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Physical Picture for the Anomalous Propagation of Ordinary Electromagnetic Waves in a Plasma

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Abstract

It is shown that the physical mechanism for the "anomalous" propagation of electromagnetic waves at frequencies below the plasma frequency, noted by several authors, is due to the deflection of particles thermal motion by the wave magnetic field, leading to a density perturbation which can be large when enhanced by some resonance. In presence of an external magnetic field, $\underset{\sim}{B}_o$, cyclotron resonance provides the enhancement for ordinary waves ($\underset{\sim}{E} \parallel \underset{\sim}{B}_o$). When $\underset{\sim}{B}_o = 0$, a wave-particle resonance can occur, again giving rise to "anomalous" propagation, if the velocity distribution is anisotropic with respect to the wave vector $\underset{\sim}{k}$, which allows "slow" electromagnetic waves, with phase velocity less than the velocity of light. The Weibel instability, which also occurs with such a distribution function, relies upon the same physical mechanism.

It is well known that electromagnetic waves cannot propagate through a cold, unmagnetized plasma if the frequency lies below the plasma frequency, the linear dispersion relation being

$$(kc)^2 = \omega^2 - \omega_p^2 . \tag{1}$$

For a "hot" plasma, i.e., one for which a Vlasov equation treatment is valid, (1) takes the form

$$(kc)^2 = \omega^2 - \omega_p^2[1 + Z'(\omega/ka)/2] ; \tag{2}$$

if we assume the unperturbed velocity distribution function to be an isotropic Maxwellian, with $a^2 = 2T/m$, Z is the usual plasma dispersion function.[1] For small values of the thermal velocity, we can use the asymptotic form of Z, and (2) gives

$$(kc)^2 = (\omega^2 - \omega_p^2)/(1 + \omega_p^2 a^2/2\omega^2 c^2) \tag{3}$$

so that, just as in the cold plasma case, a wave with frequency $\omega < \omega_p$ will be evanescent ($k^2 < 0$).

In presence of a magnetic field, $\underset{\sim}{B}_o$, one might expect similar results to obtain for ordinary electromagnetic waves, i.e., those polarized with electric field along $\underset{\sim}{B}_o$. However, Minami[2] has shown recently that this is not the case, and that, in fact, propagation is possible at frequencies ω well below ω_p provided ω is sufficiently near the cyclotron frequency, ω_c, the range of values of $\omega_c - \omega$ which give propagation increasing with $(\omega_p a/\omega c)^2$.

In this note we explain the physical reason for this unexpected propagation: magnetic deflection of the thermal motion of electrons by the wave magnetic field produces a first order velocity component along $\underset{\sim}{k}$ and hence a first order density perturbation, something which cannot happen in the cold plasma limit. Since this effect is proportional to the thermal energy, we would expect to see no qualitative changes in the limit of small a, unless some kind of wave-particle resonance occurs. In the non-magnetic case, we see from (2) that the phase velocity always exceeds c (since $Z' \geq -2$), so no resonance is possible and the cut-off at ω_p remains. (This statement no longer holds if the velocity distribution is anisotropic, as explained below.) In the magnetic case, however, cyclotron resonance can make this density perturbation so large that it reverses the sign of k^2 even for $\omega < \omega_p$, resulting in a propagation pass-band. Of course, this follows in a formal fashion from the dispersion relation, as shown by Minami, but we sketch here an elementary, particle-oriented derivation to illuminate the physical mechanism.

We choose a very simple velocity distribution, in which all particles have the same thermal velocity, a, with an isotropic distribution in direction. For those particles with unperturbed (i.e., "thermal") velocity $\underset{\sim}{v}_o$, the perturbations in velocity, $\underset{\sim}{v}_1$, and density, n_1, obey the usual equations of continuity and momentum balance,

$$\partial n_1/\partial t + \nabla \cdot (n_o \underset{\sim}{v}_1 + n_1 \underset{\sim}{v}_o) = 0 \tag{4}$$

$$\partial \underset{\sim}{v}_1/\partial t + \underset{\sim}{v}_o \cdot \nabla \underset{\sim}{v}_1 = (q/m)[\underset{\sim}{E}_1 + \underset{\sim}{v}_1 \times \underset{\sim}{B}_o + \underset{\sim}{v}_o \times \underset{\sim}{B}_1] \tag{5}$$

where n_o denotes the unperturbed density of these particles and we use units with c (velocity of light) = 1. For plane waves, $\exp[i(\underset{\sim}{k} \cdot \underset{\sim}{x} - \omega t)]$, we have

$$n_1 = n_o \underset{\sim}{k} \cdot \underset{\sim}{v}_1 / w \tag{6}$$

$$\underset{\sim}{v}_1 = (i/w)[\underset{\sim}{F} + i\underset{\sim}{F} \times \underset{\sim}{\Omega}/w - \underset{\sim}{\Omega}\underset{\sim}{F} \cdot \underset{\sim}{\Omega}/w^2](1 - \Omega^2/w^2)^{-1}$$

where $w = \omega - \underset{\sim}{k} \cdot \underset{\sim}{v}_o$, $\underset{\sim}{\Omega} = q\underset{\sim}{B}_o/m$, and

$$\underset{\sim}{F} = (q/m)[\underset{\sim}{E}_1 + \underset{\sim}{v}_o \times \underset{\sim}{B}_1]$$
$$= (q/m)[w\underset{\sim}{E}_1 + \underset{\sim}{k}\underset{\sim}{v}_o \cdot \underset{\sim}{E}_1]/\omega \tag{7}$$

