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RELATIONSHIPS BETWEEN THE CURVATURES OF TOOTH SURFACES IN THREE-DIMENSIONAL GEAR SYSTEMS

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16. Abstract

A three-dimensional gear system between crossing and intersecting axes is considered under the assumption that the first derivative of the transmission ratio is zero in the vicinity of the point of contact. The following are obtained in the article: (a) an equation that relates the normal curvatures of the tooth surfaces in the section that passes through the vector of relative velocity; (b) a relation between the principal curvatures and principal directions of the two tooth surfaces; (c) new formulas for determining the reduced curvatures of the surfaces.

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RELATIONSHIPS BETWEEN THE CURVATURES OF TOOTH SURFACES IN THREE-DIMENSIONAL GEAR SYSTEMS

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1. Initial Equations[1] /685*

The surfaces of teeth are, in relative motion, mutually enveloping surfaces, and the following equations [1] apply at their points of contact:

$$r^{(1)} = r^{(2)};\ e^{(1)} = e^{(2)};\ e^{(i)} v^{(12)} = 0;\ \ i = 1, 2 \tag{1.1}$$

Here, $r^{(i)}$ is the radius vector of surface Σ_i, and $e^{(i)}$ is the unit vector normal to this surface in the spatially fixed coordinate system; $v^{(12)}$ the velocity vector of point M, fixed with respect to surface Σ_1, relative to the corresponding point fixed with respect to the surface Σ_2.

Given that we have a three-dimensional gear system that transmits motion between crossing shafts. The following equations [1] apply in the vicinity of the point of contact of the tooth surfaces:

$$\begin{cases} \dot{r}^{(1)} = \dot{r}^{(2)};\ \dot{e}^{(1)} = \dot{e}^{(2)}; \\ \dfrac{d}{dt}(e^{(1)} \cdot v^{(12)}) = 0; \quad \omega^{(1)} = \text{const}; \quad \dot{\omega}^{(2)} = 0\,. \end{cases} \tag{1.2}$$

For $\omega^{(1)}$ = const and $\dot{\omega}^{(2)} = 0$, $d(i_{12})/dt = 0$, where $i_{12} = \omega^{(1)}/\omega^{(2)}$ is instantaneous transmission ratio.

Moreover,

$$\dot{r}^{(i)} = v_e^{(i)} + v_r^{(i)}\,. \tag{1.3}$$

[1] The reader is referred to [7] for supplementary material.

* Numbers in the margin indicate pagination in the foreign text.

Here $v_e^{(i)}$ is the velocity of a point transmitting motion, in conjunction with the surface; $v_r^{(i)}$ is the velocity of the point relative to the surface of the tooth. /686

Analogously,

$$\dot{e}^{(i)} = \dot{e}_e^{(i)} + \dot{e}_r^{(i)} = \omega^{(i)} \times e^{(i)} + \dot{e}_r^{(i)}. \tag{1.4}$$

The vector of relative velocity is calculated from

$$v^{(12)} = \omega^{(12)} \times r^{(1)} - R \times \omega^{(12)} \quad \text{mit} \quad \omega^{(12)} = \omega^{(1)} - \omega^{(2)}. \tag{1.5}$$

Here $r^{(1)}$ is the radius vector of the point of contact on the tooth surfaces (the origin 0 of radius vector $r^{(1)}$ lies on the line of application of the vector $\omega^{(1)}$); R is the radius vector of an arbitrary point on the line of application of vector $\omega^{(2)}$ originating from point 0.

If we transform equations (1.2) and (1.5), we obtain

$$v_r^{(2)} = v_r^{(1)} + v^{(12)}; \; \dot{e}_r^{(2)} = \dot{e}_r^{(1)} + \omega^{(12)} \times e^{(1)}; \; \dot{e}_r^{(1)} v^{(12)} + (\omega^{(1)}, e^{(1)}, v^{(12)}) + (e^{(1)}, \omega^{(12)}, \dot{r}^{(1)}) = 0. \tag{1.6}$$

The curvature of the enveloped surface in the normal section is determined using the following equation [2]:

$$\varkappa^{(1)} = -\frac{\dot{e}_r^{(1)} v_r^{(1)}}{|v_r^{(1)}|^2}. \tag{1.7}$$

