Tomography of core–mantle boundary and lowermost mantle coupled by geodynamics

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SUMMARY
We propose an innovative approach to mapping CMB topography from seismic P-wave traveltime inversions: instead of treating mantle velocity and CMB topography as independent parameters, as has been done so far, we account for their coupling by mantle flow, as formulated by Forte & Peltier. This approach rests on the assumption that P data are sufficiently sensitive to thermal heterogeneity, and that compositional heterogeneity, albeit important in localized regions of the mantle (e.g. within the D′ region), is not sufficiently strong to govern the pattern of mantle-wide convection and hence the CMB topography. The resulting tomographic maps of CMB topography are physically sound, and they resolve the known discrepancy between images obtained from classic tomography on the basis of core-reflected and core-refracted seismic phases. Since the coefficients of mantle velocity structure are the only free parameters of the inversion, this joint tomography–geodynamics approach reduces the number of parameters; nevertheless the corresponding mantle models fit the seismic data as well as the purely seismic ones.

Key words: Inverse theory; Body waves; Seismic tomography; Dynamics of lithosphere and mantle.

1 INTRODUCTION
As noted early on by Morelli & Dziewonski (1987), the topography of the core–mantle boundary (CMB) is a key parameter to understand the nature of deep mantle flow and magnetic field generation, and an important factor to correct for if one is to image the structure of the Earth’s core.

Seismologists have derived global maps of CMB topography based on compressional-wave traveltimes (e.g. Morelli & Dziewonski 1987; Rodgers & Wahr 1993; Obayashi & Fukao 1997; Boschi & Dziewonski 2000; Soldati et al. 2003) and the splitting of normal-mode eigenfrequencies (Ishii & Tromp 1999), to find only marginally consistent results. As Rodgers & Wahr (1993) first observed, the topography mapped on the basis of seismic phases reflected off the CMB (PcP phase) is not correlated with that derived from phases that are refracted through it (various branches of PKP). Boschi & Dziewonski (2000), Piersanti et al. (2001) and Soldati et al. (2003) show that the discrepancy between PcP- and PKP-based maps could be explained by the presence of relatively large heterogeneity in the fluid outer core, but fail to explain this unlikely feature. Fig. 6 of Ishii & Tromp (1999) shows that normal-mode based CMB topography is also not correlated with the results of traveltime imaging.

Geodynamicists have predicted the topography of the CMB caused by mantle flow (Forte et al. 1995; Lassak et al. 2007, 2010). Essentially, the CMB should be depressed under relatively dense and sinking regions of the lowermost mantle, and uplifted under less dense, rising ones. This effect can be modelled if a tomography model of seismic velocity in the mantle is interpreted in terms of equivalent density anomalies, a viscosity profile (typically based on observations of the geoid and/or postglacial rebound) is assumed, and the resulting flow is computed (e.g. Hager et al. 1985). Forte et al. (1995) found a good correlation between their geodynamical predictions and the seismic maps of CMB topography of Morelli & Dziewonski (1987). More recent studies suggest that the predictions of geodynamics are well correlated with PcP-based maps of CMB topography and not correlated at all with normal-mode- and PKP-based maps: compare, for example, fig. 1 of Soldati et al. (2003) and fig. 15 of Soldati et al. (2009). This discrepancy is partly solved if lateral heterogeneity is introduced in the outer core, but this feature lacks a clear physical explanation (Soldati et al. 2003).

Observations of Earth’s rotation pole nutation have also been used to constrain indirectly the topography of the CMB. Nutations measured via Very Long Baseline Interferometry (VLBI) have been associated with the coupling between mantle and fluid core, with the ellipticity (i.e. the degree \( l = 2 \), order \( m = 0 \) spherical harmonic component) of the CMB playing an important role. Gwinn et al. (1986) have explained VLBI data with an ellipticity of 490 ± 110 m peak-to-valley. Their results have been confirmed...
more recently by Buffett et al. (2002), although the amplitude of the excess ellipticity was estimated to be approximately 400 m.

