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MULTIPATH ERROR IN RANGE RATE MEASUREMENT BY PLL-TRANSPONDER/GRARR/TDRS

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MULTIPATH ERROR IN RANGE RATE MEASUREMENT BY PLL-TRANSPONDER/GRARR/TDRS

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ABSTRACT

Range rate errors due to specular and diffuse multipath are calculated for a Tracking and Data Relay Satellite (TDRS) using an S-Band Goddard Range and Range Rate (GRARR) system modified with a phase-locked loop transponder. The uplink and downlink multipath signals are modeled in accordance with previous work pertinent to S-band. Carrier signal processing in the coherent turn-around transponder and the GRARR receiver is taken into account. The root-mean-square (rms) range rate error is computed for the GRARR doppler extractor and N-cycle count range rate measurement. Curves of worst-case range rate error are presented as a function of grazing angle at the reflection point. The assumed parameters are: rms wave height = 0.1 meter, number of doppler count = 3,133,956, receiver PLL bandwidth = 10 Hz (one sided), spacecraft altitude from 150 miles to 1500 miles. At very low grazing angles specular scattering predominates over diffuse scattering as expected, whereas for grazing angles greater than approximately 15°, the diffuse multipath predominates. The range rate errors at different low-orbit altitudes peaked between 5° and 10° grazing angles.

TECHNICAL MEMORANDUM G-161-4

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1. INTRODUCTION

The unwanted earth-reflection propagation can occur in the uplink and downlink between a TDRS and a user spacecraft. This additional propagation to the direct path propagation can, at certain times, cause significant error in the range-rate measurement. The expected worst case errors of a TDRS/ Modified S-Band GRARR system utilizing a PLL Transponder are calculated.

In Section 2 the signal component of earth reflection will be traced through the GRARR system. The range-rate error will be expressed in terms of the parameters of electromagnetic earth reflection intensity in Sec. 3.

The earth reflections cause phase jitters in the uplink and downlink carrier components which are tracked by the spacecraft transponder and ground receiver phase-locked loops. Since the range-rate of the spacecraft is measured from the two-way doppler frequency shift of the S-band carrier, the intensity and the frequency characteristics of the reflections are examined. The effects of the reflections on the carrier phase along the GRARR systems are analyzed in order to evaluate the errors in the range-rate measurements due to the earth reflections. The earth reflection components which can cause the range-rate error must have sufficiently close frequencies to the directly propagated carriers, so that the summed carriers can be tracked by the phase-locked loops. Only when the tangential velocity of the spacecraft along the direct path is small, and the doppler frequencies of the direct and reflected components differ only slightly, will the worst case multipath range-rate error occur.

2. EARTH REFLECTION COMPONENTS IN A TDRS/GRARR UTILIZING A PLL TRANSPONDER

The range-rate measurement of a user spacecraft is accomplished by counting the doppler frequency shift of the S-band carrier created in the up and downlinks between the spacecraft and the data relay satellite (DRS). The uplink and downlink carrier frequencies are in a fixed ratio within S-band, and the two-way doppler shift is counted in a special manner at a ground receiver which is linked to the TDRS directly.

The carrier component in the uplink from TDRS to the spacecraft, when it arrives at the spacecraft, can be expressed by [1]

$$e_r(t) = \sin\left\{\omega_t t - \omega_t \dot{\tau}_d t\right\} + \langle a_s^2 \rangle \sin\left\{\omega_t t - \omega_t \dot{\tau}_d t - \phi_s - \omega_t \dot{\tau}_s t\right\}$$

$$+ \sum_k \left\{ x_{ck}(t) \sin\left(\omega_t t - \omega_t \dot{\tau}_d t - \phi_s - \omega_t \dot{\tau}_s t\right)\right.$$

$$\left. + x_{qk}(t) \cos\left(\omega_t t - \omega_t \dot{\tau}_d t - \phi_s - \omega_t \dot{\tau}_s t\right)\right\} \tag{1}$$

where ω_t is the transmitted frequency, τ_d is the propagation delay of the direct path signal, and $\dot{\tau}_d$ is its time derivative. The second term is the component of the specular reflection, where ϕ_s is its relative phase and $\dot{\tau}_s$ is relative (differential) doppler factor with respect to the direct component. The third term expresses the diffuse scatter components which have different paths close to the specular reflection path and have relative in-phase and quadrature components determined by the distributions of relative phases and relative doppler shifts.

