ESTIMATION AND IDENTIFICATION STUDY FOR FLEXIBLE VEHICLES

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BTS-TR-73-3

May 1973

Prepared under Contract No. NAS 1-11652 by
BUSINESS AND TECHNOLOGICAL SYSTEMS, INC.
Seabrook, Maryland

for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

FOREWORD

This report describes the work performed by Business and Technological Systems, Inc. under Contract No. NAS 1-11652 with the NASA/Langley Research Center. The work deals with the development and simulation of estimation and identification algorithms for flexible vehicles.

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SUMMARY

This study deals with the problems of state estimation and system parameter identification for flexible vehicles. These problems arise, for example, for a flexible, highly variable space station, for spacecraft with large flexible appendages, and for high flying aircraft with long wing spans. Knowledge of the bending states and parameters is often required for the effective control of bending or vibrations.

Flexible body single-axis attitude dynamics are modeled by adding linear approximations for the bending modes to the rigid body dynamics. A second order actuator is also included in the model. Attitude and attitude rate measurements are utilized for the estimation of the rigid body state, the bending states and for the identification of the model parameters.

A sequential and a batch estimator are studied in this estimation/identification problem. The sequential estimator tracks the rigid body and bending states very well in the presence of model parameter errors and tracks the time-varying bending frequencies and modal parameters, as well as the moment of inertia, less well. It fails to track time-varying modal damping coefficients due to lack of adequate local observability. The batch estimator is utilized to study the observability and nonlinearities of the system. The combined estimation/identification problem is found to be highly nonlinear. Given enough data and sufficient excitation, the system is found to be completely observable. A sequential version of the batch estimator might be utilized for real-time tracking and identification.

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I. INTRODUCTION

The control of bending or vibrations of a flexible body often requires the real-time estimation of the bending states to provide the necessary feedback in the control loop. In addition, when the flexible body is variable in time, it is also necessary to identify the varying parameters which describe the characteristics of the flexible body, namely the bending frequencies, damping coefficients and modal parameters. Such problems exist, for example, for a flexible, highly variable space station. They also exist for a spacecraft with large flexible appendages, and for aircraft flying at very high altitudes and possessing long wing spans. While the control problem for such flexible vehicles has received some attention (Ref. 10), the estimation and identification problems have not. In fact, the controls proposed in Ref. 10 presuppose the knowledge of the rigid body and bending states and the model parameters. Presumably, these must come from a real-time estimator and identification program if the controls are to be implemented.

The present research deals with the estimation and identification problems cited above. In particular, several estimation and identification techniques have been developed for the problem at hand and tested via computer simulations.

Briefly, the problem simulated is cast in the following mathematical framework. The flexible body single-axis attitude dynamics are modeled by adding linear approximations for the bending modes to the rigid body dynamics. A second order actuator is also included in the model. Attitude and attitude rate measurements at one point on the flexible body are simulated. The objective is to estimate the rigid body state, bending states and model parameters from these attitude and attitude rate signals. It is worth noting here that this combined state and parameter estimation problem is highly nonlinear and of quite large dimension.

The simulation and estimation models utilized in this work are presented in Sections III and IV, respectively. Sections V and VI describe the sequential and batch estimators simulated in the study. Simulation results are outlined in Section VII and conclusions and recommendations follow in Section VIII. Much of the mathematical detail is delegated to the appendices.

It is with pleasure that the authors acknowledge the support and assistance of Mr. Gerald L. Sisson of the NASA/Langley Research Center throughout this work. In addition to all the programming and simulations which he performed, Mr. Sisson lent an able and willing hand in several analytical aspects of this research.

II. SYMBOLS

English

A	$(4+2m_o) \times (4+2m_o)$ system matrix, Eqn (11)
b	$(4+2m_o) \times 1$ control coefficient vector, Eqn (11)
b_a, b_i, b_o	defined in Eqn (2)
C_e	$(4+2m_o) \times (3m_o+1)$ matrix of state and parameter e error correlations
C_f	$(4+2m_o) \times 2$ matrix of state and parameter f error correlations
D	diagonal matrix, Eqn (36)
e	$(3m_o+1) \times 1$ vector of system parameters, Eqn (12)
E	$(3m_o+1) \times (3m_o+1)$ diagonal matrix of errors in $\hat{e}$
E_{ii}	diagonal elements of E
f	2x1 vector of measurement model errors, Eqn (11)
f_1, f_2	estimator measurement model errors, Eqn (4)
F	2x2 diagonal matrix of errors in $\hat{f}$
F_{ii}	diagonal elements of F
$\mathcal{I}$	information matrix, Eqn (32)
I	rigid body moment of inertia; identity matrix
I_o	nominal inertia
J	performance index, Eqn (28)
j	time index (t_j)

k	m_ox1 vector of modal parameters, Eqn (12)
k_i	modal parameters
k_{oi}	nominal modal parameters
K_x	$(4+2m_o)$ x 2 gain matrix for state estimator
K_e	$(3m_o+1)$ x 2 gain matrix for parameter e estimator
K_f	2x2 gain matrix for parameter f estimator
M	2 x $(4+2m_o)$ measurement coefficient matrix, Eqn (11)
m_o	number of modes observed, $m_o \leq m_s$
m_s	number of modes simulated
N	number of (vector) measurements in a batch
n	iteration counter
p	parameter vector, Eqn (30)
p_o, p_{1i}, p_{2i}, p_{3i}	coefficients of parameter variations, Eqn (8)
P	$(4+2m_o)$ x $(4+2m_o)$ state estimation error covariance matrix
P_{ii}	diagonal elements of P
R	measurement noise covariance matrix, Eqn (14)
r_o, r_{1i}, r_{2i}, r_{3i}	coefficients of parameter variations, Eqn (8)
s	step function, Eqn (9)
$t_o(t_f)$	initial time (final time)
t_j	measurement time instant
t_s	step time, Eqn (8)

u	control input, Eqs (5), (6), (7)
v	2x1 measurement noise vector, Eqn (11)
v_1, v_2	measurement noise, Eqn (3)
x	$(4+2m_o) \times 1$ state vector, Eqn (11)
x_1	angle of rigid body
x_2	rate of x_1
x_3	actuator output
x_4	rate of x_3
x_{3+2i}	angle of ith mode
x_{4+2i}	rate of x_{3+2i}
y	2x1 measurement vector, Eqn (11)
y_1	simulated attitude measurement
y_2	simulated attitude rate measurement
Y	2x2 measurement residual covariance matrix

Greek

α	step-size control parameter, Eqn (36)
$\beta_o, \beta_{1i}, \beta_{2i}, \beta_{3i}$	coefficients of parameter variations, Eqn (8)
Γ	$(4+2m_o) \times 1$ vector of partial derivatives, Eqn (A9)
γ_i	elements of Γ
δ_{jk}	Kronecker delta

Δ	correction
ϵ_1, ϵ_2	tolerances, Eqs (37, 38)
λ_i	control gains
ν_o, ν_{1i}, ν_{2i}, ν_{3i}	coefficients in parameter variations, Eqn (18)
ξ	m_ox1 vector of damping coefficients, Eqn (12)
ξ_a	actuator damping coefficient
ξ_i	modal damping coefficients
ξ_{oi}	nominal modal damping coefficients
σ_1 (σ_2)	attitude (attitude rate) noise standard deviation
σ_I	standard deviation of errors in I
σ_{ω_i}	standard deviation of errors in ω_i
σ_{ξ_i}	standard deviation of errors in ξ_i
σ_{k_i}	standard deviation of errors in k_i
σ_{f_i}	standard deviation of errors in f_i
σ_{x_i}	standard deviation of errors in $\hat{x}_i(0\|0)$
τ	time interval, Eqn (A1)
Φ	$(4+2m_o)$ x $(4+2m_o)$ state transition matrix, Eqn (15)
ϕ_{ij}	elements of Φ
Ψ	$(4+2m_o)$ x $(3m_o+1)$ parameter sensitivity matrix, Eqn (15)
ψ_{ij}	elements of Ψ

Ψ^I	$(4+2m_o) \times 1$ sensititity matrix, Eqn (15)
Ψ^ω	$(4+2m_o) \times m_o$ sensitivity matrix, Eqn (15)
Ψ^ξ	$(4+2m_o) \times m_o$ sensitivity matrix, Eqn (15)
Ψ^k	$(4+2m_o) \times m_o$ sensitivity matrix, Eqn (15)
ω	m_ox1 vector of bending frequencies, Eqn (12)
ω_a	actuator frequency
ω_i	bending frequencies
ω_{oi}	nominal bending frequencies

Subscripts, Superscripts and Operators

$(\hat{\ })$	estimated value
$(\bar{\ })$	nominal value
$(\ )^T$	vector (matrix) transpose
$(\dot{\ })$	time derivative
$\mathcal{E}\{\cdot\}$	expectation (averaging) operator
$(j\|k)$	a quantity at time j, based on data up to and including time k
$(\ )^{-1}$	matrix inverse
$(\ )^+$	matrix pseudo-inverse
$\partial(\ )/\partial\eta$	partial derivative with respect to η

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III. MATHEMATICAL SIMULATION MODEL

The flexible body single-axis attitude dynamics are modeled approximately by adding linear approximations for the lowest bending modes to the rigid body dynamics. Actuator dynamics of second order are included in the model, providing an approximation to such actuators as a control moment gyro. If m_s bending modes are simulated, the dynamical equations of motion are

$$\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= b_o x_3 \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= -2\xi_a \omega_a x_4 - b_a x_3 + b_a u \\
\dot{x}_{3+2i} &= x_{4+2i} \\
\dot{x}_{4+2i} &= -2\xi_i \omega_i x_{4+2i} - \omega_i^2 x_{3+2i} + b_i x_3, \quad i = 1, \ldots, m_s
\end{aligned} \tag{1}$$

where u is the control input,

$$b_o = 1/I, \quad b_a = \omega_a^2, \quad b_i = k_i \omega_i^2 , \tag{2}$$

and the other parameters in Eqn (1) are defined in Section II. The states $(x_1 x_2)$ are the rigid body states; $(x_3 x_4)$ are the actuator states; and (x_{3+2i}, x_{4+2i}) are the bending states.

Measured attitude and attitude rates sampled at time instants j are modeled as

$$\begin{aligned}
y_1(j) &= x_1(j) + \sum_{i=1}^{m_o} x_{3+2i}(j) + f_1(j) + v_1(j) \\
y_2(j) &= x_2(j) + \sum_{i=1}^{m_o} x_{4+2i}(j) + f_2(j) + v_2(j)
\end{aligned} \tag{3}$$

where

$$f_1(j) = \sum_{i=m_o+1}^{m_s} x_{3+2i}(j), \quad f_2(j) = \sum_{i=m_o+1}^{m_s} x_{4+2i}(j) \tag{4}$$

and where $\{v_1(j)\}$, $\{v_2(j)\}$ are Gaussian, zero-mean, stationary random sequences with respective standard deviations σ_1, σ_2 and correlation σ_{12}. The latter are obtained from the independent sequences $\{r_1(j)\}$, $\{r_2(j)\}$, with respective standard deviations $\left[\sigma_1^2 - \sigma_{12}^2/\sigma_2^2\right]^{\frac{1}{2}}$, σ_2 by

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} 1 & \sigma_{12}/\sigma_2^2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r_1 \\ r_2 \end{bmatrix} \tag{5}$$

In the above equations, $m_o \leq m_s$ is the number of modes observed or modeled in the estimation process (Section IV). The functions f_1 and f_2 therefore represent modeling errors in the estimation model.

The control input u in Eqn (1) is simulated variously as a specified control $u(t)$, as a linear feedback of the states

$$u = \sum_{i=1}^{4+2m_o} \lambda_i x_i \quad , \tag{6}$$

or as a linear feedback of the estimated states

$$u = \sum_{i=1}^{4+2m_o} \lambda_i \hat{x}_i \quad . \tag{7}$$

Both continuous and sample-and-hold controls are simulated.

Variations in the system parameters are simulated via the equations

$$\begin{aligned} I &= I_o + p_o s(t-t_s) + r_o \sin(\gamma_o(t-t_o) + \beta_o) \\ \omega_i &= \omega_{oi} + p_{1i} s(t-t_s) + r_{1i} \sin(\gamma_{1i}(t-t_o) + \beta_{1i}) \\ k_i &= k_{oi} + p_{2i} s(t-t_s) + r_{2i} \sin(\gamma_{2i}(t-t_o) + \beta_{2i}) \\ \zeta_i &= \zeta_{oi} + p_{3i} s(t-t_s) + r_{3i} \sin(\gamma_{3i}(t-t_o) + \beta_{3i}) \end{aligned} \tag{8}$$

where s(•) is the step function

$$s(t-t_s) = \begin{cases} 0 & t < t_s \\ 1 & t \geq t_s \end{cases} \tag{9}$$

This permits simulation of quite general parameter variations, including steps and ramps.

