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A COMPUTER PROGRAM FOR THE CALCULATION OF THERMAL STRATIFICATION AND SELF-PRESSURIZATION IN A LIQUID HYDROGEN TANK

by R. W. Arnett and R. O. Voth

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16. Abstract

This report describes an analysis and computer program used to calculate the thermal stratification and the associated self-pressurization of a closed liquid hydrogen tank. A sample calculation is provided as well as a description and a listing of the program. Fortran-IV language is used and runs have been made on IBM 360/65 and CDC 3600 computers. Comparisons are made between the program calculations and test results from both ground and orbital coast tests of a Centaur space vehicle.

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A COMPUTER PROGRAM FOR THE CALCULATION OF THERMAL STRATIFICATION AND SELF-PRESSURIZATION IN A LIQUID HYDROGEN TANK

R. W. Arnett and R. O. Voth

1.0 Summary

This report presents a computer program developed for calculating thermal stratification and the associated self-pressurization of a closed liquid hydrogen tank. The computer program is written in Fortran IV language and has been run on both a Control Data Corporation 3600 computer and an International Business Machines 360/65 computer.

This report also presents the mathematical development of the computer program. The boundary layer equations required for the mathematical development used the classical approach for solution of the turbulent boundary layer equations, except that no assumptions were made as to the form of the thickness and velocity parameter solutions. The resulting mathematical expressions require symmetry about the vertical axis but permit axial variations of geometry and heat flux.

Comparison of the pressure rise data from both a ground test and orbital coast data is provided with the tank pressure calculation and test data agreeing within 4% or better.

Parametric data for a ground test conducted by the NASA-Lewis Research Center are used for a sample calculation with the input format and a typical program printout used to illustrate the use of the program. The program incorporates subprograms containing thermodynamic and physical properties of parahydrogen. These subprograms can be replaced by subprograms incorporating the properties of other fluids as

desired. A complete listing of the computer program is included as an appendix to the report.

2.0 Introduction

During the launch sequence or space coast of a chemical or nuclear rocket using liquid hydrogen as a propellant, it becomes necessary, at some time prior to engine ignition, to suspend venting of the hydrogen tank. This cessation of venting may be required as a safety measure and may also be necessary from an operational standpoint. Duration of this 'locked up' condition may persist for several minutes.

During the time interval that the tank is 'locked up' heat transfer through the tank walls results in energy increase in both the liquid and the ullage space vapor. Experience has shown that this energy increase is not uniformly distributed in either the liquid or vapor, but rather that higher temperatures occur in the topmost layers of both the liquid and vapor regions. This phenomenon is described as thermal stratification and has been reported by numerous observers[1, 2, 3, 4].

Associated with this temperature stratification is a rise in tank pressure due to a combination of heat transferred to the ullage vapor plus possible vaporization of a portion of the liquid. A means for predicting the amount of pressure rise as it varies with time and tank wall heat flux is needed in order to properly plan for propellant management, to determine tank structural stresses, to determine tank venting requirements and possibly to foresee the need for additional pressurant gas. The extent of liquid temperature rise, separate from its effect on the tank pressure, is also needed in order to determine the effect on pump performance. Excessive temperatures can result in cavitation due to reduction in NPSH, thus either damaging the pump or adversely affecting the pump performance.

The purpose of this report is to present an analysis and a computer program that can be used to calculate the thermal stratification and self pressurization in a liquid hydrogen tank. The program results compare well with certain space vehicle liquid hydrogen tank self pressurization data obtained from both a space coast and a ground test. The ground test is used here as a sample problem to aid the user in employing the program. The program was written using liquid hydrogen properties but the basic program should also be applicable for other fluids.

The program is written in Fortran IV for an IBM 360/65 series computer. A program listing and a description for each of the program subroutines is provided in this report.

3.0 Previous Studies

Since the early work of Huntley[1], numerous papers (see additional references) relating to stratification phenomena have appeared. Many of these papers were directed at the thermal stratification but did not consider the self-pressurization aspect. Of the reported work, that of the group at Lockheed (LMSC) was the most concerned with pressure rise in addition to the thermal aspects[5,6,7,8,9].

Schmidt, et al.[10] utilized external pressurization in a vessel with negligible sidewall heat leak to measure the liquid thermal gradient due to heat transfer at the liquid surface. This test arrangement resulted in a condensing type of heat and mass transfer at the liquid vapor interface. Good correlation of the resulting temperature distribution throughout the liquid with the error function solution for a semi-infinite solid was established.

Clark[11] reviewed the status of the generalized pressurization and stratification problem. In this review, it was pointed out that numerous assumptions are necessary in order to solve the mathematical expressions involved. The validities of some of the assumptions which are commonly made require evaluation by careful experimentation, as Clark indicated. In addition, he pointed out that the interfacial phenomena associated with mass and energy transfer, while quite complex, can probably be taken to be in thermodynamic equilibrium with little error.

Subsequent to the review of Clark[11], no comprehensive review of the situation has been published. Urquhart[12] made a survey but did not publish a comprehensive review. He concluded that the finite difference approach of Clark and Barakat[13] was thermodynamically excellent but contained restrictive assumptions, treated the laminar case only, and required excessive amounts of computer time. Urquhart further concluded that the work of the LMSC group[5, 6, 8] was representative of the boundary layer approach to stratification. He further suggested certain improvements which he felt would be beneficial.

Urquhart suggested the following improvements to the LMSC solution.

1. Utilization of both laminar and turbulent solutions to the boundary layer flow, splicing these together where the laminar to turbulent transition occurs.

   Present Authors Note: It may be pointed out that for liquid hydrogen (LH_2) in a normal earth gravity field with a relatively low heat flux of 10 mW/cm^2, a turbulent boundary layer regime is attained within 2 to 6 cm from the initiation point of boundary layer growth. Thus, this suggested addition would seem to have marginal benefit.

2. Incorporate transient effects of the establishment of the boundary layer.

Present Authors Note: It appears that, in the majority of cases, the vessel has been exposed to a constant heat flux for some time and the liquid is saturated at its ambient pressure. Stratification then begins at the time the vessel vent is closed; since steady state conditions already exist in the boundary layer,no significant transient effects occur.

3. Consider the possibility of nucleate boiling in the boundary layer.

Present Authors Note: This would seem to be a reasonable suggestion since the temperature rise in the boundary layer is likely to approach or exceed the local saturation temperature, particularly in the upper portion of the vessel. However, such boiling will likely occur only in the upper regions of the tank, thus affecting only a small portion of the boundary layer. Furthermore, since some vaporization is likely to occur anyway, the exact location of such vaporization is probably not of major importance.

4. Use variable fluid properties as the calculations proceed in real time.

Present Authors Note: This seems to have merit and could be considered, although the increased complexity does not seem to be justified at the present time in view of uncertainties in other areas.

In summary, it seems that the work of the LMSC group is representative of the most practical approach to the stratification and pressurization phenomena. The approach described in this report also utilizes the basics of the turbulent boundary layer approximate approach. However, it does not assume the vertical flat plate boundary layer, nor does it assume either a "mixed ullage" or a "saturated vapor" temperature for the ullage space.

The work reported here is an extension and refinement of a paper presented by Arnett and Millhiser in 1965[14]. Important changes from this previous development include the solution for the boundary layer thickness and the velocity parameter without assuming a particular mathematical form; the removal of modifying factors which formerly were used to force continuity of the boundary layer flow; and departure from the previously used method for forcing decay of the boundary layer in the thermally stratified layer.

4.0 Analysis

The development of the equations here will be in functional notation, except for simple relationships. More complete equations are presented in the Appendices.

Eckert and Jackson[15] developed expressions for free convection turbulent boundary layer flow on a heated vertical flat plate in a semi-infinite medium. The method of attack parallels that used by Von Karman[16, 17] for a laminar boundary layer. This same approach is used here, with modifications to adapt it to an inclined wall of finite radius, i.e., a conical shape.

4.1 Boundary Layer Equations

Prediction of thermal stratification and the associated pressure rise is based upon the assumption that heat transferred through the vessel walls is carried toward the upper portions of the vessel via a free convection boundary layer. Heat entering the boundary layer through the liquid wetted portion of the tank thus tends to accumulate in a layer adjoining the liquid surface and is manifested by a rise in temperature of this layer. Evaporation at the liquid-vapor interface usually occurs, depending on the pressure in the ullage space, this being determined in turn by the initial ullage space conditions and the amount of heat subsequently transferred to the ullage. The temperature at the liquid surface determines the tank ullage pressure; if vaporization occurs the liquid surface will be cooled due to the vaporization while if condensation occurs the temperature, and therefore the pressure, will tend to rise.

Three parameters serve to describe the boundary layer flow: (1) The temperature variation across the boundary layer, (2) thickness of the layer, and (3) the velocity distribution across the layer. For this development, assumptions of the form for the dimensionless temperature and velocity variations are the same as those used by Eckert and Jackson[15]. These are

$$\theta/\theta_w = 1 - (y/\delta)^{1/7} , \tag{1}$$

and,

$$u/U = (y/\delta)^{1/7} (1 - y/\delta)^4 . \tag{2}$$

The three parameters required then are θ_w, δ, and U. These are interdependent and are functions of vertical location in the tank, acceleration field, the wall heat flux, and the fluid properties. The manner in which θ/θ_w and u/U vary across the boundary layer, as described by (1) and (2), is shown in figure 1. Figure 2 displays the tank configuration assumed for the mathematical model.

A fluid element of thickness, dx, bounded by two planes normal to the vessel axis and a distance x from the bottom of the tank, is analyzed in order to formulate a set of equations. A section of this annular boundary layer is shown in figure 3.

The change in momentum flow, parallel to the wall, in crossing the element dx is equated to the forces acting, also parallel to the wall, on the element, i. e.,

$$\frac{d}{dx}(\dot{m}u)\,dx = \sum_{x}^{x+dx} F = F_B + F_S. \tag{3}$$

Momentum flow crossing plane "a" is expressed as

$$\sum_{y=o}^{\delta} \dot{m}u = f_u\,(u, y, R, \rho, \gamma, x), \tag{4}$$

and the change in $\dot{m}u$ in the dixtance dx is given by

$$\left[\frac{d}{dx}\sum_{y=o}^{\delta} \dot{m}u\right] dx. \tag{4a}$$

The buovant force in the direction of flow is represented by

$$F_B = f_B\,(y, R, x, \rho, \gamma), \tag{5}$$

while a shear force along the direction of flow is described as

$$F_S = f_S\,(\tau_w, R, \gamma, x). \tag{6}$$

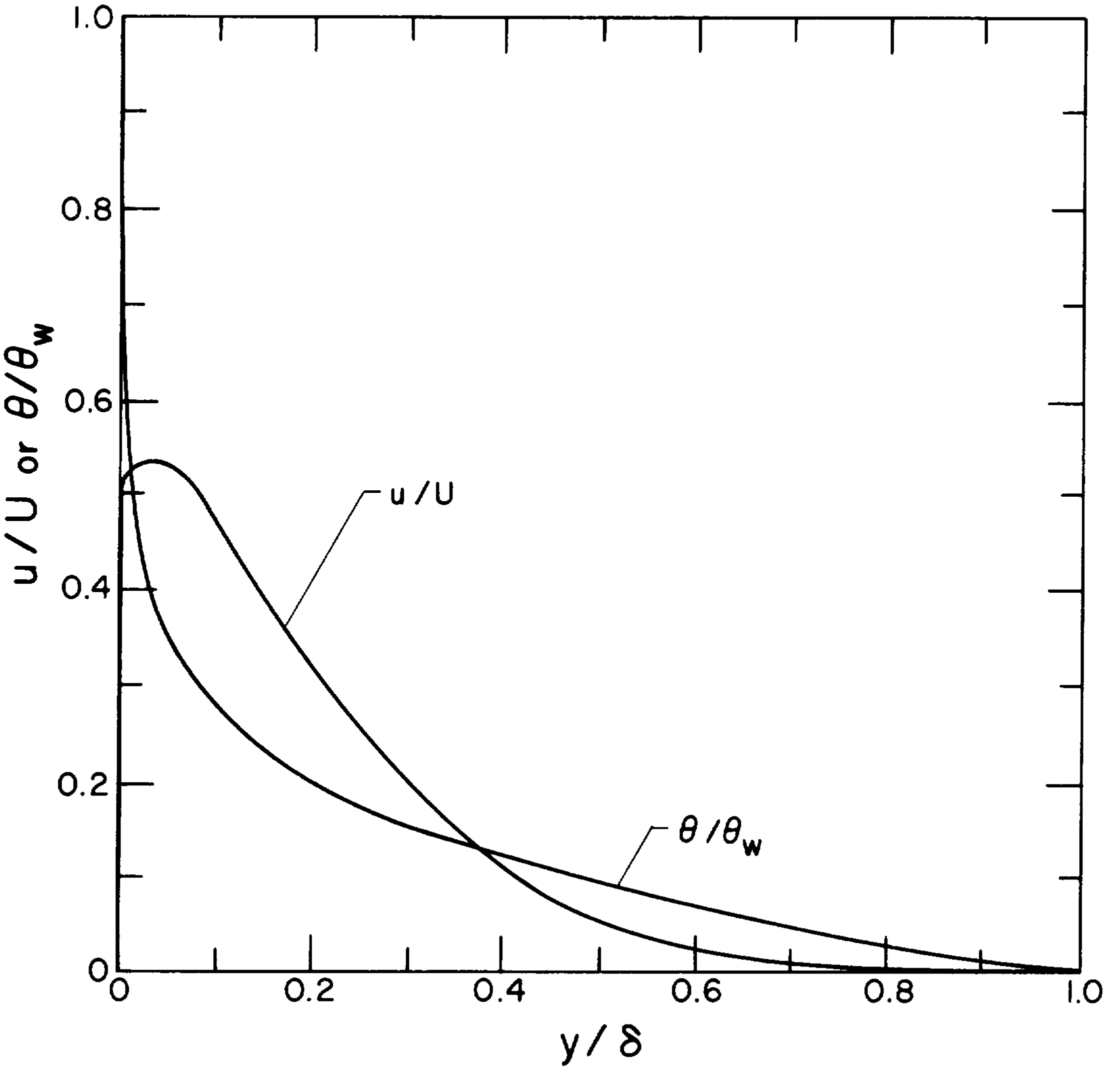

Figure 1 Assumed velocity and temperature variation across the boundary layer.

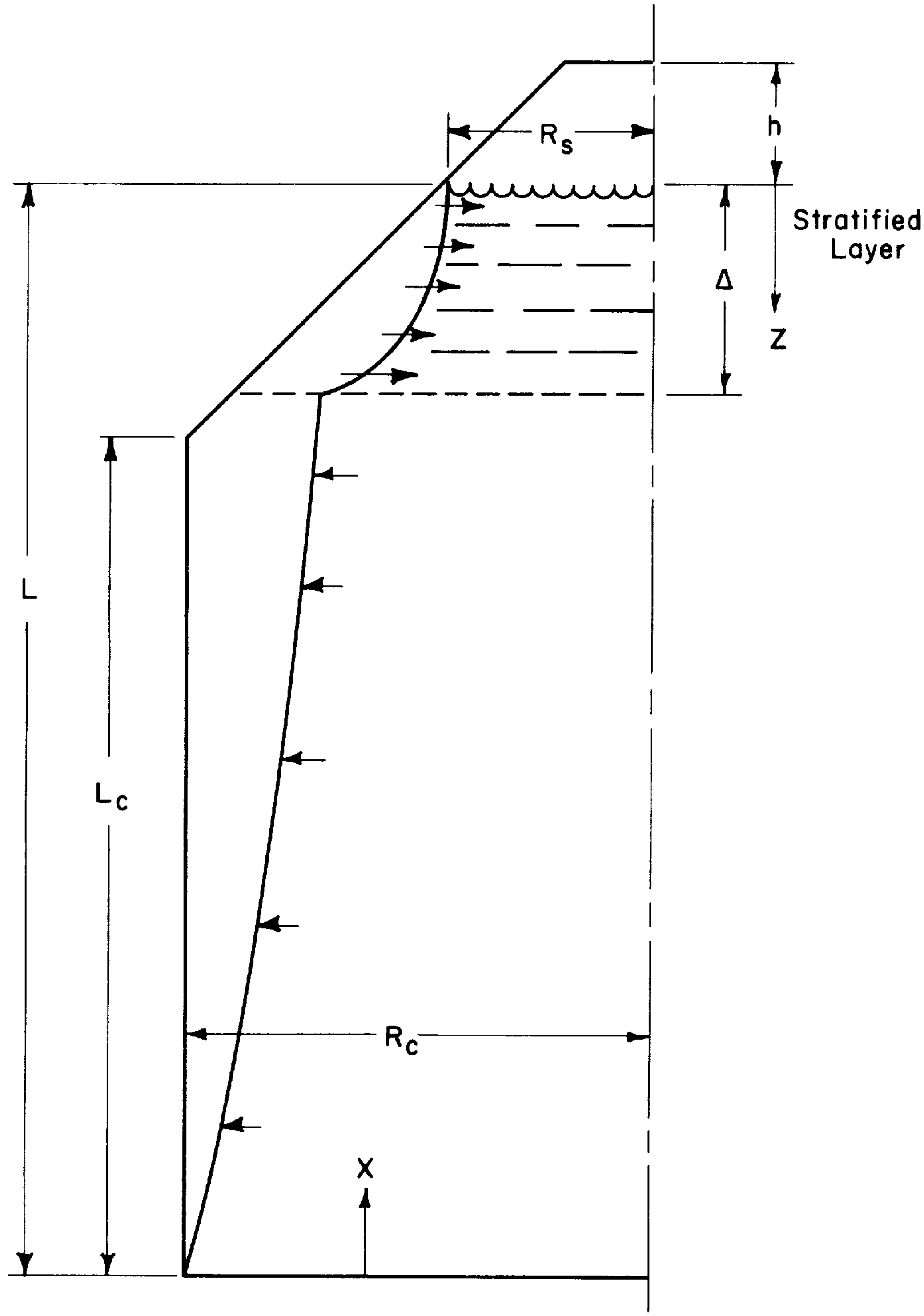

Figure 2   Assumed geometry

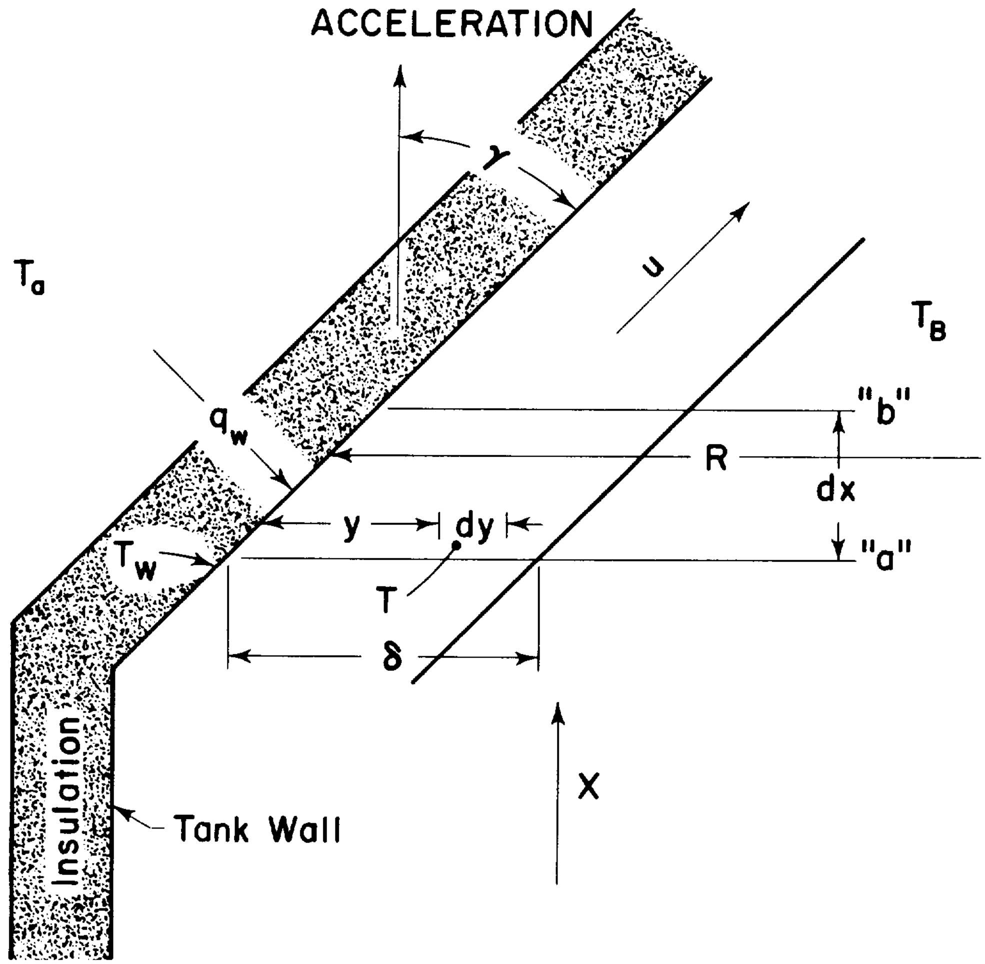

Figure 3 Boundary layer nomenclature

A wall shear stress correlation by Blasius[18, page 143] is used to substitute for τ_w in equation (6). By combining the Reynolds analogy[18, page 203], relating viscous shear stress to conductive heat transfer, with the Blasius correlation and a Prandtl number correction due to Colburn[19, page 521; 18, page 324], a suitable substitution can also be made for θ_w. The basic momentum equation is formed by equating equation 4a to the algebraic sum of equations (5) and (6). When the indicated integrations and differentiations are performed, an equation of the form

$$\frac{dU}{dx} = A(\delta, U) + B(\delta, U)\frac{d\delta}{dx} \tag{7}$$

is obtained.

A second equation can be developed by analyzing the heat transfer balance on the element under consideration. For the purpose of this analysis, the fluid in the core is considered to be at the datum condition. Since fluid entering or leaving the boundary layer at $y = \delta$ is at the datum condition, there is no heat transferred across this boundary. Thus, the only additional heat entering is that transferred through the wall at $y = o$. Equating the heat flux through the wall to the change in heat flow across the element, and performing the mathematical manipulations, results in an equation of the form

$$\frac{d\delta}{dx} = C(\delta, U) + D(\delta, U)\frac{dU}{dx} \; . \tag{8}$$

A third equation can be derived by equating the rate of heat transported across a horizontal plane located at x, to the rate of heat entering the fluid through the tank wall below this plane. Solving the resulting equation and rearranging yields an equation of the form,

$$U = E(\delta, x). \tag{9}$$

Allowance for varying heat flux over the tank height is accomplished by defining a term, q_m, such that

$$q_m \int_o^x dA = \int_o^x q_w d\,A. \qquad (10)$$

Thus, if q_w varies axially along the tank (circumferential uniformity is assumed), q_m will reflect the total heat flux pattern below the plane at x. The local heat flux, q_w, is assumed constant with time due to tank insulation. q_m is used wherever substitution for θ_w is required. Simultaneous solution of equations (7), (8), and (9) is achieved by an iteration using Newton's method.

4.2 Growth and Temperature of the Stratified Layer

An arbitrary stratified layer thickness is selected and the time for the boundary layer flow to occupy the stratified layer volume is determined. The volume flow rate parallel to the wall in the boundary layer at any horizontal plane can be determined by integration across the boundary layer. An incremental time sufficient for a boundary layer volume flow to occupy an arbitrarily small volume immediately above the horizontal plane can then be determined,

$$\Delta t = \frac{\Delta V}{\left(\int_o^\delta u dA \right)_x} .$$

A summation of these time increments for all volume increments above an arbitrary plane will supply the time required, $t = \Sigma\ \Delta t$, for the stratified layer to occupy the volume above that plane.

The problem remains as to how much energy is contained in the stratified layer, and how that energy is distributed within the layer as reflected by the temperature distribution.

Since we are considering a steady state boundary layer, i.e., the energy content of the boundary layer itself does not change with time, all of the energy entering the tank sidewall during the stratified layer growth time must be contained within the stratified layer. Thus,

$$Q = tq_m \int_o^L dA. \quad (11)$$

Based upon inspection of various experimental stratified layer temperature gradients, a vertical energy distribution of exponential form is assumed:

$$E(Z) = m\, Z^n. \quad (12)$$

We write

$$dQ = E(Z)\, dV, \quad (13)$$

and

$$Q = \int_o^\Delta E(Z)\, dV. \quad (13a)$$

Utilizing equations (11), (12), and (13) together with the tank geometry, the value of m in the energy equation can be determined and the temperature calculated from the energy and mass content of any incremental volume. The above procedure thus enables establishment of the energy distribution and the corresponding temperature gradient through the stratified layer.

In the ullage space, an iteration method is used to determine the thickness of the ullage stratified layer corresponding to the time element previously determined for the assumed thickness of the liquid stratified layer. Existence of a free convection boundary layer in the ullage space, prior to the closing of the vent, is assumed although it is recognized that a gradual change in the defining parameters of the boundary layer occurs as the pressure and temperature of the ullage vapor changes. This change is automatically included since ullage space boundary layer calculations are redone for each step change in the liquid stratified layer, with changes in the temperature, pressure, and vapor properties incorporated in the new calculations.

4.3 Ullage Pressure Calculation

A determination of the pressure existing in the ullage space is made following each calculated time increment.

Initially, the mass of gas present in the ullage space is computed, based upon a uniform temperature (assumed saturated at the initial pressure), pressure, and ullage volume. Using an iteration procedure, the amount of liquid vaporized due to the heat flux through the walls is balanced against the pressure rise due to the temperature increase in the ullage space and the increased mass due to vaporization. Vaporization is allowed in a surface layer of thickness Z_1, as illustrated in figure 4a, with the liquid in this layer being cooled to a uniform temperature of T_s by extraction of the latent heat required by the mass vaporized. Using the temperature distribution for the ullage space together with an assumed pressure, a mass for the ullage space is determined; this is then compared with the initial mass, adjusted for the amount of liquid vaporized. Successive iteration steps will result in bracketing the value for the liquid mass evaporated (or condensed) and the change in ullage mass required for equilibrium. A linear interpolation is then used to establish a final value for the mass evaporated and the corresponding ullage pressure.

