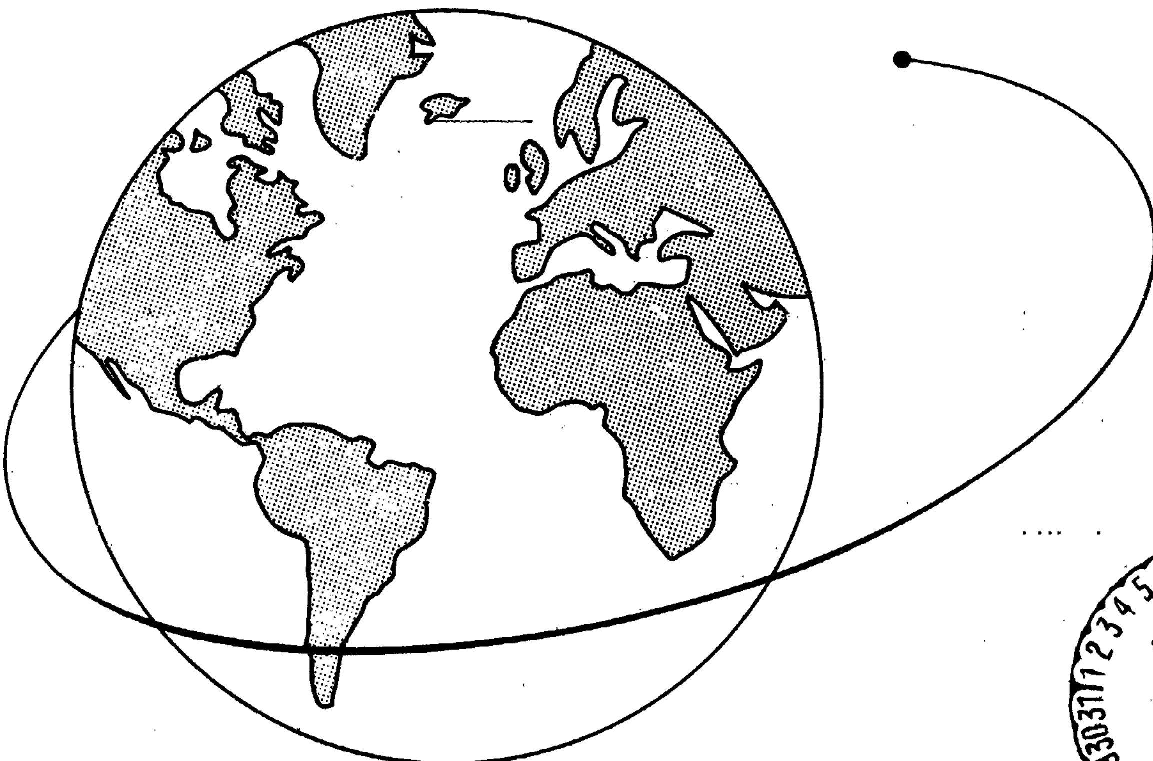
# COMPUTATION OF HANSEN COEFFICIENTS

J. R. CHERNIACK





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## ABSTRACT

This paper describes some procedures for computer development of Hansen coefficients. The method of Von Zeipel and Andoyer is found most efficient. A table extends the method from 7th to 12th order.



## RÉSUMÉ

On décrit ici certains procédés pour obtenir le développement des coefficients d'Hansen à l'aide d'un ordinateur. La méthode de Von Zeipel et Andoyer a été trouvée la plus efficace. Une table prolonge la méthode du 7ème au 12ème ordre.



## КОНСПЕКТ

В этой статье описывается процедура развития коэффициентов Гансена с помощью ЭВМ. Метод фон Зипеля и Андоера был найден наиболее эффективным. Таблица расширяет метод от 7-го до 12-го порядка.

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## 1. INTRODUCTION

Cayley's famous paper "Tables of the Development of Functions in the Theory of Elliptic Motion" (1861) contains tables of Hansen coefficients to seventh order in the eccentricity. We recently required Hansen coefficients to higher order in computer-accessible form and experimented with techniques for their computation. The method of Andoyer (1903) and Von Zeipel (1912) is an order of magnitude faster than any other technique we found. It is little known, as far as we can determine, appearing in English only in Izsak, Gerard, Efimba, and Barnett (1964) and there almost parenthetically.

In this note, we briefly describe some of the slower procedures and the Von Zeipel–Andoyer (VZA) method and extend a table of Izsak to 12th order.

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## 2. NOTATION

Our notation will be the following

r, the radius vector;

a, the semimajor axis;

v, the true anomaly;

M, the mean anomaly;

e, the eccentricity;

$i = \sqrt{-1}$;

$x = \exp(iv)$;

$z = \exp(iM)$;

the Hansen coefficients $X_j^{n,k}$, which are power series in e, and their generating function $X^{n,k}$ are defined by

$$(r/a)^n x^k = X^{n,k} = \sum_{j=-\infty}^{+\infty} X_j^{n,k} z^j \quad . \tag{1}$$

If the real and imaginary parts of (1) are separated, we can write

$$(r/a)^n \cos(kv) = \sum_{j=0}^{\infty} C_j^{n,k} \cos(jM) \quad ,$$

$$(r/a)^n \sin(kv) = \sum_{j=1}^{\infty} S_j^{n,k} \sin(jM) \quad , \tag{2}$$

where

$$C_0^{n,k} = X_0^{n,k}$$

$$C_j^{n,k} = X_{+j}^{n,k} + X_{-j}^{n,k} \qquad j = 1, 2, 3 \cdots$$

$$S_j^{n,k} = X_{+j}^{n,k} - X_{-j}^{n,k} \qquad j = 1, 2, 3 \cdots \quad .$$

## 3. SOME METHODS OF COMPUTING HANSEN COEFFICIENTS

Since Cayley's original method was designed for hand computation, it is not particularly adaptable for development on a computer. Tisserand (1888, vol. 1, p. 249) expresses Hansen coefficients as a series involving Bessel functions and hypergeometric series. Tisserand's method was programed with the symbol-manipulating system SPASM and was so slow that it stimulated the literature search that culminated in this note. One coefficient that required 180 sec to compute was eventually found in 30 msec by use of VZA.

The next method tried was direct application of Lagrange's Inversion Theorem (for a description, see Brown and Shook (1933)). Although attractive in some special cases, it is much too slow for general use.

The next method is the simplest; we compute the base set $X^{1,0}$, $X^{-1,0}$, $X^{0,1}$, and $X^{0,-1}$ by any method. Since

$$X^{\pm n, \pm k} = \left(X^{\pm 1, 0}\right)^n \left(X^{0, \pm 1}\right)^k \quad ,$$

$X^{n,k}$ can be computed from the base set by multiplication. This method is especially attractive if tables of $X^{n,k}$ are desired. Twenty seconds of computer time is required for each multiplication if 12th-order terms in e are retained. A special-purpose program could be written that is 3 to 5 times faster for the symbolic multiplication.

All the previous methods have the following defects:

a) The methods require manipulation (multiplication or differentiation) of polynomials, a rather slow class of operations.

b) The computations must be done with rational-fraction coefficients. This is about 10 times slower than is possible with integer coefficients.

The VZA method, which is next described, avoids both these defects at the cost of some added storage and complexity.

## 4. OUTLINE OF THE VON ZEIPEL–ANDOYER METHOD

In Section 5, we show how to define $J_{\rho,\sigma}(n, k)$, which are polynomials in n and k with integer coefficients. For compactness, we write $J^{n,k}_{\rho,\sigma}$ for $J_{\rho,\sigma}(n, k)$. These polynomials are computed only once and are saved for later use.

If a particular Hansen coefficient is desired to $\mathcal{O}(e^M)$, $M \leq 12$, we need only evaluate at most $(M + 1)/2$ of the J's for particular values of n, k, ρ, σ. Denominators must now be affixed to the J's, and the resulting fractions reduced to lowest terms. Since polynomial evaluation is about an order of magnitude faster for integers than for rational fractions, our ability to do most of the computation in terms of integers affords significant advantages when exact results are required.

## 5. DETAILS

### 5.1 Computation of $J^{n,k}_{\rho,\sigma}$ for $\rho \geq \sigma \geq 0$

$J^{n,k}_{\rho,\sigma}$ is a polynomial in n and k with integer coefficients. Most of the following development is given by Izsak et al. (1964) and is reprinted here only for completeness. For equations (3) through (5), $J^{n,k}_{\rho,\sigma}$ with negative $\rho$ or $\sigma$ have value 0 and can be ignored.

$$J^{n,k}_{0,0} = 1 \quad ,$$

$$J^{n,k}_{1,0} = 2k - n \quad , \tag{3}$$

$$J^{n,k}_{\rho,0} = (2k - n)\, J^{n,k+1}_{\rho-1,0} + (\rho - 1)\,(k - n)\, J^{n,k+2}_{\rho-2,0} \quad , \tag{4}$$

$$\begin{aligned} J^{n,k}_{\rho,\sigma} = {} & -(2k + n)\, J^{n,k-1}_{\rho,\sigma-1} - (\sigma - 1)\,(k + n)\, J^{n,k-2}_{\rho,\sigma-2} \\ & - \rho\,(\rho - 5\sigma + 4 + 4k + n)\, J^{n,k}_{\rho-1,\sigma-1} \\ & + \rho\,(\rho - \sigma + k) \sum_{\tau\geq 2} c_{\rho\sigma\tau}\, J_{\rho-\tau,\sigma-\tau} \quad , \end{aligned} \tag{5}$$

where

$$C_{\rho\sigma\tau} = (\rho - 1)\,(\rho - 2) \cdots (\rho - \tau + 1)\,(\sigma - 1)\,(\sigma - 2) \cdots (\sigma - \tau + 1)\, C_{\tau}$$

and

$$C_{\tau} = (-1)^{\tau} \binom{3/2}{\tau}\, 2^{2\tau - 1} = 3, 2, 3, 6, 14, 36, 99, \cdots \quad .$$

The computation begins with equations (3) and continues in the order

$$J^{n,k}_{2,0}, \quad J^{n,k}_{1,1},$$

$$J^{n,k}_{3,0}, \quad J^{n,k}_{2,1},$$

$$J^{n,k}_{4,0}, \quad J^{n,k}_{3,1}, \quad J^{n,k}_{2,2}, \quad \ldots .$$

Equation (4) is used to compute the polynomials in the first column, and equation (5) is used in all other cases.