IV. ESTIMATION MODEL

The estimation model for the system described in Section III contains m_o modes; the functions f_1 and f_2 of Eqn. (3) are treated as unknown random forcing functions when $m_o < m_s$. Then from Eqs (1) and (3), the estimation model in vector form is written as

$$\begin{aligned} \dot{x} &= Ax + bu \\ y(j) &= Mx(j) + f(j) + v(j) \end{aligned} \tag{10}$$

where the matrices (vectors) in Eqn (10) are defined by

$$x^T = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & \cdots & x_{3+2m_o} & x_{4+2m_o} \end{bmatrix}$$

$$A = \left[\begin{array}{cc|cc|c|c|c} 0 & 1 & 0 & 0 & \overset{2x2}{0} & \cdots & \overset{2x2}{0} \\ 0 & 0 & b_o & 0 & & & \\ \hline \multicolumn{2}{c|}{\overset{2x2}{0}} & 0 & 1 & \overset{2x2}{0} & \cdots & \overset{2x2}{0} \\ & & -b_a & -2\xi_a\omega_a & & & \\ \hline \multicolumn{2}{c|}{\vdots} & 0 & 0 & \begin{matrix} 0 & 1 \\ -\omega_1^2 & -2\xi_1\omega_1 \end{matrix} & \ddots & \vdots \\ & & b_1 & 0 & & & \\ \hline \multicolumn{2}{c|}{\vdots} & \multicolumn{2}{c|}{\vdots} & \overset{2x2}{0} & \ddots & \overset{2x2}{0} \\ \hline \multicolumn{2}{c|}{\overset{2x2}{0}} & 0 & 0 & \overset{2x2}{0} & \overset{2x2}{0} & \begin{matrix} 0 & 1 \\ -\omega_{m_o}^2 & -2\xi_{m_o}\omega_{m_o} \end{matrix} \\ & & b_{m_o} & 0 & & & \end{array}\right] \qquad b = \left[\begin{array}{c} 0 \\ 0 \\ \hline 0 \\ b_a \\ \hline 0 \\ 0 \\ \hline \vdots \\ \hline 0 \\ 0 \end{array}\right] \tag{11}$$

$$y^T = \begin{bmatrix} y_1 & y_2 \end{bmatrix}, \quad f^T = \begin{bmatrix} f_1 & f_2 \end{bmatrix}, \quad v^T = \begin{bmatrix} v_1 & v_2 \end{bmatrix}$$

$$M = \left[\begin{array}{c|c|c|c|c} \overset{2x2}{I} & \overset{2x2}{0} & \overset{2x2}{I} & \cdots & \overset{2x2}{I} \end{array}\right]$$

If we further define the vectors of system parameters

$$\omega^T = \left[\omega_1 \cdots \omega_{m_o}\right]$$
$$\xi^T = \left[\xi_1 \cdots \xi_{m_o}\right] \qquad (12)$$
$$k^T = \left[k_1 \cdots k_{m_o}\right]$$
$$e^T = \left[I \;\vdots\; \omega^T \;\vdots\; \xi^T \;\vdots\; k^T\right]$$

then it is seen that $A = A(e)$, a function of the parameters e.

The objective of the estimation (identification) is to estimate the state x and identify the (perhaps time-varying) parameters e and f from the data $\{y(j)\}$. The modeled estimator dynamics are given by the first of Eqs (10), while the measurements, given by the second equation in (10), are generated from the simulated dynamics. The measurement noise v is assumed to have the simulated statistics

$$\mathcal{E}\{v(j)\} = 0 \;, \quad \mathcal{E}\{v(j)v^T(k)\} = R\,\delta_{jk} \qquad (13)$$

where, of course,

$$R = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix} . \qquad (14)$$

The estimation algorithms of Sections V and VI involve certain partial derivatives or sensitivities. In particular, the following matrices of partial derivatives are required.

$$\Phi(j+1,j) = \frac{\partial x(j+1)}{\partial x(j)}$$

$$\Psi(j+1,j) = \frac{\partial x(j+1)}{\partial e(j)} = \left[\frac{\partial x(j+1)}{\partial I(j)} \;\vdots\; \frac{\partial x(j+1)}{\partial \omega(j)} \;\vdots\; \frac{\partial x(j+1)}{\partial \xi(j)} \;\vdots\; \frac{\partial x(j+1)}{\partial k(j)} \right] \qquad (15)$$
$$= \left[\Psi^I \;\vdots\; \Psi^\omega \;\vdots\; \Psi^\xi \;\vdots\; \Psi^k \right]$$

Φ above is the state transition matrix, while Ψ is the parameter sensitivity matrix. Both first order approximations and exact closed form expressions for these partial derivatives are developed in Appendices A and B.

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V. SEQUENTIAL J_o-ADAPTIVE ESTIMATOR

The J_o-Adaptive estimator[2-4] is designed for tracking the state and time-varying parameters of a dynamical system. The time-varying parameters are modeled simply as constants or low degree time polynomials between discrete measurements at time j and $j+1$. However the coefficients in these polynomial models are assumed to be uncertain and are updated by the measurements, while the uncertainties in these coefficients are not allowed to change (decrease) due to the measurement update. As a result, the coefficients in the polynomials are always allowed to change to fit time variations in the tracked parameters. The parameter filters or estimators thus in effect have a finite memory length and will track parameter variations if there is sufficient local sensitivity (or observability) to the parameter variations. The filter for the state is an extended Kalman filter[1], suitable for nonlinear problems.

Details of J-Adaptive estimators are given in the references cited above and are not repeated here. The following simple dynamical model is assumed for the parameters in the problem at hand

$$\begin{aligned} e(j+1) &= e(j) \\ f(j+1) &= f(j) \end{aligned} \tag{16}$$

The following covariance (correlation) matrices are required in the estimator

$$\begin{aligned}
E &= \mathcal{E}\left\{\left[e(j) - \hat{e}(j|k)\right]\left[e(j) - \hat{e}(j|k)\right]^T\right\} \\
F &= \mathcal{E}\left\{\left[f(j) - \hat{f}(j|k)\right]\left[f(j) - \hat{f}(j|k)\right]^T\right\} \\
P(j|k) &= \mathcal{E}\left\{\left[x(j) - \hat{x}(j|k)\right]\left[x(j) - \hat{x}(j|k)\right]^T\right\} \\
C_e(j|k) &= \mathcal{E}\left\{\left[x(j) - \hat{x}(j|k)\right]\left[e(j) - \hat{e}(j|k)\right]^T\right\} \\
C_f(j|k) &= \mathcal{E}\left\{\left[x(j) - \hat{x}(j|k)\right]\left[f(j) - \hat{f}(j|k)\right]^T\right\}
\end{aligned} \tag{17}$$

where $\hat{\eta}(j|k)$ is the estimate of η at time j, given all measurements up to and including time k. The first two covariance matrices above are held at the specified values E and F, where E and F are assumed diagonal;

$$E_{11} = \sigma_I^2;\ E_{i+1,\ i+1} = \sigma_{\omega_i}^2,\ i = 1, m_o;\ E_{i+1+m_o,\ i+1+m_o} = \sigma_{\xi_i}^2,\ i = 1, m_o;$$

$$E_{i+1+2m_o,\ i+1+2m_o} = \sigma_{k_i}^2,\ i = 1, m_o;\ F_{ii} = \sigma_{f_i}^2,\ i = 1, 2 \tag{18}$$

where the σ's (standard deviations) above are specified.

The estimator consists of (time) prediction equations which propagate or advance the estimates and covariance matrices from one measurement time to the next; and measurement update equations which modify the estimates and covariance matrices when a measurement is processed. Estimates and covariance matrices carry the arguments $(j|k)$. A prediction is an advance in the first argument, that is

$$(j|j) \xrightarrow{\text{Prediction}} (j+1|j) \tag{19}$$

while a measurement update is an advance in the second argument, that is

$$(j|j-1) \xrightarrow[\text{Update}]{\text{Measurement}} (j|j) \tag{20}$$

The prediction equations are

$$\begin{aligned} \dot{\hat{x}}(t|j) &= A(\hat{e}(j|j))\hat{x}(t|j) + bu\ , \quad t_j < t < t_{j+1}\ , \\ \hat{e}(j+1|j) &= \hat{e}(j|j) \\ \hat{f}(j+1|j) &= \hat{f}(j|j) \end{aligned} \tag{21}$$

$$\begin{aligned} P(j+1|j) &= \Phi(j+1,j)P(j|j)\Phi^T(j+1,j) + \Phi(j+1,j)C_e(j|j)\Psi^T(j+1,j) \\ &\quad + \Psi(j+1,j)C_e^T(j|j)\Phi^T(j+1,j) + \Psi(j+1,j)E\Psi^T(j+1,j) \\ C_e(j+1|j) &= \Phi(j+1,j)C_e(j|j) + \Psi(j+1,j)E \\ C_f(j+1|j) &= \Phi(j+1,j)C_f(j|j) \end{aligned} \tag{22}$$

and the measurement update equations are

$$\hat{x}(j|j) = \hat{x}(j|j-1) + K_x(j)\left[y(j) - M\hat{x}(j|j-1) - \hat{f}(j|j-1)\right]$$
$$\hat{e}(j|j) = \hat{e}(j|j-1) + K_e(j)\left[y(j) - M\hat{x}(j|j-1) - \hat{f}(j|j-1)\right] \quad (23)$$
$$\hat{f}(j|j) = \hat{f}(j|j-1) + K_f(j)\left[y(j) - M\hat{x}(j|j-1) - \hat{f}(j|j-1)\right]$$

$$P(j|j) = P(j|j-1) - K_x(j)\left[MP(j|j-1) + C_f^T(j|j-1)\right]$$
$$C_e(j|j) = C_e(j|j-1) - K_x(j)MC_e(j|j-1) \quad (24)$$
$$C_f(j|j) = C_f(j|j-1) - K_x(j)\left[MC_f(j|j-1) + F\right]$$

where

$$K_x(j) = \left[P(j|j-1)M^T + C_f(j|j-1)\right] Y^{-1}(j)$$
$$K_e(j) = C_e^T(j|j-1)M^T Y^{-1}(j) \quad (25)$$
$$K_f(j) = \left[C_f^T(j|j-1)M^T + F\right] Y^{-1}(j)$$

$$Y(j) = MP(j|j-1)M^T + MC_f(j|j-1) + C_f^T(j|j-1)M^T + F + R \quad (26)$$

In the prediction equations, the matrices Φ and Ψ are evaluated at

$$\left[\frac{\hat{x}(j+1|j) + \hat{x}(j|j)}{2}\ ,\ \hat{e}(j|j)\right]$$

in the first-order approximations, and at

$$\left[\hat{x}(j|j),\ \hat{e}(j|j)\right]$$

in the exact expressions (Appendices A and B).

The initial conditions for the estimator equations are

$\hat{x}(0|0)$ specified

$$\hat{e}^T(0|0) = \left[I_o;\ \omega_{o1},\ldots,\ \omega_{om_o};\ \zeta_{o1},\ldots,\ \zeta_{om_o};\ k_{o1},\ldots,\ k_{om_o} \right]$$

$$\hat{f}(0|0) = 0$$

$$P_{ii}(0|0) = \sigma^2_{x_i},\quad i = 1,\ldots,\ 4+2m_o\ ;\ P_{ij}(0|0) = 0,\ i \neq j \tag{27}$$

$$C_e(0|0) = 0$$

$$C_f(0|0) = 0$$

where the σ_{x_i}'s, the standard deviations of the initial state estimation errors, are specified.

A version of the J_o-Adaptive estimator is the so-called consider mode. The consider mode estimator with the parameters e "considered" is obtained by deleting the update equation of $\hat{e}$; the second of Eqs (23). In this mode, statistics of errors in the parameters e are considered in the estimator for the state in the sense that uncertainties in e are accounted for in the state gain K_x, while e itself is not estimated. Similarly, one obtains the consider mode estimator with the parameters f "considered" by deleting the third of Eqs (23).

VI. BATCH LEAST SQUARES ESTIMATOR

An estimator for the state initial condition $x(0)$ and parameters e from the batch of measurements $\{y(j)\}_1^N$ is obtained from the minimization

$$\min_{x(0),e} J \quad , \quad J = \sum_{j=1}^{N} \left[y(j) - Mx(j)\right]^T R^{-1} \left[y(j) - Mx(j)\right] \tag{28}$$

subject to the constraint

$$\delta x(j) = \Phi(j,0)\, \delta x(0) + \Psi(j,0)\, \delta e \tag{29}$$

The (constant) parameter vector f is not considered in this problem because f cannot be distinguished from the rigid body initial condition; that is, a constant f is not observable. It is assumed that the control $u(t)$ is specified in the time interval covered by the data batch.

Define the vector

$$p^T = \left[x^T(0) \quad e^T\right] \tag{30}$$

Then differentiation of J and substitution of the constraint equation (29) leads to the partial derivatives

$$-\frac{\partial J}{\partial p} = \begin{bmatrix} \sum_{j=1}^{N} \Phi^T(j,0)M^T R^{-1}\left[y(j) - Mx(j)\right] \\ \sum_{j=1}^{N} \Psi^T(j,0)M^T R^{-1}\left[y(j) - Mx(j)\right] \end{bmatrix} \tag{31}$$

$$\mathcal{J}(N) \triangleq \begin{bmatrix} \mathcal{J}_{xx}(N) & \mathcal{J}_{xe}(N) \\ \mathcal{J}^T_{xe}(N) & \mathcal{J}_{ee}(N) \end{bmatrix} = \frac{\partial^2 J}{\partial p^2} \tag{32}$$

where

$$\mathcal{J}_{xx}(N) = \sum_{j=1}^{N} \Phi^T(j,0)M^T R^{-1} M \Phi(j,0) \tag{33}$$

$$\mathcal{J}_{xe}(N) = \sum_{j=1}^{N} \Phi^T(j,0)M^T R^{-1} M \Psi(j,0) \tag{34}$$

$$\mathcal{J}_{ee}(N) = \sum_{j=1}^{N} \Psi^T(j,0)M^T R^{-1} M \Psi(j,0) \tag{35}$$

$\mathcal{J}(N)$ above is the information matrix[1] for the data batch in question.