When conditions are such that the calculated ullage pressure is greater than the surface liquid saturation pressure when no mass is evaporated, then condensation of some ullage vapor is required. Under

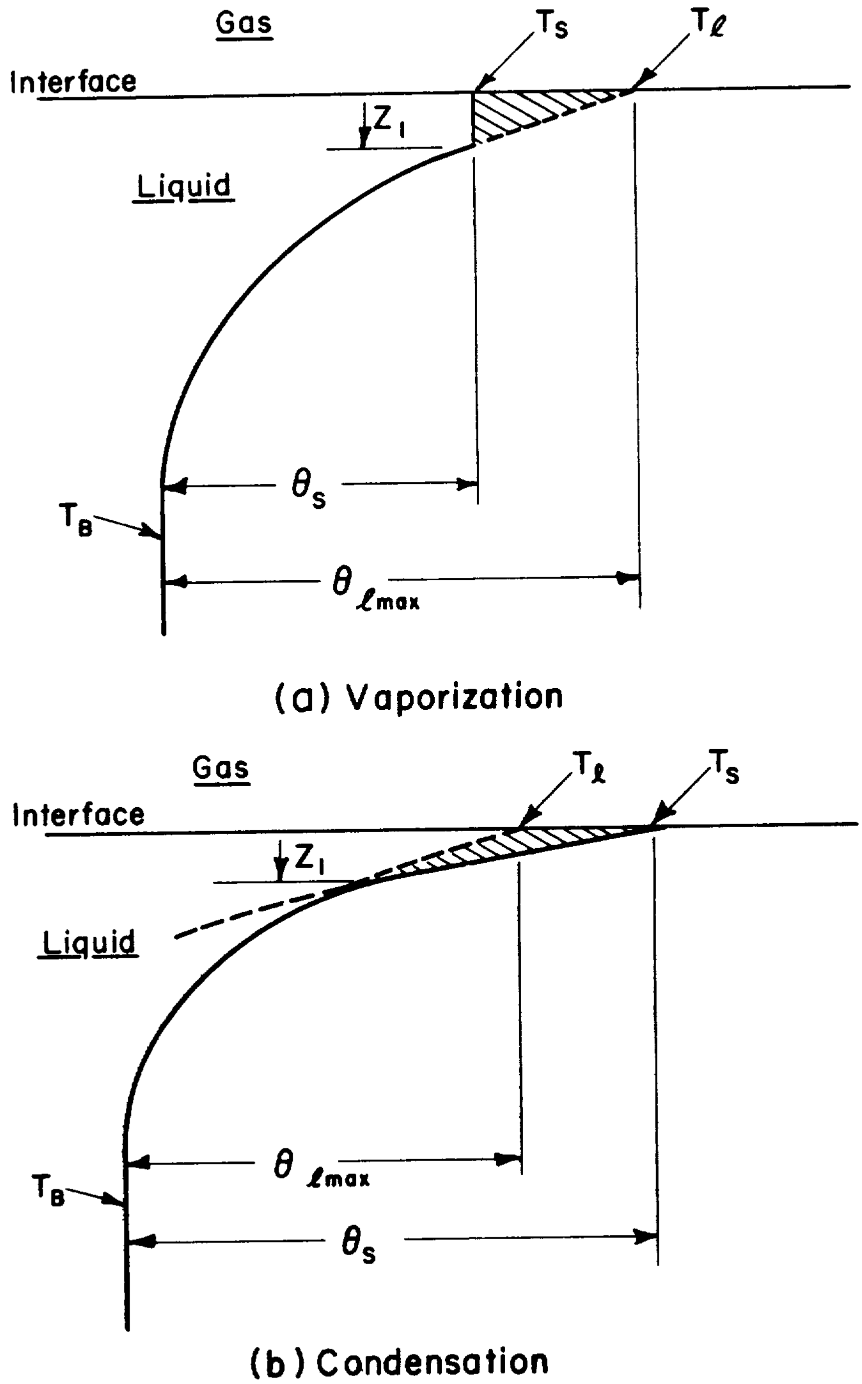

Figure 4 Temperature gradients at the liquid surface.

this condition, i. e., when the saturation temperature corresponding to the ullage pressure is greater than the liquid surface temperature determined during the stratified layer calculations, then an incremental ullage mass is condensed, with the temperature increase of the liquid layer being described by an error function as reported by Schmidt, et al.[10], figure 4b. Again successive iterations result in a pressure being established such that the ullage space pressure is in equilibrium with the liquid surface temperature.

When the saturation temperature corresponding to the ullage pressure matches the liquid surface temperature, then that pressure is recorded as the ullage pressure corresponding to the time and a new liquid stratification depth is selected to initiate a new series of calculations.

Heat transferred through the bottom of the tank is treated separately from the computer program. Application of this program has been limited to heat flux patterns where the bottom heat flux is relatively small and its affect has been treated by assuming that the thermal energy entering is uniformly distributed in the liquid. This results in a uniform increase in T_B. It is found that the increase in T_B is very small and has a negligible affect on the liquid properties. As a result, this increase in T_B can be added to the surface temperature of the liquid and the corresponding saturation pressure used as the tank pressure. In all cases encountered, this correction has amounted to only a small increase in the ullage pressure.

Complete development of the equations and a description of the computer program used to solve the equations and calculate the various outputs are included as appendices.

5.0 Results

Several calculations of pressure variation with time, using a range of parameters which were selected to bracket the probable actual parameters, have been made; in addition, various calculations were made using the best estimates of input parameters matching various actual tank pressure rise data. One of these reflects input values for an unpublished experimental pressure rise test in the Plum Brook B-2 space simulation facility, while another calculation uses input parameters telemetered to ground stations during the coast phase of the Centaur AC-8 flight[20].

The set of experimental data obtained during the Plum Brook B-2 tests represents results of the most accurately instrumented and most closely controlled experimental work so far performed on the Centaur vehicle pressure rise phenomenon.

Figure 5 shows the comparison between the experimental pressure rise data points obtained during the test conducted in the NASA-Lewis Research Center Plum Brook B-2 simulation facility and a computer prediction using the input parameter values measured during the B-2 test. The maximum deviation in absolute pressure over the measured time interval is less than 1.5%.

A comparison of the measured temperature patterns with those predicted does not reflect as close an agreement. Initial and final temperature patterns for the experimental and calculated conditions are displayed in figure 6. The initial computed temperature pattern is assumed to be uniform at the initial saturated vapor temperature; the final computed pattern results from the assumed energy distribution and is further modified by the geometry of the Centaur nose cone. In contrast, the experimental vessel displayed a significant temperature gradient both before the tank vent closure and at the end of the pressurizing period. Each of these measured gradients can be approximated

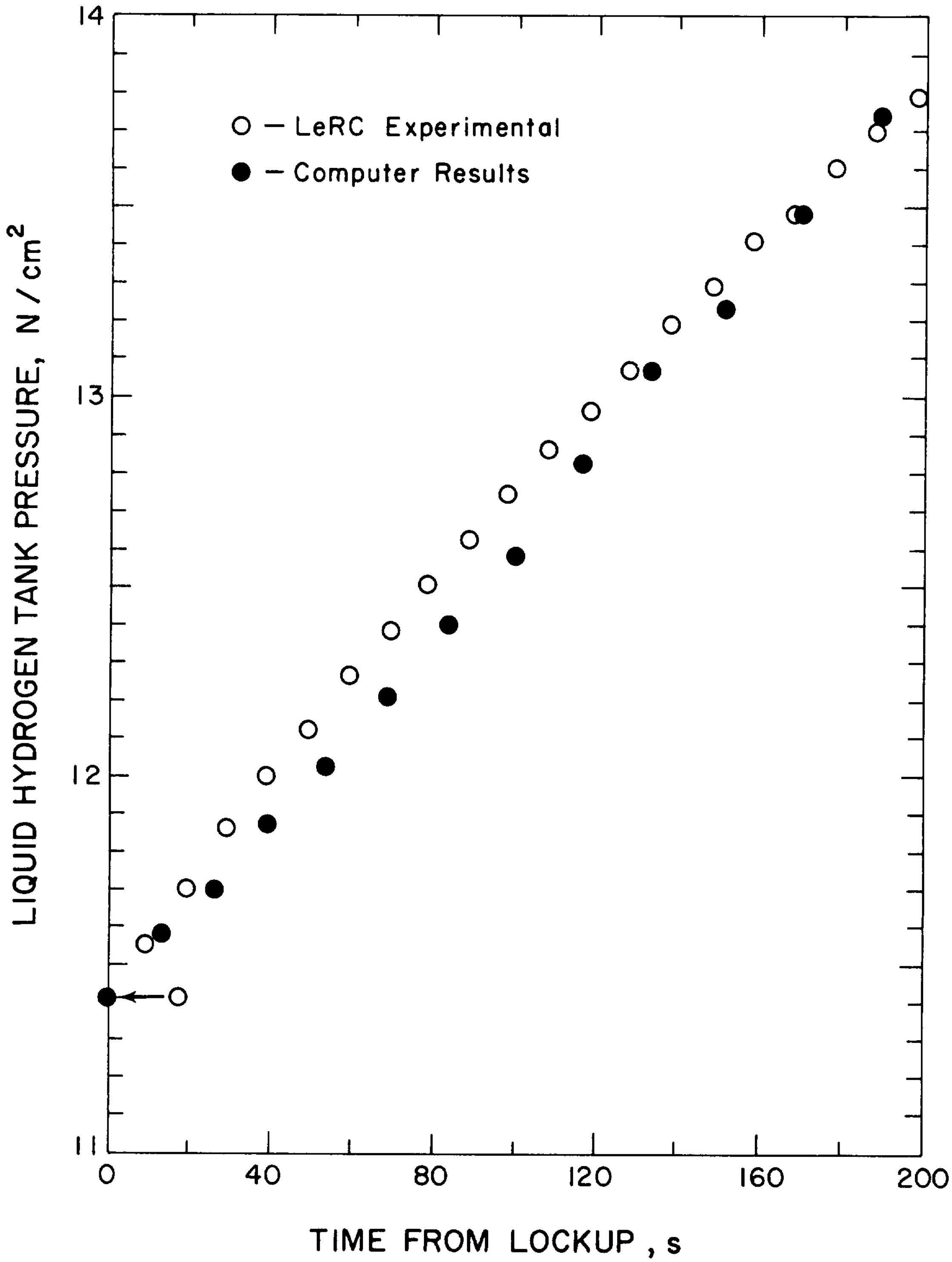

Figure 5 Self pressure rise comparison for Plum Brook B-2 test.

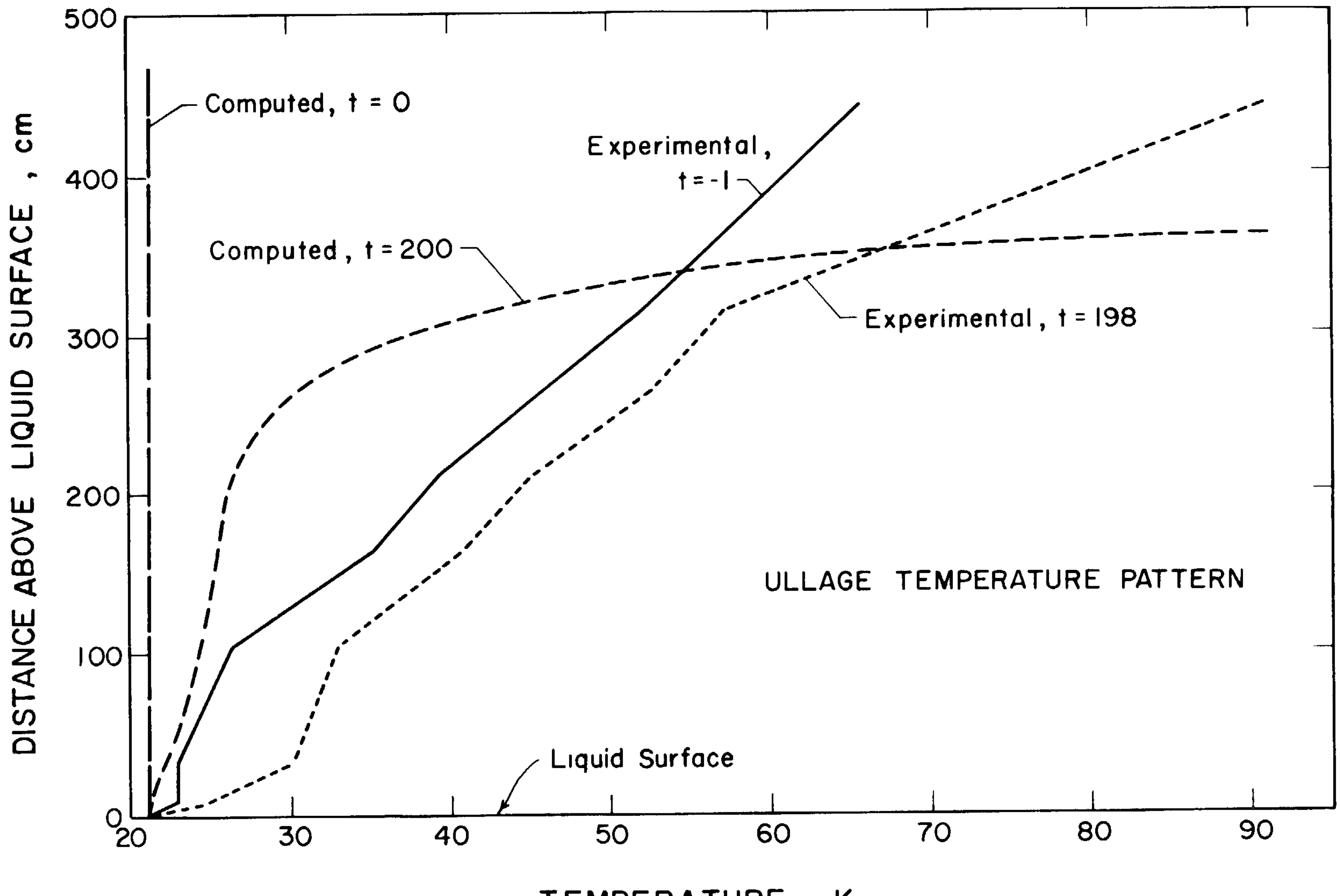

Figure 6 Ullage temperature comparison for Plum Brook B-2 test.

with a linear temperature pattern although a second or third degree function would achieve a somewhat better fit.

For the purpose of determining the ullage pressure, a precise knowledge of the temperature distribution in the ullage is not required; rather, the value of the change in internal energy is of prime importance — regardless of the distribution of that energy. For a perfect gas the change in pressure of a constant volume system is determined by the change in internal energy. Thus the nearly perfect gas behavior of hydrogen at the pressures encountered with liquid hydrogen rocket tanks results in the pressure being determined by the internal energy — and affected only slightly by the temperature distribution. A close examination of figure 6 reveals that the final calculated temperature pattern appears to have a mean value well below the final experimental temperature. This is understandable when one realizes that the assumption of saturated vapor for the initial condition results in more ullage mass than is actually the case; thus the addition of the required internal energy will result in a lower final temperature as the plot shows. However the determining factor in the pressure rise is the change in internal energy as discussed above.

Comparisons of calculated pressure rise and temperature patterns with data telemetered from the Centaur AC-8 vehicle during a low-g coast phase[20] are seen in figures 7 and 8 respectively. The calculated pressure rise is seen to be slightly lower than the telemetered data; the computed pressure is within 4% of the absolute pressure measured throughout the interval, while the pressure difference is a maximum of 15% lower than the measured pressure difference. Ullage temperature pattern comparisons again are not as well matched. The initial uniform saturated vapor temperature assumed for the calculations is seen to be a poor approximation to the actual temperature

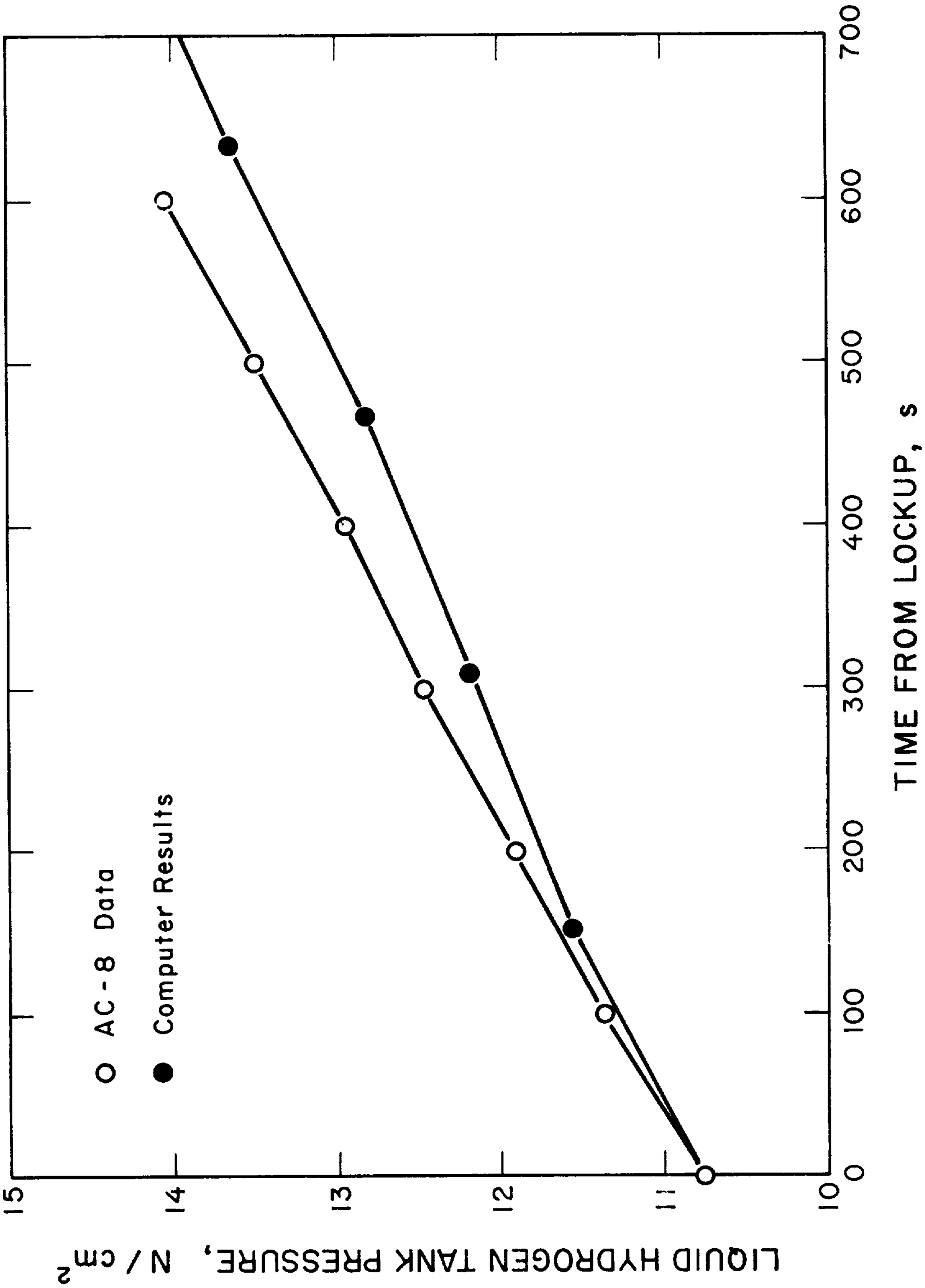

Figure 7 Self pressure rise comparison for Centaur AC-8 flight.

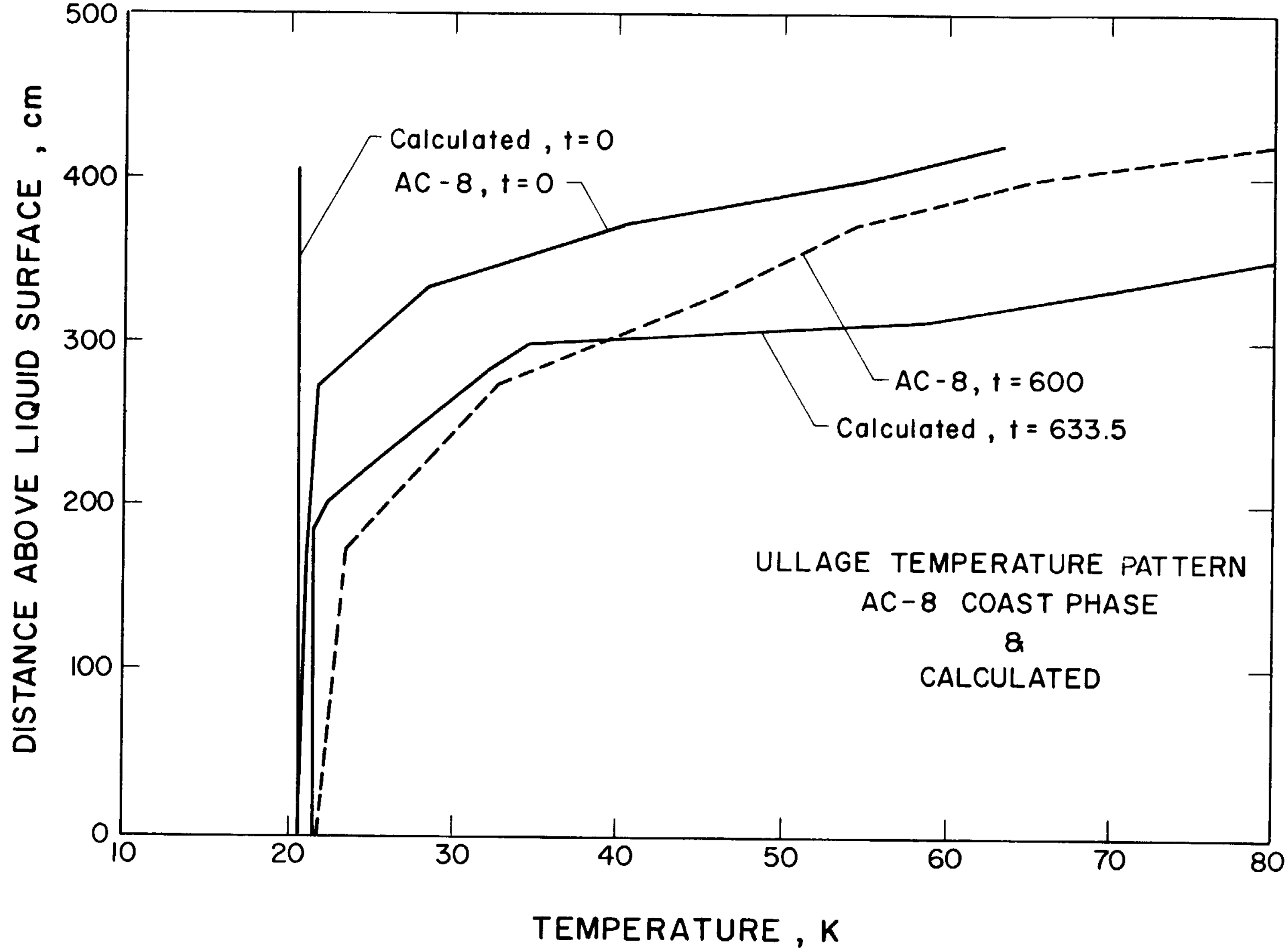

Figure 8   Ullage temperature comparison for Centaur AC-8 flight.

pattern. The final calculated temperature distribution has a shape similar to that of the telemetered data but increases more rapidly in the upper parts of the tank. As mentioned previously the temperature distribution in the ullage space does not appear to be of great importance — of more significance are the mass and total energy content and volume of the ullage vapor. A comparison of the energy content by calculation with that represented by telemetered data shows more energy is stored as sensible heat in the liquid for the actual case than is predicted from the theoretical calculations.

The influence of the location of heat input to a Centaur LH_2 tank is shown in figures 9 and 10. Figures 9 and 10 use normalized coordinates in order to compare results more easily. The normalized pressure is defined as $P_N = P/P_c$, where P_c is the maximum pressure attained for the uniform heat flux case; likewise a normalized time is used, $t_N = t/t_c$, where t_c is the time to attain the maximum pressure for the uniform heat flux case. Cases 1, 2, and 3 shown in figure 9 are for a small (8%) ullage space and have the same total heat input to the tank for each case. A uniform heat flux was used in Case 1, a wetted wall heat flux 10 times as great as the ullage wall heat flux for Case 2, and an ullage wall heat flux 10 times as great as the wetted wall heat flux for Case 3. Note that increasing the heat input to the ullage space increases the rate of pressure rise while increasing the heat into the liquid decreases the pressure rise rate, the total heat input being the same in each case. Figure 10 shows a similar result for the situation of a large (65%) ullage volume — Case 4 being uniform heat flux, Case 5 high liquid heat flux, and Case 6 high ullage heat flux, again with the same total heat flux in all three cases. The trend of pressure rise due to location of maximum heat flux is seen to be the same for both the small and the large ullage cases.

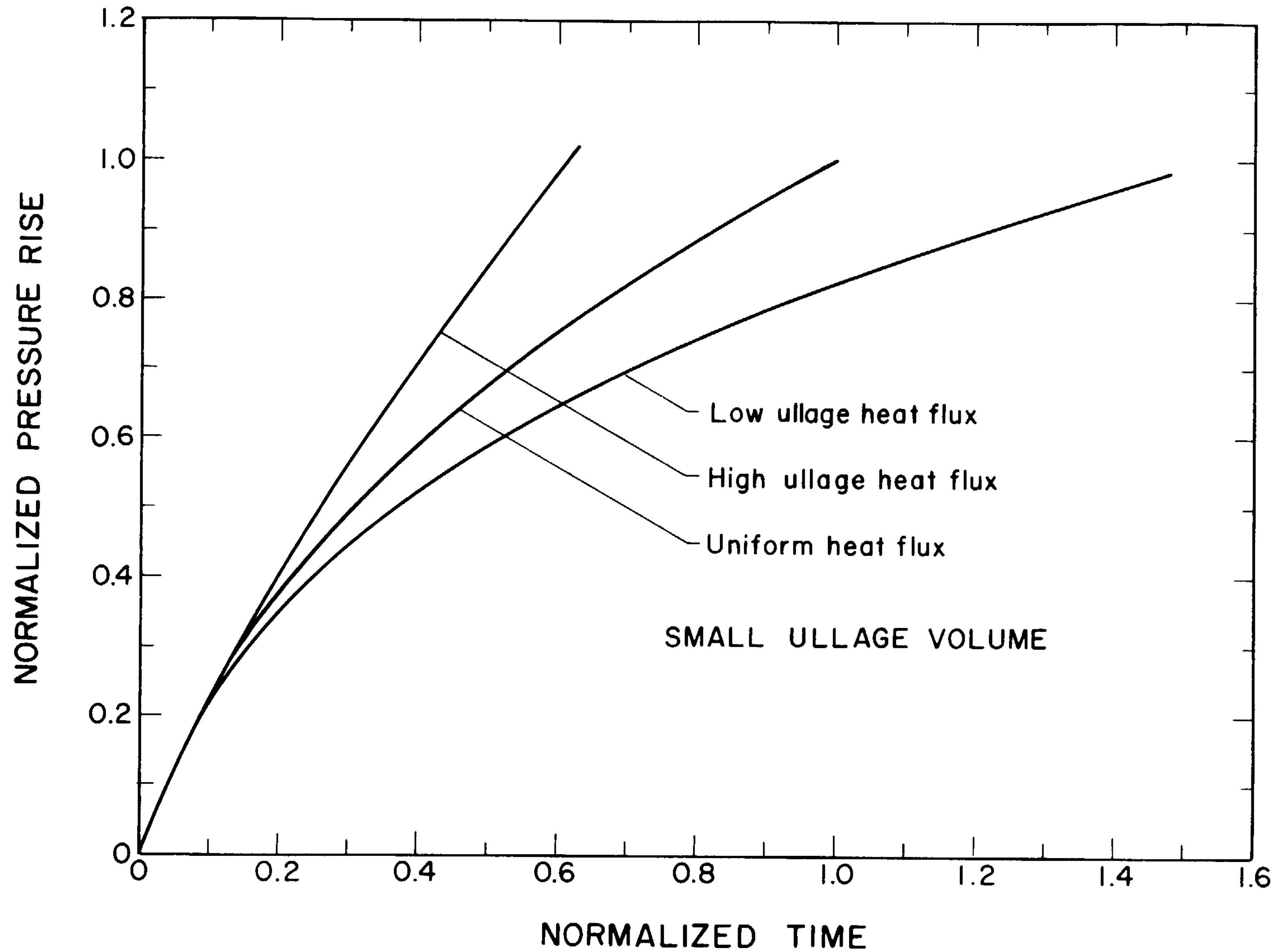

Figure 9 Effect of heat input location, small ullage volume.

