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# COVER SHEET FOR TECHNICAL MEMORANDUM

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#### ABSTRACT

A subroutine which simulates the dynamic changes in atmospheric density has been incorporated into two computer programs which were previously developed to determine ground-site visibility from orbit. The performance of both programs shows a significant increase in accuracy compared to their previous capability.

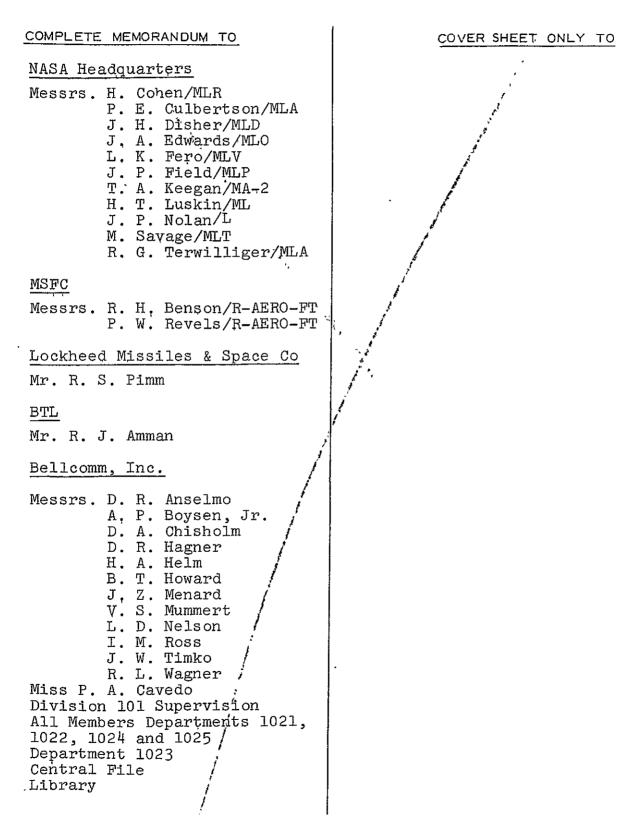
The mathematical model upon which the simulation is based was developed by MSFC. The technique requires the determination of a base value of density,  $\rho_{o}$ , from standard den-

sity tables. This density is then multiplied by factors which correct for the seasonal and diurnal variations of incident solar flux, the primary cause of dynamic density variations.

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SUBJECT:Application of a Dynamic Density<br/>Model to the Simulation of Earth<br/>Orbit Trajectories - Case 610DATE September 23, 1968FROM: A. B. Baker

#### TECHNICAL MEMORANDUM

### I. Introduction

Reference 1 describes two computer programs which can be used to study a variety of problems associated with earth orbital missions. The first, called TARGET, is based upon simple trigonometry and can be used only for circular orbits. The second is a modified version of the Bellcomm Apollo Simulation Program (BCMASP). It is significantly more accurate than the TARGET program and can be used for both circular and elliptical orbits.

Both programs generate the spacecraft ephemeris and then make additional computations at each point along the orbit. In the range of altitudes being considered for AAP (125-300 nm), the spacecraft position at any time after injection depends, in part, upon the cumulative effects of the aerodynamics drag forces which are continuously acting upon the spacecraft. Hence, the accuracy of the computer-generated ephemeris depends quite heavily upon the accuracy of the model used to simulate the effects of atmospheric drag.

In BCMASP, the drag force is defined by

$$D = \frac{1}{2} \rho V^2 C_D A \tag{1}$$

where

D is the drag force acting on the spacecraft  $\rho$  is the density at the particular altitude V is the velocity of the spacecraft relative to the atmosphere C<sub>D</sub> is the spacecraft drag coefficient A is the cross-sectional area of the spacecraft on which C<sub>D</sub> is based. The program calculates this force at each integration step, resolves it into rectangular components, and adds the components to the respective velocity derivitives which are supplied to the integrator. As a result of this drag force, the spacecraft experiences a continual loss of energy and hence a continual decrease in altitude so that the actual orbital path, illustrated in Figure 1A, is a spiral.

