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# MULTIPATH ERRORS IN RANGE RATE MEASUREMENT BY A TDRS/VHF - GRARR

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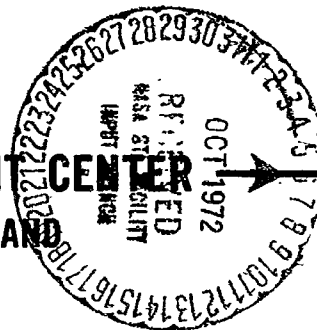
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MULTIPATH ERRORS IN RANGE RATE MEASUREMENT  
BY A TDRS/VHF - GRARR

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## ABSTRACT

Range rate errors due to multipath reflection are calculated for a Tracking and Data Relay Satellite (TDRS) system using the VHF Goddard Range and Range Rate (GRARR) system. At VHF the reflection is primarily specular, and the strength of the multipath relative to the direct path can be modeled in terms of the geometry and the surface characteristics, specifically the root-mean-square (rms) ocean wave height. The uplink and downlink multipath introduces phase jitter on the GRARR carrier and subcarrier. The derivation of these effects is reviewed leading to an expression for the rms range rate error. The derivation assumed the worst-case orbital configurations in which there was very little relative specular doppler. This means that the specular multipath interference was not attenuated by the carrier and subcarrier PLL transfer functions. Curves of range rate error are presented as a function of grazing angle with wave height 0.3 to 0.7 meters and spacecraft altitude 100 to 700 miles as parameters.

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## TECHNICAL MEMORANDUM G-161 - 5

### MULTIPATH ERRORS IN RANGE RATE MEASUREMENT BY A TDRS/VHF - GRARR

#### 1. INTRODUCTION

The VHF GRARR system which utilizes a tracking and data relay satellite (TDRS) is subject to multipath interference in the link between the user spacecraft and the TDRS. This interference arises from signal reflections off the earth's surface. The intensity of the reflected signals is sometimes sufficiently strong to cause non-negligible range-rate errors. The worst-case errors are computed and the conditions under which they occur are described in this memorandum.

#### 2. ELECTROMAGNETIC REFLECTIONS OFF THE EARTH'S SURFACE

Depending on such factors as the wavelength of the electromagnetic radiation, the grazing angle, and the roughness of the scattering surface, the scattering off the earth's surface may be either specular or diffuse. Diffuse scattering is much more likely at S-band than at VHF, particularly at low grazing angles. For low grazing angles (10° - 20°) and for calm to moderate seas, scattering off the ocean (the most serious scattering problem) is primarily specular at VHF. The Rayleigh criterion is often used to determine whether the scattering is primarily specular or diffuse. The Rayleigh criterion says that the scattering will be primarily specular if

$$\frac{8\sigma_w \sin \gamma}{\lambda_c} < 1$$

where  $\sigma_w$  is the rms surface height,  $\lambda_c$  is the wavelength of the electromagnetic radiation, and  $\gamma$  is the grazing angle.

For the specular scattering case, the ratio of the average received power from the multipath interference to that of the desired direct-path signal may be written as

$$\langle a_s^2 \rangle = \langle \rho^2 \rangle D^2 \Gamma R_o^2$$

where  $\langle \rho^2 \rangle$  is the scattering coefficient which takes into account the roughness of the scattering surface;  $D^2$  is a divergence factor which accounts for the curvature of the scattering surface resulting from the earth's sphericity;  $\Gamma$  accounts for the effects of the antenna directivity at the user satellite; and  $R_o$  takes into account the fact that the sea surface is not a perfect conductor.

The scattering coefficient, for a distribution of surface heights which may be assumed Gaussian is

$$\langle \rho^2 \rangle = e^{-\left(\frac{4\pi\sigma_w \sin \gamma}{\lambda_c}\right)^2}$$

The divergence factor  $D^2$  is given by

$$D^2 = \frac{1}{\left(1 + \frac{2r_1 r_2}{a(r_1 + r_2)}\right) \frac{1}{\sin \gamma}} \frac{1}{\left(1 + \frac{2r_1 r_2}{a(r_1 + r_2)}\right)}$$

where the distances  $a$ ,  $r_1$  and  $r_2$  are defined by the multipath geometry shown in Fig. 1.

Curves of the divergence factor for grazing angles between  $10^\circ$  and  $30^\circ$  and for user satellite orbit altitudes of 100, 400, 700, and 1000 miles are shown in Fig. 2.