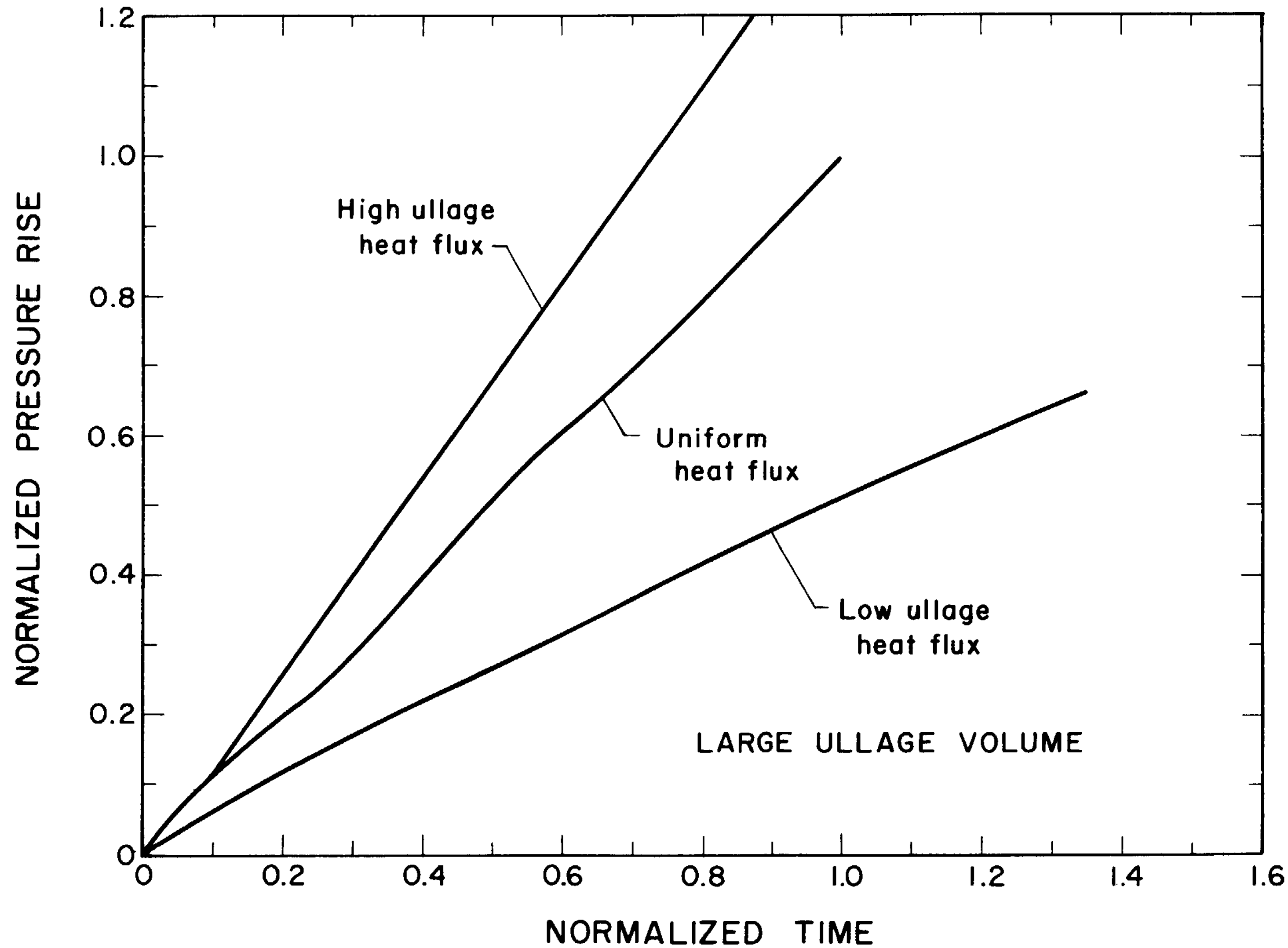

Figure 10 Effect of heat input location, large ullage volume.

6.0 Sample Calculation

A sample calculation, using the input parameters from the Plum Brook B-2 test, is presented here. Pertinent parameters from the Plum Brook B-2 test are as follows:

Liquid (hydrogen) level at station 333.3.

Normal earth gravitational field.

Initial tank pressure, 11.41 N/cm^2 (16.55 psia).

Final tank pressure, 13.79 N/cm^2 (20 psia).

Time, lockup to final pressure, 199 s.

Heat flux:

Intermediate bulkhead (bottom of tank) 440 W (1500 Btu/h).

Wetted sidewall 5568 W (19000 Btu/h).

Ullage sidewall 1495 W (5100 Btu/h).

Geometries of the actual Centaur tank and the equivalent tank used in the calculations are shown in figure 11. With the input parameters given above, the liquid level is 183.64 cm from the bottom, the cylinder-cone transition is 502.93 cm from the bottom and the top of the tank is 624.89 cm from the bottom. Tank cylinder diameter is 304.8 cm.

Heat flux rates are determined as follows:

(1) Heat flux through the bottom is given as 440 W. This heat input is assumed to be uniformly assimilated in the liquid; the method of calculation is given later.

(2) Heat flux through the liquid wetted sidewall is 5568 W. Wall area is 175847 cm^2. Heat flux is then 5568/175847 = 0.03166 W/cm^2.

(3) Heat flux into the ullage space is 1495 W. Wall area is 408 186 cm^2. Heat flux is then 1495/408186 = 0.003662 W/cm^2.

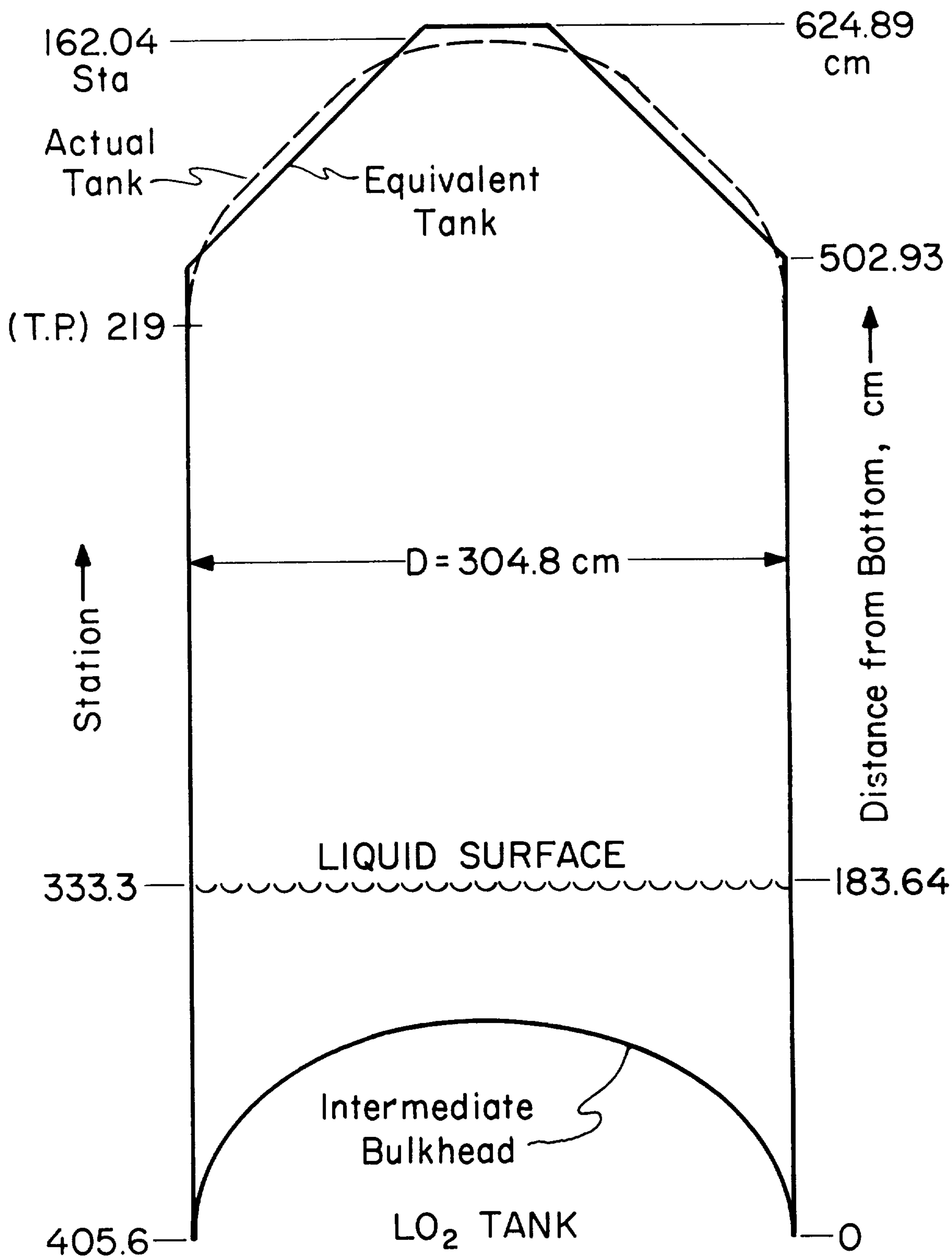

Figure 11 Tank configuration, actual and assumed, for Plum Brook B-2 test comparison.

An initial pressure of 11.41 N/cm^2 is equivalent to 1.1262 atm, the value used as the input for PS. A limiting pressure of 1.40 atm, slightly in excess of the B-2 test final pressure of 13.79 N/cm^2, is used for PL.

A summary of the input used for the computer program is as follows:

Gamma = 45.0 deg (nose cone half angle).

Acceleration field, "G" = 980.67 cm/s^2

LC = 502.93 cm

L = 183.64 cm

H = 624.89-L = 441.25 cm

RC = 152.4 cm

PS = 1.1262 atm

PL = 1.40 atm

Heat flux, given at distance χ from the bottom of tank is:

χ	q_w
0	0.03166 W/cm^2 .
183.64 cm	0.03166 W/cm^2 .
184.0 cm	0.003662 W/cm^2.
502.93 cm	0.003662 W/cm^2.
624.89 cm	0.003662 W/cm^2.

Note that a value for the heat flux <u>must</u> be given at the liquid level and at the cylinder cone transition as well as at the bottom and top of the tank. Different heat flux values may be given at any other location in addition to these.

In addition to the above inputs based upon the physical condition, additional required inputs are chosen as follows:

Liquid depth increment, DL, assigned value of 5.0 cm.

Number of increments used in stratified layer, ZINC = 50.0.

POWER = 2.0, exponent used to describe energy distribution in the stratified layer; must be non negative.

TAMB = 300.0 K.

The data cards, located at the end of the program deck, are made up of four sets. The first set, Table 1, is composed of 40 cards, in pairs, containing thermophysical properties for parahydrogen in the following order and within the punch card columns indicated: pressure, atm (1-8); liquid specific volume, cm^3/g-mol (9-16); liquid $(\partial P/\partial \rho)_T$, $cm^3 \cdot$ atm/g-mol (17-24); liquid $(\partial P/\partial T)_\rho$, atm/K (25-32); vapor specific volume, cm^3/g-mol (33-40); vapor $(\partial P/\partial \rho)_T$, $cm^3 \cdot$ atm/ g-mol (41-48); vapor $(\partial P/\partial T)_\rho$, atm/K (49-56); liquid entropy, J/g-mol-K (57-64); vapor entropy, J/g-mol-K (65-72); saturation temperature, K (73-80); next card, vapor specific heat, J/g-mol·K (1-8); liquid specific heat, J/g-mol·K (17-24).

These property data are followed by an input set of cards, Table 2, as follows: GAMMA, deg (1-10); RC, cm (11-20); LC, cm (21-30); L, cm (31-40); H, cm (41-50); next card, G, cm/s^2 (1-10); QW, W/cm^2 (11-20); DL, cm (21-30); ZINC, dimensionless (31-40); NR, dimensionless (50); (note that a value for NR in column 50 of this card tells the computer to look for an additional set of data, e.g., different geometry, liquid level, heat flux, etc. This next set of data is entered, following a blank card, after the first set of heat flux values; additional sets may also be entered so long as a number is placed in the NR column and a blank card is inserted after each set of heat flux data); next card, PS, atm (1-10); TAMB, kelvin (11-20); PL, atm (21-30); next card, POWER, dimensionless (1-10).

Table 1 Parahydrogen thermodynamic property data.

1-8	9-16	17-24	25-32	33-40	41-48	49-56	57-64	65-72	73-80
0.20	26.80	20155.	8.946	6294.2	1216.	0.013	11.95	69.51	15.861
21.81		14.79							
0.30	27.11	18605.	8.778	4397.5	1259.	0.019	12.83	67.20	16.802
22.22		15.73							
0.40	27.37	17563.	8.663	3407.8	1287.	0.025	13.51	65.58	17.532
22.59		16.48							
0.50	27.59	16845.	8.567	2795.4	1306.	0.030	14.07	64.32	18.137
22.94		17.08							
0.60	27.80	16117.	8.482	2376.6	1319.	0.036	14.56	63.29	18.659
23.27		17.67							
0.70	27.98	15566.	8.402	2071.0	1329.	0.041	15.00	62.42	19.121
23.60		18.16							
0.80	28.16	15077.	8.329	1837.1	1335.	0.047	15.39	61.66	19.538
23.91		18.62							
0.90	28.32	14626.	8.266	1652.7	1340.	0.052	15.75	60.99	19.918
24.21		19.05							
1.0	28.48	14091.	8.204	1506.86	1343.	0.057	16.08	60.41	20.268
24.50		19.53							
1.5	29.19	12268.	7.904	1040.8	1338.	0.084	17.47	58.07	21.722
25.94		21.43							
2.0	29.82	10690.	7.620	796.51	1314.	0.112	18.58	56.36	22.861
27.39		23.28							
3.0	30.98	8484.	7.088	541.72	1240.	0.169	20.36	53.85	24.632
30.46		26.52							
4.0	32.10	6787.	6.582	407.74	1147.	0.230	21.82	51.95	26.023
34.05		29.84							
5.0	33.24	5361.	6.097	323.89	1043.	0.296	23.10	50.35	27.187
38.45		33.74							
6.0	34.44	4211.	5.640	265.73	928.8	0.370	24.28	48.92	28.198
44.12		38.37							
7.0	35.74	3211.	5.203	222.51	809.0	0.453	25.39	47.59	29.097
51.79		44.67							
8.0	37.20	2387.0	4.770	188.66	680.4	0.549	26.48	46.29	29.910
62.87	53.02								
9.0	38.88	1687.0	4.337	161.07	543.4	0.662	27.58	44.98	30.651
80.41	65.56								
10.0	40.93	1109.0	3.894	137.49	405.2	0.796	27.92	28.73	31.335
86.20	67.43								
12.5	52.53	66.57	2.332	81.91	39.36	1.479	32.92	38.32	32.836
1259.25	766.30								

There then follows a set of 35 cards, Table 3, containing values of kinematic viscosity in g/cm·s for saturated liquid at 35 temperatures from 14.0 K to 32.7 K.; T, K (1-8); kinematic viscosity, g/cm·s, (9-16).

The last set of cards is indefinite in length and contains values for the sidewall heat flux at various vertical locations in the tank, Table 4. There must be at least 4 cards giving wall heat flux: at the bottom of the tank, at the liquid level, at the cylinder cone transition and at the top of the tank. As many other cards as needed may be used. If no cards are present in this location the program will use the value of QW given in the previous cards as uniform over the tank. X, cm (1-10); heat flux, W/cm^2 (11-20).

The data deck listing, Table 5, shown on the following pages shows the input for the Plum Brook B-2 test case with column numbers used in each case.

Table 2 Input data for the Plum Brook B-2 test comparison.

(1-10)	(11-20)	(21-30)	(31-40)	(41-50)
45.0	152.4	502.93	183.64	441.25
980.67	0.05	5.0	50.0	1
1.12616	300.0	1.40		
2.0				NR (if used)

Table 3 Parahydrogen kinematic viscosity data.

(1-8)	(9-16)
14.000	250.7E-6
14.500	234.1E-6
15.000	221.3E-6
15.500	207.3E-6
16.000	197.5E-6
16.500	185.6E-6
17.000	177.7E-6
17.500	168.5E-6
18.000	160.5E-6
18.500	153.8E-6
19.000	147.0E-6
19.500	141.3E-6
20.000	135.4E-6
20.500	130.6E-6
21.000	125.3E-6
21.500	120.6E-6
22.000	116.1E-6
23.000	108.1E-6
24.000	100.8E-6
25.000	93.5E-6
26.000	87.2E-6
26.500	84.1E-6
27.000	81.0E-6
27.500	78.1E-6
28.000	75.2E-6
28.500	72.4E-6
29.000	69.6E-6
29.500	67.0E-6
30.000	64.9E-6
30.500	61.2E-6
31.000	58.1E-6
31.500	55.7E-6
32.000	51.9E-6
32.500	47.5E-6
32.700	43.9E-6

Table 4 Heat flux input data for the Plum Brook B-2 test comparison

(1-10)	(11-20)
0.0	0.031666
183.64	0.031666
184.0	0.0036618
502.93	0.0036618
624.89	0.0036618

Output values are shown in the printout on the following pages. (Table 5). After listing (optional) the main calling program and the several sub-programs, the input data is listed in the order shown. The mass of vapor present in the ullage space at time t=0 is also shown. Note that PS has been truncated to two decimal places, for printing only; also that gamma has been converted to radians.

A table of some property values for parahydrogen follows, both at 1.50 atm and at pressure PS. Next is a summary line giving the thickness, DELTAL, of the liquid stratified layer, thickness, DELTAV, of the ullage stratified layer, and the time for these stratified layers to develop.

Prior to the first tabulation of temperatures in the stratified layers is a table giving the heat flux input values and the constants, AQ and BQ, used in the linear fit established between successive points. Boundary layer parameters at the bottom of the stratified layer are then listed, velocity parameter and thickness for both the liquid and vapor regions. Then follows a table of locations, temperature increases and absolute temperatures through both the liquid and vapor stratified layers.

Following this table the amount of liquid vaporized, DME, is listed together with bracketing values used in a linear interpolation. Next is listed the pressure existing after the time given earlier, the surface temperature of the true liquid surface temperature gradient and the intersection of the true liquid gradient with the gradient predicted by the tabulation above. For the vaporizing case the surface temperature and intersect temperature will always be the same.

At this point the program increases the liquid stratified layer by the amount DL and a new set of calculations is made. When the pressure

Table 5   Printout for the Plum Brook B-2 test program.

```
        DL             PL              ZINC            PS             TAMB
       5.00           1.40             50.00          1.13           300.00

      GAMMA            RC               LC              L               H
     0.78540         152.40           502.93         183.64          441.25
        G              QW
     980.670          0.050
     POWER    =     2.0000
               MASS
   4.0399+004
                                        MASS S INCREMENT
  PT          DENL        DPDRL        DPDTL        DENV        DPDRV        MUL          PRL
1.5000+000  2.9190+001  1.2268+004  7.9040+000  1.0408+003  1.3380+003  1.2917-004  1.0744+000
 DPDTV          SL          SV          TSAT         CPV          P           PRV
8.4000-002  1.7470+001  5.8070+001  2.1722+001  2.5940+001  1.1262+000  7.7790-001
     CSV          TTSAT         CSL         NUOTV        RHO          RHOV         NUL          I
  1.1100+001   2.0635+001   9.9253+000   1.1192-005   7.0344-002   1.4511-003   1.3363-003   10
       NUV         KONL         AL          KONV         CSV          AV          BETAV        BETAL       K
  7.7127-003   1.1933-003   1.7031-003   1.5970-004   1.1100+001   9.9148-003   6.6073-002   1.7090-002   15
                                        PROPERTY INCREMENT,1
                                        DTL INCREMENT

     DELTAL          DELTAV            TIME
     5.00000        127.16825         12.78504
                                        DTEMP V INCREMENT
                                        THETA L INCREMENT

       XB               Q                AQ               BQ

     0.00000         0.03167         0.0000+000       0.0000+000
   183.64000         0.03167         3.1666-002       0.0000+000
   184.00000         0.00366         1.4317+001      -7.7789-002
   502.93000         0.00366         3.6618-003       0.0000+000
   624.89000         0.00366         3.6618-003       0.0000+000
                                        THETA V INCREMENT
U,TH,UV,THV        31.184009        5.970566        97.593470        11.862800


    EVAPORATING CASE CALLED     SUB. MASS C


TOTAL ENERGY      7.453064+004
```

Table 5 Printout for the Plum Brook B-2 test program (con't)

Z	TDL	TABSL	ZV	TDV	TABSV
1.7864+002	0.0000+000	20.635	0.0000+000	0.0000+000	20.727
1.7874+002	3.5112-004	20.635	8.6804-001	2.4684-002	20.752
1.7884+002	1.4045-003	20.636	1.7361+000	4.9620-002	20.777
1.7894+002	3.1601-003	20.638	2.6041+000	8.5262-002	20.813
1.7904+002	5.6179-003	20.640	3.4722+000	1.1605-001	20.843
1.7914+002	8.7780-003	20.644	4.3402+000	1.4754-001	20.875
1.7924+002	1.2640-002	20.648	5.2083+000	1.7981-001	20.907
1.7934+002	1.7205-002	20.652	6.0763+000	2.1293-001	20.940
1.7944+002	2.2472-002	20.657	2.0347+001	2.4699-001	20.974
1.7954+002	2.8441-002	20.663	2.2890+001	2.8208-001	21.009
1.7964+002	3.5112-002	20.670	2.5434+001	3.1830-001	21.046
1.7974+002	4.2486-002	20.677	2.7977+001	3.5576-001	21.083
1.7984+002	5.0561-002	20.685	3.0520+001	3.9456-001	21.122
1.7994+002	5.9339-002	20.694	3.3064+001	4.3483-001	21.162
1.8004+002	6.8820-002	20.704	3.5607+001	4.7671-001	21.204
1.8014+002	7.9002-002	20.714	3.8150+001	5.2034-001	21.248
1.8024+002	8.9887-002	20.725	4.0694+001	5.6588-001	21.293
1.8034+002	1.0147-001	20.736	4.3237+001	6.1196-001	21.339
1.8044+002	1.1376-001	20.749	4.5781+001	6.5714-001	21.384
1.8054+002	1.2675-001	20.762	4.8324+001	7.0178-001	21.429
1.8064+002	1.4045-001	20.775	5.0867+001	7.4641-001	21.474
1.8074+002	1.5484-001	20.790	5.3411+001	7.9155-001	21.519
1.8084+002	1.6994-001	20.805	5.5954+001	8.3769-001	21.565
1.8094+002	1.8574-001	20.821	5.8497+001	8.8530-001	21.613
1.8104+002	2.0225-001	20.837	6.1041+001	9.3489-001	21.662
1.8114+002	2.1945-001	20.854	6.3584+001	9.8694-001	21.714
1.8124+002	2.3736-001	20.872	6.6127+001	1.0420+000	21.769
1.8134+002	2.5597-001	20.891	6.8671+001	1.1006+000	21.828
1.8144+002	2.7528-001	20.910	7.1214+001	1.1633+000	21.891
1.8154+002	2.9529-001	20.930	7.3758+001	1.2308+000	21.958
1.8164+002	3.1601-001	20.951	7.6301+001	1.3039+000	22.031
1.8174+002	3.3743-001	20.972	7.8844+001	1.3833+000	22.111
1.8184+002	3.5955-001	20.994	8.1388+001	1.4701+000	22.197
1.8194+002	3.8237-001	21.017	8.3931+001	1.5652+000	22.293
1.8204+002	4.0589-001	21.041	8.6474+001	1.6701+000	22.397
1.8214+002	4.3012-001	21.065	8.9018+001	1.7861+000	22.513
1.8224+002	4.5505-001	21.090	9.1561+001	1.9150+000	22.642
1.8234+002	4.8068-001	21.116	9.4105+001	2.0590+000	22.786
1.8244+002	5.0702-001	21.142	9.6648+001	2.2207+000	22.948
1.8254+002	5.3405-001	21.169	9.9191+001	2.4035+000	23.131
1.8264+002	5.6179-001	21.197	1.0173+002	2.6116+000	23.339
1.8274+002	5.9023-001	21.225	1.0428+002	2.8500+000	23.577
1.8284+002	6.1938-001	21.254	1.0682+002	3.1251+000	23.852
1.8294+002	6.4922-001	21.284	1.0936+002	3.4449+000	24.172
1.8304+002	6.7977-001	21.315	1.1191+002	3.8200+000	24.547
1.8314+002	7.1102-001	21.346	1.1445+002	4.2643+000	24.992
1.8324+002	7.4297-001	21.378	1.1699+002	4.7962+000	25.524
1.8334+002	7.7562-001	21.410	1.1954+002	5.4411+000	26.168
1.8344+002	8.0898-001	21.444	1.2208+002	6.2345+000	26.962
1.8354+002	8.4304-001	21.478	1.2462+002	7.2255+000	27.953
1.8364+002	8.7780-001	21.513	1.2717+002	8.4798+000	29.207

DME	DMV(JJ)	DMV(JJ+1)	PSAT(JJ)	PSAT(JJ+1)	DML(JJ)	DML(JJ+1)
125.9183	220.2097	99.1286	1.1452	1.1415	122.4467	126.9046

NEW P= 1.1423 ATMOSPHERES   SURFACE TEMP. 20.727   INTERSECT TEMP. 20.727 DEG K.