Now suppose that with an assumed value of $\bar{p}$ (of $\bar{x}(0)$ and $\bar{e}$), $\bar{x}(j)$ is generated by integration of the system differential equation (10). Then the gradient vector $\partial J/\partial p$ and the inforamtion matrix $\mathcal{J}(N)$ can be evaluated at $\bar{x}(j)$. The data $\{y(j)\}$ is of course generated via the simulation model of Section III. Then the standard (or almost standard) Newton-Raphson or differential correction or generalized least squares approaches[5,7] lead to the correction Δp to $\bar{p}$ given by

$$\Delta p = -\alpha \left[\mathcal{J}(N) + D\right]^{+} \frac{\partial J}{\partial p} \tag{36}$$

In the above, α is a step-size control parameter $(0 < \alpha < 1)$ and D is a diagonal nonnegative definite matrix (specified by the analyst) which may be used to condition the information matrix if required. The matrix D also effectively controls the step-size of the correction Δp. α and D are used to assure that the correction Δp is downhill; that is, that it results in a decrease in J.

The correction in Eqn (36) defines a batch lease squares iteration process for the parameter vector p. Iteration is terminated when

$$\frac{J^{n-1} - J^n}{J^{n-1}} < \epsilon_1 \tag{37}$$

and

$$\frac{|\Delta p_i^n|}{|p_i^{n-1}|} < \epsilon_2 \tag{38}$$

where n is an iteration counter and ϵ_1, ϵ_2 are specified tolerances.

Some discussion of Eqn (36) is in order. Pseudo-inversion is identical with inversion when $\mathcal{G}(N) + D$ has maximal rank. However, when $\mathcal{G}(N) + D$ is singular, the increment computed via Eqn (36) with $\alpha = 1$ gives the shortest length solution of the normal equation[6,7]

$$\left[\mathcal{G}(N) + D\right] \Delta p = - \frac{\partial J}{\partial p} \quad . \tag{39}$$

Pseudo-inversion attains this minimum norm property by changing Δp only in the subspace defined by the column space of $\left[\mathcal{G}(N) + D\right]$. This has a physical meaning for the estimation problem in terms of the observability of the system which will be described below.

The step size parameter, α, is required in all nonlinear problems because the linear approximation may call for a correction so large that the higher order terms cause the performance index to increase. That is, the minimum point predicted on a linear basis may correspond to a higher value of J when the full nonlinear function is evaluated. To avaid this, some search procedure along the vector

$$- \left[\mathcal{G}(N) + D\right]^+ \frac{\partial J}{\partial p} \tag{40}$$

is commonly used to achieve a satisfactory value for α[8].

Use of the matrix D also has the effect of limiting the step-size; however, the mathematical basis is slightly different. Use of D corresponds to the minimization of

$$J + \Delta p^T D \Delta p \tag{41}$$

which is equivalent to minimizing the estimation error of Δp with a priori variance D^{-1}. Generally, when Eqn (36) is iterated to convergence, the solution will be independent of D, although the number of iteration required to reach convergence depends strongly upon D. As an ad hoc procedure, D can be used to guarantee that $[\vartheta(N) + D]$ is nonsingular, thus avoiding the use of a pseudo-inverse.

The observability of the extended state p may be evaluated by an analysis of the matrix $\vartheta(N)$. If $\vartheta(N)$ is invertible, then all components of p can be determined from the N observations and the uncertainty in the estimate of p is given by its covariance matrix, $\vartheta^{-1}(N)$. If p is not completely observable, then $\vartheta(N)$ does not have maximal rank, indicating that the extended state space in which p lies contains a subspace about which the observations tell nothing. That is, there exists at least one vector, arbitrary multipler of which can be added to the state without affecting the observations (in the linear approximation). In such a case the recommended procedure is to use a pseudo-inverse[9],

$$\Delta p = - \alpha \vartheta^{+}(N) \frac{\partial J}{\partial p} \tag{42}$$

or to precondition the matrix by adding D before inversion. In general the pseudo-inverse procedure is preferred[9], since it guarantees that the state will not be modified in the unobservable subspace.

VII. SIMULATION RESULTS

Sequential J_o-Adaptive Estimator

A number of simulations of the sequential estimator were performed with the first order (approximate) state transition matrix and parameter sensitivity matrix. In all cases these simulations were unsatisfactory and in some cases divergence of the estimator was observed. Apparently the system is significantly nonlinear. Subsequently, all simulations employed the exact expressions for the transition and parameter sensitivity matrices given in Appendices A and B.

Figures 1-4 describe a simulation of a three mode case ($m_s = m_o = 3$). Parameter variations consist of a 10% step in all parameters at time zero. Initial state estimate errors are of the order of the measurement noise and the initial state estimation error covariance matrix (P) is consistent with these errors. The standard deviations of parameter errors in the estimator (matrix E) are 25% of the parameter variations.

Figure 1 shows the tracking (dashed lines) of the inertia and the parameters associated with the first mode. The tracking of the parameters associated with modes two and three are similar. While the tracking of inertia I, frequencies ω_i and modal parameters k_i is reasonably good, tracking of the damping coefficients ζ_i is very poor. Apparently there is insufficient local sensitivity (observability) to track the damping coefficients. The memory length of the J_o-Adaptive estimator is necessarily short so that the estimator can respond to parameter variations. Damping can only be sensed over several periods of oscillation.

Despite the poor tracking of the damping coefficients, it can be seen in Figures 2-4 that the J_o-Adaptive estimator tracks the system states quite well. The gain K_x of the state filter reflects the parameter uncertainties and, if sufficient measurements are available (sufficiently high data rates) so that several measurements are available per cycle of the highest frequency, the filter will track the states.

In view of the simulation results described above, simulations were performed in the "consider" mode described in Section V. Figures 5-7 describe such a simulation for $m_s = m_o = 5$. Parameter variations consist of a 5% step in all parameters at time zero; standard deviations of parameter errors in the estimator (matrix E) are 100% of the parameter variations. The parameters themselves are not estimated but are set at the erroneous values.

It is seen in Figures 5-7 that the estimator tracks the system states despite the parameter errors. Thus if the system parameters can be estimated approximately by some other means, the present "consider" estimator can be used for real-time tracking of the system states.

In summary, simulations indicate that the sequential J_o-Adaptive estimator cannot track variations in damping coefficients due to insufficient local observability of these coefficients. The estimator tracks other system parameters relatively well, although not extremely well. It tracks the states of the system quite well despite existing parameter errors. If approximate parameter estimates (particularly the damping coefficients) could be estimated some other way, the J_o-Adaptive estimator could be used to improve those estimates in real time (with the exception of the damping coefficients), while simultaneously tracking the system states. Or, in the consider mode, the sequential estimator will track the system states despite existing parameter errors.

Batch Estimator

In order to gain a better understanding of the system, a set of batch estimator runs was made to explore the system observability and the effects of system nonlinearities. These runs were made using the exact transition and parameter sensitivity matrices given in Appendices A and B to form the information matrix and gradient given in Section VI. No step size-control was initially utilized in the iterations. Standard conditions for these runs consisted of exciting the system initially with a large, short-time control pulse and running the batch estimator over the subsequent time period during which the control system drove the state to zero. Initial conditions for the estimator usually had zero estimates for the state and

10% errors in the parameters.

Several runs were initially made with one and two bending modes over an observation interval of about one cycle of the fundamental frequency. These runs showed very poor observability. In particular, the rigid body position and moment of inertia were difficult to estimate as evidenced by very poor conditioning of the information matrix and poor results from the estimation process.

Following this, the length of the observation interval was increased to eight seconds, encompassing over six cycles of the fundamental frequency. In this case, satisfactory convergence was obtained for the two-mode run. Let us enlarge upon this somewhat. The final values of the parameter estimates agree with the true values to at least five significant figures, and the state estimates were accurate to better than three significant figures. Observability was at all times very good; the variances were compatible with the observed estimation errors. However, a large number of iteration was required to converge, and convergence was not uniform in that the estimates and the performance index, J, did not change monotonically (see Figure 8). Notice, however, that once the iteration reaches the vicinity of the minimum (in the linear region of the minimum), the performance index, J, decreases very rapidly. The above results demonstrate the strong nonlinearities in the system and suggest that an iteration step-size control (Section VI) needs to be activated.

The three mode case was studied next. Without step-size constraints convergence never took place; the estimates bounced around in a random fashion and both input parameters k_i and damping coefficients ξ_i often assumed negative values. At this point, the step-size parameter α was introduced according to the following plan. After each change in the parameters which resulted in a reduction of J, α was set to one. After each change in the parameters which failed to reduce J, the step-size (α) was halved. When this was done the three mode case converged to values with an accuracy comparable to that which was previously described for the two mode case.

In the four mode case, terminal convergence was found to be extremely slow, with repeated halving of the step-size parameter α. This phenomenon was accompanied by a noticeable loss of significance in inverting the information matrix.

A solution to this problem was attempted by using a pseudo-inversion routine. This led to numerical catastrophe, however, in that inconsistent results were obtained during the computation of the pseudo-inverse. It was finally determined that these errors occurred because of very poor conditioning in the information matrix; an apparently unavoidable condition caused by scaling.

To circumvent the ill-conditioning problem cited above, the matrix D was introduced (Section VI) to increase the values of the diagonal elements of the information matrix. This, however, degraded performance so that the values at convergence were in error by large percentages. Recall that the use of the matrix D in the iteration does not change the location of the minimum of J only if the information matrix is nonsingular.

To eliminate this observability problem, a second control pulse was added at about 5 sec. With this additional system excitation, excellent results were obtained for both four and five modes with only the use of α to control step-size. In fact for five modes, the parameters were estimated to five significant figures even with 20% initial errors. The states were also estimated to five significant figures, except for the rigid body initial position which is exceedingly small and does not significantly affect the overall system trajectory. It should be noted that as more degrees of freedom are added (more modes estimated), there is a slight tendency to fit the noise, as evidenced by a decreasing performance index. This does not appear to materially affect the quality of the fit. Undoubtedly, the reason for the high quality of the estimation is the decoupling produced by the normal mode representation.

In summary, the batch estimator can successfully estimate the system states and simultaneously identify the (constant) parameters of the system, including moment of inertia, bending frequencies, damping coefficients and modal (input) parameters. For the estimation to be successful, however, a sufficiently large data batch must be utilized, spanning at least several cycles of the lowest frequency mode. In addition, the data rates must be sufficiently high and all the bending modes being identified must be excited by the system inputs during the observation interval. The nonlinearity of the problem does require a relatively large number of iterations for convergence.

It should be noted that the present batch estimator is suitable for the identification of constant parameters. Thus in a system where parameters are varying, such variations must be slow relative to the observability requirements described in the previous paragraph.