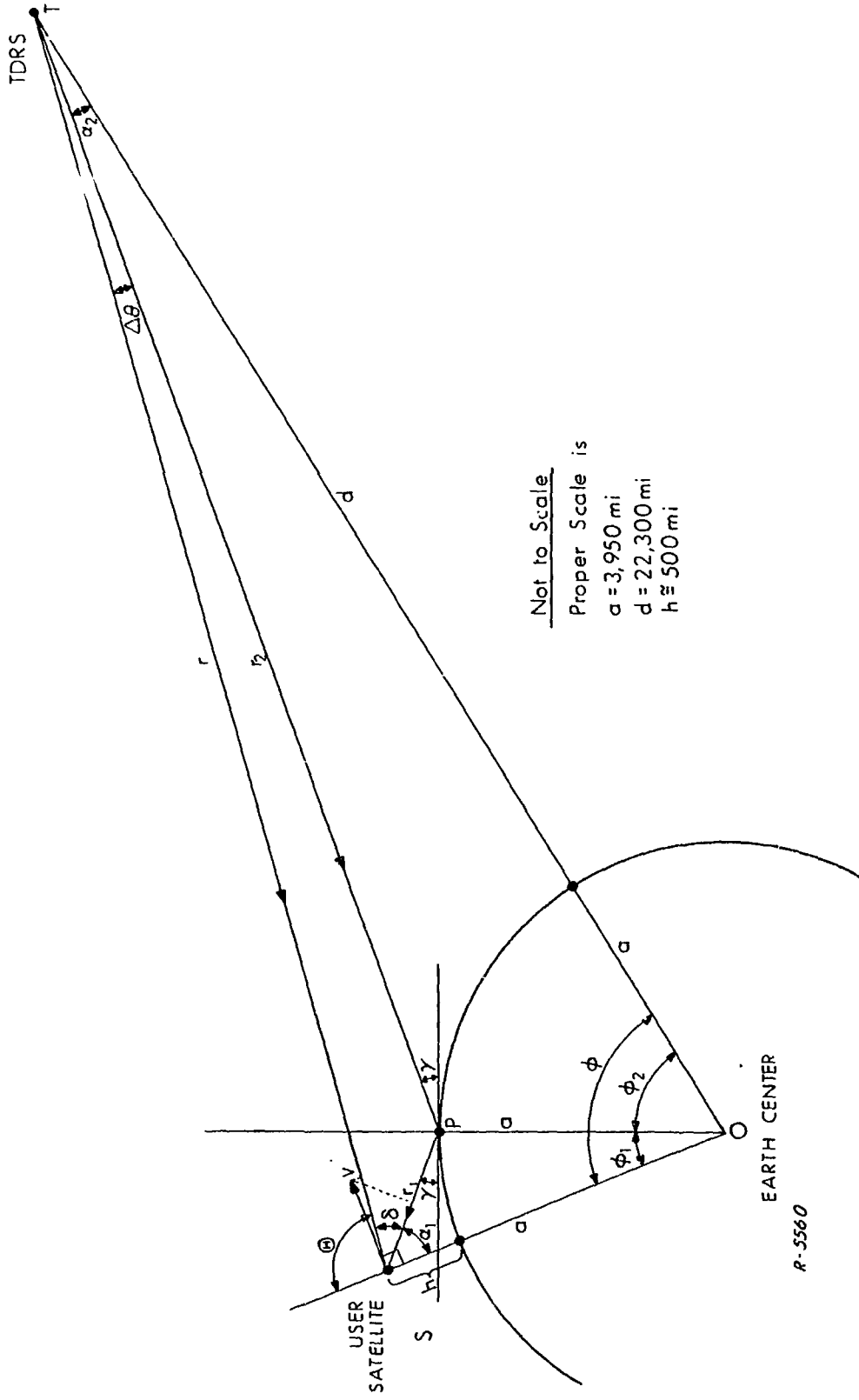


Fig. 1 Multipath Geometry

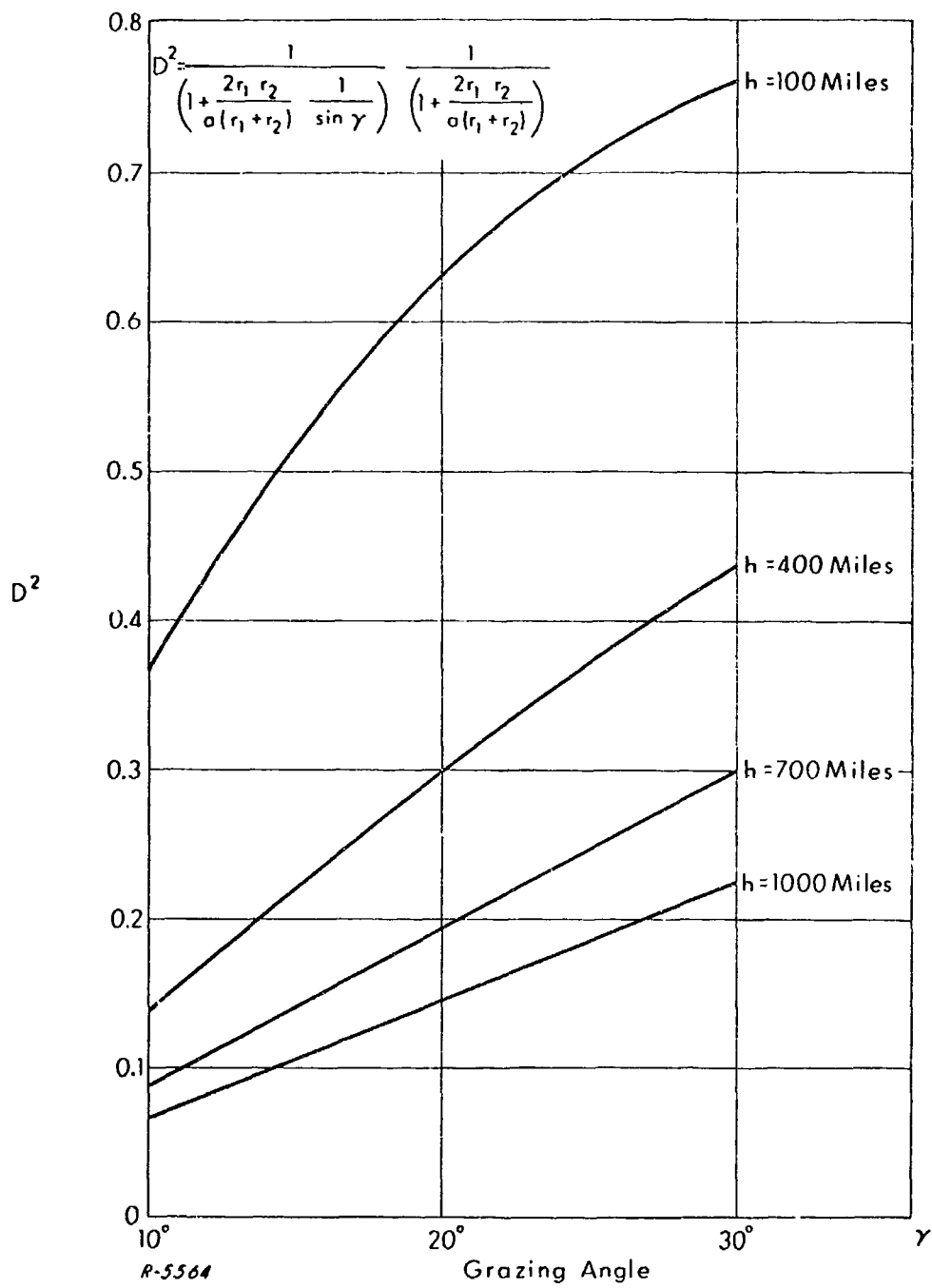


Fig. 2 Divergence Factor



The ocean wave rms height  $\sigma_w$ , of up to 1 meter can satisfy the Rayleigh criterion for specular reflections when the carrier frequency is about 150 MHz and the reflection grazing angle is within  $10^\circ$  to  $30^\circ$ . The rms ocean wave height is actually about 1 meter.

### 3. RANGE-RATE ERRORS ARISING FROM EARTH SCATTERED MULTIPATH INTERFERENCE

Expressions for the earth reflected multipath components of the phase jitters in the received carrier and subcarrier of the GRARR signal were computed by Wachsman and Ghais.<sup>1</sup>

The rms range-rate error may be shown to be

$$\epsilon_{\dot{R}} \approx \frac{c}{\sqrt{2} f_t} \frac{f_b}{2\pi N} \left\{ \langle \phi^{sc}(t)^2 \rangle + \left( \frac{13}{12} \right)^2 \langle \phi^c(t)^2 \rangle \right\}^{1/2}$$

where

$f_t$  = uplink carrier frequency

$f_b$  = bias frequency for doppler count

$N$  = number of doppler-plus-bias count

$c$  = speed of electromagnetic propagation

From Appendix A,  $\langle \phi^c(t)^2 \rangle = \frac{1}{2} a_s^2$  and  $\langle \phi^{sc}(t)^2 \rangle = \frac{1}{4} a_s^2 + \frac{1}{2} a_s^2$ . Using the results of the preceding section the curves of Fig. 3 have been computed. The relative multipath doppler is assumed to be sufficiently small to cause the multipath interference to lie well within the loop noise bandwidths. If this were not the case, the attenuation of this interference by the loop transfer function would greatly reduce the rms range-rate error.