MASS C INCREMENT

Table 5 Printout for the Plum Brook B-2 test program (con't)

PRESSURE ATMOSPHERES	TIME SECONDS	DELTAL CENTIMETERS	DELTAV CENTIMETERS	K
1.12616+000	0.00000+000	0.00000+000	0.00000+000	0
1.14231+000	1.27850+001	5.00000+000	1.27168+002	1
1.15396+000	2.60062+001	1.00000+001	1.58697+002	2
1.16896+000	3.96910+001	1.50000+001	1.87129+002	3
1.18530+000	5.38699+001	2.00000+001	2.12656+002	4
1.20314+000	6.85750+001	2.50000+001	2.35639+002	5
1.22154+000	8.38459+001	3.00000+001	2.56262+002	6
1.23890+000	9.97201+001	3.50000+001	2.74891+002	7
1.26192+000	1.16243+002	4.00000+001	2.91748+002	8
1.28272+000	1.33466+002	4.50000+001	3.06739+002	9
1.30072+000	1.51442+002	5.00000+001	3.20390+002	10
1.32514+000	1.70236+002	5.50000+001	3.32857+002	11
1.35057+000	1.89917+002	6.00000+001	3.43883+002	12
1.37131+000	2.10564+002	6.50000+001	3.53808+002	13
1.40076+000	2.32267+002	7.00000+001	3.62996+002	14

PRESSURE RISE RATE ATM/SEC	PRESSURE DIFFERENCE ATM	TIME DIFFERENCE SEC	INCREMENT
1.26349-003	1.61538-002	1.27850+001	1
8.80686-004	1.16437-002	1.32211+001	2
1.09628-003	1.50024-002	1.36849+001	3
1.15242-003	1.63400-002	1.41789+001	4
1.21341-003	1.78445-002	1.47061+001	5
1.20455-003	1.83935-002	1.52699+001	6
1.09364-003	1.73607-002	1.58742+001	7
1.39335-003	2.30228-002	1.65233+001	8
1.20785-003	2.08019-002	1.72222+001	9
1.00117-003	1.79977-002	1.79767+001	10
1.29923-003	2.44172-002	1.87936+001	11
1.29212-003	2.54297-002	1.96807+001	12
1.00484-003	2.07469-002	2.06470+001	13
1.35669-003	2.94451-002	2.17035+001	14

associated with a given stratified liquid layer exceeds PL the calculations are stopped and a summary of pressure, time, stratified layer thickness, pressure rise rate, pressure change and time difference is tabulated together with a statement that the pressure limit has been exceeded.

Adjustment of the computer-obtained temperatures and pressures for the heat flux through the bottom of the tank must be accomplished manually — if such a correction is deemed necessary. The mass of liquid contained in the tank is obtained from the actual tank geometry, the liquid level and the density of liquid at the starting pressure,

$$m_e = V_e \rho_e .$$

Assuming that all of the heat goes into the liquid, the temperature rise is then computed from $qt = m_e C_p \Delta T$, which gives $\Delta T = \frac{q\ t}{m_e C_p}$. A linear variation of pressure with temperature is assumed and the incremental pressure rise found from

$$\Delta P = \Delta T \left(\frac{\Delta P\ max}{\Delta T\ max} \right) .$$

For the Plum Brook B-2 test data the following values were obtained:

$$V_e = 6.654\ m^3 = 6.654 \times 10^6\ cm^3 ,$$

$$\rho_e = 0.03490\ g\text{-}mol/cm^3 ,$$

$$\therefore m_e = 232209\ g\text{-}mol\ of\ LH_2 .$$

q was computed previously to be 439.6 W. C_p has a value of 20.01 J/g-mol·K.

A tabulation of the adjustment for tank bottom heat input effects as a function of time follows. Tabulations are for the summary printout values.

Table 6. Adjusted pressure values

Time, seconds	Adjustment ΔP, atm	P, atm	Adjusted P, atm	Adjusted P, N/cm^2
0	0	1.12616	1.12616	11.41
12.78	0.0004	1.14231	1.1427	11.58
26.01	0.0008	1.15396	1.1548	11.70
39.69	0.0012	1.16896	1.1702	11.86
53.87	0.0017	1.1853	1.1870	12.03
68.58	0.0021	1.20314	1.2052	12.21
83.85	0.0026	1.22155	1.2241	12.40
99.72	0.0031	1.23890	1.2420	12.58
116.24	0.0036	1.26192	1.2655	12.82
133.47	0.0042	1.28272	1.2869	13.04
151.44	0.0047	1.30072	1.3054	13.23
170.24	0.0053	1.32514	1.3304	13.48
189.92	0.0059	1.35057	1.3565	13.74
210.56	0.0066	1.37131	1.3779	13.96
232.27	0.0073	1.40076	1.4081	14.27

Thus we have

$$\Delta T = 9.461 \times 10^{-5}\ t.$$

Also ΔP max = 2.7824 N/cm^2,

and ΔT max = 0.83 K.

Combining the above we get

$$\Delta P = \frac{(9.461)(10^{-5})(2.7824)}{0.83}\ t = 0.00317\ t,\ \text{N/cm}^2,$$

$$\text{or } 0.0000313\ t,\ \text{atm.}$$

7.0 SYMBOLS

A	area, cm^2.
A_ℓ	area of liquid wetted wall, cm^2.
A, B, C, D	coefficients in linear equations.
c	specific heat, J/g·K.
E	functional notation for specific energy, J/g, also linear equation constant.
F	force, g_f.
F_B	buoyant force, g_f.
F_S	viscous shear force, g_f.
f	defined function.
f'	defined function.
G_r^*	modified Grashof number, $\frac{g\beta x^4 q}{\nu^2 k}$.
g	acceleration field, cm/s^2.
L	liquid depth, cm.
m	mass, g.
m, n	constants in specific energy distribution function.
m_a	adjusted mass in ullage, g.
m_i	initial mass in ullage, g.
Δm_v	mass of fluid vaporized or condensed at liquid-vapor interface, g.
Pr	Prandtl number.
Q	thermal energy, J.

q	heat flux, W/cm^2.
q_m	mean heat flux over specified tank wall area, W/cm^2.
q_w	local wall heat flux, W/cm^2.
R	local tank radius, cm.
R_c	tank cylinder radius, cm.
R_S	tank radius at liquid surface, cm.
T	temperature, K.
T_B	bulk liquid temperature, K.
T_ℓ	liquid temperature in stratified layers, K.
T_s	temperature of liquid affected by vaporization or condensation, K.
t	time for liquid stratified layer to grow to depth Δ, s.
U	boundary layer fluid velocity parameter, cm/s.
u	local velocity in boundary layer, cm/s.
V	volume, cm^3.
x	distance from tank bottom, parallel to acceleration vector, cm.
y	distance from wall, normal to acceleration vector, cm.
Z	depth in stratified layer, cm.
Z_1	depth in stratified layer over which vaporization or condensation affects temperature, cm.

Greek Letters

α	thermal diffusivity, cm^2/s.
β	coefficient of volumetric thermal expansion, 1/K.
γ	half angle of nose cone, rad.
Δ	thickness of stratified layer, cm.
$\boldsymbol{\Delta}$	incremental value.
δ	thickness of boundary layer, cm.
$\boldsymbol{\Delta}\delta$	calculated incremental change in δ, cm.
θ	temperature rise in boundary layer, $T - T_B$, K.
θ_w	increase in wall temperature over bulk liquid, K.
λ	variable in error function integral.
ν	kinematic viscosity, cm^2/s.
ρ	density, g/cm^3.
ρ_B	density of bulk liquid, g/cm^3.
ρ_S	density of saturated vapor, g/cm^3.
τ_w	specific viscous shear force, g/cm^2.
φ	parameter in error function integral.

Subscripts

ℓ	pertains to liquid.
v	pertains to vapor.
w	pertains to wall.

8.0 References

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Appendices

A. Mathematical Development

Development of the mathematical expressions used in this analysis to determine the boundary layer descriptors, the extent of thermal stratification, and the rate of pressure rise are based on several previous works. The development does, however, include certain methods which are believed to be original for this problem. The translation[16] of Von Karman's original work[17] and the subsequent work of Eckert and Jackson[15] were primary sources for the basic premise of relating momentum flow, buoyancy force, viscous force, and thermal energy in such a way as to permit determination of the boundary layer descriptors such as thickness, velocity, and wall temperature.

This development assumes a finite geometry tank and axial symmetry of both the tank geometry and the fluid behaviour. A cone-shaped geometry is assumed which may occur over the entire height of the tank or over only the upper portion of the tank. It is assumed that the base of the cone is downward and that the acceleration force field is directed along the tank axis. Nomenclature and physical format are as shown in figures 2 and 3.

A. 1 Momentum-Force Balance Equations

Assuming a boundary layer flow parallel to the tank wall, an equation relating the change in momentum flow over a differential height, dx, to the forces acting over that same height is developed. It is assumed[15] that the temperature variation across the boundary layer is given by

$$\frac{\theta}{\theta_w} = 1 - \left(\frac{y}{\delta} \right)^{1/7} , \qquad (1)$$

and the velocity distribution by

$$\frac{u}{U} = \left(\frac{y}{\delta}\right)^{1/7} \left(1 - \frac{y}{\delta}\right)^{4} . \qquad (2)$$

It is further assumed that the thickness is the same for both the temperature boundary layer and the velocity boundary layer.

The change in momentum flow across the element is described by

$$\frac{d}{dx}(\dot{m}u)\,dx = \frac{d}{dx}\left[\int_{o}^{\delta} 2\pi\rho(R-y)u^{2}\,dy\cos\gamma\right]dx . \qquad (4b)$$

A viscous shear force acts parallel to the wall and opposite to the direction of flow,

$$F_{v} = -\tau_{w}\frac{2\pi R}{\cos\gamma}\,dx , \qquad (6a)$$

and a vertical buoyant force has a component parallel to the wall of

$$F_{B} = 2\pi g \cos\gamma \int_{o}^{\delta}(\rho_{B}-\rho)(R-y)\,dy\,dx . \qquad (5a)$$

Equating the forces acting to the change in momentum flow:

$$\frac{d}{dx}\left[2\pi\cos\gamma\int_{o}^{\delta}\rho(R-y)u^{2}\,dy\right]dx$$

$$= \left[2\pi g \cos\gamma\int_{o}^{\delta}(\rho_{B}-\rho)(R-y)\,dy\right]dx - \tau_{w}\frac{2\pi R}{\cos\gamma}\,dx . \qquad (3a)$$

Since small temperature differences are encountered, we may say $\rho/\rho_B \approx 1$, except for algebraic differences such as occur in the buoyancy term. In the buoyancy term, we let $\rho_B - \rho \approx \beta \rho_B \theta$. This results in

$$\frac{d}{dx}\left[\rho_B \int_0^\delta (R-y)\, u^2\, dy\right] = \rho_B \beta g \int_0^\delta \theta\, (R-y)\, dy - \frac{\tau_w R}{\cos^2 \gamma} . \qquad (3b)$$

Evaluating integrals gives

$$\frac{d}{dx}\left[\rho_B \;(0.052315\; R\delta U^2 - 0.0065393\; \delta^2 U^2)\right]$$

$$= \rho_B \beta g \quad (0.125\; R\delta\,\theta_w - 0.033333\,\delta^2 \theta_w) - \frac{\tau_w R}{\cos^2 \gamma} . \qquad (3c)$$

Performing the indicated differentiation, this becomes

$$0.052315\, \rho_B \left[2U(1 - \frac{\delta}{8R})\, \frac{dU}{dx} + \frac{U^2}{\delta}\, (1 - \frac{\delta}{4R})\, \frac{d\delta}{dx} + \frac{U^2}{R}\, \frac{dR}{dx}\right]$$

$$= \frac{g\, \rho_B\, \beta\, \theta_w}{8}\, (1 - 0.26667\, \frac{\delta}{R}) - \frac{\tau_w}{\delta \cos^2 \gamma} . \qquad (3d)$$

Neither θ_w or τ in this equation are known at this point. Using the Blasius correlation[18, page 143], we obtain

$$\tau_w = 0.0228\, \rho_B U^2 \left(\frac{\nu}{U \delta \cos\gamma}\right)^{1/4} . \tag{14}$$

Likewise, θ_w can be determined by combining the Reynolds analogy[18, page 203] with the above Blasius correlation plus a correction term by Colburn for Pr variation[19, page 521; 18, page 324]. This combination gives,

$$\theta_w = \frac{q_m Pr^{2/3}}{0.0228\, c \rho_B U} \left(\frac{U \delta \cos\gamma}{\nu}\right)^{1/4} . \tag{15}$$

Note that q_m is used for the heat flux term since θ_w should reflect the total heat flux pattern below the plane under consideration. Introducing these terms,

$$0.052315\, \rho_B \left[2U\left(1 - \frac{\delta}{8R}\right) \frac{dU}{dx} + \frac{U^2}{\delta} \left(1 - \frac{\delta}{4R}\right) \frac{d\delta}{dx} + \frac{U^2}{R} \frac{dR}{dx} \right]$$

$$= \frac{g \beta q_m Pr^{2/3} \delta^{1/4} \cos^{1/4}\gamma}{0.1824\, \nu^{1/4} U^{3/4}} \left(1 - 4/15 \frac{\delta}{R}\right) - \frac{0.0228\, \rho_B \nu^{1/4} U^{7/4}}{\delta^{5/4} \cos^{9/4}\gamma} . \tag{3e}$$

From this we obtain

$$\frac{dU}{dx} = \frac{52.399\, g\beta\, Pr^{2/3}\, q_m\, \delta^{1/4} \cos^{1/4}\gamma\, (1 - \frac{4}{15}\frac{\delta}{R})}{\rho_B\, c\, \nu^{1/4}\, U^{7/4}\, (1 - \frac{\delta}{8R})}$$

$$- \frac{0.2179\nu^{1/4} U^{3/4}}{\delta^{5/4} \cos^{9/4}\gamma\, (1 - \frac{\delta}{8R})} - \frac{U}{2R\,(1 - \frac{\delta}{8R})}\frac{dR}{dx} - \frac{U(1 - \frac{\delta}{4R})}{2\delta(1 - \frac{\delta}{8R})}\frac{d\delta}{dx}. \qquad (7a)$$

A. 2 Energy Balance Equation

A second equation is obtained by equating the change in thermal energy across the element to the total thermal energy entering (or leaving). In all cases, the reference temperature is the bulk fluid temperature.

Thermal energy entering in the fluid across the plane at x is given by

$$\int_o^\delta 2\pi\rho c\,\theta\,(R-y)\, u\, dy \cos\gamma,$$

and the change in thermal energy across height dx is then

$$\frac{d}{dx}\left[2\pi\rho_B c \cos\gamma \int_o^\delta \theta(R-y)\, u\, dy\right] dx.$$

Mass flow entering or leaving the element normal to the wall at the surface $y = \delta$ is at the datum temperature T_B; thus, $\theta = 0$ and no thermal energy is transported. Thermal energy is gained from the heat flux through the wall bounding the element in the amount,

$$\frac{2\pi R\, q_w}{\cos\gamma}\, dx .$$

Equating these amounts, setting $\rho = \rho_B$, and performing the indicated integration results in

$$\frac{d}{dx}\left[\theta_w U \delta\,(0.03663\,R - 0.004785\,\delta)\right] dx = \frac{R\,q_w\,dx}{c\,\rho_B \cos^2\gamma}. \quad (16)$$

Substituting for θ_w and rearranging gives

$$\frac{d}{dx}\left[R q_m U^{1/4}\,\delta^{5/4} - 0.1306\,q_m U^{1/4}\,\delta^{9/4}\right] = \frac{\nu^{1/4} q_w\,R}{1.6066\,Pr^{2/3}\cos^{9/4}\gamma}.$$

(16a)

Differentiating, collecting terms and solving for $d\delta/dx$ yields

$$\frac{d\delta}{dx} = \left[\frac{\dfrac{0.4979\,\nu^{1/4}\,q_w}{q_m Pr^{2/3} U^{1/4}\delta^{1/4}\cos^{9/4}\gamma} - \dfrac{4\,\delta}{5q_m}\left(1 - 0.1306\,\dfrac{\delta}{R}\right)\dfrac{dq_m}{dx} - \dfrac{4\delta}{5R}\,\dfrac{dR}{dx}}{\left(1 - 0.2352\,\dfrac{\delta}{R}\right)}\right]$$

$$- \left[\frac{\delta\left(1 - 0.1306\,\dfrac{\delta}{R}\right)}{5U\left(1 - 0.2352\,\dfrac{\delta}{R}\right)}\,\frac{dU}{dx}\right]. \quad (8a)$$

A.3 Gross Energy Balance Equation

Energy enters the boundary layer over the entire tank wall and is carried upward by the boundary layer. We assume that all energy entering stays in the boundary layer; therefore, the rate at which energy is carried by the boundary layer across any horizontal plane is equal to the rate at which energy is entering the boundary layer below this plane, thus:

$$2\pi \rho_B \; c \cos \gamma \int_o^{\delta} \theta \, (R-y) \, u dy = q_m A(x) , \tag{17}$$

where the plane is located a distance x above the tank bottom. Evaluating the integral and substituting for θ_w results in

$$\frac{1.6066 \; Pr^{2/3} \; RU^{1/4} \, \delta^{5/4}}{\nu^{1/4}} \left(1-0.1306 \frac{\delta}{R}\right) = \frac{A(x)}{2\pi \cos^{5/4} \gamma} . \tag{17a}$$

Solving for U,

$$U = \left[\frac{\nu^{1/4} \; A(x)}{10.095 \; Pr^{2/3} \; R \; \delta^{5/4} \left(1-0.1306 \frac{\delta}{R}\right) \cos^{5/4} \gamma} \right]^4 . \tag{9a}$$

A.4 Solution of Equations

The above developed equations are of the form:

$$\frac{dU}{dx} = A(U, \delta, x) + B(U, \delta)\,\frac{d\delta}{dx}\,, \tag{7}$$

$$\frac{d\delta}{dx} = C(U, \delta, x) + D(U, \delta)\,\frac{dU}{dx}\,, \tag{8}$$

and

$$U = U(\delta, x)\,. \tag{9}$$

Input parameters such as tank geometry, fluid properties, and heat flux to the tank are required in all these equations. Solution of the above equations is accomplished by a Newton's method iterative process. A value for δ $(=\delta_1)$ is assumed and a corresponding value for U_1 determined from equation 9a. These paired values for U_1 and δ_1 are then used in equations 7a and 8a to evaluate dU/dx and $d\delta/dx$. An incremental change in x, Δx, is then made and, using the same δ a new value U_2 is found, then $\Delta U = U_2 - U_1$. A term defined as $f_1 = \frac{\Delta U}{\Delta x} - \frac{dU}{dx}$ is then determined. δ_1 is now increased by an increment $\Delta\delta = \Delta x\, d\delta/dx$, a new set of values for U and dU/dx is calculated and an f_2 determined. The value $f' = \frac{f_2 - f_1}{\Delta\delta}$ is now found and an algebraic addition to the initial δ_1 made of the amount $-f_1/f'$. This new value for δ is then used to initiate a new series of calculations. When $|f_1|$ approaches zero within some predetermined value, then the corresponding values for δ and U are taken as the true values.

A. 5 Growth of Stratified Layer

At any plane, the volume flow rate across that plane in the boundary layer is given by

$$\dot{V} = \int_0^{\delta} 2\pi \cos \gamma (R-y)\, u\, dy$$

$$= 2\pi(0.1464\, R\delta U - 0.02723\delta^2 U) \cos \gamma . \qquad (18)$$

Since $\dot{V}\, \Delta t = \pi R^2 \Delta x$, we solve for the time increment

$$\Delta t = \frac{R}{0.2927\delta U(1-0.1860\,\frac{\delta}{R})\cos\gamma}\,\Delta Z . \qquad (19)$$

By making ΔZ sufficiently small and summing the time increments, the time, $t = \sum^{n} \Delta t$, for the stratified layer to grow to any thickness, $\Delta = \sum^{n} \Delta Z$, can be calculated.

The above developed equations apply to both the ullage volume and the liquid volume.

A.6 Temperature Distribution

During time t, thermal energy in the amount $q_m A_\ell t$ has entered the liquid and will appear in the liquid occupying the stratified layer. The question to be answered then becomes, how is this energy (as revealed by temperature) distributed in the stratified layer?

For this development, it is assumed that the energy distribution is an exponential of the form

$$E(Z) = m\ Z^n\text{, per unit volume.} \qquad (12)$$

The energy entering through the wetted wall during time t is

$$Q = q_m A_\ell\ t\ . \qquad (20)$$

The energy stored in an infinitesimal layer of liquid (in the stratified layer) is

$$dQ = \pi R^2\ E(Z)\ dZ, \qquad (13b)$$

thus

$$Q = \pi \int_0^\Delta mR^2\ Z^n\ dZ\ . \qquad (13c)$$

We assume that m takes on a series of values such that mR^2 is a constant; thus,

$$Q = \pi mR^2 \int_0^\Delta Z^n dZ = \frac{\pi\ mR^2\ \Delta^{n+1}}{n+1}\ . \qquad (13d)$$

Combining 13d and 20, we find that

$$m = \frac{q_m A_\ell \, t\,(n+1)}{\pi R^2 \Delta^{n+1}} , \tag{21}$$

and therefore that

$$dQ = \frac{q_m A_\ell \, t\,(n+1)}{\Delta} \left(\frac{Z}{\Delta}\right)^n dZ . \tag{13e}$$

Also, from the temperature of the layer:

$$dQ = \pi R^2 \rho \; c \; \theta_Z \, d\,Z . \tag{22}$$

Equating these expressions, we determine that,

$$\theta_Z = \frac{q_m A_\ell \, t\,(n+1)}{\pi R^2 \rho \; c \; \Delta} \left(\frac{Z}{\Delta}\right)^n . \tag{23}$$

θ_Z thus represents the temperature pattern over the height of the liquid stratified layer. The value for the exponent n is determined empirically and has been assigned the value 2 for most of the calculations performed. Experience has shown that the pressure rise rate is not particularly sensitive to the value chosen for the exponent.

The temperature pattern in the ullage space is determined by a different method. In the ullage space, the stratified layer depth, corresponding to the time t found for a particular depth of liquid stratified layer, is divided into small vertical increments. All of the energy entering through the ullage sidewall during the incremental time it takes for the uppermost increment to develop due to boundary layer flow is assumed to be uniformly distributed in this increment. Energy entering the ullage sidewall below this first increment during the incremental time to develop the next layer is evenly divided between the first and second layer increment; during the next time increment, the energy is evenly divided among the top three layers, then the top four, etc., continuing in this manner until the entire depth of the stratified layer has been covered. In addition, each layer accumulates all of the energy entering the sidewall area it is exposed to from the time the stratified layer depth includes that layer until the total stratified layer depth (Δ_v) is reached. Temperature in each layer is then obtained from the equation

$$\theta_{Zv} = \frac{E_{Zv}}{\pi \rho c R^2 \Delta Z_v} . \qquad (24)$$

A. 7 Ullage Mass Determination

Mass contained in the ullage space is obtained by a numerical integration. At any specific point in time, the mass contained in the ullage volume below the stratified layer is given by

$$m_1 = \rho_s V . \qquad (25)$$

In the stratified layer, a numerical integration is conducted with each of the increments containing a mass

$$\Delta m = \pi R^2 \rho(\theta_Z) \Delta Z_v , \tag{26}$$

where the density, ρ, is a function of pressure and the local temperature.

The mass contained in the ullage at t = 0 is given by

$$m_i = \rho_s V_u, \tag{27}$$

where ρ_s is the saturated vapor density at the initial pressure.

A. 8 Liquid-Vapor Interface Equilibrium

At time t = 0 it is assumed that the temperature of both liquid and vapor is uniform at the saturation temperature corresponding to the existing ullage pressure. The initial vapor mass contained in the ullage space is determined by the methods outlined in section A. 7. At time t_1, corresponding to the growth of the liquid stratified layer to depth $\Delta_{\ell 1}$ and the accompanying growth of the vapor stratified layer to depth Δ_{V1}, temperature gradients are computed for both the liquid and vapor phases by the methods of section A. 6.

Determination of the ullage pressure begins by assuming an ullage pressure corresponding to the saturation pressure for the temperature existing at the liquid surface. With this pressure and the previously established ullage space temperature gradient, a mass contained in the ullage is determined. This calculated new ullage mass is then compared with the ullage mass at time t = 0; if the new mass is larger than the initial mass, then the usual situation of liquid vaporization is required; if the calculated mass is smaller, then condensation of some vapor must occur in order to achieve equilibrium between the ullage space and the liquid surface. In the unlikely case of the new mass being equal to the initial mass, then the ullage and liquid are in equilibrium and a new series of stratified layer calculations can commence.

For the vaporizing case, a new temperature, less than the calculated surface temperature, is assumed and the corresponding saturation pressure for that temperature assumed for the ullage. An ullage mass calculated using this pressure and the previously determined temperature gradient is compared with the adjusted initial ullage mass (adjusted for the mass of liquid vaporized in order to reduce the temperature of the liquid layers adjoining the surface to the assumed temperature),

$$m_a = m_i + \Delta m_v .$$

Successive calculations permit bracketing of the equilibrium value, which is finally determined by linear interpolation between two bracketing values. The energy contained in the surface layer of liquid that is used for vaporizing liquid is found from

$$E_\ell = \pi \rho_B c \int_o^{Z_1} R^2 (T_\ell - T_s)\, dZ. \tag{28}$$

The mass vaporized is then determined to be

$$\Delta m_v = E_\ell / L_v . \tag{29}$$

For the condensing case, a similar process is followed; however, the liquid temperature gradient produced by condensation is not constant as is the case when vaporizing. Determination of the gradient proceeds by assuming a surface temperature and computing a gradient from the error function relation[10],

$$\frac{T_{s\ max} - T_s}{T_{s\ max} - T_B} = \frac{2}{\sqrt{\pi}} \int_o^{\infty} e^{-\lambda^2}\, d\lambda , \tag{30}$$

where $\varphi = \frac{Z}{2\sqrt{\alpha t}}$. $T_{s\ max}$ is the assumed surface temperature for a particular iteration. E_ℓ is calculated as before (note that E_ℓ will now be negative since $T_s > T_\ell$ and therefore Δm_v will be negative). The integration is carried only to the depth where $T_\ell = T_s$.

Adjustments to the ullage pressure for the thermal energy entering the tank bottom are made for each pressure calculation step by a manual calculation. It is assumed that all thermal energy entering is uniformly distributed in the liquid, i.e.,

$$\dot{q}t = \rho_B \, c \, V_e \, \Delta T,$$

where V_e is the volume of liquid in the tank. Thus

$$\Delta T = \frac{\dot{q}t}{\rho_B \, c \, V_e} \quad .$$

Pressure is assumed to be a linear function of temperature over the small interval considered. The maximum pressure difference and the corresponding liquid surface temperature difference are used to establish a ΔP as a function of ΔT, thus:

$$\Delta P = \Delta T \, \frac{\Delta P_{max}}{\Delta T_{max}} \quad .$$

Substituting we have

$$\Delta P = \frac{\dot{q}}{\rho_B \, c_p \, V_e} \, \frac{\Delta P_{max}}{\Delta T_{max}} \, t.$$

This value is added to each pressure in the summary using the corresponding value for t.

B. Computer Program

A computer program has been written to solve and combine solutions of the various equations developed earlier for the primary purpose of predicting the pressure history of a closed liquid hydrogen tank. The program consists of a main calling program, plus 26 subprograms, together with some standard library routines. Development and predominant use of this program have been accomplished using a Control Data Corporation* (CDC) 3600 computer; verification runs have been made on an International Business Machines*(IBM) 360/65 computer.

Writing of the program was aimed at permitting the most general input so that external parameters can be readily varied. Circumferential symmetry was assumed in the mathematical development and must be maintained. Variables which can be inserted include axial heat flux variations, tank geometry (i.e., radius, height, nose cone slope), fluid properties, starting pressure, maximum pressure, liquid quantity, and local acceleration force. When the nose of a tank is tapered, the shape is approximated with a cone.

A brief description of the functions of the main calling program and the subprograms is presented in the order that they would appear in the deck. Listings of the program and the nomenclature of the important variables are also included.