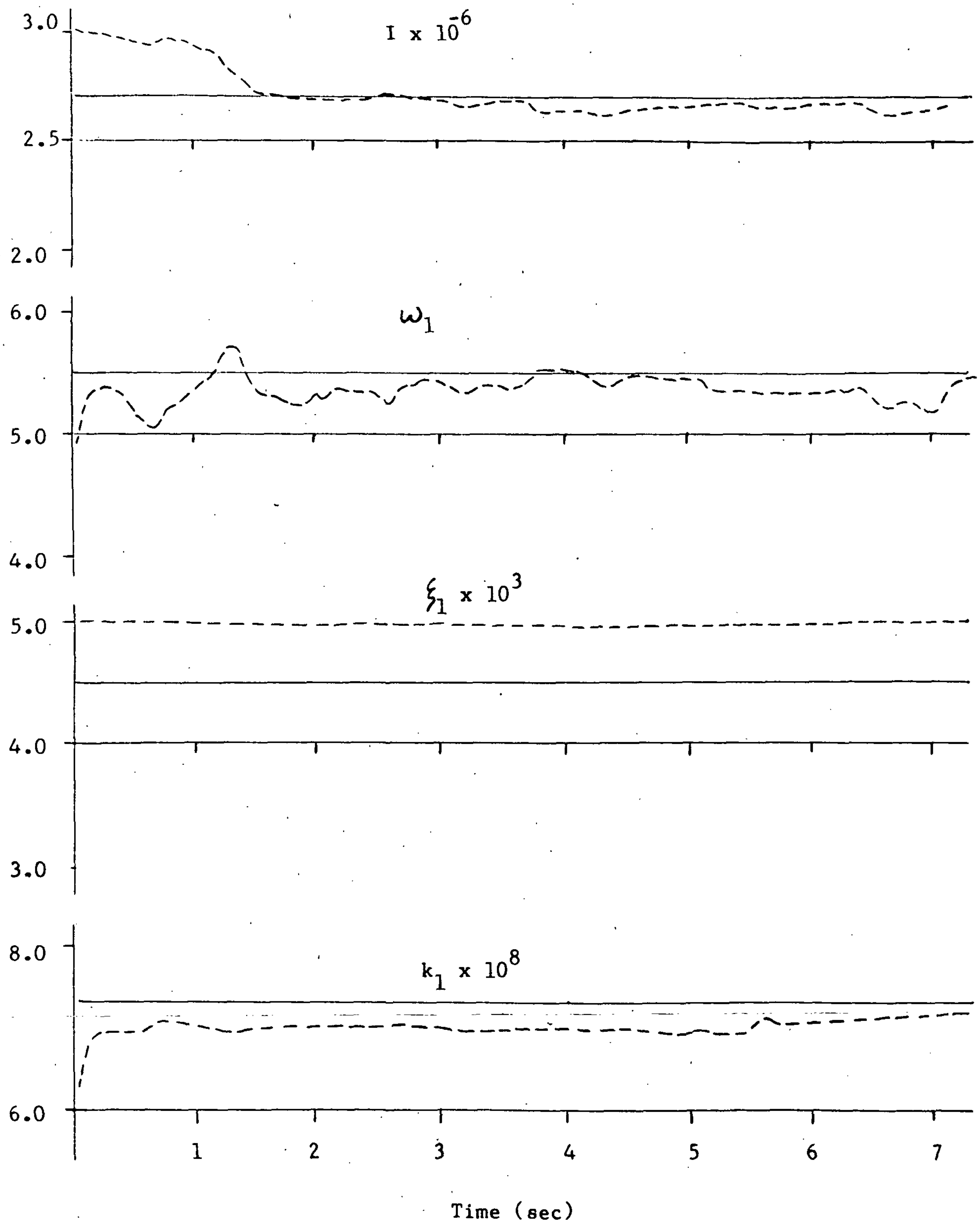

Figure 1. Parameter Estimates (Sim. 1)

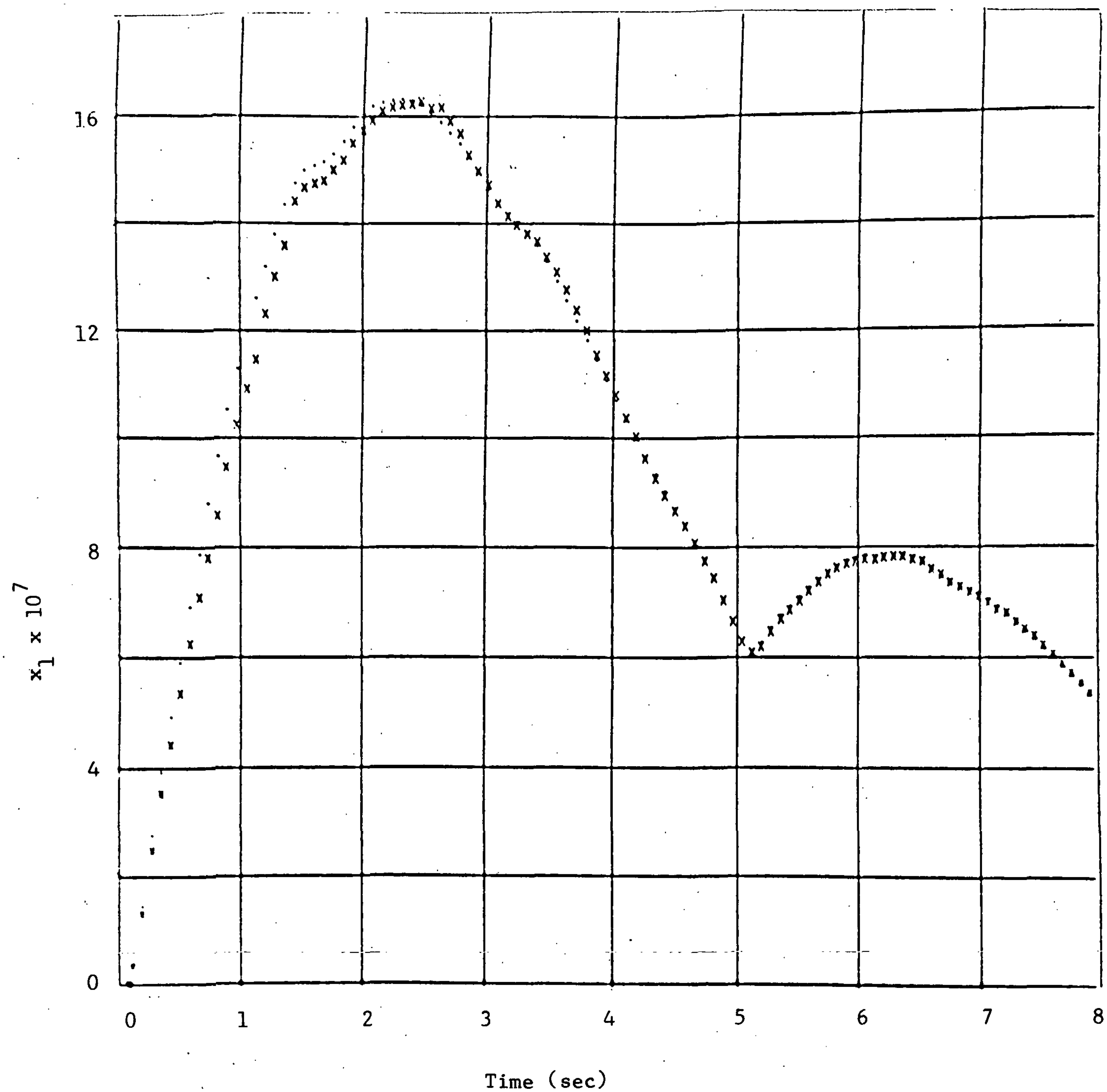

Figure 2. Angle of Rigid Body (Sim.1)

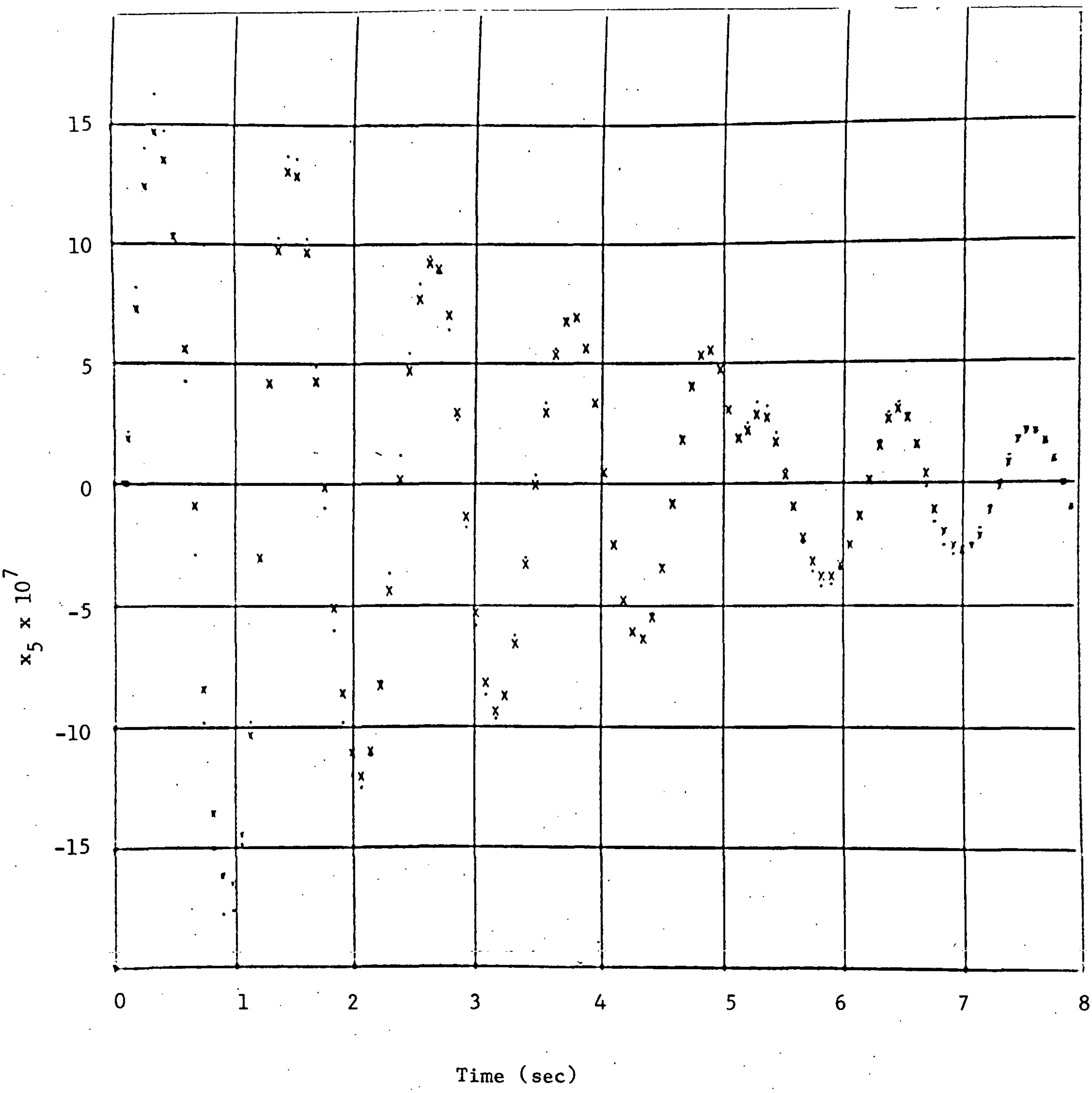

Figure 3. Angle of First Mode (Sim. 1)

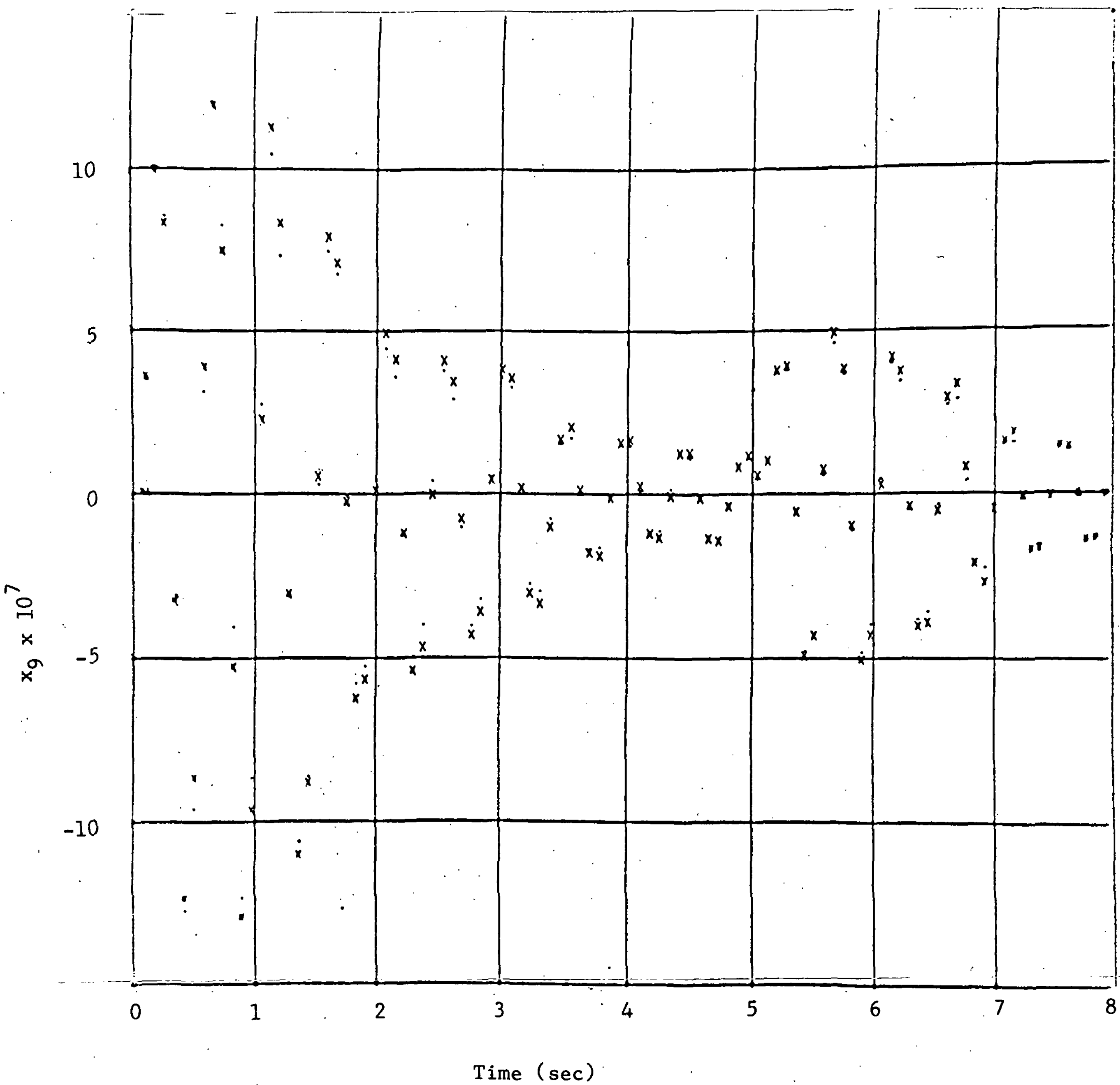

Figure 4. Angle of Third Mode (Sim. 1)