1. R. H. Wachsman and A. F. Ghais, "Multipath Effects on GRARR Tracking Utilizing a Data-Relay Satellite." Prepared under Contract No. NAS5-10780 for NASA Goddard Space Flight Center, Greenbelt, Md., 20771, Sept. 15, 1969.

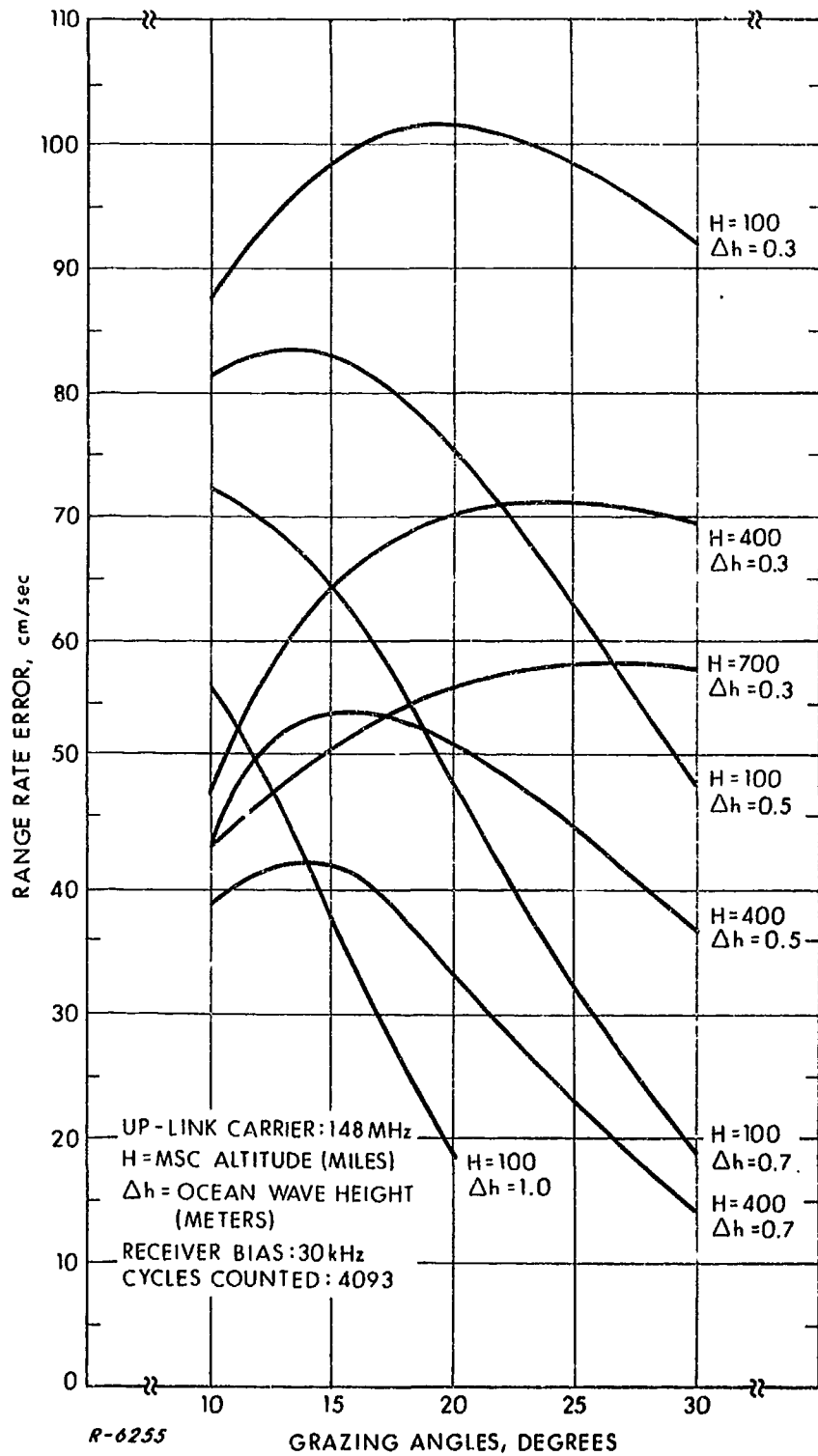


Fig. 3 The Range-Rate Errors

## APPENDIX A

### THE CARRIER AND SUBCARRIER PHASE JITTERS DUE TO THE EARTH SPECULAR REFLECTION

The carrier and the subcarrier phase jitters caused by the specular reflection of the earth along the uplink and downlink between the spacecraft and the tracking and data relay satellite (TDRS) are derived by Wachsmann and Ghais.<sup>1</sup> The derivations of the phase jitter formulae are presented here for the convenience of the readers.