The first order current is

$$\underset{\sim}{j} = \sum q(n_o \underset{\sim}{v}_1 + n_1 \underset{\sim}{v}_o) \tag{8}$$

where we sum over the isotropic distribution of directions for $\underset{\sim}{v}_o$. This must be substituted in Maxwell's equations,

$$\underset{\sim}{k} \times (\underset{\sim}{k} \times \underset{\sim}{E}) + i\omega(4\pi \underset{\sim}{j} - i\omega \underset{\sim}{E}) = 0 \tag{9}$$

For electromagnetic waves ($\underset{\sim}{E}\cdot\underset{\sim}{k} = 0$) we need only the component of (9) perpendicular to $\underset{\sim}{k}$, and hence only the perpendicular component of $\underset{\sim}{j}$. Computing this from (6), (7) and (8), and assuming "ordinary" wave polarization ($\underset{\sim}{E}_1 \parallel \underset{\sim}{B}_o$), we find

$$\underset{\sim}{j}_\perp = \sigma \underset{\sim}{E}_\perp$$

$$\sigma = (i\omega_p^2/4\pi\omega)[1 + k^2a^2/6(\omega^2 - \Omega^2)] \qquad (10)$$

where the first term in the square bracket of (10) comes from the $n_o\underset{\sim}{v}_1$ part of (8) and the second term, with the resonant denominator, comes from the $n_1\underset{\sim}{v}_o$ part. (In summing over the direction of $\underset{\sim}{v}_o$ we have neglected the $\underset{\sim}{v}_o$ dependence in w.) From (9) and (10), we find the dispersion equation

$$(kc)^2 = (\omega^2 - \omega_p^2)/[1 - (\omega_p a/\omega c)^2\nu^2/6(1 - \nu^2)] \qquad (11)$$

where $\nu = \omega/\Omega$.

From this result, which is qualitatively similar to Minami's, it is clear that propagation at frequencies below ω_p can occur provided

$$(\omega c/\omega_p a)[6(1 - \nu^2)]^{1/2}/\nu < 1 \qquad (12)$$

More to the point, from the derivation we see that although $\underset{\sim}{v}_1$ appears from (6) to have resonant terms, in fact there are none in $\underset{\sim}{v}_{1\perp}$ after we average over the directions of $\underset{\sim}{v}_o$. The only resonant term is in n_1, so that for $(1 - \nu)$ small it dominates and, for $\nu < 1$, has a phase corresponding to propagation rather than evanescence.

We note that there is a close connection between this effect and the instability associated with an anisotropic distribution for an unmagnetized plasma, first pointed out by Weibel.[3] If the velocity distribution of the plasma is anisotropic, for example, Maxwellian but with different temperatures parallel and perpendicular to k, then the dispersion relation (2) is replaced by

$$(kc)^2 = \omega^2 - \omega_p^2[1 + (R/2)Z'(\omega/ka)] \qquad (13)$$

where $R = T_\perp/T_\parallel$ is the ratio of perpendicular and parallel temperatures. If R is larger than 1, then it is possible for the phase velocity of the waves to be less than c, and, again, a resonance can occur, this time between the wave and particles travelling with the phase velocity of the wave. Unlike the magnetic case, where the field prevents particles from travelling with the wave across the magnetic field, we have not pure propagation, but rather propagation with weak Landau damping.

This is most easily seen from (13) by looking for solutions with $|\omega/ka| << 1$ so that the small argument form of Z' is appropriate,

$$Z'(s) = -2[1 + i\pi^{1/2}s + \cdots] \qquad (14)$$

Then

$$(kc)^2 = \omega^2 + \omega_p^2[R - 1 + i\pi^{1/2}R\omega/ka_{\parallel}] \tag{15}$$

and iteration gives a propagating solution with small damping:

$$k = k_o(1 + i\alpha) \tag{16}$$

where

$$k_o = [\omega_p^2(R - 1) + \omega^2]^{1/2}$$

$$\alpha = \pi^{1/2}(\omega c/\omega_p a_{\parallel})R[R - 1 + \omega^2/\omega_p^2]^{-3/2} \tag{17}$$

The condition

$$\varepsilon = (\omega c/\omega_p a_{\parallel})(R - 1)^{-1/2} << 1, \tag{18}$$

which is the analogue of (12), provides a posterior justification for the expansion (14), and (17) then shows that under these conditions the damping per cycle is of order $\varepsilon/(R - 1)$.

Here, as in the magnetic case treated above, it is the deflection of the thermal motion by the wave magnetic field, together with a wave-particle resonance, which accounts for the anomalous propagation. This same physical effect is responsible for the Weibel instability[4]; if we write (15) in the form

$$D(\omega) = \omega^2 + i\pi^{1/2}R\omega_p^2\omega/ka_{\parallel} + (R - 1)\omega_p^2 - (kc)^2 = 0 \tag{19}$$

then there is an instability (i.e., D has a root with Imω > 0) provided

$$kc < \omega_p(R - 1)^{1/2} \tag{20}$$

This is not surprising, since the anisotropic distribution function provides a source of free energy to drive the instability. It would, however, be disturbing if there were a similar instability in the magnetic case, where we have assumed an isotropic, and therefore stable, Maxwellian. In fact, it can readily be seen that solving (11) for ω with k real gives only real roots.

In summary, propagation of electromagnetic waves below the plasma frequency can occur in two cases: 1) in a magnetized plasma, when the wave polarization corresponds to an ordinary wave and the condition (11) is satisfied; 2) in an unmagnetized plasma, with an anisotropic velocity distribution, when the condition (18) is satisfied. In both cases, the physical mechanism responsible is the deflection of the thermal motion of particles in the wave magnetic field, together with the existence of a wave-particle resonance which makes the resulting density perturbation the dominant term in the perturbed current density. In the first case, the propagation can be undamped (k purely real) and the plasma is, as expected, stable. In the second case, there is a small Landau damping and, for certain ranges of k, the plasma will be unstable against growth of the wave in question.

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