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MULTIPATH PERFORMANCE OF A TDRS SYSTEM EMPLOYING WIDEBAND FM VHF SIGNALS

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MULTIPATH PERFORMANCE OF A TDRS SYSTEM EMPLOYING WIDEBAND FM VHF SIGNALS

by

Sung J. Sohn
Ahmad F. Ghais

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by

ADCOM
A Teledyne Company
808 Memorial Drive
Cambridge, Massachusetts 02139

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ABSTRACT

This report provides an approximate theoretical analysis of the effects of specular reflection multipath on the performance of a one-way Tracking and Data Relay Satellite-to-user link. The analysis pertains to the wideband FM system employing a sinusoidal subcarrier to achieve spectrum spreading. Bounds on multipath effects are derived for receivers with and without limiters and for data modulated on the carrier or the subcarrier.

For data modulation, performance is evaluated in terms of an additive lowpass signal at the data detection filter. Doppler and range tracking performance is evaluated in terms of root-mean-square (rms) error of carrier frequency in a carrier PLL and rms phase jitter in a subcarrier PLL. The work reported here is superseded by more detailed analyses in later reports.

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LIST OF MATHEMATICAL SYMBOLS AND NOTATIONS

t	= time
ω_c	= carrier frequency
ω_{sc}	= subcarrier frequency
δ	= carrier FM modulation index
$x_{sc}(t)$	= uplink data signal, including command and ranging signals, modulating the subcarrier
$y_{sc}(t)$	= downlink data signal, including telemetry and ranging signals, modulating the subcarrier
$x_c(t)$	= uplink data signal modulating the carrier
$y_c(t)$	= downlink data signal modulating the carrier
τ_d	= direct path time delay
τ_s	= additionaltime delay for the specular reflection path
$\dot{\tau}_d$	= time derivative of τ_d
$\dot{\tau}_s$	= time derivative of τ_s
β	= time dilation factor caused by relative motion of MS with respect to TDRS
t_d	$= \beta_d t \approx (1 - \dot{\tau}_d)t$ = time base of direct path signal component
t_s	$= \beta_d t - \beta_s t + \tau_s \approx \tau_s + \dot{\tau}_s t$
$A(t)$	= envelope of the received multipath signal
$\phi(t)$	= phase of the received multipath signal
$\phi_c(t)$	= phase error of the carrier PLL due to the multipath
$\phi_{sc}(t)$	= phase error of the subcarrier PLL due to the multipath
θ_c	= carrier PLL phase error in the absence of the multipath
θ_{sc}	= subcarrier PLL phase error in the absence of the multipath
$\hat{\delta}$	= estimate of the carrier FM modulation index in a receiver

a_s = coefficient of specular reflection

$e_{sc}(t)$ = input to the subcarrier PLL in the receiver

$\phi_{BPsc}(t)$ = bandpass component of $\phi(t)$ at ω_{sc}

$e_{sc\ell}(t)$ = input to the subcarrier VCO

K_c = amplifier gain of the carrier PLL

K_{sc} = amplifier gain of the subcarrier PLL

e_{ac} = input to the matched filter in a receiver for the data modulation of the carrier

e_{asc} = input to the matched filter in a receiver for the data modulation of the subcarrier

$\alpha(t)$ = sinusoidal angle of $\phi(t)$

ν, γ, ϵ = variables used in the analysis of the received signal phase $\phi(t)$

c = speed of electromagnetic propagation

r = data bit rate

B_{nc} = double-sided noise bandwidth of the carrier loop in Hz

B_{nsc} = double-sided noise bandwidth of the subcarrier loop in Hz

$h_{LPF}(t)$ = impulse response function of a lowpass filter at the input to a VCO

$\Delta\dot{R}$ = range rate error caused by the multipath

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MULTIPATH PERFORMANCE OF A TDRS SYSTEM EMPLOYING WIDEBAND FM VHF SIGNALS

1. INTRODUCTION

A part of the electromagnetic power transmitted from the TDRS or a user satellite reflects off the earth surface and reaches the receiver at the TDRS or the satellite. At VHF the specular reflection is strong when the specular reflection region is on the ocean. The diffuse scatter is relatively very weak. Hence, the wave propagation mainly consists of the direct path and the specular reflection path. The reflection coefficient, the relative delay of the reflection signal and its differential doppler frequency with respect to the direct signal's are functions of the satellite orbit, the altitude and position of the TDRS, and the earth surface condition for reflection.

The function of the communication systems studied here is to send the commands (up-link) from a ground station to the user satellites via a synchronous-orbit TDRS and to send back telemetry (down-link) on the return links. The systems also track the range and range-rate of the satellites. The signal carriers in the up- and down-links are sinusoidally FM modulated in order to provide immunity from the disturbance by the multipath propagation. The wide-band FM, in effect, suppresses the signal components reflected off the earth. Two different types of wide-band FM systems are studied. In one type, the signal carrier is PM modulated by the binary data. In the other, the sub-carrier which FM modulates the carrier is PM modulated by the data.

The signal transmitted from the TDRS is in the form

$$e_t(t) = \sin[\omega_c t + \delta \sin\{\omega_{sc} t + x_{sc}(t)\} + x_c(t)] , \qquad (1.1)$$

where

ω_c = carrier frequency

ω_{sc} = subcarrier frequency

δ = carrier FM modulation index

$x_{sc}(t)$ = uplink data signal, including command and ranging signals, modulating the subcarrier

$x_c(t)$ = uplink data signal modulating the carrier

In this equation, only $x_{sc}(t)$ in one type of system, and $x_c(t)$ in the other, actually exist. Both of them are included for convenience. An equivalent functional block diagram of the transponder at a user spacecraft is shown in Fig. 1. A transponder detects the uplink command and transmits the downlink telemetry. The subcarrier and the carrier are coherently transponded for range and range rate tracking, respectively. The equivalent functional block diagram of the command transmitter and the telemetry receiver at the TDRS are the same as the transmitter and receiver parts, respectively, of the transponder.

