DAMPING OF HIGH FREQUENCY WAVES IN THE SOLAR WIND

M. L. GOLDSTEIN
L. A. FISK

JUNE 1973

GODDARD SPACE FLIGHT CENTER
GREENBELT, MARYLAND
DAMPING OF HIGH FREQUENCY WAVES IN THE SOLAR WIND

M. L. Goldstein*
Laboratory for Extraterrestrial Physics

and

L. A. Fisk
Laboratory for High Energy Astrophysics

June 1973

* NAS-NRC Postdoctoral Resident Research Associate

Goddard Space Flight Center
Greenbelt, Maryland
DAMPING OF HIGH FREQUENCY WAVES IN THE SOLAR WIND

ABSTRACT

Cyclotron damping by suprathermal fluxes of protons and electrons in the interplanetary medium will greatly attenuate high-frequency Alfvén waves and whistler waves within distances $< 1$ AU of the sun. Electrons with energies between 50 eV to 2 KeV are heated as a result of damping interplanetary whistler waves with frequencies $2 < \omega/2\pi < 30$ Hz in the frame of the solar wind. This heating may account, in part, for the observed suprathermal tail of solar wind electrons. Protons with energies $< 50$ KeV damp Alfvén waves with frequencies $10^{-3} < \omega/2\pi < 10^{-2}$ Hz. This damping mechanism may explain several features of a "scatter-free" solar electron events and high-intensity, anisotropic solar-proton streams.
I. INTRODUCTION

In this report, we consider the consequences of cyclotron (or magnetic-Landau) damping of high-frequency waves in the interplanetary medium. The frequency range of interest is from $10^{-2} \Omega_p \lesssim \omega \lesssim 10^{-1} |\Omega_e|$ in the plasma (solar wind) frame, where $\Omega_p$ and $\Omega_e$ are the gyrofrequencies of protons and electrons, respectively. In the frame of a spacecraft, the frequency range is from about $10^{-2}$ to $10^2$ Hz with a magnetic field, $B$, of $5\gamma$, and a solar wind speed of $V_{sw} = 300$ km/s.

Waves in this frequency range have wavelengths comparable to the Larmor radii of protons with energy $\lesssim 500$ KeV and electrons $\lesssim$ several MeV, and thus will resonantly scatter these low-energy particles. In fact, in current diffusion theories (Jokipii 1966, 1967, 1968) the diffusion coefficient $\kappa_\parallel$, parallel to the mean field is quite small ($\kappa_\parallel \lesssim 10^{20}$ cm$^2$/s for protons, and $\kappa_\parallel \lesssim 10^{21}$ cm$^2$/s for electrons) provided that the spectral index of the power spectrum of magnetic field fluctuations is less than about two. This small value of $\kappa_\parallel$ reflects the efficiency with which high-frequency magnetic field fluctuations can resonantly scatter low-energy particles. Thus a mechanism that attenuates these waves, such as cyclotron damping, will produce observable consequences in the spectra and anisotropies of low-energy particles.

In Section II we outline briefly some of the observed features of low-energy particle behavior that may be the direct result of cyclotron damping. Section III contains a quantitative discussion of the theory, and a calculation of the appropriate damping lengths.
II. Observed Behavior of Low Energy Particles

Listed below are some of the observed features of the behavior of low-energy particles that may be the result of cyclotron damping of high-frequency waves. We provide this list principally to motivate the theory reviewed in Section III.

1) Recent observations have shown that the thermal distribution of solar-wind electrons has an extended suprathermal tail in the energy range 50 eV to \( \sim \) 2 KeV, as can be seen in the composite electron spectrum shown in Figure 1 (Montgomery, et al., 1968; Ogilvie et al., 1971; Anderson, et al., 1972; Lin, et al., 1972; Scudder, et al., 1973). Such a suprathermal tail can result from heating nonthermal electrons through the damping of high frequency-waves in the solar wind. In the next section we argue that sufficient energy is probably available in high-frequency field fluctuations to supply the required energy to 50 eV-2 KeV electrons. One should note that at energies below \( \sim \)100 eV the suprathermal tail may be a consequence of the heat flux of the solar wind. However, it is not well understood why the heat flux is carried by \( \sim \)100 eV electrons, rather than by electrons with energies of only a few eV (Montgomery, 1972).

