NASA Technical Memorandum 80100

NASA-TM-80100 19790023169

# THE RIM INERTIAL MEASURING SYSTEM (RIMS)

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AUGUST 1979



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## SUMMARY

The concept of the Rim Inertial Measuring System (RIMS) is introduced and an approach for extracting angular rate and linear acceleration information from an RIMS unit is presented and discussed. The RIMS consists of one or more small Annular Momentum Control Devices (AMCDs), mounted in a strapped down configuration, which are used to measure angular rates and linear accelerations of a moving vehicle. An AMCD consists of a spinning rim, a set of noncontacting magnetic bearings for supporting the rim, and a noncontacting electromagnetic spin motor. The approach for extracting angular rate and linear acceleration information is for a single spacecraft mounted RIMS unit.

## INTRODUCTION

The Rim Inertial Measuring System (RIMS) consists of small Annular Momentum Control Devices (AMCDs), mounted in a strapped down configuration, which are used to measure angular rates and linear accelerations of a moving vehicle such as a spacecraft or aircraft. An AMCD, shown schematically in figure 1, consists of a spinning rim (no central hub section), a set of noncontacting magnetic suspension stations for supporting the rim, and a noncontacting electromagnetic motor for spinning the rim. A description of the AMCD concept and its potential applications are presented in reference 1. The measured angular rates and linear accelerations are processed to yield the attitude and position and time derivatives of attitude and position of the vehicle.

Using either magnetic bearing force information or magnetic bearing position sensor information an RIMS with one AMCD can provide linear acceleration measurements along three orthogonal axes and angular rate measurements about two orthogonal axes. In order to obtain rate measurements about three orthogonal axes a minimum of two AMCDs would be required. This paper presents the equations of motion for a spacecraft-mounted RIMS which contains one AMCD. An approach for extracting angular rate and linear acceleration information from the RIMS magnetic suspension system parameters is also presented and discussed. The magnetic bearing system considered consists of three equally spaced magnetic bearing suspension stations which provide active control in both the axial and radial directions.

N79-31340 #

## SYMBOLS

| | |
|---|---|
| $A$ | defined by equation (15) |
| $a_{S1}, a_{S2}, a_{S3}$ | acceleration along the spacecraft 1, 2, and 3 axes, respectively |
| $E_S$ | external disturbance torque acting on the spacecraft |
| $F$ | force produced by a given magnetic bearing actuator |
| $F_A$ | translational force on the AMCD rim |
| $\bar{F}_A$ | defined by equation (10) |
| $F_C$ | force command for a given magnetic bearing actuator |
| $F_R$ | radial force on the AMCD rim produced by the magnetic suspension system |
| $\bar{F}_{R1}, \bar{F}_{R2}$ | radial forces on the AMCD rim resolved along the 1 and 2 axes, respectively |
| $F_S$ | translational force on the spacecraft |
| $F_X$ | axial force on the AMCD rim produced by the magnetic suspension system |
| $G_A$ | total moment acting about the center of mass of the AMCD rim |
| $G_\varepsilon$ | error torque |
| $G_S$ | reaction torque on the spacecraft caused by motion of the AMCD rim acting through the magnetic suspension system |
| $g$ | gap displacement about a nominal operating point |
| $g_C$ | gap command |
| $g_R$ | gap measured at a given radial magnetic bearing actuator |
| $g_{RC}$ | gap command for a givan radial magnetic bearing actuator |

| | |
|---|---|
| $g_X$ | gap measured at a given axial magnetic bearing actuator |
| $g_{XC}$ | gap command for a given axial magnetic bearing actuator |
| $H_A$ | stored angular momentum of AMCD rim about the spin axis |
| $i$ | current in magnetic actuator electromagnet |
| $I_A$ | moment of inertia of the AMCD rim about a transverse axis |
| $I_S$ | moment of inertia of the spacecraft about a principal axis |
| $K_A$ | axial position gain |
| $\bar{K}_A$ | defined by equation (25) |
| $K_{AR}$ | radial position gain |
| $\bar{K}_{AR}$ | radial equivalent of $\bar{K}_A$ (see eqn. (36)) |
| $K_B$ | equivalent electromagnet gain |
| $K_m$ | equivalent permanent magnetic stiffness |
| $K_R$ | axial rate gain |
| $\bar{K}_R$ | defined by equation (25) |
| $K_{RR}$ | radial rate gain |
| $\bar{K}_{RR}$ | radial equivalent of $\bar{K}_R$ (see eqn. (36)) |
| $M$ | defined by equation (20) |
| $m_A$ | mass of the AMCD rim |
| $m_S$ | mass of the spacecraft |
| $r_{CA}$ | position of the AMCD rim center of mass with respect to inertial space |
| $r_{CAS}$ | defined by equations (4) and (6) |
| $r_{CS}$ | position of the spacecraft center of mass with respect to inertial space |
| $r_m$ | mean radius of the AMCD rim |
| $s$ | laplace variable |

| | |
|---|---|
| $T$ | defined by equation (16) |
| $\dot{\theta}_A$ | AMCD Euler rates |
| $\theta_A$ | AMCD Euler angles with respect to inertial axes (3, 2, 1 rotation sequence) |
| $\theta_{AS}$ | defined by equation (4) |
| $\dot{\theta}_S$ | spacecraft Euler rates |
| $\theta_S$ | spacecraft Euler angles with respect to inertial axes (3, 2, 1 rotation sequence) |
| $\Omega_S$ | laplace transform of $\dot{\theta}_S$ |
| 1, 2, 3 | orthogonal axis system |

Notation:

| | |
|---|---|
| [ ] | rectangular matrix |
| $[\ ]^{-1}$ | inverse of [ ] |
| $[\ ]^T$ | transpose of [ ] |
| $[\ ]^{\#}$ | generalized inverse of [ ] |
| { } | column vector |

A single dot over a symbol denotes a first derivative with respect to time. Double dots denote a second derivative.