The communication and range and range-rate tracking performances of the systems are evaluated. The receivers of two types, one with a limiter for the carrier phase detection, the other without, are treated. For the receiver with the limiter the phase of the signal carrier at the output of the limiter is approximately analyzed. For the receiver without the limiter the received carrier phase is accurately analyzed. Then, the choice of proper subcarrier frequency is given that minimizes the multipath disturbance. The worst situation performances which can occur at certain values of differential delay and doppler were evaluated at the subcarrier frequency thus chosen.

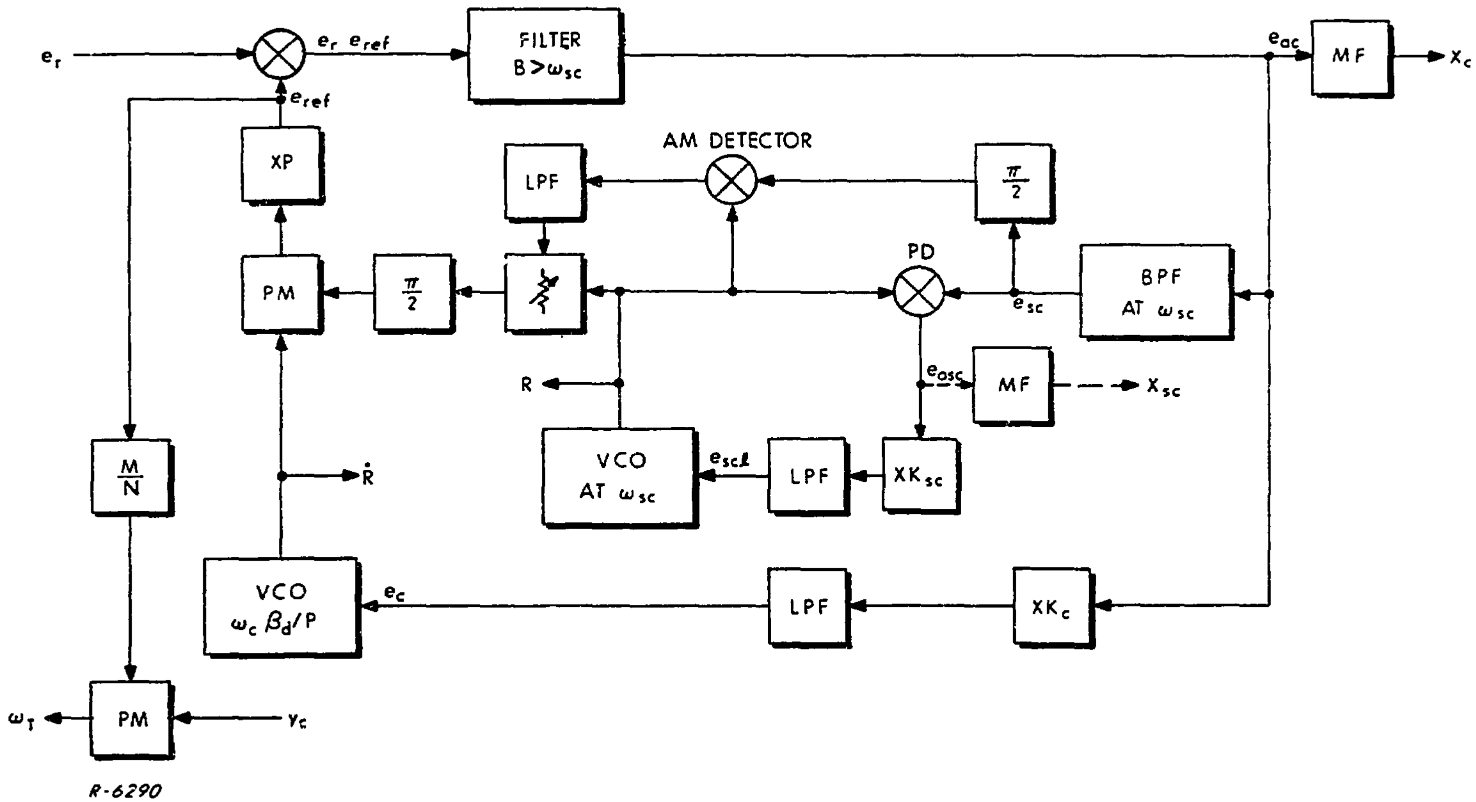

Fig. 1 The Functional Block Diagram of the Transponder (by Ghais and Lippincott)

2. ANALYSIS OF TWO-PATH SIGNAL COMPONENTS IN RECEIVERS

2.1 Two-Path Signal Components in a Receiver with a Limiter for Carrier Phase Detection

The received signal at a mission satellite for the uplink command and the received signal at a TDRS for the downlink telemetry consists of direct path components and the specular reflection components. A direct path signal at the input to a receiver of an uplink can be expressed as

$$e_d(t) = \sin[\omega_c t_d + \delta \sin\{\omega_{sc} t_d + x_{sc}(t_d)\} + x_c(t_d)] , \qquad (2.1)$$

where

ω_c = carrier frequency (radians per second)

ω_{sc} = subcarrier frequency

δ = carrier FM modulation index

$x_{sc}(t)$ = uplink data signal, including command and ranging signals, modulating the subcarrier

$x_c(t)$ = uplink data signal modulating the carrier

$t_d = t(1 - \dot{\tau}_d)$

t = time variable

$\dot{\tau}_d$ = time derivative of the direct path time delay

The direct path signal of a downlink can also be expressed by the equation after the substitution of $x_{sc}(t)$ and $x(t)$ by

$y_{sc}(t)$ = downlink data signal, including telemetry and ranging signals, modulating the subcarrier

$y_c(t)$ = downlink data signal modulating the carrier.

In the following analysis, whenever a statement applies equally to the uplink and the downlink, only the uplink will be discussed. In Eq. (2.1) only $x_{sc}(t)$ or $x_c(t)$ actually exists. Both of them will be included in the equations, whenever it is convenient.

