NASA CR-122467

Goddard

PLASMA PHYSICS GROUP

Get DRA

(NASA-CR-122467) LASER AMPLIFICATION IN AN INHOMOGENEOUS PLASMA R.B. White (California Univ.) Jan. 1972 12 p CSCL 20E
N72-31516
Unclas
G3/16 40305

UNIVERSITY OF CALIFORNIA
LOS ANGELES

ENGINEERING PHYSICS METEOROLOGY

Laser Amplification in an

Inhomogeneous Plasma

by

Roscoe B. White

January, 1972

PPG-109

Plasma Physics Group
Department of Physics
University of California
Los Angeles, California 90024

This work was partially supported by the Office of Naval Research, Grant #N00014-69-A-0200-4023; the National Science Foundation, Grant #GP-22817; the Atomic Energy Commission, Contract AT(04-3)-34, Project #157; and the National Aeronautics and Space Administration, Contract NGR-05-007-190 and NGR-05-007-116.

I. Introduction

The propagation of an electromagnetic wave into an inhomogeneous plasma was first studied by Budden[1]. In a plasma with large density gradients a QTX mode propagating perpendicular to the magnetic field can encounter a resonance and a cutoff separated by a distance comparable to the incident wave length. In this region the wave is evanescent, and in general there will be a reflected and a transmitted wave, and amplification will occur in the region near the resonance. The amplification is important for the study of nonlinear phenomena and for feedback stabilization applications.

Consider the propagation of a QTX mode in the x-direction. We begin with the differential equation for $E_y(x)$

$$\frac{d^2E}{dx^2} + k^2(x)E = 0 \qquad \text{with } k^2 = k_0^2 \left[1 + \frac{\alpha(1-\alpha)}{\beta+\alpha-1} \right]$$

and $k_0 = \frac{\omega}{c}$ $\qquad \alpha = \left(\frac{\omega_p}{\omega} \right)^2 \qquad \beta = \frac{\Omega_e^2}{\omega^2} < 1$.

Cutoff occurs when $\alpha = 1 - \sqrt{\beta}$ and resonance when $\alpha = 1 - \beta$. Assume the special case of Budden, i.e.

$$1 + \frac{\alpha(1-\alpha)}{\beta+\alpha-1} = 1 + \frac{x_0}{x} \quad ,$$ which leads to Whittaker's equation.

---

[1] K.G. Budden, Radio Waves in the Ionosphere (Cambridge University Press), 1966.

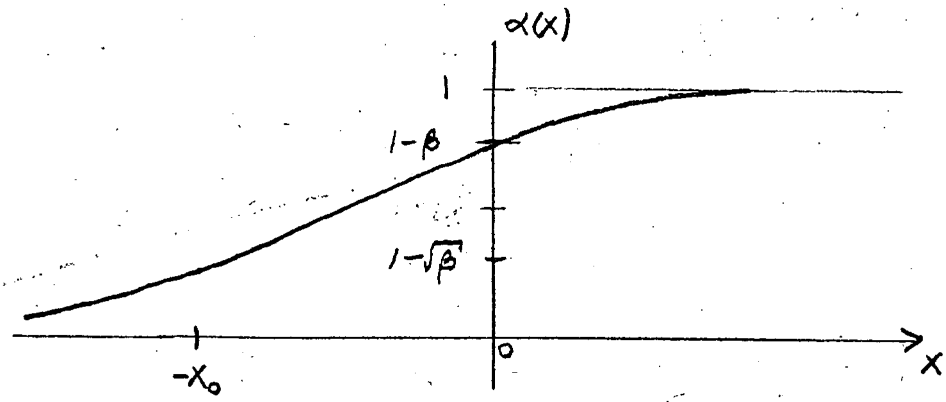


Fig. 1. Plasma density $\alpha(x) = (\omega_p/\omega)^2$ as a function of position.

