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FINAL REPORT

A COMPUTER CODE FOR PREDICTING GAMMA PRODUCTION CROSS SECTIONS BY NEUTRON INELASTIC SCATTERING FROM LIGHT NUCLEI

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By

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DEPARTMENT OF PHYSICS
ALABAMA A & M UNIVERSITY

JUNE 1972

Prepared for the National Aeronautics and Space Administration
Washington, D.C.

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SUMMARY

Gamma-ray production cross section by the inelastic scattering of neutrons form light nuclei are considered. The applicability of optical model potential is discussed. Based on experimental data, a cascade approach is developed to calculate the inelastic gamma production cross sections. $\sigma(n,n'\gamma)$ cross sections are evaluated in the case of O^{16} using computer code "LINGAP" in conjunction with ABACUS-2, and are compared with reported values.

INTRODUCTION

Inelastic collisions of neutrons with nuclei are of considerable interest, since these lead to emission of particles and gamma rays. At intermediate energies (above few MeV) reactions lead to (n,p) , (n,n') , (n,α) and (n, 2n) channels each of which would generally be followed by gamma emission from the product nuclei. It is of considerable interest to determine the cross section for each of these reactions as a function of the neutron energy.

The reaction mechanisms are described by adopting appropriate nuclear models. One of the most successful techniques adopted in the study of the neutron-nucleus inelastic collisions involves the compound nucleus concept. The theoretical formalism for this is worked out by Hauser and Fesbach[1]. The compound nucleus, formed by the absorption of the projectile nucleon, does not "remember" the manner in which it was formed so that its decay will depend only upon the properties of the exit channels. The parameters that control the exit channels are E the energy, J the spin and the parity of the states of the nuclei. The compound nucleus formation is usually schematically represented as shown in fig. 1.

In the compound nucleus concept it is assumed that the incoming particle suffers multiple collisions with the nucleons of the target, thus losing its energy successively until finally equilibrium is reached. It is probable that suff-

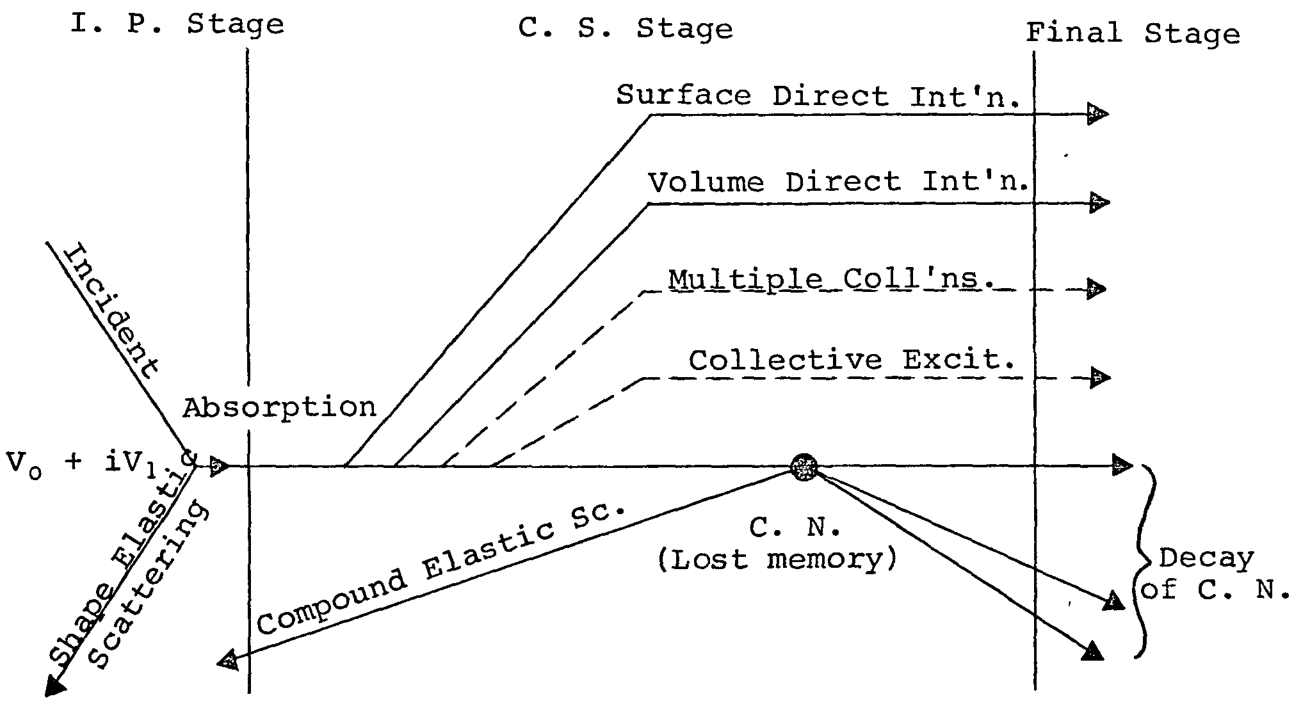

Fig. 1.

Independent particle stage, compound state stage, and final stage of a nuclear reaction.

(Ref. 9.)

icient energy can be imparted just to one nucleon so that it escapes the nucleus.

On the other hand it is possible for the reaction to take place via the direct interaction process. At higher projectile energies, when the time the nucleon requires to traverse the nucleus is several orders of magnitude less than the typical compound nucleus life-time of the order 10^{-14} sec, the reaction proceeds through direct interaction.

It is of interest in this study to develop a computer code to determine the neutron induced gamma ray production cross sections in light nuclei. The inelastic collision cross section as developed by Hauser and Fesbach is found to be extremely successful in the case medium and heavy weight nuclei. This has been observed by several groups[2-4].

THEORETICAL CONSIDERATIONS

Neutron as a projectile to bombard on nuclei of all masses has served as a means to understand the structure and properties of nuclei to a considerable extent. Neutrons react with many nuclei from very low neutron bombarding energies of the order of ev to very high energies of the order of several MeV leading to various types of reactions.

The energy balance is given by:

$$M_A + M_n + E_n = M_{A+1} + E_O \qquad (1)$$

where E_n is the incident neutron energy and E_O the energy evolved in the reaction. M_A, M_{A+1}, and M_n are the target nucleus, product nucleus and neutron mass respectively. The product nucleus which is generally referred to as the compound nucleus may decay by emitting photons, particles or even by fission.

At low incident energies, because of the centrifugal barrier for high angular momenta, $\ell = 0$, i. e., S wave neutron capture is several times more probable than P wave ($\ell = 1$) or higher order neutrons.

Thermal Neutron Capture

For $E_n \sim$ few ev, where S wave capture dominates, the compound nucleus spin is $J = I \pm \ell$ where I is the target nucleus spin. In this case parity is conserved. If E_n is such that the compound nucleus energy is close to one of its natural levels, resonance capture is said to occur and de-excitation takes place through gamma ray emission, the details of which are determined by the spin, parity and energy values of the levels involved. On the other hand, if the energy of the compound nucleus lies between two resonances, capture can take place to both the levels. The gamma rays emitted is this case will be mixtures of various multiple orders.

The compound nucleus, formed in an excited state, can decay either by emitting one or several gamma rays or through the emmission of one or several particles. Each mode of decay can be characterized by a meanlife τ_i where the values of i denote the various modes of decay. Obviously the mean-life of the excited state is

$$\frac{1}{\tau} = \sum_i \frac{1}{\tau_i} . \qquad (2)$$

τ is related through the uncertainity principle with level width Γ and is given by

$$\Gamma = \frac{\hbar}{\tau} . \qquad (3)$$

Thus the partial widths for each of the possible modes of decay can be written as

$$\Gamma_i = \frac{\hbar}{\tau_i} \qquad (4)$$

For the case in which the possible excited states are very close, the continuum model can predict the transition spectra with appreciable accuracy. Troubetskoy[5] has used such an approach for selected nuclei. Yost[6] has developed a general formalism to include the transitions from and to discrete levels where parity and spin dependence are also taken into account. For the resonance reaction the Briet-Wigner approach provides satisfactory results. Accordingly the cross section for (n,a) reaction is given by

$$\sigma(n,a) = \sigma_c \frac{\Gamma_a}{\Gamma} , \qquad (5)$$

where 'a' could mean a particle or photon emission. Γ_a is the particle/photon width. In the case where only one level of the compound is involved σ_c is given by

$$\sigma_c = \pi \lambda^2 f \frac{\Gamma_a \Gamma}{(E-E_r)^2 + \Gamma^2/4} \tag{6}$$

Here E_r is the neutron kinetic energy for which the excitation energy is equal to the difference between the ground state (of the target nucleus) and the energy of that level in the compound nucleus into which absorption takes place. The statistical weighting factor f is given by

$$f = \frac{(2J+1)}{(2I+1)(2S+1)} = \frac{(2J+1)}{2(2I+1)} \tag{7}$$

I being the spin of the target nucleus, ℓ, the orbital angular momentum of the neutron and J the spin of the compound nucleus level involved.