All published global seismic maps of CMB topography so far have been derived by calculating the partial derivatives of seismic observations with respect to topography perturbations, which are then used to set up a linear inverse problem (Morelli & Dziewonski 1987; Rodgers & Wahr 1993; Obayashi & Fukao 1997; Ishii & Tromp 1999; Boschi & Dziewonski 2000; Soldati et al. 2003). This is based on some linearized equation relating seismic observations $\delta s$ to mantle seismic velocity (and/or density) $\delta v$ and CMB topography $\delta c$, that is,

$$\delta s = \int_V K_s(r, \theta, \phi) \delta v(r, \theta, \phi) dV + \int_\Omega K_c(\theta, \phi) \delta c(\theta, \phi) d\Omega, \quad (1)$$

where $V$ denotes the volume of the Earth, $\Omega$ the solid angle, $K_s$ and $K_c$ the velocity and CMB-topography partial derivatives (depending on the nature of the observation, and on the theoretical formulation of wave propagation/the Earth’s free oscillations) and $r, \theta, \phi$ are radius, colatitude and longitude, respectively. The left-hand side $\delta s$ is an observation (traveltime, normal-mode eigenfrequency, ...), while $\delta v$ and $\delta c$ are the unknown functions to be determined. Both $\delta v$ and $\delta c$ are then written as linear combinations of selected basis functions, and, in the presence of a large database of $\delta s$, a mixed-determined linear inverse problem (Menke 1989) is defined (e.g. Boschi & Dziewonski 1999).

With this study we propose an alternative approach, consisting of replacing $\delta c$ in eq. (1) with an expression describing its dependence on $\delta v$ through the physics of mantle convection. An analytical expression for $\delta c$ in terms of $\delta v$ is given, for example, by Forte et al. (1994), assuming (i) that mantle heterogeneity and the associated flow are of purely thermal origin (no compositional heterogeneity), so that density and velocity heterogeneity are proportional to one another, and (ii) that mantle viscosity is estimated reasonably well. The only unknown of the inverse problem is then the function $\delta v$, which is parameterized and inverted for as before. Although $\delta c$ is not a free parameter of the inversion, the resulting models of $\delta v$ implicitly account for the sensitivity of seismic data to $\delta c$. $\delta c$ itself can be derived from $\delta s$ after the inversion, using again the theory of Forte et al. (1994).

The neglect of compositional heterogeneity introduces a potential inaccuracy to our procedure, as a number of studies has indicated that lowermost mantle is chemically heterogeneous (e.g. Karato 2003; Deschamps & Trampert 2003; Trampert et al. 2004; Della Mora et al. 2011). Our results should be seen as preliminary, before a way to account for compositional heterogeneity in our formulation is found. It is, however, not unlikely that the introduction of compositional heterogeneity will perturb only marginally the coupling between flow and CMB, and hence our models of the latter: it has been shown by Simmons et al. (2009), and earlier by Forte & Mitrovica (2001), that even at the African superplume, where thermal and compositional heterogeneity have opposite effects on mantle flow, compositional effects are too weak to inhibit upward buoyancy forces.

In the following, we employ the geodynamical relationship between $\delta c$ and $\delta s$ in our implementation of the tomography inverse problem. Our goal is to employ our physical, a priori knowledge of the mantle-CMB system to reduce the non-uniqueness of tomography models.

### 2 Seismology and Geodynamics

#### 2.1 Dynamic relationship between discontinuity topography and mantle density

If heterogeneities in the lower mantle are mainly thermal in origin, areas of high velocity can be associated to denser than average mantle, and low velocities to less dense mantle. With this assumption, analytical models of thermal convection can be developed (e.g. Hager et al. 1985; Forte & Peltier 1991), and a physical relationship between density heterogeneity and the undulation of global discontinuities in the Earth’s interior can be established. In particular, Forte et al. (1994) showed that a simple relationship exists between perturbations in the density structure of the mantle $\delta \rho$, and the topography $\delta c$ of the CMB, with $\delta \rho$, $\delta c$ treated as small perturbations from a spherically symmetric reference model.

Let us introduce the spherical harmonic expansion of CMB topography

$$\delta c(\theta, \phi) = \sum_{l,m} \delta c_{lm} Y_{lm}(\theta, \phi), \quad (2)$$

where $Y_{lm}$ is the spherical harmonic function of degree $l$ and order $m$, and $\delta c_{lm}$ is the corresponding spherical-harmonic coefficient of $\delta c$.