Setting $z = k_0 x$ and $z_0 = k_0 x_0$ we find

$$\frac{d^2E}{dz^2} + \left(1 + \frac{z_0}{z}\right) E = 0 \quad .$$

Thus the problem has essentially only one parameter, z_0. In terms of physical variables

$$\left.\frac{d\alpha}{dx}\right)_0 \simeq \frac{\alpha(0) - \alpha(-x_0)}{x_0} = \frac{\sqrt{\beta}-\beta}{x_0}$$

And thus

$$z_0 = +k_0 \frac{(\sqrt{\beta}-\beta)}{\left.\frac{d\alpha}{dx}\right)_0} \quad .$$

Also

$$\frac{d\alpha}{dx} = \frac{1}{\omega^2}\frac{4\pi e^2}{m}\frac{dn}{dx} = \frac{\omega_p^2}{\omega^2}\left(\frac{dn}{dx}\frac{1}{n}\right) = \frac{\omega_p^2}{\omega^2}\frac{1}{L} \quad ,$$

where L is the scale length of the density gradient.

II. Integral Solution

Given $\frac{d^2E}{dz^2} + \left(1 + \frac{z_0}{z}\right) E = 0$, we have as an integral solution[2]

[2] A. Baños (unpublished); G.M. Weyl, Phys. Rev. Lett. 25, 1417 (1970); H.L. Berk and L.D. Perlstein, UCRL Preprint 72536.

$$E(z) = z \int_c e^{-izt}(t-1)^{+i(z_0/2)}(t+1)^{-i(z_0/2)}dt \quad .$$

Equivalently, let $\quad 2w = t + 1$

$$2w - 2 = t - 1$$

$$2(w - 1) = t - 1$$

$$E(z) \cong z \int_c e^{-iz(2w-1)}(w-1)^{+i(z_0/z)}w^{-i(z_0/z)}dw$$

Proof

$$E = z \int e^{-izt}(t-1)^{a}(t+1)^{-a}dt \qquad a = + i(z_0/z)$$

$$E'' = \int_c -it(2 - izt)e^{izt}(t-1)^{a}(t+1)^{-a}dt$$

$$E'' + \left(1 - \frac{2ia}{z}\right) E = \int_c (-2it - 2ia - zt^2 + z)e^{izt}(t-1)^{a}(t+1)^{-a}dt$$

Let $\quad F(t) = -ie^{-izt}(t-1)^{a+1}(t+1)^{1-a}$

$$\frac{dF}{dt} = \left(-iz + \frac{a+1}{t-1} + \frac{1-a}{t+1} \right) F$$

$$\frac{dF}{dt} = [-iz(t^2-1) + (a+1)(t+1) + (1-a)(t-1)] \cdot [ie^{-izt}(t-1)^{a}(t+1)^{-a}]$$

Thus

$$E'' + \left(1 - \frac{2ia}{z} \right)E = \int_c \frac{dF}{dt}\, dt \quad ,$$

and the integral representation will give a solution provided that F(t) (the bilinear concomittant) vanishes at the end points of the contour, i.e. for

$z > 0 \quad t \to -i\infty \quad , \quad \pm 1$

$z < 0 \quad t \to +i\infty \quad , \quad \pm 1$

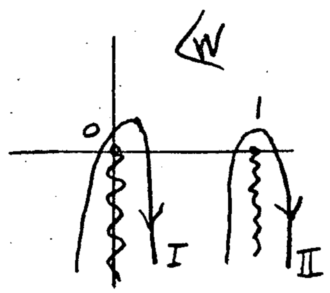
We thus have an integral representation for the field

$$E(z) = ze^{+iz} \int e^{s(w)} dw$$

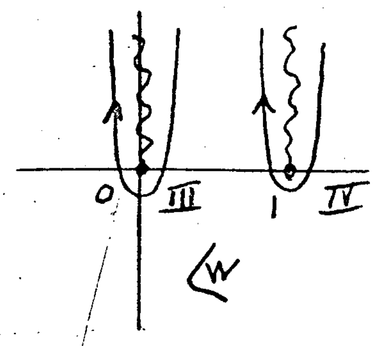
$$s(w) = -i \left[2w - \frac{z_0}{2} \ln \frac{w-1}{w} \right]$$

with possible contours of integration given by

z > 0



z < 0



Further notice that for $|z| \to \infty$ the major contribution to E comes from Im w $\approx$ 0 and thus (asymptotic behavior will be calculated exactly later)