B.1 Program Description

RAVE

This is the main calling program; it calls the subprograms in the correct order, detects when the pressure limit is exceeded

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* Precise specification of the computer employed has been necessary to make the description sufficiently meaningful. Identification of this computer or its manufacturer by the National Bureau of Standards in no way implies a recommendation or endorsement by the Bureau.

and prints out a summary of the results. Input required by the program consists of 40 cards containing fluid properties, plus four cards containing tank geometry, heat flux, initial condition, limiting conditions, and exponent to be used for liquid temperature variation.

The general calling sequence of RAVE is MASSS, PROPERTY, DTL, DTEMPV, THETAL, THETAV, and MASSC. This sequence is used repetitively starting with an increment, Δ, of liquid stratified layer thickness and successively increases this thickness by the same increment until the ullage pressure exceeds the pressure limit established as part of the input data. The printed summary includes the stratified layer thickness, elapsed time, tank pressure, pressure rise rate, pressure difference, and time difference.

Subprograms

MASSS

Calculates the initial mass of vapor in the ullage volume under the assumption that the vapor is saturated at the initial pressure. Subprogram FINDD is called by MASSS to determine the vapor density.

FINDD

Calculates the density of parahydrogen when given the pressure and temperature. An initial estimate of density is needed when saturated conditions exist in order to guide the result toward the liquid or vapor value as desired. Note that this subprogram is for parahydrogen only; if another fluid is used, then a different program will be required.

PROPERTY

Uses input cards of temperature and viscosity to calculate liquid and vapor properties for use by other subprograms.

DTL

Calculates the time for the stratified layer to occupy a specified thickness, Δ. Calls subprogram ULIQ for use in calculations.

ULIQ

This subprogram calculates the liquid boundary layer thickness and a velocity parameter. It calls DQML, UCALCL and DUCALL for use in the calculation.

DQML

Calculates the mean heat flux, q_m, over the tank wall below location x and the rate of change of q_m with height at x, i.e., dq_m/dx. q_w is drawn into the program from common via the values for the constants in the linear equation for q_w (see QWL).

UCALCL

Calculates the velocity parameter U when given a boundary layer thickness, δ. Used in the iteration procedure of DTL.

DUCALL

Given a paired set of values for U and δ (from UCALCL) this subprogram calculates a value for dU/dx. QWL is used in this subprogram.

QWL

From the set of heat flux values given as input data, this subprogram fits a linear function between adjacent locations. Where a change in heat flux occurs, and at the liquid-vapor interface, a finite distance must separate heat flux input values, i.e., the heat flux at any specific location must be unique.

DTEMPV

Calculates the time for the ullage space stratified layer to reach a certain depth. Determines a stratified layer thickness corresponding to the time calculated in DTL by an iteration procedure. Calls UVAP and INTEGRAT.

UVAP

Calculates, via an iteration procedure, a consistent set of values for U_v and δ_v. Uses subprograms DQMV, UCALCV, and DUCALV.

INTEGRAT

Uses the Newton-Cotes numerical integration method to integrate over a set of values using a minimum of seven evenly spaced points.

DQMV

Calculates values for the mean heat flux, q_{mv}, and the rate of change of q_{mv} with height, dq_{mv}/dx. Uses output of QWV.

UCALCV

Calculates a value for the velocity parameter, U_v, in the ullage space when given a boundary layer thickness, δ_v.

DUCALV

Calculates the rate of change of U_v with height, dU_v/dx, when given a paired set of values for U_v and δ_v. Uses QWV in the calculations.

QWV

Uses the same set of input data for heat flux as QWL and supplies a linear fit between successive values.

THETAL

Determines the temperature distribution in the liquid stratified layer based on the assumed exponential variation of specific energy. The exponent used is part of the input data. Subprograms ULIQ, DQML, and INTEGRAT are used.

THETAV

Determines the temperature distribution in the ullage stratified layer by the method described by equation 24, appendix A. 6. Uses PTCP in the calculations.

PTCP

Calculates specific heats for either the liquid or vapor state using input data from CSUBP or CSUBV. Values produced are for parahydrogen only and substitution must be made if another liquid is used.

MASSC

Determines equilibrium value for liquid vapor interface. Subprograms VP, MASSU, INTEGRAT, ERF and DMF are used. Iterates on a final value that assures pressure and temperature equilibrium at the interface as well as a mass and energy balance.

VP

Calculates a saturated vapor pressure corresponding to a given temperature. Gives values for only parahydrogen as written; a corresponding program would be needed for a different fluid.

MASSU

Calculates the mass contained in the ullage volume when given a temperature distribution and a pressure. Uses subprograms FINDD and INTEGRAT.

ERF

Calculates values of the error function for values of the argument supplied by the using subprogram. Used by MASSC to treat the condensing case.

DMF

Calculates the latent heat of vaporization at an average pressure between the pressure of the previous step and the current try. Used in MASSC to compute the mass vaporized or condensed. Values are for parahydrogen only; a corresponding program would be needed for a different fluid.

CSUBP

A tabular set of data of constant pressure specific heats for parahydrogen only. Used by PTCP.

CSUBV

A tabular set of data of constant volume specific heats for parahydrogen only. Used by PTCP.

In addition to the above described programs, certain other library programs are used in various of the subprograms. These are:

TANF

Returns the tangent of an angle when given that angle.

COSF

Returns the cosine of an angle when given that angle.

SQRTF

Extracts the square root of a number.

EXPF

Returns the value e^n when given n.

B.2 Program Nomenclature

Symbol	Description
AL	Thermal diffusivity, liquid, cm^2/s.
AQ	Constant in linear equation for q_w.
AV	Thermal diffusivity, vapor, cm^2/s.
BETAL	Thermal coefficient of volumetric expansion, liquid, 1/K.
BETAV	Thermal coefficient of volumetric expansion, vapor, 1/K.
BQ	Constant in linear equation for q_w.
COSA COSG	Cosine of GAMMA.
CPL	Constant pressure specific heat, liquid, J/g-mol·K.
CPV	Constant pressure specific heat, vapor, J/g-mol·K.
DELTAL	Stratified layer thickness, liquid, cm.
DELTAV	Stratified layer thickness, vapor, cm.
DENL	Specific volume, saturated liquid, $cm^3/g\text{-}mol$.
DENV	Specific volume, saturated vapor, $cm^3/g\text{-}mol$.
DPDRL	Value of $(\partial P/\partial \rho)_T$, saturated liquid, $atm\cdot cm^3/g\text{-}mol$.
DPDRV	Value of $(\partial P/\partial \rho)_T$, saturated vapor, $atm\cdot cm^3/g\text{-}mol$.

DPDTL	Value of $(\partial P/\partial T)_\rho$, saturated liquid, atm/K.
DPDTV	Value of $(\partial P/\partial T)_\rho$ saturated vapor, atm/K.
DRDX	Rate of change of tank radius with height, dimensionless.
GAMMA	Half angle at nose cone, rad.
G	Axial acceleration field, cm/s^2.
H	Distance, liquid surface to top of tank, cm.
LC	Height of cylindrical portion of tank, cm.
L	Height of liquid in tank, cm.
MASSI	Initial mass of vapor in ullage space, g.
NUL	Kinematic viscosity, liquid, cm^2/s.
NUV	Kinematic viscosity, vapor, cm^2/s.
PL	Tank pressure upper limit, atm.
POWER	Exponent used for specific energy distribution in liquid stratified layer.
PR	Prandtl number, liquid.
PRV	Prandtl number, vapor.
P	Current tank pressure, atm.
PS	Initial tank pressure, atm.
Q	Wall heat flux at discrete points, W/cm^2.

QW	Linearized value for wall heat flux over tank height, W/cm^2.
RC	Cylindrical radius of tank, cm.
RHO	Density used during current calculation, liquid, g/cm^3.
RHOV	Density used during current calculations, saturated vapor, g/cm^3.
R	Local tank radius, cm.
RS	Tank radius at liquid surface, cm.
SL	Entropy, saturated liquid, J/g-mol· K.
SV	Entropy, saturated vapor, J/g-mol· K.
TAMB	Ambient temperature, K.
TANG	Tangent of GAMMA.
TB	Bulk temperature, liquid, K.
TDL	Local temperature increase in stratified layer, liquid, K.
TDV	Local temperature increase in stratified layer, vapor, K.
TH	Boundary layer thickness, liquid, cm.
THV	Boundary layer thickness, vapor, cm.
TIME	Current time to achieve DELTAL, s.