## 5.2 Computation of $X^{n,k}_{\rho,\sigma}$

$$X^{n,k}_{\rho,\sigma} = X^{n,-k}_{\sigma,\rho} \qquad \rho < \sigma$$

$$X^{n,k}_{\rho,\sigma} = \frac{J^{n,k}_{\rho,\sigma}}{(2^{\rho+\sigma}\rho!\sigma!)} \qquad \rho \geq \sigma \quad . \tag{6}$$

## 5.3 Computation of $X^{n,k}_{j}$

$$X^{n,k}_{j} = \sum_{\substack{\rho-\sigma=j+k \\ \rho,\,\sigma\geq 0}} X^{n,k}_{\rho,\sigma}\, e^{\rho+\sigma} \quad . \tag{7}$$

## 6. THE PROGRAMING

Izsak published $J^{n,k}_{\rho,\sigma}$ for $\rho, \sigma \geq 0$, $\rho + \sigma \leq 7$. We continued the calculation to $\rho + \sigma \leq 12$, using the general-purpose symbol-manipulating system SPASM (Hall and Cherniack, 1969). The computation to order 8 took 1.5 min, and the extension of the computation to order 12 took an additional 13.5 min of CDC 6400 time.

The output of our program was in the form of Fortran DATA statements, which became the heart of a Fortran program that computes integer values of $J^{n,k}_{\rho,\sigma}$ from specific values of n, k, $\rho$, and $\sigma$.

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## APPENDIX

This Appendix contains a brief example, as well as a table of $J^{n,k}_{\rho,\sigma}$ for $0 \le \rho + \sigma \le 12$.

The example is the computation of r/a to $\mathcal{O}(e)$. From equation (1), we have

$$r/a = X^{1,0} \quad ,$$

$$= \sum_{j=-\infty}^{\infty} X_j^{1,0} z^j \quad ,$$

$$= \sum_{j=-\infty}^{\infty} z^j \sum_{\substack{\rho-\sigma=j \\ \rho,\sigma\ge 0}} X^{1,0}_{\rho,\sigma} e^{\rho+\sigma} \quad , \quad \text{(by equation (7))} \quad ,$$

$$= X^{1,0}_{0,0} + \left(X^{1,0}_{0,1} z^{-1} + X^{1,0}_{1,0} z^{1}\right) e + \mathcal{O}(e^2) \quad . \qquad \text{(A-1)}$$

From Section 5.2, it follows that

$$X^{1,0}_{0,0} = J^{1,0}_{0,0} \quad ,$$

$$= 1 \quad .$$

Similarly,

$$X^{1,0}_{0,1} = X^{1,0}_{1,0} \quad ,$$

$$= J^{1,0}_{1,0}/2 \quad ,$$

$$= (2k - n)/2 \Big|^{n=1}_{k=0} \quad ,$$

$$= -1/2 \quad .$$

Substituting in equation (A-1) gives

$$r/a = 1 - e(z^{-1} + z)/2 + \mathcal{O}(e^2) \quad ,$$

$$= 1 - e \cos M + \mathcal{O}(e^2) \quad .$$

## Table A-1. Polynomials in n and k for $J^{n,k}_{\rho,\sigma}$ tabulated for $0 \le \rho + \sigma \le 12$, $\rho \ge \sigma$.

$J(0,0) = 1$

$J(1,0) = -1*N+2*K$

$J(2,0) = -3*N+N^2+5*K-4*K*N+4*K^2$

$J(1,1) = N+N^2-4*K^2$

$J(3,0) = -17*N+9*N^2-1*N^3+26*K-33*K*N+6*K*N^2+30*K^2-12*K^2*N+8*K^3$

$J(2,1) = 3*N+N^2-1*N^3-2*K+5*K*N+2*K*N^2-10*K^2+4*K^2*N-8*K^3$

$J(4,0) = -142*N+95*N^2-18*N^3+N^4+206*K-330*K*N+102*K*N^2-8*K*N^3+283*K^2-192*K^2*N+24*K^2*N^2+120*K^3-32*K^3*N+16*K^4$

$J(3,1) = 22*N-1*N^2-6*N^3+N^4-22*K+47*K*N+3*K*N^2-4*K*N^3-64*K^2+48*K^2*N-60*K^3+16*K^3*N-16*K^4$

$J(2,2) = 2*N-1*N^2-2*N^3+N^4-9*K^2-8*K^2*N^2+16*K^4$

$J(5,0) = -1569*N+1220*N^2-305*N^3+30*N^4-1*N^5+2194*K-4080*K*N+1660*K*N^2-230*K*N^3+10*K*N^4+3360*K^2-2995*K^2*N+660*K^2*N^2-40*K^2*N^3+1790$
$*K^3-840*K^3*N+80*K^3*N^2+400*K^4-80*K^4*N+32*K^5$

$J(4,1) = 231*N-68*N^2-41*N^3+14*N^4-1*N^5-258*K+572*K*N-76*K*N^2-42*K*N^3+6*K*N^4-648*K^2+617*K^2*N-60*K^2*N^2-8*K^2*N^3-614*K^3+296*K^3*N-16$
$*K^3*N^2-240*K^4+48*K^4*N-32*K^5$

$J(3,2) = 3*N-8*N^2-5*N^3+6*N^4-1*N^5+10*K-10*K*N-12*K*N^2+2*K*N^3+2*K*N^4-12*K^2-9*K^2*N-36*K^2*N^2+8*K^2*N^3+26*K^3-24*K^3*N-16*K^3*N^2+80*K^4$
$-16*K^4*N+32*K^5$

$J(6,0) = -21578*N+18694*N^2-5595*N^3+745*N^4-45*N^5+N^6+29352*K-60752*K*N+29535*K*N^2-5530*K*N^3+435*K*N^4-12*K*N^5+48538*K^2-51615*K^2*N$
$+15345*K^2*N^2-1680*K^2*N^3+60*K^2*N^4+29835*K^3-18860*K^3*N+3240*K^3*N^2-160*K^3*N^3+8660*K^4-3120*K^4*N+240*K^4*N^2+1200*K^5-192*K^5*N$
$+64*K^6$

## Table A-1 (Cont.)

$$
\begin{aligned}
J(5,1) = {} & 3096N-1466N^2-255N^3+185N^4-25N^5+N^6-3608K+8454KN-2280KN^2-340KN^3+130KN^4-\text{[illegible]}KN^5-8588K^2+9535K^2N-1765K^2N^2 \\
& -80K^2N^3+20K^2N^4-8200K^3+5280K^3N-640K^3N^2-3740K^4+1360K^4N-80K^4N^2-800K^5+128K^5N-64K^6
\end{aligned}
$$

$$
\begin{aligned}
J(4,2) = {} & -72N-58N^2+5N^3+41N^4-13N^5+N^6+136K-344KN-115KN^2+66KN^3+17KN^4-4KN^5+314K^2-399K^2N-255K^2N^2+112K^2N^3-4K^2N^4 \\
& +593K^3-452K^3N-136K^3N^2+32K^3N^3+788K^4-304K^4N-16K^4N^2+400K^5-64K^5N+64K^6
\end{aligned}
$$

$$
J(3,3) = 24N-26N^2-15N^3+25N^4-9N^5+N^6-172K^2+39K^2N-93K^2N^2+48K^2N^3-12K^2N^4+196K^4-48K^4N+48K^4N^2-64K^6
$$

$$
\begin{aligned}
J(7,0) = {} & -355081N+334369N^2-113974N^3+18515N^4-1540N^5+63N^6-1N^7+472730K-1062285KN+587230KN^2-133945KN^3+14490KN^4-735KN^5 \\
& +14KN^6+828758K^2-1000727K^2N+361935K^2N^2-54425K^2N^3+3570K^2N^4-84K^2N^5+563486K^3-432775K^3N+101990K^3N^2 \\
& -9240K^3N^3+280K^3N^4+193130K^4-95340K^4N+13440K^4N^2-560K^4N^3+35560K^5-10416K^5N+672K^5N^2+3360K^6-448K^6N+128K^7
\end{aligned}
$$

$$
\begin{aligned}
J(6,1) = {} & 50195N-30679N^2+326N^3+2515N^4-520N^5+39N^6-1N^7-59570K+147149KN-55026KN^2-175KN^3+2270KN^4-285KN^5+10KN^6 \\
& -138970K^2+172821K^2N-45045K^2N^2+995K^2N^3+570K^2N^4-36K^2N^5-134246K^3+104095K^3N-19030K^3N^2+600K^3N^3+40K^3N^4 \\
& -66790K^4+33140K^4N-3840K^4N^2+80K^4N^3-17800K^5+5232K^5N-288K^5N^2-2400K^6+320K^6N-128K^7
\end{aligned}
$$

$$
\begin{aligned}
J(5,2) = {} & -2229N-55N^2+446N^3+275N^4-160N^5+23N^6-1N^7+2850K-8045KN-182KN^2+1295KN^3+50KN^4-75KN^5+6KN^6+8262K^2-10755 \\
& K^2N-1485K^2N^2+1675K^2N^3-150K^2N^4-4K^2N^5+12246K^3-9695K^3N-810K^3N^2+680K^3N^3-40K^3N^4+11610K^4-5900K^4N+80K^4 \\
& N^3+5960K^5-1776K^5N+32K^5N^2+1440K^6-192K^6N+128K^7
\end{aligned}
$$

$$
\begin{aligned}
J(4,3) = {} & 127N-159N^2-46N^3+147N^4-76N^5+15N^6-1N^7+214K+109KN+26KN^2+57KN^3-10KN^4-9KN^5+2KN^6-842K^2+481K^2N-441K^2 \\
& N^2+351K^2N^3-126K^2N^4+12K^2N^5-494K^3-89K^3N+90K^3N^2+24K^3N^3-24K^3N^4+874K^4-364K^4N+384K^4N^2-48K^4N^3-40K^5+48 \\
& K^5N+96K^5N^2-480K^6+64K^6N-128K^7
\end{aligned}
$$

