THEORETICAL AND EXPERIMENTAL STUDIES OF THE NATURE AND CHARACTERISTICS OF SPACE-RELATED PLASMA RESONANCE PHENOMENA

Semiannual Report No. 11
NASA Research Grant NGL 05-020-176
SU-IPR Report No. 492
September 1972

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NASA Research Grant NGL 05-020-176
for the period
1 January - 30 June 1972

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FOREWORD

The subject of this research grant is the theoretical and experimental study in the laboratory of space-related plasma wave propagation and resonance phenomena. Research under the grant has been proceeding under the direction of Prof. F. W. Crawford since its inception on 1 December 1966. Up to 30 June 1970, the work was related primarily to whistler propagation phenomena. Since then, the program has been expanded to include projects proceeding previously under NASA Grant NGR 05-020-077, also under the direction of Prof. Crawford, on "Alouette" resonance phenomena. This is the eleventh semiannual report on NASA Grant NGL 05-020-176, and covers the period from 1 January to 30 June 1972.

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I. INTRODUCTION

The research program proceeding under this grant is concerned primarily with plasma wave and resonance phenomena, in both the small-signal and nonlinear régimes, which are observable in the ionosphere and may be amenable to simulation in laboratory experiments. Although we shall deal in detail with our recent work in later sections, the motivation behind it may be stressed here: most of the topics have stemmed from observations of puzzling plasma phenomena made by space-probing vehicles and ground-based transmitters and receivers. In general, the testing of theoretical explanations of these phenomena, and predictions based on them, requires development of new generations of equipment which may prove costly, time-consuming, and sometimes inadequate. An alternative approach, when the analysis shows that scaling is feasible, is to check the validity of the theoretical analyses by suitable laboratory plasma experiments. The most significant scientific advantages offered by this method of attack are the ready accessibility of the plasma compared to the magnetosphere, and the possibility of varying its parameters over wide ranges. Additional advantages of relative speed and cheapness also commend laboratory experimentation.

It should be said that the potentialities of laboratory simulation of space plasma phenomena are often oversold. In fact, only a limited number of effects can be studied easily in this way. In our work, we have confined ourselves to a small number of wave propagation and resonance phenomena which can be demonstrated to be amenable to scaling. These have included, for example, whistler propagation and "Alouette" resonances, and it has proved feasible to investigate their properties under well-controlled conditions. Apart from the possibility of explaining phenomena already observed in space plasmas, laboratory experimentation offers methods of developing new experiments and diagnostic techniques to be used on future space-probing vehicles, and with ground-based equipment. In particular, it is hoped and anticipated that the work being carried out under this grant will be able to make a material contribution to NASA activities such as the PPEPL/Shuttle, and other satellite projects involving study and use of the properties of space plasmas.

Specifically, the projects carried out during the six-month reporting period from 1 January to 30 June 1972, and to be described in later sections, were as follows: Section II describes two theoretical analyses of whistlers, the first dealing with triggered VLF emissions, and the second with modulational instabilities. Section III discusses low-frequency instabilities observed in the hollow cathode arc discharge originally constructed for whistler damping measurements, and describes a theory for the steady state discharge. Section IV is concerned with topics in nonlinear wave interactions other than those associated with triggered VLF emissions (Section IIB). We have been contributing to the development of a general Lagrangian formalism which avoids much of the tedium of conventional iterative methods, and have extended our studies of the specific case of nonlinear interaction in a plasma column. Section V extends our previous work on the Herlofson Paradox to the related phenomenon of the "infinity catastrophe" associated with the resonance cone of a dipole antenna in a cold magnetoplasma.

The report closes with some comments in Section VI on the research program planned for the next six months, and a bibliography of reports, conference papers and publications resulting from the program during the reporting period.

II. WHISTLERS

The major part of our program has been devoted for several years to the elucidation of certain properties of whistler propagation in the magnetosphere. In particular, we have been concerned with developing theories of phenomena such as whistler damping, amplification, and triggering, and subsequently testing the theoretical predictions by means of laboratory plasma experiments in which the parameters are scaled so as to be comparable to those in the magnetosphere. Most recently, we have been concentrating our effort on the theoretical description of two observed magnetospheric whistler phenomena: triggered VLF emissions, and modulational instabilities.

A. Triggered VLF Emissions

In SAR 10, we divided the study of triggered emissions into consideration of the triggering signal, onset of the emission, and the stimulated whistler. The general triggering signal is an oblique whistler whose stability characteristics are described in detail in a report prepared during the last reporting period.[13] Here we shall discuss briefly the main features of the mechanism envisaged for the emission onset: a more comprehensive analysis has been written up for publication in a recent report.[1]

The distortion caused by a large amplitude whistler to the nonthermal electron population of the (homogeneous) equatorial region suggests the possible formation of whistler sidebands. This phenomenon has characteristics which fit observed properties of the onset of stimulated emissions, i.e. the frequency offset and triggering delay, and may provide the mechanism underlying the triggering process.

At time $t = 0$, a large amplitude whistler of frequency ω_0 and wavenumber k_0 is applied to a homogeneous magnetoplasma of electron cyclotron frequency Ω. The ions are taken to be stationary. We assume that at the same time small amplitude test waves with $(\omega - \Omega)/k \approx (\omega_0 - \Omega)/k_0$ are impressed on the plasma, and study their temporal evolution in this initial value problem. It is found that whistler test waves located within two narrow bands centered on the frequency ω_0 may experience large consecutive growth at the early stages of the interaction provided that the unperturbed nonthermal electron distribution, $f_0(v_\parallel, v_\perp)$, satisfies two conditions: first, f_0 must have a certain degree of $v_\perp$-monochromaticity, i.e. a well defined peak for some $v_\perp > 0$; second, f_0 must be either linearly unstable for the original whistler or, if stable, satisfy

$$2(\Omega/k_0)\int_0^\infty f_0 v_\perp dv_\perp \approx \left|\int_0^\infty v_\perp^3(\partial f_0/\partial v_\parallel)dv_\perp\right| , \qquad (1)$$

where the integrals are evaluated at $v_\parallel = (\omega_0 - \Omega)/k_0$.

