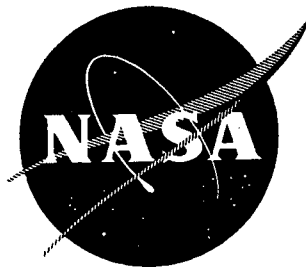


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EXPERIMENTAL DETERMINATION OF THE TURBULENCE
IN A LIQUID ROCKET COMBUSTION CHAMBER

by

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The authors wish to express their appreciation for the contribution of several individuals to the work described in this report. Mr. J. R. Huerkamp spent countless hours analyzing hundreds of gas samples and Mr. J. D. Campbell and Mr. J. P. Savant contributed numerous helpful suggestions while fabricating the experimental apparatus.

ABSTRACT

The intensity of turbulence and the Lagrangian correlation coefficient for a liquid rocket combustion chamber have been determined experimentally using the tracer gas diffusion method. The results indicate that the turbulent diffusion process can be adequately modeled by the one-dimensional Taylor theory; however, the numerical values show significant disagreement with previously accepted values. The intensity of turbulence is higher than had been previously reported by a factor of about two, while the Lagrangian correlation coefficient which has been assumed to be unity in the past is much less than unity.

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SUMMARY

The intensity of turbulence and the Lagrangian correlation coefficient in a liquid rocket combustion chamber have been determined experimentally using a small rocket engine which operated at 300 psia chamber pressure and produced approximately 250 pounds thrust. The method used consisted of injecting a tracer gas at a point along the chamber centerline while taking samples along a diameter at a downstream station. Several injection points were used for each of three sampling stations.

The gas samples were analyzed using a combination of Orsat analyzer and gas chromatograph to determine the products of combustion and the tracer gas concentration profiles. The turbulence parameters were then calculated from the concentration data using G. I. Taylor's diffusion theory.

The results indicate that turbulent diffusion in a combustion chamber can be adequately modeled by the one-dimensional Taylor theory which assumes that the intensity of turbulence is a function of the longitudinal coordinate only and the Lagrangian correlation is of the form $e^{-\alpha\tau}$ where τ is the dispersion time. The numerical values obtained from this investigation; however, show significant disagreement with previously accepted values. The intensity of turbulence is higher than had been previously reported by a factor of about two, varying from a maximum of approximately 30% near the injector to 10% at the nozzle entrance. On the other hand, the

Lagrangian correlation coefficient which has been assumed to be unity by previous investigators, was found to be much smaller than unity. Using the $e^{-\alpha r}$ expression, the correlation decreases to 0.1 in less than 2-1/2 inches of chamber length.

INTRODUCTION

Gas-phase mixing is one of the processes which is generally assumed to influence rocket engine performance. Turbulent flow theory indicates that the rate of gas-phase mixing is a function of the intensity of turbulence and the Lagrangian correlation coefficient. There is very little information available regarding these quantities and only one previous attempt has been made to experimentally measure the intensity of turbulence under the actual environment of a rocket combustion chamber.

Hersch¹ experimentally determined the intensity of turbulence in a two-dimensional hydrogen-liquid oxygen rocket engine using a photographically visible tracer material. Hersch made the assumption that the Lagrangian correlation was unity and used the equations developed by Bittker² in order to reduce the photographic data to values of intensity of turbulence.

During the work reported herein both the intensity of turbulence and the Lagrangian correlation coefficient in a liquid rocket combustion chamber were determined. An indirect experimental method was used to determine these quantities. The method consisted of injecting helium tracer gas at an upstream point in the combustion chamber while taking samples along a diameter at a downstream station. These samples were analyzed to determine the helium concentration, thus determining the spreading of the tracer gas. The intensity of

turbulence and Lagrangian correlation coefficient could be calculated from turbulent flow theory and the measured helium tracer gas distributions by means of the analytical methods set forth herein.

SYMBOLS

D_t	turbulent diffusion coefficient
L	lateral scale of turbulence
l	longitudinal distance in nozzle
l_{mer}	distance from injector to end of zone of maximum energy release rate
\bar{M}	mean molecular weight of products of combustion
n	number of moles of a gaseous combustion or tracer component
\bar{P}	mean free stream pressure of products of combustion
R	radius of combustion chamber
R_L	Lagrangian correlation coefficient
r	radial cylindrical coordinate
\bar{r}^2	mean square dispersion radius
\bar{T}	mean free stream temperature of products of combustion
t, t', t''	dispersion times
\bar{U}_z	mean flow in axial direction
$\overline{u'^2}, \overline{v'^2}, \overline{w'^2}$	instantaneous mean square turbulent velocity in $\theta, r,$ and z directions respectively
X	mole fraction
\bar{Y}^2	mean square dispersion distance
Z	longitudinal distance between sampling station and helium injection point
z	longitudinal cylindrical coordinate

α	constant defined in equation 9
ϵ	contraction parameter in equation 14
θ	angular cylindrical coordinate
$\bar{\rho}$	mean free stream density of products of combustion
T	intensity of turbulence
τ	dispersion time
$\bar{\omega}_{He}$	mass concentration of helium tracer gas

Subscripts

A	start of nozzle contraction
B	arbitrary longitudinal station along nozzle
I	helium injector location
S	sampling location
w	chamber wall

Superscripts

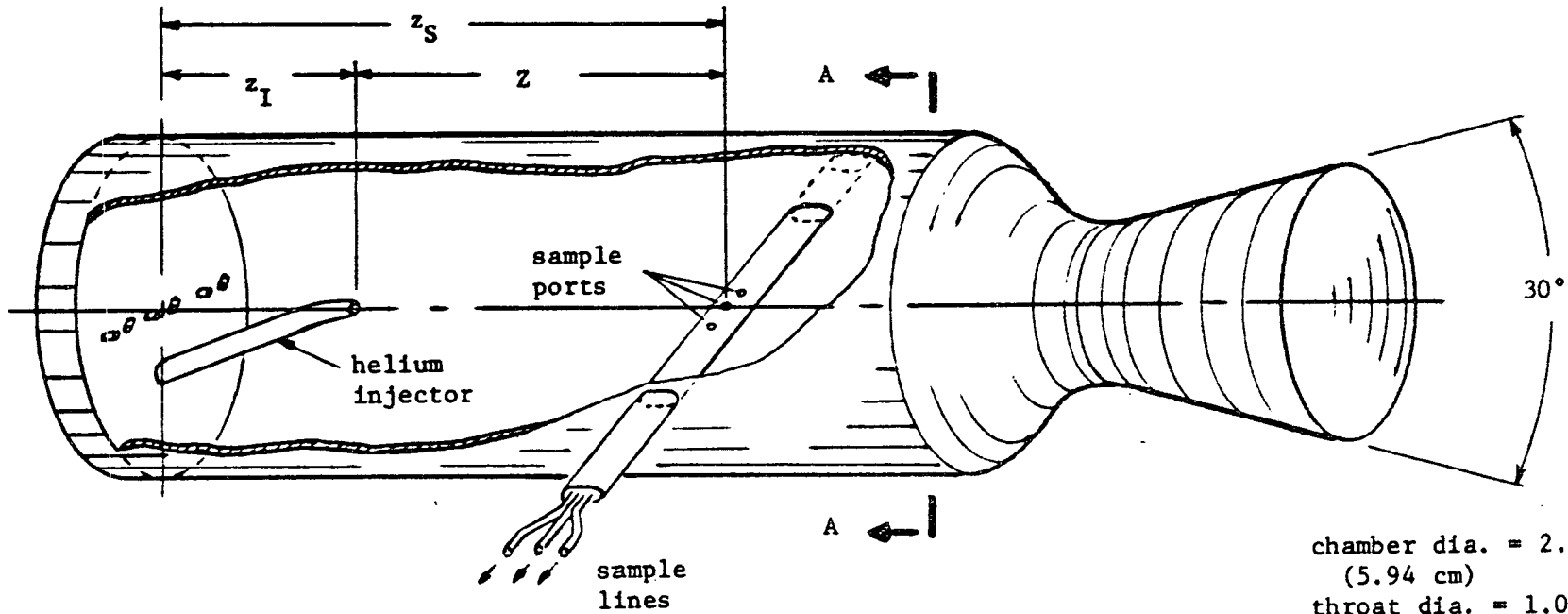
$\bar{\quad}$ (over bar)	mean or average values
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EXPERIMENTAL APPARATUS AND PROCEDURE

A small rocket engine using liquid oxygen and heptane operating at a nominal chamber pressure of 300 psia was used in these experiments (see Figure 1). Probes of various lengths could be inserted through the injector to allow the helium tracer gas to be released at various points along the center line of the combustion chamber. Another probe passing across the chamber along a diameter was used to withdraw gas samples. This probe which contained three (3) sample ports could be moved laterally across the chamber or rotated circumferentially about the chamber center line. In addition, the sample probe could be placed at several longitudinal stations.

A like-on-like impingement injector with dimensions as shown by Figure 1 was used. The drop distribution for the injector had been precisely determined by Heidmann and Foster³. Further information regarding the experimental apparatus is given by Partus⁴.