In TARGET, this path is approximated by a series of concentric circles as shown in Figure 1B. The loss of altitude during one revolution can be approximated by <sup>(2)</sup>

$$\Delta h = 4\pi B \rho (R+h)^2$$
(2)

where

Δh is the change in altitude over one revolution
ρ is the density at altitude h
B is the ballistic coefficient
R is the radius of the earth
h is the altitude of the spacecraft.

Note that

$$B = \frac{C_D A}{2m}$$
(3)

where

m is the mass of the spacecraft.

All of the variables used in equations (1) and (2) have an effect upon the accuracy of the approximation. A major source of error stems from the fact that these variables are not constant over the entire mission but are continuously changing. The uncertainties in atmospheric density, however, are by far the most significant. No direct measurements of density have been made in the region above 100 nm and so the basis for density models above this altitude have been analyses of the effects of drag on the motion of artificial satellites. The results of these analyses can, at best, be considered speculative. In addition to a variation with altitude, several classes of density fluctuations have been observed in the upper atmosphere. These fluctuations all have one common feature: they are caused by variations in the heating of the earth's atmosphere which, in turn, results from variations in energy coming from the sun. The causes of these variations can be classified into three general categories:

- a) those caused by the variation of the subsolar point
- b) those caused by the variations in the ultraviolet flux from the sun
- c) those caused by variations in the corpuscular flux from the sun.

The variations caused by the earth's rotation result in a "diurnal bulge" which occurs approximately  $30^{\circ}$  eastward in longitude from the subsolar point. At this point, the atmospheric temperature is 40% higher than the minimum temperature in the dark hemisphere and results in a density variation of more than 9:1. The fluctuations caused specifically by variations in ultraviolet flux are not easily distinguishable from those caused by corpuscular variations. However, distinct activity cycles of 27 days (corresponding to the period of the sun's rotation), 6 months, and 1-2 years have been noted. In addition there is a slowly varying fluctuation which correlates with the sun's ll-year cycle.

The original versions of both earth-orbit trajectory programs contained a simple time-invariant, altitude-dependent density profile based upon the 1962 U. S. Standard Atmosphere. The latter depicts idealized, middle-latitude, year-round conditions for the range of solar activity that occurs between sunspot minimum and sunspot maximum. Results using this static model were reasonable for short-lifetime missions but ephemeris errors increased markedly as the spacecraft lifetime increased. To improve the accuracy of the computer-generated ephemerides therefore, the static density profile in each program has been replaced by a dynamic model which reflects the additional density variations in time and position that are dependent upon sunspot activity.

### II. Mathematical Representation of a Dynamic Density Model

The mathematical techniques for generating a time and position dependent density model were derived by personnel at the MSFC Aero-Astrodynamics Laboratory and have been successfully incorporated into an Earth Orbital Lifetime Prediction Model (Reference 3). As one would expect, the accuracy of lifetime predictions is limited by our ability to predict sunspot activity. The validity of the mathematical techniques has therefore been verified by using the model in retrospective analyses of the lifetimes of previously launched spacecrafts so that accurate physical dimensions and actual observed solar activity levels could be used. Under these conditions, lifetime calculations using the dynamic density model have given highly accurate results (as low as 0.25 percent deviation from the spacecraft's actual lifetime) while predictions using the corresponding static model have been in error by as much as 50 to 100 percent. The same computational techniques have been used to derive the dynamic density model for the TARGET and BCMASP programs. The remainder of this section contains a brief summary of these techniques. A more complete discussion can be found in References 3 and 4.

The basic approach to generating a time and position dependent value of density is to first determine an altitude-dependent value of density,  $\rho_0$ , from the 1962 U.S. Standard Atmosphere and then to multiply that value by two dimensionless factors,  $C_1$  and  $C_2$ . The former represents the effect of the density variation resulting from daily and seasonal fluctuations in the solar and geomagnetic activity while  $C_2$  describes the effect of the diurnal bulge.  $C_1$  is defined by the expression

$$C_{1} = \left(\frac{S}{S_{0}}\right)^{K(h,\psi)}$$

(4)

where

S is a dimensionless heating parameter

- S is an empirically derived constant which varies with the density profile. A value of 200 is used for the 1962 U. S. Standard Atmosphere
- K is a known function (Reference 3) of the altitude and the spacecraft position relative to the diurnal bulge
- $\psi$  is the geocentric angle between the spacecraft and the center of the diurnal bulge.