Table A-1 (Cont.)

$$\begin{aligned} J(8,0) = {} & -6805296*N+6852460*N^{2}-2581964*N^{3}+464729*N^{4}-49840*N^{5}+2842*N^{6}-84*N^{7}+N^{8}+8902448*K-21370136*K*N+13040916*K*N^{2}-3433388*K*N^{3} \\ & +458640*K*N^{4}-32396*K*N^{5}+1148*K*N^{6}-16*K*N^{7}+16352684*K^{2}-21771428*K^{2}*N+9077782*K^{2}*N^{2}-1883780*K^{2}*N^{3}+153650*K^{2}*N^{4}-6720*K^{2}*N^{5} \\ & +112*K^{2}*N^{6}+12000604*K^{3}-10613820*K^{3}*N+3082100*K^{3}*N^{2}-388080*K^{3}*N^{3}+21840*K^{3}*N^{4}-448*K^{3}*N^{5}+4628057*K^{4}-2812320*K^{4}*N+550480*K^{4} \\ & *N^{2}-42560*K^{4}*N^{3}+1120*K^{4}*N^{4}+1023120*K^{5}-415744*K^{5}*N+49728*K^{5}*N^{2}-1792*K^{5}*N^{3}+130592*K^{6}-3[illegible]256*K^{6}*N+1792*K^{6}*N^{2}+8960*K^{7}-1024 \\ & *K^{7}*N+256*K^{8} \end{aligned}$$

$$\begin{aligned} J(7,1) = {} & 952848*N-687996*N^{2}+84280*N^{3}+33649*N^{4}-10640*N^{5}+1162*N^{6}-56*N^{7}+N^{8}-1139472*K+2950800*K*N-1351154*K*N^{2}+114793*K*N^{3}+35525*K \\ & *N^{4}-7693*K*N^{5}+525*K*N^{6}-12*K*N^{7}-2643828*K^{2}+3594304*K^{2}*N-1171023*K^{2}*N^{2}+89250*K^{2}*N^{3}+10745*K^{2}*N^{4}-1680*K^{2}*N^{5}+56*K^{2}*N^{6} \\ & -2590518*K^{3}+2298653*K^{3}*N-546875*K^{3}*N^{2}+36260*K^{3}*N^{3}+980*K^{3}*N^{4}-112*K^{3}*N^{5}-1366792*K^{4}+830480*K^{4}*N-138880*K^{4}*N^{2}+6720*K^{4}*N^{3} \\ & -415380*K^{5}+168896*K^{5}*N-17808*K^{5}*N^{2}+448*K^{5}*N^{3}-72464*K^{6}+17920*K^{6}*N-896*K^{6}*N^{2}-6720*K^{7}+768*K^{7}*N-256*K^{8} \end{aligned}$$

$$\begin{aligned} J(6,2) = {} & -55728*N+15052*N^{2}+8612*N^{3}+889*N^{4}-2000*N^{5}+442*N^{6}-36*N^{7}+N^{8}+65968*K-191916*K*N+31906*K*N^{2}+22074*K*N^{3}-2560*K*N^{4}-1062*K*N^{5} \\ & +190*K*N^{6}-8*K*N^{7}+192608*K^{2}-269444*K^{2}*N+18306*K^{2}*N^{2}+26580*K^{2}*N^{3}-4290*K^{2}*N^{4}+16*K^{2}*N^{6}+262622*K^{3}-230926*K^{3}*N+11530*K^{3}*N^{2} \\ & +12360*K^{3}*N^{3}-1560*K^{3}*N^{4}+32*K^{3}*N^{5}+218177*K^{4}-131680*K^{4}*N+8400*K^{4}*N^{2}+2240*K^{4}*N^{3}-160*K^{4}*N^{4}+109320*K^{5}-44544*K^{5}*N+2592*K^{5}*N^{2} \\ & +128*K^{5}*N^{3}+30944*K^{6}-7680*K^{6}*N+256*K^{6}*N^{2}+4480*K^{7}-512*K^{7}*N+256*K^{8} \end{aligned}$$

$$\begin{aligned} J(5,3) = {} & 1296*N-1084*N^{2}-168*N^{3}+1009*N^{4}-720*N^{5}+202*N^{6}-24*N^{7}+N^{8}+688*K+2032*K*N+642*K*N^{2}+275*K*N^{3}-385*K*N^{4}-79*K*N^{5}+47*K*N^{6}-4*K*N^{7} \\ & -6164*K^{2}+8768*K^{2}*N-1671*K^{2}*N^{2}+2610*K^{2}*N^{3}-1535*K^{2}*N^{4}+240*K^{2}*N^{5}-8*K^{2}*N^{6}-9586*K^{3}+3919*K^{3}*N+2695*K^{3}*N^{2}-180*K^{3}*N^{3}\,[illegible]*K^{3}*N^{4} \\ & +48*K^{3}*N^{5}-1352*K^{4}-880*K^{4}*N+4480*K^{4}*N^{2}-960*K^{4}*N^{3}-4380*K^{5}+1856*K^{5}*N+1488*K^{5}*N^{2}-192*K^{5}*N^{3}-6032*K^{6}+1536*K^{6}*N+128*K^{6}*N^{2} \\ & -2240*K^{7}+256*K^{7}*N-256*K^{8} \end{aligned}$$

## Table A-1 (Cont.)

$$J(4,4) = 720 N - 1044 N^2 - 140 N^3 + 889 N^4 - 560 N^5 + 154 N^6 - 20 N^7 + N^8 - 7140 K^2 + 2012 K^2 N - 3674 K^2 N^2 + 2268 K^2 N^3 - 958 K^2 N^4 + 192 K^2 N^5 - 16 K^2 N^6$$
$$+ 7801 K^4 - 2720 K^4 N + 2128 K^4 N^2 - 576 K^4 N^3 + 96 K^4 N^4 - 2272 K^6 + 512 K^6 N - 256 K^6 N^2 + 256 K^8$$

$$J(9,0) = -148869153 N + 158479488 N^2 - 64727522 N^3 + 13541116 N^4 - 1632099 N^5 + 116172 N^6 - 4830 N^7 + 105 N^8 - 1 N^9 + 191915874 K - 486603432 K N$$
$$+ 321495288 K N^2 - 94568292 K N^3 + 14740026 K N^4 - 1298556 K N^5 + 64680 K N^6 - 1692 K N^7 + 18 K N^8 + 366387696 K^2 - 527559990 K^2 N$$
$$+ 245509992 K^2 N^2 - 53085016 K^2 N^3 + 6036660 K^2 N^4 - 370818 K^2 N^5 + 11592 K^2 N^6 - 144 K^2 N^7 + 285672572 K^3 - 281715336 K^3 N + 95275824$$
$$K^3 N^2 - 14937300 K^3 N^3 + 1179780 K^3 N^4 - 45360 K^3 N^5 + 672 K^3 N^6 + 120489264 K^4 - 85196937 K^4 N + 20747160 K^4 N^2 - 2249520 K^4 N^3$$
$$+ 110880 K^4 N^4 - 2016 K^4 N^5 + 30357474 K^5 - 15334704 K^5 N + 2570400 K^5 N^2 - 173376 K^5 N^3 + 4032 K^5 N^4 + 4711392 K^6 - 1629600 K^6 N$$
$$+ 169344 K^6 N^2 - 5376 K^6 N^3 + 442176 K^7 - 94464 K^7 N + 4608 K^7 N^2 + 23040 K^8 - 2304 K^8 N + 512 K^9$$

$$J(8,1) = 20708079 N - 16863440 N^2 + 3397822 N^3 + 354996 N^4 - 220899 N^5 + 32508 N^6 - 2254 N^7 + 76 N^8 - 1 N^9 - 24825794 K + 67033480 K N - 35394136 K N^2$$
$$+ 5252716 K N^3 + 418390 K N^4 - 193428 K N^5 + 19768 K N^6 - 868 K N^7 + 14 K N^8 - 57696576 K^2 + 84361490 K^2 N - 32280380 K^2 N^2 + 3912174 K^2$$
$$N^3 + 104020 K^2 N^4 - 54586 K^2 N^5 + 3754 K^2 N^6 - 80 K^2 N^7 - 57407276 K^3 + 56561344 K^3 N - 16246384 K^3 N^2 + 1615460 K^3 N^3 - 9940 K^3 N^4$$
$$- 6160 K^3 N^5 + 224 K^3 N^6 - 31660832 K^4 + 22292543 K^4 N - 4755240 K^4 N^2 + 362320 K^4 N^3 - 5600 K^4 N^4 - 224 K^4 N^5 - 10522834 K^5 + 5299504 K^5$$
$$N - 798112 K^5 N^2 + 40768 K^5 N^3 - 448 K^5 N^4 - 2152864 K^6 + 743904 K^6 N - 70784 K^6 N^2 + 1792 K^6 N^3 - 264768 K^7 + 56576 K^7 N - 2560 K^7 N^2$$
$$- 17920 K^8 + 1792 K^8 N - 512 K^9$$