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[1] R.H. Wachsman and A. F. Ghais, "Multipath Effects on GRARR Tracking Utilizing a Data-Relay Satellite." Final report to Goddard Space Flight Center, by ADCOM, a Teledyne Company, under Contract NAS5-10780, September, 1969.

Equation (1) can also be expressed as

$$e_r(t) = A_r(t)\left\{\sin \omega_t t - \omega_t \dot{\tau}_d t - \phi_r(t)\right\} \tag{2}$$

and

$$\phi_r(t) \approx \sqrt{\langle a_s^2 \rangle}\, \sin\left(\phi_s + \omega_t \dot{\tau}_s t\right)$$
$$+ X_c(t) \sin\left(\phi_s + \omega_t \dot{\tau}_s t\right)$$
$$- X_q(t) \cos\left(\phi_s + \omega_t \dot{\tau}_s t\right) \tag{3}$$

where the large X's are the summed small x_k's in Eq. (1). The quantity $\phi_r(t)$ is the uplink carrier phase jitter.

The transponder in the user spacecraft will coherently transpond the carrier frequency in the turn around ratio of

$$\frac{n}{m} = \frac{5}{4} \quad .$$

The transmitted downlink carrier is then,

$$\sin\left\{\frac{n}{m}\left(1 - \dot{\tau}_d\right)\omega_t t - \frac{n}{m}\phi_r(t)\right\} ,$$

and the carrier frequency can now be defined as

$$\omega_T = \frac{n}{m}\left(1 - \dot{\tau}_d\right)\omega_t \, . \tag{4}$$

The amplitude modulation factor of Eq. (2) disappears when the signal goes through the limiter of the transponder.

As in the case of uplink, the downlink received carrier has the form

$$\begin{aligned} e_R = & \sin\left[\omega_T\left(1 - \dot{\tau}_d\right)t - \frac{n}{m}\,\phi_r\left(\left\{1 - \dot{\tau}_d\right\}t\right)\right] \\ & + \sqrt{\langle a_s^2\rangle}\,\sin\left[\omega_T\left(1 - \dot{\tau}_d\right)t - \frac{n}{m}\,\phi_r\left(\left\{1 - \dot{\tau}_d - \dot{\tau}_s\right\}t - \tau_s\right) - \omega_T\tau_s - \omega_T\dot{\tau}_s t\right] \\ & + X_c(t)\sin\left[\quad '' \quad\right] \\ & + X_q(t)\cos\left[\quad '' \quad\right] \end{aligned} \tag{5}$$

The received carrier component can also be expressed by

$$e_R(t) = A_R(t)\left\{\omega_T\,(1-\dot{\tau}_d)t - \frac{n}{m}\,\phi_r\left(\left[1 - \dot{\tau}_d\right]t\right) - \phi_R(t)\right\} \tag{6}$$

and

$$\begin{aligned} \phi_R(t) \approx & \sqrt{\langle a_s^2\rangle}\,\sin\left(\phi_s + \omega_T\dot{\tau}_s t\right) \\ & + X_c(t)\sin\left(\phi_s + \omega_T\dot{\tau}_s t\right) \\ & - X_q(t)\cos\left(\phi_s + \omega_T\dot{\tau}_s t\right). \end{aligned} \tag{7}$$

The quantity $\phi_R(t)$ is the downlink carrier phase jitter. Figure 1 illustrates phase jitters of the carrier in the range-rate system.