<sup>\*</sup>These techniques can be used with any density profile. However, the most accurate results have been obtained by using the 1962 U. S. Standard Atmosphere.

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The heating parameter is defined in Reference 3 as

$$S = \overline{S}e^{g(t)}$$
(5)

where

$$e^{g(t)}$$
 is a correction for seasonal effects.

 $\overline{S}$  is defined as

$$\overline{S} = 25 + 0.8 \overline{F}_{10.7} + 0.4 (F_{10.7} - \overline{F}_{10.7}) + 10A_{p}$$
 (6)

where

$$F_{10.7}$$
 is the daily value of 10.7 cm solar flux  
 $\overline{F}_{10.7}$  is a 365 day running average of  $F_{10.7}$   
 $A_p$  is an index of geomagnetic activity.

The effect of the diurnal bulge varies with both altitude and spacecraft position. This effect is defined in Reference 5 by the expression

$$C_{2} = \frac{1 + f(h) \left(\frac{1 + \cos \psi}{2}\right)^{3}}{1 + f(h) \left(\frac{1 + \cos 75^{\circ}}{2}\right)^{3}}$$
(7)

where f(h) is defined as

$$f(h) = 0.19(e^{0.0055h} - 1.9)$$
(8)

Note that the expression for  $C_2$  assumes that the base atmosphere represents a mean diurnal effect so that when  $\psi$  is set equal to a value of 75°,  $C_2$  becomes equal to one regardless of altitude.

In summary, the value of density used in equations (1) and (2) is expressed by

$$\rho = \rho_0 C_1 C_2 = \rho_0 \left(\frac{S}{S_0}\right)^K \left(\frac{1 + f(h)\left(\frac{1 + \cos\psi}{2}\right)^3}{1 + f(h)\left(\frac{1 + \cos75^\circ}{2}\right)^3}\right)$$
(9)

This value of density will vary with both absolute time (Julian date) and position relative to the earth-sun line as well as altitude.

#### III. Computational Approach

The 1962 U. S. Standard Atmosphere was used as the reference for the time-invariant density profiles in both the TARGET and BCMASP programs and therefore only the means for computing the factor F had to be added to complete the dynamic model. This was most easily accomplished by adding a new subroutine called DYNAMC to each program. In BCMASP, DYNAMC is called by subroutine DENSTY immediately after a value of density,  $\rho_{o}$ , has

been determined from the tables. A flow diagram of this subroutine, as it is used in BCMASP, appears in Figure 2.

The computational approach used in this subroutine follows the mathematical discussion in Section II. There are six inputs to the subroutine: the three components of the spacecraft position vector in true rectangular coordinates (coordinates oriented with the true equator of date), the magnitude of this vector, the geodetic altitude, and the current Julian date. The Julian date is converted to the equivalent Gregorian date which in turn is used to find the magnitude of the average 10.7 cm solar flux ( $\overline{F}_{10.7}$ ). A table of predicted values of  $\overline{F}_{10.7}$  for

the period 1968.00 to 1978.75 was obtained from MSFC<sup>\*</sup> and is contained in a DATA statement at the beginning of the subroutine. After determining the decimal value of the current year, linear interpolation is used to determine the correlating value of solar flux.

<sup>\*</sup>This table is updated every three months by the Space Environment Branch at MSFC. The latest available version should be used for all investigations.