$$J(7,2) = -1421085 N + 647572 N^2 + 118014 N^3 - 30912 N^4 - 24059 N^5 + 8288 N^6 - 994 N^7 + 52 N^8 - 1 N^9 + 1649210 K - 4900542 K N + 1464816 K N^2 + 326102 K$$
$$N^3 - 103586 K N^4 - 10570 K N^5 + 4480 K N^6 - 380 K N^7 + 10 K N^8 + 4724220 K^2 - 7052444 K^2 N + 1361500 K^2 N^2 + 407652 K^2 N^3 - 114660 K^2 N^4$$
$$+ 4284 K^2 N^5 + 560 K^2 N^6 - 32 K^2 N^7 + 6167864 K^3 - 5984090 K^3 N + 859488 K^3 N^2 + 206080 K^3 N^3 - 47040 K^3 N^4 + 2240 K^3 N^5 + 4826500 K^4$$
$$- 3332399 K^4 N + 409640 K^4 N^2 + 43120 K^4 N^3 - 7840 K^4 N^4 + 224 K^4 N^5 + 2381134 K^5 - 1186416 K^5 N + 120288 K^5 N^2 + 3136 K^5 N^3 - 448 K^5 N^4$$
$$+ 728000 K^6 - 250880 K^6 N + 17920 K^6 N^2 + 131712 K^7 - 28160 K^7 N + 1024 K^7 N^2 + 12800 K^8 - 1280 K^8 N + 512 K^9$$

## Table A-1 (Cont.)

$$
\begin{aligned}
J(6,3) = {} & 31851N-19836N^{2}-4274N^{3}+7992N^{4}-7419N^{5}+2808N^{6}-474N^{7}+36N^{8}-1N^{9}-17514K+9731[illegible]KN+[illegible]440KN^{2}-10290KN^{3}-6742KN^{4} \\
& +318KN^{5}+752KN^{6}-132KN^{7}+6KN^{8}-137724K^{2}+226232K^{2}N-5484K^{2}N^{2}+7816K^{2}N^{3}-[illegible]K^{2}N^{4}+4704K^{2}N^{5}-288K^{2}N^{6}-244016 \\
& K^{3}+199254K^{3}N+45824K^{3}N^{2}-17400K^{3}N^{3}-5800K^{3}N^{4}+1440K^{3}N^{5}-64K^{3}N^{6}-196068K^{4}+96085K^{4}N+60360K^{4}N^{2}-20560K^{4}N^{3} \\
& +480K^{4}N^{4}+96K^{4}N^{5}-160134K^{5}+69744K^{5}N+21408K^{5}N^{2}-5952K^{5}N^{3}+192K^{5}N^{4}-112896K^{6}+38336K^{6}N+2304K^{6}N^{2}-512K^{6}N^{3} \\
& -43008K^{7}+9216K^{7}N-7680K^{8}+768K^{8}N-512K^{9}
\end{aligned}
$$

$$
\begin{aligned}
J(5,4) = {} & 5223N-8392N^{2}+142N^{3}+6628N^{4}-5059N^{5}+1732N^{6}-310N^{7}+28N^{8}-1N^{9}+12434K+4756KN+808KN^{2}+1704KN^{3}-558KN^{4}-64KN^{5} \\
& +120KN^{6}-28KN^{7}+2KN^{8}-44616K^{2}+25326K^{2}N-28096K^{2}N^{2}+20186K^{2}N^{3}-9340K^{2}N^{4}+2482K^{2}N^{5}-328K^{2}N^{6}+16K^{2}N^{7}-24748K^{3} \\
& -7955K^{3}N-1712K^{3}N^{2}-380K^{3}N^{3}-500K^{3}N^{4}+240K^{3}N^{5}-32K^{3}N^{6}+56936K^{4}-28101K^{4}N+17720K^{4}N^{2}-6640K^{4}N^{3}+1440K^{4}N^{4}-96 \\
& K^{4}N^{5}+11674K^{5}+976K^{5}N+800K^{5}N^{2}-576K^{5}N^{3}+192K^{5}N^{4}-16352K^{6}+5920K^{6}N-2944K^{6}N^{2}+256K^{6}N^{3}-1344K^{7}+256K^{7}N-512K^{7} \\
& N^{2}+2560K^{8}-256K^{8}N+512K^{9}
\end{aligned}
$$

$$
\begin{aligned}
J(10,0) = {} & -3660215680N+4085349936N^{2}-1783995060N^{3}+409987640N^{4}-55545735N^{5}+4654713N^{6}-243390N^{7}+7710N^{8}-135N^{9}+N^{10}+46[illegible]5680 \\
& K-12377921472KN+8734581300KN^{2}-2809439920KN^{3}+493632405KN^{4}-51160368KN^{5}+3202710KN^{6}-118620KN^{7}+2385KN^{8}-20 \\
& KN^{9}+9192571536K^{2}-14123569320K^{2}N+7180416540K^{2}N^{2}-1748744970K^{2}N^{3}+233775150K^{2}N^{4}-18036270K^{2}N^{5}+797790K^{2}N^{6} \\
& -18720K^{2}N^{7}+180K^{2}N^{8}+7532982780K^{3}-8108335880K^{3}N+3086200950K^{3}N^{2}-568383060K^{3}N^{3}+56346150K^{3}N^{4}-3063480K^{3}N^{5} \\
& +85680K^{3}N^{6}-960K^{3}N^{7}+3411371620K^{4}-2712557295K^{4}N+775397805K^{4}N^{2}-105453600K^{4}N^{3}+7345800K^{4}N^{4}-252000K^{4}N^{5}+3360K^{4} \\
& N^{6}+349614645K^{5}-562674756K^{5}N+118223280K^{5}N^{2}-11262720K^{5}N^{3}+493920K^{5}N^{4}-8064K^{5}N^{5}+169650516K^{6}-73566720K^{6}N \\
& +10782240K^{6}N^{2}-645120K^{6}N^{3}+13440K^{6}N^{4}+19551840K^{7}-5892480K^{7}N+541440K^{7}N^{2}-15360K^{7}N^{3}+1407360K^{8}-264960K^{8}N \\
& +11520K^{8}N^{2}+57600K^{9}-5120K^{9}N+1024K^{10}
\end{aligned}
$$

## Table A-1 (Cont.)

$$J(9,1) = 506758528N - 452619408N^2 + 118669404N^3 - 1274344N^4 - 4588899N^5 + 904953N^6 - 81942N^7 + 3966N^8 - 99N^9 + N^{10} - 607421824K$$

$$+1701902832KN - 1001042568KN^2 + 198381004KN^3 - 2154768KN^4 - 4461660KN^5 + 666708KN^6 - 43104KN^7 + 1332KN^8 - 16KN^9$$

$$-1419152736K^2 + 2205711756K^2N - 954721752K^2N^2 + 152322618K^2N^3 - 3868074K^2N^4 - 1485162K^2N^5 + 170898K^2N^6 - 7200K^2N^7 + 108$$

$$K^2N^8 - 1434245504K^3 + 1538216832K^3N - 510136872K^3N^2 + 66287592K^3N^3 - 2290260K^3N^4 - 205296K^3N^5 + 18144K^3N^6 - 384K^3N^7$$

$$-819824536K^4 + 647206833K^4N - 164987991K^4N^2 + 16937760K^4N^3 - 573720K^4N^4 - 10080K^4N^5 + 672K^4N^6 - 291164496K^5 + 171464832$$

$$K^5N - 32792256K^5N^2 + 2489088K^5N^3 - 64512K^5N^4 - 66533124K^6 + 28719264K^6N - 3898272K^6N^2 + 193536K^6N^3 - 2688K^6N^4 - 9782976$$

$$K^7 + 2943744K^7N - 253440K^7N^2 + 6144K^7N^3 - 893568K^8 + 168192K^8N - 6912K^8N^2 - 46080K^9 + 4096K^9N - 1024K^{10}$$

$$J(8,2) = -38673792N + 22935600N^2 + 288332N^3 - 1257544N^4 - 219975N^5 + 155001N^6 - 25662N^7 + 1950N^8 - 71N^9 + N^{10} + 44645760K - 135410208KN$$

$$+54185036KN^2 + 2601920KN^3 - 3007837KN^4 + 55328KN^5 + 92330KN^6 - 12868KN^7 + 663KN^8 - 12KN^9 + 125538288K^2 - 198236236K^2N$$

$$+56105132K^2N^2 + 4594870K^2N^3 - 2986354K^2N^4 + 239442K^2N^5 + 10654K^2N^6 - 1824K^2N^7 + 52K^2N^8 + 159781428K^3 - 168816584K^3N$$

$$+36397466K^3N^2 + 2510676K^3N^3 - 1317190K^3N^4 + 105784K^3N^5 - 1456K^3N^6 - 64K^3N^7 + 121424484K^4 - 93589727K^4N + 16449405K^4N^2$$

$$+435686K^4N^3 - 274680K^4N^4 + 16800K^4N^5 - 224K^4N^6 + 59562867K^5 - 34433756K^5N + 4922064K^5N^2 - 26880K^5N^3 - 26208K^5N^4 + 896K^5$$

