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A High Frequency Correction to the Kirchoff Approximation, with Application to Rough Surface EM Wave Scattering

by

Frederick C. Jackson

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Page 28, second para. Delete 3rd through 5th sentences.

Page 35, eqn. for σ_{HV}. In first line of integrand, for

$$4B^2_{\xi\eta} - (k\gamma)^{-2} R_{\xi\xi\eta\eta}$$

read

$$4B^2_{\xi\eta} + (k\gamma)^{-2} R_{\xi\xi\eta\eta}$$

and in fourth line of integrand, for

$$-\frac{1}{2} R_{\xi} R_{\xi\xi\eta\eta}$$

read $+ R_{\xi} R_{\xi\xi\eta\eta}$.

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Abstract

A high frequency correction to the Kirchoff approximation is developed for application to rough surface scattering. An approximate solution to the Magnetic Field Integral Equation for perfect conductivity and plane-wave excitation yields a perturbed surface current expressed as a linear function of the second derivatives of surface height. The corrected surface current vector is substituted into the far-field Stratton-Chu integral and average backscattered powers for the four polarization combinations are computed on the assumption that the surface is describable as a stationary Gaussian random process. The strength of this scattering solution is that it can account for height-curvature correlation without requiring small height and slope.

I. Introduction

The problem of electromagnetic wave scattering from rough surfaces has received increasing attention in the last several years. Yet, little new of substance has been added to the theoretical foundations laid down in the 1950's. Most of the work in the last decade has been based on one or the other of the two theoretical pillars -- the Rayleigh-Rice and the randomized Kirchoff methods. The randomized Kirchoff method developed by Beckmann [1] and others uses the physical optics integral with the so-called Kirchoff or tangent-plane boundary values of the field. The Kirchoff method is good for softly undulating surfaces where the curvature is everywhere small compared to the microwave propagation constant. An advantage of the Kirchoff method is that the surface height variations do not necessarily have to be small compared to the radar wavelength. In application to radar sea-return, Kirchoff theory is most suitable for predicting return (backscatter) from near-vertical incidence, where the scattering mechanism is dominated by specular reflection. At larger angles of incidence, Kirchoff theory fails to represent the scattering process. A shortcoming of Kirchoff theory is that it cannot account for depolarization in the plane of incidence.

The Rayleigh-Rice method has gained increasing favor in the last few years. Essentially a small perturbation method, it requires small surface heights and slopes. The Rayleigh-Rice method is especially applicable to large angles of incidence where much of the surface height variation is effectively small compared to the radar wavelength. Second-order Rice theory is capable of predicting depolarization in the plane of incidence (Valenzuela, [2]). Rice's theory has been refined by Wait [3] among others.

Fung and Chan [4] have developed a 'composite surface model' which combines Kirchoff and Rayleigh-Rice theories. The model requires a surface with small irregularities superposed on a larger, softly undulating surface. The problem with the model is that for a surface like the sea-surface there is no separation between the scales of roughness. The sea-height spectrum decays monotonically toward higher wave number; there is no spectral gap or 'quiet' zone between long wave components and short wave components. This is not to dispose of the composite surface model. The 'spectral gap' required to separate the Kirchoff regime from the Rice regime may not be very large for the sea-surface; ultimately the width of the gap depends on the slope of the roughness spectrum.

The model developed in this paper is an alternative to the composite surface model; it applies especially to describing the scattering process between Kirchoff and Rice regimes. That is, it applies to predicting scattering from undulations too large in height for Rice theory and too large in curvature for Kirchoff theory. The approach is essentially an extension of the randomized Kirchoff method. The integral equation for the magnetic field is used to supply corrections to the Kirchoff boundary values, and the corrected values are substituted into the Stratton-Chu far-field integral.

II. Development of the Integral Equation

The scattering surface is assumed to be perfectly conducting. Perfect conductivity is not too bad an assumption for microwaves incident on sea water. In section VI, a means of circumventing the limitation of this assumption will be offered. Let us also assume that the surface is free from singular curves (cusps). This assumption is in line with the development to follow, namely, it will be assumed that the surface possesses a 'good bit' of smoothness. For a perfectly conducting surface free from singular curves, the Magnetic Field Integral Equation is according to Mittra [5]:

$$\underline{J}_s = \hat{n} \times 2\underline{H}^i + \hat{n} \times \frac{1}{2\pi} \int_S \underline{J}_s' \times \nabla' G \, dS' \qquad (2.1)$$

where $\underline{J}_s = \hat{n} \times \underline{H}$ is the surface current density and

$\underline{H}$ is the magnetic field on the surface, S.

$\hat{n}$ is the unit surface normal vector, directed outward from the conducting volume.

$\underline{H}^i$ is the incident magnetic field evaluated on the surface.

(') (prime) denotes source point coordinates $\underline{x}'$ as opposed to field point coordinates, $\underline{x}$.

G is the Green's function for homogeneous space,

$G = \dfrac{e^{-ik|\underline{x} - \underline{x}'|}}{|\underline{x} - \underline{x}'|}$; k is the microwave propagation constant; $\underline{x}'$ and $\underline{x}$ are respectively the source and field point coordinates of the surface S.

The Magnetic Field Integral Equation formulation (2.1) is as Mittra points out ideally suited to near-planar, or smooth geometries. For, when the surface is approximately planar, the vector $\underline{J}_s' \times \nabla' G$ is oriented nearly parallel to the field point normal vector, $\hat{n}$. Hence, the integral contribution is small compared to the 'forcing function' (the Kirchoff boundary value), $\hat{n} \times 2\underline{H}^i$. The integral can then be regarded as a 'small perturbation' on the Kirchoff boundary value, $\hat{n} \times 2\underline{H}^i$. We shall find this 'small perturbation' approach very convenient.

The integral equation is developed for a quasi-horizontal, single-valued wavy surface described by $z = f(x, y)$, where x and y are horizontal Cartesian coordinates, and z is the vertical coordinate. In the (x, y, z) Cartesian system, the following applies:

$$\underline{x} = (x, y, f), \quad \underline{x}' = (x', y', f')$$

$$\underline{\rho} = \underline{x}' - \underline{x} = (\xi, \eta, \zeta)$$

$$\rho = |\underline{\rho}| = (\xi^2 + \eta^2 + \zeta^2)^{\frac{1}{2}}$$

$$\xi = x' - x, \quad \eta = y' - y, \quad \zeta = f' - f .$$

The surface normal vector is given by

$$\hat{n} = (-f_x, \; -f_y, \; 1) \cos \omega,$$
$$\cos \omega = (1 + f_x^2 + f_y^2)^{-\frac{1}{2}} \qquad (2.2)$$