An automatic timing device was used to fire the engine and take the samples. About 1-1/2 seconds was required for the engine to reach equilibrium during which time the sample valves were closed and the lines between the sample valves and engine were purged with argon. The helium tracer gas was injected during start-up so that it would also reach an equilibrium condition. The purge was then stopped and samples were withdrawn for about 1-1/2 seconds. Each sample would thus consist of products of combustion, the helium, and a considerable amount of argon. The samples were collected in sample bottles for later analysis.



chamber dia. = 2.34 in
 (5.94 cm)
 throat dia. = 1.00 in
 (2.54 cm)
 exit dia. = 2.00 in (5.08 cm)
 mass flow rate LOX =
 .89 lb/sec (.39 Kg/sec)
 mass flow rate heptane =
 .45 lb/sec (.20 Kg/sec)

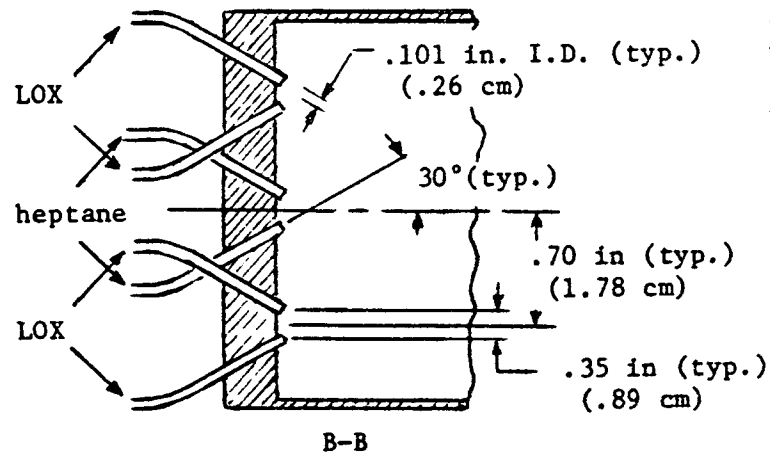
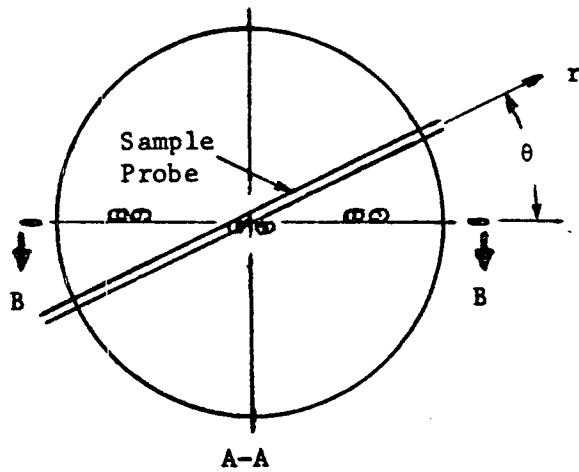


Figure 1 Sampling Configuration

ANALYSIS OF SAMPLE

A method combining absorption and chromatographic techniques was used to analyze the samples for carbon dioxide (CO_2), oxygen (O_2), carbon monoxide (CO), hydrogen (H_2), helium (He), and argon (A). The water content was not measured. The amount of CO_2 , O_2 , CO , and most of the H_2 were measured by selective absorption using a Orsat analyzer⁵. The absorption method was used because it is well suited for analyzing these gases in large concentrations. Also, all of these gases, CO_2 , O_2 , CO , and H_2 , tend to contaminate the chromatograph column.

The remaining sample which contained A, He, and a small amount of H_2 was analyzed using a gas chromatograph to determine the concentration of He and H_2 in a manner similar to that given by Villalobos⁶. A Linde 5A molecular sieve column was used with argon as the carrier gas.

EXPERIMENTAL DATA

Gas samples were taken from three longitudinal stations along the combustion chamber. For each of these stations a range of helium injector points along the chamber centerline were investigated. Also different circumferential angles of the sample probe with respect to the propellant injector were used. Three of the many helium concentration profiles are shown in Figure 2 to give an idea of the experimental scatter. The curves also show the effect of the walls which will be discussed later.

The experimental results did not show any variation of helium concentration with circumferential angle. The mean square dispersion radius could be calculated assuming axial symmetry by

$$\overline{r^2} = \frac{\int_0^{\infty} \overline{\omega}_{\text{He}} r^3 dr}{\int_0^{\infty} \overline{\omega}_{\text{He}} r dr} \quad (1)$$

Values of $\overline{r^2}$ were calculated by numerically integrating the helium concentration curves to give the experimental data points shown on Figure 3. These curves show the mean square dispersion radius versus the distance between the injection point and the sampling station for the three sampling stations investigated. Note that for a 2.34" diameter combustion chamber the maximum value which $\overline{r^2}$ can be is 0.685 in.², i.e., a flat helium distribution curve.