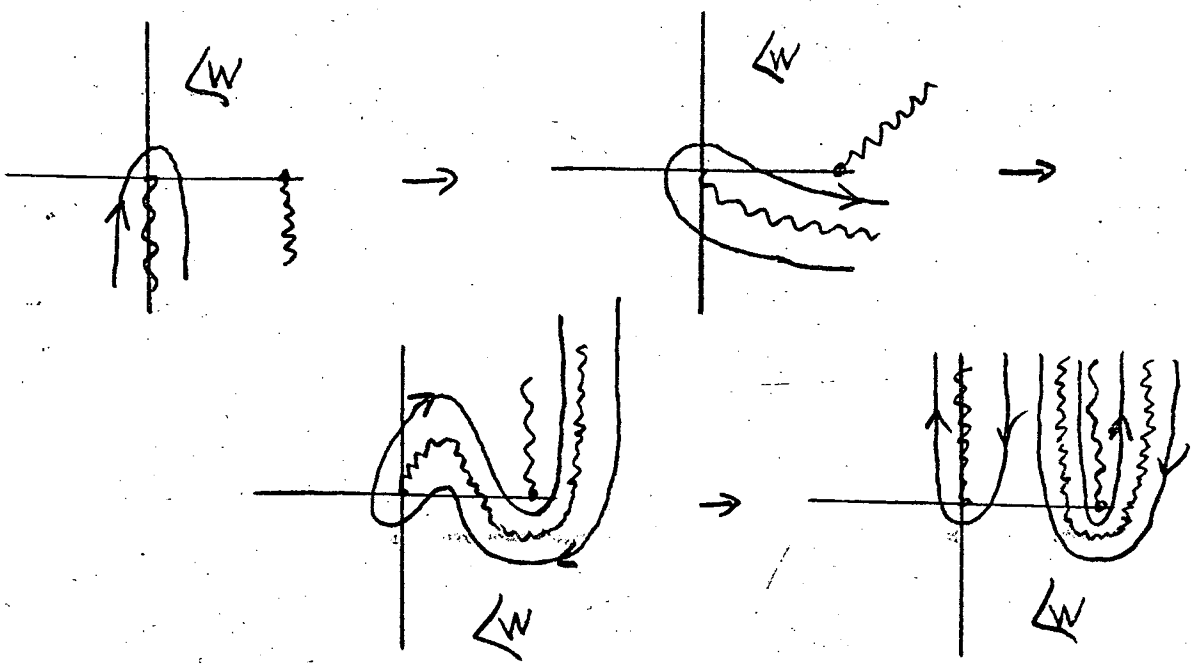
$$z > 0 \begin{cases} E_I(z) \to c_I e^{+iz} & \text{right moving} \\ E_{II}(z) \to c_{II} e^{-iz} & \text{left moving} \end{cases}$$

$$z < 0 \begin{cases} E_{III}(z) \to c_{III} e^{+iz} & \text{right moving} \\ E_{IV}(z) \to c_{IV} e^{-iz} & \text{left moving} \end{cases}$$

There is also one contour in the finite plane encircling both branch points which gives a solution. However a second independent finite-plane contour does not exist and this representation is therefore not useful.

For boundary conditions we choose for $z > 0$ $E_I(z)$, i.e. a transmitted right moving wave. Thus for $z > 0$ we have the contour **I**.

To analytically continue to $z < 0$ we are restricted to the LHP in z. (This can be demonstrated to be necessary by including a small collision frequency in the problem). Thus to keep the integral convergent we must rotate the contour ccw as z is rotated clockwise. Thus the contour II becomes successively



and thus E_I becomes after continuation to $z < 0$:

$$E_I \rightarrow E_{IV}(1 - e^{\pi z_0}) + E_{III}$$

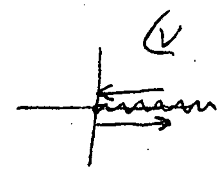
III. Asymptotic Values

$$E(z) = z e^{iz} \int e^{-2iwz} \left(\frac{w-1}{w}\right)^{\frac{iz_0}{2}} dw$$

A. $z > 0$: We are interested in E_I. The major contribution comes for $\text{Im } w \approx 0$ let $w = -iv$. It is trivial to show that the semi-circle part of the contour gives no contribution. We are then left with

$$E_I(z) \rightarrow z e^{iz} \int e^{-2vz} (-iv)^{-\frac{iz_0}{2}} -i\,dv$$

where the contour in the v-plane is:



This gives

$$|E_I(z)| \rightarrow e^{-\frac{\pi z_0}{4}} \left(\frac{e^{\pi z_0} - 1}{2}\right) \left|\Gamma\left(1 - \frac{iz_0}{2}\right)\right|$$

B. $z < 0$ We calculate first E_{III}, the incoming amplitude. $w = +iv$

$$E_{III}(z) \rightarrow z e^{iz} \int e^{2vz} (iv)^{-\frac{iz_0}{2}} i\,dv$$

$$|E_{III}(z)| \rightarrow e^{\frac{\pi z_0}{4}} \left(\frac{e^{\pi z_0} - 1}{2}\right) \left|\Gamma\left(1 - \frac{iz_0}{2}\right)\right|$$

Finally to calculate E_{IV} take $w - 1 = +iv$

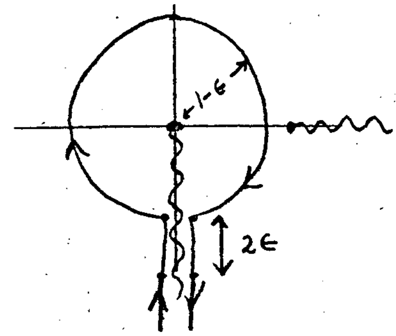
$$|E_{IV}(z)| \rightarrow e^{-\frac{\pi z_0}{4}} \left(\frac{1 - e^{-\pi z_0}}{2}\right) \left|\Gamma\left(1 + \frac{iz_0}{2}\right)\right|$$

Thus the transmission coefficient $|T| = \dfrac{|E_I|}{|E_{III}|} = e^{-\frac{\pi z_0}{2}}$

and reflection $|R| = \dfrac{|E_{IV}|(e^{\pi z_0} - 1)}{|E_{III}|} = 1 - e^{-\pi z_0}$

which are the values given by Budden.

C. $z \approx 0$ We calculate the value of the field E_I near $z = 0$. Distort the contour as shown and break the integral into three parts:



1. A circle of radius $1 - \varepsilon$ about $w = o$, ε a small positive number.
2. The two pieces of contour with $-1-\varepsilon < \text{Im}w < -1+\varepsilon$
3. $\text{Im}w < -1+\varepsilon$

Then it can easily be shown that the contribution to the second part is bounded by εzM, M a fixed number, and the contribution from the circular part of the integral is bounded by zN, N a fixed number. The third part of the contour yields, for ε arbitrarily small

$$E_I(o) = \frac{-i}{2}(1-e^{\pi z_0})$$

It is then easy to calculate $E_x(z)$, which becomes infinite at $z = 0$ in the absence of collisions.

Including a collision frequency we have

$$k^2(z) = k_0^2\left[1 + \frac{\alpha(1-\alpha+is)}{\beta+\alpha-1+is(\alpha-2)}\right]$$

$$s = \frac{\nu}{\omega} << 1$$

The collision frequency produces an insignificant change in E_y, but as E_x is given by

$$iE_x = \frac{\sqrt{\beta}\,\alpha E_y}{1-\alpha-\beta-i(s-2)}$$

we have, for $z = 0$

$$E_x(0) = -\frac{\sqrt{\beta}(1-\beta)E_y(0)}{s(1+\beta)}$$

IV. Application to a specific problem

We calculate the field in the vicinity of the resonance for physical parameters corresponding to experiments presently under way at UCLA under the direction of F. Chen.

We take as initial data

Laser	$\lambda = 3.37 \times 10^{-2}$ cm
Density	$n = 2 \times 10^{16}$
Electron	$T_e = 2$ volts
Scale length	$L = .5$ cm
Arc	5 K gauss

Thus

$$\omega_p \approx 7 \times 10^{12}$$

$$\omega_c \approx 8 \times 10^{10}$$

$$\omega = 6 \times 10^{12}$$

$$\beta = 1.8 \times 10^{-4}$$

$$\frac{d\alpha}{dx} = 2.6/\text{cm}$$

$$z_0 \cong 1$$

A direct on-line calculation of the field using the integration contours shown was carried out. The data is plotted in Fig. 2 on a scale where the initial incoming field is normalized to 1.

