0015              DO 5 II=1,3
0016              DO 4 JJ=1,3
0017              LM18(II,JJ)=0.0
0018              LM19(II,JJ)=0.0
0019            4 LM20(II,JJ)=0.
0020            5 LM20(II,II)=250000.
0021           10 CONTINUE
            C     MODIFY FOR A TWO ARMED ROTOR
0022              IF(P.EQ.3.)GO TO 20
0024              DO 12 II=1,3
```

```
      DO 12 JJ=1,3
      DO 12 KK=1,3
      LM10(II,JJ,KK,3)=0.
      LM11(II,JJ,KK,3)=0.
      LM12(II,JJ,KK,3)=0.
      LM13(II,JJ,KK,3)=0.
      LM16(II,JJ,KK,3)=0.
      LM27(II,JJ,KK,3)=0.0
   12 LM17(II,JJ,KK,3)=0.0
   20 CONTINUE
C SET UP MASS MATRIX
      DO 25 II=1,84
      DO 25 JJ=1,84
   25 LAM(II,JJ)=0.0
      M=84.
C  CALCULATE LM27
      KK=8
      CALL GMPRD(NTHET,PITHET,WCK1,3,3,3)
      DO 35 JJ=1,KK
      DO 35 II=1,3
   35 CALL GMPRD(LM16B(1,1,II,JJ),WCK1,LM27(1,1,II,JJ),3,3,3)
      CALL  STACK(LM1,LAM,3,3,84,1,1)
      CALL  STACK(LM2,LAM,3,3,84,1,4)
      CALL  STACK(LM6,LAM,3,3,84,1,7)
      CALL  STACK(LM3,LAM,3,9,84,1,10)
      CALL  STACK(LM4,LAM,3,3,84,1,28)
      CALL  STACK(LM5,LAM,3,9,84,1,31)
      CALL  STACK(LM5(1,1,1,2),LAM,3,9,84,1,42)
      CALL  STACK(LM5(1,1,1,3),LAM,3,9,84,1,67)
      CALL  STACK(LM24,LAM,3,3,84,4,1)
      CALL  STACK(LM25,LAM,3,3,84,4,4)
      CALL  STACK(LM26,LAM,3,3,84,4,7)
      CALL  STACK(LM18,LAM,3,3,84,7,1)
      CALL  STACK(LM19,LAM,3,3,84,7,4)
      CALL  STACK(LM20,LAM,3,3,84,7,7)
      DO 50 JJ=1,3
      DO 50 KK=1,3
      DO 50 II=1,3
      NN=II+3*(KK-1)+9
      LAM(NN,JJ)=LM7(II,JJ,KK)
      LAM(NN,JJ+3)=LM8(II,JJ,KK)
      LAM(NN+9,JJ+3)=LM14(II,JJ,KK)
      MM=3*(KK-1)+9+JJ
      LAM(NN,MM)=LM9(II,JJ,KK)
   50 LAM(NN+9,MM+9)=LM15(II,JJ,KK)
      CALL  STACK(LM21,LAM,3,3,84,28,1)
      CALL  STACK(LM22,LAM,3,3,84,28,4)
      CALL  STACK(LM23,LAM,3,3,84,28,28)
      DO 60 IJ=1,3
      DO 60 JJ=1,3
      DO 60 KK=1,3
      DO 60 II=1,3
      NN=II+3*(KK-1)+18*(IJ-1)+30
      LAM(NN,JJ)=LM10(II,JJ,KK,IJ)
      LAM(NN,JJ+3)=LM11(II,JJ,KK,IJ)
      LAM(NN,JJ+27)=LM13(II,JJ,KK,IJ)
      LAM(NN+9,JJ+27)=LM16(II,JJ,KK,IJ)
      LAM(NN+9,JJ+3)=LM27(II,JJ,KK,IJ)
```

FIGURE 10.9 (Cont'd)

```
      MM=JJ+3*(KK-1)+18*(IJ-1)+30
      LAM(NN,MM)=LM12(II,JJ,KK,IJ)
   60 LAM(NN+9,MM+9)=LM13(II,JJ,KK,IJ)
      CALL GENM
C FORM REDUCED MASS MATRIX - TRANS(MODE SHAPES).LAM.(MODE SHAPES)
C     FIRST CALCULATE  TRANS(MODE SHAPES).LAM , STORE IN WOK2
C       NOTE THE MATRIX MULT. IS CARRIED OUT BELOW TO
C             AVOID UNNECESSARY ZERO MULTIPLICATIONS
      N=8.+NROT+NHUB
      DO 70 II=1,N
      DO 70 JJ=1,84
   70 WOK2(II,JJ)=0.0
C ROW 1 THRU 8
      DO 75 II=1,3
      DO 71 JJ=1,18
   71 WOK2(II,JJ)=LAM(II,JJ)
      DO 72 JJ=28,39,1
   72 WOK2(II,JJ)=LAM(II,JJ)
      DO 73 JJ=49,57,1
   73 WOK2(II,JJ)=LAM(II,JJ)
      DO 75 JJ=67,75,1
   75 WOK2(II,JJ)=LAM(II,JJ)
      DO 76 II=4,6,1
      DO 76 JJ=1,9
   76 WOK2(II,JJ)=LAM(II,JJ)
      DO 77 II=4,6,1
      DO 77 JJ=10,84,1
   77 WOK2(II,JJ)=LAM(II,JJ)
C  ROWS 9 THRU 8+NO.OF HUB MODES=8+NHUB
      IF(NHUB.EQ.0)GO TO 100
      DO 85 II=1,NHUB
      DO 80 JJ=1,3
      DO 80 KK=1,9
   80 WOK2(II+8,JJ)=WOK2(II+8,JJ)+PHIMH(KK,II)*LAM(KK+9,JJ)
      DO 85 JJ=4,6,1
      DO 85 KK=1,18
   85 WOK2(II+8,JJ)=WOK2(II+8,JJ)+PHIMH(KK,II)*LAM(KK+9,JJ)
      NN=0
      DO 95 LL=1,6
      DO 90 II=1,NHUB
      DO 90 JJ=1,3
      DO 90 KK=1,3
   90 WOK2(II+8,JJ+9+NN)=WOK2(II+8,JJ+9+NN)+PHIMH(KK+NN,II)*LAM(KK+9+N
     *JJ+9+NN)
   95 NN=NN+3
C  ROWS 9+NHUB THRU N
  100 IF(NROT.EQ.0)GO TO 140
      NN=NHUB+8
      LL=57
      IF(P.EQ.2.)LL=39
      DO 115 II=1,NROT
      DO 110 JJ=1,6
      DO 105 KK=1,LL
  105 WOK2(II+NN,JJ)=WOK2(II+NN,JJ)+PHIMR(KK,II)*LAM(KK+27,JJ)
  110 CONTINUE
      DO 111 JJ=7,27,1
      DO 111 KK=1,3
  111 WOK2(II+NN,JJ)=WOK2(II+NN,JJ)+PHIMR(KK,II)*LAM(KK+27,JJ)
```

FIGURE 10.9 (Cont'd)

```
      DO 112 JJ=31,84,1
      DO 112 KK=1,3
  112 WOK2(II+NN,JJ)=WOK2(II+NN,JJ)+PHIMR(KK,II)*LAM(KK+27,JJ)
      DO 115 JJ=28,30,1
      DO 115 KK=1,LL
  115 WOK2(II+NN,JJ)=WOK2(II+NN,JJ)+PHIMR(KK,II)*LAM(KK+27,JJ)
      LL=0
      DO 135 IJ=1,18
      DO 130 II=1,NROT
      DO 130 JJ=1,3
      DO 130 KK=1,3
  130 WOK2(II+NN,JJ+LL+30)=WOK2(II+NN,JJ+LL+30)+PHIMR(KK+LL+3,II)*LAM(
     *+30+LL,JJ+LL+30)
  135 LL=LL+3
  140 CONTINUE
C
C     MULTIPLY MODE SHAPES BY WOK2  TO COMPLETE THE REDUCED
C     MASS MATRIX-MASS(N,N)
      DO 205 II=1,N
      DO 205 JJ=1,N
  205 MASS(II,JJ)=0.0
C   COL. 1 THRU 8
      DO 207 JJ=1,8
      DO 207 II=1,N
  207 MASS(II,JJ)=WOK2(II,JJ)
C   COL. 9 THRU NHUB PLUS 8
      NN=NHUB+8
      IF(NHUB.EQ.0)GO TO 215
      DO 210 II=1,N
      DO 210 JJ=1,NHUB
      DO 210 KK=1,18
  210 MASS(II,JJ+8)=MASS(II,JJ+8)+WOK2(II,KK+9)*PHIMH(KK,JJ)
  215 IF(NROT.EQ.0)GO TO 225
      LL=39
      IF(P.EQ.3.)LL=57
C   COL.(NHUB+8) THRU N
      DO 220 II=1,6
      DO 220 JJ=1,NROT
      DO 216 KK=1,LL
  216 MASS(II,JJ+NN)=MASS(II,JJ+NN)+WOK2(II,KK+27)*PHIMR(KK,JJ)
  220 CONTINUE
      DO 222 II=1,NROT
      DO 222 JJ=1,NROT
      DO 222 KK=1,LL
  222 MASS(II+NN,JJ+NN)=MASS(II+NN,JJ+NN)+WOK2(II+NN,KK+27)*PHIMR(KK,JJ
  225 CONTINUE
C
C INVERT MASS AND STORE IN MASS
      CALL SPEVN1(MASS,N,WOK3,M,NIX)
C SET UP RIGHT SIDE FORCING FUNCTION
      DO 200 II=1,3
      RTS(II)=SIG1(II)
      RTS(II+3)=SIG8(II)
      RTS(II+6)=SIG6(II)*NS
      DO 190 JJ=1,3
      NN=JJ+9+3*(II-1)
      RTS(NN)=SIG2(JJ,II)
  190 RTS(NN+9)=SIG4(JJ,II)
```

FIGURE 10.9 (Cont'd)

```
      RTS(II+27)=SIG7(II)
      DO 195 JJ=1,3
      NN=30+JJ+3*(II-1)
      RTS(NN)=SIG3(JJ,II,1)
      RTS(NN+9)=SIG5(JJ,II,1)
      RTS(NN+18)=SIG3(JJ,II,2)
      RTS(NN+27)=SIG5(JJ,II,2)
      IF(P.EQ.2.)GO TO 192
      RTS(NN+36)=SIG3(JJ,II,3)
      RTS(NN+45)=SIG5(JJ,II,3)
      GO TO 195
  192 RTS(NN+36)=0.0
      RTS(NN+45)=0.0
  195 CONTINUE
  200 CONTINUE
      CALL GENR
C     FORM REDUCED RIGHT SIDE AND STORE IN WOK3
      DO 3000 II=1,N
 3000 WOK3(II)=0.0
      DO 300 II=1,8
  300 WOK3(II)=RTS(II)
      IF(NHUR.EQ.0)GO TO 320
      DO 310 II=1,NHUR
      DO 310 KK=1,18
  310 WOK3(II+8)=WOK3(II+8)+FHINH(KK,II)*RTS(KK+9)
  320 NN=NHUR+8
      IF(NROT.EQ.0)GO TO 350
      LL=39
      IF(P.EQ.3.)LL=57
      DO 330 II=1,NROT
      DO 330 KK=1,LL
  330 WOK3(II+NN)=WOK3(II+NN)+FHIMF(KK,II)*RTS(KK+27)
  350 CONTINUE
C     CALCULATE THE SECOND DERIVATIVES
      DO 4000 II=1,N
 4000 XVECDD(II)=0.0
      DO 400 II=1,N
      DO 400 KK=1,N
  400 XVECDD(II)=XVECDD(II)+MASS(II,KK)*WOK3(KK)
      RETURN
      END
```

FIGURE 10.10 INTIG

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20.1 (AUG 71)                          OS/360  FORTRAN H

      COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                        SOURCE,BCD,NOLIST,DECK,LOAD,MAP,NOEDIT,ID,NOXREF
0002                    SUBROUTINE INTIG
0003                    IMPLICIT REAL (A-Z)
0004                    COMMON/TIME/T,DT,NSYM,ANTTWC
0005                    COMMON/FLX/
                       *AALPHA(3,3,3),ALPHA(3),BETA(3),DEL1(3),DEL2(3,3),DEL3(3,3,3),
                       *DEL4(3,3),DEL5(3,3,3),DEL6(3),DEL7(3),DEL8(3),DELTB,
                       *CR(57,57),CH(18,18),CHG(6,18),DEL(3),DELD(3),
                       *EIH(3),EIR(3),EH,ER,ETA,FC(3,3),FD(3,3,3),FG(3),GAM(3),GAMD(3),G
                       *GJH(3),GJR(3),GPSID(3,3),GR,GYGB(3,3),GXL(3,3),DD(500),
                       *GRH(3,3),GWR(3,3),GWX(3,3),GWXRA(3,3,3),GWY(3,3),GWYC(3,3,3),
                       *GWYS(3,3),GYB(3,3),GYBM(3,3),GYH(3,3),HCMASS(3),HI,HJ,HMASS
0006                    COMMON/FLX/
                       *IA(3,3),IB(3,3),IC(3,3,3),ID(3,3,3,3),IH(3,3),JT(3,3),KBS1,KBS2,
                       *KBVH(3,3),KBVR(3,3),KHS(3,3),KHV(3,3),LAM(84,84),LM1(3,3),
                       *LM2(3,3),LM3(3,3,3),LM4(3,3),LM5(3,3,3,3),LM6(3,3),LM7(3,3,3),
                       *LM8(3,3,3),LM9(3,3,3),LM10(3,3,3,3),LM11(3,3,3,3),LM12(3,3,3,3),
                       *LM13(3,3,3,3),LM14(3,3,3),LM15(3,3,3),LM16(3,3,3,3),LM17(3,3,3,3)
                       *LM18(3,3),LM19(3,3),LM20(3,3),LM21(3,3),LM22(3,3),LM23(3,3),
                       *LM24(3,3),LM25(3,3),LM26(3,3),LM4B(3,3),LM13B(3,3,3,3),KHG(6,18),
                       *LM16B(3,3,3,3),KF(57,57),KH(18,18)
0007                    COMMON/FLX/
                       *M,M1(3,3),M2(3,3,3),M3(3,3,3,3),M4(3,3),M5(3,3),MAST,MASS(84,84),
                       *MB(3),MBEAR(3),MC(3,3),MD(3,3,3),MDOCK(3),MG(3),NROT,NHUB,
                       *N,NS,P,PASS,PIBETA(3,3),PIGAM(3,3),PIPHIC(3,3,3),PIPHID(3,3,3,3),
                       *PIPSI(3,3),PITHET(3,3),PHIC(3,3),PHICD(3,3),PHID(3,3,3),
                       *PHIDD(3,3,3),PSI(3),PSID(3),PHIMR(57,57),PHIMH(18,18),NTHET(3,3)
0008                    COMMON/FLX/
                       *RDMASS(3,3),RH(3),RI,RJ,RMASS,RRMASS,RTS(84),SI1(3,3),SI2(3),
                       *SI3(3,3,3),SI4(3),SIG1(3),SIG2(3,3),SIG3(3,3,3),SIG4(3,3),
                       *SIG5(3,3,3),SIG6(3),SIG7(3),SIG8(3),SGAM(3,3),SMASS,STHET(3,3),
                       *SUM1(3,3,3),TAUA(3),TAUC(3,3),TAUD(3,3,3),TAUL(3),TAUS(3),
                       *THET(3),THETD(3),PHC(3),STHE(3,3),SPIN
0009                    COMMON/FLX/
                       *UC(3,3),UCD(3,3),UD(3,3,3),UDD(3,3,3),WOK1(3,3),WOK2(84,84),
                       *WOK3(84),WOK4(3,3),WOK5(3,3),WOK6(3,3),WR(3),WXR(3),WXRA(3,3),
                       *WXY(3),WYC(3,3),WYH(3),WZ(3,3,3),XC(84),XCD(84),XD(3,3,3),XN(3,3)
                       *XL(3),XLL(84),XVEC(84),XVECD(84),XVECDD(84),YC(3,3),YB(3),YE3(3),
                       *YG(3),YH(3),ZA(3),ZAD(3),ZV3(3,3)
0010                    DIMENSION NP(84)
0011                    DIMENSION NPASS(4),XS(4),XSL(4),XCDI(84),XCI(84)
0012                    INTEGER NPASS,NROT,NHUB,NSYM
0013                    INTEGER II,JJ,KK,LL,NN,NF
0014                    INTEGER IJ,N,M
                C    PERFORM INTEGRATIONS
0015                    IF(PASS.GT.0.)GO TO 10
0017                    DO 5 II=1,N
0018                  5 NP(II)=0.0
0019                    DO 6 II=1,4
0020                  6 NPASS(II)=0.0
0021                    DO 100 II=1,84
                C      INITIAL PHYSICAL COORDINATES
0022                    XC(II)=DD(300+II)
0023                100 XCD(II)=DD(400+II)
0024                    XC(28)=DEL(1)
0025                    XC(29)=DEL(2)
0026                    XC(30)=DEL(3)
```

FIGURE 10.10 (Cont'd)

```
0027            XCD(28)=DELD(1)
0028            XCD(29)=DELD(2)
0029            XCD(30)=DELD(3)
0030            IF(ANTTWO.EQ.0.)GO TO 108
0032            DO 109 II=1,57
0033        109 XCD(II+27)=PHIMR(II,NSYM+2)*DD(430)/PHIMR(3,NSYM+2)
0034        108 CONTINUE
          C    INITIAL CONDITIONS OF MODAL COORDINATES
          C    COMPUTE THE INVERSE OF  (PHI(TRANS)PHI) AND STORE IN WOK2
0035            DO 110 JJ=1,N
0036            DO 110 II=1,N
0037        110 WOK2(II,JJ)=0.0
0038            DO 115 II=1,8
0039        115 WOK2(II,II)=1.0
0040            IF(NHUB.EQ.0)GO TO 125
0042            DO 120 II=1,NHUB
0043            DO 120 JJ=1,NHUB
0044            DO 120 KK=1,18
0045        120 WOK2(II+8,JJ+8)=WOK2(II+8,JJ+8)+PHIMH(KK,II)*PHIMH(KK,JJ)
0046        125 IF(NROT.EQ.0)GO TO 135
0048            NN=NHUB+8
0049            LL=39
0050            IF(P.EQ.3.)LL=57
0052            DO 130 II=1,NROT
0053            DO 130 JJ=1,NROT
0054            DO 130 KK=1,LL
0055        130 WOK2(II+NN,JJ+NN)=WOK2(II+NN,JJ+NN)+PHIMR(KK,II)*PHIMR(KK,JJ)
0056        135 CONTINUE
          C    INVERT
0057            CALL SREVN1(WOK2,N,WOK3,M,NIX)
          C    COMPUTE  (PHI(TRANS).XC) AND STORE IN WOK3
0058            DO 1400 II=1,N
0059       1400 WOK3(II)=0.0
0060            DO 140 II=1,8
0061        140 WOK3(II)=XC(II)
0062            IF(NHUB.EQ.0)GO TO 150
0064            DO 145 II=1,NHUB
0065            DO 145 KK=1,18
0066        145 WOK3(II+8)=WOK3(II+8)+PHIMH(KK,II)*XC(KK+9)
0067        150 IF(NROT.EQ.0)GO TO 160
0069            DO 155 II=1,NROT
0070            DO 155 KK=1,LL
0071        155 WOK3(II+NN)=WOK3(II+NN)+PHIMR(KK,II)*XC(KK+27)
0072        160 CONTINUE
          C  CALCULATE XVEC(0)=WOK2.WOK3
0073            DO 1700 II=1,N
0074       1700 XVEC(II)=0.0
0075            DO 170 II=1,N
0076            DO 170 KK=1,N
0077        170 XVEC(II)=XVEC(II)+WOK2(II,KK)*WOK3(KK)
          C
          C  COMPUTE  (PHI(T).XCD) AND STORE IN WOK3
0078            DO 2400 II=1,N
0079       2400 WOK3(II)=0.0
0080            DO 240 II=1,8
0081        240 WOK3(II)=XCD(II)
0082            IF(NHUB.EQ.0)GO TO 250
0084            DO 245 II=1,NHUB
```

```
      DO 245 KK=1,18
  245 WOK3(II+8)=WOK3(II+8)+PHINH(KK,II)*XCD(KK+9)
  250 IF(NROT.EQ.0)GO TO 260
      DO 255 II=1,NROT
      DO 255 KK=1,LL
  255 WOK3(II+NN)=WOK3(II+NN)+PHINR(KK,II)*XCD(KK+27)
  260 CONTINUE
C  CALCULATE XVECD(0)=WOK2.WOK3
      DO 2700 II=1,N
 2700 XVECD(II)=0.0
      DO 270 II=1,N
      DO 270 KK=1,N
  270 XVECD(II)=XVECD(II)+WOK2(II,KK)*WOK3(KK)
C
   10 DO 20 II=1,N
   20 CALL PIEMAN(XVEC(II),XVECD(II),XVECDD(II),DT,NP(II),XCDI(II),XCI(I
     *I),XLL(II))
C
C  TRANSFORM MODAL COORDINATES INTO PHYSICAL COORDINATES
C      FIRST DERIVATIVES
C CALCULATE (PHI.XVECD) AND STORE IN WOK3
      DO 3000 II=1,84
 3000 WOK3(II)=0.0
      DO 300 II=1,8
  300 WOK3(II)=XVECD(II)
      WOK3(9)=0.
      IF(NHUB.EQ.0)GO TO 320
      DO 310 II=1,18
      DO 310 KK=1,NHUB
  310 WOK3(II+9)=WOK3(II+9)+PHINH(II,KK)*XVECD(KK+8)
      GO TO 330
  320 DO 325 II=1,18
  325 WOK3(II+9)=0.0
  330 IF(NROT.EQ.0)GO TO 340
      NN=NHUB+8
      DO 335 II=1,LL
      DO 335 KK=1,NROT
  335 WOK3(II+27)=WOK3(II+27)+PHINR(II,KK)*XVECD(KK+NN)
      GO TO 345
  340 DO 343 II=27,84,1
  343 WOK3(II)=0.0
  345 DO 50 II=1,3
      ZAD(II)=WOK3(II)
      WYH(II)=WOK3(II+3)
      PSID(II)=WOK3(II+6)
      DO 40 JJ=1,3
      NN=9+JJ+3*(II-1)
      UCD(JJ,II)=WOK3(NN)
   40 PHICD(JJ,II)=WOK3(NN+9)
      DELD(II)=WOK3(II+27)
      DO 45 JJ=1,3
      NN=30+JJ+3*(II-1)
      UDD(JJ,II,1)=WOK3(NN)
      PHIDD(JJ,II,1)=WOK3(NN+9)
      UDD(JJ,II,2)=WOK3(NN+18)
      PHIDD(JJ,II,2)=WOK3(NN+27)
      IF(P.EQ.2.)GO TO 45
      UDD(JJ,II,3)=WOK3(NN+36)
```

FIGURE 10.10 (Cont'd)

```
      PHIDD(JJ,II,3)=WOK3(NN+45)
   45 CONTINUE
   50 CONTINUE
C
C     STATE VECTOR
C   CALCULATE (PHI.XVEC) AND STORE IN WOK3
      DO 3500 II=1,84
 3500 WOK3(II)=0.0
      DO 350 II=1,8
  350 WOK3(II)=XVEC(II)
      WOK3(9)=0.
      IF(NHUB.EQ.0)GO TO 370
      DO 360 II=1,18
      DO 360 KK=1,NHUB
  360 WOK3(II+9)=WOK3(II+9)+PHINH(II,KK)*XVEC(KK+8)
      GO TO 380
  370 DO 375 II=1,18
  375 WOK3(II+9)=0.0
  380 IF(NROT.EQ.0)GO TO 390
      NN=NHUB+8
      DO 385 II=1,LL
      DO 385 KK=1,NROT
  385 WOK3(II+27)=WOK3(II+27)+PHINR(II,KK)*XVEC(KK+NN)
      GO TO 395
  390 DO 393 II=27,84,1
  393 WOK3(II)=0.0
  395 DO 70 II=1,3
      ZA(II)=WOK3(II)
      PSI(II)=WOK3(II+6)
      DEL(II)=WOK3(II+27)
      DO 60 JJ=1,3
      NN=9+JJ+3*(II-1)
      UC(JJ,II)=WOK3(NN)
      PHIC(JJ,II)=WOK3(NN+9)
      UD(JJ,II,1)=WOK3(NN+21)
      PHID(JJ,II,1)=WOK3(NN+30)
      UD(JJ,II,2)=WOK3(NN+39)
      PHID(JJ,II,2)=WOK3(NN+48)
      IF(P.EQ.2.)GO TO 60
      UD(JJ,II,3)=WOK3(NN+57)
      PHID(JJ,II,3)=WOK3(NN+66)
   60 CONTINUE
   70 CONTINUE
C   INTEGRATE THETD(1)
      CALL STLJES(THET(1),THETD(1),DT,NPASS(1),XS(1),XSL(1))
C   INTEGRATE GAMD
      DO 112 II=1,3
  112 CALL STLJES(GAM(II),GAMD(II),DT,NPASS(II+1),XS(II+1),XSL(II+1))
C   FORM THETD,THET(2),THET(3)
      CALL GMPRD(PITHET,WYH,WOK1,3,3,1)
      THETD(1)=DELD(1)+SPIN-WOK1(1,1)
      THETD(2)=DELD(2)
      THETD(3)=DELD(3)
      THET(2)=DEL(2)
      THET(3)=DEL(3)
C   FORM GAMD
      CALL GMPRD(SGAM,WYH,GAMD,3,3,1)
      RETURN
```

FIGURE 10.11 GENM

```
20.1 (AUG 71)                          OS/360   FORTRAN H

   COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                     SOURCE,BCD,NOLIST,DECK,LOAD,MAP,NOEDIT,ID,NOXREF
0002             SUBROUTINE GENM
0003             IMPLICIT REAL (A-Z)
0004             COMMON/FLX/
                *AALPHA(3,3,3),ALPHA(3),BETA(3),DEL1(3),DEL2(3,3),DEL3(3,3,3),
                *DEL4(3,3),DEL5(3,3,3),DEL6(3),DEL7(3),DEL8(3),DELTB,
                *CP(57,57),CH(18,18),CHG(6,18),DEL(3),DELD(3),
                *EIH(3),EIR(3),EH,ER,ETA,FC(3,3),FD(3,3,3),FG(3),GAM(3),GAMD(3),GH,
                *GJH(3),GJR(3),GPSID(3,3),GR,GYGB(3,3),GXL(3,3),DD(500),
                *GRH(3,3),GWR(3,3),GWX(3,3),GWXRA(3,3,3),GWY(3,3),GWYC(3,3,3),
                *GWYS(3,3),GYB(3,3),GYRN(3,3),GYH(3,3),HCMASS(3),HI,HJ,HMASS
0005             COMMON/FLX/
                *IA(3,3),IB(3,3),IC(3,3,3),ID(3,3,3,3),IH(3,3),JT(3,3),KBS1,KBS2,
                *KBVH(3,3),KBVP(3,3),KHS(3,3),KHV(3,3),LAM(84,84),LM1(3,3),
                *LM2(3,3),LM3(3,3,3),LM4(3,3),LM5(3,3,3,3),LM6(3,3),LM7(3,3,3),
                *LM8(3,3,3),LM9(3,3,3),LM10(3,3,3,3),LM11(3,3,3,3),LM12(3,3,3,3),
                *LM13(3,3,3,3),LM14(3,3,3),LM15(3,3,3),LM16(3,3,3,3),LM17(3,3,3,3),
                *LM18(3,3),LM19(3,3),LM20(3,3),LM21(3,3),LM22(3,3),LM23(3,3),
                *LM24(3,3),LM25(3,3),LM26(3,3),LM4B(3,3),LM13B(3,3,3,3),KHG(6,18),
                *LM16B(3,3,3,3),KR(57,57),KH(18,18)
0006             COMMON/FLX/
                *M,M1(3,3),M2(3,3,3),M3(3,3,3,3),M4(3,3),M5(3,3),MAST,MASS(84,84),
                *MB(3),MBEAR(3),MC(3,3),MD(3,3,3),MDOCK(3),MG(3),NROT,NHUB,
                *N,NS,P,PASS,PIBETA(3,3),PIGAM(3,3),PIPHIC(3,3,3),PIPHID(3,3,3,3),
                *PIPSI(3,3),PITHET(3,3),PHIC(3,3),PHICD(3,3),PHID(3,3,3),
                *PHIDD(3,3,3),PSI(3),PSID(3),PHIMR(57,57),PHIMH(18,18),NTHET(3,3)
0007             COMMON/FLX/
                *RDMASS(3,3),RH(3),RI,RJ,RMASS,RRMASS,RTS(84),SI1(3,3),SI2(3),
                *SI3(3,3,3),SI4(3),SIG1(3),SIG2(3,3),SIG3(3,3,3),SIG4(3,3),
                *SIG5(3,3,3),SIG6(3),SIG7(3),SIG8(3),SGAM(3,3),SMASS,STHET(3,3),
                *SUM1(3,3,3),TAUA(3),TAUC(3,3),TAUD(3,3,3),TAJL(3),TAUS(3),
                *THET(3),THETD(3),RHO(3),STHE(3,3),SPIN
0008             COMMON/FLX/
                *UC(3,3),UCD(3,3),UD(3,3,3),UDD(3,3,3),WOK1(3,3),WOK2(84,84),
                *WOK3(84),WOK4(3,3),WOK5(3,3),WOK6(3,3),WR(3),WXR(3),WXRA(3,3),
                *WXY(3),WYC(3,3),WYH(3),WZ(3,3,3),XC(84),XCD(84),XD(3,3,3),XN(3,3),
                *XL(3),XLL(84),XVEC(84),XVECD(84),XVECDD(84),YC(3,3),YB(3),YE3(3),
                *YG(3),YH(3),ZA(3),ZAD(3),ZV3(3,3)
0009             INTEGER II,JJ,KK,LL,NN
0010             DIMENSION A(3,3),B(3,3),D(3,3,3),C(3,3),E(3,3,3),F(3,3,3,3),
                *G(3,3,3,3),H(3,3,3),GX(3,84),GY(3,57)
0011             CALL GMTRA(PIGAM,A,3,3)
0012             S2=SIN(GAM(2))
0013             S3=SIN(GAM(3))
0014             C2=COS(GAM(2))
0015             C3=COS(GAM(3))
0016             C(1,1)=C3*C2
0017             C(2,1)=S3
0018             C(3,1)=0.
0019             C(1,2)=-S3*C2
0020             C(2,2)=C3
0021             C(3,2)=0.
0022             C(1,3)=S2
0023             C(2,3)=0.
0024             C(3,3)=1.
0025             CALL GMPRD(C,GYP,B,3,3,3)
          C   FW UOD0      0.0000      0.0000
```

FIGURE 10.11 (Cont'd)

```
      DO 1 II=1,3
      CALL GMADD(YC(1,II),UC(1,II),WOK1,3,1)
      CALL GMSUB(WOK1,YB,WOK4,3,1)
      CALL GAMMA(WOK1,WOK4)
    1 CALL GMPRD(C,WOK1,D(1,1,II),3,3,3)
C  FORM E
      KK=P
      DO 5 II=1,KK
      CALL GMTRA(PITHET,WOK1,3,3)
      CALL GMTRA(AALPHA(1,1,II),WOK4,3,3)
      CALL GMPRD(WOK1,WOK4,WOK5,3,3,3)
      CALL GMPRD(C,WOK5,E(1,1,II),3,3,3)
C  FORM H
      CALL GMTRA(STHE,WOK6,3,3)
      CALL GMPRD(WOK6,WOK4,H(1,1,II),3,3,3)
C  FORM F
      CALL GMPRD(AALPHA(1,1,II),PITHET,WOK4,3,3,3)
      CALL GMPRD(WOK4,YB,WOK6,3,3,1)
      CALL GMPRD(B,WOK5,WOK1,3,3,3)
      DO 5 JJ=1,3
      CALL GMADD(XD(1,JJ,II),UD(1,JJ,II),WOK4,3,1)
      CALL GMADD(WOK6,WOK4,WOK5,3,1)
      CALL GAMMA(WOK2,WOK5)
      CALL GMPRD(E(1,1,II),WOK2,WOK5,3,3,3)
      CALL GMSUB(WOK5,WOK1,F(1,1,JJ,II),3,3)
C  FORM G
      CALL GAMMA(WOK5,WOK4)
    5 CALL GMPRD(H(1,1,II),WOK5,G(1,1,JJ,II),3,3,3)
C CREATE THE NECESSARY PORTION OF THE TRANSFORMATION MATRIX
      CALL GMTRA(PITHET,WOK1,3,3)
      CALL GMPRD(C,WOK1,WOK4,3,3,3)
      DO 50 JJ=1,3
      DO 50 II=1,3
      GX(II,JJ)=B(II,JJ)
      GX(II,JJ+3)=C(II,JJ)
      GX(II,JJ+6)=0.
      GX(II,JJ+27)=WOK4(II,JJ)
      DO 10 LL=1,3
      NN=9+3*(JJ-1)
      GX(LL,II+NN)=D(LL,II,JJ)
      GX(LL,II+NN+9)=C(LL,II)
      GX(LL,II+NN+21)=F(LL,II,JJ,1)
      GX(LL,II+NN+30)=E(LL,II,1)
      GX(LL,II+NN+39)=F(LL,II,JJ,2)
      GX(LL,II+NN+48)=E(LL,II,2)
      IF(KK.EQ.2)GO TO 15
      GX(LL,II+NN+57)=F(LL,II,JJ,3)
      GX(LL,II+NN+66)=E(LL,II,3)
   15 GY(LL,II+NN-6)=G(LL,II,JJ,1)
      GY(LL,II+NN+3)=H(LL,II,1)
      GY(LL,II+NN+12)=G(LL,II,JJ,2)
      GY(LL,II+NN+21)=H(LL,II,2)
      IF(KK.EQ.2)GO TO 10
      GY(LL,II+NN+30)=G(LL,II,JJ,3)
      GY(LL,II+NN+39)=H(LL,II,3)
   10 CONTINUE
   50 GY(II,JJ)=STHE(JJ,II)
C
```

FIGURE 10.11 (Cont'd)

```
C     TRANSFORM MASS MATRIX
      LL=84
      NN=57
      IF(KK.EQ.3)GO TO 80
      LL=66
      NN=39
   80 DO 100 JJ=1,LL
      DO 100 II=1,LL
  100 MASS(II,JJ)=LAM(II,JJ)
      DO 150 JJ=1,LL
      DO 150 II=1,3
      LAM(II,JJ)=0.
      DO 150 KK=1,3
  150 LAM(II,JJ)=LAM(II,JJ)+A(II,KK)*MASS(KK,JJ)
      DO 200 JJ=1,LL
      DO 200 II=1,3
      LAM(II+3,JJ)=0.
      DO 200 KK=1,LL
  200 LAM(II+3,JJ)=LAM(II+3,JJ)+GX(II,KK)*MASS(KK,JJ)
      DO 300 JJ=1,LL
      DO 300 II=1,3
      LAM(II+27,JJ)=0.0
      DO 300 KK=1,NN
  300 LAM(II+27,JJ)=LAM(II+27,JJ)+GY(II,KK)*MASS(KK+27,JJ)
      RETURN
      ENTRY GENR
C     TRANSFORM RIGHT SIDE
      DO 500 II=1,LL
  500 XC(II)=RTS(II)
      DO 600 JJ=1,3
      RTS(JJ)=0.0
      DO 600 KK=1,3
  600 RTS(JJ)=RTS(JJ)+A(JJ,KK)*XC(KK)
      DO 700 JJ=1,3
      RTS(JJ+3)=0.0
      DO 700 KK=1,LL
  700 RTS(JJ+3)=RTS(JJ+3)+GX(JJ,KK)*XC(KK)
      DO 800 JJ=1,3
      RTS(JJ+27)=0.0
      DO 800 KK=1,NN
  800 RTS(JJ+27)=RTS(JJ+27)+GY(JJ,KK)*XC(KK+27)
      RETURN
      END
```

FIGURE 10.12 CMG

```
20.1 (AUG 71)                              OS/360  FORTRAN H

    COMPILER OPTIONS - NAME=  MAIN,OPT=02,LINECNT=60,SIZE=0000K,
                         SOURCE,BCD,NOLIST,DECK,LOAD,MAP,NOEDIT,ID,NOXREF
 0002                 SUBROUTINE CMG
 0003                 IMPLICIT REAL (A-Z)
 0004                 COMMON/TIME/T,DT,NSYM,ANTTWO
 0005                 COMMON/FLX/
                     *AALPHA(3,3,3),ALPHA(3),BETA(3),DEL1(3),DEL2(3,3),DEL3(3,3,3),
                     *DEL4(3,3),DEL5(3,3,3),DEL6(3),DEL7(3),DEL8(3),DELTB,
                     *CR(57,57),CH(18,18),CHG(6,18),DEL(3),DELD(3),
                     *EIH(3),EIR(3),EH,ER,ETA,FC(3,3),FD(3,3,3),FG(3),GAM(3),GAMD(3)
                     *GJH(3),GJR(3),GPSID(3,3),GR,GYGB(3,3),GXL(3,3),DD(500),
                     *GRH(3,3),GWR(3,3),GWX(3,3),GWXRA(3,3,3),GWY(3,3),GWYC(3,3,3),
                     *GWYS(3,3),GYB(3,3),GYBM(3,3),GYH(3,3),HCMASS(3),HI,HJ,HMASS
 0006                 COMMON/FLX/
                     *IA(3,3),IB(3,3),IC(3,3,3),ID(3,3,3,3),IH(3,3),JT(3,3),KBS1,KBS
                     *KBVH(3,3),KBVR(3,3),KHS(3,3),KHV(3,3),LAM(84,84),LM1(3,3),
                     *LM2(3,3),LM3(3,3,3),LM4(3,3),LM5(3,3,3,3),LM6(3,3),LM7(3,3,3),
                     *LM8(3,3,3),LM9(3,3,3),LM10(3,3,3,3),LM11(3,3,3,3),LM12(3,3,3,3
                     *LM13(3,3,3,3),LM14(3,3,3),LM15(3,3,3),LM16(3,3,3,3),LM17(3,3,3
                     *LM18(3,3),LM19(3,3),LM20(3,3),LM21(3,3),LM22(3,3),LM23(3,3),
                     *LM24(3,3),LM25(3,3),LM26(3,3),LM4B(3,3),LM13B(3,3,3,3),KHG(6,1
                     *LM16B(3,3,3,3),KB(57,57),KH(18,18)
 0007                 COMMON/FLX/
                     *M,M1(3,3),M2(3,3,3),M3(3,3,3,3),M4(3,3),M5(3,3),MAST,MASS(84,8
                     *MB(3),MBEAR(3),MC(3,3),MD(3,3,3),MDOCK(3),MG(3),NROT,NHUB,
                     *N,NS,P,PASS,PIBETA(3,3),PIGAM(3,3),PIPHI((3,3,3),PIPHID(3,3,3,
                     *PIPSI(3,3),PITHET(3,3),PHIC(3,3),PHICD(3,3),PHID(3,3,3),
                     *PHIDD(3,3,3),PSI(3),PSID(3),PHIMR(57,57),PHIMH(18,18),NTHET(3,
 0008                 COMMON/FLX/
                     *RDMASS(3,3),RH(3),RI,RJ,RMASS,RRMASS,RTS(84),SI1(3,3),SI2(3),
                     *SI3(3,3,3),SI4(3),SIG1(3),SIG2(3,3),SIG3(3,3,3),SIG4(3,3),
                     *SIG5(3,3,3),SIG6(3),SIG7(3),SIG8(3),SGAM(3,3),SMASS,STHET(3,3)
                     *SUM1(3,3,3),TAUA(3),TAUC(3,3),TAUD(3,3,3),TAJL(3),TAUS(3),
                     *THET(3),THETD(3),RHC(3),STHE(3,3),SPIN
 0009                 COMMON/FLX/
                     *UC(3,3),UCD(3,3),UD(3,3,3),UDD(3,3,3),WOK1(3,3),WOK2(84,84),
                     *WOK3(84),WOK4(3,3),WOK5(3,3),WOK5(3,3),WR(3),WXR(3),WXRA(3,3),
                     *WXY(3),WYC(3,3),WYH(3),WZ(3,3,3),XC(84),XCD(84),XD(3,3,3),XN(3
                     *XL(3),XLL(84),XVEC(84),XVECD(84),XVECDD(84),YC(3,3),YB(3),YE3(
                     *YG(3),YH(3),ZA(3),ZAD(3),ZV3(3,3)
 0010                 COMMON/CONT/
                     *TC(3),TAUF,START,STOP,H,WNG,ZETG,BDR,B,FE,BDE,BDEF,BDEDD,BDED,
                     *BIAS,WAIT
 0011                 DIMENSION XCOR(3),XLAS(3),NF(3)
 0012                 INTEGER II,NP
                C     THIS SUBROUTINE DEVELOPS THE CONTROL TORQUES,TC,FOR WOBBLE DAMPI
                C     THE WOBBLE DAMPING TECHNIQUE CONSISTS FO A SGCMG THAT IS DRIVEN
                C     THAT IT'S ANGULAR MOMENTUM VECTOR LAGS THE WOBBLE VECTOR BY 90 DI
                C ANGULAR RATES OF BODY THAT CMG IS WITHIN
 0013                 WR1=WXR(1)
 0014                 WR2=WXR(2)
 0015                 WR3=WXR(3)
 0016                 IF(PASS.GT.0.)GO TO 400
 0018                 DONE=1.
 0019                 AL=0.
 0020                 BDED=-WR1
 0021                 BDEF=0.
 0022                 B=0.0
```

FIGURE 10.12 (Cont'd)

```
        DO 1 II=1,3
      1 NP(II)=0.0
C TEST FOR START AND STOP
    400 IF(DONE.EQ.0.)GO TO 431
        IF(SQRT(WR2**2+WR3**2)-START)430,430,432
    432 DONE=0.
        GO TO 41
    431 IF(SQRT(WR2**2+WR3**2)-STOP)410,410,41
    410 DONE=1.
    430 BDP=-WR1
        XCOR(1)=BDP
        XLAS(1)=0.
        BDEDD=0.
        IF(B.GE.0.)GO TO 415
        B=6.2831853+B
        XCOR(2)=6.2831853+XCOR(2)
    415 CONTINUE
        GO TO 40
     41 IF(ABS(WR2)-.0000001)42,42,15
     15 XY=ABS(WR3)/ABS(WR2)
        FE=ATAN(XY)
        GO TO 43
     42 FE=1.5707963
     43 IF(WR2)18,17,17
     17 IF(WR3)50,50,48
     48 BDE=FE
C TRANSFER FOM QUAD4 TO QUAD1
        IF(AL.EQ.1.)GO TO 480
        AL=0.
        GO TO 36
    480 BDEF=BDEF-6.2831853
        XCOR(3)=XCOR(3)-6.2831853
        AL=0.
        GO TO 36
     50 BDE=6.2831853-FE
C TRANSFER FROM OLADU TO QUAD4
        IF(AL.EQ.0.)GO TO 490
        AL=1.
        GO TO 36
    490 BDEF=BDEF+6.2831853
        XCOR(3)=XCOR(3)+6.2831853
        AL=1.
        GO TO 36
     18 IF(WR3)32,31,31
     31 BDE=3.1415926-FE
        AL=-1.
        GO TO 36
     32 BDE=3.1415926+FE
        AL=-1.
C LOW PASS FILTER
     36 IF(TAUF)601,601,605
    601 BDEF=BDE
        BDEFF=BDEF
        BDEFD=0.0
        GO TO 610
    605 BDEFD=(BDE-BDEF)/TAUF
C  BIAS THE COMMANDED ANGLE(BY BIA)TO COMPENSATE FOR FILTER LAG.
C  BIAS=0. FOR NO BIAS.  BIAS=-CR+WOBBLE FREQ.  MINUS FOR MINI
```

FIGURE 10.12 (Cont'd)

```
          BIA=BIAS*TAUF
          BDEB=BDEF+BIA
C
CONSTRAIN FILTER OUTPUT BTWN 0 AND 2PI
          IF(BDEB.GT.6.2831853)GO TO 495
          IF(BDEB.LT.0.)GO TO 496
          BDEFF=BDEB
          GO TO 610
      495 BDEFF=BDEB-6.2831853
          GO TO 610
      496 BDEFF=6.2831853+BDEB
C DYNAMICS-TORQUER MOTOR+GIMBAL
      610 IF(B)530,532,532
      530 B=6.2831853+B
          XCOR(2)=XCOR(2)+6.2831853
          GO TO 535
      532 IF(B.LE.6.2831853)GO TO 535
          B=B-6.2831853
          XCOR(2)=XCOR(2)-6.2831853
      535 EB=BDEFF-B
          IF(ABS(EB).LE.3.1415926)GO TO 305
          EB1=6.2831853-ABS(EB)
          EB=-SIGN(EB1,EB)
C   WAIT OPTION FOR ACTIVE CONTROL
      305 IF(WAIT.EQ.0.)GO TO 30
          IF(ABS(EB).GT.WAIT)GO TO 430
       30 BDEDD=EB*WNG**2-(2.*ZETG*WNG)*BDED
      332 IF(ABS(BDED)-1.) 34,34,33
       33 BDR=SIGN(1.,BDED)
          GO TO 40
       34 BDR=BDED
       40 CONTINUE
C LIMIT NEGATIVE RATE TO MINUS SPIN
      620 IF(BDED.LT.-WR1)BDR=-WR1
C 90 DEG H-LAG-LAW     B=ANGLE BTWN INNER GMBL  AXIS  AND THE 2 AXIS
      420 TC(2)=(-WR1-BDR)*H*COS(B)
          TC(3)=(-WR1-BDR)*H*SIN(B)
          TC(1)=(WR3*SIN(B)+WR2*COS(B))*H
          CALL STLJES(BDED,BDEDD,DT,NP(1),XCOR(1),XLAS(1))
          CALL STLJES(B,BDR,DT,NP(2),XCOR(2),XLAS(2))
          CALL STLJES(BDEF,BDEFD,DT,NP(3),XCOR(3),XLAS(3))
C  APPLY TORQUES TO BODY THAT CMG IS LOCATED WITHIN
          TAUL(1)=TC(1)
          TAUL(2)=TC(2)
          TAUL(3)=TC(3)
          RETURN
          END
```

FIGURE 10.13 RIEMAN

```
      SUBROUTINE RIEMAN(Y,Y1,Y2,DT,NPASS,Y1C,YC,Y2L)
C
C       THIS SUBROUTINE IS A MODIFIED-ADAMS INTEGRATION FOR A SECOND
C     ORDER DIFFERENTIAL EQUATION. THE VARIABLES USED ARE AS FOLLOWS --
C
C    *      Y- DEPENDENT VARIABLE AS PREDICTED BY THIS INTEGRATION
C    *     Y1- FIRST DERIVATIVE OF Y (ALSO PREDICTED HERE)
C          Y2- SECOND DERIVATIVE OF Y FROM MAIN PROGRAM (PER DIF. EQ.)
C          DT- INDEPENDENT VARIABLE
C    *  NPASS- 0 INITIALLY AND CHANGED TO 1 BY THIS SUBROUTINE
C    *    Y1C- CORRECTED VALUE OF Y1 COMPUTED FROM Y2L AND Y2
C    *     YC- CORRECTED VALUE OF Y COMPUTED FROM Y1L AND Y1
C    *    Y1L- LAST VALUE OF Y1
C    *    Y2L- LAST VALUE OF Y2
C
C    *NOTE- ALL THE VARIABLES WITH AN ASTERISK MUST BE SAVED IN THE
C         CALLING PROGRAM. THE CALLING PROGRAM MUST INTIALIZE NPASS,
C         Y,Y1 AND DT. Y2 IS COMPUTED IN THE CALLING PROGRAM FROM THE
C         GOVERNING DIFFERENTIAL EQUATION. A SAMPLE CALLING SEQUENCE IS.
C
C     DO 1 I=1,10
C     X2DOT(I)=-2.*ZETA(I)*OMEGAN(I)*X1DOT(I)-OMEGAN(I)**2*X(I)+U(I)
C   1 CALL RIEMAN (X(I),X1DOT(I),X2DOT(I),.001,NP(I),XCOR(I),XC(I),X1L(I
C    1 ),X2L(I) )
C     SOLVING THE TEN DIFFERENTIAL EQUATIONS...
C
C         D2X          DX      2
C         ---I +  2Z O --I +   O  X  = U     FOR I=1,10
C         DT2      I I DT      I  I    I
C
C
      IF(NPASS)5,3,5
    3 Y2L=Y2
      Y1C=Y1
      YC=Y
      NPASS=1
      GO TO 6
    5 YC=YC+Y1C*DT+(Y2+2.*Y2L)/6.*DT**2
      Y1C=Y1C+DT*(Y2+Y2L)/2.
    6 Y=YC+Y1C*DT+DT**2*(4.*Y2-Y2L)/6.
      Y1=Y1C+DT*( 3.*Y2-Y2L)/2.
      Y2L=Y2
      RETURN
      END
```

FIGURE 10.14 STLJES

```
      SUBROUTINE STLJES(Y,Y1,DT,NPASS,YC,Y1L)
      IF ( NPASS .NE. 0 ) GO TO 5
      Y1L=Y1
      YC=Y
      NPASS=1
      GO TO 6
    5 YC=YC+DT*(Y1+Y1L)/2.
    6 Y=YC+DT*( 3.*Y1-Y1L)/2.
      Y1L=Y1
      RETURN
      END
```