For, $\ell=o$ neutrons which is valid assumption for slow neutrons, and for the case where particle emission energetically not possible, one can then write the cross section for gamma production as

$$\sigma(n,\gamma) = \pi \lambda^2 f_o \frac{\Gamma_n \Gamma_\gamma}{(E-E_r)^2 + \Gamma^2/4} . \tag{8}$$

However, as neutron energy increases collisions become inelastic leading to particle and gamma ray emission. In this case compound nucleus formalism is found to be the appropriate concept to explain the reactions.

The generalized compound nucleus concept is dependent on the appropriate nuclear potential, of the target nucleus, which makes the absorption of the nucleon possible. However such an absorption which leads to the compound nucleus formation would not account for the resonance effects which are often seen and are intimately connected with the quantum states of the compound system. Thus in the compound nucleus concept one can at best get informations averaged over the resonances. This brings in the limitation that the validity of the concept is dependent on the density of the available states. The situation is rather favorable in the medium to heavy weight nuclei where the nuclear levels are more closely spaced than in lighter nuclei. Also as the projectile energy increases the higher energy states of the entrance channel become available and this also leads to better accuracy in the averaging over resonances. At still higher energies where the nuclear levels are so close that they can be treated as a continuum, the cross section for the formation of the compound nucleus is the same as the average reaction cross section.

The various cross sections that enter into nuclear reactions are given below.

$$\sigma_t = \sigma_{e\ell} + \sigma_r \quad . \tag{9}$$

Here, σ_t is the total cross section, $\sigma_{e\ell}$ is the elastic scattering cross section, which is the case where the quantum states of the target nucleus are not disturbed by the reaction. The elastic cross section is made up of, σ_{se} which is that type of scattering that takes place without the formation of the compound nucleus and σ_{ce} which is that part of elastic scattering that takes place after the formation of the compound nucleus. Thus, if one considers another cross section σ_c which denotes the cross section for the formation of compound nucleus, the latter can be split as

$$\sigma_c = \sigma_{ce} + \sigma_r \tag{10}$$

Obviously, from equations (9) and (10) one can conclude that

$$\begin{aligned} \sigma_t &= \sigma_{e\ell} + \sigma_r \\ &= \sigma_{se} + \sigma_{ce} + \sigma_r \\ &= \sigma_{se} + \sigma_c . \end{aligned} \tag{11}$$

Any potential adopted to solve Schrödinger's equation for this problem of nucleon-nucleus interactions should predict reasonable values of the elastic and reaction cross sections.

A simple form of potential given by Fesbach et al[7] is written as

$$V = V_o + iV_1 \tag{12}$$

The incoming neutron sees a potential of the form given in equation 12. The real part V_o of the potential accounts for the shape elastic scattering and the imaginary part V_1 is responsible for the formation of the compound nucleus. It is reasonable to expect the cross section for compound nucleus formation to increase with increase in the energy of the incoming neutrons. Therefore one can expect a proportionate dependence of V on the neutron energy. Also V will be dependent upon the mass number A and radius r of the nucleus.

However any such potential cannot satisfactorily explain the resonances in the cross sections in the low energy region. Resonances are the strong fluctuations in the nuclear cross sections that are found in the low energy range. On the other hand at higher energies, particularly in the medium and heavy weight nuclei level widths are smaller and cross sections become smooth functions of energy.

Following Weisskopf[8] one can obtain the cross sections in the following fashion.

Let the vector distance between the center of the nucleus and the incident particle be denoted by r_α and let the incident particle be described by a plane wave of the form $e^{ik_\alpha \cdot r_\alpha}$. If k_α is parallel to the $\bar{Z}$ axis one can then rewrite the plane wave by e^{ikz}.

Let the velocity of the particle = $\bar{v}_\alpha = \dot{\bar{r}}_\alpha$.

$$k_\alpha = \frac{M_\alpha v_\alpha}{k.} \tag{13}$$

Here the reduced mass is given by $M = \dfrac{M_\alpha \, Mx}{M_\alpha + Mx}$

where M_α is the projectile mass and Mx the target mass

Channel wave number $K_\alpha = \left(\dfrac{2 M\alpha \, \varepsilon\alpha}{\hbar} \right)^{1/2}$ (14)

where ε_α is the entrance channel energy.

As usual one expands the plane wave into spherical harmonics and write

$$e^{ikz} \simeq \frac{\pi^{1/2}}{kr} \sum_{\ell=0}^{\infty} (2\ell+1)^{1/2} \left[i^{\ell+1} \exp\left(-i\left(kr - \frac{\pi\ell}{2}\right)\right) - \exp\left(+i\left(kr + \frac{\pi\ell}{2}\right)\right) \right] Y_{\ell,0} \tag{15}$$

Equation (15) describes the undisturbed plane wave The nuclear reaction changes this so that the outgoing wave is written as

$$\psi(r) \cong \frac{\pi^{1/2}}{kr} \sum_{\ell=0}^{\infty} (2\ell+1)^{1/2}\, i^{\ell+1} \{\exp \left| -i(kr- \frac{\pi\ell}{2} \right|$$

$$-\eta_\ell \exp \left| + i(kr+\frac{\pi\ell}{2} \right| \} Y\ell,o \tag{16}$$

Here η_ℓ is the coefficient of the outgoing wave and is the complex reflection factor.

Then the scattered wave is given by

$$\psi(\bar{r})-e^{ikz} = \frac{\pi^{1/2}}{kr} \sum_{\ell=0}^{\infty} (2\ell+1)^{1/2} i^{\ell+1}$$

$$(1-\eta\ell) \exp \ +ikr+ \frac{\pi\ell}{2} \ \ Y\ell,o \tag{17}$$

One usually defines the scattering across section as

$$\sigma = \frac{\text{number of events per unit time per nucleus}}{\text{number of incident particles per unit area per unit time}}$$

Then one obtains N_{sc}/N which represents σ_{sc} by determining the flux ψ_{sc} through a sphere of radius r_o with the target at its center.

This is written as

$$N_{sc} = \frac{\hbar}{2iM} \int \left(\frac{\partial \psi sc}{\partial r} \psi^*_{sc} - \frac{\partial \psi^*_{sc}}{\partial r} \psi_{sc} \right) r^2 \sin \theta \, d\theta \, d\phi \qquad (18)$$

The orthogonality of spherical harmonics leads to

$$N_{sc} = \frac{v\pi}{k^2} \sum_{\ell=0}^{\infty} (2\ell+1) \left|1-\eta\ell\right|^2 \qquad (19)$$

Here v is the speed of the particles and the value of the flux N in a plane wave is given by the speed. Therefore $\sigma_{sc}\ell$ is obtained by dividing equation 19 by N or its equivalent v and for a particular valve of ℓ this quantity becomes

$$\sigma_{sc}\ell = \frac{\pi}{k^2} (2\ell+1) \left|1-\eta\ell\right|^2 \qquad (20)$$

In a similar manner the reaction cross section is found to be

$$\sigma_{r,\ell} = \pi \lambda\!\!\!^{-2} (2\ell+1) \left(1-\left|\eta\ell\right|^2 \right) \qquad (21)$$

It is necessary to average these cross sections and the phase shift term η over the energy. This is done by defining

$$\bar{\eta}_{\ell}= \frac{1}{\Delta E}\int_{E-\frac{\Delta E}{2}}^{E+\frac{\Delta E}{2}} \eta_{\ell}(E')\,dE'. \tag{22}$$

Similarly, one can average the cross sections also. This is a brief survey of the gross-structure problem which is solved by choosing an appropriate complex potential that will give satisfactory values for the phase shift $\bar{\eta}_{\ell}$.

NUCLEAR POTENTIAL FORMS

It is necessary to choose an appropriate potential to describe the reactions which include elastic scattering and compound nucleus formation, where the latter amounts to an absorption. The problem remains to write a two body potential which when used to solve the Schrödinger equation will lead to satisfactory values of the phase shift η_{ℓ} for each partial wave. In order to explain the phenomena of scattering and absorption, one adopts a potential of the complex form

$$V(r)=Uf(r)+iWg(r). \tag{23}$$

In order to accomodate the complexity introduced due to the spin of the incident particle, one introduces spin-orbit terms through U_s and W_s which are the real and imaginary parts of the spin-dependent potential. Thus one writes

$$V(r)=Uf(r)+iWg(r)+(U_s+iW_s)h(r)\vec{\ell}\cdot\vec{\sigma} \tag{24}$$

where h(r) is the radial variation of the spin-dependent part, σ the Pauli-spin operator and ℓ the angular momentum. Finally, if the incident particle is charged, the Coulomb part of the potential is introduced through another term $V_C(r)$ so that the complete optical potential is written as

$$V(r)=V_C(r)+Uf(r)+iWg(r)+(U_S+iW_S)h(r)\vec{\ell}\cdot\vec{\sigma} \tag{25}$$

Various forms of potentials are listed by Hodgson[9]. There have been efforts made to derive the optical potential from elementary nucleon-nucleon interaction. Perry and Buck have carried out some successful calculations using such non-local potentials.

