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A NEW APPROACH TO COSMIC RAY DIFFUSION THEORY

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A NEW APPROACH TO COSMIC RAY DIFFUSION THEORY

We have investigated a new approach to deriving a diffusion equation for charged particles in a static, random magnetic field. Our approach differs from the usual, quasi-linear one, in that we replace particle orbits in the average field by particle orbits in a partially averaged field. In this way the fluctuating component of the field significantly modifies the particle orbits in those cases where the orbits in the average field are unrealistic. This method allows us to calculate a finite value for the pitch angle diffusion coefficient for particles with a pitch angle of 90° rather than the divergent or ambiguous results obtained by quasi-linear theories. Results of this new approach are compared with results of computer simulations using Monte Carlo techniques.

We propose a new scheme for deriving a kinetic equation for the one particle cosmic ray distribution function $\langle f \rangle$ averaged over an ensemble of static, random magnetic fields $\underset{\sim}{B}$ The essence of our new method is that the zeroth order partic orbits partially contain the effects of the fluctuating magnetic component $\delta\underset{\sim}{B}$. The result of our theory is that $\langle f \rangle$ satisfies a diffusion equation in $\mu=\cos\theta$, where θ is the particle pitch angle measured with respect to the average field $\langle\underset{\sim}{B}\rangle$. So long as μ is not too small, the diffusion coefficient $D(\mu)$ is the same as that derived from quasi-linear theories[1-5]. When $\mu \simeq o$ ($\theta \simeq \pi/2$), a regime in which consid erable controversy has existed[5,6], we obtain a D which is finite and markedly different from previous incorrect result The reason for this difference near $\theta \simeq \pi/2$ is that our thec adequately describes the motion of such particles over a coherence time of the fluctuations, while quasi-linear theory follows the motion of particles as if $\langle \underset{\sim}{B} \rangle$ only existed and is hence a very poor approximation to their actual motion. The correctness of our theory is substantiated by comparison with the results of a Monte Carlo analysis.

For computational simplicity we consider the slab model[4] in which $\underset{\sim}{B} = \hat{e}_z \langle B \rangle + \hat{e}_x \, \delta B(z)$, $\langle B \rangle$ being spatially homogeneous and δB depending only on the single spatial variable z. The method can be generalized to more complex geometries.

The theory begins from the continuity equation for F, the cosmic ray distribution function in the phase space whose dimensions are z, μ, speed v, and gyrophase ϕ. It proceeds by a formalism analogous to Weinstock's[7] plasma turbulence theory to a diffusion equation for $\langle f \rangle = (2\pi)^{-1} \int_o^{2\pi} d\phi \langle F \rangle$. The assumptions made in the derivation are a) $\delta F(t=o) \equiv o$, b) $\langle F \rangle$ has only a weak ϕ-dependence and c) $\langle f \rangle$ has a slow phase space evolution so that the usual adiabatic approximation is valid. The diffusion coefficient $D(\mu,t)$ is given by

$$D(\mu,t) = \frac{q^2}{2\pi m^2 c^2} (1-\mu^2)^{1/2} \int_o^{2\pi} d\phi \sin\phi \int_o^t d\tau \langle \delta B(z) U(t,\tau) (1-\mu^2)^{1/2} \sin\phi \, \delta B(z) \rangle \tag{1}$$

The operator U operates on everything to its right. In its exact form[7], U is U_A, a complicated non-linear operator which has no physical interpretation nor algorithm for constructing it.

Quasi-linear theory approximates U_A by U_O, a propagator which propagates particles along helical trajectories in the field $<\underset{\sim}{B}>$. $\theta \simeq \pi/2$ particles thus execute nearly circular orbits and remain in a correlated region of field for arbitrarily long times. Hence the true orbits are incorrectly described to an arbitrarily large degree.