Figure 1 shows the growth rates, $\gamma(t)$, and the maximal consecutive growths, Λ_M , of the test waves. The magnetoplasma is made up of a cold component with density $\omega_p = 10\ \Omega$ permeated by a dilute nonthermal electron population of fractional number density β , and velocity distribution

$$f_0(v_\parallel, v_\perp) = \beta\ A_N\ \exp\ (-v_\parallel^2/v_p^2)\ \exp\ [-(v_\perp - \bar{v}_\perp)^2/v_c^2] \ , \qquad (2)$$

where A_N is a normalization factor, and $v_p, v_\perp$ and v_c are constants. The test waves are labeled with the value of w ,

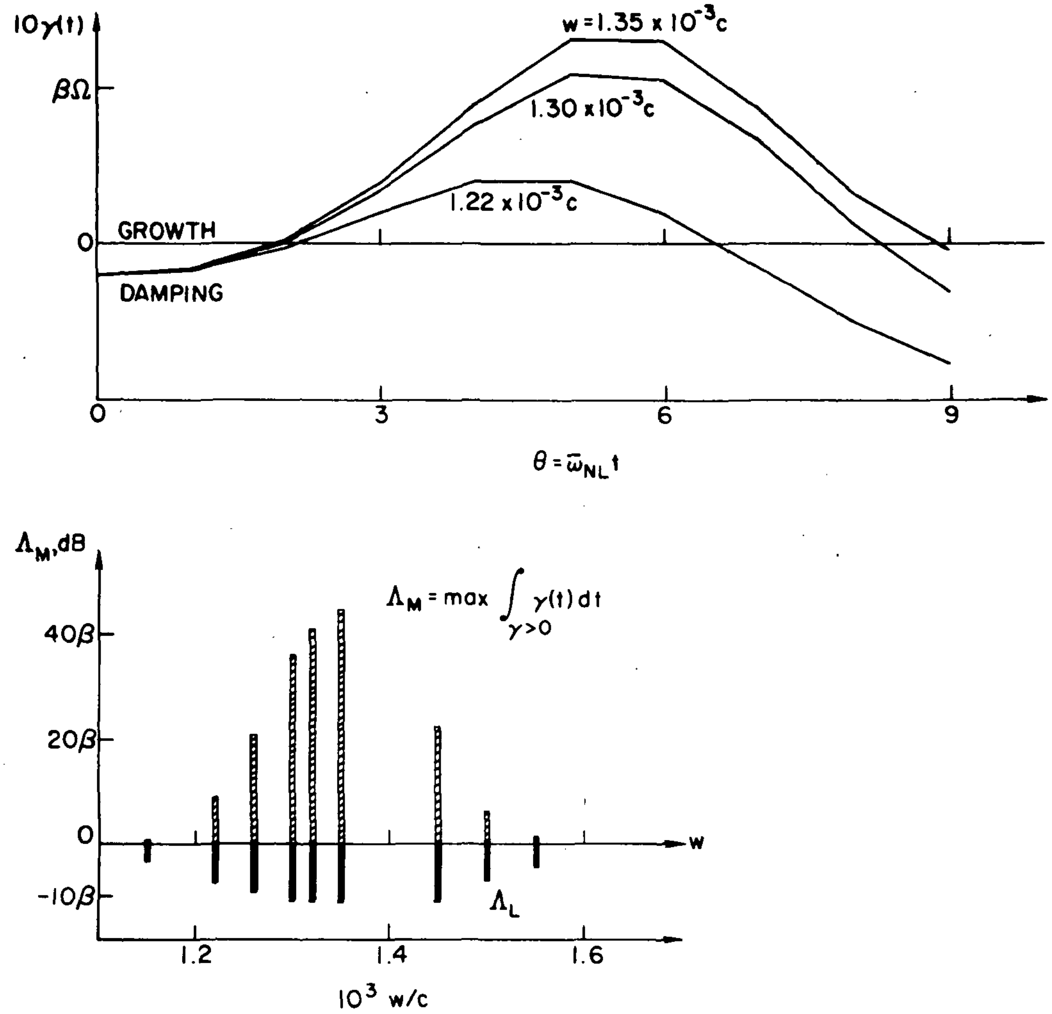
$$w = \frac{\omega - \Omega}{k} - \frac{\omega_0 - \Omega}{k_0} , \qquad (3)$$

and the time scale is normalized with respect to $1/\bar{\omega}_{NL}$,

$$\bar{\omega}_{NL} = a\ k_0\ \bar{v}_\perp , \qquad (4)$$

where a is the electron gyrofrequency defined by the triggering wave magnetic field of amplitude $a/\Omega = 4 \times 10^{-4}$.

Comparison of these results with the observed characteristics of the onset of stimulated emissions yields satisfactory agreement,[1,14] although the observed suppression of one of the sidebands is not accounted for in this onset theory. Presumably, the quenching of one of the



$$f_0(v_\parallel, v_\perp) = \beta A_N \exp(-v_\parallel^2/v_p^2) \exp\left[-(v_\perp - \bar{v}_\perp)^2/v_c^2\right]$$

$\bar{v}_\perp/c = v_p/c = 0.025$ $\bar{v}_\perp/v_c = 4$

$\omega_0/\Omega = 0.5$ $\omega_p/\Omega = 10$ $B/B_0 = 10^{-4}$

FIG. 1. Characteristics of test waves in the resonance region: growth rates (γ), maximal consecutive growths (Λ_M), and linear attenuation (Λ_L) in the absence of the large amplitude whistler (β is the fractional number density of the hot plasma). [$\bar{v}_\perp/v_c = 4$].

sidebands is due to further nonlinear effects brought about by the finite amplitude of the predominant sideband and/or the actual inhomogeneity of the magnetosphere.