A specular reflection signal arrives at a receiver with an additional time delay. The sp lar reflection component can be expressed as

$$e_s(t) = a_s \sin[\omega_c(t_d - t_s) + \delta \sin\{\omega_{sc}(t_d - t_s) + x_{sc}(t_d - t_s)\} + x_c(t_d - t_s)] , \qquad (2.2)$$

where

$t_s = \tau_s + \dot{\tau}_s t$

τ_s = additional time delay of the specular reflection component with respect to the direct path component

$\dot{\tau}_s$ = time derivative of τ_s

The sum of the direct and specular reflection signals is

$$\begin{aligned} e_r(t) &= e_d(t) + e_s(t) \\ &= A(t)\sin[\omega_c t_d + \delta \sin\{\omega_{sc} t_d + x_{sc}(t_d)\} + x_c(t_d) - \phi(t)] \end{aligned} \qquad (2.3)$$

where

$$A(t) = \left\{1 + a_s^2 + 2a_s \cos \alpha(t)\right\}^{1/2}$$

$$\phi(t) = \tan^{-1} \frac{a_s \sin \alpha(t)}{1 + a_s \cos \alpha(t)}$$

$$\begin{aligned} \alpha(t) = {} & \omega_c t_s + \delta \sin\{\omega_{sc} t_d + x_{sc}(t_d)\} + x_c(t_d) \\ & -\delta \sin\{\omega_{sc}(t_d - t_s) + x_{sc}(t_d - t_s)\} - x_c(t_d - t_s) \end{aligned}$$

The reference signal for the phase detection of the received signal can be expressed as

$$e_{ref}(t) = \sqrt{2}\cos\{\omega_c t_d + \theta_c + \hat{\delta}\sin(\omega_{sc} t_d + \theta_{sc})\} , \tag{2.4}$$

where θ_c and θ_{sc} are phase errors with respect to the phases of the direct path signal. $\hat{\delta}$ is the estimate of the carrier frequency deviation which is determined by the amplification of the subcarrier VCO output.

The received signal and the reference signal are put into the multiplier. Then the output of the multiplier is put into an amplitude limiter. The output from the limiter can be expressed as

$$e_r(t)e_{ref}(t)A^{-1}(t) - \text{high frequency term} = \frac{1}{\sqrt{2}}\sin[-\theta_c + \delta\sin\{\omega_{sc} t_d + x_{sc}(t_d)\} - \hat{\delta}\sin\{\omega_{sc} t_d + \theta_{sc}\} + x_c(t_d) - \phi(t)] . \tag{2.5}$$

The input to the subcarrier PLL which is indicated as $e_{sc}(t)$ in Fig. 1 can be expressed as

$$e_{sc}(t) \approx \frac{1}{\sqrt{2}}[\delta\sin\{\omega_{sc} t_d + x_{sc}(t_d)\} - \hat{\delta}\sin\{\omega_{sc} t_d + \theta_{sc}\} - \phi_{BPsc}(t)] \tag{2.6}$$

where the approximately-equal sign applies when the reflection coefficient, a_s, is sufficiently small and all loops are tracking with small error. $\phi_{BPsc}(t)$ is the bandpass component of $\phi(t)$.

The input to the subcarrier VCO, $e_{sc\ell}(t)$, can be expressed as

$$\begin{aligned} e_{sc\ell}(t) &\approx \frac{1}{\sqrt{2}} K_{sc} h_{LP}(t) \oplus \left[\{\delta\sin[\omega_{sc} t_d + x_{sc}(t_d)] - \phi_{BPsc}(t)\}\sqrt{2}\cos(\omega_{sc} t_d + \theta_{sc})\right] \\ &= \frac{1}{\sqrt{2}} K_{sc} h_{LP}(t) \oplus \left\{\frac{1}{\sqrt{2}}\delta[x_{sc}(t_d) - \theta_{sc}] - \phi_{BPsc}(t)\sqrt{2}\cos(\omega_{sc} t_d + \theta_{sc})\right\} \\ &= \frac{1}{\sqrt{2}} K_{sc}\left\{\frac{1}{\sqrt{2}}\delta(-\theta_{sc}) - \phi_{BPsc}(t)\sqrt{2}\cos(\omega_{sc} t_d + \theta_{sc})\right\} , \end{aligned} \tag{2.7}$$

where

K_{sc} = the amplitude gain of the subcarrier PLL

$h_{LP}(t)$ = the impulse response function of the LPF in subcarrier PLL

2.2 Two-Path Signals in a Receiver without a Limiter

The approximate expressions of the signals in different places in the receiver which has no limiter can be derived in the same procedure that the signals were derived for a receiver which has a limiter in Sec. 2.1. In this case of the receiver with no limiter the amplitude factor A(t) must be retained in the signal expressions. In this section the directly propagated signal and the reflected signal over the earth surface are separately traced in a receiver without a limiter until the signals arrive at the non-linear components in the receiver.

The input to the subcarrier PLL of the direct signal component can be expressed, again assuming small tracking errors, as

$$e_{sc}(t) = \frac{1}{\sqrt{2}} [\delta \sin\{\omega_{sc} t_d + x_{sc}(t_d)\} - \hat{\delta} \sin\{\omega_{sc} t_d + \theta_{sc}\}] . \qquad (2.8)$$

This equation has two differences from Eq. (2.6). This equation has the equality sign, and does not have the addition of the phase error $\phi_{BPsc}(t)$. The input to the subcarrier VCO is

$$e_{sc\ell}(t) = \frac{1}{\sqrt{2}} K_{sc} h_{LP}(t) \oplus \frac{1}{\sqrt{2}} \delta \{x^{\bullet}_{sc}(t_d) - \theta_{sc}\}$$

$$= - \frac{1}{2} K_{sc} \delta \theta_{sc} . \qquad (2.9)$$

The reflection signal component at the output of the multiplier is

$$e_s(t)e_{ref}(t) = \frac{a_s}{\sqrt{2}} \sin[-\omega_c t_s - \theta_c + \delta \sin\{\omega_{sc}(t_d - t_s) + x_{sc}(t_d - t_s)\}$$

$$- \hat{\delta} \sin(\omega_{sc} t_d + \theta_{sc}) + x_c(t_d - t_s)] \, . \qquad (2.10)$$

In the analysis of the input to the subcarrier PLL which is the bandpass component of $e_s(t)e_{ref}(t)$ the phase lock errors θ_c and θ_{sc} can be ignored without loss of accuracy in the systems performance evaluation.