FIGURE 10.15 GAMMA

```
C SUBROUTINE   GAMMA                                                    FLX00870
      SUBROUTINE GAMMA (GAM,ANG)                                        FLX00880
C     THIS ROUTINE SETS UP THE GAMMA ARRAY FROM THREE ANGLES            FLX00890
C                                                                       FLX00900
C     PARAMETERS ARE AS FOLLOWS...                                      FLX00910
C                                                                       FLX00920
C     GAM                               3X3 GAMMA ARRAY                 FLX00930
C     ANG                               3X1 ANGLE ARRAY                 FLX00940
C                                                                       FLX00950
      DIMENSION GAM(9),ANG(3)                                           FLX00960
      DO 5 I=1,9,4                                                      FLX00970
    5 GAM(I)=0.0                                                        FLX00980
      GAM(2)=ANG(3)                                                     FLX00990
      GAM(3)=-ANG(2)                                                    FLX01000
      GAM(4)=-ANG(3)                                                    FLX01010
      GAM(6)=ANG(1)                                                     FLX01020
      GAM(7)=ANG(2)                                                     FLX01030
      GAM(8)=-ANG(1)                                                    FLX01040
      RETURN                                                            FLX01050
      END                                                               FLX01060
```

FIGURE 10.16 STACK

```
C SUBROUTINE STACK                                                      FLX01310
      SUBROUTINE STACK(MATIN,MATOUT,MINR,MINC,MOUTOR,MC1,MC2)           FLX01320
C                                                                       FLX01330
C     THIS ROUTINE STACKS SMALL MATRICIES IN A LARGE MATRIX             FLX01340
C     PARAMETERS ARE AS FOLLOWS...                                      FLX01350
C                                                                       FLX01360
C     MATIN       INPUT SMALL MATRIX                                    FLX01370
C     MATOUT      OUTPUT MATRIX                                         FLX01380
C     MINR        NUMBER OF ROWS IN INPUT MATRIX                        FLX01390
C     MINC        NUMBER OF COLUMNS IN INPUT MATRIX                     FLX01400
C     MOUTOR      ORDER OF OUTPUT MATRIX                                FLX01410
C     MC1         ROW COORDINATE OF STARTING LOCATION                   FLX01420
C     MC2         COLUMN COORDINATE OF STARTING LOCATION                FLX01430
C                                                                       FLX01440
      DIMENSION MATIN(MINR,MINC),MATOUT(MOUTOR,MOUTOR)                  FLX01450
      DO 10 J=1,MINC                                                    FLX01460
      DO 10 I=1,MINR                                                    FLX01470
   10 MATOUT(MC1-1+I,MC2-1+J)=MATIN(I,J)                                FLX01480
      RETURN                                                            FLX01490
      END                                                               FLX01500
```

FIGURE 10.17 ABCP1

```
C SUBROUTINE ABCPI                                                  FLX0001
      SUBROUTINE ABCPI(A,PI,ANG,K)
C     THIS ROUTINE GENERATES THE 3X3 MATRICIES A AND PI
C                                                                   FLX0010
C     A                               3X3 REAL MATRIX A             FLX0015
C     B                               3X3 REAL MATRIX B             FLX0016
C     C                               3X3 REAL MATRIX C             FLX0017
C     PI                              3X3 REAL MATRIX PI            FLX0018
C     K - CONTROL LOGIC
C       - 0 CALCULATE A MATRIX
C       - 1 CALCULATE A,PI MATRIX
C                                                                   FLX0019
C     ANG                             3 ELEMENT ARRAY OF ANGLES     FLX0020
C                                                                   FLX0014
      DIMENSION TEMP(9),A(9),B(9),C(9),ANG(3),PI(9)
      DO 1 I=2,4
    1 A(I)=0.                                                       FLX0053
      C1=DCOS(DBLE(ANG(1)))
      S1=DSIN(DBLE(ANG(1)))
      A(7)=0.                                                       FLX0056
      A(1)=1.                                                       FLX0057
      A(5)=C1                                                       FLX0058
      A(6)=-S1                                                      FLX0059
      A(8)=S1                                                       FLX0060
      A(9)=C1                                                       FLX0061
      IF(K.EQ.0) GO TO 100
      DO 2 I=2,8,2
    2 B(I)=0.                                                       FLX0064
      C2=DCOS(DBLE(ANG(2)))
      S2=DSIN(DBLE(ANG(2)))
      B(1)=C2                                                       FLX0067
      B(3)=S2                                                       FLX0068
      B(5)=1.                                                       FLX0069
      B(7)=-S2                                                      FLX0070
      B(9)=C2                                                       FLX0071
      DO 3 I=6,8
    3 C(I)=0.                                                       FLX0074
      C3=DCOS(DBLE(ANG(3)))
      S3=DSIN(DBLE(ANG(3)))
      C(3)=0.                                                       FLX0077
      C(1)=C3                                                       FLX0078
      C(2)=-S3                                                      FLX0079
      C(4)=S3                                                       FLX0080
      C(5)=C3                                                       FLX0081
      C(9)=1.0                                                      FLX0082
      CALL GMPRD (C,B,TEMP,3,3,3)
      CALL GMPRD (TEMP,A,PI,3,3,3)                                  FLX0085
  100 RETURN                                                        FLX0051
      END
```

FIGURE 10.18 SMPY

```
C       SUBROUTINE SMPY
C       PURPOSE
C          MULTIPLY EACH ELEMENT OF A MATRIX BY A SCALAR TO FORM A
C          RESULTANT MATRIX
C
C       USAGE
C          CALL SMPY(A,C,R,N,M)
C
C       DESCRIPTION OF PARAMETERS
C          A - NAME OF INPUT MATRIX
C          C - SCALAR
C          R - NAME OF OUTPUT MATRIX
C          N - NUMBER OF ROWS IN MATRIX A AND R
C          M - NUMBER OF COLUMNS IN MATRIX A AND R
C
        SUBROUTINE SMPY(A,C,R,N,M)
        DIMENSION A(1),R(1)
        IT=N*M
        DO 1 I=1,IT
      1 R(I)=A(I)*C
        RETURN
        END
```

FIGURE 10.19 GMADD

```
C       SUBROUTINE GMADD
C       PURPOSE
C          ADD TWO GENERAL MATRICES TO FORM RESULTANT GENERAL MATRIX
C
C       USAGE
C          CALL GMADD(A,B,R,N,M)
C
C       DESCRIPTION OF PARAMETERS
C          A - NAME OF FIRST INPUT MATRIX
C          B - NAME OF SECOND INPUT MATRIX
C          R - NAME OF OUTPUT MATRIX
C          N - NUMBER OF ROWS IN A,B,R
C          M - NUMBER OF COLUMNS IN A,B,R
C
        SUBROUTINE GMADD(A,B,R,N,M)
        DIMENSION A(1),B(1),R(1)
        NM=N*M
        DO 10 I=1,NM
     10 R(I)=A(I)+B(I)
        RETURN
        END
```

FIGURE 10.20 SCLA

```
C        SUBROUTINE SCLA
C
C        PURPOSE
C           SET EACH ELEMENT OF A MATRIX EQUAL TO A GIVEN SCALAR
C
C        USAGE
C           CALL SCLA(A,C,N,M)
C
C        DESCRIPTION OF PARAMETERS
C           A - NAME OF INPUT MATRIX
C           C - SCALAR
C           N - NUMBER OF ROWS IN MATRIX A
C           M - NUMBER OF COLUMNS IN MATRIX A
C
      SUBROUTINE SCLA(A,C,N,M)
      DIMENSION A(1)
      IT=N*M
      DO 1 I=1,IT
    1 A(I)=C
      RETURN
      END
```

FIGURE 10.21 GMTRA

```
C        SUBROUTINE GMTRA
C        PURPOSE
C           TRANSPOSE A GENERAL MATRIX
C
C        USAGE
C           CALL GMTRA(A,R,N,M)
C
C        DESCRIPTION OF PARAMETERS
C           A - NAME OF MATRIX TO BE TRANSPOSED
C           R - NAME OF RESULTANT MATRIX
C           N - NUMBER OF ROWS OF A AND COLUMNS OF R
C           M - NUMBER OF COLUMNS OF A AND ROWS OF R
C
      SUBROUTINE GMTRA(A,R,N,M)
      DIMENSION A(1),R(1)
      IR=0
      DO 10 I=1,N
      IJ=I-N
      DO 10 J=1,M
      IJ=IJ+N
      IR=IR+1
   10 R(IR)=A(IJ)
      RETURN
      END
```

FIGURE 10.22 GMPDR

```
C        SUBROUTINE GMPRD
C        PURPOSE
C           MULTIPLY TWO GENERAL MATRICES TO FORM A RESULTANT GENERAL
C           MATRIX
C
C        USAGE
C           CALL GMPRD(A,B,R,N,M,L)
C
C        DESCRIPTION OF PARAMETERS
C           A - NAME OF FIRST INPUT MATRIX
C           B - NAME OF SECOND INPUT MATRIX
C           R - NAME OF OUTPUT MATRIX
C           N - NUMBER OF ROWS IN A
C           M - NUMBER OF COLUMNS IN A AND ROWS IN B
C           L - NUMBER OF COLUMNS IN B
C
         SUBROUTINE GMPRD(A,B,R,N,M,L)
         DIMENSION A(1),B(1),R(1)
         IR=0
         IK=-M
         DO 10 K=1,L
         IK=IK+M
         DO 10 J=1,N
         IR=IR+1
         JI=J-N
         IB=IK
         R(IR)=0
         DO 10 I=1,M
         JI=JI+N
         IB=IB+1
      10 R(IR)=R(IR)+A(JI)*B(IB)
         RETURN
         END
```

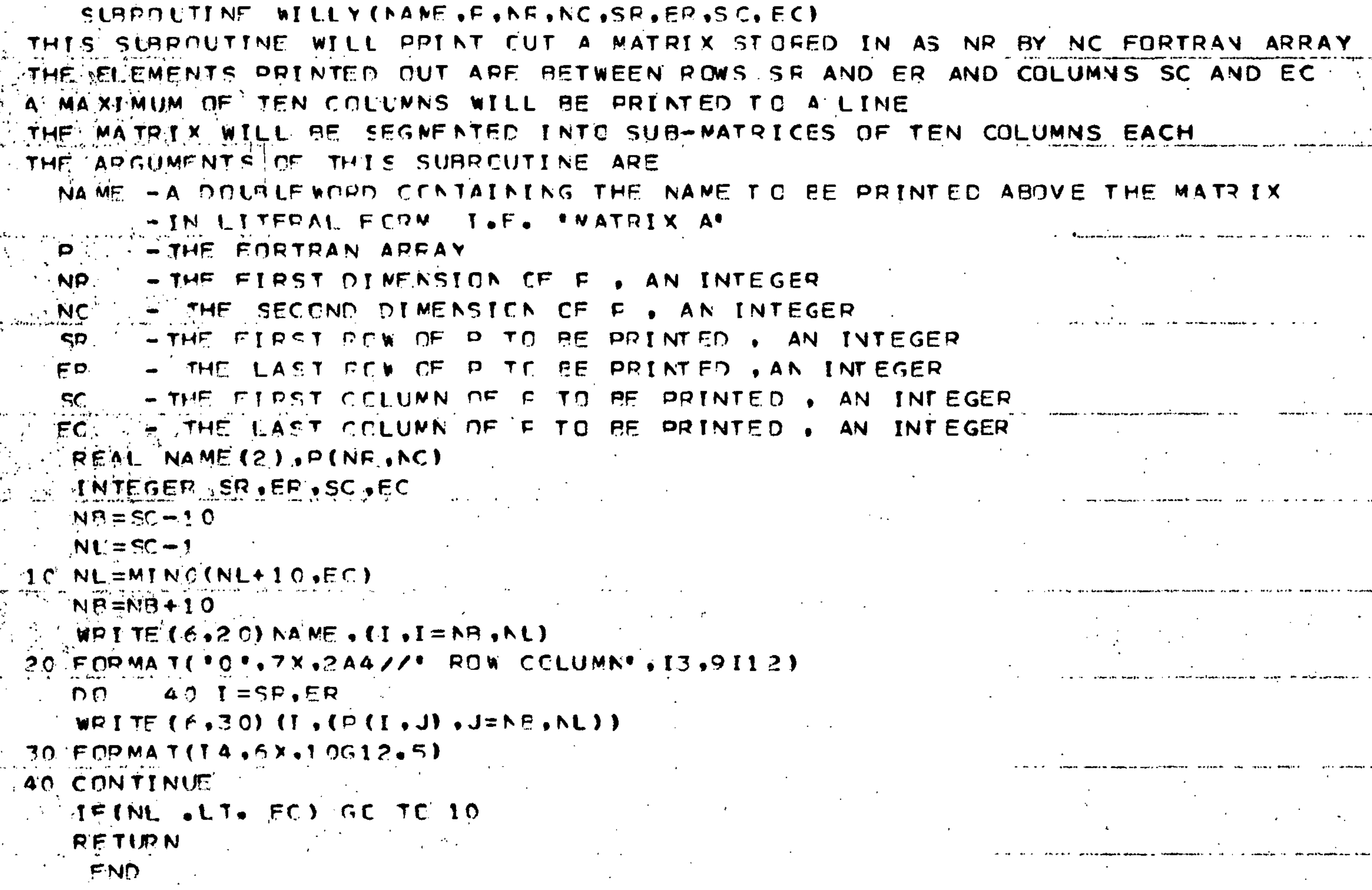
FIGURE 10.23 GMSUB

```
C        SUBROUTINE GMSUB
C        PURPOSE
C           SUBTRACT ONE GENERAL MATRIX FROM ANOTHER TO FORM RESULTANT
C           MATRIX
C
C        USAGE
C           CALL GMSUB(A,B,R,N,M)
C
C        DESCRIPTION OF PARAMETERS
C           A - NAME OF FIRST INPUT MATRIX
C           B - NAME OF SECOND INPUT MATRIX
C           R - NAME OF OUTPUT MATRIX
C           N - NUMBER OF ROWS IN A,B,R
C           M - NUMBER OF COLUMNS IN A,B,R
C
         SUBROUTINE  GMSUB(A,B,R,N,M)
         DIMENSION A(1),B(1),R(1)
         NM=N*M
         DO 10 I=1,NM
      10 R(I)=A(I)-B(I)
         RETURN
         END
```

FIGURE 10.24 SREVN1

```
      SUBROUTINE SREVN1(A,M,LOC,MID,NIX)
C
C   MATRIX INVERSION
C
C  A-CONVENTIONAL FORTRAN DOUBLE ARRAY CONTAINING MATRIX TO BE INVERTED
C  M- MATRIX ORDER
C  LOC-  SINGLE ARRAY DIMENSIONED AT LEAST TO M
C  MID-  FIRST DIMENSION OF A,NOT LESS THAN M
C  NIX-  ERROR INDICATOR, SET TO ZERO AFTER SUCCESSFUL EXECUTION.
C
      DIMENSION A(MID,1)
      INTEGER LOC(1)
  100 N = M
      DO 190  K = 1,N
      PIVOT = 0.0
      DO 120  I = K,N
      IF (PIVOT -  ABS(A(I,K))) 110,110,120
  110 PIVOT =  ABS(A(I,K))
      L = I
  120 CONTINUE
      IF (PIVOT) 140,130,140
  130 NIX = -1
      GO TO 210
  140 LOC(K) = L
      DO 150  J = 1,N
      TEMP1 = A(K,J)
      A(K,J) = A(L,J)
  150 A(L,J) = TEMP1
      TEMP1 = A(K,K)
      A(K,K) = 1.0
      DO 160  J = 1,N
  160 A(K,J) = A(K,J)/TEMP1
      DO 190  I = 1,N
      IF (I - K) 170,190,170
  170 TEMP1 = -A(I,K)
      A(I,K) = 0.0
      DO 180   J = 1,N
  180 A(I,J) = A(I,J) + TEMP1*A(K,J)
  190 CONTINUE
      DO 200  K = 1,N
      NK = N - K
      L = LOC(NK+1)
      DO 200  I = 1,N
      TEMP1 = A(I,NK+1)
      A(I,NK+1) = A(I,L)
  200 A(I,L) = TEMP1
      NIX = 0
  210 RETURN
      END
```

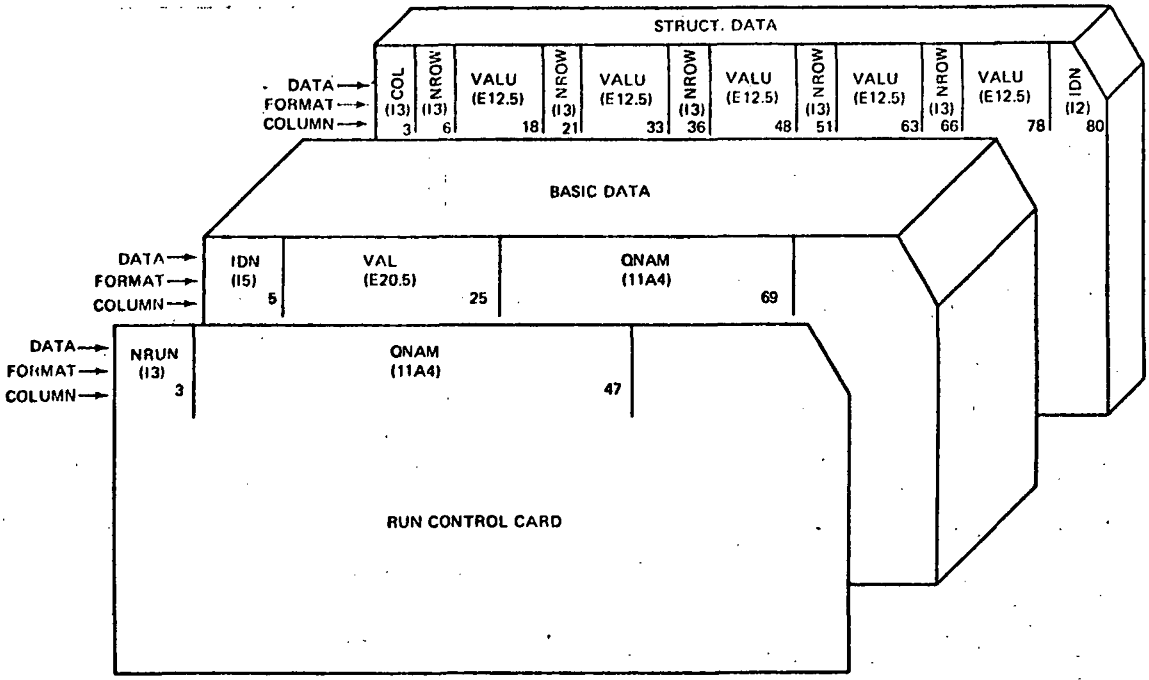
FIGURE 10.25 WILLY



```
      SUBROUTINE WILLY(NAME,P,NR,NC,SR,ER,SC,EC)
C     THIS SUBROUTINE WILL PRINT OUT A MATRIX STORED IN AS NR BY NC FORTRAN ARRAY
C     THE ELEMENTS PRINTED OUT ARE BETWEEN ROWS SR AND ER AND COLUMNS SC AND EC
C     A MAXIMUM OF TEN COLUMNS WILL BE PRINTED TO A LINE
C     THE MATRIX WILL BE SEGMENTED INTO SUB-MATRICES OF TEN COLUMNS EACH
C     THE ARGUMENTS OF THIS SUBROUTINE ARE
C        NAME -A DOUBLEWORD CONTAINING THE NAME TO BE PRINTED ABOVE THE MATRIX
C             -IN LITERAL FORM  I.E. 'MATRIX A'
C        P    -THE FORTRAN ARRAY
C        NR   -THE FIRST DIMENSION OF P , AN INTEGER
C        NC   - THE SECOND DIMENSION OF P , AN INTEGER
C        SR   -THE FIRST ROW OF P TO BE PRINTED , AN INTEGER
C        ER   - THE LAST ROW OF P TO BE PRINTED ,AN INTEGER
C        SC   -THE FIRST COLUMN OF P TO BE PRINTED , AN INTEGER
C        EC   - THE LAST COLUMN OF P TO BE PRINTED , AN INTEGER
       REAL NAME(2),P(NR,NC)
       INTEGER SR,ER,SC,EC
       NB=SC-10
       NL=SC-1
   10  NL=MIN0(NL+10,EC)
       NB=NB+10
       WRITE(6,20) NAME,(I,I=NB,NL)
   20  FORMAT('0',7X,2A4//' ROW COLUMN',I3,9I12)
       DO   40 I=SR,ER
       WRITE(6,30) (I,(P(I,J),J=NB,NL))
   30  FORMAT(I4,6X,10G12.5)
   40  CONTINUE
       IF(NL .LT. EC) GO TO 10
       RETURN
        END
```

10.1.3 Operating Instructions

Program operation is governed by three groups of input data cards: RUN CONTROL CARD, BASIC DATA CARDS, and STRUCTURAL DATA CARDS. The required order for these groups is shown below along with the required format and corresponding information provided by each data card.



RUN CONTROL CARD - The first data card for each set of data representing a single run. NRUN is an integer variable defining the number of the run to be executed. If NRUN should equal zero all running is terminated. Multiple runs are accomplished by stacking data sets. The last data set must contain a RUN CONTROL CARD only, with NRUN = 0; this will terminate the day's running. NRUN also serves another logical function and that is to control the input of STRUCTURAL DATA. Specifically, if NRUN = 1 a complete set of STRUCTURAL DATA must be read in; if NRUN > 1 then the STRUCTURAL DATA used for the previous run is acceptable.

QNAM is an alphanumeric variable used to identify the run to be executed.

BASIC DATA - Defines: vehicle geometry, mass properties, initial state, CMG control logic and hardware characteristics, and general simulation timing logic for printing, plotting and integration.

Each data card defines a single input variable whose value is VAL, and is identified by it's identification number IDN. All BASIC DATA are initialized to zero internal to the program, so that only values other than zero need be read-in. Subsequent runs require data cards only to define changes to BASIC DATA. A blank card is required at the end of the BASIC DATA group so as to terminate data read-in. Subsequent runs that do not require changes to BASIC DATA still require a blank card in the BASIC DATA position of the data set.

Table 10.2 provides a correlation between identification numbers and the corresponding Basic Data variables. These variables have been defined in Table 10.1, however, a remarks column is added for additional clarification. For the case of a multiple dimensioned input array, the range of IDN is given. It should be noted that a multiple dimensioned array is internally stored in a stacked column arrangement; for example, a triple array A(3,3,3) will have an IDN range from (n + 1) to (n + 27) where the A(3,1,1) element will have a corresponding (n + 3) identification number, and the A(1,1,3) element will have a corresponding (n + 19) IDN, etc.

QNAM is an alphanumeric variable used to describe the BASIC DATA variable.

STRUCTURAL DATA - Defines: Modal Matrix, Stiffness Matrix, and Damping Matrix for the rotor and flexible portion of hub. This data is provided by the Modal Service Program described in Section 10.2. All the Structrual Data matrices are initialized to zero; therefore, only those elements with non-zero values require input data. Each card can contain data for as much as five elements of the matrix that belong to a common column (COL). VALU represents the data value contained in element (NROW, COL); where NROW is the row number that immediately precedes it's corresponding VALU on the data card.

TABLE 10.2

BASIC DATA IDENTIFICATION

NOTE: The range of all indices is 3.

IDN	Program Variable	Remarks
1	DTPLOT	IF DTPLOT = 0, no plots will be generated IF DTPLOT > 0, calcomp plots are scaled at DTPLOT seconds per inch.
2	DT	Iteration Time (sec). Should not be greater than 1/20 the period of the largest freq.
3	TEND	Run Duration (seconds)
4	DTPRNT	Print every DTPRNT sec.
5	NS	= 0. Shuttle not attached 1. Shuttle attached
6	P	Number of rotor arms. 2 for T, 3 for Y.
7	HMASS	Weight (lbs) of each body. Note: Only the weight of the lumped masses in the first arm are read-in RDMASS(I, 1). Masses of other arms are set equal to arm #1.
8	SMASS	
9	RRMASS	
10-12	HCMASS (I)	
13-15	RDMASS (I, 1)	
16-24	IA (I, J)	Inertia (slug-ft^2) of each body. Note: Inertia properties of each arm is assumed identical.
25-33	IH (I, J)	
34-42	IB (I, J)	
43-69	IC (I, J, K)	
70-96	ID (I, J, K, 1)	
97-99	YH (I)	Vehicle Geometry. (feet)
100-102	RH (I)	
103-105	YB (I)	
106-108	YG (I)	
109-111	YE3 (I)	
112-114	XL (I)	
115-117	XN (I 1)	
118-120	ZV3 (I, 1)	
201-209	YC (I, J)	
121-129	XD (I, J, 1)	
130	α^2	Locate second arm (rad), locate third arm (rad). Note: first arm is the zero reference.
131	α^3	
132-134	β	Locate shuttle (radians)

IDN	Program Variable	Remarks	
135	KBS1	Stiffness and Damping at bearning and Shuttle connection, (feet, lb, rad, sec)	
136	KBS2		
137	DELTB		
138	ETA		
139-147	KBVR (I, J)		
148-156	KBVH (I, J)		
157-165	KHS (I, J)		
166-174	KHV (I, J)		
200	SPIN	(RAD/SEC)	
210	NROT	-minimum is one for spin.	Number of flexible modes.
211	NHUB		
212	NSYM		
213	ANTTWO	-normally set to zero.	
214	PBLOCK	-3 variables plotted per block	
220	CONTR		CMG Variables
221	STOP	(rad/sec)	
222	START	(rad/sec)	
223	TAUF	(sec)	
224	H	(ft-lb-sec)	
225	WNG	(rad/sec)	
226	ZETG		
227	BIAS	= wobble freq. (rad/sec). + for MaxI,-for MinI. For no bias set = 0.	
228	WAIT	(radians)	
301-302	ZA (I)	Initial vehicle state (feet, RAD, SEC)	
370-309	PSI (I)		
310-318	UC (I, J)		
319-327	PHIC (I, J)		
331-339	UD (I, J, 1)		
340-348	PHID (I, J. 1)		
349-357	UD (I, J, 2)		
358-366	PHID (I, J, 2)		
367-375	UD (I, J, 3)		
376-384	PHID (I, J, 3)		
401-402	ZAD (I)		
404-406	WYH (I)		
407-409	PSID (I)		
410-418	UCD (I, J)		
419-427	PHICD (I, J)		
428-430	WXY (I)		
431-439	UDD (I, J, 1)		
440-448	PHIDD (I, J, 1)		
449-457	UDD (I, J,2)		
458-466	PHIDD (I, J, 2)		
467-475	UDD (I, J, 3)		
476-484	PHIDD (I, J, 3)		
485-487	GAM (I)		
488-490	THET (I)		

There should be no blank fields between one NROW/VALU combination and another. In addition, although all the NROW/VALU fields on one card need not be filled, data must be placed on the card from left to right.

The data found on a particular card belongs to the matrix identified by IDN. The correlation between IDN and the corresponding structural data matrix, is shown in Table 10.3. It is noted that IDN will always appear in column 80. A zero in column 80 will terminate STRUCTURAL DATA read-in; hence, a blank card is required at the end of the STRUCTURAL DATA group.

For subsequent runs that do not require new structural data, a blank card is needed in place of the STRUCTURAL DATA group. If new data is desired, then an entire STRUCTURAL DATA group is required along with NRUN set to 1. There is one exception to the above outlined multiple run precedure and that is when the number of rotor modes (either total or symmetric) is changed. For this case, only new rotor modal data (PHIMR) need be read-in, provided NRUN is not set to 1.

TABLE 10.3

STRUCTURAL DATA IDENTIFICATION

IDN	Structural Data Matrix	Description
1	PHIMR	Rotor Modal Matrix
2	PHIMH	Hub Modal Matrix
3	KTA or KYA	Rotor Stiffness Matrix
4	CT or CY	Rotor Damping Matrix
5	KH	Hub Stiffness Matrix
6	CH	Hub Damping Matrix
7	KHG	Hub Stiffness About Point G.
8	CHG	Hub Damping About Point G.

10.1.4 SAMPLE RUN

A CMG controlled "Y" space station with a shuttle docked to its hub is the set of conditions chosen to demonstrate THP operation. This condition was chosen since it is one that yields the maximum number of print variables per print cycle.

The RUN CONTROL CARD and the BASIC DATA group is printed at the beginning of the run as shown in Table 10.4. Referring to Table 10.4, the argument of the DD array is IDN of the corresponding Basic Data variable. Any Basic Data omitted from the print indicates a zero value. For futher clarification of the variable definitions refer to Tables 10.1 and 10.2.

TABLE 10.4 SAMPLE RUN; INPUT DATA

```
RUN NUMBER=  1          Y  CMG CONTROL

     DD(     1)=             0.20000E 02      DTPLOT      TIME PER INCH OF PLT
     DD(     2)=             0.10000E 00     DT
     DD(     3)=             0.20000E 03     TEND
     DD(     4)=             0.50000E 01     DTPRNT
     DD(     5)=             0.10000E 01     NS  SHUTTLE ATTACHED
     DD(     6)=             0.30000E 01     P - NO. OF ARMS
     DD(     7)=             0.23000E 05     HMASS
     DD(     8)=             0.31694E 06      SMASS   SHUTTLE MASS
     DD(     9)=             0.19000E 05     RRMASS - ROTOR RING MASS (LBS)
     DD(    10)=             0.45000E 04      HCMASS
     DD(    11)=             0.35000E 04      HCMASS
     DD(    12)=             0.12500E 06       HCMASS
     DD(    13)=             0.72500E 04     RDMASS(1) - LUMPED ROTOR MASSES(LBS)
     DD(    14)=             0.72500E 04     RDMASS(2) - LUMPED ROTOR MASSES(LBS)
     DD(    15)=             0.12300E 06     RDMASS(3) - LUMPED ROTOR MASSES(LBS)
     DD(    16)=             0.20076E 07      IA
     DD(    20)=             0.26869E 07      IA
     DD(    24)=             0.68264E 06      IA
     DD(    25)=             0.18038E 09      IH(1,1)   SHUTTLE INERTIA
     DD(    29)=             0.17877E 09     IH(2,2)    SHUTTLE INERTIA
     DD(    33)=             0.27250E 07     IH(3,3)    SHUTTLE INERTIA
     DD(    34)=             0.12692E 05     IB(1,1) - ROTOR RING INERTIA
     DD(    38)=             0.32781E 05     IB(2,2) - ROTOR RING INERTIA
     DD(    42)=             0.32781E 05     IB(3,3) - ROTOR RING INERTIA
     DD(    43)=             0.14160E 04     IC(1,1,1) - INERTIA OF LUMPED HUB MASS
     DD(    47)=             0.57580E 04     IC(2,2,1) - INERTIA OF LUMPED HUB MASS
     DD(    51)=             0.57580E 04     IC(3,3,1) - INERTIA OF LUMPED HUB MASS
     DD(    52)=             0.10050E 04     IC(1,1,2) - INERTIA OF LUMPED HUB MASS
     DD(    56)=             0.31200E 04     IC(2,2,2) - INERTIA OF LUMPED HUB MASS
     DD(    60)=             0.31200E 04     IC(3,3,2) - INERTIA OF LUMPED HUB MASS
     DD(    61)=             0.16441E 07     IC(1,1,3) - INERTIA OF LUMPED HUB MASS
     DD(    65)=             0.20349E 07     IC(2,2,3) - INERTIA OF LUMPED HUB MASS
     DD(    69)=             0.48576E 06     IC(3,3,3) - INERTIA OF LUMPED HUB MASS
     DD(    70)=             0.22731E 05     ID(1,1,1) - INERTIA OF LUMPED ROTOR MASS
     DD(    74)=             0.22731E 05     ID(2,2,1) - INERTIA OF LUMPED ROTOR MASS
     DD(    78)=             0.20830E 04     ID(3,3,1) - INERTIA OF LUMPED ROTOR MASS
     DD(    79)=             0.22731E 05     ID(1,1,2) - INERTIA OF LUMPED ROTOR MASS
     DD(    83)=             0.22731E 05     ID(2,2,2) - INERTIA OF LUMPED ROTOR MASS
     DD(    87)=             0.20830E 04     ID(3,3,2) - INERTIA OF LUMPED ROTOR MASS
     DD(    88)=             0.46948E 06     ID(1,1,3) - INERTIA OF LUMPED ROTOR MASS
     DD(    92)=             0.20187E 07     ID(2,2,3) - INERTIA OF LUMPED ROTOR MASS
     DD(    96)=             0.16426E 07     ID(3,3,3) - INERTIA OF LUMPED ROTOR MASS
     DD(    97)=             0.59000E 01     YH(1) - LOCATE SHUTTLE
     DD(   102)=            -0.12700E 03     RH(3) - LOCATE SHUTTLE
     DD(   103)=            -0.58150E 02       YB(1)
     DD(   106)=            -0.68150E 02     YG(1) - LOCATE G
     DD(   109)=             0.18600E 02     YE3(1) - LOCATE E
     DD(   117)=             0.62500E 01     XN(3) - LOCATE ROTOR ARM - PT.N
     DD(   120)=            -0.32000E 02     ZV3(3) - LOCATE ARM - PT.V
     DD(   123)=             0.23330E 02     XD(3,1) - LOCATE ROTOR MASSES
     DD(   126)=             0.57330E 02     XD(3,2) - LOCATE ROTOR MASSES
     DD(   129)=             0.10625E 03     XD(3,3) - LOCATE ROTOR MASSES
     DD(   133)=             0.15708E 01     BETA(2)
     DD(   157)=             0.21193E 09     KHS(1,1)
     DD(   161)=             0.21193E 09     KHS(2,2)
     DD(   166)=             0.13500E 07     KHV
     DD(   170)=             0.13500E 07     KHV
     DD(   201)=            -0.80920E 02     YC(1,1) - LOCATE HUB MASSES
     DD(   204)=            -0.10015E 03     YC(1,2) - LOCATE HUB MASSES
```

TABLE 10.4 (Cont'd)

```
DD(  207)=        -0.13955E 03     YC(1,3) - LOCATE HUB MASSES
DD(  130)=         0.20944E 01     ALPHA(2) - LOCATE ARM 2
DD(  131)=         0.41887E 01     ALPHA(1) - LOCATE ARM 3
DD(  135)=         0.21156E 09     KBS1 - BEARING SPRING CONSTANT
DD(  137)=         0.10000E 04     DELTB - DISCONTINUITY DEFLECTION OF BEARIN
DD(  143)=         0.26950E 08     KBVR(2,2) - OUTER RACE DAMPING
DD(  147)=         0.26950E 08     KBVR(3,3) - OUTER RACE DAMPING
DD(  200)=         0.26200E 00     SPIN - NOMINAL SPIN RATE (RAD/SEC)
DD(  210)=         0.60000E 01     NROT
DD(  211)=         0.20000E 01       NHUB  NUMBER OF HUB MODES
DD(  212)=         0.30000E 01      NSYM
DD(  214)=         0.40000E 01       PBLOCK
DD(  221)=         0.11000E-04     STOP
DD(  222)=         0.15000E-04     START
DD(  223)=         0.20000E 01     TAUF
DD(  224)=         0.10000E 05      H- CMG WHEEL SIZE
DD(  225)=         0.30000E 01     WNG- CMG NATURAL FREQ
DD(  226)=         0.80000E 00     ZETG- CMG DAMPING
DD(  406)=         0.59000E-03      WYH(3)   DOCKING DISTURBANCE
DD(  428)=         0.26200E 00     WXY(1) - INITIAL SPIN RATE (RAD/SEC)
DD(  220)=         0.10000E 01     CONTR
```

The Structural Data matrices used during the run are part of the namelist TESTC, which is printed once at the beginning of each run. The printed output of TESTC for this sample run is not shown here because it is felt to be too lengthy for the purpose that it would serve. However, the stiffness data used was twice baseline stiffness (i.e. nominal stiffness as described in Section 4.3) with 1% structrual damping for both rotor and hub.

At the end of each print cycle the variables in the following four namelists are printed:

- TIM
- STATE
- CMGPR
- SHUT

TIM - Contains only T, the problem time (in seconds).

STATE - Contains physical displacements and rates (in feet, rad, and seconds), modal displacements, rates and accelerations (i.e. XVEC, XVECD, XVECDD), and the maximum rotor ring transverse angular rate over one print cycle (i.e. WTMM in rad/sec). The latter variable is used as an indicator for stability.

CMGPR - Contains the simulation variables associated with CMG control. This namelist is printed only when there is active CMG wobble damping (i.e. CONTR ≠0). The units are: feet, lbs, rad, sec.

SHUT - Contains the simulation variables associated with a flexibly attached shuttle. This namelist is printed only when NS ≠ 0. The units are: feet, lbs, rad, sec.

As indicated by the basic data variable DTPRNT, the above namelists are printed every 5 sec. The printed output at the 100 second time point is shown in Table 10.5. As seen in table 10.5, the name of each data array is printed followed by the values presently stored in the array. Each output variable is printed in row column order; for example, the elements of a N by M array will be printed as follows: (1,1), (2,1)...(N,1) (1,2)...(N,2)...(1,M)...(N,M). For arrays that are not filled (e.g. XVEC, XVECD, and XVECDD) "garbage" is printed in place of the unused areas, and should be ignored. It should be noted that although some arrays are not filled for a particular run, these arrays may be filled during a future run.

Selected time history plots, for the above described sample run, are shown in Figure 10.26. Four plot blocks are shown (PBLOCK = 4) with three variables per block. The particular variables chosen for plotting are determined by the following statements that appear in MAIN:

```
WRITE (1)   T, WXR (2), WXR (3), WYH (2)
WRITE (1)   WYH (3), WYC (3,3) UCD (2,3)
WRITE (1)   PSID (1), UDD (1,3,2), UDD (2,3,2)
WRITE (1)   WZ (2,3,1) BDR, B
```

TABLE 10.5 THP SAMPLE OUTPUT

```
&TIM
T = 100.09221
&END
&STATE
ZAD=-0.42243119E-05,-0.81469351E-03, 0.37135044E-03,ZA=-0.41771011E-04,-0.68797112E-01, 0.93632527E-02,WYH=-0.30900657E-06,
-0.35000755E-03,-0.10131979E-03,GAM= 0.10882439E-03,-0.75873062E-02, 0.44365600E-02,UCD= 0.0          , 0.87796880E-05,
-0.12559058E-05, 0.0          , 0.50080474E-04,-0.72405337E-05, 0.0          , 0.19321924E-03,-0.28575800E-04,UC= 0.0
-0.38465063E-04,-0.12203099E-04, 0.0          ,-0.21540973E-03,-0.70353140E-04, 0.0          ,-0.84652100E-03,-0.27765869E-03,
PHICD= 0.0          ,-0.19128350E-06,-0.13328063E-05, 0.0          ,-0.41826388E-06,-0.28629293E-05, 0.0          ,
-0.58051734E-06,-0.38715334E-05,PHIC= 0.0          ,-0.18586188E-05, 0.58392134E-05, 0.0          ,-0.40640889E-05,
 0.12542901E-04, 0.0          ,-0.56445224E-05, 0.16961727E-04,DELD= 0.30469106E-07,-0.53122048E-05, 0.65816794E-05,DEL=
-0.33244879E-06,-0.34725002E-04,-0.24760859E-04,WXR= 0.26199991    ,-0.24978840E-03, 0.26165787E-03,UDD=-0.48675836E-04,
-0.40212490E-05, 0.0          ,-0.38448768E-03,-0.31555406E-04, 0.0          ,-0.11542665E-02,-0.93914932E-04, 0.0          ,
 0.77007688E-04, 0.24318259E-04, 0.0          , 0.60827634E-03, 0.19082952E-03, 0.0          , 0.18261618E-02, 0.56794682E-03,
 0.0          ,-0.27818431E-04,-0.20296997E-04, 0.0          ,-0.21973772E-03,-0.15927413E-03, 0.0          ,-0.65969606E-03,
-0.47402829E-03, 0.0          ,UD=-0.31956611E-03,-0.25901027E-04, 0.0          ,-0.25242518E-02,-0.20324976E-03, 0.0
-0.75780712E-02,-0.60491008E-03, 0.0          ,-0.37791659E-04, 0.45936875E-04, 0.0          ,-0.29854174E-03, 0.36047422E-03,
 0.0          ,-0.89633139E-03, 0.10728526E-02, 0.0          , 0.35657408E-03,-0.20035848E-04, 0.0          , 0.28165274E-02,
-0.15722460E-03, 0.0          , 0.84557012E-02,-0.46791858E-03, 0.0          ,PHIDD= 0.45621965E-06,-0.55304781E-05,
 0.13953115E-06, 0.11071625E-05,-0.13576495E-04, 0.41653016E-06, 0.13069193E-05,-0.16155056E-04, 0.55414733E-06,-0.27589613E-05,
 0.87496801E-05, 0.28326895E-07,-0.66955317E-05, 0.21479049E-04, 0.85437819E-07,-0.79035781E-05, 0.25558598E-04, 0.11359219E-06,
 0.23027414E-05,-0.31608024E-05,-0.16617264E-06, 0.55883120E-05,-0.77595914E-05,-0.49572361E-06, 0.65965460E-05,-0.92330492E-05,
-0.65975991E-06,PHID= 0.29385301E-05,-0.36308687E-04,-0.52492970E-06, 0.71312834E-05,-0.89133231E-04,-0.15670266E-05,
 0.84179264E-05,-0.10606278E-03,-0.20847556E-05,-0.52116429E-05,-0.42942265E-05, 0.89500026E-06,-0.12647897E-04,-0.10544172E-04,
 0.26712705E-05,-0.14930008E-04,-0.12545148E-04, 0.35550038E-05, 0.22731110E-05, 0.40514147E-04,-0.37641126E-06, 0.55162509E-05,
 0.99454424E-04,-0.11277170E-05, 0.65113591E-05, 0.11834590E-03,-0.15002697E-05,WZ= 0.26200026    ,-0.25518006E-03, 0.25228481E-03
 0.26200092    ,-0.26295194E-03, 0.23872306E-03, 0.26200110    ,-0.26539457E-03, 0.23442546E-03, 0.26199704    , 0.36001066E-03,
 0.84399187E-04, 0.26199311    , 0.37227408E-03, 0.82821440E-04, 0.26199192    , 0.37612231E-03, 0.82326122E-04, 0.26200211    ,
-0.10477108E-03,-0.33670268E-03, 0.26200539    ,-0.10917417E-03,-0.32158964E-03, 0.26200640    ,-0.11055036E-03,-0.31680381E-03,
XVEC=-0.41771011E-04,-0.68797112E-01, 0.93632527E-02,-0.21184402E-04,-0.75862780E-02, 0.44356622E-02,-0.23476270E-04,
-0.37858617E-05,-0.27765869E-03,-0.84652100E-03, 0.75698197E-02, 0.82537963E-05,-0.77038561E-03,-0.33244879E-06, 0.46760179E-02,
 0.60491008E-03, 0.43191878E-47,-0.92721176E 24,-0.92780003E 24, 0.13978862E 32, 0.18317901E 65, -6.5289307    , -15.396561    ,
-0.71942763E 24,-0.66275808E 24,-0.64386516E 24, 0.14436852E 20,-0.28834567E 22,-0.10726782E 23, 0.14416585E 20, 0.15179363E 09,
 0.50494187E 28, 0.16706001E 32, 0.12128405E 19, 0.25719897E 27, 0.59888199E 17, 0.38146973E-05, 0.59879403E 17,  4.0000038    ,
 0.44822870E 29, 0.34919312E 29, 0.25718008E 27, 0.65886457E 29, 0.59910190E 17, 0.32043457E-03, 0.59914588E 17, 0.68664551E-04,
 0.59883801E 17, 0.49774933E 29, 0.12742344E 32, 0.25016000E 29, 0.64041491E 31, 0.11630703E-49,-0.90817010E 24,-0.98388246E 24,
-0.98388477E 24,-0.10204779E 25, 0.38688963E 31, 0.37495650E 67, 0.26614859E 72, 0.11319707E-48,-0.36628247E 25, 0.18139380E 65,
 0.26012795E 31, 0.84499619E 57, 0.10207091E 32,-0.91245225E-69, 0.25719897E 27, 0.65896129E 29, 0.59883801E 17, 0.25733119E 27,
 0.65886457E 29, 0.59914588E 17, 0.25719897E 27, 0.41176013E 28, 0.21890042E 09, 0.25718008E 27, 0.29507322E-47,-0.88944666E 24,
-0.92690854E 24, -96.344971    ,-0.71942763E 24,-0.66275808E 24,-0.64386516E 24,XVECD=-0.42243119E-05,-0.81469351E-03,
 0.37135044E-03,-0.30900657E-06,-0.35000755E-03,-0.10131979E-03,-0.87953021E-06,-0.19656682E-05,-0.28575800E-04, 0.19321924E-03,
 0.11580253E-02,-0.37586906E-05,-0.52098767E-03, 0.30469106E-07,-0.12429289E-02, 0.93914932E-04, 0.39916527E-48,-0.70010928E 24,
 0.44277231E-79,-0.60044152E 22,-0.88946511E 24,-0.10781753E 25,-0.10401197E 25, 0.77021298E-77,-0.90832114E 24,-0.60069372E 22,
-0.10731537E 23,-0.44289848E 22, 0.10207043E 32, 0.25016000E 29, 0.64041104E 31, 0.11630703E-49,-0.90817010E 24,-0.10027673E 25,
-0.10027719E 25,-0.10393672E 25, 0.38688818E 31, 0.37561470E 67, 0.23863386E 72, 0.10908428E 70, 0.99211576E 57, 0.76717901E 31,
-0.33292361E-69, 0.25517780E 27, 0.65151430E 29, 0.59413210E 17, 0.25719897E 27, 0.65915471E 29, 0.59883801E 17, 0.25517780E 27,
 0.28001744E 28, 0.21890429E 09, 0.25718008E 27, 0.29507322E-47,-0.88944666E 24,-0.33457413E 25, -1317.5195    ,-0.71942763E 24,
-0.66275808E 24,-0.64344088E 24, 0.44277231E-79,-0.35701160E 22, 0.49774631E 29, 0.34919312E 29,  4.0000038    , 0.95367432E-06,
 0.59413210E 17, 0.10158660E 29, 0.26006461E 31,-0.74184194E-67, 0.23657224E 19, 0.26011877E 31, 0.23657576E 19, 0.21743774E-03,
 0.59883801E 17, 0.15110458E 29, 0.38682967E 31, 0.66574930E 33, 0.35107704E-48,-0.88943398E 24,-0.90836034E 24,-0.96380577E 24,
 0.65842936E 29, 0.59905792E 17,XVECDD= 0.63204971E-06,-0.10923020E-03,-0.20887773E-03, 0.10735526E-07, 0.88046290E-05,
-0.27061382E-04,-0.13368071E-05,-0.61256442E-05, 0.51874202E-04,-0.16711606E-03,-0.58652600E-03,-0.33025008E-05, 0.20742499E-03,
 0.38684963E-07, 0.15069775E-02,-0.10338146E-04,-0.60055681E 22,-0.12214567E-57, -14424.000    ,-0.36455378E 22,-0.24407348E 22,
-0.36443849E 22,-0.12214567E-57, -4184.3125    ,-0.71942763E 24,-0.66275808E 24,-0.64344203E 24,-0.16297221E 22,-0.85013542E 24,
  2816.0000    , 0.95367432E-06, 0.59404414E 17, 0.38146973E-05, 0.59896995E 17, 0.68265813E 67, 0.10143231E 68,  5.0000029    ,
  4.0000010    , 0.34919350E 29, 0.69581898E 29, 0.48936084E 67, 0.11391934E 58, 0.48941291E 67, 0.11391979E 58, 0.48940905E 67,
 0.74533412E 29, 0.22734818E 72, 0.11392024E 58, 0.92659453E-55,-0.35695260E 22, 0.93059253E-45,-0.92678402E 24, 0.41694393E-78,
-0.36340806E 22,-0.94609661E 24,-0.26582049E 22, 0.59883801E 17, 0.26819719E-48,-0.90790493E 24, 0.30679752E-78, 0.28974273E-46,
-0.92607397E 24, 0.36760433E-48, 0.83301380E-79, 0.11876469E 67, 0.48457354E 64, 0.72073742E 67, 0.57683238E 64,-0.26660871E 22,
 -304526.00    ,-0.96557969E 24, 0.24899849E 68, 0.16596456E 65,-0.26628163E 22,-0.94594904E 24,-0.90832114E 24,-0.90714429E 24,
 0.49774612E 29, 0.10250601E-48,-0.34743695E 22,-0.92607397E 24, 0.36760433E-48, 0.83301380E-79, 0.14245996E 67,RTS=-0.16778376E-0
 0.17160183    ,  1.7147627    ,  67.381180    ,  4104.7500    , -10592.082    ,  4976.9742    ,  805.00439    ,-0.55535607E-01,
-0.15030510E-02, 0.23860257E-01,-0.34721319E-02,-0.14481503E-02, 0.10744762    , 0.33719756E-01,-0.72214067E-01,  13.182028    ,
  3.7661238    ,-0.14149169E-03,-0.20889796E-01,-0.31877148    , 0.34663099E-04, 0.11768218E-01,-0.73400760    , 0.55489581E-01,
```

```
  40.345169    , -34.576782    ,  5.1743793    , -63.654370    , -3363.3611    ,-0.44968057    , 0.92609897E-02,  360.86890    ,
 -0.69154054   , 0.35996619E-02,  886.78613    , -10.475311    ,  3.6843698    ,  27882.648    ,-0.16847956    ,-0.39163208    ,
  0.28197765E-02, 0.60173601   , -2.0940180    ,-0.52989662E-01, -2.9392776   ,  229.22549    ,  55.375931    ,-0.18427843    ,
 -0.17812993E-01,  360.86572   ,-0.51046836    ,-0.28460167E-01,  886.76025   , -12.868697    ,-6.5324135     ,  27881.359    ,
  0.32154864   ,-0.28316456    ,-0.54457963E-01,-0.87675583   ,-0.60660845    , 0.16466689    ,  5.8356698    ,  11.630569    ,
 -93.527512    , 0.63770443    , 0.44715591E-02,  360.87061   ,  1.5533247    ,-0.16172446E-01,  886.79996   ,  22.996506    ,
  2.8714132    ,  27883.383    ,-0.10885251    , 0.21761894    ,-0.14848733    , 0.67432165    ,  6.3839073    , 0.63229084E-01,
 -1.4456899    , -225.68520    ,  38.589096    ,WTMM= 0.36805263E-03
&END
&CMGPR
WXR= 0.26199991     ,-0.24978840E-03, 0.26165787E-03,FE= 0.80860132     ,BDE=  2.3329906     ,BDEF=  2.6666317     ,B=  2.7608681     ,
EDEDD= 0.70283413E-02,BDED=-0.17799848     ,BDR=-0.17869216     ,TC=  3.2640266     ,  778.82422     , -295.72046
&END
&SHUT
PSID=-0.67953021E-06,-0.19656682E-05, 0.0              ,FSI=-0.23476270E-04,-0.37858617E-05, 0.0              ,WR= 0.10044024E-03,
-0.35197288E-03,-0.31725222E-06,JT= 0.28507930E-06,-0.23476241E-04, 0.99999982     , 0.88877836E-10, 0.99999994     , 0.23476241E-04
-0.99999988    , 0.82185217E-10, 0.28508020E-06,MDCCK=  4976.5547     ,  804.99878     , 0.0              ,PIPSI= 0.99999994     ,
 0.0           ,-0.37858608E-05, 0.88877836E-10, 0.99999994     , 0.23476241E-04, 0.37858599E-05,-0.23476256E-04, 0.99999988     ,
TAUS= 0.0           , 0.0           , 0.0
&END
```

These plot variables are defined as follows:

T - Time (must always appear as first variable in first plot block)

WXR (2), WXR (3) - Inertial angular rate of rotor ring projected on the two and three axes in the rotor ring.