Subscripts:

| | |
|---|---|
| 1, 2, 3 | orthogonal components along 1, 2, 3 axes, respectively |
| A | AMCD coordinate system |
| a, b, c | magnetic bearing station a, b, or c |
| S | spacecraft coordinate system |
| C | commanded quantity |

## Equations of Motion

The equations of motion presented here are taken from reference 1. The location of the magnetic bearing suspension stations with respect to the AMCD axis system is shown in figure 2. From reference 1, the rotational equations of motion for the AMCD-spacecraft system (using small angle and rate assumptions) are

$$\begin{Bmatrix} \ddot{\theta}_{A1} \\ \ddot{\theta}_{A2} \\ \ddot{\theta}_{S1} \\ \ddot{\theta}_{S2} \\ \ddot{\theta}_{S3} \end{Bmatrix} = \begin{Bmatrix} (1/I_A)(G_{A1} - \dot{\theta}_{A2} H_A) \\ (1/I_A)(G_{A2} + \dot{\theta}_{A1} H_A) \\ (1/I_{S1})(G_{S1} + E_{S1}) \\ (1/I_{S2})(G_{S2} + E_{S2}) \\ (1/I_{S3})(E_{S3}) \end{Bmatrix} \tag{1}$$

where $\theta_{A1}$ and $\theta_{A2}$ are AMCD Euler angles with respect to inertial space; $\theta_{S1}$, $\theta_{S2}$, and $\theta_{S3}$ are spacecraft Euler angles with respect to inertial space; $I_A$ is the transverse moment of inertia of the AMCD rim; $I_{S1}$, $I_{S2}$, and $I_{S3}$ are spacecraft moments of inertia; $G_{A1}$ and $G_{A2}$ are torques acting about the center of mass of the AMCD rim produced by the magnetic bearings; $G_{S1}$ and $G_{S2}$ are reaction torques on the spacecraft produced by motion of the AMCD rim acting through the magnetic bearings; $E_{S1}$, $E_{S2}$, and $E_{S3}$ are external disturbance torques acting on the spacecraft; and $H_A$ is the stored momentum of the AMCD rim about the spin axis. The translational equations are

$$\begin{Bmatrix} \ddot{r}_{CA1} \\ \ddot{r}_{CA2} \\ \ddot{r}_{CA3} \\ \ddot{r}_{CS1} \\ \ddot{r}_{CS2} \\ \ddot{r}_{CS3} \end{Bmatrix} = \begin{Bmatrix} (1/m_A)F_{A1} \\ (1/m_A)F_{A2} \\ (1/m_A)F_{A3} \\ (1/m_S)F_{S1} \\ (1/m_S)F_{S2} \\ (1/m_S)F_{S3} \end{Bmatrix} \tag{2}$$

where $r_{CA1}$, $r_{CA2}$, and $r_{CA3}$ are components of the vector which locates the AMCD rim center of mass with respect to the origin of the inertial coordinate system; $r_{CS1}$, $r_{CS2}$, and $r_{CS3}$ are components of the vector which locates the spacecraft center of mass with respect to the origin of the inertial coordinate system; $F_{A1}$, $F_{A2}$, and $F_{A3}$ are translational forces on the AMCD rim (in AMCD axes) produced by the magnetic bearings; $F_{S1}$, $F_{S2}$, $F_{S3}$ are the total translational forces on the spacecraft (in spacecraft axes) which includes the forces produced by the magnetic bearings; and $m_A$ and $m_S$ are the masses of the AMCD rim and spacecraft, respectively. In the RIMS, the AMCD is used as a sensing device only and consequently will be made as small as possible. The reaction torques on the spacecraft ($G_{S1}$ and $G_{S2}$), and translational forces (elements of $F_{S1}$, $F_{S2}$, and $F_{S3}$), produced by motion of the AMCD rim acting through the magnetic bearings can be ignored in this case and the spacecraft equations of motion become uncoupled from the AMCD equations of motion. By assuming the center of mass of the AMCD to be coincident with the center of mass of the spacecraft, the axial magnetic bearing gaps, in terms of rotations and translations, can be written as (from ref. 1)

$$\begin{Bmatrix} g_{Xa} \\ g_{Xb} \\ g_{Xc} \end{Bmatrix} = \begin{bmatrix} (\sqrt{3}/2)r_m & -(1/2)r_m & 1 \\ -(\sqrt{3}/2)r_m & -(1/2)r_m & 1 \\ 0 & r_m & 1 \end{bmatrix} \begin{Bmatrix} \theta_{AS1} \\ \theta_{AS2} \\ r_{CAS3} \end{Bmatrix} \tag{3}$$

where $g_{Xa}$, $g_{Xb}$, and $g_{Xc}$ are the axial gaps for bearing stations a, b, and c respectively; $r_m$ is the radius of the AMCD rim; and $\theta_{AS1}$, $\theta_{AS2}$, and $r_{CAS3}$ are defined as

$$\begin{pmatrix} \theta_{AS1} \triangleq [\theta_{A1} - \theta_{S1}] \\ \theta_{AS2} \triangleq [\theta_{A2} - \theta_{S2}] \\ r_{CAS3} \triangleq [r_{CA3} - r_{CS3}] \end{pmatrix} \tag{4}$$

The radial gaps in terms of translations can be written as

$$\begin{Bmatrix} g_{Ra} \\ g_{Rb} \\ g_{Rc} \end{Bmatrix} = \begin{bmatrix} 1/2 & \sqrt{3}/2 \\ 1/2 & -\sqrt{3}/2 \\ -1 & 0 \end{bmatrix} \begin{Bmatrix} r_{CAS1} \\ r_{CAS2} \end{Bmatrix} \tag{5}$$

where $g_{Ra}$, $g_{Rb}$, and $g_{Rc}$ are the radial gaps for bearing stations a, b, and c respectively and $r_{CAS1}$ and $r_{CAS2}$ are defined as

$$\left(\begin{array}{l} r_{CAS1} \triangleq [r_{CA1} - r_{CS1}] \\ r_{CAS2} \triangleq [r_{CA2} - r_{CS2}] \end{array}\right) \tag{6}$$