In order to clarify this question, and to test the creation of sidebands through the proposed mechanism, a computer simulation of this complex problem is now being considered, starting with the simpler, but not dissimilar, phenomenon of the creation of sidebands of large amplitude electrostatic Landau waves.[2,10]

B. Modulational Instability

Recent observations of the amplitude spectrum of whistlers originally emitted as constant amplitude monochromatic waves have revealed the development of marked amplitude modulation.[15,16] We have analyzed the possible relevance to these observations of a self-action effect known as "modulational instability" which can occur for propagation of large amplitude waves in nonlinear dispersive media. The modulational instability, as sketched in Fig. 2, may bring about a large amplitude modulation in an initially constant amplitude wave-packet.

Previous work on this problem is restricted to cold plasmas, and either neglects ion motion,[17,18] or disregards relativistic effects.[17,19-21] We have found for the cold plasma case that simultaneous consideration of these two factors alters the modulational stability spectrum of the wave trains in a fundamental way:[5] the unstable band is contiguous to $\Omega/4$ and lies above or below that frequency, depending on the plasma density. The importance of considering simultaneously the influence of ion motion and relativistic dynamics is shown in Fig. 3, where the instability spectra obtained using simplifications are contrasted with the actual spectrum.

Because the growth rate is proportional to $(B/B_0)^2$ where B and B_0 are the wave and static magnetic fields, it is not likely that this instability will be responsible for the recent observations[15,16] of amplitude modulation. However, when consideration is made of the nonthermal population of the magnetosphere, we expect the growth rates to become proportional to $(B/B_0)^{1/2}$. Under these circumstances, it is conceivable that the modulational instability should become observable

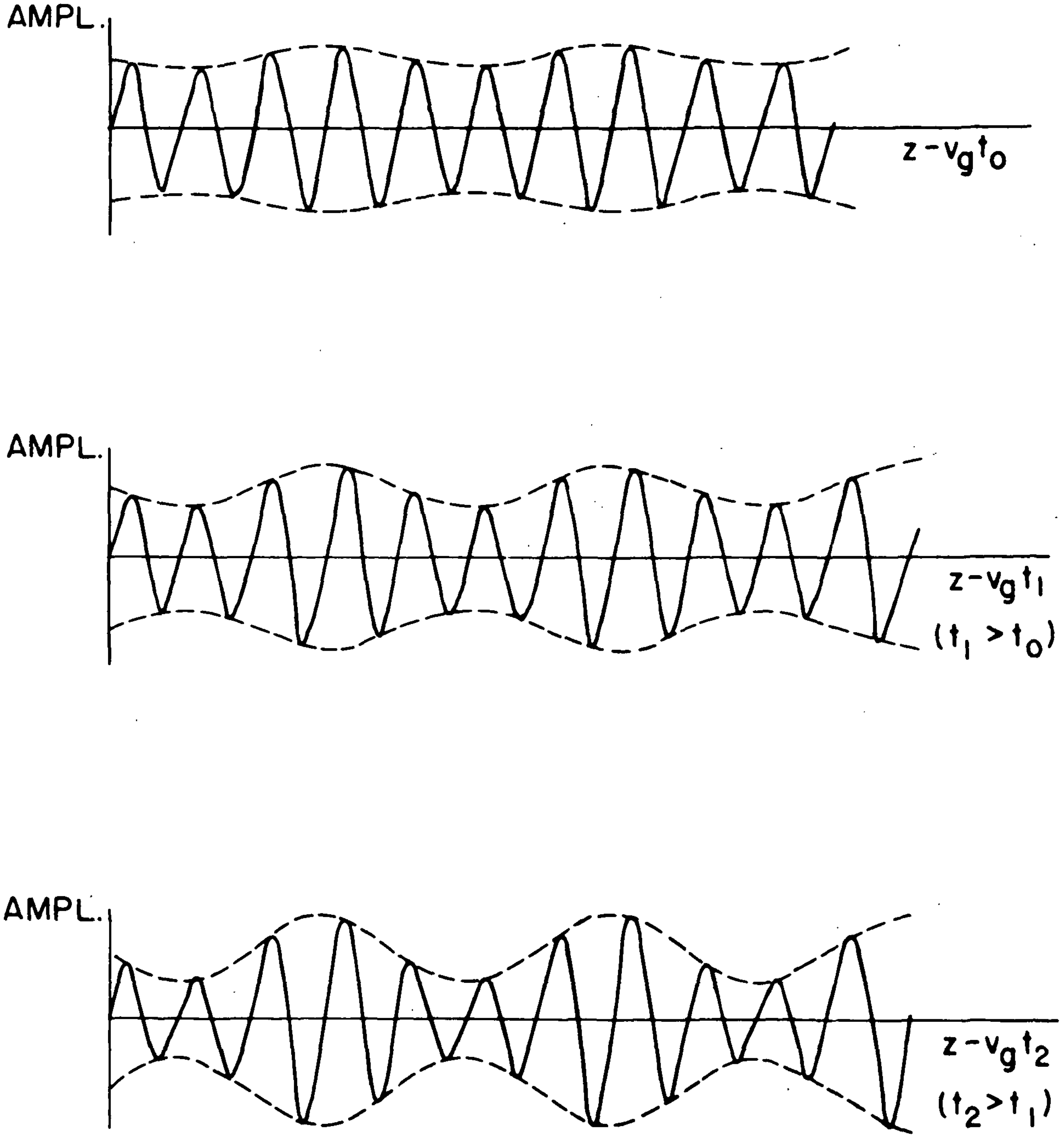


FIG. 2. Possible evolution of a wave packet with group velocity v_g in a modulationally unstable medium.

