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DEVELOPMENT OF A HIGH-RELIABILITY ROTARY ACTUATOR FOR SPACEFLIGHT USE

Volume II - Appendices to Technical Report

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TABLE OF CONTENTS

LIST OF ILLUSTRATIONS

APPENDIX A

BEARING LOAD ANALYSES

A.1 BREADBOARD ACTUATOR MODEL

This analysis presents the calculations by which the maximum allowable overhung load capability of the breadboard actuator Model EH-818-U1 was determined. The conclusions of this analysis are as follows:

Model EH-818-U1 Dynavector actuator with bearing configuration described above can safely support an overhung load of 3440 pounds. The actuator will require a bearing preload of 1130 pounds and the preload will be obtained by stacking two schrorr disc springs in parallel and rotating a 3/8 - 24 spring preload screw 100 degrees.

The friction torque required to drive through the preloaded bearings is 6.18 in-lbs at the output shaft and the calculated bearing life at rated conditions is $2.5 \cdot 10^6$ hours (minimum).

A.2 OVERHUNG-LOAD ANALYSIS

Based on the freebody diagram of Figure A-1, the following nomenclature is used:

<u>Nomenclature</u>

F_G = overhung load to be solved

T_L = 2400 in-lbs stall torque

α = 20 degrees mesh pressure angle

R_{B1} = outboard bearing load

R_{B2} = inboard bearing load

F_o = output gear force

R_o = output gear pitch radius

$$\underline{\Sigma F_x' = 0}$$

$$R_{B1} + F_o + F_G = R_{B2} \qquad \text{(A-1)}$$

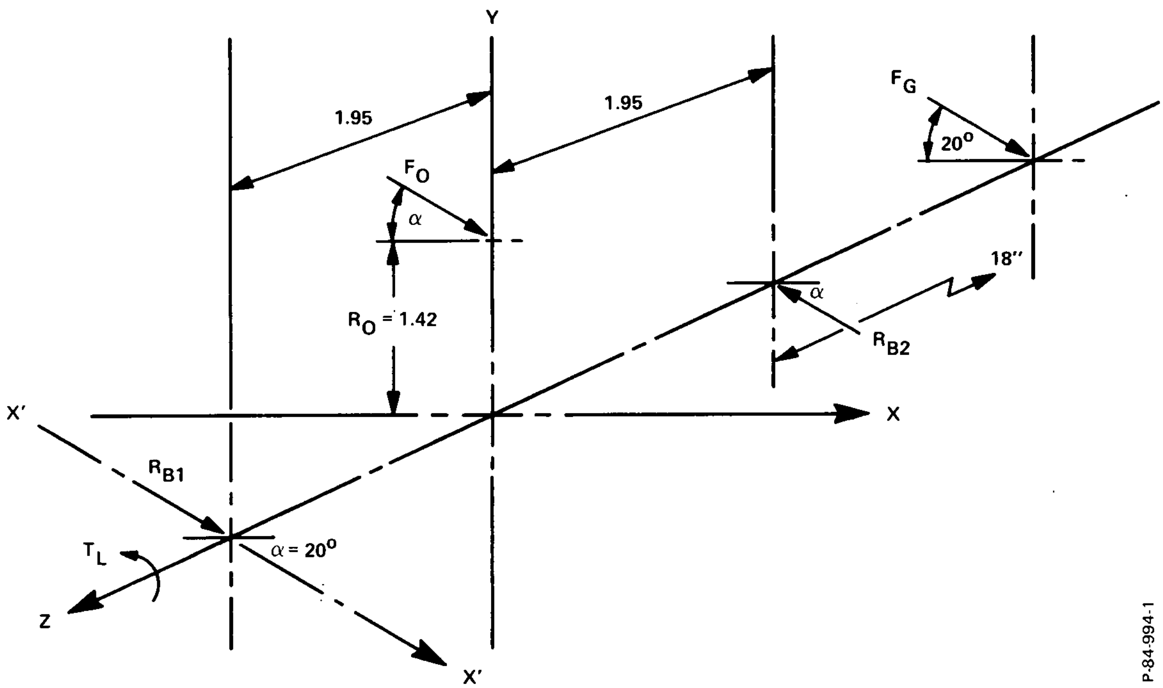

Figure A-1 - Actuator Model EH-818-U1 Bearing Load Free Body Diagram

$$\underline{\Sigma M_{Fo}} = 0$$

$$-1.95 R_{B1} - 1.95 R_{B2} + 19.95 F_B = 0 \qquad (A-2)$$

$$\Sigma M_Z = 0$$

$$F_o \cos 20 \text{ deg } R_o = T_L \qquad (A-3)$$

Solve (A-2) for $R_{B1} + R_{B2}$

$$R_{B1} + R_{B2} = \frac{19.95}{1.95} F_G$$

$$R_{B1} + R_{B2} = 10.25 F_G \qquad (A-4)$$

Solve (A-1) and (A-3) for $R_{B1} - R_{B2}$

$$R_{B1} - R_{B2} = -F_G - F_o$$

$$R_{B1} - R_{B2} = -F_G - \frac{T_L}{R_o \cos 20 \text{ deg}} \qquad \text{(A-5)}$$

Sum (A-4) plus (A-5) to obtain F_G

$$2 R_{B1} = 9.25 F_G - \frac{T_L}{1.335} \qquad \text{(A-6)}$$

The maximum value of F_G is determined from the static capacity of the selected bearings. As defined by AFBMA and FAFNIR, the basic static capacity is defined as the load which limits the race deformation to 0.1 x ball diameter and the bearing functions well after the loadup. The bearings selected for the breadboard design are:

Outboard Bearing R_{B1} - Series 9112

Inboard Bearing R_{B2} - Series 211W

From Manufacturer's Data:

R_{B1} static capacity = 15,000 pounds

R_{B2} static capacity = 22,000 pounds

Solve (A-6) for F_G maximum given T_L = 2400 in-lb

$$R_{B1} = 4.625 F_G - \frac{T_L}{2.67}$$

$$R_{B1} = 4.625 F_G - \frac{2400}{2.67}$$

$$15{,}000 = 4.625 F_G - 898$$

$$F_G = \frac{15{,}898}{4.625} = 3440 \text{ pounds}$$

Check to see if R_{B2} is overloaded at F_G = 3440 pounds. From equation (A-4)

$$R_{B2} = 10.25 F_G - R_{B1}$$

$$= 10.25 (3440) - 15,000$$

$$= 35,300 - 15,000$$

$$R_{B2} = 20,300$$

R_{B2} limited static capacity is 22,000 pounds. Therefore F_G = 3440 pounds is acceptable.

A.3 BEARING PRELOAD ANALYSES (Figure A-2)

Assuming F_G reaches maximum values of 3440 pounds, calculate preload required to offset bearing separating loads that result from the 12-degree race contact angle of the selected angular contact bearing. Given:

F_{S1}, F_{S2} = thrust force developed through 12-degree race contact angle

R_{B1}, R_{B2} = bearing radial loads

ΔF = required preload force

Condition I: <u>Actuator is at Stall with the Overhung Load F_G = 0</u>

Then

$$R_{B1} = R_{B2} = 900 \text{ pounds at } T_L = 2400 \text{ in-lb stall torque}$$

the bearing separating forces are:

$$R_{B1} \tan 12 \text{ deg} = 900 \tan 12 \text{ deg} = 192 \text{ pounds}$$

The required bearing preload at 2400 in-lb stall torque and with overhung load, F_G = 0 is 192 pounds.

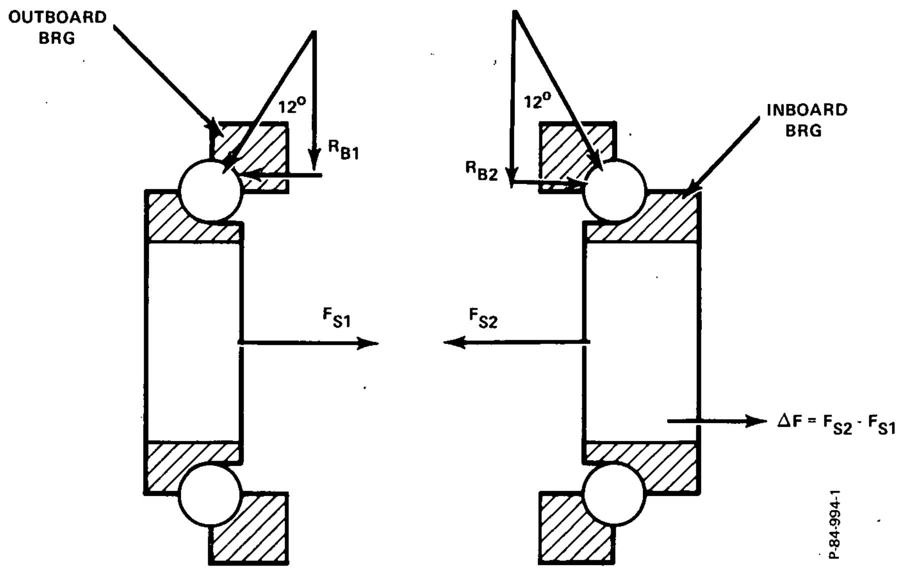

Figure A-2 - Bearing Preload Free Body Diagram

Condition II: <u>Actuator Subject to 3440-Pound Overhung Load and T_L = 0 in-lb</u>

Given:

$$R_{B1} = 15{,}000 \text{ pounds} \quad \text{and} \quad R_{B2} = 20{,}300 \text{ pounds}$$

Then:

$$F_{S1} = 15{,}000 \tan 12 \text{ deg} \qquad F_{S2} = 20{,}300 \tan 12 \text{ deg}$$

$$F_{SL} = 3190 \text{ pounds} \qquad F_{S2} = 4320 \text{ pounds}$$

$$\Delta F = F_{S2} - F_{S1} = 4320 - 3190$$

$$\Delta F = +1130 \text{ pounds}$$

A.4 BEARING FRICTION TORQUE ANALYSIS

1130-pound bearing preload force is required when overhung load F_G = 3440 pounds is acting on the actuator bearings.

Assume F_S preload = 1130 pounds and normal operating condition. Calculate friction torque when actuator is loaded to 2400 in-lb. Given:

$$T_f = f\ R\ w_e \qquad \text{(A-7)}$$

where

T_f = friction torque

n = no. of balls/bearing

f = 0.0015 bearing coefficient friction (mfg. data)

R = bearing race pitch radius

d = ball diameter

w_e = equivalent radial load

The following data was obtained from FAFNIR Bearing Design Manual

Bearing No. 2MM9112W — 60 MM ID; 95 MM OD; 18 MM wide

or 2.3622 inches ID — 3.7402 inches OD — 0.7087 inches wide

$$nd^2 = 3.14$$

Bearing No. 2MM211W — 55 MM ID; 100 MM OD; 21 MM wide

or 2.1654 inches ID — 3.9370 inches OD — 0.8268 inches wide

$$nd^2 = 4.43$$

and given

$$w_e = X_2\ V\ R + Y_2\ T \qquad \text{(A-8)}$$

where

X_2 = 0.45 (FAFNIR radial load correction factor)

V = 1.2 (load and inner ring rotating)

R = bearing radial load 900 pounds at T_L = 2400 in-lbs

T = spring preload - 1130 pounds

Y_2 = thrust correction factor

Solve Y_2 for Bearing No. 2MM9112W1

$$Y_2 = f\left(\frac{T}{n_d^2}\right)$$

$$Y_2 = f\left(\frac{1130}{3.14}\right)$$

$$Y_2 = f\ (360)$$

$$Y_2 = 1.14$$

obtained from Bearing Manufacturers Thrust Correction Table.

Solve Y_2 for Bearing No. 2MM211W1

$$Y_2 = f\left(\frac{T}{n_d^2}\right)$$

$$Y_2 = f\left(\frac{1130}{4.43}\right)$$

$$Y_2 = f\ (255)$$

$$Y_2 = 1.23$$

obtained from Bearing Manufacturers Thrust Correction Table.

Solve (A-8) for w_e

For Bearing No. 2MM9112W1 (outboard bearing)

$$w_e = (0.45)\ (1.2)\ (900) + 1.14\ (1130)$$

$$w_e = 486 + 1290$$

$$w_e = 1776 \text{ pounds}$$

and

For Bearing No. 2MM211W1 (inboard bearing)

$$w_e = (0.45)\ (1.2)\ (400) + 1.23\ (1130)$$

$$w_e = 486 + 1390$$

$$w_e = 1876 \text{ pounds}$$

Solving (A-7) for friction torque T_f

$$T_f = f\ R\ w_e$$

For Bearing No. 2MM211W1 (outboard bearing)

$$T_f = (0.0015)\ (1.18)\ (1776)$$

$$T_f = 3.14 \text{ in-lbs}$$

For Bearing No. 2MM211W1 (inboard bearing)

$$T_f = (0.0015)\ (1.08)\ (1876)$$

$$T_f = 3.04 \text{ in-lbs}$$

With the actuator operating near stall, the combined bearing friction torque is 6.18 in-lbs.

A.5 BEARING PRELOAD SPRING AND SCREW ANALYSES

Analyses to determine the number of degrees the preload screw must be rotated to obtain a 1130-pound bearing preload.

A Schrorr disc spring load-deflection curve, a spring rate of 100,000 lb/in is shown in Figure A-3.

Given a 3/4 - 24 screw thread - calculate the number of degrees required to preload bearings to 1130 pounds.

$$\text{Lead} = \frac{1}{24} = 0.0413 \text{ in/rev}$$

Rate = 100,000 lb/in

Required spring deflection = 1130/100,000 = 0.0113 inch and the number of degrees screw rotation is determined from

$$\theta_s = \frac{0.0113}{0.0413} (360)$$

$$\theta_s = 98 \text{ deg}$$

The preload screw shall be rotated 100 degrees to obtain 1130-pound bearing preload.

A.6 BEARING B-10 LIFE ANALYSIS

Calculate Bearing B-10 life with 1130-pound preload. At rated conditions - 150 ft-lbs output torque and 1 rpm the bearing reaction forces = 150/200 (900) = 765 pounds/bearing.

