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Department of Physics and Astronomy
University of Maryland
College Park, Maryland 20742

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James A. Earl
Department of Physics and Astronomy, University of Maryland
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The origin and time variations of the steep spectrum of electrons observed below 20 MeV may be explained by a simple model in which the spectrum of interplanetary cosmic rays is decomposed, at low energies, into two independently varying components.

Recent experimental results (L'Heureux, Fan and Meyer, 1972) (McDonald, Cline and Simnett, 1972) have identified variations of the steep spectrum of electrons below 20 MeV as an aspect of cosmic ray modulation. But these short term fluctuations around a stable average are qualitatively different from long term changes of the electron spectrum seen above 20 MeV (Meyer, Schmidt and L'Heureux, 1972) which are analogous to the modulation of protons and helium nuclei at comparable energies (Rygg and Earl, 1971). Fisk and Van Hollebeke (1972) have explained this difference as the effect of a modulating region far beyond the orbit of Earth. Computer calculations (Lezniak and Webber, 1971) (Goldstein, Ramaty and Fisk, 1970) have succeded in reproducing most features of the observed electron spectrum. However these treatments involve a somewhat arbitrary specification of parameters and are cumbersome to apply to short term variations. As an alternative, this letter presents an analytical approach which provides qualitative insight into both observations and numerical calculations and which embodies, as a natural feature of a specific model, two cosmic ray components whose temporal variations are uncorrelated.

Cosmic ray modulation results from two factors: an equilibrium between diffusion and convection in which interstellar particles gain access to the inner solar system by diffusing upstream through a scattering medium moving outward with the solar wind velocity V (Parker, 1958) and adiabatic deceleration in which particles lose energy in collisions with scattering centers in the expanding medium. (Laster, Lenchek and Singer, 1962). Parker's (1965) equation, which describes these effects, can be formulated as:

$$\frac{1}{r^2}\frac{\partial}{\partial r} r^2 S = V\frac{\partial}{\partial r} (C-1)F \qquad (1)$$

$$S = VCF - K(\partial F/\partial r) \qquad (2)$$

where S is the radial streaming flux, r is radial distance from the sun, and where the particle density in phase space F is related to the differential intensity J,

$$F = Jc^2/(E^2 - E_o^2) \qquad (3)$$

where E is total energy, E_o is rest energy and c is the velocity of light (Gleeson and Axford, 1967) (Jokipii, 1971). The parameter C, discussed by Forman (1970), is defined by, $CF = -(\alpha T/3)(\partial F/\partial T)$, where T is kinetic energy and $\alpha = (T+2E_o)/(T+E_o)$. The cosmic ray diffusion coefficient K is related to the power spectrum of interplanetary magnetic irregularites (Jokipii and Coleman, 1968) (Sari and Ness, 1969). (See Fig. 1.)

Fig. 2a illustrates the model. The dotted box encloses a region in r, T space, $T < T_o$ and $r < D$, within which K is constant. Elsewhere, particles diffuse freely ($K = \infty$) with the interstellar spectrum $F(\infty,T) = PT^{-\sigma}$, a power law in kinetic energy for which $C = \alpha\sigma/3$. The density F of cosmic rays within the box is given by the sum of two components: first, a constant independent of r and T, $F_1 = PT_o^{-\sigma}$, which matches the interstellar density at the $T = T_o$ boundary and which is evidently a solution of Eq. 1 and Eq. 2 and, second, a solution F_2 which matches at the $r = D$ boundary the spectrum $F_2(D,T) = P(T^{-\sigma} - T_o^{-\sigma})$ for $T < T_o$. Just below T_o, F_2 is difficult to compute but, for $T \ll T_o$, a solution derived by Parker (1965) applies