ION \ REL	YES	NO
YES	A_i	B
NO	C	D

$\omega_{pe}^2/\bar{\omega}_p^2$	<1	=1	>1
A_i	$A_<$	$A_=$	$A_>$

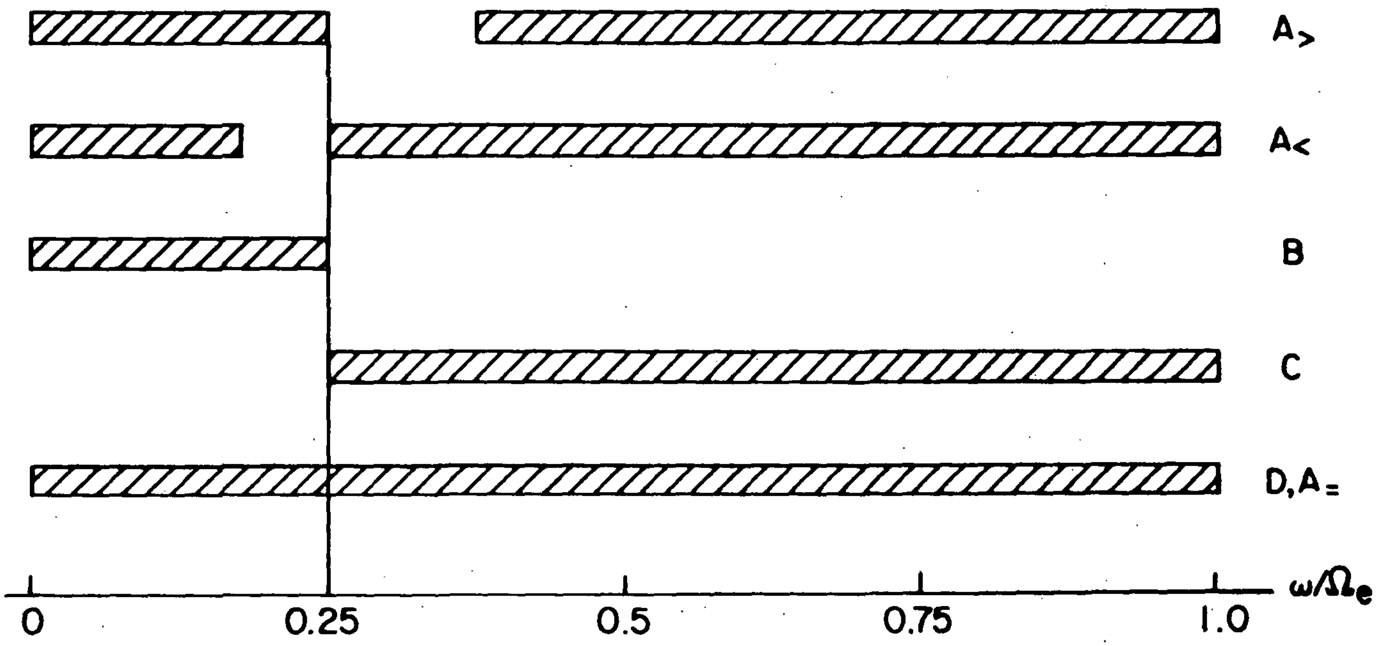


FIG. 3. Whistler modulational instability spectrum in cold plasmas (stable bands are hatched). (REL - Relativistic force equation used; ION - Ion motion considered.) $\left[\omega \gg \Omega_i\ ,\ \omega_{pe}^2 \gg \Omega_e^2\right]$.

in the outer shells of the magnetosphere. We are currently working on the whistler modulational instability in hot plasmas.[8] By adapting the theory to the magnetospheric plasma, we expect to find out whether this instability might indeed by observed in the magnetosphere.

III. LOW-FREQUENCY INSTABILITIES

Earlier SAR have described our attempts to verify the plane wave whistler dispersion relation under conditions where the collisionless cyclotron damping due to electron thermal velocities exceeds that due to Coulomb and electron-neutral collisions. Although considerable success was obtained in our early studies in the collisional régime,[22,23] we have had great difficulty in producing a suitable hot dense plasma inexpensively for the measurements of collisionless damping. Most recently, we have been working with a hollow cathode arc discharge (HCD). Although this has many desirable features, we have found it to be too strongly subject to low-frequency instabilities to be of use in an experiment requiring a high degree of discharge quiescence, and we have abandoned our attempts to measure collisionless cyclotron damping.

The low-frequency instabilities are of considerable interest in their own right; we have reported in earlier SAR our systematic studies of low-frequency magnetoplasma instabilities which may be responsible for the phenomenon of spread-F. Most of our previous work has been carried out with positive columns. In the HCD, new instabilities are observed due to radial electric fields. During the reporting period, an experimental study of these instabilities has been concluded, and progress has been made on an analysis of the steady state of the HCD.

A. Experiment

During the reporting period, agreement has been obtained between the experimental characteristics of the instabilities and the theory described in SAR 10. This has been achieved by solving numerically the differential equation which arises in our theory, in order to account for the radial variation of the DC electric field observed experimentally. The work is currently being written up in report form for publication,[24] and for inclusion in a Ph.D. thesis,[25] so only the salient features need be mentioned here.

In summary, two instabilities are observed: Mode I has azimuthal mode number $m = 1$, and occurs when the radial electric field is negative (directed inwards); Mode II has $m = -1$, and occurs when the radial electric field is positive. The radial electric field is controlled by varying the potential of a secondary anode cylinder located close to the outer discharge radius. To explain the instabilities, a linear perturbation analysis has been performed on the two-fluid moment equations. The model is of a low-β, collisionless, cylindrical plasma column, immersed in a uniform axial magnetic field, having a Gaussian electron density profile and an arbitrary radial electric field profile. Reasonable correlation between theory and experiment has been demonstrated for both modes by comparing the calculated and measured frequencies, mode numbers, and also the eigenfunctions of the density and potential fluctuations. The instabilities are basically centrifugal flute modes, driven by the $\underset{\sim}{E} \times \underset{\sim}{B}$ drift in the presence of a density gradient, and modified by the velocity shear due to nonuniform $\underset{\sim}{E} \times \underset{\sim}{B}$ rotation.

B. Theory

A steady state theory of the HCD should include both the effects of the finite length of the discharge and the spatially varying electron and ion temperatures. We have shown that an earlier theory based on a two-moment description of the positive column[26] may be generalized to two dimensions,[27] and thus adequately describe a column in a magnetic field which is collisional in the radial direction, but essentially collisionless axially. Unfortunately, it has not been possible to find a separable solution to the problem which includes spatially varying electron and ion temperatures, and which should be more directly applicable to the HCD. Instead, we are developing a one-dimensional theory which includes varying electron and ion temperatures. The model calls for the existence of an axial current and generation of electron-ion pairs by collisions of electrons and neutrals, where the axial drift of electrons is included in the generation rate.