<u>Outboard Bearing No. 2MM211W1 - Given Basic Dynamic Capacity C_B</u>

$$C_B = 6100 \text{ pounds at } 33\text{-}1/3 \text{ rpm}$$

$$w_e = X_2 \; V \; R + Y_2 \; T$$

$$= (0.45)\;(1.2)\;(675) + 1.14\;(1130)$$

C

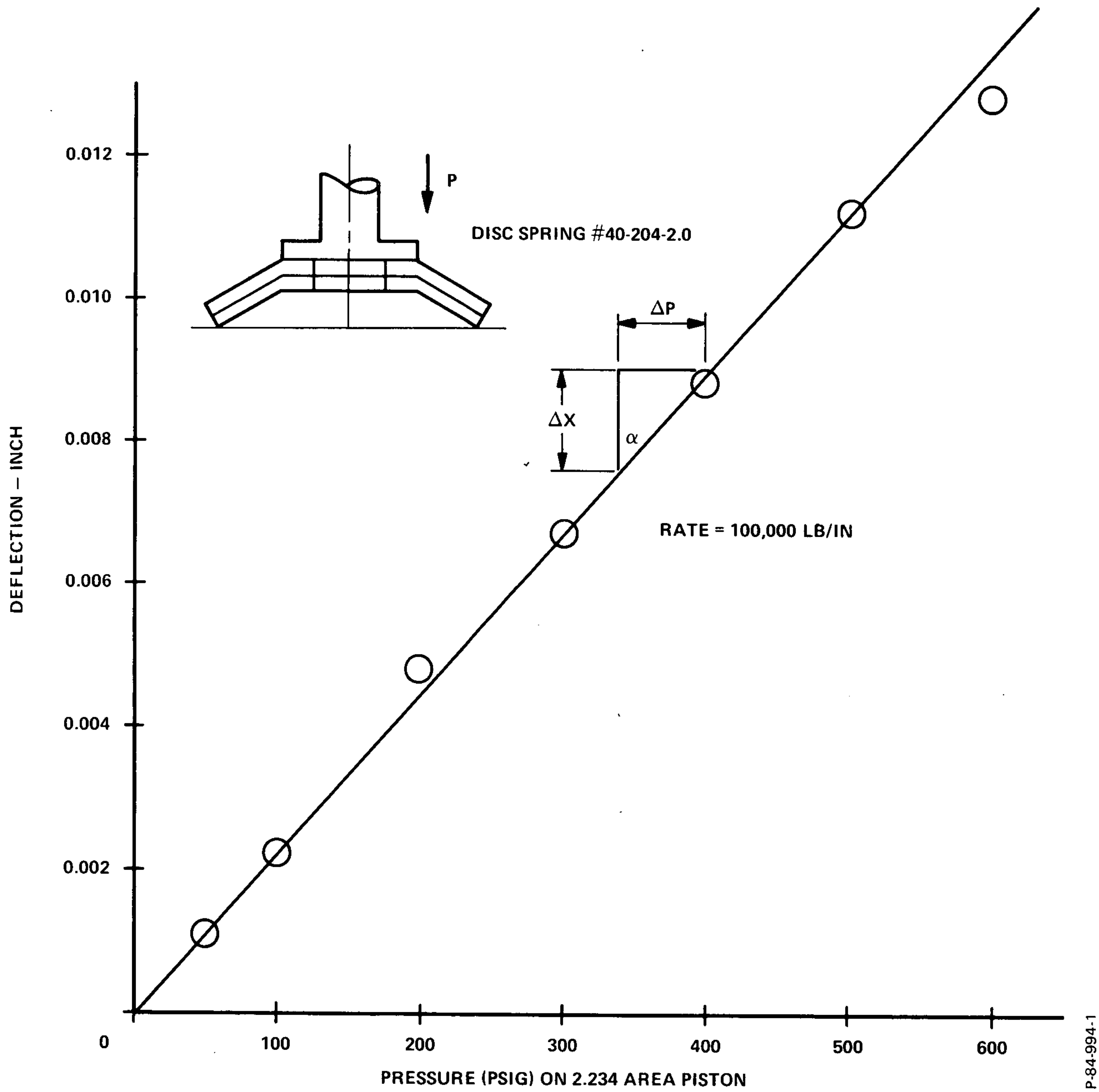

Figure A-3 - Schnorr Disc Spring Deflection Curve

$$= 364 + 1290$$

$$w_e = 1654 \text{ pounds}$$

$$L_{10} = \frac{50,000}{N}\left(\frac{C_B}{R_E}\right)^3 = \left(\frac{50,000}{1}\right)\left(\frac{6100}{1654}\right)^3 = 50,000\ (3.69)^3 = 50,000\ (50.4)$$

$$L_{10} = 2.50 \cdot 10^6 \text{ hours}$$

<u>Inboard Bearing 2MM211W1 - Given Basic Dynamic Capacity C_B</u>

$$C_B = 9250 \text{ pounds at } 33\text{-}1/3 \text{ rpm}$$

$$w_e = X_2\ V\ R + Y_2\ T$$

$$= 364 + 1390$$

$$w_e = 1754$$

$$L_{10} = \left(\frac{50,000}{1}\right)\left(\frac{9250}{1754}\right)^3 = 50,000\ (146)$$

$$L_{10} = 7.3 \cdot 10^6 \text{ hours}$$

A.7 FLIGHTWEIGHT ACTUATOR MODEL EH-818-U2

The Bearing Load Analyses for the flightweight actuator Model EH-818-U2 is identical to the breadboard analysis and as such will not be repeated. In the flightweight design, the magnitude of the allowable overhung load must be reduced from 3440 pounds to 2828 pounds to avoid static brinelling of the inboard bearing. At NASA's request, this bearing was resized to be identical to the Series 9112 outboard bearing. This trade-off in overhung load capacity resulted in simplifying the assembly, reducing the shaft seal size and overall actuator weight and volume. Although the bearing axial preload may now be reduced from 1130 pounds to 600 pounds, it is suggested that the 1130-pound preload remain. This additional preload will insure assembly stiffness.

APPENDIX B

STRUCTURAL STIFFNESS ANALYSIS

B.1 FLIGHTWEIGHT ACTUATOR Model EH-818-U2

The following analyses format was used to calculate the stresses at 2400 in-lb stall torque, angular windup at 1200 in-lb output torque, and deflections (as required) of each load carrying member for Actuator Model EH-818-U2.

1. Sketch and dimension each detail.
2. Determine peak loads transmitted through member being analyzed.
3. Calculate section modulus I and polar inertia J.
4. Calculate torsional stress and angular windup.

Given torque at 2400 in-lbs (stall) calculate the output gear load F_o transmitted. (Ref. Figure B-1.)

$$F_o \cos 20° \, r_o = 2400 \text{ in-lbs}$$

where

$$r_o = 1.399$$

then

$$F_o = \frac{2400}{1.399 \cos 20°} = \frac{2400}{1.32} = 1820 \text{ lbs}$$

$$F_o = 1820 \text{ lbs}$$

and the ground reaction gear load F_G is calculated by the ratio of ground to output gear pitch radii.

$$F_G = \frac{r_g}{r_o} F_o$$

$$= \frac{2.797}{2.906} (1820)$$

$$\underline{F_G = 1750 \text{ lbs}}$$

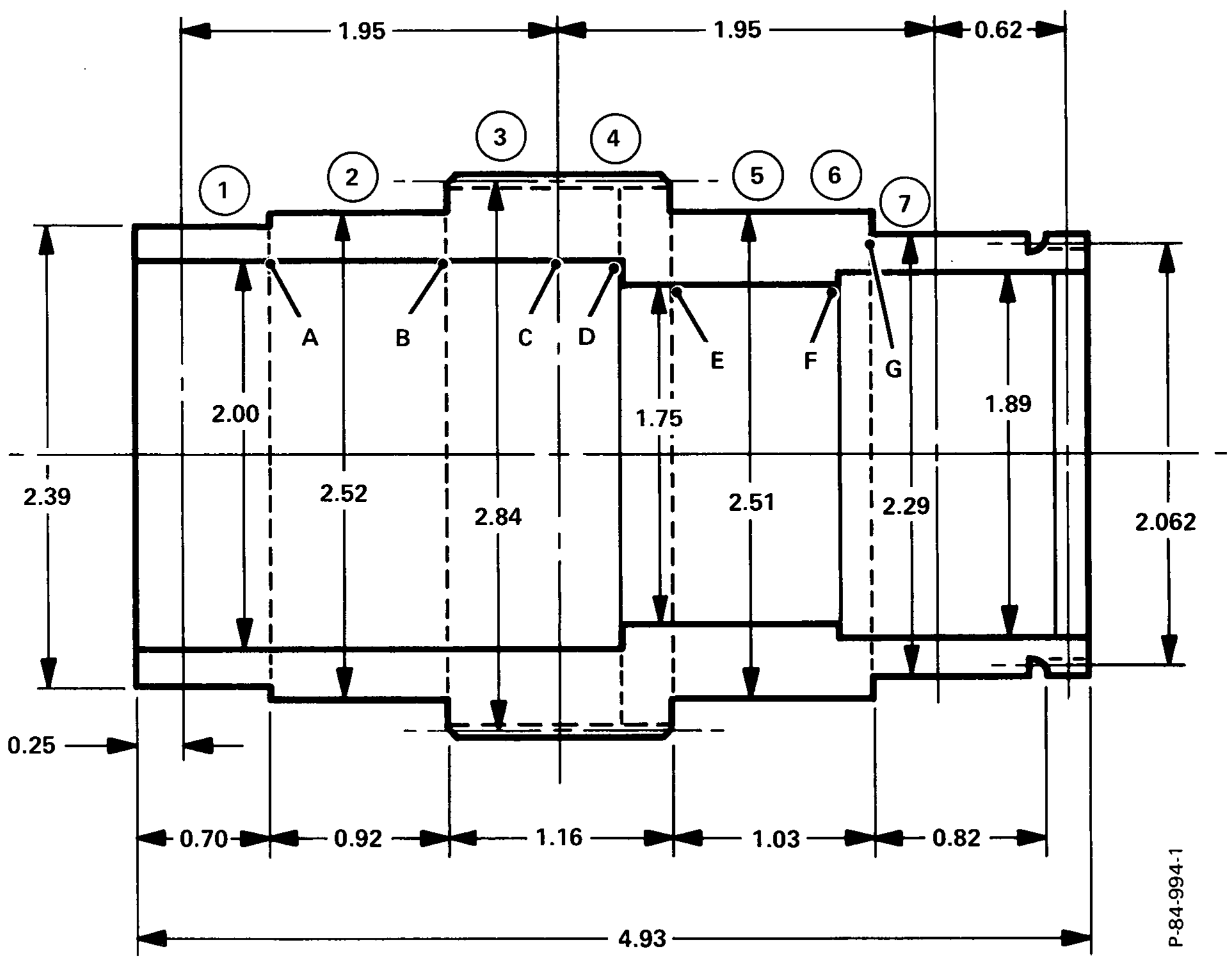

Figure B-1 - Actuator Output Gear (Flightweight)

As the actuator design contains a split ground gear mesh, each side transmits:

Split design F_G/side = 875 lbs at 2400 in-lb stall

Calculate output gear torsional stress at 2400 in-lbs (ref. Figure B-1). Given

$$S = \frac{T(c)}{J} \qquad \text{(B-1)}$$

Calculate J Sections (1) through (5) in Figure B-1

$$J_1 = \frac{\pi}{32}(2.39^4 - 2.0^4) = 0.098\ (32.8 - 16.0) = 1.65$$

$$J_2 = \frac{\pi}{32}(2.52^4 - 2.0^4) = 0.098\ (41.2 - 16.0) = 2.47$$

$$J_3 = \frac{\pi}{32}(2.84^4 - 2.0^4) = 0.098\ (65 - 16) = 4.8$$

$$J_4 = \frac{\pi}{32}(2.84^4 - 2.75^4) = 0.098\ (65 - 9.4) = 5.44$$

$$J_5 = \frac{\pi}{32}(2.51^4 - 1.75^4) = 0.098\ (39.8 - 9.4) = 2.98$$

$$J_6 = \frac{\pi}{32}(2.51^4 - 1.45^4) = 0.098\ (39.8 - 4.4) = 3.47$$

$$J_7 = \frac{\pi}{32}(2.29^4 - 1.89^4) = 0.098\ (32.8 - 12.8) = 0.098\ (20) = 1.96$$

Solving (B-1) for torsional stress S

$$S = \frac{T \cdot c}{J}$$

and at stall T = 2400 in-lbs

$$S_1 = 0\left(\frac{C_1}{J_1}\right) = 0\left(\frac{1.195}{1.65}\right) = 0$$

$$S_2 = 0\left(\frac{C_2}{J_2}\right) = 0\left(\frac{1.26}{2.47}\right) = 0$$

Sections (1) and (2) do not transmit torque, therefore T = 0

$$S_3 = 2400\left(\frac{C_3}{J_3}\right) = 2400\left(\frac{1.42}{4.8}\right) = 2400\ (0.296) = 710$$

$$S_4 = 2400\left(\frac{C_4}{J_4}\right) = 2400\left(\frac{1.42}{5.44}\right) = 2400\ (0.261) = 625$$

$$S_5 = 2400\left(\frac{C_5}{J_5}\right) = 2400\left(\frac{1.26}{2.98}\right) = 2400\ (0.422) = 1015$$

$$S_6 = 2400\left(\frac{C_6}{J_6}\right) = 2400\left(\frac{1.26}{3.47}\right) = 2400\ (0.363) = 871$$

$$S_7 = 2400\left(\frac{C_7}{J_7}\right) = 2400\left(\frac{1.06}{1.96}\right) = 2400\ (0.542) = 1300$$

Analyses to determine shaft windup at 1200 in-lb load torque (NASA Spec.). Given

$$\theta = \frac{T\ L}{G\ J} \qquad \text{(B-2)}$$

where

θ = windup - radians

T = 1200 in-lbs (specified load torque)

G = $10.35 \cdot 10^6$ torsional modulus for 18 Ni 350 steel

Solving

$$\theta = \frac{T}{G} \sum_{n=3}^{n=7} \frac{L_n}{J_n} \qquad \text{(B-3)}$$

$$\theta = \frac{1200}{10.35} \cdot 10^{-6} \sum_{n=3}^{n=7} \frac{L_n}{J_n}$$

$$\theta = 116 \cdot 10^{-6} \sum_{n=3}^{n=7} \frac{L_n}{J_n} \qquad \text{(B-4)}$$

And from Figure B-1

$L_1 = 0.7\ J_1 = 1.65$ $\quad$ $L_4 = 0.25\ J_4 = 5.44$

$L_2 = 0.92\ J_2 = 2.47$ $\quad$ $L_5 = 0.85\ J_5 = 2.98$

$L_3 = 1.16\ J_3 = 4.8$ $\quad$ $L_6 = 0.2\ J_6 = 3.47$

Using (B-4), calculate windup from mid-position of L_3 to end position of L_7.

$$\theta = 116 \cdot 10^{-6} \sum_{n=3}^{n=7} \frac{L_n}{J_n}$$

$$= 116 \cdot 10^{-6} \left[\frac{0.58}{4.8} + \frac{0.25}{5.44} + \frac{0.85}{2.98} + \frac{0.2}{3.47} + \frac{1.03}{1.95}\right]$$

$$= 116 \cdot 10^{-6} \,[0.121 + 0.046 + 0.285 + 0.0577 + 0.526]$$

$$= 116 \cdot 10^{-6} \,[1.036]$$

$$\theta = 120 \cdot 10^{-6} \text{ radians}$$

Convert θ radians to θ arc-minutes

$$\left(\theta_{radians}\right)\left(\frac{rev}{2\pi\ rad}\right)\left(\frac{360\ degrees}{rev}\right)\left(\frac{60\ minute}{degree}\right) = \theta_{minute}$$

$$\theta_{minutes} = \frac{180}{\pi}(60)\ \theta_{radians}$$

$$\theta_{minute} = 3440\ \theta_{radians} \qquad \text{(B-5)}$$

therefore

$$\hat{\theta}_{minutes} = (3.44 \cdot 10^3)(120 \cdot 10^{-6})$$

$$= 4.13 \cdot 10^{-3}$$

$$\hat{\theta}_{minutes} = 0.41 \text{ arc minutes}$$

Determine Output Gear Shaft Deflection at 2400 in-lb Load Torque. Using Figures B-1, B-2, and B-3 calculate $\frac{M}{EI}$ for each shaft segment.