WYH (2), WYH (3) - Inertial angular rate of rigid hub section projected on the two and three axes in the rigid hub section.

WYC (3,3) - Inertial angular rate of the third flexible hub mass, projected on the three axis in the third lumped mass in the hub.

UCD (2,3) - The linear velocity of the third flexible hub mass relative to the rigid hub section, projected on the two axis in the third hub mass.

PSID (1) - Shuttle angular rate relative to the rigid hub section, projected on the one axis in the Shuttle.

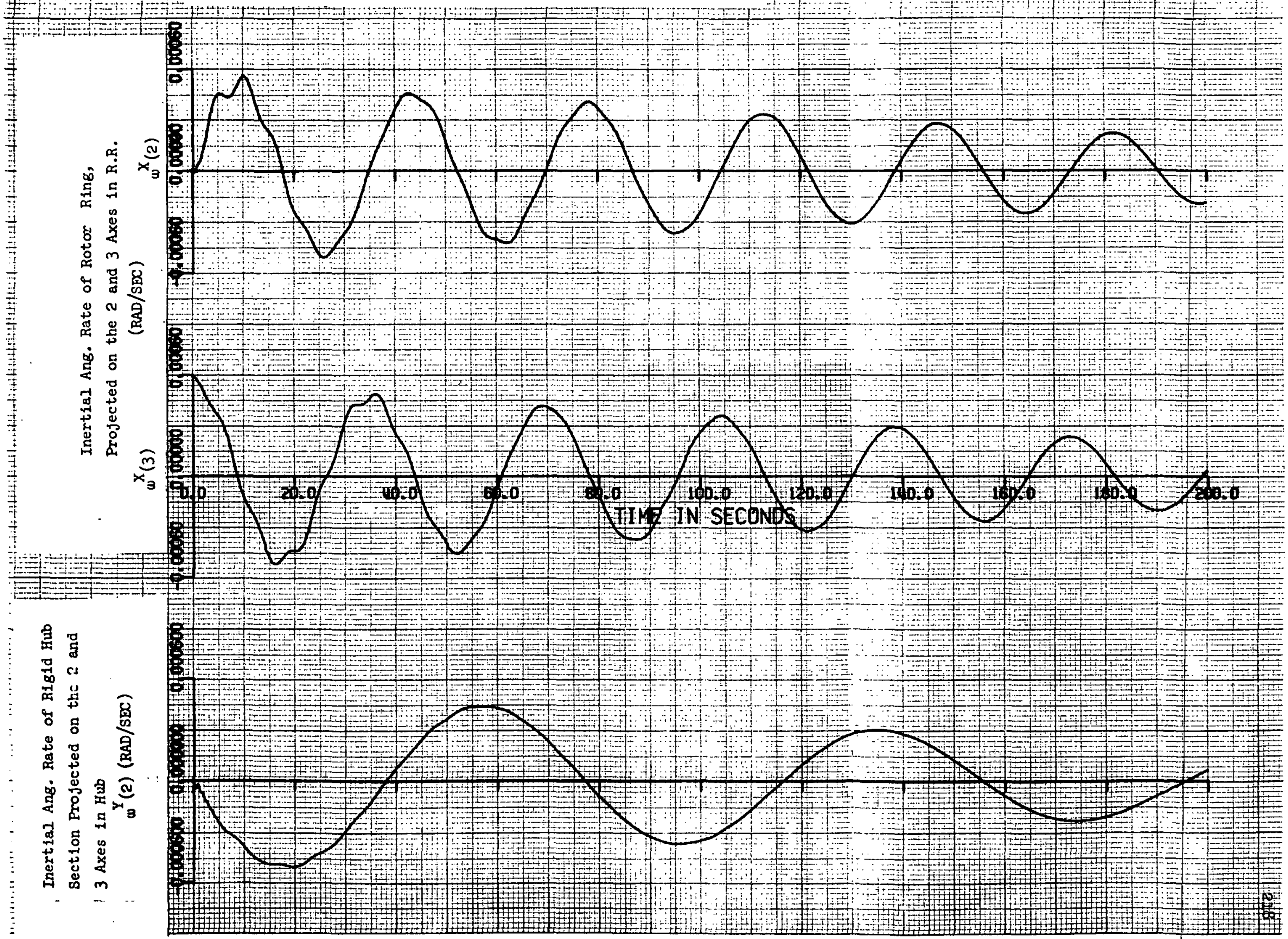
UDD (1,3,2), UDD(2,3,2) - Linear velocity of the third lumped mass in the second rotor arm, projected on the mass' one and two axes.

WZ (2,3,1) - Inertial angular rate of the third lumped mass in the first rotor arm, projected on the mass' two axis.

BDR, B - Gimbal rate, gimbal angle.

It should be noted that the CALCOMP subroutines that are included as part of the program package have been designed specifically for operation at the Grumman facility. Modification to these routines would most likely be required for use at another facility.

FIGURE 10.26 "Y"/SHUTTLE - CMG CONTROL
(H = 10,000 FT-LB-SEC)



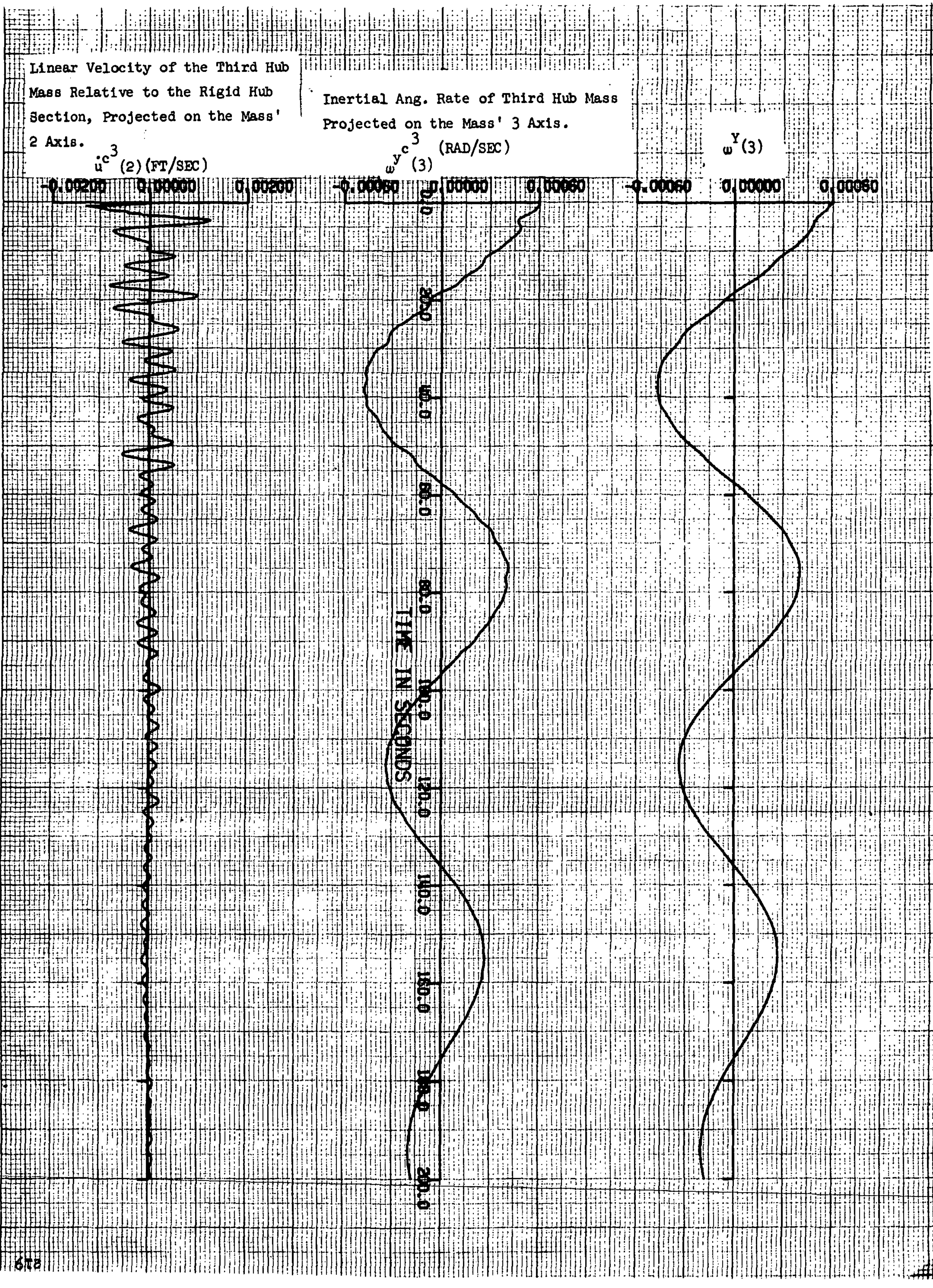


FIGURE 10.26 (Con't)

FIGURE 10.26 (Con't)

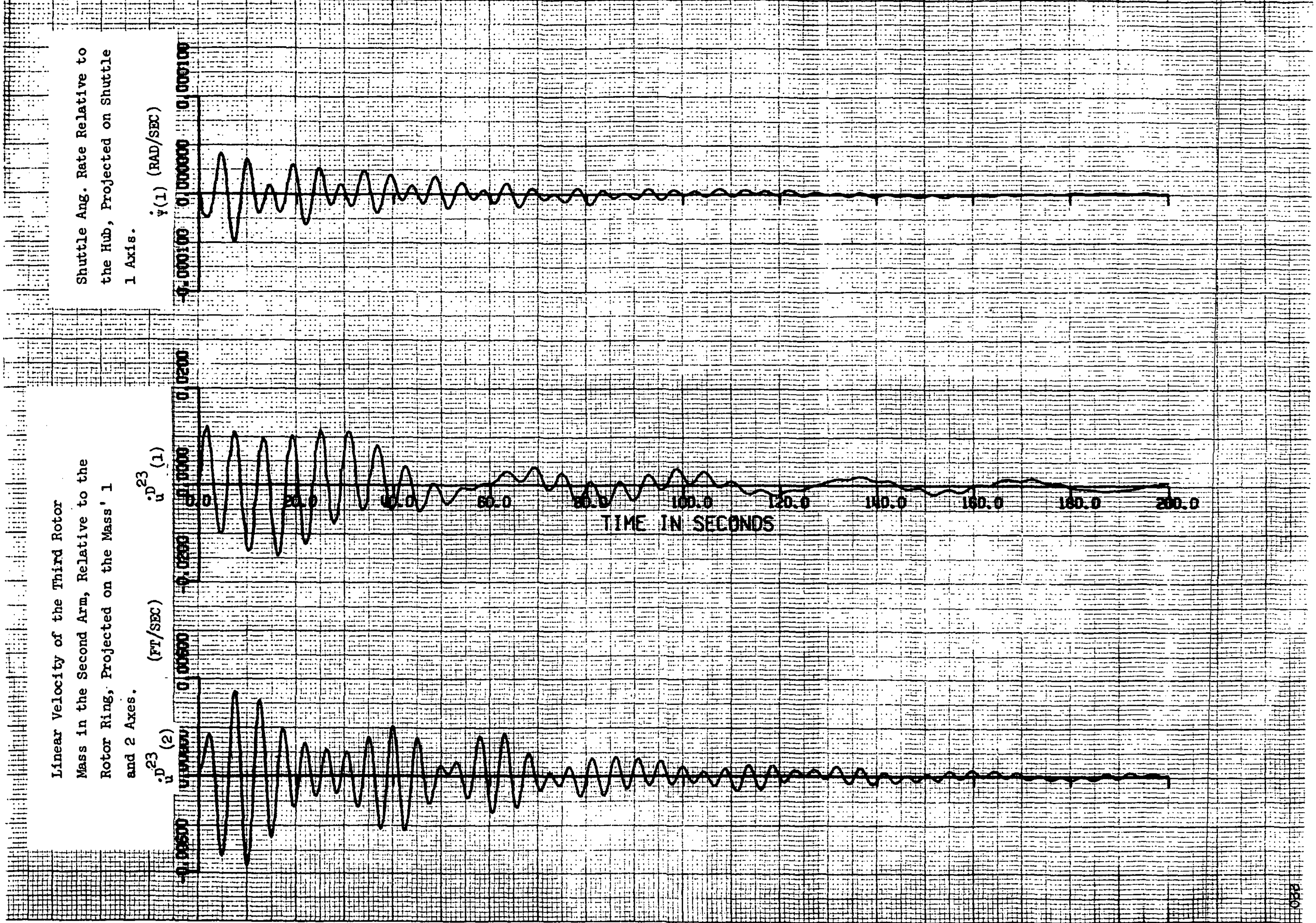
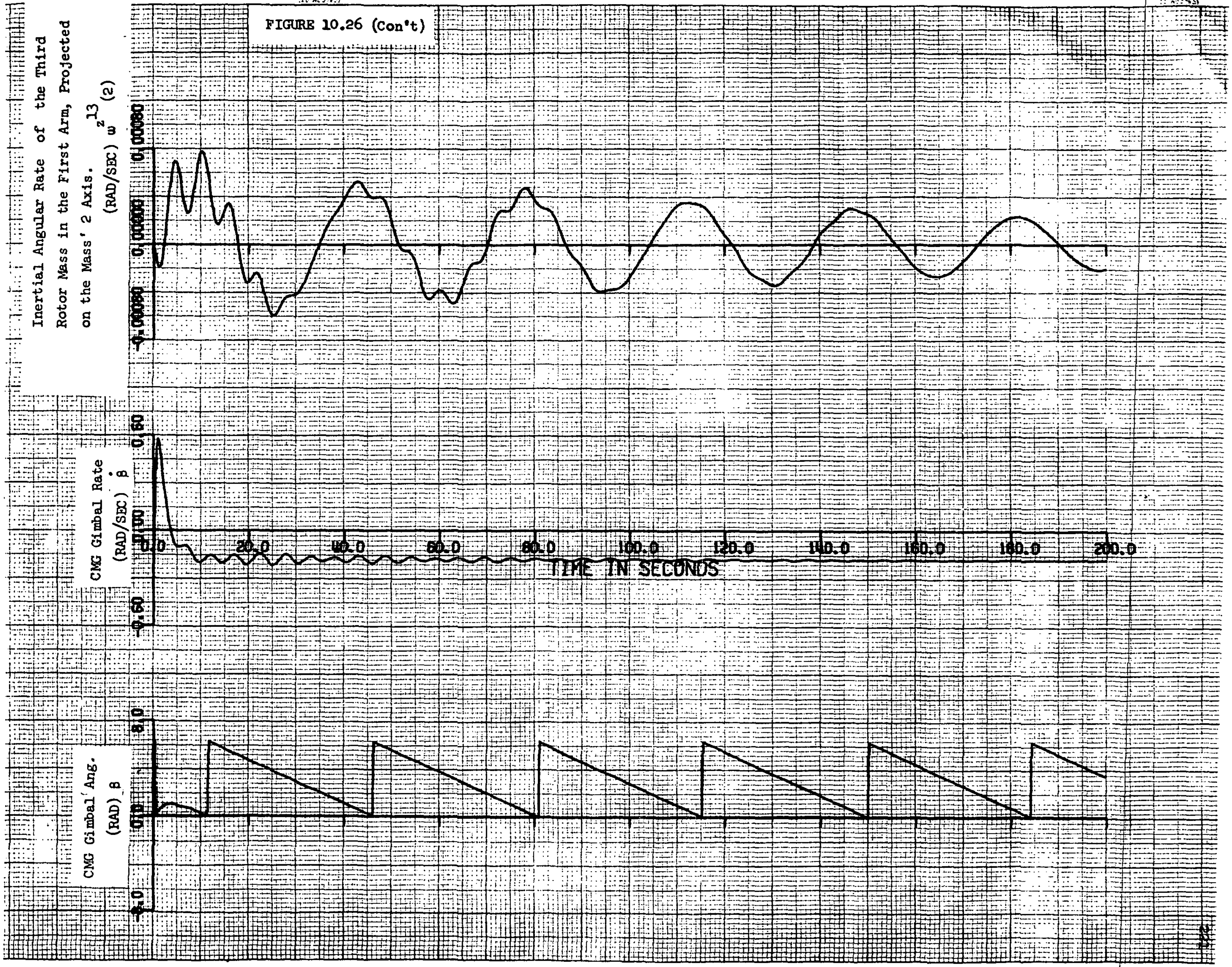


FIGURE 10.26 (Con't)



Inertial Angular Rate of the Third Rotor Mass in the First Arm, Projected on the Mass' 2 Axis. (RAD/SEC) $\omega_z^{13}(2)$

CMG Gimbal Rate (RAD/SEC) $\dot{\beta}$

CMG Gimbal Ang. (RAD) β

10.2 Modal Service Program

The Modal Service Program (MSP) of the General Flexible-Body Stage II digital simulation establishes the following Structural Data for use by the Time History Program:

- Mode Shapes
- Stiffness Matrices
- Damping Matrices

The MSP is special purpose in that it provides the above data for only the "T" and "Y" space station configurations. The method used for formulating this data is outlined in Section 6.2. Structural data for the hub and rotor may be obtained during one program execution. Printed output of the above structural data is provided as well as punched data cards with the proper format for direct input to the Time History Program. In addition, modal frequencies are provided so as to aid the user of the Time History Program in choosing the signficant mode shapes.

Operating insutructions for the MSP are found in Section 10.2.2.

10.2.1 Program Description (with Listings)

As in the case of the Time History Program, program functions are accomplished by several subroutines. Figure 10.27 presents the logical flow between operational subroutines. The logical tests along with the calling of these subroutines is accomplished by the MAIN (or Executive) routine.

A brief description of each routine is presented below.

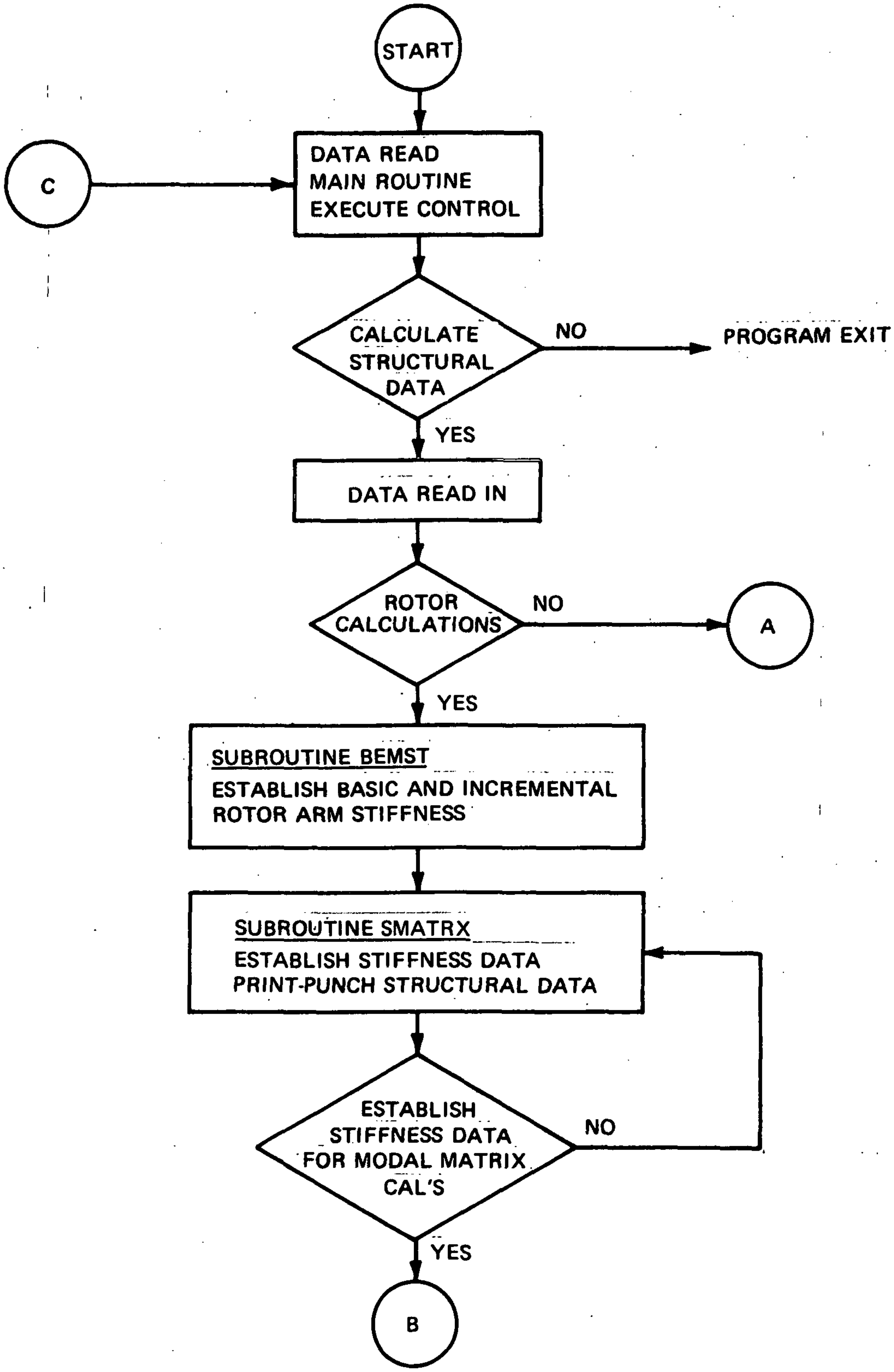


Figure 10-27. Model Service Program, Flow Chart

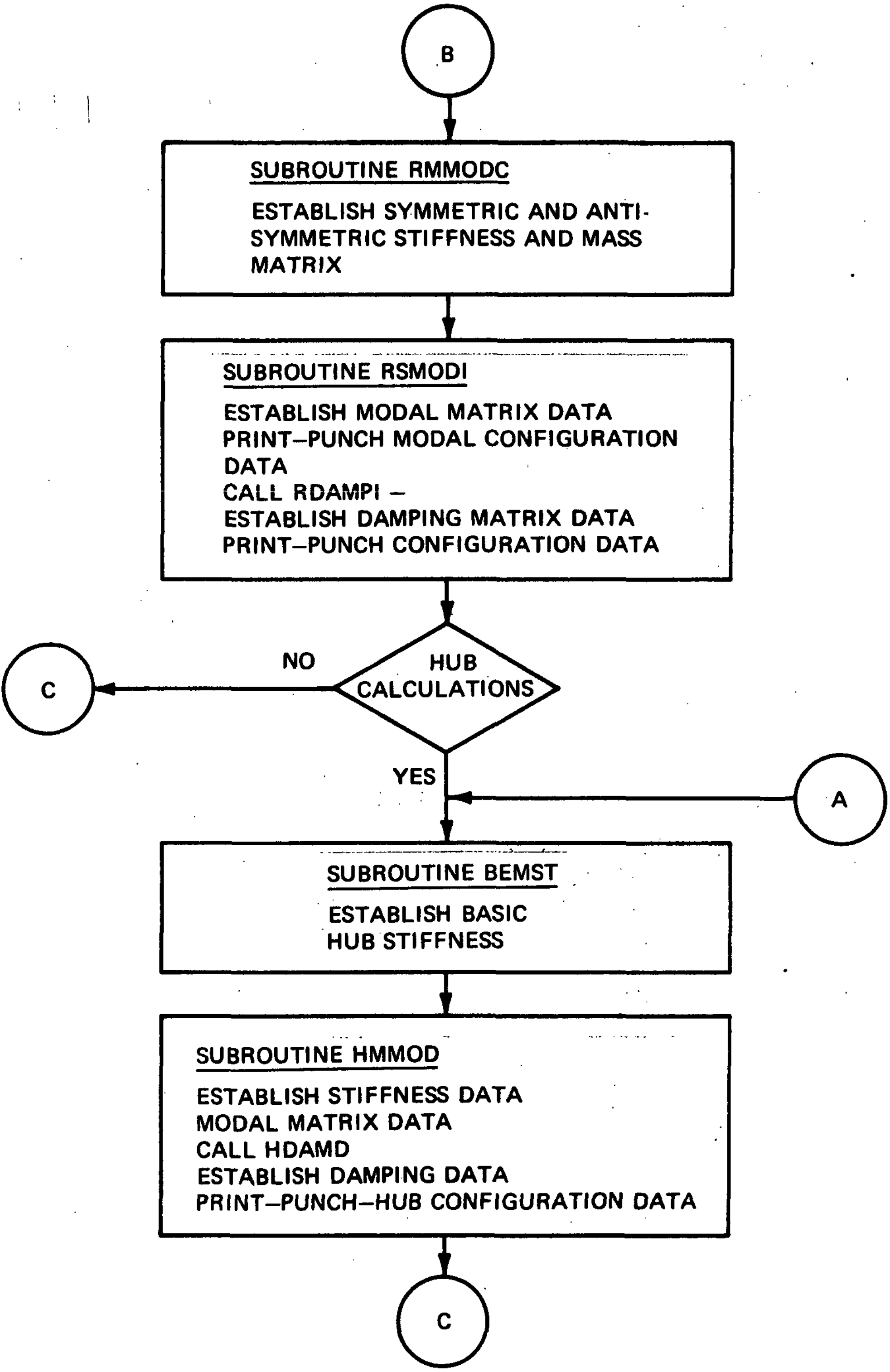


Figure 10-27. Model Service Program, Flow Chart (Cont.)

Main Routine (listing is found in Figure 10.28)

The main program accepts input data and identifies the data with the program variables. This routine establishes the calling sequence for the subroutines in which the Rotor and/or Hub stiffness, damping and modal matrices are formed.

Subroutine BEMST (figure 10.29)

Establishes an intermediate basic and incremental stiffness matrix for the rotor and an intermediate stiffness matrix for the Hub.

Subroutine SMATRX (figure 10.30)

Establishes the final rotor stiffness matrix for use by the THP, and an intermediate stiffness matrix that is used to formulate mode shapes.

Subroutine RMMODC (figure 10.31)

Establishes the rotor symmetric and antisymmetric stiffness and mass data arrays which are utilized in the computation of system modes and mode shapes.

Subroutine RSMODI (figure 10.32)

Computes the rotor modal matrix to be used by the THP.

Subroutine RDAMP (figure 10.33)

Establishes the rotor damping matrix to be used by the THP.

Subroutine HMMOD (figure 10.34)

Computes the Hub stiffness matrix and modal matrix to be used by the THP.

Subroutine HDAMP (figure 10.35)

Computes the Hub damping matrix to be used by the THP. This subroutine is called from HMMOD.

Subroutine LATE (figure 10.36)

Generates Rotor and Hub modal frequencies and mode shapes in the following manner:

The problem to be solves is,

where

$$[M][\ddot{\Phi}] = \lambda[K][\Phi] \qquad (10\text{-}5)$$

$$\lambda = \frac{1}{\omega^2}$$

and $[K]$ - Strifness Matrix - (Positive Definite Symmetric)

$[M]$ - Mass Matrix (Symmetric)

$[\phi]$ - Matrix of eigen vectors (mode shapes)

$[\omega]$ - Frequency (Rad/sec)

$[\lambda]$ - Eigen value.

The first program operation is to factor the stiffness matrix $[K]$ into the form of a lower triangular matrix $[L]$ such that

$$[K] = [L]\,[L]^T \tag{10 -6}$$

This above factorization is performed in subroutine "FUTILE" (figure 10.37). Subroutine "DAGGER" (figure 10.38) is then called repeatedly to form,

$$[L]^{-1}[M][L]^{-T} = [C] \tag{10 -7}$$

where array $[C]$ is defined by the following standard eigen value problem,

$$[C]\,Y = [D]\,[Y] \tag{10 -8}$$

and $[Y] = [L]^T[\phi]$

$[D]$ = Diagonal array composed of eigen values $\lambda_1, \lambda_2, \lambda_3 \ldots$

Matrix $[C]$ is then converted in subroutine "SWITCH" (figure 10.39) to a form acceptable to subroutine "SYMEIG" (figure 10.40). SYMEIG solves the standard eigen value problem. Subroutine TRIEQ (figure 10.41) is used to transform the eigenvectors to obtain $[\phi]$. Additional supporting routines required for subroutine (LATE) are:

- TFORM (figure 10.42)
- STURM (figure 10.43)
- PREP (figure 10.44)
- QSVEC (figure 10.45)
- AND (figure 10.46)
- DOTPRO (figure 10.47)
- RDM (figure 10.48)

It should be noted that mode shapes will not be calculated if the stiffness matrix is not positive definite. Under these circumstances an error message "Stiffness Array not positive definite - Eigenvalues-modal vectors not computed-check input data" is printed. At this point the program run is terminated and the next data set is read.

An error message indicating a negative eigenvalue may occur. If the printed magnitude of the error indicator "CHEC" is close to zero, this error can be ignored and attributed to computer round-off

Subroutine COMPR (Figure 10.49)

Converts double dimensioned array into a column vector (or a single dimensioned array).

Subroutine ABCP (Figure 10.50)

Generates the transformation matrix for a single angle rotation.

Subroutine "Anorm" (Figure 10.51)

Normalizes each column of a matrix.

Subroutine "PWRIT" (Figure 10.52)

Generates punched data in the proper format for direct input to the THP.

Refer to section 10.1.2 for a description of the following utility subroutines:

- WILLY
- STACK
- GAMMA
- SREVN1

10.2.2 Operating Instructions

Program operation is controlled by input data cards exactly as described in Section 10.1.3 for the THP. The only difference is that the STRUCTURAL DATA block is not used. Specifically, a RUN CONTROL card is required followed by the BASIC DATA group. The required format for these data groups is identical to the THP.

ROTOR BASIC DATA

All arms are assumed to have identical geometric and mass properties. The rotor arm geometric properties that have to be considered are illustrated in Figure 10.53. Input data card idntification (ID) numbers are included in Figure 10.53. Table 10.6 defines all rotor input data variables and associated ID numbers. A sample input card set-up to obtain rotor configuration data is illustrated in Table 10.7.

HUB BASIC DATA

The hub geometrical properties that must be considered along with their corresponding ID numbers are shown in Figure 10.54. Table 10.8 identifies all hub input data parameters and associated ID numbers. A sample input data set-up to obtain Hub structural data is illustrated in Table 10.9.

```
      INTEGER P,ER,EC
      COMMON /PUNCH/ IRIT,TREIG,TPHHF,TSMAT,TCTYA,TK7H,TCH,TK6HG,TCHG
      COMMON /BEAM1/
     *HL(3),AL(3),PSS(3),HUB,EI,AE,GJ
      COMMON /RHMST/
     *WORK6(72,72),WORK8(72,60),WCRK9(72,60),WORK11(57,57)
      COMMON /RRMID/
     *RRIB(3,3),DI1(3,3),DI2(3,3),DI3(3,3),DM1,DM2,DM3
      COMMON /RHMST1/
     *XRN(3),XRD1(3),XRD2(3),ZV(3),ALPH1(3,3),GXN(3,3),GZP(3,3),ANG(2),
     *PT1,PT2,PT3,P,BKS1,TEST,QARRA(54,3),XRD3(3)
      COMMON /EIGVEC/
     *COLK(435),COLM(435),EIG(29,29),QMEGA1(28),QMEGA2(29),TOL1,FREQ
      COMMON /HUBID/
     *CI1(3,3),CI2(3,3),CI3(3,3),YHE(3),CM1,CM2,CM3,YHA(3),YHG(3),
     *YHC1(3),YHC2(3),YHC3(3)
      COMMON /HDAMPC/
     *DAMPH
      COMMON /RDAMPC/
     *DAMPR
      COMMON /PRINT/
     *IOUT,IOIN
      COMMON /TERUP/BRAK
      DIMENSION DD(200)
      DIMENSION QNAM(7)
      DIMENSION XRP(3)
      WRITE(6,901)
  901 FORMAT(' TYPE FILE NUMBER TO BE USED')
      READ(5,900) IOIN
  900 FORMAT(I1)
      IOUT=8
      DO 1 I=1,200
    1 DD(I)=0.
   21 READ(IOIN,22) NRUN,QNAM
   22 FORMAT(I3,7A4)
      WRITE(IOUT,23) NRUN,QNAM
   23 FORMAT(1H1,'RUN NUMBER=',I3,5X,7A4/)
      IF(NRUN.GT.0) GO TO 25
      CALL EXIT
   25 READ(IOIN,26) ID,VAL,QNAM
   26 FORMAT(I5,E20.5,7A4)
      IF(ID.LE.0) GO TO 27
      DD(ID)=VAL
      WRITE(IOUT,28) ID,DD(ID),QNAM
   28 FORMAT(5X,'DD(',I5,')=',G20.7,5X,7A4)
      GO TO 25
   27 CONTINUE
C
C     INPUT DATA ARRAY
C
      DO 40 I=1,3
      XRD1(I)=DD(I+1)
      XRD2(I)=DD(I+4)
      XRD3(I)=DD(I+7)
```

```
AN IV G LEVEL  1, MOD 1              MAIN              DATE = 71363              13.16.

          40 XRP(I)=DD(I+10)
             P=DD(14)
             I=15
             DO 29 J=1,3
             DO 29 L=1,3
             DI1(L,J)=DD(I)
          29 I=I+1
             I=24
             DO 30 J=1,3
             DO 30 L=1,3
             DI2(L,J)=DD(I)
          30 I=I+1
             I=33
             DO 31 J=1,3
             DO 31 L=1,3
             DI3(L,J)=DD(I)
          31 I=I+1
             I=42
             DO 32 J=1,3
             DO 32 L=1,3
             RRIB(L,J)=DD(I)
          32 I=I+1
             AREA=DD(51)
             AREA=ARFA/144.
             AI=DD(52)
             AI=AI/(144.**2)
             GBE=DD(53)
             GBE=GBE*144.
             BE=DD(54)
             BE=BE*144.
             PMJ=DD(55)
             PMJ=PMJ/(144.**2)
             FREQ=DD(56)
             TOL1=DD(57)
             XRN(1)=DD(58)
             XRN(2)=DD(59)
             XRN(3)=DD(60)
             DM1=DD(61)
             DM2=DD(62)
             DM3=DD(63)
             PR=DD(64)
             ANG(1)=DD(65)
             ANG(2)=24.
             IF(P.EQ.2) ANG(1)=10.
             BKS1=DD(67)
             DAMPR=DD(68)
      C
      C      TAPE-CARD PUNCH CONTROL-TOL. DATA
      C
             IRIT=DD(90)
             TREIG=DD(91)
             TPHHF=DD(92)
             TSMAT=DD(93)
             TCTYA=DD(94)
```

```
            TK7H=DD(95)
            TCH=DD(96)
            TK6HG=DD(97)
            TCHG=DD(98)
      C
      C     HUB INPUT DATA
      C
            HUBCAL=DD(100)
            DO 35 I=1,3
            YHA(I)=DD(100+I)
            YHG(I)=DD(103+I)
            YHC1(I)=DD(106+I)
            YHC2(I)=DD(109+I)
            YHC3(I)=DD(112+I)
         35 YHE(I)=DD(115+I)
            I=119
            DO 36 J=1,3
            DO 36 L=1,3
            CI1(L,J)=DD(I)
         36 I=I+1
            I=128
            DO 37 J=1,3
            DO 37 L=1,3
            CI2(L,J)=DD(I)
         37 I=I+1
            I=137
            DO 38 J=1,3
            DO 38 L=1,3
            CI3(L,J)=DD(I)
         38 I=I+1
            CM1=DD(146)
            CM2=DD(147)
            CM3=DD(148)
            DAMPH=DD(149)
            AE=AREA*BE
            EI=BE*AI
            GJ=GBE*PMJ
      C
      C     ARM ELEMENT CALCULATIONS
      C
            AL(1)=XRD1(3)-XRN(3)
            AL(2)=XRD2(3)-XRD1(3)
            AL(3)=XRP(3)-XRD2(3)
            ZV(1)=0.
            ZV(2)=0.
            ZV(3)=XRP(3)-XRD3(3)
            HL(1)=YHC1(1)-YHG(1)
            HL(2)=YHC2(1)-YHC1(1)
            HL(3)=YHE(1)-YHC2(1)
            DO 60 J=1,3
            AL(J)=ABS(AL(J))
         60 HL(J)=ABS(HL(J))
            HUB=0.
            TEST=1.
```

```
      C
      C     STIFFNESS CALCULATIONS
      C
            PRS=PR**2
            PT1=DM1*XRD1(3)*PRS
            PT2=DM2*XRD2(3)*PRS
            PT3=DM3*XRD3(3)*PRS
            PSS(1)=PT1+PT2+PT3
            PSS(2)=PT2+PT3
            PSS(3)=PT3
            IF(HUBCAL.EQ.1.) GO TO 45
            BRAK=0.
            CALL BEMST
            CALL SMATRX
            CALL RMMODC
            CALL RSMOD1
            IF(HUBCAL.EQ.0.) GO TO 50
      C
      C     HUB CALCULATIONS
      C
         45 CONTINUE
            HUB=1.
            BRAK=0.
            CALL BEMST
            CALL HMMOD
         50 CONTINUE
            IF(IRIT.LE.6) GO TO 21
            IV=0
            WRITE(IRIT,499) IV
        499 FORMAT(78X,I2)
            GO TO 21
            END
```

FIGURE 10.29 BEMST

```
IV G LEVEL  1, MOD 1            BEMST             DATE = 71363          13.19.27
      SUBROUTINE BEMST
      COMMON /BEAM1/
     *HL(3),AL(3),PSS(3),HUB,EI,AE,GJ
      COMMON /RHMST/
     *WORK6(72,72),WORK8(72,60),WCRK9(72,60),WORK11(57,57)
      DIMENSION BK1(3),BK2(3),BK3(3),BK4(3),BK6(3),POL(3),SK(12,12),SSK(
     *12,12),TSK(12,12),BK5(3)
      DIMENSION ARY(3,3)
      IF(HUB.EQ.1.) GO TO 40
      DO 110 J=1,3
      BK1(J)=AE/AL(J)
      BK2(J)=12.*EI/AL(J)**3
      BK3(J)=6.*EI/AL(J)**2
      BK4(J)=GJ/AL(J)
      BK5(J)=4.*EI/AL(J)
      BK6(J)=2.*EI/AL(J)
  110 CONTINUE
      GO TO 43
   40 DO 42 J=1,3
      BK1(J)=AE/HL(J)
      BK2(J)=12.*EI/HL(J)**3
      BK3(J)=6.*EI/HL(J)**2
      BK4(J)=GJ/HL(J)
      BK5(J)=4.*EI/HL(J)
      BK6(J)=2.*EI/HL(J)
   42 CONTINUE
   43 CONTINUE
      DO 111 L=1,36
      DO 111 J=1,12
  111 WORK6(L,J)=0.
      DO 112 L=1,12
      DO 112 J=1,12
      SK(L,J)=0.
  112 SSK(L,J)=0.
C
C     INCREMENTAL + BASIC STIFFNESS ARRAY CALS
C
C
C     ARRAY K* CALS
C
      DO 116 J=1,3
      SK(1,1)   =BK2(J)
      SK(5,1)   =BK3(J)
      SK(7,1)   =-BK2(J)
      SK(11,1)   =BK3(J)
      SK(2,2)   =BK2(J)
      SK(4,2)   =-BK3(J)
      SK(8,2)   =-BK2(J)
      SK(10,2)   =-BK3(J)
      SK(3,3)   =BK1(J)
      SK(9,3)   =-BK1(J)
      SK(4,4)   =BK5(J)
      SK(8,4)   =BK3(J)
      SK(10,4)   =BK6(J)
```

```
IV G LEVEL  1, MOD  1                BEMST                  DATE = 71363              13.19.27
      SK(5,5)   =BK5(J)
      SK(7,5)   =-BK3(J)
      SK(11,5)   =BK6(J)
      SK(6,6)   =BK4(J)
      SK(12,6)   =-BK4(J)
      SK(7,7)   =BK2(J)
      SK(11,7)   =-BK3(J)
      SK(8,8)   =BK2(J)
      SK(10,8)   =BK3(J)
      SK(9,9)   =BK1(J)
      SK(10,10)   =BK5(J)
      SK(11,11)   =BK5(J)
      SK(12,12)   =BK4(J)
      IF(HUB.EQ.1.) GO TO 80
C
C     ARRAY K** CALS
C
      POL(J)=PSS(J)/(10.*AL(J))
      SSK(1,1)   =12.*POL(J)
      SSK(5,1)   =AL(J)*POL(J)
      SSK(7,1)   =-SSK(1,1)
      SSK(11,1)   =SSK(5,1)
      SSK(2,2)   =SSK(1,1)
      SSK(4,2)   =-SSK(5,1)
      SSK(8,2)   =-SSK(2,2)
      SSK(10,2)   =SSK(4,2)
      SSK(4,4)   =4.*AL(J)**2/3.*POL(J)
      SSK(8,4)   =SSK(5,1)
      SSK(10,4)   =-SSK(4,4)/4.
      SSK(5,5)   =SSK(4,4)
      SSK(7,5)   =SSK(4,2)
      SSK(11,5)   =SSK(10,4)
      SSK(7,7)   =SSK(1,1)
      SSK(11,7)   =SSK(4,2)
      SSK(8,8)   =SSK(1,1)
      SSK(10,8)   =SSK(5,1)
      SSK(10,10)   =SSK(4,4)
      SSK(11,11)   =SSK(4,4)
   80 CONTINUE
      KM=1
      DO 114 K=1,12
      DO 113 L=KM,12
      SK(K,L)=SK(L,K)
      SSK(K,L)=SSK(L,K)
  113 CONTINUE
  114 KM=KM+1
      DO 115 L=1,12
      DO 115 K=1,12
  115 TSK(L,K)=SK(L,K)+SSK(L,K)
      GO TO (101,102,103),J
  101 L=1
      K=1
      GO TO 104
  102 L=13
```

```
      K=1
      GO TO 104
  103 L=25
      K=1
  104 CALL STACK(TSK,WORK6,12,12,72,L,K)
  116 CONTINUE
      IF(HUB.EQ.0.) GO TO 200
      DO 149 L=1,12
      DO 149 J=20,31
  149 WORK6(L,J)=0.
      DO 150 L=1,3
      DO 150 J=1,3
  150 ARY(L,J)=0.
      ARY(3,1)=-1.
      ARY(2,2)=1.
      ARY(1,3)=1.
      DO 151 L=1,10,3
      K=L+19
  151 CALL STACK(ARY,WORK6,3,3,72,L,K)
C
C     WORK6(1-12,20-31)=ARRAY C'
C
      N=0
  152 CONTINUE
      DO 153 L=1,12
      LN=L+N
      DO 153 J=1,12
      J39=J+39
      J19=J+19
      WORK6(LN,J39)=0.
      DO 153 K=1,12
  153 WORK6(LN,J39)=WORK6(LN,J39)+WORK6(LN,K)*WORK6(K,J19)
      N=N+12
      IF(N.LE.24) GO TO 152
C
C     WORK6(1-36,40-51)=ARRAY(KSTAR*Q')
C
      DO 154 L=1,12
      L19=L+19
      DO 154 J=20,31
      J13=J-7
  154 WORK6(J13,L19)=WORK6(L,J)
C
C     WORK6(13-24,20-31)=ARRAY C'(TRANSPOSE)
C
      N=0
  155 CONTINUE
      DO 156 L=1,12
      LN=L+N
      L12=L+12
      DO 156 J=1,12
      J39=J+39
      WORK6(LN,J)=0.
      DO 156 K=1,12
```

FIGURE 10.29 (Cont'd)

```
IV G LEVEL  1, MOD  1                BEMSI                DATE = 71363              13.19.27

          KN=K+N
          K19=K+19
      156 WORK6(LN,J)=WORK6(LN,J)+WCRK6(L12,K19)*WCRK6(KN,J39)
          N=N+12
          IF(N.LE.24) GO TO 155
      200 CONTINUE
          DO 117 L=1,24
          DO 117 J=25,48
      117 WORK6(L,J)=0.
          DO 118 L=1,12
          DO 118 J=1,12
      118 WORK6(L,24+J)=WORK6(L,J)
          DO 119 L=7,18
          DO 119 J=7,18
      119 WORK6(L,24+J)=WORK6(L,24+J)+WCRK6(L+6,J-6)
          DO 120 L=13,24
          DO 120 J=13,24
      120 WORK6(L,24+J)=WORK6(L,24+J)+WCRK6(L+12,J-12)
    C
    C     WORK6(1-24,25-48)=ARRAY K4
    C
          RETURN
          END
```

FIGURE 10.30 SMATRX

```
FILED SMATRX   FORTRAN  P1                                  CALLDATA  TIME - SH
      SUBROUTINE  SMATRX
      INTEGER P,ER,EC
      COMMON /PUNCH/ IRIT,TREIG,TPHHF,TSMAT,TCTYA,TK7H,TCH,TK6HG,TCHG
      COMMON /BEAM1/
     *HL(3),AL(3),PSS(3),HUB,EI,AE,GJ
      COMMON /RHMST/
     *WORK6(72,72),WORK8(72,6C),WORK9(72,60),WCRK11(57,57)
      COMMON /RHMST1/
     *XRN(3),XRD1(3),XRD2(3),ZV(3),ALPH1(3,3),GXN(3,3),GZP(3,3),ANG(2),
     *PT1,PT2,PT3,P,BKS1,TEST,QARRA(54,3),XRD3(3)
      COMMON /PRINT/
     *IOUT,IOIN
      DIMENSION XRNM(3),ZVM(3)
      DO 122 L=1,24
      DO 121 J=1,24
  121 WORK6(L,J)=0.
  122 WORK6(L,L)=1.
      DO 108 L=1,3
      XRNM(L)=-XRN(L)
  108 ZVM(L)=-ZV(L)
      CALL GAMMA(GXN,XRNM)
      CALL GAMMA(GZP,ZVM)
      CA_L STACK(GXN,WORK6,3,3,72,1,4)
      CA_L STACK(GZP,WORK6,3,3,72,19,22)
C
C     WORK6(1-24,1-24)=ARRAY Q1
C
      DO 123 L=1,24
      DO 123 J=1,24
      WORK6(L,J+48)=0.
      DO 123 K=1,24
  123 WORK6(L,J+48)=WORK6(L,J+48)+WORK6(L,K+24)*WCRK6(K,J)
C
C     WORK6(1-24,49-72)=ARRAY(K4*Q1)
C
      DO 124 L=1,24
      DO 124 J=1,24
  124 WORK6(L,J+24)=WORK6(J,L)
C
C     WORK6(1-24,25-48)=ARRAY Q1(TRANSPOSE)
C
      DO 125 L=1,24
      DO 125 J=1,24
      WORK6(L,J)=0.
      DO 125 K=1,24
  125 WORK6(L,J)=WORK6(L,J)+WORK6(L,K+24)*WCRK6(K,J+48)
C
C     WORK6(1-24,1-24)=ARRAY K5
C
      WORK6(4,4)=WORK6(4,4)+PSS(1)*XRN(3)
      WORK6(5,5)=WORK6(5,5)+PSS(1)*XRN(3)
      WORK6(22,22)=WORK6(22,22)-PT3*ZV(3)
      WORK6(23,23)=WORK6(23,23)-PT3*ZV(3)
C
C     WORK6(1-24,1-24)=ARRAY K5*
```