At the receiver the reference frequency (which does not have doppler shift) is subtracted from the received carrier frequency and the bias frequency is added to it. The output of the receiver is

$$\begin{aligned} e_D(t) = & \sin\left\{\frac{n}{m}\,\omega_b t + \omega_T\left(1 - \dot{\tau}_d\right)t - \frac{n}{m}\,\phi_r\left(\left[1 - \dot{\tau}_d\right]t\right) - \phi_R(t) - \frac{n}{m}\omega_t t\right\} \\ \approx & \sin\left\{\frac{n}{m}\,\omega_b t - 2\,\frac{n}{m}\,\dot{\tau}_d\omega_t t - \frac{n}{m}\,\phi_r\left(\left[1 - \dot{\tau}_d\right]t\right) - \phi_R(t)\right\} \end{aligned} \tag{8}$$

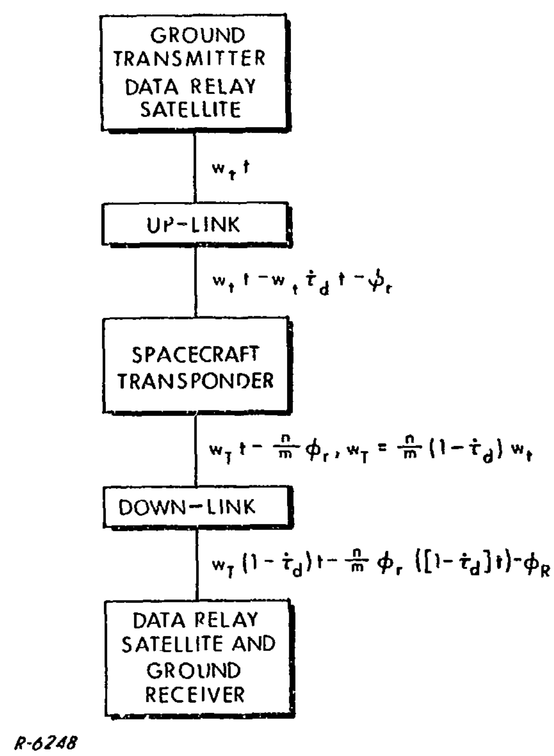

Fig. 1 Phase Jitters in GRARR Carrier with PLL Transponder

where $\omega_B = \frac{n}{n'} \omega_b$ is the bias frequency. In Eq. (8) the first two terms in the parenthesis are the required terms for the range-rate measurement and the latter two terms are the phase jitters produced by the earth multipath reflections.

The measured range rate with the system and the actual range rate are different by the amount

$$\left(\dot{R}_{meas} - \dot{R}_{true}\right) \approx \frac{-c}{2\omega_t T} \left\{ \frac{m}{n} \left[\phi_R\left(t_1 + T\right) - \phi_R\left(t_1\right) \right. \right.$$

$$\left. \left. + \left[\phi_r\left\{\left(1 - \dot{\tau}_d\right)\left(t_1 + T\right)\right\} - \phi_r\left\{\left(1 - \dot{\tau}_d\right) t_1\right\}\right]\right] \right\} \tag{9}$$

where c is the velocity of light, T is the duration of the doppler measurement, and t_1 is the instant the measurement started. A convenient index of the range-rate error is the statistical root-mean-square of the error,

$$\epsilon_{\dot{R}} = \left(\left\langle\left(\dot{R}_{meas} - \dot{R}_{true}\right)^2\right\rangle\right)^{\frac{1}{2}}$$

$$\approx \frac{c\omega_B}{\sqrt{2}\,\omega_t\, 2\pi N} \left\{ \frac{m^2}{n^2} \langle \phi_R(t)^2 \rangle + \langle \phi_r\left(\left[1 - \dot{\tau}_d\right] t\right)^2 \rangle \right\}^{\frac{1}{2}} \tag{10}$$

An approximate measurement time

$$T \approx \frac{2\pi N}{\omega_B}$$

is used for the equation. In the equation $\phi_r(t)$ and $\phi_R(t)$ are the uplink and downlink carrier phase jitters, respectively, as introduced in Eqs. (3) and (7). In the next section the root-mean-square of the error will be estimated in terms of the physical earth reflection parameters.