Where

M = 1775 in-lb at max. load position

Note subscript ($\pm$) implies section inertia taken to left (-) or to right (+) of section being analyzed.

$$\frac{M}{EI}_{A-} = \frac{410}{(0.825)\ 30 \times 10^6} = 16.6 \times 10^{-6}$$

$$\frac{M}{EI}_{A+} = \frac{410}{(1.235)\ 30 \times 10^6} = 11.1 \times 10^{-6}$$

$$\frac{M}{EI}_{B-} = \frac{1250}{(1.235)\ 30 \times 10^6} = 33.8 \times 10^{-6}$$

$$\frac{M}{EI}_{B+} = \frac{1250}{(2.4)\ 30 \times 10^6} = 17.35 \times 10^{-6}$$

$$\frac{M}{EI}_{C} = \frac{1775}{(2.4)\ 30 \times 10^6} = 24.7 \times 10^{-6}$$

$$\frac{M}{EI}_{d-} = \frac{1540}{72 \times 10^6} = 21.4 \times 10^{-6}$$

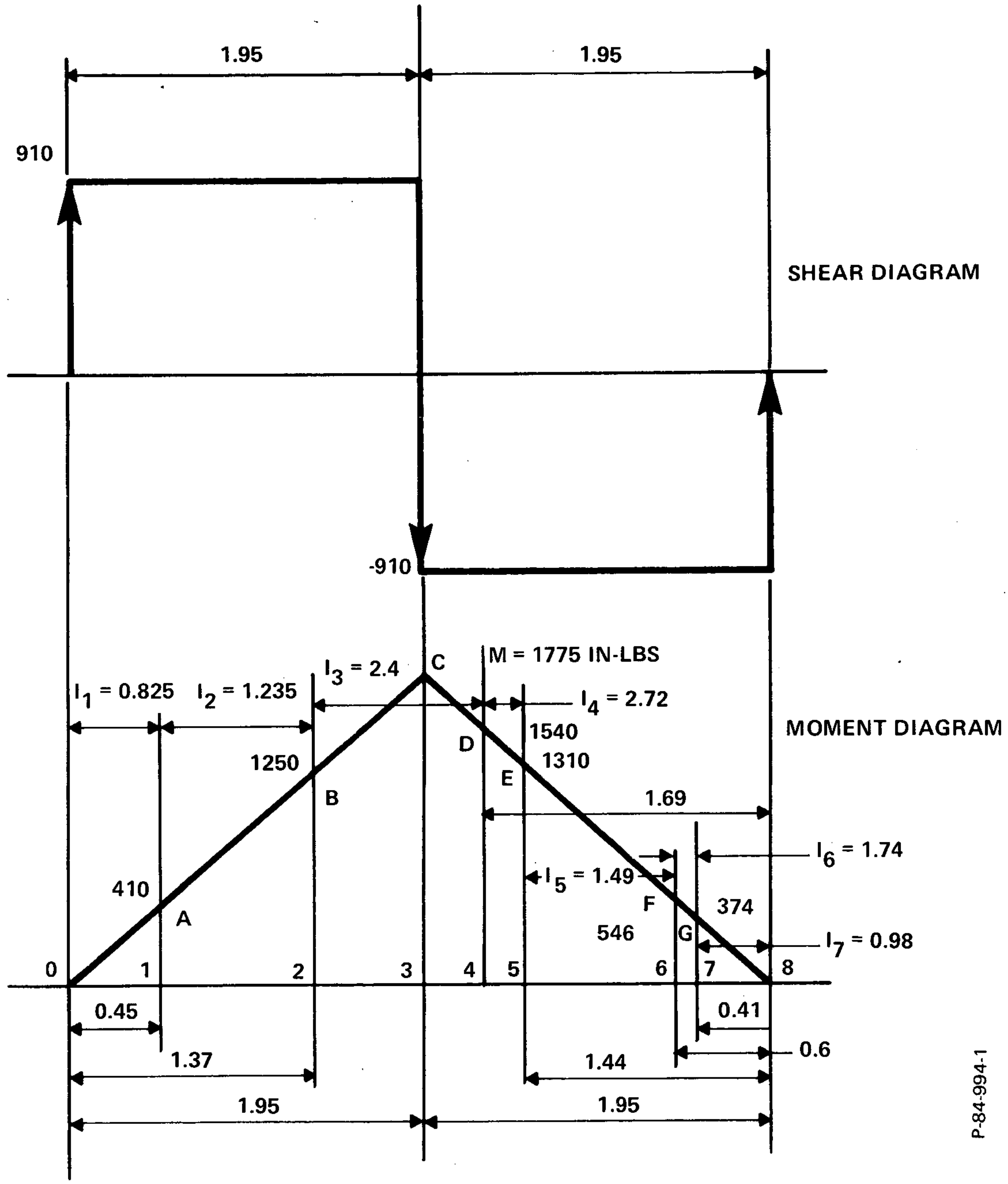

Figure B-2 - Output Gear Shear and Moment Diagrams

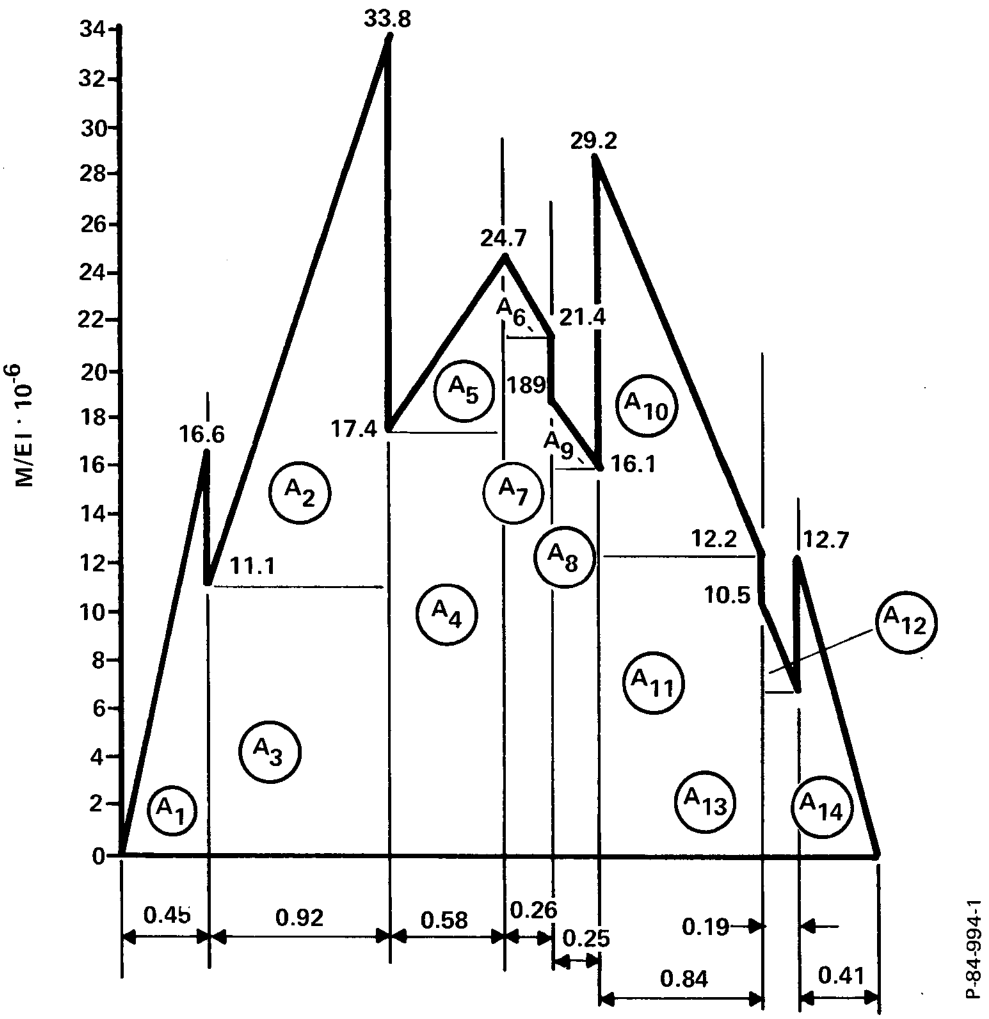

Figure B-3 - Output Gear M/EI Diagram

$$\frac{M}{EI}_{d+} = \frac{1540}{(2.72)\ (30 \times 10^6)} = 18.9 \times 10^{-6}$$

$$\frac{M}{EI}_{e-} = \frac{1310}{81.6 \times 10^6} = 16.1 \times 10^{-6}$$

$$\frac{M}{EI}_{e+} = \frac{1310}{(1.49)\ 30 \times 10^6} = 29.2 \times 10^{-6}$$

$$\frac{M}{EI}_{f-} = \frac{546}{44.8 \times 10^6} = 12.2 \times 10^{-6}$$

$$\frac{M}{EI}_{f+} = \frac{546}{(1.74)\ 30 \times 10^6} = 10.45 \times 10^{-6}$$

$$\frac{M}{EI}_{G-} = \frac{374}{52.2 \times 10^6} = 7.15 \times 10^{-6}$$

$$\frac{M}{EI}_{G+} = \frac{374}{30 \times 10^6\ (0.98)} = 12.7 \times 10^{-6}$$

Calculate deflection Y_2 from $A\overline{X}$ about Y_2. The following segment areas and $\overline{X}$ distances are taken from Figure B-3