There are several difficulties in adopting an appropriate set of parameters for optical potential for any given nucleus. When applying the model to any particular nucleus by attempting to fit the angular distribution of the elastic scattering data or by other means, one often finds that there may be several sets of values of the parameters which would give good fits of the cross section data. An additional criterion one may use is to fit measured reaction cross section with predicted values of such cross sections. And this may narrow down the ambiguities.

The difficulty is more accute in the case of light nuclei because of the difficulty in averaging over resonances. However, in the case of O^{16} which is considered as a sample problem in this work, it is possible to fit the data for neutron energies above 12 MeV[10]. Total cross sections measured, in the case of oxygen for $E_n<16$ MeV by Barschall[11] and elastic angular distributions measured by Chase etal at energies of 6.02, 6.53, 8.0, and 11.6 MeV[12] by Bauer, Anderson and Christensen[13] at 14 MeV provide reasonable amounts of information to attempt a fit to the cross sections. Johnson and Wallin have concluded that above 12 MeV, the total cross section is a smoothly fluctuating function of energy around 1.5 barns. Therefore, one can conclude the optical model should work well for neutron energies above 12 MeV in the case of O^{16}. Johnson and Wallin and several other groups have attempted to fit the reaction cross sections with optical model calculations for the case of O^{16} and N^{14}. Different model parameters for O^{16} which have been used by several groups are given in table 1. Unless one uses a non-local potential as developed Perry and Buck,[14] it will become necessary to use different sets of optical model parameters for different neutron energies due to the energy dependence of the optical potential. Preston[15] has concluded that for the case of local optical potential, among the constants R, a, b, Vc, Wc, VSo which enter the expression for the potential function, apart from the coulomb term, the difference between protons and neutrons is only in b. For neutrons b ∿ 1.2fm. On these basis it seems reasonable to adopt the optical model parameters for O^{16} from Duke[16]

His values are also given in table 1 .

The other functions which are given in equation 24 are given below. The function f(r) is usually adopted to be the Wood - Saxon form:

$$f(r) = \frac{1}{1 + \exp\left(\frac{r-R}{a}\right)} \tag{26}$$

and g(r) is the surface centered imaginary potential used by Fesbach and Bjorklund,

$$g(r) = \exp\left[-\left(\frac{r-R}{b}\right)^2\right]. \tag{27}$$

The spin orbit term is given by

$$h(r) = -\left(\frac{\hbar}{m_\pi c}\right)^2 \frac{1}{r} \frac{df(r)}{dr} \tag{28}$$

When m_π is pion mass. Nuclear radius R is given by $r_o A^{1/3}$ where A is the mass number. In these calculations a constant value of b = 1 is adopted. Also a fixed value of 1.2 is used for r_o. The important variations are in V, VS and W.

OPTICAL POTENTIAL PARAMETERS FOR

O^{16}

En MeV	r_o fm	V MeV	W MeV	a fm	b fm	V MeV	Ref
14	1.2	48.46	7.06	.71	1.0	5	Lutz[18]
14.5	1.2	52.2	3.2	.53	.7	4.2	Duke*
15.2	1.25	47.8	1.8	.57	1.8	4.6	
15.6	1.25	49.2	2.9	.62	1.1	6.6	
16	1.25	52.3	14.2	.46	.2	12.1	
16.4	1.25	45.7	22.6	.61	.2	11.7	
17	1.25	47.5	28.0	.60	.2	12.3	
17.4	1.25	47.6	5.7	.52	.6	4.0	
18	1.25	44.7	3.6	.55	1.0	2.3	
18.4	1.25	46.4	4.8	.57	.9	2.4	
19.2	1.30	46.1	13.1	.56	.4	3.1	

Table 1

*The value of b is adapted for neutrons

These considerations show that an extensive search of available experimental and theoretical data will provide satisfactory parameters for the optical model potential in the case of a light nucleus such as O^{16} which is of interest in this work.

INELASTIC CROSS SECTIONS

In the light of the above discussions it is reasonable to assume that the Hauser-Fesbach formalism can be applied to evaluate the inelastic cross sections in the case of selected light nuclei for neutron energies above a minimum value. Hauser-Fesbach formalism calculates the reaction cross section from an initial state i to a final state i' of the nucleus in terms of the transmission coefficients $T_\ell(E)$,

$$T_\ell(E) = \left| 1 - |\eta_\ell|^2 \right| \tag{29}$$

Also,

$$\sigma_c^{(\ell)} = (2\ell+1)\,\pi \lambda\!\!\!^{-2}\, T_\ell(E) \,. \tag{30}$$

Here $\sigma_c^{(\ell)}$ is the cross section for the formation of compound nucleus for neutrons of angular momentum ℓ. In a similar manner the cross section for the production of neutrons of energy E', of angular momentum ℓ', channel spin j and moving in a direction θ is given by Hauser and Fesbach as

$$\sigma(\ell,j\,|\ell',j'|\,\theta) = \pi\lambda^2(2\ell+1)\; T_\ell(E) \sum_j \frac{A_j(\ell,j\;\ell',j'\;\;\theta)}{1+\sum' T_p(E_{q'})/T_v(E)} \tag{31}$$

By using the sum rule and integrating over θ, one obtains the inelastic cross section to be.

$$\sigma(\ell,j\,|\ell',j') = \pi\lambda^2 T_\ell(E)\sum_J \left[(2J+1)/\left(1+\sum_{pqr}{}' \frac{T_p(E_{q'})}{T_\ell{}'(E')}\right)\right] \tag{32}$$

In equations 29 to 32, i is the target spin (in units of h), i'spin of the residual nucleus, ℓ&ℓ' are the initial final angular momentum of the neutron, and E and E' are the corresponding initial and final energies. J denotes the spin of a level in the compound nucleus, r refers to possible channel spins, p to possible neutron angular momenta and $E_{q'}$ to possible final neutron energies.

ABACUS-2[(17)] as developed by Auerbach uses scattering by an optical potential. The phase shifts $\eta_{\ell j}$ are related to the transmission coefficients through equation 29. These transmission coefficients are used to evaluate the partial inelastic scattering cross section for each excited level. The gamma transition probabilities can be then calculated from the cascade approach which is given in the next section.

CASCADE FORMALISM

Let i and j be two arbitrary levels above the ground state of nucleus as shown in fig. (2).

It is required to evaluate $\sigma_{(\gamma)}{}^{i}_{j}$ when the nucleus is subjected to an inelastic collision by neutron reaction.

The excitation of level i depends on $\sigma_n(E)$ the partial inelastic collision cross section for neutron energy $n(E) \geq i(E)$ where $i(E)$ is the energy of the level i.

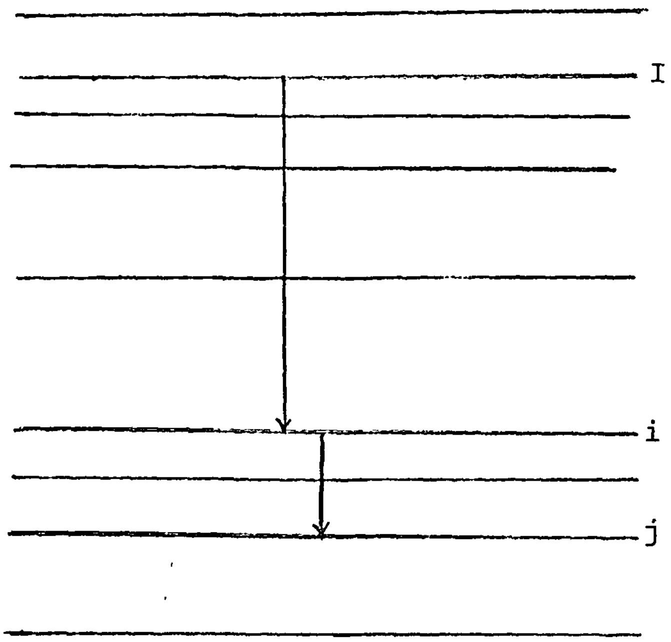

FIG. 2 NUCLEAR CASCADE.

The quantity can be calculated by Hauser-Fesbach formalism with the aid of ABACUS-2. In addition to this type of excitation the level i can be fed by allowed gamma transitions from states I > i and the cross section for this be denoted by $\sigma^{I>i}_{i(\gamma)}$.