$$N^5 + 19077268K^6 - 8152480K^6N + 886816K^6N^2 - 14336K^6N^3 - 896K^6N^4 + 3919776K^7 - 1175168K^7N + 85248K^7N^2 - 1024K^7N^3 + 493952$$

$$K^8 - 92928K^8N + 3328K^8N^2 + 34560K^9 - 3072K^9N + 1024K^{10}$$

$$J(7,3) = 1105792N - 329744N^2 - 164068N^3 + 91384N^4 - 80563N^5 + 40761N^6 - 9142N^7 + 990N^8 - 51N^9 + N^{10} - 1036160K + 3972112KN - 385484KN^2$$

$$-498706KN^3 - 63056KN^4 + 32074KN^5 + 9170KN^6 - 3152KN^7 + 282KN^8 - 8KN^9 - 4459008K^2 + 7522912K^2N - 547420K^2N^2 - 405822K^2$$

$$N^3 - 228690K^2N^4 + 95382K^2N^5 - 9086K^2N^6 + 96K^2N^7 + 12K^2N^8 - 7537872K^3 + 7472574K^3N + 280924K^3N^2 - 61849[illegible]K^3N^3 [illegible]70K^3N^4$$

$$+34216K^3N^5 - 3024K^3N^6 + 64K^3N^7 - 7061704K^4 + 4678597K^4N + 663453K^4N^2 - 484960K^4N^3 + 29960K^4N^4 + 3360K^4N^5 [illegible]24K^4N^6$$

$$-4975328K^5 + 2542400K^5N + 239904K^5N^2 - 156800K^5N^3 + 10752K^5N^4 - 2723028K^6 + 1105440K^6N + 9184K^6N^2 - 21504K^6N^3 + 896K^6N^4$$

$$-985824K^7 + 291712K^7N - 6912K^7N^2 - 1024K^7N^3 - 208512K^8 + 39168K^8N - 768K^8N^2 - 23040K^9 + 2048K^9N - 1024K^{10}$$

## Table A-1 (Cont.)

$$J(6,4) = 44672*N-82640*N^{2}+11980*N^{3}+59512*N^{4}-53543*N^{5}+21753*N^{6}-4862*N^{7}+606*N^{8}-39*N^{9}+N^{10}+86656*K-15472*K*N-17132*K*N^{2}+21336*K*N^{3}$$

$$-10839*K*N^{4}+712*K*N^{5}+1862*K*N^{6}-716*K*N^{7}+43*K*N^{8}-4*K*N^{9}-240864*K^{2}+305548*K^{2}*N-262444*K^{2}*N^{2}+215702*K^{2}*N^{3}-104642*K^{2}*N^{4}$$

$$+34578*K^{2}*N^{5}-6082*K^{2}*N^{6}+480*K^{2}*N^{7}-12*K^{2}*N^{8}-272844*K^{3}-81680*K^{3}*N-31474*K^{3}*N^{2}-436*K^{3}*N^{3}-3290*K^{3}*N^{4}+4936*K^{3}*N^{5}-1104*K^{3}*N^{6}$$

$$+64*K^{3}*N^{7}+477796*K^{4}-385943*K^{4}*N+155557*K^{4}*N^{2}-82720*K^{4}*N^{3}+23880*K^{4}*N^{4}-2400*K^{4}*N^{5}+32*K^{4}*N^{6}+246849*K^{5}-32564*K^{5}*N-7056*K^{5}$$

$$*N^{2}-9472*K^{5}*N^{3}+5088*K^{5}*N^{4}-364*K^{5}*N^{5}-72396*K^{6}+65376*K^{6}*N-44768*K^{6}*N^{2}+6144*K^{6}*N^{3}+128*K^{6}*N^{4}+4704*K^{7}+1664*K^{7}*N-11520*K^{7}*N^{2}$$

$$+1024*K^{7}*N^{3}+37248*K^{8}-6912*K^{8}*N-768*K^{8}*N^{2}+11520*K^{9}-1024*K^{9}*N+1024*K^{10}$$

$$J(5,5) = 40320*N-69264*N^{2}+8540*N^{3}+50840*N^{4}-44835*N^{5}+17913*N^{6}-3990*N^{7}+510*N^{8}-35*N^{9}+N^{10}-512736*K^{2}+160060*K^{2}*N-261320*K^{2}*N^{2}+175610$$

$$*K^{2}*N^{3}-89610*K^{2}*N^{4}+27030*K^{2}*N^{5}-4910*K^{2}*N^{6}+480*K^{2}*N^{7}-20*K^{2}*N^{8}+554280*K^{4}-225055*K^{4}*N+174425*K^{4}*N^{2}-63200*K^{4}*N^{3}+17000*K^{4}*N^{4}$$

$$-2400*K^{4}*N^{5}+160*K^{4}*N^{6}-165316*K^{6}+53920*K^{6}*N-27040*K^{6}*N^{2}+5120*K^{6}*N^{3}-640*K^{6}*N^{4}+19840*K^{8}-3840*K^{8}*N+1280*K^{8}*N^{2}-1024*K^{10}$$

$$J(11,0) = -99920609601*N+116193701393*N^{2}-53705023251*N^{3}+13303809750*N^{4}-1987186025*N^{5}+189142107*N^{6}-11717013*N^{7}+469590*N^{8}-11715*N^{9}$$

$$+165*N^{10}-1*N^{11}+125858034202*K-347995531169*K*N+259675332774*K*N^{2}-89956697050*K*N^{3}+17413394570*K*N^{4}-2048510079*K*N^{5}$$

$$+151839534*K*N^{6}-7111170*K*N^{7}+203610*K*N^{8}-3245*K*N^{9}+22*K*N^{10}+255381306686*K^{2}-414552563370*K^{2}*N+226805535990*K^{2}*N^{2}$$

$$-60810543860*K^{2}*N^{3}+9221275140*K^{2}*N^{4}-841869336*K^{2}*N^{5}+47061630*K^{2}*N^{6}-1571790*K^{2}*N^{7}+28710*K^{2}*N^{8}-220*K^{2}*N^{9}+218228403492*K^{3}$$

$$-252579287070*K^{3}*N+105760197620*K^{3}*N^{2}-22079772330*K^{3}*N^{3}+2588599860*K^{3}*N^{4}-177770670*K^{3}*N^{5}+7073220*K^{3}*N^{6}-150480*K^{3}*N^{7}$$

$$+1320*K^{3}*N^{8}+104765098900*K^{4}-91577104025*K^{4}*N+22655441585*K^{4}*N^{2}-4766840925*K^{4}*N^{3}+419195700*K^{4}*N^{4}-20448120*K^{4}*N^{5}+517440$$

$$*K^{4}*N^{6}-5280*K^{4}*N^{7}+31573269530*K^{5}-21179770689*K^{5}*N+5256540366*K^{5}*N^{2}-631849680*K^{5}*N^{3}+39380880*K^{5}*N^{4}-1219680*K^{5}*N^{5}+14784$$

$$*K^{5}*N^{6}+6282779118*K^{6}-3213687708*K^{6}*N+594464640*K^{6}*N^{2}-50524320*K^{6}*N^{3}+1995840*K^{6}*N^{4}-29568*K^{6}*N^{5}+840209832*K^{7}-319160150*K^{7}$$

$$*N+41638080*K^{7}*N^{2}-2238720*K^{7}*N^{3}+42240*K^{7}*N^{4}+74859840*K^{8}-20000640*K^{8}*N+1647360*K^{8}*N^{2}-42240*K^{8}*N^{3}+4266240*K^{9}-718080*K^{9}*N$$