3. VHF WIDE-BAND-FM COMMUNICATIONS BETWEEN TDRS AND SATELLITES; PERFORMANCE OF WIDE-BAND FM THROUGH TIME-VARIANT TWO-PATH CHANNELS

3.1 Systems Performances with Limiters in Receivers

3.1.1 PI Modulation of the Carrier

The communication systems analyzed here have limiters in receivers for carrier phase detection. The carrier in the uplink or downlink is frequency modulated by a subcarrier and also is phase modulated by binary data. Since the data modulate the carrier, the arrived signal at a receiver has the form of Eq. (2.3) except that $x_{sc}(t_d)$ does not exist. The input to the matched filter, e_{ac} in Fig. 1, is a lowpass component of the limiter output expressed in Eq. (2.5). The matched filter input is

$$e_{ac}(t) = \frac{1}{\sqrt{2}} [x_c(t_d) - \phi_{ac}(t)] , \tag{3.1}$$

where $\phi_{ac}(t)$ is the lowpass component of $\phi(t)$.

It is necessary to examine the phase, $\phi(t)$, of the received multipath signal in detail. If the reflection coefficient a_s is sufficiently small, i.e., $a_s < \frac{1}{3}$,

$$\phi(t) \approx a_s \sin \alpha(t) . \tag{3.2}$$

The angle $\alpha(t)$ is, from Eq. (2.3),

$$\alpha(t) = \omega_c t_s + \delta\{\sin \omega_{sc} t_d - \sin \omega_{sc}(t_d - t_s)\} + x_c(t_d) - x_c(t_d - t_s)$$

$$= \delta\sqrt{2}\,(1 - \cos \omega_{sc} t_s)^{1/2} \sin\left(\omega_{sc} t_d + \tan^{-1} \frac{\sin \omega_{sc} t_s}{1 - \cos \omega_{sc} t_s}\right) + \omega_c t_s + x_c(t_d) - x_c(t_d - t_s). \tag{3.3}$$

Simple expression of $\alpha(t)$ is possible with the new notations,

$$\nu \equiv \delta\sqrt{2}\,(1 - \cos\omega_{sc}t_s)^{1/2}$$

$$\gamma \equiv \tan^{-1}\frac{\sin\omega_{sc}t_s}{1 - \cos\omega_{sc}t_s} \tag{3.4}$$

$$\epsilon \equiv \omega_c t_s + x_c(t_d) - x_c(t_d - t_s)\,.$$

$\sin\alpha(t)$ can now be expanded as

$$\begin{aligned}\sin\alpha(t) &= \sin\{\nu\sin(\omega_{sc}t_d + \gamma) + \epsilon\} \\ &= J_0(\nu)\sin\epsilon \\ &\quad + J_1(\nu)[\sin\{\epsilon + (\omega_{sc}t_d + \gamma)\} - \sin\{\epsilon - (\omega_{sc}t_d + \gamma)\}] \\ &\quad + J_2(\nu)[\sin\{\epsilon + 2(\omega_{sc}t_d + \gamma)\} + \sin\{\epsilon - 2(\omega_{sc}t_d + \gamma)\}] \\ &\quad + J_3(\nu)[\sin\{\epsilon + 3(\omega_{sc}t_d + \gamma)\} - \sin\{\epsilon - 3(\omega_{sc}t_d + \gamma)\}] \\ &\quad \vdots \\ &\quad + J_n(\nu)[\sin\{\epsilon + n(\omega_{sc}t_d + \gamma)\} \pm \sin\{\epsilon - n(\omega_{sc}t_d + \gamma)\}]\,, \\ &\quad \vdots\end{aligned} \tag{3.5}$$

where $J_0(\nu)$, $J_1(\nu)$, $J_2(\nu)$, ... are the Bessel functions of the first kind, of order 0, 1, 2, ..., respectively.

Since the purpose of the subcarrier is to decrease the multipath disturbance, it is proper to choose the value of ω_{sc} which can minimize $\phi_{ac}(t)$. Such a choice is possible along the following considerations:

a) Assume that the differential doppler frequency of the subcarrier, $\omega_{sc}\dot{\tau}_s$ is much smaller than the bandwidth of the binary data $x_c(t)$.

b) The frequency bandwidth of $J_i(\nu)$ for i = 0, 1, 2, ... is approximately the frequency $\omega_{sc}\dot{\tau}_s$. Hence, the bandwidth is much smaller than the data bandwidth.

c) The frequency bandwidth of sin ϵ is the bandwidth of the binary data plus $\omega_c\dot{\tau}_s$ when $\omega_c\dot{\tau}_s$ is smaller than the data bandwidth.

d) The bandwidth of sin γ is on the order of magnitude of $\omega_{sc}\dot{\tau}_s$.

e) The bandwidth of each term in the series of Eq. (3.5) is, from b), c), and d), the data bandwidth plus $\omega_c\dot{\tau}_s$ and several times of $\omega_{sc}\dot{\tau}_s$. The centers of the frequency bands of the terms in the series are separated by ω_{sc}.

f) With the assumption of a), by choosing ω_{sc} to be larger than the binary data bandwidth, it is possible to make the disturbance to be

$$\phi_{ac}(t) = a_s J_0(\nu)\sin\epsilon \ . \tag{3.6}$$

g) When ω_{sc} is thus chosen, the matched filter output of $\phi_{ac}(t)$ is large in case $\omega_c\dot{\tau}_s$ is small. In other words, after the choice of the system parameter ω_{sc}, the worst disturbance occurs when $\omega_c\dot{\tau}_s$ is small.

The advantage of the wideband FM can be determined by comparing the multipath disturbance just derived with the multipath disturbance in a communication system which does not have the subcarrier, i.e., $\omega_{sc} = 0$. If $\omega_{sc} = 0$ is substituted into Eq. (3.2),

$$\phi(t) \approx a_s \sin\{\omega_c t_s + x_c(t_d) - x_c(t_d - t_s)\}$$

$$= a_s \sin\epsilon \ . \tag{3.7}$$

Thus the advantage of the wideband FM can be by the factor $J_0(\nu)$.