$$F_2(r,T) = PT^{-\sigma} \frac{M(2C,2,Vr/K)}{M(2C,2,VD/K)} \qquad (4)$$

where M is a confluent hypergeometric function (Abromowitz and Stegun, 1964). As is indicated by the contour lines of Fig. 2a, F_2 decreases rapidly with distance inward from the boundary. Thus the predicted spectrum for $r < D$, as indicated in Fig. 2b, consists, at high energies ($T > T_o$), of the interstellar spectrum, at intermediate energies ($T \lesssim T_o$), of a region of constant density F_1 populated by particles adiabatically decelerated from the boundary at $T = T_o$ and, at low energies ($T \ll T_o$), of an attenuated power law representing the contribution of particles diffusing inward from the boundary at $r = D$. In the more general case when K varies with r and T within the box, the primitive solution, F_1 = const., remains valid for any choice of T_o, while the F_2 component is described at low energies, $(Vr/K) \gg 1$, by an approximate solution extensively discussed by Fisk and Axford (1969),

$$F_2(r,T) = F(\infty,T) \exp - \int_r^D (V/K)\, dr \qquad (5)$$

which gives a steep but attenuated spectrum similar to that of Eq. 4. Eq. 5 is identical to that originally derived by Parker (1958) neglecting deceleration because the time required for particles to diffuse one scale length from the boundary K/V^2 is much smaller than the time for significant energy loss r/V. This restriction implies that Eq. 5 must be used with caution when the attenuation is large, but, in this case, the F_1 component will dominate.

An important feature of the model is that the cosmic ray density within the broad range of energy where F_1 dominates depends upon magnetic spectral power at relatively low frequencies (below 10^{-4} Hz) corresponding to energies above T_o. (See Fig. 1.) In contrast, the density in regions dominated by F_2 is affected by power at much higher frequencies (above 10^{-3} Hz). This means that variations in the two components could be nearly independent. The fact that short term variations of the magnetic spectrum are observed at high frequencies (Siscoe, et. al., 1968) (Sari and Ness, 1969) but not at low frequencies (Mathews, Quenby and Sear, 1971) implies that the F_1 component at a given energy could remain nearly constant in the presence of rapid variations of F_2 at the same energy. The exponential relationship in Eq. 5 implies that the F_2 component will show large fluctuations in response to relatively small changes in the parameters V, D and K. On the other hand, the point made by Mathews et al., (1971), that observed long term variations in these parameters appear to be too small to account for the modulation of protons above 1 GeV, also applies to the electron F_1 component. This contradiction can be explained by assuming that T_o corresponds to a fixed value of K and that $K = K_o T^a$, in which case fractional changes $\Delta F_1/F_1$ are sensitively dependent upon changes in K_o resulting from long term variations in the magnetic power spectrum at low frequencies,

$$(\Delta F_1/F_1) = (\sigma/a)(\Delta K_o/K_o) \qquad (6)$$

If $\sigma = 3.6$ ($J \propto T^{-2.6}$) and if $a = 0.75$ (see Fig. 1), $(\Delta F_1/F_1) = 5\ (\Delta K_o/K_o)$.

In the observed spectra of protons presented in Fig. 3a, the region of constant density from 30 to 300 MeV, which implies the relationship $J = AT$ where A is constant (See Eq. 3.), has been identified by Rygg and Earl (1971) as an F_1 component. For electrons (Fig. 2b), the analogous behavior $J = BT^2$, corresponding to F_1 = const., is not conspicuous. Luhman has demonstrated that electron intensities published before 1971 are marginally consistent with the

$J = BT^2$ law, but the data of Meyer, et. al., (1971) do not display the striking plateau that marks the proton F_1 component. This absence may be a result of experimental difficulties in resolving the narrow trough expected in the intensity spectrum between 100 MeV and 200 MeV, or the F_1 and F_2 components may be inherently less distinctly separated for electrons than for protons, either because the electron diffusion coefficient is less strongly dependent on energy (See Fig. 1) or because the interstellar spectrum of electrons is steeper.