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Referring again to Figure 2, the newly determined value of solar flux is used to determine the geomagnetic activity index  $A_p$  by using the following relationship<sup>(6)</sup>

$$A_{p} = 1.8 \qquad 0 < \overline{F}_{10.7} < 80$$

$$A_{p} = 2.2 \qquad 80 \leq \overline{F}_{10.7} < 130 \qquad (10)$$

$$A_{p} = 2.8 \qquad 130 \leq \overline{F}_{10.7}$$

The next step in the program is the calculation of  $\overline{S}$  and S using equations (6) and (5). However the daily fluctuations of 10.7 cm flux are usually ignored (actually considered equal to  $\overline{F}_{10.7}$ ) so that equation (6) becomes

$$\overline{S} = 25 + 0.8 \overline{F}_{10.7} + 10A_{p}$$
 (11)

The direction cosines of the diurnal bulge are computed from the direction cosines of the earth-sun line. The angle  $\psi$  is then computed from the relation

$$\cos \psi = \mathbf{i}_{\mathrm{S}} \mathbf{i}_{\mathrm{B}} + \mathbf{j}_{\mathrm{S}} \mathbf{j}_{\mathrm{B}} + \mathbf{k}_{\mathrm{S}} \mathbf{k}_{\mathrm{B}}$$
(12)

where

permitting the variable  $C_2$  to be evaluated using equation (7). The functions K and  $C_2$  are evaluated in the final part of the subroutine and control is returned to subroutine DENSTY.

The sequence of calculations is almost identical in the version of DYNAMC used in the TARGET program. There is however one significant difference. The TARGET program assumes that the spacecraft moves with constant velocity in a circular orbit and therefore does not utilize an inertial coordinate - 8 -

system. In order to remain consistent with the program's level of complexity, a mean value ( $\psi = 75^{\circ}$ ,  $C_2 = 1$ ) is assumed for the position of the diurnal bulge.

#### IV. Simulator Performance Comparison

As in Reference 1, the performance of the drag models was investigated by comparing spacecraft altitude as a function of time as generated by the three different simulators. In order to make a proper comparison, it is important that each program be supplied with the same initial conditions. Therefore, BCMASP was used to obtain a set of injection conditions at a radial distance of 3592 nm. It is this radial distance which shall be compared as a function of time since it is independent of altitude variations resulting from geodetic differences and hence is a more accurate measure of the solitary effects of the drag model. For convenience however, the data is discussed in terms of "geocentric altitude", which is nothing more than the radial distance less a constant; in this case the equatorial radius of the earth. Therefore the geocentric altitude corresponding to the radial distance of 3592 nm is 150 nm.

In both the BCMASP and the MSFC Lifetime Program, the accuracy of the spacecraft ephemeris varies inversely with the magnitude of the differential time step used to generate the trajectory. In the Lifetime Program, the accuracy is not very sensitive to step size, however a relatively small step (0.2 day) was used to obtain the maximum possible accuracy. The reverse is true for BCMASP, however. In this case, both the accuracy and the running time are particularly sensitive to step size. A tangential effort (Appendix A) showed that an interval of 60 seconds was the largest interval in which the resulting error accumulation could be ignored and so a 60 second step size was used for all subsequent BCMASP runs.

Two runs were made with each program; one using the static density model, the other using the corresponding dynamic model. The results of all six runs are shown in Figure 3 and a performance comparison is tabulated in Table 1. The table compares all of the results to those obtained from the MSFC Lifetime Program with the dynamic model since the latter program has already proved to be highly accurate.

The table shows that there is a marked improvement in the performance of both TARGET and BCMASP when using the dynamic model. The deviation of the BCMASP-generated altitude profile from the one generated by the MSFC Lifetime Program was reduced to three-tenths of one percent. Figure 3 shows the two profiles to be almost identical. Though the TARGET program is still much less accurate than the BCMASP, its error is reduced from 30 to 17 percent. - 9 -

It is also interesting to note that results from the BCMASP and MSFC Lifetime Program are in close agreement regardless of which density model is used, indicating that the computational approach taken by the MSFC program (i.e., using the respective product of the apogee and perigee decay rates and the time step) can, in some cases, be a useful substitute for the repetitive integration of the differential equations of motion.