2) High-intensity proton streams at low-energies (\( \gtrsim 0.1 \) protons/(cm\(^2\) s ster KeV) at \( \sim 500 \) KeV) are observed to be highly anisotropic (cf. Krimigis, et al., 1971). Such large proton fluxes could damp the magnetic irregularities that normally scatter these low-energy particles, and thus permit the large anisotropies.
3) At times, solar electrons in the energy range 50 KeV to 1 MeV are observed to propagate from the sun to earth with little or no scattering (Lin, 1970; Wang, et al., 1971). These "scatter-free" events can be understood if interplanetary conditions change at the time of the events and the waves that normally scatter these electrons are damped.

III. Theory

Cyclotron damping is a resonant interaction in which a particle moving with velocity $V$ interacts with a transverse wave whose Doppler-shifted frequency in the frame moving with velocity $V_{||} = \mathbf{V} \cdot \mathbf{B}/B$ is $\Omega_{\nu}$ ($\nu = p, e$ for protons and electron, respectively). Thorough treatments of cyclotron damping can be found in Stix (1962), Scarf (1962), and Tidman and Jaggi (1962). We outline below only those parts of the theory that pertain to the damping of waves by a nonthermal distribution of particles (Tidman and Jaggi, 1962; Tidman, 1966). We also consider only the case of parallel propagating waves $k_{\parallel} B$. However, waves propagating at angles less than $\cos^{-1} |\Omega_{\nu}|/c$ ($c$ is the speed of light) also experience cyclotron damping (Tidman, 1966). For the waves considered here damping can be important within a fairly large cone of angles $\sim 60^\circ$ about the mean field.

The damping decrement, $\gamma$, of transverse waves propagating parallel to the ambient magnetic field is found by expanding the dispersion relation (Montgomery and Tidman, 1964) in powers of $\gamma/k_{\parallel}$ and equating the real and imaginary parts
to zero. Defining the particle distribution function $f_\nu$, so that $\int f_\nu \, dv = 1$, one finds (Tidman and Jaggi, 1962) that the real part, $\omega$, of the frequency satisfies

$$\omega^2 - c^2 k^2 - \omega \sum_\nu \frac{\pi \omega^2_\nu}{k} \int_{-\infty}^{\infty} dv_\parallel \frac{\Psi(v_\parallel)}{v_\parallel - \left(\frac{\omega \pm \Omega_\nu}{k}\right)} = 0$$

and the imaginary part is given by

$$\gamma = \frac{\pi \omega \sum_\nu \frac{\omega^2_\nu}{k} \left(\frac{\omega \pm \Omega_\nu}{k}\right)}{2\omega - \sum_\nu \frac{\pi \omega^2_\nu}{k} \left[\frac{\Psi(v_\parallel)}{v_\parallel - \left(\frac{\omega \pm \Omega_\nu}{k}\right)} + \frac{\omega_\nu}{k} \int_{-\infty}^{\infty} dv_\parallel \frac{\psi^{(1)}}{v_\parallel - \left(\frac{\omega \pm \Omega_\nu}{k}\right)}\right]}$$

where $\omega^2 = 4\pi N_\nu e^2/M_\nu$, $N$ is the density of the ambient plasma, and $P$ denotes the Cauchy principal-part. Also,

$$\psi^{(n)}(v_\parallel) = \frac{d^n}{dv_\parallel^n} \psi(v_\parallel) = \frac{d^n}{dv_\parallel^n} \left[\int_0^{\infty} v_\parallel^2 dv_\parallel \frac{\partial f_\nu}{\partial v_\parallel} + \chi(v_\parallel)\right]$$

where

$$\chi(v_\parallel) = -\frac{k}{\omega} \int_0^{\infty} v_\parallel^2 dv_\parallel \left(v_\parallel \frac{\partial f_\nu}{\partial v_\parallel} - v_\parallel \frac{\partial f_\nu}{\partial v_\parallel}\right)$$