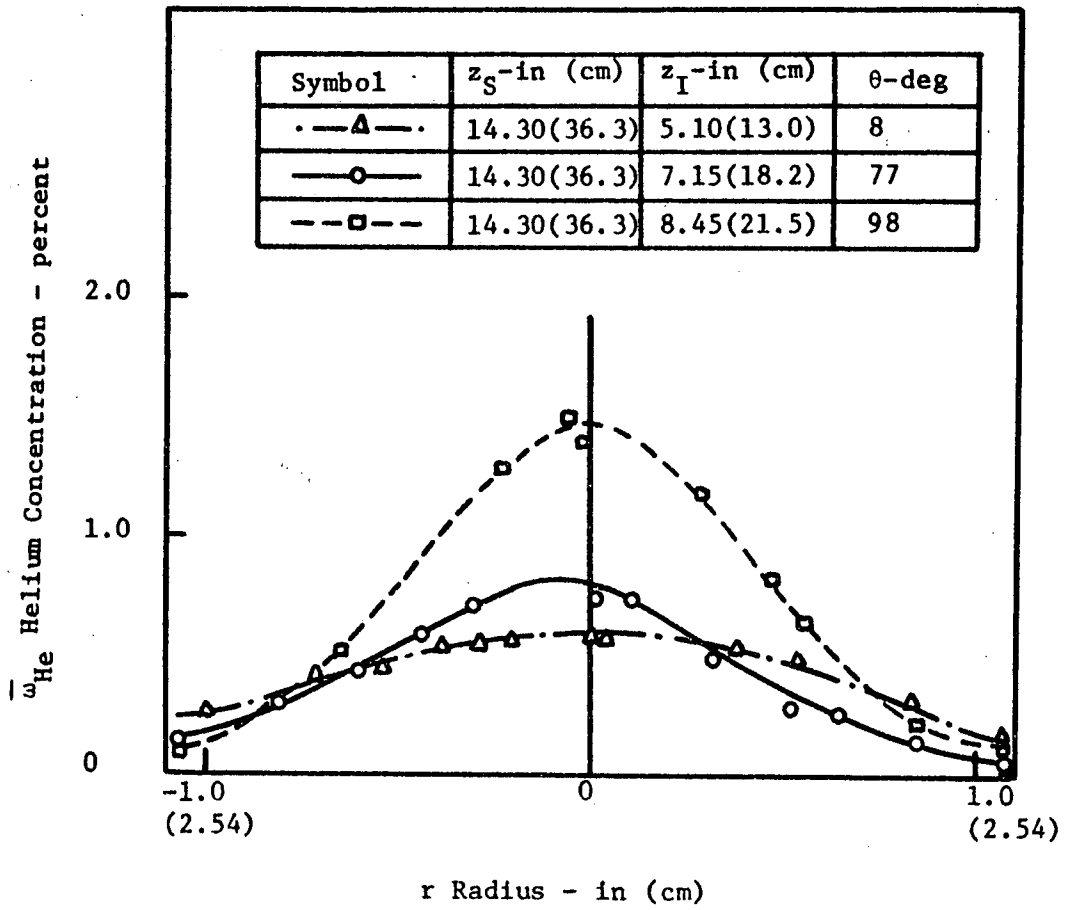


Figure 2 Typical Helium Concentration Profiles

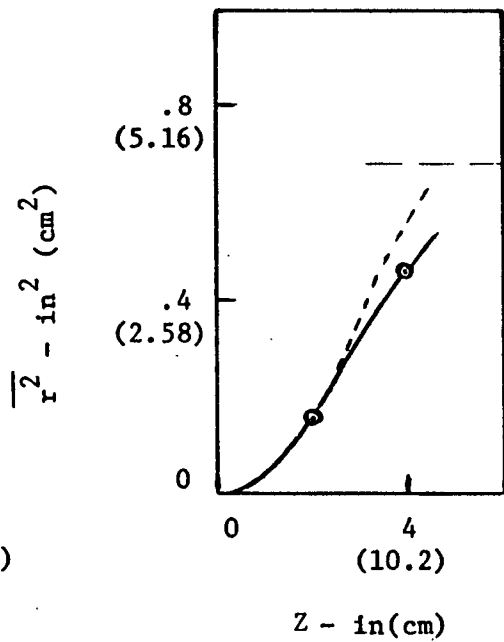
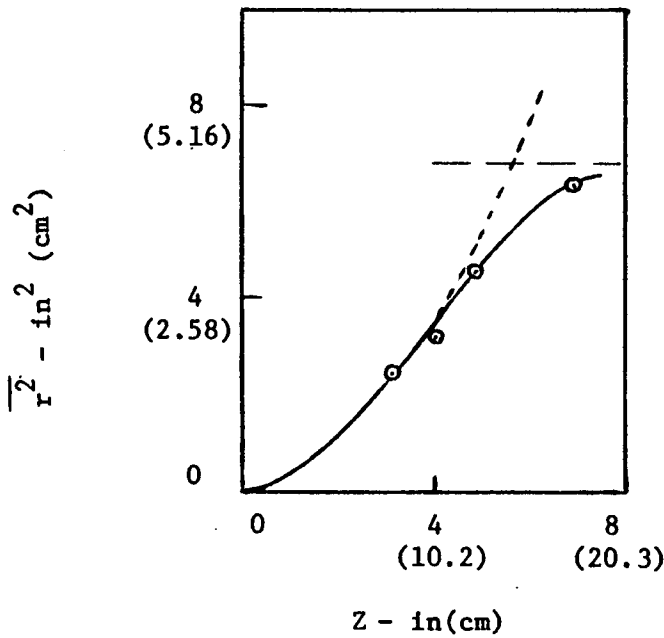
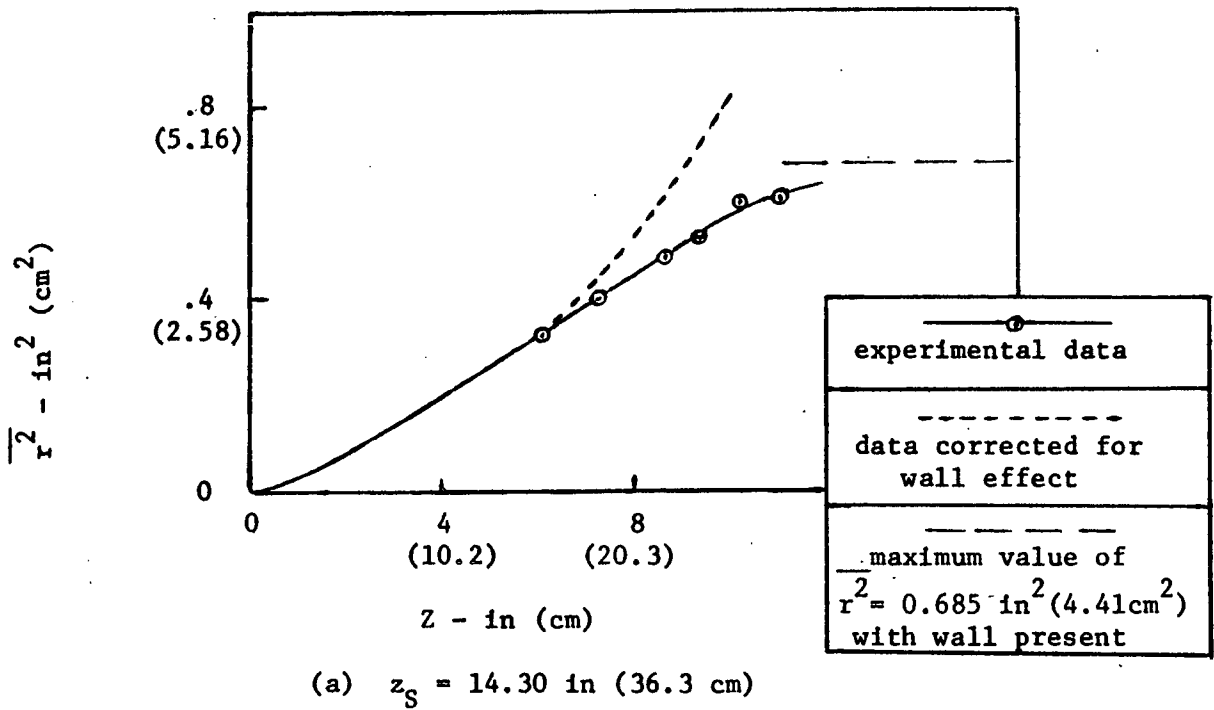
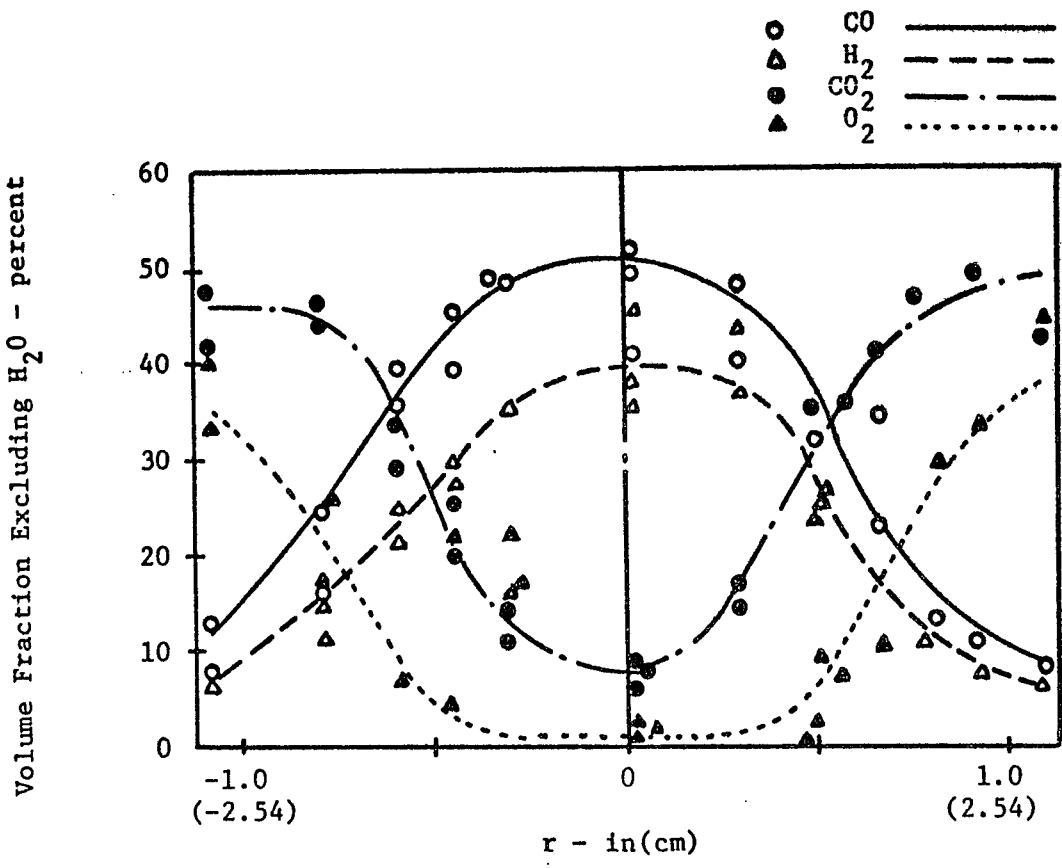


Figure 3 Experimental Mean Square Dispersion Radius

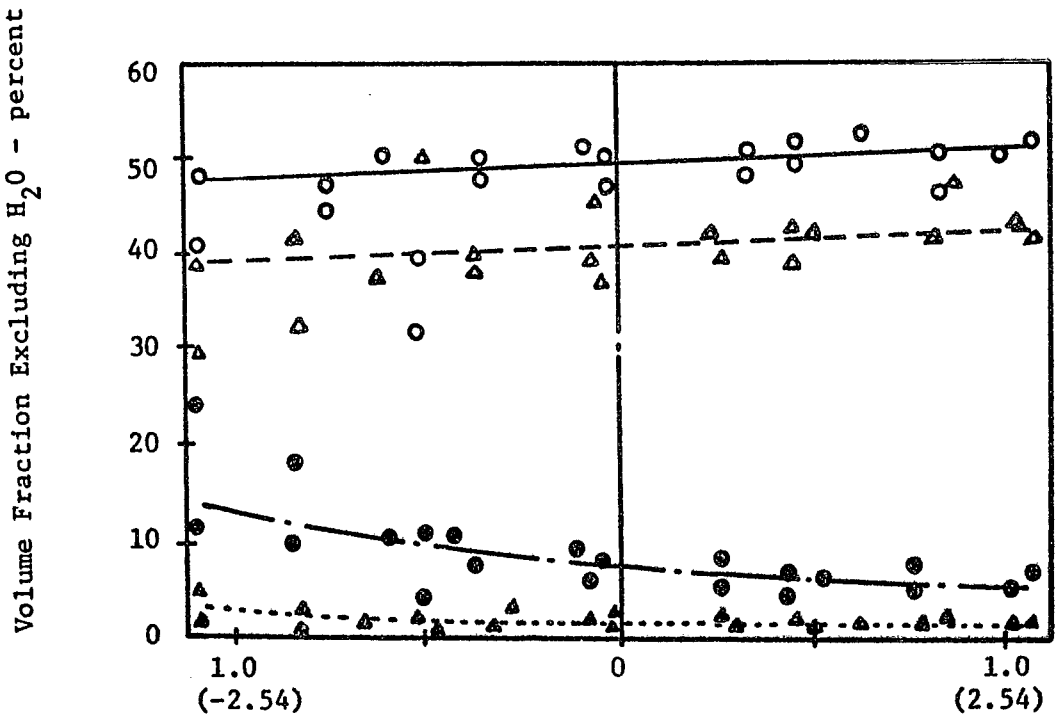
A by-product of analyzing the sample for the tracer gas concentration is that the concentrations of CO_2 , O_2 , CO , and H_2 in the products of combustion are determined. The H_2O content was not measured. Figure 4 shows the mole fractions of each of the products of combustion, excluding water, for the three sampling stations. The mole fraction (or volume fraction) is defined as

$$x_j = \frac{n_j}{\sum_i n_i} \quad (2)$$

where i is CO_2 , O_2 , CO , and H_2 , j is the component being considered and n is the number of moles of each gas. No analyses have been made of the product of combustion concentration data except as required to show that they are consistent and reasonable.