The author would like to acknowledge helpful discussions with A. Baños and F. Chen.

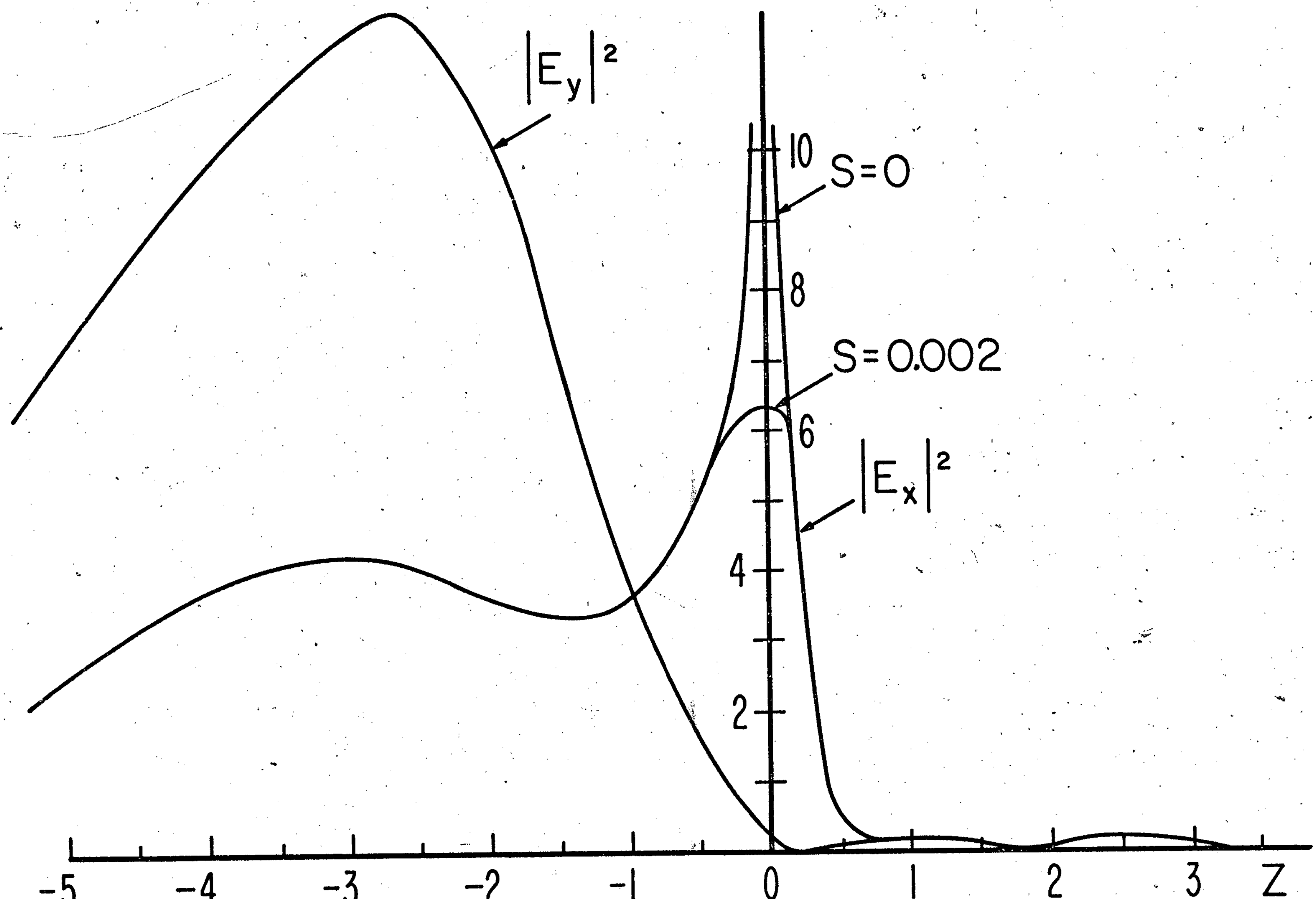


Fig. 2. $|E_x|^2$ and $|E_y|^2$ as functions of z, $z_o = +1$, $s = \frac{\nu_{eff}}{\omega}$. Incoming $|E_y|$ is normalized to 1.