The equations of the enveloping surfaces are usually significantly more complex than the equations of the enveloped surface. For this reason, it is desirable to express the curvature of the enveloping surface in the normal section by means of the curvature of the enveloped surface and the parameters of relative motion [3]:

$$\varkappa^{(2)} = -\frac{\dot{e}_r^{(2)} v_r^{(2)}}{|v_r^{(2)}|^2} = -\frac{[\dot{e}_r^{(1)} + \omega^{(12)} \times e][v_r^{(1)} + v^{(12)}]}{[v_r^{(1)} + v^{(12)}]^2}. \tag{1.8}$$

In equations (1.7) and (1.8), the sign of the curvature is positive if the surface's radius of curvature and the normal unit vector are in the same direction. The vectors $\dot{e}_r^{(i)}$ and $v_r^{(i)}$ are collinear in the principal direction of the surface.

2. The Relationship Between the Curvatures of the Mutually Enveloped Surfaces in the Common Normal Section Passing Through the Vector $v^{(12)}$

L. V. Korostelev [4] and A. M. Pavlov [5] suggested determining the curvatures of the mutually enveloping surfaces of the teeth in the normal section that passes through the vector of relative velocity $v^{(12)}$. In this section, the vectors $v_r^{(1)}$, $v_r^{(2)}$ and $v^{(12)}$ are collinear.

We now seek a relationship between the normal curvatures in the section that passes through vector $v^{(12)}$. The vectors $\dot{e}_r^{(i)}$, $v_r^{(i)}$, $v^{(12)}$ and $\omega^{(12)} \times e^{(1)}$ lie in the plane that is tangent to the surfaces of the teeth at their point of contact. We resolve these vectors in the two directions containing the unit vectors i_t and i_n corresponding to the direction of $v^{(12)}$ and the direction perpendicular to it.

Curvature in the normal section passing through i_t is determined by the equation

$$\varkappa_t^{(i)} = -\frac{\dot{e}_{rt}^{(i)}}{v_{rt}^{(i)}} \quad (i = 1; 2) \tag{2.1}$$

On the basis of equations (1.6)-(2.1) we obtain

$$(\varkappa_t^{(1)} - \varkappa_t^{(2)})\, v_{rt}^{(1)} - \varkappa_t^{(2)}\, v_t^{(12)} - (\omega^{(12)}, e^{(1)}, i_t) = 0\,, \tag{2.2}$$

$$[(e^{(1)}, \omega^{(12)}, i_t) - \varkappa_t^{(1)}\, v_t^{(12)}]\, v_{rt}^{(1)} + v_t^{(12)}\, (\omega^{(1)}, e^{(1)}, i_t) + (e^{(1)}, \omega^{(12)}, v_s^{(1)}) = 0\,. \tag{2.3}$$

By factoring $v_{rt}^{(1)}$ out of equations (2.2) and (2.3), we obtain

$$(2.4)\quad (\varkappa_t^{(1)} - \varkappa_t^{(2)})\,[v_t^{(12)}(\omega^{(1)}, e^{(1)}, i_t) - (\omega^{(12)}, e^{(1)}, v_e^{(1)})] - \\ - \varkappa_t^{(1)}\varkappa_t^{(2)}(v_t^{(12)})^2 - (\varkappa_t^{(1)} + \varkappa_t^{(2)})\, v_t^{(12)}(\omega^{(12)}, e^{(1)}, i_t) - (\omega^{(12)}, e^{(1)}, i_t)^2 = 0\,.$$

Relation (2.4) connects the curvatures of the tooth surfaces in the normal section passing through vector $v^{(12)}$. This relation can be viewed as a generalization of the Euler-Savary equation for three-dimensional gear systems. We know that this equation relates the tooth profiles in planar gear systems when the gears of the mechanism have parallel axes of rotation. The normal curvature $\kappa_t^{(2)}$ of the enveloping surface can be determined from relation (2.4) if the normal curvatures $\kappa_t^{(1)}$ of the enveloped surface, the coordinates of the point of contact and the parameters of relative motion and transmitted motion are considered known.