(a) $z_s = 6.05$ in (15.4 cm); $\theta = 0^\circ$



(b) $z_s = 6.05$ in (15.4 cm); $\theta = 90^\circ$

Figure 4 Products of Combustion Excluding Water

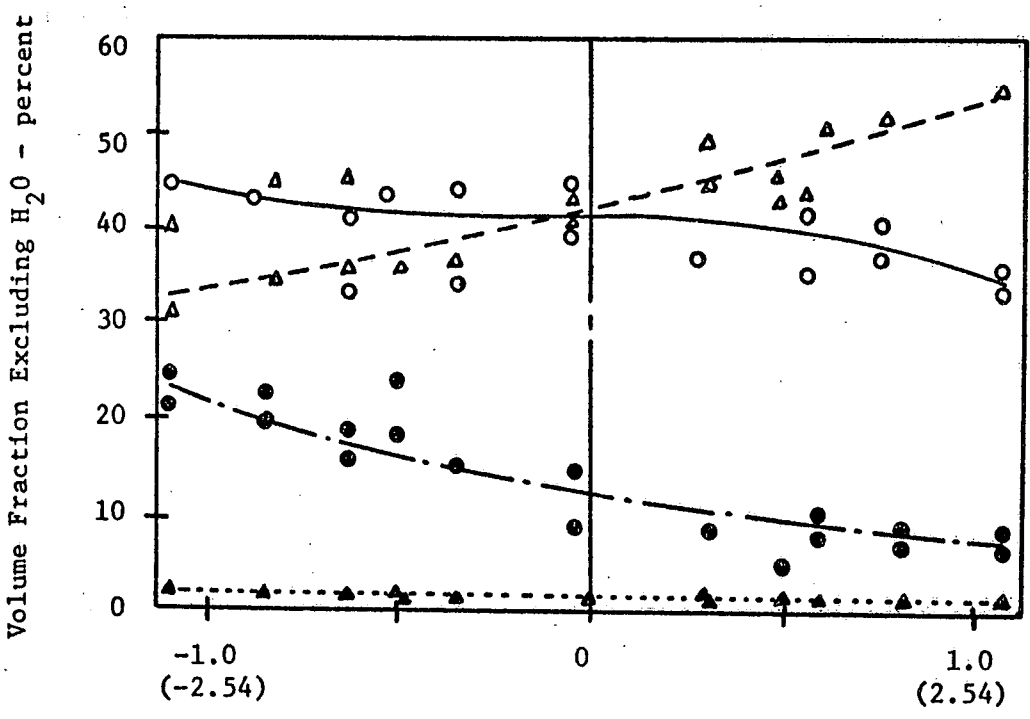
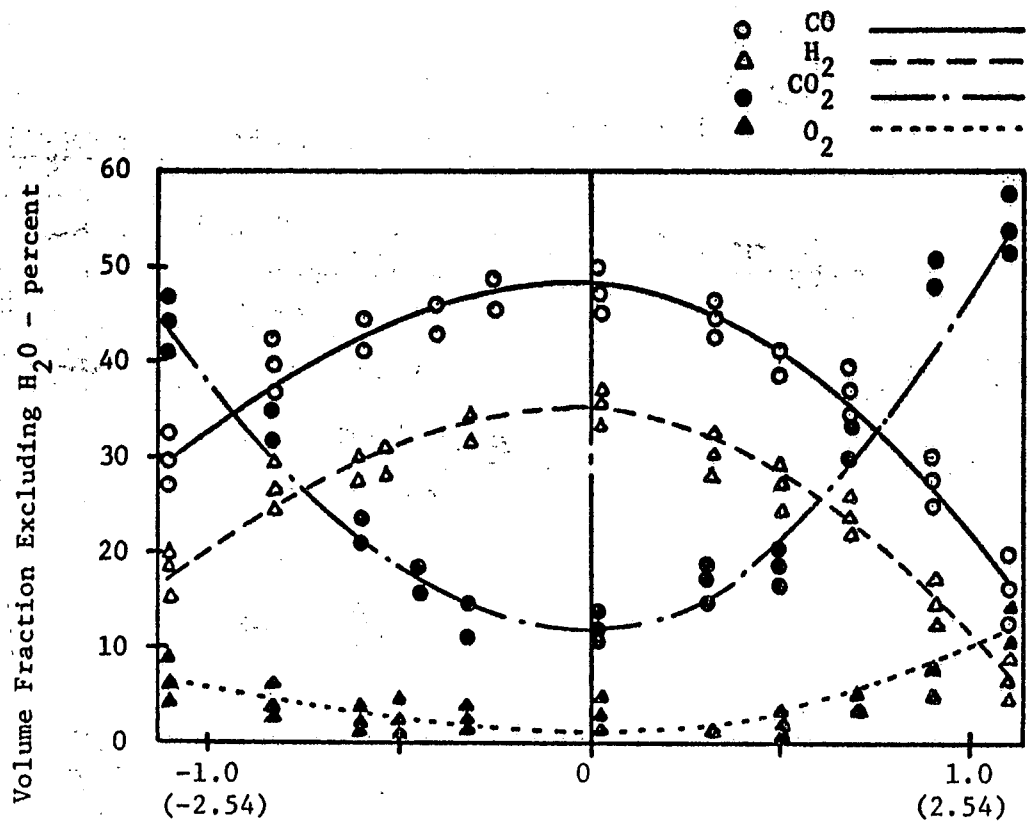
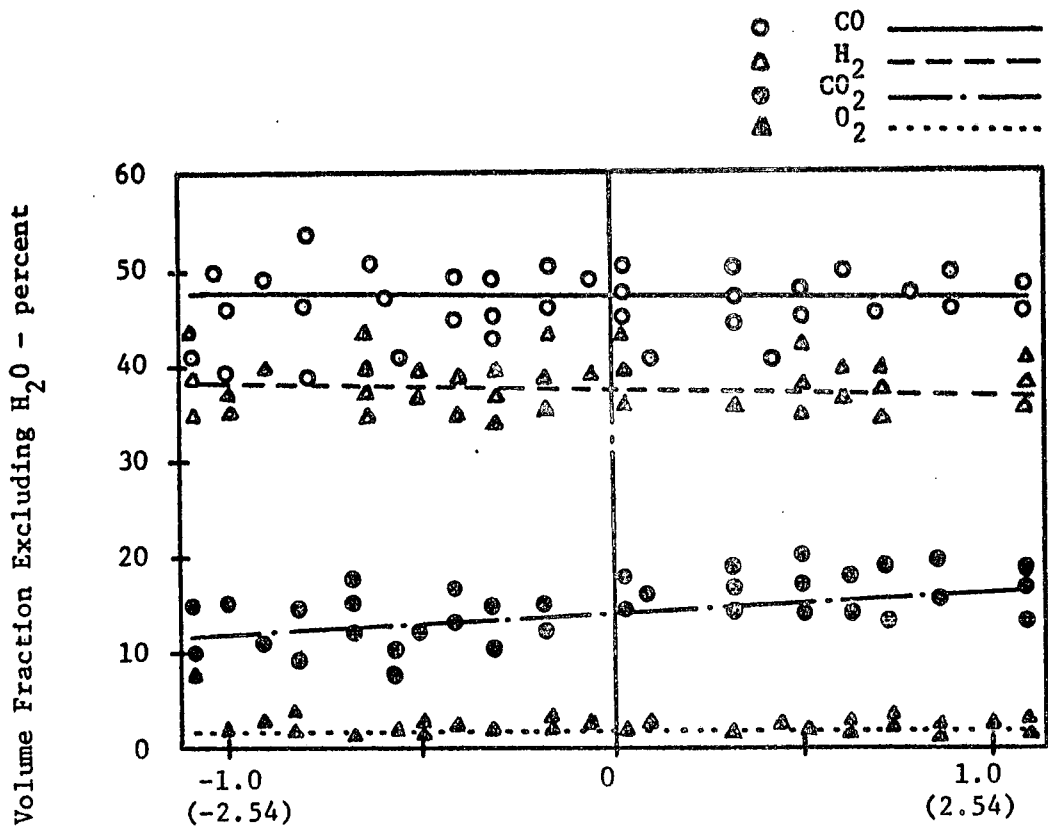
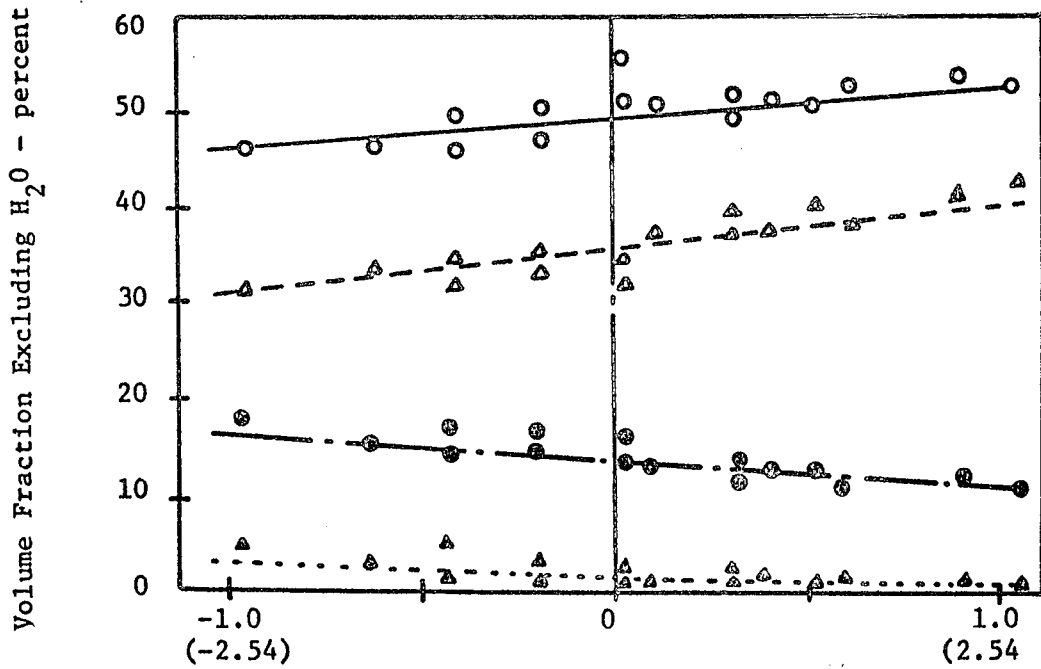


Figure 4 Products of Combustion Excluding Water (continued)



(e) $z_s = 14.30$ in (36.3 cm); $\theta = 8^\circ$



(f) $z_s = 14.30$ in (36.3 cm); $\theta = 98^\circ$

Figure 4 Products of Combustion Excluding Water (concluded)

ANALYSIS OF RESULTS

Theoretical Development

The task is now to develop sufficient equations to allow the determination of the intensity of turbulence and the Lagrangian correlation coefficient from the mean square dispersion data available. Two approaches are available for solving turbulent diffusion problems. These are the Lagrangian approach first developed by G. I. Taylor and the Eulerian approach sometimes referred to as the turbulent Fick equation. Both of these approaches will be used in the analysis of the experimental data.

The equation for turbulent motion developed by Taylor⁷ can be written

$$\overline{r^2} = 4 \int_0^t dt' \int_0^{t'} d\tau R_L(\tau) \sqrt{v'^2(t')} \sqrt{v'^2(t' - \tau)} \quad (3)$$

where $\overline{r^2}$, the mean square dispersion in cylindrical coordinates, is twice $\overline{Y^2}$, the mean square dispersion in cartesian coordinates as given by Taylor. The Lagrangian correlation, $R_L(\tau)$, is defined as:

$$R_L(\tau) \equiv \frac{v'(t') v'(t' - \tau)}{\sqrt{v'^2(t')} \sqrt{v'^2(t' - \tau)}} \quad (4)$$

It is usually assumed that the turbulence is homogenous, i.e., $\overline{v'^2}$ is constant and thus can be taken outside the integral in equation (3).

Equation (3) can then be integrated by parts as was done by Kampe de Feriet^{8,9} to get an equation of one variable. In the case of the rocket chamber, it cannot be assumed a priori that $\overline{v'^2}$ is constant; thus the double integral must be retained. It will be assumed that the mean square turbulent velocity $\overline{v'^2}$ is a function of only the longitudinal coordinates, z . Also, it will be assumed that the turbulence is isotropic; i.e., $\overline{u'^2} = \overline{v'^2} = \overline{w'^2}$ at every point (r, θ, z) .

The time t required for a particle of fluid to traverse the longitudinal distance from the point of release z_I to any point of interest z is related to the longitudinal space coordinate by

$$t = \int_{z_I}^z \frac{dz}{\overline{U}_z(z)} \quad (5)$$

where t is zero when the particle is released. The Taylor equation is applicable for free isotropic turbulence, where the turbulence can be a function of only the longitudinal coordinate, z . Also, the Lagrangian correlation coefficient has been assumed to be a function of the dispersion time only.

The presence of the chamber walls had a significant effect on the experimental data and the data must be corrected to give an equivalent free turbulence condition before the Taylor equation can be used. In order to correct for this effect it is necessary to use the turbulent Fick equation which can be written in cylindrical coordinates as¹⁰

$$\bar{\rho} \bar{U}_z \frac{\partial \bar{\omega}_{\text{He}}}{\partial z} + \bar{\omega}_{\text{He}} \frac{\partial(\bar{\rho} \bar{U}_z)}{\partial z} = \frac{D_t}{r} \frac{\partial}{\partial r} \left(r \bar{\rho} \frac{\partial \bar{\omega}_{\text{He}}}{\partial r} \right) \quad (6)$$

This equation gives the concentration of the tracer gas, $\bar{\omega}_{\text{He}}$, as a function of two variables, r and z . It has been assumed that the flow is steady and circumferentially symmetrical. Also, it has been assumed that diffusion in the axial direction is negligible.