Here the subscripts stand for partial differentiation. We shall freely use this convention for any quantities involving the surface height, f. On vector quantities, however, such as the current vector, subscripts will denote Cartesian components. E.g. $\underline{J}_s = (J_x, J_y, J_z)$. We calculate the gradient of the Green's function,

$$\nabla' G = \psi \underline{\rho} \;,$$
$$\psi = -(1 + ik\rho) \frac{e^{-ik\rho}}{\rho^3} \; . \qquad (2.3)$$

Now, let us expand the triple cross product,

$$\hat{n} \times \underline{J}_s' \times \nabla' G = (\hat{n} \cdot \nabla' G) \underline{J}_s' - (\hat{n} \cdot \underline{J}_s') \nabla' G \; . \qquad (2.4)$$

Using the preceding relations we find

$$\hat{n} \cdot \nabla' G = (-\xi f_x - \eta f_y + \zeta) \psi \cos \omega$$

$$\hat{n} \cdot \underline{J}_s' = (-f_x J_x' - f_y J_y' + J_z') \cos \omega$$

From the condition that $\underline{J}_s$ is tangential to the surface it follows that

$$\hat{n} \cdot \underline{J}_s = 0$$

so that J_z is related to J_x and J_y as

$$J_z = f_x J_x + f_y J_y \quad . \tag{2.5}$$

Hence, $\hat{n} \cdot \underline{J}_s'$ can be written as

$$\hat{n} \cdot \underline{J}_s' = \left[(f_y' - f_x) J_x' + (f_y' - f_y) J_y' \right] \cos \omega$$

Thus, the triple cross product (2.4) becomes

$$\hat{n} \times \underline{J}_s' \times \nabla' G = (-\xi f_x - \eta f_y + \zeta) \psi \cos \omega \, \underline{J}_s' - \left[(f_x' - f_x) J_x' + (f_y' - f_y) J_y' \right] \psi \cos \omega \, \underline{\rho} .$$

The first two components of this vector can be expressed as the matrix product

$$\psi \cos \omega \, \underline{\underline{M}} \, \underline{J}' \tag{2.6}$$

where $\underline{J}'$ is the two-vector

$$\underline{J}' = \begin{bmatrix} J_x' \\ J_y' \end{bmatrix} \tag{2.6a}$$

and $\underline{\underline{M}}$ is the two by two matrix

$$\underline{\underline{M}} = \begin{bmatrix} -\xi f'_x - \eta f_y + \zeta & -\xi (f'_y - f_y) \\ -\eta (f'_x - f_x) & -\eta f'_y - \xi f_x + \zeta \end{bmatrix} \tag{2.6b}$$

$\underline{\underline{M}}$ can be expressed rather neatly if we note that $\zeta = f' - f$ can be written as

$$\zeta = \zeta\,(x, y;\ \xi, \eta)$$

so that

$$\frac{\partial \zeta}{\partial \xi} = f'_x \qquad (x \rightarrow y)$$

and

$$\frac{\partial \zeta}{\partial x} = f'_x - f_x \qquad (x \rightarrow y)\ .$$

Thus, (2.6b) can be written as

$$\underline{\underline{M}} = \begin{bmatrix} -\xi \dfrac{\partial \zeta}{\partial \xi} - \eta f_y + \zeta & -\xi \dfrac{\partial \zeta}{\partial y} \\ -\eta \dfrac{\partial \zeta}{\partial x} & -\eta \dfrac{\partial \zeta}{\partial \eta} - \xi f_x + \zeta \end{bmatrix} \tag{2.7}$$

For the sake of simplicity, let $\underline{F}$ stand for the x and y, components of $\hat{n} \times 2\underline{H}^{i}$. On setting $dS' = \sec \omega' \, d\xi \, d\eta$, the first two component equations of (2.1) become

$$\underline{J} = \underline{F} + \frac{1}{2\pi} \iint \psi \cos \omega \, \underline{\underline{M}} \, \underline{J}' \sec \omega' \, d\xi \, d\eta$$

Or, if we multiply both sides of the equation by sec ω, and we permit ourselves to change the names of $\underline{F}$ and $\underline{J}$ so that

$$\begin{aligned} \underline{F} \sec \omega &\rightarrow \underline{F} \\ \underline{J} \sec \omega &\rightarrow \underline{J} \, , \end{aligned} \tag{2.8}$$

then we get the compact expression:

$$\underline{J} = \underline{F} + \frac{1}{2\pi} \iint \psi \, \underline{\underline{M}} \, \underline{J}' \, d\xi \, d\eta \tag{2.9}$$

If equation (2.9) can be solved for J_x and J_y, then J_z can be found readily from (2.5).

III. The High Frequency Approximation

The Kirchoff approximation ($\underline{J} \sim \underline{F}$) is a high frequency approximation. For a high enough microwave frequency (wave number), the surface can be considered to be locally flat. The integral in (2.9) is then negligible and $\underline{J} \sim \underline{F}$ obtains. The Kirchoff approximation means simply that everywhere on the surface there is a perfect reflection of the incident wave--the surface is everywhere like a mirror.

Let us see what equation (2.9) implies if the high frequency (small curvature) condition is relaxed somewhat. Since we are still dealing with a 'smooth' surface, we can assume that the surface height f has a Taylor series expansion about every point $\underline{x}_o = (x_o, y_o)$:

$$f(\underline{x}) = f + f_x u + f_y v + f_{xx} \frac{u^2}{2} + f_{xy} u v + f_{yy} \frac{v^2}{2} + \cdots$$

From now on it will be understood that f and its derivatives are to be evaluated at the local origin $\underline{x}_o$. We shall have $\underline{u} = (u, v)$ stand for the relative (horizontal) position of a field point. The relative position vector of a source point shall be given by $\underline{u} + \underline{\xi} = (u + \xi, v + \eta)$. If we form the difference $\zeta = f' - f$, we get

$$\zeta = \xi f_x + \eta f_y + f_{xx} (u \xi + \frac{\xi^2}{2}) + f_{xy} (u\eta + v \xi + \xi \eta)$$

$$+ f_{yy} (v \eta + \frac{\eta^2}{2}) + \cdots$$

A little algebra will show that the matrix $\underline{\underline{M}}$ given by (2.7) has the expansion,

$$\underline{\underline{M}} = \begin{bmatrix} -f_{xx}\frac{\xi^2}{2} + f_{yy}\frac{\eta^2}{2} & -f_{xy}\,\xi^2 - f_{yy}\,\xi\,\eta \\ -f_{xy}\,\eta^2 - f_{xx}\,\xi\,\eta & f_{xx}\frac{\xi^2}{2} - f_{yy}\frac{\eta^2}{2} \end{bmatrix} \tag{3.1}$$

$$+ \text{ higher order terms in } \underline{u} \text{ and } \underline{\xi}.$$

Thus, to a first approximation, $\underline{\underline{M}}$ is independent of the local field point coordinate $\underline{u}$ and can be expressed in terms of the separation vector $\underline{\xi}$ alone. Call the matrix given explicitly in (3.1) $\underline{\underline{M}}^{(2)}$.