In our theory we approximate U_A by U_p, a propagator which propagates along trajectories in a <u>partially averaged</u> magnetic field. The partial averaging is over a subset of realizations of the full ensemble, the subset being all those realizations which have a given value of $\delta B(z)$ at the field point z. We assume that δB is a Gaussian process so that U_p propagates along particle trajectories in the partially averaged field $B_p(z',z)=\hat{e}_z<B> + \hat{e}_x\delta B(z)C(z-z')$. Here C is the normalized correlation function for the fluctuations.

Addition of effects of the partially averaged field removes $\theta \simeq \pi/2$ particles from the correlation region in a time short enough that deviations from their true trajectories are not catastrophic. Further, our method of partial averaging accounts for the effects of the fluctuations most accurately at the spatial point where they are most important, viz. at z.

We have thus far considered the guiding center limit, where $\delta B(z) r_g / \langle B \rangle z_c \ll 1$, r_g being the particle gyro-radius in B_p and z_c the correlation length of the fluctuations. In this limit the z-motion of the guiding center is that of a particle on a potential hill of height $v^2[\delta B(z)/\langle B \rangle]^2$. So long as the guiding center has any (even infinitesimal) speed at z=z', it moves a distance z_c in a finite time interval of order $\langle B \rangle z_c / \delta B(z) v$ in magnitude. The particle motion in B_p is reasonably described by the approximate form

$$z(\tau) \simeq z + v(\mu^2 + \alpha \delta B^2(z)/\langle B \rangle^2)^{1/2} (t-\tau) \equiv z + v(t-\tau) G(\mu, \delta B) \qquad (2)$$

with α a numerical constant of order unity.

The orbit given by Eq. (2) is used in evaluating Eq. (1). We further assume a) the effect of δB in propagating μ and ϕ can be neglected and b) a spatially homogeneous ensemble. The diffusion coefficient which we obtain

$$D(\mu,t) = q^2 (1-\mu^2)(4\Pi m^2 c^2)^{-1} \langle \delta B^2(z) \int_{-\infty}^{+\infty} dk P(k) \sin[(\langle\omega\rangle + kvG)t] (\langle\omega\rangle + kvG)^{-1} \rangle \qquad (3)$$

is exactly of the form of Jokipii's diffusion coefficient but with G appearing in place of simply μ. (Here P(k) is the power spectrum corresponding to C and $\langle\omega\rangle$ is the gyro-frequency $q\langle B \rangle/mc$.) $D(\mu,t)$ consists of a transient term which decays with the characteristic time $z_c/v\langle G \rangle$ and is thus negligible after at most a few deflection times in the rms fluctuating field plus a resonant time independent term. Because of the presence of G rather than μ in Eq. (3) the resonance is broadened in a fashion similar to that found in the theory of strong plasma turbulence[8].

For an exponential correlation function, the resonant contribution is

$$D(\mu,\infty)=(2\Pi\alpha)^{-1/2}[2\langle B\rangle(\langle\omega^2\rangle z_c{}^2+\mu^2v^2)]^{-1}\mu^2(1-\mu^2)\langle\omega^2\rangle v z_c\langle\delta B^2\rangle^{1/2}$$

$$\exp\left(\frac{\langle B\rangle^2\mu^2}{4\langle\delta B^2\rangle\alpha}\right)\quad K_1\left(\frac{\langle B\rangle^2\mu^2}{4\langle\delta B^2\rangle\alpha}\right) \tag{4}$$

where K_1 is the modified Bessel function. When $\langle B\rangle^2\mu^2/4\langle\delta B^2\rangle\alpha >> 1$, Eq. 4 approaches the quasi-linear result. In the opposite extreme, however, we differ markedly from quasi-linear diffusion theory: $D(0,\infty) = 0$ in quasi-linear theory; we find $D(0,\infty) = (2\alpha)^{1/2}v\langle\delta B^2\rangle^{3/2}\Pi^{-1/2}z_c{}^{-1}\langle B\rangle^{-3}$. Our theory allows free diffusion through 90^o pitch angles.