It can be shown¹¹ that the turbulent diffusion coefficient is related to the dispersion by

$$D_t = \frac{\bar{U}_z}{4} \frac{d\bar{r}^2}{dz} \quad (7)$$

Correction for Wall Effects

The mass transport equation (6) was solved by finite difference methods on a digital computer¹² for the case of helium being injected into a stream with a mean velocity field $\bar{U}_z(z)$. The program calculates the helium concentration profile at any specified downstream station for two different boundary conditions. These boundary conditions are:

(i) free turbulence

$$\bar{\omega}_{\text{He}} = 0 \text{ @ } r = \infty$$

(ii) with wall at $r = R$

(8)

$$\frac{\partial \bar{\omega}_{\text{He}}}{\partial r} = 0 \text{ @ } r = R$$

The mean velocity $\bar{U}_z(z)$ was calculated using the steady state combustion computer program given by Breen, et.al.¹³. This is a program based on

the one-dimension vaporization model of Priem and Heidmann¹⁴. The program was modified to use the experimental drop distribution for the injector used for these experiments. Figure 5 shows the mean flow conditions which were used for this work.

Concentration profiles were calculated for the cases of free turbulence and with the wall for a range of $D_t(z)$'s and several different injection points. It was found that the ratio of the mean square dispersion with the wall to the mean square dispersion for free turbulence can be correlated with the ratio of the wall to centerline helium concentrations calculated using the wall boundary conditions. This correlation, shown by Figure 6, was found to be essentially independent of diffusion coefficient, injection point or mean velocity. The experimental data corrected for wall effect to give equivalent "free turbulence" values is shown on Figure 3.

Determination of Turbulent Parameters

The objective of this section is to find the turbulent velocity as a function of z only, i.e., $\sqrt{v'^2(z)}$, and a constant value of α , defined below, which will satisfy the experimental data, provided they exist. In order to get a first approximation of these quantities the combustion chamber was divided into 28 sections in the axial direction and the turbulent velocity $\sqrt{v'^2}$ was assumed constant over each section. The correlation was assumed to be of the form

$$R_L(\tau) = e^{-\alpha\tau} \quad (9)$$

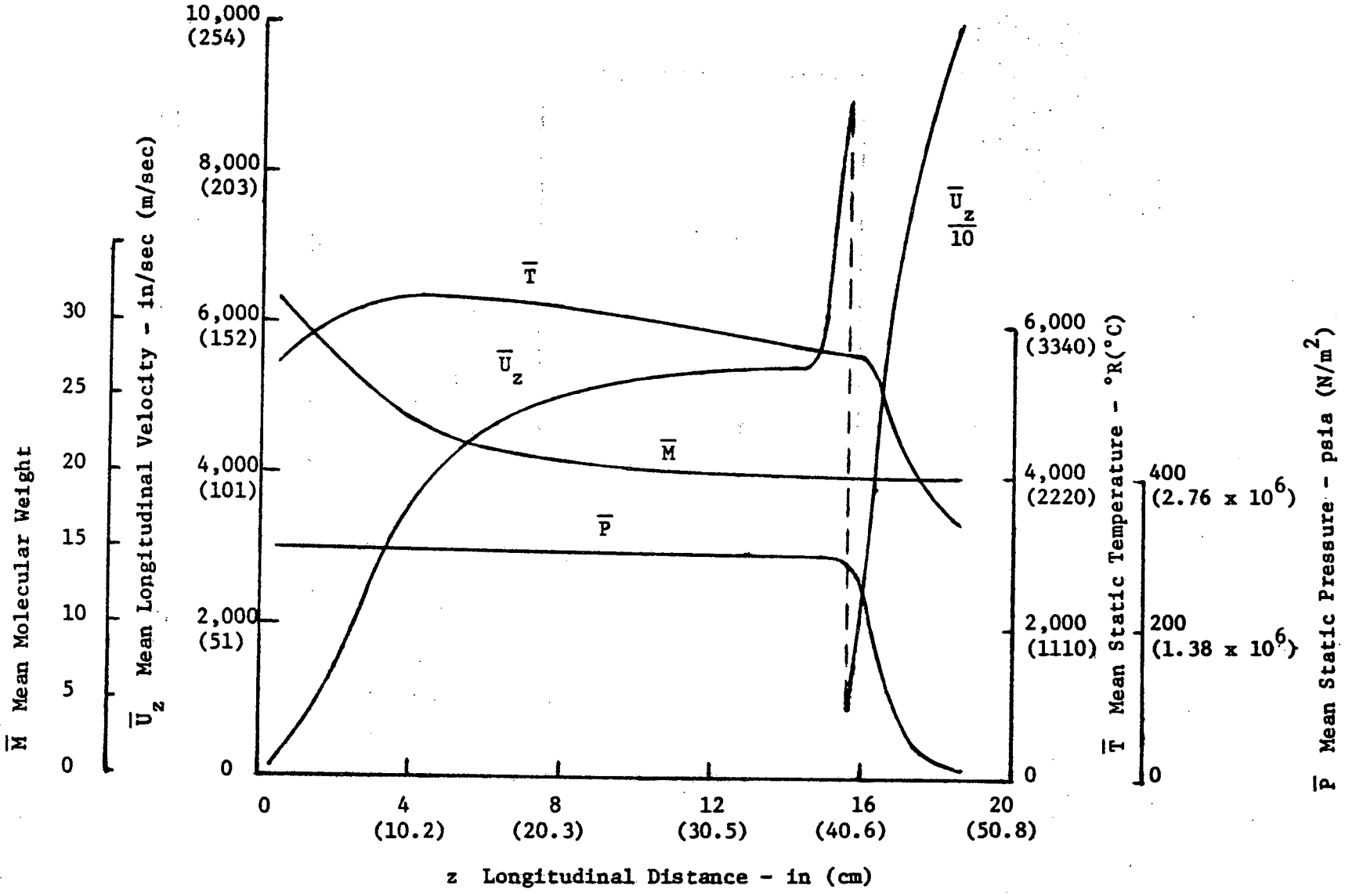


Figure 5 Combustion Vapor Properties

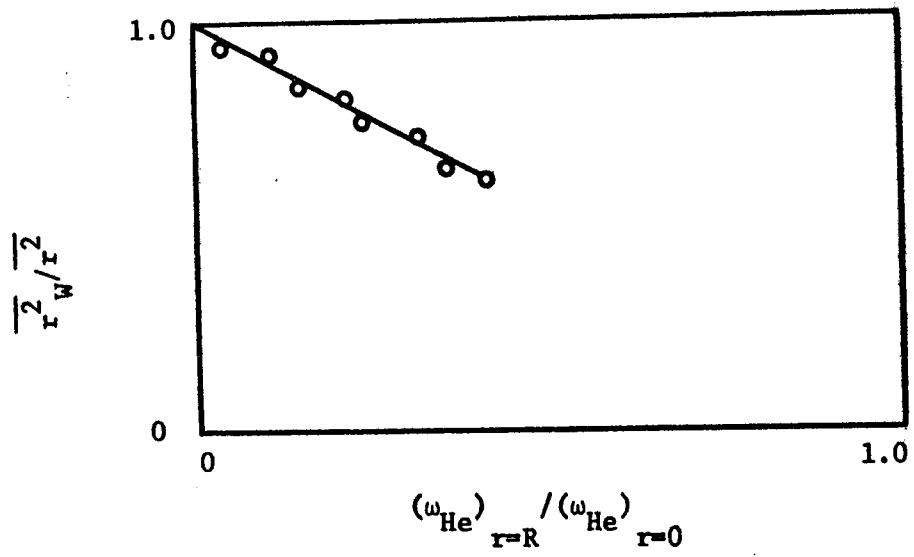


Figure 6 Wall Effect Correlation

Using these assumptions and the experimental data from the 14.3" sampling station, curves of $\sqrt{v'^2}$ can be calculated for a range of values of α as outlined below. The 14.3" station is chosen because the experimental data for this station spans a greater portion of the combustion chamber.

Let the injection point be a multiple of 0.5" upstream of the sample station. Equation (3) can then be written as the sum of the integrals over each 0.5" increment of the distance between the injection point and the sample station. When the injection point is only 0.5" from the sample station, there will be only one value of turbulent velocity, $\sqrt{v'^2}$ and the value of \bar{r}^2 at that distance can be read from Figure 3a for $Z = .5$ ". Then for any value of α , $\sqrt{v'^2(28)}$ can be calculated. Next consider the injection point is 1" or 2 steps away from the sample station. $\sqrt{v'^2(27)}$ will be unknown, \bar{r}^2 can be read from the corrected curve of Figure 3a, and $\sqrt{v'^2(28)}$ has been precisely calculated. Thus, for the same values of α , $\sqrt{v'^2(27)}$ can be calculated. This process can be repeated to find the remaining $\sqrt{v'^2}$ as long as \bar{r}^2 values are available.