FIGURE 10.30 (Cont'd)

```
FILE: SMATRX   FORTRAN  P1                                  CALLDATA  TIME - SH

C
      DO 220 L=1,24
      DO 220 J=1,24
  220 WORK11(L,J)=WORK6(L,J)
  221 CONTINUE
      CALL ABCP(ALPH1,ANG(1))
      DO 126 L=1,24
      DO 126 J=1,24
      WORK6(L+24,J)=0.
  126 CONTINUE
      CALL STACK(ALPH1,WORK6,3,3,72,25,1)
      CALL STACK(ALPH1,WORK6,3,3,72,28,4)
      DO 127 L=7,24
  127 WORK6(L+24,L)=1.
C
C     WORK6(25-48,1-24)=ARRAY Q2
C
      DO 128 L=1,24
      DO 128 J=1,24
      WORK6(L+48,J)=0.
      DO 128 K=1,24
  128 WORK6(L+48,J)=WORK6(L+48,J)+WORK6(L,K)*WORK6(K+24,J)
C
C     WORK6(49-72,1-24)=ARRAY(K5*Q2)
C
      DO 129 L=1,24
      DO 129 J=1,24
  129 WORK6(J+24,L+48)=WORK6(L+24,J)
C
C     WORK6(25-48,49-72)=ARRAY Q2(TRANSPOSE)
C
      DO 130 L=1,24
      DO 130 J=1,24
      WORK6(L+24,J+24)=0.
      DO 130 K=1,24
  130 WORK6(L+24,J+24)=WORK6(L+24,J+24)+WORK6(L+24,K+48)*WORK6(K+48,J)
C
C     WORK6(25-48,25-48)=ARRAY K6
C
      IF(P.EQ.2) GO TO 145
C
C     THREE ARM ROTOR CALCULATIONS
C
      CALL ABCP(ALPH1,ANG(2))
      DO 131 L=1,24
      DO 131 J=1,24
      WORK6(L+48,J)=0.
  131 CONTINUE
      CALL STACK(ALPH1,WORK6,3,3,72,49,1)
      CALL STACK(ALPH1,WORK6,3,3,72,52,4)
      DO 132 L=7,24
  132 WORK6(L+48,L)=1.
C
C     WORK6(49-72,1-24)=ARRAY Q2*
C
```

```
      DO 133 L=1,24
      DO 133 J=1,24
      WORK6(L+48,J+24)=0.
      DO 133 K=1,24
  133 WORK6(L+48,J+24)=WORK6(L+48,J+24)+WORK6(L,K)*WORK6(K+48,J)
C
C     WORK6(49-72,24-48)=ARRAY(K5*Q2*)
C
      DO 134 L=1,24
      DO 134 J=1,24
  134 WORK6(J+24,L)=WORK6(L+48,J)
C
C     WORK6(25-48,1-24)=ARRAY Q2*(TRANSPOSE)
C
      DO 135 L=1,24
      DO 135 J=1,24
      WORK6(L+48,J+48)=0.
      DO 135 K=1,24
  135 WORK6(L+48,J+48)=WORK6(L+48,J+48)+WORK6(L+24,K)*WORK6(K+48,J+24)
C
C     WORK6(49-72,49-72)=ARRAY K6*
C
      DO 136 L=1,24
      DO 136 J=1,24
      WORK6(L+24,J)=0.
      WORK6(L+48,J)=0.
      WORK6(L,J+24)=0.
      WORK6(L,J+48)=0.
      WORK6(L+48,J+24)=0.
  136 WORK6(L+24,J+48)=0.
C
C     WORK6(1-72,1-72)=ARRAY K7Y
C
      DO 137 L=1,72
      DO 137 J=1,60
  137 WORK8(L,J)=0.
      DO 138 L=1,24
  138 WORK8(L,L)=1.
      DO 139 L=1,18
      WORK8(L+30,L+24)=1.
  139 WORK8(L+54,L+42)=1.
      DO 140 L=1,6
      WORK8(L+24,L)=1.
  140 WORK8(L+48,L)=1.
C
C     WORK8=ARRAY Q3Y
C
      DO 141 L=1,72
      DO 141 J=1,60
      WORK9(L,J)=0.
      DO 141 K=1,72
  141 WORK9(L,J)=WORK9(L,J)+WORK6(L,K)*WORK8(K,J)
      DO 142 L=1,72
      DO 142 J=1,60
  142 WORK6(J,L)=WORK8(L,J)
```

```
C
C       WORK6( 1-60,1-72)=ARRAY Q3Y(TRANSPOSE)
C
        DO 143 L=1,60
        DO 143 J=1,60
        WORK8(L,J)=0.
        DO 143 K=1,72
  143   WORK8(L,J)=WORK8(L,J)+WORK6(L,K)*WORK9(K,J)
C
C       WORK8( 1-60,1-60)=ARRAY K8Y
C
        DO 144 L=1,57
        DO 144 J=1,57
  144   WORK9(L,J)=WOPK8(L+3,J+3)
        WORK9(2,2)=WORK9(2,2)+BKS1
        WORK9(3,3)=WORK9(3,3)+BKS1
        DO 240 L=58,72
        DO 240 J=1,60
  240   WORK9(L,J)=0.
        DO 241 L=1,57
        DO 241 J=58,60
  241   WORK9(L,J)=0.
C
C       WORK9( 1-57,1-57)=ARRAY K10Y
C
        GO TO 155
C
C       TWO ARM ROTOR CALCULATIONS
C
  145   DO 146 L=1,24
        DO 146 J=1,24
        WORK6(L+24,J)=0.
  146   WORK6(L,J+24)=0.
C
C       WORK6( 1-48,1-48)=ARRAY K7T
C
        DO 147 L=1,48
        DO 147 J=1,42
  147   WORK8(L,J)=0.
        DO 148 L=1,24
  148   WORK8(L,L)=1.
        DO 149 L=1,18
  149   WORK8(L+30,L+24)=1.
        DO 150 L=1,6
  150   WORK8(L+24,L)=1.
C
C       WORK8( 1-48,1-42)=ARRAY Q3T
C
        DO 151 L=1,48
        DO 151 J=1,42
        WORK9(L,J)=0.
        DO 151 K=1,48
  151   WORK9(L,J)=WORK9(L,J)+WORK6(L,K)*WORK8(K,J)
C
C       WORK9( 1-48,1-42)=ARRAY(K7T*Q3T)
```

```
FILEO SMATRX   FORTRAN  P1                                    CALLDATA  TIME - SH

C
      DO 152 L=1,48
      DO 152 J=1,42
  152 WORK6(J,L)=WORK8(L,J)
C
C     WORK6(1-42,1-48)=ARRAY Q3T(TRANSPOSE)
C
      DO 153 L=1,42
      DO 153 J=1,42
      WORK8(L,J)=0.
      DO 153 K=1,48
  153 WORK8(L,J)=WORK8(L,J)+WORK6(L,K)*WORK9(K,J)
C
C     WORK8(1-42,1-42)=ARRAY K8T
C
      DO 154 L=1,39
      DO 154 J=1,39
  154 WORK9(L,J)=WORK8(L+3,J+3)
      WORK9(2,2)=WORK9(2,2)+BKS1
      WORK9(3,3)=WORK9(3,3)+BKS1
      DO 230 L=40,72
      DO 230 J=1,60
  230 WORK9(L,J)=0.
      DO 231 L=1,39
      DO 231 J=40,60
  231 WORK9(L,J)=0.
C
C     WORK9(1-39,1-39)=ARRAY K10T
C
  155 IF(TEST.EQ.2) GO TO 160
      WORK9(2,2)=WORK9(2,2)-BKS1
      WORK9(3,3)=WORK9(3,3)-BKS1
C
C     WORK9(1-39,1-39)=ARRAY K9TA    2-ARM ROTCR
C     WORK9(1-57,1-57)=ARRAY K9YA    3-ARM RCTCR
C
      DO 156 L=1,24
      DO 156 J=1,24
  156 WORK6(L,J)=WORK11(L,J)
      WORK6(2,2)=WORK6(2,2)-PT1/XRD1(3)-PT2/XRD2(3)-PT3/XRD3(3)
      WORK6(2,8)=WORK6(2,8)+PT1/XRD1(3)
      WORK6(8,2)=WORK6(8,2)+PT1/XRD1(3)
      WORK6(8,8)=WORK6(8,8)-PT1/XRD1(3)
      WORK6(2,14)=WORK6(2,14)+PT2/XRD2(3)
      WORK6(14,2)=WORK6(14,2)+PT2/XRD2(3)
      WORK6(14,14)=WORK6(14,14)-PT2/XRD2(3)
      WORK6(2,20)=WORK6(2,20)+PT3/XRD3(3)
      WORK6(20,2)=WORK6(20,2)+PT3/XRD3(3)
      WORK6(20,20)=WORK6(20,20)-PT3/XRD3(3)
      TEST=2.
C
C     WORK6(1-24,1-24)=ARRAY K5**
C
C     STIFFNESS MATRIX  CALCULATIONS
C
```

FIGURE 10.30 (Cont'd)

```
FILE: SMATRX   FORTRAN  P1                                  CALLDATA  TIME - SH
      DO 158 L=1,57
      DO 157 J=1,57
  157 WORK11(L,J)=0.
  158 WORK11(L,L)=1.
      CALL GAMMA(GXN,XRD1)
      CALL STACK(GXN,WORK11,3,3,57,4,1)
      DO 259 L=1,3
      WORK11(L+6,L)=-1.
      WORK11(L+12,L)=-1.
      WORK11(L+18,L)=-1.
  259 CONTINUE
      CALL GAMMA(GXN,XRD2)
      CALL STACK(GXN,WORK11,3,3,57,10,1)
      CALL GAMMA(GXN,XRD3)
      CALL STACK(GXN,WORK11,3,3,57,16,1)
C
C     WORK11(4-21,1-3)=ARRAY Q4
C
      CALL ABCP(ALPH1,ANG(1))
      DO 159 L=1,18
      DO 159 J=1,3
      WORK11(L+21,J)=0.
      DO 159 K=1,3
  159 WORK11(L+21,J)=WORK11(L+21,J)+WORK11(L+3,K)*ALPH1(K,J)
C
C     WORK11(22-39,1-3)=ARRAY Q5
C
      DO 235 L=1,36
      DO 235 J=1,3
  235 QARRA(L,J)=WORK11(L+3,J)
      IF(P.EQ.2) GO TO 186
C
C     THREE ARM ROTOR CALCULATIONS
C
      CALL ABCP(ALPH1,ANG(2))
      DO 181 L=1,18
      DO 181 J=1,3
      WORK11(L+39,J)=0.
      DO 181 K=1,3
  181 WORK11(L+39,J)=WORK11(L+39,J)+WORK11(L+3,K)*ALPH1(K,J)
C
C     WORK11(40-57,1-3)=ARRAY Q5*
C
      DO 236 L=1,54
      DO 236 J=1,3
  236 QARRA(L,J)=WORK11(L+3,J)
      DO 182 L=4,57
      DO 182 J=1,3
  182 WORK11(L,J)=-WORK11(L,J)
C
C     WORK11(4-57,1-3)=-ARRAY(Q4,Q5,Q5*)
C     WORK11(1-57,1-57)=ARRAY QY7
C
      DO 183 L=1,57
      DO 183 J=1,57
```

```
FILE: SMATRX   FORTRAN  P1                                   CALLDATA  TIME - S

      WORK8(L,J)=0.
      DO 183 K=1,57
  183 WORK8(L,J)=WORK8(L,J)+WORK9(L,K)*WORK11(K,J)
C
C     WORK8(1-57,1-57)=ARRAY((K11Y*)QY7)
C
      DO 184 L=1,57
      DO 184 J=1,57
  184 WORK11(L,J)=WORK8(L,J)
C
C     WORK11(1-57,1-57)=ARRAY KYA
C
      GO TO 191
C
C     TWO ARM ROTOR CALCULATIONS
C
  186 DO 187 L=4,39
      DO 187 J=1,3
  187 WORK11(L,J)=-WORK11(L,J)
C
C     WORK11(4-39,1-3)=-ARRAY(Q4,Q5)
C     WORK11(1-39,1-39)=ARRAY QT7
C
      DO 188 L=1,39
      DO 188 J=1,39
      WORK8(L,J)=0.
      DO 188 K=1,39
  188 WORK8(L,J)=WORK8(L,J)+WORK9(L,K)*WORK11(K,J)
C
C     WORK8(1-39,1-39)=ARRAY(K11T*)*QT7)
C
      DO 189 L=1,39
      DO 189 J=1,39
  189 WORK11(L,J)=WORK8(L,J)
C
C     WORK11(1-39,1-39)=ARRAY KTA
C
C
C     STIFFNESS MATRIX PRINTOUT
C
  191 IF(P.EQ.2) GO TO 194
      WRITE(IOUT,192)
  192 FORMAT('0','THREE ARM ROTOR STIFFNESS MATRIX')
      ER=57
      EC=57
      GO TO 196
  194 WRITE (IOUT,195)
  195 FORMAT('0','TWO ARM ROTOR STIFFNESS MATRIX')
      ER=39
      EC=39
  196 CONTINUE
      DO 279 L=1,ER
  279 WORK11(L,1)=0.
      K=53
      IF(P.EQ.2) K=35
```

FIGURE 10.30 (Cont'd)

```
FILEO SMATRX   FORTRAN  P1                                  CALLDATA  TIME - SH
      L=5
      M=17
  280 CONTINUE
      WORK11(L,1)=-PT1
      WORK11(L+6,1)=-PT2
      WORK11(M,1)=-PT3
      M=M+18
      L=L+18
      IF(M.LE.K) GO TO 280
      CALL WILLY('S-MATRIX',WORK11,57,57,1,ER,1,EC)
      CALL PWRIT(WORK11,57,57,1,ER,1,EC,3,TSMAT,IRIT)
      GO TO 221
  160 RETURN
      END
```

FIGURE 10.31 RMMODC

```
FILEO RMMODC   FORTRAN  P1                                 CALLDATA  TIME - SH

      SUBROUTINE RMMODC
      INTEGER P,ER,EC
      COMMON /RHMST/
     *WORK6(72,72),WORK8(72,60),WORK9(72,60),WORK11(57,57)
      COMMON /RHMST1/
     *XRN(3),XRD1(3),XRD2(3),ZV(3),ALPH1(3,3),GXN(3,3),GZP(3,3),ANG(2),
     *PT1,PT2,PT3,P,BKS1,TEST,QARRA(54,3),XRD3(3)
      COMMON /RRMID/
     *RRIB(3,3),DI1(3,3),DI2(3,3),DI3(3,3),DM1,DM2,DM3
      COMMON /EIGVEC/
     *COLK(435),COLM(435),EIG(29,29),OMEGA1(28),OMEGA2(29),TCL1,FREQ
      COMMON /PRINT/
     *IOUT,IOIN
C
C     ROTOR MASS MATRIX CALCULATIONS
C
      DO 161 L=1,57
      DO 161 J=1,57
  161 WORK8(L,J)=0.
      DO 163 L=1,3
      DO 162 J=1,3
      WORK8(L,J)=RRIB(L,J)
      WORK8(L+6,J+6)=DI1(L,J)
      WORK8(L+12,J+12)=DI2(L,J)
      WORK8(L+18,J+18)=DI3(L,J)
      WORK8(L+24,J+24)=DI1(L,J)
      WORK8(L+30,J+30)=DI2(L,J)
  162 WORK8(L+36,J+36)=DI3(L,J)
      WORK8(L+3,L+3)=DM1
      WORK8(L+9,L+9)=DM2
      WORK8(L+15,L+15)=DM3
      WORK8(L+21,L+21)=DM1
      WORK8(L+27,L+27)=DM2
  163 WORK8(L+33,L+33)=DM3
C
C     WORK8(1-39,1-39)=ARRAY M2T
C
      IF(P.EQ.2) GO TO 200
C
C     THREE ROTOR SYMMETRIC AND ANTISYMMETRIC MOTION CALCULATIONS
C
      DO 165 L=1,3
      DO 164 J=1,3
      WORK8(L+42,J+42)=DI1(L,J)
      WORK8(L+48,J+48)=DI2(L,J)
  164 WORK8(L+54,J+54)=DI3(L,J)
      WORK8(L+39,L+39)=DM1
      WORK8(L+45,L+45)=DM2
  165 WORK8(L+51,L+51)=DM3
C
C     WORK8(1-57,1-57)=ARRAY M2Y
C
      CALL RDAMP
      DO 166 L=1,57
      DO 166 J=1,57
```

```
FILEO RMMODC   FORTRAN  P1                              CALLDATA  TIME - SH
  166 WORK6(L,J)=0.
      WORK6(2,1)=1.
      WORK6(1,29)=1.
      WORK6(3,30)=1.
      L=4
      DO 167 J=1,9
      WORK6(L,J+1)=1.
      WORK6(L+1,J+30)=1.
  167 L=L+2
      DO 168 L=1,18
      WORK6(L+21,L+10)=1.
  168 WORK6(L+21,L+39)=1.
      DO 169 L=1,18,2
      WORK6(L+39,L+10)=1.
      WORK6(L+40,L+11)=-1.
      WORK6(L+39,L+39)=-1.
  169 WORK6(L+40,L+40)=1.
C
C     WORK6(1-57,1-28)=ARRAY YSYM2
C     WORK6(1-57,29-57)=ARRAY YANT2
C
      DO 170 L=1,57
      DO 170 J=1,28
      WORK11(L,J)=0.
      DO 170 K=1,57
  170 WORK11(L,J)=WORK11(L,J)+WORK9(L,K)*WORK6(K,J)
      DO 171 L=1,57
      DO 171 J=29,57
      WORK11(L,J)=0.
      DO 171 K=1,57
  171 WORK11(L,J)=WORK11(L,J)+WORK9(L,K)*WORK6(K,J)
C
C     WORK11(1-57,1-28)=ARRAY(K10Y*YSYM2)
C     WORK11(1-57,29-57)=ARRAY(K10Y*YANT2)
C
      DO 172 L=1,57
      DO 172 J=1,28
      WORK9(L,J)=0.
      DO 172 K=1,57
  172 WORK9(L,J)=WORK9(L,J)+WORK8(L,K)*WORK6(K,J)
      DO 173 L=1,57
      DO 173 J=29,57
      WORK9(L,J)=0.
      DO 173 K=1,57
  173 WORK9(L,J)=WORK9(L,J)+WORK8(L,K)*WORK6(K,J)
C
C     WORK9(1-57,1-28)=ARRAY(M2Y*YSYM2)
C     WORK9(1-57,29-57)=ARRAY(M2Y*YANT2)
C
      DO 174 L=1,57
      DO 174 J=1,57
  174 WORK8(J,L)=WORK6(L,J)
C
C     WORK8(1-28,1-57)=ARRAY YSYM2(TRANSPOSE)
C     WORK8(29-57,1-57)=ARRAY YANT2(TRANSPOSE)
```

```
C
      DO 175 L=1,72
      DO 175 J=1,72
  175 WORK6(L,J)=0.
      DO 176 L=1,28
      DO 176 J=1,28
      WORK6(L,J)=0.
      DO 176 K=1,57
  176 WORK6(L,J)=WORK6(L,J)+WORK8(L,K)*WORK11(K,J)
      DO 177 L=1,28
      DO 177 J=1,28
      WORK6(L,J+39)=0.
      DO 177 K=1,57
  177 WORK6(L,J+39)=WORK6(L,J+39)+WORK8(L,K)*WORK9(K,J)
C
C     WORK6( 1-28, 1-28)=ARRAY KYSYM
C     WORK6( 1-28,40-67)=ARRAY MYSYM
C
      DO 178 L=1,29
      DO 178 J=1,29
      WORK6(L+39,J)=0.
      DO 178 K=1,57
  178 WORK6(L+39,J)=WORK6(L+39,J)+WORK8(L+28,K)*WORK11(K,J+28)
      DO 179 L=1,29
      DO 179 J=1,29
      WORK6(L+39,J+39)=0.
      DO 179 K=1,57
  179 WORK6(L+39,J+39)=WORK6(L+39,J+39)+WORK8(L+28,K)*WORK9(K,J+28)
C
C     WORK6( 40-68,1-29)=ARRAY KYANT
C     WORK6( 40-68,40-68)=ARRAY MYANT
C
      DO 180 L=1,57
      DO 180 J=1,57
  180 WORK11(J,L)=WORK8(L,J)
C
C     WORK11( 1-57, 1-28)=ARRAY YSYM2
C     WORK11( 1-57,29-57)=ARRAY YANT2
C
      GO TO 250
C
C     TWO ROTOR SYMMETRIC AND ANTISYMMETRIC MOTION CALCULATIONS
C
  200 CONTINUE
      CALL RDAMP
      DO 201 L=1,39
      DO 201 J=1,39
  201 WORK6(L,J)=0.
      WORK6( 3, 1)=1.
      WORK6( 1,20)=1.
      WORK6( 2,21)=1.
      DO 202 L=1,18
      WORK6(L+3,L+1)=1.
  202 WORK6(L+3,L+21)=1.
      DO 203 L=1,18,2
```

```
FILEO RMMODC   FORTRAN  P1                              CALLDATA  TIME - SH
      WORK6(L+21,L+1)=1.
      WORK6(L+22,L+2)=-1.
      WORK6(L+21,L+21)=-1.
  203 WORK6(L+22,L+22)=1.
C
C     WORK6( 1-39,1-19)=ARRAY TSYM2
C     WORK6( 1-39,20-39)=ARRAY TANT2
C
      DO 204 L=1,39
      DO 204 J=1,19
      WORK11(L,J)=0.
      DO 204 K=1,39
  204 WORK11(L,J)=WORK11(L,J)+WORK9(L,K)*WORK6(K,J)
      DO 205 L=1,39
      DO 205 J=20,39
      WORK11(L,J)=0.
      DO 205 K=1,39
  205 WORK11(L,J)=WORK11(L,J)+WORK9(L,K)*WORK6(K,J)
C
C     WORK11( 1-39,1-19)=ARRAY(K10T*TSYM2)
C     WORK11( 1-39,20-39)=ARRAY(K10T*TANT2)
C
      DO 206 L=1,39
      DO 206 J=1,19
      WORK9(L,J)=0.
      DO 206 K=1,39
  206 WORK9(L,J)=WORK9(L,J)+WORK8(L,K)*WORK6(K,J)
      DO 207 L=1,39
      DO 207 J=20,39
      WORK9(L,J)=0.
      DO 207 K=1,39
  207 WORK9(L,J)=WORK9(L,J)+WORK8(L,K)*WORK6(K,J)
C
C     WORK9( 1-39,1-19)=ARRAY(M2T*TSYM2)
C     WORK9( 1-39,20-39)=ARRAY(M2T*TANT2)
C
      DO 208 L=1,39
      DO 208 J=1,39
  208 WORK8(J,L)=WORK6(L,J)
C
C     WORK8( 1-19,1-39)=ARRAY TSYM2(TRANSPOSE)
C     WORK8( 20-39,1-39)=ARRAY TANT2(TRANSPOSE)
C
      DO 209 L=1,72
      DO 209 J=1,72
  209 WORK6(L,J)=0.
      DO 210 L=1,19
      DO 210 J=1,19
      WORK6(L,J)=0.
      DO 210 K=1,39
  210 WORK6(L,J)=WORK6(L,J)+WORK8(L,K)*WORK11(K,J)
      DO 211 L=1,19
      DO 211 J=1,19
      WORK6(L,J+39)=0.
      DO 211 K=1,39
```

```
FILEO RWMODC   FORTRAN  P1                                   CALLDATA  TIME - SH

  211 WORK6(L,J+39)=WORK6(L,J+39)+WORK8(L,K)*WCRK9(K,J)
C
C     WORK6(1-19,1-19)=ARRAY KTSYM
C     WORK6(1-19,40-58)=ARRAY MTSYM
C
      DO 212 L=1,20
      DO 212 J=1,20
      WORK6(L+39,J)=0.
      DO 212 K=1,39
  212 WORK6(L+39,J)=WORK6(L+39,J)+WORK8(L+19,K)*WCRK11(K,J+19)
      DO 213 L=1,20
      DO 213 J=1,20
      WORK6(L+39,J+39)=0.
      DO 213 K=1,39
  213 WORK6(L+39,J+39)=WORK6(L+39,J+39)+WORK8(L+19,K)*WCRK9(K,J+19)
C
C     WORK6(40-59,1-20)=ARRAY KTANT
C     WORK6(40-59,40-59)=ARRAY MTANT
C
      DO 214 L=1,39
      DO 214 J=1,39
  214 WORK11(J,L)=WORK8(L,J)
C
C     WORK11(1-39,1-19)=ARRAY TSYM2
C     WORK11(1-39,20-39)=ARRAY TANT2
C
  250 CONTINUE
      RETURN
      END
```

FIGURE 10.32 RSMOD1

```
IV G LEVEL 1, MOD 1              RSMOD1               DATE = 71363              13.18.49

      SUBROUTINE RSMOD1
      INTEGER P,ER,EC
      COMMON /PUNCH/ IRIT,TREIG,TPHHF,TSMAT,TCTYA,TK7H,TCH,TK6HG,TCHG
      COMMON /RHMST/
     *WORK6(72,72),WORK8(72,60),WORK9(72,60),WORK11(57,57)
      COMMON /RHMST1/
     *XRN(3),XRD1(3),XRD2(3),ZV(3),ALPH1(3,3),GXN(3,3),GZP(3,3),ANG(2),
     *PT1,PT2,PT3,P,BKS1,TEST,QARRA(54,3),XRD3(3)
      COMMON /RRMID/
     *RRIB(3,3),DI1(3,3),DI2(3,3),DI3(3,3),DM1,DM2,DM3
      COMMON /EIGVEC/
     *COLK(435),COLM(435),EIG(29,29),QMEGA1(28),QMEGA2(29),TOL1,FREQ
      COMMON /PRINT/
     *IOUT,IOIN
      COMMON /TERUP/BRAK
      RAD2=6.2831853
      QMEG=RAD2*FREQ
      QMEG2=QMEG**2
      IF(P.EQ.2) GO TO 260
C
C     THREE ARM ROTOR K-BAR CALCULATIONS
C     FOR ANTI-SYMMETRIC PROBLEM ONLY
C
      DO 255 L=1,29
      DO 255 J=1,29
  255 WORK6(L+39,J)=WORK6(L+39,J)+QMEG2*WORK6(L+39,J+39)
C
C     WORK6(40-68,1-29)=ARRAY KYANT-BAR
C
      GO TO 265
C
C     TWO ARM ROTOR-K-BAR CALCULATIONS
C     FOR ANTI-SYMMETRIC PROBLEM ONLY
C
  260 DO 261 L=1,20
      DO 261 J=1,20
  261 WORK6(L+39,J)=WORK6(L+39,J)+QMEG2*WORK6(L+39,J+39)
C
C     WORK6(40-59,1-20)=ARRAY KTANT-BAR
C
      GO TO 270
C
C     THREE ARM ROTOR EIGENVECTOR-EIGENVALUE CALCULATIONS
C
C     SYMMETRIC PROBLEM CALCULATIONS
C
  265 CALL COMPR(WORK6,COLK,72,1,1,28)
      CALL COMPR(WORK6,COLM,72,1,40,28)
      CALL LATE(28,COLK,28,TOL1,QMEGA1,NIK1,COLM,WORK6,72,1,QMEG2,EIG)
      IF(BRAK.EQ.1.) RETURN
C
C     WORK6(1-28,1-28)=ARRAY EIG-YSYM1
C
C     ANTI-SYMMETRIC PROBLEM CALCULATIONS
```

```
      C
            CALL COMPR(WORK6,COLK,72,40,1,29)
            CALL COMPR(WORK6,COLM,72,40,40,29)
            CALL LATE(29,COLK,29,TOL1,QMEGA2,NIX2,COLM,WORK6,72,40,QMEG2,EIG)
            IF(BRAK.EQ.1.) RETURN
      C
      C     WORK6(40-68,1-29)=ARRAY EIG-YANT1
      C
            DO 266 L=1,57
            DO 266 J=1,28
            WORK8(L,J)=0.
            DO 266 K=1,28
        266 WORK8(L,J)=WORK8(L,J)+WORK11(L,K)*WORK6(K,J)
            DO 267 L=1,57
            DO 267 J=1,29
            WORK8(L,J+28)=0.
            DO 267 K=29,57
        267 WORK8(L,J+28)=WORK8(L,J+28)+WORK11(L,K)*WORK6(K+11,J)
      C
      C     WORK8(1-57,1-28)=ARRAY EIG-YSYM2
      C     WORK8(1-57,29-57)=ARRAY EIG-YANT2
      C
            GO TO 275
      C
      C     TWO ARM ROTOR EIGENVECTOR-EIGENVALUE CALCULATIONS
      C
      C     SYMMETRIC PROBLEM CALCULATIONS
      C
        270 CALL COMPR(WORK6,COLK,72,1,1,19)
            CALL COMPR(WORK6,COLM,72,1,40,19)
            CALL LATE(19,COLK,19,TOL1,QMEGA1,NIX1,COLM,WORK6,72,1,QMEG2,EIG)
            IF(BRAK.EQ.1.) RETURN
      C
      C     WORK6(1-19,1-19)=ARRAY EIG-TSYM1
      C
      C     ANTI-SYMMETRIC PROBLEM CALCULATIONS
      C
            CALL COMPR(WORK6,COLK,72,40,1,20)
            CALL COMPR(WORK6,COLM,72,40,40,20)
            CALL LATE(20,COLK,20,TOL1,QMEGA2,NIX2,COLM,WORK6,72,40,QMEG2,EIG)
            IF(BRAK.EQ.1.) RETURN
      C
      C     WORK6(40-59,1-20)=ARRAY EIG-TANT1
      C
            DO 271 L=1,39
            DO 271 J=1,19
            WORK8(L,J)=0.
            DO 271 K=1,19
        271 WORK8(L,J)=WORK8(L,J)+WORK11(L,K)*WORK6(K,J)
            DO 272 L=1,39
            DO 272 J=1,20
            WORK8(L,J+19)=0.
            DO 272 K=20,39
        272 WORK8(L,J+19)=WORK8(L,J+19)+WORK11(L,K)*WORK6(K+20,J)
```

FIGURE 10.32 (Cont'd)

```
C
C      WORK8(1-39,1-19)=ARRAY EIG-TSYM2
C      WORK8(1-39,20-39)=ARRAY EIG-TANT2
  275 CONTINUE
      DO 279 L=1,57
      DO 276 J=1,57
  276 WORK9(L,J)=0.
  279 WORK9(L,L)=1.
      J=29
      K=54
      IF(P.EQ.3) GO TO 400
      J=20
      K=36
  400 CONTINUE
      DO 401 L=1,3
  401 WORK8(L,J)=WORK9(L,1)
      DO 402 L=1,K
  402 WORK8(L+3,J)=-QARRA(L,1)
C
C      RELATIVE COORDINATES MODAL MATRIX CALCULATIONS
C
      IF(P.EQ.2) GO TO 280
C
C      THREE ARM ROTOR CAL'S
C
      DO 277 L=1,54
      DO 277 J=1,3
  277 WORK9(L+3,J)=QARRA(L,J)
C
C      WORK9(1-57,1-57)=ARRAY QY6
C
      DO 278 L=1,57
      DO 278 J=1,57
      WORK11(L,J)=0.
      DO 278 K=1,57
  278 WORK11(L,J)=WORK11(L,J)+WORK9(L,K)*WORK8(K,J)
C
C      WORK11(1-57,1-57)=MODAL MATRIX EIG-YR
C
      CALL ANORM(WORK6,WORK11,57,72,57)
      GO TO 285
C
C      TWO ARM ROTOR CAL'S
C
  280 DO 281 L=1,36
      DO 281 J=1,3
  281 WORK9(L+3,J)=QARRA(L,J)
C
C      WORK9(1-39,1-39)=ARRAY QT6
C
      DO 282 L=1,39
      DO 282 J=1,39
      WORK11(L,J)=0.
      DO 282 K=1,39
```

```
  282 WORK11(L,J)=WORK11(L,J)+WORK9(L,K)*WORK8(K,J)
C
C     WORK11(1-39,1-39)=MODAL MATRIX EIG-TR
C
      CALL ANORM(WORK6,WORK11,57,72,39)
  285 CONTINUE
      DO 297 L=1,3
      DO 296 J=1,3
  296 ALPH1(L,J)=0.
  297 ALPH1(L,L)=1.
C
C     ALPH1(1-3,1-3)=ARRAY I3
C
      DO 298 L=1,57
      DO 298 J=1,57
  298 WORK11(L,J)=0.
      CALL STACK(ALPH1,WORK11,3,3,57,1,1)
      CALL STACK(ALPH1,WORK11,3,3,57,10,4)
      CALL STACK(ALPH1,WORK11,3,3,57,4,7)
      CALL STACK(ALPH1,WORK11,3,3,57,13,10)
      CALL STACK(ALPH1,WORK11,3,3,57,7,13)
      CALL STACK(ALPH1,WORK11,3,3,57,16,16)
C
C     WORK11(1-18,1-18)=ARRAY Q8
C
      DO 299 L=1,18
      DO 299 J=1,18
  299 WORK11(L+21,J+21)=WORK11(L+21,J+21)+WORK11(L,J)
      DO 305 L=4,18
      DO 305 J=4,18
  305 WORK11(L,J)=0.
      DO 306 L=1,18
      DO 306 J=1,18
  306 WORK11(L+3,J+3)=WORK11(L+3,J+3)+WORK11(L+21,J+21)
C
C     WORK11(1-39,1-39)=ARRAY Q9T
C
      IF(P.EQ.2) GO TO 310
C
C     THREE ARM ROTOR CALCULATIONS
C
      DO 307 L=1,18
      DO 307 J=1,18
  307 WORK11(L+39,J+39)=WORK11(L+39,J+39)+WORK11(L+21,J+21)
C
C     WORK11(1-57,1-57)=ARRAY Q9Y
C
      DO 308 L=1,57
      DO 308 J=1,57
      WORK9(L,J)=0.
      DO 308 K=1,57
  308 WORK9(L,J)=WORK9(L,J)+WORK11(L,K)*WORK6(K,J)
C
C     WORK9(1-57,1-57)=ARRAY EIG-YRF
```

```
C
      GO TO 315
C
C     TWO ARM ROTOR CALCULATIONS
C
  310 CONTINUE
      DO 311 L=1,39
      DO 311 J=1,39
      WORK9(L,J)=0.
      DO 311 K=1,39
  311 WORK9(L,J)=WORK9(L,J)+WORK11(L,K)*WORK6(K,J)
C
C     WORK9(1-39,1-39)=ARRAY EIG-TRF
C
  315 CONTINUE
      ER=57
      IF(P.EQ.2) ER=39
      EC=ER
      DO 316 L=1,ER
      DO 316 J=1,EC
  316 WORK11(L,J)=WORK9(L,J)
      CALL ANORM(WORK11,WORK11,57,57,ER)
      IF(P.EQ.2) GO TO 300
      WRITE(IOUT,286)
  286 FORMAT('0','THREE ARM ROTOR MODAL MATRIX')
      NB=-9
      NL=0
      NUMM=28
      NUM1=29
      NBB=19
      NLL=28
      NLL1=57
      GO TO 287
  300 WRITE(IOUT,301)
  301 FORMAT('0','TWO ARM ROTOR MODAL MATRIX')
      NB=-9
      NL=0
      NUMM=19
      NUM1=20
      NBB=10
      NLL=19
      NLL1=39
  287 WRITE(IOUT,288)
  288 FORMAT('0','SYMMETRIC MODAL MATRIX EIGENVALUES')
  289 NL=NL+10
      IF(NL.GT.NUMM) NL=NUMM
      NB=NB+10
      WRITE(IOUT,290) (I,I=NB,NL)
  290 FORMAT('0',7X,'OMEGA1'//' EIG. VAL-NO.',I3,9I12)
      WRITE(IOUT,291) (OMEGA1(K),K=NB,NL)
  291 FORMAT(12X,10E12.5)
      IF(NL.LT.NUMM) GO TO 289
      WRITE(IOUT,292)
  292 FORMAT('0','ANTISYMMETRIC MODAL MATRIX EIGENVALUES')
```

```
IV G LEVEL 1, MOD 1              RSMOD1              DATE = 71363              13.18.49

      NL=0
      NB=-9
  293 NL=NL+10
      NLL=NLL+10
      IF(NL.GT.NUM1) NL=NUM1
      IF(NLL.GT.NLL1)  NLL=NLL1
      NB=NB+10
      NBB=NBB+10
      WRITE(IOUT,294)  (I,I=NBB,NLL)
  294 FORMAT('0',7X,'QMEGA2'//' EIG. VAL-NO.',I3,9I12)
      WRITE(IOUT,295)  (QMEGA2(K),K=NB,NL)
  295 FORMAT(12X,10E12.5)
      IF(NL.LT.NUM1) GO TO 293
      WRITE(IOUT,350)
  350 FORMAT('0','ROTOR MODAL MATRIX')
      CALL WILLY('EIG-TYRP',WORK11,57,57,1,ER,1,EC)
C
C     INSERT PUNCH-TAPE ROUTINE HERE
      CALL PWRIT(WORK11,57,57,1,ER,1,EC,1,TREIG,IRIT)
C
      CALL RDAMP1
      RETURN
      END
```

FIGURE 10.33 RDAMP

```
IV G LEVEL  1, MOD  1                RDAMP                  DATE = 71363            13.19.1

          SUBROUTINE RDAMP
          COMMON /PUNCH/ IRIT,TREIG,TFHHF,TSMAT,TCTYA,TK7H,TCH,TK6HG,TCHG
          COMMON /PRINT/IOUT,IOIN
          COMMON /RDAMPC/
         *DAMPR
          COMMON /RHMST/
         *WORK6(72,72),WORK8(72,60),WCRK9(72,60),WORK11(57,57)
          COMMON /RHMST1/
         *XRN(3),XRD1(3),XRD2(3),ZV(3),ALPH1(3,3),GXN(3,3),GZP(3,3),ANG(2),
         *PT1,PT2,PT3,P,BKS1,TEST,QARRA(54,3),XRD3(3)
          COMMON /EIGVEC/
         *COLK(435),COLM(435),EIG(29,29),QMEGA1(28),QMEGA2(29),TOL1,FREQ
          INTEGER P
          DIMENSION AINV(57,57),LCC(57)
          M=57
          IF(P.EQ.2) M=39
          DO 10 L=1,M
          DO 10 J=1,M
       10 AINV(L,J)=WORK8(L,J)
      C
      C   AINV(1-39/57,1-39/57)=ARRAY W2T/W2Y
      C
          RETURN
      C
      C   ENTRY POINT
      C
          ENTRY RDAMP1
          QMEGA2(1)=0.
          LZ=0
          LL=28
          LM=LL+1
          LK=57
          IF(P.EQ.3) GO TO 15
          LL=19
          LM=LL+1
          LK=39
       15 CONTINUE
          DO 20 L=1,M
          DO 20 J=1,M
       20 WORK9(L,J)=0.
          DO 21 L=1,LL
       21 WORK9(L,L)=QMEGA1(L)
          DO 22 L=LM,LK
          LZ=LZ+1
       22 WORK9(L,L)=QMEGA2(LZ)
      C
      C   WORK9(1-39/57,1-39/57)=ARRAY OMEGA-T/Y
      C
          DO 35 L=1,M
          DO 35 J=1,M
       35 WORK6(L,J)=WORK8(L,J)
          CALL ANORM(WORK6,WORK6,72,72,M)
          DO 23 L=1,M
          DO 23 J=1,M
```

FIGURE 10.33 (Cont'd)

```
   23 WORK8(L,J)=AINV(L,J)
      DO 24 L=1,M
      DO 24 J=1,M
   24 AINV(L,J)=WORK6(L,J)
C
C     AINV(1-39/57,1-39/57)=ARRAY EIGVEC(NORMALIZED)T/Y
C
      CALL SREVN1(AINV,M,LOC,57,NNN)
      DO 25 L=1,M
      DO 25 J=1,M
      WORK11(L,J)=0.
      DO 25 K=1,M
   25 WORK11(L,J)=WORK11(L,J)+WORK9(L,K)*AINV(K,J)
C
C     WORK11(1-39/57,1-39/57)=ARRAY(OMEGA-T/Y*EIGVEC-NORM( INVERSE))=A
C
      DO 26 L=1,M
      DO 26 J=1,M
      WORK9(L,J)=0.
      DO 26 K=1,M
   26 WORK9(L,J)=WORK9(L,J)+WORK6(L,K)*WORK11(K,J)
C
C     WORK9(1-39/57,1-39/57)=ARRAY(EIGVEC(NORM)*A)=B
C
      DAMPR2=2.*DAMPR
      DO 27 L=1,M
      DO 27 J=1,M
      WORK11(L,J)=0.
      DO 27 K=1,M
   27 WORK11(L,J)=WORK11(L,J)+DAMPR2*WORK8(L,K)*WORK9(K,J)
C
C     WORK11(1-39/57,1-39/57)=ARRAY CT/Y
C
      DO 29 L=1,M
      DO 28 J=1,M
   28 WORK9(L,J)=0.
   29 WORK9(L,L)=1.
      MM=M-3
      DO 30 L=1,MM
      DO 30 J=1,3
   30 WORK9(L+3,J)=-QARRA(L,J)
C
C     WORK9(1-39/57,1-39/57)=ARRAY CT7/CY7
C
      DO 31 L=1,M
      DO 31 J=1,M
      AINV(L,J)=0.
      DO 31 K=1,M
   31 AINV(L,J)=AINV(L,J)+WORK11(L,K)*WORK9(K,J)
      DO 40 L=1,M
   40 AINV(L,1)=0.
C
C     AINV(1-39/57,1-39/57)=ARRAY CTA/CYA
C
```

FIGURE 10.33 (Cont'd)

```
I IV G LEVEL  1, MOD  1                 RDAMP               DATE = 71363              13.19.1
          WRITE(IOUT,60)
       60 FORMAT('0','ROTOR DAMPING MATRIX')
          CALL WILLY('ARY-CTYA',AINV,57,57,1,M,1,M)
    C
    C     INSERT TAPE-PUNCH ROUTINE HERE
          CALL PWRIT(AINV,57,57,1,M,1,M,4,TCTYA,IRIT)
    C
          RETURN
          END
```

FIGURE 10.34 HMMOD

```
AN IV G LEVEL  1, MOD 1              HMMOD              DATE = 71363            13.21.5

            SUBROUTINE HMMOD
            INTEGER P,ER,EC
            COMMON /PUNCH/ IRIT,TREIG,TFHHF,TSMAT,TCTYA,TK7H,TCH,TK6HG,TCHG
            COMMON /BEAM1/
           *HL(3),AL(3),PSS(3),HUB,EI,AE,GJ
            COMMON /RHMST/
           *WORK6(72,72),WORK8(72,60),WORK9(72,60),WORK11(57,57)
            COMMON /EIGVEC/
           *COLK(435),COLM(435),EIG(29,29),OMEGA1(28),OMEGA2(29),TCL1,FREQ
            COMMON /HUBID/
           *CI1(3,3),CI2(3,3),CI3(3,3),YHE(3),CM1,CM2,CM3,YHA(3),YHG(3),
           *YHC1(3),YHC2(3),YHC3(3)
            COMMON /PRINT/
           *IOUT,IOIN
            COMMON /TERUP/BRAK
            DIMENSION YHE3(3),GYHL(3,3)
            DO 10 L=1,24
            DO 9  J=1,24
          9 WORK6(L,J)=0.
         10 WORK6(L,L)=1.
            YHE3(3)=0.
            YHE3(2)=0.
            YHE3(1)=-(YHE(1)-YHC3(1))
            CALL GAMMA(GYHL,YHE3)
            CALL STACK(GYHL,WORK6,3,3,72,19,22)
      C
      C     WORK6(1-24,1-24)=ARRAY Q1H
      C
            DO 12 L=1,24
            DO 12 J=1,24
            WORK6(L,J+48)=0.
            DO 12 K=1,24
         12 WORK6(L,J+48)=WORK6(L,J+48)+WORK6(L,K+24)*WORK6(K,J)
      C
      C     WORK6(1-24,49-72)=ARRAY(K4H*Q1H)
      C
            DO 13 L=1,24
            DO 13 J=1,24
         13 WORK6(L,J+24)=WORK6(J,L)
      C
      C     WORK6(1-24,25-48)=ARRAY Q1H(TRANSPOSE)
      C
            DO 14 L=1,24
            DO 14 J=1,24
            WORK6(L,J)=0.
            DO 14 K=1,24
         14 WORK6(L,J)=WORK6(L,J)+WORK6(L,K+24)*WORK6(K,J+48)
      C
      C     WORK6(1-24,1-24)=ARRAY K5H
      C
            DO 15 L=1,24
            DO 15 J=1,18
         15 WORK8(L,J)=WORK6(L,J+6)
      C
```

```
C     WORK8(1-24,1-18)=ARRAY K6H
C     WORK8(1-6,1-18)=ARRAY K6HG
C
      DO 16 L=1,72
      DO 16 J=1,72
   16 WORK6(L,J)=0.
      DO 17 L=1,18
      DO 17 J=1,18
   17 WORK6(L,J)=WORK8(L+6,J)
C
C     WORK6(1-18,1-18)=ARRAY K7H
C
      DO 18 L=1,3
      WORK6(L,L+39)=CM1
      WORK6(L+6,L+45)=CM2
   18 WORK6(L+12,L+51)=CM3
      CALL STACK(CI1,WORK6,3,3,72,4,43)
      CALL STACK(CI2,WORK6,3,3,72,10,49)
      CALL STACK(CI3,WORK6,3,3,72,16,55)
C
C     WORK6(1-18,40-57)=ARRAY MH
C
      CALL COMPR(WORK6,COLK,72,1,1,18)
      CALL COMPR(WORK6,COLM,72,1,40,18)
      CALL LATE(18,COLK,18,TOL1,OMEGA1,NIX3,COLM,WORK6,72,1,OMEG2,EIG)
      IF(BRAK.EQ.1.) RETURN
C
C     WORK6(1-18,1-18)=ARRAY EIGVEC-HUB
C
      CALL ANORM(WORK11,WORK6,72,57,18)
C
C     WORK11(1-18,1-18)=NORMALIZE ARRAY EIGVEC-HUB
C     HUB DAMPING MATRIX CALCULATIONS
C
      CALL HDAMP
C
C     INSERT-TAPE-PUNCH ROUTINE HERE
      CALL PWRIT(WORK8,72,60,1,6,1,18,7,TK6HG,IRIT)
      CALL PWRIT(WORK8,72,60,7,24,1,18,5,TK7H,IRIT)
C
      DO 19 L=1,18
      DO 19 J=1,18
   19 WORK6(L,J)=0.
      DO 20 L=1,3
      WORK6(L,L)=1.
      WORK6(L+9,L+3)=1.
      WORK6(L+3,L+6)=1.
      WORK6(L+12,L+9)=1.
      WORK6(L+6,L+12)=1.
   20 WORK6(L+15,L+15)=1.
C
C     WORK6(1-18,1-18)=ARRAY Q8
C
      DO 21 L=1,18
```

FIGURE 10.34 (Cont'd)

```
AN IV G LEVEL  1, MOD  1              HMMOD              DATE = 71363              13.21.5
      DO 21 J=1,18
      WORK6(L,J+18)=0.
      DO 21 K=1,18
   21 WORK6(L,J+18)=WORK6(L,J+18)+WORK6(L,K)*WORK11(K,J)
      DO 22 L=1,18
      DO 22 J=1,18
   22 WORK11(L,J)=WORK6(L,J+18)
      CALL ANORM(WORK11,WORK11,57,57,18)
C
C     WORK11(1-18,1-18)=ARRAY FH-HF
C
      ER=18
      EC=18
      NB=-9
      NL=0
      NUMM=18
      WRITE(IOUT,24)
   24 FORMAT('0','HUB-MODAL MATRIX EIGENVALUES')
   26 NL=NL+10
      IF(NL.GT.NUMM) NL=NUMM
      NB=NB+10
      WRITE(IOUT,28) (I,I=NB,NL)
   28 FORMAT('0',7X,'OMEGA'//' EIG. VAL-NO.',I3,9I12)
      WRITE(IOUT,30) (OMEGA1(K),K=NB,NL)
   30 FORMAT(12X,10E12.5)
      IF(NL.LT.NUMM) GO TO 26
      WRITE(IOUT,40)
   40 FORMAT('0','HUB MODAL MATRIX')
      CALL WILLY('MODE-MAT',WORK11,57,57,1,ER,1,EC)
C
C     INSERT TAPE-PUNCH ROUTINE HERE
      CALL PWRIT(WORK11,57,57,1,18,1,18,2,TPHHF,IRIT)
C
      RETURN
      END
```

FIGURE 10.35 HDAMP

```
FILE: HDAMP     FORTRAN  P1                              CALLDATA  TIME - SHA
      SUBROUTINE HDAMP
      COMMON /PUNCH/ IRIT,TREIG,TPHHF,TSMAT,TCTYA,TK7H,TCH,TK6HG,TCHG
      COMMON /PRINT/IOUT,IOIN
      COMMON /HDAMPC/
     *DAMPH
      COMMON /RHMST/
     *WORK6(72,72),WORK8(72,60),WORK9(72,60),WORK11(57,57)
      COMMON /EIGVEC/
     *COLK(435),COLM(435),EIG(29,29),OMEGA1(28),OMEGA2(29),TOL1,FREQ
      DIMENSION AINV(18,18),LOC(18)
      DO 51 L=1,18
      DO 50 J=1,18
   50 WORK11(L+19,J+19)=0.
   51 WORK11(L+19,L+19)=OMEGA1(L)
C
C     WORK11(20-37,20-37)=ARRAY OMEGA-H (WH)
C
      DO 52 L=1,18
      DO 52 J=1,18
   52 AINV(L,J)=WORK11(L,J)
C
C     AINV(1-18,1-18)=ARRAY EIGVEC(NORMALIZED)
C
      CALL SREVN1(AINV,18,LOC,18,NNX)
      DO 53 L=1,18
      DO 53 J=1,18
      WORK11(L+19,J)=0.
      DO 53 K=1,18
   53 WORK11(L+19,J)=WORK11(L+19,J)+WORK11(L+19,K+19)*AINV(K,J)
C
C     WORK11(20-37, 1-18)=ARRAY(WH*OH*(INVERSE))=A
C
      DO 54 L=1,18
      DO 54 J=1,18
      WORK11(L+39,J)=0.
      DO 54 K=1,18
   54 WORK11(L+39,J)=WORK11(L+39,J)+WORK11(L,K)*WORK11(K+19,J)
C
C     WORK11(40-57,1-18)=ARRAY(EIGVEC(NORM)*A)=B
C
      DAMP2=2.*DAMPH
      DO 55 L=1,18
      DO 55 J=1,18
      WORK11(L+19,J+39)=0.
      DO 55 K=1,18
   55 WORK11(L+19,J+39)=WORK11(L+19,J+39)+DAMP2*WORK6(L,K+39)*WORK11(K+3
     *9,J)
C
C     WORK11(20-37,40-57)=ARRAY CH
C
      DO 60 L=1,18
      DO 60 J=1,18
   60 AINV(L,J)=WORK8(L+6,J)
C
C     AINV(1-18,1-18)=ARRAY K7H
```