The choice of which program to use depends upon the application and upon the user's requirements. If ground site visibility is to be investigated, then either TARGET or the BCMASP Earth-Orbit Simulator must be used. Table 1 shows the accuracy of the BCMASP to be more than an order of magnitude better than the accuracy of the TARGET program, however the same mission may be simulated by the TARGET program in a much shorter time (again, as much as an order of magnitude, depending upon the number of target sites being investigated). The final choice depends upon the accuracy required and how much computation time one is willing to spend.

If the particular investigation includes only a study of the spacecraft altitude profile, then the MSFC Lifetime Program should be used. Its accuracy has been shown to be equivalent to the BCMASP for this application and it requires significantly less computation time.

### V. Summary

The inaccuracies of the time-invariant density model used in the original versions of TARGET and the BCMASP Earth-Orbit Simulator are sufficient to cause significant errors in the determination of the drag force and hence in the spacecraft ephemeris. The static model has therefore been replaced with a dynamic model, one in which the density is both altitude and time dependent. The resulting increase in accuracy is quite noticeable. When compared to the altitude profile generated by the MSFC Lifetime Program, the deviation of the BCMASPgenerated profile was three-tenths of one percent and the deviation in the profile generated by the TARGET program was reduced to 17%.

The high degree of agreement between the MSFC Program and BCMASP (Figure 3) indicates that either program could be used to investigate spacecraft altitude profiles. The MSFC Program however is specifically designed to perform these types of studies and should be used for this application. It affords more flexibility and also requires significantly less computation BELLCOMM, INC.

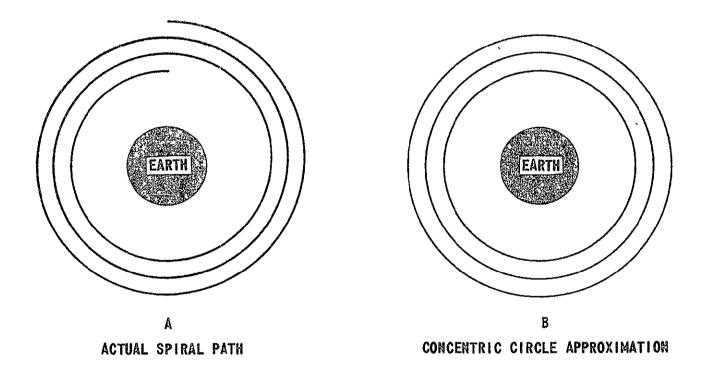
time than BCMASP. When studying ground site visibility however, the TARGET program is sufficiently inaccurate to negate its advantages in computation time over BCMASP. BCMASP therefore should be used for ground site visibility investigations whenever the computation time is not prohibitive.

B. Baker

1025-ABB-dcs

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- R. Benson, E. Fleishman, and R. Hill
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# FIGURE I - EFFECT OF DRAG ON THE SPACECRAFT ORBITAL PATH