The worst case of the multipath effect to the wideband FM happens when $\omega_c\dot{\tau}_s$ is smaller than the carrier PLL loop bandwidth. In this case the input to the matched filter is

$$\sqrt{2}\, e_{ac}(t) = x_c(t_d) - a_s J_0(\nu)\cos\omega_c t_s \sin\{x_c(t_d) - x_c(t_d - t_s)\}$$

$$\approx x_c(t_d) - a_s J_0(\nu)\{\cos\omega_c t_s\}\{x_c(t_d) - x_c(t_d - t_s)\} \ . \tag{3.8}$$

$\cos \omega_c t_s$ can be regarded as a constant of value from -1 to 1 in the performance evaluation if the data rate is larger than $\omega_c \dot{\tau}_s$. Since this is the real situation, the input to the matched filter in the worst case is

$$\sqrt{2}\, e_{ac}(t) \approx \{1 - a_s J_0(\nu)\} x_c(t_d) + a_s J_0(\nu) x_c(t_d - t_s) . \qquad (3.9)$$

The minimum value of the matched filter output occurs when

$$x_c(t_d) = -x_c(t_d - t_s) .$$

The minimum matched filter output implies a reduction by the factor

$$1 - 2a_s J_0(\nu) . \qquad (3.10)$$

When ν is large (larger than one, as far as this performance evaluation is concerned),

$$J_0(\nu) \approx \sqrt{\frac{2}{\pi\nu}} \cos\left(\nu - \frac{\pi}{4}\right). \qquad (3.11)$$

When $\nu = 0$, $J_0(\nu) = 1$ which is the maximum.

3.1.2 PM Modulation of the Subcarrier

The advantage of wideband FM in a multipath communication channel is studied here when the subcarrier is PM modulated by the binary data. From Eqs. (2.6) and (2.7), the input to the matched filter in the receiver, $e_{asc}(t)$ in Fig. 1, can be expressed as

$$e_{asc}(t) = \frac{1}{2}\, \delta x_{sc}(t_d) - \phi_{BPsc}(t) \cos(\omega_{sc} t_d + \theta_{sc}) . \qquad (3.12)$$

The second term in the right side of Eq. (3.12) is the multipath disturbance.

The advantage of wideband FM of this communication system can be calculated using a similar method to that in Sec. 3.1.1 for another system. The subcarrier frequency can be properly chosen in order to minimize the multipath disturbance. The phase of the received multipath signal, $\phi(t)$, can be expressed as Eq.(3.2). The angle $\alpha(t)$ is, from Eq.(2.3),

$$\alpha(t) = \omega_c t_s + \delta \sin\{\omega_{sc} t_d + x_{sc}(t_d)\} - \delta \sin\{\omega_{sc}(t_d - t_s) + x_{sc}(t_d - t_s)\} .$$

Equation (3.5) also applies to this case. The variables used in Eq.(3.5) are, in this case,

$$\nu \equiv \delta\sqrt{2}\,[1 - \cos\{\omega_{sc} t_s + x_{sc}(t_d) - x_{sc}(t_d - t_s)\}]^{1/2}$$

$$\gamma \equiv x_{sc}(t_d) + \tan^{-1} \frac{\sin\{\omega_{sc} t_s + x_{sc}(t_d) - x_{sc}(t_d - t_s)\}}{1 - \cos\{\omega_{sc} t_s + x_{sc}(t_d) - x_{sc}(t_d - t_s)\}}$$

$$\epsilon \equiv \omega_c t_s . \qquad (3.13)$$

When $\delta > 3$, the bandwidth of $J_n(\nu)$ is about δr, where r is the data bandwidth in Hz. Therefore, if the subcarrier frequency is larger than $r(1 + \delta)$ plus a few times of the maximum subcarrier differential-doppler $\omega_{sc}\dot{\tau}_s$, the bandpass component (at ω_{sc}) of the received signal phase is

$$\phi_{BPsc}(t) = a_s J_1(\nu)[\sin\{\epsilon + (\omega_{sc} t_d + \gamma)\} - \sin\{\epsilon - (\omega_{sc} t_d + \gamma)\}]$$

$$= a_s J_1(\nu) 2(\cos \epsilon)\sin(\omega_{sc} t_d + \gamma) , \qquad (3.14)$$

when the carrier-frequency differential-doppler, $\omega_c\dot{\tau}_s$, is much smaller than the data bandwidth.

From Eqs.(3.12) and (3.14), the input to the matched filter is

$$e_{asc}(t) = \frac{1}{2}\,\delta x_{sc}(t_d) - a_s J_1(\nu)(\cos \epsilon)\sin(\gamma - \theta_{sc}) . \qquad (3.15)$$

In the calculation of the maximum effect of the multipath, $\cos \epsilon$ in Eq.(3.15) should be substitubed by one. Since the bandwidth of $J_1(\nu)\sin(\gamma - \theta_{sc})$ is

$r(1+\nu)$, and since the maximum of $J_1(\nu)$ is 0.58,

$$\text{rms value of } \{J_1(\nu)\sin(\gamma - \theta_{sc})\} \le \frac{1}{2} \times 0.58 \approx \frac{1}{4} \,,$$

and a lower bound of the normalized output from the matched filter is

$$1 - \frac{a_s}{2\delta(1+\delta)^{1/2}} \tag{3.16}$$

3.2 Systems Performance without Limiters in Receivers

3.2.1 PM Modulation of the Carrier

When the carrier is modulated by the data and a receiver does not have a limiter, the input to the matched filter for the binary data is, from Eq. (2.10)

$$e_{ac}(t) = \frac{1}{\sqrt{2}} x_c(t_d) + \frac{a_s}{\sqrt{2}} \sin[-\omega_c t_s + \delta \sin\{\omega_{sc}(t_d - t_s)\} - \hat{\delta} \sin\omega_{sc} t_d + x_c(t_d - t_s)] \,, \tag{3.17}$$

where θ_c and θ_{sc} are ignored. The second term in Eq. (3.17) can be expanded in the form of Eq. (3.5) in which the variables are

$$\nu \cong \delta\sqrt{2}\,(1 - \cos\omega_{sc} t_s)^{1/2\,*}$$

$$\gamma \equiv -\tan^{-1} \frac{\sin\omega_{sc} t_s}{\cos\omega_{sc} t_s - \hat{\delta}/\delta} \tag{3.18}$$

$$\epsilon \equiv -\omega_c t_s + x_c(t_d - t_s) \,.$$

Using the same argument used in Sec. 3.1.1, for a sufficiently large ω_{sc} the multipath disturbance which is added to the direct signal component $x_c(t_d)$ is

$$a_s J_0(\nu) \sin\epsilon \tag{3.19}$$

* The difference between the accurate value of ν and this approximate expression has been actually verified to be inconsequential to the continuing analysis.

