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Final Report
For the period March 15, 1978 - September 30, 1979

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INTRODUCTION

This report summarizes work done in lap joint analyses under the Composites for Advanced Space Transportation Systems (CASTS) program during the period from March 15, 1978 to September 30, 1979. The purpose of this research was to introduce photoelasticity as a quantitative tool in the experimental stress analysis efforts of the CASTS program. Work was directed towards two goals: (1) developing experimental techniques and confirming experimentally the analysis of various single lap joint configurations and (2) developing the transmission photoelastic methods as applied to anisotropic birefringent composite model materials.

The results of the first part of the investigation, dealing with photoelastic analysis of lap joint configurations, are given in detail in "An Investigation of the Effect of Adhesive Stiffness on the Adherends of Single Lap Joints," a proposed NASA TP by Paul A. Cooper and Stephen A. Hann. This report will elaborate the work relating to birefringent anisotropic models. The work done in this phase of the investigation can be divided into three parts:

(1) separation of principal stresses or strains,

(2) photoelastic calibration, and

(3) development of model-prototype relations.

The following section will describe each of the above studies.

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WORK PERFORMED

Separation of Principal Stresses or Strains

In orthotropic birefringent models, the isochromatic fringes do not give the principal stress (or strain) difference. For a meaningful photoelastic analysis of such models, the individual values of principal stresses or strains ha'e to be determined. The methods suggested so far are not satisfactory because they require either successive integration (shear difference method, a method requiring numerical solution of the compatibility equation), leading to an accumulation of error, or a not-so-easy experimental technique (holography).

It was desired, therefore, to consider methods which were completely experimental and relatively simple. The first method examined was the possibility of using oblique incidence on birefringent composite models. When light is incident normally on such a model, the isochromatic fringe order is related to the principal strains as

$$N_o = \frac{h\varepsilon_x}{f_{\varepsilon L}} \left\{ \left[\left(\cos^2 \beta + \frac{\varepsilon_y}{\varepsilon_x} \sin^2 \beta \right) - \left(\sin^2 \beta + \frac{\varepsilon_y}{\varepsilon_x} \cos^2 \beta \right) \frac{f_{\varepsilon L}}{f_{\varepsilon T}} \right]^2 + \left[\left(1 - \frac{\varepsilon_y}{\varepsilon_x} \right) \frac{f_{\varepsilon L}}{f_{\gamma LT}} \sin 2\beta \right]^2 \right\}^{1/2} \qquad (\ )$$

where ε_x is the major principal strain, ε_y is the minor principal strain, β is the angle between ε_x and the material symmetry axis L, and $f_{\varepsilon L}$, $f_{\varepsilon T}$, $f_{\gamma LT}$ are the principal material-strain-fringe values. If the light beam is incident at an angle θ_x, the isochromatic fringe order for oblique incidence is given by

$$N_{\theta_x} = \frac{1}{\cos \theta_x} \frac{\varepsilon_x}{f_{\varepsilon L}} \left\{ \left[\left(\cos^2 \beta + \frac{\varepsilon'_y}{\varepsilon_x} \sin^2 \beta \right) - \left(\sin^2 \beta + \frac{\varepsilon'_y}{\varepsilon_x} \cos^2 \beta \right) \frac{f_{\varepsilon L}}{f_{\varepsilon T}} \right]^2 + \left[\left(1 - \frac{\varepsilon'_y}{\varepsilon_x} \right) \frac{f_{\varepsilon L}}{f_{\gamma LT}} \sin 2\beta \right]^2 \right\}^{1/2} \qquad (2)$$

where ε'_y is the secondary principal strain and is given by

$$\varepsilon'_y = \varepsilon_y \cos^2 \theta_x + \varepsilon_z \sin^2 \theta_x \tag{3}$$

For a model under a state of plane stress, it is possible to show that the strain normal to the plane of the model is related to the in-plane principal strains through the following relation:

$$\varepsilon_z = -\varepsilon_x \left[\frac{\cos^2 \beta(\nu_{L3} + \nu_{T3}\nu_{LT}) + \sin^2 \beta(\nu_{T3} + \nu_{L3}\nu_{TL})}{1 - \nu_{LT}\nu_{TL}}\right]$$

$$- \varepsilon_y\left[\frac{\sin^2 \beta(\nu_{L3} + \nu_{T3}\nu_{LT}) + \cos^2 \beta(\nu_{T3} + \nu_{L3}\nu_{TL})}{1 - \nu_{LT}\nu_{TL}}\right] \tag{4}$$