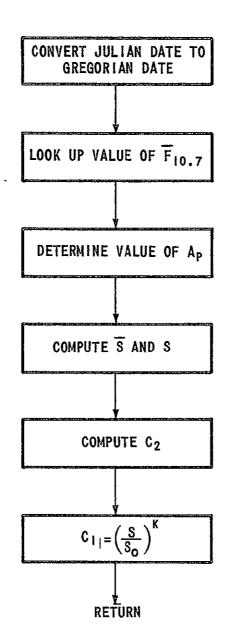


FIGURE 2 - FLOW DIAGRAM FOR SUBROUTINE DYNAMC

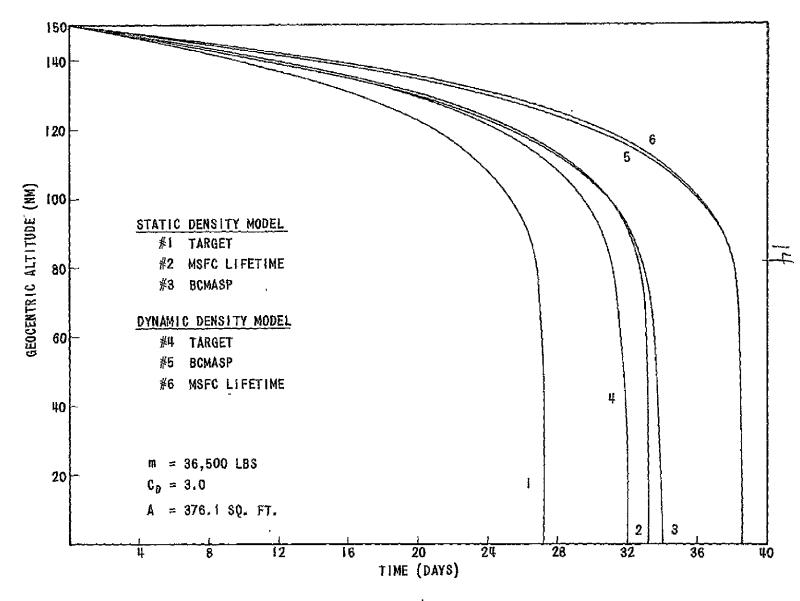


FIGURE 3 - CM/SM LIFETIME

# TABLE 1

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# Drag Model Performance Comparison

Model	$\frac{\texttt{Lifetime}}{(\texttt{Days})}$	Deviation (Percent)
MSFC Dynamic	38.6	
BCMASP Dynamic	38.5	0.3
BCMASP Static	34.0	11.9
MSFC Static	33.2	14.0
TARGET Dynamic	32.0	17.1
TARGET Static	27.2	29.5

15

### Appendix A

#### Computation Errors in the BCMASP Earth-Orbit Simulator

One of the major obstacles to evaluating the drag model in BCMASP is the inability to differentiate between those changes in altitude which result from the effects of physical forces (i.e., drag and gravity) and those which result from computational error (i.e., truncation and round-off errors). In BCMASP, the magnitude of the error varies with the size of the integration step and the effect is cumulative. Analysis of this error variation would indicate the largest integration step size which would produce a negligible error over the total simulation. This step size would be used in the evaluation of the drag model and would also place an upper limit on the step size used for ground site visibility investigations.

To determine the magnitude of the errors and their variation with step size, a series of computer runs were made with the BCMASP Earth-Orbit Simulator using identical input data for a 150 nm circular orbit. Two sets of runs were made. The first used Cowell integration without any perturbations; the second also used Cowell integration but included the orbital perturbations caused by solar and lunar gravity as well as the earth's oblateness. In addition, one control run was made for each set using the automatic step size control, HSMAX, set equal to .04. The integration step will be increased or decreased to the point where the integration error indicator is just  $\leq$  HSMAX.

The results of the control run for the first set are tabulated in Table A-1 and are illustrated graphically in Figure A-1. The latter shows a linearly increasing deviation from the initial altitude, an indication that the deviation can be attributed to computational error. As the table indicates, the maximum deviation is well below two tenths of a mile.

In order to make a valid comparison of the effects of step size on the truncation error, it is necessary to compare the data to the control value of altitude at the appropriate mission time rather than to the initial reading at time zero because the effect of the physical forces acting upon the spacecraft causes the magnitude of its radius vector to oscillate around some mean value. Hence the only accurate measure of the deviation is to compare the value of altitude for a particular mission time with a value known to be correct to within a controlled tolerance (i.e., the corresponding value in the control run) at that same mission time. These results are shown in Table A-2. Note that events were inserted into the Events List for the control run at 5, 10, 20, and 25 days to insure that a value of altitude would be calculated at the precise mission time despite the use of the step size control.

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Appendix A (contd.)

The table shows the deviation from the corresponding control reading for runs with different integration step sizes. (Step sizes greater than two minutes were found to give results which far exceeded usable limits). The deviation listed at each data point represents the additional truncation error induced by increasing the step size. As would be expected the computational errors are significantly greater when the effects of perturbations are considered. In both sets however the maximum errors which result from using a one minute step size are far below the uncertainties associated with the density profile whereas the errors resulting from the use of the two minute step size are approximately equal to those uncertainties. Hence it was concluded that a step size no larger than 60 seconds should be used in all subsequent analyses.