Total inelastic excitation cross section for level i is given by,

$$\sigma_{T_i}(E) = \sigma_n(E) + \sigma_i(\gamma)^{I>i} \qquad (33)$$

Then

$$\sigma\gamma^i_j\,(E) = \sigma_{T_i}(E)\,\frac{\Gamma^i_j}{\Gamma} \qquad (34)$$

where the left hand side quantity in equation (34) denotes the cross section for the production of gamma ray by a transition from level i to level j, Γ^i_j is the partial level width for this process and Γ is the total radiative width of the level i. One can evaluate P^i_j from measured values of the relative intensities of the transitions originating from the level i. Thus one writes

$$P^i_j = \frac{\Gamma^i_j}{\Gamma}$$

$$= \frac{\text{Intensity of } \gamma^i_j}{\sum_{j=o}^{i-1} \text{Intensities of } \gamma^i_j}$$

In order to evaluate $\sigma_i^{I>i}(\gamma)$, for any given neutron energy n(E), one chooses all levels I in the target nucleus for which $I(E) \leq n(E)$, where $I(E_k)$ denotes the various energy levels in the nucleus. Obviously the levels of interest are those for which k goes from 1 to k max. Therefore one lists all the inelastic partial cross sections for each of these levels from an ABACUS-2 computation for the selected nucleus. The index k is set at k max where k max will denote the highest level chosen for which $I(E_{k_{max}}) \leq n(E)$ and k=1 will correspond to the ground state. Obviously $\sigma_n(E)^{k\ max}$ will denote the partial inelastic cross section for exciting the highest state. Then the cross section for exciting a level i by gamma transition form k max is given by

$$\sigma_i(\gamma)^{k\ max} = \sigma_n(E)^{k\ max} \frac{\Gamma_i^{k\ max}}{\sum_{j=o}^{k\ max\ -1} \Gamma_j^{k\ max}} \tag{36}$$

$$= \sigma_n(E)^{k\ max}\ P_i^{k\ max}$$

Then one considers the next lower level ie, the one corresponding to $(k_{max}-1)$. This level will be excited by the inelastic neutron collision as the one above and the corresponding partial cross section is given by $\sigma_n(E)^{k\ max-1}$. This level can also be excited by a gamma transition from the level at k max. The cross section for exciting the level at $(k\ max - 1)$ is given by

$$\sigma_T^{(k\ max-1)} = \left[\sigma_n(E)^{(k\ max-1)} + \sigma_n(E)^{k\ max}\ P_{k\ max-1}^{k\ max}\right] \tag{37}$$

Then the cross section for gamma transitions from this level to level i is written as

$$\sigma(\gamma)_i^{k\ max-1} = \sigma_T^{(k\ max-1)}\ P_i^{k\ max-1} \tag{38}$$

Carrying out similar calculations for levels below down to (i+1), one can write the total cross section for exciting level i as

$$\sigma_{T_i}(E) = \sigma_n(E)^i + \sigma_i^{i+1}(\gamma) + \sigma_i^{i+2}(\gamma) + \ldots\ldots\sigma_i^{k\ max-1}(\gamma) + \sigma_i^{k\ max}(\gamma) \qquad (39)$$

Therefore the cross section for gamma-ray transition from level i to level j as a result of a neutron inelastic collision is given by

$$\sigma\ \gamma_j^i(E) = \left[\sigma_n(E)^i + \sum_{k=i+1}^{k\ max} \sigma_i^{\ k}(\gamma)\right] \frac{\Gamma_j^i}{\Gamma} \qquad (40)$$

Γ_j^i / Γ is the relative transition probability for a transition from level i to level j .

Computer code LINGAP as given in appendix calculates the gamma production cross section using equation (40).

The inelastic cross sections $\sigma\ n(E)^i$ in the equation (40) are calculated for various neutron energies by using ABACUS-2. The transition probability for each gamma transition is separately found out from measured intensity values of all the gamma rays proceeding from a given level.

GAMMA RAYS BY NEUTRON INELASTIC COLLISION O^{16}.

In the case of oxygen, the optical model parameters used are given in Table 1. However, fixed values of $b = 1fm$ and $r_0 = 1.2fm$ were used. The relative gamma-ray transition probabilities for each level were calculated from the reported experimental values of relative intensities from ref [19]. These are given in Fig. 4.

Calculations have been made for neutron energies from 15 MeV to 19 MeV at intervals of 0.5 MeV. In the light of the discussions in the earlier part of this report, it is concluded that optical model can be applied for this region in the case of O^{16}.

Gamma production cross sections are calculated for transitions from energy levels $I(E) \leq 11.07$ MeV. The calculated values are given in tables 2 through 11. The cross sections are plotted as a function of neutron energy, E_n in figs. 5 through 9.

It seems reasonable to disregard the transitions from levels for which $I(E) > 11.62$ MeV. In Figure 3, which is taken from Morgan et al [20], one observes that above 11.62 MeV the $(n,n'\alpha)$ channel is the dominant mode of decay. This is also verified by Fig. 3.

Results of the calculations compare favorably with the experimental values reported by Stehn et al [21], for 6.13 MeV gamma-ray.

Gamma production cross sections are illustrated graphically and in tabular forms for the dominant transitions in O^{16}. There are not enough experimental values reported to compare these results. Joanou and Fench [22] have compiled $\sigma(n,n')$, in the case of O^{16}, for E_n up to 15 MeV. They have given a value of 500 mb at 15 MeV. Present calculations show that $\sigma(n,n'\gamma)$ at this energy is about 200 mb. The cross sections for 6.14, 6.92, and 7.12 MeV gamma-rays, as reported by Engesser and Thompson[23] for neutron energy of 14.7 MeV, are 120, 60, and 80 mb respectively. These values compare favorably with present calculations.

Code "Lingap" can be used for evaluating $\sigma(n,n'\gamma)$ for any nucleus for which $\sigma(n,n')$ can be evaluated using optical potential model or in any other manner. It will also be necessary to obtain the relative gamma transition probabilities to determine the gamma production cross sections.

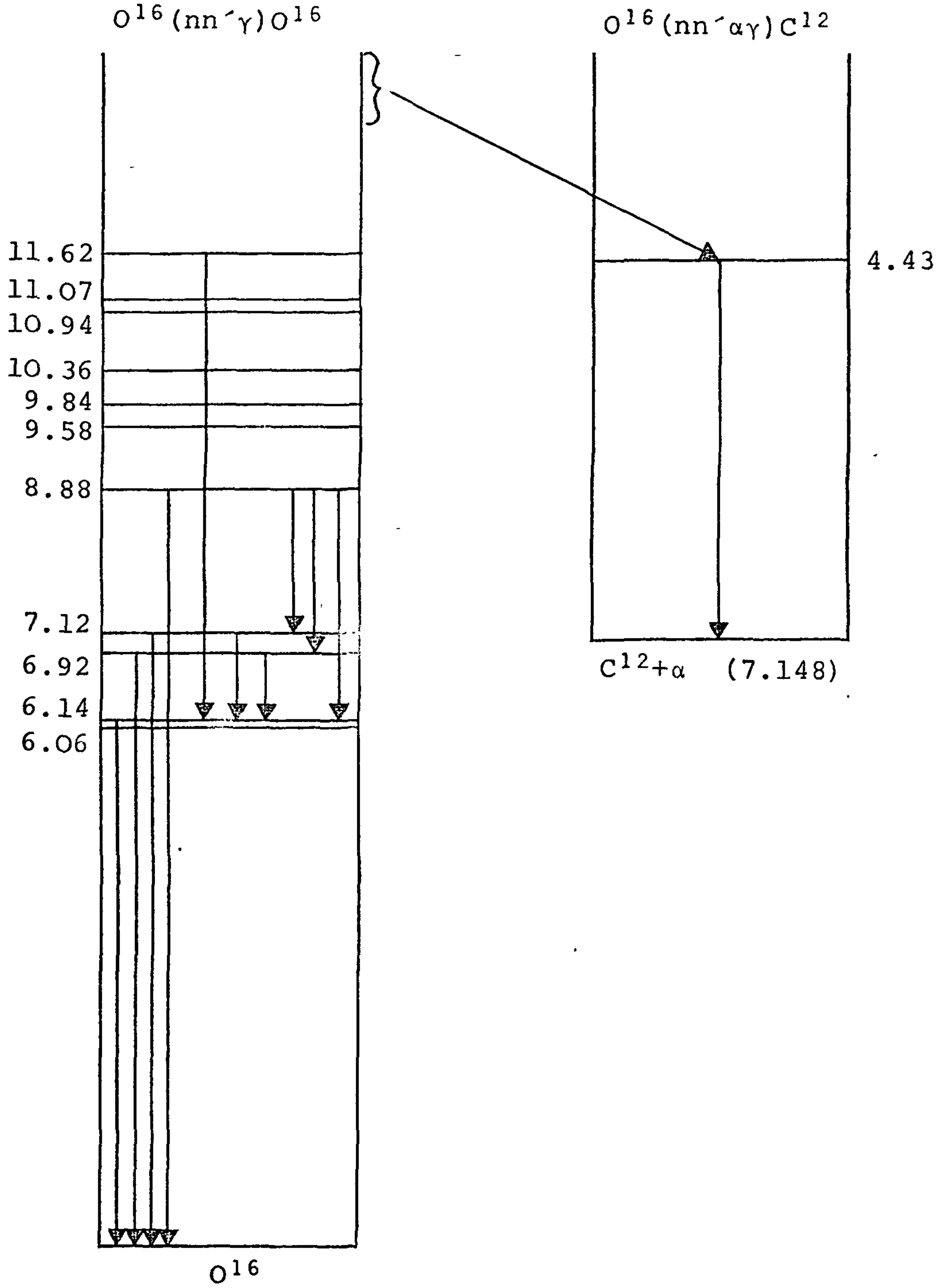

Fig. (3)
Inelastic neutron reactions in O^{16}
(Ref. 20.)