We have

$$\psi = -(1 + i k \rho)\,\frac{e^{-ik\rho}}{\rho^3}$$

where $\rho^2 = \xi^2 + \eta^2 + \zeta^2$. We can write $\zeta = f_x\,\xi + f_y\,\eta + \epsilon\,(\underline{u}\,;\,\underline{\xi})$. And we can express ρ as

$$\rho = \rho_1 + \delta\,(\,\underline{u}\,;\,\underline{\xi}\,)$$

where

$$\rho_1 = (\,a^2\,\xi^2 + b^2\,\eta^2)^{\frac{1}{2}} \tag{3.2}$$

and

$$a^2 = 1 + f_x^{\;2}\;, \quad b^2 = 1 + f_y^{\;2}\,.$$

We see that ρ_1 is the distance on the $\underline{x}_o$ tangent plane corresponding to the separation vector $\underline{\xi}$. The 'first-order' approximation to ψ is

$$\psi^{(1)} = -(1 + i k \rho_1) \frac{e^{-ik\rho_1}}{\rho_1^3} . \tag{3.3}$$

This approximation is good so long as $\delta^2 << \rho_1^2$ and $k\,\delta << 1$. The leading error term is proportional to the curvature.

If the approximation $\psi \underline{\underline{M}} = \psi^{(1)} \underline{\underline{M}}^{(2)}$ is made, the leading error term is proportional to third derivatives of surface height and the product of two second derivatives. What we are going to assume is that the bulk of the integral $\iint \psi^{(1)} \underline{\underline{M}}^{(2)} \underline{J}' \, d\xi \, d\eta$ is formed in the neighborhood of $\underline{x}_o$ where the error terms are small. Brekhovskikh [6] has shown that the Kirchoff approximation is valid if

$$4 \pi r_c \cos \theta >> \lambda \tag{3.4}$$

where r_c is the radius of curvature, θ is the 'local' angle of incidence and λ is the radar wavelength. Let us follow Brekhovskikih's example by assuming a similar criterion for the applicability of our approximation, namely

$$(4 \pi r_c \cos \theta)^2 >> \lambda^2 . \tag{3.5}$$

Of course, these inequalities cannot be interpreted in a strict sense since large third derivatives can exist even if second derivatives are small. We should really interpret r_c as a root-mean-square value for the surface. Let us drop any further discussion of error and see just what results obtain on the basis of the approximation.

Then, assuming $\psi \underline{\underline{M}} = \psi^{(1)} \underline{\underline{M}}^{(2)}$, the integral equation (2.9) becomes

$$\underline{J}(\underline{u}) = \underline{F}(\underline{u}) + \frac{1}{2\pi} \iint \psi^{(1)}(\rho_1) \underline{\underline{M}}^{(2)}(\underline{\xi}) \underline{J}(\underline{u} + \underline{\xi})\, d\xi\, d\eta . \qquad (3.6)$$

Equation (3.6) can be solved exactly by Fourier transformation techniques. Fourier transformation is now a common method for solving two dimensional (plane) diffraction problems, where integral equations of this type occur [7]. But remember, unlike a true two-dimensional equation, equation (3.6) is only approximately correct. It would make little sense to solve (3.6) exactly. The older method of interated kernels is a more appropriate method of solution.

The integral operation takes an O (1) quantity into an O (λ/r_c) quantity, an O (λ/r_c) quantity into an O (λ^2/r_c^2) quantity, and so on. Since $\underline{J}$ differs from $\underline{F}$ by an O (λ/r_c) quantity we can set

$$\underline{J} = \underline{F} + \underline{J}^{(1)} \qquad (3.7)$$

where $\underline{F}$ = O (1) and $\underline{J}^{(1)} = O(\lambda/r_c)$. Equation (3.6) then yields for $\underline{J}^{(1)}$:

$$\underline{J}^{(1)}(\underline{u}) = \frac{1}{2\pi} \iint \psi^{(1)}(\rho_1) \underline{\underline{M}}^{(2)}(\underline{\xi}) \underline{F}(\underline{u} + \underline{\xi})\, d\xi\, d\eta + O(\lambda^2/r_c^2) \qquad (3.8)$$

We have lost nothing here since equation (3.6) was only accurate to $O(\lambda/r_c)$ to start with. Realize that the above equation is most accurate at the local origin, $\underline{u} = 0$. And since we no longer need the convolution properties of the original equation because $\underline{F}$ is a known function, all we need calculate is

$$\underline{J}^{(1)} = \frac{1}{2\pi} \iint \psi^{(1)} \underline{\underline{M}}^{(2)} \underline{F} \, d\xi \, d\eta \tag{3.9}$$

The last step is to make some simplifying approximations to $\underline{F}$.

The incident wave is taken to be a plane-polarized plane wave making an angle of incidence θ to the z axis. The plane of incidence is the x - z plane, and the angle θ is counted positive when the wave comes from the negative x direction. Define the direction cosines $\alpha = \sin\theta$ and $\gamma = \cos\theta$. From the definition of $\underline{F} = \hat{n} \times 2\,\underline{H}^i \sec\omega$, it follows that $\underline{F}$ has the form

$$\underline{F} = 2\,\underline{A}\, e^{-ik(\alpha x - \gamma f)} \tag{3.10}$$

where for a unit magnetic field and for vertical polarization ($\underline{E}^i$ - vector in the x - z plane)

$$\underline{A}^v = \begin{bmatrix} 1 \\ \\ 0 \end{bmatrix} \quad ; \tag{3.11}$$

and for horizontal polarization ($\underline{E}^i$ - vector parallel to the y-axis)

$$\underline{A}^H = \begin{bmatrix} -\alpha f_y \\ \\ \gamma + \alpha f_x \end{bmatrix} \tag{3.12}$$

It is entirely consistent with the development of equation (3.9) to expand $\underline{F}$ in terms of $\underline{\xi}$ about $\underline{x}_o$ and to neglect terms of $O(\lambda/r_c)$. We can do this because when multiplied by $\psi^{(1)}\ \underline{\underline{M}}^{(2)}$, $O(\lambda/r_c)$ terms become $O(\lambda^2/r_c^2)$ terms. We then have, approximately, $\underline{F} = \underline{F}^{(1)}$,

$$\underline{F}^{(1)} = 2\,\underline{A}^{(1)}\, e^{-ik(\alpha x - \gamma f)} \cdot e^{-i\underline{\ell}\cdot\underline{\xi}} \tag{3.13}$$

$\underline{A}^{(1)}$ is given by $\underline{A}$ with f_x and f_y evaluated at the local origin, x. (The sub-nought notation is abandoned). And $\underline{\ell}$ is the wave number,

$$\underline{\ell} = k\,(\alpha - \gamma f_x\,,\ -\gamma f_y)\ . \tag{3.14}$$

Putting $\underline{F}^{(1)}$ in the integral (3.9) we get

$$\underline{J}^{(1)} = 2\, e^{-ik(\alpha x - \gamma f)}\ \underline{\underline{S}}\ \underline{A} \tag{3.15}$$

where $\underline{\underline{S}}$ is defined by the Fourier integral

$$\underline{\underline{S}} = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int \psi^{(1)} \underline{\underline{M}}^{(2)} e^{-i \underline{\ell} \cdot \underline{\xi}} \, d\xi \, d\eta . \tag{3.16}$$

The infinite limits have been applied just to make the integral definite.