in the worst multipath situation. When $\omega_c \dot{\tau}_s$ is smaller than the carrier PLL bandwidth, the input to the matched filter is

$$\sqrt{2}\, e_{ac}(t) \approx x_c(t_d) + a_s J_0(\nu)\{x_c(t_d - t_s)\} \; . \tag{3.20}$$

The minimum output of the matched filter is approximately by the factor

$$1 - a_s J_0(\nu) \; . \tag{3.21}$$

3.4.2 PM Modulation of the Subcarrier

When the binary data is PM modulating the subcarrier, the input to the matched filter has the direct signal component which is the bandpass component of the product of $e_{sc}(t)$ in Eq. (2.8) and the subcarrier reference

$$\sqrt{2}\cos(\omega_{sc} t_d + \theta_{sc}) \; . \tag{3.22}$$

The input to the matched filter of the direct path signal is, from Eq. (2.8),

$$\begin{aligned} e_{asc}(t) &= \frac{1}{2}\,\delta\, \sin\{x_{sc}(t_d) - \theta_{sc}\} \\ &\approx \frac{1}{2}\,\delta x_{sc}(t_d) \; . \end{aligned} \tag{3.23}$$

The bandpass signal $e_{sc}(t)$ of the reflection signal component is the bandpass of $e_s(t)e_{ref}(t)$ of Eq. (2.10). Equation (2.10) can be expanded in the form of Eq. (3.5), where the variables are

$$\begin{aligned} \nu &\equiv \delta\sqrt{2}\,[1 - \cos\{\omega_{sc} t_s - x_{sc}(t_d - t_s)\}]^{1/2} \\ \gamma &\equiv -\tan^{-1}\frac{\cos\{\omega_{sc} t_s - x_{sc}(t_d - t_s)\} - \hat{\delta}/\delta}{\sin\{\omega_{sc} t_s - x_{sc}(t_d - t_s)\}} \\ \epsilon &\equiv -\omega_c t_s \; . \end{aligned} \tag{3.24}$$

If ω_{sc} is larger than $2\pi r(1 + \delta)$ plus a few times of $\omega_{sc}\dot{\tau}_s$, the bandpass component $e_{sc}(t)$ of the reflection signal is

$$e_{sc}(t) = \frac{a_s}{\sqrt{2}} J_1(\nu)2(\cos \epsilon)\sin(\omega_{sc}t_d - \gamma) , \qquad (3.25)$$

when $\omega_c\dot{\tau}_s$ is much smaller than ω_{sc}.

When this $e_{sc}(t)$ is multiplied by the subcarrier reference of Eq. (3.22), the result is the matched filter input

$$e_{asc}(t) = a_s J_1(\nu)(\cos \epsilon)\sin(-\gamma - \theta_{sc}) . \qquad (3.26)$$

In the calculation of the maximum effect of the multipath, $\cos \epsilon$ in Eq. (3.26) shall be substituted by one. Since the bandwidth of $J_1(\nu)\sin(-\gamma - \theta_{sc})$ is $r(1 + \delta)$, and since the maximum of $J_1(\nu)$ is 0.58,

$$\text{rms value of } \{J_1(\nu)\sin(-\gamma - \theta_{sc})\} \lesssim \frac{1}{4} , \qquad (3.27)$$

and a lower bound to the matched filter output is

$$1 - \frac{a_s}{2\delta(1 + \delta)^{1/2}} \qquad (3.28)$$

4. SATELLITE TRACKING VIA TDRS USING VHF WIDE-BAND FM SYSTEMS

4.1 Systems Performance with Limiters in Receivers

4.1.1 Error in Satellite Range Rate Measurement from Carrier Doppler Frequency

The doppler frequency of the received carrier in the uplink or downlink is not accurately proportional to the relative velocity of a user satellite with respect to the TDRS. The error in the range rate measurement due to the multipath links is studied here.

In a receiver with a limiter for the carrier phase detection the error in the range rate $\dot{R}$ as indicated in Fig. 1 is determined by the error in the phase detection of the signal carrier. The error in the carrier phase detection which is induced by the multipath is the LPF output of $\phi(t)$,

$$\phi_c(t) = h_{LPF}(t) \oplus \phi(t) , \tag{4.1}$$

where $h_{LPF}(t)$ is the impulse response function of the lowpass filter, LPF. Applying the same argument for low multipath disturbance written in Sec. 3.1.1, for a sufficiently large ω_{sc}, the LPF output of $\phi(t)$ is

$$\phi_c(t) = h_{LPF}(t) \oplus a_s J_0(\nu) \sin \epsilon , \tag{4.2}$$

where

$$\nu \equiv \delta\sqrt{2}\,(1 - \cos \omega_{sc} t_s)^{1/2}$$

$$\epsilon \equiv \omega_c t_s + x_c(t_d) - x_c(t_d - t_s) ,$$

when the data modulate the carrier phase, and

$$\nu \equiv \delta\sqrt{2}\,[1 - \cos\{\omega_{sc} t_s + x_{sc}(t_d) - x_{sc}(t_d - t_s)\}]^{1/2}$$

$$\epsilon \equiv \omega_c t_s ,$$

when the data modulate the subcarrier phase. The parameters ν and ϵ are as previously defined in Eqs. (3.4) and (3.13).