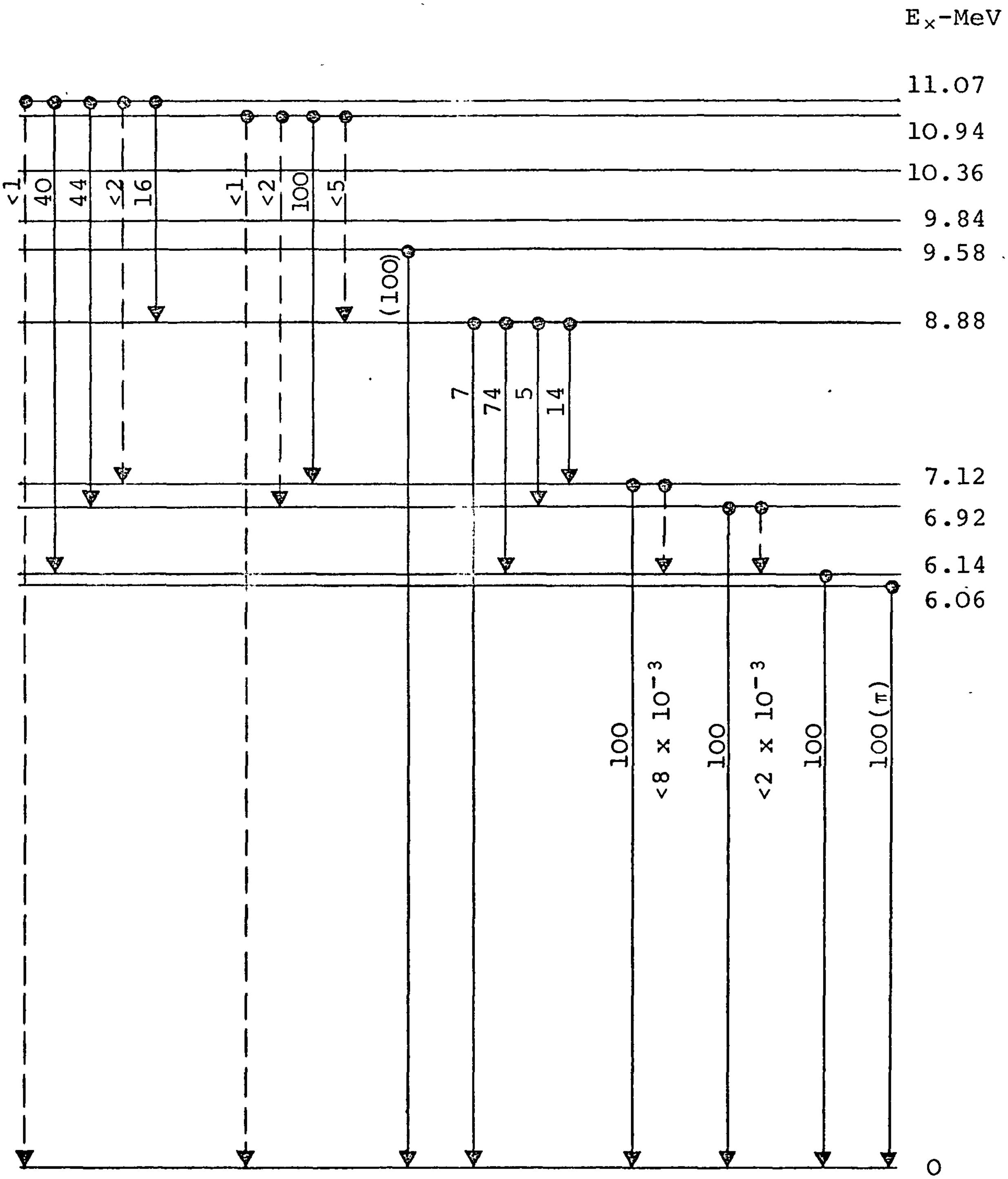

Fig. 4. Gamma-ray transitions in O^{16}
(Ref. 19)

$E\gamma$ = 6.06 MeV (6.06 MeV→0)

Neutron Energy in MeV	$\sigma(n, n' \gamma)$ in mb
15.0	16.21
15.5	13.35
16.0	9.93
16.5	26.89
17.	16.26
17.5	14.65
18.0	21.24
18.5	5.99
19.0	17.0

Table 2

$E\gamma$ = 6.14 MeV (6.14 MeV→0)

Neutron Energy in MeV	$\sigma(n, n', \gamma)$ in mb
15	52.26
15.5	48.55
16	29.13
16.5	41.53
17	39.38
17.5	29.75
18	28.55
18.5	42.11
19	31.15

Table 3

Cross Sections for the Production of Neutron Induced Gamma Rays in O^{16}

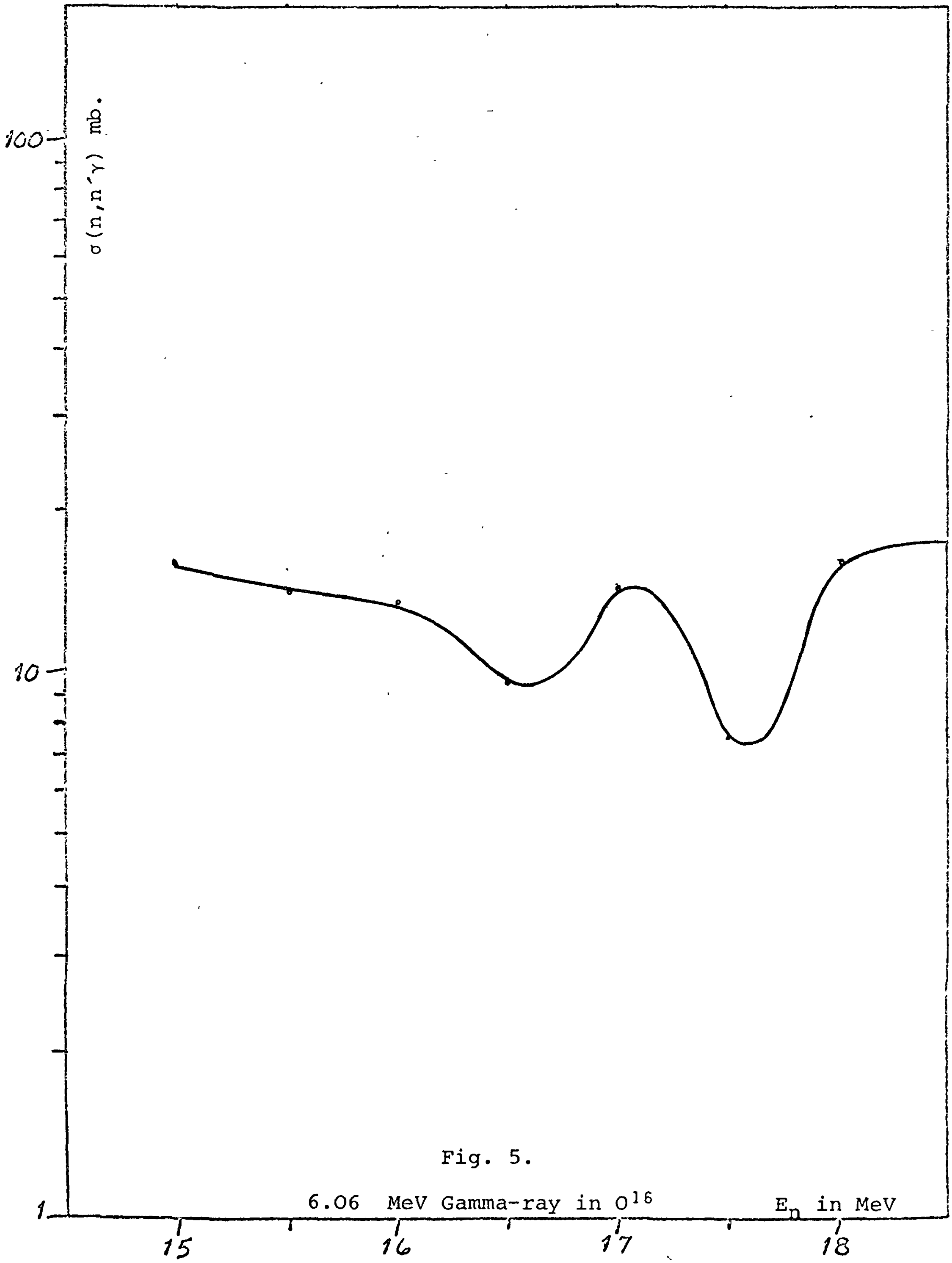

Fig. 5.