The Fourier integral involves three types of integrals, viz.

$$\int_{-\infty}^{\infty} \int \left\{ \begin{matrix} \xi^2 \\ \eta^2 \\ \xi\eta \end{matrix} \right\} \frac{(1 + i k \rho_1)}{\rho_1^3} e^{-i (k \rho_1 + \underline{\ell} \cdot \underline{\xi})} d\xi \, d\eta \tag{3.17}$$

which on transforming to the elliptical coordinates defined by

$$\xi = \frac{\rho_1}{a} \cos \varphi$$

$$\eta = \frac{\rho_1}{b} \sin \varphi$$

become

$$\frac{1}{a\,b} \int_0^{\infty} \int_0^{2\pi} \left\{ \begin{matrix} \frac{1}{a^2} \cos^2 \varphi \\ \frac{1}{b^2} \sin^2 \varphi \\ \frac{1}{2ab} \sin 2\varphi \end{matrix} \right\} (1 + i k \rho_1) e^{-i[k\rho_1 + L\rho_1 \cos(\Theta - \varphi)]} d\rho_1 \, d\varphi \tag{3.18}$$

where $$L = k\left[\frac{(a - \gamma f_x)^2}{a^2} + \frac{\gamma^2 f_y^2}{b^2}\right]^{\frac{1}{2}}$$

and $$\Theta = \tan^{-1}\left[\frac{-\gamma f_y/b}{(a - \gamma f_x)/a}\right]$$

Integrating over the polar angle we get

$$\frac{\pi}{ab}\int_0^\infty \left[\begin{Bmatrix} \frac{1}{a^2} \\ \frac{1}{b^2} \\ 0 \end{Bmatrix} \cdot J_o(L\rho_1) - \begin{Bmatrix} \frac{1}{a^2}\cos 2\Theta \\ -\frac{1}{b^2}\cos 2\Theta \\ \frac{1}{ab}\sin 2\Theta \end{Bmatrix} \cdot J_2(L\rho_1)\right] \cdot (1 + ik\rho_1)\, e^{-ik\rho_1} d\rho_1$$

where J_o and J_2 are Bessel functions. The 'radiation' integrals (i.e. those involving $ik\rho_1$) are a bit troublesome to evaluate. Their absence from any tables of Fourier/Bessel transforms which the author could find seemed to confirm the author's suspicion that they were divergent integrals. Recourse was made to the device of a weighting function. And--lo! --it was found that in the limit as the weighting function went to unity, the integrals did indeed exist! It was later discovered that a 'simple' consideration of the Fourier transform as a Laplace transform evaluated at the imaginary argument $s = -ik$ yielded the same answer.* We find that the integrals (3.18) are equal to

*Credit for this 'discovery' is due Mrs. Neptune Rodriguez. It was she who suggested the substitution of the Laplace variable for the Fourier variable.

$$- \frac{2\pi i}{k\,a\,b\,\nu^3} \begin{Bmatrix} (1 - m^2)/a^2 \\ (1 - \ell^2)/b^2 \\ \ell m/a\,b \end{Bmatrix} \tag{3.19}$$

where again we have

$$a^2 = 1 + f_x^{\ 2} \ , \ b^2 = 1 + f_y^{\ 2}$$

and where we have defined

$$\ell = (\alpha - \gamma f_x)/\,a$$

$$m = -\,\gamma f_y\,/b$$

and where ν is equal to

$$\nu = (1 - \ell^2 - m^2)^{\frac{1}{2}} \tag{3.20}$$

Referring to (3.1), we write the elements of $\underline{\underline{S}}$:

$$S_{11} = \frac{-\,i}{2\,k\,a\,b\,\nu^3} \left[(1-m^2)\frac{f_{xx}}{a^2} - (1 - \ell^2)\frac{f_{yy}}{b^2}\right] \tag{3.21a}$$

$$S_{12} = \frac{-\,i}{k\,a\,b\,\nu^3} \left[(1-m^2)\,\frac{f_{xy}}{a^2} + \ell m \frac{f_{yy}}{ab}\right] \tag{3.21b}$$

$$S_{21} = \frac{-i}{k\,a\,b\,\nu^3}\left[(1-\ell^2)\,\frac{f_{xy}}{b^2} + \ell\,m\,\frac{f_{xx}}{ab}\right] \qquad (3.21c)$$

$$S_{22} = -\,S_{11} \qquad (3.21d)$$

the wave numbers $k\,\ell$ and $k\,m$ are the projections of the propagation vector $\underline{k}$ on to the tangent plane in the x and y directions respectively. The quantity ν is a bit difficult to interpret, but it can be written in terms of the 'tilt angles' of the surface, ψ and δ, defined by $\tan\psi = f_x$ and $\tan\delta = f_y$:

$$\nu^2 = \cos^2(\theta - \psi) - \cos^2\theta\,\sin^2\delta \quad .$$

At (locally) normal incidence, $\psi = \theta$ and $\delta = 0$ so that $\nu = 1$ (its maximum value). For local incidence near grazing ($\psi = \theta - \pi/2$), ν may go to zero causing $\underline{\underline{S}}$ to blow up. Since $\underline{\underline{S}}$ must be small if it is to be a good approximation, we must avoid large angles of incidence. The failure of representation at large angles of incidence is a consequence of the perfect conductivity assumption.

It is interesting to compare our derived results with Brekhovskikh's criterion (cf. eqn. 3.4). For the one dimensional case, $f = f(x)$ alone, say, the condition that $\underline{\underline{S}}$ be small gives

$$\frac{1}{2\,k\,\nu^3}\;\frac{|f_{xx}|}{a^3} << 1 \,.$$

Or, since $|f_{xx}|/a^3 = r_c^{-1}$, $\nu = \cos(\theta - \psi)$, and $k = 2\pi/\lambda$

this means

$$4\pi r_c \cos^3(\theta - \psi) \gg \lambda \tag{3.22}$$

Thus, the applicability of our method for large incidence angles is more limited than we supposed or the basis of Brekhovskikh's criterion. Wait and Conda [8] have used the criterion,

$$\pi r_c \cos^3(\theta - \psi) \gg \lambda \tag{3.23}$$

Note that $\underline{\underline{S}}$ is purely imaginary in number. This means that the perturbed current density $\underline{J}^{(1)}$ is 90° out of phase with the zeroth-order current. Exactly how this phase shift will determine the scattered field will depend on the height-curvature correlation properties of the surface.