The uplink and downlink carrier phase jitters are developed within the phase-lock loops of the spacecraft transponder and the ground receiver respectively. In practice, the transponder loop bandwidth is much larger than that of the ground receiver, and the uplink carrier phase jitter component may be attenuated by the ground receiver phase-lock loop.

3. COMPUTATION OF WORST - CASE RANGE-RATE ERROR

There are two different types of wave reflections at the surface of the earth which will produce the phase jitters of the uplink and downlink carriers of the PLL Transponder TDRS/GRARR. One is called specular reflection, the other is called diffuse scatter. Specular reflection is the mirror-like reflection which happens when there is a relatively smooth surface at the reflection point. Diffuse scatter is a collection of many relatively small area reflections having reflection intensities and relative phases different from each other.

A surface could be considered sufficiently smooth for a specular reflection when the "Rayleigh criterion,"

$$h < \frac{\lambda}{8 \sin \gamma}$$

can be satisfied where h is the height of the surface irregularities, λ is the wavelength, and γ is the incident and also the reflecting angle.[2] The area of the reflection is the same as if the reflecting surface were perfectly smooth. The relative power of the specular reflection can be set as

$$\langle a_s^2 \rangle = \left(\rho_s D R_o\right)^2 \tag{11}$$

[2] P. Beckman and A. Spizzichino, "The Scattering of Electromagnetic Waves from Rough Surfaces," A Pergamon Press Book, The Macmillan Company, New York, 1963.

where R_o is the reflection coefficient of a smooth and plane earth. D is the divergence coefficient caused by the curvature of the earth, and ρ_s is a coefficient derived from the effect of the earth surface irregularities.

For a surface whose random heights are distributed in a Gaussian distribution, and for which the correlation between the random heights of two points on the surface is both isotropic and diminishing as a Gaussian function of the distance between the two points, the scattering coefficient may be shown to be given by

$$\langle |\rho_s|^2 \rangle = e^{-\left(\frac{4\pi \sigma_w \sin \gamma}{\lambda}\right)} \qquad (12)$$

where σ_w is the standard deviation of the normal distribution of heights, γ the grazing angle, and λ the wavelength of the electromagnetic radiation. In practical cases the equation is in good agreement with experiments of earth surface reflections.

An approximate ratio of the average scattered power to the directly propagated power was evaluated by Durrani and Staras[3] as

$$K = \frac{\langle P_s \rangle}{P_d} = D^2 |R_o|^2 Q \qquad (13)$$

where

$$Q = \frac{\tan A}{\tan B}\left[1 - H \tan^2 A + \frac{4\theta}{\sin 2A}\right]^{-1}$$

$A = V - 2\theta$

$B = V - \theta$

V = target aspect angle

θ = incident and reflection angle at the specular point

[3] S. H. Durrani and H. Staras, "Multipath Problems in Communications between Low-Altitude Spacecraft and Stationary Satellites," RCA Review, March, 1968.

Equation (13) is based on the form o the coefficient of diffuse scatter calculated by Beckmann,[2] and it applies under the following assumptions:

a) The surface of the earth is rough and cannot satisfy the Rayleigh criterion of smoothness.

b) The heights are normally distributed and its horizontal autocorrelation function is in the normal form.

c) The rms slope is fairly small.

d) Shadowing of one part of the surface by another is unimportant. Multiple scattering is also unimportant.

The target aspect angle, $\dot{V}$, in this case, is the angle made by the local vertical direction at the spacecraft and the line segment defined by the spacecraft and the TDRS.