6.06 MeV Gamma-ray in O^{16}

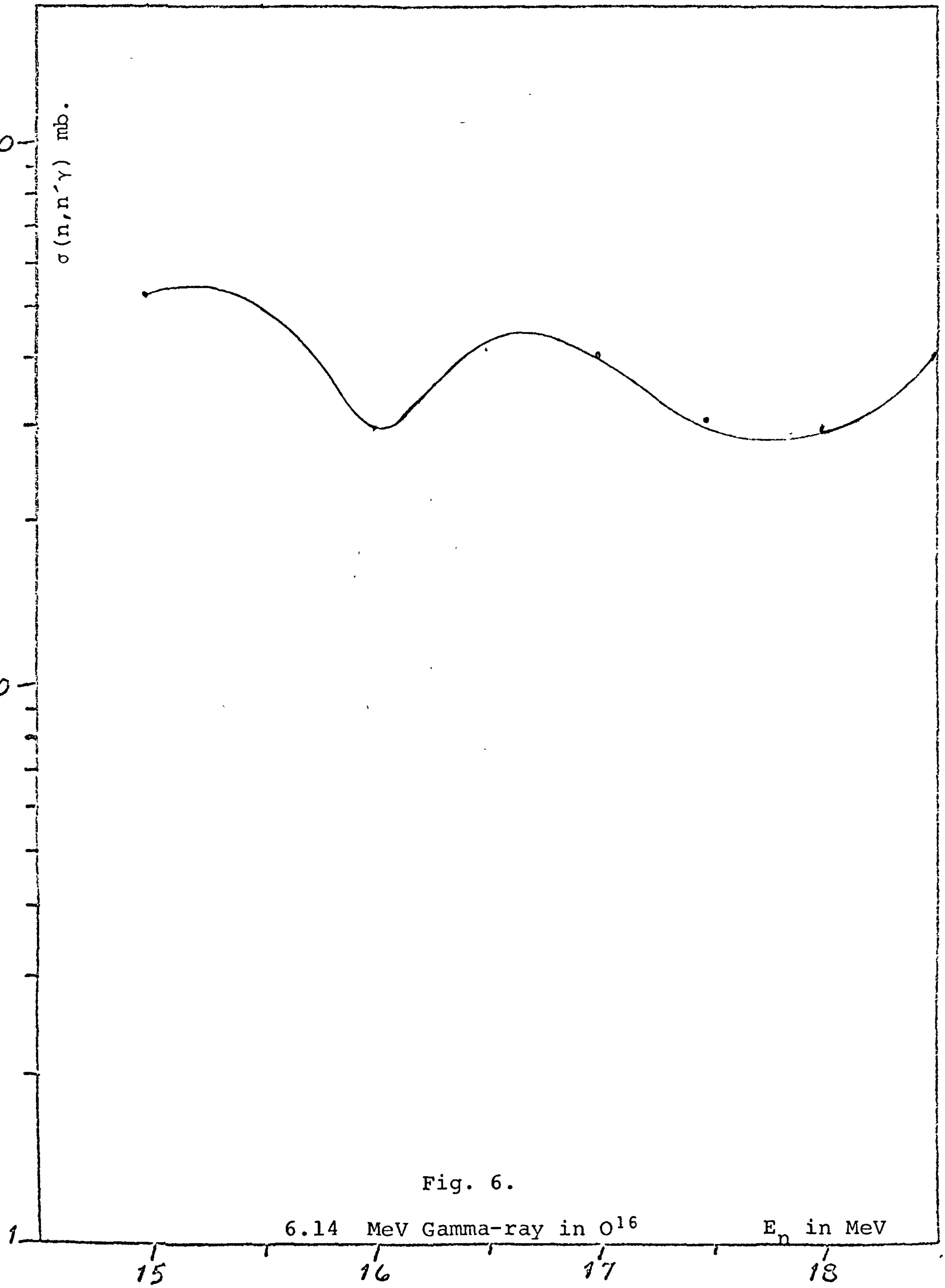

Fig. 6.

Eγ = 6.92 MeV (6.92 MeV→O)

Neutron Energy in MeV	σ(n, n' γ) in mb
15.0	40.5
15.5	38.15
16.0	34.89
16.5	23.17
17.0	19.89
17.5	25.37
18.0	13.53
18.5	25.01
19.0	26.0

Table 4

Eγ = 7.12 MeV (7.12 MeV→O)

Neutron Energy in MeV	σ(n, n' γ) in mb
15.0	22.27
15.5	21.88
16.0	29.70
16.5	31.20
17.0	43.2
17.5	32.68
18.0	46.36
18.5	24.62
19.0	37.95

Table 5

Cross Sections for the Production of
Neutron Induced Gamma Rays in O^{16}

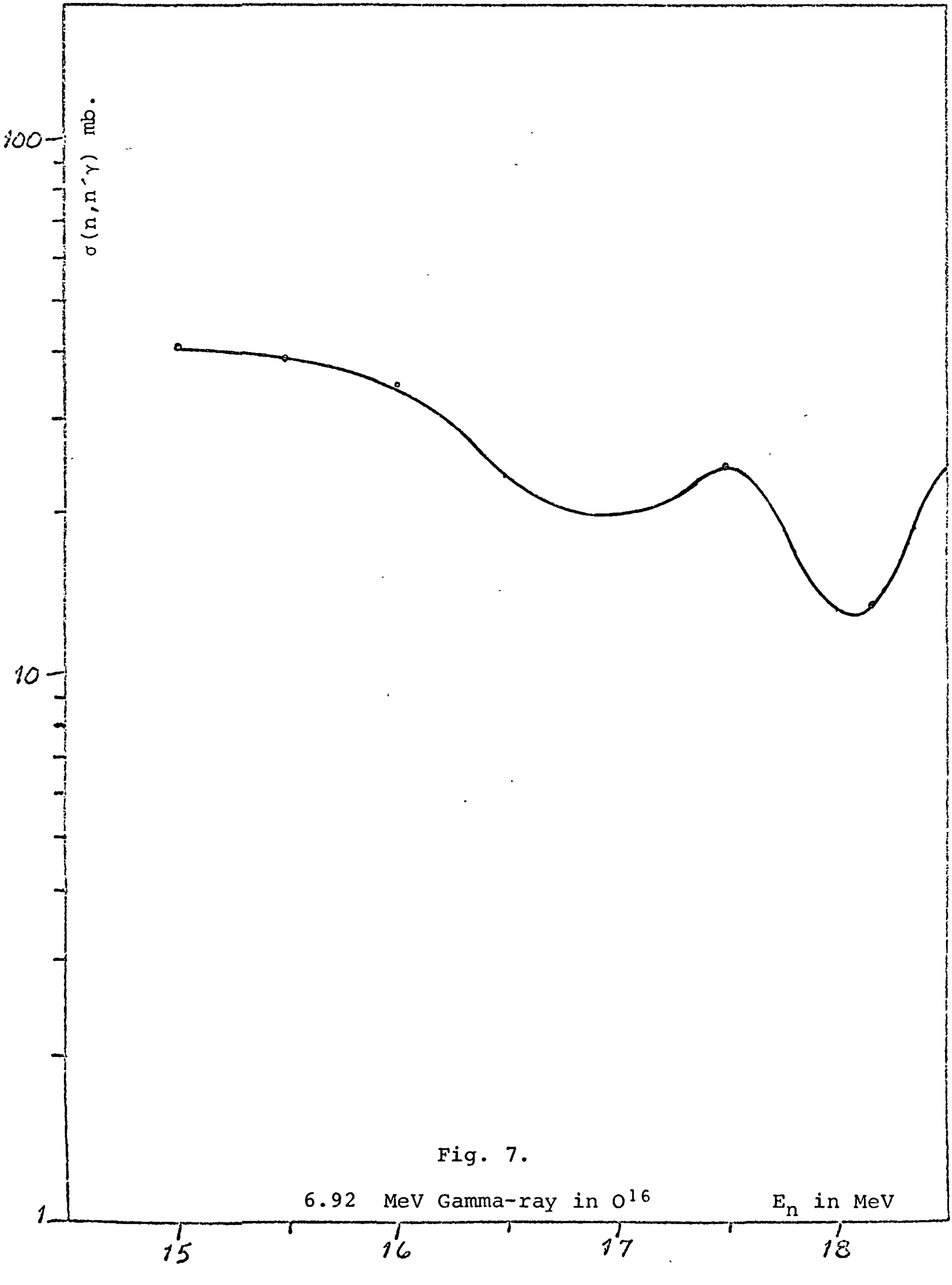

Fig. 7.