Let us summarize our results by putting together equations (3.10) and (3.15) in (3.7). The total current density times sec ω can then be expressed as

$$\underline{J} = 2(\underline{\underline{I}} + \underline{\underline{S}})\underline{A}\ e^{-ik(\alpha x - \gamma f)} \tag{3.24}$$

where $\underline{\underline{I}}$ is the two by two identity matrix.

IV. Correspondence with Rice's Theory

In this section we show that our derived surface currents and those implied by Rice's theory [9] are identical provided that the assumptions of both theories are satisfied. To keep things simple, consider only the one-dimensional case, $f = f(x)$ alone. And consider only the case of horizontal polarization.

With

$$\underline{A} = \underline{A}^H = \begin{bmatrix} 0 \\ \\ \gamma + \alpha f_x \end{bmatrix}$$

and

$$\underline{\underline{S}} = \begin{bmatrix} \frac{-i}{2k\nu^3} \frac{f_{xx}}{a^3} & 0 \\ & \\ 0 & +\frac{i}{2k\nu^3} \frac{f_{xx}}{a^3} \end{bmatrix}$$

equation (3.24) yields (on multiplying by $\cos\omega = \cos\psi$) :

$$J_x = 0$$

$$J_y = 2\left(1 + \frac{i}{2 k \nu^3} \frac{f_{xx}}{a^3}\right)(\gamma + \alpha f_x)\, e^{-i k (\alpha x - \gamma f)} \cos \psi \tag{4.1}$$

And from (2.5) it follows that

$$J_z = 0.$$

If we are to compare this result with Rice's theory, we must impose the same constraints Rice imposed on the surface structure, namely, that surface heights and slopes must be small,

$$|k f| << 1 \quad \text{and} \quad |f_x| << 1 \;.$$

Under these conditions our J_y expression (4.1) becomes, approximately

$$J_y = 2\left(\gamma + i k \gamma^2 f + \alpha f_x + \frac{i f_{xx}}{2 k \gamma^2}\right) e^{-i k \alpha x}. \tag{4.2}$$

Rice's first-order electric field is given by

$$E_x = E_z = o$$

$$E_y = E_y^a - 2 i k \gamma \Sigma P (m-\nu)\, e^{-i(a m x + b(m) z)}$$

where

$E_y^a = 2\, i \sin k\, \gamma\, z\, e^{-i a \nu x}$ is the 'regular' or specular field.

$a = 2\pi/L$ is the fundamental wave number of the surface, L being the 'period' of the surface.

ν = is an integer chosen to make $a\,\nu \sim k\,\alpha$

P = is the (complex) Fourier coefficient in the Fourier series representation of f, $f = \Sigma\, P\,(m)\, e^{-i a m x}$.

$b\,(m) = (k^2 - a^2 m^2)^{\frac{1}{2}}$

From the electric field we find the current density. The 'regular' current density is simply

$$\underline{J}^a = \hat{z} \times \underline{H}^a \big|_{z=o} = i\,(k\,\eta)^{-1}\, \hat{z} \times \nabla \times \underline{E}^a \big|_{z=o}$$

where η is the impedance of free space. Putting in $\underline{E}^a = (0,\ E_y^a,\ 0)$ we get $J_x^a = J_z^a = 0$ and

$$J_y^a = 2\,\gamma\, e^{-i k \alpha x} \tag{4.4}$$

The factor of η^{-1} has been omitted since it arises from taking a unit electric vector rather than a unit magnetic vector. The perturbed (first-order) current is given by

$$\underline{J}^{(1)} = i\,(k\,\eta)^{-1}\,\hat{n} \times \nabla \times \underline{E}^{(1)}\big|_{z=f}$$

where $\underline{E}^{(1)} = (0,\ E_y^{(1)},\ 0)$ and $E_y^{(1)}$ is given by the summation term in (4.3).

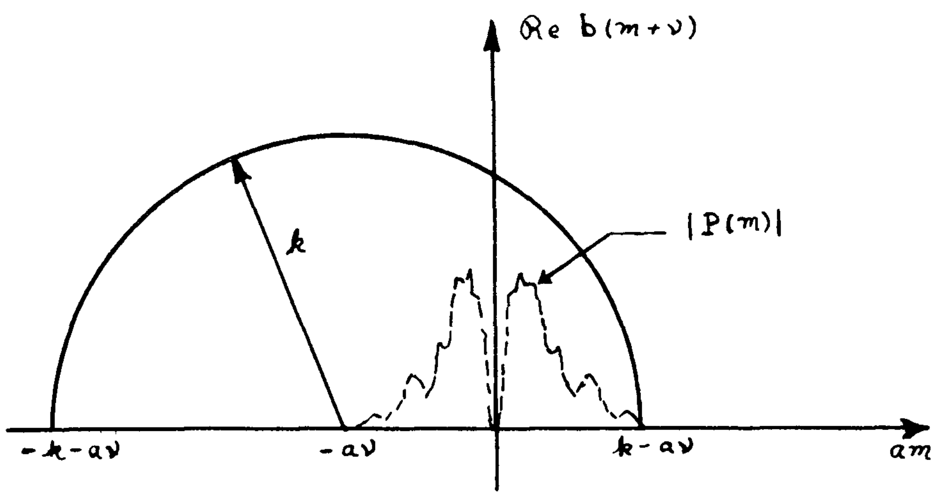
Putting in $\underline{E}^{(1)}$, we get $J_x^{(1)} = J_z^{(1)} = 0$ and

$$J_y^{(1)} = 2\,i\,\gamma\ \Sigma\, b\,(m)\ P\,(m-\nu)\ e^{-i\,a\,m\,x} \tag{4.5}$$

In arriving at (4.5) we used the condition $|b\,f| << 1$. Terms of second order, e.g. of $O\,(f_x^2)$, are neglected. And again we have dropped the η^{-1} factor. Since the summation in (4.5) extends from $-\infty$ to $+\infty$ (4.5) can be transformed to

$$J_y^{(1)} = 2\,i\,\gamma\, e^{-i\,a\,\nu\,x}\ \Sigma\, b\,(m+\nu)\ P(m)\ e^{-i\,a\,m\,x} \tag{4.6}$$

In order to satisfy the conditions of our theory, the P(m) must be tightly distributed about m = 0. In particular, we assume that the P(m) fall away rapidly to zero in the interval $|a\,m| < |k - a\,\nu|$.