In the case in which a motion is transmitted between parallel or intersecting gear axes with the aid of a mechanism, the relative motion can be reduced to a rotation about the instantaneous axis. For the point of contact of the tooth surfaces that lies on the instantaneous axis of rotation, we have

$$v^{(12)} = 0; \quad v_{rt}^{(1)} = v_{rt}^{(2)},$$

where unit vector i_t coincides, in direction, with vector $v_{rt}^{(1)}$. /687
Relation (2.4) assumes the following form for the case under consideration:

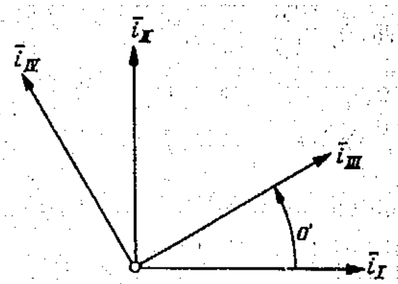
$$(2.5)\quad \varkappa_t^{(1)} - \varkappa_t^{(2)} = -\frac{(\omega^{(12)}, e^{(1)}, i_t)^2}{(\omega^{(12)}, e^{(1)}, v_e^{(1)})}.$$

In planar gear systems (with parallel axes of rotation for the gears), unit vector i_t coincides in direction with the unit vector of the tangent to the profiles. In the case of intersecting axes, unit vector i_t can have any direction.

3. The Relationships Between Principal Curvatures in the Case of Point and Linear Contact Between Surfaces

Let us first consider the case in which the tooth surfaces make contact with one another at one point at every instant. We wish to adopt the convention of calling the totality of points of contact on the tooth surface the "line of operation."

We now determine, for the moving point of contact, a relation between the direction of the tangent to the line of operation, the principal curvatures and the principal directions of the surfaces in contact with one another. Let us designate the unit vectors in the principal directions on surface Σ_1 as i_I and i_{II}; the unit vectors in the principal directions on surface Σ_2 as i_{III} and i_{IV}; the angle between directions I and III (Fig. 1) as σ; the principal curvatures of surface Σ_1 as κ_I and κ_{II}; and the principal curvatures of surface Σ_2 as κ_{III} and κ_{IV}. Let us turn to equation (1.6) and resolve the vectors in this equation in the directions of unit vectors i_I and i_{II}. We observe (Fig. 1) that



$$v_{rI}^{(2)} = v_{rIII}^{(2)} \cos\sigma - v_{rIV} \sin\sigma; \quad v_{rII}^{(2)} = v_{rIII}^{(2)} \sin\sigma + v_{rIV}^{(2)} \cos\sigma;$$
$$\dot{e}_{rI}^{(2)} = \dot{e}_{rIII}^{(2)} \cos\sigma - \dot{e}_{rIV}^{(2)} \sin\sigma; \quad \dot{e}_{rII}^{(2)} = \dot{e}_{rIII}^{(2)} \sin\sigma + \dot{e}_{rIV}^{(2)} \cos\sigma$$

Let us also take into consideration that the following applies to the principal directions:

$$\dot{e}_{ri}^{(m)} = -\varkappa_i v_{ri}^{(m)}; \qquad m = 1 \qquad (i = I, II)$$
$$m = 2 \qquad (i = III, IV).$$

After several transformations, we obtain a system of linear equations of the following type:

$$
\begin{cases} a_{11}\xi_1 + a_{12}\xi_2 = b_1 \\ a_{21}\xi_1 + a_{22}\xi_2 = b_2 \\ a_{31}\xi_1 + a_{32}\xi_2 = b_3 , \end{cases} \tag{3.1}
$$

$$\xi_1 = v_{rI}^{(1)}; \quad \xi_2 = v_{rII}^{(1)};$$

$$a_{11} = -\varkappa_I + \frac{1}{2}[(\varkappa_{III} + \varkappa_{IV}) + (\varkappa_{III} - \varkappa_{IV})\cos 2\sigma];$$

$$a_{12} = a_{21} = \frac{1}{2}(\varkappa_{III} - \varkappa_{IV})\sin 2\sigma;$$

$$a_{22} = -\varkappa_{II} + \frac{1}{2}[(\varkappa_{III} + \varkappa_{IV}) - (\varkappa_{III} - \varkappa_{IV})\cos 2\sigma];$$

$$a_{31} = (\mathbf{e}^{(1)}, \boldsymbol{\omega}^{(12)}, \mathbf{i}_I) - \varkappa_I v_I^{(12)};$$