On the other hand, low energy electrons exhibit the steep spectrum (Simnett and McDonald, 1969) and short term fluctuations (L'Heureux, Fan and Meyer, 1972) expected for an F_2 component. These fluctuations include quiet time increases (McDonald, Cline and Simnett, 1972) which embody the anticorrelation with solar activity implied by Eq. 5. The fact that long term changes are almost imperceptible indicates that the magnetic spectral density at frequencies above 10^{-2} Hz fluctuates around a stable average level. The steep and intense interstellar spectrum of electrons deduced at low energies from radio data (Goldstein, Ramaty and Fisk, 1970)(Alexander, et. al., 1969) suggests that the factor of 200 displacement in Fig. 2b between the solid line extending the high energy spectrum and the dotted line representing the modulated spectrum can be identified with the attenuation factor in Eq. 4 giving D = 4 a.u. ($K = 3 \times 10^{20}$ cm^2 sec^{-1}, $V = 4 \times 10^7$ cm/sec). Although this estimate for D is sensitive to the radial dependence of K, it is in fair agreement with values based upon observations of solar flare particles (Burlaga, 1967) and with those based upon calculated rates of turbulent wave damping (Jokipii and Davis, 1969). The observed 3 to 14 day duration of the quiet time increases can be interpreted in terms of the integral appearing in Eq. 5 as the time required for a spatial region of abnormally large diffusion coefficient to be convected from r = 1 a.u. to r = D. This approach puts D at 2 to 5 a.u., consistent with the above estimates.

In view of the similarity between the observed proton spectrum (Fig. 3a) and the predicted spectrum (Fig. 2b) and in view of the fact that the model does explain electron observations, it is appropriate to attempt to interpret the steep upturn in protons below 30 MeV (Fan, et. al., 1968) as an F_2 component. There are two objections to this construction. First, the large energy density implied by a steep and intense spectrum of galactic protons is difficult to reconcile with current understanding of the dynamics and heating of the interstellar medium. Second, numerical analysis demonstrates that clearly separated F_1 and F_2 components are present when K is a continuous function of energy (J. Luhmann, private communication), but plausible values of the parameters give a much smaller intensity for the proton F_2 component at Earth than is observed (Lezniak and Webber, 1971). However, the argument based upon energy density, while powerful, is indirect, and not all choices of interplanetary parameters consistent with a proton F_2 component are ruled out by existing data. Consequently, it is worth taking note, in the paragraph that follows, of the fact that direct observations of interplanetary protons and helium are, at least, consistent with the model presented here.

Kinsey (1970) argues that the high degree of variability he observed for protons below 30 MeV indicates a solar origin. While solar particles are undoubtedly present during events, the fluctuations seen during quiet times could also be interpreted as those expected for a galactic F_2 component. In this picture, Eq. 2 and Eq. 5 predict for the F_2 component an outward flux $S = V(C-1)F_2$ which is consistent with those observed for protons below 10 MeV (Gleeson, et. al., 1971)(Rao, et. al., 1967). The observations summarized by O'Gallagher (1972) are in qualitative agreement with Fig. 2a. Here, the small gradients seen near Earth can be identified with the zero gradient of the F_1 component while the single observation of a large positive gradient at low energies was carried out on Mariner 4 which is the only mission that may have sampled the large F_2 component expected beyond 1 A.U. The negative gradients seen by Gleeson, et. al., (1971) may be due to solar particles. However, the presence within 1 A.U. of a solar component whose intensity is decreasing with distance from the sun, is not necessarily inconsistent with the simultaneous existence of a galactic F_2 component which dominates at and beyond Earth.

The questions raised here will soon be resolved when results from the Pioneer 10 mission to Jupiter confirm or deny the existence of the large positive gradients expected at low energies for F_2 components.