The results of a computer run with constant ion temperature and variable electron temperature are shown in Fig. 4 (n, T_e, Φ and ϵ^I denote electron density, electron temperature, potential, and ionization energy, respectively). These are to be compared with measurements of steady-state quantities reported in SAR 9, and in the comprehensive report on HCD instabilities which is in preparation.[24] It is found that the general features of the HCD are reproduced, i.e. fall-off of

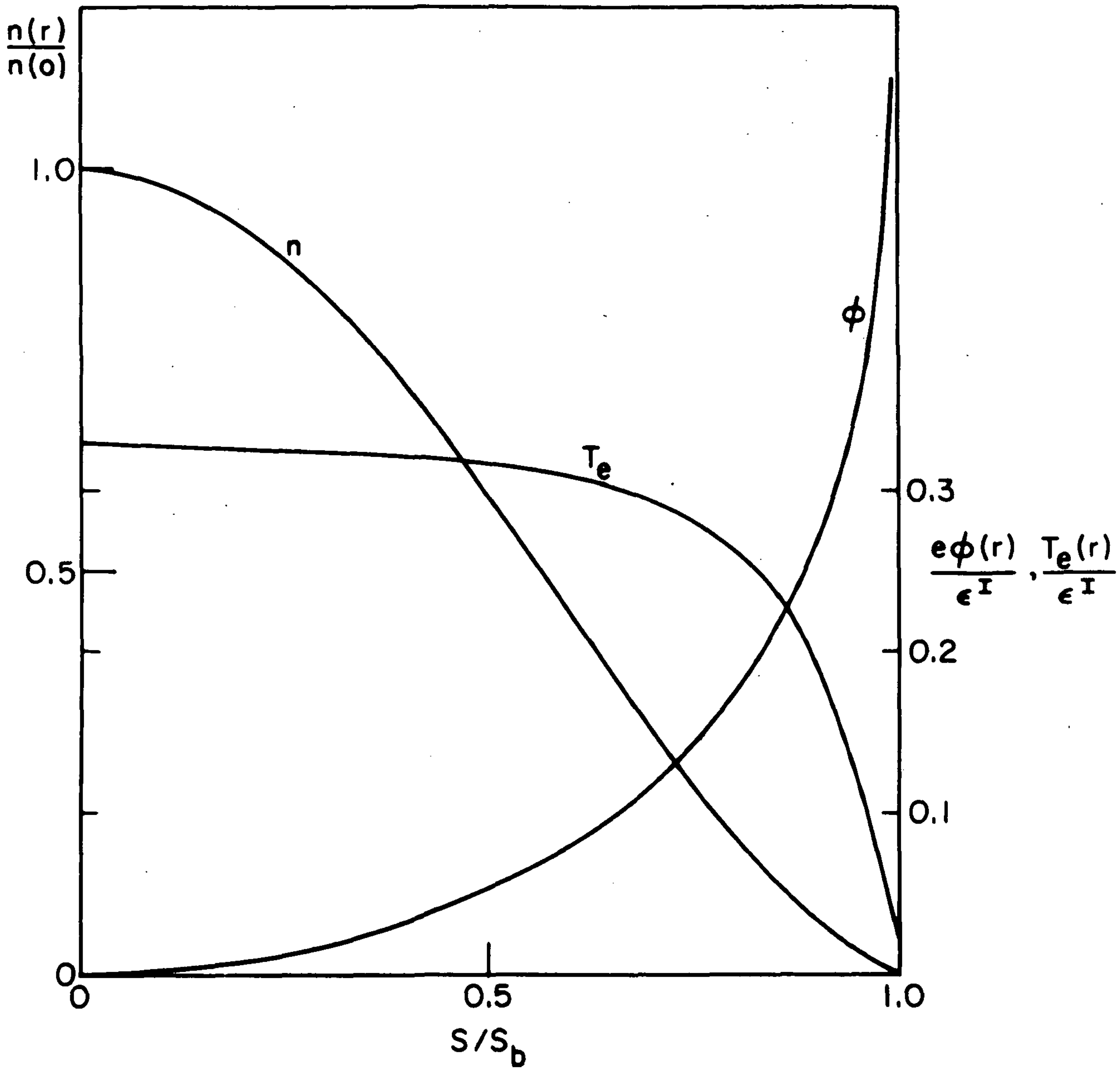


FIG. 4. The radial variations of density, electron temperature and potential in a positive column with strong axial magnetic field and relatively high constant ion temperature.

electron density and electron temperature, and increase of potential, with radius. To reproduce the abrupt fall-off of density and temperature at the edge of the bright core of the HCD (see SAR 9) it was necessary to decrease the axial electric field to zero at some point on the radius corresponding physically to the edge of the bright column of the HCD. This procedure resulted in the expected abrupt drop in electron temperature and density at that point in the computer results.

Our current efforts are centered on improving the ion energy equation in such a way that direct application to experiment will be possible. The ion temperature is an extremely important parameter which is difficult to measure experimentally with high spatial resolution, so that a theoretical model of ion motion would be very useful.

Although the present one-dimensional theory can reproduce many of the measured features of the steady state parameters of the HCD qualitatively, for quantitative agreement it is necessary to include the effects of Coulomb collisions, since the degree of ionization of the arc is 20 - 50%. The possibility of such an extension will be studied during the next reporting period.

IV. NONLINEAR WAVE INTERACTIONS

In several previous SAR, and Section IIB of this one, we have discussed nonlinear wave-wave and wave-particle interactions. This is currently a rapidly growing area of both space and laboratory plasma physics, and two relevant topics have received support under the grant during the reporting period. First, we have been developing a Lagrangian formalism for describing nonlinear wave interactions concisely, and second, we have been extending our work on nonlinear scattering from a cold plasma column to take account of a warm inhomogeneous plasma. Progress is as follows.