At particle speeds several times the thermal speed, one can show that thermal effects are unimportant in the numerator of (2). Furthermore, because
the density of the thermal plasma is assumed to be much greater than that of the energetic particles, one can ignore the contribution of the energetic component in (1) and in the denominator of (2). If we define the phase velocity, \( V_{ph} = \omega/k \), (1) becomes

\[
V_{ph} = c \left[ 1 - \frac{\omega_e^2}{\omega (\omega + \Omega_e)} - \frac{\omega_p^2}{\omega (\omega + \Omega_p)} \right]^{-1/2}
\]

In the frequency range \( \Omega_p < \omega < |\Omega_e| \) only \( V_{ph} \) of the extraordinary mode ('+') is less than \( c \) and can be damped by this mechanism. The phase velocity of the ordinary mode ('-') is greater than \( c \) and will not be considered further. For \( \omega < \Omega_p \), both modes can be damped and \( V_{ph} = V_A = B/(4\pi N M_p)^{1/2} \) the Alfvén speed.

The Whistler Mode.

We consider the first the damping of the whistler mode \( (\Omega_p < \omega < |\Omega_e|) \). These waves have wavelengths comparable to the gyroradii of suprathermal solar-wind electrons (50 eV to ~ 2 KeV), and can provide an energy source for heating these particles. We use an isotropic distribution function under the assumption that the particles are scattered and thus isotropized while damping the waves. However, the assumption of isotropy is not essential to our argument because we show below that an anisotropic distribution function enhances the damping of the whistler mode. After the waves are damped electron fluxes are expected to be anisotropic because less high-frequency power will be available to produce isotropy via pitch-angle scattering.
We let \( f_\nu = f_{\nu 1} + f_{\nu 2} \) where 1 and 2 refer to thermal and suprathermal particle distributions, respectively. We have defined \( f_{\nu 2} = M_j j_\nu / (N \nu^2) \) where \( j_\nu (v) = A_\nu (1/2 M_j v^2)^{-\gamma} \) is the differential energy spectrum measured in units of particles/cm\(^2\) s ster KeV. In this frequency range the protons do not contribute to the damping because the velocities involved are a few times the thermal velocity and at quiet times the observed velocity distribution in this range is a good fit to a Maxwellian (Ogilvie, private communication). Consequently,

\[
\gamma \approx \frac{\pi^2 V_{ph} \Omega_e j_e \left( \frac{\Omega_e}{\omega} V_{ph} \right)}{N \Gamma_e} \tag{6}
\]

We define a damping length \( L_{||} = (V_{sw} + V_{ph}) / |\gamma| \). Substituting numbers appropriate to 1 AU (\( B = 5 \times 10^{-5} \) G, \( N = 10 \) cm\(^{-3}\), \( V_{sw} = 3 \times 10^7 \) cm/s, \( A_e = 10^5, \Gamma_e = 3 \)) we find that \( L_{||} \lesssim 3 \times 10^{12} \) cm for electrons between 50 eV and 2 KeV (\( 20 \Omega_p < \omega < |\Omega_e| / 5 \)). Thus, suprathermal electrons near 1 AU will absorb wave energy within a characteristic distance \( \sim 0.2 \) AU.

In order to heat sufficiently the suprathermal electrons via this mechanism one must be able to find a region of interplanetary space between the sun and 1AU in which the energy density in the magnetic field fluctuations is comparable to the energy density in the particles. In the energy range 50 eV - 2 KeV most of the particle energy is found in 50 eV electrons. However, these electrons damp the highest frequency waves, which contain the least magnetic energy.

Thus, if there is a heliocentric distance less than 1 AU at which the energy
density in 50 eV electrons is comparable to that in the waves that they damp, we will have shown that the suprathermal electrons between 50 eV and 3 KeV can be, at least in part, produced by this wave damping mechanism. Of course, an element of conjecture will remain in this discussion because the required high-frequency fluctuations cannot be observed at 1 AU let alone at other heliocentric distances.

We assume that the power spectrum of magnetic fluctuations near the sun can be estimated by extrapolating to smaller heliocentric distances the power spectra that are observed near 1 AU. We preserve the spectral shape in this extrapolation, and therefore we assume that the power spectrum for fluctuations along the mean field direction is approximately given by

\[ P_{zz}(\hbar) = \frac{\langle B'_z^2 \rangle}{\hbar_c^3} \left( \frac{\hbar_c}{\hbar} \right)^{3/2}, \quad \hbar > \hbar_c \]  

(7)

where \( \hbar \) is wave number and \( \hbar_c = 2\pi/\lambda \) with \( \lambda \) the correlation length of the field fluctuations. The z-component of the field strength has a mean square value denoted by \( \langle B'_z^2 \rangle \).