6.92 MeV Gamma-ray in O^{16}

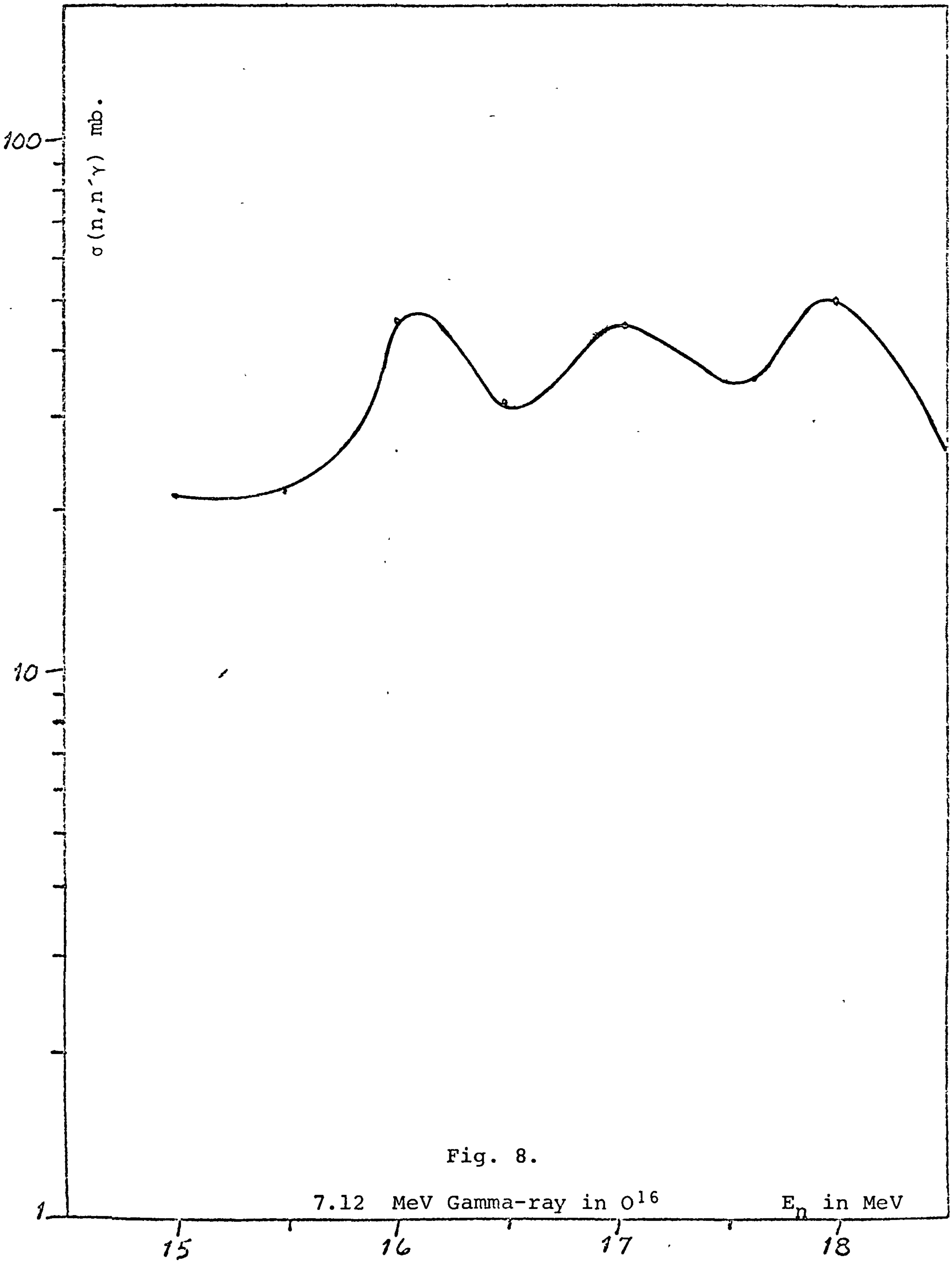

Fig. 8.

7.12 MeV Gamma-ray in O^{16}

$E\gamma$ = 2.74 MeV (8.88 MeV → 6.14)

Neutron Energy in MeV	$\sigma(n, n'\ \gamma)$ in mb
15.0	16.2
15.5	15.2
16.0	14.5
16.5	10.2
17.0	14.6
17.5	7.3
18.0	16.85
18.5	17.0
19.	16.0

Table 6

$E\gamma$ = 1.76 MeV (8.88 MeV → 7.12 MeV)

Neutron Energy in MeV	$\sigma(n, n'\ \gamma)$ in mb
15.0	3.0
15.5	3.0
16.0	2.7
16.5	2.0
17.0	2.7
17.5	1.5
18.0	3.0
18.5	3.5
19.0	3.0

Table 7

Cross Sections for the Production of Neutron Induced Gamma Rays in O^{16}

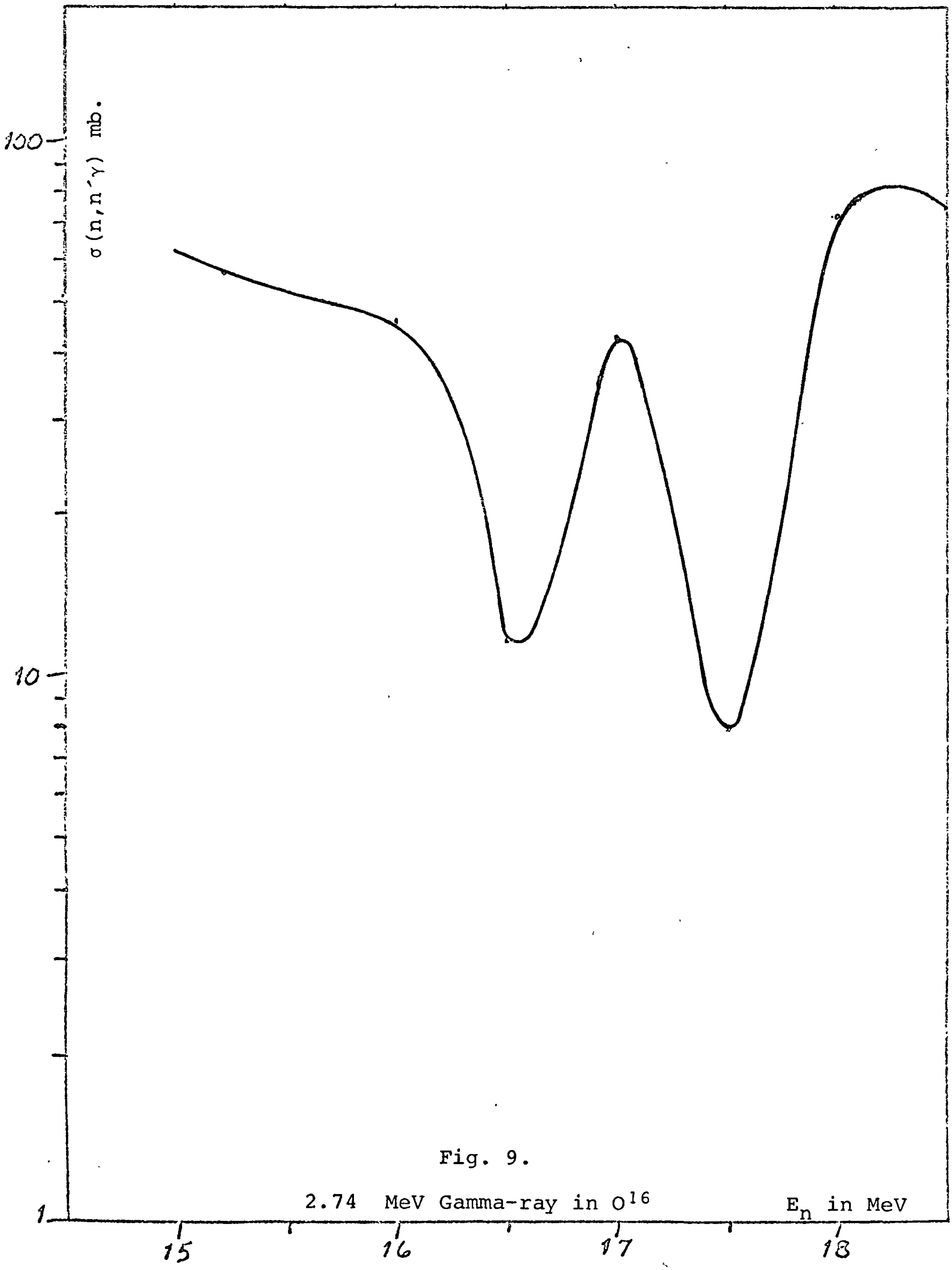

Fig. 9.