$$a_{32} = (\mathbf{e}^{(1)}, \boldsymbol{\omega}^{(12)}, \mathbf{i}_{II}) - \varkappa_{II} v_{II}^{(12)}.$$

$$b_1 = (\mathbf{e}^{(1)}, \boldsymbol{\omega}^{(12)}, \mathbf{i}_I) + \frac{1}{2} v_I^{(12)} [(\varkappa_{III} + \varkappa_{IV}) + (\varkappa_{III} - \varkappa_{IV})\cos 2\sigma] - \frac{v_{II}^{(12)}}{2}(\varkappa_{III} - \varkappa_{IV})\sin 2\sigma;$$

$$b_2 = (\mathbf{e}^{(1)}, \boldsymbol{\omega}^{(12)}, \mathbf{i}_{II}) - \frac{1}{2} v_I^{(12)} (\varkappa_{III} - \varkappa_{IV})\sin 2\sigma - \frac{v_{II}^{(12)}}{2}[(\varkappa_{III} + \varkappa_{IV}) - (\varkappa_{III} - \varkappa_{IV})\cos 2\sigma];$$

$$b_3 = \mathbf{e}^{(1)}[(\boldsymbol{\omega}^{(2)} \times \mathbf{v}_s^{(1)}) - (\boldsymbol{\omega}^{(1)} \times \mathbf{v}_s^{(2)})].$$

In the case of point contact of the surfaces, $v_{rI}^{(1)}$ and $v_{rII}^{(1)}$ must possess certain values. For this reason, the system of linear equations must be consistent. As is known from linear algebra, this requires that /688

$$
\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix} = 0 . \tag{3.2}
$$

By solving expression (3.2), we obtain a relationship of the following type (the parameters of motion are assumed to be given):

$$f(\varkappa_I, \varkappa_{II}, \varkappa_{III}, \varkappa_{IV}, \sigma) = 0. \tag{3.3}$$

Relation (3.3) applies for arbitrary directions of the tangent to the line of operation (for arbitrary ratio $v_{rI}^{(1)} : v_{rII}^{(1)}$). The following problem frequently arises: It is necessary to determine κ_{III}, κ_{IV} and σ, it being assumed that κ_I and κ_{II} are known at the point of contact between the surfaces. We can make use of system of equations (3.1) for this if we initially specify the ratio $v_{rI}^{(1)} : v_{rII}^{(1)}$. Once we have eliminated $v_{rI}^{(1)}$ and $v_{rII}^{(1)}$, we obtain a system of two independent equations that relate κ_{III} and κ_{IV} and σ. The missing third equation is obtained if the dimensions of the deformed contact surfaces are given, which we know to be functions of the "magnitude" of osculation of the surfaces, their principle curvatures and angle σ [6].

In the case of linear contact between the tooth surfaces, the system of linear equations must be valid for various directions of $v_r^{(1)}$. The system must be consistent, but the values $v_{rI}^{(1)}$ and $v_{rII}^{(1)}$ become indeterminate. This is possible if the rank of the matrix

$$\begin{Vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{Vmatrix}$$

is equal to one.

From this it follows that

$$\frac{a_{11}}{a_{21}} = \frac{a_{12}}{a_{22}} = \frac{b_1}{b_2}; \quad \frac{a_{21}}{a_{31}} = \frac{a_{22}}{a_{32}} = \frac{b_2}{b_3}.$$

If we note that $a_{21} = a_{12}$, we can write the derived equations in the form

$$\frac{a_{11}}{a_{21}} = \frac{a_{12}}{a_{22}} = \frac{a_{31}}{a_{32}} = \frac{b_1}{b_2}, \tag{3.4}$$

$$\frac{a_{31}}{a_{31}} = \frac{b_2}{b_3} \tag{3.5}$$

Only two independent equations can be obtained from equations (3.4), since it must be taken into consideration that

$$b_1 = a_{31} - \nu_{\mathrm{I}}^{(12)} a_{11} - \nu_{\mathrm{II}}^{(12)} a_{12}; \qquad b_2 = a_{32} - \nu_{\mathrm{I}}^{(12)} a_{12} - \nu_{\mathrm{II}}^{(12)} a_{22}$$