FIGURE 10.35 (Cont'd)

```
C
      CALL SREVN1(AINV,18,LOC,18,NX1)
      DO 61 L=1,18
      DO 61 J=1,18
      WORK8(L,J+19)=0.
      DO 61 K=1,18
   61 WORK8(L,J+19)=WORK8(L,J+19)+AINV(L,K)*WCRK11(K+19,J+39)
C
C     WORK8(1-18,20-37)=ARRAY(K7H(INV)*CH)
C
      DO 62 L=1,6
      DO 62 J=1,18
      WORK8(L,J+39)=0.
      DO 62 K=1,18
   62 WORK8(L,J+39)=WORK8(L,J+39)+WORK8(L,K)*WORK8(K,J+19)
C
C     WORK8(1-6,40-57)=ARRAY CHG
C
      WRITE(IOUT,80)
   80 FORMAT('0','HUB STIFFNESS MATRIX')
      CALL WILLY('ARRY-K7H',WORK8,72,60,7,24,1,18)
      WRITE(IOUT,81)
   81 FORMAT('0','HUB DAMPING MATRIX')
      CALL WILLY('ARRY-CH ',WORK11,57,57,20,37,40,57)
      WRITE(IOUT,82)
   82 FORMAT('0','HUB (ATTACHMENT POINT) STIFFNESS MATRIX')
      CALL WILLY('ARY-K6HG',WORK8,72,60,1,6,1,18)
      WRITE(IOUT,83)
   83 FORMAT('0','HUB (ATTACHMENT POINT) DAMPING MATRIX')
      CALL WILLY('ARRY-CHG',WORK8,72,60,1,6,40,57)
C
C     INSERT-TAPE-PUNCH ROUTINE HERE
      CALL PWRIT(WORK11,57,57,20,37,40,57,6,TCH,IRIT)
      CALL PWRIT(WORK8,72,60,1,6,40,57,8,TCHG,IRIT)
C
      RETURN
      END
```

FIGURE 10.36 LATE

```
FILE: LATE     FORTRAN  P1                                  CALLDATA  TIME - SHA

      SUBROUTINE LATE(N,B,NUMEIG,TOL,QMEGA,NIX,A,ARRAY,NA,IR,COR,EIG)
      COMMON/PRINT/IOUT,IOIN
      COMMON /TERUP/BRAK
C     SOLVE THE EIGENVALUE PROBLEM
C     (A) (X) =(LAMBDA) (B) (X)
C     WHERE-
C     A  = THE NXN MASS MATRIX
C     B  = THE NXN STIFFNESS MATRIX
C     Q = THE EIGENVALUES
C     QMEGA = THE FREQUENCIES
C     EIG  = THE EIGENVECTORS
C     NUMEIG= NUMBER OF EIGENVALUES
C     N  = ORDER OF A,B
C     TOL = TOLERANCE LIMIT  0.LE.TOL.LT.1E-9
C     NIX =    B ARRAY POSITIVE-DEFINITE TEST
C         =0 ARRAY OK
C         =+ OVERFLOW
C         =- NOT POSITIVE DEFINITE
C     ARRAY - EIGENVECTOR ARRAY OUTPUT
C     NA - ORDER OF ARRAY
C     IR - STARTING ROW OF EIGENVECTOR ARRAY
C     COR - FREQ. SHIFT FACTOR ANTISYMMETRIC PROB. ONLY
C
      DIMENSION A(1),B(1),EIG(N,1),Q(210)
      DIMENSION ARRAY(NA,NA),QMEGA(NUMEIG)
      EQUIVALENCE (DOUBLE,Q(1))
      DOUBLE PRECISION DOUBLE
      COMMON/WINTER/INDIC8
      COMMON/BOND/M,L
      INDIC8=-1
      M=N
      L=1
      INDEX=(N*N+N)/2+1
      INDEX1=INDEX-1
    6 NIX=0
C
C     FORM CHOLESKY DECOMPOSITION OF B MATRIX
C
      CALL FUTILE(B,N,NIX)
      IF(NIX)2,7,7
    2 WRITE(IOUT,41)
   41 FORMAT('0',10X,'STIFFNESS ARRAY NOT POSITIVE DEFINITE',//,11X,'EIG
     *ENVALUES-MODAL VECTORS NOT COMPUTED',//,11X,'CHECK INPUT DATA')
      BRAK=1.
      RETURN
C
C     FORM A-MATRIX COLUMN INVERSES - STORE IN Q
C
    7 IK=0
      DO 10 I=1,N
      DO 11 I1=1,I
      IL=IK+I1
   11 Q(I1)=B(IL)
      IK=IK+I
      X=-Q(I)
```

```
FILE: LATE     FORTRAN  P1                          CALLDATA  TIME - SHAR

      Q(I)=-1.                                                               I
      DO 20 J=1,I                                                            I
   20 Q(J)=Q(J)/X                                                            I
   10 CALL DAGGER(A,N,Q,I ,EIG)                                              L
C                                                                            L
C     STORE(L)(A)(L-TRANSPOSE) IN A FORM SUITABLE FOR SYMEIG                 L
C     WHERE L IS THE INVERSE OF THE CHOLESKY MATRIX                          L
C                                                                            L
      CALL SWITCH(A,N)                                                       L
      CALL SYMEIG(A,N,1,NUMEIG,Q,TOL)                                        L
C                                                                            L
C     EIGENVALUES ARE IN Q                                                   L
C                                                                            L
      CALL SYMVEC(EIG,1,N,N)                                                 L
      DO 24 I=1,N                                                            L
   24 CALL TRIEQ(B,EIG(1,I))                                                 L
      IF(IR.EQ.1)  GO TO 26                                                  L
      DO 25 I=1,NUMEIG                                                       L
      CHEC=1./Q(I)-COR                                                       L
      IF(CHEC.GE.0.) GO TO 25                                                L
      WRITE(IOUT,50) I,Q(I),CHEC                                             L
   50 FORMAT('0',7X,'NEGATIVE EIGENVALUE(ANTI-MODE)-SUBROUTINE LATE'//1      L
     *X,'Q(',I3,')=',E14.7,5X,'CHEC=',E14.7)                                 L
      CHEC=ABS(CHEC)                                                         L
   25 QMEGA(I)=SQRT(CHEC)                                                    L
      GO TO 29                                                               L
   26 DO 27 I=1,NUMEIG                                                       L
   27 QMEGA(I)=SQRT(1./Q(I))                                                 L
   29 CONTINUE                                                               L
      DO 30 L=1,N                                                            L
      DO 30 J=1,N                                                            L
      JJ=J+IR-1                                                              L
   30 ARRAY(JJ,L)=EIG(J,L)                                                   L
      RETURN                                                                 L
      END                                                                    L
```

FIGURE 10.37 FUTILE

```
N IV G LEVEL 1, MOD 1              FUTILE              DATE = 71363          13.17.28

            SUBROUTINE FUTILE(A,N,NIX)
            DIMENSION A(1)
            DOUBLE PRECISION SUM
            EQUIVALENCE (SUN,SUM)
            K1 = 1
            KK = 0
            DO 210  K = 1,N
            KK = KK + K
            IK = KK
            KK1 = KK - 1
            IF (KK1) 60,50,60
         50 ASSIGN 100 TO LEAP
            GO TO 70
         60 ASSIGN  80 TO LEAP
         70 I1 = K1
            DO 140  I = K,N
            SUM = -A(IK)
            GO TO LEAP, (80,100)
         80 IJ = I1
            DO 90 KJ = K1,KK1
            SUM = SUM + A(IJ)*A(KJ)
         90 IJ = IJ + 1
        100 I1 = I1 + I
            IF (I - K) 120,105,120
        105 DENOM = -SUM
            IF (DENOM) 980,980,110
        110 DENOM = -SQRT(DENOM)
            A(IK) = -DENOM
            GO TO 130
        120 A(IK) = SUM / DENOM
        130 IK = IK + I
        140 CONTINUE
            K1 = K1 + K
        210 CONTINUE
            NIX = 0
        220 RETURN
        980 NIX = -K
            GO TO 220
             END
```

FIGURE 10.38 DAGGER

```
FILE: DAGGER   FORTRAN  P1                                     CALLDATA  TIME - S

      SUBROUTINE DAGGER(A,M,P,K0,Q)
      DIMENSION A(1),P(1),Q(1)
      DOUBLE PRECISION SUM
      EQUIVALENCE (SUN,SUM)
      K = K0
      K1 = K + 1
      L=1
      LL = 0
      INDEX = 1
      DO 130  I = 2,K1
      LJ = INDEX
      SUM = 0.
      DO 90  J = 1,L
      SUM = SUM + A(LJ)*P(J)
   90 LJ = LJ + 1
      IF (K - L) 100,120,100
  100 LL = LL + L
      LJ = LL + L
      DO 110  J = I,K
      SUM = SUM + A(LJ)*P(J)
  110 LJ = LJ + J
  120 Q(I-1) = SUN
      INDEX = INDEX + L
  130 L = I
      SUM = 0.
      DO 140  I = 1,K
      SUM = SUM + P(I)*Q(I)
      A(LL+1) = Q(I)
  140 LL = LL + 1
      A(LL) = SUN
      IF (M - K)  150, 200, 150
  150 DO 190  L = K1,M
      JL = LL+L-K
      SUM = 0.
      DO 180  J = 1,K
      SUM = SUM + P(J)*A(JL)
  180 JL=JL+1
      LL = LL + L - 1
  190 A(LL) = SUM
  200 CONTINUE
      RETURN
      END
```

FIGURE 10.39 SWITCH

```
IV G LEVEL 1, MOD 1                SWITCH              DATE = 71363            13.17.36

          SUBROUTINE SWITCH(A,M)
          DIMENSION A(1)
          N = IABS(M)
          IF (N - 2) 190,190,90
       90 L = (N*(N+1)) / 2
          KEY = 1
          LOCK = N/2 + 1
          IF (M) 100,190,160
      100 IF (N - 3) 110,140,110
      110 KKT = 3
          NKP = N - 1
          IMAGE = L
          INTO = L - 3
          I = 3
          DO 130  K = 2,LOCK
          DO 120  IK = KKT,NKP
          X = A(IK)
          A(IK) = A(INTO)
          A(INTO) = X
          INTO = INTO - I
      120 I = I + 1
          KKT = NKP + K
          NKP = NKP + N - K
          IMAGE = IMAGE - K
          INTO = IMAGE
      130 I = K
      140 IF (KEY) 150,190,150
      150 KEY = 0
      160 LOV2 = L / 2
          K = L - 2
          DO 170  I = 3,LOV2
          X = A(I)
          A(I) = A(K)
          A(K) = X
      170 K = K - 1
          IF (KEY) 180,190,180
      180 KEY = 0
          GO TO 100
      190 RETURN
          END
```

FIGURE 10.40 SYMEIG

```
IV G LEVEL  1, MOD  1                SYMEIG               DATE = 71363               13.16.35

         SUBROUTINE SYMEIG(A,M,INDEX,NUMBR,R,TOL)
C   MASTER ROUTINE FOR EIGENVALUES AND (OPTIONALLY) EIGENVECTORS OF A
C     REAL SYMMETRIC MATRIX STORED TRIANGULARLY IN CORE. THE HOUSEHOLDER
C     REDUCTION TO TRIDIAGONAL FORM (SUBROUTINE TFORM) IS FOLLOWED BY A
C     BISECTION TECHNIQUE (SUBROUTINE STURM) FOR THE ROOTS AND THEN IN-
C     VERSE ITERATION (SUBROUTINE TRIVEC) FOR THE VECTORS.
C
C
C   MEANING OF THE PARAMETERS.
C     A         -  THE TRIANGULAR ARRAY CONTAINING THE MATRIX. DURING EXEC-
C                    CUTION THE ARRAY IS CHANGED.
C
C     M         -  ORDER OF A.
C
C     INDEX     -  INDEX OF FIRST EIGENVALUE REQUIRED.
C
C     NUMBR     - NUMBER OF EIGENVALUES REQUIRED.
C
C     R         - REAL ARRAY FOR THE ROOTS AND WORKING STORAGE. R MUST CON-
C                   TAIN AT LEAST 8*M WORDS AND START ON A D.P. BOUNDARY.
C
C     TOL       - TOLERANCE. IF B IS THE MAG. OF LARGEST ROOT, THE ROOTS
C                   WILL USUALLY HAVE ERRORS AS LARGE AS B*TOL.
C
      DIMENSION A(1),R(1)
      LD = 1 + M
      LO = LD + M
      LS = LO + M
      LP = LS + M
      LQ = LP + M
      LR = LQ + M
      CALL TFORM(A,M,R(LD),R(LO),R(LS),R(LP))
      EPS = AMIN1(TOL,1.E-3)
      CALL STURM(M,INDEX,NUMBR,R(LD),R(LO),R(LS),R(LP),R,EPS)
      RETURN
      ENTRY SYMVEC(X,LOW,KOUNT,MID)
      DIMENSION X(MID,1)
      CALL  QSVEC(A,R(LD),R(LO),R(LP),R(LQ),R(LR),R(LS),M)
      ROOT = R(LS-1)
      EPS = ROOT * 3.E-7
      K = LOW
      DO 200  I = 1,KOUNT
      ROOT = AMIN1(ROOT-EPS,R(K))
      CALL  QWIEL(ROOT,X(1,I))
  200 K = K + 1
      RETURN
      END
```

FIGURE 10.41 TRIEQ

```
V G LEVEL 1, MOD 1              TRIEQ             DATE = 71363          13.17.45

      SUBROUTINE TRIEQ(A,Y)
      REAL A(1),Y(1)
      COMMON /WINTER/INDIC8
      COMMON /BOND/ M,L
      DOUBLE PRECISION SUM
      EQUIVALENCE (SUN,SUM)
      L1 = L
      LM1 = L1 - 1
      MM1 = M - 1
      IF (INDIC8) 130,100,100
  100 Y(L1) = Y(L1) / A(L1)
      IF (MM1) 105,125,105
  105 I1 = L1
      DO 120  I = L1,MM1
      I1 = I1 + I
      SUM = -Y(I+1)
      IJ = I1
      DO 110  J = L1,I
      SUM = SUM + A(IJ)*Y(J)
  110 IJ = IJ + 1
      II = IJ
  120 Y(I+1) = -SUM / A(II)
  125 IF (INDIC8) 170,140,170
  130 II = (M*M + M) / 2 -  LM1
  140 I = M
  145 Y(I) = Y(I) / A(II)
      II = II - I
      I = I - 1
      IF (I - L1) 170,150,150
  150 SUN = -Y(I+1)
      IJ = II + L1
      DO 160  J = L1,I
      Y(J) = Y(J) + SUN*A(IJ)
  160 IJ = IJ + 1
      GO TO 145
  170 RETURN
       END
```

FIGURE 10.42 TFORM

```
 IV G LEVEL 1, MOD 1              TFORM              DATE = 71363          13.16.45

              SUBROUTINE TFORM(A,N,D,O,S,P)
              DIMENSION A(1),D(1),O(1),S(1)
              BL = 0.
              BU = 0.
              OLD = 0.
              D(1) = A(1)
              K1K1 = 1
              N1 = N - 1
              DO 230  K = 1,N1
              KP1 = K + 1
              KK = K1K1
              KKP1 = KK + 1
              NK = N - K
              KN = KK + NK
              K1K1 = KN + 1
              SUM = 0.
              DO 100  KJ = KKP1,KN
          100 SUM = SUM + A(KJ)*A(KJ)
              S(K) = SUM
              RHO = SQRT(SUM)
              RAD = OLD + RHO
              BL = AMIN1(BL,D(K)-RAD)
              BU = AMAX1(BU,D(K)+RAD)
              IF (K - N1) 120,230,230
          120 OLD = RHO
              IF (A(KKP1)) 140,140,130
          130 RHO = -RHO
          140 O(K) = RHO
              IF (SUM) 150,230,150
          150 A(KKP1) = A(KKP1) - RHO
              RHO =  1. / (RHO*A(KKP1))
              A(KK) = RHO
              IJ = KK
              DO 160  J = KP1,N
              IJ = IJ + 1
              O(J) = A(IJ)
          160 D(J) = 0.
              II = K1K1
              NI = NK
              DO 190  I = KP1,N
              D(I) = D(I) + A(II)*O(I)
              IJ = II
              II = II + NI
              NI = NI - 1
              IF (NI) 170,190,170
          170 X = O(I)
              DO 180  J = I,N1
              IJ = IJ + 1
              D(J+1) = D(J+1) + A(IJ)*X
          180 D(I) = D(I) + A(IJ)*O(J+1)
          190 D(I) = D(I) * RHO
              SUM = 0.
              DO 200  I = KP1,N
          200 SUM = SUM + D(I)*O(I)
```

FIGURE 10.42 (Cont'd)

```
IV G LEVEL 1, MOD 1                TFORM                  DATE = 71363           13.16.45

         TAU = RHO * SUM * .5
         DO 210  I = KP1,N
     210 D(I) = D(I) + TAU*O(I)
         II = K1K1
         NI = NK
         DO 220  I = KP1,N
         RHO = D(I)
         TAU = O(I)
         IJ = II
         II = II + NI
         NI = NI - 1
         DO 220  J = I,N
         A(IJ) = A(IJ) + RHO*O(J) + TAU*D(J)
     220 IJ = IJ + 1
     230 D(K+1) = A(K1K1)
         O(N1) = A(KKP1)
         O(N) = AMIN1(BL,D(N)-RHO)
         S(N) = AMAX1(BU,D(N)+RHO)
         RETURN
         END
```

FIGURE 10.43 STRUM

```
IV G LEVEL 1, MOD 1                 STURM                  DATE = 71363            13.16.56

         SUBROUTINE STURM (N,LIM1,NUMB,D,OFFD,SEC,PFFD,SIGMA,EPS)
         DIMENSION D(1),OFFD(1),SEC(1),SIGMA(1),PFFD(1)
         DATA HALF / .5/
         BL = OFFD(N)
         BU = SEC(N)
         LIM2 = LIM1 + NUMB - 1
         CALL PREP(N,D,SEC,ROOT,LORD)
         N1 = N - 1
         IF (N1) 16,200,200
     200 TOL = AMAX1(-BL,BU)
         OFFD(N) = TOL
         TOL = TOL * AMAX1(1.E-15,EPS)
         DO 2  I = LIM1,LIM2
         SIGMA(I) = BL
       2 PFFD(I) = BU
         LORD = 0
         L = LIM1 - 1
         RUTE = 1.E20
         GO TO 3
     300 DO 400   I = K,L
     400 SIGMA(I) = ROOT
       3 K = L + 1
         IF (K - LIM2) 4,4,16
       4 BU = PFFD(K)
         ROOT = BU  +  HALF * (SIGMA(K) - BU)
         IF (K - L) 5,7,5
       5 DO 6  I = K,LIM2
         IF (BU - PFFD(I)) 7,6,7
       6 L = I
       7 IF (ABS(ROOT - RUTE) - TOL) 300,300,8
       8 CALL DET(LORD)
         DO 11  I = K,L
         IF(I -LORD) 9,9,10
       9 SIGMA(I) = ROOT
         GO TO 11
      10 PFFD(I) = ROOT
      11 CONTINUE
         RUTE = ROOT
         GO TO 4
      16 RETURN
         END
```

FIGURE 10.44 PREP

```
IV G LEVEL  1, MOD 1              PREP              DATE = 71363          13.17.05

          SUBROUTINE PREP(N,D,SEC,/ROOT/,/LORD/)
          DIMENSION D(1),SEC(1)
          EQUIVALENCE (RD2,RE2),(RD4,RE4)
          N1 = N - 1
          RETURN
          ENTRY DET
          RD0 = ROOT
          LOW = 0
          LAWD = 0
      100 RD2 = 0.D0
          RD4 = 1.D0
          DO 120  I = LOW,N1
          RD4 = D(I+1) - RD0 - RD2
          IF (RD4) 120,140,110
      110 LAWD = LAWD + 1
      120 RE2 = SEC(I+1) / RE4
      130 LORD = LAWD
          RETURN
      140 LAWD = LAWD + 1
          IF (RE2) 150,160,150
      150 I = I + 1
      160 LOW = I + 1
          IF (LOW - N1) 100,100,130
          END
```

FIGURE 10.45 QSVEC

```
FILE: QSVEC    FORTRAN  P1                                   CALLDATA  TIME - SH

      SUBROUTINE  QSVEC(A,D,OFFD,P,Q,R,S,N)
C  SYMMETRIC MATRIX EIGENVECTOR CALCULATION.
C  GIVEN THE ENTRIES (D AND OFFD) OF THE HOUSEHOLDER TRI-DIAGONAL FORM
C  B OF A REAL SYMMETRIC MATRIX A, AND GIVEN A GOOD APPROXIMATE ROOT OF
C    B (AND A) THIS FORTRAN 4 SUBROUTINE COMPUTES A UNIT EIGENVECTOR X
C    OF B, THEN TRANSFORMS IT TO A UNIT VECTOR OF A, USING THE VECTORS
C  wSTORED IN THE A ARRAY.
      DIMENSION A(1),D(1),OFFD(1),P(1),Q(1),R(1),S(1),X(1)
      DOUBLE PRECISION SUM
      COMMON /INFO/ SUM,M,IX,IA
C
C  PART 1.  PRELIMINARIES.
C
      IX = 1
      IA = 1
      N1 = N - 1
      N2 = N - 2
      RETURN
      ENTRY  QWIEL(ROOT,X)
      ASSIGN 170 TO KOUNT
      TOL = 0.
      DO 100  I = 1,N
      P(I)  = D(I)  - ROOT
      Q(I)  = OFFD(I)
      R(I)  = 0.
      TOL = AMAX1(TOL,ABS(D(I)))
  100 X(I)  = RDM(X)  + .1
      TOL = (TOL + 1.E-15)  * 1.E-15
C
C  PART 2.  MATRIX DECOMPOSITION.
C
      DO 150  I = 1,N1
      T = ABS  (P(I))
      U = ABS  (OFFD(I))
      IF  (T + U - TOL)  110,120,120
  110 P(I)  = TOL
      T = P(I)
  120 IF  (T - U)  130,140,140
  130 S(I)  = P(I)/OFFD(I)
      IF(ABS(S(I)).LT.TOL)  S(I)=TOL
      S(I)  = OR(S(I), 1)
      TEMP = Q(I)
      P(I)  = OFFD(I)
      Q(I)  = P(I+1)
      R(I)  = Q(I+1)
      P(I+1)  = TEMP - S(I)*Q(I)
      Q(I+1)  = -S(I)*R(I)
      GO TO 150
  140 S(I)  =  OFFD(I)/P(I)
      S(I)  =   AND(S(I),-2)
      P(I+1)  = P(I+1)  - S(I)*Q(I)
  150 CONTINUE
      IF (ABS(P(N))  .LT.  TOL)   P(N)  = TOL
      GO TO 210
C
```

FIGURE 10.45 (Cont'd)

```
FILE: QSVEC    FORTRAN  P1                                     CALLDATA  TIME - SH

C  PART 3.  RIGHT SIDE MODIFICATION.
C
  170 ASSIGN 330 TO KOUNT
      DO 200  I = 1,N1
      TEMP = AND(S(I), 1)
      IF (TEMP) 180,190,180
  180 T = X(I)
      X(I) = X(I+1)
      X(I+1) = T - S(I)*X(I)
      GO TO 200
  190 X(I+1) = X(I+1) - S(I)*X(I)
  200 CONTINUE
C
C  PART 4.  TRIANGULAR SYSTEM SOLUTION.
C
  210 X(N) = X(N)/P(N)
      X(N1) = (X(N1) - Q(N1)*X(N)) / P(N1)
      DO 220  I = 2,N1
      K = N - I
  220 X(K) = (X(K) - Q(K)*X(K+1) - R(K)*X(K+2)) / P(K)
C
C  PART 5. SCALING TO UNIT VECTOR.
C
  230 SUM = 0.D0
      M = N
      SCALAR = SQRT(DOTPRO(X,X))
      DO 250  I = 1,N
  250 X(I) = X(I)/SCALAR
      GO TO KOUNT, (170,330,370)
C
C  PART 6.  TRANSFORMATION BY ORTHOGONAL MATRICES.
C
  330 L = (N*(N+1))/2 - 4
      DO 360  I = 1,N2
      NI = N - I
      SUM = 0.D0
      M = I + 1
      SCALAR =  A(L-1) * DOTPRO(X(NI),A(L))
      IJ = L
      DO 350  J = NI,N
      X(J) = X(J) + SCALAR*A(IJ)
  350 IJ = IJ + 1
  360 L = L - I - 3
      ASSIGN 370 TO KOUNT
      GO TO 230
  370 RETURN
      END
```

FIGURE 10.46 AND

```
FILE: AND       FORTRAN  P1                                   CALLDATA  TIME -

      FUNCTION AND(X,Y)
      LOGICAL UND,JA,NEIN
      REAL NO
      EQUIVALENCE(UND,E),(JA,SI),(NEIN,NO)
      IFLAG=0
      GO TO 1
      ENTRY OR(X,Y)
      IFLAG=1
    1 SI=X
      NO=Y
      UND=JA.AND.NEIN
      IF(IFLAG.EQ.1)UND=JA.OR.NEIN
      AND=E
      IF(IFLAG.EQ.1)OR=E
      RETURN
      END
```

FIGURE 10.47 DOTPRO

```
      FUNCTION DOTPRO(X,Y)
      DIMENSION X(1),Y(1)
      DOUBLE PRECISION S,DOTPRO
      COMMON /INFO/ S,N,IX,IY
      IF (N) 120,120,100
  100 JX = 1
      JY = 1
      DO 110  J = 1,N
      S = S + X(JX)*Y(JY)
      JX = JX + IX
  110 JY = JY + IY
  120 DOTPRO = S
      RETURN
      END
```

FIGURE 10.48 RDM

```
      FUNCTION RDM(IX)
      DATA IY/5757403/
      IY = IY*65539
      IF (IY.GE.0) GO TO 6
      IY = IY + 2147483647 + 1
    6 YFL = IY
      RDM = YFL*.4656613E-9
      RETURN
      ENTRY RDMIN(IX)
      IY = IX
      RDMIN = IX
      RETURN
      ENTRY RDMOUT(IX)
      IX = IY
      RDMOUT = IX
      RETURN
      END
```

FIGURE 10.49 COMPR

```
AN IV G LEVEL 1, MOD 1                COMPR                DATE = 71363                13.20.18

      C       SUBROUTINE COMPR(A,AC,N,IR,IC,K)
      C
      C       PURPOSE
      C       ROUTINE RESTRUCTURES LOWER TRAINGLE K-ORDER ARRAY TO COMPRESSED
      C       STORAGE. K-ORDER ARRAY STORED AS SUB-ARRAY IN A
      C
      C       A-INPUT ARRAY
      C       N-ORDER OF A
      C       AC-OUTPUT COLUMN VECTOR
      C       IR-INITIAL ROW ELEMENT LOCATION-ARRAY A
      C       IC-INITIAL COLUMN ELEMENT LOCATION-ARRAY A
      C       K-ORDER OF SUBARRAY STORED IN A
      C
      C       USAGE
      C       CALL SUBROUTINE COMPR(A,AC,N,IR,IC,K)
      C
              SUBROUTINE COMPR(A,AC,N,IR,IC,K)
              DIMENSION A(N,N),AC(435)
              IK=0
              DO 10 I=1,K
              II=IR+I-1
              DO 10 J=1,I
              IK=IK+1
              JJ=IC+J-1
           10 AC(IK)=A(II,JJ)
              RETURN
              END
```

FIGURE 10.50 ABCP

```
AN IV G LEVEL 1, MOD 1              ABCP                DATE = 71363          13.20.2

            SUBROUTINE ABCP(A,ANG)
      C
      C     THIS ROUTINE GENERATES 3X3 MATRIX A
      C     ANG - INPUT ANGLE
      C
      C     A - 3X3 REAL MATRIX A
      C
            DIMENSION A(9)
            IF(ANG.EQ.10.) GO TO 10
            DO 1 I=2,4
          1 A(I)=0.
            C1=-.5
            S1=.866025403784
            IF(ANG.EQ.24.) S1=-S1
            A(7)=0.
            A(1)=1.
            A(5)=C1
            A(6)=-S1
            A(8)=S1
            A(9)=C1
            GO TO 12
         10 DO 11 I=1,9
         11 A(I)=0.
            A(1)=1.
            A(5)=-1.
            A(9)=-1.
         12 RETURN
            END
```

FIGURE 10.51 ANORM

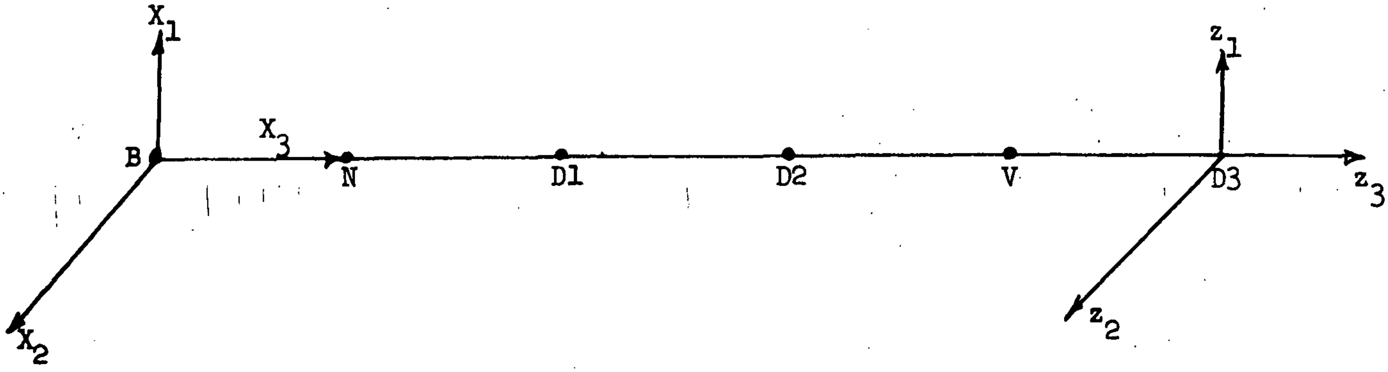
```
 IV G LEVEL 1, MOD 1              ANORM              DATE = 71363              13.21.14

            SUBROUTINE ANORM(AOUT,AIN,NA,NB,N)
            DIMENSION AOUT(NB,NB),AIN(NA,NA)
            DO 10 L=1,N
            AMX=0.
            DO 5 J=1,N
          5 AMX=AMAX1(ABS(AIN(J,L)),AMX)
            IF(AMX.EQ.0.) AMX=1.
            DO 10 J=1,N
         10 AOUT(J,L)=AIN(J,L)/AMX
            RETURN
            END
```

FIGURE 10.52 PWRIT

```
N IV G LEVEL 1, MOD 1                PWRIT               DATE = 71363          13.21.46

            SUBROUTINE PWRIT(P,NR,NC,SR,ER,SC,EC,IV,TOL,IRIT)
            REAL P(NR,NC)
            DIMENSION KC(5),KA(5)
            INTEGER SR,ER,SC,EC
            IF(IRIT.EQ.0) RETURN
            J=SC
            MOD=1
          5 N=0
            DO 30 L=SR,ER
            IF(ABS(P(L,J)).LT.TOL) GO TO 16
            N=N+1
            KC(N)=L
            KA(N)=L-(SR-1)
            IF(N.EQ.5) GO TO 18
         16 IF(L.EQ.ER.AND.N.NE.0) GO TO 18
            GO TO 30
         18 CONTINUE
            WRITE(IRIT,25) MOD,IV,(KA(I),P(KC(I),J),I=1,N)
         25 FORMAT(I3,T79,I2,T4,5(I3,E12.5))
            N=0
         30 CONTINUE
            J=J+1
            MOD=MOD+1
            IF(J.LE.EC) GO TO 5
            RETURN
            END
```



X_1, X_2, X_3 - Fixed Frame in rotor ring. X_3 COORDINATE is aligned with the #1 ARM OF (T or Y) ROTOR CONFIGURATION

z_1, z_2, z_3 - Fixed FRAME IN END module

GEOMETRICAL DATA MEASURED WITH RESPECT TO X_1, X_2, X_3 COORDINATES

Location Point	Definition	ID
N	Attachment point of rotor arm to rotor ring (X_3 coord in feet)	60
D1	Mass center location of first mass module in rotor arm (X_3 coord in feet)	4
D2	Mass center location of second mass module in rotor (X_3 coord in feet)	7
V	Connection point between rotor arm and associated end module (X_3 coord in feet)	13
D3	Mass center location of end module (X_3 coord in feet)	10
B	Fixed point in hub about which rotor rotates	Not utilized in program

FIGURE 10.53 ROTOR ARM GEOMETRICAL DATA INPUT PROPERTIES

TABLE 10.6

ROTOR INPUT DATA GUIDE

POINT LOCATION	COORDINATE REFERENCE	ID NUMBER	DATA UNITS	DEFINITION
D1	X	2	FT	First mass center location (see Figure 10.53)
	Y	3		
	Z	4		
D2	X	5	FT	Second mass center location (see Figure 10.53)
	Y	6		
	Z	7		
D3	X	8	FT	End module location (see Figure 10.53)
	Y	9		
	Z	10		
V	X	11	FT	Rotor Arm - Associated end module Intersection - Location (see Figure 10.53)
	Y	12		
	Z	13		
—	—	14	—	Number of arms on rotor 2. - T Configuration 3. - Y Configuration
D1	XX	15	Slug-Ft^2	(I) Inertia property data - first mass center location (Coordinate frame nominally aligned to X1, X2, X3 (rotor ring) coordinates - Figure 10.53
	YX	16		
	ZX	17		
	XY	18		
	YY	19		
	ZY	20		
	XZ	21		
	YZ	22		
	ZZ	23		
D2	XX	24	Slug-Ft^2	(I) Inertia property data - second mass center location (Coordinate frame nominally aligned to X1, X2, X3 (rotor ring) coordinates - Figure 10.53
	YX	25		
	ZX	26		
	XY	27		
	YY	28		
	ZY	29		

POINT LOCATION	COORDINATE REFERENCE	ID NUMBER	DATA UNITS	DEFINITION
D2	XZ	30	Slug-Ft^2	(I) Inertia property data - second mass center location
	YZ	31		
	ZZ	32		(Coordinate frame nominally alligned to X1, X2, X3 (rotor ring) coordinates - Figure 10.53
D3	XX	33	Slug-Ft^2	(I) Inertia property data - end module
	XX	34		(Coordinate frame nominally aligned to X1, X2, X3 (rotor ring) coordinates - Figure 10.53
	ZX	35		
	XY	36		
	YY	37		
	ZY	38		
	XZ	39		
	YZ	40		
	ZZ	41		
B	XX	42	Slug-Ft^2	(I) Rotor ring inertia property data
	YX	43		
	ZX	44		
	XY	45		
	YY	46		
	ZY	49		
	XZ	50		
	YZ	51		
	ZZ	52		
--	--	51	In^2	Cross-sectional area of beam (rotor arm and/or hub arm)
--	--	52	In^4	Area moment of inertia
--	--	53	Lbs/In^2	Modulus of rigidity (G)
--	--	54	Lbs/In^2	Modulus of elasticity (E)
--	--	55	In^4	Polar moment of inertia
--	--	56	CPS	Freq shift Utilized in anti-symmetric Elgen-value calculations - value set to 5.
--	--	57	--	Tolerance factor - Utilized in modal matrix calculations - value set to 0. Limits 0 to 1.E-9

POINT LOCATION	COORDINATE REFERENCE	ID NUMBER	DATA UNITS	DEFINITION
N	X	58	FT	Rotor ring - rotor arm intersection location
	Y	59		
	Z	60		
D1	--	61	Slugs	Point D1 mass
D2	--	62	Slugs	Point D2 mass
D3	--	63	Slugs	End module mass
--	--	64	Rad/Sec	Rotor spin rate
--	--	67	In-Lbs/Rad	Bearing spring constant
--	--	68	--	Structural damping ratio
--	--	100	--	Program control - 0. - Calculate rotor structural data only 1. - Calculate hub data only 2. - Calculate rotor and Hub structural data
		90	--	Program control - punch output data 0. - Do not punch data 7. - Punch data Note: 7. is IBM 360 system output device for punch output
		91	--	Punch output data lower (magnitude) limit; control modal matrix output
--	--	93	--	Punch output data lower (magnitude) limit; control stiffness matrix output
--	--	94	--	Punch output data lower (magnitude) limit; control damping matrix output

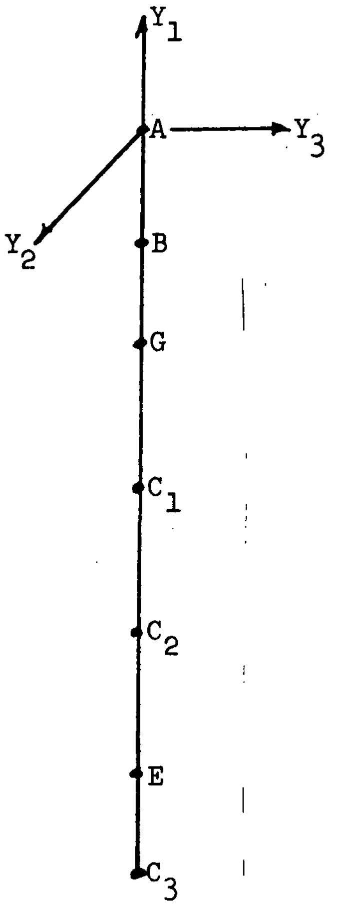
TABLE 10.7

SAMPLE ROTOR INPUT DATA SETUP

CARD	FORMATTED DATA INPUT			TYPE
1	1 ROTOR T CONF. RUN NO. 1			Program Control Card
2	4	23.3	XRD1(3)	
3	7	57.3	XRD2(3)	R
4	10	105.23	XRD3(3)	O
5	13	74.25	XRP(3)	T
6	14	2.	P	O
7	15	22731.3	DI1(1,1)	R
8	19	22731.3	DI1(2,2)	
9	23	2083.	DI1(3,3)	D
10	24	22731.3	DI2(1,1)	A
11	28	22731.3	DI2(2,2)	T
12	32	2083.	DI2(3,3)	A
13	33	2034929.53	DI3(1,1)	
14	37	485759.91	DI3(2,2)	I
15	41	1644085.22	DI3(3.3)	N
16	42	11296.	RRIB(1,1)	P
17	46	15586.	RRIB(2,2)	U
18	50	15586.	RRIB(3,3)	T
19	51	15.823	AREA	
20	52	13446.	AI	
21	53	4.0385	E+6GBE	
22	54	1.05	E+7BE	
23	55	26892.	PMJ	
24	56	5.	FREQ	
25	60	6.25	XRN(3)	
26	61	225.16	DM1	
27	62	225.16	DM2	
28	63	3881.99	DM3	
29	64	.2618	PR	
30	65	3.14159	ANG(2)*	
31	67	2.1156	E+9BSK1	
32	68	.01	ROTOR DAMPING FACTOR	

*Note: Program data card not required - angle data for rotor arms stored in program

CARD	FORMATTED DATA INPUT			TYPE
33	90	7.	IRIT	PUNCH OUTPUT CONTROL
34	91	1.	E-5TREIG	MODAL MATRIX TOL. LIMIT
35	93	10.	TSMAT	STIFFNESS MATRIX TOL. LIMIT
36	94	10.	TCTYA	DAMPING MATRIX TOL. LIMIT
37	BLANK CARD			PROGRAM CONTROL CARD



Y_1, Y_2, Y_3 - Coordinate frame fixed in rigid section of hub at A, the Y_1 axis is parallel to the nominal spin axis of hub.

Geometrical data is measured with respect to Y_1, Y_2, Y_3 coordinates.

Location Point	Definition	Card Data Input Format - I5 I.D. Number
A	Mass center of rigid section of hub	Data not required for program
B	Fixed Point in hub about which rotor rotates	Data not required for program
G	Location of connection between flexible and rigid hub sections	104
C_1	Location - of first module mass center	107
C_2	Location - of second module mass center	110
E	Location of connection between hub second module and end module	116
C_3	Location - of end module	113

FIGURE 10.54 HUB GEOMETRICAL DATA INPUT PROPERTIES

TABLE 10.8

HUB INPUT DATA GUIDE

POINT LOCATION	COORDINATE REFERENCE	ID NUMBER	DATA UNITS	DEFINITION
G	Y_1	104	Ft	Intersection of hub flexible and rigid sections
	Y_2	105		
	Y_3	106		
C_1	Y_1	107	Ft	First mass center location
	Y_2	108		
	Y_3	109		
C_2	Y_1	110	Ft	Second mass center location
	Y_2	111		
	Y_3	112		
C_3	Y_1	113	Ft	End mass center location
	Y_2	114		
	Y_3	115		
E	Y_1	116	Ft	Hub second module - end module intersection
	Y_2	117		
	Y_3	118		
C_1	XX	119	Slug-Ft^2	(I) Inertia property data - first mass
	YX	120		
	ZX	121		Coordinate frame nominally aligned to Y_1, Y_2, Y_3 (hub rigid body) coordinates Figure 10.54
	XY	122		
	YY	123		
	ZY	124		
	XZ	125		
	YZ	126		
	ZZ	127		
C_2	XX	128	Slug-Ft^2	(I) Inertia property data - second mass
	YX	129		
	ZX	130		Coordinate frame nominally aligned to Y_1, Y_2, Y_3 (hub rigid body) coordinates
	XY	131		
	YY	132		
	ZY	133		
	XZ	134		
	YZ	135		
	ZZ	136		

POINT LOCATION	COORDINATE REFERENCE	ID NUMBER	DATA UNITS	DEFINITION
C_3	XX	137	Slug-Ft^2	(I) Inertia property data - end mass
	YX	138		
	ZX	139		Coordinate frame nominally aligned to
	XY	140		Y_1, Y_2, Y_3 (hub rigid body) coordinates
	YY	141		
	ZY	142		
	XZ	143		
	YZ	144		
	ZZ	145		
C_1	--	146	Slugs	Point C1 mass
C_2	--	147	Slugs	Point C2 mass
C_3	--	148	Slugs	Point C3 mass
--	--	149	--	Structural damping ratio
--	--	51	In^2	Cross-sectional area of beam (hub arm and/ or rotor arm)
--	--	52	In^4	Area moment of inertia
--	--	53	Lbs/In^2	Modulus of rigidity (G)
--	--	54	Lbs/In^2	Modulus of elasticity (E)
--	--	55	In^4	Polar moment of inertia
--	--	57	--	Tolerance factor-utilized in Eigen value matrix calculations - value set to 0. Limits 0 to 1. E-9
--	--	100	--	Program control 0. - Calculate only rotor structural data 1. - Calculate only hub data 2. - Calculate both rotor and hub
--	--	90	--	Program control - punch output data 0. - Do not punch data 7. - Punch data Note: 7. is IBM 360 system output device for punch output.

POINT LOCATION	COORDINATE REFERENCE	ID NUMBER	DATA UNITS	DEFINITION
--	--	92	--	Punch output data lower magnitude limit; control modal matrix output
--	--	95	--	Punch output data lower magnitude limit; control hub stiffness matrix
--	--	96	--	Punch output data lower magnitude limit; control hub damping matrix
--	--	97	--	Punch output data lower magnitude limit; control hub (attachment point) stiffness matrix
--	--	98	--	Punch output data lower magnitude limit; control hub (attachment point) damping matrix

TABLE 10.9

SAMPLE HUB INPUT DATA SETUP

CARD	FORMATTED DATA INPUT			TYPE
1	1HUB	T CONF. RUN NO. 1		PROGRAM CONTROL CARD
2	100	1.	HUB CALCULATIONS	H
3	104	-27.8	YHG(1)	U
4	107	-63.	YHC1(1)	B
5	110	-122.	YHC2(1)	
6	113	-175.1	YHC3(1)	I
7	116	-156.5	YHE(1)	N
8	119	1735329.45	CI1(1,1)	P
9	123	2012994.74	CI1(2,2)	U
10	127	292228.82	CI1(3,3)	T
11	128	1733430.57	CI2(1,1)	
12	132	1829283.28	CI2(2,2)	D
13	136	98517.36	CI2(3,3)	A
14	137	1644085.2	CI3(1,1)	T
15	141	2034929.53	CI3(2,2)	A
16	145	485759.91	CI3(3,3)	
17	146	741.8022	CM1	
18	147	536.5137	CM2	
19	148	3881.9876	CM3	
20	149	.01	DAMPH	
21	51	15.823	AREA	
22	52	13446.	AI	
23	53	4.0385	E+6GBE	
24	54	1.05	E+7BE	
25	55	26892.	PMJ	
26	90	7.	IRIT	PUNCH OUTPUT CONTROL
27	92	1.	E-5TPHHF	MODAL MATRIX TOL. LIMIT
28	95	10.	TK7H	STIFFNESS MATRIX TOL. LIMIT
29	96	10.	TCH	DAMPING MATRIX TOL. LIMIT
30	97	10.	TK6HG	ATT. PT. STIF. MAT. TOL. LIMIT
31	98	10.	TCHG	ATT. PT. DAMP MAT. TOL. LIMIT
32	BLANK CARD			PROGRAM CONTROL CARD
33	BLANK CARD			PROGRAM CONTROL CARD

10.2.3 Sample Run

Printed output of Rotor and Hub "T" structural data is illustrated in Tables 10.10 and 10.11, respectively. At the start of each printed output the RUN CONTROL CARD along with the BASIC input data are printed.

Rotor Structural Data output will be printed in the following order:

- Input Card Data
- Stiffness Matrix
- Modes (rad/sec)
- Modal Matrix
- Damping Matrix

The order of printing hub structural data output is as follows:

- Input Card Data
- Hub Stiffness Matrix
- Hub Damping Matrix
- Hub (attachment point) Stiffness Matrix
- Hub (attachment point) damping Matrix
- Hub Modes (rad/sec)
- Hub Modal Matrix

The modes are printed in ascending order of frequency (in radians per second) with printed titles designating symmetric and antisymmetric modes. A mode shape, defined by the Modal Matrix columns, is linked with a corresponding frequency whose Eigen value number is identical with the column number.

The modal, stiffness, and damping matricies will be punched on cards in the proper format for direct input to the Time History Program. The last card of each data set of punched output will only contain a zero in column 80. Structural data that is less than the tolerance limit specified by the basic input data will be omitted from the punched output.