The frequency band spread of the scattered signal will be needed for the range-rate error calculation. An approximate value of the frequency band was derived by Durrani and Staras.[3] A similar formula of the frequency band is[1]

$$B_m \approx \frac{v_o}{\lambda}\left(\frac{1}{\sqrt{2}} \tan \beta_o\right) \sin \gamma \text{ Hz} \tag{14}$$

where

v_o = spacecraft velocity $\qquad \left(\frac{1}{\sqrt{2}} \tan \beta_o\right)$ = rms sea surface slope

If the slopes of the sea surfaces are considered to be nearly normally distributed, the doppler frequency spectrum also has a normal distribution. Such an assumption of the frequency spectrum is acceptable in the calculation of the carrier phase jitters within the receiver phase-lock loop bandwidth. The spectrum of the phase jitter in the uplink carrier has the form

$$\Psi_M^c\left(\omega_t \dot{\tau}_s\right) = \frac{2\sigma_m^2}{\sqrt{2\pi}\, B_m}\, e^{-\frac{(\omega_t \dot{\tau}_s)^2}{2(2\pi B_m)^2}} \tag{15}$$

where $2\sigma_m^2$ is the average power scatter ratio and B_m is the rms doppler spread. Replacement of the subscript m by M can produce the spectrum of the downlink carrier phase jitter.

In case where the doppler frequency spread is much wider than the receiver phase-lock loop bandwidth, the scatter power spectrum can be assumed to be constant within the bandwidth.

The actual numerical constants of the TDRS/GRARR system which are required in the calculation of the range-rate errors are the following:

uplink carrier frequency = 1750-1850 MHz = f_t

downlink carrier frequency = 2200-2300 MHz = f_T

transponder ratio = $\frac{n}{m} = \frac{5}{4}$

bias frequency for doppler measurement = f_b = 500 kHz

one-sided carrier-tracking-PLL bandwidth = B_L = 10, 30, 100, 300, 1000, 3000 Hz.

Three-fourths of the earth's surface is covered with water and the largest expected reflections over the seas are as strong as the largest expected reflections over some flat lands. The average rms sea wave height is about 1 meter. It is not very rare to have rms wave height of 0.1 meter. And this wave height is chosen here as the representative case of producing a strong specular reflection.

Using the preceding results, the worst case expected range-rate errors were plotted in Fig. 2 with respect to reflection grazing angle for different user spacecraft altitudes. The errors occur when the doppler frequency is very small. At such an instant the spacecraft, the earth center, TDRS and the center of earth reflection are on a plane and the spacecraft velocity is perpendicular to the plane. It can be shown[1] that in the worst case

$$\langle \phi_r(t)^2 \rangle = \langle \phi_R(t)^2 \rangle = \frac{1}{2} a_s^2 + B_L \Psi_M^c(0)$$