Substituting the expression for ε_z from equation (4) into equation (3) and again substituting the expression for ε'_y into equation (2) and simplifying, it is possible to write

$$N^2_{\theta_x} = \frac{h^2}{\cos^2 \theta_x} \frac{1}{f^2_{EL}} \left\{[C\varepsilon_x + D\varepsilon_y]^2 + [E\varepsilon_x + F\varepsilon_y]^2\right\} \tag{5}$$

where

$$C = \cos^2 \beta - A \sin^2 \theta_x \sin^2 \beta \cos^2 \beta - B \sin^2 \theta_x \sin^4 \beta - \frac{f_{EL}}{f_{ET}} \sin^2 \beta$$

$$+ \frac{f_{EL}}{f_{ET}} A \sin^2 \theta_x \cos^4 \beta + \frac{f_{EL}}{f_{ET}} B \sin^2 \theta_x \sin^2 \beta \cos^2 \beta \tag{6}$$

(Cont'd)

$$D = \cos^2 \theta_x \sin^2 \beta - A \sin^2 \theta_x \sin^4 \beta - B \sin^2 \theta_x \sin^2 \beta \cos^2 \beta$$

$$- \frac{f_{EL}}{f_{ET}} \cos^2 \theta_x \cos^2 \beta = \frac{f_{EL}}{f_{ET}} A \sin^2 \theta_x \sin^2 \beta \cos^2 \beta + \frac{f_{EL}}{f_{ET}} B \sin^2 \theta_x \cos^4 \beta$$

$$E = \frac{f_{EL}}{f_{\gamma LT}} \sin 2\beta \; (1 + A \sin^2 \theta_x \cos^2 \beta + B \sin^2 \theta_x - \sin^2 \beta)$$

$$F = \frac{f_{EL}}{f_{\gamma LT}} \sin 2\beta \; (-\cos^2 \theta_x + A \sin^2 \theta_x \sin^2 \beta + B \sin^2 \theta_x \cos^2 \beta) \qquad (6)$$

(Concl'd)

In equations (6), A and B are given by

$$A = \frac{\nu_{L3} + \nu_{T3}\nu_{LT}}{1 - \nu_{LT}\nu_{TL}}$$

$$B = \frac{\nu_{T3} + \nu_{L3}\nu_{TL}}{1 - \nu_{LT}\nu_{TL}} \qquad (7)$$

It is possible to solve for the principal strains ε_x and ε_y from equations (1) and (5), knowing the normal incidence-isochromatic fringe order, the oblique incidence-isochromatic fringe order, and the isoclinic parameter. It is, of course, necessary to measure the principal material properties, elastic and photoelastic. These are the Poisson's ratios ν_{LT}, ν_{TL}, and the strain-fringe values $f_{\varepsilon L}$, $f_{\varepsilon T}$, $f_{\gamma LT}$; the constants A and B can be obtained by a calibration procedure which also yields the angle of oblique incidence, θ_x, for the combination of the available oblique incidence adapter (using a chopped prism) and the model material.

Three calibration specimens, for instance tensile specimens, are chosen with the angle between the L-axis and the specimen axis (the latter being the loading direction) equal to 0°, 90°, and an intermediate value. The angle between the major principal strain direction and the L-axis, β, is known for each specimen; it is 0° for the first two and can be computed for the third. For each specimen, the normal and oblique incidence fringe orders are measured and substituted in equations (1) and (2). The resulting set of three equations for normal incidence can be solved for the three principal strain-fringe values $f_{\varepsilon L}$, $f_{\varepsilon T}$, and $f_{\gamma LT}$. The set of three equations for oblique incidence can be solved for A, B, and θ_x.

The procedure for obtaining ε_x and ε_y from equations (1) and (5) can be simplified if the angle θ_x can be chosen so that D = F = 0 in equation (5). In this case, equation (5) reduces to

$$\varepsilon_x = \frac{N_{\theta_x} \cos \theta_x f_{\varepsilon L}}{h\sqrt{C^2 + E^2}} \tag{8}$$

The above value of ε_x can be substituted in equation (1) to determine ε_y. The angle θ_x needed to make D = F = 0 is given by

$$\tan^2 \theta_x = \frac{1}{A \sin^2 \beta + B \cos^2 \beta} \tag{9}$$

Thus, the angle for oblique incidence will vary from point to point, depending upon the isoclinic parameter β.