2.74 MeV Gamma-ray in O^{16}

$E\gamma$ = 9.58 MeV (9.58 MeV → 0)

Neutron Energy in MeV	$\sigma(n, n' \gamma)$ in mb
15.0	10.0
15.5	9.9
16.0	18.9
16.5	16.9
17.0	9.3
17.5	23.5
18.0	13.85
18.5	16.2
19.0	23.0

Table 8

E_γ = 3.82 MeV(10.94 MeV → 0)

Neutron Energy in MeV	Cross Section in mb
15	2.7
15.5	2.3
16.0	13.0
16.5	8.99
17.0	23.5
17.5	22.3
18.0	20.7
18.5	8.4
19.	19.5

Table 9

$E\gamma = 4.93$ MeV $(11.07 \rightarrow 6.14$ MeV$)$

Neutron energy in MeV	Cross Section in mb
15.0	5.1
15.5	4.9
16.0	3.9
16.5	10.2
17.0	6.2
17.5	6.3
18.0	9.1
18.5	2.0
19.0	6.0

Table 10

$E\gamma = 4.15$ MeV $(11.07 \rightarrow 6.92$ MeV$)$

Neutron Energy in MeV	Cross Section in mb
15.0	5.6
15.5	0.2
16.0	4.2
16.5	11.3
17.0	6.8
17.5	7.0
18.0	10.0
18.5	2.4
19.0	7.0

Table 11

REFERENCES

1- Hauser, W. and Fesbach, H. Phys. Rev 87 pp 366-373 1952.

2- Goldman, D. T. and Lubitz, C. R. Calculation of Inelastic Neutron Scattering, KAPL-2163

3- Snow, R. and George, M. C. Nuclear Science and Engineering. 35 p 409, 1969

4- Dunford, C. L. Francis, N. C. Lubitz, C. R. and Rapaport, J. Proceedings of Gatlinberg American Physical Society Meeting Oct. 10-12, 1963

5- Troubetzkoy, E. S. Phy. Rev. 122 p 212, 1960

6- Yost, K. J. "A Formalism for the Calculation of Neutron Capture Gamma-Ray Spectra." ORNL-TM-1867 Preliminary Oakridge National Laboratory (1967)

7- Fesbach, H. , Porter, C. E. and Weisskopf Phys. Rev. 96 pp 448-464, 1959

8- Blatt, J. M. and Weisskopf, V. F. Theoretical Nuclear Physics, John Wiley & Sons, London, 1963 pp312-365.

9- Hodgson, P. E. The Optical Model of Elastic Scattering. pp 187-189. Oxford University press. London 1963

10- Johnson, B. and Wallin, L. NP-14056. Research Institute of National Defence, 1966, Stockholm, Sweden.

11- Coon, J. H., Graves, E. R. and Barschall, H. H. Phys. Rev. 88, 562 (1952).

12- Chase, L. R. , Johnson, R. G. , Smith, R. V. Vaugh, F. T and Walt, M. AFSWC-TR-61-15 (1961). Lockheed.

13- Bauer, R. W., Anderson, J. D. and Christensen, L. J. Nuclear Phy. 47, 241 (1963)

14- Perry, F. and Buck, B. Nuclear Physics 32 (1962) pp 353-380.

15- Preston, M. A., Physics of the Nucleus (Addison-Wesley Publishing Company, Reading, Massachusetts), 1963. pp. 546-549.

16- Duke, C. B., Physical Review, 129(1963). pp. 581-591.

17- Auerbach, E. H., ABACUS-2 Program Operation and Input Description. BNL 6562. 1962.

18- Lutz, H. F., Mason, J. B., and Karvelis, M. D., Nuclear Physics 47(1963). pp. 521-524.

19- Ajzenberg-Selove, F., and Lauritsen, Energy Levels of Light Nuclei. Nuclear Physics. 11(1959). pp. 1-340.

20- Morgan, I. L., Ashe, J. B., Benjamin, R. W., Hudson, O. M., Mathur, R. L., Nellis, D. O., and Parker, C. V., Annual Progress Report TID20658. Texas Nuclear Corporation. 1963.

21- Stehn, J. R., Goldberg, M. D., Magurno, B. A., and Chasman, R. W., Neutron Cross Sections BNL 325. 1964.

22- Joanou, C. D., and Fench, H., Nuclear Science and Technology. 17(1963). pp. 425-434.

23- Engesser, F. C., and Thompson, W. E., Journal of Nuclear Energy. 21(1967). pp. 487-507.

COMPUTER CODE LINGAP DESCRIPTION

Computer code LINGAP which is given in the appendix. The code evaluates the gamma production cross section as given in equation (40)

E(K) = Energies of the states of the nucleus in MeV

EN(I) = Neutron energies in MeV.

SIGI(I,K) = Neutron inelastic cross section for levels E(K) and for energies EN(I).

PROB - Gamma transition probabilities evaluated from radiation width.

GASIG - Gamma production cross section for the particular neutron energy.

The first input card contains the values of N1, Mx, and M1. This enables one to vary the dimensions of EN and SIGI. In order to do a multirun for different values of projectile energy the following procedure is to be followed. The first set of data cards contain E in sequence followed by EN in sequence. SIGI values for each level for EN(1) are given on separate cards. PROB values follow these cards. The remaining SIGI values are punched in order and these values follow the PROB values.

APPENDIX

CODE-LINGAP

```
      DIMENSION E(25),EN(25),SIGI(25,25),PROB(25,25),SIG(25,25)
      DIMENSION GASIG(25,25)
      INTEGER P
      INTEGER R
      READ(1,333) N1,MX,M1
      ME = MX-1
  333 FORMAT (I2,I2,I2)
      DO 17 R=1,N1
   17 CONTINUE
      DO 15 J=1,20
   15 PROB (R,J)=0.0
    5 CONTINUE
   99 FORMAT (8F10.7)
      READ(1,99) (E(N),N=1,N1)
      READ(1,99) (EN(N),N=1,MX)
   90 P = 0
  101 P = P+1
   92 IMAX = N1
   98 I = 1
  100 I=I+1
      S=EN(P)
      U=E(I)
      M=P
      WRITE (3,300)
  300 FORMAT(1H1,6X,2HEN,10X,1HE)
  999 WRITE (3,998) S,U
  998 FORMAT(1X,F10.7,1X,F10.7)
      IF (S-U)103,104,106
  103 GO TO 101
  104 GO TO 109
  106 I2 = I+1
  107 U1 = E(I2)
      WRITE(3,998)S,U1
      IF (S-U1)112,112,114
  111 I = I2-1
      GO TO 109
  112 I = I2
  113 GO TO 109
  114 IF (IMAX-I2) 112,112,115
  115 I2 = I2+1
      GO TO 107
  109 READ (1,165)(SIGI(N,M),N=1,I)
  165 FORMAT(F10.7)
      WRITE (3,167)
```

```
167  FORMAT (///,44X,4HSIGI)
     WRITE(3,166)(SIGI(N,M),N=1,I)
166  FORMAT (44X,F10.7)
     KC = I-1
     R = I
     KK = R-1
     IF (P-2) 997,995,995
997  CONTINUE
     WRITE(3,301)
301  FORMAT(1H1,3X,1HJ,4X,1HR,47X,9HPROB(R,J))
     DO 76 K = 1,KC
     READ(1,180) (PROB(R,J),J=1,KK)
180  FORMAT(F10.7)
     WRITE(3,2000) K,R,(PROB(R,J),J=1,KK)
2000 FORMAT(1X,2I5,10F10.7)
     KK = KK-1
76   R = R-1
188  R=I
189  K1 = R - 1
995  CONTINUE
     WRITE(3,302)
302  FORMAT(1H1,1X,1HR,2X,1HJ,12X,8HSIG(R,J))
     DO 222 J=1, K1
     SIG(R,J)=SIGI(R,P)*PROB(R,J)
222  WRITE (3,996) R,J,SIG(R,J)
223  R = R-1
     IF (R-2) 184,187,187
187  GO TO 189
184  CONTINUE
C GAMMA TRANSITION CROSS SECTIONS
238  L = 0
239  L =L+1
     R=L+1
     SUM=0
     IF (I-L) 392,242,240
240  GAIEL=SIG(R,L)
     SUM=SUM+GAIEL
242  IF (I-R) 370,370,360
360  R=R+1
     GO TO 240
370  TGAIEL=SUM
     IF (L-1) 239,239,372
372  K2 = L-1
     WRITE(3,303)
303  FORMAT(1H1,1X,1HL,2X,1HJ,12X,10HGASIG(L,J))
373  DO 380 J=1,K2
     GASIG(L,J) = (TGAIEL + SIGI(L,P))*(PROB(L,J))
380  WRITE (3,996) L,J,GASIG(L,J)
996  FORMAT(1X,I2,1X,I2,1X,F18.12)
     IF (I-L) 395,395,390
```

```
390  GO TO 239
392  CONTINUE
395  IF (P-ME) 400,450,450
400  GO TO 101
450  CONTINUE
     END
```