$$A_1 = \frac{1}{2}\ (0.45)\ (16.6) = 3.74 \times 10^{-6}$$

$$\overline{r}_1 = \frac{1}{3}\ (0.45) + (3.90 - 0.45)$$

$$\overline{r}_1 = 0.15 + 3.45 = 3.6''$$

$$\overline{r}_1 = 3.6$$

$$A_2 = \frac{1}{2} (0.92) \ (22.7) = 10.45 \times 10^{-6}$$

$$\bar{r}_2 = \frac{1}{3} (0.92) + (1.95 + 0.58)$$

$$A_3 = (0.92) \ (11.1 \times 10^{-6}) = 10.22 \times 10^{-6}$$

$$\bar{r}_3 = 0.46 + 2.53$$

$$\bar{r}_3 = 2.99$$

$$A_4 = (0.58) \ (17.4 \times 10^{-6}) = 10.1 \times 10^{-6}$$

$$\bar{r}_4 = 1.95 + 0.29$$

$$\bar{r}_4 = 2.14$$

$$A = \frac{1}{2} (0.58) \ (7.3 \times 10^{-6}) = 2.12 \times 10^{-6}$$

$$\bar{r}_5 = \frac{1}{3} (0.58) + 1.95$$

$$\bar{r}_5 = 2.14$$

$$A_6 = \frac{1}{2} (0.26) \ (3.3 \times 10^{-6})$$

$$A_6 = 0.429 \times 10^{-6}$$

$$\bar{r}_6 = 1.95 - \frac{1}{3} (0.26)$$

$$= 1.95 - 0.086$$

$$\bar{r}_6 = 1.86$$

b

$$A_7 = (0.26)\ (21.4 \times 10^{-6}) = 5.56 \times 10^{-6}$$

$$\bar{r}_7 = 1.95 - 0.13$$

$$\bar{r}_7 = 1.82$$

$$A_8 = 0.25\ (16.1) \times 10^{-6} = 4.025 \times 10^{-6}$$

$$\bar{r}_8 = \frac{0.25}{2} + (0.84 + 0.19 + 0.41)$$

$$= 0.125 + 1.44$$

$$\bar{r}_8 = 1.565$$

$$A_9 = \frac{1}{2}\ (0.25)\ (2.8 \times 10^{-6})$$

$$A_9 = 0.35 \times 10^{-6}$$

$$\bar{r}_9 = \frac{2}{3}\ (0.25) + 1.44$$

$$= 0.167 + 1.44$$

$$\bar{r}_9 = 1.607$$

$$A_{10} = \frac{1}{2}\ (0.84)\ (17 \times 10^{-6})$$

$$A_{10} = 7.14 \times 10^{-6}$$

$$\bar{r}_{10} = \frac{2}{3}\ (0.84) + (0.17 + 0.41)$$

$$= 0.562 + 0.60$$

$$\bar{r}_{10} = 1.162$$

$$A_{11} = 0.84\ (12.2 \times 10^{-6})$$

$$A_{11} = 10.25 \times 10^{-6}$$

$$\bar{r}_{11} = \frac{0.84}{2} + 0.60$$

$$= 0.42 + 0.6$$

$$\bar{r}_{11} = 1.02$$

$$A_{12} = \frac{1}{2}\ (0.19)\ (3.3 \times 10^{-6})$$

$$A_{12} = 0.314 \times 10^{-6}$$

$$\bar{r}_{12} = \frac{2}{3}\ (0.19) + 0.41$$

$$= 0.127 + 0.41$$

$$\bar{r}_{12} = 0.537$$

$$A_{13} = 0.19\ (7.2 \times 10^{-6}) = 1.37 \times 10^{-6}$$

$$\bar{r}_{13} = 0.095 + 0.41$$

$$\bar{r}_{13} = 0.505$$

$$A_{14} = \frac{1}{2}\ (0.41)\ (12.7 \times 10^{-6}) = 2.6 \times 10^{-6}$$

$$\bar{r}_{14} = 0.205$$

Summing $A\bar{r}$

$$y_2 = A_1 \bar{r}_1 + A_2 \bar{r}_2 + \dots A_n r_n \qquad (B\text{-}6)$$

$$A_1 \bar{r}_1 = (3.6 \times 10^{-6})(3.74) = 13.5 \times 10^{-6}$$

$$A_2 \bar{r}_2 = (10.45 \times 10^{-6})\ (2.837) = 29.6 \times 10^{-6}$$

$$A_3 \bar{r}_3 = (10.22 \times 10^{-6})\ (2.99) = 30.5 \times 10^{-6}$$

$$A_4 \bar{r}_4 = (10.1 \times 10^{-6})\ (2.14) = 21.6 \times 10^{-6}$$

$$A_5 \bar{r}_5 = (2.12 \times 10^{-6})\ (2.14) = 4.54 \times 10^{-6}$$

$$A_6 \bar{r}_6 = (0.429 \times 10^{-6})\ (1.86) = 0.8 \times 10^{-6}$$

$$A_7 \bar{r}_7 = (5.56 \times 10^{-6})\ (1.82) = 10.1 \times 10^{-6}$$

$$A_8 \bar{r}_8 = (4.025 \times 10^{-6})\ (1.565) = 6.29 \times 10^{-6}$$

$$A_9 \bar{r}_9 = (0.35 \times 10^{-6})\ (1.607) = 0.562 \times 10^{-6}$$

$$A_{10} \bar{r}_{10} = (7.14 \times 10^{-6})\ (1.162) = 8.3 \times 10^{-6}$$

$$A_{11} \bar{r}_{11} = (10.25 \times 10^{-6})\ (1.02) = 10.5 \times 10^{-6}$$

$$A_{12} \bar{r}_{12} = (0.314 \times 10^{-6})\ (0.537) = 0.168 \times 10^{-6}$$

$$A_{13} \bar{r}_{13} = (1.37 \times 10^{-6})\ (0.505) = 0.69 \times 10^{-6}$$

$$A_{14} \bar{r}_{14} = (2.6 \times 10^{-6})\ (0.205) = 0.52 \times 10^{-6}$$

then

$$y_2 = 137.7 \times 10^{-6} \text{ inch}$$

Calculate deflection Y_1 from $A\overline{X}$ about Y_2

$A_1 = 3.74 \times 10^{-6}$ $\quad \bar{r}_1 = 2/3\ (0.45) = 0.30$

$\bar{r}_1 = 0.3$

$A_2 = 10.45 \times 10^{-6}$ $\quad \bar{r}_2 = 0.45 + 2/3\ (0.92)$

$= 0.45 + 0.616$

$\bar{r}_2 = 1.066$

$A_3 = 10.22 \times 10^{-6}$ $\quad \bar{r}_3 = 0.45 + 0.46$

$\bar{r}_3 = 0.91$

$A_4 = 10.1 \times 10^{-6}$ $\quad \bar{r}_4 = 1.37 + 0.29$

$\bar{r}_4 = 1.66$

$A_5 = 2.12 \times 10^{-6}$ $\quad \bar{r}_5 = 1.37 + 2/3\ (0.58)$

$= 1.37 + 0.39$

$\bar{r}_5 = 1.76$

$A_6 = 0.429 \times 10^{-6}$ $\quad \bar{r}_6 = 1.95 + 1/3\ (0.26)$

$= 1.95 + 0.086$

$\bar{r}_6 = 2.036$

$A_7 = 5.56 \times 10^{-6}$ $\quad \bar{r}_7 = 1.95 + 0.13$

$\bar{r}_7 = 2.08$

$A_8 = 4.025 \times 10^{-6}$ $\bar{r}_8 = 1.95 + 0.26 + 1/2\ (0.25)$

$= 2.21 + 0.125$

$\bar{r}_8 = 2.335$

$A_9 = 0.35 \times 10^{-6}$ $\bar{r}_9 = 2.21 + 0.083 = 2.293$

$A_{10} = 7.14 \times 10^{-6}$ $\bar{r}_{10} = 1.95 + 0.51 + 1/3\ (0.84)$

$= 2.46 + 0.28$

$\bar{r}_{10} = 2.74$

$A_{11} = 10.25 \times 10^{-6}$ $\bar{r}_{11} = 2.46 + 0.42$

$\bar{r}_{11} = 2.88$

$A_{12} = 0.314 \times 10^{-6}$ $\bar{r}_{12} = 2.46 + 0.84 + 1/3\ (0.19)$

$= 3.30 + 0.06$

$\bar{r}_{12} = 3.36$

$A_{13} = 1.37 \times 10^{-6}$ $\bar{r}_{13} = 3.30 + 0.095$

$\bar{r}_{13} = 3.395$

$A_{14} = 2.6 \times 10^{-6}$ $\bar{r}_{14} = 3.90 - 2/3\ (0.41)$

$= 3.90 - 0.275$

$\bar{r}_{14} = 3.62$

$A_1\ r_1 = (3.74 \times 10^{-6})\ (0.3) = 1.12 \times 10^{-6}$

$A_2\ r_2 = (10.45 \times 10^{-6})\ (1.066) = 11.12 \times 10^{-6}$

$A_3\ r_3 = (10.22 \times 10^{-6})\ (0.91) = 9.3 \times 10^{-6}$

$$A_4\, r_4 = (10.1 \times 10^{-6})\ (1.66) = 16.8 \times 10^{-6}$$

$$A_5\, r_5 = (2.12 \times 10^{-6})\ (1.76) = 3.73 \times 10^{-6}$$

$$A_6\, r_6 = (0.429 \times 10^{-6})\ (2.036) = 0.874 \times 10^{-6}$$

$$A_7\, r_7 = (5.56 \times 10^{-6})\ (2.08) = 11.55 \times 10^{-6}$$

$$A_8\, r_8 = (4.025 \times 10^{-6})\ (2.335) = 9.37 \times 10^{-6}$$

$$A_9\, \bar{r}_9 = (0.35 \times 10^{-6})\ (2.293) = 0.804 \times 10^{-6}$$

$$A_{10}\, \bar{r}_{10} = (7.14 \times 10^{-6})\ (2.74) = 19.6 \times 10^{-6}$$

$$A_{11}\, \bar{r}_{11} = (10.25 \times 10^{-6})\ (2.88) = 29.6 \times 10^{-6}$$

$$A_{12}\, \bar{r}_{12} = (0.314 \times 10^{-6})\ (3.36) = 1.055 \times 10^{-6}$$

$$A_{13}\, \bar{r}_{13} = (1.37 \times 10^{-6})\ (3.395) = 4.65 \times 10^{-6}$$

$$A_{14}\, \bar{r}_{14} = (2.6 \times 10^{-6})\ (3.62) = 9.42 \times 10^{-6}$$

Solve (B-6) for Y_1, given $Y_1 = A_1\, \bar{r}_1 + A_2\, \bar{r}_2 + \ldots A_n\, r_n$, then

$$Y_1 = 128.79 \times 10^{-6}$$

The deflection at the applied gear load is determined from

$$Y = \left(\frac{Y_2}{2} - \frac{Y_1}{2}\right) \tag{B-7}$$

Solving

$$Y = \frac{(137.7 - 128.8)}{2} \times 10^{-6}$$

$$Y = 4.5 \times 10^{-6} \text{ in. at } T_L = 2400 \text{ in-lbs}$$

A graphical approximation of the shaft deflection is presented in Figure B-4.

Analysis to Determine Torsional Windup and Stresses - Rotor Ring Gear

The windup from centerline output gear to centerline of ground gear may be found as follows. From Figure B-5;

Polar inertia - J

$$J = \frac{\pi}{32} (3.325^4 - 3.05^4)$$

$$= 0.0984 (121 - 86)$$

$$= 0.0984 (35)$$

$$J = 3.45 \text{ in}^4$$

and solving for θ

$$\theta = \frac{T\ L}{G\ J} \text{ radians}$$

where

T = 600 in-lb (split path)

L = 1.12 inch

$G = 10.35 \times 10^6$ lb/in^2 (maraging steel)

$J = 3.45$ in^4

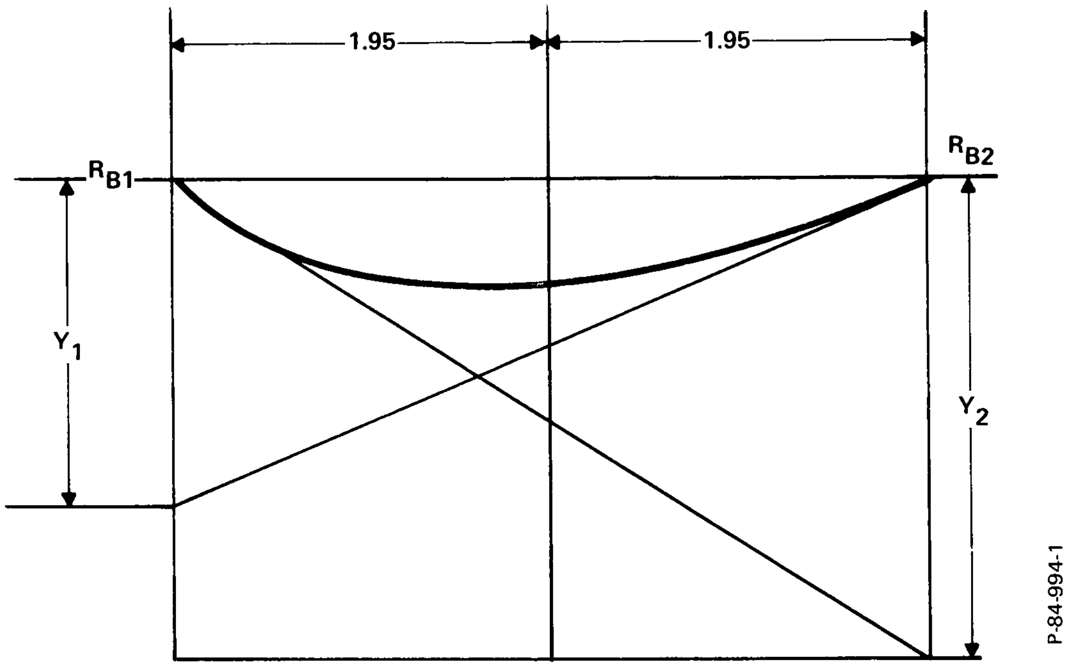

Figure B-4 - Area Moment Method Diagram to Determine Output Gear Deflection

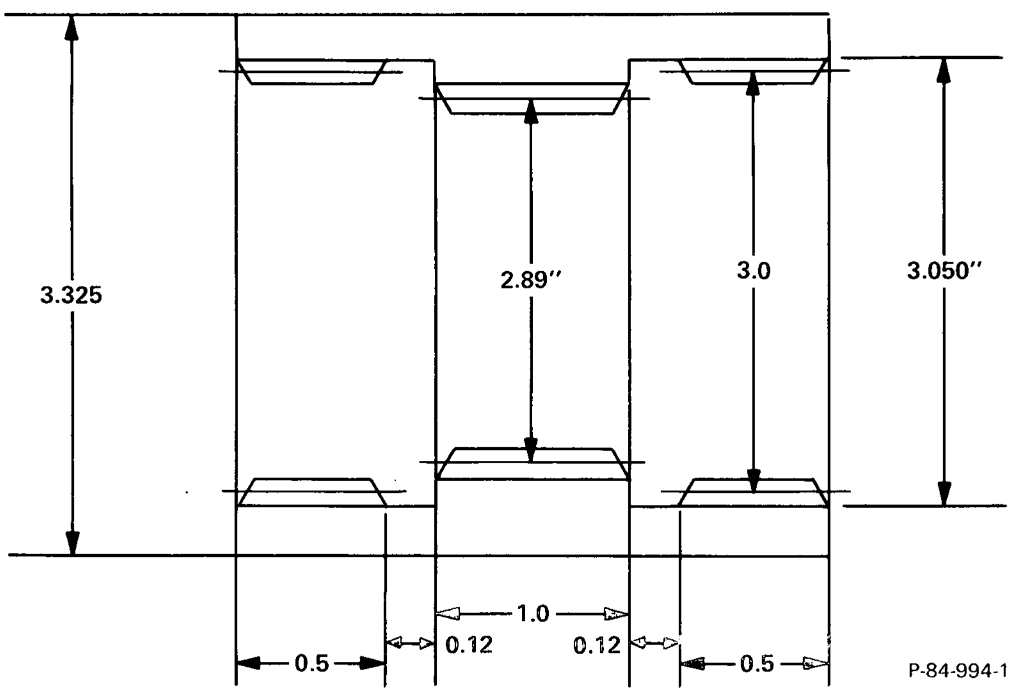

Figure B-5 - Rotor Ring Gear (Flightweight) then

$$\theta = \frac{(600)\ (1.12)}{(10.35)\ (3.45)}$$

$$\theta = 18.82 \times 10^{-6} \text{ radians}$$

or

$$\hat{\theta} = 0.065 \text{ arc min (each way)}$$

Calculate rotor torsional stress, τ, at 1200 in-lb load torque.

$$\tau = \frac{T(c)}{J}$$

$$\tau = \frac{(600)\ (1.662)}{3.45}$$

$$\tau = 290 \text{ psi}$$

and at 2400 in-lb stall torque

$$\tau = 580 \text{ psi}$$

<u>Analyses to Determine Torsional Windup and Stress of the Inboard and the Outboard Ground Gear - Figure B-6</u>

The inboard and outboard ground gear torsional windup is calculated from the ground gear mesh centerline to the stator housing flange interface. The windup is calculated at 1200 in-lb load torque.