$$+28160*K^{9}*N^{2}+140800*K^{10}-11264*K^{10}*N+2048*K^{11}$$

## Table A-1 (Cont.)

```
J(10, 1) = 13785644259*N-13259178423*N^2 +4128271029*N^3 -326187530*N^4 -89671805*N^5 +25467603*N^6 -2894493*N^7 +181110*N^8 -2495*N^9 +125*N^10 -1
         *N^11 -16497587682*K+47775667437*K*N-30615474150*K*N^2 +7323888930*K*N^3 -478028010*K*N^4 -94168389*K*N^5 +21493122*K*N^6 -1866990
         *K*N^7 +84030*K*N^8 -1935*K*N^9 +18*K*N^10 -36821161114*K^2 +63577543518*K^2 *N-30368621490*K^2 *N^2 +5858300060*K^2 *N^3 -365258460*K^2 *N^4
         -32503464*K^2 *N^5 +6653430*K^2 *N^6 -428070*K^2 *N^7 +12510*K^2 *N^8 -140*K^2 *N^9 -39840547572*K^3 +45865705670*K^3 *N-17056828780*K^3 *N^2
         +2691585330*K^3 *N^3 -159590340*K^3 *N^4 -3961650*K^3 *N^5 +943740*K^3 *N^6 -41520*K^3 *N^7 +600*K^3 *N^8 -23472637020*K^4 +20329328715*K^4 *N
         -5951864415*K^4 *N^2 +758268315*K^4 *N^3 -39662700*K^4 *N^4 +81480*K^4 *N^5 +60480*K^4 *N^6 -1440*K^4 *N^7 -8782171970*K^5 +5840837709*K^5 *N
         -1332433494*K^5 *N^2 +132184080*K^5 *N^3 -5505360*K^5 *N^4 +50400*K^5 *N^5 +1344*K^5 *N^6 -2179299402*K^6 +1107735636*K^6 *N-191022720*K^6 *N^2
         +13839840*K^6 *N^3 -396480*K^6 *N^4 +2688*K^6 *N^5 -363392232*K^7 +137516640*K^7 *N-16916160*K^7 *N^2 +794880*K^7 *N^3 -11520*K^7 *N^4 -40248000*K^8
         +10734720*K^8 *N-840960*K^8 *N^2 +19200*K^8 *N^3 -2837760*K^9 +477440*K^9 *N-17920*K^9 *N^2 -115200*K^10 +9216*K^10 *N-2048*K^11

J( 9, 2) = -1135530669*N+797886217*N^2 -75561963*N^3 -34044138*N^4 +1140595*N^5 +2810451*N^6 -652701*N^7 +65910*N^8 -3471*N^9 +93*N^10 -1*N^11
         +1309298242*K-4053023165*K*N+1952204934*K*N^2 -105129218*K*N^3 -77514326*K*N^4 +8682261*K*N^5 +1595454*K*N^6 -383250*K*N^7 +30114
         *K*N^8 -1057*K*N^9 +14*K*N^10 +3629966662*K^2 -6019133010*K^2 *N+2129033886*K^2 *N^2 -49485332*K^2 *N^3 -75575388*K^2 *N^4 +10074456*K^2 *N^5
         -21546*K^2 *N^6 -67590*K^2 *N^7 +4158*K^2 *N^8 -76*K^2 *N^9 +4549538772*K^3 -5156717622*K^3 *N+1417657676*K^3 *N^2 -24008754*K^3 *N^3 -35203644*K^3
         *N^4 +4342602*K^3 *N^5 -140028*K^3 *N^6 -3024*K^3 *N^7 +168*K^3 *N^8 +3407955908*K^4 -2879422453*K^4 *N+641488785*K^4 *N^2 -17311413*K^4 *N^3
         -8313900*K^4 *N^4 +858312*K^4 *N^5 -25536*K^4 *N^6 +96*K^4 *N^7 +1672967042*K^5 -1088051853*K^5 *N+197893206*K^5 *N^2 -7192080*K^5 *N^3 -988848*K^5
         *N^4 +78624*K^5 *N^5 -1344*K^5 *N^6 +553869750*K^6 -277207980*K^6 *N+39930240*K^6 *N^2 -1471008*K^6 *N^3 -52416*K^6 *N^4 +2688*K^6 *N^5 +123646824*K^7
         -46390368*K^7 *N+4950720*K^7 *N^2 -143616*K^7 *N^3 -768*K^7 *N^4 +18236736*K^8 -4846464*K^8 *N+338688*K^8 *N^2 -5376*K^8 *N^3 +1694976*K^9 -284928
         *K^9 *N+9728*K^9 *N^2 +89600*K^10 -7168*K^10 *N+2048*K^11
```

## Table A-1 (Cont.)

$$
\begin{aligned}
J(8,3) = {} & 40962255*N-16377071*N^{2}-4696051*N^{3}+2071670*N^{4}-929305*N^{5}+608027*N^{6}-178773*N^{7}+25430*N^{8}-1875*N^{9}+65*N^{10}-1*N^{11}-43059130*K \\
& +153430161*K*N-35964934*K*N^{2}-14992710*K*N^{3}+1338550*K*N^{4}+927759*K*N^{5}+41874*K*N^{6}-65150*K*N^{7}+9350*K*N^{8}-515*K*N^{9}+10*K*N^{10} \\
& -156653090*K^{2}+271136998*K^{2}*N-44680186*K^{2}*N^{2}-16726308*K^{2}*N^{3}-909692*K^{2}*N^{4}+1921640*K^{2}*N^{5}-278754*K^{2}*N^{6}+8338*K^{2}*N^{7}+512*K^{2}*N^{8} \\
& -28*K^{2}*N^{9}-250400388*K^{3}+274296206*K^{3}*N-23607572*K^{3}*N^{2}-17765398*K^{3}*N^{3}+1016652*K^{3}*N^{4}+727790*K^{3}*N^{5}-106820*K^{3}*N^{6}+4560*K^{3}*N^{7} \\
& -40*K^{3}*N^{8}-235739980*K^{4}+181987911*K^{4}*N-4539423*K^{4}*N^{2}-11904557*K^{4}*N^{3}+1361780*K^{4}*N^{4}+69384*K^{4}*N^{5}-12992*K^{4}*N^{6}+352*K^{4}*N^{7} \\
& -155971226*K^{5}+92142561*K^{5}*N-1940750*K^{5}*N^{2}-4023600*K^{5}*N^{3}+455280*K^{5}*N^{4}-6048*K^{5}*N^{5}-448*K^{5}*N^{6}-76527570*K^{6}+35968996*K^{6}*N \\
& -1626240*K^{6}*N^{2}-670880*K^{6}*N^{3}+60480*K^{6}*N^{4}-896*K^{6}*N^{5}-26501160*K^{7}+9655648*K^{7}*N-533696*K^{7}*N^{2}-50944*K^{7}*N^{3}+2816*K^{7}*N^{4}-6025920 \\
& *K^{8}+1585280*K^{8}*N-72960*K^{8}*N^{2}-1280*K^{8}*N^{3}-837888*K^{9}+140544*K^{9}*N-3584*K^{9}*N^{2}-64000*K^{10}+5120*K^{10}*N-2048*K^{11}
\end{aligned}
$$

$$
\begin{aligned}
J(7,4) = {} & 255879*N-975951*N^{2}+284157*N^{3}+640198*N^{4}-655073*N^{5}+303387*N^{6}-60661*N^{7}+12630*N^{8}-1131*N^{9}+53*N^{10}-1*N^{11}+1053162*K-1405017*K \\
& *N-386058*K*N^{2}+480182*K*N^{3}-125398*K*N^{4}+28889*K*N^{5}+20174*K*N^{6}-13570*K*N^{7}+2666*K*N^{8}-213*K*N^{9}+6*K*N^{10}-957042*K^{2}+1242318*K^{2} \\
& *N-3988114*K^{2}*N^{2}+2923972*K^{2}*N^{3}-1298108*K^{2}*N^{4}+508648*K^{2}*N^{5}-116466*K^{2}*N^{6}+12450*K^{2}*N^{7}-522*K^{2}*N^{8}+4*K^{2}*N^{9}-214668*K^{3}-3054758 \\
& *K^{3}*N-1001020*K^{3}*N^{2}+324758*K^{3}*N^{3}+53172*K^{3}*N^{4}+68978*K^{3}*N^{5}-27132*K^{3}*N^{6}+2864*K^{3}*N^{7}-88*K^{3}*N^{8}+7816452*K^{4}-7406681*K^{4}*N \\
& +1300537*K^{4}*N^{2}-913269*K^{4}*N^{3}+437780*K^{4}*N^{4}-60088*K^{4}*N^{5}+1344*K^{4}*N^{6}+96*K^{4}*N^{7}+6298682*K^{5}-2444673*K^{5}*N-555506*K^{5}*N^{2}-35280*K^{5} \\
& *N^{3}+110992*K^{5}*N^{4}-15456*K^{5}*N^{5}+448*K^{5}*N^{6}+1773870*K^{6}+85988*K^{6}*N-837760*K^{6}*N^{2}+160608*K^{6}*N^{3}+3136*K^{6}*N^{4}-896*K^{6}*N^{5}+1100904*K^{7} \\
& -218208*K^{7}*N-240960*K^{7}*N^{2}+40704*K^{7}*N^{3}-768*K^{7}*N^{4}+815424*K^{8}-200576*K^{8}*N-23808*K^{8}*N^{2}+2816*K^{8}*N^{3}+266495*K^{9}-44288*K^{9}*N-512 \\
& *K^{9}*N^{2}+38400*K^{10}-3072*K^{10}*N+2048*K^{11}
\end{aligned}
$$

## Table A-1 (Cont.)

J( 6, 5) =

$$375771*N-689335*N^{2}+154469*N^{3}+465190*N^{4}-473125*N^{5}+215155*N^{6}-56253*N^{7}+8950*N^{8}-855*N^{9}+45*N^{10}-1*N^{11}+1131758*K+413653*K*N$$

$$+98922*K*N^{2}+105170*K*N^{3}-44170*K*N^{4}+5555*K*N^{5}+6306*K*N^{6}-2990*K*N^{7}+590*K*N^{8}-55*K*N^{9}+2*K*N^{10}-3967786*K^{2}+2390326*K^{2}*N$$

$$-2509210*K^{2}*N^{2}+1917460*K^{2}*N^{3}-1013660*K^{2}*N^{4}+347160*K^{2}*N^{5}-74970*K^{2}*N^{6}+9770*K^{2}*N^{7}-690*K^{2}*N^{8}+20*K^{2}*N^{9}-2332932*K^{3}-577010*K^{3}$$

$$*N-1451800*K^{3}*N^{2}-24110*K^{3}*N^{3}-28420*K^{3}*N^{4}+15790*K^{3}*N^{5}-5060*K^{3}*N^{6}+720*K^{3}*N^{7}-40*K^{3}*N^{8}+5008660*K^{4}-2729605*K^{4}*N+1846345*K^{4}*N^{2}$$

$$-771325*K^{4}*N^{3}+227700*K^{4}*N^{4}-41720*K^{4}*N^{5}+4160*K^{4}*N^{6}-160*K^{4}*N^{7}+1033150*K^{5}+80349*K^{5}*N+110730*K^{5}*N^{2}-27120*K^{5}*N^{3}+15280*K^{5}*N^{4}$$