It should be noted that the assumption that the AMCD and spacecraft centers of mass are coincident was made only to simplify the equations for the present development. The general AMCD-spacecraft equations of motion given in reference 1 are for an arbitrary location of the AMCD center of mass with respect to the spacecraft center of mass. The torques on the rim due to axial bearing forces are

$$\begin{Bmatrix} G_{A1} \\ G_{A2} \end{Bmatrix} = \begin{bmatrix} (\sqrt{3}/2)r_m & -(\sqrt{3}/2)r_m & 0 \\ -(1/2)r_m & -(1/2)r_m & r_m \end{bmatrix} \begin{Bmatrix} F_{Xa} \\ F_{Xb} \\ F_{Xc} \end{Bmatrix} \tag{7}$$

The radial forces resolved along the 1 and 2 axes are

$$\begin{Bmatrix} \bar{F}_{R1} \\ \bar{F}_{R2} \end{Bmatrix} = \begin{bmatrix} 1/2 & 1/2 & -1 \\ \sqrt{3}/2 & -\sqrt{3}/2 & 0 \end{bmatrix} \begin{Bmatrix} F_{Ra} \\ F_{Rb} \\ F_{Rc} \end{Bmatrix} \tag{8}$$

where $F_{R1}$ and $F_{R2}$ are forces on the rim along the 1 and 2 axes and $F_{Ra}$, $F_{Rb}$, and $F_{Rc}$ are the radial forces produced by bearing stations a, b, and c respectively. The rim rotational and axial translational dynamics are (from equations 1 and 2)

$$\begin{Bmatrix} \ddot{\theta}_{A1} \\ \ddot{\theta}_{A2} \\ \ddot{r}_{CA3} \end{Bmatrix} = \begin{Bmatrix} (1/I_A) & (G_{A1} - \dot{\theta}_{A2} H_A) \\ (1/I_A) & (G_{A2} + \dot{\theta}_{A1} H_A) \\ (1/m_A) & \bar{F}_A \end{Bmatrix} \tag{9}$$

where

$$\bar{F}_A = F_{Xa} + F_{Xb} + F_{Xc} \tag{10}$$

Finally, the radial translational dynamics become (from eqn. 2)

$$\begin{Bmatrix} \ddot{r}_{CA1} \\ \ddot{r}_{CA2} \end{Bmatrix} = 1/m_A \begin{Bmatrix} \bar{F}_{R1} \\ \bar{F}_{R2} \end{Bmatrix} \tag{11}$$

## MAGNETIC BEARING SUSPENSION SYSTEM

The AMCD utilized in the present RIMS application has a magnetic bearing suspension system which provides active positioning control of the rim in both the axial and radial directions. The axial and radial suspension systems are independent and are designed separately.

Magnetic bearings.- The magnetic bearing actuators selected for the present development utilize permanent magnet flux-biasing. This is the technique used for a laboratory test model AMCD (ref. 3). For small motions about a given operating point, the magnetic bearing force as a function of electromagnet current and rim displacement with respect to the operating point can be written as (ref. 3)

$$F = K_B\ i + K_m g \tag{12}$$

where $K_B$ is an equivalent electromagnet gain, $i$ is electromagnet current, $K_m$ is an equivalent permanent magnet stiffness, and $g$ is gap displacement about a nominal operating point.

Axial system.- The axial magnetic suspension control system approach selected for the present development is one that uses independent control loops for each suspension station. At zero rim spin speed (zero momentum), for three magnetic bearing suspension stations spaced equidistantly around the rim and for theoretical rim inertia distribution, it can shown that axial motions of the rim in each of the bearing stations are uncoupled. That is, axial motion of the rim in one bearing produces no motion in the other two bearings. Consequently, at zero momentum the axial magnetic bearing control system can be represented as three identical independent systems and a single design, using a simplified suspended mass model, can be performed. Using this design approach, the closed loop magnetic bearing control system parameters required to produce desired system performance at a given rim momentum are obtained by analyses similar to those in reference 2. The independent station design approach was taken for a laboratory test model AMCD which is described in reference 3.

In order to illustrate this approach, assume that the electromagnet current in equation (12) is a function of rim position error and error rate. The force as a function of gap error can then be written as

$$F = K_B\ (K_A + K_R s)(g_C - g) + K_m g \tag{13}$$

where $K_A$ is a position gain, $K_R$ is a rate gain, and $(g_C - g)$ is the rim position error signal where $g_C$ is a gap command and $g$ is the actual signal. Taking the Laplace transform of equation (9) and rearranging results in

$$\begin{bmatrix} I_A\ s^2 & H_A s & 0 \\ -H_A s & I_A\ s^2 & 0 \\ 0 & 0 & m_A\ s^2 \end{bmatrix} \begin{Bmatrix} \theta_{A1} \\ \theta_{A2} \\ r_{CAS3} \end{Bmatrix} = \begin{Bmatrix} G_{A1} \\ G_{A2} \\ \bar{F}_A \end{Bmatrix} \tag{14}$$

In order to simplify the terms in the following development, define

$$[A] \triangleq \begin{bmatrix} I_A\ s^2 & H_A s & 0 \\ -H_A s & I_A\ s^2 & 0 \\ 0 & 0 & m_A\ s^2 \end{bmatrix} \tag{15}$$

and

$$[T] \triangleq \begin{bmatrix} (\sqrt{3}/2)r_m & -(1/2)r_m & 1 \\ -(\sqrt{3}/2)r_m & -(1/2)r_m & 1 \\ 0 & r_m & 1 \end{bmatrix} \tag{16}$$

Equation (14) becomes

$$[A] \begin{Bmatrix} \theta_A \\ \hline r_{CA3} \end{Bmatrix} = \begin{Bmatrix} G_A \\ \hline \bar{F}_A \end{Bmatrix} = [T]^T \ \{F_X\} \tag{17}$$