TABLE 10.10 SAMPLE OUTPUT; "T," BASELINE ROTOR

```
RUN NUMBER=  1      ROTOR T CONF. RUN NO. 1

     DD(    4)=        23.29999          XRD1(3)
     DD(    7)=        57.29999          XRD2(3)
     DD(   10)=        105.2300          XRD3(3)
     DD(   13)=        74.25000          XRP(3)
     DD(   14)=        2.000000          P
     DD(   15)=        22731.30          DI1(1,1)
     DD(   19)=        22731.30          DI1(2,2)
     DD(   23)=        2083.000          DI1(3,3)
     DD(   24)=        22731.30          DI2(1,1)
     DD(   28)=        22731.30          DI2(2,2)
     DD(   32)=        2083.000          DI2(3,3)
     DD(   33)=        2034929.          DI3(1,1)
     DD(   37)=        485759.9          DI3(2,2)
     DD(   41)=        1644085.          DI3(3,3)
     DD(   42)=        11296.00          RRIB(1,1)
     DD(   46)=        15586.00          RRIB(2,2)
     DD(   50)=        15586.00          RRIB(3,3)
     DD(   51)=        15.82300          AREA
     DD(   52)=        13446.00          AI
     DD(   53)=        4038500.          GBE
     DD(   54)=      0.1050000E 08       BE
     DD(   55)=        26892.00          PMJ
     DD(   56)=        5.000000          FREQ
     DD(   59)=        6.250000          XRN(3)
     DD(   61)=        225.1600          DM1
     DD(   62)=        225.1600          DM2
     DD(   63)=        3881.990          DM3
     DD(   64)=      0.2618000           PR
     DD(   65)=        3.141589          ANG(2)
     DD(   67)=      0.2115600E 09       BKS1
     DD(   68)=      0.9999998E-02       ROTOR DAMPING FACTOR
     DD(   90)=        7.000000          IRIT
     DD(   91)=      0.1000000E-04       TREIG
     DD(   93)=        10.00000          TSMAT
     DD(   94)=        10.00000          TCTYA
```

TWO ARM ROTOR STIFFNESS MATRIX

TABLE 10.10 (Cont'd)

```
      S-MATRIX

ROW COLUMN  1             2          3          4              5              6              7              8              9              10
  1         0.0           0.0        0.0        0.0            0.35087E 08    0.0            0.24148E 09    0.0            0.0            0.0
  2         0.0           -1616.0    0.0        -0.35087E 08   0.0            0.0            0.0            0.24148E 09    0.0            0.0
  3         0.0           0.0        -128.00    0.0            0.0            0.0            0.0            0.0            -0.44234E 08   0.0
  4         0.0           394.00     0.0        0.26761E 07    0.0            0.0            0.0            -0.15147E 08   0.0            -0.30036E 06
  5         -359.57       0.0        0.0        0.0            0.26761E 07    0.0            0.15147E 08    0.0            0.0            0.0
  6         0.0           0.0        0.0        0.0            0.0            0.14631E 08    0.0            0.0            0.0            0.0
  7         0.0           0.0        0.0        0.0            0.15147E 08    0.0            0.34556E 09    0.0            0.0            0.0
  8         0.0           272.00     0.0        -0.15147E 08   0.0            0.0            0.0            0.34556E 09    0.0            -0.50917E 07
  9         0.0           0.0        0.0        0.0            0.0            0.0            0.0            0.0            0.66416E 08    0.0
 10         0.0           736.00     0.0        -0.30036E 06   0.0            0.0            0.0            -0.50917E 07   0.0            0.27183E 07
 11         -884.27       0.0        0.0        0.0            -0.30036E 06   0.0            0.50917E 07    0.0            0.0            0.0
 12         0.0           0.0        0.0        0.0            0.0            -0.48865E 07   0.0            0.0            0.0            0.0
 13         0.0           0.0        0.0        0.0            -0.50917E 07   0.0            0.57640E 08    0.0            0.0            0.0
 14         0.0           -256.00    0.0        0.50917E 07    0.0            0.0            0.0            0.57640E 08    0.0            0.15386E 08
 15         0.0           0.0        0.0        0.0            0.0            0.0            0.0            0.0            -0.22182E 08   0.0
 16         0.0           28080.     0.0        0.0            0.0            0.0            0.0            0.0            0.0            -0.24180E 07
 17         -27998.       0.0        0.0        0.0            0.0            0.0            0.0            0.0            0.0            0.0
 18         0.0           0.0        0.0        0.0            0.0            0.0            0.0            0.0            0.0            0.0
 19         0.0           0.0        0.0        0.0            0.0            0.0            0.0            0.0            0.0            0.0
 20         0.0           -7424.0    0.0        0.0            0.0            0.0            0.0            0.0            0.0            0.95386E 08
 21         0.0           0.0        0.0        0.0            0.0            0.0            0.0            0.0            0.0            0.0
 22         0.0           -394.00    0.0        0.0            0.0            0.0            0.0            0.0            0.0            0.0
 23         -359.57       0.0        0.0        0.0            0.0            0.0            0.0            0.0            0.0            0.0
 24         0.0           0.0        0.0        0.0            0.0            0.0            0.0            0.0            0.0            0.0
 25         0.0           0.0        0.0        0.0            0.0            0.0            0.0            0.0            0.0            0.0
 26         0.0           -272.00    0.0        0.0            0.0            0.0            0.0            0.0            0.0            0.0
 27         0.0           0.0        0.0        0.0            0.0            0.0            0.0            0.0            0.0            0.0
 28         0.0           -736.00    0.0        0.0            0.0            0.0            0.0            0.0            0.0            0.0
 29         -884.27       0.0        0.0        0.0            0.0            0.0            0.0            0.0            0.0            0.0
 30         0.0           0.0        0.0        0.0            0.0            0.0            0.0            0.0            0.0            0.0
 31         0.0           0.0        0.0        0.0            0.0            0.0            0.0            0.0            0.0            0.0
 32         0.0           256.00     0.0        0.0            0.0            0.0            0.0            0.0            0.0            0.0
 33         0.0           0.0        0.0        0.0            0.0            0.0            0.0            0.0            0.0            0.0
 34         0.0           -28080.    0.0        0.0            0.0            0.0            0.0            0.0            0.0            0.0
 35         -27998.       0.0        0.0        0.0            0.0            0.0            0.0            0.0            0.0            0.0
 36         0.0           0.0        0.0        0.0            0.0            0.0            0.0            0.0            0.0            0.0
 37         0.0           0.0        0.0        0.0            0.0            0.0            0.0            0.0            0.0            0.0
 38         0.0           7424.0     0.0        0.0            0.0            0.0            0.0            0.0            0.0            0.0
 39         0.0           0.0        0.0        0.0            0.0            0.0            0.0            0.0            0.0            0.0
```

S-MATRIX

TABLE 10.10 (Cont'd)

```
                                                                                                        5
ROW COLUMN 11         12           13           14           15           16           17           18           19           20
  1      0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
  2      0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
  3      0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
  4      0.0          0.0          0.0          0.50917E 07  0.0          0.0          0.0          0.0          0.0          0.0
  5     -0.30036E 06  0.0         -0.50917E 07  0.0          0.0          0.0          0.0          0.0          0.0          0.0
  6      0.0         -0.48865E 07  0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
  7      0.50917E 07  0.0          0.57640E 08  0.0          0.0          0.0          0.0          0.0          0.0          0.0
  8      0.0          0.0          0.0          0.57640E 08  0.0          0.0          0.0          0.0          0.0          0.0
  9      0.0          0.0          0.0          0.0         -0.22182E 08  0.0          0.0          0.0          0.0          0.0
 10      0.0          0.0          0.0          0.15386E 08  0.0         -0.24180E 07  0.0          0.0          0.0          0.95386E 08
 11      0.27183E 07  0.0         -0.15386E 08  0.0          0.0          0.0         -0.24180E 07  0.0         -0.95386E 08  0.0
 12      0.0          0.14683E 08  0.0          0.0          0.0          0.0          0.0         -0.98019E 07  0.0          0.0
 13     -0.15386E 08  0.0          0.34691E 09  0.0          0.0          0.0          0.20478E 08  0.0          0.75008E 09  0.0
 14      0.0          0.0          0.0          0.34691E 09  0.0         -0.20478E 08  0.0          0.0          0.0          0.75008E 09
 15      0.0          0.0          0.0          0.0          0.66677E 08  0.0          0.0          0.0          0.0          0.0
 16      0.0          0.0          0.0         -0.20478E 08  0.0          0.24180E 07  0.0          0.0          0.0         -0.95386E 08
 17     -0.24180E 07  0.0          0.20478E 08  0.0          0.0          0.0          0.24180E 07  0.0          0.95386E 08  0.0
 18      0.0         -0.98019E 07  0.0          0.0          0.0          0.0          0.0          0.98019E 07  0.0          0.0
 19     -0.95386E 08  0.0          0.75008E 09  0.0          0.0          0.0          0.95386E 08  0.0          0.38218E 10  0.0
 20      0.0          0.0          0.0          0.75008E 09  0.0         -0.95386E 08  0.0          0.0          0.0          0.38218E 10
 21      0.0          0.0          0.0          0.0         -0.44495E 08  0.0          0.0          0.0          0.0          0.0
 22      0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
 23      0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
 24      0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
 25      0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
 26      0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
 27      0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
 28      0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
 29      0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
 30      0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
 31      0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
 32      0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
 33      0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
 34      0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
 35      0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
 36      0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
 37      0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
 38      0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
 39      0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
```

TABLE 10.10 (Cont'd)

```
S-MATRIX
ROW COLUMN 21           22           23           24           25           26           27           28           29           30
  1        0.0          0.0          0.35087E 08  0.0          0.24148E 09  0.0          0.0          0.0          0.0          0.0
  2        0.0          0.35087E 08  0.0          0.0          0.0         -0.24148E 09  0.0          0.0          0.0          0.0
  3        0.0          0.0          0.0          0.0          0.0          0.0          0.44234E 08  0.0          0.0          0.0
  4        0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
  5        0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
  6        0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
  7        0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
  8        0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
  9        0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
 10        0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
 11        0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
 12        0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
 13        0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
 14        0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
 15       -0.44495E 08  0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
 16        0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
 17        0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
 18        0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
 19        0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
 20        0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
 21        0.44495E 08  0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
 22        0.0          0.26761E 07  0.0          0.0          0.0         -0.15147E 08  0.0         -0.30036E 06  0.0          0.0
 23        0.0          0.0          0.26761E 07  0.0          0.15147E 08  0.0          0.0          0.0         -0.30036E 06  0.0
 24        0.0          0.0          0.0          0.14631E 08  0.0          0.0          0.0          0.0          0.0         -0.48865E 07
 25        0.0          0.0          0.15147E 08  0.0          0.34556E 09  0.0          0.0          0.0          0.50917E 07  0.0
 26        0.0         -0.15147E 08  0.0          0.0          0.0          0.34556E 09  0.0         -0.50917E 07  0.0          0.0
 27        0.0          0.0          0.0          0.0          0.0          0.0          0.66416E 08  0.0          0.0          0.0
 28        0.0         -0.30036E 06  0.0          0.0          0.0         -0.50917E 07  0.0          0.27183E 07  0.0          0.0
 29        0.0          0.0         -0.30036E 06  0.0          0.50917E 07  0.0          0.0          0.0          0.27183E 07  0.0
 30        0.0          0.0          0.0         -0.48865E 07  0.0          0.0          0.0          0.0          0.0          0.14688E 08
 31        0.0          0.0         -0.50917E 07  0.0          0.57640E 08  0.0          0.0          0.0         -0.15386E 08  0.0
 32        0.0          0.50917E 07  0.0          0.0          0.0          0.57640E 08  0.0          0.15386E 08  0.0          0.0
 33        0.0          0.0          0.0          0.0          0.0          0.0         -0.22182E 08  0.0          0.0          0.0
 34        0.0          0.0          0.0          0.0          0.0          0.0          0.0         -0.24180E 07  0.0          0.0
 35        0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0         -0.24180E 07  0.0
 36        0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0         -0.98019E 07
 37        0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0         -0.95386E 08  0.0
 38        0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.95386E-08  0.0          0.0
 39        0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0          0.0
```

S-MATRIX

TABLE 10.10 (Cont'd)

ROW	COLUMN 31	32	33	34	35	36	37	38	39
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
5	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
6	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
7	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
14	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
17	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
18	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
19	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
20	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
21	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
22	0.0	0.50917E 07	0.0	0.0	0.0	0.0	0.0	0.0	0.0
23	-0.50917E 07	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
24	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
25	0.57640E 08	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
26	0.0	0.57640E 08	0.0	0.0	0.0	0.0	0.0	0.0	0.0
27	0.0	0.0	-0.22182E 08	0.0	0.0	0.0	0.0	0.0	0.0
28	0.0	0.15386E 08	0.0	-0.24180E 07	0.0	0.0	0.0	0.95386E 08	0.0
29	-0.15386E 08	0.0	0.0	0.0	-0.24180E 07	0.0	-0.95386E 08	0.0	0.0
30	0.0	0.0	0.0	0.0	0.0	-0.98019E 07	0.0	0.0	0.0
31	0.34691E 09	0.0	0.0	0.0	0.20478E 08	0.0	0.75008E 09	0.0	0.0
32	0.0	0.34691E 09	0.0	-0.20478E 08	0.0	0.0	0.0	0.75008E 09	0.0
33	0.0	0.0	0.66677E 08	0.0	0.0	0.0	0.0	0.0	-0.44495E 08
34	0.0	-0.20478E 08	0.0	0.24180E 07	0.0	0.0	0.0	-0.95386E 08	0.0
35	0.20478E 08	0.0	0.0	0.0	0.24180E 07	0.0	0.95386E 08	0.0	0.0
36	0.0	0.0	0.0	0.0	0.0	0.98019E 07	0.0	0.0	0.0
37	0.75008E 09	0.0	0.0	0.0	0.95386E 08	0.0	0.38218E 10	0.0	0.0
38	0.0	0.75008E 09	0.0	-0.95386E 08	0.0	0.0	0.0	0.38218E 10	0.0
39	0.0	0.0	-0.44495E 08	0.0	0.0	0.0	0.0	0.0	0.44495E 08

TABLE 10.10 (Cont'd)

```
TWO ARM ROTOR MODAL MATRIX

SYMMETRIC MODAL MATRIX EIGENVALUES

       QMEGA1

EIG. VAL-NO.  1           2           3           4           5           6           7           8           9           10
          0.86054E 00 0.92989E 00 0.24699E 01 0.88844E 01 0.14604E 02 0.24638E 02 0.53133E 02 0.56732E 02 0.10384E 03 0.10502E 03

       QMEGA1

EIG. VAL-NO. 11           12          13          14          15          16          17          18          19
          0.10625E 03 0.14362E 03 0.14504E 03 0.15093E 03 0.16344E 03 0.16709E 03 0.21162E 03 0.21257E 03 0.29717E 03

ANTISYMMETRIC MODAL MATRIX EIGENVALUES

       QMEGA2

EIG. VAL-NO. 20           21          22          23          24          25          26          27          28          29
          0.71603E-01 0.81744E 00 0.25968E 01 0.56783E 01 0.12317E 02 0.24638E 02 0.32570E 02 0.44683E 02 0.76819E 02 0.84573E 02

       QMEGA2

EIG. VAL-NO. 30           31          32          33          34          35          36          37          38          39
          0.11201E 03 0.11383E 03 0.14599E 03 0.15087E 03 0.16706E 03 0.20641E 03 0.21133E 03 0.29576E 03 0.31660E 03 0.34149E 03
```

TABLE 10.10 (Cont'd)

```
ROTOR MODAL MATRIX

      EIG-TYRP

ROW COLUMN  1           2           3           4           5           6           7           8           9           10
  1        0.0         0.0         0.0         0.0         0.0         0.0         0.0         0.0         0.0         0.0
  2        0.0         0.0         0.0         0.0         0.0         0.0         0.0         0.0         0.0         0.0
  3        0.48204E-14 0.16047E-12 0.10495     0.30674E-09-0.15040E-08 0.86863E-12-0.21421E-05-0.18308E-05 0.64847E-03-0.99071E-03
  4        0.33215E-07 0.43656E-01 0.89379E-06-0.11042E-04-0.24449      0.14343E-05 0.46491E-02 -1.0000     -0.69522E-01 -1.0000
  5        0.42930E-01-0.47507E-07 0.19144E-05-0.22743     -0.34037E-05-0.15929E-05  1.0000      0.34105E-02 -1.0000      0.68526E-01
  6        0.81526E-08-0.60791E-09 0.26379E-10-0.12922E-05 0.22712E-05-0.25608      0.23604E-03 0.24194E-03-0.16738E-02 0.13165E-02
  7        0.83795E-07 0.34176     0.27605E-08-0.17222E-04 -1.0000     -0.78583E-06 0.14891E-02-0.43786      0.51916E-01 0.81679
  8        0.33855     -0.14593E-06 0.25199E-08 -1.0000     0.18919E-04 0.57529E-07 0.78897     0.38672E-02 0.92102     -0.64116E-01
  9        0.43074E-12-0.23329E-13-0.77523E-12-0.32450E-08 0.15911E-07-0.75958     -0.26367E-06 0.25353E-05-0.10265E-03 0.83804E-04
 10        0.27977E-06  1.0000     -0.14800E-08-0.41345E-06-0.24751E-01-0.47531E-07-0.10883E-03 0.31835E-01-0.10907E-02-0.16297E-01
 11         1.0000     -0.42573E-06-0.19878E-08-0.15615     0.49691E-05 0.18249E-06-0.48274E-01-0.20075E-03-0.23248E-01 0.16121E-02
 12        0.42417E-12-0.25292E-13-0.15038E-11-0.27308E-08 0.17064E-07 -1.0000     -0.44489E-05-0.44835E-05 0.30550E-04-0.24179E-04
 13       -0.48807E-02 0.75674E-08-0.12383E-09 0.23253E-01-0.60768E-06 0.64041E-07-0.87450E-01-0.38612E-03 0.60357E-01-0.46279E-02
 14       -0.15390E-08 0.49550E-02 0.98324E-10-0.33888E-06-0.24736E-01-0.23195E-07 0.24862E-03-0.85726E-01-0.27871E-02-0.61263E-01
 15        0.94818E-14 0.38312E-12 0.25087     0.72924E-09-0.35382E-08 0.19866E-11-0.40579E-05-0.33418E-05 0.31895E-03-0.44435E-03
 16       -0.11929E-01 0.50778E-08 0.25167E-10 0.94213E-02-0.30881E-06-0.59397E-07 0.94594E-01 0.44358E-03 0.73103E-01-0.50693E-02
 17        0.28266E-08 0.11958E-01-0.13287E-10 0.28000E-07-0.63756E-02-0.64555E-07-0.37011E-03 0.10305     -0.60129E-02-0.93207E-01
 18       -0.23378E-10 0.22762E-11 0.75094     0.50531E-08-0.75765E-08-0.64715E-10-0.28205E-06-0.11845E-05-0.17290E-03 0.25711E-03
 19       -0.14173E-01 0.59871E-08 0.10530E-09-0.23915E-01 0.41383E-06 0.12999E-07-0.46326E-02-0.19612E-04-0.19458E-02 0.13508E-03
 20        0.43306E-08 0.14083E-01-0.10514E-09 0.42667E-06 0.26305E-01 0.33776E-07 0.53759E-04-0.14386E-01 0.41135E-03 0.62099E-02
 21        0.18089E-12 0.65077E-12  1.0000     -0.59416E-10 0.15040E-08-0.56009E-12 0.21178E-05 0.18009E-05-0.64938E-03 0.99216E-03
 22        0.33215E-07 0.43656E-01 0.89379E-06-0.11042E-04-0.24449      0.14343E-05 0.46491E-02 -1.0000     -0.69522E-01 -1.0000
 23       -0.42930E-01 0.47507E-07-0.19144E-05 0.22743      0.34037E-05 0.15929E-05 -1.0000     -0.34105E-02  1.0000     -0.68526E-01
 24        0.81526E-08-0.60791E-09 0.26379E-10-0.12922E-05 0.22712E-05-0.25608      0.23604E-03 0.24194E-03-0.16738E-02 0.13165E-02
 25        0.83795E-07 0.34176     0.27605E-08-0.17222E-04 -1.0000     -0.78583E-06 0.14891E-02-0.43786      0.51916E-01 0.81679
 26       -0.33855      0.14593E-06-0.25199E-08  1.0000     -0.18919E-04-0.57529E-07-0.78897     -0.38672E-02-0.92102      0.64116E-01
 27        0.43074E-12-0.23329E-13-0.77523E-12-0.32450E-08 0.15911E-07-0.75958     -0.26367E-06 0.25353E-05-0.10265E-03 0.83804E-04
 28        0.27977E-06  1.0000     -0.14800E-08-0.41345E-06-0.24751E-01-0.47531E-07-0.10883E-03 0.31835E-01-0.10907E-02-0.16297E-01
 29       -1.0000      0.42573E-06 0.19878E-08 0.15615     -0.49691E-05-0.18249E-06 0.48274E-01 0.20075E-03 0.23248E-01-0.16121E-02
 30        0.42417E-12-0.25292E-13-0.15038E-11-0.27308E-08 0.17064E-07 -1.0000     -0.44489E-05-0.44835E-05 0.30550E-04-0.24179E-04
 31        0.48807E-02-0.75674E-08 0.12383E-09-0.23253E-01 0.60768E-06-0.64041E-07 0.87450E-01 0.38612E-03-0.60357E-01 0.46279E-02
 32       -0.15390E-08 0.49550E-02 0.98324E-10-0.33888E-06-0.24736E-01-0.23195E-07 0.24862E-03-0.85726E-01-0.27871E-02-0.61263E-01
 33       -0.94818E-14-0.38312E-12-0.25087     -0.72924E-09 0.35382E-08-0.19866E-11 0.40579E-05 0.33418E-05-0.31895E-03 0.44435E-03
 34        0.11929E-01-0.50778E-08-0.25167E-10-0.94213E-02 0.30881E-06 0.59397E-07-0.94594E-01-0.44358E-03-0.73103E-01 0.50693E-02
 35        0.28266E-08 0.11958E-01-0.13287E-10 0.28000E-07-0.63756E-02-0.64555E-07-0.37011E-03 0.10305     -0.60129E-02-0.93207E-01
 36        0.23378E-10-0.22762E-11-0.75094     -0.50531E-08 0.75765E-08 0.64715E-10 0.28205E-06 0.11845E-05 0.17290E-03-0.25711E-03
 37        0.14173E-01-0.59871E-08-0.10530E-09 0.23915E-01-0.41383E-06-0.12999E-07 0.46326E-02 0.19612E-04 0.19458E-02-0.13508E-03
 38        0.43306E-08 0.14083E-01-0.10514E-09 0.42667E-06 0.26305E-01 0.33776E-07 0.53759E-04-0.14386E-01 0.41135E-03 0.62099E-02
 39       -0.18089E-12-0.65077E-12 -1.0000      0.59416E-10-0.15040E-08 0.56009E-12-0.21178E-05-0.18009E-05 0.64938E-03-0.99216E-03
```

EIG-TYRP

TABLE 10.10 (Cont'd)

```
ROW COLUMN 11          12          13          14          15          16          17          18          19          20
  1      0.0         0.0         0.0         0.0         0.0         0.0         0.0         0.0         0.0         1.0000
  2      0.0         0.0         0.0         0.0         0.0         0.0         0.0         0.0         0.0         0.0
  3      0.99827    -0.11185E-03-0.38076E-03 0.48687E-04 0.27507     0.41099E-04 0.26999E-03-0.17865     0.11799E-05 0.0
  4     -0.16678E-01-0.72970     1.0000     -0.14831E-01 0.17737E-02 0.70190E-01 0.13640E-01 0.11411E-01-0.19379E-01 0.0
  5      0.14431E-01-1.0000     -0.71040     0.13793    -0.15726E-02-0.13014E-02-0.97461E-02-0.61268E-02 0.14530E-01 0.0
  6      0.51857E-04 0.22929E-02-0.16092E-01 0.97289E-03-0.13587E-03-0.61634E-03 1.0000      0.89064    -0.98472     0.0
  7      0.14461E-01-0.59006E-01 0.74284E-01-0.10310E-02-0.53674E-03 1.0000      0.44348E-03 0.29578E-02-0.34693E-03 0.0
  8     -0.14687E-01-0.81021E-01-0.21762E-01 1.0000     -0.15035E-03 0.41933E-03 0.66109E-03 0.36683E-03-0.12607E-02 0.0
  9      0.53836E-05 0.36242E-03-0.26249E-02 0.18397E-03-0.33607E-04-0.17607E-03 0.94630     0.84771     1.0000      0.0
 10     -0.28357E-03-0.77115E-03 0.11917E-02-0.73800E-05 0.33931E-04-0.60480E-01-0.10881E-04-0.16178E-03 0.95921E-05 0.0
 11      0.36629E-03-0.18654E-02-0.28648E-02-0.62031E-01 0.95263E-05-0.12842E-04-0.22618E-04-0.14341E-04 0.55645E-04 0.0
 12     -0.10491E-05-0.50726E-04 0.35896E-03-0.22782E-04 0.35221E-05 0.17034E-04-0.56543E-01-0.50576E-01-0.29431E-01 0.0
 13     -0.11837E-02-0.13808    -0.92981E-01 0.84472E-03-0.14484E-03-0.19754E-04-0.47565E-03-0.21932E-03 0.53175E-03 0.0
 14     -0.12424E-02 0.97248E-01-0.13076     0.18461E-02-0.18372E-03 0.58332E-05-0.12270E-02-0.10218E-02 0.15721E-02 0.0
 15      0.40087     0.13897E-03 0.50069E-03-0.78962E-04-0.63694    -0.10389E-03-0.14857E-02 1.0000     -0.15531E-04 0.0
 16     -0.11376E-02-0.16101E-01-0.12410E-01-0.12135     0.27940E-04 0.84779E-05 0.10161E-03 0.70495E-04-0.14188E-03 0.0
 17     -0.16333E-02 0.90897E-02-0.11933E-01 0.16668E-03-0.56229E-04 0.10774    -0.24494E-04 0.23937E-03 0.47207E-04 0.0
 18     -0.28077    -0.24434E-05 0.54384E-03-0.69204E-04-1.0000     -0.18779E-03 0.67082E-03-0.48665     0.44511E-04 0.0
 19      0.30486E-04-0.11847E-03-0.19412E-03-0.45233E-02 0.70261E-06-0.79050E-06-0.73142E-06-0.29706E-06 0.16570E-05 0.0
 20      0.10787E-03 0.19581E-03-0.30240E-03 0.17669E-05-0.10488E-04 0.18689E-01 0.35843E-05 0.50107E-04-0.31562E-05 0.0
 21     -1.0000      0.11236E-03 0.38133E-03-0.48648E-04-0.27434    -0.40936E-04-0.27054E-03 0.17905    -0.12241E-05 0.0
 22     -0.16678E-01-0.72970     1.0000     -0.14831E-01 0.17737E-02 0.70190E-01 0.13640E-01 0.11411E-01-0.19379E-01 0.0
 23     -0.14431E-01 1.0000      0.71040    -0.13793     0.15726E-02 0.13014E-02 0.97461E-02 0.61268E-02-0.14530E-01 0.0
 24      0.51857E-04 0.22929E-02-0.16092E-01 0.97289E-03-0.13587E-03-0.61634E-03 1.0000      0.89064    -0.98472     0.0
 25      0.14461E-01-0.59006E-01 0.74284E-01-0.10310E-02-0.53674E-03 1.0000      0.44348E-03 0.29578E-02-0.34693E-03 0.0
 26      0.14687E-01 0.81021E-01 0.21762E-01-1.0000      0.15035E-03-0.41933E-03-0.66109E-03-0.36683E-03 0.12607E-02 0.0
 27      0.53836E-05 0.36242E-03-0.26249E-02 0.18397E-03-0.33607E-04-0.17607E-03 0.94630     0.84771     1.0000      0.0
 28     -0.28357E-03-0.77115E-03 0.11917E-02-0.73800E-05 0.33931E-04-0.60480E-01-0.10881E-04-0.16178E-03 0.95921E-05 0.0
 29     -0.36629E-03 0.18654E-02 0.28648E-02 0.62031E-01-0.95263E-05 0.12842E-04 0.22618E-04 0.14341E-04-0.55645E-04 0.0
 30     -0.10491E-05-0.50726E-04 0.35896E-03-0.22782E-04 0.35221E-05 0.17034E-04-0.56543E-01-0.50576E-01-0.29431E-01 0.0
 31      0.11837E-02 0.13808     0.92981E-01-0.84472E-03 0.14484E-03 0.19754E-04 0.47565E-03 0.21932E-03-0.53175E-03 0.0
 32     -0.12424E-02 0.97248E-01-0.13076     0.18461E-02-0.18372E-03 0.58382E-05-0.12270E-02-0.10218E-02 0.15721E-02 0.0
 33     -0.40087    -0.13897E-03-0.50069E-03 0.78962E-04 0.63694     0.10389E-03 0.14857E-02-1.0000      0.15531E-04 0.0
 34      0.11376E-02 0.16101E-01 0.12410E-01 0.12135    -0.27940E-04-0.84779E-05-0.10161E-03-0.70495E-04 0.14188E-03 0.0
 35     -0.16333E-02 0.90897E-02-0.11933E-01 0.16668E-03-0.56229E-04 0.10774    -0.24494E-04 0.23937E-03 0.47207E-04 0.0
 36      0.28077     0.24434E-05-0.54384E-03 0.69204E-04 1.0000      0.18779E-03-0.67082E-03 0.48665    -0.44511E-04 0.0
 37     -0.30486E-04 0.11847E-03 0.19412E-03 0.45233E-02-0.70261E-06 0.79050E-06 0.73142E-06 0.29706E-06-0.16570E-05 0.0
 38      0.10787E-03 0.19581E-03-0.30240E-03 0.17669E-05-0.10488E-04 0.18689E-01 0.35843E-05 0.50107E-04-0.31562E-05 0.0
 39      1.0000     -0.11236E-03-0.38133E-03 0.48648E-04 0.27434     0.40936E-04 0.27054E-03-0.17905     0.12241E-05 0.0
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EIG-TYRP

TABLE 10.10 (Cont'd)

ROW	COLUMN 21	22	23	24	25	26	27	28	29	30
1	-0.17298E-07	0.88861E-10	-0.97426E-02	-0.31229E-07	0.23182E-10	0.94371E-02	-0.11722E-07	0.95362E-02	0.11332E-06	0.94702E-02
2	-0.31742E-02	0.10891E-07	0.29607E-07	-0.96003E-02	0.12576E-07	0.24858E-08	-0.94552E-02	0.97540E-07	-0.95255E-02	-0.26182E-06
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	-0.43701E-01	-0.44621E-06	-0.28237E-07	-0.78610E-01	-0.42788E-06	-0.28098E-07	-0.31833E-02	-0.16718E-05	0.11753	0.71956E-05
5	0.22232E-08	-0.97309E-10	-0.12584E-01	-0.10580E-06	0.28734E-08	0.46232E-01	-0.17397E-06	0.14479	0.17434E-05	0.27968
6	-0.33666E-06	-0.60099E-06	-0.22360E-07	0.10594E-07	0.25608	0.22367E-08	-0.67255E-08	-0.22908E-07	-0.29501E-07	0.19060E-06
7	-0.34193	-0.21534E-05	-0.10054E-05	-0.14308	-0.45558E-06	-0.14542E-06	0.52289	-0.63498E-05	0.62485	0.12131E-04
8	0.37222E-06	-0.16617E-08	-0.21160	-0.75145E-06	-0.11137E-07	0.50423	-0.62332E-06	0.64237	0.77272E-05	0.38802
9	-0.71733E-07	0.36031E-05	-0.82459E-09	-0.33054E-09	0.75958	-0.65224E-11	-0.41812E-08	-0.14230E-08	0.86018E-07	0.25894E-07
10	-1.0000	-0.29854E-05	-0.31712E-05	1.0000	-0.16467E-05	-0.26207E-06	1.0000	-0.10239E-04	1.0000	0.27598E-04
11	0.18666E-05	-0.80541E-08	-1.0000	-0.24486E-05	-0.24843E-07	1.0000	-0.10741E-05	1.0000	0.12011E-04	1.0000
12	0.54976E-07	-0.55228E-08	-0.77067E-09	-0.10870E-09	1.0000	-0.48673E-10	-0.66187E-10	0.45572E-09	0.25102E-08	-0.43502E-08
13	-0.32479E-09	0.14115E-10	0.18211E-02	0.15251E-07	-0.41717E-09	-0.64833E-02	0.24638E-07	-0.19721E-01	-0.23459E-06	-0.37859E-01
14	-0.49594E-02	-0.28053E-07	-0.99362E-08	-0.72637E-02	-0.60340E-07	-0.24360E-08	0.30701E-02	-0.18217E-06	0.19051E-01	0.88742E-06
15	0.11645E-06	-0.25085	-0.34543E-11	0.16429E-07	-0.66899E-07	-0.18678E-14	-0.23003E-07	-0.20982E-12	-0.98517E-08	0.34148E-11
16	-0.22867E-07	0.84218E-10	0.10980E-01	0.20581E-07	-0.64634E-09	-0.16587E-01	-0.12258E-07	-0.26458E-03	-0.14329E-07	-0.21267E-01
17	-0.11958E-01	-0.14949E-06	-0.35172E-07	0.92406E-02	-0.29707E-07	-0.37886E-08	0.21921E-01	0.16191E-07	-0.12625E-02	0.57164E-06
18	-0.29399E-08	-0.75092	-0.10381E-10	0.10851E-09	-0.12494E-06	-0.26959E-14	-0.11715E-08	-0.73355E-13	-0.18626E-08	0.38137E-11
19	-0.33499E-07	0.14221E-09	0.17658E-01	0.39226E-07	0.53041E-09	-0.87041E-02	0.12747E-07	-0.98454E-02	-0.11648E-06	-0.91832E-02
20	-0.14079E-01	-0.62539E-07	-0.47180E-07	0.27053E-01	-0.12189E-07	-0.21171E-08	0.69766E-02	-0.10727E-06	0.10467E-01	0.24315E-06
21	0.58642E-08	-1.0000	-0.13774E-10	-0.84279E-10	0.18885E-09	-0.89393E-15	0.15105E-10	-0.11176E-15	0.37369E-11	-0.38495E-14
22	0.43701E-01	0.44621E-06	0.28237E-07	0.78610E-01	0.42788E-06	0.28098E-07	0.31833E-02	0.16718E-05	-0.11753	-0.71956E-05
23	0.22232E-08	-0.97309E-10	-0.12584E-01	-0.10580E-06	0.28734E-08	0.46232E-01	-0.17397E-06	0.14479	0.17434E-05	0.27968
24	0.33666E-06	0.60099E-06	0.22360E-07	-0.10594E-07	-0.25608	-0.22367E-08	0.67255E-08	0.22908E-07	0.29501E-07	-0.19060E-06
25	0.34193	0.21534E-05	0.10054E-05	0.14308	0.45558E-06	0.14542E-06	-0.52289	0.63498E-05	-0.62485	-0.12131E-04
26	0.37222E-06	-0.16617E-08	-0.21160	-0.75145E-06	-0.11137E-07	0.50423	-0.62332E-06	0.64237	0.77272E-05	0.38802
27	0.71733E-07	-0.36031E-05	0.82459E-09	0.33054E-09	-0.75958	0.65224E-11	0.41812E-08	0.14230E-08	-0.86018E-07	-0.25894E-07
28	1.0000	0.29854E-05	0.31712E-05	-1.0000	0.16467E-05	0.26207E-06	-1.0000	0.10239E-04	-1.0000	-0.27598E-04
29	0.18666E-05	-0.80541E-08	-1.0000	-0.24486E-05	-0.24843E-07	1.0000	-0.10741E-05	1.0000	0.12011E-04	1.0000
30	-0.54976E-07	0.55228E-08	0.77067E-09	0.10870E-09	-1.0000	0.48673E-10	0.66187E-10	-0.45572E-09	-0.25102E-08	0.43502E-08
31	-0.32479E-09	0.14115E-10	0.18211E-02	0.15251E-07	-0.41717E-09	-0.64833E-02	0.24638E-07	-0.19721E-01	-0.23459E-06	-0.37859E-01
32	0.49594E-02	0.28053E-07	0.99362E-08	0.72637E-02	0.60340E-07	0.24360E-08	-0.30701E-02	0.18217E-06	-0.19051E-01	-0.88742E-06
33	0.11645E-06	-0.25085	-0.34543E-11	0.16429E-07	-0.66899E-07	-0.18678E-14	-0.23003E-07	-0.20982E-12	-0.98517E-08	0.34148E-11
34	-0.22867E-07	0.84218E-10	0.10980E-01	0.20581E-07	-0.64634E-09	-0.16587E-01	-0.12258E-07	-0.26458E-03	-0.14329E-07	-0.21267E-01
35	0.11958E-01	0.14949E-06	0.35172E-07	-0.92406E-02	0.29707E-07	0.37886E-08	-0.21921E-01	-0.16191E-07	0.12625E-02	-0.57164E-06
36	-0.29399E-08	-0.75092	-0.10381E-10	0.10851E-09	-0.12494E-06	-0.26959E-14	-0.11715E-08	-0.73355E-13	-0.18626E-08	0.38137E-11
37	-0.33499E-07	0.14221E-09	0.17658E-01	0.39226E-07	0.53041E-09	-0.87041E-02	0.12747E-07	-0.98454E-02	-0.11648E-06	-0.91832E-02
38	0.14079E-01	0.62539E-07	0.47180E-07	-0.27053E-01	0.12189E-07	0.21171E-08	-0.69766E-02	0.10727E-06	-0.10467E-01	-0.24315E-06
39	0.58642E-08	-1.0000	-0.13774E-10	-0.84279E-10	0.18885E-09	-0.89393E-15	0.15105E-10	-0.11176E-15	0.37369E-11	-0.38495E-14

EIG-TYRF

TABLE 10.10 (Cont'd)

ROW	COLUMN 31	32	33	34	35	36	37	38	39
1	0.21419E-06	0.70734E-10	0.14390E-02	-0.43658E-07	0.22766E-11	0.25782E-07	0.71182E-07	0.17299E-06	-0.95030E-02
2	0.94837E-02	0.32456E-06	0.67532E-07	0.16923E-02	-0.64767E-06	-0.18477E-05	-0.23738E-04	-0.95030E-02	-0.10060E-06
3	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
4	-0.25962	-0.55009E-05	-0.59207E-05	0.59922E-02	0.95460E-05	0.47687E-04	0.60010E-03	0.23919	0.25090E-05
5	0.63399E-05	-0.13451E-08	-0.39224E-01	-0.15584E-05	-0.89647E-09	0.10873E-05	0.21410E-05	0.33499E-05	-0.23626
6	0.62814E-07	-0.16387E-05	-0.36914E-05	0.10877E-04	0.28578E-03	-1.0000	-0.96482	0.61941E-05	-0.10022E-07
7	-0.41806	-0.18409E-04	-0.18680E-04	1.0000	-0.44709E-04	0.12276E-03	0.13433E-02	0.54460	0.57681E-05
8	0.84408E-05	-0.41826E-07	-1.0000	-0.20101E-04	-0.10645E-07	0.38420E-05	0.74727E-05	0.10163E-04	-0.54457
9	-0.22077E-07	-0.18196E-05	-0.70509E-06	0.23154E-04	0.23749E-03	-0.93614	1.0000	-0.47688E-05	0.65863E-08
10	-1.0000	-0.35499E-04	-0.62997E-05	-0.24452	0.74495E-04	0.19388E-03	0.24994E-02	1.0000	0.10586E-04
11	0.22675E-04	0.10304E-07	0.21885	-0.38643E-05	0.99016E-09	0.26520E-05	0.76087E-05	0.18148E-04	-1.0000
12	0.20115E-06	0.40212E-05	0.89238E-07	-0.11831E-05	-0.10200E-04	0.56096E-01	-0.29726E-01	0.12100E-06	-0.13114E-09
13	-0.85428E-06	0.23063E-09	0.64652E-02	0.19949E-06	0.13262E-09	-0.13106E-06	-0.16422E-06	-0.16042E-06	0.84100E-02
14	-0.37911E-01	-0.11561E-05	0.34132E-06	0.16508E-02	0.17672E-05	0.16563E-05	0.19816E-04	0.81765E-02	0.86624E-07
15	0.19933E-06	-1.0000	0.67518E-08	0.95973E-05	0.99351	0.30125E-04	0.32411E-05	0.14268E-07	0.22357E-12
16	-0.40967E-06	0.54969E-08	0.13083	0.19747E-05	0.13928E-08	-0.42633E-06	-0.51975E-06	-0.10362E-06	0.95088E-02
17	-0.23135E-01	-0.94020E-06	-0.10285E-05	0.11679	-0.73190E-05	0.36253E-05	0.21917E-04	0.95110E-02	0.99479E-07
18	-0.64389E-07	-0.99287	0.88996E-08	-0.98817E-05	-1.0000	-0.28376E-04	-0.43803E-05	0.41320E-08	-0.22901E-13
19	-0.20574E-06	0.13690E-09	0.34768E-02	0.12159E-06	0.46949E-10	-0.40679E-07	-0.95850E-07	-0.17229E-06	0.95030E-02
20	-0.87105E-02	-0.28594E-06	-0.33462E-06	0.18837E-01	-0.10397E-05	0.21738E-05	0.23425E-04	0.95033E-02	0.10058E-06
21	-0.10780E-10	0.12623E-02	-0.10662E-10	0.29464E-08	0.63554E-03	0.11491E-07	0.76956E-07	-0.90330E-10	0.23525E-15
22	0.25962	0.55009E-05	0.59207E-05	-0.59922E-02	-0.95460E-05	-0.47687E-04	-0.60010E-03	-0.23919	-0.25090E-05
23	0.63399E-05	-0.13451E-08	-0.39224E-01	-0.15584E-05	-0.89647E-09	0.10873E-05	0.21410E-05	0.33499E-05	-0.23626
24	-0.62814E-07	0.16387E-05	0.36914E-05	-0.10877E-04	-0.28578E-03	1.0000	0.96482	-0.61941E-05	0.10022E-07
25	0.41806	0.18409E-04	0.18680E-04	-1.0000	0.44709E-04	-0.12276E-03	-0.13433E-02	-0.54460	-0.57681E-05
26	0.84408E-05	-0.41826E-07	-1.0000	-0.20101E-04	-0.10645E-07	0.38420E-05	0.74727E-05	0.10163E-04	-0.54457
27	0.22077E-07	0.18196E-05	0.70509E-06	-0.23154E-04	-0.23749E-03	0.93614	-1.0000	0.47688E-05	-0.65863E-08
28	1.0000	0.35499E-04	0.62997E-05	0.24452	-0.74495E-04	-0.19388E-03	-0.24994E-02	-1.0000	-0.10586E-04
29	0.22675E-04	0.10304E-07	0.21885	-0.38643E-05	0.99016E-09	0.26520E-05	0.76087E-05	0.18148E-04	-1.0000
30	-0.20115E-06	-0.40212E-05	-0.89238E-07	0.11831E-05	0.10200E-04	-0.56096E-01	0.29726E-01	-0.12100E-06	0.13114E-09
31	-0.85428E-06	0.23063E-09	0.64652E-02	0.19949E-06	0.13262E-09	-0.13106E-06	-0.16422E-06	-0.16042E-06	0.84100E-02
32	0.37911E-01	0.11561E-05	-0.34132E-06	-0.16508E-02	-0.17672E-05	-0.16563E-05	-0.19816E-04	-0.81765E-02	-0.86624E-07
33	0.19933E-06	-1.0000	0.67518E-08	0.95973E-05	0.99351	0.30125E-04	0.32411E-05	0.14268E-07	0.22357E-12
34	-0.40967E-06	0.54969E-08	0.13083	0.19747E-05	0.13928E-08	-0.42633E-06	-0.51975E-06	-0.10362E-06	0.95088E-02
35	0.23135E-01	0.94020E-06	0.10285E-05	-0.11679	0.73190E-05	-0.36253E-05	-0.21917E-04	-0.95110E-02	-0.99479E-07
36	-0.64389E-07	-0.99287	0.88996E-08	-0.98817E-05	-1.0000	-0.28376E-04	-0.43803E-05	0.41320E-08	-0.22901E-13
37	-0.20574E-06	0.13690E-09	0.34768E-02	0.12159E-06	0.46949E-10	-0.40679E-07	-0.95850E-07	-0.17229E-06	0.95030E-02
38	0.87105E-02	0.28594E-06	0.33462E-06	-0.18837E-01	0.10397E-05	-0.21738E-05	-0.23425E-04	-0.95033E-02	-0.10058E-06
39	-0.10780E-10	0.12623E-02	-0.10662E-10	0.29464E-08	0.63554E-03	0.11491E-07	0.76956E-07	-0.90330E-10	0.23525E-15

ROTOR DAMPING MATRIX

TABLE 10.10 (Cont'd)

APY-CTYA

ROW COLUMN	1	2	3	4	5	6	7	8	9	10
1	0.0	-0.34884E-01	-0.11478E-04	0.62342E-02	1839.0	-0.95430E-04	10779.	-0.47031E-01	-0.17644E-05	0.12575E-04
2	0.0	20431.	-0.15104E-04	-1878.2	0.63232E-02	-0.60481E-01	0.32693E-02	11118.	-0.10174E-01	-29.294
3	0.0	0.21809E-02	33739.	-0.59604E-01	0.10396	0.10250E-01	0.82285	0.47153	-2912.2	0.33562E-02
4	0.0	1789.1	-0.73161E-01	406.18	-1.2345	4.8219	-8.2540	-1248.5	-0.14603	-58.627
5	0.0	0.11155E-01	0.97475E-01	-1.5162	381.52	-3.8700	1149.7	16.050	0.38186	-0.41046E-01
6	0.0	-0.41236E-01	-0.76514	1.1200	-0.74337	1133.8	-5.1696	-11.783	1.5705	0.16105E-01
7	0.0	0.18931E-01	0.27054	-2.2961	1149.0	-7.6248	51891.	24.229	2.0806	0.23520
8	0.0	-1310.4	0.27229	-1239.9	8.1252	-32.447	54.367	52181.	0.76330	-691.36
9	0.0	0.63254E-01	1959.8	-0.11084	0.19710	0.64806E-02	1.6706	0.92324	7252.5	0.20296E-02
10	0.0	905.52	-0.77035E-01	-58.643	-0.48290E-01	0.10425	-0.34618	-691.78	0.18863	395.27
11	0.0	-0.78728E-02	-0.90358E-02	0.86904E-01	-57.967	0.28848	656.15	-0.80445	-0.56133E-01	0.57467E-02
12	0.0	-0.10412E-02	-0.43386	-0.26130	0.17188	-198.47	1.2482	2.7982	0.87692	-0.54893E-02
13	0.0	-0.11108	-0.28442	0.87276	-686.39	2.6102	6483.0	-8.8242	-0.45638	0.12677E-01
14	0.0	-1920.0	-1.0499	654.99	0.82588	-2.1250	6.1320	6383.1	1.5965	1190.6
15	0.0	-0.14457E-01	594.17	-0.40532E-01	-0.32840E-02	-0.71969E-01	-0.65657E-01	0.38907	-1275.3	0.26486E-02
16	0.0	6621.7	0.91070E-01	-8.5245	0.64313E-02	-0.77915E-01	-0.83970	-114.61	-0.16586	-304.60
17	0.0	-0.46193E-01	-0.80404E-02	-0.14797	-18.027	-0.38097	174.47	1.4090	0.15100E-01	-0.64018E-03
18	0.0	0.20908E-01	0.52781	-0.81909E-01	0.55429E-01	-111.16	0.35062	0.81077	-1.0666	0.83528E-03
19	0.0	-0.27620	-0.26546	-2.3217	-949.32	-5.2718	8232.2	7.3837	0.32283	0.75428
20	0.0	-19422.	-3.4674	471.98	-0.72199	2.4643	8.4766	5313.5	6.6247	11467.
21	0.0	0.32561E-01	78659.	0.44608	-0.66188	-0.12022	-5.3749	-2.2263	-938.97	-0.11063
22	0.0	-1789.1	-0.73164E-01	59.292	-1.2345	4.8204	-8.2562	-251.06	-0.14463	4.5261
23	0.0	0.13445E-01	-0.97478E-01	1.5112	-82.624	3.8698	-315.40	-16.014	-0.38186	0.41056E-01
24	0.0	0.40497E-01	-0.76498	1.1157	-0.74327	3.3083	-5.1722	-11.751	1.5764	0.15935E-01
25	0.0	0.57943E-01	-0.27053	2.2944	-314.71	7.6311	-1682.3	-24.187	-2.0806	-0.23567
26	0.0	1311.1	0.27230	-242.46	8.1231	-32.446	54.356	1268.5	0.73117	-3.7301
27	0.0	0.62464E-01	-1959.8	0.11089	-0.19710	-0.16263E-01	-1.6706	-0.91395	78.933	-0.73961E-02
28	0.0	-905.50	-0.77034E-01	4.5144	-0.48267E-01	0.10106	-0.34536	-4.1583	0.19464	1.4408
29	0.0	-0.75134E-02	0.90363E-02	-0.85871E-01	-6.7947	-0.28986	-9.3448	0.79341	0.56131E-01	-0.60704E-02
30	0.0	0.10643E-02	-0.43389	-0.26059	0.17186	-0.83974	1.2486	2.7989	0.87671	-0.33433E-02
31	0.0	-0.95954E-01	0.28446	-0.84466	-62.785	-2.5894	-156.35	8.6315	0.45642	-0.93754E-02
32	0.0	1920.3	-1.0499	-39.021	0.82554	-2.1598	6.1130	84.239	1.6489	-6.4913
33	0.0	-0.14780E-01	-594.16	0.40869E-01	0.32844E-02	0.82343E-01	0.65659E-01	-0.39894	15.516	0.25975E-02
34	0.0	-6621.7	0.91070E-01	1.3946	0.64428E-02	-0.65868E-01	-0.84013	3.0026	-0.19202	1.2054
35	0.0	-0.45754E-01	0.80410E-02	0.13721	-6.7106	0.37781	8.4221	-1.3470	-0.15097E-01	0.93215E-03
36	0.0	-0.20720E-01	0.52780	-0.81162E-01	0.55421E-01	-0.20273	0.35089	0.83659	-1.0812	-0.22294E-02
37	0.0	-0.25404	0.26548	2.3534	-324.20	5.3958	197.92	-7.5548	-0.32274	-0.73763
38	0.0	19422.	-3.4674	-78.893	-0.72144	2.3582	8.4813	-61.397	7.0792	-42.741
39	0.0	0.33260E-01	-78659.	-0.45071	0.66188	-0.21683	5.3749	2.2615	72.911	0.10240

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TABLE 10.10 (Cont'd)

ROW COLUMN	11	12	13	14	15	16	17	18	19	20
1	31.766	0.32617E-04	387.57	-0.14905E-02	-0.50569E-06	0.68685E-04	19.443	-0.54759E-06	1085.8	-0.24036E-02
2	-0.66490E-03	0.30445E-01	-0.10452	353.36	0.38629E-02	-4.2665	-0.25290E-02	-0.78564E-02	-0.95029E-01	353.50
3	-0.59087E-02	0.78737E-02	-0.20284	-0.11435	-299.40	0.62656E-03	-0.10731E-02	-0.86078E-02	-0.14068E-01	-0.96216E-02
4	0.13864	-2.5108	2.1081	655.68	-0.41374E-01	-8.5163	-0.25470E-01	0.67265	-1.1577	471.14
5	-58.152	2.0034	-689.33	-0.47255E-01	-0.13184	0.81746E-02	-18.140	-0.53095	-949.93	-0.49277
6	0.81218E-01	-199.40	1.2077	0.75866E-01	-1.2921	0.41880E-02	-0.17631E-01	-110.46	-0.79090	-0.87521E-01
7	655.70	4.2881	6490.7	2.4692	-0.94649	-0.28579	173.68	-1.3045	8215.6	11.076
8	-0.77597	17.304	-14.482	6370.0	0.21161	-114.68	0.40710E-01	-4.8349	2.4004	5323.9
9	-0.67119E-02	-0.17044E-02	-0.37484	-0.24303	-1276.0	0.48476E-02	-0.38146E-02	-0.23498E-03	-0.92165E-01	-0.14522
10	0.19308E-02	-0.50374E-01	0.79488E-01	1190.9	-0.13760	-304.60	0.14795E-02	0.11487E-01	0.56843E-01	11468.
11	435.85	-0.15798	-1365.4	0.17266	0.24926E-01	-0.81549E-02	-357.26	0.46115E-01	-13902.	0.30679
12	-0.17209E-01	1120.4	-0.27069	-0.19862E-01	-0.70657	-0.35363E-03	0.37594E-02	-715.91	0.17266	-0.10779E-02
13	-1365.6	-1.4419	52817.	0.12032	0.85229E-01	-0.41063E-02	2181.7	0.42690	73304.	0.28680
14	0.78066E-01	0.99744	-2.3959	51964.	-1.4819	-2006.5	-0.13416	-0.21366	-5.2031	65778.
15	-0.13267E-02	0.25038E-01	0.69407E-02	-0.80135E-02	7340.7	-0.22830E-02	0.66353E-03	-0.23657E-03	0.21999E-01	0.88057E-01
16	-0.19288E-01	0.35348E-01	-0.40415	-2006.7	0.14111	378.04	-0.54599E-02	-0.71251E-02	-0.19771	-12858.
17	-357.27	0.20705	2181.7	-0.49418E-01	-0.10612E-01	0.25593E-03	421.28	-0.59821E-01	16976.	-0.19375E-02
18	-0.71839E-02	-715.67	-0.96630E-01	-0.11362E-01	0.84626	-0.10106E-02	0.15853E-02	2485.0	0.68660E-01	0.35331E-01
19	-13902.	2.4511	73316.	-7.6958	-0.25592	-0.29180	16977.	-0.51277	0.84635E 06	13.504
20	0.28129	-1.2083	7.9950	65786.	-5.4069	-12858.	0.15577	0.28360	5.8978	0.58711E 06
21	0.14943	0.16534	0.18196	1.3822	-5137.6	0.37978E-01	-0.10861	-0.95082E-01	-4.4011	-1.7790
22	0.13829	-2.5105	2.1121	-38.362	-0.41956E-01	1.4064	-0.25049E-01	0.67275	-1.1357	-79.891
23	-6.6098	-2.0034	-59.854	0.47510E-01	0.13184	-0.82150E-02	-6.5954	0.53097	-323.58	0.49424
24	0.81531E-01	-1.7813	1.1998	0.75257E-01	-1.2973	0.45911E-02	-0.18106E-01	0.50593	-0.81023	-0.92186E-01
25	-8.9047	-4.2902	-164.17	-2.4412	0.94649	0.28529	9.1868	1.3045	215.39	-11.060
26	-0.77495	17.319	-14.468	70.968	0.23313	2.9218	0.40167E-01	-4.8483	2.3735	-51.039
27	0.67111E-02	0.30394E-02	0.37485	0.19769	16.216	0.11290E-02	0.38156E-02	0.19905E-02	0.92211E-01	-0.67770E-01
28	0.20474E-02	-0.43504E-01	0.78644E-01	-6.2572	-0.14415	1.1980	0.13843E-02	0.72828E-02	0.53667E-01	-42.469
29	-2.0852	0.15886	-11.934	-0.18086	-0.24925E-01	0.86071E-02	-4.9049	-0.46427E-01	-164.41	-0.32290
30	-0.17294E-01	0.46622	-0.26897	0.42442E-02	-0.70293	-0.26209E-02	0.38743E-02	-0.13907	0.17724	0.88414E-01
31	-11.670	1.4324	-101.56	-0.71376E-01	-0.85242E-01	0.52234E-03	-10.503	-0.42503	-638.69	-0.15631
32	0.85622E-01	1.0686	-2.4784	41.146	-1.5453	-2.4127	-0.14203	-0.25892	-5.4989	148.97
33	0.13270E-02	-0.32248E-01	-0.69445E-02	0.55802E-01	3.3736	-0.30488E-02	-0.66364E-03	0.37626E-02	-0.21989E-01	0.11430
34	-0.19570E-01	0.27094E-01	-0.40158	-2.5954	0.15183	4.3957	-0.51685E-02	-0.34873E-02	-0.18501	-40.918
35	-4.8980	-0.20482	-10.517	0.25195E-01	0.10611E-01	0.31735E-04	-38.449	0.58884E-01	-392.93	-0.15150E-02
36	-0.71926E-02	0.10381	-0.96084E-01	-0.13065E-01	0.89528	0.96660E-03	0.16087E-02	-0.29470E-01	0.69593E-01	-0.51710E-01
37	-164.44	-2.5276	-650.95	7.9476	0.25589	0.27169	-393.16	0.53950	-19147.	-12.813
38	0.28371	-0.93490	7.9550	155.99	-5.8076	-40.924	0.15247	0.10900	5.7338	1521.2
39	-0.14943	0.22511	-0.18197	-1.4674	20.563	-0.29351E-01	0.10861	-0.11574	4.4009	1.4529

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TABLE 10.10 (Cont'd)

ROW	COLUMN 21	22	23	24	25	26	27	28	29	30
1	0.16114E-05	-0.61869E-02	1839.0	0.97881E-04	10779.	0.46455E-01	0.22713E-05	-0.11190E-04	31.767	-0.36644E-04
2	0.28573E-03	1878.2	0.61753E-02	0.60473E-01	0.34127E-02	-11118.	-0.10165E-01	29.294	-0.64753E-03	-0.30431E-01
3	-774.72	-0.59590E-01	-0.10396	0.10261E-01	-0.82320	0.47169	2912.2	0.33478E-02	0.59027E-02	0.78644E-02
4	0.71306E-01	59.292	1.2344	4.8204	8.2574	-251.07	0.14463	4.5259	-0.13826	-2.5105
5	-0.23373E-02	-1.5112	-82.624	-3.8698	-315.41	16.011	-0.38186	-0.40975E-01	-6.6094	2.0034
6	0.61581	1.1157	0.74327	3.3091	5.1721	-11.751	-1.5768	0.15935E-01	-0.81533E-01	-1.7814
7	0.17322	-2.2939	-314.71	-7.6312	-1682.4	24.150	-2.0805	0.23658	-8.9002	4.2902
8	-0.36425	-242.47	-8.1212	-32.447	-54.349	1268.5	-0.73111	-3.7261	0.77436	17.319
9	-932.17	-0.11089	-0.19710	0.16255E-01	-1.6706	0.91397	78.935	0.73936E-02	0.67087E-02	-0.30350E-02
10	0.59689E-01	4.5143	0.48252E-01	0.10107	0.34513	-4.1576	-0.19464	1.4410	-0.20355E-02	-0.43504E-01
11	-0.41111E-02	0.85873E-01	-6.7945	0.28986	-9.3489	-0.79376	0.56136E-01	0.60675E-02	-2.0856	-0.15885
12	0.33541	-0.26059	-0.17186	-0.84043	-1.2486	2.7989	-0.87673	-0.33430E-02	0.17293E-01	0.46649
13	0.66022E-01	0.84462	-62.787	2.5895	-156.36	-8.6333	0.45646	0.95034E-02	-11.666	-1.4324
14	0.76271	-39.021	-0.82493	-2.1598	-6.1106	84.250	-1.6489	-6.4922	-0.85767E-01	1.0686
15	-5139.0	-0.40867E-01	0.32851E-02	-0.82335E-01	0.65631E-01	0.39895	15.506	-0.25981E-02	0.13264E-02	0.32247E-01
16	-0.68581E-01	1.3948	-0.64445E-02	-0.65868E-01	0.84001	3.0026	0.19202	1.2049	0.19567E-01	0.27093E-01
17	0.46150E-02	-0.13721	-6.7106	-0.37781	8.4252	1.3471	-0.15099E-01	-0.92498E-03	-4.8976	0.20482
18	-0.39692	-0.81163E-01	-0.55421E-01	-0.20251	-0.35087	0.83656	1.0813	-0.22296E-02	0.71937E-02	0.10366
19	0.12570	-2.3534	-324.19	-5.3958	198.05	7.5615	-0.32281	0.73790	-164.42	2.5275
20	2.5639	-78.900	0.72117	2.3582	-8.4745	-61.409	-7.0793	-42.718	-0.28360	-0.93489
21	87528.	0.45071	0.66188	0.21683	5.3750	-2.2615	72.919	-0.10240	-0.14943	-0.22511
22	0.71469E-01	406.18	1.2343	4.8220	8.2548	-1248.5	0.14603	-58.627	-0.13861	-2.5108
23	0.23377E-02	1.5162	381.53	3.8700	1149.7	-16.047	0.38186	0.40964E-01	-58.152	-2.0034
24	0.61957	1.1200	0.74337	1133.8	5.1695	-11.782	-1.5702	0.16105E-01	-0.81220E-01	-199.40
25	-0.17322	2.2949	1149.0	7.6249	51891.	-24.183	2.0805	-0.23614	655.70	-4.2881
26	-0.37193	-1239.9	-8.1236	-32.447	-54.374	52181.	-0.76326	-691.35	0.77528	17.304
27	66.197	0.11084	0.19710	-0.64739E-02	1.6706	-0.92326	7252.5	-0.20278E-02	-0.67102E-02	0.16988E-02
28	0.63861E-01	-58.643	0.48283E-01	0.10425	0.34626	-691.78	-0.18863	395.27	-0.19153E-02	-0.50373E-01
29	0.41110E-02	-0.86924E-01	-57.967	-0.28849	656.15	0.80498	-0.56137E-01	-0.57410E-02	435.85	0.15798
30	0.32862	-0.26131	-0.17188	-198.47	-1.2482	2.7981	-0.87705	-0.54890E-02	0.17208E-01	1120.4
31	-0.66025E-01	-0.87322	-686.39	-2.6103	6482.9	8.8307	-0.45645	-0.12749E-01	-1365.6	1.4418
32	0.80602	654.99	-0.82575	-2.1250	-6.1351	6383.1	-1.5965	1190.6	-0.78160E-01	0.99743
33	21.404	0.40529E-01	-0.32852E-02	0.71964E-01	-0.65633E-01	-0.38907	-1275.3	-0.26478E-02	-0.13260E-02	-0.25033E-01
34	-0.71168E-01	-8.5244	-0.64198E-02	-0.77915E-01	0.83970	-114.61	0.16586	-304.60	0.19283E-01	0.35348E-01
35	-0.46109E-02	0.14796	-18.027	0.38098	174.47	-1.4090	0.15101E-01	0.63107E-03	-357.27	-0.20705
36	-0.42754	-0.81910E-01	-0.55430E-01	-111.16	-0.35060	0.81074	1.0666	0.83505E-03	0.71850E-02	-715.67
37	-0.12571	2.3214	-949.32	5.2718	8232.1	-7.3858	0.32286	-0.75465	-13902.	-2.4511
38	2.7831	471.98	0.72125	2.4643	-8.4741	5313.4	-6.6248	11467.	-0.28110	-1.2083
39	1919.6	-0.44608	-0.66188	0.12022	-5.3750	2.2263	-938.98	0.11063	0.14943	-0.16534

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TABLE 10.10 (Cont'd)

ROW	COLUMN 31	32	33	34	35	36	37	38	39
1	387.57	0.14663E-02	0.59205E-06	-0.67252E-04	19.443	0.29808E-05	1085.8	0.23437E-02	-0.15406E-05
2	-0.10455	-353.36	0.38641E-02	4.2664	-0.25272E-02	0.78473E-02	-0.94953E-01	-353.48	0.28108E-03
3	0.20299	-0.11426	299.37	0.60609E-03	0.10552E-02	-0.86020E-02	0.14408E-01	-0.97696E-02	774.73
4	-2.1126	-38.358	0.41951E-01	1.4063	0.25035E-01	0.67275	1.1348	-79.899	-0.71471E-01
5	-59.854	-0.46910E-01	0.13186	0.81894E-02	-6.5950	-0.53097	-323.58	-0.49389	0.23291E-02
6	-1.1998	0.75254E-01	1.2980	0.45899E-02	0.18106E-01	0.50607	0.81021	-0.92086E-01	-0.62004
7	-164.14	2.4523	0.94644	-0.28552	9.1788	-1.3045	215.04	11.062	-0.17318
8	14.470	70.934	-0.23320	2.9175	-0.39381E-01	-4.8484	-2.3326	-50.891	0.37197
9	0.37485	-0.19765	16.229	-0.11320E-02	0.38127E-02	-0.19923E-02	0.92262E-01	0.67706E-01	66.187
10	-0.78598E-01	-6.2550	0.14416	1.1979	-0.14049E-02	0.72817E-02	-0.54603E-01	-42.466	-0.63866E-01
11	-11.930	0.18101	-0.24935E-01	-0.86020E-02	-4.9047	0.46423E-01	-164.39	0.32277	0.41196E-02
12	0.26897	0.42442E-02	0.70329	-0.26206E-02	-0.38739E-02	-0.13983	-0.17722	0.88389E-01	-0.32894
13	-101.59	0.72593E-01	-0.85324E-01	-0.55567E-03	-10.507	0.42501	-638.84	0.15802	-0.65967E-01
14	2.4747	41.181	1.5453	-2.4157	0.14221	-0.25892	5.5090	149.08	-0.80602
15	-0.69252E-02	-0.55806E-01	3.3768	0.30472E-02	-0.66634E-03	-0.37631E-02	-0.22053E-01	-0.11432	21.420
16	0.40163	-2.5965	-0.15184	4.3957	0.51702E-02	-0.34865E-02	0.18494	-40.923	0.71171E-01
17	-10.521	-0.25219E-01	0.10615E-01	-0.37011E-04	-38.449	-0.58882E-01	-392.94	0.16597E-02	-0.46150E-02
18	0.96077E-01	-0.13065E-01	-0.89567	0.96668E-03	-0.16089E-02	-0.28285E-01	-0.69609E-01	-0.51716E-01	0.42785
19	-651.07	-7.9486	0.25607	-0.27188	-393.17	-0.53941	-19147.	12.818	-0.12587
20	-7.9569	156.01	5.8078	-40.934	-0.15248	0.10897	-5.7268	1521.3	-2.7832
21	-0.18199	1.4674	20.559	0.29354E-01	0.10862	0.11574	4.4009	-1.4529	1919.5
22	-2.1087	655.68	0.41369E-01	-8.5163	0.25453E-01	0.67266	1.1567	471.14	-0.71309E-01
23	-689.33	0.46662E-01	-0.13186	-0.81474E-02	-18.141	0.53095	-949.94	0.49235	-0.23294E-02
24	-1.2077	0.75862E-01	1.2923	0.41867E-02	0.17630E-01	-110.46	0.79087	-0.87421E-01	-0.61603
25	6490.7	-2.4797	-0.94644	0.28600	173.69	1.3046	8215.9	-11.076	0.17318
26	14.483	6370.0	-0.21167	-114.69	-0.39999E-01	-4.8350	-2.3623	5324.0	0.36429
27	-0.37486	0.24302	-1276.0	-0.48442E-02	-0.38112E-02	0.23800E-03	-0.92191E-01	0.14526	-932.16
28	-0.79349E-01	1190.9	0.13760	-304.61	-0.14993E-02	0.11485E-01	-0.57767E-01	11468.	-0.59693E-01
29	-1365.4	-0.17284	0.24935E-01	0.81481E-02	-357.26	-0.46112E-01	-13902.	-0.30659	-0.41194E-02
30	0.27069	-0.19862E-01	0.70702	-0.35328E-03	-0.37591E-02	-715.92	-0.17264	-0.11024E-02	-0.33577
31	52817.	-0.12204	0.85317E-01	0.41013E-02	2181.7	-0.42686	73304.	-0.28682	0.65972E-01
32	2.3940	51964.	1.4818	-2006.5	0.13428	-0.21366	5.2104	65778.	-0.76272
33	0.69255E-02	0.80134E-02	7340.7	0.22845E-02	0.66618E-03	0.23331E-03	0.22060E-01	-0.88037E-01	-5139.0
34	0.40413	-2006.7	-0.14111	378.04	0.54636E-02	-0.71245E-02	0.19774	-12858.	0.68584E-01
35	2181.7	0.49471E-01	-0.10616E-01	-0.25043E-03	421.28	0.59819E-01	16976.	0.17768E-02	0.46192E-02
36	0.96623E-01	-0.11362E-01	-0.84661	-0.10105E-02	-0.15856E-02	2485.0	-0.68676E-01	0.35324E-01	0.39721
37	73316.	7.6982	-0.25610	0.29201	16977.	0.51269	0.84635E 06	-13.510	0.12587
38	-7.9944	65786.	5.4072	-12858.	-0.15585	0.28358	-5.8939	0.58711E 06	-2.5640
39	0.18198	-1.3822	-5137.6	-0.37981E-01	-0.10862	0.95084E-01	-4.4011	1.7790	87528.

TABLE 10.11 SAMPLE OUTPUT; "T," BASELINE HUB