$$-3360*K^{5}*N^{5}+320*K^{5}*N^{6}-1583658*K^{6}+674836*K^{6}*N-312960*K^{6}*N^{2}+79840*K^{6}*N^{3}-12480*K^{6}*N^{4}+640*K^{6}*N^{5}-209832*K^{7}+15200*K^{7}*N-22720$$

$$*K^{7}*N^{2}+6400*K^{7}*N^{3}-1280*K^{7}*N^{4}+194880*K^{8}-58240*K^{8}*N+19200*K^{8}*N^{2}-1280*K^{8}*N^{3}+19200*K^{9}-3840*K^{9}*N+2560*K^{9}*N^{2}-12800*K^{10}+1024$$

$$*K^{10}*N-2048*K^{11}$$

J(12, 0) =

$$-2998836525312*N+3615197586912*N^{2}-1755078270264*N^{3}+463397746636*N^{4}-75078221130*N^{5}+7923937307*N^{6}-560544138*N^{7}+26750823$$

$$*N^{8}-848430*N^{9}+17105*N^{10}-198*N^{11}+N^{12}+3741844896000*K-10717581772224*K*N+8393058481896*K*N^{2}-3096688261216*K*N^{3}$$

$$+649773013890*K*N^{4}-84711184470*K*N^{5}+7166361510*K*N^{6}-399457740*K*N^{7}+14534190*K*N^{8}-331430*K*N^{9}+4290*K*N^{10}-24*K*N^{11}$$

$$+7783475191776*K^{2}-13249010287656*K^{2}*N+7714550650488*K^{2}*N^{2}-2240602530540*K^{2}*N^{3}+376310458425*K^{2}*N^{4}-39190475016*K^{2}*N^{5}$$

$$+2606200674*K^{2}*N^{6}-110561220*K^{2}*N^{7}+2888325*K^{2}*N^{8}-42240*K^{2}*N^{9}+264*K^{2}*N^{10}+6894817995384*K^{3}-8486856824512*K^{3}*N$$

$$+3846935531340*K^{3}*N^{2}-888997172580*K^{3}*N^{3}+118828155600*K^{3}*N^{4}-9703037652*K^{3}*N^{5}+490158900*K^{3}*N^{6}-14908080*K^{3}*N^{7}+249480*K^{3}*N^{8}$$

$$-1760*K^{3}*N^{9}+3476543668444*K^{4}-3287596654650*K^{4}*N+1177749560625*K^{4}*N^{2}-215722521630*K^{4}*N^{3}+22545633495*K^{4}*N^{4}-1395646560*K^{4}$$

$$*N^{5}+50468880*K^{4}*N^{6}-982080*K^{4}*N^{7}+7920*K^{4}*N^{8}+1118393357730*K^{5}-829466196174*K^{5}*N+234456232626*K^{5}*N^{2}-33476810136*K^{5}*N^{3}$$

$$+2646650160*K^{5}*N^{4}-117089280*K^{5}*N^{5}+2705472*K^{5}*N^{6}-25344*K^{5}*N^{7}+242570121475*K^{6}-141235284960*K^{6}*N+31018633800*K^{6}*N^{2}$$

$$-3342588480*K^{6}*N^{3}+188532960*K^{6}*N^{4}-5322240*K^{6}*N^{5}+59136*K^{6}*N^{6}+36373036392*K^{7}-16396377888*K^{7}*N+2710952640*K^{7}*N^{2}-208032000$$

$$*K^{7}*N^{3}+7476480*K^{7}*N^{4}-101376*K^{7}*N^{5}+3785307504*K^{8}-1281139200*K^{8}*N+150543360*K^{8}*N^{2}-7349760*K^{8}*N^{3}+126720*K^{8}*N^{4}+268773120*K^{9}$$

$$-64514560*K^{9}*N+4815360*K^{9}*N^{2}-112640*K^{9}*N^{3}+12432640*K^{10}-1892352*K^{10}*N+67584*K^{10}*N^{2}+337920*K^{11}-24576*K^{11}*N+4096*K^{12}$$

## Table A-1 (Cont.)

$$J(11, 1) = +12623724800*N-421916611040*N^{2}+148783553336*N^{3}-18812277924*N^{4}-1304692070*N^{5}+721017275*N^{6}-102222582*N^{7}+7861623*N^{8}$$

$$-363330*N^{9}+10065*N^{10}-154*N^{11}+N^{12}-492623724800*K+1469567988800*K*N-1010419269112*K*N^{2}+277061070748*K*N^{3}-28865901130*K*N^{4}$$

$$-1359324505*K*N^{5}+672283227*K*N^{6}-76347678*K*N^{7}+4514400*K*N^{8}-150865*K*N^{9}+2695*K*N^{10}-20*K*N^{11}-1168651377760*K^{2}$$

$$+2003036946856*K^{2}*N-1037725976628*K^{2}*N^{2}+230770846460*K^{2}*N^{3}-21123259850*K^{2}*N^{4}-297948420*K^{2}*N^{5}+235335408*K^{2}*N^{6}-21500160$$

$$*K^{2}*N^{7}+932910*K^{2}*N^{8}-20240*K^{2}*N^{9}+176*K^{2}*N^{10}-1217301231080*K^{3}+1488723544308*K^{3}*N-608284119520*K^{3}*N^{2}+111468336650*K^{3}*N^{3}$$

$$-9047343690*K^{3}*N^{4}+82569102*K^{3}*N^{5}+39156810*K^{3}*N^{6}-2860440*K^{3}*N^{7}+82500*K^{3}*N^{8}-880*K^{3}*N^{9}-736306360504*K^{4}+688967085730*K^{4}*N$$

$$-225587553785*K^{4}*N^{2}+33926048770*K^{4}*N^{3}-2371594995*K^{4}*N^{4}+46126080*K^{4}*N^{5}+3123120*K^{4}*N^{6}-179520*K^{4}*N^{7}+2640*K^{4}*N^{8}$$

$$-287487219470*K^{5}+211018937495*K^{5}*N-55253317581*K^{5}*N^{2}+6683071164*K^{5}*N^{3}-381806040*K^{5}*N^{4}+7835320*K^{5}*N^{5}+96096*K^{5}*N^{6}-4224*K^{5}$$

$$*N^{7}-76108323280*K^{6}+43944852336*K^{6}*N-9048561984*K^{6}*N^{2}+847862400*K^{6}*N^{3}-36664320*K^{6}*N^{4}+591360*K^{6}*N^{5}-13950612060*K^{7}$$

$$+6251965104*K^{7}*N-979022880*K^{7}*N^{2}+66718080*K^{7}*N^{3}-1921920*K^{7}*N^{4}+16896*K^{7}*N^{5}-1772481744*K^{8}+597780480*K^{8}*N-67077120*K^{8}*N^{2}$$

$$+2956800*K^{8}*N^{3}-42240*K^{8}*N^{4}-153183360*K^{9}+36706560*K^{9}*N-2632960*K^{9}*N^{2}+56320*K^{9}*N^{3}-8588800*K^{10}+1306624*K^{10}*N-45056*K^{10}*N^{2}$$

$$-281600*K^{11}+20480*K^{11}*N-4096*K^{12}$$

# Table A-1 (Cont.)

$$J(10,\ 2) = -36009967872 N + 28590400736 N^{2} - 4942940792 N^{3} - 719748084 N^{4} + 167373190 N^{5} + 44477243 N^{6} - 16505370 N^{7} + 2151783 N^{8} - 148350 N^{9}$$

$$+ 5745 N^{10} - 118 N^{11} + N^{12} + 41512894720 K - 131104118176 K N + 72074274944 K N^{2} - 9010026688 K N^{3} - 1745694260 K N^{4} + 409103740 K N^{5}$$

$$+ 14196420 K N^{6} - 10532616 K N^{7} + 1182660 K N^{8} - 61700 K N^{9} + 1580 K N^{10} - 16 K N^{11} + 113959973056 K^{2} - 197266981048 K^{2} N + 81321773[illegible]72$$

$$K^{2} N^{2} - 7406296060 K^{2} N^{3} - 1767362675 K^{2} N^{4} + 383328456 K^{2} N^{5} - 14945910 K^{2} N^{6} - 1922580 K^{2} N^{7} + 216225 K^{2} N^{8} - 8000 K^{2} N^{9} + 104 K^{2} N^{10}$$

$$+ 141531046576 K^{3} - 170573013264 K^{3} N + 55450374920 K^{3} N^{2} - 4098227640 K^{3} N^{3} - 863017680 K^{3} N^{4} + 167264328 K^{3} N^{5} - 8978760 K^{3} N^{6}$$

$$- 14880 K^{3} N^{7} + 14640 K^{3} N^{8} - 320 K^{3} N^{9} + 105346198564 K^{4} - 96220355290 K^{4} N + 25485362945 K^{4} N^{2} - 1737620430 K^{4} N^{3} - 215990985 K^{4} N^{4}$$

$$+ 37742880 K^{4} N^{5} - 1839600 K^{4} N^{6} + 24000 K^{4} N^{7} + 240 K^{4} N^{8} + 51953289340 K^{5} - 37233058276 K^{5} N + 8118586140 K^{5} N^{2} - 526673616 K^{5} N^{3}$$

$$- 26863200 K^{5} N^{4} + 4515840 K^{5} N^{5} - 163968 K^{5} N^{6} + 1536 K^{5} N^{7} + 17659392355 K^{6} - 10005612960 K^{6} N + 1771695912 K^{6} N^{2} - 103360320 K^{6} N^{3}$$

$$- 1226400 K^{6} N^{4} + 268800 K^{6} N^{5} - 5376 K^{6} N^{6} + 4183718448 K^{7} - 1851334083 K^{7} N + 256594560 K^{7} N^{2} - 12172800 K^{7} N^{3} + 38400 K^{7} N^{4} + 6144 K^{7}$$