These calculations were made to obtain curves of the root mean square turbulent velocities for a range of values of α , from $\alpha = 5 \text{ sec}^{-1}$, which gives $R_L \approx 1$, to a value of $\alpha = 20,000 \text{ sec}^{-1}$, which gives $R_L \approx 0$. After a first approximation of these curves were obtained by the method described above, the curves were adjusted by trial and error to get a better fit of the experimental data. The resulting turbulent velocities are shown on Figure 7. It should be noted that so far no determination has been made as to which $\sqrt{v'^2}$ curve and associated α

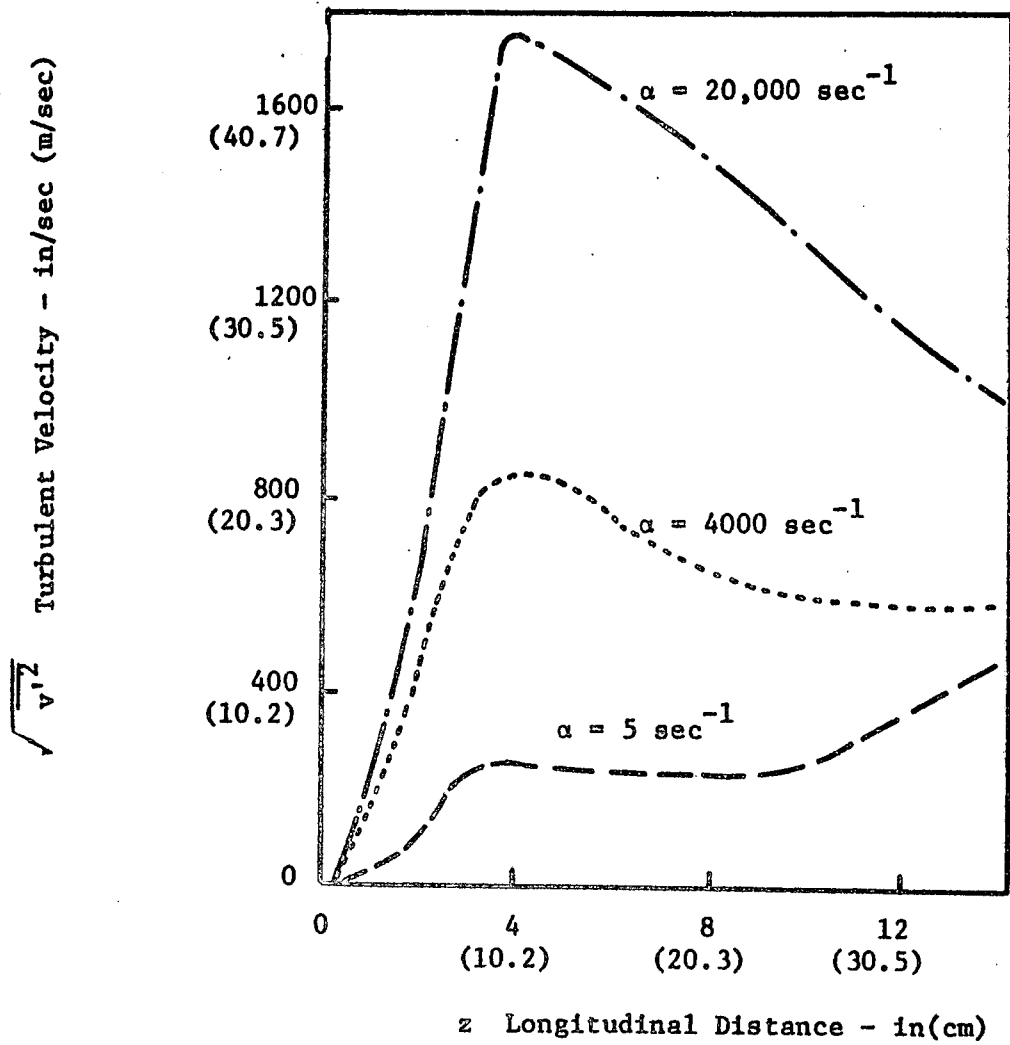


Figure 7 Turbulent Velocities for Various Correlation Coefficients

are correct. Any one of these curves can be used in either the Fick equation (6) or the Taylor equation (3) to recalculate $\overline{r^2}$ and the corrected curve of Figure 3a will be approximately duplicated.

Equation (3) can now be solved in the forward direction by numerical methods to give accurate values of $\overline{r^2}$ for each of the $\sqrt{v'^2}$ curves and constant α 's. This was done for a range of injection points and the three sample stations for which experimental data were available. The result of the calculations are shown on Figure 8. Note that all of the cases approximately duplicate the data for the 14.30" sampling station (Figure 8a); however, there is a large variation for the other two sampling stations (Figure 8b and c). The curves for $\alpha = 4000 \text{ sec}^{-1}$ appears to give the best fit for the experimental data.

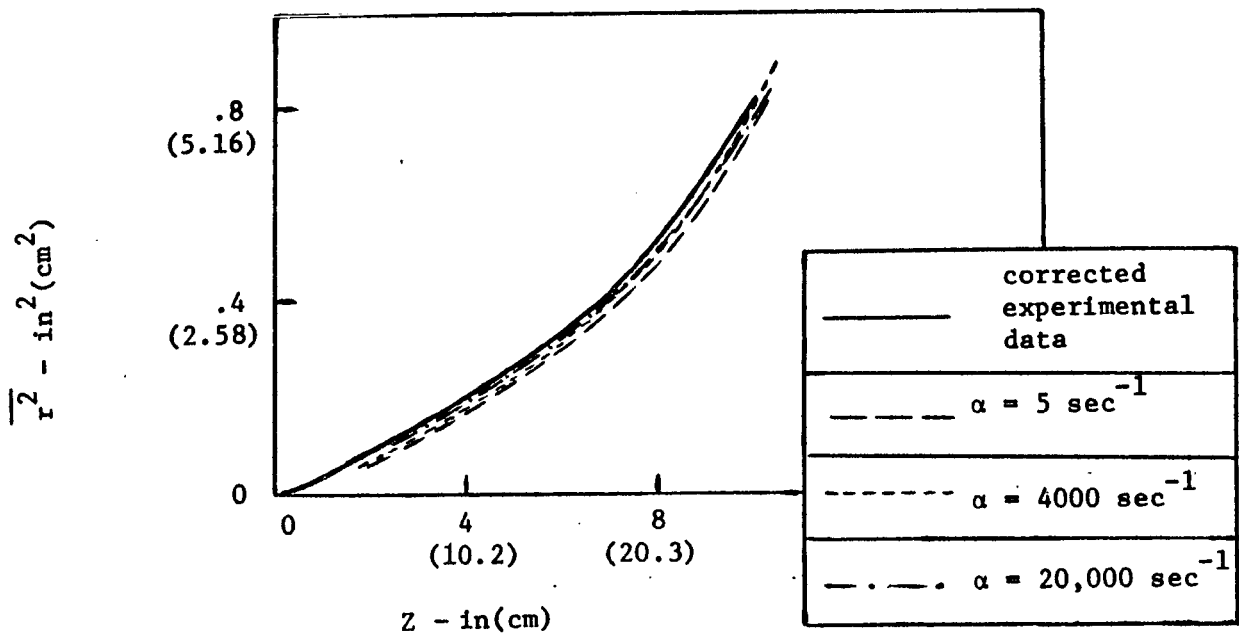
The above analysis shows that the turbulent diffusion in the rocket engine configuration used for these tests can be best represented using the curve for v'^2 from Figure 7 labeled $\alpha = 4000 \text{ sec}^{-1}$.

Application of Results

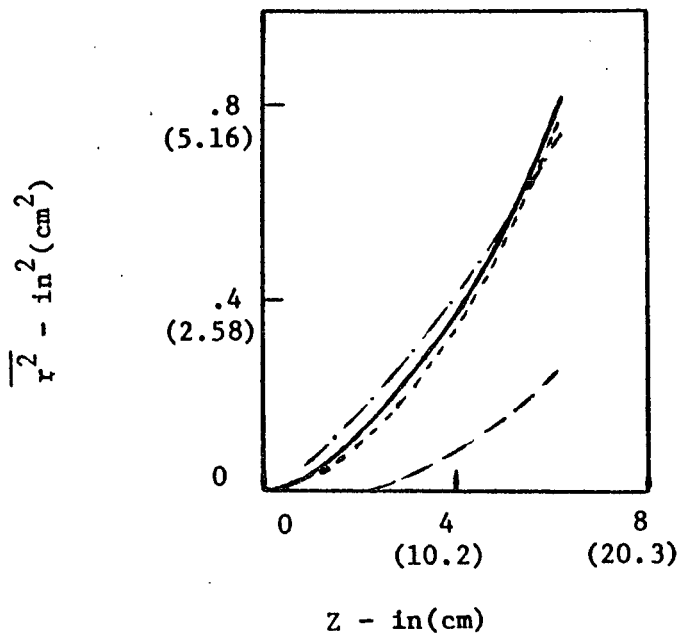
The turbulent intensity, defined as

$$T \equiv \frac{\sqrt{\overline{v'^2}}}{\overline{U}_z} \times 100 \quad (10)$$

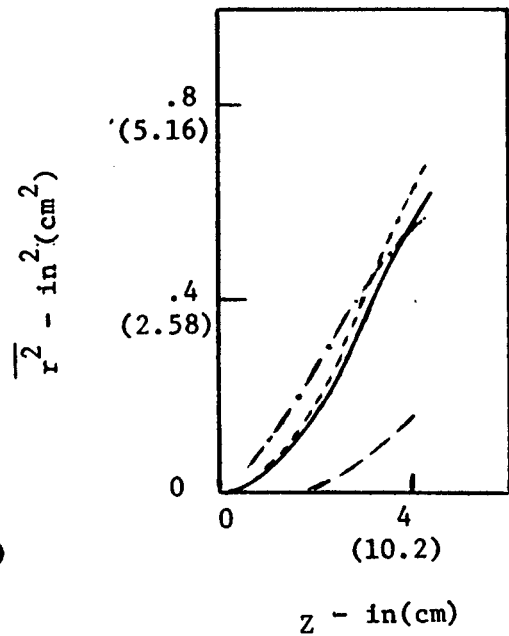
has been calculated using the "best fit" turbulent velocity curve found above and is shown on Figure 9. In addition to showing the turbulent



(a) $z_S = 14.30$ in (36.3 cm)



(b) $z_S = 8.90$ in (22.6 cm)



(c) $z_S = 6.05$ in (15.4 cm)