<u>Solve J (from Figure B-6)</u>

$$J_1 = \frac{\pi}{64}\ (2.861^4 - 2.661^4) = \frac{\pi}{64}\ (67 - 50) = \frac{\pi}{64}\ (17) = 0.834$$

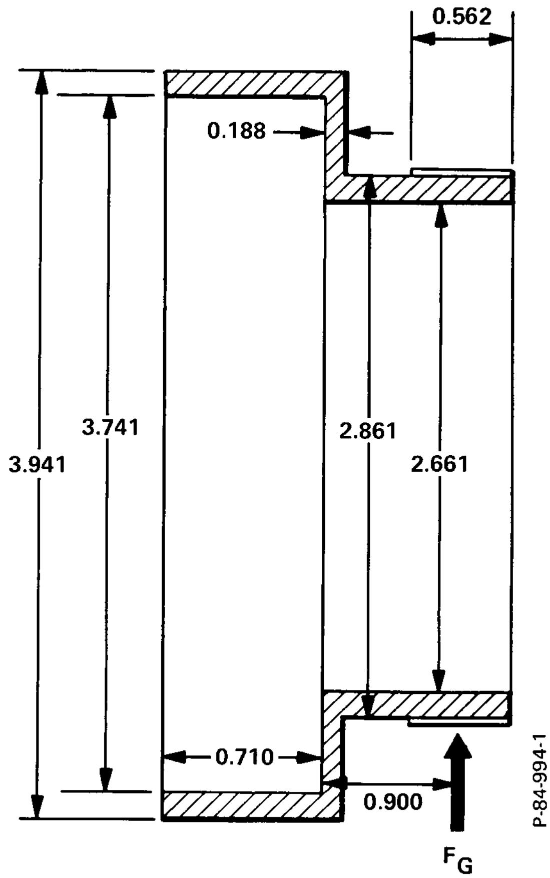

Figure B-6 - Ground Gear (Flightweight)

$$J_2 = \frac{\pi}{64}(3.941^4 - 2.661^4) = \frac{\pi}{64}(240 - 50) = \frac{\pi}{64}(190) = 9.34$$

<u>For Steel</u>

$G = 10.5 \times 10^6$, $T = 600$ (split path) $\ell_1 = 0.75$ $\ell_2 = 0.188$

For the section shown in Figure B-6

$$\theta = \frac{T}{G}\left(\frac{L_1}{J_1} + \frac{L_2}{J_2}\right) \text{ radians}$$

$$\theta = \frac{600}{10 \times 10^6}\left(\frac{0.75}{0.834} + \frac{0.188}{9.34}\right)$$

$$\theta = 60 \times 10^{-6}\ (0.90 + 0.020)$$

$$\theta = 60 \times 10^{-6}\ (0.92)$$

$$\theta = 55 \times 10^{-6} \text{ radians}$$

and

$$\hat{\theta} = 0.189 \text{ arc min.}$$

Calculate Torsional Stress, τ, at 1200 in-lb Load Torque

$$\tau = \frac{T(c)}{J_1}$$

$$\tau = \frac{(600)\ (1.43)}{0.834} = 1030 \text{ psi}$$

and

$$= 2060 \text{ psi at } 2400 \text{ in-lb stall}$$

Calculate shear stress, S_s at 1200 in-lbs given F_o = torque force

$$S_s = \frac{F_o}{A_1} = \frac{438}{0.785}\ (2.861^2 - 2.661^2)$$

$$S_s = \frac{438}{0.872}$$

and

$$S_s = 1004 \text{ psi at 2400 in-lb stall}$$

Calculate bending stress at 1200 in-lbs

$$S_b = \frac{M(c)}{I} = \frac{(550)\ (1.43)}{J/2} = \frac{(1100)\ (1.43)}{0.834}$$

$$S_b = 1890 \text{ psi}$$

and

$$S_b = 3780 \text{ psi at 2400 in-lb stall}$$

Calculate ground gear mesh radial deflection induced by torque reaction load.

<u>At 1200 in-lb Load Torque</u>

Given

$$y = \frac{w\ \ell^3}{3EI}$$

$$y = \frac{(438)\ (1.081)^3}{3(30 \times 10^6)\ (0.417)} = \frac{1.26\ (438)}{90 \times 10^6\ (0.417)}$$

$$y = \frac{550}{37.5 \times 10^6} = 14.7 \times 10^{-6}$$

$$y = 14.7 \times 10^{-6} \text{ inches}$$

and

$$y = 29.4 \times 10^{-6} \text{ inches at 2400 in-lb torque}$$

Analyses to Determine Torsional Windup and Stresses in the Stator Housing

Given stator housing is a thin wall tubular structure with the following dimensions:

OD = 7.45 inches

ID = 7.25 inches

Wall = 0.10 inch

Length = 2.8 inches

and solving J

$$J = \pi \left(\frac{r_o^4 - r_i^4}{2} \right)$$

$$J = \frac{\pi}{2} [(3.72)^4 - (3.625)^4]$$

$$J = 31.191 \text{ in}^4$$

and the torsional windup θ at 1200 in-lb load torque is

$$\theta = \frac{T \ell}{G J}$$

where

$T = 600$ (split path)

$\ell = 2.8$ inches

$G = 12 \times 10^6$

$$\theta = \frac{(1200)\ (2.8)}{(12 \times 10^6)\ (31.19)}$$

$$\theta = 4.49 \times 10^{-6} \text{ radians}$$

or

$$\hat{\theta} = 0.016 \text{ arc minutes}$$

Solve for torsional stress, τ, at 1200 in-lbs, given

$$\tau = \frac{T(c)}{J}$$

where

$T = 600$ in-lb

$c = 3.72$ in.

$J = 39.19$ in^4

$$\tau = \frac{(600)\ (3.72)}{39.19}$$

$\tau = 57$ psi

and at 2400 in-lbs stall torque

$$\tau = 114 \text{ psi}$$

WINDUP SUMMARY

The windup was calculated at 1200 in-lb load torque

I. Loop to ground via outboard ground gear

Output gear	0.410 arc min
Motor ring gear	0.065 arc min
Outboard ground gear	0.189 arc min
Stator housing	0.016 arc min
Total	0.680 arc min

II. Loop to ground via inboard ground gear

Output gear	0.410 arc min
Rotor ring gear	0.065 arc min
Inboard ground gear	0.189 arc min
Total	0.664 arc min

The structural rigidity is determined by the ratio of load torque to windup and is

$$k_s = \frac{T_L}{\Sigma\theta} = \frac{1200}{0.680}$$

$$k_s = 1770 \text{ in-lb/arc min.}$$

or

$$k_s = 6.00 \times 10^6 \text{ in-lb/radian}$$

B.2 BREADBOARD ACTUATOR MODEL EH-818-U1

These analyses were conducted to determine the breadboard actuator windup and stresses at 1200 in-lb output load torque

Output Gear Analysis Reference Figure B-7

Given

$$\theta = \frac{T\ L}{G\ J}$$

where

T = load torque, in-lbs

L = section length - inch

G = torsional modulus of elasticity, psi

J = polar moment of inertia, in^4

Solving for J from Figure B-7

$$J_1 = \frac{\pi}{32}\ (2.82^4 - 0.754) = 0.098\ (63.5 - 0.32) = 6.20,\ L_1 = 0.7$$

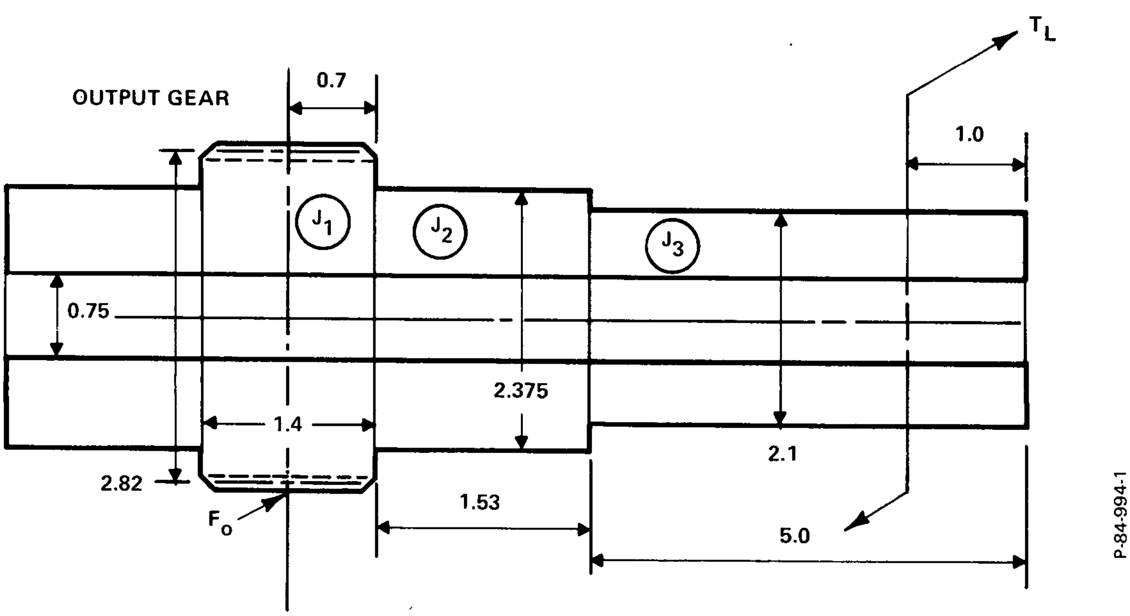

Figure B-7 - Output Gear (Breadboard)

$$J_2 = \frac{\pi}{32} (2.375^4 = 0.75^4) = 0.098 (31.0 - 0.32) = 3.01, L_2 = 1.53$$

$$J_3 = \frac{\pi}{32} (2.1^4 = 0.75^4) = 0.098 (19.5 - 0.32) = 1.782, L_3 = 4.0$$

Solve

$$\theta = \frac{T\ L}{J\ G} = \frac{T}{J}\left(\frac{L_1}{J_1} + \frac{L_2}{J_2} + \frac{L_3}{J_3} + \frac{L_4}{J_4}\right)$$

where

$G = 10.8 \times 10^6$

$G = 1200$ in-lb

Solving

$$\theta = \frac{1200}{10.8 \times 10^6}\left(\frac{0.7}{6.2} + \frac{1.53}{3.01} + \frac{4.0}{1.78}\right)$$

$$\theta = 111 \times 10^{-6}$$

$$\theta = 319 \times 10^{-6} \text{ radians}$$

and converting to arc minutes from

$$\hat{\theta}_{min} = 3440\ \theta_{rad}$$

then

$$\hat{\theta} = 1.1 \text{ arc minutes}$$

Ring Gear Windup Analysis Reference Figure B-8

Split Path load torque in direction is 600 in-lbs

Solve for J

$$J_1 = \frac{\pi}{32} (3.9^4 - 2.903^4) = 0.098 (232 - 71.5) = 15.74$$

$$J_2 = \frac{\pi}{32} (3.9^4 - 3.12^4) = 0.098 (232 - 95) = 13.44$$

$$J_3 = \frac{\pi}{32} (3.9^4 - 3.03^4) = 0.098 (232 - 85) = 14.4$$

and given

$$\theta = \frac{T\ L}{G\ J} = \frac{T}{G} \sum_{n=1}^{n=3} \left(\frac{L}{J}\right)$$

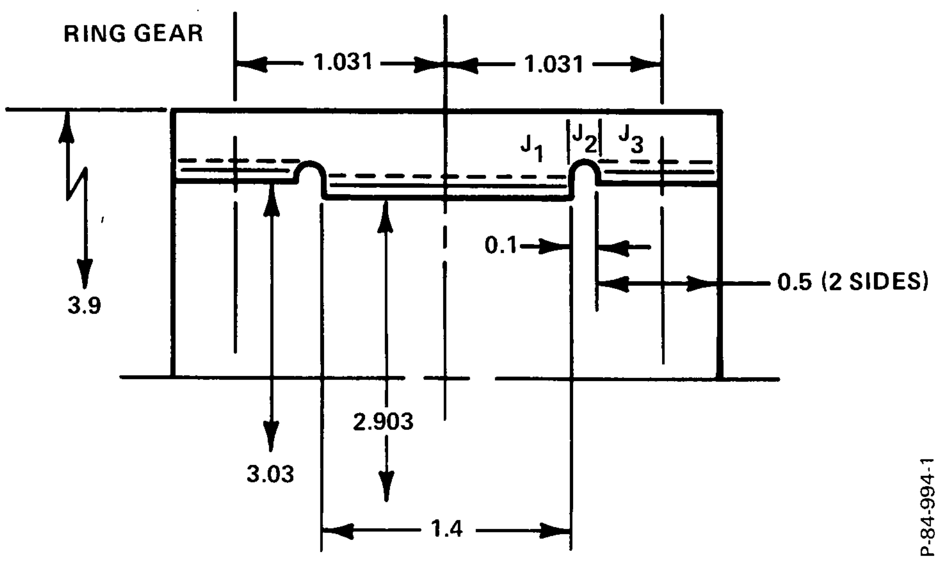

Figure B-8 - Ring Gear (Breadboard)

where

$L_1 = 0.7$

$L_2 = 0.1$

$L_3 = 0.25$

$$\theta_{rad} = \frac{600}{10.8 \times 10^6}\left(\frac{0.7}{15.74} + \frac{0.1}{13.44} + \frac{0.25}{14.4}\right)$$

$$\theta_{rad} = (55.5 \times 10^{-6})\ (0.069)$$

$$\theta_{rad} = 3.82 \times 10^{-6} \text{ radians}$$

and

$$\hat{\theta}_{min} = 0.014 \text{ arc min.}$$

<u>Outboard Ground Gear windup analyses Reference Figure B-9</u>

Solving for J

$$J_1 = \frac{\pi}{32}\ (2.936^4 - 2.47^4) = 0.098\ (74 - 37) = 3.630$$

$$J_2 = \frac{\pi}{32}\ (2.83^4 - 2.47^4) = 0.098\ (64.0 - 37) = 2.65$$

$$\theta = \frac{T}{G}\left(\frac{L_1}{J_1} + \frac{L_2}{J_2}\right) = \left(\frac{600}{10.0 \times 10^6}\right)\left(\frac{0.25}{3.63} + \frac{0.375}{2.65}\right)$$

$$\theta = 11.6 \times 10^{-6} \text{ radians}$$

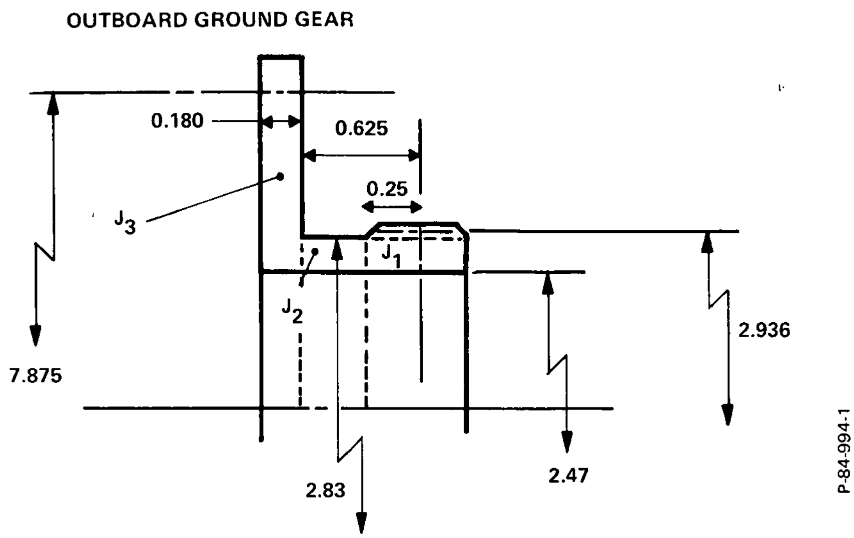

Figure B-9 - Outboard Ground Gear (Breadboard)

then

$$\hat{\theta}_{min} = 0.040 \text{ arc minutes}$$

Outboard ground gear flange section

Solve by calculating stress-strain relationship. Shear area = $\pi D \ell = \pi (2.83) (0.188) = 1.67 \text{ in}^2$ and the distributed force F.