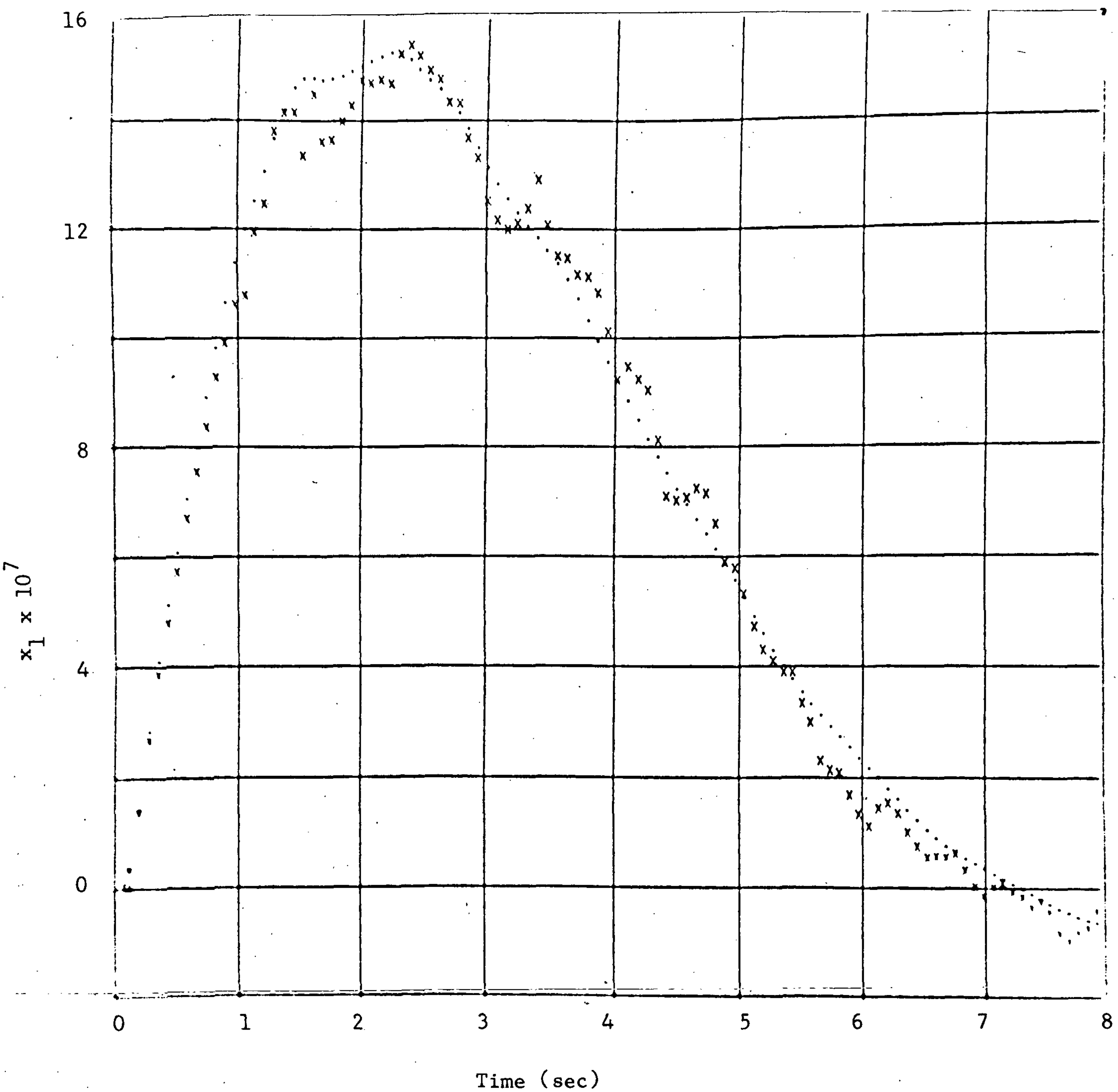

Figure 5. Angle of Rigid Body (Sim. 2)

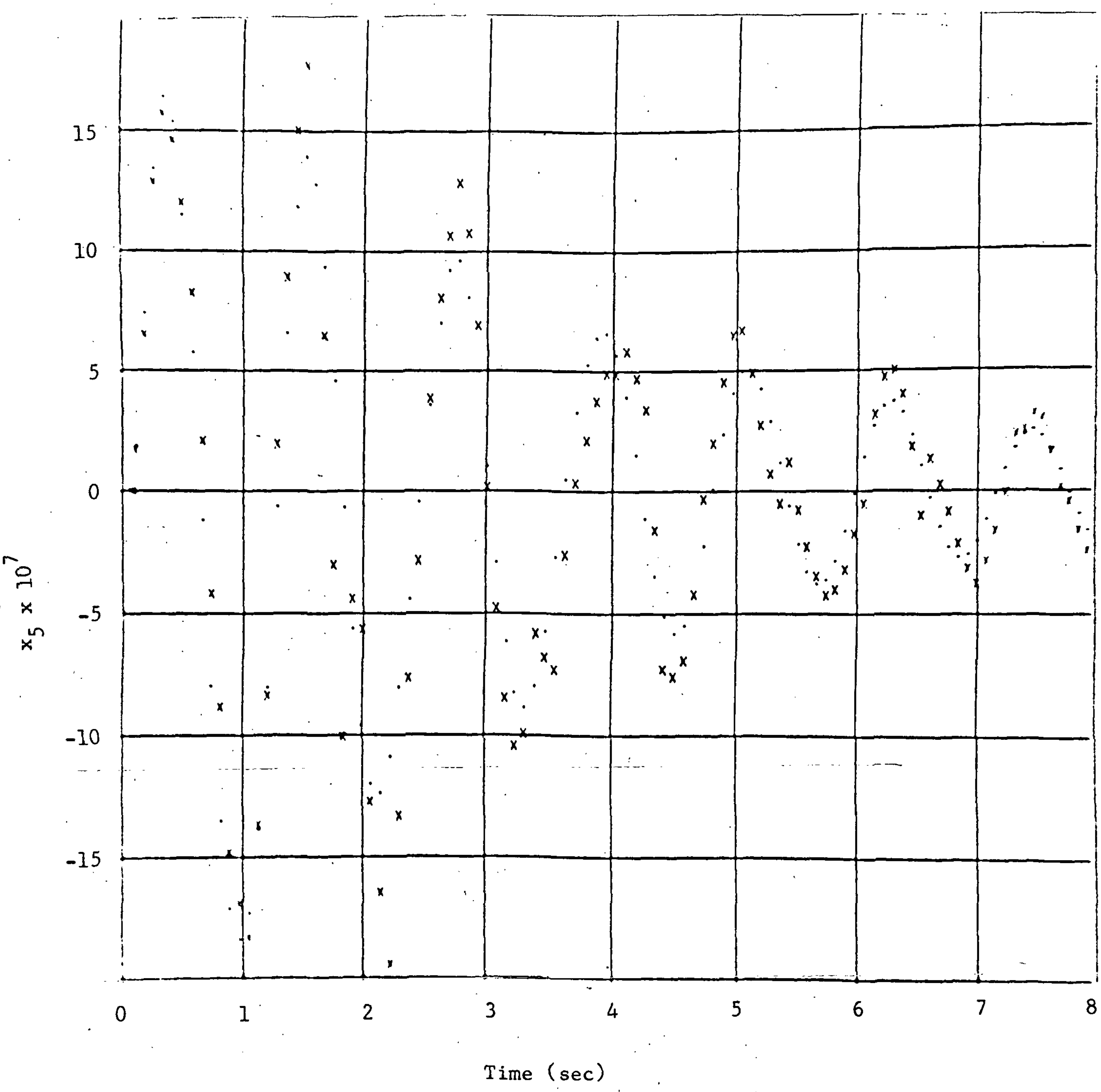

Figure 6. Angle of First Mode (Sim. 2)

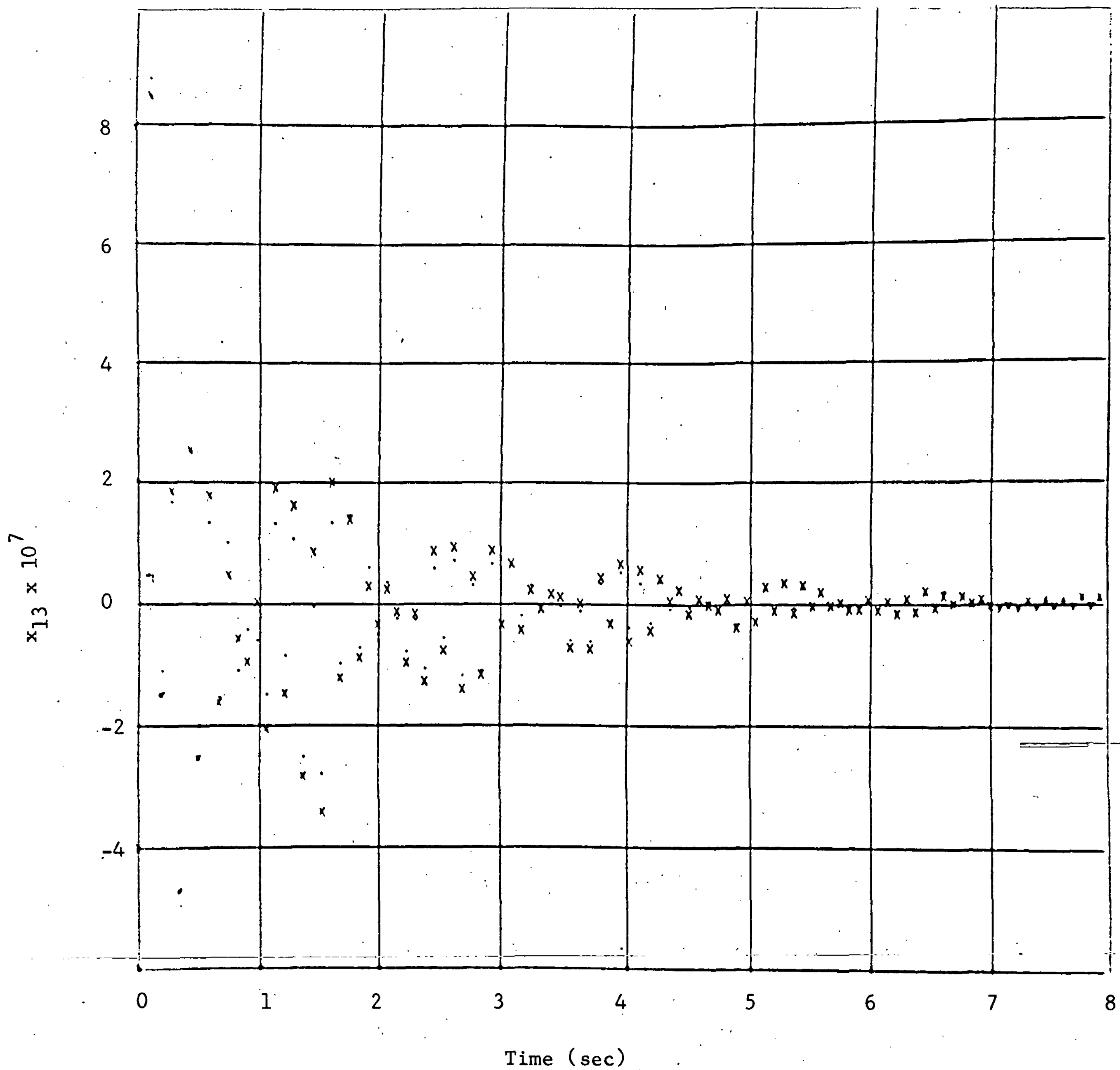

Figure 7. Angle of Fifth Mode (Sim. 2)