TSAT	Saturation temperature corresponding to P, K.
U	Boundary layer velocity parameter, liquid, cm/s.
UV	Boundary layer velocity parameter, vapor, cm/s.
X	Vertical distance variable, cm.
ZINC	Number of increments of DELTAL and DELTAV.
Z	Vertical distance in stratified layer, liquid, cm.
ZV	Vertical distance in stratified layer, vapor, cm.

```
      PROGRAM  RAVE
      DIMENSION TABSL(101),TABSV(101),ZY(101)
    1  COMMON AL,AQ(100),AV,BETAL,BETAV,BQ(100),COSA,COSG,CPL(20),CSL,
     1 CPV(20),CSLA,CSV(20),DELTAL(50),DELTAV(50),DENL(20),DENV(20),DL,
     2 DPDRL(20),DPDRV(20),DPDTL(20),DPDTV(20),DRDX,DVP,FP,G,GAMMA,H,
     3 I,IDX,IU,IXV,IZ,JERROR,JTR,KB,KD,L,LC,MASS,MASS1,NPT,NTR,NUL,
     4 NULA,NUV,NX,P,PA(50),POWER,PR,PRV,PT(20),Q(100),QW,R,RC,RHO,
     5 RHOV,RS,SL(20),SLC,SV(20),TAMB,TANG,TB,TDL(101),TDV(101),TH,THV,
     6 TIME(50),TSAT(20),U,UV,X,XB(100),Y(101),Z(101),ZINC,ZV(101)
    2  FORMAT(10E8.0/2E10.0)
    3 FORMAT(4F10.0)
    4 FORMAT(5F10.0)
    5 FORMAT(4F10.0,9X,I1)
            REAL  L,LC,MASS,NUL,LV,NUV
       NTR = 1
      NPT=1
       JTR = 1
      READ(5,2)(PT(I),DENL(I),DPDRL(I),DPDTL(I),DENV(I),DPDRV(I),DPDTV(I
     1) ,SL(I ),SV(I),TSAT(I),CPV(I),CPL(I),I=1,20)
    8  READ(5,4)     GAMMA, RC, LC, L, H
    9 READ(5,5) G,QW,DL,ZINC,NR
   10  READ(5,3)      PS, TAMB, PL
      READ (5,1111) POWER
 1111 FORMAT(F10.0)
      GAMMA= 3.1415926535*GAMMA/180.0
      JJ=ZINC
      NPTS=JJ+1
       JERROR = 1
       KB = 1
       KD = 1
      DO 600 II=1,50
  600 DELTAL(I)=0.0
   11  DELT LO = 0.0
   12  DELT VO = 0.0
   13  DVP = 0.0
   14  KR = 1
   16  P = PS
   17 CONTINUE
      WRITE(6,171) DL,PL  ,ZINC,PS,TAMB
  171 FORMAT(18X,2HDL,17X,3HPL  ,16X,4HZINC,17X,2HPS,16X,4HTAMB/5F20.2)
  172 FORMAT(10X,     91H      GAMMA                        RC
     1LC                     L                         H /F20.5,4F20.2)
      WRITE(6,172)        GAMMA,RC,LC,L,H
      WRITE(6,173)        G,QW
  173 FORMAT(16X,1HG,20X,2HQW/ 2F20.3)
      WRITE (6,1112) POWER
 1112 FORMAT(12X,12H POWER    =   ,F10.4)
   18 CONTINUE
   19  CALL MASS S
      WRITE(6,180)
  180  FORMAT(50X,16HMASS S INCREMENT)
   20  CALL PROP  TY
  190  FORMAT(50X,20HPROPERTY INCREMENT,1)
      WRITE(6,190)
```

```
 21   GO TO 27
 22   DO 24 I = 2,20
 23   IF(PT(I)-P) 24,25,25
 24   CONTINUE
 25   CALL PROP  TY
210   FORMAT(50X,20HPROPERTY INCREMENT 2)
     WRITE(6,210)
 27   CALL DTL
      GO TO (221,42),JERROR
221  WRITE(6,220)
220   FORMAT(50X,13HDTL INCREMENT)
 28   CALL  DTEMP V
      GO TO (231,42),JERROR
231  WRITE(6,230)
230   FORMAT(50X,17HDTEMP V INCREMENT)
 29   CALL THETA L
      GO TO (241,42),JERROR
241  WRITE(6,240)
240   FORMAT(50X,17HTHETA L INCREMENT)
 30   CALL THETA V
      GO TO (244,42),JERROR
244  GO TO (245,251),NPT
245  IF(NX.LT.3) GO TO 251
     WRITE(6,500)(XB(JJ),Q(JJ),AQ(JJ  ),BQ(JJ  ),JJ=1,NX)
500  FORMAT(16X,    2HXB,20X,2HQ ,16X,2HAQ,19X,2HBQ//(2X,2F20.5,2E20.4)
    1)
250   FORMAT(50X,17HTHETA V INCREMENT)
251  WRITE(6,250)
     NPT=2
     WRITE(6,502)
502  FORMAT(14X,1HZ,15X,3HTDL,13X,5HTABSL,14X,2HZV,14X,3HTDV,
    1        13X,5HTABSV /)
     DO 505 JJ=1,NPTS
     TABSL(JJ) = TB + TDL(JJ)
     TABSV(JJ) = TB + TDV(JJ)
504  ZY(JJ) = L + Z(JJ) - DELTAL(KB)
505  WRITE(6,510)ZY(JJ),TDL(JJ),TABSL(JJ),ZV(JJ),TDV(JJ),TABSV(JJ)
510  FORMAT(2(4X,2E16.4,5X,F10.3))
 31   CALL MASSC
      GO TO (261,42),JERROR
260   FORMAT(50X,16HMASS C INCREMENT)
261  WRITE(6,260)
 33   JTR = 2
 35   IF(KR.GT.1) GO TO 39
 36   PA0 = PS
      PA(KR) = P
      IF(P.GT.PL) GO TO 42
      TIME0 = 0.0
 37   KR = 2
 38   GO TO 22
 39   PA(KR) = P
      IF(P.GT.PL) GO TO 42
 40   KR = KR+1
 41   GO TO 22
```

```
42 WRITE(6,45)
    KO = 0
   WRITE (6,46)     PAO,TIMEO,DELT LO,DELT VO,KO
43 WRITE(6,46)(PA(K),TIME(K),DELTAL(K  ),DELTAV(K  ), K   ,K=1,KR)
45  FORMAT(1H1,6X,8HPRESSURE,11X,4HTIME,9X,6HDELTAL,9X,6HDELTAV,9X,
  1  1HK/5X,11HATMOSPHERES,8X,7HSECONDS,5X,11HCENTIMETERS,4X,11HCENTI
  2METERS/)
46  FORMAT( 4E15.5,I10)
47 WRITE(6,55)
    DELPA = PA(1)-PAO
    DELTB = TIME(1)-TIMEO
    PRISEA = DELPA/DELTB
    IDA = 1
   WRITE(6,54)       PRISEA,DELPA,DELTB,IDA
48  DO 53 ID = 2,KR
49  IF( KR.GT.50) GO TO 59
50  DELP = PA(ID) - PA(ID-1)
51  DELT = TIME(ID) - TIME(ID-1)
52  PRISE = DELP/DELT
53 WRITE(6,54)       PRISE,DELP,DELT,ID
54  FORMAT (3E30.5,I20)
55  FORMAT(1H0,15X,18HPRESSURE RISE RATE,12X,19HPRESSURE DIFFERENCE,
  1 13X,15HTIME DIFFERENCE,13X,9HINCREMENT/22X,7HATM/SEC,24X,3HATM,
  2  28X,3HSEC/)
56 WRITE(6,57)
57  FORMAT(3X, 59HTANK PRESSURE HAS EXCEEDED UPPER LIMIT, SCRATCH ONE
  1 CENTAUR)
58  GO TO 61
59 WRITE(6,60)
60  FORMAT(5X,37HINCREMENTS OF PRESSURE RISE EXCEED 50)
61 IF(NR.EQ.0) GO TO 62
   NTR=NTR+1
   NPT = 1
   GO TO 8
62 CONTINUE
    END
```

```
   SUBROUTINE  MASS S
    THIS ROUTINE COMPUTES MASS CONTAINED IN ULLAGE GAS
       SPACE WHEN NO TEMPERATURE GRADIENT EXSISTS
 1   COMMON AL,AQ(100),AV,BETAL,BETAV,BQ(100),COSA,COSG,CPL(20),CSL,
    1 CPV(20),CSLA,CSV(20),DELTAL(50),DELTAV(50),DENL(20),DENV(20),DL,
    2 DPDRL(20),DPDRV(20),DPDTL(20),DPDTV(20),DRDX,DVP,FP,G,GAMMA,H,
    3 I,IDX,IU,IXV,IZ,JERROR,JTR,KB,KD,L,LC,MASS,MASS1,NPT,NTR,NUL,
    4 NULA,NUV,NX,P,PA(50),POWER,PR,PRV,PT(20),Q(100),QW,R,RC,RHO,
    5 RHOV,RS,SL(20),SLC,SV(20),TAMB,TANG,TB,TDL(101),TDV(101),TH,THV,
    6 TIME(50),TSAT(20),U,UV,X,XB(100),Y(101),Z(101),ZINC,ZV(101)
         REAL L,LC,MASS,MASS1
 3   DO 5  I = 2,20
4   IF (PT(I)-P) 5,6,6
 5   CONTINUE
 6   TB = FINT(TSAT)
 7   RHOG = (FIND D(P,0.50,TB))*0.00201572
 8   COSG = COSF(GAMMA)
 9  TANG = SIN(GAMMA)/COS(GAMMA)
10   IF (L.LT.LC) GO TO 14
11   RS = RC-(L-LC)*TANG
12   MASS = RHOG*3.14159*((RS**2)*H-RS*(H**2)*TANG+(((H**3)*(TANG
  1   **2))/3.0))
    WRITE(6,22) MASS
     MASS1 = MASS
13   RETURN
14   RS = RC
15   H1 = H
16   H = H-(LC-L)
17   MASSA=RHOG*3.14159*((RS**2)*H-RS*(H**2)*TANG+(((H**3)*(TANG
  1   **2))/3.0))
18   H = LC-L
19   MASSB = RHOG*3.14159*(RC**2)*H
20   MASS = MASSA+MASSB
21  WRITE(6,22) MASS
22   FORMAT(24X,                    4HMASS /E20.4)
23   MASS1 = MASS
24   H = H1
25   RETURN
     END
```

```
      FUNCTION FINDD(PRES,TG,TEMP)
      P=PRES
      T=TEMP
      IF(TG.NE.0.0) GO TO 6
      DF=((T/13.803)**0.3104277)/26.176
      IF(T.GT.100.) GO TO 6
      IF(T.GT.34.) GO TO 7
      IF(P.GT.12.759)GO TO 4
      IF(T.LT.13.803) T=13.803
      IF(T.LT.20.268) GO TO 1
      V=2.00062-50.09708/(T+1.0044)+0.01748495*T
      VP=10.0**V
      IF(T.LE.29.0) GO TO 2
      A=T-29.0
      TTT=A*A*A
      TF=TTT*A*A
      TS=TTT*A*TTT
      VP=VP+0.001317*TTT-0.00005926*TF+0.000003913*TS
      GO TO 2
    1 VP=10.0**(1.772454-44.36888/T+0.02055468*T)
    2 IF(VP-P)3,6,6
    3 D=.0417-0.0000154*T*T
      IF(0.0695+(T-13.803)*(30.3312*EXP(-5.693/T)+(T+T)/3.)-P)5,5,8
    4 IF(0.0695+(T-13.803)*(30.3312*EXP(-5.693/T)+(T+T)/3.)-P)5,5,7
    5 FINDD=DF *1000.028
      RETURN
    6 D=P/T/80.
      GO TO 8
    7 D=(DF*(100.-T)+(T-22.)*P/8206.)/78.
    8 R=26.176*D
      RR=R*R
      RRR=R*RR
      RRRR=RR*RR
      X=13.803/T
      SSX=SQRT(SQRT(X))
      S=D/.01544
      A=T/32.976
      A=A*A
      A=A*A
      T8=A*A
      SMO=S-1.0
      PHI=0.0
      Z=0.0
      C=(((((((10.72108689-0.77025578*RRRR)*RRR-67.14588122)*RR+
     1239.11305168)*R-294.2851871)*R+128.32512808)*R-13.98447635
     2)*R-2.01760851)*R
      BEXP=   ((((1.83199451-0.94457676*R)*RR-0.24100077)*R-2.2337918
     1)*R+1.96823016)
      BEXP=EXP(BEXP)
      B=R*(BEXP-1.14271)
      AEXP=EXP(-1.42684*R)
      A=2.13502896*R+1.01704119*RR*AEXP
      PI=(81.*R-697.)*RR
      PEXP=EXP(A*SSX+B*X+C*X*X*X+Z)
```

```
      P=PI+D*T*82.0597*PEXP
      DPI=(243.*R-1394.)*R
      DADR=2.13502896+(2.03408238-1.45115505*R)*R*AEXP
      DBDR=-1.14271+((((7.32797804-4.7228838*R)*RR-0.48200154)*R
     1-2.2337918)*R+1.)*BEXP
      DCDR=((((((107.2108689-10.78358092*RRRR)*RRR-470.02116854)
     1*RR+1195.5652584)*R-1177.1407484)*R+384.97538424)*R-27.9689527)
     2*R-2.01760851
      DZDS=((((0.15360072*S-0.5492768736)*S+0.5995350712)*S-0.1811562976
     1)*S-0.01617496)*PHI
      DPDR=26.176*DPI+82.0597*T*(1.+26.176*D*(DADR*SSX+DBDR*X+DCDR*X*X*X
     1+2.4742833*DZDS))*PEXP
      DN=D-(P-PRES)/DPDR
      IF(DN)9,9,10
    9 DN=1.0E-7
   10 PCT=ABS(DN-D)/D
      D=DN
      IF(PCT-0.00005)11,8,8
   11 FINDD=D *1000.028
      RETURN
      END
```

```
      FUNCTION FINT(PU)
 1    COMMON AL,AQ(100),AV,BETAL,BETAV,BQ(100),COSA,COSG,CPL(20),CSL,
     1 CPV(20),CSLA,CSV(20),DELTAL(50),DELTAV(50),DENL(20),DENV(20),DL,
     2 DPDRL(20),DPDRV(20),DPDTL(20),DPDTV(20),DRDX,DVP,FP,G,GAMMA,H,
     3 I,IDX,IU,IXV,IZ,JERROR,JTR,KB,KD,L,LC,MASS,MASSI,NPT,NTR,NUL,
     4 NULA,NUV,NX,P,PA(50),POWER,PR,PRV,PT(20),Q(100),QW,R,RC,RHO,
     5 RHOV,RS,SL(20),SLC,SV(20),TAMB,TANG,TB,TDL(101),TDV(101),TH,THV,
     6 TIME(50),TSAT(20),U,UV,X,XB(100),Y(101),Z(101),ZINC,ZV(101)
      DIMENSION PU(101)
10    FINT      =PU(I)-(PU(I)-PU(I-1))*(PT(I)-P)/(PT(I)-PT(I-1))
      RETURN
      END
```

```
      SUBROUTINE PROP   TY
      THIS ROUTINE COMPUTES THE FLUID PROPERTIES OF SUBJECT LIQUID
     SOURCES OF CORRELATIONS AND TABLES USED BELOW
       LIQUID PROPERTIES ' NBS REPORT 7987,RODER,WEBER AND GOODWIN
       CSV     7987 AND DEFINITION
       NUL AND NUV       CDC R'349    DILLER
       KONL       COMPENDIUM AND 7987+CALCULATION
      KONV   COMPENDIUM + CALCULATIONS
       LATENT HEAT       7987+DEFINITION
    1 COMMON AL,AQ(100),AV,BETAL,BETAV,BQ(100),COSA,COSG,CPL(20),CSL,
     1 CPV(20),CSLA,CSV(20),DELTAL(50),DELTAV(50),DENL(20),DENV(20),DL,
     2 DPDRL(20),DPDRV(20),DPDTL(20),DPDTV(20),DRDX,DVP,FP,G,GAMMA,H,
     3 I,IDX,IU,IXV,IZ,JERROR,JTR,KB,KD,L,LC,MASS,MASS1,NPT,NTR,NUL,
     4 NULA,NUV,NX,P,PA(50),POWER,PR,PRV,PT(20),Q(100),QW,R,RC,RHO,
     5 RHOV,RS,SL(20),SLC,SV(20),TAMB,TANG,TB,TDL(101),TDV(101),TH,THV,
     6 TIME(50),TSAT(20),U,UV,X,XB(100),Y(101),Z(101),ZINC,ZV(101)
           REAL  L,LC,MASS,NUL,LV,NUV,KONL,KONB,KONV,NUOTV,NULA,NU,MUL
      DIMENSION SATT(35),NU(35)
    6 FORMAT (2E8.0)
    7 GO TO (8,9), JTR
    8 READ(5,6)(SATT(K),NU(K),K=1,35)
    9 TTSAT = FINT(TSAT)
      GO TO(10,27),JTR
   10 CSL = FINT(CPL)/2.016
   11 CSLA=CSL
   12 RHO = 2.016/FINT(DENL)
   13 DO 15 K=2,35
   14 IF(SATT(K)-TTSAT)15,16,16
   15 CONTINUE
   16 NUL=NU(K)-(NU(K)-NU(K-1))*(SATT(K)-TTSAT   )/(SATT(K)-SATT(K-1))
   17 MUL = NUL
   18 NUL = NUL / RHO
   19 NULA = NUL
   20 KONL = ((1.702+0.05573*TTSAT)*10.0**(-4))*4.184
   21 AL = KONL/(RHO*CSL)
   22 DEL = FINT(DENL)
   23 DPL = FINT(DPDTL)
   24 DRL = FINT(DPDRL)
   25 BETAL = DEL*(DPL/DRL)
   26 PR L  = NUL/AL
      SLC = FINT(SL)
   27 NUOTV=8.5558*((TTSAT**1.5)/(TTSAT+19.55))*((TTSAT+650.39)
     1 /(TTSAT+1175.9))*(10.0**(-6   ))
   28 RHOV = 2.016/FINT(DENV)
   32 NUV = NUOTV/RHOV
   33 KONV = (7.40*TTSAT+7.0)*(10.0**(-6))
   35 CSV(1) = 0.9*(FINT(CPV))/2.016
   36 AV=KONV/(RHOV*CSV(1))
   37 DEV = FINT(DENV)
   38 DPV = FINT(DPDTV)
   39 DRV = FINT(DPDRV)
   40 BETAV = DEV*(DPV/DRV)
   41 PRV = NUV/AV
     WRITE(6,130)PT(I),DENL(I),DPDRL(I),DPDTL(I),DENV(I),DPDRV(I),MUL,
```

```
   1PRL
130   FORMAT(1H0,3X,2HPT,8X,4HDENL,7X,5HDPDRL,7X,5HDPDTL,8X,4HDENV,7X,
    1   5HDPDRV,9X,3HMUL,9X,3HPRL/8E12.4)
132   FORMAT(3X,5HDPDTV,10X,2HSL,10X,2HSV,8X,4HTSAT,9X,3HCPV,11X,1HP,
    1  9X,3HPRV/7E12.4)
 42   FORMAT(7E14.4,I5)
142   FORMAT (8E14.4,I5)
 43 FORMAT(9X,3HCSV   ,9X,5HTTSAT,11X,3HCSL,9X,5HNUOTV,11X,3HRHO,10X
    1     ,4HRHOV,11X,3HNUL,13X,1HI)
    WRITE(6,132)DPDTV(I),SL(I),SV(I),TSAT(I),CPV(I),P,PRV
 44 WRITE(6,43)
 45 WRITE(6,42)CSV(1),TTSAT,CSL,NUOTV,RHO,RHOV,NUL,I
 46 FORMAT(11X,3HNUV,10X,4HKONL,12X,2HAL,             10X,4HKONV,11X,
    1     3HCSV,12X,2HAV,9X,5HBETAV,9X,5HBETAL,4X,1HK)
 47 WRITE(6,46)
 48 WRITE(6,142)NUV,KONL,AL,       KONV,CSV(1),AV,BETAV    ,BETAL    ,K
 50   RETURN
      END
```

```
      SUBROUTINE DT L
C     THIS ROUTINE COMPUTES THE GROWTH RATE OF THE STRATIFIED LIQUID
C     LAYER
    1 COMMON AL,AQ(100),AV,BETAL,BETAV,BQ(100),COSA,COSG,CPL(20),CSL,
     1 CPV(20),CSLA,CSV(20),DELTAL(50),DELTAV(50),DENL(20),DENV(20),DL,
     2 DPDRL(20),DPDRV(20),DPDTL(20),DPDTV(20),DRDX,DVP,FP,G,GAMMA,H,
     3 I,IDX,IU,IXV,IZ,JERROR,JTR,KB,KD,L,LC,MASS,MASS1,NPT,NTR,NUL,
     4 NULA,NUV,NX,P,PA(50),POWER,PR,PRV,PT(20),Q(100),QW,R,RC,RHO,
     5 RHOV,RS,SL(20),SLC,SV(20),TAMB,TANG,TB,TDL(101),TDV(101),TH,THV,
     6 TIME(50),TSAT(20),U,UV,X,XB(100),Y(101),Z(101),ZINC,ZV(101)
      DIMENSION F(7)
            REAL  L,LC,MASS,NUL,LV,NUV,KONL,KONB,KONV,NUOTV,NULA,NU
    3 IDX = 1
    4 B = BETAL
    5 PR=NUL/AL
    6 GO TO (9,57),NPT
    9 NX=0
   10 READ (5,11) XB(NX+1),Q(NX+1)
   11 FORMAT(2F10.0)
      IF(Q(NX+1)) 13,13,12
   12 NX=NX+1
      GO TO 10
   13 IF(NX)14,14,17
   14 NX=3
      XB(1)=0.0
      XB(2)=LC
      XB(3)=L+H
      BQ(1)=0.0
      BQ(2)=0.0
      BQ(3)=0.0
      AQ(1)=0.0
      AQ(2)=QW
      AQ(3)=QW
      GO TO 22
   17 BQ(1)=0.0
      AQ(1)=0.0
      DO 21 K=2,NX
      BQ(K)=(Q(K-1)-Q(K))/(XB(K-1)-XB(K))
      AQ(K)=Q(K-1)-BQ(K)*XB(K-1)
   21 CONTINUE
   22 CONTINUE
      DO 25 J=2,NX
      IF(XB(J).GT.L) GO TO 26
   25 CONTINUE
   26 CONTINUE
      IXV=J
      TIMEP=0.0
      DELTAL(1)=DL
      DELTAL(1)=DL
      GO TO 58
   57 DELTAL(KB)=DELTAL(KB-1)+DL
   58 X=L-DELTAL(KB)
      IF(X.LE.L) GO TO 61
   59 WRITE (6,60) X
```

```
60 FORMAT(8H1FOR X = F9.2,20H, X IS OUT OF RANGE.   )
   JERROR=2
   RETURN
61 F(4)=X+DL/2.
   F(3)=F(4)-DL*0.2029225757
   F(5)=F(4)+DL*0.2029225757
   F(2)=F(4)-DL*0.3707655928
   F(6)=F(4)+DL*0.3707655928
   F(1)=F(4)-DL*0.4745539562
   F(7)=F(4)+DL*0.4745539562
   ASSIGN 63 TO KTR
   DO 66 N=1,7
   X=F(N)
   GO TO 70
63 CONTINUE
66 F(N)=(R-TH*(2.0-TH/R))/(2.0*U*TH*COSA*(.1464-.02723*TH/R))
   TIME(KB)=TIMEP+(F(4)*0.4179591836+(F(3)+F(5))*0.3818300505+(F(2)+
  1F(6))*0.2797053915+(F(1)+F(7))*0.1294849662)*DL/2.0
   TIMEP=TIME(KB)
   X=L-DELTAL(KB)
   ASSIGN 67 TO KTR
   GO TO 70
67 CONTINUE
   RETURN
70 CONTINUE
72 IF(X-LC)73,73,74
73 COSA=1.0
   DRDX=0.
   GO TO 75
74 COSA=COSG
   DRDX=-TANG
75 CONTINUE
   R=RC+(X-LC)*DRDX
   CALL ULIQ
   GO TO KTR, (63,67)
   END
```

```
      SUBROUTINE U LIQ
    1  COMMON AL,AQ(100),AV,BETAL,BETAV,BQ(100),COSA,COSG,CPL(20),CSL,
     1 CPV(20),CSLA,CSV(20),DELTAL(50),DELTAV(50),DENL(20),DENV(20),DL,
     2 DPDRL(20),DPDRV(20),DPDTL(20),DPDTV(20),DRDX,DVP,FP,G,GAMMA.H,
     3 I,IDX,IU,IXV,IZ,JERROR,JTR,KB,KD,L,LC,MASS,MASS1,NPT,NTR,NUL,
     4 NULA,NUV,NX,P,PA(50),POWER,PR,PRV,PT(20),Q(100),QW,R,RC,RHO,
     5 RHOV,RS,SL(20),SLC,SV(20),TAMB,TANG,TB,TDL(101),TDV(101),TH,THV,
     6 TIME(50),TSAT(20),U,UV,X,XB(100),Y(101),Z(101),ZINC,ZV(101)
      COMMON /VAR/NUV14, NUL14, PRV23, PRL23, COSA54
           REAL NUV 14, NUL14, LC, NUL, NUV
      PRL23 = PR**(2.0/3.0)
      NUL14 = NUL**0.25
      COSA54=COSA**1.25
C     GUESS FOR TH
   10 TH1 = 0.1*X**(5.0/7.0)
   30 CALL DQML(X, AREAX1, QM1, DQMDX1)
   60 X2 = X +      1.
   70 CALL DQML(X2, AREAX2, QM2, DQMDX2)
   20 DO 170KK=1,20
   40 U1 = U CALC L(AREAX1, TH1)
   50 DUDX1 = DU CAL L(QM1, DQMDX1, TH1, U1, X)
   80 U2 = U CALC L(AREAX2,TH1)
   90 DEL U1 = (U2 - U1)/     1.
  100 F1 = DUDX1 - DEL U1
  110 IF(ABSF(F1) -    .001*DUDX1)180,180,120
  120 TH2 = TH1*1.1
  130 U3 = U CALC L(AREAX1, TH2)
  140 DUDX2 = DU CAL L(QM1, DQMDX1, TH2, U3, X)
  150 U4 = U CALC L(AREA X2, TH2)
  160 DEL U2 = (U4 - U3)/     1.
  170 TH1 = TH1 - F1/((F1 - (DUDX2 - DEL U2))/(TH1 - TH2))
      WRITE(6, 300)
  300 FORMAT(1H1//////////19H * * * * * * * * * ///24H U LIQ DID NOT CON
     1VERGE. )
      WRITE (6,301) X, TH1, DUDX1, DEL U1
  301 FORMAT( 4H X =, E20.10/6H TH1 =, E20.10/ 8H DUDX1 =, E20.10/9H DEL
     1 U1 =, E20.10,1H1)
  180 TH = TH1
  190 U = U1
      IF (TH/R.LT. 0.2) GO TO 200
      TH = 0.2*R
      U=U CALC L(AREAX1,TH )
  200 RETURN
      END
```

```
      SUBROUTINE DQML   (A8,A,QM,DQM)
    1  COMMON AL,AQ(100),AV,BETAL,BETAV,BQ(100),COSA,COSG,CPL(20),CSL,
     1 CPV(20),CSLA,CSV(20),DELTAL(50),DELTAV(50),DENL(20),DENV(20),DL,
     2 DPDRL(20),DPDRV(20),DPDTL(20),DPDTV(20),DRDX,DVP,FP,G,GAMMA,H,
     3 I,IDX,IU,IXV,IZ,JERROR,JTR,KB,KD,L,LC,MASS,MASS1,NPT,NTR,NUL,
     4 NULA,NUV,NX,P,PA(50),POWER,PR,PRV,PT(20),Q(100),QW,R,RC,RHO,
     5 RHOV,RS,SL(20),SLC,SV(20),TAMB,TANG,TB,TDL(101),TDV(101),TH,THV,
     6 TIME(50),TSAT(20),U,UV,X,XB(100),Y(101),Z(101),ZINC,ZV(101)
            REAL LC
      QMI=0.0
      XX=A8
      QT=0.0
      AT=0.0
      DO 20 J =2,NX
      IJ = J
      IF(XB(J).GT.LC) GO TO 10
      CO=1.0
      TA=0.0
      GO TO 15
   10 CO=COSA
      TA=-DRDX
   15 CONTINUE
      IF(XB(J).GE.XX) GO TO 30
      XB1=XB(J)-XB(J-1)
      XB2=XB(J)**2-XB(J-1)**2
      XB3=XB(J)**3.-XB(J-1)**3.
      A1=2.*RC*XB1-XB2*TA   +2.*LC*XB1*TA
      AX=A1*3.14159265/CO
      AT=AT+AX
      A1=A1*AQ(J)
      A2=XB2*RC-(2.*XB3*TA   /3.)+XB2*TA   *LC
      A2=BQ(J)*A2
      QMI=3.14159265/CO*(A1+A2)
      QT=QT+QMI
   20 CONTINUE
      J=IJ
   30 CONTINUE
      XB1=XX-XB(J-1)
      XB2=XX**2-XB(J-1)**2
      XB3=XX**3.-XB(J-1)**3.
      A1=2.*RC*XB1-XB2*TA   +2.*LC*XB1*TA
      AX=A1*3.14159265/CO
      AT=AT+AX
      A1=A1*AQ(J)
      A2=XB2*RC-(2.*XB3*TA   /3.)+XB2*TA   *LC
      A2=BQ(J)*A2
      QMI=3.14159265/CO*(A1+A2)
      QT=QT+QMI
      A1=RC+LC*TA
      DU=2.*3.14159265/CO*(AQ(J)*A1+XX*(BQ(J)*A1-AQ(J)*TA)-XX*XX*
     1BQ(J)*TA)
      DV=2.0*3.14159265/CO*(A1-XX*TA)
      A=AT
      QM=QT/AT
```

```
DQM=(AT*DU-QT*DV)/(AT*AT)
RETURN
END
```

```
      FUNCTION U CALC L(AN, THA)
1     COMMON AL,AQ(100),AV,BETAL,BETAV,BQ(100),COSA,COSG,CPL(20),CSL,
     1 CPV(20),CSLA,CSV(20),DELTAL(50),DELTAV(50),DENL(20),DENV(20),DL,
     2 DPDRL(20),DPDRV(20),DPDTL(20),DPDTV(20),DRDX,DVP,FP,G,GAMMA,H,
     3 I,IDX,IU,IXV,IZ,JERROR,JTR,KB,KD,L,LC,MASS,MASS1,NPT,NTR,NUL,
     4 NULA,NUV,NX,P,PA(50),POWER,PR,PRV,PT(20),Q(100),QW,R,RC,RHO,
     5 RHOV,RS,SL(20),SLC,SV(20),TAMB,TANG,TB,TDL(101),TDV(101),TH,THV,
     6 TIME(50),TSAT(20),U,UV,X,XB(100),Y(101),Z(101),ZINC,ZV(101)
      COMMON /VAR/NUV14, NUL14, PRV23, PRL23, COSA54
            REAL NUV 14, NUL14, LC, NUL, NUV
      U CALC L = (NUL14*AN/(10.0946*PRL23*THA**1.25*R*COSA54*
     1            (1.0 - 0.1306*THA/R)))**4
      RETURN
      END
```

```
      FUNCTION DU CAL L(QM, DQMDX, THA, UA, XA)
1     COMMON AL,AQ(100),AV,BETAL,BETAV,BQ(100),COSA,COSG,CPL(20),CSL,
     1 CPV(20),CSLA,CSV(20),DELTAL(50),DELTAV(50),DENL(20),DENV(20),DL,
     2 DPDRL(20),DPDRV(20),DPDTL(20),DPDTV(20),DRDX,DVP,FP,G,GAMMA,H,
     3 I,IDX,IU,IXV,IZ,JERROR,JTR,KB,KD,L,LC,MASS,MASS1,NPT,NTR,NUL,
     4 NULA,NUV,NX,P,PA(50),POWER,PR,PRV,PT(20),Q(100),QW,R,RC,RHO,
     5 RHOV,RS,SL(20),SLC,SV(20),TAMB,TANG,TB,TDL(101),TDV(101),TH,THV,
     6 TIME(50),TSAT(20),U,UV,X,XB(100),Y(101),Z(101),ZINC,ZV(101)
      COMMON /VAR/NUV14, NUL14, PRV23, PRL23, COSA54
           REAL NUV 14, NUL14, LC, NUL, NUV
      TH14 = THA**0.25
      AA=52.394*G*BETAL*PRL23*QM*TH14*COSA**(1.0/4.0)
      AA=AA/(RHO*CSL*NUL14*UA**(7.0/4.0))
      AA = AA*(1.0 - 4.0*THA/(15.0*R))
      AA = AA - 0.2179*NUL14*UA**0.75/(THA**1.25*COSA**(9.0/4.0))
      AA = AA - UA*DRDX/(2.0*R)
      AA = AA/(1.0 - THA/(8.0*R))
      BB = (-UA/(2.0*THA)*(1.0 - THA/(4.0*R)))/(1.0 - THA/(8.0*R))
      CC = 0.4979*NUL14*QWL(XA)
      CC = CC/(PRL23*QM*UA**0.25*TH14*COSA**(9.0/4.0))
      CC = CC - 4.0*THA*DQMDX*(1.0 - 0.1306*THA/R)/(5.0*QM)
      CC = CC - 4.0*THA*DRDX/(5.0*R)
      CC = CC/(1.0 - 0.2351*THA/R)
      DD = -THA*(1.0 - 0.1306*THA/R)/(5.0*UA)
      DD = DD/(1.0 - 0.2351*THA/R)
      DU CAL L = (AA + CC*BB)/(1.0 - BB*DD)
      RETURN
      END
```

```
      FUNCTION QWL(XX)
    1  COMMON AL,AQ(100),AV,BETAL,BETAV,BQ(100),COSA,COSG,CPL(20),CSL,
     1 CPV(20),CSLA,CSV(20),DELTAL(50),DELTAV(50),DENL(20),DENV(20),DL,
     2 DPDRL(20),DPDRV(20),DPDTL(20),DPDTV(20),DRDX,DVP,FP,G,GAMMA,H,
     3 I,IDX,IU,IXV,IZ,JERROR,JTR,KB,KD,L,LC,MASS,MASS1,NPT,NTR,NUL,
     4 NULA,NUV,NX,P,PA(50),POWER,PR,PRV,PT(20),Q(100),QW,R,RC,RHO,
     5 RHOV,RS,SL(20),SLC,SV(20),TAMB,TANG,TB,TDL(101),TDV(101),TH,THV,
     6 TIME(50),TSAT(20),U,UV,X,XB(100),Y(101),Z(101),ZINC,ZV(101)
      DO 27 J=2,IXV
      IF(XB(J).GT.XX) GO TO 30
   27 CONTINUE
      J=IXV
   30 QWL=AQ(J)+BQ(J)*XX
      RETURN
      END
```

```
      SUBROUTINE DTEMP V
C     THIS ROUTINE COMPUTES THE GROWTH RATE OF THE STRATIFIED VAPOR
C      LAYER
    1 COMMON AL,AQ(100),AV,BETAL,BETAV,BQ(100),COSA,COSG,CPL(20),CSL,
     1 CPV(20),CSLA,CSV(20),DELTAL(50),DELTAV(50),DENL(20),DENV(20),DL,
     2 DPDRL(20),DPDRV(20),DPDTL(20),DPDTV(20),DRDX,DVP,FP,G,GAMMA,H,
     3 I,IDX,IU,IXV,IZ,JERROR,JTR,KB,KD,L,LC,MASS,MASS1,NPT,NTR,NUL,
     4 NULA,NUV,NX,P,PA(50),POWER,PR,PRV,PT(20),Q(100),QW,R,RC,RHO,
     5 RHOV,RS,SL(20),SLC,SV(20),TAMB,TANG,TB,TDL(101),TDV(101),TH,THV,
     6 TIME(50),TSAT(20),U,UV,X,XB(100),Y(101),Z(101),ZINC,ZV(101)
      DIMENSION F(128),TIMEV(128)
            REAL  L,LC,MASS,NUL,LV,NUV,KONL,KONB,KONV,NUOTV,NULA,NU
      DV=H/127.0
      IDX=2
   12 DO 25 K=1,127
      FK=K-1
      X=H-FK*DV
   16 IF(X+L-LC)18,18,20
   18 COSA=1.0
      DRDX=0.0
      GO TO 22
   20 COSA=COSG
      DRDX=-TANG
   22 CONTINUE
      R=RC+DRDX*(X+L-LC)
      CALL UVAP
   25 F(K)=(R-THV*(2.0-THV/R))/(2.0*UV*THV*COSA*(0.1464-0.02723*THV/R))
      F(128) = 1.E+70
      CALL INTEG   (128,DV,F,TIMEV)
      J=1
      K=64
      DO 30 KNT=1,8
      IF(TIME(KB).GE.TIMEV(J+K))J=J+K
   30 K=(K+1)/2
      FK=J-1
      X=FK*DV+DV*(TIME(KB)-TIMEV(J))/(TIMEV(J+1)-TIMEV(J))
      DELTAV(KD)=X
      X=H-X
   34 IF(X+L-LC)36,36,38
   36 COSA=1.0
      DRDX=0.0
      GO TO 40
   38 COSA=COSG
      DRDX=-TANG
   40 CONTINUE
      R=RC+DRDX*(X+L-LC)
      CALL UVAP
       WRITE(6,41) DELTAL(KB),DELTAV(KD),TIME(KB)
   41  FORMAT(14X,    6HDELTAL,14X,6HDELTAV,16X,4HTIME/3F20.5)
      RETURN
      END
```

```
      SUBROUTINE UVAP
    1  COMMON AL,AQ(100),AV,BETAL,BETAV,BQ(100),COSA,COSG,CPL(20),CSL,
     1 CPV(20),CSLA,CSV(20),DELTAL(50),DELTAV(50),DENL(20),DENV(20),DL,
     2 DPDRL(20),DPDRV(20),DPDTL(20),DPDTV(20),DRDX,DVP,FP,G,GAMMA,H,
     3 I,IDX,IU,IXV,IZ,JERROR,JTR,KB,KD,L,LC,MASS,MASS1,NPT,NTR,NUL,
     4 NULA,NUV,NX,P,PA(50),POWER,PR,PRV,PT(20),Q(100),QW,R,RC,RHO,
     5 RHOV,RS,SL(20),SLC,SV(20),TAMB,TANC,TB,TDL(101),TDV(101),TH,THV,
     6 TIME(50),TSAT(20),U,UV,X,XB(100),Y(101),Z(101),ZINC,ZV(101)
      COMMON /VAR/NUV14, NUL14, PRV23, PRL23, COSA54
            REAL NUV 14, NUL14, LC, NUL, NUV
      NUV 14 = NUV**0.25
      PRV 23 = PRV**(2.0/3.0)
      COSA54=COSA**1.25
C     GUESS FOR TH
   10 TH1 = 0.1*X**(5.0/7.0)
   30 CALL DQMV(X, AREAX1, QM1, DQMDX1)
   60 X2 = X +      1.
   70 CALL DQMV(X2, AREAX2, QM2, DQMDX2)
   20 DO 170KK=1,20
   40 U1 = U CALC V(AREAX1, TH1)
   50 DUDX1 = DU CAL V(QM1, DQMDX1, TH1, U1, X)
   80 U2 = U CALC V(AREAX2,TH1)
   90 DEL U1 = (U2 - U1)/     1.
  100 F1 = DUDX1 - DEL U1
  110 IF(ABSF(F1) -   .001*DUDX1)180,180,120
  120 TH2 = TH1*1.1
  130 U3 = U CALC V(AREAX1, TH2)
  140 DUDX2 = DU CAL V(QM1, DQMDX1, TH2, U3, X)
  150 U4 = U CALC V(AREA X2, TH2)
  160 DEL U2 = (U4 - U3)/     1.
  170 TH1 = TH1 - F1/((F1 - (DUDX2 - DEL U2))/(TH1 - TH2))
  180 THV=TH1
  190 UV=U1
      IF (THV/R.LT. 0.2) GO TO  200
      THV= 0.2* R
      UV= UCALCV(AREAX1 ,THV)
  200 RETURN
      END
```

```
      SUBROUTINE INTEG   (I, DZ, A, B)
      DIMENSION A(128), B(128)
C     INTEGRATES BY NEWTON-COTES FORMULAS.  A IS THE INPUT TABLE AND
C     B IS THE OUTPUT TABLE OF THE VALUES OF THE INTEGRAL.  VALUES
C     MUST BE EVENLY SPACED BY DZ.  THERE ARE I POINTS TO BE OPERATED
C     ON.  AT LEAST SEVEN POINTS ARE REQUIRED.
C
C     INITIALIZE.
    1 X14 = (3.0*DZ/8.0)*(A(1) + 3.0*(A(2) + A(3)) + A(4))
    2 X15 = (2.0*DZ/45.0)*(7.0*(A(1) + A(5)) + 32.0*(A(2) + A(4)) +
     1      12.0*A(3))
    3 X16 = (5.0*DZ/288.0)*(19.0*(A(1) + A(6)) + 75.0*(A(2) + A(5)) +
     1       50.0*(A(3) + A(4)))
    4 X17 = (DZ/140.0)*(41.0*(A(1) + A(7)) + 216.0*(A(2) + A(6)) +
     1       27.0*(A(3) + A(5)) + 272.0*A(4))
    5 X27 = (5.0*DZ/288.0)*(19.0*(A(2) + A(7)) + 75.0*(A(3) + A(6)) +
     1      50.0*(A(4) + A(5)))
    6 X37 = (2.0*DZ/45.0)*(7.0*(A(3) + A(7)) + 32.0*(A(4) + A(6)) +
     1      12.0*A(5))
    7 B(1) = 0.0
    8 B(2) = X17 - X27
    9 B(3) = X17 - X37
   10 B(4) = X14
   11 B(5) = X15
   12 B(6) = X16
   13 B(7) = X17
   14 DO 15 K=8,I
   15 B(K) = B(K-5) + (5.0*DZ/288.0)*(19.0*(A(K-5) + A(K)) +
     1       75.0*(A(K-4) + A(K-1)) + 50.0*(A(K-3) + A(K-2)))
   16 RETURN
      END
```

```
      SUBROUTINE DQMV  (A8,A,QM,DQM)
   1  COMMON AL,AQ(100),AV,BETAL,BETAV,BQ(100),COSA,COSG,CPL(20),CSL,
     1 CPV(20),CSLA,CSV(20),DELTAL(50),DELTAV(50),DENL(20),DENV(20),DL,
     2 DPDRL(20),DPDRV(20),DPDTL(20),DPDTV(20),DRDX,DVP,FP,G,GAMMA,H,
     3 I,IDX,IU,IXV,IZ,JERROR,JTR,KB,KD,L,LC,MASS,MASS1,NPT,NTR,NUL,
     4 NULA,NUV,NX,P,PA(50),POWER,PR,PRV,PT(20),Q(100),QW,R,RC,RHO,
     5 RHOV,RS,SL(20),SLC,SV(20),TAMB,TANG,TB,TDL(101),TDV(101),TH,THV,
     6 TIME(50),TSAT(20),U,UV,X,XB(100),Y(101),Z(101),ZINC,ZV(101)
            REAL LC,L
      XX=A8+L
      AT=0.0
      QT=0.0
      DO 20 J=IXV,NX
      IJ = J
      IF(XB(J).GT.LC) GO TO 10
      CO=1.0
      TA=0.0
      GO TO 15
   10 CO=COSA
      TA=-DRDX
   15 CONTINUE
      VALX=XB(J-1)
      IF(VALX.LT.L) VALX=L
      IF(XB(J).GE.XX) GO TO 30
      XB1=XB(J)-VALX
      XB2=XB(J)**2-VALX**2
      XB3=XB(J)**3-VALX**3
      A1=2.*RC*XB1-XB2*TA  +2.*LC*XB1*TA
      AX=A1*3.14159265/CO
      AT=AT+AX
      A1=A1*AQ(J)
      A2=XB2*RC-(2.*XB3*TA  /3.)+XB2*TA  *LC
      A2=BQ(J)*A2
      QMI=3.14159265/CO*(A1+A2)
      QT=QT+QMI
   20 CONTINUE
      VALX=XB(IJ)
      J=IJ
   30 CONTINUE
      XB1=XX-VALX
      XB2=XX**2-VALX**2
      XB3=XX**3-VALX**3
      A1=2.*RC*XB1-XB2*TA  +2.*LC*XB1*TA
      AX=A1*3.14159265/CO
      AT=AT+AX
      A1=A1*AQ(J)
      A2=XB2*RC-(2.*XB3*TA  /3.)+XB2*TA  *LC
      A2=BQ(J)*A2
      QMI=3.14159265/CO*(A1+A2)
      QT=QT+QMI
      A1=RC+LC*TA
      DU=2.*3.14159265/CO*(AQ(J)*A1+XX*(BQ(J)*A1-AQ(J)*TA)-XX*XX*
     1BQ(J)*TA)
      DV=2.0*3.14159265/CO*(A1-XX*TA)
```

```
A=AT
QM=QT/AT
DQM=(AT*DU-QT*DV)/(AT*AT)
RETURN
END
```

```
      FUNCTION U CALC V(AN, THA)
1     COMMON AL,AQ(100),AV,BETAL,BETAV,BQ(100),COSA,COSG,CPL(20),CSL,
     1 CPV(20),CSLA,CSV(20),DELTAL(50),DELTAV(50),DENL(20),DENV(20),DL,
     2 DPDRL(20),DPDRV(20),DPDTL(20),DPDTV(20),DRDX,DVP,FP,G,GAMMA,H,
     3 I,IDX,IU,IXV,IZ,JERROR,JTR,KB,KD,L,LC,MASS,MASS1,NPT,NTR,NUL,
     4 NULA,NUV,NX,P,PA(50),POWER,PR,PRV,PT(20),Q(100),QW,R,RC,RHO,
     5 RHOV,RS,SL(20),SLC,SV(20),TAMB,TANG,TB,TDL(101),TDV(101),TH,THV,
     6 TIME(50),TSAT(20),U,UV,X,XB(100),Y(101),Z(101),ZINC,ZV(101)
      COMMON /VAR/NUV14, NUL14, PRV23, PRL23, COSA54
            REAL NUV 14, NUL14, LC, NUL, NUV
      U CALC V = (NUV14*AN/(10.0946*PRV23*THA**1.25*R*COSA54*
     1            (1.0 - 0.1306*THA/R)))**4
      RETURN
      END
```

```
      FUNCTION DU CAL V(QM, DQMDX, THA, UA, XA)
1      COMMON AL,AQ(100),AV,BETAL,BETAV,BQ(100),COSA,COSG,CPL(20),CSL,
     1 CPV(20),CSLA,CSV(20),DELTAL(50),DELTAV(50),DENL(20),DENV(20),DL,
     2 DPDRL(20),DPDRV(20),DPDTL(20),DPDTV(20),DRDX,DVP,FP,G,GAMMA,H,
     3 I,IDX,IU,IXV,IZ,JERROR,JTR,KB,KD,L,LC,MASS,MASS1,NPT,NTR,NUL,
     4 NULA,NUV,NX,P,PA(50),POWER,PR,PRV,PT(20),Q(100),QW,R,RC,RHO,
     5 RHOV,RS,SL(20),SLC,SV(20),TAMB,TANG,TB,TDL(101),TDV(101),TH,THV,
     6 TIME(50),TSAT(20),U,UV,X,XB(100),Y(101),Z(101),ZINC,ZV(101)
      COMMON /VAR/NUV14, NUL14, PRV23, PRL23, COSA54
           REAL NUV 14, NUL14, LC, NUL, NUV
      TH14 = THA**0.25
      AA = 52.394*G*BETAV*PRV23*QM*TH14*COSA**(1.0/4.0)
      AA=AA/(RHOV*CSV(1)*NUV14*UA**(7.0/4.0))
      AA = AA*(1.0 - 4.0*THA/(15.0*R))
      AA = AA - 0.2179*NUV14*UA**0.75/(THA**1.25*COSA**(9.0/4.0))
      AA = AA - UA*DRDX/(2.0*R)
      AA = AA/(1.0 - THA/(8.0*R))
      BB = (-UA/(2.0*THA)*(1.0 - THA/(4.0*R)))/(1.0 - THA/(8.0*R))
      CC = 0.4979*NUV14*QWV(XA)
      CC = CC/(PRV23*QM*UA**0.25*TH14*COSA**(9.0/4.0))
      CC = CC - 4.0*THA*DQMDX*(1.0 - 0.1306*THA/R)/(5.0*QM)
      CC = CC - 4.0*THA*DRDX/(5.0*R)
      CC = CC/(1.0 - 0.2351*THA/R)
      DD = -THA*(1.0 - 0.1306*THA/R)/(5.0*UA)
      DD = DD/(1.0 - 0.2351*THA/R)
      DU CAL V = (AA + CC*BB)/(1.0 - BB*DD)
      RETURN
      END
```

```
      FUNCTION QWV(XXX)
 1     COMMON AL,AQ(100),AV,BETAL,BETAV,BQ(100),COSA,COSG,CPL(20),CSL,
      1 CPV(20),CSLA,CSV(20),DELTAL(50),DELTAV(50),DENL(20),DENV(20),DL,
      2 DPDRL(20),DPDRV(20),DPDTL(20),DPDTV(20),DRDX,DVP,FP,G,GAMMA,H,
      3 I,IDX,IU,IXV,IZ,JERROR,JTR,KB,KD,L,LC,MASS,MASS1,NPT,NTR,NUL,
      4 NULA,NUV,NX,P,PA(50),POWER,PR,PRV,PT(20),Q(100),QW,R,RC,RHO,
      5 RHOV,RS,SL(20),SLC,SV(20),TAMB,TANG,TB,TDL(101),TDV(101),TH,THV,
      6 TIME(50),TSAT(20),U,UV,X,XB(100),Y(101),Z(101),ZINC,ZV(101)