A second method that is proposed to be tested is based on the dispersion of birefringence in composite models. If the variations of the fundamental photoelastic constants with the wavelength are different, then the principal stresses or strains can be determined from the isochromatic fringe patterns obtained for two different wavelengths. Specimens have been designed and fabricated for testing the feasibility of this method.

Photoelastic Calibration

Beam, compression and tension specimens have been conventionally used as photoelastic calibration specimens. In the case of orthotropic model materials, the circular disk specimen has not been used as a calibration specimen because a closed form solution is not available. However, in view of the difficulty and expense in fabricating such model materials, it is preferable to minimize the amount of material required for calibration.

A numerical solution for the stresses in an anisotropic circular disk under diametral compression has been utilized in an attempt to employ the circular disk as the calibration specimen. A nonlinear least-squares method is being applied to a single isochromatic fringe pattern in order to determine the three fundamental photoelastic constants and the residual isochromatic birefringence due to the fabrication procedure.

The isochromatic fringe order can be written as

$$N = h\left\{\left[\frac{1}{f_L}(\sigma_1\cos^2\beta + \sigma_2\sin^2\beta) - \frac{1}{f_T}(\sigma_1\sin^2\beta + \sigma_2\cos^2\beta)\right]^2 + \left[\frac{1}{f_{LT}}(\sigma_1 - \sigma_2)\sin 2\beta\right]^2\right\}^{1/2} + Nr \qquad (10)$$

where Nr is the residual fringe order, known to be distributed uniformly throughtout the model; h is the model thickness

let

$$g_k = \left[\frac{1}{f_L}(\sigma_{1}k\cos^2\beta_k + \sigma_{2_k}\sin^2\beta_k] - \frac{1}{f_T}(\sigma_{1_k}\sin^2\beta_k + \sigma_2\cos^2\beta_k)\right]^2 + \left[\frac{1}{f_{LT}}(\sigma_{1_k} - \sigma_{2_k})\sin^2\beta_k\right]^2 - \left[\frac{N-Nr}{h}\right]^2 \qquad (11)$$

where $k = 1, 2, 3, 4, \ldots\ldots\ldots\ldots M \; (M > 4)$

It is possible to write

$$\{g\} = [b] \; \{\Delta k\} \tag{12}$$

where

$$\{g\} = \begin{Bmatrix} -g_1 \\ -g_2 \\ \cdot\cdot \\ \cdot\cdot \\ -g_M \end{Bmatrix} \tag{13}$$

$$[b] = \begin{bmatrix} \frac{\partial g1}{\partial f_L} & \frac{\partial g_1}{\partial f_T} & \frac{\partial g_1}{\partial f_{LT}} & \frac{\partial g_1}{\partial Nr} \\ - - - & - - - & - - - & - - - \\ \frac{\partial gm}{\partial f_L} & \frac{\partial gm}{\partial f_T} & \frac{\partial gm}{\partial f_{LT}} & \frac{\partial gm}{\partial N_r} \end{bmatrix} \tag{14}$$

$$\{\Delta k\} = \begin{Bmatrix} \Delta f_L \\ \Delta f_T \\ \Delta f_{LT} \\ \Delta Nr \end{Bmatrix} \tag{15}$$

then, if

$$[d] = [b]^T \; [b] \tag{16}$$

is evaluated, we can write

$$\{k\} = [d]^{-1} [b]^T \{g\} \tag{17}$$

Assuming initial values of f_L, f_T, f_{LT}, Nr, by an iterative procedure, the photoelastic constants and the residual birefringence can be determined. The method is being applied to a unidirectionally reinforced circular disk.

Development of Model-Prototype Relations

The glass-fiber reinforced polymeric model materials being used as photoelastic materials incorporate commerical fibers in a commercial matrix. Thus they can be considered as structural composite materials. However, many of the structural composite materials incorporate graphite fibers, boron fibers, etc., in various matrices, such as aluminum. Therefore, an attempt was made in this investigation to treat the glass fiber-polymeric composite as a material to model another composite.

CONCLUSIONS

It appears that once the stresses or strains in the photoelastic composite are determined, the corresponding stresses in any other structural composite (having the same axes of symmetry) can be found by employing scaling factors which are functions of the elastic constants. It is proposed to test these scaling relationships by studying a suitable configuration.