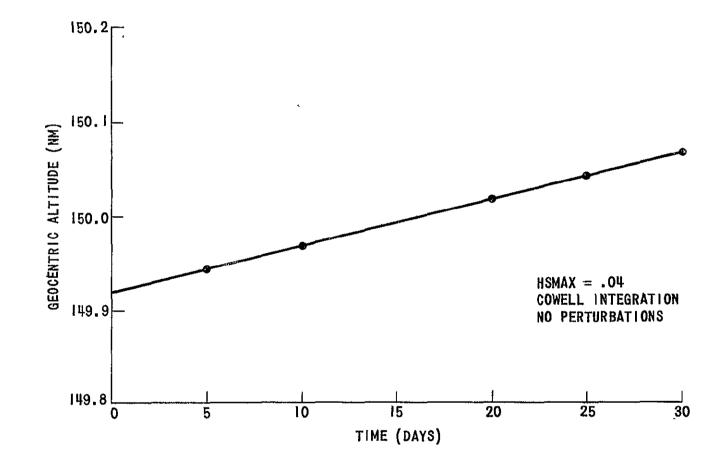


FIGURE A-I - DEVIATION FROM INITIAL ALTITUDE

19.

# TABLE A-1

# Results of Set 1 Control Run\*

Day	Geocentric Altitude (nm)	Altitude Deviation (nm)
0	149.918	
5	149.942	.024
10	149.968	.05
20	150.018	.10
25	150.043	.125
30	150.070	.152

HSMAX = .04 Cowell Integration No Perturbations

### TABLE A-2

20

### Deviation from Control Altitude

Day	Control Altitude	Integration Step Size		
	(nm)	0.5 min	1.0 min	2.0 min
		Set l <sup>*</sup>		
		Set 1		
0 5 10 20 25 30	149.918 149.942 149.968 150.018 150.043 150.070	0.0 0.0 0.0 0.0 0.0 0.0	0.0 0.084 0.169 0.337 0.421 0.506	0.0 2.674 5.362 10.832 13.548 16.300
		Set 2 <sup>**</sup>		
0- 5 10 20 25 30	149.918 148.599 147.077 145.166 145.415 146.572	0.0 0.0 0.0 0.0 0.0 0.0	0.0 0.077 0.147 0.567 1.145 2.034	0.0 2.408 2.488 3.856 14.551 7.213

<sup>\*</sup>Set 1 - Cowell integration No - Perturbations

<sup>\*\*</sup>Set 2 - Cowell integration including solar gravity, lunar gravity, and Earth oblateness perturbations

B1 - CONTROL NUMBER: B4 - ACCESSION NUMBER:	U077502
B3 - COLLECTION CODE:	4
B6 - MANAGEMENT CODE:	XF
B7 - PROCESS ACTNG CD: B8 - PRODUCT MANAGER:	01 B7A-REGISTRATION FEE: 00000 H
B9 - RECEIPT TYPE:	1 B9A-LOAN DUE OUT:
<b>B10- TRANSACTION:</b>	TN B10A-DUPE/PRIOR/SUPER:
B12- RETURNS:	0 B12A-RETURN DATE:
B13- PROCESSING:	1
B14- PAT. POTENTIAL: B17- FORM/PRICE:	0 12001 ,
B18- ANNOUNCE:	0000
B19- PUBLICATION-1:	u8112 B20- PUBLICATION-2:
B21- LIMITATION:	0
B23- PC BIN:	000
B24- STOCK: B25- PAGES/SHEETS:	0000 B24A-STOCK TYPE CODES: N 00021
B26- PC PRICE CODE:	A02
B27- DOMESTIC PRICE:	0000000 B28- FOREIGN PRICE: 0000000
B29- ACTION CODES:	BX
B33- MN PRICE CODE:	X00
B34- DOMESTIC PRICE:	
B36- ACTION CODES: B37- RELEASABILITY CD:	XM A
B38- MF PRINT:	D
B39- ADDITIONAL INFO:	n
B40- PRINT PC:	n
B41- PC DUE:	n
B42- SOURCE ORDER:	n
B42A-GENERATE RDP: B42B-SUPPLIER SRC CD:	0
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