In the Monte Carlo analysis we integrate on a computer Newton's equations of motion for a single particle in 200 realizations of the random field $\underset{\sim}{B} = \langle B\rangle\hat{e}_z + \delta B(z)\hat{e}_x$. In each realization the particle starts at t=o with random phase ϕ but at the same $z=z_o$ and with the same $\mu=\mu_o$. The statistics of δB are Gaussian with an exponential correlation function. Absorbing boundaries are placed at $\mu_1<o$ and $\mu_2>o$. The probability distribution $\langle\bar{f}\rangle(\mu,t_i) = \int_{-\infty}^{+\infty}dz\langle f\rangle(z,\mu,t_i)$ is examined stroboscopically at regular intervals t_i. We also follow the time development of the time summed function

$$\langle\bar{f}\rangle_s(\mu,t_i) = \sum_{j=o}^{i}\langle\bar{f}\rangle(\mu,t_j).$$

$\langle \bar{f} \rangle$ is identified as the distribution function corresponding to steady (in time) injection at z_o, μ_o. This identification is based on the fact that $\langle \bar{f} \rangle$ corresponds to impulsive injection and thus is the Green's function corresponding to arbitrary injection.

Because of the absorbing boundaries there ultimately results a steady state in which there are constant fluxes j_ℓ and j_r away from μ_o and toward μ_1 and μ_2 respectively. Our computation routine evaluates j_ℓ and j_r as well as $\langle \bar{f} \rangle_s(\mu,\infty)$. Since

$$j = -D(\mu,\infty)\ \partial \langle \bar{f} \rangle_s(\mu,\infty)/\partial\mu \qquad (5)$$

we are thus able to evaluate D from the Monte Carlo method.

The histogram in Figure 1 shows $\langle \bar{f} \rangle_s(\mu,\infty)$ for a simulation with the typical values $\mu_o=0.2, \mu_1=-0.2$, and $\mu_2=0.6$. The statistical fluctuations in $\langle \bar{f} \rangle_s$ result from using a finite ensemble. Note that particles move freely through $\mu=o$. The curves result from integrating Eq. 5 using the known values of j_r and j_ℓ and D as given by our theory, Eq. 4 with $\alpha=1$. The boundary conditions used here are that $\langle \bar{f} \rangle_s=o$ a mean free path in μ outside the absorbing walls[9].

Figure 2 shows plots of $D(\mu,\infty)z_c/v$. The solid curve is again obtained from Eq. 4 with $\alpha=1$. A smaller $\langle \delta B^2 \rangle^{1/2}/\langle B \rangle$ would result in a larger (in μ) region of agreement between our theory and quasi-linear theory. Shown also are values of

$D(\mu,\infty)$ obtained from the computer work and Eq. 5, using the known j's and measured values of the slope $\partial\langle\bar{f}\rangle_s(\mu,\infty)/\partial\mu$. Error diamonds reflect our estimate of the uncertainties in these "experimental" quantities.

The agreement between theory and numerical simulation shown in Figures (1) and (2) clearly illustrates our conclusion. For small μ, quasi-linear theory is grossly inadequate. One must develop a theory which adequately describes the physics of cosmic rays in all μ-regimes. The theory which we propose evidently does so.