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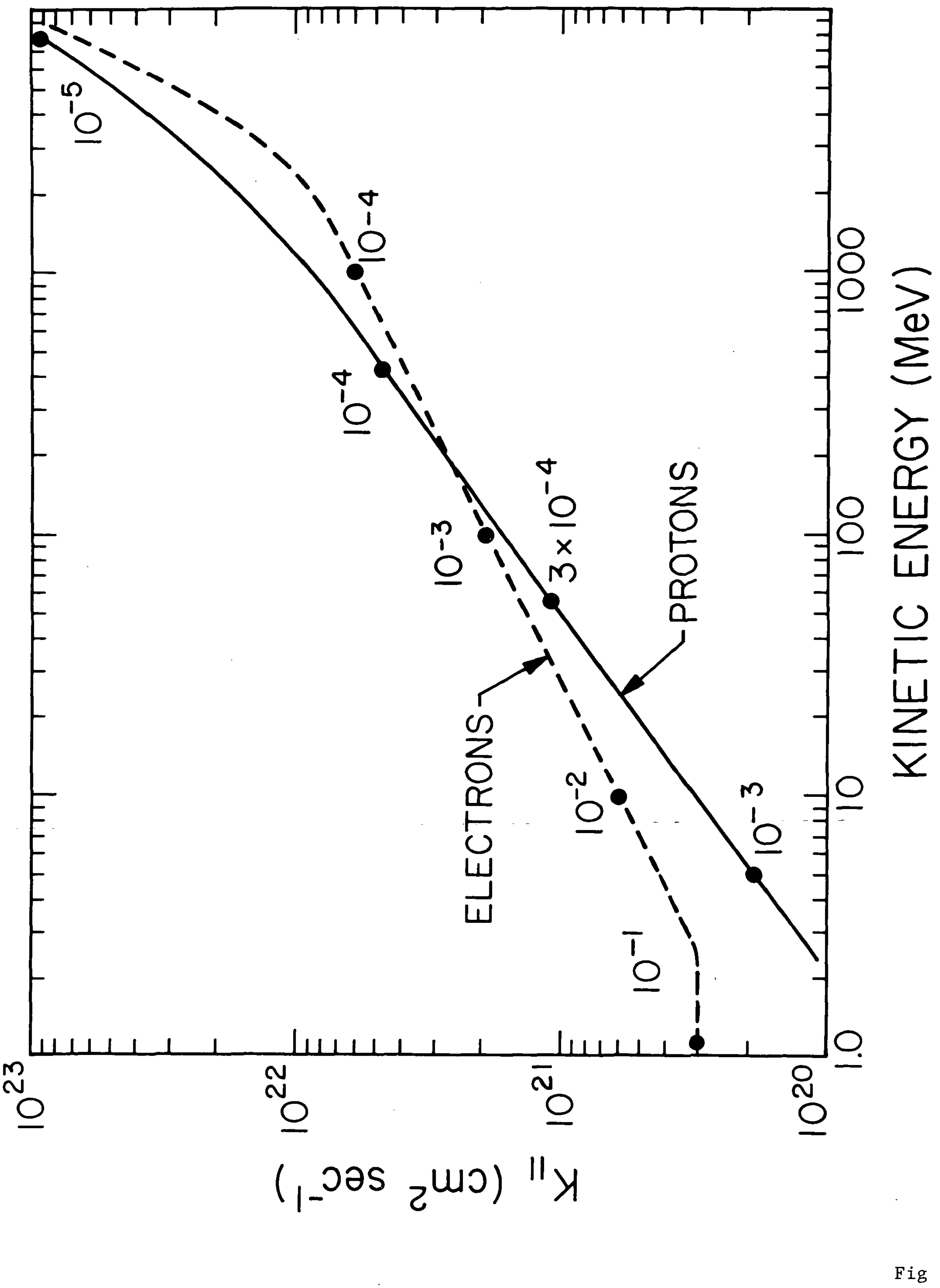
FIGURE CAPTIONS

Figure 1. Cosmic ray diffusion coefficient K plotted as a function of kinetic energy after Jokipii and Coleman (1968). Circles indicate the frequency in Hz of magnetic fluctuations responsible for scattering particles of the given energies. The region of constant K for low energy electrons is based on the results of Sari and Ness (1969).

Figure 2. Within the dotted box, the spectrum consists of two components F_1 = const., dominant in the cross hatched region, and F_2 a spectrum varying rapidly with energy and radius as indicated by the sloping contour lines just inside the boundary. At a fixed radius, these components appear in the spectrum at right as a plateau just below T_o and a steep spectrum at low energies.

Figure 3. The observed proton spectra exhibit the qualitative behavior of Figure 2b, but the plateau is not visible in the electron spectra.

Proton data are those of Rygg and Earl (1971) (circles) and of Hsieh (1970) and Mason and Simpson (1971) (crosses). Electron data were reported by Meyer, Schmidt and L'Heureaux (1971).