```
RUN NUMBER=  1      HUB T CONF. RUN NO. 1.

     DD(   100)=        1.000000        HUB CALCULATIONS
     DD(   104)=      -27.79999         YHG(1)
     DD(   107)=      -63.00000         YHC1(1)
     DD(   110)=     -122.0000          YHC2(1)
     DD(   113)=     -175.1000          YHC3(1)
     DD(   116)=     -156.5000          YHE(1)
     DD(   119)=      1735329.          CI1(1,1)
     DD(   123)=      2012994.          CI1(2,2)
     DD(   127)=      292228.8          CI1(3,3)
     DD(   128)=      1733430.          CI2(1,1)
     DD(   132)=      1829283.          CI2(2,2)
     DD(   136)=      98517.31          CI2(3,3)
     DD(   137)=      1644085.          CI3(1,1)
     DD(   141)=      2034929.          CI3(2,2)
     DD(   145)=      485759.9          CI3(3,3)
     DD(   146)=      741.8020          CM1
     DD(   147)=      536.5137          CM2
     DD(   148)=      3881.988          CM3
     DD(   149)=    0.9999998E-02       DAMPH
     DD(    51)=      15.82300          AREA
     DD(    52)=      13446.00          AI
     DD(    53)=      4038500.          GBE
     DD(    54)=    0.1050000E 08       BE
     DD(    55)=      26892.00          PMJ
     DD(    90)=      7.000000          IRIT
     DD(    92)=    0.1000000E-04       TPHHF
     DD(    95)=      10.00000          TK7H
     DD(    96)=      10.00000          TCH
     DD(    97)=      10.00000          TK6HG
     DD(    98)=      10.00000          TCHG
```

```
HUB STIFFNESS MATRIX
                                                  TABLE 10.11 (Cont'd)
      ARRY-K7H

ROW COLUMN  1             2             3             4             5             6             7             8             9             10
  1        0.75359E 07   0.0           0.0           0.0           0.0           0.0          -0.28160E 07   0.0           0.0           0.0
  2        0.0           0.32704E 06   0.0           0.0           0.0           0.30578E 07   0.0          -57286.        0.0           0.0
  3        0.0           0.0           0.32704E 06   0.0          -0.30578E 07   0.0           0.0           0.0          -57286.        0.0
  4        0.0           0.0           0.0           0.34209E 08   0.0           0.0           0.0           0.0           0.0          -0.12783E 08
  5        0.0           0.0          -0.30578E 07   0.0           0.17788E 09   0.0           0.0           0.0          -0.16899E 07   0.0
  6        0.0           0.30578E 07   0.0           0.0           0.0           0.17788E 09   0.0           0.16899E 07   0.0           0.0
  7       -0.28160E 07   0.0           0.0           0.0           0.0           0.0           0.76317E 07   0.0           0.0           0.0
  8        0.0          -57286.        0.0           0.0           0.0           0.16899E 07   0.0           0.34380E 06   0.0           0.0
  9        0.0           0.0          -57286.        0.0          -0.16899E 07   0.0           0.0           0.0           0.34380E 06   0.0
 10        0.0           0.0           0.0          -0.12783E 08   0.0           0.0           0.0           0.0           0.0           0.34643E 08
 11        0.0           0.0           0.16899E 07   0.0           0.33235E 08   0.0           0.0           0.0           0.32524E 07   0.0
 12        0.0          -0.16899E 07   0.0           0.0           0.0           0.33235E 08   0.0          -0.32524E 07   0.0           0.0
 13        0.0           0.0           0.0           0.0           0.0           0.0          -0.48157E 07   0.0           0.0           0.0
 14        0.0           0.0           0.0           0.0           0.0           0.0           0.0          -0.28651E 06   0.0           0.0
 15        0.0           0.0           0.0           0.0           0.0           0.0           0.0           0.0          -0.28651E 06   0.0
 16        0.0           0.0           0.0           0.0           0.0           0.0           0.0           0.0           0.0          -0.21861E 08
 17        0.0           0.0           0.0           0.0           0.0           0.0           0.0           0.0           0.10271E 08   0.0
 18        0.0           0.0           0.0           0.0           0.0           0.0           0.0          -0.10271E 08   0.0           0.0

      ARRY-K7H

ROW COLUMN  11            12            13            14            15            16            17            18
  1        0.0           0.0           0.0           0.0           0.0           0.0           0.0           0.0
  2        0.0          -0.16899E 07   0.0           0.0           0.0           0.0           0.0           0.0
  3        0.16899E 07   0.0           0.0           0.0           0.0           0.0           0.0           0.0
  4        0.0           0.0           0.0           0.0           0.0           0.0           0.0           0.0
  5        0.33235E 08   0.0           0.0           0.0           0.0           0.0           0.0           0.0
  6        0.0           0.33235E 08   0.0           0.0           0.0           0.0           0.0           0.0
  7        0.0           0.0          -0.48157E 07   0.0           0.0           0.0           0.0           0.0
  8        0.0          -0.32524E 07   0.0          -0.28651E 06   0.0           0.0           0.0          -0.10271E 08
  9        0.32524E 07   0.0           0.0           0.0          -0.28651E 06   0.0           0.10271E 08   0.0
 10        0.0           0.0           0.0           0.0           0.0          -0.21861E 08   0.0           0.0
 11        0.18014E 09   0.0           0.0           0.0          -0.49423E 07   0.0           0.14876E 09   0.0
 12        0.0           0.18014E 09   0.0           0.49423E 07   0.0           0.0           0.0           0.14876E 09
 13        0.0           0.0           0.48157E 07   0.0           0.0           0.0           0.0           0.0
 14        0.0           0.49423E 07   0.0           0.28651E 06   0.0           0.0           0.0           0.10271E 08
 15       -0.49423E 07   0.0           0.0           0.0           0.28651E 06   0.0          -0.10271E 08   0.0
 16        0.0           0.0           0.0           0.0           0.0           0.21861E 08   0.0           0.0
 17        0.14876E 09   0.0           0.0           0.0          -0.10271E 08   0.0           0.39665E 09   0.0
 18        0.0           0.14876E 09   0.0           0.10271E 08   0.0           0.0           0.0           0.39665E 09
```

TABLE 10.11 (Cont'd)

```
HUB DAMPING MATRIX

      ARRY-CH

ROW  COLUMN  1            2            3            4            5            6            7            8            9            10
  1         1459.8      -0.14182E-04-0.48549E-04 0.25974E-07 0.46197E-03-0.19926E-03 -270.47      0.11271E-04 0.57674E-05-0.41705E-08
  2        -0.92266E-03  289.98      -0.33141E-01-0.44794E-02 0.22745      1631.3      0.20631E-02 -45.658     0.15926E-01 0.71949E-02
  3        -0.82652E-04-0.37401E-02  394.21      0.86874E-03 -2390.9      1.3285     -0.31671E-02-0.20960E-01 -32.652     -0.87556E-02
  4        -0.25224E-04-0.14846      0.62892E-01 0.15036E 06 0.36099     -4.1539      0.12809E-03-0.29020     -0.11532     -32676.
  5        -0.74243E-02-0.10580      -2390.9     0.47509E-01 0.33469E 06 -16.973      0.26368E-01 0.11601E-01 -1619.6      0.15517
  6        -0.63217E-03  1630.4      1.1202      -0.24887     -12.060      0.13653E 06 0.63033E-02  1070.5     -0.43606E-01 0.31421
  7         -270.47     -0.12834E-04-0.24715E-03 0.13556E-06 0.23367E-02-0.49626E-03  1231.0      0.51901E-04 0.32607E-04-0.35748E-07
  8        -0.14455E-03 -45.665      0.23367E-01 0.12877E-01-0.18940      1070.3      0.55958E-03  208.84     -0.11151E-01-0.81955E-02
  9        -0.12842E-03 0.13684E-01 -32.666     -0.14664E-02 -1619.5      0.11062      0.45781E-03-0.10149E-01  241.60     0.34825E-02
 10         0.86474E-05 0.17981     -0.64737E-01 -32677.      1.3424      3.5823     -0.43914E-04 0.12207     -0.11806      0.13711E 06
 11         0.15676E-01 0.23941      1365.6     -0.59032E-01  56283.      16.376     -0.72159E-01-0.31996E-01  1517.7      0.26595
 12         0.41795E-02 -664.79      0.43641E-01-0.40269E-01-0.40856      12465.     -0.17692E-01 -611.48     -0.51193E-01-0.48342E-01
 13         -144.92     -0.45250E-03 0.21293E-03-0.43113E-06-0.17851E-02-0.57963E-02 -708.81      0.18818E-03-0.38291E-04 0.30343E-06
 14         0.21904E-04-0.60090      0.19552E-02-0.14664E-01-0.43794E-01  79.488      0.16464E-05 -140.61      0.51457E-02 0.12832E-01
 15        -0.81968E-04-0.88455E-02 -14.663      0.75719E-03 -1281.0     -0.37416      0.36159E-03 0.10249E-02 -176.94     -0.29545E-02
 16         0.90636E-06-0.17243      0.11658     -7932.3     -0.92208     -3.7557     -0.39672E-05-0.54956E-01-0.24586     -62785.
 17        -0.35282E-02-0.36183      510.73      0.13519      39146.     -8.2102      0.19769E-01-0.52379      5828.9     -0.15850
 18        -0.49533E-02 -138.51      0.54177      0.75766     -6.2347      4000.9      0.27637E-01 -4281.6      0.16431     -0.33602

      ARRY-CH

ROW  COLUMN 11            12           13           14           15           16           17           18
  1        -0.34960E-03-0.34024E-03 -144.93     -0.11263E-04 0.31196E-05-0.54732E-08-0.94742E-04-0.30430E-03
  2        -0.88585E-01 -665.02     -0.15634E-02-0.59456     -0.81546E-02-0.44409E-02 0.28622      -137.89
  3         1365.7      0.61939      0.24590E-02 0.21760E-02 -14.684      0.69773E-02  510.04      0.81061E-01
  4        -0.67247      1.6533     -0.92833E-04 0.27632      0.85720E-01 -7932.7     -3.0339       12.596
  5         56285.      -11.834     -0.18037E-01-0.26734     -1280.9     -0.16292      39158.      -9.1743
  6         8.7428       12456.     -0.59710E-02  79.502     -0.13715     -0.17770      4.3503      4010.8
  7        -0.17383E-02-0.20997E-02 -708.80     -0.62935E-04 0.12952E-04-0.16760E-07-0.37724E-03-0.18000E-02
  8         0.49212E-01 -811.56     -0.34206E-03 -140.62      0.62452E-02 0.24019E-02-0.23122      -4281.8
  9         1517.3     -0.52463E-01-0.31171E-03 0.65911E-02 -176.93     -0.32531E-02  5829.2      0.27422
 10        -1.5709      -3.6138      0.31599E-04-0.14093      0.11388     -62785.     -4.3272      -5.0562
 11         0.31547E 06  3.8601      0.51480E-01 0.18920     -4621.5     -0.21912      87731.       8.7651
 12        -0.52567      80507.      0.12261E-01  1604.4      0.42384E-01 0.55054E-01 -1.4564      39793.
 13         0.16105E-02 0.42859E-03  1932.0     -0.12655E-03-0.51905E-05-0.10718E-06 0.25235E-04-0.49973E-02
 14         0.88417E-01  1604.3     -0.32841E-04  166.30     -0.50911E-02-0.54898E-02 0.18424      4649.0
 15         -4621.0     -0.24356     -0.25680E-03-0.48451E-02  244.81      0.31747E-02 -7557.4     -0.12748
 16        -3.1555       3.0929      0.30525E-05 0.67313E-01 0.21982      0.10285E 06 -7.7748       1.5766
 17         87722.       8.0988     -0.14499E-01 0.51261     -7557.2      0.55533E-01 0.39153E 06  19.321
 18         7.1383       39793.     -0.20887E-01  4649.0     -0.24777      0.25882E-01  8.8463      0.22345E 06
```

TABLE 10.11 (Cont'd)

```
HUB ( ATTACHMENT POINT)  STIFFNESS MATRIX

      ARY-K6HG

ROW  COLUMN  1             2             3             4             5             6             7             8             9             10
  1       -0.47199E 07  0.0           0.0           0.0           0.0           0.0           0.0           0.0           0.0           0.0
  2        0.0         -0.26976E 06   0.0           0.0           0.0          -0.47477E 07   0.0           0.0           0.0           0.0
  3        0.0          0.0          -0.26976E 06   0.0           0.47477E 07   0.0           0.0           0.0           0.0           0.0
  4        0.0          0.0           0.0          -0.21426E 08   0.0           0.0           0.0           0.0           0.0           0.0
  5        0.0          0.0          -0.47477E 07   0.0           0.55707E 08   0.0           0.0           0.0           0.0           0.0
  6        0.0          0.47477E 07   0.0           0.0           0.0           0.55707E 08   0.0           0.0           0.0           0.0

      ARY-K6HG

ROW  COLUMN 11             12            13            14            15            16            17            18
  1        0.0          0.0           0.0           0.0           0.0           0.0           0.0           0.0
  2        0.0          0.0           0.0           0.0           0.0           0.0           0.0           0.0
  3        0.0          0.0           0.0           0.0           0.0           0.0           0.0           0.0
  4        0.0          0.0           0.0           0.0           0.0           0.0           0.0           0.0
  5        0.0          0.0           0.0           0.0           0.0           0.0           0.0           0.0
  6        0.0          0.0           0.0           0.0           0.0           0.0           0.0           0.0

HUB ( ATTACHMENT POINT)  DAMPING MATRIX

      ARRY-CHG

ROW  COLUMN  1             2             3             4             5             6             7             8             9             10
  1        -1044.4       0.47952E-03  0.82768E-04  0.26959E-06-0.10136E-02  0.64918E-02 -251.76      -0.25135E-03-0.83102E-07-0.26351E-06
  2        0.10453E-02 -243.71        0.78193E-02  0.62622E-02  0.57455E-02 -2781.0      -0.26243E-02 -22.565      -0.99205E-02-0.11829E-01
  3        0.29303E-03-0.10993E-02 -256.88       -0.15934E-03  5291.5      -1.0650       0.23477E-02-0.11837E-01 -32.008       0.82276E-02
  4        0.15670E-04  0.14108     -0.11473     -0.10976E 06-0.78129      4.3272      -0.80205E-04  0.22309      0.47925     -41645.
  5        0.22357E-01  0.37376    -4956.7       -0.12764    -4708.0        6.7361       0.41116E-01  0.61146    -1273.2        0.15273
  6       -0.41461E-01  4590.0      -0.38304     -1.5730       2.4169      16949.        0.10933      1375.7       0.19859      1.4415

      ARRY-CHG

ROW  COLUMN 11             12            13            14            15            16            17            18
  1        0.47742E-03  0.20114E-02 -1078.3       0.20075E-03-0.10881E-04  0.12942E-06  0.44675E-03  0.71015E-02
  2       -0.49043E-01 -127.76        0.19382E-02 -25.079       0.70005E-02  0.75277E-02-0.23924     -229.30
  3        1738.1      -0.32341     -0.18905E-02-0.39212E-02 -53.183      -0.68989E-02  1218.2      -0.22774
  4        5.3989      -1.1325       0.58181E-04-0.20270     -0.41941    -32131.        15.136      -9.1163
  5        30201.       18.890      -0.38311E-01-0.41826    -5416.5      -0.80286E-01  27719.       -28.817
  6       -0.81409      3704.9      -0.77492E-01  4895.7      -0.10613     -0.64192      3.6928      9340.8
```

TABLE 10.11 (Cont'd)

```
HUB-MODAL MATRIX EIGENVALUES

      OMEGA

EIG. VAL-NO.  1             2             3             4             5             6             7             8             9             10
          0.45785E 00 0.47574E 00 0.14248E 01 0.35451E 01 0.42664E 01 0.56755E 01 0.57238E 01 0.61003E 01 0.11097E 02 0.14880E 02

      OMEGA

EIG. VAL-NO.  11            12            13            14            15            16            17            18
          0.17455E 02 0.21486E 02 0.26970E 02 0.30044E 02 0.30214E 02 0.50155E 02 0.87929E 02 0.13262E 03

HUB MODAL MATRIX

      MODE-MAT

ROW  COLUMN  1             2             3             4             5             6             7             8             9             10
  1        0.59903E-14 0.67253E-12 0.10778E-13 0.30127E-11 0.15487E-11 0.19931E-08-0.19561E-09 0.75385E-09 0.36700E-08 0.22766E-06
  2        0.13110E-07 0.79529E-01-0.68377E-06 0.35622E-06-0.31872E-03 0.39905      -0.16838E-01-0.73214E-05-0.57577E-04 1.0000
  3        0.78731E-01-0.24985E-09 0.18103E-05-0.39728      0.22814E-04-0.51476E-06 0.36507E-03 1.0000      -0.21158E-01 0.33728E-03
  4        0.73242E-14 0.17984E-11 0.24450E-13 0.80109E-11 0.41353E-11 0.53096E-08-0.52312E-09 0.20099E-08 0.97019E-08 0.58863E-06
  5       -0.40928E-08 0.48490      -0.14443E-05-0.17815E-05-0.94907E-03 1.0000      -0.41912E-01-0.33368E-04 0.10663E-04-0.25714
  6        0.48272     -0.97904E-09 0.45339E-05 -1.0000      0.10095E-04-0.25144E-05-0.39870E-04-0.46009      1.0000      0.13211E-04
  7       -0.60228E-11 0.62231E-11 0.10833E-11 0.14603E-06 0.13808E-09 0.98665E-08-0.10248E-08 0.13580E-07 0.35798E-07 0.74538E-06
  8        0.22419E-08 1.0000      0.16927E-05 0.42843E-07 0.63031E-05-0.44949E-01 0.19427E-02 0.18458E-05 0.17979E-05-0.36568E-01
  9        1.0000     -0.19845E-08-0.16191E-06-0.33455E-01 0.64533E-05 0.18043E-06 0.36841E-05-0.10629E-01 0.25647     -0.11811E-04
 10       -0.14299E-09-0.68497E-08 0.35297     -0.84556E-08 1.0000     -0.75802E-05-0.56445      0.17177E-05-0.15723E-06-0.65415E-06
 11        0.42880E-02-0.18304E-10 0.95088E-07-0.20029E-01-0.29770E-06 0.41225E-07 0.15273E-04 0.53264E-01-0.40538E-01-0.19277E-05
 12        0.10355E-08-0.43184E-02 0.17705E-07 0.76307E-07 0.15550E-04-0.18209E-01 0.76160E-03 0.14195E-05 0.14090E-05-0.35291E-01
 13       -0.57376E-11-0.11029E-09 0.84732      0.44138E-07 0.20504      0.15002E-04 1.0000     -0.31291E-05-0.14209E-06 0.99940E-07
 14        0.88930E-02-0.16897E-10-0.21219E-08 0.45534E-02-0.67858E-06-0.13348E-06-0.27645E-04-0.96414E-01-0.36497E-01 0.13793E-06
 15        0.45598E-11-0.88683E-02-0.16248E-08-0.16400E-07-0.62534E-06 0.47381E-02-0.20028E-03-0.52543E-06-0.20620E-05 0.41237E-01
 16       -0.52202E-11-0.12277E-09 1.0000     -0.16444E-07-0.55574     -0.10552E-04-0.68308      0.21075E-05 0.33688E-06 0.13577E-06
 17        0.10071E-01-0.19693E-10-0.31676E-08 0.24929E-01-0.10252E-05 0.13182E-06 0.13709E-04 0.59078E-01-0.15115E-01 0.42810E-06
 18       -0.20601E-09-0.10015E-01-0.10843E-06-0.74283E-07-0.26641E-04 0.26321E-01-0.11029E-02-0.65993E-06 0.13656E-05-0.29054E-01