A. Lagrangian Formalism

Both wave-wave and wave-particle interactions affect wave evolution in a plasma, but only the latter affect the averaged properties of the plasma itself, i.e. the "background state." The background concept has been discussed previously by Dougherty,[28] who showed how the perturbation of the background could be accounted for in the Lagrangian formalism for certain simple examples. He did not, however, outline a procedure which could deal in general with problems of the complexity encountered in plasma physics. In SAR 9, we described how the Lagrangian analysis could be extended to describe the evolution of the plasma background in a specific example, that of electrostatic waves propagating in a Vlasov plasma. During the reporting period, we have proceeded with a more general and systematic extension of the Lagrangian approach to account for the background evolution in a manner consistent with the nonlinear wave theory developed earlier.

Like its predecessors, our extended Lagrangian method is based on a perturbation expansion of the Lagrangian density, $\mathcal{L}$, about a given reference state, R. The perturbation variables are scalars, denoted by $\{q^i: \ i=1,\ldots M\}$. The independent variables (coordinates) are $\underset{\sim}{v}$, $\underset{\sim}{x}$, and t , denoting velocity, position, and time, respectively. The parameters of the reference state are assumed to change on a slow scale in position and time, by which we mean that they change significantly only over intervals which are large in comparison with the wave periods. This slow-scale dependence is characterized by a dimensionless parameter,

ϵ , as in $R(\underline{v}, \epsilon \underline{x}, \epsilon t)$. In the presence of weak nonlinear coupling, the wave and background parameters will also be perturbed on the slow scale. Our interaction picture of the state of the medium may thus be stated symbolically as

$$\text{Medium} \equiv \overbrace{\left\{ q_w^i = \sum_\eta q_\eta^i \right\}} + \overbrace{\left\{ q_0^i \right\} + \text{Reference State}} , \qquad (5)$$

where, in this usage, "$\equiv$" means "is defined by," and "+" means "together with". In the above expression, the scalars $\{q_0^i(\underline{v}, \epsilon \underline{x}, \epsilon t)\}$ are the slow-scale, background perturbation components. The wave perturbation components, $\{q_w^i\}$, are sums of the contributions of each wave present, where the contributions of Wave η are the $\{q_\eta^i\}$.

In keeping with the concepts expressed above, the interaction processes in the Lagrangian description have been classified as wave-wave and wave-background interactions. The latter interactions include the wave-particle interactions of plasma theory as a special case, since the averaged particle distribution functions and the averaged fields constitute the plasma background parameters. The concept of wave-background interaction has proved convenient in the Lagrangian theory because it applies not only to plasmas which have statistical descriptions, but also to hydrodynamic plasmas, and a wide variety of other media.

The extension of the previous Lagrangian methods to account for the nonlinear background evolution requires an ordering scheme which relates the relative magnitudes of the $\{q_\eta^i\}$, the $\{q_0^i\}$, and the parameter ϵ . The ordering problem has been solved by the use of a dual Lagrangian expansion as the basis of the extended Lagrangian method. In the dual expansion, the terms are ordered separately in powers of the wave and background perturbation components. Thus, we have

$$\mathcal{L} = \sum_m \mathcal{L}_m = \sum_m \sum_n \mathcal{L}_{m,n} , \qquad (6)$$

where the indices m and n denote the order of the terms $\mathcal{L}_{m,n}(q_w^i, q_0^i, \underset{\sim}{v}, \epsilon\underset{\sim}{x}, \epsilon t)$ in powers of the $\{q_w^i\}$, and the $\{q_0^i\}$, respectively.

For the analysis, we simplify the expressions in the dual expansion by averaging them in a restricted sense; over intervals in $\underset{\sim}{x}$ and t which are large in comparison with the wave periods, but small in comparison with the intervals over which significant perturbation of the waves or background can occur. The average is denoted by an overhead bar. The components $\{\bar{\mathcal{L}}_{m,n}\}$ are calculated by means of the small-signal forms of the wave components, in which the relations between wave variables are adequately described by the mode-structure coefficients obtained from the linear theory. We make a trial approximation to $\bar{\mathcal{L}}$ by neglecting all but a number of lower order terms in the corresponding dual expansion. We then apply an averaged-Hamilton's principle in which, for consistency with the averaging process, only slow-scale variations in the wave parameters of the $\{q_0^i\}$ are allowed. Separate variations of wave and background components yield the corresponding interaction equations. The effects of the various terms of the dual expansion on these equations are indicated in Fig. 5. The primary expansion, in joint powers of the wave components, is shown. Each term in the primary expansion has a secondary expansion, in joint powers of the background (non-oscillatory) perturbation components. Through the averaged Hamilton's principle, each term in the primary expansion of $\bar{\mathcal{L}}$ generates a particular class of terms in the wave and background equations. (Note that in SAR 10, a similar diagram was given for the description of wave evolution only, by means of a simple perturbation expansion of $\mathcal{L}$).

After the equations of wave and background evolution are obtained from the trial approximation to $\bar{\mathcal{L}}$, the equations are used to determine the relative order of all perturbation components, and to determine whether the terms neglected in the trial approximation are indeed insignificant. If not, the process is repeated until a self-consistent approximation is obtained. A detailed description of the analysis is currently being written up for inclusion in a Ph.D. thesis.[29]