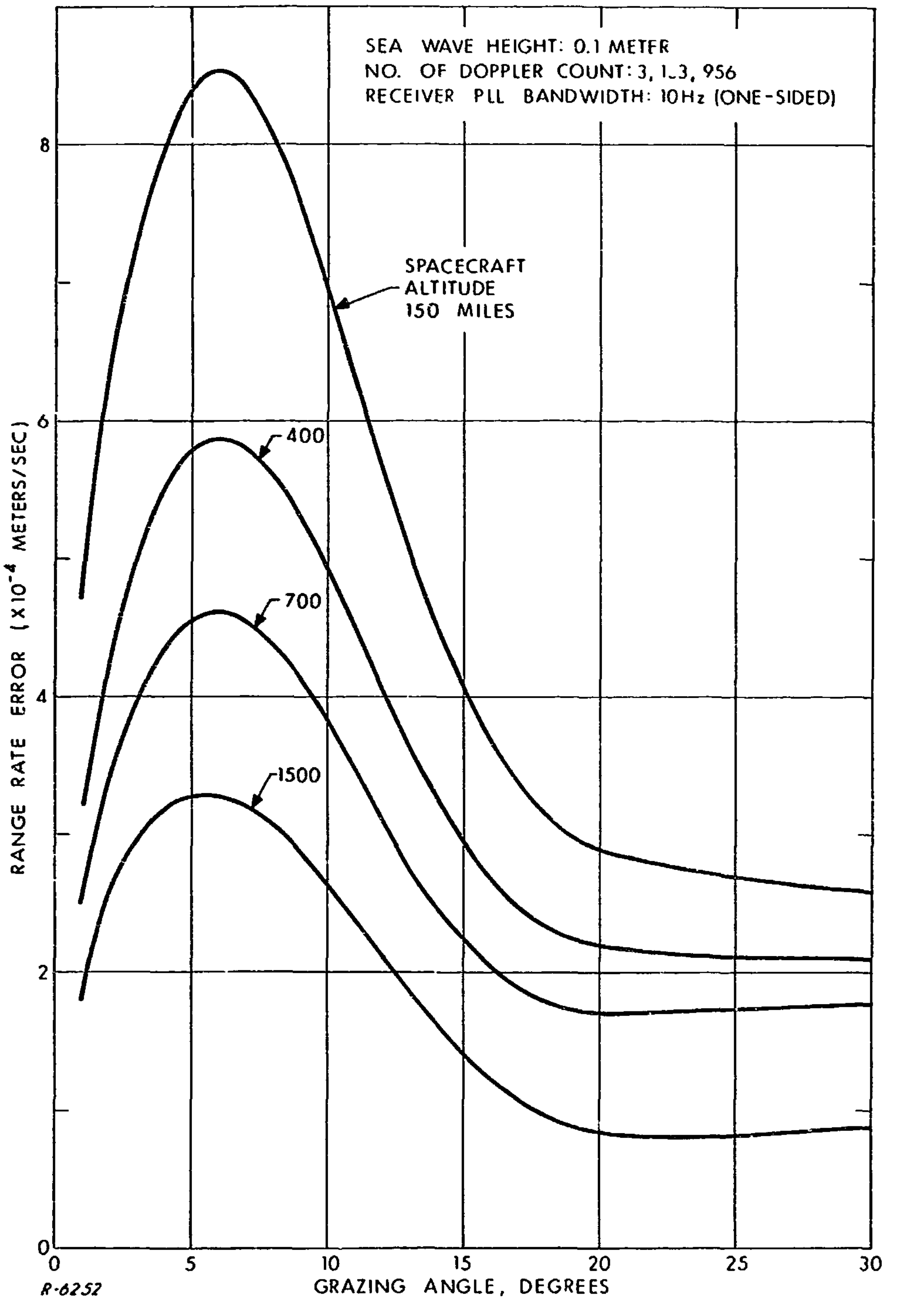

Fig. 2 Worst Case Range-rate Error as a Function of the Grazing Angle of Reflection

Since the diffuse scatter multipath components will be regarded as strong noise by the noise sensing system at the receiver, and small doppler dynamics is expected when the range-rate errors are present, the narrowest phase-lock loop is expected to be in operation at the receiver. One-sided tracking bandwidth of 10 Hz was used in the computation of range-rate errors. The largest number of doppler count, $N = 3.133956 \times 10^6$, was used in the computation. The largest count number is expected in use when the doppler change is small and also doppler itself is, in fact, small. The rms expected range-rate is inversely proportional to the doppler count number. The numbers of doppler count of a modified S-band GRARR system are given in Table 1.

Table 1 Counting and Sampling Rates

Recording Rate	S-Band (N Counter)
4/sec	65,503
2/sec	131,007
1/sec	229,263
6/min	3,133,956

The numerical calculations of the range-rate error which resulted in Fig. 2 make it possible to analyze the error in terms of contributing factors in error amounts. The error due to the specular reflection decreases sharply as the grazing angle increases over 10 degrees, and the diffuse scatter remains as the error generating factor. For grazing angles smaller than 5 degrees the divergence of the rays reflected off the spherical earth reduces the received reflected power and determines the manner in which the error changes with grazing angle. The intensity of the specular reflection is determined not only by the grazing angle, but also by the smoothness of the earth surface as can be realized from Eq. (12) The computed range-rate errors for grazing angles of more than 15 degrees in the figure, which are mostly due to diffuse scatter, are merely a weak bound, and the actual errors are, in fact, smaller.