Eq. (5) of Forte et al. (1994) then stipulates that $\delta c_{lm}$ are related to the $r$-dependent harmonic coefficients $\delta \rho_{lm}$ of density perturbation through

$$\delta c_{lm} = \frac{1}{\Delta \rho_{lm}} \int_c^{\infty} B_l(r) \delta \rho_{lm}(r) dr, \quad (3)$$

with $c$ and $a$ denoting the reference, mean radii of the CMB and Earth’s surface, respectively, and $\Delta \rho_{lm} = -4.43 \text{ g cm}^{-3}$ the density jump across the CMB according to PREM (Dziewonski & Anderson 1981).

The ‘kernels’ $B_l$ are $l$-dependent partial derivatives describing the response of CMB topography to density perturbations in the mantle. These kernels are calculated as explained by Forte & Peltier (1991), once the Earth’s radial density and viscosity profiles are known. Constraining Earth’s viscosity is subject to substantial uncertainties and non-uniqueness. We may neglect the effect of large-scale lateral viscosity variations (Moucha et al. 2007), and first compute $B_l$ based on the viscosity profile of Mitrovica & Forte (1997), shown to explain observations of both gravity and postglacial deformation; most of this study is based on this viscosity model. We next modify the profile of Mitrovica & Forte (1997) to include a very low viscosity layer at the base of the mantle (Section 5.3), simulating the rheology of post-perovskite Yamazaki et al. (2006); Tosi et al. (2009); Ammann et al. (2010), expected to be the dominant phase in this depth range (D’ layer) (e.g. Murakami et al. 2004; Oganov & Ono 2004). In an additional set of experiments, we make use of the simpler model of Soldati et al. (2009) (Section 5.3). Fig. 1 includes all the viscosity profiles we experimented with.

Although the average density profile of the Earth is relatively well known (Dziewonski & Anderson 1981), lateral variations in density can in practice be estimated from those of seismic velocity. In establishing this scaling between seismic velocity and density anomalies, it is common to begin with the assumption that there are no lateral variations in the Earth’s composition, nor pressure- or temperature-induced phase transformations; and in this case the velocity–density scaling may be estimated on the basis of laboratory measurements (e.g. Forte et al. 1994). This assumption of a
depth-dependent velocity–density scaling has been relaxed in recent studies that allow for a fully 3-D relationship between seismic and density anomalies (Simmons et al. 2009, 2010). In this study, we simply assume that relative density anomalies are proportional to compressional-velocity ones, at all depths, through the constant factor $\delta \ln \rho / \delta \ln v_P = 0.45$, found by averaging the profile of Karato (1993). We repeated our experiment using a depth-dependent scaling in accordance with Karato & Karki (2001), and found that the resulting mantle and CMB models are only marginally affected. This is due to the relative weight of the seismic and geodynamic parts of the matrix to invert: the former is dominant for realistic values of the density–velocity scaling, and becomes comparable to the product of the geodynamic matrix by the scaling factor only for scalings one order of magnitude larger. For brevity we do not illustrate this test in detail.

We show in Fig. 2 (solid lines) the kernels $B_l$ resulting, at several different harmonic degrees $l$, from the viscosity profile of Mitrovica & Forte (1997). Low-$l$ kernel values are non-negligible throughout the mantle, growing steadily from the Earth’s surface to the 660 km discontinuity and remaining constant down to the CMB. High-$l$ kernels are zero in the top 2000 km of the mantle, and grow quickly starting at a depth that itself grows with increasing $l$. The presence of a low-viscosity, post-perovskite layer (dashed lines) results in an additional quick growth of low-degree $B_l$ with depth at the base of the mantle; the depth-dependence of high-degree $B_l$ is more similar to that found from the viscosity model of Mitrovica & Forte (1997), except that their growth is confined to larger depths.

2.2 Mapping between spherical-harmonic and pixel parameterizations

In this study, we adhere to the approximately equal-area voxel parametrization of Boschi & Dziewonski (1999), Boschi & Dziewonski (2000) and Soldati et al. (2003). To make use of the results of Forte et al. (1994) in a voxel formulation, we need an operator that converts spherical-harmonic to pixel coefficients of a given function.