$$F = \frac{T}{r} = \frac{600}{1.415} = 424\#$$

Solve shear stress G for F/A

$$G = \frac{424}{1.67} = 254 \text{ psi}$$

given stress-strain relationship

$$G = \frac{\sigma}{\varepsilon}$$

Solve ε

$$\varepsilon = \frac{254}{28 \times 10^6}$$

$$\varepsilon = 9.08 \times 10^{-6} \text{ in/in}$$

Find windup θ in flange

From:

$$S = r\,\theta$$

$$\theta_{rad} = \varepsilon$$

Therefore

$$\theta = 9.08 \times 10^{-6} \text{ rad}$$

and

$$\hat{\theta} = 0.031 \text{ arc min}$$

Stator Tube Windup Analyses Reference Figure B-10

Assume stator is not integral part of structure. Solve for J and θ

$$J = \frac{\pi}{32}\,(7.5^4 - 7.27^4) = 0.098\,(3150 - 2780) = 85.2$$

$$\theta = \frac{T\,L}{G\,J} = (55.5 \times 10^{-6})\,(0.0454)$$

$$\theta = 2.52 \times 10^{-6} \text{ rad}$$

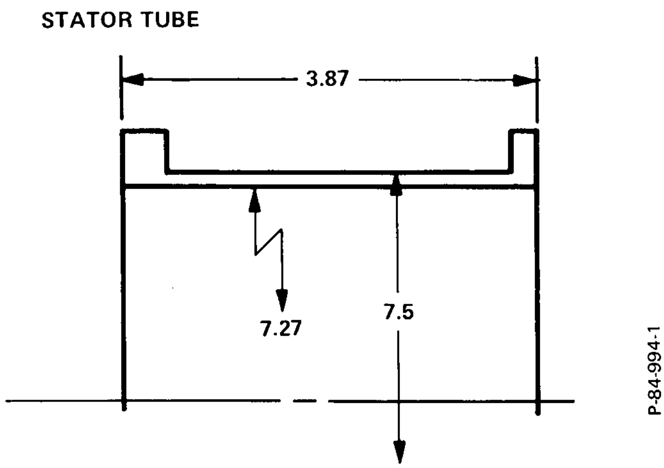

Figure B-10 - Stator Housing (Breadboard)

and

$$\hat{\theta}_{min} = 0.009 \text{ arc min}$$

Assume stator is an integral part of the structure. Solve for J and θ

$$J = \frac{\pi}{32}(7.5^4 - 4.00^4) = 0.098\ (3150 - 256) = 284$$

$$\theta = \left(\frac{85.2}{284}\right)(0.009)$$

$$\hat{\theta} = 0.0027 \text{ arc min}$$

Inboard Ground Gear Windup Analyses Reference Figure B-11

Solve for J

$$J_1 = \frac{\pi}{32}(2.936^4 - 2.47^4) = 3.630$$

$$J_2 = \frac{\pi}{32}(2.83^4 - 2.47^4) = 2.650$$

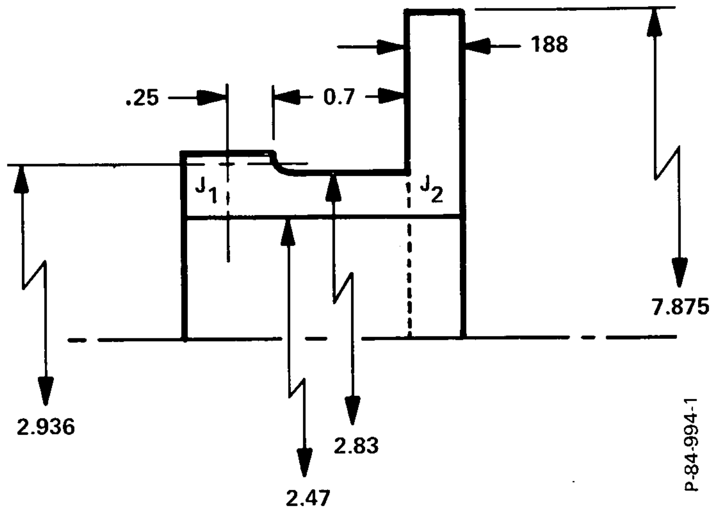

Figure B-11 - Inboard Ground Gear (Breadboard)

and

$$L_1 = 0.25$$

$$L_2 = 0.7$$

Solve for θ

$$\theta = \frac{T}{G}\left(\frac{L_1}{J_1} + \frac{L_2}{J_2}\right)$$

$$\theta = \frac{600}{10.8 \times 10^6}\left(\frac{0.25}{3.65} + \frac{0.7}{2.65}\right)$$

$$\theta = 21.25 \times 10^{-6} \text{ radians}$$

and

$$\hat{\theta} = 0.073 \text{ arc minutes}$$

The inboard ground gear flange windup is the same as that for the outboard ground gear. Therefore

$$\hat{\theta} = 0.031 \text{ arc min.}$$

<u>Summation of Breadboard Actuator Windup</u> (calculated at 1200 in-lbs)

I. Loop to ground via outboard ground gear

Output gear	1.10
Rotor ring	0.014
Outboard ground	0.040
Outboard flange	0.031
Stator housing	0.009 (max)
	1.194 arc minutes

II. Loop to ground via inboard ground gear

Output gear	1.10
Rotor ring	0.014
Inboard ground	0.108
	1.218 arc minutes

The actuator structural stiffness K_s is calculated from

$$K_s = \frac{T_L}{\varepsilon\theta} = \frac{1200}{1.218}$$

$$K_s = 985 \text{ in-lb/arc min}$$

or

$$K_s = 3.4 \times 10^6 \text{ in-lb/radian}$$

Stress Analysis

The breadboard actuator torsional, bending and shear stress analyses were completed but not included because the values were found to be small, (less than 5,000 psi).

APPENDIX C
GEAR MESH PV ANALYSES

The following analysis is for both the breadboard actuator Model EH-818-U1, and flightweight actuator Model EH-818-U2.

NOTE: The maximum PV loading occurs at the output mesh.

Given:

Output Mesh No. Teeth = 185

from layout analyses the number of teeth sharing the load = 16 teeth.

Therefore:

$$\text{deg/tooth} = \frac{360}{185} = 1.95 \text{ deg/tooth}$$

$$\text{let deg/tooth} = 2.0 \text{ degrees}$$

<u>Sliding Velocity</u>

Gear mesh sliding velocity is determined from

$$\dot{X} = e\ \omega_m \cos(\omega_m t) \qquad \text{(C-1)}$$

where

$\dot{X}$ = linear velocity in/sec

ω_m = (TR) ω_o - motor speed in radian/sec

and

TR = transmission ratio

ω_o = output speed rad/sec

Given the following design parameters

rated speed = 1 rpm

TR = 818:1

then

$$\omega_o = 1 \left(\frac{\pi}{30}\right) \text{ rad/sec}$$

$$\omega_o = 0.105 \text{ rad/sec}$$

and motor speed ω_m

$$\omega_m = 818 \ (0.105)$$

$$\omega_m = 85.6 \text{ rad/sec}$$

Solve (C-1) for $\dot{X}$

$$\dot{X} = e \ \omega_m \cos (\omega_m t)$$

where

$$e = \text{eccentricity} = 0.048 \text{ inch}$$

$$\dot{X} = (0.048)\ (85.6) \cos (\omega_m t)$$

$$\dot{X} = 4.11 \cos (\omega_m t) \text{ in/sec}$$

or

$$\dot{X} = 20.55 \cos (\omega_m t) \text{ ft/min}$$

Force Distribution

From layout analyses we have established 17 teeth are in mesh and of these, 16 teeth are driving the load. Given the load distribution shown in Figure C-1, solve for summation torque about output gear longitudinal axis.

$$F_1 \ r_o \cos \alpha + F_2 \ r_o \cos \alpha + \ldots . \ F_{16} \ r_o \cos \alpha = T_L$$

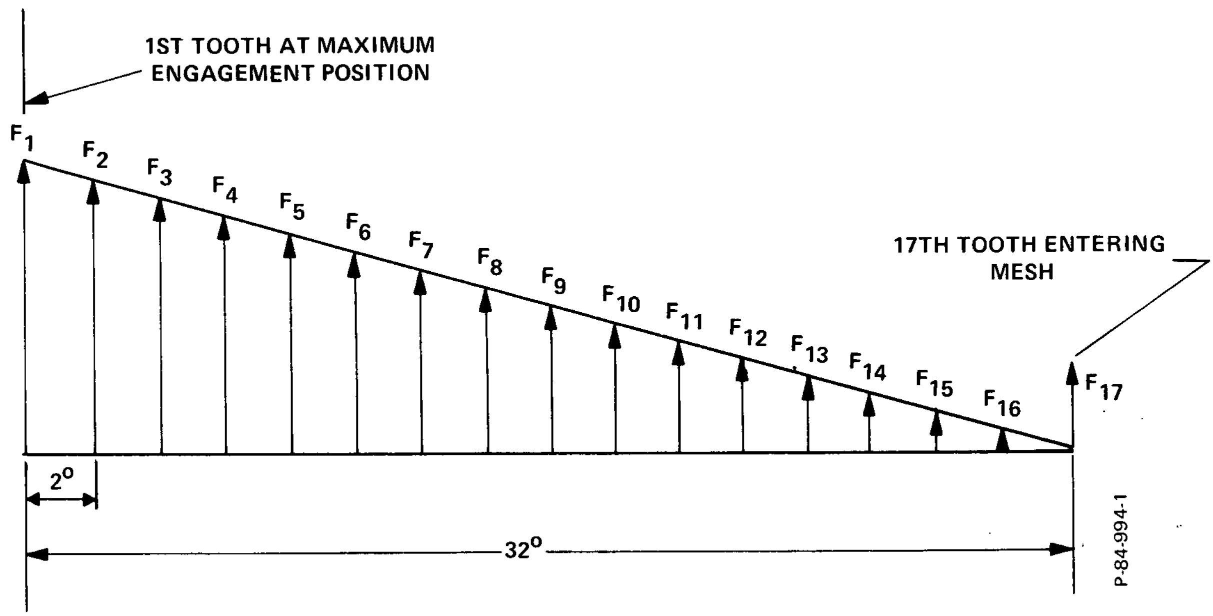

Figure C-1 - Output Mesh Load Distribution Diagram

or

$$\sum_{n=1}^{n=16} F_n \, r_o \cos \alpha = T_L \qquad \text{(C-2)}$$

where

F_n = load per tooth - pounds

and

$$T_L = \frac{\text{Rated torque}}{\text{Transmission efficiency}}$$

and at rated conditions

$$T_L = \frac{1800}{0.9}$$

T_L = 2000 in-lb

Given the load distribution shown in Figure C-1, solve for tooth force F_1.

$$F_1 = F_1$$

$$F_2 = 15/16\ F_1$$

$$F_3 = 14/16\ F_1$$

.

.

.

.

$$F_{16} = 1/16\ F_1$$

$$F_{17} = 0\ F_1$$

Summing in (C-2)

$$F_1\ r_o \cos \alpha\ [1 + 15/16 + 14/16 + 13/16 + \ldots .\ 1/16] = T_L$$

$$F_1\ r_o \cos \alpha\ [1 + \frac{120}{16}] = T_L$$

and given

$$r_o = 1.447 \text{ inch}$$

$$T_L = 2000 \text{ in-lb}$$

$$\alpha = 20 \text{ tooth pressure angle}$$

then

$$8.5\ F_1\ r_o \cos \alpha = T_L$$

Substituting and solving for F_1

$$F_1\ (1.447)\ (\cos 20 \text{ deg})\ (8.5) = 2000$$

$$(11.55)\ F_1 = 2000$$

$$F_1 = 173.5 \text{ pounds}$$

Given:	Then:
$F_1 = F_1$	$F_1 = 173.5$ pounds
$F_2 = 15/16\ F_1 = 0.937\ F_1$	$F_2 = 162.5$
$F_3 = 14/16\ F_1 = 0.875\ F_1$	$F_3 = 152$
$F_4 = 13/16\ F_1 = 0.812\ F_1$	$F_4 = 141$
$F_5 = 12/16\ F_1 = 0.75\ F_1$	$F_5 = 130$
$F_6 = 11/16\ F_1 = 0.689\ F_1$	$F_6 = 119.5$
$F_7 = 10/16\ F_1 = 0.625\ F_1$	$F_7 = 108.5$
$F_8 = 9/16\ F_1 = 0.562\ F_1$	$F_8 = 97.5$
$F_9 = 8/16\ F_1 = 0.5\ F_1$	$F_9 = 86.8$
$F_{10} = 7/16\ F_1 = 0.438\ F_1$	$F_{10} = 77$
$F_{11} = 6/16\ F_1 = 0.375\ F_1$	$F_{11} = 65$
$F_{12} = 5/16\ F_1 = 0.312\ F_1$	$F_{12} = 54.2$
$F_{13} = 4/16\ F_1 = 0.210\ F_1$	$F_{13} = 43.4$
$F_{14} = 3/16\ F_1 = 0.188$	$F_{14} = 32.4$
$F_{15} = 2/16\ F_1 = 0.125$	$F_{15} = 21.7$
$F_{16} = 1/16\ F_1 = 0.0625$	$F_{16} = 10.9$
$F_{17} = 0\ F_1$	$F_{17} = 0$

The sliding velocity for each tooth position is calculated from (C-2) and the results are summarized in Table C-1. Tooth force F and product $F\dot{X}$ is also tabulated in Table C-1.

As shown in Table C-1 the maximum $F\dot{X}$ position occurs at tooth No. 9, where F = 86.8 pounds and X = 5.70 ft/min.