With the Fourier coefficients tightly distributed about $m = 0$ (see illustration), it is permissible to develop $b(m + \nu)$ in a power series in m, stopping at m^2:

$$b(m + \nu) = (k^2 - a^2 (m + \nu)^2)^{\frac{1}{2}}$$

$$b(m + \nu) \sim k\gamma - \frac{\alpha}{\gamma} a m - \frac{1}{2k\gamma^2} a^2 m^2$$

Here we have set $a\nu = k\alpha$ and $b(\nu) = (k^2 - a^2 \nu^2)^{\frac{1}{2}} = k\gamma$. Putting this approximation to $b(m + \nu)$ into (4.6) we get approximately

$$\begin{aligned} J_y^{(1)} = 2 i \gamma e^{-ik\alpha x} \Big\{ & k\gamma \Sigma P(m) e^{-iamx} \\ & - i \frac{\alpha}{\gamma} \Sigma(-iam) P(m) e^{-iamx} \\ & + \frac{1}{2k\gamma^3} \Sigma(-a^2 m^2) P(m) e^{-iamx} \Big\} \end{aligned} \qquad (4.7)$$

But, we have

$$f = \Sigma P(m) e^{-iamx}$$

$$f_x = \Sigma(-iam) P(m) e^{-iamx}$$

$$f_{xx} = \Sigma(-a^2 m^2) P(m) e^{-iamx}$$

So that (4.7) is equal to

$$J_y^{(1)} = 2\left(i k \gamma^2 f + \alpha f_x + \frac{i}{2 k \gamma^2} f_{xx}\right) e^{-i k \alpha x} \tag{4.8}$$

Finally, combining (4.4) and (4.8) we get for the total current density, $J_y = J_y^a + J_y^{(1)}$,

$$J_y = 2 (\gamma + i k \gamma^2 f + \alpha f_x + \frac{i}{2 k \gamma^2} f_{xx}) e^{-i k \alpha x} \tag{4.9}$$

which agrees with our result (4.2).

V. Calculation of the Scattered Power

We apply our results to the calculation of the average power backscattered from a random rough surface. In the case of perfect conductivity, the far-field Stratton-Chu integral [10] reduces to:

$$\underline{E}(\underline{R}) = \frac{-ik\,e^{-ikR}}{4\pi R} \hat{R} \times \iint_{A_o} \hat{R} \times (-\eta \underline{J}_s \sec\omega)\, e^{ik\hat{R}\cdot\underline{x}}\, dx\, dy \, . \tag{5.1}$$

$\underline{E}$ is the electric field vector, $\underline{R} = R\hat{R}$ is the position vector of the far field (Fraunhoffer zone) point; $\underline{x}$ is the position vector of the source points on the surface, and A_o is the illuminated area.

For backscatter, the unit vector $\hat{R}$ is directed toward the source of incident radiation and so is given by

$$\hat{R} = (-\alpha,\ 0,\ \gamma) \, .$$

Since in the far field the E-vector oscillates transversely to the propagation vector $k\hat{R}$, we have

$$\underline{E} \cdot \hat{R} = 0 \, ;$$

hence only two components are needed to specify $\underline{E}$. In practice the 'horizontal' and 'vertical' components are used,

$$E^H = E_y$$

and

$$E^V = -\gamma^{-1} E_x \, .$$

For a unit incident electric field, we have

$$\eta \underline{J}_s \sec\omega = \begin{bmatrix} J_x \\ J_y \\ f_x J_x + f_y J_y \end{bmatrix} ;$$

and J_x and J_y are given by equation (3.24). Expressing the amplitude vector of the incident field $\underline{A}$ explicitly for H and V polarizations, but keeping the symbols S_{ij} for the elements of $\underline{\underline{S}}$, equation (5.1) yields for the four polarization combinations:

$$E^{HH} = C \iint [(\gamma + \alpha f_x)(1 - S_{11}) - \alpha f_y S_{21}] e^{-i2k(\alpha x - \gamma f)} \, dx \, dy \tag{5.2a}$$

$$E^{HV} = -C \iint [(\gamma + \alpha f_x)^2 S_{12} - 2\alpha f_y (\gamma + \alpha f_x) S_{11} - (\alpha f_y)^2 S_{21}] e^{-i2k(\alpha x - \gamma f)} \, dx \, dy \tag{5.2b}$$

$$E^{VH} = C \iint S_{21} \, e^{-i2k(\alpha x - \gamma f)} \, dx \, dy \tag{5.2c}$$

$$E^{VV} = -C \iint [(\gamma + \alpha f_x)(1 + S_{11}) + \alpha f_y S_{21}] e^{-i2k(\alpha x - \gamma f)} \, dx \, dy \tag{5.2d}$$

The first H or V stands for a horizontally or vertically polarized incident wave; the second H or V stands for the horizontal or vertical component of the backscattered wave. In the above, we have used the fact that $S_{22} = -S_{11}$; we have let C stand for $-ik \exp(-ikR)/4\pi R$.

The scattered power is proportional to $|E|^2$. The usual way to calculate $|E|^2$ is to form the two-fold integral from $|E|^2 = E E^*$. The integrals (5.2) are of the form

$$E = C \iint (K + P)\, e^{-i2k(\alpha x - \gamma f)}\, dx\, dy \tag{5.3}$$

where K is the Kirchoff term, equal to $\gamma + \alpha f_x$ for co-polarization (HH, VV) and equal to zero for cross-polarization (HV, VH). P is the perturbation part. The forms (5.3) yield for the co-polarized EE^*:

$$EE^* = CC^* \iiiint (KK' + K'P^* + KP')\, e^{-i2k[\alpha(x'-x) - \gamma(f'-f)]}\, dx\, dy\, dx'\, dy' \ ; \tag{5.4a}$$

and for the cross-polarized EE^*:

$$EE^* = CC^* \iiiint P'P^* e^{-i2k[\alpha(x'-x) - \gamma(f'-f)]}\, dx\, dy\, dx'\, dy' \tag{5.4b}$$

In the co-polarized returns (5.4a), the products $P'P^*$ are discarded since they are not significantly different in magnitude from the errors in the interaction or cross-product terms. In the cross-polarized returns (5.4b), the cross-product terms are identically zero, so that the $P'P^*$ terms are significant.

We consider a rough surface $z = f(x, y)$ to be a realization of a stationary (homogeneous) random process. The return powers are then random variables. For an illuminated area A_o large compared to the scale of roughness, the variability in the power return is small, the return being nearly a deterministic quantity. Thus, if A_o is large, any realization can be inserted into the equations (5.4) and an average power calculated. But to proceed this way is to ignore available information on the statistics of the process. Also, generality is lost by having to make A_o large. We proceed rather by taking the mathematical expectation, denoted by the corner brackets $\langle \cdots \rangle$. Since expectation and integral operations are commutative, the average power return in (5.4a) can be written as

$$\langle |E|^2 \rangle = CC^* \iiiint \langle (K'K + K'P^* + KP') \cdot$$

$$\cdot e^{i2k\gamma(f'-f)} \rangle e^{-i2k\alpha(x'-x)} \, dx\, dy\, dx'\, dy' \quad ,$$

and similarly for (5.4b).