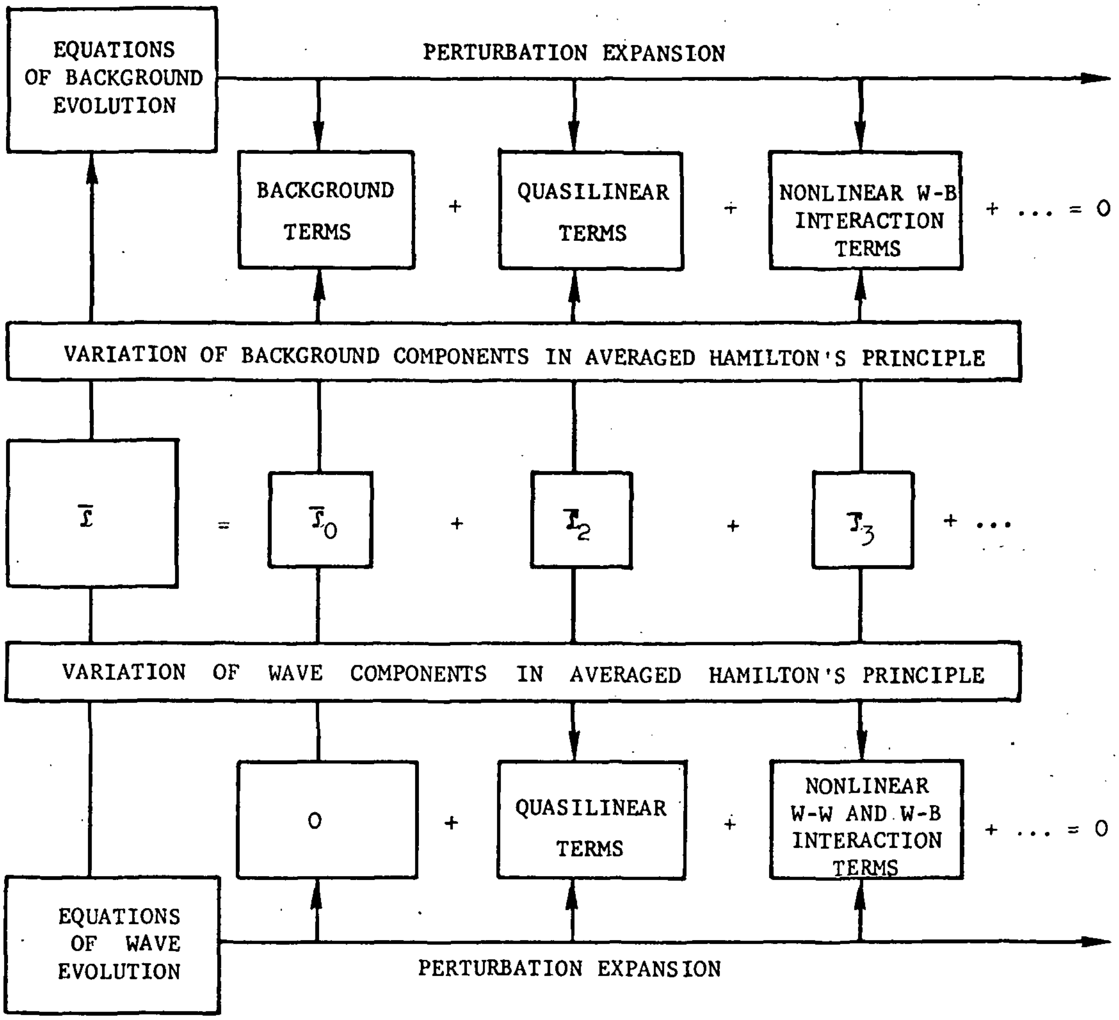


FIG. 5. Perturbation expansion of the averaged Lagrangian density, and the interaction equations.

B. Scattering from a Plasma Column

This work was originally stimulated by the apparent observation of nonlinear effects in radar returns from meteor trails. In previous SAR and reports we have modelled the problem as a cold plasma column irradiated by an incident plane wave. This theory can be regarded as completed for the case in which the participating frequencies all exceed the plasma frequency at the column axis. However, if one of the frequencies lies below the axial plasma frequency, the solution for the electric field has a nonphysical infinity at the radius where the plasma permittivity passes through zero. A theory taking electron temperature effects into account was presented in SAR 10. A computer program to calculate the nonlinear scattered power in the case of a parabolic electron density profile has been written and is currently being tested. This task is complicated by the fact that the zero-temperature limit, obtained previously from cold plasma theory, cannot be reached due to numerical difficulties. The results of the calculations will be presented and evaluated in a thesis which will be completed during the next reporting period.[30] This thesis will also present for comparison our experimental results on nonlinear scattering.

V. RESONANCE CONE

In SAR 10 and a recent report[7] we discussed the Herlofson paradox. Briefly, this denotes the phenomenon in which a cold, collisionless, inhomogeneous plasma exhibits loss when driven by a sinusoidal signal. This result follows when one analyzes a cold, collisional plasma and then lets the collision frequency tend to zero. The paradox is associated with the presence in the plasma of a resonant region where the plasma and signal frequency are equal, giving rise to a singularity in the mathematical formalism. The problem can be resolved by the introduction of temperature or by performing a transient analysis of the buildup process.[7] The transient analysis shows that the amount of stored energy in the resonant region rises linearly with time at a rate corresponding to the collisionless loss of the steady state calculation. When temperature is introduced, it can be shown that the energy storage rate is exactly matched by the outflow of energy in plasma waves, so that the steady state condition is attained. It can also be shown that the rate of energy outflow in the plasma waves exactly agrees again with the collisional loss rate of the cold plasma case as the temperature tends to zero.

During the reporting period, we have been examining the relation of this work to the "infinity catastrophe" associated with the resonance cone of a dipole antenna immersed in a cold overdense magnetoplasma. Although there are parallels in several important aspects, for the resonance cone the difficulty arises in assuming that the plasma is collisionless from the start, i.e., one does not assume a collisional plasma and then let the collision frequency tend to zero. The electric fields along the resonance cone become infinite and the radiation resistance varies as $1/\ell$ (and therefore becomes infinite as ℓ passes to zero). Lee and Mittra[31] have studied the transient behavior of this system and have shown, just as in the case of the Herlofson paradox, that the fields in the resonance cone rise linearly with time, with consequent energy storage. The paradox of infinite electric field is no more than the passage to the limit $t \to \infty$. Singh and Gould[32] have introduced temperature, and again have shown that the steady state is restored by the matching of the rate of energy storage in the resonance cone region to the rate of outflow of energy in plasma waves. In the limit $\ell \rightarrow 0$ they showed that the radiation resistance varied as ℓ^2.