Substituting from equation (13) results in

$$[A] \begin{Bmatrix} \theta_A \\ \hline r_{CA3} \end{Bmatrix} = K_B(K_A + K_R\ s)\ [T]^T\ (\{g_{Xc}\} - \{g_X\}) + K_m[T]^T\ \{g_X\} \tag{18}$$

Radial system.- The equations of motion for the radial system are somewhat simpler than the axial system since momentum coupling is not involved. From equation (2)

$$\begin{Bmatrix} \ddot{r}_{CA1} \\ \ddot{r}_{CA2} \end{Bmatrix} = (1/m_A) \begin{Bmatrix} \bar{F}_{R1} \\ \bar{F}_{R2} \end{Bmatrix} \tag{19}$$

where $\bar{F}_{R1}$ and $\bar{F}_{R2}$ are defined in equation (8). In order to simplify the terms in the following development define

$$[M] = \begin{bmatrix} 1/2 & \sqrt{3}/2 \\ 1/2 & -\sqrt{3}/2 \\ -1 & 0 \end{bmatrix} \tag{20}$$

Making the electromagnet current a function of rim position error and error rate, the force produced by a given radial bearing can be written as

$$F_R = K_{BR} \, (K_{AR} + K_{RR} \, s) \, (g_{RC} - g_R) + K_{mR} \, g_R \tag{21}$$

where $K_{AR}$ is radial position gain, $K_{RR}$ is radial rate gain, $K_{BR}$ is the equivalent electromagnet gain, $K_{mR}$ is the equivalent permanent magnet stiffness, and $(g_{RC} - g_R)$ is the rim radial position error signal where $g_{RC}$ is a radial gap command and $g_R$ is the actual radial gap signal. Using equation (6), the radial accelerations in terms of bearing forces become

$$\begin{Bmatrix} \ddot{r}_{CA1} \\ \ddot{r}_{CA2} \end{Bmatrix} = \left(\frac{1}{m_A}\right) [M]^T \begin{Bmatrix} F_{Ra} \\ F_{Rb} \\ F_{Rc} \end{Bmatrix} \tag{22}$$

which, from equation (21), becomes

$$\begin{Bmatrix} \ddot{r}_{CA1} \\ \ddot{r}_{CA2} \end{Bmatrix} = \left(\frac{1}{m_A}\right) K_{BR}(K_{AR} + K_{RR} \, s) \; [M]^T \left\{ g_{RC} - g_R \right\} + K_{mR} [M]^T \left\{ g_R \right\} \tag{23}$$

## ANGULAR RATE AND LINEAR ACCELERATION MEASUREMENT

This section presents a possible implementation of an RIMS consisting of a single AMCD. Using this system, angular rate about the AMCD transverse axes and linear acceleration along all three axes can be obtained. The rates and accelerations are obtained from magnetic bearing position sensor information. The resulting information is in spacecraft body axes in the manner of a strapped down inertial measuring system.

Measurements from the axial system.- It is assumed that the AMCD rim will be operated in a "centered" position about a nominal operating point. The gap command term in equation (18) would be zero in this case and the equation can be written as

$$[A]\left\{\begin{array}{c}\theta_A\\ \hline r_{CA3}\end{array}\right\} = -(KB\ K_R s + [K_B K_A - K_m])\ [T]^T \{g_X\} \qquad (24)$$

For the purposes of simplification, the following definitions are made

$$\left(\begin{array}{l}\bar{K}_R \triangleq K_B\ K_R\\ \bar{K}_A \triangleq [K_B\ K_A - K_m]\end{array}\right) \qquad (25)$$

Using these definitions and the transformation of equation (3), equation (24) becomes

$$[A]\left\{\begin{array}{c}\theta_A\\ \hline r_{CA3}\end{array}\right\} = -\ (\bar{K}_R\ s + \bar{K}_A)\ [T]^T\ [T]\left\{\begin{array}{c}\theta_{AS}\\ \hline r_{CAS3}\end{array}\right\} \qquad (26)$$

Equation (26) rearranged and expanded becomes

$$\left(\begin{array}{c}I_A\ s^2\ \theta_{A1} + H_A s\ \theta_{A2} + 1.5r_m^{\ 2}\ (\bar{K}_R s + \bar{K}_A)\ \theta_{AS1} = 0\\ I_A\ s^2\ \theta_{A2} - H_A s\ \theta_{A1} + 1.5r_m^{\ 2}\ (\bar{K}_R s + \bar{K}_A)\ \theta_{AS2} = 0\\ m_A s^2\ r_{CA3} + 3\ (\bar{K}_R s + \bar{K}_A)\ r_{CAS3} = 0\end{array}\right) \qquad (27)$$

Substituting from equation (4) results in

$$\left(\begin{array}{c}I_A\ s^2\ (\theta_{AS1} + \theta_{S1}) + H_A s\ (\theta_{AS2} + \theta_{S2}) + 1.5r_m^{\ 2}\ (\bar{K}_R s + \bar{K}_A)\ \theta_{AS1} = 0\\ I_A\ s^2\ (\theta_{AS2} + \theta_{S2}) - H_A s\ (\theta_{AS1} + \theta_{S1}) + 1.5r_m^{\ 2}\ (\bar{K}_R s + \bar{K}_A)\ \theta_{AS2} = 0\\ m_A s^2\ (r_{CAS3} + r_{CS3}) + 3\ (K_R s + K_A)\ r_{CAS3} = 0\end{array}\right) \qquad (28)$$

Solving for $\theta_{AS1}$ gives

$$\theta_{AS1} = \frac{-H_A s\theta_{S2} - H_A s\theta_{AS2} - I_A s^2\theta_{S1}}{I_A s^2 + 1.5r_m^{\ 2}\ \bar{K}_R s + 1.5r_m^{\ 2}\ \bar{K}_A} \qquad (29)$$