The direct path component of the uplink carrier when it arrives at the spacecraft can be written as

$$\sin(\omega_t t - \omega_t \dot{\tau}_d t) , \quad (\text{A.1})$$

where  $\omega_t$  is the transmitted carrier frequency and  $\dot{\tau}_d$  is the time derivative of the time delay of the signal to reach the spacecraft. The specular reflection component of the carrier can be written as

$$a_s \sin(\omega_t t - \omega_t \dot{\tau}_d t - \phi_s - \omega_t \dot{\tau}_s t) , \quad (\text{A.2})$$

where  $\phi_s$  is the phase difference,  $\dot{\tau}_s$  is the delay time difference with respect to the direct path component, and  $a_s$  is the reflection coefficient. When the direct and reflected components are added together and have gone through an amplitude limiter at the spacecraft, the carrier component becomes

$$e_r(t) = \sin\{\omega_t t - \omega_t \dot{\tau}_d t - \phi_m^c(t)\} , \quad (\text{A.3})$$

where

$$\phi_m^c(t) \approx a_s \sin(\phi_s + \omega_t \dot{\tau}_s t) .$$

The approximately equal sign holds when the reflection coefficient,  $a_s$ , is sufficiently smaller than one.

At the spacecraft transponder, the received signal is heterodyned down to the intermediate frequency after being mixed with a multiple (M) of a locally generated reference signal. The resultant signal will phase-modulate the new carrier which is another multiple (N) of the local reference signal. The transmitted downlink can be written as

$$e_T(t) = \sin[\omega_T t + K \sin\{\omega_{sc} t + \phi_m^c(t)\}] , \quad (A.4)$$

where

$$\omega_T = N\omega_o$$

$$\omega_{sc} = M\omega_o - \omega_t(1 - \dot{\tau}_d)t ,$$

$$\omega_o = \text{transponder oscillator frequency}$$

When the downlink signal reaches the TDRS, it is in the form (approximately)

$$e_R(t) \approx \sin[\omega_T t - \omega_T \dot{\tau}_d t + K \sin\{\omega_{sc} t - \omega_{sc} \dot{\tau}_d t - \phi_m^c(t)\}] \\ + a_S \sin[\omega_T t - \omega_T \dot{\tau}_d t - \phi_S - \omega_T \dot{\tau}_s t + K \sin\{\omega_{sc} t - \omega_{sc} \dot{\tau}_d t - \omega_{sc} \tau_\varepsilon - \omega_{sc} \dot{\tau}_s t\}] . \quad (A.5)$$

This signal can be separated into the carrier

$$e_R^c(t) = \sin\{\omega_T t - \omega_T \dot{\tau}_d t - \phi^c(t)\} , \quad (A.6)$$

where

$$\phi^c(t) \approx a_S \sin(\phi_S + \omega_T \dot{\tau}_s t)$$

and the subcarrier

$$e_R^{sc}(t) \approx \sin\{\omega_{sc} t - \omega_{sc} \dot{\tau}_d t + \phi_m^c(t) - \phi_M^{sc}(t)\} , \quad (A.7)$$

where

$$\phi_M^{sc}(t) \approx a_S \cos(\phi_S + \omega_T \dot{\tau}_s t) \sin(\omega_{sc} \tau_\varepsilon + \omega_{sc} \dot{\tau}_s t) .$$

The net subcarrier phase jitter is

$$\phi^{sc}(t) \equiv \phi_M^{sc}(t) - \phi_m^c(t) \quad (\text{A.8})$$

The function of the doppler extractor at the receiver is to generate a signal of the form

$$e_D(t) \approx \sin\{\omega_b t + 2\omega_t \dot{\tau}_d t - \frac{M}{N} \phi^c(t) - \phi^{sc}(t)\}, \quad (\text{A.9})$$

where  $\omega_b$  is the doppler bias frequency.

For the VHF GRARR, the ratio  $M/N$  is (13/12).