           REAL L
      XX=XXX+L
      DO 27 J=IXV,NX
      IF(XB(J).GT.XX) GO TO 30
   27 CONTINUE
      J = NX
   30 QWV=AQ(J)+BQ(J)*XX
      RETURN
      END
```

```
      SUBROUTINE THETA L
    1 COMMON AL,AQ(100),AV,BETAL,BETAV,BQ(100),COSA,COSG,CPL(20),CSL,
     1 CPV(20),CSLA,CSV(20),DELTAL(50),DELTAV(50),DENL(20),DENV(20),DL,
     2 DPDRL(20),DPDRV(20),DPDTL(20),DPDTV(20),DRDX,DVP,FP,G,GAMMA,H,
     3 I,IDX,IU,IXV,IZ,JERROR,JTR,KB,KD,L,LC,MASS,MASS1,NPT,NTR,NUL,
     4 NULA,NUV,NX,P,PA(50),POWER,PR,PRV,PT(20),Q(100),QW,R,RC,RHO,
     5 RHOV,RS,SL(20),SLC,SV(20),TAMB,TANG,TB,TDL(101),TDV(101),TH,THV,
     6 TIME(50),TSAT(20),U,UV,X,XB(100),Y(101),Z(101),ZINC,ZV(101)
      DIMENSION RZ(101), TERM 1(101), TERM 1 I(101), TERM 2(101),
     1           TERM 2 I(101)
      COMMON /VAR/NUV14, NUL14, PRV23, PRL23, COSA54
            REAL NUV 14, NUL14, LC, NUL, NUV, L, NULA
      XX=TH
      XXX=U
   10 CSL = CSLA
   20 NUL = NULA
   30 PR = NUL/AL
   40 NUL14 = NUL**0.25
   50 PRL23 = PR**(2.0/3.0)
   60 PI = 3.1415926535
   70 CON 1 = PRL23/(0.0228*CSL*RHO*NUL14)
  100 DZ = DELTAL(KB)/ZINC
  110 IZ = ZINC
  111 NPTS = IZ + 1
  120 IU = IZ + 1
  130 Z(1) = 0.0
  140 DO 150 J=2,NPTS
  150 Z(J) = Z(J-1) + DZ
  151 X = L - DELTAL(KB) - DZ
  160 DO 420 J=1,NPTS
  170 X = X + DZ
  180 IF(X - LC)190,190,250
  190 COSA = 1.0
  200 COSA54 = 1.0
  205 TANA=0.0
  210 R = RC
  220 RZ(J) = RC
  230 DRDX = 0.0
  240 GO TO 290
  250 COSA = COSG
  255 TANA=TANG
  260 COSA54 = COSA**1.25
  270 DRDX = -TANG
  280 R = RC - (X - LC)*TANG
  290 RZ(J) = R
  300 CALL U LIQ
  310 CALL DQML(X, AREAZ, QMZ, DQMDX)
  320 THETA 1 = CON1*(QMZ*(TH*COSA)**0.25)/U**0.75
  330 YINC = 10.0
  340 NPTS Y = YINC + 1.0
  350 D TH = TH/YINC
  360 Y TH = 0.0
  370 DO 400 JJ=1,NPTS Y
  380 THETA 2 = THETA 1*(1.0 - (Y TH/TH)**(1.0/7.0))
```

```
390 TERM 1(JJ) = ((R - Y TH)*THETA 2)/(1.0 + BETAL*THETA 2)
400 Y TH = Y TH + D TH
410 CALL INTEG   (NPTS Y, D TH, TERM 1, TERM 1 I)
420 TERM 2(J) = TERM 1 I(NPTS Y)
430 CALL INTEG   (NPTS, DZ, TERM 2, TERM 2 I)
440 TERM 4 = QMZ*AREAZ*TIME(KD) + 2.0*PI*RHO*CSL*TERM 2 I(NPTS)
450 TERM 4 =(POWER+1.)*TERM 4/(DELTA L(KD)*PI*RHO*CSL)
470 DO 493 J=1,NPTS
480 IF(J-1)490,490,492
490 TDL(J) = 0.0
491 GO TO 493
492 TDL(J) = (Z(J)/DELTAL(KB))**(POWER      )*TERM 4/RZ(J)**2
493 CONTINUE
    TH=XX
    U=XXX
500 RETURN
    END
```

```
      SUBROUTINE THETA V
    1 COMMON AL,AQ(100),AV,BETAL,BETAV,BQ(100),COSA,COSG,CPL(20),CSL,
     1 CPV(20),CSLA,CSV(20),DELTAL(50),DELTAV(50),DENL(20),DENV(20),DL,
     2 DPDRL(20),DPDRV(20),DPDTL(20),DPDTV(20),DRDX,DVP,FP,G,GAMMA,H,
     3 I,IDX,IU,IXV,IZ,JERROR,JTR,KB,KD,L,LC,MASS,MASS1,NPT,NTR,NUL,
     4 NULA,NUV,NX,P,PA(50),POWER,PR,PRV,PT(20),Q(100),QW,R,RC,RHO,
     5 RHOV,RS,SL(20),SLC,SV(20),TAMB,TANG,TB,TDL(101),TDV(101),TH,THV,
     6 TIME(50),TSAT(20),U,UV,X,XB(100),Y(101),Z(101),ZINC,ZV(101)
      COMMON /VAR/NUV14, NUL14, PRV23, PRL23, COSA54
            REAL NUV 14, NUL14, LC, NUL, NUV, L, NULA
      DIMENSION TERM 1(101), TERM 2(101), TERM 4(101), TERM 5(101)
      SOLUTION OF DIFFERENTIAL EQUATION IS BY THE RUNGE-KUTTA METHOD.
      YY=THV
      YYY=UV
   10 IV = ZINC
   20 NPTS = IV + 1
   30 DV = DELTA V(KD)/ZINC
   40 ZV(1) = H - DELTA V(KD)
   50 CSVV = CSV(1)
   60 COSA5 = COSG**1.25
   70 PI = 3.1415926535
   80 THETA A = T AMB - TB
  200 DO 210 J=2,NPTS
  210 ZV(J) = ZV(J-1) + DV
  300 DO 480 J=1,NPTS
  310 X = ZV(J)
  320 IF(X + L - LC)330,330,380
  330 COSA = 1.0
  340 COSA54 = 1.0
  350 R = RC
  360 DRDX = 0.0
  370 GO TO 411
  380 COSA = COSG
  390 COSA54 = COSA5
  400 DRDX = -TANG
  410 R = RC - (X + L - LC)*TANG
  411 TDV(J) = TB
  420 CALL U VAP
  430 CALL DQMV(X, AREA, QM, DQMDX)
  440 QWVA = QWV(X)
  441 IF(J-NPTS)450,442,442
  442 TERM 1(J) = QM*AREA/(H - X + DV/10.0)
  443 GO TO 460
  450 TERM 1(J) = QM*AREA/(H - X)
  460 TERM 2(J) = 2.0*PI*QWVA/COSA*R
  470 TERM 4(J) = R*(1.0 - THV/R)**2/(0.2927*UV*THV*COSA*
     1             (1.0 - 0.18605*THV/R))
  480 TERM 5(J) = PI*R*R*RHOV
  500 DO 970 J=1,NPTS
  510 JJJ = NPTS - J + 1
  520 IF(J - NPTS)530,970,970
  530 TERM 2 A = TERM 2(JJJ)
  540 TERM 5 A = TERM 5(JJJ)
  550 TPSI = TB
```

```
570 DO 950 JJ=1,JJJ
580 KK = JJJ - JJ + 1
590 IF(JJ - JJJ)600,950,950
600 TERM 1 A = TERM 1(KK)
610 TERM 4 A = TERM 4(KK)
611 TPSIA = TPSI
620 KOUNT = 1
630 GO TO 900
640 SUM = XK
650 TERM 1 A = (TERM 1(KK) + TERM 1(KK-1))/2.0
660 TERM 4 A = (TERM 4(KK) + TERM 4(KK-1))/2.0
670 TPSIA = TPSI + XK/2.0
680 KOUNT = 2
690 GO TO 900
700 SUM = SUM + XK*2.0
710 TPSIA = TPSI + XK/2.0
720 KOUNT = 3
730 GO TO 900
740 SUM = SUM + XK*2.0
750 TERM 1 A = TERM 1(KK-1)
760 TERM 4 A = TERM 4(KK-1)
770 TPSIA = TPSI + XK
780 KOUNT = 4
790 GO TO 900
800 SUM = SUM + XK
810 TPSI = TPSI + SUM/6.0
820 GO TO 950
900 CSV(1) = PTCP(P, TPSIA)
910 TERM 3 = (THETA A + TB - TPSIA)/THETA A
920 XK = (TERM 1 A + TERM 2 A*TERM 3)*DV*TERM 4 A/(TERM 5 A*CSV(1))*DV
930 GO TO(640, 700, 740, 800),KOUNT
950 CONTINUE
960 TDV(JJJ) = TPSI
970 CONTINUE
980 TDV(1) = 0.0
    CSV(1) = CSVV
    ZV(1) = 0.0
    DO 990 J=2,NPTS
    TDV(J) = TDV(J) - TB
990 ZV(J) = ZV(J-1) + DV
    THV=YY
    UV=YYY
    RETURN
    END
    FUNCTION COSF(A)
```

```
COSF = COS(A)
RETURN
END
```

```
FUNCTION ABSF(A)
ABSF = ABS(A)
RETURN
END
```

```
FUNCTION PTCP(PRES,TEMP)
PTCP=PTHEAT(PRES,TEMP,1)
RETURN
END
```

```
FUNCTION PTCV(PRES,TEMP)
PTCV=PTHEAT(PRES,TEMP,2)
RETURN
END
```

```
      FUNCTION PTGAMM(PRES,TEMP)
      PTGAMM=PTHEAT(PRES,TEMP,3)
      RETURN
      END
```

```
      FUNCTION PTHEAT(PRES,TEMP,KTRANS)
      COMMON/SPHEAT/CP(823),CV(823)
      DIMENSION LOC(23),JP(19),MX(19),BP(19),DP(19),BT(23),DT(19),PS(12)
     1,TS(12),TL(10)
      DATA PS/1.022,2.,4.,8.,14.,25.,43.,69.,99.,128.,151.,160./
      DATA TS/24.845,27.07,29.81,33.07,36.18,39.96,44.12,48.33,51.97,
     154.79,56.72,57.46/
      DATA LOC/1,50,71,113,133,141,170,165,228,264,348,372,426,492,528,
     1584,626,638,734,654,682,694,718/
      DATA JP/7,3,6,5,2,4,9,9,4,7,4,9,6,6,7,7,2,2,5/
      DATA MX/5,1,4,3,0,2,2,4,2,5,2,7,4,4,5,5,0,0,3/
      DATA BP/0.,20.,0.,1000.,100.,-1000.,1469.6,0.,1469.6,587.84,587.84
     1,0.,100.,100.,40.,40.,0.,0.,1./
      DATA BT/2600.,2600.,2600.,2600.,800.,300.,120.,120.,25.2,27.,72.,
     181.,25.,56.,26.,41.,25.,25.,2600.,5000.,5000.,5000.,5000./
      DATA DP/5.,40.,200.,1000.,4900.,2000.,1175.68,293.92,1175.68,
     1146.96,146.96,73.48,100.,100.,20.,20.,40.,40.,1./
      DATA DT/400.,400.,400.,800.,600.,100.,30.,30.,12.6,9.,3.6,9.,5.,
     15.,4.,8.,4.,8.,200./
      DATA TL/24.846,27.175,29.310,31.299,33.176,34.962,36.672,38.317,
     139.904,41.456/
    1 P=PRES*14.696
      IF(P.LT.1.0) P=1.0
      T=TEMP*1.8
      KTR=KTRANS
      IF(T.LT.126.) GO TO 9
      IF(T.LT.2600.) GO TO 5
      IF(T.GE.6000.) T=5999.99999
      IF(P.GE.100.) GO TO 3
      IF(P.GE.30.) GO TO 2
      IF(P.GE.5.)GO TO 20
      N=19
      N1=19
      GO TO 33
   20 N=1
      N1=20
      GO TO 33
    2 N=2
      N1=21
      GO TO 33
    3 IF(P.GE.1000.) GO TO 4
      N=3
      N1=22
      GO TO 33
    4 N=4
      N1=23
      GO TO 33
    5 IF(T.LT.300.)GO TO 7
      IF(T.LT.800.)GO TO 6
      N=5
      GO TO 33
    6 N=6
      GO TO 33
    7 IF(P.LT.1469.6) GO TO 8
```

```
      N=7
      GO TO 33
    8 N=8
      GO TO 33
    9 IF(P.LT.587.84) GO TO 12
      IF(P.LT.1469.6) GO TO 10
      N=9
      GO TO 30
   10 IF(P.LT.1028.72.AND.T.GE.72.0.AND.T.LT.90.0) GO TO 11
      N=10
      GO TO 30
   11 N=11
      GO TO 33
   12 IF(T.LT.81.) GO TO 13
      N=12
      GO TO 33
   13 IF(P.LT.160.) GO TO 15
      TM=((.86867647E-7*P-.12613701E-3)*P+.10353383)*P+43.8056878
      IF(T.GT.TM) GO TO 14
      N=13
      GO TO 30
   14 N=14
      GO TO 33
   15 DO 16 I=2,12
      IF(P-PS(I))17,17,16
   16 CONTINUE
      I=12
   17 TM=TS(I-1)+(TS(I)-TS(I-1))*(P-PS(I-1))/(PS(I)-PS(I-1))
      IF(T.GE.TM) GO TO 18
      N=15
      IF(P.LT.40.) N=17
      GO TO 30
   18 N=16
      IF(P.LT.40.) N=18
      GO TO 33
   30 F=P/587.84
      I=F
      IF(I.GT.8) I=8
      FI=I
      F=F-FI
      TQ=(1.0-F)*TL(I+1)+F*TL(I+2)
      IF(T.LT.TQ) T=TQ
   33 IF(T.LE.5000.)N1=N
      FP=(P-BP(N))/DP(N)
      IP=FP
      IF(IP.GT.MX(N)) IP=MX(N)
      FI=IP
      F=FP-FI
      FP=1.0-F
      FT=(T-BT(N1))/DT(N)
      IT=FT
      FI=IT
      FF=FT-FI
      FT=1.0-FF
```

```
      I=IT*JP(N)+IP+LOC(N1)
      J=I+JP(N)
      IF(KTR.EQ.2) GOTO 37
      CTCP=FP*FT*CP(I)+F*FT*CP(I+1)+FP*FF*CP(J)+F*FF*CP(J+1)
      IF(N.LT.13.OR.N.GE.17) GO TO 36
      IF(N.LT.15) GO TO 35
      CTCP=CTCP/(187.506-P+ABS (T-TM)*28.13)
      GO TO 36
   35 CTCP=CTCP/(ABS (T-TM)/1.8+ABS (P-187.506)*.008008982)
   36 IF(KTR.GE.2) GO TO 37
      PTHEAT=CTCP*4.18674
      RETURN
   37 PTHEAT=FP*FT*CV(I)+F*FT*CV(I+1)+FP*FF*CV(J)+F*FF*CV(J+1)*4.18674
      IF(KTR.LT.3) RETURN
      PTHEAT=CTCP/PTHEAT
      RETURN
      END
```

```
 BLOCK DATA
 COMMON/SPHEAT/AA(111),AB(111),AC(111),AD(111),AE(116),AF(112),
1AG(111),AH(40),
2          AI(110),AJ(111),AK(111),AL(111),AM(116),AN(112),AO(111)
3,AP(41)
 DATAAA/3.804,3.796,3.794,3.794,3.793,3.793,3.793,4.025,3.951,3.937
1,3.931,3.927,3.924,3.922,4.655,4.259,4.184,4.15,4.13,4.117,4.107,6
2.495,5.032,4.755,4.632,4.559,4.509,4.472,10.98,6.839,6.054,5.706,5
3.498,5.356,5.252,20.06,10.48,8.646,7.834,7.35,7.02,6.776,35.35,16.
483,13.18,11.55,10.58,9.919,9.429,3.793,3.792,3.792,3.924,3.917,3.9
513,4.117,4.076,4.059,4.51,4.359,4.298,5.36,4.931,4.757,7.028,6.027
6,5.62,9.936,7.923,7.103,3.793,3.791,3.791,3.791,3.791,3.79,3.916,3
7.91,3.908,3.907,3.906,3.905,4.075,4.043,4.031,4.025,4.022,4.02,4.3
86,4.236,4.192,4.172,4.161,4.183,4.933,4.581,4.457,4.401,4.369,4.34
96,6.03,5.21,4.92,4.791,4.715,4.759,7.93,6.276,5.692,5.433,5.279/
 DATAAB/5.173,3.79,3.79,3.789,3.789,3.789,4.02,4.014,4.011,4.01,4.0
108,4.346,4.29,4.265,4.25,4.24,5.173,4.912,4.796,4.726,4.679,3.47,3
2.511,3.504,3.507,3.624,3.622,3.792,3.789,3.744,4.108,4.297,4.346,3
3.658,3.842,3.961,4.015,3.537,3.637,3.708,3.749,3.479,3.535,3.579,3
4.608,3.461,3.495,3.523,3.542,3.461,3.478,3.494,3.511,2.586,2.981,3
5.396,3.693,3.806,3.785,3.434,3.187,3.036,2.861,3.089,3.309,3.498,3
6.628,3.693,3.64,3.501,3.379,3.2,3.342,3.474,3.584,3.673,3.737,3.8,
73.744,3.681,3.519,3.633,3.738,3.826,3.903,3.957,4.03,4.041,4.015,3
8.744,3.838,3.921,3.994,4.06,4.111,4.223,4.248,4.243,3.877,3.951,4.
9018,4.079,4.133,4.179,4.292,4.333,4.343,3.926,3.98,4.033,4.087/
 DATAAC/4.125,4.152,4.263,4.317,4.346,1.334,1.264,1.217,1.2,1.884,1
1.742,1.667,1.587,2.387,2.151,2.025,1.947,2.871,2.506,2.337,2.232,3
2.311,2.792,2.584,2.461,3.668,3.035,2.789,2.652,3.846,3.23,2.968,2.
3821,3.84,3.37,3.113,2.963,3.767,3.475,3.25,3.104,1.543,1.515,1.491
4,1.468,1.447,1.428,1.414,2.003,1.96,1.922,1.89,1.862,1.835,1.812,2
5.51,2.426,2.357,2.301,2.253,2.21,2.173,3.151,2.967,2.836,2.734,2.6
652,2.585,2.529,4.07,3.651,3.383,3.198,3.063,2.956,2.871,5.417,4.48
76,3.997,3.69,3.477,3.318,3.196,5.793,5.081,4.509,4.119,3.844,3.64,
83.484,4.855,4.893,4.637,4.339,4.084,3.879,3.714,4.14,4.37,4.391,4.
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239.24,30.92,26.61,23.87,21.93,44.14,36.29,31.69,28.62,26.38,46.18,
340.53,36.27,33.17,30.8,44.47,42.64,39.57,36.91,34.71/
 END
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      SUBROUTINE MASSC
C     THIS SUBROUTINE COMPUTES ULLAGE SPACE MASS AFTER
C     STRATIFICATION OCCURS
    1 COMMON AL,AQ(100),AV,BETAL,BETAV,BQ(100),COSA,COSG,CPL(20),CSL,
     1 CPV(20),CSLA,CSV(20),DELTAL(50),DELTAV(50),DENL(20),DENV(20),DL,
     2 DPDRL(20),DPDRV(20),DPDTL(20),DPDTV(20),DRDX,DVP,FP,G,GAMMA,H,
     3 I,IDX,IU,IXV,IZ,JERROR,JTR,KB,KD,L,LC,MASS,MASS1,NPT,NTR,NUL,
     4 NULA,NUV,NX,P,PA(50),POWER,PR,PRV,PT(20),Q(100),QW,R,RC,RHO,
     5 RHOV,RS,SL(20),SLC,SV(20),TAMB,TANG,TB,TDL(101),TDV(101),TH,THV,
     6 TIME(50),TSAT(20),U,UV,X,XB(100),Y(101),Z(101),ZINC,ZV(101)
      DIMENSION Y1(101),PSAT(101),E (101),DML(101),DMV(101),VOL(101),
     1 T(101),THETS(101), E1(101),EL(101),Y2(101),Y3(101),E2(101),
     2 VOL1(101),TXX(101)
            REAL L,LC,MASS,LV,       MASS1
    3 NPTS = IZ +1
    4 KJ = 1
      WRITE(6,2000) U,TH,UV,THV
 2000 FORMAT(1X,11HU,TH,UV,THV   ,4F20.6)
      DML(1)=0.0
      PSAT(1)=VP(TDL(NPTS)+TB)
      PRESS=PSAT(1 )
      CALL MASSU(PRESS,CMT,TDL(NPTS)+TB)
       Y(1) = (3.14159*(RS**2)*TDL(NPTS)*RHO*CSLA)
      Y1(1)=3.14159*RS**2
      DMV(1 )=CMT      -MASS1
      IF(DMV(1).LT.0.0) GO TO 29
       WRITE (6,250)
    7  DO 20 JJ = 1,IZ
    8  J=NPTS  -JJ
C     FIND SATURATED PRESSURE FOR EACH MASS
      PSAT(JJ+1)=VP(TDL(J)+TB)
C     DMASS TABLE FOR LIQUID
   17 R=RS+(DELTAL(KB)-Z(J))*TANG
   18 IF(R.GT.RC) R=RC
   19 Y(JJ+1)= (3.14159*(R**2)*           TDL(J)      *RHO*CSLA)
      Y1(JJ+1)=3.14159*R**2
   20 CONTINUE
      DZZ=Z(NPTS)-Z(IZ)
   21  CALL INTEG    (IU  ,DZZ,Y,E )
      CALL INTEG    (IU,DZZ,Y1,VOL)
      IF(IU.EQ.NPTS) GO TO 23
      IL=NPTS+1-IU
      DO 800 JJ=1,IL
      J=IU+JJ-1
      Y2(JJ)=Y(J)
      Y3(JJ)=Y1(J)
  800 CONTINUE
      DZZ=Z(2)
      CALL INTEG    (IL,DZZ,Y2,E2)
      CALL INTEG    (IL,DZZ,Y3,VOL1)
      DO 900 JJ=2,IL
      J=IU+JJ-1
      E(J)=E(IU)+E2(JJ)
      VOL(J)=VOL(IU)+VOL1(JJ)
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900 CONTINUE
 23  DO 25 JJ = 1,IZ
      J = NPTS   -JJ
    ZN=DELTAL(KB)-Z(J)
    ES=TDL(J)*RHO*CSLA*VOL(JJ+1)
 24   EL(JJ+1) =  E(JJ+1)- ES
 25 CONTINUE
    WRITE(6,3000) E(NPTS)
3000 FORMAT(16HOTOTAL ENERGY    ,E15.6)
 28 GO TO 145
 29  WRITE (6,200)
 30 X=2.0*SQRT (AL*TIME(KD))
 32 Y1(1)=3.14159*RS**2*RHO*CSLA
 34 JJ=IZ/8*8       +1
 36 DZ=(Z(NPTS)-Z(IZ))/16.0
 38 KJ=KJ+1
 42 JKI=IZ
 46 Z1=DZ
 48 DO 94 III=2,JJ
 50 PHE=Z1/X
 52 ERZ=ERF(PHE)
 53 IF((DELTAL(KB)-Z1).LT.Z(JKI)) JKI=JKI-1
 54 TXX(III)=TDL(JKI+1)-(TDL(JKI+1)-TDL(JKI))*(Z(JKI+1)-(DELTAL(KB)
   1-Z1))/(Z(JKI+1)-Z(JKI))
 56 THETS(III)=TXX(III)/(1.0-ERZ)
 58 IF((THETS(III)+TB)-32.5)  70,70,59
 59 IF(III-8) 65,60,60
 60 NINT=III-1
 62 GO TO 106
 65 DZ=DZ/2.0
 66 GO TO 38
 70 PSAT(III)=VP(THETS(III)+TB)
 78 R=RS+Z1*TANG
 80 IF(R.GT.RC) R=RC
 82 Y(III)=3.14159*R**2*RHO*CSLA*TXX(III)
 84 Y1(III)=3.14159*R**2*RHO*CSLA*    (1.0-ERZ)
 86 Z1=Z1+DZ
 94 CONTINUE
104 NINT=III
106 CALL INTEG   (NINT,DZ,Y1, E)
108 CALL INTEG   (NINT,DZ,Y, E1)
110 DO 114 III=2,NINT
112  EL(III)= E1(III)- E(III)*THETS(III)
114 CONTINUE
116 PRESS=PSAT(NINT)
117 DML(NINT)=DMF(PRESS,THETS(NINT)+TB,EL(NINT))
118 CALL MASSU(PRESS,CMT,THETS(NINT)+TB)
120 DMV(NINT)=CMT-MASS1
121 CONTINUE
122 IF(DML(NINT)-DMV(NINT)) 132,132,124
124 IF(2*NINT-JJ)126,126,128
126 IF(KJ.GT.10) GO TO 173
    DZ=DZ/2.0
127 GO TO 38
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128 DZ=DZ*2.0
    IF(KJ.GT.10) GO TO 173
130 GO TO 38
132 CONTINUE
133 III=1
134 KA=NINT-2
135 KA=(KA+1)/2
136 JJ=III+KA
    IF(JJ.GT.NPTS) JJ=NPTS
137 PRESS=PSAT(JJ)
     DML(JJ)=DMF(PRESS,THETS(JJ)+TB,EL(JJ))
138 CALL MASSU(PRESS,CMT,THETS(JJ)+TB)
139 DMV(JJ)=CMT-MASS1
140 IF(DML(JJ).GE.DMV(JJ)) III=III+KA
    IF(KA.GT.1) GO TO 135
    TS=THETS(III+1)+TB
    TS1=THETS(III)+TB
    TS2=THETS(III+1)+TB
    TI3=TXX(III)+TB
    TI4=TXX(III+1)+TB
143 GO TO 154
145 III=1
    KA=NPTS-2
146 KA=(KA+1)/2
    JJ=III+KA
    IF(JJ.GT.NPTS) JJ=NPTS
    PRESS=PSAT(JJ)
    TDLL=TDL(NPTS-JJ+1)+TB
    DML(JJ)=DMF(PRESS,TDLL,EL(JJ))
    CALL MASSU(PRESS,CMT,TDLL)
    DMV(JJ)=CMT-MASS1
147 IF(DMV(JJ).GE.DML(JJ)) III=III+KA
    IF(KA.GT.1) GO TO 146
    TS=TDL(NPTS-III)+TB
    TS1=TDL(NPTS-III+1)+TB
    TS2=TDL(NPTS-III)+TB
    TI3=TS1
    TI4=TS2
154 JJ=III
    PRESS=PSAT(JJ+1)
    DML(JJ+1)=DMF(PRESS,TS,EL(JJ+1))
156 CALL MASSU(PRESS,CMT,TS)
158 DMV(JJ+1)=CMT-MASS1
160 PN=PSAT(JJ)*(DML(JJ+1)-DMV(JJ+1))-PSAT(JJ+1)
   1*(DML(JJ)-DMV(JJ))
162 PD=DMV(JJ)-DMV(JJ+1)+DML(JJ+1)-DML(JJ)
164 P=PN/PD
    TSURF=TS1+(TS2-TS1)*(P-PSAT(JJ))/(PSAT(JJ+1)-PSAT(JJ))
    TINT=TI3+(TI4-TI3)*(P-PSAT(JJ))/(PSAT(JJ+1)-PSAT(JJ))
166 DME=DMV(JJ+1)+(DMV(JJ)-DMV(JJ+1))*(P-PSAT(JJ+1))
   1 /(PSAT(JJ)-PSAT(JJ+1))
168 KB=KB+1
170 KD=KD+1
171 WRITE(6,264) DME,DMV(JJ),DMV(JJ+1),PSAT(JJ),PSAT(JJ+1),
```

```
   1 DML(JJ),DML(JJ+1)
172 WRITE(6,260) P,TSURF,TINT
    GO TO 174
173 WRITE(6,220)
     JERROR = 2
    P=12.759
174  RETURN
200  FORMAT(///'  CONDENSING CASE CALLED     SUB MASS C'/   )
220 FORMAT(120H ******************ERROR IN MASSC PRESSURE IN ULLAGE A
   1PPROACHING OR IS IN EXCESS OF CRITICAL PRESSURE ***************
   2 //     120H ******************************************* CRITICAL
   3 POINT VALUE IS RETURNED ***************************************)
250  FORMAT(///6X, 40HEVAPORATING CASE CALLED     SUB. MASS C /   )
260 FORMAT(1H0,10X,6HNEW P=,F7.4,12H ATMOSPHERES,10X,
   113HSURFACE TEMP.,F7.3,10X,15HINTERSECT TEMP.,
   2F7.3,2X,6HDEG K.)
264 FORMAT(15X,3HDME,10X,7HDMV(JJ),7X,9HDMV(JJ+1),7X,8HPSAT(JJ) ,
   16X,10HPSAT(JJ+1),9X,7HDML(JJ),6X,9HDML(JJ+1)/5X,7F15.4)
     END
```

```
      FUNCTION VP(TEMP)
      T=TEMP
      IF(T.LT.13.803) GO TO 4
      IF(T.LT.20.268) GO TO 3
      IF(T.GT.32.976) GO TO 6
    8 V=2.00062-50.09708/(T+1.0044)+0.01748495*T
      P=10.0**V
      IF(T-29.0)1,1,2
    1 VP=P
      RETURN
    2 A=T-29.0
      TTT=A*A*A
      TF=TTT*A*A
      TS=TTT*A*TTT
      VP=P+0.001317*TTT-0.00005926*TF+0.000003913*TS
      RETURN
    3 VP=10.0**(1.772454-44.36888/T+0.02055468*T)
      RETURN
    4 WRITE(6,5)
    5 FORMAT(120H ******************************************* TEMPERATURE
     1 IS BELOW TRIPLE POINT *********************************************
     2 / /   120H ********************************************* TRIPLE P
     3OINT VALUE RETURNED **********************************************)
      T=13.803
      GO TO 3
    6 WRITE (6,7)
    7 FORMAT(120H ****************************************** TEMPERATURE
     1IS ABOVE CRITICAL POINT *****************************************
     2 //    120H ********************************************* CRITICAL
     3 POINT VALUE IS RETURNED *****************************************)
      T=32.976
      GO TO 8
      END
```

```
      SUBROUTINE MASSU(PRESS,CMT,TS)
C        THIS ROUTINE COMPUTES AN ULLAGE MASS GIVEN A PRESSURE = PRESS
C             AND RETURNS A MASS = CMT
    1 COMMON AL,AQ(100),AV,BETAL,BETAV,BQ(100),COSA,COSG,CPL(20),CSL,
     1 CPV(20),CSLA,CSV(20),DELTAL(50),DELTAV(50),DENL(20),DENV(20),DL,
     2 DPDRL(20),DPDRV(20),DPDTL(20),DPDTV(20),DRDX,DVP,FP,G,GAMMA,H,
     3 I,IDX,IU,IXV,IZ,JERROR,JTR,KB,KD,L,LC,MASS,MASS1,NPT,NTR,NUL,
     4 NULA,NUV,NX,P,PA(50),POWER,PR,PRV,PT(20),Q(100),QW,R,RC,RHO,
     5 RHOV,RS,SL(20),SLC,SV(20),TAMB,TANG,TB,TDL(101),TDV(101),TH,THV,
     6 TIME(50),TSAT(20),U,UV,X,XB(100),Y(101),Z(101),ZINC,ZV(101)
      DIMENSION CMU(101),CM(101),CML(101),ZV1(101),CMU2(101),CMU1(101),
     1   Y1(101)
         REAL L,LC
    5 NPTS = IZ+1
   31 RO=(FIND D(PRESS,0.50,TS))*0.00201572
   32 IF(L.LT.LC) GO TO 57
C       CASE 1  STRATIFIED LAYER ENTIRELY IN NOSE CONE
   34 CML(1)=RO*3.14159*((RS**2   )*(H-DELTAV(KD))-RS*((H-DELTAV(KD))
     1   **2)*TANG+(((H-DELTAV(KD))**3)/3.0)*(TANG**2))
   37 TANA = TANG
   38 DZ = DELTAV(KD)/ZINC
   39 ZV(1) = 0.0
   41 DO 42   LK = 2,NPTS
   42 ZV(LK) = ZV(LK-1) + DZ
   45 DO49   K = 1,NPTS
   46 R = RC-(L+H-DELTAV(KD)+ZV(K)-LC)*TANA
      T = TDV(K)+TS
   47 RHOG=(FIND D(PRESS,0.50,T))*0.00201572
   48 Y(K) = 3.14159*RHOG*(R**2)
   49 CONTINUE
   51 CALL INTEG   ( NPTS, DZ, Y, CMU)
     CMT=CML(1)+CMU(NPTS)
   56 RETURN
C      CASE 2  STRATIFIED LAYER CROSSES TANK TRANSITION
C      PROPORTION STRATIFIED LAYER ABOVE AND BELOW TANK TRANSITION
   57 IF (L+H-DELTAV(KD).GT.LC) GO TO 105
   59 DU = L+H-LC
   60 DLL = DELTAV(KD)-DU
   61 FRACTL = DLL/DELTAV(KD)
   62 NPTSL = FRACTL *ZINC+1.0
   63 IF(NPTSL.GE.7) GO TO 67
   64 NPTSL = 7
   65 NPTSU = NPTS-6
   66 GO TO 71
   67 NPTSU = NPTS-NPTSL+1
   68 IF(NPTSU.GE.7) GO TO 76
   69 NPTSU = 7
   70 NPTSL = NPTS-6
   71 IF(NPTS.GE.15) GO TO 76
   72 PRINT 73
   73 FORMAT(48H1 NOT ENOUGH INCREMENTS FOR ZV IN CMU---IN MASSC)
   74 JERROR = 2
   75 RETURN
   76 DZU = DU/(NPTSU-1)
```

```
 77  DZL = DLL/(NPTSL-1)
       TABLE OF Y(K) FOR CYLINDER SECTION
 78  ZV(1) = 0.0
 79  TANA = 0.0
 80  DO 84 KK = 1,NPTSL
 81  R = RC-(L+H-DELTAV(KD)+ZV(KK)-LC)*TANA
     T = TDV(KK) + TS
 82 RHOG=(FIND D(PRESS,0.50,T))*0.00201572
 83 Y(KK) = 3.14159*RHOG*(R**2)
 84  ZV(KK+1) = ZV(KK)+DZL
 85  CALL INTEG   ( NPTSL, DZL, Y, CMU1)
     MASS IN CYLINDER SECTION BELOW STRATIFIED LAYER
 86  CL1 = 3.14159*RO*(RC**2)*(   H-DELTAV(KD))
     MASS IN PORTION OF ULLAGE BELOW TANK TRANSITION
 88 CML(NPTSL)=CL1+CMU1(NPTSL)
       MASS IN PORTION OF ULLAGE ABOVE TANK TRANSITION
 91  TANA =TANG
 92  ZV1(1) = ZV(NPTSL)
 93  DO 97 JI = 1,NPTSU
 94  R = RC-(L+H-DELTAV(KD)+ZV1(JI)-LC)*TANA
    N=NPTSL-1+JI
    T=TDV(N)+TS
 95 RHOG=(FIND D(PRESS,0.50,T))*0.00201572
 96 Y1(JI)= 3.14159*RHOG*(R**2)
 97  ZV1(JI+1) = ZV1(JI)+DZU
 98  CALL INTEG   ( NPTSU,DZU, Y1, CMU2)
      TOTAL MASS IN ULLAGE SPACE
    CMT=CML(NPTSL)+CMU2(NPTSU)
104  RETURN
       CASE 3  UNSTRATIFIED PORTION OF ULLAGE CROSSES
                    TANK TRANSITION
      MASS IN CYLINDER PORTION
105   CL1 = 3.14159*RO*(RC**2)*(LC-L)
       MASS IN NOSE BELOW STRATIFIED LAYER
    H2=(L+H-DELTAV(KD)-LC)
106 R1=RS-H2*TANG
107 CL2=3.14159*H2/3.*(RS**2+R1**2+RS*R1)*RO
        MASS BELOW STRATIFIED LAYER
108  CML(1) = CL1 + CL2
109  DZ = DELTAV(KD)/ZINC
110  TANA = TANG
112  ZV(1) = 0.0
113  DO  114  K  = 2,NPTS
114  ZV(K ) = ZV(K -1) + DZ
115 CONTINUE
116  DO120 JC = 1,NPTS
117  R = RC-(L+H-DELTAV(KD)+ZV(JC)-LC)*TANA
     T = TDV(JC)+TS
118 RHOG=(FIND D(PRESS,0.50,T))*0.00201572
119 Y(JC) = 3.14159*RHOG*(R**2)
120 CONTINUE
121  CALL INTEG   (NPTS, DZ, Y, CMU)
       TOTAL MASS IN ULLAGE
126 CMT=CML(1)+CMU(NPTS)
```

```
127  RETURN
     END
```

```
      FUNCTION ERF(X)
      DIMENSION B(10,10),V(5),VINCR(5),Y(256),NVALUS(5)
      DIMENSION AA(50),AB(45),AC(45),AD(45),AE(45),AF(26)
      EQUIVALENCE(AA,Y),(AB,Y(51)),(AC,Y(96)),(AD,Y(141)),(AE,Y(186))
      EQUIVALENCE(AF,Y(231))
       DATA AA/0.0,.02256457469,.04511110615,.06762159439,.09007812584,
     1 .11246291602,.13475835182,.15694703306,.17901181320,.20093583902,
     2 .22270258921,.24429591160,.26570005895,.28689972322,.30788006803,
     3 .32862675946,.34912599480,.36936452934,.38932970113,.40900945342,
     4 .42839235505,.44746761843,.46622511528,.48465539000,.50274967069,
     5 .52049987781,.53789863048,.55493925046,.57161576382,.58792290038,
     6 .60385609085,.61941146190,.63458582912,.64937668796,.66378220274,
     7 .67780119384,.69143312314,.70467807785,.71753675281,.73001043130,
     8 .74210096471,.75381075087,.76514271145,.77610026832,.78668731917,
     9 .79690821242,.80676772155,.81627101898,.82542364964,.83423150434/
                                                              DATA AB/
     1 .84270079295,.84883447667,.85478421145,.86055291752,.86614358664,
     2 .87155927596,.87680310194,.88187823430,.88678789017,.89153532819,
     3 .89612384294,.90055675926,.90483742692,.90896921523,.91295550797,
     4 .91679969838,.92050518430,.92407536352,.92751362930,.93082336599,
     5 .93400794494,.93707072048,.94001502616,.94284417109,.94556143656,
     6 .94817007278,.95067329581,.95307428466,.95537617864,.95758207481,
     7 .95969502564,.96171803684,.96365406541,.96550601777,.96727674813,
     8 .96896905700,.97058568986,.97212933600,.97360262746,.97500813820,
     9 .97634838334,.97762581857,.97884283967,.98000178222,.98110492131/
                                                              DATA AC/
     1 .98215447153,.98315258690,.98410136103,.98500282736,.98585895940,
     2 .98667167122,.98744281785,.98817419593,.98886754427,.98952454462,
     3 .99014682240,.99073594755,.99129343540,.99182074761,.99231929313,
     4 .99279042924,.99323546256,.99365565017,.99405220070,.99442627546,
     5 .99477898959,.99511141320,.99542457260,.99571945145,.99599699195,
     6 .99625809604,.99650362661,.99673440867,.99695123054,.99715484503,
     7 .99734597064,.99752529257,.99769346440,.99785110820,.99799881667,
     8 .99813715370,.99826665559,.99838783206,.99850116734,.99860712112,
     9 .99870612960,.99879860642,.99888494363,.99896551257,.99904066481/
                                                              DATA AD/
     1 .99911073297,.99917603160,.99923685799,.99929349295,.99934620158,
     2 .99939523401,.99944032612,.99948320022,.99952256573,.99955911981,
     3 .99959304798,.99962452471,.99965371398,.99968076986,.99970583698,
     4 .99972905108,.99975053947,.99977042149,.99978880894,.99980580653,
     5 .99982151225,.99983601774,.99984940871,.99986176523,.99987316207,
     6 .99988366905,.99989335129,.99990226950,.99991048029,.99991803636,
     7 .99992498681,.99993137728,.99993725025,.99994264521,.99994759883,
     8 .99995214516,.99995631582,.99996014013,.99996364527,.99996685644,
     9 .99996979696,.99997248844,.99997495086,.99997790950,.99998053361/
                                                              DATA AE/
     1 .99998285914,.99998491842,.99998674048,.99998835134,.99998977436,
     2 .99999103043,.99999213826,.99999311455,.99999397424,.99999473065,
     3 .99999539565,.99999597982,.99999649258,.99999694229,.99999733640,
     4 .99999768150,.99999798344,.99999824741,.99999847801,.99999867928,
     5 .99999885482,.99999900780,.99999914100,.99999925690,.99999935766,
     6 .99999944518,.99999952115,.99999958704,.99999964414,.99999969358,
     7 .99999973635,.99999977333,.99999980528,.99999983285,.99999985663,
     8 .99999987712,.99999989476,.99999990995,.99999992300,.99999993920,
```

```
     9 .99999995208,.99999996230,.99999997039,.99999997678,.99999998183/
                                                               DATA AF/
     1 .99999998580,.99999998893,.99999999138,.99999999330,.99999999480,
     2 .99999999597,.99999999689,.99999999760,.99999999815,.99999999858,
     3 .99999999891,.99999999916,.99999999936,.99999999951,.99999999969,
     4 .99999999980,.99999999988,.99999999992,.99999999995,.99999999997,
     5 .99999999998,.99999999999,.99999999999,1.0,1.0,1.0/
      DATA NVALUS/51,183,224,244,256/
      DATA VINCR/0.02,0.015,0.02,0.03,0.05/
      DATA V/1.0,2.98,3.8,4.4,5.0/
      LV=5
      KK=7
      TEST=.1E-10
      PI=3.1415926536
      IF(X)5,15,7
    5 X=-X
      SIGN=-1.0
      GO TO 11
    7 SIGN=1.0
      GO TO 11
   15 ERF=0.0
      GO TO 101
   11 IF(X-5.75 )17,17,21
   21 ERF=SIGN
      X=X*SIGN
      GO TO 101
   17 DO 25 L=1,LV
      IF(X-V(L))27,57,25
   25 CONTINUE
      YY=Y(256)
      DELTAX=X-5.7
      X1=5.7
      GO TO 59
   57 N=NVALUS(L)
      ERF=Y(N)*SIGN
      X=X*SIGN
      GO TO 101
   27 IF(L-1)31,31,33
   31 INTPT=X/VINCR(1)
      IF(INTPT)63,63,64
   63 DELTAX=X
      YY=0.0
      FACTOR=1.1283791671
      DENOM=1.0
   60 AK=DELTAX
      SUM=AK*FACTOR
      DO 62 J=2,KK
      QJ=J
      AK=DELTAX*DELTAX*AK*(2.0*QJ-3.0)/(2.0*QJ-1.0)
      DENOM=(QJ-1.0)*DENOM
      TERMK=FACTOR*AK*((-1.0)**(J-1))/DENOM
      SUM=SUM+TERMK
      IF( ABS(TERMK)-TEST)41,41,62
   62 CONTINUE
```

```
      GO TO 41
   64 QINTPT=INTPT
      X1=QINTPT*VINCR(1)
      DELTAX=X-X1
      N=1
      GO TO 35
   33 INTPT=(X-V(L-1))/VINCR(L)
      QINTPT=INTPT
      X1=(QINTPT*VINCR(L))+V(L-1)
      DELTAX=X-X1
      N=NVALUS(L-1)
   35 NJ=INTPT+N
      YY=Y(NJ)
   59 FACTOR=1.1283791671*EXP(-X1*X1)
      DENOM=1.0
   61 U=DELTAX/X1
      Q=DELTAX*X1
      B(1,1)=DELTAX
      SUM=B(1,1)*FACTOR
      DO 37 K=2,KK
      QK=K
      B(K,1)=2.0*Q*(QK-1.0)*B(K-1,1)/QK
      AK=B(K,1)
      DO 39 I=2,K
      QI=I
      B(K,I)=(U*(QK+QI-2.0)*(QK-QI+1.)*B(K,I-1))/(2.*(QK+QI-1.)*(QI-1.))
   39 AK=AK+B(K,I)
      DENOM=(QK-1.0)*DENOM
      TERMK=FACTOR*AK*((-1.0)**(K-1))/DENOM
      SUM=SUM+TERMK
      IF( ABS(TERMK)-TEST)41,41,37
   37 CONTINUE
   41 ERF=(YY+SUM)*SIGN
      X=X*SIGN
  101 RETURN
      END
```

```
      FUNCTION DMF(PRESS,T,E)
 1    COMMON AL,AQ(100),AV,BETAL,BETAV,BQ(100),COSA,COSG,CPL(20),CSL,
     1 CPV(20),CSLA,CSV(20),DELTAL(50),DELTAV(50),DENL(20),DENV(20),DL,
     2 DPDRL(20),DPDRV(20),DPDTL(20),DPDTV(20),DRDX,DVP,FP,G,GAMMA,H,
     3 I,IDX,IU,IXV,IZ,JERROR,JTR,KB,KD,L,LC,MASS,MASS1,NPT,NTR,NUL,
     4 NULA,NUV,NX,P,PA(50),POWER,PR,PRV,PT(20),Q(100),QW,R,RC,RHO,
     5 RHOV,RS,SL(20),SLC,SV(20),TAMB,TANG,TB,TDL(101),TDV(101),TH,THV,
     6 TIME(50),TSAT(20),U,UV,X,XB(100),Y(101),Z(101),ZINC,ZV(101)
      PX=(P+PRESS)/2.0
      DO 5 IJ=2,20
      IF(PT(IJ)-PX)5,8,8
 5    CONTINUE
       WRITE (6,15)
      GO TO 10
 8    S=SV(IJ)-(SV(IJ)-SV(IJ-1))*(PT(IJ)-PRESS)/(PT(IJ)-PT(IJ-1))
      VL=(S-SLC)*T/2.01572
      DMF=E/VL
      RETURN
10    VL=(SV(20)-SLC)*T/2.01572
      DMF=E/VL
      RETURN
15     FORMAT( 53HNO BRACKETING VALUES IN PROPERTIES TABLE  SUB. MASSC )
      END
```