The term $s\theta_{S2}$ represents the spacecraft rate, $\Omega_{S2}$, about the 2 axis and the term $s\theta_{S2}$ represents the relative rate between the AMCD rim and spacecraft about the 2 axis. The relative rate between rim and spacecraft is a transient term and its effects can be minimized by proper selection of magnetic bearing control loop parameters (i.e., $\bar{K}_R$ and $\bar{K}_A$). The term $I_A s^2\theta_{S1}$ represents a torque on the rim due to a spacecraft acceleration about the 1 axis. This term represents an error torque which is characteristic of rate gyros. The terms $H_A s\theta_{AS2}$ and $I_A s^2\theta_{S1}$ can be lumped together as an error torque $G_{\varepsilon 1}$. Equation (29) then can be written as

$$\theta_{AS1} = \frac{-H_A \Omega_{S2} - G_{\varepsilon 1}}{I_A s^2 + 1.5r_m^2 \bar{K}_R s + 1.5r_m^2 \bar{K}_A} \tag{30}$$

Assuming negligible errors the steady state response of equation (30) becomes

$$\theta_{AS1} = -\left(\frac{H_A}{1.5r_m^2 \bar{K}_A}\right)\Omega_{S2} \tag{31}$$

Solving for $\theta_{AS2}$ in a similar fashion results in

$$\theta_{AS2} = \frac{H_A \Omega_{S1} + G_{\varepsilon 2}}{I_A s^2 + 1.5r_m^2 \bar{K}_R s + 1.5r_m^2 \bar{K}_A} \tag{32}$$

which has a steady state response of (neglecting errors)

$$\theta_{AS2} = \left(\frac{H_A}{1.5r_m^2 \bar{K}_A}\right)\Omega_{S1} \tag{33}$$

Turning next to $r_{CAS3}$

$$r_{CAS3} = \frac{-m_A s^2 r_{CS3}}{m_A s^2 + 3\bar{K}_R s + 3\bar{K}_A} \tag{34}$$

where $s^2 r_{CS3}$ represents the spacecraft acceleration, $a_{S3}$, along the 3 axis. Equation (34) has a steady state response given by

$$r_{CAS3} = -\left(\frac{m_A}{3\bar{K}_A}\right) a_{S3} \tag{35}$$

A block diagram showing a proposed implementation of angular rate measurements about the spacecraft 1 and 2 axes and linear acceleration along the 3 axis is shown in figure 3. Basically, the scheme uses axial magnetic bearing position sensor information which is transformed, using equation 3, into relative angular and linear displacements of the rim with respect to the spacecraft. The sensors could be the same type as used on the laboratory model AMCD (ref. 3) and the gain and summing portions of the block diagram could be analog operational amplifiers.

Measurements from the radial system.- The radial position sensor outputs can be used to determine spacecraft accelerations along the 1 and 2 axes in a manner similar to that of obtaining acceleration along the 3 axis with the axial sensors. Assuming a rim centered mode, equation (23) can be written as

$$\begin{Bmatrix} m_A s^2 & r_{CA1} \\ m_A s^2 & r_{CA2} \end{Bmatrix} = -(\bar{K}_{RR}\, s + \bar{K}_{AR})\ [M]^T \{g_R\} \tag{36}$$

where $\bar{K}_{RR} = K_{BR}K_{RR}$ and $\bar{K}_{AR} = (K_{BR}K_{AR} - K_{MR})$

Making the substitution $\{g_R\} = [M]\begin{Bmatrix} r_{CAS1} \\ r_{CAS2} \end{Bmatrix}$, equation (36) expanded becomes

$$\begin{pmatrix} m_A s^2 \quad r_{CA1} = -(\bar{K}_{RR}\, s + \bar{K}_{AR})\ r_{CAS1} \\ m_A s^2 \quad r_{CA2} = -(\bar{K}_{RR}\, s + \bar{K}_{AR})\ r_{CAS2} \end{pmatrix} \tag{37}$$

Substituting from equation (6) and rearranging terms results in

$$\begin{pmatrix} (m_A s^2 + \bar{K}_{RR}\, s + \bar{K}_{AR})\ r_{CAS1} = -\ m_A s^2\ r_{CS1} \\ (m_A s^2 + \bar{K}_{RR}\, s + \bar{K}_{AR})\ r_{CAS2} = -\ m_A s^2\ r_{CS2} \end{pmatrix} \tag{38}$$

Solving for $r_{CAS1}$ gives

$$r_{CAS1} = \frac{-m_A s^2\ r_{CS1}}{m_A s^2 + \bar{K}_{RR}\, s + \bar{K}_{AR}} \tag{39}$$

where the term $s^2\ r_{CS1}$ represents spacecraft acceleration, $a_{S1}$, along the 1 axis. Equation (39) has a steady state solution of

$$r_{CAS1} = \left(\frac{-m_A}{\bar{K}_{AR}}\right) a_{S1} \tag{40}$$

Solving for $r_{CAS2}$ gives

$$r_{CAS2} = \frac{-m_A s^2\ r_{CS2}}{m_A s^2 + \bar{K}_{RR}\, s + \bar{K}_{AR}} \tag{41}$$

where the term $s^2 r_{CS2}$ represents spacecraft acceleration, $a_{S2}$, along the 2 axis. Equation (41) has a steady state response of

$$r_{CAS2} = \left(\frac{-m_A}{\bar{K}_{AR}}\right) a_{S2} \tag{42}$$

A block diagram showing a proposed implementation of linear acceleration measurements along the spacecraft 1 and 2 axes using radial magnetic bearing position sensor information is shown in figure 4. The transformation shown in this figure is obtained by taking the generalized inverse of [M]. That is, since

$$\{g_R\} = [M] \begin{Bmatrix} r_{CAS1} \\ r_{CAS2} \end{Bmatrix} \tag{43}$$

then

$$\begin{Bmatrix} r_{CAS1} \\ r_{CAS2} \end{Bmatrix} = [M]^{\#} \{g_R\} \tag{44}$$

where $[M]^{\#}$ represents the generalized inverse of [M] and is defined as

$$[M]^{\#} = ([M]^T [M])^{-1} [M]^T \tag{45}$$

Computing $[M]^{\#}$ yields

$$[M]^{\#} = \begin{bmatrix} 1/3 & 1/3 & -2/3 \\ 1/\sqrt{3} & -1/\sqrt{3} & 0 \end{bmatrix} \tag{46}$$