$K_{\parallel}$ (cm^2 sec^{-1})
10^{23}
10^{22}
10^{21}
10^{20}
KINETIC ENERGY (MeV)
1.0
10
100
1000
ELECTRONS
PROTONS
10^{-5}
10^{-4}
10^{-4}
3×10^{-4}
10^{-3}
10^{-3}
10^{-2}
10^{-1}

Fig 1

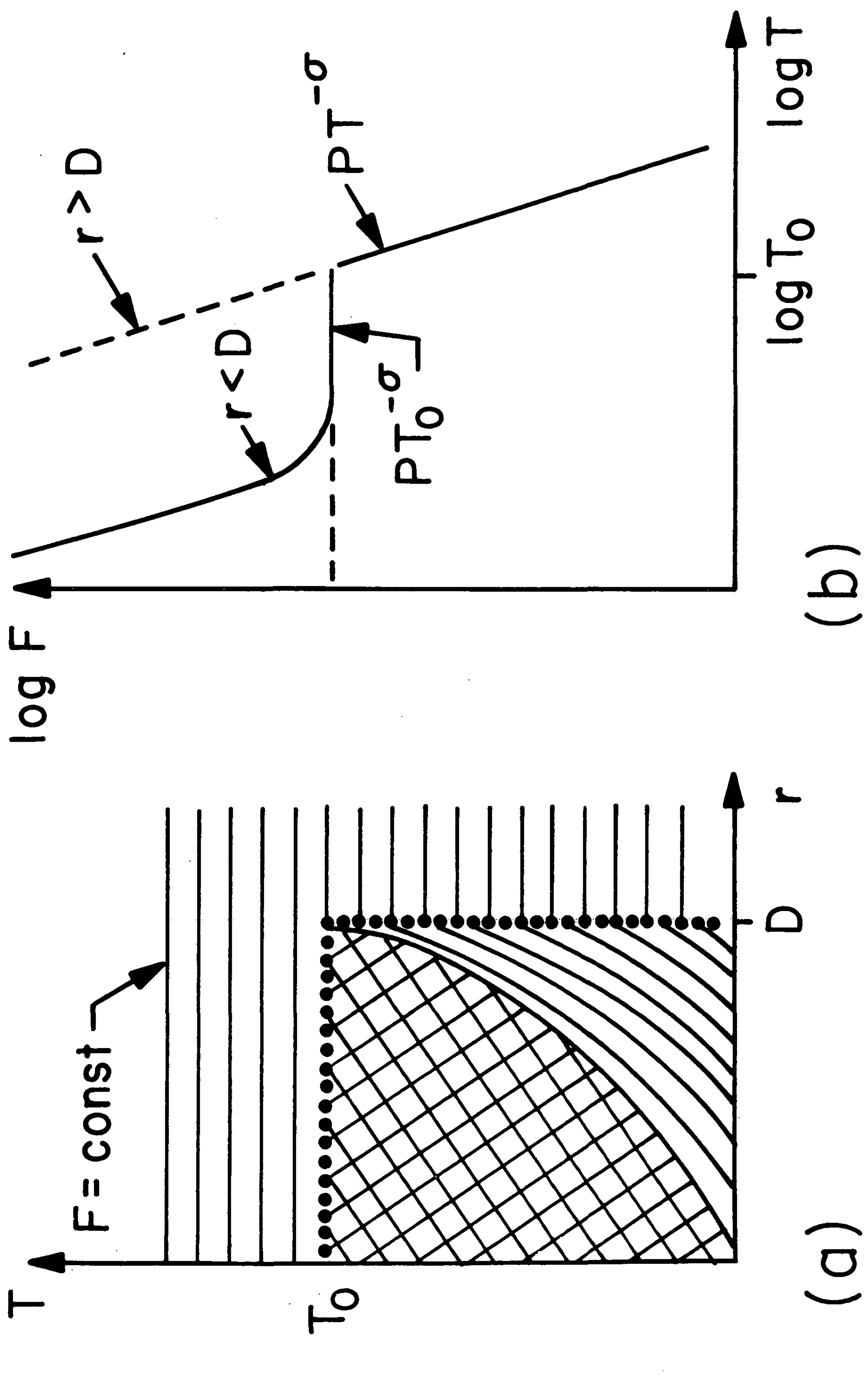