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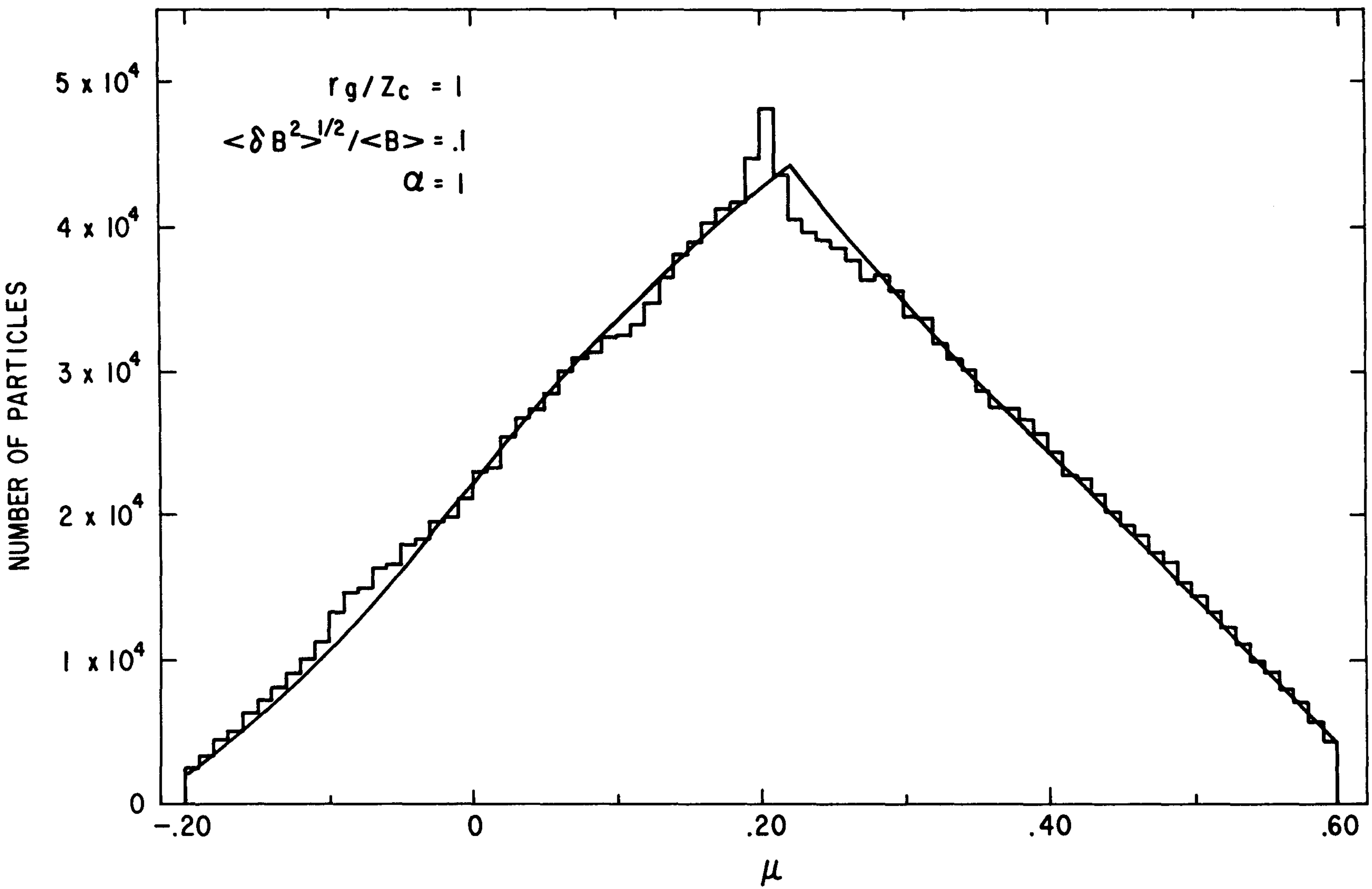
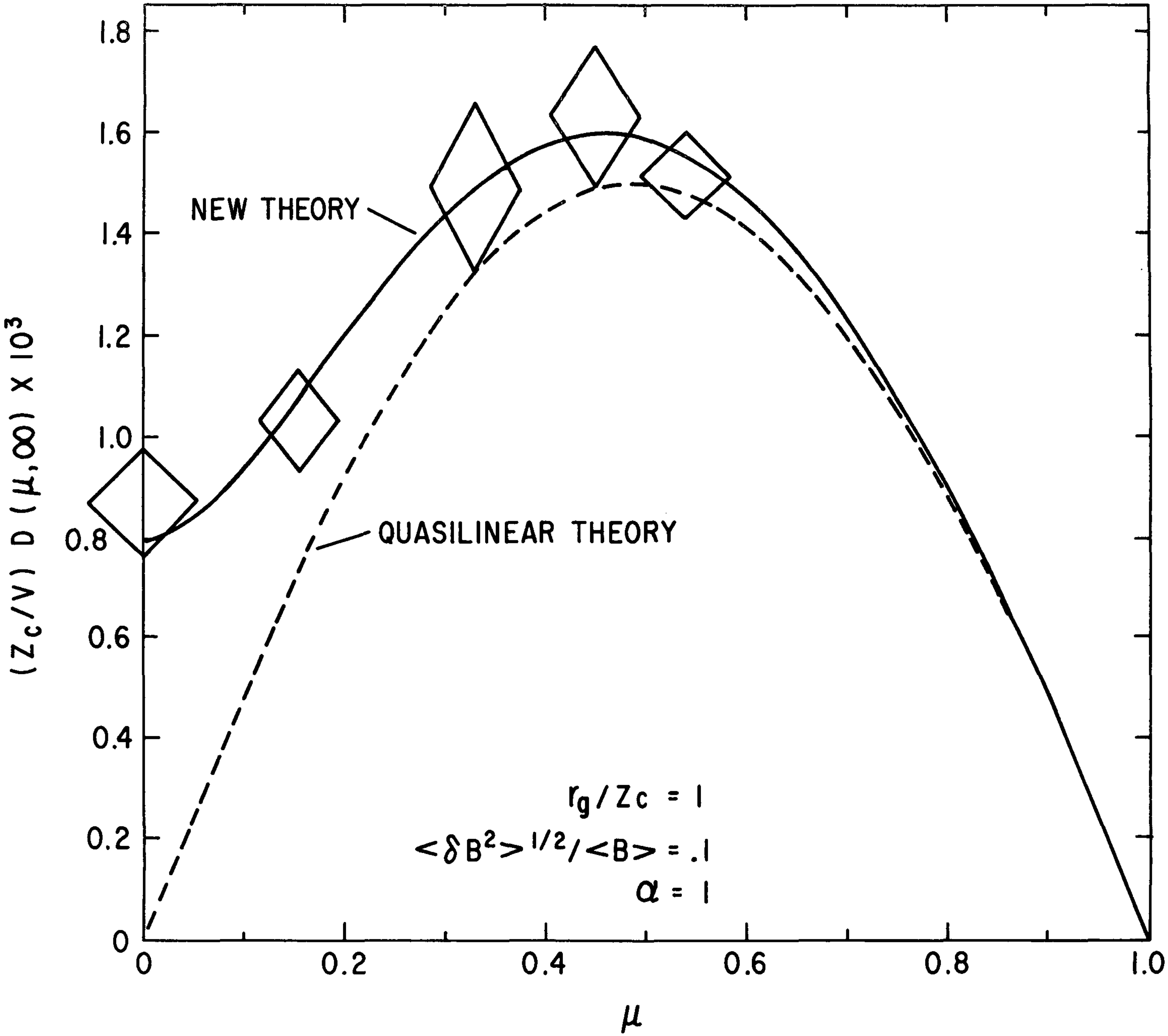
FIGURE CAPTIONS

Figure 1. Histogram of $<\bar{f}>_s(\mu,\infty)$ for $\mu_o=0.2, \mu_1=-0.2$, and $\mu_2=0.6$ produced by computer "experiment". Smooth curves result fror. integrating Eq. 5 with D given by Eq. 4 and j_r and j_ℓ taken from "experiment".

Figure 2. Pitch angle diffusion coefficient $D(\mu,\infty)$ computed from Eq. 4 as compared with that given by quasi-linear theory. "Data" points are obtained from the computer simulation of $<\bar{f}>_s(\mu,\infty)$ shown in Figure 1 by means of Eq. 5.


rg / Zc = 1
<δB²>^1/2 / <B> = .1
α = 1
NUMBER OF PARTICLES
5 x 10^4
4 x 10^4
3 x 10^4
2 x 10^4
1 x 10^4
0
-.20
0
.20
.40
.60
μ



NEW THEORY
QUASILINEAR THEORY
$r_g / Z_c = 1$
$\langle \delta B^2 \rangle^{1/2} / \langle B \rangle = .1$
$\alpha = 1$
$(Z_c/V)\, D(\mu, \infty) \times 10^3$
μ

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