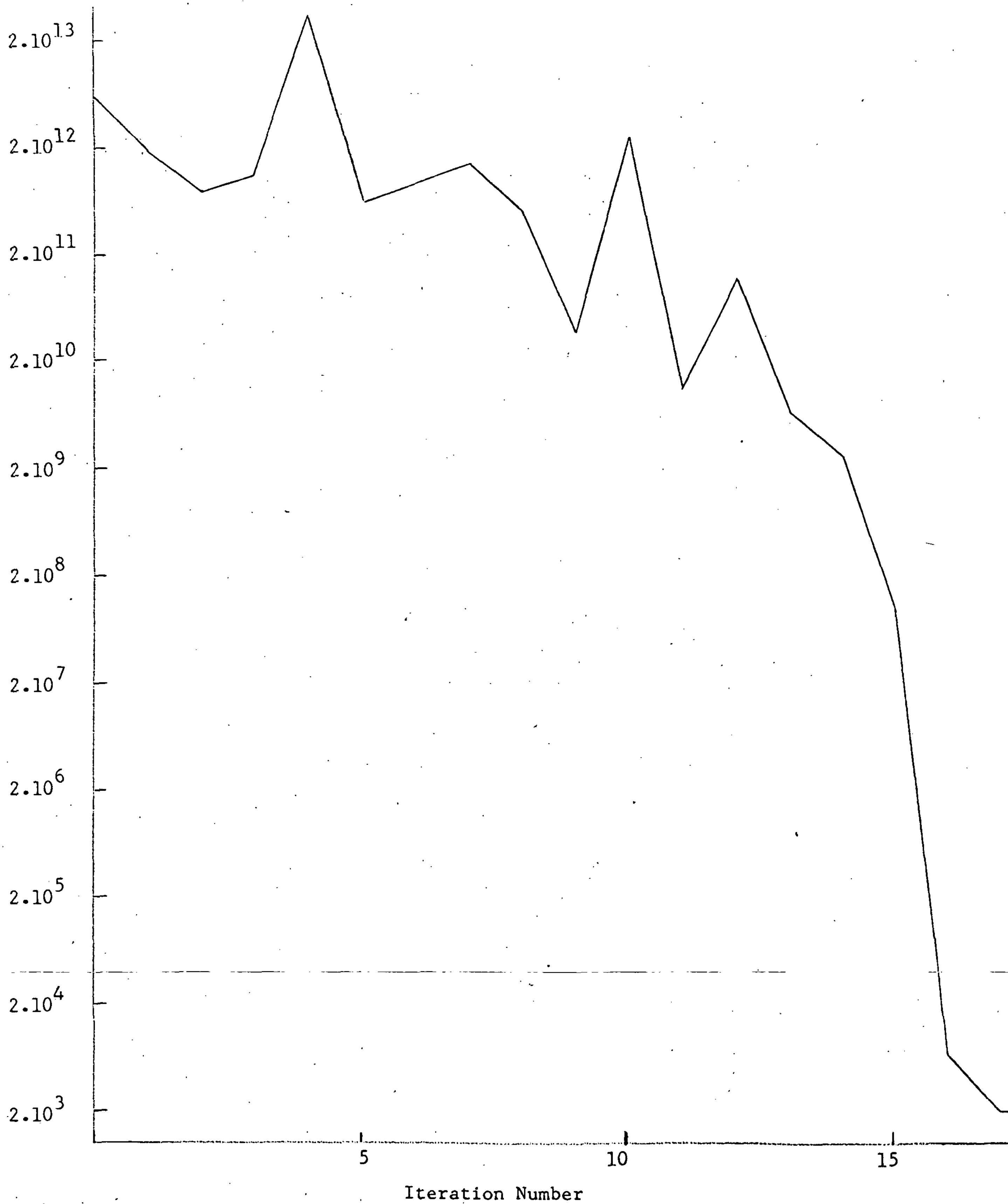

Figure 8A. Performance Index, J

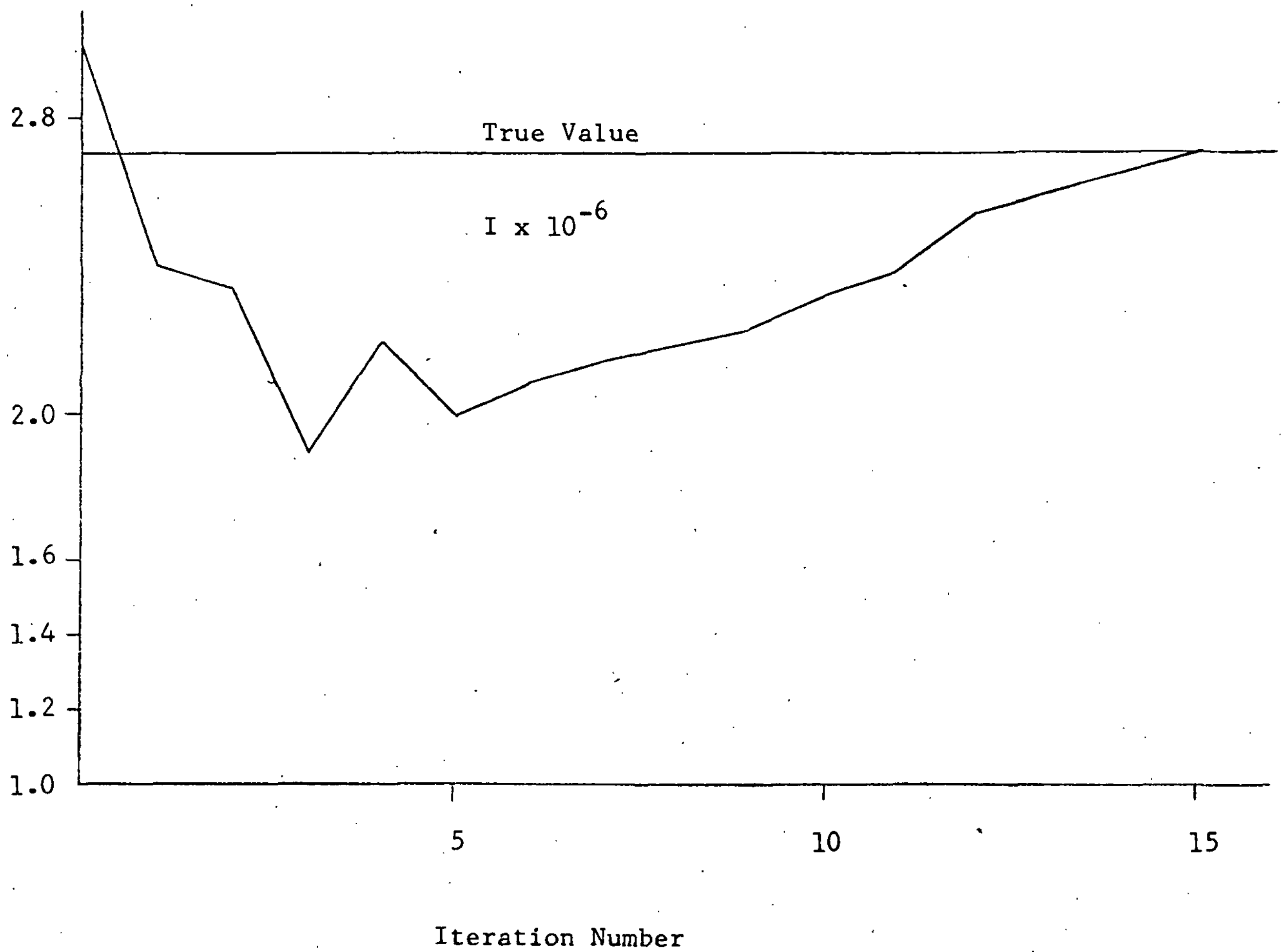

Figure 8B. Moment of Inertia

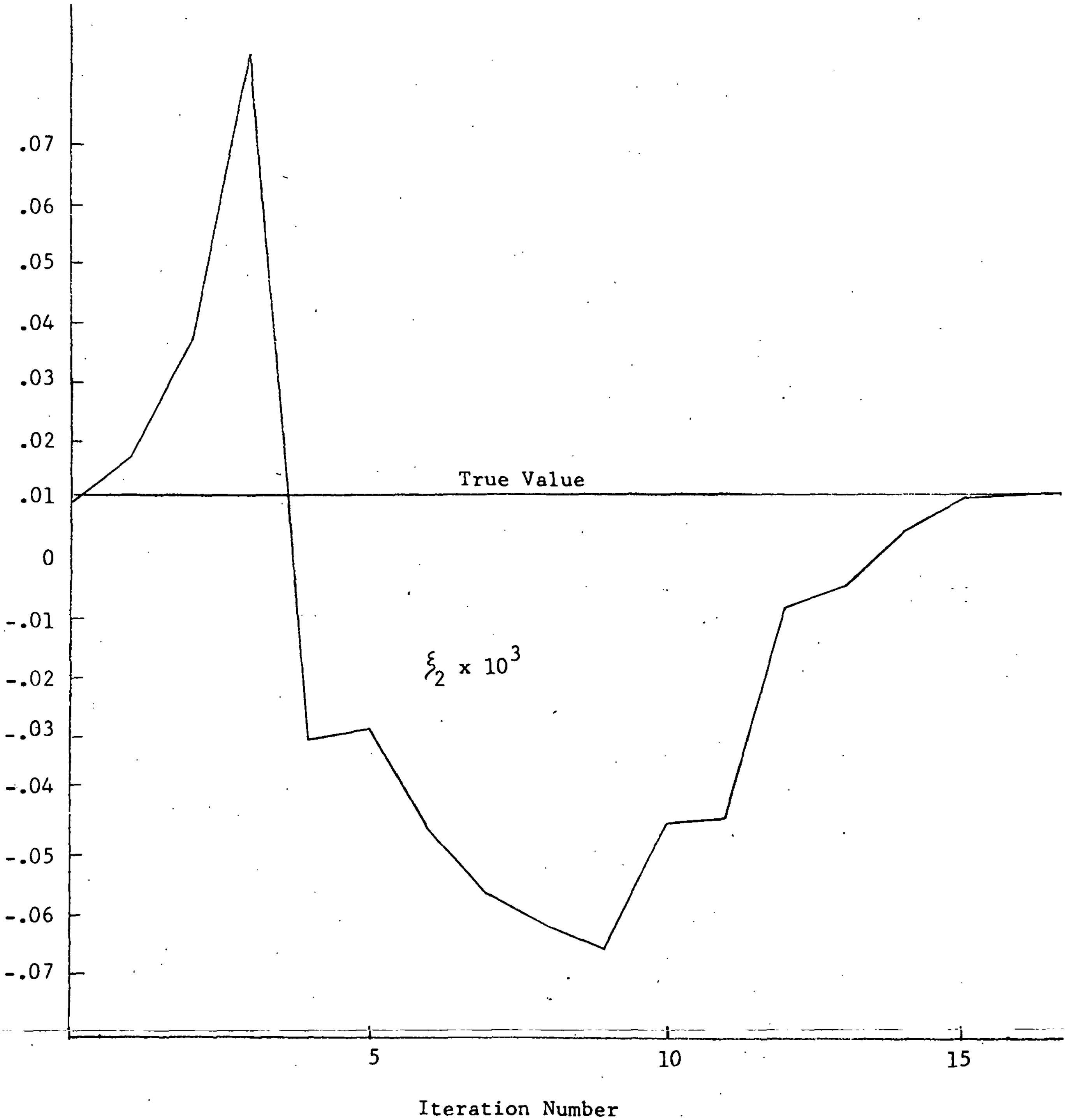

Figure 8C. Damping Coefficient

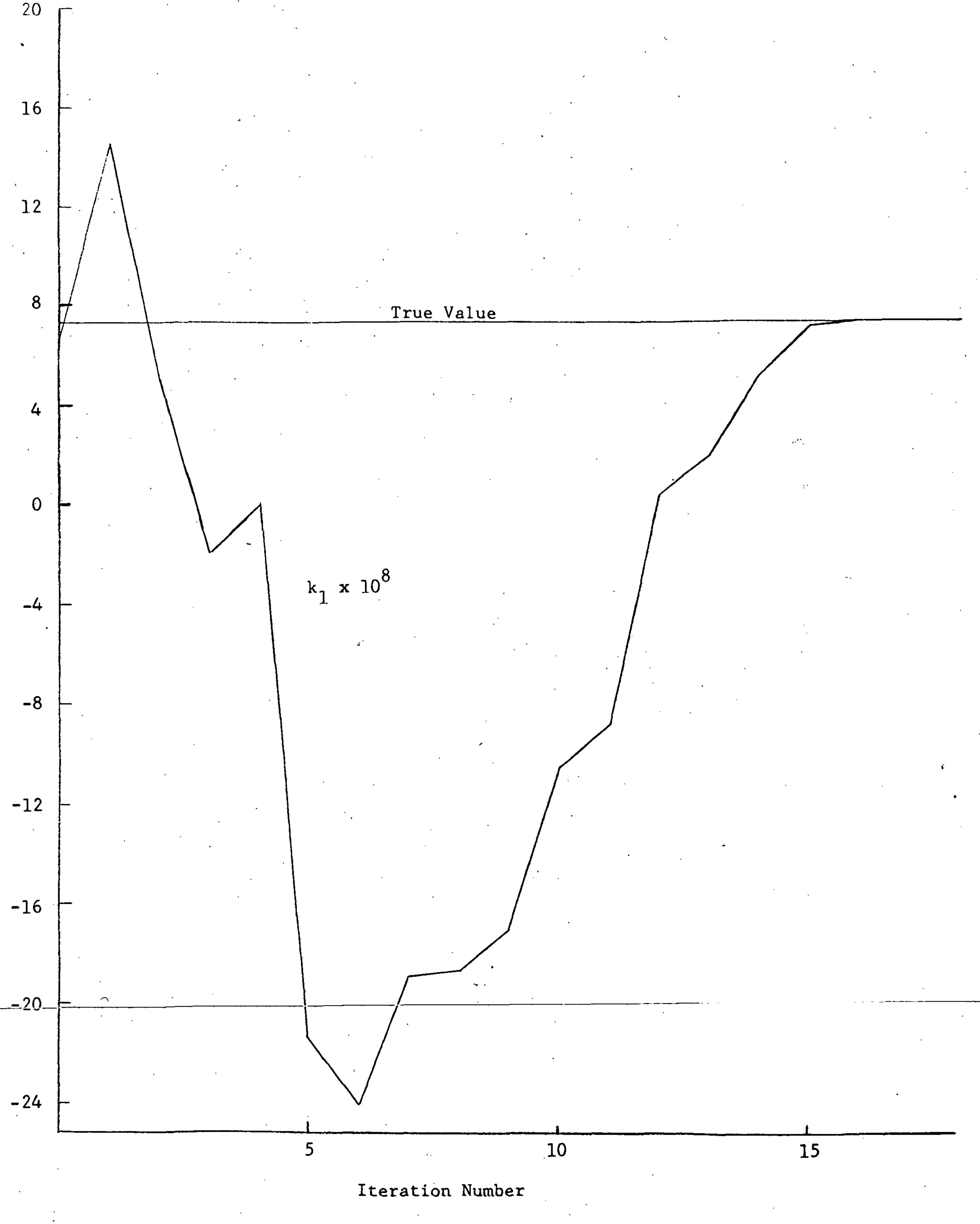

Figure 8D. Input Coefficient

VIII. CONCLUSIONS AND RECOMMENDATIONS

Simulations indicate that the J_o-Adaptive sequential estimator will not track the modal damping coefficients due to a lack of local observability of these coefficients in the data. This sequential estimator, designed to track time-varying system parameters, has a relatively short memory length to enable the tracking of those varying parameters. However, the J_o-Adaptive sequential estimator does track the rigid body and bending states in the presence of parameter errors quite well and can, simultaneously, improve the estimate of the moment of inertia, bending frequencies and modal parameters k_i. If model parameters are to be estimated via this sequential estimator, it is important that those modes whose parameters are being estimated are excited during the estimation process. In summary, the J_o-Adaptive sequential estimator is an effective tracker for the rigid body and bending states, but cannot be depended upon as the primary parameter identification tool, particularly as far as the damping coefficients are concerned.