0.20	26.80	20155.	8.946	6294.2	1216.	0.013	11.95	69.51	15.861
21.81		14.79							
0.30	27.11	18605.	8.778	4397.5	1259.	0.019	12.83	67.20	16.802
22.22		15.73							
0.40	27.37	17563.	8.663	3407.8	1287.	0.025	13.51	65.58	17.532
22.59		16.48							
0.50	27.59	16845.	8.567	2795.4	1306.	0.030	14.07	64.32	18.137
22.94		17.08							
0.60	27.80	16117.	8.482	2376.6	1319.	0.036	14.56	63.29	18.659
23.27		17.67							
0.70	27.98	15566.	8.402	2071.0	1329.	0.041	15.00	62.42	19.121
23.60		18.16							
0.80	28.16	15077.	8.329	1837.1	1335.	0.047	15.39	61.66	19.538
23.91		18.62							
0.90	28.32	14626.	8.266	1652.7	1340.	0.052	15.75	60.99	19.918
24.21		19.05							
1.0	28.48	14091.	8.204	1506.86	1343.	0.057	16.08	60.41	20.268
24.50		19.53							
1.5	29.19	12268.	7.904	1040.8	1338.	0.084	17.47	58.07	21.722
25.94		21.43							
2.0	29.82	10690.	7.620	796.51	1314.	0.112	18.58	56.36	22.861
27.39		23.28							
3.0	30.98	8484.	7.088	541.72	1240.	0.169	20.36	53.85	24.632
30.46		26.52							
4.0	32.10	6787.	6.582	407.74	1147.	0.230	21.82	51.95	26.023
34.05		29.84							
5.0	33.24	5361.	6.097	323.89	1043.	0.296	23.10	50.35	27.187
38.45		33.74							
6.0	34.44	4211.	5.640	265.73	928.8	0.370	24.28	48.92	28.198
44.12		38.37							
7.0	35.74	3211.	5.203	222.51	809.0	0.453	25.39	47.59	29.097
51.79		44.67							
8.0	37.20	2387.0	4.770	188.66	680.4	0.549	26.48	46.29	29.910
62.87	53.02								
9.0	38.88	1687.0	4.337	161.07	543.4	0.662	27.58	44.98	30.651
80.41	65.56								
10.0	40.93	1109.0	3.894	137.49	405.2	0.796	27.92	28.73	31.335
86.20	67.43								
12.5	52.53	66.57	2.332	81.91	39.36	1.479	32.92	38.32	32.836
1259.25	766.30								

45.0	152.4	502.93	183.64	441.25
980.67	0.05	5.0	50.0	
1.12616	300.0	1.40		
2.0				

14.000	250.7E-6
14.500	234.1E-6
15.000	221.3E-6
15.500	207.3E-6
16.000	197.5E-6
16.500	185.6E-6
17.000	177.7E-6
17.500	168.5E-6
18.000	160.5E-6
18.500	153.8E-6
19.000	147.0E-6
19.500	141.3E-6
20.000	135.4E-6
20.500	130.6E-6
21.000	125.3E-6
21.500	120.6E-6
22.000	116.1E-6
23.000	108.1E-6
24.000	100.8E-6
25.000	93.5E-6
26.000	87.2E-6
26.500	84.1E-6
27.000	81.0E-6
27.500	78.1E-6
28.000	75.2E-6
28.500	72.4E-6
29.000	69.6E-6
29.500	67.0E-6
30.000	64.9E-6
30.500	61.2E-6
31.000	58.1E-6
31.500	55.7E-6
32.000	51.9E-6
32.500	47.5E-6
32.700	43.9E-6

0.0	0.031666
183.64	0.031666
184.0	0.0036618
502.93	0.0036618
624.89	0.0036618