## CONCLUDING REMARKS

The basic Rim Inertial Measuring System (RIMS) has been described and a possible implementation presented. The purpose of the RIMS is to provide measurements of angular rates and linear accelerations which can be processed to yield the attitude and position and time derivatives of attitude and position of a moving vehicle such as a spacecraft or aircraft. The implementation presented is that of a spacecraft mounted RIMS utilizing a single AMCD to provide linear acceleration along three spacecraft axes and angular rate about two spacecraft axes. Angular rate information about the third spacecraft axis would require an additional AMCD utilized in the same manner. The magnetic bearing and bearing control system design approach for the AMCD used in the implementation presented is similar to that used for a laboratory test model AMCD. It should be obvious that any number of bearing and bearing control system design approaches could be taken. For example, bearings without permanent magnets could be used and may be desirable (ref. 4). Also, instead of independent station control for the axial system, the rim rotations about the transverse axes could be controlled separately from the axial translation. Also, optimal estimators could be used for estimating the desired parameters. Another variation could be more suspension stations. For example, the complexity of the bearing gap transformation could be reduced at the cost of adding another suspension station which would bring the total to four. In the area of electronics, a digital system could be used instead of the analog system presented. Finally, the bearing forces could be used as parameters to compute the desired rates and accelerations as can be seen from an examination of the suspension system equations. Bearing forces can be computed from electromagnet current and bearing gap information (eqn. 12).

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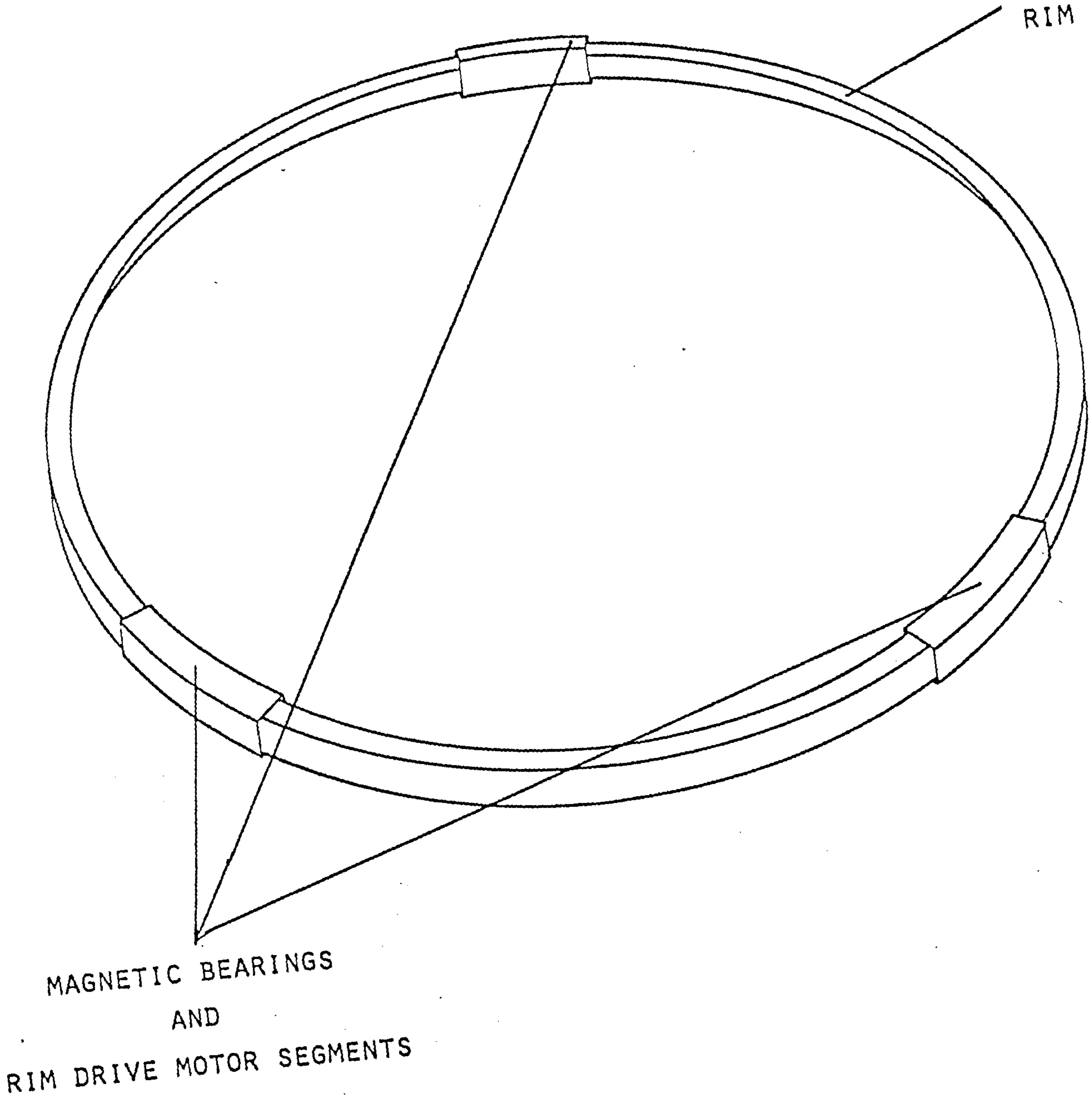


Figure 1.- Basic AMCD concept.