Table C-1

Tooth Position	$(\omega_m t)$	$\cos(\omega_m t)$	$\dot{X} = 20.55 \cos \omega_m t$	F	$F\dot{X}$
1	90	0	0	173.5	0
2	88	0.035	0.72	162.5	117
3	86	0.070	1.44	152	219
4	84	0.105	2.16	141	308
5	82	0.139	2.86	130	372
6	80	0.174	3.58	119.5	429
7	78	0.208	4.28	108.5	465
8	76	0.242	4.97	97.5	485
9	74	0.277	5.70	86.8	495
10	72	0.309	6.36	77	489
11	70	0.342	7.04	65	458
12	68	0.375	7.71	54.2	417
13	66	0.407	8.37	43.4	364
14	64	0.438	9.0	32.4	292
15	62	0.469	9.64	21.7	209
16	60	0.500	10.27	10.9	112
17	58	0.530	10.9	0	0

Contact Stress (S_c)

In the worst case, the internal gear mesh behaves as a cylinder of radius r_1 bearing on a cylinder of radius r_2. And the hertz contact stress, S_c, is determined from

$$S_c = 0.591 \sqrt{p\ E\ \frac{(D_1 + D_2)}{D_1\ D_2}} \qquad \text{(C-3)}$$

where

S_c = hertz stress, psi

p = unit loading, lb/in

E = modulus of elasticity, psi

D_1 & D_2 = diameter of bearing surfaces (2 times the tooth radius of curvature)

Substituting into (C-3)

Given

$E = 30 \cdot 10^6$ psi

$D_1 = D_2 = 0.290$ inch (gear geometry)

$$S_c = 0.591\sqrt{\frac{p\ (30 \cdot 10^6)\ (0.290 + 0.290)}{(0.290)\ (0.290)}}$$

$$S_c = 0.591\sqrt{\frac{p\ 30 \cdot 10^6 (0.580)}{0.084}}$$

P is determined from tooth load analysis

p = unit load/inch

at tooth No. 9 (max. velocity)

$p = F/\ell$

where

ℓ = tooth length 1.0 inch

Therefore

p = 86.8 lb/in

And solving (C-3) for S_c

$$S_c = 0.591\sqrt{(86.8)\ (30 \cdot 10^6)\ (6.9)}$$

$$S_c = 0.591\sqrt{18 \cdot 10^9}$$

$$S_c = 0.591 \sqrt{180 \cdot 10^8}$$

$$S_c = 0.591 \; 13.4 \times 10^4$$

$$S_c = 78{,}400 \text{ psi}$$

And PV being defined as $S_c \cdot \dot{X}$ psi ft/min we calculate:

$$PV_{rated} = (78.4 \cdot 10^3)\ (5.70) = 447{,}000 \text{ psi ft/min}$$

and from NASA spec, the long life operation will occur at 10 percent rated. Therefore:

$$PV_{life} = 44{,}700 \text{ psi ft/min}$$

Table C-2 summarizes the PV loading for each tooth at rated condition.

Summary

As shown in summary, Table C-2, the teeth in position 11 and 12 are subjected to the highest PV loaded - 447,000 psi ft/min. This value is approximately 15 percent of the PV value that a pneumatic Dynavector actuator was successfully tested for $1 \cdot 10^6$ endurance cycles. The test was run unlubricated and final inspection indicated no measurable wear.

Output Gear Mesh Bending Stress

The gear mesh bending stress, S_b, is determined from

$$S_b = \frac{4\ T_o}{Y\ b\ c\ d^2} \qquad \text{(C-4)}$$

where

T_o = load torque, in-lb

Y = Lewis form factor

b = face width, inch

c = % teeth driving the load

d = pitch diameter, inch

Table C-2

Tooth Position	F (Lbs)	$\overset{\circ}{X}$ (Ft/Min)	S_c (Psi)	PV (Psi Ft/Min)
1	173.0	0	111,000	0
2	162.5	0.72	107,500	77,500
3	152	1.44	104,000	150,000
4	141	2.16	100,000	216,000
5	130	2.86	96,000	274,000
6	119.5	3.58	92,200	330,000
7	108.5	4.28	87,600	375,000
8	97.5	4.97	83,200	413,000
9	86.8	5.70	78,400	447,000
10	77	6.36	73,800	470,000
11	65	7.04	67,800	477,000
12	54.2	7.71	62,000	477,000
13	43.4	8.37	55,500	465,000
14	32.4	9.0	47,800	430,000
15	21.7	9.64	39,200	378,000
16	10.9	10.27	27,800	284,000
17	0	10.9	0	0

and given

T_o = 2400 in-lb

b = 1.0 inch

Y = 0.5

c = 0.09 (layout value)

d = 2.82 inches

Solving:

$$S_b = \frac{4(2400)}{(0.5)(1.0)(0.09)(2.82)^2}$$

$$S_b = 26,800 \text{ psi}$$

Summary

The tooth bending stress is 8 percent the material's (18 N_i 350 steel) yield strength. The mesh, designed for minimum PV with optimum rigidity, is structurally sound.

Output Gear Tooth Deflection at Stall Torque

In the previous mesh loading analyses it was determined that the most highly loaded tooth transmits 154 lbs at 2400 in-lb stall torque. The following analysis is presented to determine the tooth beam deflection. Given

$$Y = \frac{w \ell^3}{15 E I} \qquad \text{(C-5)}$$

where

Y = beam deflection, inch

w = load on tooth, pounds

ℓ = tooth height, inch

E = material modulus of elasticity, psi

I = beam section modulus, in.

and given

w = 154 lbs

ℓ = 0.036 inch

E = $30 \cdot 10^6$ psi

$$I = \frac{b t^3}{12} \qquad \text{(C-6)}$$

where

t = 0.037 inch (tooth thickness)

b = 1.0 inch (tooth length)

Solving (C-6) for I

$$I = \frac{(1)(0.037)^3}{12}$$

$$I = 4.18 \cdot 10^6 \text{ in}^4$$

and solving (C-5) for Y

$$Y = \frac{(154)(0.036)^3}{(15)(30 \cdot 10^6)(4.18 \cdot 10^{-6})}$$

$$Y = 3.84 \cdot 10^{-6} \text{ inch}$$

Find the resulting angular change in shaft position due to tooth deflection. Given

$$\theta = Y/r \qquad \text{(C-7)}$$

where

θ = angular change, radian

Y = deflection, inch

r = output gear pitch radius, inch

and given

$Y = 3.84 \cdot 10^{-6}$

r = 1.447 inch

Solve (C-7) for θ

$$\theta = \frac{3.84 \cdot 10^{-6}}{1.447}$$

$$\theta = 2.66 \cdot 10^{-6} \text{ radian}$$

and converting θ_{radian} to θ arc minutes from the following relationship

$$\theta = 3440 \ \frac{\text{arc min}}{\text{radian}}$$

then

$$\theta = 3440 \ (2.66 \cdot 10^{-6})$$

$$\theta = 0.00913 \text{ arc-minutes}$$

Summary

The tooth beam deflection at stall and the resulting angular change in shaft position is negligible.

APPENDIX D

NITRIDING OF NITROLOY N STEEL

GENERAL

A composite output gear containing an Alnico V permanent magnet to increase the holding force between the rotor ring gear and the output gear was fabricated for the breadboard actuator. The gear was machined from a Nitroloy N grade steel whose magnetic properties are better than all of the maraging grade steels. Its mechanical properties, although less than maraging grade steels, are acceptable. Nitroloy N grade steel has two drawbacks:

(1) Unpredictable dimensional control during nitriding.

(2) Brittle white layer buildup.

A heat treated test sample machined to the part configuration was used to evaluate the geometrical changes of the heat treated sample.

HEAT TREATMENT OF NITROLOGY N NITRIDING GRADE STEEL

Nitroloy N grade steels may shrink or grow during the nitriding cycle. The shrinkage or growth is totally dependent upon the part shape and its cross-sectional area. For example, it was anticipated that the diameter of the output gear test sample would grow 0.005 in/in radially and the length 0.0015 in/in.

A test sample machined to the shape of the composite output gear was heat treated as per PS 998 Rev A. The heat treat procedure and the results are summarized below.

<u>Heat Treat Procedure</u>

(1) Quench and temper for R_c 35 max. prior to machining.

(2) Allow 0.0005/0.0015 in/in stock for final grind after nitriding.

(3) Floe-process nitride to minimize white layer (pure nitride) formation.

(4) Final grind. Do not exceed 0.002/0.003 in per side stock removal.

<u>Results</u>

(1) Part did not grow or shrink - all planes.

(2) Case was 0.005/0.007 in. thick.

CONCLUSION

The test sample did not shrink or grow. Therefore, the allowance of 0.0005/0.0015 in/in grind stock is not required.

PROJECT NO.	BENDIX RESEARCH LABORATORIES SOUTHFIELD, MICHIGAN	CODE IDENT.	SPECIFICATION NO.	REV.
2870-2113		11272	PS-998	A

ENGINEERING SPECIFICATION

TITLE
Process and Heat Treat Procedure for Nitrided Nitroloy N Steel

CUSTOMER	CONTRACT NO.	DATE RELEASED
NASA Goddard Space Flight Center	NAS-521142	8-19-70

REVISION STATUS OF SHEETS
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REV.																									
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REV.																									

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PREPARED BY	N. L. Sikora	DATE 8-12-70	CHECKED BY		DATE:

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CONFIG. MGR			CHIEF DRAFTSMAN		

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2870-2113	SOUTHFIELD, MICHIGAN	11272	PS-998	A

ENGINEERING SPECIFICATION

General

This process specification is intended for Nitroloy N steel grade bar stock in the fully annealed condition and includes the machining sequences necessary to provide finished parts within the drawing tolerances.

Procedure

Pre-Nitriding

1. Rough machine annealed bar removing all decarburized layer. Leave .030 to .060 inch stock for finish machining after heat-treatment.
2. Harden part at 1650 to 1750°F. Oil or water quench.
3. Temper at 1150 to 1250°F. Hold two hours at heat.
4. Finish machine all over. Note: Allow .0005 in/in stock all length dimensions and .0015 in/in on all radial dimensions for growth during nitriding.

Nitriding

1. Floe process nitride all over for $^{.005}/_{.006}$ in case depth. White layer after nitriding to be less than .0005 inches.
2. Surface hardness to be R_c 67 min.

Post Nitriding

Light grind or lap to finish dimensions. Total stock removal shall not exceed .002/.003 inch per side.

Test Specimen

A test-specimen of 1/2 inch diameter bar one and one-half (1 1/2) inch long centerless ground and copper plated for half the length to the thickness of .001/.0015 inch shall accompany the parts to determine hardness characteristics.

REVISIONS
WHEN THIS CONTINUATION SHEET IS REVISED, THE REVISION LETTER SHALL ALSO BE RECORDED ON THE TITLE SHEET. SEE ENGINEERING CHANGE NOTICE (ECN) IDENTIFIED BY THIS SPECIFICATION NUMBER, FOR DESCRIPTION OF EACH REVISION.

APPENDIX E

18 Ni 350 MARAGING STEEL PROCESS SPECIFICATION

GENERAL

The 18 Ni 350 maraging grade steels have been selected for all internal and external gearing on the flightweight actuator design because of its high yield strength characteristics, its consistent shrink rate during the aging-nitriding heat treat process, and its excellent anti-wear characteristics.

Heat Treatment of 18 Ni 350 Maraging Steel

Because the 18 Ni 350 grade maraging steel is a new grade of material, not much data has been published regarding the actual shrink rate and case/core hardness which can be expected during the aging-nitriding cycle. International Nickel Company (INCo.) suggested that we process a sample part through the heat treat cycle and measure the exact shrink rates and case thickness. Three sample parts were fabricated to the same cross-sectional area as the orbiting rotor ring gear, the most critical part within the flightweight design. A procedural description and the results of the heat treat tests are summarized below.

TEST SAMPLE 1

The first test sample was heat treated per BRL process specification PS-996 Rev. A. This specification requires a 24 hour heat treat cycle at 825 to 850°F. The expected shrink rate was 0.0008 in/in with a case hardness of R_c 67, a case depth of 0.004 to 0.006 inch, and a core hardness of R_c 55 min.

The actual test data results were:

(1) The shrink rate was 0.0009/0.0011 in/in - all planes.

(2) The case and core hardness were the same - R_c 58/59.

(3) There was not any measurable case thickness.

Conclusion

The 24 hours at 825-850°F temperature was inadequate. The actual time at temperature should have been 48 hours.

TEST SAMPLE 2

The second test sample was heat treated per PS-996 Rev. B. This specification requires a 48 hour heat treat cycle at 825 to 850°F. We expected to see a shrink rate of 0.001 in/in nominal and in addition, a total surface growth due to nitriding of 0.0006/0.0008 inch on the diameter. (The occurrence of surface growth was suggested by INCO. It is independent of the numerical magnitude of the diameter, i.e., a 1 inch diameter will grow the same amount as a 10 inch diameter.)

The actual test data results were:

(1) The shrink rate was 0.0009/0.0011 inch - all planes.

(2) The case hardness was R_c 63.

(3) The core hardness was R_c 56-57.

(4) The case thickness was 0.0018/0.002 inch.

(5) A white layer (pure nitride) of 0.000032 inch was measured.

(6) No measurable surface growth after nitriding.

Conclusions

The shrink rate is consistent and the core hardness is acceptable. However, the case hardness of R_c 63 is 2-3 points low and the case thickness is 40 percent of the 0.005 inch desired.

Prior to heat treat of test sample 3, the test results of test sample 2 were discussed with INCO. They agreed that a higher temperature would improve the case characteristics and recommended that we heat treat test sample 3 at 875°F for 48 hours. INCO also indicated that the commercial grade of maraging steel we were using (certified stock purchased from the Vanadium Alloy Steel Co. (VASCOMAX), Latrobe, Pennsylvania) contained 1.3 percent titanium instead of the 1.7 percent that INCO uses in their steel.They recommended that we talk to our material vendor VASCOMAX, to confirm the 875°F heat treat temperature. On 21 October 1970 VASCOMAX Corporation was contacted. They concurred that a 875°F heat treat for the 18 Ni 350 grade steel is acceptable. Both INCO and VASCOMAX cautioned that nitriding above the 875°F temperature may result in a loss of material strength.

TEST SAMPLE 3

The third test sample was heat treated per PS-996 Rev. C. This specification requires 48 hour heat treat cycle at 875°F. The actual test data results were:

(1) The shrink rate was 0.0009/0.0011 in/in - all planes.

(2) The case hardness was R_c 63.

(3) The core hardness was R_c 59.

(4) The case thickness was 0.0026/0.003 inch.

(5) A white layer (pure nitride) of 0.0002/0.0003 inch.

(6) No measurable surface growth after nitriding.

(7) The case structure and bond was "excellent."

Conclusion

The shrink rate remains consistent and the core properties increased in hardness to R_c 59, 5 points above the required value. The case hardness remained at R_c 63, 2-3 points below the required hardness. However, the case thickness increased to 0.003 or 60 percent of the desired value.

Each of the test samples, 1 through 3, were anlyzed by Detroit Testing Laboratory.

SUMMARY AND RECOMMENDATIONS

The heat treat performed on the sample test specimens indicates that 18 Ni 350 maraging steel has a predictable shrink rate of 0.0009/0.0011 in/in with no measurable surface growth on diameters up to 4.0 inches and that the materials core properties at R_c 59 are better than required. The R_c 63 case hardness and 0.003 in thickness although less than desired, is acceptable because the R_c 59 core provides good backup for the 0.003 in case layer.

A fabrication procedure which will eliminate final machining after heat is presented below.

FABRICATION PROCEDURE

(1) Finish machine the internal gears to print dimensions allowing 0.0009/0.0011 in/in stock for shrinkage during heat treat.

(2) Finish machine mating external gears to obtain smooth rolling mesh action. The radial separation shall not exceed 0.005 inch maximum.

(3) Heat treat mesh assemblies as per PS-996 Rev. C.

(4) After heat treat, inspect mating mesh assemblies for smoothness of rolling. Inspect and record dimensions over wires and radial dropout.

   (a) Radial dropout should not exceed 0.002/0.004 inch at ground gear mesh.