By making use of (3.5) and the equations

$$\frac{a_{11}}{a_{21}} = \frac{a_{12}}{a_{22}} = \frac{a_{31}}{a_{32}}. \tag{3.6}$$

we obtain three relations of the following type for determining κ_{III}, κ_{IV} and σ:

$$\tan 2\sigma = \frac{2F}{\varkappa_{\mathrm{I}} - \varkappa_{\mathrm{II}} + G}, \tag{3.7}$$

$$\varkappa_{\mathrm{III}} + \varkappa_{\mathrm{IV}} = \varkappa_{\mathrm{I}} + \varkappa_{\mathrm{II}} + S, \tag{3.8}$$

$$\varkappa_{\mathrm{III}} - \varkappa_{\mathrm{IV}} = \frac{\varkappa_{\mathrm{I}} - \varkappa_{\mathrm{II}} + G}{\cos 2\sigma}, \tag{3.9}$$

where /689

$$F = \frac{a_{31} a_{32}}{b_3 + \nu_{\mathrm{I}}^{(12)} a_{31} + \nu_{\mathrm{II}}^{(12)} a_{32}},$$

$$G = \frac{a_{31}^2 - a_{32}^2}{b_3 + \nu_{\mathrm{I}}^{(12)} a_{31} + \nu_{\mathrm{II}}^{(12)} a_{32}},$$

$$S = \frac{a_{31}^2 + a_{32}^2}{b_3 + \nu_{\mathrm{I}}^{(12)} a_{31} + \nu_{\mathrm{II}}^{(12)} a_{32}}.$$

4. Reduced Normal Curvature

Reduced normal curvature refers to the difference between the normal curvatures of the two surfaces Σ_1 and Σ_2 in their common section. We shall designate it as

$$\varkappa^{(p)} = \varkappa^{(1)} - \varkappa^{(2)} \tag{4.1}$$

We now seek an expression for reduced normal curvature. For this purpose we designate the angle formed by the vector $v_r^{(2)}$ and unit vector i_I in the principal direction on surface Σ_1 as q. As in the preceding section, we resolve the vectors in equation (1.6) in the principal directions containing unit vectors i_I and i_{II}. We take into consideration here the fact that

$$\tan q = \frac{v_{rII}^{(2)}}{v_{rI}^{(2)}} = \frac{v_{rII}^{(1)} + v_{II}^{(12)}}{v_{rI}^{(1)} + v_{I}^{(12)}} \tag{4.2}$$

After several transformations, we obtain a system of two linear equations of the following type:

$$\begin{cases} a_{31}\xi_1 + a_{32}\xi_2 = b_3 \\ a_{41}\xi_1 + a_{42}\xi_2 = b_4 , \end{cases} \tag{4.3}$$

where

$$\xi_1 = v_{rI}^{(1)}, \quad \xi_2 = v_{rII}^{(1)}, \quad a_{41} = \sin q, \quad a_{42} = -\cos q;$$

$$b_4 = v_{II}^{(12)} \cos q - v_{I}^{(12)} \sin q .$$

The coefficients of the unknowns, a_{31} and a_{32}, and the free term b_3 have already been discussed in Section 3.

If we make use of equations (4.3), we obtain

$$v_{rI}^{(1)} = \frac{b_3 + a_{32}(v_{II}^{(12)} - v_{I}^{(12)} \tan q)}{a_{31} + a_{32} \tan q} . \tag{4.4}$$

Consequently,

$$v_{rI}^{(1)} + v_{I}^{(12)} = \frac{b_3 + a_{31} v_{I}^{(12)} + a_{32} v_{II}^{(12)}}{a_{31} + a_{32} \tan q} . \tag{4.5}$$

We thus obtain the following for normal curvature:

$$\varkappa^{(2)} = -\frac{\dot{c}_{rI}^{(2)} v_{rI}^{(2)} + \dot{c}_{rII}^{(2)} v_{rII}^{(2)}}{(v_{rI}^{(2)})^2 + (v_{rII}^{(2)})^2} . \tag{4.6}$$