Much was learned from simulations of the batch estimator. First, the combined state and parameter estimation problem is highly nonlinear, requiring a relatively large number of iterations for convergence. Second, provided data rates are sufficiently high and the measurements cover a sufficiently long interval (containing several cycles of the lowest frequency mode), and provided all system modes are excited during the entire observation interval, then the system is completely observable. It is then possible to identify the moment of inertia, all bending frequencies, all damping coefficients, and all modal (input) parameters with very high accuracy. The batch estimator is of course suitable only for the identification of constant parameters. Therefore, in case the system parameters are time-varying such variations must be slow relative to the observability criteria noted above.

The above conclusions are of necessity qualitative in nature. Specific quantitative conclusions can only be drawn with reference to a well defined physical problem. For example, estimation accuracy depends on the attitude and attitude rate measurement noise statistics. Whether or not the J_o-Adaptive sequential estimator tracking of the parameters is adequate depends upon the utilization of the estimates of these parameters, as, for example the required accuracy in these parameters for adequate controller performance. The data rates required for adequate identification and tracking (sequential or batch) may or may not be feasible for a given real-time operation.

Certain aspects of this estimation/identification problem were not exhausted in the present study, and it is recommended that these be pursued further.

1. The difficulties in the pseudo-inverse application which, it is believed, stem from poor scaling in the problem, should be pursued. Possibly, the problem might be re-scaled. This is of more than academic importance since it is not known a priori whether or not all system modes are in fact being excited and thus are observable. Attempts at identification of unobservable modes leads to overall poor estimator performance. The use of the pseudo-inverse should reduce this problem since unobservable parameters are automatically eliminated from the parameter set via the pseudo-inverse.

2. It would be worthwhile to recast the batch estimator into a sequential one. This would probably reduce computing time because of the step-wise relinearization inherent in the extended sequential estimator. It is usually agreed that sequential estimators are more suitable for nonlinear problems and for real-time operation. Such a sequential estimator could be re-initialized to handle parameter variations.

3. A study of the estimation process in the presence of model errors provided for through the function f (see Eqn (4)) should be pursued further. As presently modeled in the sequential estimator, the function f cannot be distinguished from the rigid body initial condition. A more appropriate model for f is a harmonic oscillator forced by noise, with fixed frequency which is higher than the frequencies of the modeled modes.

Finally, it is recommended that the techniques and experience gained in this study be applied to a concrete aeronautical or astronautical system. The estimation/identification problem studied here is relevant for spacecraft with flexible appendages and for high altitude aircraft, as well as for a flexible space station.

Appendix A

State Transition Matrix

A first order approximation to the dynamics in Eqn (10) is given by

$$x(j+1) = x(j) + A(e)x(j)\tau + bu\tau\ , \quad \tau = t_{j+1} - t_j \tag{A1}$$

From Eqn (A1), a first order approximation to the state transition matrix of Eqn (15) is available as

$$\Phi(j+1,j) = I + A(e)\tau \tag{A2}$$

However, an exact state transition matrix can be constructed for Eqn (10), as given below.

The 2x2 diagonal blocks of Φ are easily obtained. For the rigid body,

$$\begin{aligned} \phi_{11} &= 1 \\ \phi_{12} &= \tau \\ \phi_{21} &= 0 \\ \phi_{22} &= 1 \end{aligned} \tag{A3}$$

For the actuator,

$$\phi_{33} = e^{-\xi_a \omega_a \tau}\left[\cos \nu_a \tau + \frac{\xi_a}{\sqrt{1-\xi_a^2}} \sin \nu_a \tau\right],$$

where

$$\nu_a = \omega_a \sqrt{1-\xi_a^2}\ ,$$

$$\phi_{34} = \frac{1}{\nu_a} e^{-\zeta_a \omega_a \tau} \sin \nu_a \tau$$

$$\phi_{43} = -\omega_a^2 \phi_{34} \tag{A4}$$

$$\phi_{44} = \phi_{33} - 2\zeta_a \omega_a \phi_{34} \quad .$$

The modal terms of course have the same form as the actuator; thus for $i=1,\ldots, m_s$ we have

$$\phi_{2i+3,2i+3} = e^{-\zeta_i \omega_i \tau} \left[\cos \nu_i \tau + \frac{\zeta_i}{\sqrt{1-\zeta_i^2}} \sin \nu_i \tau \right] \quad ,$$

where

$$\nu_i = \omega_i \sqrt{1-\zeta_i^2} \quad ,$$

$$\phi_{2i+3,2i+4} = \frac{1}{\nu_i} e^{-\zeta_i \omega_i \tau} \sin \nu_i \tau \tag{A5}$$

$$\phi_{2i+4,2i+3} = -\omega_i^2 \phi_{2i+3,2i+4}$$

$$\phi_{2i+4,2i+4} = \phi_{2i+3,2i+3} - 2\zeta_i \omega_i \phi_{2i+3,2i+4} \quad .$$

Because the normal modes are decoupled, all other elements of the transition matrix are zero except for the third and fourth column, which describe the effects of the actuator. For the rigid body,

$$\phi_{13} = \phi_{24} + 2\zeta_a \omega_a \phi_{14}$$

$$\phi_{14} = \frac{1}{b_a} (b_o \phi_{12} - \phi_{23}) \quad , \quad b_a = \omega_a^2$$

$$\phi_{23} = b_o \phi_{34} + 2\zeta_a\omega_a \phi_{24} \tag{A6}$$

$$\phi_{24} = \frac{b_o}{b_a} (\phi_{22} - \phi_{33}) \quad .$$

For the modal terms, the expressions are more complicated, requiring several auxiliary definitions. Let

$$\begin{aligned}
A &= 2\omega_a\omega_i(\zeta_a\omega_i - \zeta_i\omega_a) \\
B &= \omega_i^2 - \omega_a^2 \\
C &= 2(\zeta_i\omega_i - \zeta_a\omega_a) \\
\Delta &= AC - B^2 \\
D &= b_i(\phi_{33} - \phi_{2i+3,2i+3}) \\
E &= b_i(\phi_{34} - \phi_{2i+3,2i+4}) \quad .
\end{aligned} \tag{A7}$$

Then,

$$\begin{aligned}
\phi_{2i+3,4} &= \frac{1}{\Delta}(CD - BE) \\
\phi_{2i+4,4} &= \frac{1}{\Delta}(-BD + AE) \\
\phi_{2i+3,3} &= 2\zeta_a\omega_a \phi_{2i+3,4} + \phi_{2i+4,4} \\
\phi_{2i+4,4} &= b_i \phi_{2i+3,2i+4} - \omega_a^2 \phi_{2i+3,4} \quad .
\end{aligned} \tag{A8}$$

This completes the definition of the transition matrix $\Phi(\tau)$.

In order to generate the parameter sensitivity matrix Ψ in Appendix B, it is convenient to have the vector

$$\Gamma = \frac{\partial x(j+1)}{\partial u(j)} \tag{A9}$$

which gives the effect of control on the state. For all k we have

$$\gamma_{2k-1} = b_a \int_o^t \phi_{2k-1,4}(s)ds$$

$$\gamma_{2k} = b_a \phi_{2k-1,4} \quad . \tag{A10}$$

It follows then that

$$\gamma_1 = b_o \frac{\tau^2}{2} - \phi_{13}(\tau)$$

$$\gamma_2 = b_a \phi_{14}(\tau) \quad .$$

$$\gamma_3 = 1 - \phi_{33} \tag{A11}$$

$$\gamma_4 = b_a \phi_{34}(\tau) \quad .$$

For the modal terms we use the facts that

$$\Gamma = A^{-1}(\Phi - I)b$$

(when A^{-1} exists) and that the system is decoupled, to compute

$$\gamma_{2i+3} = - b_a \frac{2\xi_a k_i}{\omega_a} \phi_{34} - b_a \frac{k_i}{\omega_a^2} (\phi_{44} - 1)$$

$$- b_a \frac{2\xi_i}{\omega_i} \phi_{2i+3,4} - \frac{b_a}{\omega_i^2} \phi_{2i+4,4} \tag{A12}$$

$$\gamma_{2i+4} = b_a \phi_{2i+3,4}$$

for $i = 1, \ldots, m_o$.

Appendix B

Parameter Sensitivity Matrix

First order approximations to the parameter sensitivities (partial derivatives) of Eqn (15) are obtained by explicitly differentiating Eqn (A1). These are

$$\Psi^{I} = \left[0, \quad -x_3 \tau / I^2 , \quad \overset{(2+2m_o)x1}{0} \right]^T \tag{B1}$$

$$\Psi^{\omega} = \left[\begin{array}{c:c:c} \multicolumn{3}{c}{\overset{4xm_o}{0}} \\ \hdashline \Psi_1^{\omega} & \overset{2x1}{0} & \cdots \quad \overset{2x1}{0} \\ \hdashline \overset{2x1}{0} & \Psi_2^{\omega} & \\ \hdashline \vdots & & \ddots \\ \overset{2x1}{0} & & \Psi_{m_o}^{\omega} \end{array} \right] \tag{B2}$$

where

$$\overset{2x1}{\Psi_i^{\omega}} = \left[\begin{array}{c} 0 \\ 2\tau \left[k_i \omega_i x_3 - \xi_i x_{4+2i} - \omega_i x_{3+2i} \right] \end{array} \right] \tag{B3}$$

$$\Psi^{\xi} = \begin{bmatrix} \multicolumn{4}{c}{\overset{4\times m_o}{0}} \\ \hline \Psi_1^{\xi} & & & \\ \hline & \Psi_2^{\xi} & & \\ \hline & & \ddots & \\ & & & \Psi_{m_o}^{\xi} \end{bmatrix} \tag{B4}$$

where

$$\overset{2\times 1}{\Psi_i^{\xi}} = \begin{bmatrix} 0 \\ -2\tau\nu_i x_{4+2i} \end{bmatrix} \tag{B5}$$

$$\Psi^{k} = \begin{bmatrix} \multicolumn{4}{c}{\overset{4\times m_o}{0}} \\ \hline \Psi_1^{k} & & & \\ \hline & \Psi_2^{k} & & \\ \hline & & \ddots & \\ & & & \Psi_{m_o}^{k} \end{bmatrix} \tag{B6}$$

where

$$\overset{2\times 1}{\Psi_i^{k}} = \begin{bmatrix} 0 \\ \tau\omega_i^2 x_3 \end{bmatrix} \tag{B7}$$

The exact expressions for Ψ are obtained by differentiation of the solution

$$x(t_{j+1}) = \Phi(\tau)x(t_j) + \Gamma(\tau)u(t_j)$$

with respect to the parameters of the system. This leads easily to

$$\Psi^I = -b_o \left[\phi_{13}x_3 + \phi_{14}x_4 + \gamma_1 u \ , \ \phi_{23}x_3 + \phi_{24}x_4 + \gamma_2 u \ , \ 0,\ldots,0 \right]^T, \qquad \text{(B8)}$$

$$\Psi_i^k = \begin{bmatrix} \phi_{2i+3,3}x_3/k_i + \phi_{2i+3,4}x_4/k_i + \gamma_{2i+3}u/k_i \\ \phi_{2i+4,3}x_3/k_i + \phi_{2i+4,4}x_4/k_i + \gamma_{2i+4}u/k_i \end{bmatrix}, \qquad \text{(B9)}$$

and with somewhat more difficulty to the formulae which follow.