Figure 8 Comparison of Calculated and Experimental Data

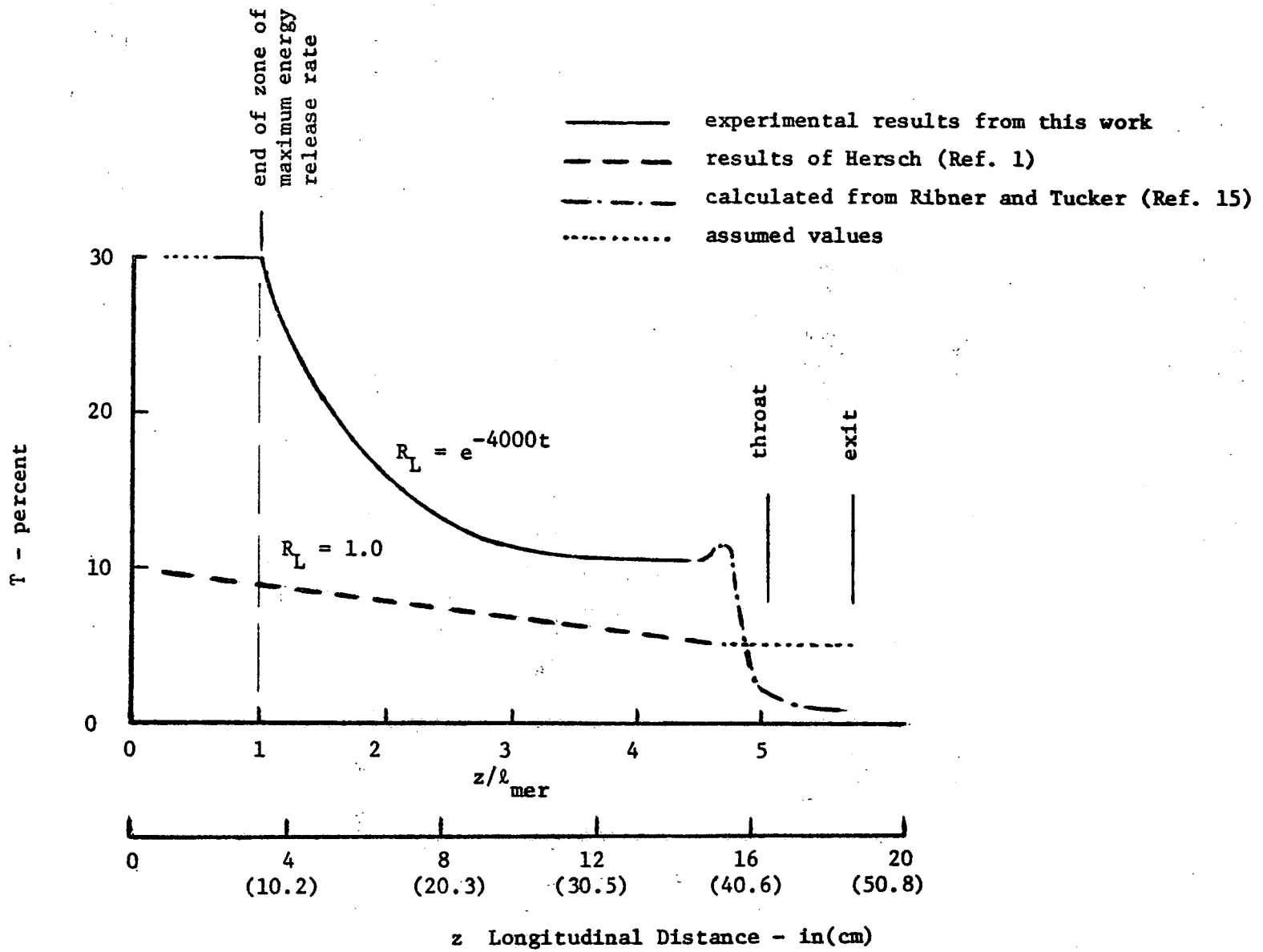


Figure 9 Recommended Intensity of Turbulence

intensity as a function of the longitudinal distance, z , Figure 9 also shows the intensity as a function of the dimensionless ratio z/ℓ_{mer} where ℓ_{mer} is the distance from the injector to the end of the zone of maximum energy release.

Only one operating configuration was investigated in the course of this work, so that no experimental information was obtained with regard to how the results given herein may be scaled to other configurations. However, the dimensionless ratio, z/ℓ_{mer} , is believed to be more significant than the dimensional distance, z , for scaling purposes. It is also believed that over a limited range the intensity of turbulence should be proportional to the volumetric energy release rate and inversely proportional to the mean gas density and velocity. It is further believed that the correlation coefficient is relatively insensitive to changes in operating conditions and in no case will it approach unity. Again, none of these scaling relations have been proven.

No experimental results were obtained in the region from the injector to about 2" downstream so that in this region the intensity of turbulence is taken to be 30% from an extrapolation of the experimental data. Also the mixing process is assumed to start 0.3" downstream from the injector face.

Likewise, no experimental results were obtained for the nozzle region and a continuation of the intensity of turbulence curve was

obtained by using the theory of Ribner and Tucker¹⁵. Ribner and Tucker treat the case of changes in lateral turbulent velocity in an accelerating flow with viscous decay. They found that the ratio of the mean square lateral turbulent velocity, $\overline{v'_B{}^2}$, at any point in the contracting section to the mean square lateral turbulent velocity, $\overline{v'_A{}^2}$, at the entrance to the contracting section could be expressed as a function of the longitudinal distance, ℓ , between the two stations by the expression

$$\frac{\overline{v'_B{}^2}}{\overline{v'_A{}^2}} = C(\ell) D(\ell) \quad (11)$$

where $C(\ell)$ is the contribution due to the contraction and is given by the expression

$$C(\ell) = \frac{3}{8} \left(\frac{R_A}{R_B} \right)^2 \left[\frac{2 - \epsilon}{1 - \epsilon} - \frac{\epsilon^2}{(1 - \epsilon)^{3/2}} \tanh^{-1} \sqrt{1 - \epsilon} \right] \quad (12)$$

and $D(\ell)$ is the contribution due to viscous decay and is given by the empirical expression

$$D(\ell) = \frac{1}{1 + 0.58 \left(\sqrt{\overline{v'_A{}^2}} / L_A \right) \tau(\ell)} \quad (13)$$

The contraction parameter, ϵ , is given by

$$\epsilon = \left(\frac{R_B}{R_A} \right)^2 \left(\frac{\overline{U}_{zA}}{\overline{U}_{zB}} \right)^2 \quad (14)$$

and the decay time, $\tau(\ell)$, is the time required to traverse the distance ℓ . The decay time can be calculated from the relation:

$$\tau(\ell) = \int_A^B \frac{dz}{\bar{U}_z} \quad (15)$$

The lateral scale of turbulence at point A was calculated using the definition of scale of turbulence

$$L_A = \int_0^\infty R_L dz \quad (16)$$

where the correlation coefficient was taken to be e^{-4000t} and $dz = \bar{U}_{zA} dt$. This yielded a value of $L_A = 1.4''$.

The results of Hersch are also shown on Figure 9 for comparison. The continuation of this curve into the nozzle section at a constant value of $T = 5\%$ was not recommended by Hersch; however, in practice this extrapolation has been made.

The data of Figure 9 were used to calculate the mean square dispersion for a gas injected at the injector face ($z = 0.3''$) and allowed to disperse for the complete length of the engine using the findings from the present work as well as the results of Hersch. The values were obtained by solving the Taylor equation in the double integral form

$$\overline{r^2} = \frac{4}{10^4} \left(\frac{R_B}{R} \right)^2 \int_0^t dt' T(t') \bar{U}_z(t') \int_0^{t'} dt R_L(\tau) T(t' - \tau) \bar{U}_z(t' - \tau) \quad (17)$$

on a digital computer. The ratio $(R_B/R)^2$ is included to account for the change in cross-sectional area of the stream tube as the flow passes through the nozzle. In order to obtain values of T and \bar{U}_z it is first necessary to obtain a functional relationship between dispersion time and longitudinal position. This was done by using the equation

$$t'' = \int_{z_I}^z \frac{dz}{\bar{U}_z(z)} \quad (18)$$

Then letting $t' = t''$ or $t' - \tau = t''$ as appropriate, the corresponding value for z can be determined which in turn yields values of T and \bar{U}_z . Appendix A gives a computer program written in FORTRAN IV for calculating values for r^2 .

Figure 10 shows the resulting mean square dispersion curves. Near the injector the present work gives values slightly higher than those due to Hersch; however, in the downstream region near the nozzle the values are considerably less than those due to Hersch. Thus the present results predicts that the overall gas phase mixing due to turbulent diffusion is significantly less than that predicted by the results due to Hersch.

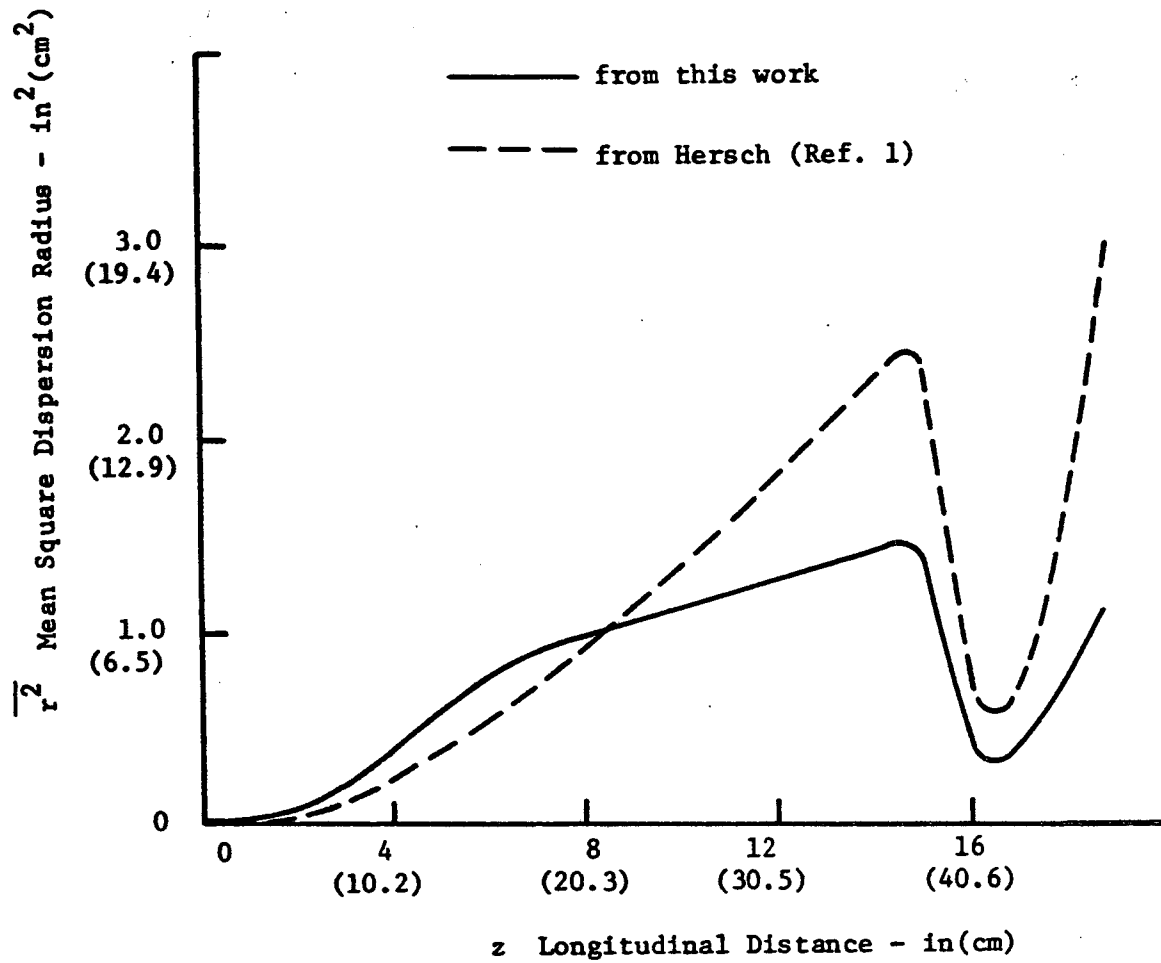


Figure 10 Mean Square Dispersion Radius for Complete Engine

SUMMARY OF RESULTS

The diffusion of a tracer gas in a liquid rocket combustion chamber has been measured and the data analyzed using existing turbulence theories to find the intensity of turbulence and the Lagrangian correlation coefficient. Although the method is indirect and lengthy, turbulent mixing information can be obtained which is unobtainable by other methods.