An immediate consequence of stationarity is that expectations of the type

$$\langle (K'P^* + KP') e^{i2k\gamma(f'-f)} \rangle$$

can be expressed as

$$\Phi(\underline{\xi}) + \Phi^*(-\underline{\xi})$$

where

$$\Phi = \langle K'P^* e^{i2k(f'-f)} \rangle$$

and

$$\underline{\xi} = \underline{x}' - \underline{x} \quad .$$

The expectations are computed on the assumption that f is a stationary Gaussian random process of zero mean, $\langle f \rangle = 0$. Define the twelve dimensional random vector $\underline{Y}$ whose first six elements are f, f_x, f_y, f_{xx}, f_{xy}, f_{yy} and whose second six elements are f', f_x', etc. The mean of any derivative of a stationary process is zero; hence

$$\langle \underline{Y} \rangle = \underline{0} \, . \tag{5.5}$$

Since the mean of the vector is zero the covariance matrix $\underline{\underline{\Lambda}}$ can be written as

$$\underline{\underline{\Lambda}}(\underline{\xi}) = (\lambda_{ij}) = \langle Y_i Y_j \rangle \tag{5.6}$$

The multivariate Gaussian distribution with the covariance $\underline{\underline{\Lambda}}$ has the probability density function,

$$p(\underline{y}) = \frac{1}{(2\pi)^6 (\det \Lambda)^{\frac{1}{2}}} \exp\left\{ -\frac{1}{2} \underline{y}^T \Lambda^{-1} \underline{y} \right\} \tag{5.7}$$

where $\underline{y}^T$ is the transpose of $\underline{y}$ and $\underline{\underline{\Lambda}}^{-1}$ is the inverse of $\underline{\underline{\Lambda}}$. Define the characteristic function of $\underline{Y}$,

$$\phi(t) = \langle e^{i\underline{t}\cdot\underline{Y}} \rangle \quad ; \tag{5.8a}$$

$$\phi(\underline{t}) = \int_{-\infty}^{\infty} dy_1 \int_{-\infty}^{\infty} dy_2 \cdots \int_{-\infty}^{\infty} dy_{12}\, e^{i\underline{t}\cdot\underline{y}}\; p(\underline{y}) \ . \tag{5.8b}$$

If $p(\underline{y})$ is the multivariate normal distribution (5.7), then ϕ has the form [11]:

$$\phi(\underline{t}) = \exp\left\{ -\frac{1}{2} \underline{t}^T \underline{\underline{\Lambda}}\, \underline{t} \right\} .$$

Or, in terms of elements t_i,

$$\phi(t_1, t_2, \cdots, t_{12}) = \exp\left\{ -\frac{1}{2} \Sigma \lambda_{ij} t_i t_j \right\} \tag{5.9}$$

Now, the required expectations could be generated in a simple manner from the characteristic function if only the slope-dependent coefficients in $\underline{\underline{S}}$ were expressible as polynomials in f_x and f_y. We expand the (three) slope-coefficients in (3.21) in a Taylor series about $f_x = f_y = 0$. (Expansion about the rms values λ_{22}, λ_{33} might be more sensible, but it is a good bit more difficult.) We can truncate at first, second, or third order in slope. With the multipliers of the phasor $\exp\{i2k\gamma(f' - f)\}$ expressed as polynomials in the Y-elements we can compute term by term the expectations of the forms:

$$\langle e^{i2k\gamma(Y_7 - Y_1)} \rangle$$
$$\langle Y_p e^{i2k\gamma(Y_7 - Y_1)} \rangle \qquad (5.10)$$
$$\langle Y_p Y_q e^{i2k\gamma(Y_7 - Y_1)} \rangle$$
$$\vdots$$

These averages are computed from the characteristic function in the manner outlined: Define the twelve dimensional vector $\underline{t}^*$ (star does not mean complex conjugate) all of whose elements are zero except for t_1^* and t_7^* which have the values,

$$t_1^* = -2k\gamma$$
$$t_7^* = +2k\gamma \qquad (5.11)$$

The expectations (5.10) can then be written as

$$\langle e^{i\underline{t}^* \cdot \underline{Y}} \rangle$$
$$\langle Y_p e^{i\underline{t}^* \cdot \underline{Y}} \rangle$$
$$\langle Y_p Y_q e^{i\underline{t}^* \cdot \underline{Y}} \rangle$$
$$\vdots$$

From the definition of the characteristic function (5.8), we find

$$\langle e^{i\underline{t}^* \cdot \underline{Y}} \rangle = \phi(\underline{t}^*)$$

$$\langle Y_p e^{i\underline{t}^* \cdot \underline{Y}} \rangle = i^{-1} \frac{\partial \phi}{\partial t_p}\bigg|_{\underline{t} = \underline{t}^*}$$

$$\langle Y_p Y_q e^{i\underline{t}^* \cdot \underline{Y}} \rangle = i^{-2} \frac{\partial^2 \phi}{\partial t_p \partial t_q}\bigg|_{\underline{t} = \underline{t}^*} \tag{5.12}$$

$$\vdots$$

From (5.9) and the definition of $\underline{t}^*$ (5.11) we find

$$\phi(\underline{t}^*) = \chi = \exp\{-4k^2\gamma^2(\lambda_{11} - \lambda_{17})\}$$

$$\frac{\partial \phi}{\partial t_p}\bigg|_{\underline{t}^*} = 2k\gamma\chi(\lambda_{p1} - \lambda_{p7})$$

$$\frac{\partial^2 \phi}{\partial t_p \partial t_q}\bigg|_{\underline{t}^*} = \chi[4k^2\gamma^2(-\lambda_{q1} + \lambda_{q7})(-\lambda_{p1} + \lambda_{p7}) - \lambda_{pq}]$$

$$\vdots$$

In the manner outlined, the required expectations can be calculated. The last step is to find the covariances λ_{ij} as a function of the lag $\underline{\xi} = \underline{x}' - \underline{x}$. All 72 covariances can be expressed as partial derivatives of the <u>covariance function,</u>

$$R(\underline{\xi}) = \langle ff' \rangle .$$

In accordance with our 'smoothness' condition, $R(\underline{\xi})$ possesses continuous partial derivatives of all orders.

If the illuminated area A_o is large compared to the scales of roughness in the x and y directions ('correlation lengths') , then the scattering integrals of the form

$$\left(\frac{k}{2\pi R}\right)^2 \iiiint C(\underline{\xi})\, e^{-i2k\alpha(x'-x)}\, dx\, dy\, dx'\, dy' \ ,$$

are nearly equal to

$$\left(\frac{k}{2\pi R}\right)^2 A_o \int_{-\infty}^{\infty} \int_{-Y}^{Y} C(\underline{\xi})\, e^{-i2k\alpha\xi}\, d\xi\, d\eta \ .$$

$C(\underline{\xi})$ stands for the expectation $\langle \{\cdots\} e^{i2k\gamma(f'-f)} \rangle$ and Y is a large distance in the η direction. Because of the behavior of the exponential χ †, Y can go to infinity only in the sense of the limit

$$\lim_{Y\to\infty} \int_{-\infty}^{\infty} \int_{-Y}^{Y} C(\underline{\xi})\, e^{-i2k\alpha\xi}\, d\xi\, d\eta \ ;$$

and it is in this limiting sense that the infinite limits of integration in the final formulas have been applied.