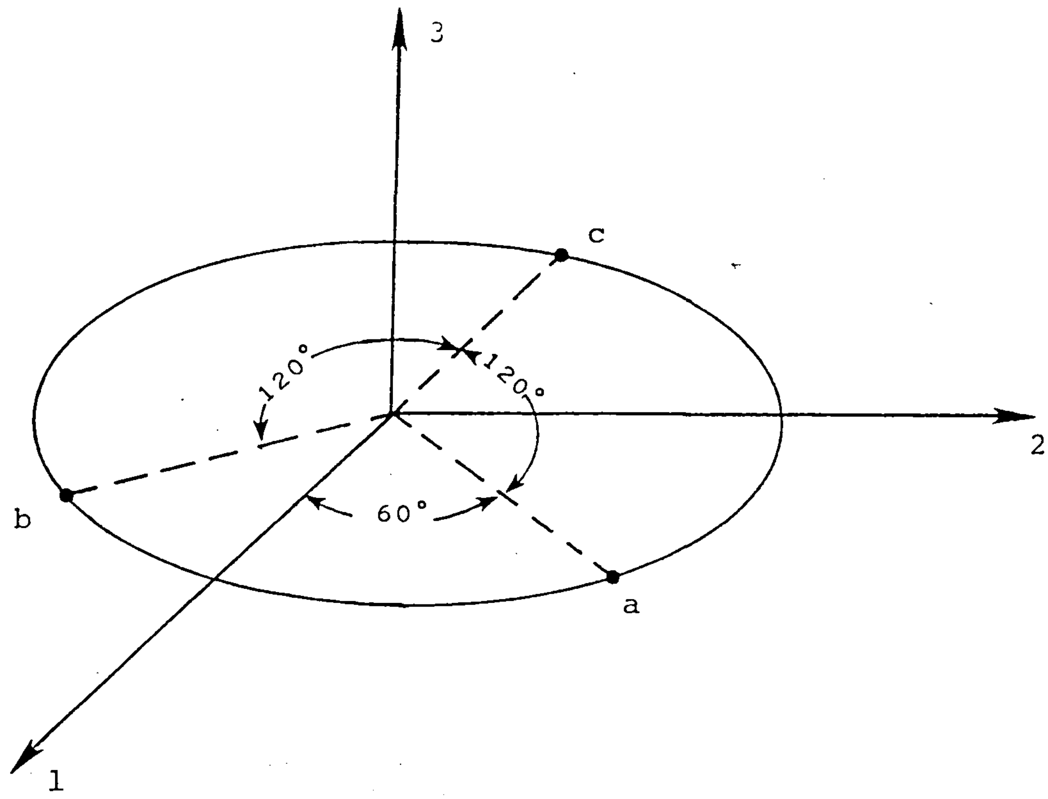


Figure 2.- Locations of magnetic bearing suspension stations with respect to AMCD axes.

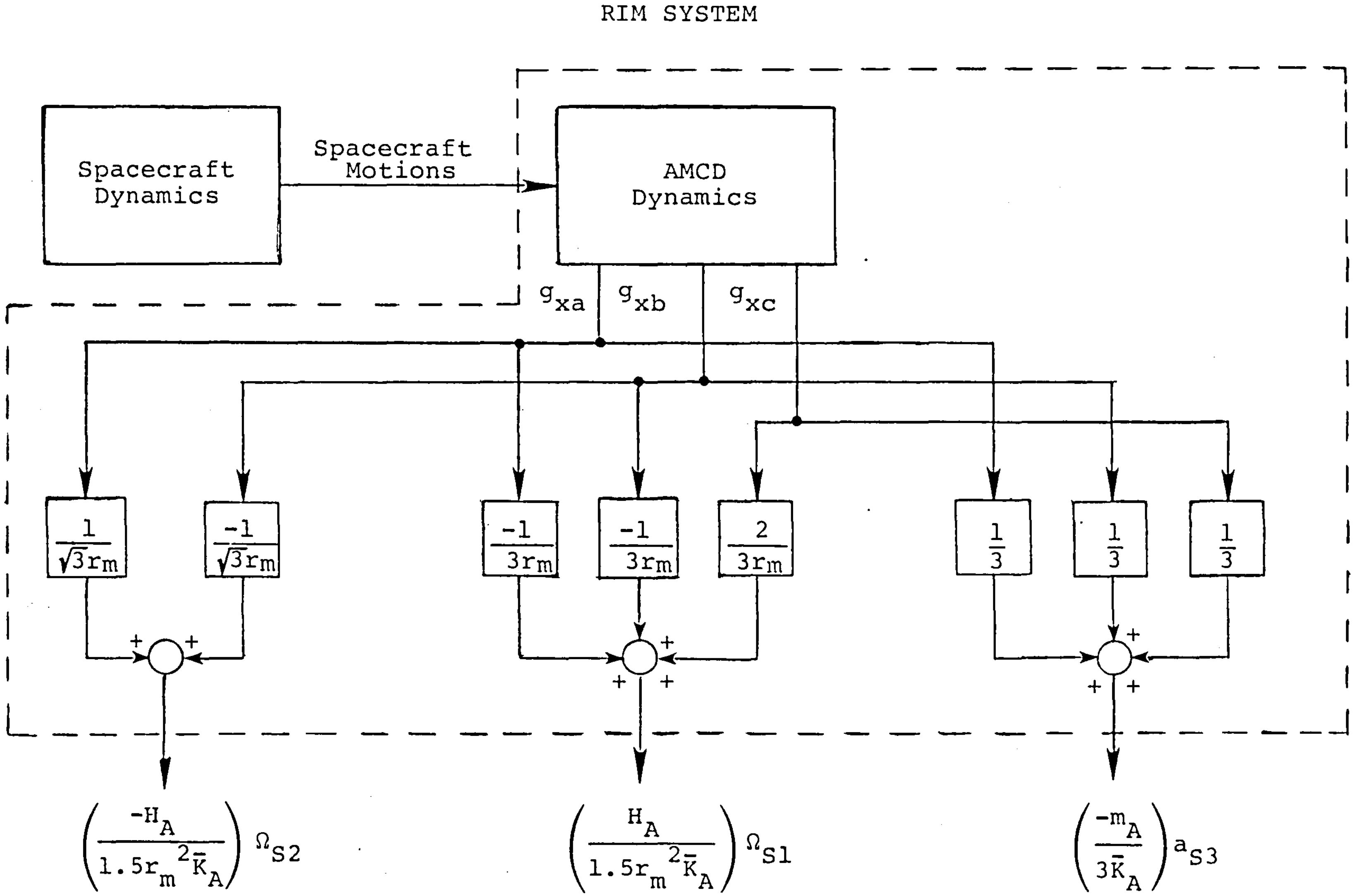


Figure 3.- Measurements from axial components of RIM system.