Since the frequency error at the output of the carrier VCO is the input voltage error, $K_c \phi_c(t)$, the rms error in the range rate can be expressed as

$$\Delta\dot{R}_{rms} = \frac{2\pi c}{\omega_c} K_c \left[\overline{\{\phi_c(t)\}^2}\right]^{1/2} , \tag{4.3}$$

where

c = speed of electromagnetic propagation

K_c = amplifier gain of carrier PLL.

From Eq. (4.2) it is easy to learn that in the case of the modulated carrier, the largest range rate error occurs when $\omega_{sc}\dot{\tau}_s$ is smaller than the double-sided noise bandwidth of the carrier loop and $\omega_c\dot{\tau}_s/2\pi$ is smaller than the data bit rate. In such a situation the maximum range rate occurs when $J_0(\nu) = 1$, i.e., when $\cos \omega_{sc} t_s = 1$. The maximum error is

$$\Delta\dot{R}_{max} = \frac{2\pi c}{\omega_c} K_c a_s \left\{\overline{\left(h_{LPF}(t) \oplus \sin \epsilon\right)^2}\right\}^{1/2} . \tag{4.4}$$

The lowpass filter reduces the power in sin ϵ by the ratio B_{nc}/r, where

r = the data bit rate

B_{nc} = double-sided noise bandwidth of the carrier loop in Hz

The total power of sin ϵ is $\overline{\sin^2 \epsilon} = 1/2$. Therefore

$$\Delta\dot{R}_{max} = \frac{\sqrt{2}\,\pi c}{\omega_c} K_c a_s \left(\frac{B_{nc}}{r}\right)^{1/2} . \tag{4.5}$$

In the case of the modulated subcarrier, the bandwidth of $J_0(\nu)$ is δr, and the bandwidth of sin ϵ is much narrower than the PLL noise bandwidth. Therefore, since the maximum value of sin ϵ is one, the rms range rate error is

$$\Delta \dot{R}_{max} = \frac{2\pi c}{\omega_c} kK_c a_s \left(\frac{B_{nc}}{\delta r} \right)^{1/2} , \tag{4.6}$$

where k is the rms value of $J_0(\nu)$, and is a decreasing function in δ, and is bounded as

$$\frac{2^{-1/4}}{\sqrt{\pi\delta}} < k < \frac{1}{\sqrt{2}} , \quad \text{for } \delta > 3 . \tag{4.7}$$

4.1.2 Error in Range Measurement from Subcarrier Phase

The satellite range can be measured from the sinusoidal phase of the subcarrier VCO output. The mean square error in the range measurement is derived in this section for a receiver having a limiter in carrier phase detection.

At the sufficiently large subcarrier frequency that was discussed in Sec. 3.1, the noise caused by the multipath, at the input to the subcarrier VCO, is from Eq. (2.7) the lowpass component of

$$K_{sc} \frac{2}{\delta} \phi_{BPsc}(t) \cos \omega_{sc} t_d . \tag{4.8}$$

Substituting $\phi_{BPsc}(t)$ defined in Eq. (3.14) into this expression, the noise caused by the multipath is the lowpass component of

$$K_{sc} \frac{a_s}{\delta} J_1(\nu) 2(\cos \epsilon)(\sin \gamma) . \tag{4.9}$$

The mean square phase error of the VCO output is the product of the power spectrum density of the input noise by the noise bandwidth of the subcarrier PLL. When the data modulate the carrier phase, the root-mean-square phase error, caused by the multipath is, in the worst multipath disturbance, from Eq. (4.9),

$$\left\{\overline{\phi_{sc}^2}\right\}^{1/2} = K_{sc}\,\frac{a_s}{\delta}\,J_1(\nu)\sqrt{2}\left(\frac{B_{nsc}}{r}\right)^{1/2} \text{ (radians)}, \qquad (4.1$$

where $J_1(\nu)$ should be substituted by the maximum value, 0.58, in order to compute the maximum phase error.

When the data modulate the subcarrier phase, cos ϵ should be substituted by its maximum, one, and the bandwidth of $J_1(\nu)\sin\gamma$ is $r(1+\delta)$. Therefore, since the maximum of $J_1(\nu)$ is 0.58,

$$\text{rms value of } \{J_1(\nu)\sin\gamma\} \lesssim \frac{1}{4},$$

and a bound to the rms phase error is

$$\left\{\overline{\phi_{sc}^2}\right\}^{1/2} \lesssim K_{sc}\,\frac{a_s}{2\delta}\left\{\frac{B_{nsc}}{r(1+\delta)}\right\}^{1/2} \quad \text{radians}.$$

4.2 Systems Performance without Limiters

4.2.1 Error in Satellite Range Rate Measurement from Carrier Doppler Frequency

The error in the input to the carrier VCO is the error in the output frequency from the VCO. The error in the VCO output frequency is proportional to the error in the range rate measurement. The error at the input to the VCO is an additive noise, and is the lowpass component of

$$e_s(t)e_{ref}(t)$$

which is expressed in Eq. (2.10), and it is also the lowpass component of

$$a_s J_0(\nu)\sin\epsilon,$$

where ν and ϵ are as defined in Eq. (3.18) in the case of the modulated carrier, and as defined in Eq. (3.24) in the case of the modulated subcarrier.

The double-sided frequency bandwidth of $J_0(\nu)$ is smaller than the PLL noise bandwidth in the worst disturbance case when carrier is modulated. When the subcarrier is modulated, the double-sided bandwidth is

$$\delta r \; .$$

The double-sided frequency bandwidth of sin ϵ is r when the carrier is modulated. And it is much narrower than the PLL noise bandwidth when the subcarrier is modulated.

The rms values of the additive noise are

$$\frac{1}{\sqrt{2}} K_c a_s \left(\frac{B_{nc}}{r}\right)^{1/2} \quad ; \text{ modulated carrier case}^*$$

$$k K_c a_s \left(\frac{B_{nc}}{\delta r}\right)^{1/2} \quad ; \text{modulated subcarrier case,}$$

where $2^{-1/4}/\sqrt{\pi\delta} < k < 1/\sqrt{2}$, for $\delta > 3$, and k is a decreasing function of δ. The rms range rate errors are the products of the additive phase noises and c/ω_c.