$$N^{5} + 686527344 K^{8} - 229870080 K^{8} N + 23362560 K^{8} N^{2} - 775680 K^{8} N^{3} + 3840 K^{8} N^{4} + 76208640 K^{9} - 18201600 K^{9} N + 1203200 K^{9} N^{2} - 20480 K^{9}$$

$$N^{3} + 5443840 K^{10} - 827392 K^{10} N + 26624 K^{10} N^{2} + 225280 K^{11} - 16384 K^{11} N + 4096 K^{12}$$

## Table A-1 (Cont.)

```
J( 9, 3)   = 1559699712*N-805973472*N^2 -91214280*N^3 +70659676*N^4 -13682214*N^5 +9070907*N^6 -3552822*N^7 +645495*N^8 -62082*N^9 +3281*N^10 -90*N^11
             +N^12 -1700787456*K+5944219776*K*N-2052412776*K*N^2 -340363676*K*N^3 +114982890*K*N^4 +17773713*K*N^5 -2590371*K*N^6 -1155858*K*N^7
             +270288*K*N^8 -22615*K*N^9 +849*K*N^10 -12*K*N^11 -5740668576*K^2 +10231655400*K^2*N-2569111236*K^2*N^2 -441286692*K^2*N^3 +85599510*K^2
             *N^4 +35300412*K^2*N^5 -8363376*K^2*N^6 +467136*K^2*N^7 +14814*K^2*N^8 -2064*K^2*N^9 +48*K^2*N^10 -8788241592*K^3 +10336977596*K^3*N
             -1820576448*K^3*N^2 -443636122*K^3*N^3 +85066506*K^3*N^4 +12725538*K^3*N^5 -3403386*K^3*N^6 +229080*K^3*N^7 -4644*K^3*N^8 -16*K^3*N^9
             -8107894072*K^4 +6952409634*K^4*N-845710905*K^4*N^2 -284199678*K^4*N^3 +55197261*K^4*N^4 +193536*K^4*N^5 -511056*K^4*N^6 +29376*K^4*N^7
             -432*K^4*N^8 -5164244994*K^5 +3432299841*K^5*N-336572523*K^5*N^2 -98873628*K^5*N^3 +17811864*K^5*N^4 -588672*K^5*N^5 -26208*K^5*N^6 +1152*K^5
             *N^7 -2401250512*K^6 +1279346544*K^6*N-119972160*K^6*N^2 -17974656*K^6*N^3 +2903040*K^6*N^4 -96768*K^6*N^5 -809492292*K^7 +344682960*K^7*N
             -30504672*K^7*N^2 -1557888*K^7*N^3 +229248*K^7*N^4 -4608*K^7*N^5 -190902480*K^8 +62687232*K^8*N-4654080*K^8*N^2 -41472*K^8*N^3 +6912*K^8*N^4
             -30125952*K^9 +7139072*K^9*N-375552*K^9*N^2 +1024*K^9*N^3 -2997760*K^10 +454656*K^10*N-12288*K^10*N^2 -168960*K^11 +12288*K^11*N-4096*K^12

J( 8, 4)   = -11811072*N-10813984*N^2 +7893192*N^3 +7954892*N^4 -9303242*N^5 +4686107*N^6 -1428490*N^7 +268135*N^8 -30382*N^9 +2001*N^10 -70*N^11 +N^12
             +28653824*K-84729792*K*N-926664*K*N^2 +17240032*K*N^3 -1561514*K*N^4 +187406*K*N^5 +140770*K*N^6 -224580*K*N^7 +64762*K*N^8 -7650*K
             *N^9 +406*K*N^10 -8*K*N^11 +59202016*K^2 -114628264*K^2*N-56051912*K^2*N^2 +57868180*K^2*N^3 -18347303*K^2*N^4 +7581560*K^2*N^5 -2295902*K^2
             *N^6 +323516*K^2*N^7 -18971*K^2*N^8 +256*K^2*N^9 +8*K^2*N^10 +134270888*K^3 -193845376*K^3*N-21665340*K^3*N^2 +20806068*K^3*N^3 +1978928*K^3*N^4
             +439908*K^3*N^5 -576324*K^3*N^6 +94320*K^3*N^7 -5416*K^3*N^8 +96*K^3*N^9 +257566492*K^4 -233851706*K^4*N+10606257*K^4*N^2 -3146654*K^4*N^3
             +8050903*K^4*N^4 -1635424*K^4*N^5 +63504*K^4*N^6 +5056*K^4*N^7 -272*K^4*N^8 +219252086*K^5 -120861274*K^5*N-14656474*K^5*N^2 +4777080*K^5*N^3
             +2133264*K^5*N^4 -494592*K^5*N^5 +27328*K^5*N^6 -256*K^5*N^7 +110667427*K^6 -37444064*K^6*N-16352952*K^6*N^2 +4081856*K^6*N^3 -21280*K^6*N^4
             -46592*K^6*N^5 +1792*K^6*N^6 +52908408*K^7 -16240224*K^7*N-4901312*K^7*N^2 +1361152*K^7*N^3 -55552*K^7*N^4 -1024*K^7*N^5 +23852400*K^8
             -7004672*K^8*N-517120*K^8*N^2 +154112*K^8*N^3 -4352*K^8*N^4 +7212288*K^9 -1658368*K^9*N-512*K^9*N^2 +6144*K^9*N^3 +1250560*K^10 -188416*K^10
             *N+2048*K^10*N^2 +112640*K^11 -8192*K^11*N+4096*K^12
```

## Table A-1 (Cont.)

```
J( 7, 5)  = +161792*N-8068064*N^2 +2535352*N^3 +4964316*N^4 -5794950*N^5 +2942715*N^6 -874454*N^7 +163703*N^8 -19490*N^9 +1425*N^10 -58*N^11 +N^12
            +9263360*K-574816*K*N-641576*K*N^2 +1460796*K*N^3 -1106730*K*N^4 +196075*K*N^5 +104895*K*N^6 -70662*K*N^7 +18040*K*N^8 -2365*K*N^9
            +155*K*N^10 -4*K*N^11 -26900416*K^2 +36978952*K^2*N-28829452*K^2*N^2 +24257100*K^2*N^3 -13385850*K^2*N^4 +5030620*K^2*N^5 -1242528*K^2*N^6
            +196000*K^2*N^7 -18250*K^2*N^8 +880*K^2*N^9 -16*K^2*N^10 -34998904*K^3 -1886996*K^3*N-550320*K^3*N^2 +21170*K^3*N^3 -498330*K^3*N^4 +343350*K^3
            *N^5 -130270*K^3*N^6 +25800*K^3*N^7 -2380*K^3*N^8 +80*K^3*N^9 +43433096*K^4 -38757150*K^4*N+23037735*K^4*N^2 -10771390*K^4*N^3 +3397485*K^4*N^4
            -752640*K^4*N^5 +94640*K^4*N^6 -5440*K^4*N^7 +80*K^4*N^8 +18367130*K^5 -599901*K^5*N+2276127*K^5*N^2 -699860*K^5*N^3 +318920*K^5*N^4 -103040*K^5
            *N^5 +14560*K^5*N^6 -640*K^5*N^7 -16002640*K^6 +10069360*K^6*N-3900736*K^6*N^2 +1321600*K^6*N^3 -250880*K^6*N^4 +17920*K^6*N^5 -4820172*K^7
            +757104*K^7*N-353440*K^7*N^2 +176000*K^7*N^3 -45440*K^7*N^4 +2560*K^7*N^5 +1590576*K^8 -911360*K^8*N+343040*K^8*N^2 -33280*K^8*N^3 -1280*K^8
            *N^4 +255360*K^9 -101120*K^9*N+75520*K^9*N^2 -5120*K^9*N^3 -202240*K^10 +28672*K^10*N+4096*K^10*N^2 -56320*K^11 +4096*K^11*N-4096*K^12

J( 6, 6)  = 3628800*N-6999840*N^2 +2124936*N^3 +4344076*N^4 -4992570*N^5 +2514875*N^6 -744282*N^7 +139623*N^8 -16830*N^9 +1265*N^10 -54*N^11 +N^12
            -56427840*K^2 +18781704*K^2*N-28772480*K^2*N^2 +20334660*K^2*N^3 -11544995*K^2*N^4 +4248840*K^2*N^5 -1046102*K^2*N^6 +167340*K^2*N^7 -16815
            *K^2*N^8 +960*K^2*N^9 -24*K^2*N^10 +60878404*K^4 -27148890*K^4*N+21271745*K^4*N^2 -8914830*K^4*N^3 +2921015*K^4*N^4 -618720*K^4*N^5 +85520*K^4
            *N^6 -6720*K^4*N^7 +240*K^4*N^8 -18617945*K^6 +7296864*K^6*N-3874136*K^6*N^2 +1041600*K^6*N^3 -210080*K^6*N^4 +23040*K^6*N^5 -1280*K^6*N^6
            +2392944*K^8 -698880*K^8*N+261120*K^8*N^2 -38400*K^8*N^3 +3840*K^8*N^4 -147200*K^10 +24576*K^10*N-6144*K^10*N^2 +4096*K^12
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## BIOGRAPHICAL NOTE

JEROME R. CHERNIACK graduated from City College of New York in 1958 and has been with the Observatory since 1959.

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## NOTICE

This series of Special Reports was instituted under the supervision of Dr. F. L. Whipple, Director of the Astrophysical Observatory of the Smithsonian Institution, shortly after the launching of the first artificial earth satellite on October 4, 1957. Contributions come from the Staff of the Observatory.

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