Combining the Orsat analyzer and the gas chromatograph proved to be an excellent method of determining the concentration of a tracer gas mixed with products of combustion. The resulting helium concentration profiles were consistent and repeatable.

The results indicate that turbulent diffusion in a combustion chamber can be adequately modeled by the one-dimensional Taylor theory assuming that the turbulent velocity is a function of longitudinal coordinate only and the Lagrangian correlation is of the form $e^{-\alpha t}$.

The intensity of the turbulence was found to be higher than previously thought. The present work indicates a maximum value of about 30% where about 10% had been the accepted value. This increase in turbulence level is offset by the decrease in correlation coefficient. The Lagrangian correlation was found to be much smaller than the value of unity assumed in previous work. In the work herein the correlation was found to drop to about .4 in one inch of chamber length.

The results obtained herein predict that the gas phase mixing due to turbulent diffusion is significantly less than would be obtained using the results of Hersch.

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APPENDIX A

COMPUTER PROGRAM TAYLOR

This section describes the use of the computer program for calculating the mean square dispersion radius as a function of combustion chamber length. This is accomplished by using a double summation technique to solve the Taylor Equation (Eq. 17). Values of \bar{U}_z , T, R, and z are input as tabular arrays and the subroutine TABLE is used to determine values from these arrays. The subroutine linearly interpolates between points. After the input data has been read in, the program generates another tabular array of z and t" (Eq. 18) in order to convert from dispersion time to chamber coordinate.

This program is written in FORTRAN IV to run on an IBM 7044. Very little effort has been made to optimize the program with regard to running time or user convenience.

C
C
C

*****SAMPLE INPUT*****

Card	Column										
	1	6	11	16	21	26	31	36	41	46	51
1	011		A 4000								
2	0.3		100.		30.0		1.17				
3	1.3		650.		30.0		1.17				
4	2.3		1500.		30.0		1.17				
5	3.3		2700.		30.0		1.17				
6	3.8		3300.		25.4		1.17				
7	4.3		3800.		22.6		1.17				
8	4.8		4050.		21.0		1.17				
9	6.3		4600.		16.3		1.17				
10	8.3		5000.		12.8		1.17				
11	10.3		5220.		11.1		1.17				
12	14.3		5350.		10.8		1.17				
13	0.05		.4E-04		.4E+04		0.1		0.03		
14	0.3		6.4								

- (1) Program will normally go back and read another set of values for ZINJ & ZSAM (similar to card 14).
- (2) One blank card will cause program to go back and read another turb. curve (similar to card 1).
- (3) Two blank cards will cause program to exit.

C
C
C

*****SAMPLE OUTPUT*****

ZINJ = 0.30(IN) ZSAM = 6.40(IN)
ALPHA = 0.400E 04(SEC**-1) TURB CURVE A 4000

Z (IN)	RSQB (IN**2)	DELTAU (SEC)
0.31	0.000	0.400E-04
0.60	0.004	0.400E-04
0.90	0.015	0.400E-04
1.20	0.027	0.400E-04
1.50	0.044	0.400E-04
1.81	0.069	0.400E-04
2.12	0.096	0.400E-04
2.40	0.128	0.400E-04
2.76	0.178	0.400E-04
3.03	0.220	0.400E-04
3.33	0.275	0.100E-04
3.63	0.331	0.100E-04
3.92	0.387	0.100E-04
4.20	0.439	0.100E-04
4.51	0.493	0.100E-04
4.82	0.548	0.100E-04
5.11	0.597	0.100E-04
5.41	0.644	0.100E-04
5.71	0.690	0.100E-04
6.02	0.735	0.100E-04
6.34	0.778	0.100E-04

C
C
C
C

*****FCRTRAN LISTING*****

```
15 DIMENSION ZT(30),UT(30), TIT(30) ,ZZT(50),TT(50),RT(30)
COMMON ZT,UT, TIT ,ZZT,TT,RT
CONTINUE
1 READ(5,1) NZ,TIC
FORMAT(I3,7X,A6)
IF(NZ.EQ.0) GO TO 16
DO 2 J=1,NZ
3 READ(5,3) ZT(J),UT(J), TIT(J),RT(J)
4 FORMAT(4F10.3)
5 CONTINUE
6 READ(5,4) DELZ,DELTZ,ALPHA
7 FORMAT(F10.3,E10.3,E10.3)
8 CONTINUE
9 READ(5,13) ZINJ,ZSAM
10 FORMAT(2F10.3)
11 IF(ZSAM.EQ.0.) GO TO 12
12 WRITE(6,31) ZINJ,ZSAM,ALPHA,TIC
31 FORMAT(1H1,7H ZINJ =,F8.2,22H(IN)
14H(IN),/,/, 8H ALPHA =,E10.3,24H(SEC**-1)
2,A6,/,/)
13 WRITE(6,14)
14 FORMAT(12H Z (IN) ,16H RSQB (IN**2)
1,14H DELTAU (SEC) ,/,/)
Z=ZINJ
CALL TABLE(Z,R,ZT,RT,DRDZ,NZ)
RINT=R
ZSTOP=ZSAM
T=0.
I=1
KK=0
C THIS SECTION GENERATES THE DISPERSION TIME VS DISTANCE
C TABLE. EVERY TENTH VALUE THAT IS CALCULATED IS
C ENTERED INTO THE TABLE.
5 CONTINUE
K=(KK/10)*10-KK
IF(K.NE.0) GO TO 6
ZZT(I)=Z
TT(I)=T
IF(Z.GT.ZSTOP) GO TO 7
I=I+1
6 CONTINUE
Z=Z+DELZ/2.
CALL TABLE(Z,U,ZT,UT,DUDZ,NZ)
T=T+DELZ/U
Z=Z+DELZ/2.
KK=KK+1
GO TO 5
7 CONTINUE
NT=I
ZPR=ZINJ
ZZZ=ZINJ
```

ZSAM =,F8.2,
TURB CURVE

```

Z=ZINJ
SUM=0.
TPR=0.
CALL TABLE(ZSAM,TSAM,ZZT,TT,DZDT,NT)
8 CONTINUE
SUMA=0.
C THIS SECTION DECREASES THE TIME STEP
C AS THE MEAN VELOCITY INCREASES.
IF(Z.LT.3.) GO TO 26
IF(Z.LT.15.4) GO TO 27
IF(Z.LT.16.4) GO TO 28
DELT=DELTZ/32.
GC TC 30
26 DELT=DELTZ
GO TC 30
27 DELT=DELTZ/4.
GC TC 30
28 DELT=DELTZ/16.
30 CCNTINUE
TAU=0.
TPR=TPR+DELT/2.
IF(TPR.GE.TSAM) GO TO 9
CALL TABLE(TPR,ZZ,TT,ZZT,CZDT,NT)
CALL TABLE(ZZ,TIOTPR,ZT,TIT,DTDZ,NZ)
CALL TABLE(ZZ,UOTPR,ZT,UT,DUDZ,NZ)
TPR=TPR+DELT/2.
20 CCNTINUE
TAU=TAL+DELT/2.
TMTAU=TPR-TAU
CALL TABLE(TMTAU,ZZ,TT,ZZT,DZDT,NT)
CALL TABLE(ZZ,TIOTMT,ZT,TIT,DTDZ,NZ)
CALL TABLE(ZZ,UOTMT,ZT,UT,DUDZ,NZ)
IF(ALPHA.EG.0.) GO TO 22
RL=EXP(-ALPHA*TAU)
GO TC 23
22 CCNTINUE
RL=1.
23 CCNTINUE
SUMA=SUMA+RL*TIOTMT*UOTMT*DELT
IF(RL.LT..0005) GO TO 21
TAU=TAL+DELT/2.
IF(TAU.GT.TPR) GO TO 21
GO TC 20
21 CCNTINUE
SUM=SUM+TIOTPR*UOTPR*DELT*SUMA
CALL TABLE(TPR,Z,TT,ZZT,DZDT,NT)
C THIS SECTION CAUSES OUTPUT TO BE PRINTED EVERY 0.3 INCH
C AS CONTROLLED BELOW BY STATEMENT ZZZ=ZZZ+0.3.
IF(Z.GE.ZZZ) GO TO 24
GC TC 25
24 CONTINUE
CALL TABLE(Z,R,ZT,RT,DRDZ,NZ)

```



```

      RSB=4.*SUM/10,**4*(R/RINT)**2
      WRITE(6,10) Z,RSB,DELT
10    FORMAT(2X,F8.2,6X,F10.3,7X,E10.3)
      ZZZ=ZZZ+.3
25    CONTINUE
      GO TO 8
9     CONTINUE
      GO TO 11
12    CONTINUE
      GO TO 15
16    CONTINUE
      CALL EXIT
      END

```

```

      SUBROUTINE TABLE(X,Y,XT,YT,DYDX,N)
      DIMENSION XT(1),YT(1)
      IF(N.NE.1) GO TO 1
      Y=YT(1)
      DYDX=0.
      RETURN
1     IF(XT(N).LT.XT(1)) GO TO 3
      DO 2 J=2,N
      IF(XT(J).GE.X) GO TO 6
2     CONTINUE
      GO TO 5
3     DO 4 J=2,N
      IF(XT(J).LE.X) GO TO 6
4     CONTINUE
5     J=N
6     I=J-1
      DYDX=(YT(J)-YT(I))/(XT(J)-XT(I))
      Y=YT(I)+DYDX*(X-XT(I))
      RETURN
      END

```