The power returns are usually given in terms of the normalized isotropic radar backscatter cross sections, $\sigma°$, defined by

$$\sigma° = \frac{4\pi R^2}{A_o} \langle |E|^2 \rangle \ .$$

---

† χ has the asymptotic value $\exp[-4k^2\gamma^2 R(0,0)]$. In the asymptotic state, the integral over η increases monotonically as $\gamma^2\chi(0,0)\cdot\eta$. Except in the case of vertical incidence, $\theta = 0$, the phasor $e^{-i2k\alpha\xi}$ nullifies the constant contribution from large η. At $\theta = 0$ we have a Dirac spike.

$\langle |E|^2 \rangle$ is the quantity we have calculated, namely, the ratio of $\langle |E|^2 \rangle$ at the receiver to $|E|^2$ incident. Here, we do not consider realistic antenna gain patterns. The incident field is taken to be of constant amplitude over the area A_o .

On the following page, formulas for σ° for the four polarization combinations are presented. The perturbation integrals have been calculated only to first order in slope. Calculation of higher-order slope terms is straightforward but tedious.

Note that the VV perturbation is the negative of the HH perturbation. The Kirchoff integral for HH and VV is the same integral arrived at in the scalar Kirchoff theory [12]. To first-order in slope, cross-polarized returns are equal.

$$\sigma^\circ_{HH} = K + P$$

$$\sigma^\circ_{VV} = K - P$$

$$K = \frac{2k^2}{\pi} \int_0^\infty d\xi \int_{-\infty}^\infty d\eta \left\{ [\gamma^2 - a^2(R_{\xi\xi} + 4k^2 R_\xi^2)] \cos 2ka\xi - 4k\gamma^2 a R_\xi \sin 2ka\xi \right\} e^{4k^2\gamma^2 B}$$

$$P = \frac{2k^2}{\pi} \int_0^\infty d\xi \int_{-\infty}^\infty d\eta \left\{ 2(B_{\xi\xi} - \gamma^2 B_{\eta\eta}) \cos 2ka\xi + [2ka(2R_\xi B_{\xi\xi} - 4\gamma^2 R_\eta B_{\xi\eta} + 2\gamma^2 R_\xi B_{\eta\eta}) - k^{-1}\gamma^{-2} a R_{\xi\xi\xi} + k^{-1} a R_{\xi\eta\eta}] \sin 2ka\xi \right\} e^{4k^2\gamma^2 B}$$

$$\sigma^\circ_{HV} = \sigma^\circ_{VH} = \frac{2k^2}{\pi} \int_0^\infty d\xi \int_{-\infty}^\infty d\eta \left\{ (4B^2_{\xi\eta} - (k\gamma)^{-2} R_{\xi\xi\eta\eta}) \cos 2ka\xi + [16ka B_{\xi\eta}(R_\xi B_{\xi\eta} + R_\eta B_{\xi\xi}) + 4k^{-1}\gamma^{-2} a (B_{\xi\xi} R_{\xi\eta\eta} + B_{\xi\eta} R_{\xi\xi\eta} + R_\eta B_{\xi\xi\xi\eta} - \frac{1}{2} R_\xi R_{\xi\xi\eta\eta})] \sin 2ka\xi \right\} e^{4k^2\gamma^2 B}$$

where $B = -R(0,0) + R(\xi, \eta)$

and $B_{\xi\xi} = -R_{\xi\xi}(0,0) + R_{\xi\xi}(\xi, \eta)$, etc.

VI. A Note on the Infinite Conductivity Assumption

In the beginning of Section II, it was suggested that there was a way to correct for the perfect conductivity approximation. Reasoning from a physical sense, we know that the Kirchoff formulation means that the incident wave is reflected according to the geometrical optics Law of Reflection. Thus, the zeroth-order backscattered field is composed of waves which have been reflected at locally vertical incidence. The 'backscatterers' are those area elements (facets) which happen to be oriented perpendicular to the incident ray. This has been pointed out by Barrick [13]. As the Law of Reflection is true for finite conductivity as well as for infinite conductivity, the only effect finite conductivity has is to reduce the amplitude and shift the phase of the back-reflected waves (other than the 180° prescribed by infinite conductivity). Since the reflection coefficient--the Fresnel reflection coefficient at vertical incidence, $\Re(0)$--is the same for every back-reflected wave, the net effect on the sum of the waves is to multiply the field strength by the factor $\Re(0)$. And this is so regardless of polarization, since at vertical incidence the distinction between H and V polarization disappears. Thus, the effect of finite conductivity on the zeroth-order (Kirchoff) power return is simply accounted for by multiplying σ° by $|\Re(0)|^2$.

A recent paper by Kaufman [14] lends support to the point taken here. Using a vector Kirchoff formulation for an arbitrary dielectric constant, Kaufman calculated the scattered power patterns for different polarizations. His $\sigma^\circ_{HH}(\theta)$ curve shows that the only effect of finite conductivity is to lower the db power by a constant amount equal to $1/|\Re(0)|^2$ in decibels. The slight departure from a constant difference can be attributed entirely to errors in the approximations made.

Away from the vertical ($|\theta| > 30°$, say) where the Kirchoff contribution to the backscattered power becomes vanishingly small, our argument does not apply. For larger angles of incidence, HH and HV perturbation integrals without the $|\Re(0)|^2$ correction may be good for sea water. The vertical-transmit perturbation integrals, however, may become increasingly unreliable as the Brewster angle is approached.

VII. Concluding Remarks

Unlike the zeroth-order field which is an interference field set up by specular reflectors, the first-order field we have calculated is a true diffraction field. The wave normals are actually bent in the vicinity of the surface. As shown in section IV, Rice's results become identical to ours in the high frequency limit. Thus, this theory provides a connection between the Kirchoff and Rayleigh-Rice theories.

The power of this theory is that it takes into account the correlation between surface height and surface curvature without requiring the height to be small compared to the radar wavelength. This is a distinct advantage of the physical optics approach over the Rayleigh-Rice approach.

As with Kirchoff theory, this theory is most suitable for small angles of incidence. However, we can expect this theory to predict returns from angles maybe twice as large. In the case of radar sea-return, Kirchoff theory gives reasonable predictions for copolarized returns up to 30°. This theory may provide good predictions for copolarized and cross-polarized returns for angles of incidence as large as 60°.

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