On assuming isotropic fluctuations, the energy density in field fluctuations above \( \hbar_1 \) is given by (Batchelor, 1956)

\[ C_{\hbar_1} = \frac{1}{4\pi} \int_{\hbar_1}^{\infty} \hbar^{-3} \frac{d}{d\hbar} \left( \frac{1}{\hbar} \frac{dP_{zz}}{d\hbar} \right) d\hbar \]  

(8)

We choose \( \hbar_1 \) to be the wave number of fluctuations damped by \(~50\) eV electrons.
We further assume that \( B_z^2 \approx 1/4 B^2 \), \( B = B_0/R^2 \), \( \kappa_c = \kappa_{c_0}/R \), and \( \kappa_i = \kappa_i_0/R^2 \). The subscript "0" denotes quantities evaluated at 1 AU, and \( R \) is heliocentric distance in AU. We take \( B_0 = 5 \) AU, \( \kappa_{c_0} = 3 \times 10^{-11} \) cm\(^{-1} \), and \( \kappa_{i_0} = 10^{-5} \) cm\(^{-1} \). The energy density in magnetic field fluctuations that is available to heat 50 eV electrons is then given by
\[
\mathcal{E}_{\kappa_i} \approx 0.6 R^{-7/2} \text{eV/cm}^3
\] (9)

The energy density in electrons above \( E = 50 \) eV is
\[
\mathcal{E}_E \approx \frac{4\pi j_e(E) E^2}{R^2 \nu (\Gamma_e - 1/2)(\Gamma_e - 3/2)} \approx 20 R^{-2} \text{eV/cm}^3
\] (10)

where we have taken \( \Gamma_e = 3 \), \( j_e (> 50 \text{ eV}) = 10^9 \) (cm\(^2\) s ster keV\(^{-1} \)) and \( \nu = 4 \times 10^8 \) cm s\(^{-1} \). Therefore at \( R \approx 0.09 \) AU \( \mathcal{E}_{\kappa_i} \approx \mathcal{E}_E \), as required.

From Eqns. (5) and (6) it is easy to show that \( L_{||} \approx R^2 \), so that 50 eV electrons damp waves even more effectively at the sun than at 1 AU.

The Alfven Mode

We consider next the damping of high frequency Alfven waves (\( \omega < \Omega_p \)). One can show that electrons alone cannot damp these waves within any reasonable heliocentric distance because the required electrons are of quite high energy (\( > \) several hundred keV) and are few in number. Thus the damping must be done by suprathermal protons above \( \sim 100 \) eV. Summing (2) over polarization, the relevant damping decrement is (cf. Tidman, 1966)
\[
\gamma \approx -2\pi^2 \left( \frac{V_A}{c} \right)^2 \frac{\omega_e^2}{|\Omega_e|} \frac{M_p}{\Omega_p} \left( \frac{\Omega_p}{\omega} V_A \right) j_p \left( \frac{\Omega_p}{\omega} V_A \right).
\] (11)
The quasi-steady interplanetary proton flux is variable and at times can reach intensities \( \gtrsim 10^2 \) protons/cm\(^3\) s ster MeV at 1 MeV. At these times, protons with energies up to \( \sim 1 \) MeV can effectively damp fluctuations and consequently propagate in a highly anisotropic flux, in agreement with the observations of Krimigis, et al. (1971).

The damping of high-frequency Alfvén waves by low energy protons may also affect electrons that resonate with these fluctuations. During most times substantial proton fluxes, \( \sim 0.5 \) protons/cm\(^2\) s ster KeV have been observed at 60 KeV (Lin, private communication). If we assume here that \( j_p = 10^5 E^{-3} \) (protons/cm\(^2\) s ster KeV) consistent with Lin's average observations (as opposed to the quietest periods observed), then Alfvén waves in the frequency range \( 10^{-2} \Omega_p < \omega \lesssim 10^{-1} \Omega_p \) will be damped within 1 AU. This damping could affect the scattering of electrons with energies between \( \sim 100 \) KeV and several MeV.