      MODE-MAT

ROW  COLUMN  11            12            13            14            15            16            17            18
  1        0.29062     -0.11459E-06 0.35793E-06 0.31106E-07-0.76811E-07 0.20639E-06 1.0000      0.51099
  2       -0.10054E-05 0.55130E-03 -1.0000      0.55826E-03 0.14444     -0.12918      0.64147E-06-0.16263E-05
  3       -0.79842E-06 -1.0000     -0.62928E-03 0.14478     -0.28022E-02 0.21804E-03-0.24492E-05 0.20252E-05
  4        0.75441     -0.29272E-06 0.90107E-06 0.75861E-07-0.18560E-06 0.42914E-06 0.63942     -1.0000
  5       -0.31066E-06 0.19436E-04 0.28484     -0.59356E-02 -1.0000     -1.0000      0.33639E-06-0.49223E-06
  6       -0.51055E-08-0.43733E-01 0.40611E-03 -1.0000      0.13662E-01 0.95159E-04 0.38254E-06-0.50188E-06
  7        1.0000      0.62784E-09 0.10920E-05 0.63232E-08-0.13291E-06-0.34023E-06-0.12220      0.75887E-01
  8        0.13364E-06-0.23034E-04-0.28529E-01 0.41608E-03 0.69985E-01 0.16218     -0.14123E-07 0.15393E-07
  9        0.11393E-07 0.35992E-01-0.37376E-04 0.12074     -0.16148E-02-0.24081E-04 0.27239E-07-0.40544E-07
 10       -0.71994E-11-0.81556E-07 0.23005E-06 0.17776E-06 0.74551E-06 0.73276E-06 0.23402E-10-0.33213E-10
 11        0.42808E-08 0.40175E-02 0.23721E-05 0.69477E-03-0.61358E-05-0.17536E-05 0.50590E-08-0.69192E-08
 12        0.13420E-06 0.44093E-06-0.50368E-01-0.24390E-03-0.38556E-01 0.59215E-02 0.11772E-07-0.13920E-07
 13        0.36106E-11 0.17531E-06-0.49821E-06 0.14681E-06-0.18168E-06-0.62761E-06-0.80440E-11 0.11418E-10
 14        0.14228E-08-0.33697E-02-0.97105E-06-0.32795E-02 0.42227E-04 0.11757E-05-0.13464E-07 0.18842E-07
          -0.12483E-06 0.28314E-04 0.24847E-01-0.38950E-03-0.65735E-01 0.16219     -0.92440E-07 0.11707E-06
          -0.10671E-11-0.13658E-06 0.55190E-06 0.39928E-06 0.19735E-07 0.43330E-06-0.88970E-12 0.12278E-11
          -0.26405E-08-0.24352E-02 0.41566E-05-0.83348E-02 0.11295E-03 0.22098E-05 0.33489E-08-0.45693E-08
           0.70863E-07-0.15854E-04-0.11057E-01 0.15459E-03 0.25926E-01 0.43708E-01 0.15922E-07-0.23235E-07
```

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APPENDIX A

MODES, FREQUENCIES, STIFFNESS AND DAMPING MATRICES

This appendix contains an outline of the procedure used to obtain the structural characteristics for both simulation vehicles. For each rotor a stiffness matrix will be formed for use in the time-history simulation. This matrix is applicable for a rotor in which tensile forces are present. In the time-history formulation the tensile forces naturally arise from centrifugal effects. However, to obtain more accurate modes, equivalent external forces were added to the nonrotating idealization used for the eigenvalue problem. As a result slightly different stiffness matrices are formed below for the time-history and eigenvalue problems.

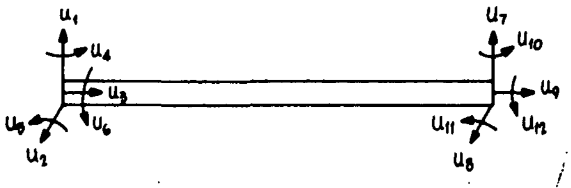
I. Rotor

Basic beam element stiffness matrix (circular cross-section)

We begin with the stiffness matrix for a beam segment located between two node points. This basic matrix does not include the effects of tension due to centrifuged forces.

A = cross-sectional area
I = moment of inertia of cross-section
J = polar moment of inertia of cross-section
L = length of beam
E = modulus of elasticity
G = modulus of rigidity

displacement convention =



$$[K] = \left[\begin{array}{cccccccccccc}
k_2 & & & & & & & & & & & \\
 & k_2 & & & & & & \text{(symmetric)} & & & & \\
 & & k_1 & & & & & & & & & \\
 & -k_5 & & k_5 & & & & & & & & \\
k_3 & & & & k_5 & & & & & & & \\
 & & & & & k_4 & & & & & & \\
-k_2 & & & & -k_3 & & k_2 & & & & & \\
 & -k_2 & & k_3 & & & & k_2 & & & & \\
 & & -k_1 & & & & & & k_1 & & & \\
 & -k_3 & & k_6 & & & & k_3 & & k_5 & & \\
k_3 & & & & k_6 & & -k_3 & & & & k_5 & \\
 & & & & & -k_4 & & & & & & k_4
\end{array}\right]$$

where

$$k_1 = \frac{AE}{L} \qquad k_4 = \frac{GJ}{L}$$

$$k_2 = \frac{12EI}{L^3} \qquad k_5 = \frac{4EI}{L}$$

$$k_3 = \frac{6EI}{L^2} \qquad k_6 = \frac{2EI}{L}$$

Incremental Beam Element Stiffness Matrix

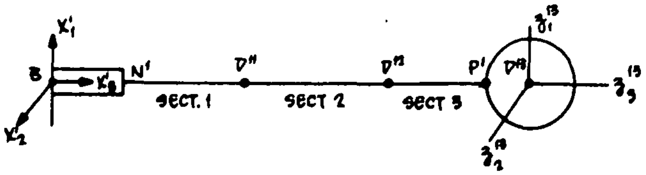
The following incremental stiffness matrix will be added to $[K^*]$ to account for the stiffening effect due to the centrifugal forces

P** = axial tensile load applied to the beam; the direction of this load does not change as beam bends

$$[K^{**}] = \frac{P^{**}}{10L}\left[\begin{array}{cccccccccccc}
12 & & & & & & & & & & & \\
 & 12 & & & & & & & & & & \\
 & & & & & & \multicolumn{5}{c}{\text{(SYMMETRIC)}} & \\
 & -L & & \frac{4L^2}{3} & & & & & & & & \\
L & & & & \frac{4L^2}{3} & & & & & & & \\
 & & & & & & & & & & & \\
-12 & & & & -L & & 12 & & & & & \\
 & -12 & & L & & & & 12 & & & & \\
 & & & & & & & & & & & \\
 & -L & & -\frac{L^2}{3} & & & & L & & \frac{4L^2}{3} & & \\
L & & & & -\frac{L^2}{3} & & -L & & & & \frac{4L^2}{3} & \\
 & & & & & & & & & & &
\end{array}\right]$$

Stiffness Of A Rotor Arm

Next the element stiffness matrices will be assembled to obtain the stiffness matrix for a rotor arm.



$[K1^*]$, $[K2^*]$ and $[K3^*]$ denote basic stiffness for elements 1, 2 and 3, respectively.

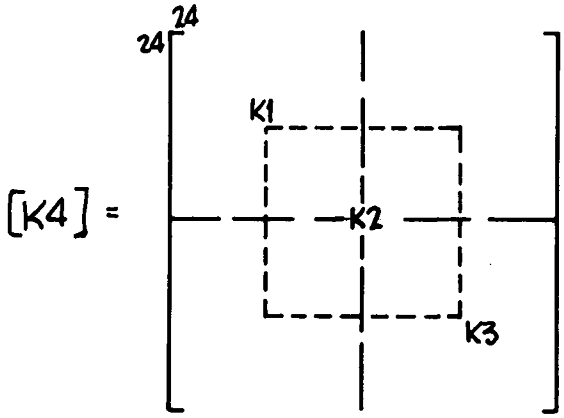
If P_1, P_2, P_3 are centrifugal forces due to $M_D 11$, $M_D 12$ and $M_D 13$, respectively, then

$$P_1 = m_{D11}\,\Omega^2[0,0,1]\{X_{D11}\}, \quad P_2 = m_{D12}\,\Omega^2[0,0,1]\{X_{D12}\}, \quad P_3 = m_{D13}\,\Omega^2[0,0,1]\{X_{D13}\}$$

For the linear elastic analysis it is sufficiently accurate to express the axial tensile loads in Section 1, 2 and 3 as $(P_1 + P_2 + P_3)$, $(P_2 + P_3)$ and (P_3), respectively. These loads can now be used to form $[K1^{**}]$, $[K2^{**}]$ and $[K3^{**}]$ which are the incremental stiffness of elements 1, 2 and 3 respectively. Next the total element stiffness matrix is obtained by combining the basic and incremental stiffnesses:

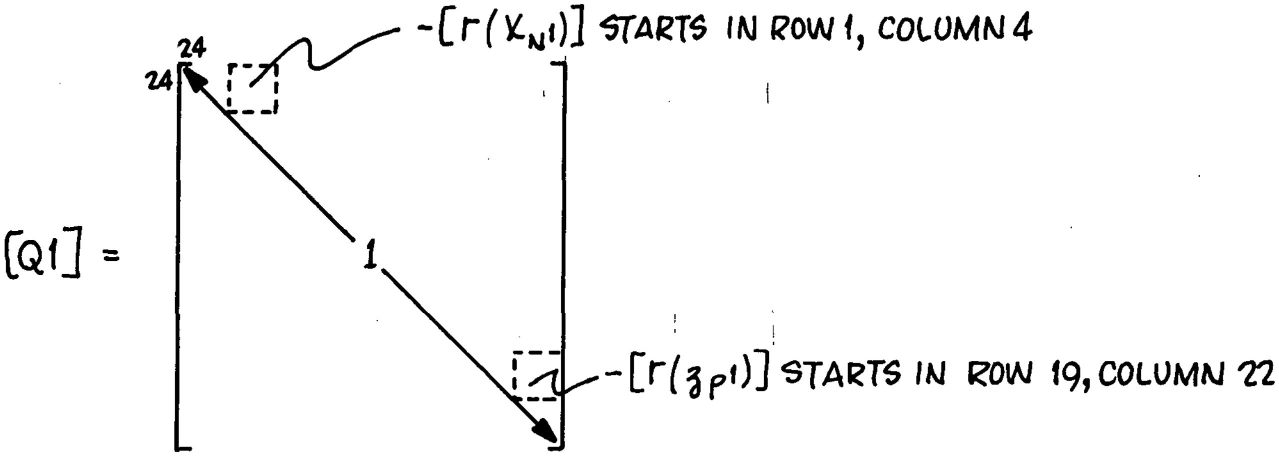
$$\begin{aligned} [K1] &= [K1^*] + [K1^{**}] \\ [K2] &= [K2^*] + [K2^{**}] \\ [K3] &= [K3^*] + [K3^{**}] \end{aligned}$$

The above matrices are now stacked to obtain an assembled stiffness matrix for the three beam elements.



[K1] STARTS IN ROW 1, COLUMN 1
[K2] STARTS IN ROW 7, COLUMN 7
[K3] STARTS IN ROW 13, COLUMN 13

Matrix [K4] operates on displacements at points N^1, D^{11}, D^{12} and P^1. Matrix [Q1] is formed to transform to displacements at B, D^{11}, D^{12} and D^{13}.



The transformed stiffness matrix is [K5]:

$$[K5] = [Q1]^T [K4] [Q1]$$

Adjustments To [K5]:

The previous transformation does not include all linear terms involving the axial tensile loads; thus, the following correction terms are provided:

$$k_7 = (P_1 + P_2 + P_3)[0,0,1]\{X_{N^1}\}$$

$$k_8 = -(P_3)[0,0,1]\{z_{P^1}\}$$

k_7 is added to the (4, 4) and (5, 5) elements of [K5]; also, k_8 is added to the (22, 22) and (23, 23) elements of [K5]. The result is denoted as [K5A]; this matrix will lead to the stiffness matrix supplied to the time history program.

Addition of External Loads Simulating Centrifugal Effects For Eigenvalue Problem

In order to properly form the stiffness matrix for the eigenvalue problem centrifugal loads P_1, P_2 and P_3 are now added to the vehicle idealization. As was mentioned previously the incremental stiffness matrix $[K^{**}]$ is based on the assumption that the axial load does not change direction as the beam bends. Since centrifugal forces pass through the axis of rotation, they retain their direction only when a rotor mass moves out of the spin plane; the following correction terms are needed for rotor motion in the spin plane:

$$k_9 = P_1/[0,0,1]\{X_{D^{11}}\}, \quad k_{10} = P_2/[0,0,1]\{X_{D^{12}}\}, \quad k_{11} = P_3/[0,0,1]\{X_{D^{13}}\}$$

k_9, k_{10} and k_{11} are added to [K5A] as follows:

k_9 to the (2, 8) and (8, 2) elements

k_{10} to the (2, 14) and (14, 2) elements

k_{11} to the (2, 20) and (20, 2) elements

$-(k_9 + k_{10} + k_{11})$ to the (2, 2) element

$-k_9$ to the (8, 8) element

$-k_{10}$ to the 14, 14) element

$-k_{11}$ to the (20, 20) element

The result is denoted as [K5B]; this matrix will lead to the stiffness matrix used in the eigenvalue problem.

The static axial displacement due to centrifugal loads is not included in the eigenvalue problem. These static displacements are as follows:

$$\text{POINT } D^{11}: \frac{(P_1+P_2+P_3)L_1}{AE} + \frac{(P_2+P_3)L_2}{AE} + \frac{P_3L_3}{AE}$$

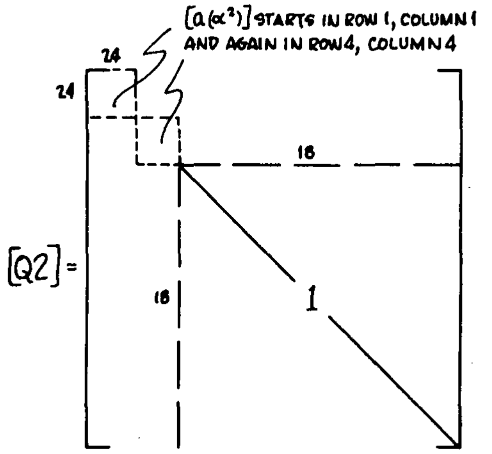
$$\text{POINT } D^{12}: \frac{(P_2+P_3)L_3}{AE} + \frac{P_3L_3}{AE}$$

$$\text{POINT } D^{13}: \frac{P_3L_3}{AE}$$

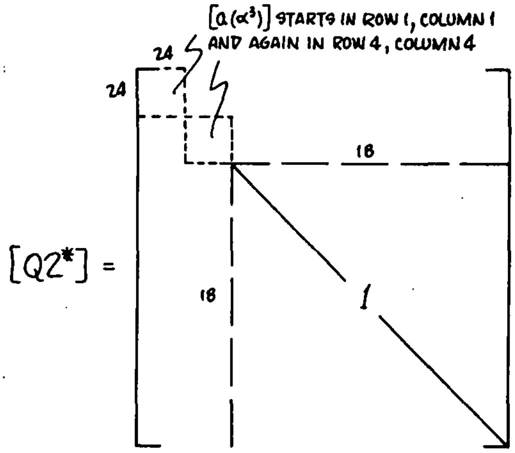
Stiffness of Entire Rotor

As all arms in a rotor are identical, matrices [K5A] and [K5B] can represent the stiffness of any arm in local (not global) coordinates; i.e., in coordinates aligned with that arm. In order to assemble the rotor, the displacement at point B (which is common to all arms) must be transformed to a common coordinate system; let the displacements at B be measured in $\underset{\sim}{x}^1$ coordinates. The stiffness matrix of the first arm is already in $\underset{\sim}{x}^1$ coordinates and, therefore, requires no transformation. The second and third arm stiffness matrices will be transformed using the following matrices:

Second arm:



Third arm (if present):



$$[K6A] = [Q2]^T[K5A][Q2]$$

$$[K6B] = [Q2]^T[K5B][Q2]$$

$$[K6A^*] = [Q2^*]^T[K5A][Q2^*]$$

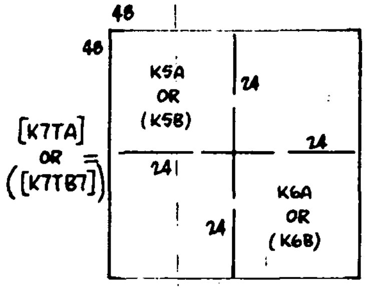
$$[K6B^*] = [Q2^*]^T[K5B][Q2^*]$$

Assemble Rotor

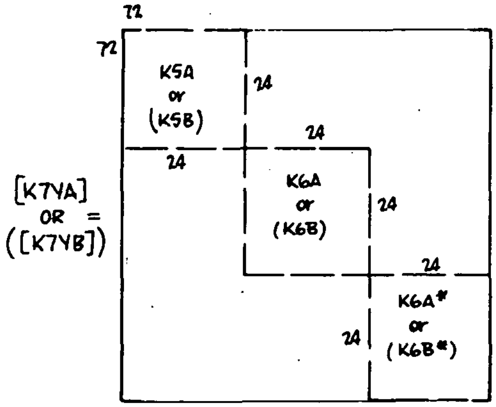
Next the stiffness matrix of the entire rotor is formed by noting that the displacements at point B are common to all arms. This is accomplished by carrying out the following steps:

The previously obtained arm stiffness matrices are first stacked.

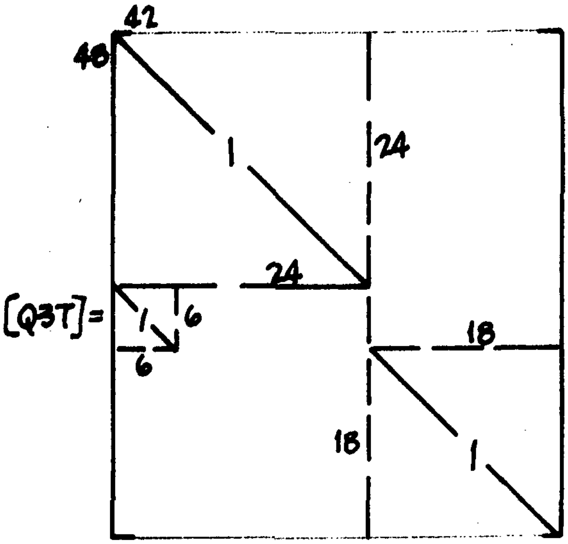
T Rotor

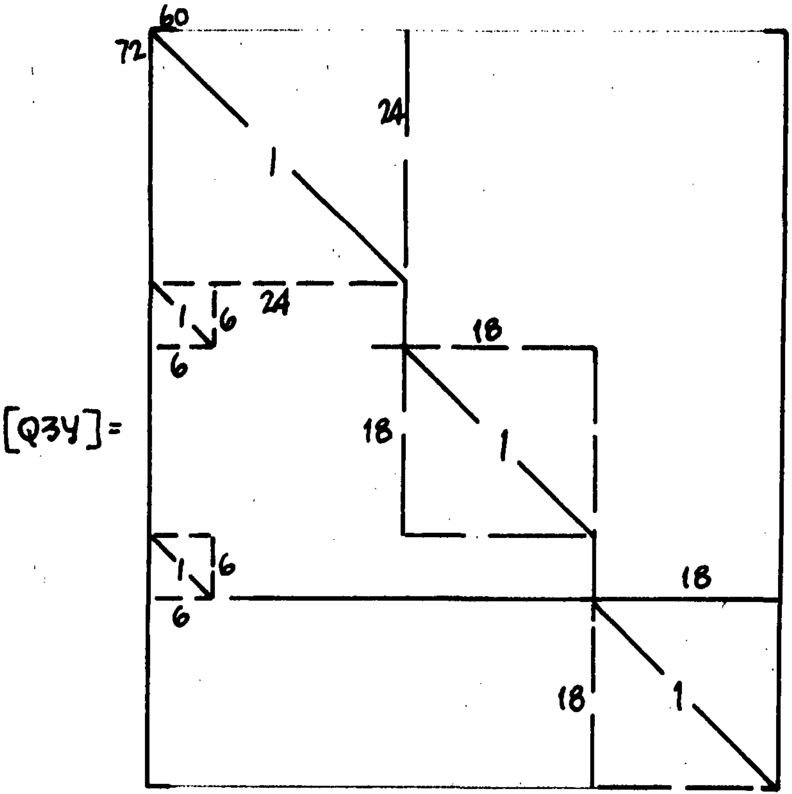


Y Rotor



Next the following matrices are formed to superimpose the displacements at point B.





The matrices are now transformed as follows:

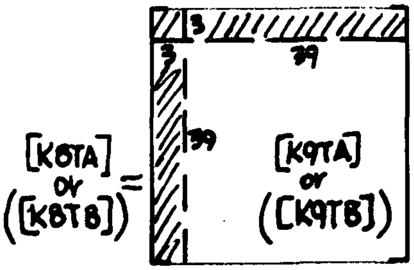
$$[K8TA] = [Q3T]^T [K7TA][Q3T]$$

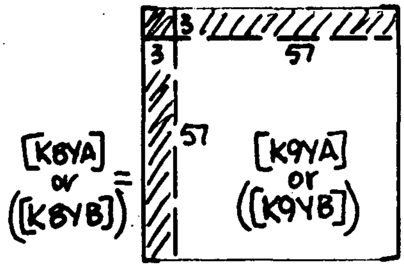
$$[K8TB] = [Q3T]^T [K7TB][Q3T]$$

$$[K8YA] = [Q3Y]^T [K7YA][Q3Y]$$

$$[K8YB] = [Q3Y]^T [K7YB][Q3Y]$$

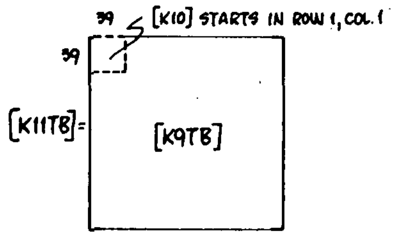
Next the rotor is pinned at point B by deleting the first three rows and columns.

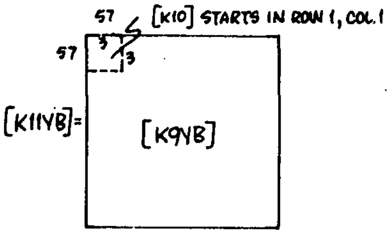




Except for a transformation to relative coordinates, stiffness matrices [K9 TA] and [K9 YA] are the rotor stiffness matrices for the time-history simulation. Whereas the bearing spring already appears in the time-history equations of Section 6.3, it must now be incorporated into the eigenvalue problem. This is accomplished as follows:

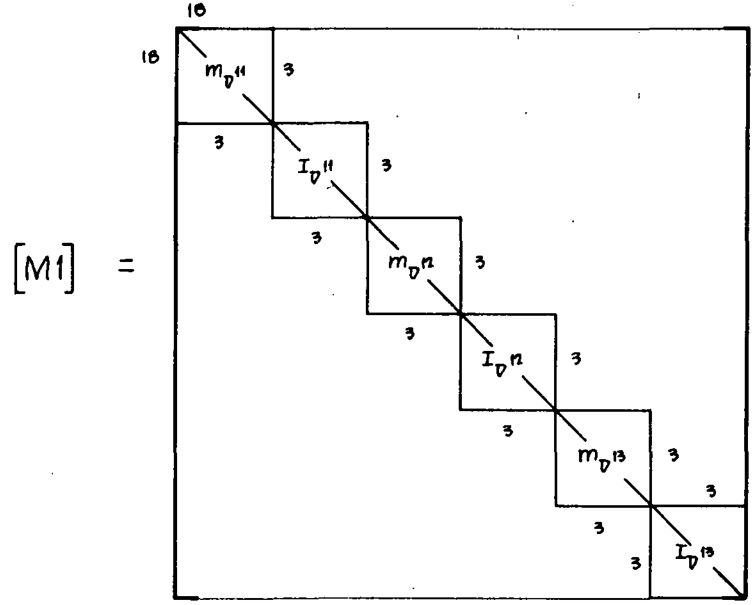
$$[K10] = K_{891}\begin{bmatrix}0 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}$$



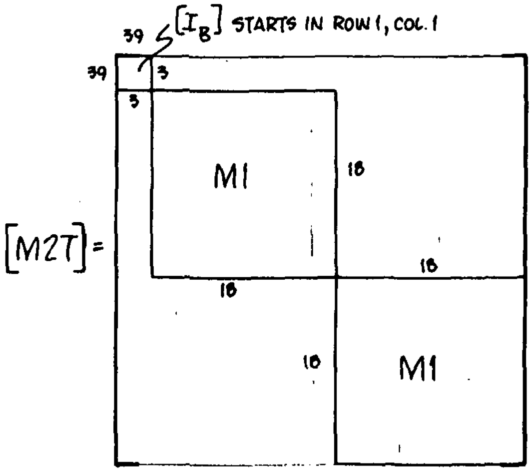


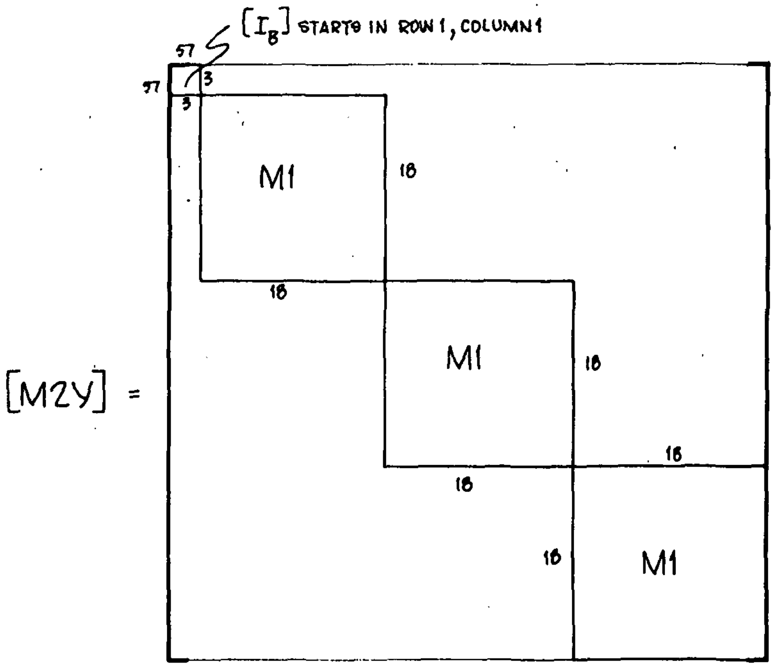
Mass Matrix

The mass matrix for one rotor arm is as follows:



In order to obtain the mass matrix for the entire rotor the matrices for each arm and for the rotor ring are stacked as follows:





Rotor Eigenvalue Problem

The eigenvalue problem for each simulation vehicle is formulated in this section.

The equations of motion follow:

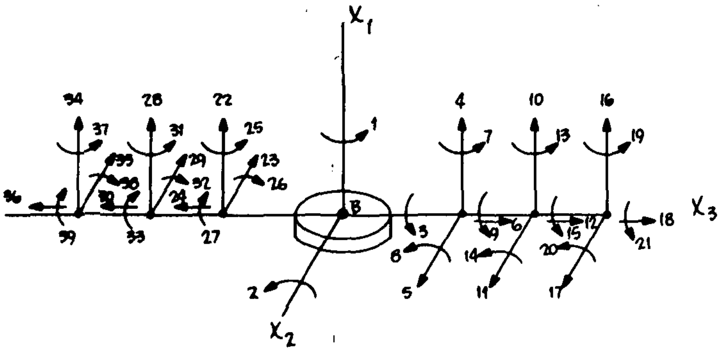
T Rotor

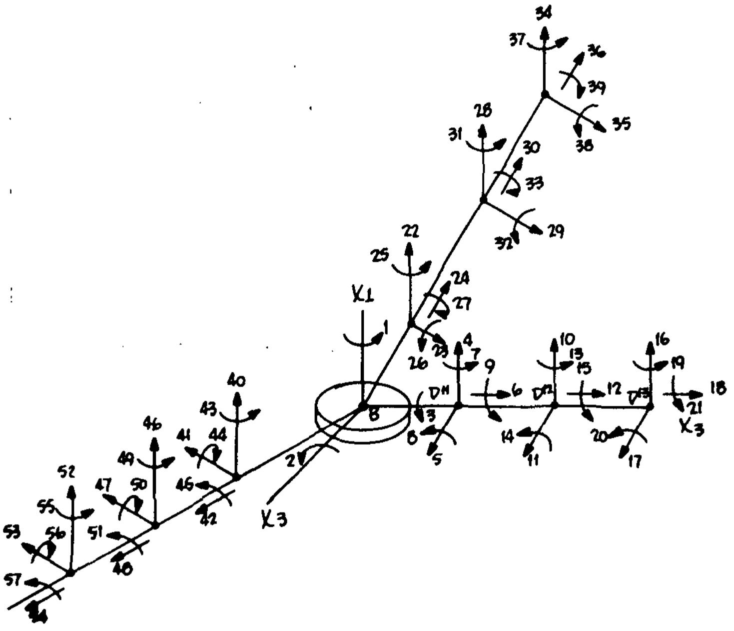
$$[M2T]\{\ddot{q}_T\}+[K11TB]\{q_T\}=0$$

Y Rotor

$$[M2Y]\{\ddot{q}_Y\}+[K11YB]\{q_Y\}=0$$

The displacements $\{q_T\}$ and $\{q_Y\}$ are shown below:





T Rotor

Symmetric motion (with respect to the X_1-X_2 plane) is characterized by:

ROTOR RING

$q_1 = 0$
$q_2 = 0$
$q_3 = q_3$

ARMS

$q_4 = q_{22}$
$q_5 = -q_{23}$
$q_6 = q_{24}$
$q_7 = -q_{25}$
$q_8 = q_{26}$
$q_9 = -q_{27}$
$q_{10} = q_{28}$
$q_{11} = -q_{29}$
$q_{12} = q_{30}$
$q_{13} = -q_{31}$
$q_{14} = q_{32}$
$q_{15} = -q_{33}$
$q_{16} = q_{34}$
$q_{17} = -q_{35}$
$q_{18} = q_{36}$
$q_{19} = -q_{37}$
$q_{20} = q_{38}$
$q_{21} = -q_{39}$

19 DEGREES OF FREEDOM

Antisymmetric motion (with respect to the X_1-X_2 plane) is characterized by:

ROTOR RING

$q_1 = q_1$
$q_2 = q_2$
$q_3 = 0$

ARMS

$q_4 = -q_{22}$
$q_5 = q_{23}$
$q_6 = -q_{24}$
$q_7 = q_{25}$
$q_8 = -q_{26}$
$q_9 = q_{27}$

$q_{10} = -q_{28}$
$q_{11} = q_{29}$
$q_{12} = -q_{30}$
$q_{13} = q_{31}$
$q_{14} = -q_{32}$
$q_{16} = q_{33}$

$q_{16} = -q_{34}$
$q_{17} = q_{35}$
$q_{18} = -q_{36}$
$q_{19} = q_{37}$
$q_{20} = -q_{38}$
$q_{21} = q_{39}$

20 DEGREES OF FREEDOM

Y Rotor

Symmetric motion (with respect to the X_1-X_3 plane) is characterized by:

ROTOR RING

$q_1 = 0$
$q_2 = q_2$
$q_3 = 0$

FIRST ARM	SECOND AND THIRD ARMS
$q_4 = q_4$	$q_{22} = q_{40}$
$q_5 = 0$	$q_{23} = -q_{41}$
$q_6 = q_6$	$q_{24} = q_{42}$
$q_7 = 0$	$q_{25} = -q_{43}$
$q_8 = q_8$	$q_{26} = q_{44}$
$q_9 = 0$	$q_{27} = -q_{45}$
$q_{10} = q_{10}$	$q_{28} = q_{46}$
$q_{11} = 0$	$q_{29} = -q_{47}$
$q_{12} = q_{12}$	$q_{30} = q_{48}$
$q_{13} = 0$	$q_{31} = -q_{49}$
$q_{14} = q_{14}$	$q_{32} = q_{50}$
$q_{15} = 0$	$q_{33} = -q_{51}$
$q_{16} = q_{16}$	$q_{34} = q_{52}$
$q_{17} = 0$	$q_{35} = -q_{53}$
$q_{18} = q_{18}$	$q_{36} = q_{54}$
$q_{19} = 0$	$q_{37} = -q_{55}$
$q_{20} = q_{20}$	$q_{38} = q_{56}$
$q_{21} = 0$	$q_{39} = -q_{57}$

28 DEGREES OF FREEDOM

Antisymmetric motion (with respect to the X_1-X_3 plane) is characterized by:

ROTOR RING

$q_1 = q_1$
$q_2 = 0$
$q_3 = q_3$

ARM 1

$q_4 = 0$
$q_5 = q_5$
$q_6 = 0$
$q_7 = q_7$
$q_8 = 0$
$q_9 = q_9$

$q_{10} = 0$
$q_{11} = q_{11}$
$q_{12} = 0$
$q_{13} = q_{13}$
$q_{14} = 0$
$q_{16} = q_{15}$

$q_{16} = 0$
$q_{17} = q_{17}$
$q_{18} = 0$
$q_{19} = q_{19}$
$q_{20} = 0$
$q_{21} = q_{21}$

ARMS 2 & 3

$q_{22} = -q_{40}$
$q_{23} = q_{41}$
$q_{24} = -q_{42}$
$q_{25} = q_{43}$
$q_{26} = -q_{44}$
$q_{27} = q_{45}$

$q_{28} = -q_{46}$
$q_{29} = q_{47}$
$q_{30} = -q_{48}$
$q_{31} = q_{49}$
$q_{32} = -q_{50}$
$q_{33} = q_{51}$

$q_{34} = -q_{52}$
$q_{35} = q_{53}$
$q_{36} = -q_{54}$
$q_{37} = q_{55}$
$q_{38} = -q_{56}$
$q_{39} = q_{57}$

29 DEGREES OF FREEDOM

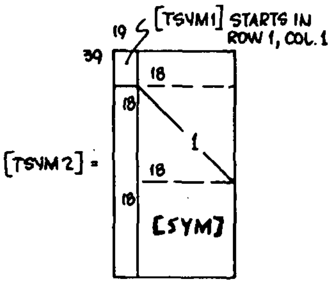
The matrices used to convert the basic equations into symmetric and anti-symmetric motion (viz [TSYM2], [YSYM2], [TANT2] and [YANT2]) are formed below:

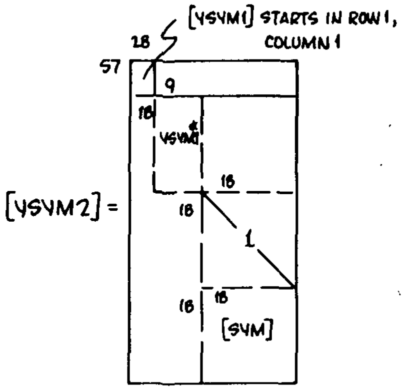
$$[\text{TSYM1}] = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$[\text{YSYM1}] = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$[\text{YSYM1}^*] \quad \begin{array}{cc} & 9 \\ 18 & \begin{bmatrix}
1 & & & & & & & & \\
0 & & & & & & & & \\
 & 1 & & & & & & & \\
 & 0 & & & & & & & \\
 & & 1 & & & & & & \\
 & & 0 & & & & & & \\
 & & & 1 & & & & & \\
 & & & 0 & & & & & \\
 & & & & 1 & & & & \\
 & & & & 0 & & & & \\
 & & & & & 1 & & & \\
 & & & & & 0 & & & \\
 & & & & & & 1 & & \\
 & & & & & & 0 & & \\
 & & & & & & & 1 & \\
 & & & & & & & 0 & \\
 & & & & & & & & 1 \\
 & & & & & & & & 0
\end{bmatrix} \end{array}$$

$$[\text{SYM}] \quad \begin{array}{cc} & 18 \\ 18 & \begin{bmatrix}
1 & & & & & & & & & & & & & & & & & \\
 & -1 & & & & & & & & & & & & & & & & \\
 & & 1 & & & & & & & & & & & & & & & \\
 & & & -1 & & & & & & & & & & & & & & \\
 & & & & 1 & & & & & & & & & & & & & \\
 & & & & & -1 & & & & & & & & & & & & \\
 & & & & & & 1 & & & & & & & & & & & \\
 & & & & & & & -1 & & & & & & & & & & \\
 & & & & & & & & 1 & & & & & & & & & \\
 & & & & & & & & & -1 & & & & & & & & \\
 & & & & & & & & & & 1 & & & & & & & \\
 & & & & & & & & & & & -1 & & & & & & \\
 & & & & & & & & & & & & 1 & & & & & \\
 & & & & & & & & & & & & & -1 & & & & \\
 & & & & & & & & & & & & & & 1 & & & \\
 & & & & & & & & & & & & & & & -1 & & \\
 & & & & & & & & & & & & & & & & 1 & \\
 & & & & & & & & & & & & & & & & & -1
\end{bmatrix} \end{array}$$



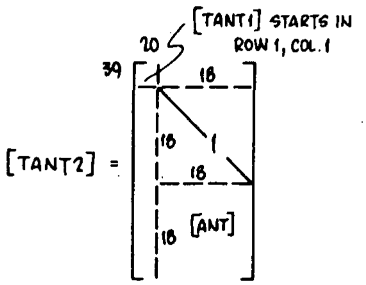


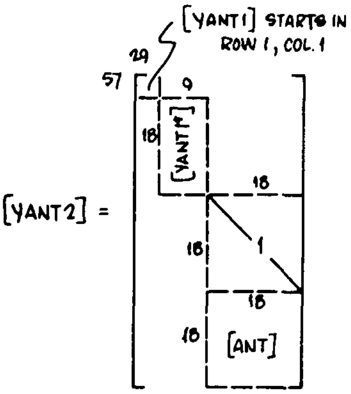
$$[TANT1] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$[YANT1] = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$$

$$[YANT1^{*}] = {}_{18}\overset{9}{\begin{bmatrix} 0 & & & & & & & & \\ 1 & & & & & & & & \\ & 0 & & & & & & & \\ & 1 & & & & & & & \\ & & 0 & & & & & & \\ & & 1 & & & & & & \\ & & & 0 & & & & & \\ & & & 1 & & & & & \\ & & & & 0 & & & & \\ & & & & 1 & & & & \\ & & & & & 0 & & & \\ & & & & & 1 & & & \\ & & & & & & 0 & & \\ & & & & & & 1 & & \\ & & & & & & & 0 & \\ & & & & & & & 1 & \\ & & & & & & & & 0 \\ & & & & & & & & 1 \end{bmatrix}}$$

$$[ANT] = -[SYM]$$





The symmetric and antisymmetric eigenvalue problem are now set up as indicated below:

${}^{19}\overset{19}{[\mathrm{MTSYM}]} = [\mathrm{TSYM2}]^T[\mathrm{M2T}][\mathrm{TSYM2}]$ — SYMMETRIC MASS — ${}^{28}\overset{28}{[\mathrm{MYSYM}]} = [\mathrm{YSYM2}]^T[\mathrm{M2Y}][\mathrm{YSYM2}]$

${}^{19}\overset{19}{[\mathrm{KTSYM}]} = [\mathrm{TSYM2}]^T[\mathrm{K11TB}][\mathrm{TSYM2}]$ — SYMMETRIC STIFFNESS — ${}^{28}\overset{28}{[\mathrm{KYSYM}]} = [\mathrm{YSYM2}]^T[\mathrm{K11YB}][\mathrm{YSYM2}]$

$[\mathrm{MTSYM}]\{\ddot{q}_T\}_S + [\mathrm{KTSYM}]\{q_T\} = 0$ — BASIC SYMMETRIC EQUATION — $[\mathrm{MYSYM}]\{\ddot{q}_Y\}_S + [\mathrm{KYSYM}]\{q_Y\}_S = 0$

${}^{20}\overset{20}{[\mathrm{MTANT}]} = [\mathrm{TANT2}]^T[\mathrm{M2T}][\mathrm{TANT2}]$ — ANTISYMMETRIC MASS — ${}^{29}\overset{29}{[\mathrm{MYANT}]} = [\mathrm{YANT2}]^T[\mathrm{M2Y}][\mathrm{YANT2}]$

${}^{20}\overset{20}{[\mathrm{KTANT}]} = [\mathrm{TANT2}]^T[\mathrm{K11TB}][\mathrm{TANT2}]$ — ANTISYMMETRIC STIFFNESS — ${}^{29}\overset{29}{[\mathrm{KYANT}]} = [\mathrm{YANT2}]^T[\mathrm{K11YB}][\mathrm{YANT2}]$

$[\mathrm{MTANT}]\{\ddot{q}_T\}_A + [\mathrm{KTANT}]\{q_T\}_A = 0$ — BASIC ANTISYMMETRIC EQUATION — $[\mathrm{MYANT}]\{\ddot{q}_Y\}_A + [\mathrm{KYANT}]\{q_Y\}_A = 0$

In order to obtain the mode shapes [ø] and frequencies ω, the above eigenvalue problems must be solved. The results are denoted as follows:

SYMMETRIC MODES: ${}^{19}\overset{19}{[\mathrm{\emptyset TSYM1}]}$ SYMMETRIC MODES: ${}^{28}\overset{28}{[\mathrm{\emptyset YSYM1}]}$

ANTISYMMETRIC MODES: ${}^{20}\overset{20}{[\mathrm{\emptyset TANT1}]}$ ANTISYMMETRIC MODES: ${}^{29}\overset{29}{[\mathrm{\emptyset YANT1}]}$

Next the modal vectors are expanded to full size

${}^{39}\overset{19}{[\mathrm{\emptyset TSYM2}]}\ {}^{39}\overset{19}{[\mathrm{TSYM2}]}\ {}^{19}\overset{19}{[\mathrm{\emptyset TSYM1}]}$ ${}^{57}\overset{28}{[\mathrm{\emptyset YSYM2}]} = [\mathrm{YSYM2}][\mathrm{\emptyset YSYM1}]$

${}^{39}\overset{20}{[\mathrm{\emptyset TANT2}]} = {}^{39}\overset{20}{[\mathrm{TANT2}]}\ {}^{20}\overset{10}{[\mathrm{\emptyset TANT1}]}$ ${}^{57}\overset{29}{[\mathrm{\emptyset YANT2}]} = [\mathrm{YANT2}][\mathrm{\emptyset YANT1}]$

The modal matrices are now partitioned into modes to be retained and modes to be ignored in the modal reduction procedure.

$$[\phi TSYM2] = [\overset{(retain)}{\phi TSYM3} \,\vdots\, \overset{(ignore)}{\phi TSYM4}] \qquad [\phi YSYM2] = [\overset{(retain)}{\phi YSYM3} \,\vdots\, \overset{(ignore)}{\phi YSYM4}]$$

$$[\phi TANT2] = [\overset{(retain)}{\phi TANT3} \,\vdots\, \overset{(ignore)}{\phi TANT4}] \qquad [\phi YANT2] = [\overset{(retain)}{\phi YANT3} \,\vdots\, \overset{(ignore)}{\phi YANT4}]$$

The symmetric and antisymmetric modal matrices of the n_t modes to be retained are:

The symmetric and antisymmetric modal matrices of the n_y modes to be returned are:

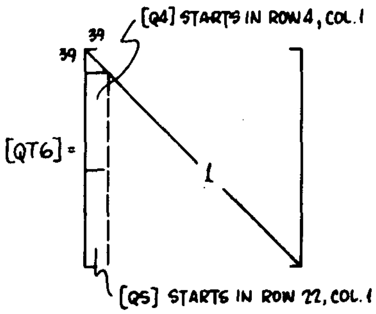
$${}^{39}\overset{n_t}{[\phi T]} = [\phi TSYM3 \,\vdots\, \phi TANT3] \qquad {}^{57}\overset{n_y}{[\phi Y]} = [\phi YSYM3 \,\vdots\, \phi YANT3]$$

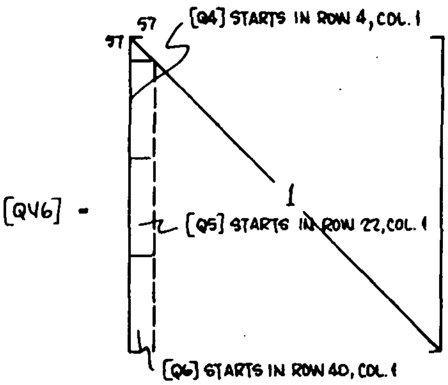
The above mode shapes are expressed in local absolute coordinates, whereas the equations of motion in the time history program are in local relative coordinates. The mode shapes can be converted into relative coordinates by use of transformation matrices [QT6] and [QY6] defined as follows:

$$[Q4] = {}^{18}\overset{3}{\left[\begin{array}{c} [r(x_{p11})] \\ \\ \hline \begin{matrix} -1 & & \\ & -1 & \\ & & -1 \end{matrix} \\ \hline [r(x_{p12})] \\ \\ \hline \begin{matrix} -1 & & \\ & -1 & \\ & & -1 \end{matrix} \\ \hline [r(x_{p13})] \\ \\ \hline \begin{matrix} -1 & & \\ & -1 & \\ & & -1 \end{matrix} \end{array}\right]}$$

$$[Q5] = [Q4][a(\alpha^2)]$$

$$[Q5^*] = [Q4][a(\alpha^3)]$$

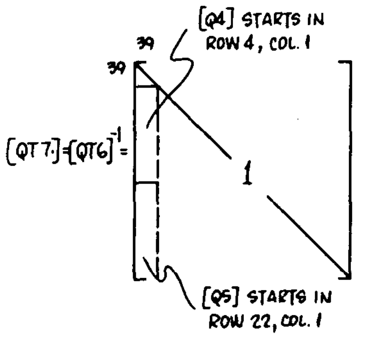




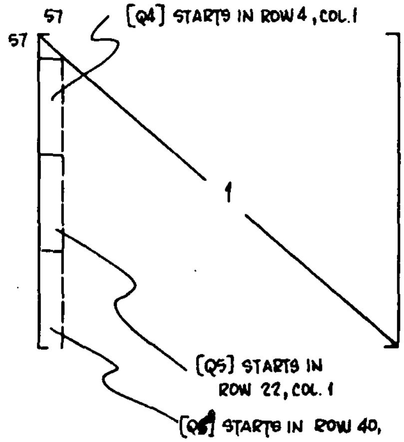
Next the modal matrices are transformed to relative coordinates:

$$\overset{n_t}{{}_{39}[\phi TR]} = [QT6][\phi T] \qquad \overset{n_y}{{}_{57}[\phi VR]} = [QY6][\phi Y]$$

The stiffness matrices needed for the time history program must also be converted into relative coordinates.



(ALTERNATIVELY, $[QT7] = 2[I] - [QT6]$)



(ALTERNATIVELY, $[QY7] = 2[I] - [QY6]$)

$$[KTA] = [K9TA][QT7] \qquad [KYA] = [K9YA][QY7]$$

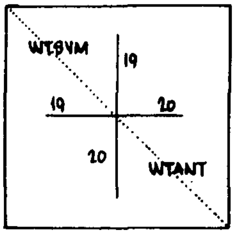
Rotor Damping

It was decided to use modal damping to represent energy dissipation in the structure. Since inversion of the square modal matrix is used in manipulations below. all modes are now assembled.

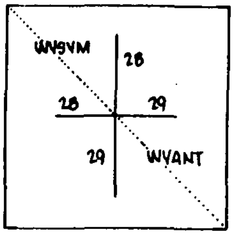
$$ {}^{39}\overset{39}{[\phi TALL]} = [\,\phi TSYM2 \mid \phi TANT2\,] \qquad {}^{57}\overset{57}{[\phi YALL]} = [\,\phi YSYM2 \mid \phi YANT2\,] $$

The following frequency matrices will also be required.

$[WT] =$



$[WY] =$



The following physical damping matrix is based upon a fraction, γ, of critical modal damping:

$$[CT] = 2\gamma[M2T][\phi TALL][\omega T][\phi TALL]^{-1} \qquad [CY] = 2\gamma[M2Y][\phi YALL][\omega Y][\phi YALL]^{-1}$$

Recalling that the bearing springs were incorporated into the stiffness matrices used for the modal analyses, it follows that the physical damping effects, as given by [CT] and [CY], will be transmitted to the hub. This unrealistic effect is deleted as follows:

$$[CT^*] = [QT7]^T[CT][QT7] \qquad [CY^*] = [QY7]^T[CY][QY7]$$

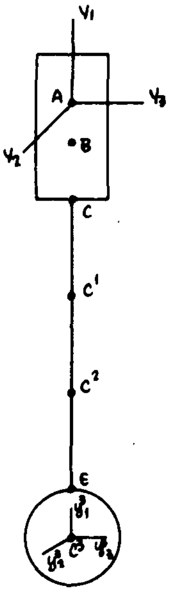
The first 3 rows and columns of $[CT^*]$ are deleted, and the result is called $[CT^{**}]$. The final damping matrix is:

$$[CTA] = [QT7]^{-T}[CT^{**}] = [QT6]^T[CT^{**}]$$

The first 3 rows and columns of $[CY^*]$ are deleted and the result is called $[CY^{**}]$. The final damping matrix is:

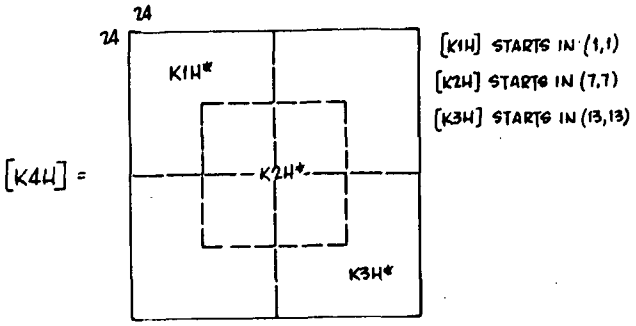
$$[CYA] = [QY7]^{-T}[CY^{**}] = [QY6]^T[CY^{**}]$$

II. Hub

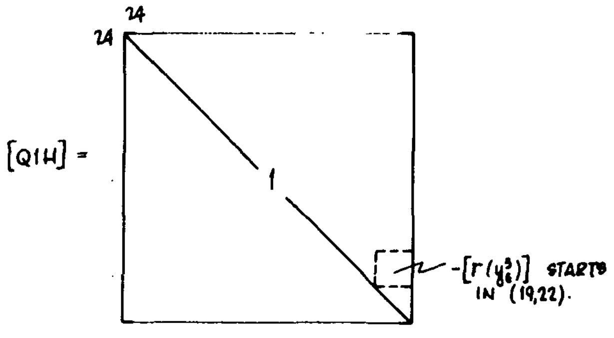


The basic beam stiffness matrix, $[K^*]$, presented in the rotor section is used to compute $[K1H^*]$, $[K2H^*]$ and $[K3H^*]$ for hub elements 1, 2 and 3, respectively. Note that the coordinate systems are not the same; that is, the Y_1 axis is the torsional axis for the hub, whereas the X_3 axis was the torsional axis in the rotor arm.

Next the stiffness elements are stacked to form the stiffness matrix of the total hub.

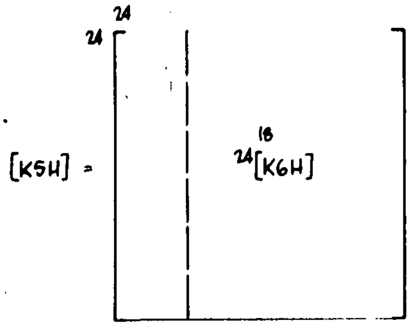


Matrix [K4H] operates on displacements at points G, C^1, C^2 and E. The transformation matrix [Q1H] is now formed to convert to displacements of G, C^1, C^2 and C^3; call result [K5H].

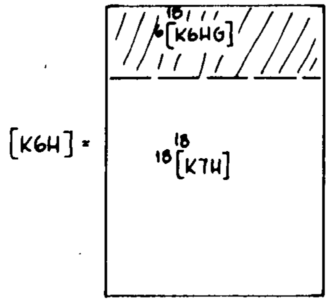


$$[K5H] = [Q1H]^T[K4H][Q1H]$$

To cantlever the hub at point G, the first six columns of [K5H] are deleted below. The result is called [K6H].

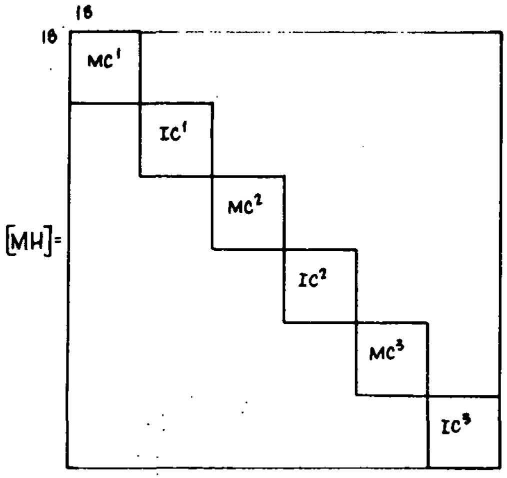


[K6H] is partitioned into [K6HG] and[K7H] as shown below:



[K7H] is the stiffness matrix for the flexible section of hub. [K6HG] can be used to obtain the elastic loads transmitted to the rigid section of hub at point G.

The hub mass matrix is now stacked as follows:



[MH] and [K7H] are used to formulate the eigenvalue problem, i.e.,

$$[MH]\{\ddot{q}_H\} + [K7H]\{q_H\} = 0$$

The solution of the above problem provides the modal matrix $[\phi H^{*}]$ and frequencies, ω.

$[\phi H^{*}]$ is partitioned into n_h modes to be retained for modal reduction and modes to be ignored.

$$[\phi H] = [\overset{(retain)}{\phi H} \,\vdots\, \overset{(ignore)}{\phi H^{**}}]$$

Hub Damping

As was the case for the rotor, modal damping is used in the hub. The complete modal matrix $[\phi H^{*}]$ as well as the following frequency matrix are required to obtain the hub damping matrix:

$$[WH] = \begin{array}{c|c} & 18 \\ \hline 18 & \begin{bmatrix} w_{H1} & & & \\ & w_{H2} & & \\ & & \ddots & \\ & & & w_{H18} \end{bmatrix} \end{array}$$

Finally the physical damping matrix is computed based upon a fraction, δ, of critical modal damping. The damping matrix for the flexible section of hub is:

$${}_{18}^{\;\;18}[CH] = 2\delta[MH][\phi H^{*}][W_H][\phi H^{*}]^{-1}$$

and the damping forces transmitted to the rigid section of hub can be computed using [CHG].

$${}_{6}^{\;\;18}[CHG] = [K6HG][K7H]^{-1}[CH]$$

Combined Modal Matrix - Total Vehicle

The order of the displacements in the mode shapes must be made compatible with the order of the variables in the time history equations (Equations 6-43). The following matrix is used to reorder these displacements:

$$[Q8] = {}^{18}\overset{18}{\begin{bmatrix} I3 & & & & & \\ & & I3 & & & \\ & & & & I3 & \\ & I3 & & & & \\ & & & I3 & & \\ & & & & & I3 \end{bmatrix}} \qquad \text{WHERE } [I3] = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

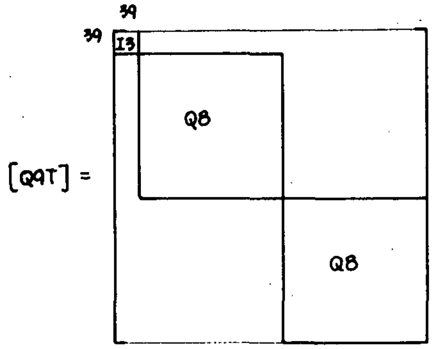
The hub modes, as required in the time history simulation, are

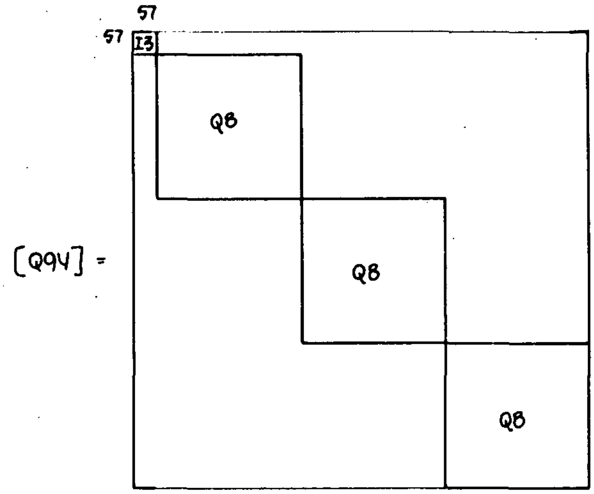
$${}^{18}\overset{n_h}{[\phi HF]} = {}^{18}\overset{18}{[Q8]}[\phi H]$$

similarly for rotor modes,

For T Rotor Modes

For Y Rotor Modes





The rotor modes, as required in the time history simulation, are

$${}^{39}\overset{n_t}{[\phi TRF]} = [Q9T][\phi TR] \qquad {}^{57}\overset{n_y}{[\phi YRF]} = [Q9Y][\phi YR]$$

Assemble Total Reduction Matrix For Total Vehicle

In the time-history simulation, the hybrid coordinates used for numerical integration are the physical coordinates for the rigid section of hub $\{Z_A\}$ and $\{W^Y\}$, the two bending rotation of the shuttle Ψ_1 and Ψ_2, the n_h hub modal coordinates, and the n_T or n_Y rotor modal coordinates. In order to rigidize the shuttle torsional coordinate Ψ_3, the following constraint matrix is formed:

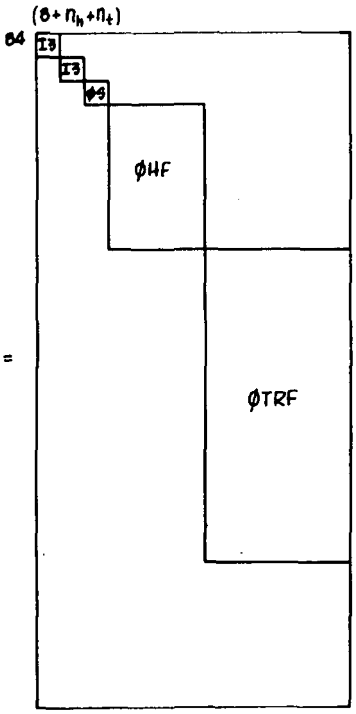
$$[\phi S] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

Then, the total matrix used to reduce the number of coordinates for numerical integration is:

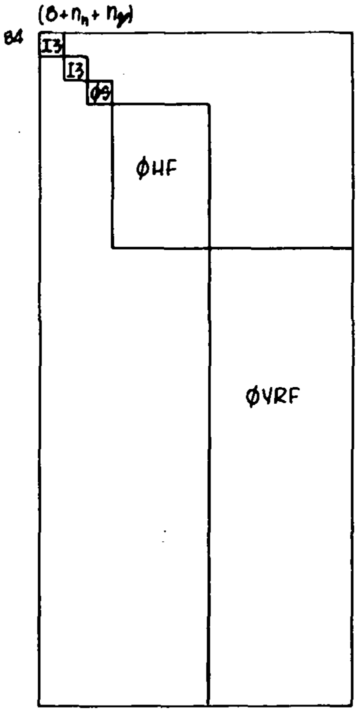
For T Configuration

For Y Configuration

$[\phi] =$



or $[\phi] =$



APPENDIX B

FREQUENCY SHIFT TECHNIQUE

The fundamental vibration equation may be written as

$$\omega_i^2 [M] \{\phi_i\} = [K] \{\phi_i\} \tag{B-1}$$

where ω_i is the frequency and $\{\phi_i\}$ is the associated mode shape. Since the eigenvalue algorithm used in this study cannot treat a zero frequency, this frequency was removed from the equations in the following manner. Let $\bar{\omega}_i$ be a new frequency which is related to ω_i by the small constant shift frequency ω_o by

$$\bar{\omega}_i^2 = \omega_i^2 + \omega_o^2 \tag{B-2}$$

The fundamental equation (B-1) may then be written as

$$\bar{\omega}_i^2 [M] \{\phi_i\} = ([K] + \omega_o^2 [M]) \{\phi_i\} \tag{B-3}$$

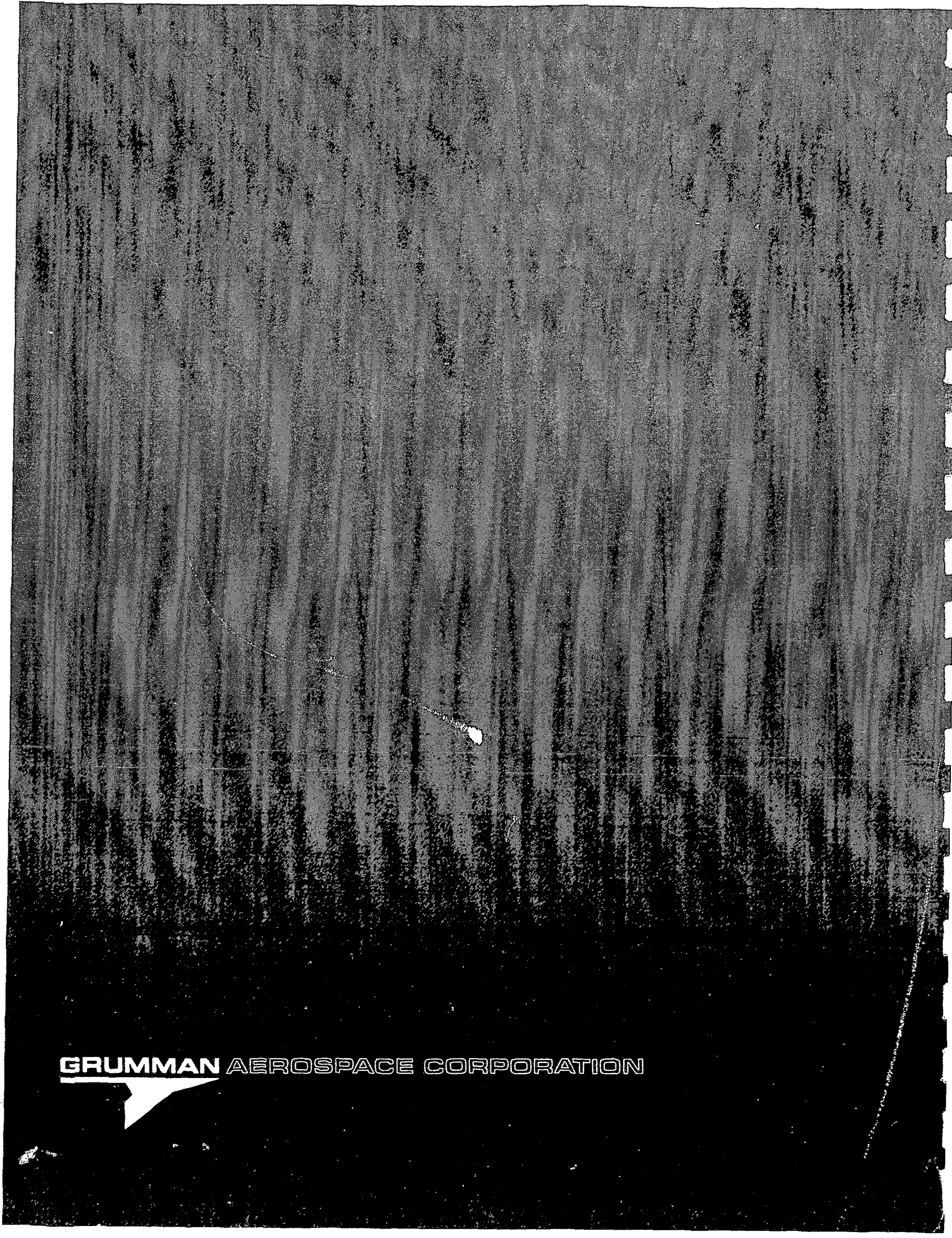
or

$$\bar{\omega}_i^2 [M] \{\phi_i\} = [\bar{K}] \{\phi_i\} \tag{B-4}$$

where the new "stiffness" matrix $[\bar{K}]$ is given by

$$[\bar{K}] = [K] + \omega_o^2 [M] \tag{B-5}$$

Note that Equation (B-4) is the same form as (B-1) and also that the mode shapes $\{\phi_i\}$ have not been effected. Accordingly, the eigenvalue algorithm may now be applied to Equation (B-4), yielding the mode shapes $\{\phi_i\}$ and shifted frequencies $\bar{\omega}_i$; the original frequencies may then be recovered by using Equation (B-2).



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