Fig. 2

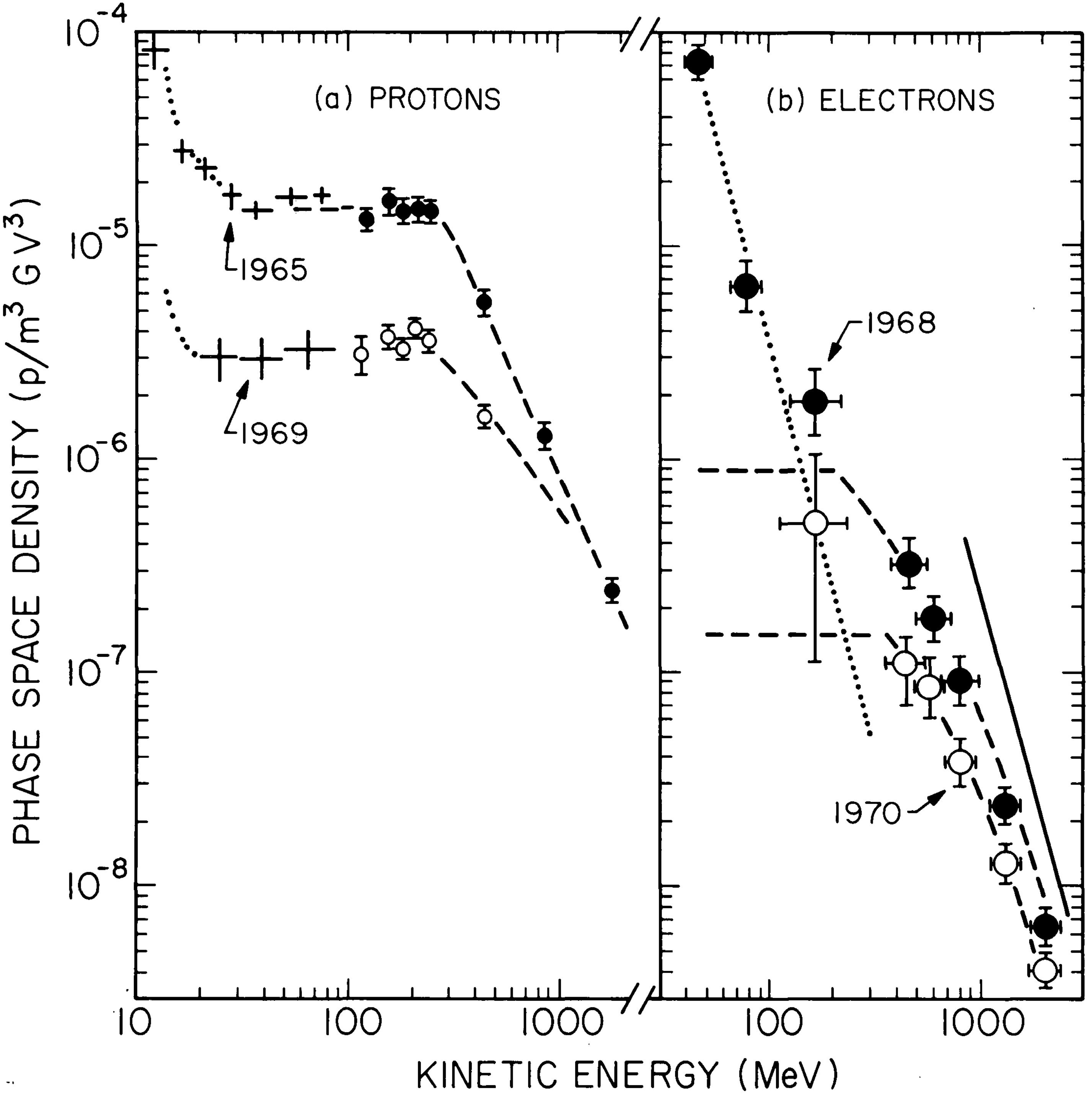

(a) PROTONS
(b) ELECTRONS
1965
1969
1968
1970
PHASE SPACE DENSITY (p/m^3 GV^3)
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10
100
1000
100
1000
KINETIC ENERGY (MeV)

Fig 3