To transform expression (4.6), we make use of the following relations:

$$(4.7)\qquad \begin{cases} v_{rI}^{(2)} = v_{rI}^{(1)} + v_{I}^{(12)}; \quad v_{rII}^{(2)} = (v_{rI}^{(1)} + v_{I}^{(12)}) \tan q; \\ \dot{e}_{rI}^{(2)} = \dot{e}_{rI}^{(1)} + (\omega^{(12)}, e^{(1)}, i_{I}) = -\varkappa_{I} v_{rI}^{(1)} + (\omega^{(12)}, e^{(1)}, i_{I}); \\ \dot{e}_{rII}^{(2)} = -\varkappa_{II} v_{rII}^{(1)} + (\omega^{(12)}, e^{(1)}, i_{II}). \end{cases}$$

We also note that

$$(4.8)\qquad \begin{aligned} (\omega^{(12)}, e^{(1)}, i_{I}) &= -(a_{31} + \varkappa_{I} v_{I}^{(12)}); \\ (\omega^{(12)}, e^{(1)}, i_{II}) &= -(a_{32} + \varkappa_{II} v_{II}^{(12)}) \end{aligned}$$

If we substitute relations (4.7) and (4.8) into expression (4.6) /690
and transform the latter, we obtain

$$(4.9)\qquad \varkappa^{(p)} = -\frac{(a_{31} + a_{32} \tan q)^2 \cos^2 q}{b_3 + a_{31} v_{I}^{(12)} + a_{32} v_{II}^{(12)}}.$$

The reduced curvature reaches extreme values in two mutually perpendicular directions [7]:

a) in the direction coinciding with the tangent τ to the line of contact of surfaces Σ_1 and Σ_2,

b) in the direction perpendicular to τ.

In the first direction, $\kappa^{(p)} = \kappa^{(p)}_{min} = 0$ since $\kappa^{(1)} = \kappa^{(2)}$. Because of this, we obtain

$$(4.10)\qquad \tan q_{\tau} = -\frac{a_{31}}{a_{32}} = -\frac{(\omega^{(12)}, e^{(1)}, i_{I}) + \varkappa_{I} v_{I}^{(12)}}{(\omega^{(12)}, e^{(1)}, i_{II}) + \varkappa_{II} v_{II}^{(12)}}.$$

For the direction m perpendicular to tangent τ,

$$(4.11)\qquad \tan q_{m} = -\cot q_{\tau} = \frac{a_{32}}{a_{31}}.$$

After substituting (4.11) into (4.9), we obtain

$$\varkappa^{(p)}_{\max} = -\frac{a_{31}^2 + a_{32}^2}{b_3 + a_{31} v_{\mathrm{I}}^{(12)} + a_{32} v_{\mathrm{II}}^{(12)}} . \qquad (4.12)$$

In the case in which a rotary motion is transmitted between parallel or intersecting axes and the point of contact lies on the instantaneous axis of rotation,

$$\varkappa^{(p)}_{\max} = -\frac{(\omega^{(12)} \times e^{(i)})^2}{(\omega^{(12)}, e^{(i)}, n^{(i)})} \quad (i = 1; 2) . \qquad (4.13)$$

Let us now consider the case of point contact between surfaces Σ_1 and Σ_2. It is desirable here to write the expression for reduced curvature in the form of Euler's equation, which relates the normal curvature with the principal curvatures of the surface:

$$\varkappa^{(p)} = \varkappa^{(1)} - \varkappa^{(2)} = \varkappa_{\mathrm{I}} \cos^2 q + \varkappa_{\mathrm{II}} \sin^2 q - [\varkappa_{\mathrm{III}} \cos^2 (q - \sigma) + \varkappa_{\mathrm{IV}} \sin^2 (q - \sigma)] . \qquad (4.14)$$

Here q, as before, is in the plane of contact of the angle between the principal direction containing unit vector i_{I} and the direction in which $\kappa^{(p)}$ is considered.

The reduced curvature also reaches extreme values in two mutually perpendicular directions in the case of point contact between the surfaces [7]. It is apparent that $\kappa^{(p)} = \kappa^{(p)}_{max}$ if $d\kappa^{(p)}/dq = 0$. From this it follows that

$$\tan 2q = \frac{(\varkappa_{\mathrm{III}} - \varkappa_{\mathrm{IV}}) \sin 2\sigma}{(\varkappa_{\mathrm{II}} - \varkappa_{\mathrm{I}}) + (\varkappa_{\mathrm{III}} - \varkappa_{\mathrm{IV}}) \cos 2\sigma} = \frac{2 a_{12}}{a_{11} - a_{22}} . \qquad (4.15)$$

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