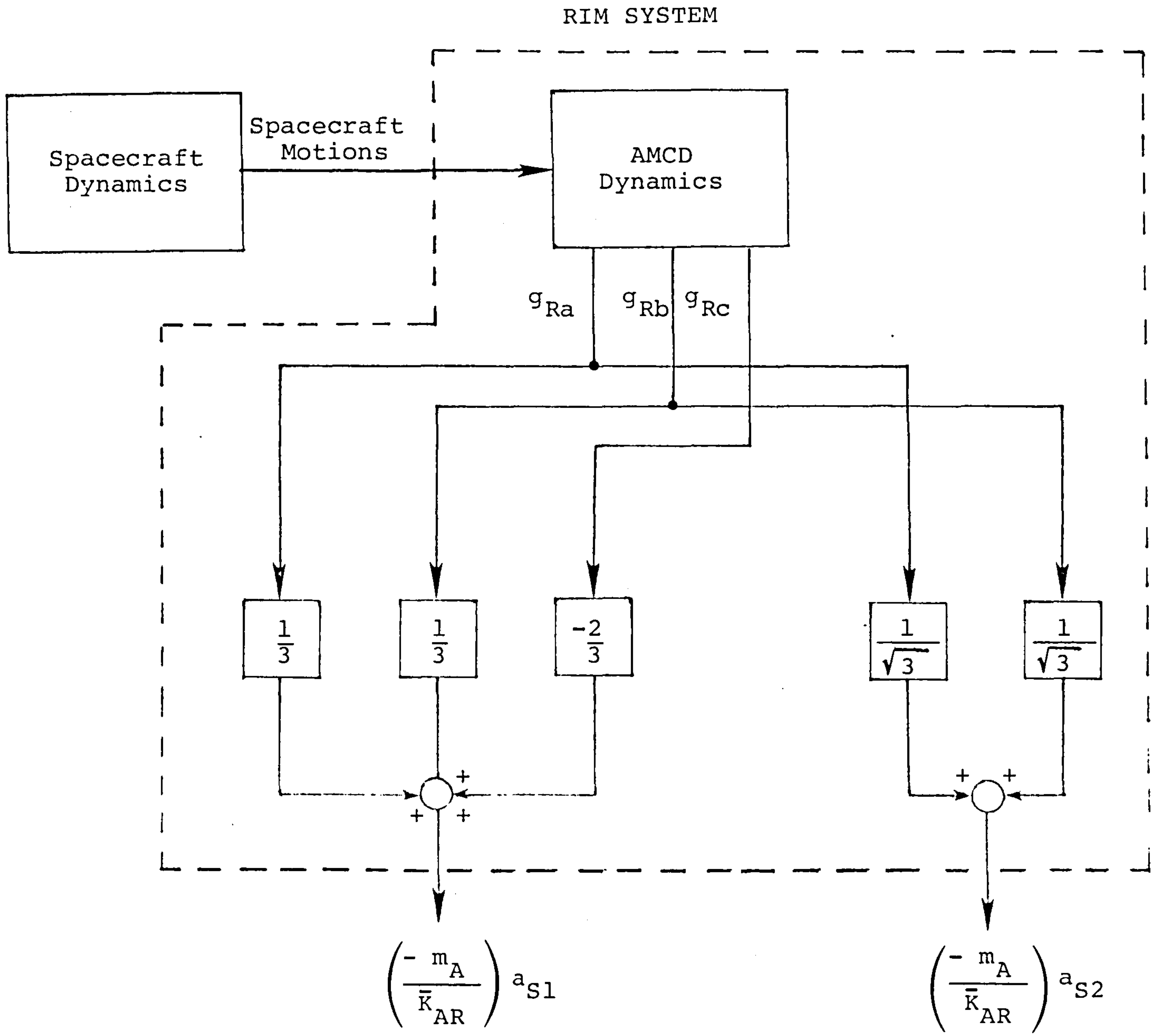


Figure 4.- Measurements from radial components of RIM system.

| 1. Report No. NASA TM-80100 | 2. Government Accession No. | 3. Recipient's Catalog No. |
|---|---|---|
| 4. Title and Subtitle The Rim Inertial Measuring System (RIMS) | | 5. Report Date August 1979 |
| | | 6. Performing Organization Code |
| 7. Author(s) Nelson J. Groom | | 8. Performing Organization Report No. |
| | | 10. Work Unit No. 506-19-13-01 |
| 9. Performing Organization Name and Address NASA Langley Research Center Hampton, Virginia 23665 | | 11. Contract or Grant No. |
| | | 13. Type of Report and Period Covered Technical Memorandum |
| 12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 20546 | | 14. Sponsoring Agency Code |
| 15. Supplementary Notes | | |

16. Abstract

The concept of the Rim Inertial Measuring System (RIMS) is introduced and an approach for extracting angular rate and linear acceleration information from a RIMS unit is presented and discussed. The RIMS consists of one or more small Annular Momentum Control Devices (AMCDs), mounted in a strapped down configuration, which are used to measure angular rates and linear accelerations of a moving vehicle. An AMCD consists of a spinning rim, a set of noncontacting magnetic bearings for supporting the rim, and a noncontacting electromagnetic spin motor. The approach for extracting angular rate and linear acceleration information is for a single spacecraft mounted RIMS unit.

| 17. Key Words (Suggested by Author(s)) | | 18. Distribution Statement | |
|---|---|---|---|
| Inertial Measuring System<br>Magnetic suspension<br>Annular Momentum Control Device | | Unclassified - Unlimited<br><br>Subject Category 19 | |
| 19. Security Classif. (of this report) | 20. Security Classif. (of this page) | 21. No. of Pages | 22. Price* |
| Unclassified | Unclassified | 19 | $4.00 |

* For sale by the National Technical Information Service, Springfield, Virginia 22161

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