   (b) Radial dropout should not exceed 0.003/0.005 inch at output gear mesh.

(5) Mesh lap assemblies to insure conjugate mesh action exists and to remove fine burrs and surface irregularities due to heat treat. Light vapor blast prior to mesh lapping is optional.

(6) Re-inspect and record:

   (a) dimensions over wires - internal and external.

   (b) radial dropout - all meshes.

   (c) tooth alignment between meshes.

   (d) smoothness of the mesh rolling action.

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2870-2113	SOUTHFIELD, MICHIGAN	11272	PS-996	C

ENGINEERING SPECIFICATION

TITLE

Process and Heat Treat Procedure for Nitrided Maraging Steel Type 18 N_i 350

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NASA Goddard Space Flight Center	NAS 5-21142	7-29-70

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REV.	C	C																							
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REV "B" SEPT 14, 1970

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PREPARED BY N. L. Sikora	DATE 7-21-70	CHECKED BY	DATE:

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2870-2113	SOUTHFIELD, MICHIGAN	11272	PS 996	C

ENGINEERING SPECIFICATION

General

This process specification is intended for Maraging Steel Type 18 Ni 350 grade billet, bar or sheet in the as received condition and includes the machining sequences necessary to provide finished parts within drawing tolerances.

Procedure

1. If the billet, bar or sheet are not in the solution annealed condition, solution anneal at 1650 degrees Fahrenheit for one (1) hour and air cool to room temperature. Then solution anneal for one (1) hour at 1450 degrees Fahrenheit and cool. Hardness should be Rockwell 'C' 30 - 35.

2. Machine the billet, bar or sheet forging to finish dimensions, leaving only sufficient material to lap or dust grind the part. Allowance must be made for shrinkage of the part during the nitriding-aging process. Allow 0.001 in/in for shrinkage during aging.

3. Ultrasonically degrease in a trichlorethylene solution, followed by a deionized water rinse. Prepare surface with a 200 grain aluminum oxide blast, using air as the medium to a light matte finish. Surface must not be contaminated during this operation.

4. Nitride at 875 - 880 degrees Fahrenheit for forty-eight (48) hours in 25 - 30 percent dissociated ammonia.

5. Hardness to be:

   Case Rockwell 'C' 66-67

   Superficial Rockwell '15N' 92-93

   Core Rockwell 'C' 55 MIN

6. Dust grind or lap to finish dimensions.

7. A test specimen of 1/2 inch diameter bar one and one-half (1-1/2) inches long centerless ground and copper plated for half the length to a thickness of 0.001/0.0015 inches shall accompany the parts to determine hardness and case characteristics.

REVISIONS

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BRL-910-32 2171595 ORIGINAL FILED IN PRODUCT DESIGN SECTION

APPENDIX F

ALUMINUM-TO-STEEL SHRINK FIT ANALYSES

Flightweight Actuator Model EH-818-U2

To minimize weight, the actuator mounting and output pivot flanges were fabricated from aluminum alloy for a shrink fit assembly onto their mating steel parts. The following analyses was conducted to determine the required interference fit and resulting stresses.

Output Pivot Flange Analyses

The operating temperature range is:

+158°F to -40°F

Given a stall torque of 2400 in-lbs and a minimum safety factor = 2, the torque-to-slip will be 5000 in-lb. The coefficient of friction of aluminum on steel is is 0.6 (SAE data).

The torque required to cause complete slippage of an interference fit is given by

$$T = 1/2 \, \pi \, f \, p \, \ell \, d^2 \qquad \text{(F-1)}$$

where

T = load torque, in-lb

f = coefficient friction - 0.6

d = interference diameter, inches - 2.36 inches

ℓ = interference length, inches - 0.5 inch

p = contact pressure, psi

Substituting and solving (F-1) for p

$$5000 \text{ in-lbs} = \left(\frac{1}{2}\,\pi\right)(0.6\,p)\;(0.5)\;(2.36)^2$$

$$p = \frac{5000}{(0.942)\;(2.79)}$$

$$p = 1908 \text{ unit contact pressure}$$

Analyses to determine magnitude of radial interference fit to provide 5000 in-lb torque at 158°F. Given the interference fit solution:

$$\delta = \delta_{outer\ tube} + \delta_{inner\ tube} \qquad \text{(F-2)}$$

where δ = radial interference - inch, and the following radial dimensions for the inner and outer members are

a = 0.805 inch μ_o (poisons modulus for aluminum) = 0.36

b = 1.18 inch μ_i (poisons modulus for steel) = 0.26

c = 1.555 inch

Rewriting (F-2)

$$\delta = \frac{b\ p}{E}\left(\frac{C^2 + b^2}{C^2 - b^2} + \mu_o\right) + \frac{b\ p}{E}\left(\frac{b^2 + a^2}{b^2 - a^2} - \mu_i\right) \qquad \text{(F-3)}$$

$$\delta = \left(\frac{1.18 \times 2000}{10 \times 10^6}\right)\left(\frac{1.555^2 + 1.18^2}{1.555^2 - 1.18^2} + 0.36\right)$$

$$+ \left(\frac{1.18 \times 2000}{30 \times 10^6}\right)\left(\frac{1.18^2 + 0.805^2}{1.18^2 - 0.805^2} - 0.26\right)$$

δ = 0.0012-inch radial and the required diametral interference fit is 0.0024 inch. This value insures 5000 in-lb no-slip torque at 158°F.

At these conditions, determine the tangential stress at the inner surface of the outer member.

$$S_{ot} = P\left(\frac{C^2 + b^2}{C^2 - b^2}\right) = 1908\ (3.715) = 7120 \text{ psi}$$

Analysis to determine the effects of operating temperature range on interference fit, contact pressure and stress are as follows;

Given the coefficient of expansion, ε:

$$\varepsilon_{\text{aluminum (2024-T3 grade)}} = 12.9 \times 10^{-6} \text{ in/in°F}$$

and

$$\varepsilon_{\text{steel (Maraging 18 }N_i\text{ 350 grade)}} = 5.6 \times 10^{-6} \text{ in/in°F}$$

Calculate the difference in expansion, Δ:

$$\Delta = \varepsilon_{\text{aluminum}} - \varepsilon_{\text{steel}}$$

$$\Delta = 7.3 \cdot 10^{-6} \text{ in/in°F}$$

Note the change in shaft diameter = ΔT times the nominal shaft diameter - d.

Case I

Given the operating temperature range, 70°F < T < 158°F, solve for additional radial interference required at 70°F room temperature assembly.

The increase in operating temperature will be 88°F, and the addition interference required at room temperature assembly is:

$$\Delta\varepsilon = \Delta \cdot \Delta T$$

$$\Delta\varepsilon = (7.3 \cdot 10^{-6}) (88) = 642 \cdot 10^{-6} \text{ in./in.}$$

The additional change in interference $\Delta\delta$ is calculated from $\Delta\delta = \Delta\varepsilon \cdot d$.

$$\Delta\delta = (642 \cdot 10^{-6}) (2.36)$$

$$\Delta\delta = 0.001530$$

and the required room temperature assembly diametral interference:

$$\delta = 0.0024 + 0.0015$$

$$\delta = 0.0039 \text{ inch}$$

Analyses to determine increased contact pressure, p, at 70°F room temperature assembly. Solve equation (F-3) for p:

$$\delta = \frac{b\ p}{E}\left(\frac{C^2 + b^2}{C^2 - b^2} + \mu_o\right) + \frac{b\ p}{E}\left(\frac{b^2 + a^2}{b^2 - a^2} - \mu_i\right)$$

$$0.0019 = \frac{1.18\ p}{10 \cdot 10^6}(4.075) + \frac{1.18\ p}{30 \cdot 10^6}(2.481)$$

$$0.0019 = \frac{4.80\ p}{10 \cdot 10^6} + \frac{2.93\ p}{30 \cdot 10^6}$$

$$p = 3300 \text{ psi}$$

Calculate outer member tangential stress at 70°F temperature:

$$S_{ot} = P\left(\frac{C^2 + b^2}{C^2 - b^2}\right) = 3300 \cdot (3.715)$$

$$S_{ot} = 12{,}250$$

Calculate unit contact pressure, p, at -40°F:

$$\Delta T = 70°F - (-40°F)$$

$$\Delta T = 110°F$$

Diametral change = $(7.3 \cdot 10^{-6})$ in/in°F) (-110°F) (2.36 shaft diameter)

$$\text{Diametral change} = -0.0019 \text{ inch}$$

The total shaft interference at -40°F is equal to initial assembly interference at 70°F plus shrink due to temperature drop.

$$\delta = 0.0039 + 0.0019$$

$$\delta = 0.0058 \text{ inch}$$

and the contact pressure, p, at -40°F

$$p = \frac{0.0058}{0.0039} (3300)$$

$$p = 4920 \text{ psi}$$

and the outer member tangential stress is

$$S_{ot} = 18{,}450 \text{ psi}$$

Summary

The interference fit design is safe for all modes of operation. A 0.0039-inch interference fit at 70°F room temperature will insure a minimum of 5000 in-lbs output torque capacity at the dissimilar metal joint prior to slip. At -40°F, the aluminum's tensile stress level is 18,450 psi which is 74 percent the minimum tensile strength of a 2000 series aluminum alloy.

Mounting Flange Analyses

The following is an interference fit analysis for the aluminum mounting flange on the inboard ground mounting gear.

Given the radial interference diameters

$a = 3.65$ inches (steel - inside radius)

$b = 3.750$ inches (steel - outside radius)

$c = 3.95$ inches (aluminum - outside radius)

and the operating temperature range is +158°F to -40°F. Assume a 2400 in-lbs stall torque times a minimum safety factor = 2.0. The design torque shall be 5000 in-lbs. Also assume a coefficient of friction, steel on aluminum - 0.6. Calculate unit contact pressure to transmit 5000 in-lbs. Using (F-1) solve for p. Given

$T = 5000$ in-lbs

$\ell = 0.40$ inch

$d = 7.5$ inches

$$T = \frac{1}{2} \pi \delta p \ell d^2$$

$$5000 = \left(\frac{\pi}{2}\right) (0.6) (p) (0.4) (7.5)^2$$

$$p = \frac{10{,}000}{\pi (0.6) (0.4) (56.25)}$$

$$p = 236 \text{ psi}$$

Solve for 158°F interference fit of aluminum mounting flange on steel

Given $p = 236$ psi,

$$\delta = \frac{3.750 \cdot 236}{10 \cdot 10^6} \left(\frac{3.95^2 + 3.75^2}{3.95^2 - 3.75^2} + \mu_o \right) + \frac{3.75 \cdot 236}{30 \cdot 10^6} \left(\frac{3.75^2 + 3.65^2}{3.75^2 - 3.65^2} - \mu_i \right)$$

where

$\mu_{o(aluminum)} = 0.36$, $\mu_{i(steel)} = 0.26$

$$\delta = \frac{885}{10 \cdot 10^6} \left(\frac{15.60 + 14.063}{15.60 - 14.063} + 0.36 \right) + \frac{885}{30 \cdot 10^6} \left(\frac{14.063 + 13.323}{14.063 - 13.323} - 0.26 \right)$$

$\delta = 0.002833$ inch .

The 158°F tangential stress at inner surface of outer member is found from

$$S_{ot} = p \left(\frac{c^2 + b^2}{c^2 - b^2} \right)$$

$$S_{ot} = 236 \ (19.36) = 4560 \text{ psi}$$

Find the required interference at 70°F room temperature.

$\Delta e = 7.3 \cdot 10^{-6}$ in/in°F , $\Delta T = 88$°F , steel outside radius - 3.75 inch

then:

$$\Delta\delta = (7.3 \cdot 10^{-6}) \ (88) \ (3.75) = 0.00241$$

Summing

$$\delta = 0.0024 + 0.0028$$

$\delta = 0.0052$-inch radial interference at 70°F .

Substituting into (F-3) and solving for p

$$0.0052 = \frac{3.75}{10 \cdot 10^6} \ p \ (19.72) + \frac{3.75}{30 \cdot 10^6} \ p \ (36.74)$$

$$5.2 \cdot 10^3 = 7.39 \ p + 4.6 \ p$$

$p = 434$ psi at room temperature

The slip torque is calculated by:

$$T_{slip} = \frac{434}{236}(5000) = 9210 \text{ in-lbs}$$

and outer member tangential stress:

$$S_{ot} = \frac{434}{236}(4560) = 8400 \text{ psi}$$

Find δ at T = -40°F

$\Delta T = -198°F$ (from +158°F to -40°F)

$\Delta\delta = \frac{198}{88}(0.00241) = 0.0054$ in. radial change in interference diameter.

Summing

$$\Delta\delta = 0.0054 + 0.0028$$

$$\Delta\delta = 0.0082 \text{ in. at } -40°F$$

and

$$p = \frac{0.0082 \cdot 10^6}{11.99} = 684 \text{ psi}$$

then the torque-to-slip is calculated from

$$T_{slip} = \frac{684}{236}(5000) = 14{,}500 \text{ in-lbs}$$

and the tangential stress is

$$S_{ot} = 2.895\ (4560) = 13{,}200 \text{ psi}$$

Summary

The interference fit design is safe for all modes of operation. A 0.0052-inch interference fit at 70°F room temperature will insure a minimum of 5000 in-lbs output torque capacity at the dissimilar metal joint prior to slip. At -40°F, the aluminum's tensil stress is 13,200 psi and as such, 53 percent the minimum tensil strength of a 2000 series aluminum alloy.

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15. Supplementary Notes

16. Abstract

The program objective was to develop a space-qualified, high reliability, 150 ft-lb rated torque rotary actuator based on the Bendix Dynavector® drive concept. This drive is an integrated variable reluctance orbit motor-epicyclic transmission actuator. The performance goals were based on future Control Moment Gyro torquer applications and represent a significant advancement in the torque-to-weight ratio, backlash, inertia and response characteristics of electric rotary drives.

The program accomplishments have been in two areas (1) the development of two high ratio (818:1) actuator configurations (breadboard and flightweight) and (2) the invention of a reliable proximity switch sensor system for self-commutation without use of optical or electrical brush techniques.

Other significant accomplishments used in the actuator and controller hardware include: (1) Design of a 818:1 single pass orbital epicyclic transmission using a difference of 6 teeth between working meshes; (2) Procedures for fabricating precision gearing from nitrided maraging steel; (3) Development of a low inertia, responsive actuator which requires only two bearings and two moving parts; (4) Energy transfer techniques for optimum coil energization; (5) Controller logic analyses by which a 8-pole motor is commutated by 4 proximity sensors; (6) Test results indicating zero backlash and stiffness of 3.9 to 4.2 arc-min/100 lb-ft; and (7) Frequency response tests and analyses to predict performance with gimbal inertias up to 500 slug-ft^2.

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