Lee and Mittra[31] point out that it would be desirable to make further studies of the resonance cone phenomena by including loss in the formalism. This has never been done to our knowledge, and we plan to undertake this important task during the next reporting period to determine whether letting loss tend to zero gives the same results as are obtained by letting temperature pass to zero, or by considering transient solutions. This would be very valuable to know, since in studying cold plasma effects it is generally simpler to approach the cold collisionless limit by letting the loss tend to zero rather than the temperature.

VI. FUTURE PROGRAM

During the next six months, the program will continue essentially along the lines suggested in Sections II-V, with considerable effort devoted to writing up recent work; one Ph.D. thesis was completed during the reporting period,[5] and four more are expected shortly.[25,29,30,33] The whistler studies of Section II will be extended to determine the role of nonlinearly-excited sidebands in producing the spectral shapes, e.g. hooks, risers, etc., associated with triggered VLF emissions. The experimental studies of HCD instabilities described in Section III are regarded as completed, and it only remains to bring the steady state HCD theory to a satisfactory conclusion. The nonlinear studies of Section IV have resulted in a powerful formalism for tackling specific wave-wave and wave-particle interaction problems which we now intend to apply to cases of interest. One such case, scattering from a positive column (see Section IVB) will be concluded soon. The resonance cone studies of Section V will be extended analogously to our previous elucidation of the Herlofson Paradox.

One or two new items will be added to the program. In particular we wish to develop some plasma projects suitable for the NASA PPEPL/Shuttle Program, and also wish to re-examine the theory of incoherent scatter as a classical three-wave interaction problem.

REPORTS, CONFERENCE PAPERS, AND PUBLICATIONS RESULTING FROM NASA GRANT NGL 05-020-176 (1 January - 30 June 1972)

1. Brinca, A. L. "Whistler Sideband Growth due to Nonlinear Wave-Particle Interaction"
   I.P.R. 448 (January 1972)
   J. Geophys. Res. 77, 3508 (July 1972).

2. Brinca, A. L., "Landau and Whistler Sideband Growth due to Wave-Particle Interaction"
   I.P.R. 457 (January 1972).

3. Rognlien, T. D., and Self, S. A., "Interpretations of Dispersion Relations for Bounded Systems"
   J. Plasma Phys. 7, 13 (February 1972).

4. Harker, K. J., "A Study and Classification of Nonlinear High Frequency Ionospheric Instabilities by Coupled Mode Theory"
   Int. J. Elect. 32, 297 (March 1972).

5. Brinca, A. L., "Modulational Instability of Whistlers in Cold Plasmas"
   I.P.R. 464 (March 1972).

6. Crawford, F. W., "Nonlinear Wave-Wave and Wave-Particle Interactions"
   *Symposium on Plasma Waves, Instabilities and Interactions, Spåtind, Norway, April 1972 [Invited paper].

7. Crawford, F. W., and Harker, K. J., "Energy Absorption in Cold Inhomogeneous Plasmas: The Herlofson Paradox"
   *Symposium on Plasma Waves, Instabilities and Interactions, Spåtind, Norway, April 1972
   J. Plasma Phys. (to be published).

8. Brinca, A. L., "The Whistler Modulational Instability"
   I.P.R. 475 (April 1972).

9. Kim, H., "Lagrangian Description of Warm Plasmas"
   I.P.R. 470 (May 1972) [Ph.D. Thesis].

10. Brinca, A. L., "Sideband Growth in Nonlinear Landau Wave-Particle Interaction"
    J. Plasma Phys. 7, 385 (June 1972).

I.P.R. = Stanford University Institute for Plasma Research Report.
* = Conference presentation.

Semiannual Reports

11. No. 10 (1 July - 31 December 1972)
    I.P.R. 456 (January 1972).

12. No. 11 (1 January - 30 June 1972)
    I.P.R. 492 (September 1972).

REFERENCES

13. Brinca, A. L., Stanford University Institute for Plasma Research Report No. 442 (November 1971).

14. Lee, B. G., Stanford University Radioscience Laboratory Technical Report No. 3412-10 (August 1968).

15. Bell, T. F. and Helliwell, R. A., J. Geophys. Res. 76, 8414, (1971).

16. Likhter, Ya. I., Molchenov, O. A., and Chmyrev, V. M., JETP Letters 14, 325 (1971).

17. Tam, C. K. W., Phys. Fluids 12, 1028 (1969).

18. Tang, T. and Sivasubramanian, Phys. Fluids 14, 444 (1971).

19. Taniuti, T. and Washimi, H., Phys. Rev. Letters 22, 454 (1969).

20. Hasegawa, A., Phys. Rev. Letters 21, 1165 (1970).

21. Hasegawa, A., Phys. Fluids 15, 870 (1972).

22. Lee, J. C., Stanford University Institute for Plasma Research Report No. 313 (May 1969) [Ph.D. Thesis].

23. Lee, J. C., Fessenden, T. J., and Crawford, F. W., Proc. Ninth International Conference on Phenomena in Ionized Gases, Bucharest, Rumania, September 1969 (Acad. Soc. Rep. Rumania, 1969), p. 475.

24. Ilić, D. B., Rognlien, T. D., Self, S. A., and Crawford, F. W., Stanford University Institute for Plasma Research Report (in preparation).

25. Rognlien, T. D., Stanford University Institute for Plasma Research Report (in preparation).

26. Ewald, H. N., Crawford, F. W., and Self, S. A., J. Appl. Phys. 38, 2753 (1967).

27. Ilić, D. B. and Self, S. A., J. Appl. Phys. 43, 2620 (1972).

28. Dougherty, J. P., J. Plasma Phys. 4, 761 (1970).

29. Galloway, J. J., Stanford University Institute for Plasma Research Report (in preparation).

30. Larsen, J. M., Stanford University Institute for Plasma Research Report (in preparation).

31. Lee, S. W. and Mittra, R., Radio Science 2, 813 (1967).

32. Singh, N. and Gould, R. W., Radio Science 6, 1151 (1971).

33. Brinca, A. L., Stanford University Institute for Plasma Research Report (in preparation).