$$(\Psi_i^\omega)_1 = x_{2i+3} \frac{\partial \phi_{2i+3,2i+3}}{\partial \omega_i} + x_{2i+4} \frac{\partial \phi_{2i+3,2i+4}}{\partial \omega_i}$$

$$+ x_3(2\zeta_a\omega_a \frac{\partial \phi_{2i+3,4}}{\partial \omega_i} + \frac{\partial \phi_{2i+4,4}}{\partial \omega_i}) + x_4 \frac{\partial \phi_{2i+3,4}}{\partial \omega_i}$$

$$+ u \frac{\partial \Gamma_{2i+3}}{\partial \omega_i} \qquad \text{(B10)}$$

$$(\Psi_i^\omega)_2 = -\omega_i x_{2i+3} (2\phi_{2i+3,2i+4} + \omega_i \frac{\partial \phi_{2i+3,2i+4}}{\partial \omega_i})$$

$$+ x_{2i+4}(\frac{\partial \phi_{2i+3,2i+3}}{\partial \omega_i} - 2\zeta_i \phi_{2i+3,2i+4} - 2\zeta_i\omega_i \frac{\partial \phi_{2i+3,2i+4}}{\partial \omega_i})$$

$$+ x_3(2k_i\omega_i \phi_{2i+3,2i+4} + k_i\omega_i^2 \frac{\partial \phi_{2i+3,2i+4}}{\partial \omega_i} - \omega_a^2 \frac{\partial \phi_{2i+3,4}}{\partial \omega_i})$$

$$+ x_4 \frac{\partial \phi_{2i+4,4}}{\partial \omega_i} + u\, b_a \frac{\partial \phi_{2i+3,4}}{\partial \omega_i}$$

where

$$\frac{\partial \phi_{2i+3,2i+3}}{\partial \omega_i} = -\omega_i \tau \phi_{2i+3,2i+4}$$

$$\frac{\partial \phi_{2i+3,2i+4}}{\partial \omega_i} = \frac{\tau}{\omega_i} \phi_{2i+3,2i+3} - \frac{1+2\xi_i \omega_i \tau}{\omega_i} \phi_{2i+3,2i+4}$$

(B11)

$$\frac{\partial \phi_{2i+3,4}}{\partial \omega_i} = - \frac{\phi_{2i+3,4}}{\Delta} \frac{\partial \Delta}{\partial \omega_i} + \frac{1}{\Delta} \left(C \frac{\partial D}{\partial \omega_i} + D \frac{\partial C}{\partial \omega_i} - B \frac{\partial E}{\partial \omega_i} - E \frac{\partial B}{\partial \omega_i} \right)$$

$$\frac{\partial \phi_{2i+4,4}}{\partial \omega_i} = - \frac{\phi_{2i+4,4}}{\Delta} \frac{\partial \Delta}{\partial \omega_i} + \frac{1}{\Delta} \left(-B \frac{\partial D}{\partial \omega_i} - D \frac{\partial B}{\partial \omega_i} + A \frac{\partial E}{\partial \omega_i} + E \frac{\partial A}{\partial \omega_i} \right)$$

and

$$A = 2\omega_a \omega_i (\xi_a \omega_i - \xi_i \omega_a)$$

$$B = \omega_i^2 - \omega_a^2$$

$$C = 2(\xi_i \omega_i - \xi_a \omega_a)$$

(B12)

$$\Delta = AC - B^2$$

$$D = b_i(\phi_{33} - \phi_{2i+3,2i+3})$$

$$E = b_i(\phi_{34} - \phi_{2i+3,2i+4}) ,$$

and therefore

$$\frac{\partial A}{\partial \omega_i} = 2\omega_a (2\xi_a \omega_i - \xi_i \omega_a)$$

$$\frac{\partial B}{\partial \omega_i} = 2\omega_i$$

$$\frac{\partial C}{\partial \omega_i} = 2\zeta_i$$

(B13)

$$\frac{\partial \Delta}{\partial \omega_i} = 2\zeta_i A + C \frac{\partial A}{\partial \omega_i} - 4\omega_i B$$

$$\frac{\partial D}{\partial \omega_i} = \frac{2D}{\omega_i} - b_i \frac{\partial \phi_{2i+3,2i+3}}{\partial \omega_i}$$

and

$$\frac{\partial E}{\partial \omega_i} = \frac{2E}{\omega_i} - b_i \frac{\partial \phi_{2i+3,2i+4}}{\partial \omega_i} .$$

In addition,

$$\frac{\partial \Gamma_{2i+3}}{\partial \omega_i} = + 2\zeta_i \frac{\omega_a^2}{\omega_i^2} \phi_{2i+3,4} + 2 \frac{\omega_a^2}{\omega_i^3} \phi_{2i+4,4}$$

(B14)

$$- 2\zeta_i \frac{\omega_a^2}{\omega_i} \frac{\partial \phi_{2i+3,4}}{\partial \omega_i} - \frac{\omega_a^2}{\omega_i^2} \frac{\partial \phi_{2i+4,4}}{\partial \omega_i}$$

Finally, we have

$$(\Psi_i^{\zeta})_1 = x_{2i+3} \frac{\partial \phi_{2i+3,2i+3}}{\partial \zeta_i} + x_{2i+4} \frac{\partial \phi_{2i+3,2i+4}}{\partial \zeta_i}$$

$$+ x_3 (2\zeta_a \omega_a \frac{\partial \phi_{2i+3,4}}{\partial \zeta_i} + \frac{\partial \phi_{2i+4,4}}{\partial \zeta_i}) + x_4 \frac{\partial \phi_{2i+3,4}}{\partial \zeta_i}$$

$$+ u \frac{\partial \Gamma_{2i+3}}{\partial \zeta_i}$$

(B15)

$$(\Psi_i^{\zeta})_2 = -\omega_i^2 x_{2i+3} \frac{\partial \phi_{2i+3,2i+4}}{\partial \zeta_i} +$$

$$+ x_{2i+4}\left(\frac{\partial\phi_{2i+3,2i+3}}{\partial\xi_i} - 2\omega_i \phi_{2i+3,2i+4} - 2\xi_i\omega_i \frac{\partial\phi_{2i+3,2i+4}}{\partial\xi_i} \right)$$

$$+ x_3\left(k_i\omega_i^2 \frac{\partial\phi_{2i+3,2i+4}}{\partial\xi_i} - \omega_a^2 \frac{\partial\phi_{2i+3,4}}{\partial\xi_i}\right) + x_4 \frac{\partial\phi_{2i+4,4}}{\partial\xi_i}$$

$$+ ub_a \frac{\partial\phi_{2i+3,4}}{\partial\xi_i}$$

where

$$\frac{\partial\phi_{2i+3,2i+3}}{\partial\xi_i} = e^{-\xi_i\omega_i\tau}\left[- \frac{\omega_i\tau}{1-\xi_i^2}\cos\nu_i\tau + \frac{1}{(1-\xi_i^2)^{3/2}} \sin\nu_i\tau \right] \tag{B16}$$

$$\frac{\partial\phi_{2i+3,2i+4}}{\partial\xi_i} = - \frac{e^{-\xi_i\omega_i\tau}}{\nu_i^2}\left[\xi_i\omega_i^2\tau\cos\nu_i\tau + \left(\frac{-\xi_i\omega_i + \nu_i^2\tau}{\sqrt{1-\xi_i^2}}\right)\sin\nu_i\tau \right]$$

and

$$\frac{\partial\phi_{2i+3,4}}{\partial\xi_i} = - \frac{\phi_{2i+3,4}}{\Delta}\frac{\partial\Delta}{\partial\xi_i} + \frac{1}{\Delta}\left(C\frac{\partial D}{\partial\xi_i} + D\frac{\partial C}{\partial\xi_i} - B\frac{\partial E}{\partial\xi_i} \right) \tag{B17}$$

$$\frac{\partial\phi_{2i+4,4}}{\partial\xi_i} = - \frac{\phi_{2i+4,4}}{\Delta}\frac{\partial\Delta}{\partial\xi_i} + \frac{1}{\Delta}\left(-B\frac{\partial D}{\partial\xi_i} + A\frac{\partial E}{\partial\xi_i} + E\frac{\partial A}{\partial\xi_i} \right)$$

where,

$$\frac{\partial A}{\partial\xi_i} = -2\omega_a^2\omega_i$$

$$\frac{\partial B}{\partial\xi_i} = 0$$

$$\frac{\partial C}{\partial \xi_i} = 2\omega_i \tag{B18}$$

$$\frac{\partial \Delta}{\partial \xi_i} = 2\omega_i A - 2\omega_a^2 \omega_i C$$

$$\frac{\partial D}{\partial \xi_i} = - b_i \frac{\partial \phi_{2i+3,2i+3}}{\partial \xi_i}$$

$$\frac{\partial E}{\partial \xi_i} = - b_i \frac{\partial \phi_{2i+3,2i+4}}{\partial \xi_i}$$

In addition,

$$\frac{\partial \Gamma_{2i+3}}{\partial \xi_i} = - 2 \frac{\omega_a^2}{\omega_i} \phi_{2i+3,4} - 2\xi_i \frac{\omega_a^2}{\omega_i} \frac{\partial \phi_{2i+3,4}}{\partial \xi_i} - \frac{\omega_a^2}{\omega_i^2} \frac{\partial \phi_{2i+4,4}}{\partial \xi_i} \tag{B19}$$

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Appendix C

Model Characteristics

The parameters of the simulation model are given below:

$I_o = 3.0 \times 10^6$ slug-ft^2

$\omega_a = 15.0$ rad/sec $\quad \zeta_a = 0.4$

$\omega_{o1} = 5.00$ rad/sec	$\zeta_{o1} = 0.005$	$k_{o1} = 0.66667 \times 10^{-7}$ rad/ft-lb
$\omega_{o2} = 8.61$ rad/sec	$\zeta_{o2} = 0.01$	$k_{o2} = 0.44965 \times 10^{-7}$ rad/ft-lb
$\omega_{o3} = 15.00$ rad/sec	$\zeta_{o3} = 0.01$	$k_{o3} = 0.19259 \times 10^{-7}$ rad/ft-lb
$\omega_{o4} = 26.04$ rad/sec	$\zeta_{o4} = 0.01$	$k_{o4} = 0.34411 \times 10^{-7}$ rad/ft-lb
$\omega_{o5} = 45.00$ rad/sec	$\zeta_{o5} = 0.01$	$k_{o5} = 0.13169 \times 10^{-7}$ rad/ft-lb
$\omega_{o6} = 50.50$ rad/sec	$\zeta_{o6} = 0.01$	$k_{o6} = 0.12417 \times 10^{-7}$ rad/ft-lb
$\omega_{o7} = 56.65$ rad/sec	$\zeta_{o7} = 0.01$	$k_{o7} = 0.12464 \times 10^{-7}$ rad/ft-lb
$\omega_{o8} = 63.56$ rad/sec	$\zeta_{o8} = 0.01$	$k_{o8} = 0.10726 \times 10^{-7}$ rad/ft-lb
$\omega_{o9} = 71.32$ rad/sec	$\zeta_{o9} = 0.01$	$k_{o9} = 0.10484 \times 10^{-7}$ rad/ft-lb
$\omega_{o10} = 80.02$ rad/sec	$\zeta_{o10} = 0.01$	$k_{o10} = 0.11453 \times 10^{-7}$ rad/ft-lb

$\lambda_1 = -0.1 \times 10^7$	$\lambda_5 = 0.482472 \times 10^5$	$\lambda_9 = 0.259771 \times 10^7$
$\lambda_2 = -0.235094 \times 10^7$	$\lambda_6 = -0.272896 \times 10^6$	$\lambda_{10} = -0.521888 \times 10^5$
$\lambda_3 = -0.300177$	$\lambda_7 = 0.706988 \times 10^6$	$\lambda_{11} = 0.338741 \times 10^7$
$\lambda_4 = -0.209141 \times 10^{-1}$	$\lambda_8 = -0.194223 \times 10^6$	$\lambda_{12} = 0.121428 \times 10^6$
$\lambda_i = 0, \; i > 14$		$\lambda_{13} = 0.125113 \times 10^7$
		$\lambda_{14} = 0.737513 \times 10^5$

$$\sigma_1 = 1.0\times10^{-9}\ \text{rad} \qquad \sigma_2 = 2.0\times10^{-8}\ \text{rad/sec} \qquad \sigma_{12} = 0.0$$

The system is set into motion by a control pulse at t_o, and is further excited in some simulations by control pulses at intermediate times t_i. Sample-and-hold control is used in all the simulations. Estimated state feedback (Eqn (7)) is used in simulations involving the sequential estimator, while a specified control $u(t)$ is used in batch estimator simulations. Various parameter variations (Eqn (8)) are utilized (see Section VII). The system equations are integrated via a fourth order Runge-Kutta routine at a fixed step of 0.0078125 sec. Measurements are sampled at each integration step.

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ABSTRACT

Techniques are studied for the estimation of rigid body and bending states and the identification of model parameters associated with the single-axis attitude dynamics of a flexible vehicle. This problem is highly nonlinear but completely observable provided sufficient attitude and attitude rate data is available and provided all system bending modes are excited in the observation interval. A sequential estimator tracks the system states in the presence of model parameter errors. A batch estimator identifies all model parameters with high accuracy.