With normal interplanetary conditions, cyclotron damping cannot attenuate waves that scatter electrons below \( \sim 100 \) KeV. This damping, then, cannot account for electrons observed in scatter-free electron events (Lin, 1970; Wang et al., 1971). However, it can be shown that the damping will be effective to lower energies during periods of low Alfvén speed. For example, if \( V_A \) is lowered by a factor of order two to \( \sim 20 \) km/s, the damping is effective for electrons with energy down to \( \sim 25 \) KeV. Thus, low \( V_A \) together with particularly effective cyclotron damping may be necessary in order that scatter-free events occur.
In the above calculations, we used isotropic distribution functions; however, we can readily include particle anisotropies. Let $F = f_{v,2}(v)(1 + \zeta \cos \alpha)$ be the suprathermal component, where $\zeta \lesssim 1$ is a measure of the anisotropy, and $\alpha$ is the particle pitch angle defined so that $\alpha = 0$ is directed away from the sun. Then one can show that the denominator of (2) becomes

$$-\pi \omega \sum_{\nu} \frac{\omega^2 M_{\nu}}{k N_{\nu}} j_\nu \left( \frac{\omega \pm \Omega_{\nu}}{k} \right) \left[ \frac{1}{\Gamma_{\nu}} + \frac{2 \zeta}{2 \Gamma_{\nu} + 1} - \frac{2 \zeta (\omega \pm \Omega_{\nu})}{(4 \Gamma_{\nu}^2 - 1) k V_{ph}} \right]$$

(12)

For electrons that are damping whistlers,

$$\frac{\omega \pm \Omega_{\nu}}{k} \to \frac{\Omega_{e}}{\omega} V_{ph} < 0$$

so that the anisotropy enhances the damping. However, an anisotropic flux of protons will damp only left-hand circularly polarized Alfvén waves ($\Omega_p/k < 0$), while allowing the righthand mode to grow, provided that

$$\frac{1}{\Gamma_p} + \frac{2 \zeta}{2 \Gamma_p + 1} < \frac{2 \zeta (\Omega_p/\omega)}{(4 \Gamma_p^2 - 1)}$$

(13)

For $\Omega_p/\omega \lesssim 70$ (13) is satisfied when $\zeta \gtrsim 0.1$. However, even in this case, the right-hand mode cannot grow indefinitely. The presence of helium in the solar wind will cause a mode coupling, changing the polarization from the right to the left hand; the latter would then be damped (Smith and Brice, 1964; Scarf and Fredricks, 1968). The problem is highly nonlinear and to our knowledge the equilibrium state has not been calculated. Independent of this mode coupling
mechanism, the right hand mode could only grow until it becomes nonlinear. It would then be damped via a variety of mechanisms (Hollweg, 1971; Lee and Volk, 1973). In fact, this right hand mode together with the undamped, higher-frequency, ion-cyclotron wave can produce at least part of the high frequency power sometimes observed up to several Hz (Holzer, et. al., 1966; Fairfield, private communication).

IV. Conclusions

In this report, we have argued that cyclotron damping of high-frequency waves by suprathermal particles in the solar wind can account for several features of the behavior of low-energy protons and electrons. Electrons in the energy range 50 eV-2 KeV damp whistler waves, and as a result may be heated, contributing to the observed suprathermal tail of the solar wind (Montgomery, et al., 1968; Anderson, et al., 1972; Ogilvie, et al., 1971). Low energy protons damp high-frequency Alfvén waves. In quasi-steady, high-intensity proton streams, the damping attenuates waves that scatter ~1 MeV protons and may account for the large anisotropies that are seen (cf., Krimigis, et. al., 1971). During more average time periods, protons with energies in the frame of the solar wind between 100 eV and 50 KeV attenuate the fluctuations that scatter electrons with energies between ~100 KeV to several MeV. During periods of low Alfvén speed, this damping attenuates waves that scatter electrons with energies down to ~25 KeV and, provided that the damping is particularly effective, may account for some features of scatter-free electron events.
References


Figure 1. A compilation of quiet-time interplanetary electron data (after Lin, et al., 1972)