4.2.2 Error in Range Measurement from Subcarrier Phase

From Eqs. (3.23) and (3.26), and the error signal due to the specular reflection component at the input to the subcarrier VCO is the lowpass component of

$$K_{sc} \frac{2}{\delta} a_s J_1(\nu)(\cos \epsilon)(\sin \gamma) \; .$$

The bandwidth of $J_1(\nu)$ is narrower than the PLL noise bandwidth when the carrier is modulated, and δr when the subcarrier is modulated.

* This maximum error is the same as the error of the tracking system without the subcarrier.

The bandwidth of cos ϵ is r when the carrier is modulated, and narrower than the PLL noise bandwidth when the subcarrier is modulated.

The bandwidth of sin γ is narrower than the PLL noise bandwidth when the carrier is modulated, and r when the subcarrier is modulated.

When the carrier is modulated, the root-mean-square phase error caused by the multipath is, in the worst situation,

$$\left\{\overline{\phi_{sc}^2}\right\}^{1/2} = K_{sc} \frac{\sqrt{2}}{\delta} a_s (0.58) \left(\frac{B_{nsc}}{r}\right)^{1/2} ,$$

where the factor 0.58 is the maximum of $J_1(\nu)$. When the subcarrier is modulated,

$$\text{rms value of } \{J_1(\nu) \sin \nu\} \lesssim \frac{1}{4} ,$$

and the rms range error is bounded as

$$\left\{\overline{\phi_{sc}^2}\right\}^{1/2} \lesssim K_{sc} \frac{a_s}{2\delta} \left(\frac{B_{nsc}}{r(1+\delta)}\right)^{1/2} .$$

5. CONCLUSIONS

The performances of the TDRS systems employing wideband FM VHF signals are evaluated. The carrier is FM modulated by a sinusoidal subcarrier in order to suppress the multipath disturbance. The systems' configurations are explained in detail in another memorandum under the contract.

Two types of communication signals are considered. In one type the signal carrier is PM modulated by the binary data. In the other, the subcarrier is PM modulated. Two types of receivers, one with a limiter for carrier phase detection, the other without, are treated. The system performance is approximately evaluated in the case where the receiver has a limiter, and accurately evaluated in the case where the receiver does not have a limiter. The minimum subcarrier frequencies that are required in order to effectively suppress the multipath disturbances have been derived. If the data modulate the carrier, the minimum subcarrier frequency is the data bandwidth plus a few times of the maximum subcarrier differential-doppler. If the data modulate the subcarrier, the minimum subcarrier frequency is $r(1 + \delta)$ plus a few times of the maximum subcarrier differential-doppler where r is the data bit rate and δ is the FM modulation index. The worst situation systems-performances which can occur at certain values of differential delay and doppler have been evaluated at the subcarrier frequencies thus chosen.

If a PM communication signal has no subcarrier, and if the receiver has an amplitude limiter before the carrier phase detection, the minimum (in the worst multipath disturbance) amplitude of the matched filter output can be derived to be

$$1 - 2a_s , \qquad (5.1)$$

where a_s is the coefficient of specular reflection. The second term in Eq.(5.1) is caused by the multipath.

If the signal is wide-band-FM modulated by a sinusoidal subcarrier, and if the receiver has the limiter, the minimum output from the matched filter is

$$1 - 2a_s J_0(\nu) \tag{5.2}$$

as derived in Sec. 3.1.1. Comparing Eqs. (5.1) and (5.2) it can be noticed that the term of the multipath disturbance is in the ratio of one-to-$J_0(\nu)$, $J_0(\nu)$ being the Bessel function of the first kind, of order zero, where ν is, as defined in Eq. (3.4),

$$\nu \equiv \delta\sqrt{2}\,(1 - \cos\omega_{sc} t_s)^{1/2} , \tag{5.3}$$

where

δ = FM modulation index

$t_s \approx \tau_s + \dot{\tau}_s t$

τ_s = additional delay for the specular reflection path .

$\dot{\tau}_s$ = time derivative of τ_s

The total time during which $J_0(\nu)$ is larger than a specific value is actually determined by δ.

If a PM communication has no subcarrier, and if the receiver has no amplitude limiter, the minimum matched filter output can be derived to be

$$1 - a_s . \tag{5.4}$$

If the signal is wide-band-FM modulated by a sinusoidal subcarrier, and if the receiver has no limiter, the minimum matched filter output is

$$1 - a_s J_0(\nu) \tag{5.5}$$

as derived in Sec. 3.2.1.

If the subcarrier, instead of the carrier, is PM modulated by the binary data, a lower bound to the matched filter output is, when $\delta > 3$,

$$1 - \frac{a_s}{2\delta(1 + \delta)^{1/2}} \tag{5.6}$$

This is an approximate bound if the receiver has an amplitude limiter, and it is always right if the receiver does not have the limiter.

The rms errors in the satellite range tracking from the subcarrier phase and the range rate tracking from the carrier doppler frequency are derived. In both the types of systems that the data modulate the carrier phase and the data modulate the subcarrier phase, the approximate errors when the receivers have limiters are the same as when the receivers do not have limiters. The approximate errors when the receivers have limiters are valid if the reflection coefficient a_s is smaller than 1/3.

If the data modulate the carrier phase, the rms error in the range rate tracking is reduced by the ratio, $J_0(\nu)$ to one, where ν is as defined in Eq. (5.3), compared with a system which has no subcarrier (not wideband FM).

If the data modulate the subcarrier, the rms range rate error is a function of δ, and it decreases in δ faster than $1/\sqrt{\delta}$, and at $\delta = \sqrt{2}$ the error is about the same as if there were no subcarrier.

If the data modulate the carrier phase, the maximum rms error in the range tracking decreases with δ like $1/\delta$.

If the data modulate the subcarrier phase, the rms range error is smaller than in the case of modulating the carrier. The rms range error decreases with δ faster than

$$\frac{1}{\delta(1 + \delta)^{1/2}} .$$