We define the characteristic function $p_i(\theta, \phi)$ of the $i$th surface pixel in our grid,

$$p_i(\theta, \phi) = \begin{cases} 1 & \text{if } (\theta, \phi) \text{ lies within the } i\text{th pixel} \\ 0 & \text{elsewhere.} \end{cases}$$

A horizontal cross-section through a voxel-parametrized tomographic model of relative seismic velocity perturbation $\delta v/v(r, \theta, \phi)$ can be thought of as a linear combination of pixel functions

$$\frac{\delta v}{v}(r, \theta, \phi) = \sum_i x_i(r) p_i(\theta, \phi)$$

through the $r$-dependent coefficients $x_i(r)$.

Provided that spherical harmonic coefficients are computed up to a sufficiently high $l$, one can accurately describe $p_i(\theta, \phi)$ as a linear combination of spherical harmonics: we denote $p_{l,m}$ the coefficient of degree $l$ and order $m$ of the $i$th pixel function, so that

$$p_i(\theta, \phi) = \sum_{l,m} p_{l,m} Y_{lm}(\theta, \phi).$$
Let us now substitute eq. (6) into (5). We find
\[ \frac{\delta v}{v}(r, \theta, \phi) = \sum_l x_l(r) \sum_m p_{lm} Y_{lm}(\theta, \phi), \] (7)
and after changing the order of summation
\[ \frac{\delta v}{v}(r, \theta, \phi) = \sum m \left[ \sum_l x_l(r) p_{lm} \right] Y_{lm}(\theta, \phi). \] (8)

We infer that pixel \( x_l(r) \) and spherical-harmonic \( (x_{lm}(r)) \) coefficients of \( \delta v/v \) are related by
\[ x_{lm}(r) = \sum_l x_l(r) p_{lm}, \] (9)
and the required operator simply consists of the harmonic coefficients of the pixel functions forming our parametrization grid. Depending on pixel size, the summation in eq. (6) will have to be extended to a different maximum degree \( l \). Given the pixel size, we determine the maximum \( l \) by trial and error. Fig. 3 shows that, for the 5\(^{\circ}\) \times 5\(^{\circ}\) parametrization used here (Section 3), all harmonic degrees up to \( l = 89 \) must be considered.

2.3 Geodynamically self-consistent mantle/CMB tomography

Let us consider a spherically symmetric reference model \( v(r) \), \( c \) of \( P \)-wave velocity and CMB topography respectively, with laterally varying perturbations \( \delta v(r, \theta, \phi)/v(r) \) and \( \delta c(\theta, \phi)/c \). The corresponding traveltime perturbation associated with a \( P \)-wave-reflected by the CMB (\( PcP \) phase) can then be determined approximately through the linear equation (consistent with the general expression (1) of Section 1)
\[ \delta t = - \int_{\text{path}} \frac{\delta v(r(s), \theta(s), \phi(s))}{v^2(r)} ds + K_{PcP} \frac{\delta c(\theta_b, \phi_b)}{c} \] (10)
(Boschi & Dziewonski 2000), where \( (\theta_b, \phi_b) \) are the coordinates at which the \( PcP \) raypath is reflected (‘bounces’) off the CMB, \( r = r(s), \theta = \theta(s), \phi = \phi(s) \) is the ray-path (‘path’) equation, and the sensitivity \( K_{PcP} \) of \( \delta t \) to CMB undulations depends on ray path geometry and is defined, for example, by Dziewonski & Gilbert (1976).

Although Boschi & Dziewonski (2000) and Soldati et al. (2003), among others, treat \( \delta c/c \) as a free parameter of their seismic inversions, we choose to rewrite it in terms of mantle \( \delta v/v \) through eq. (3) of Section 2.1. We replace \( \delta c \) in (10) with its spherical harmonic expansion (2),
\[ \delta t = - \int_{\text{path}} \frac{\delta v}{v^2} ds + \frac{K_{PcP}}{c} \sum_{l,m} \delta x_{lm} Y_{lm}(\theta_b, \phi_b) \] (11)
(where, in the interest of brevity, we have omitted the spatial-dependence of \( \delta v \) and \( v \)), so that eq. (3) can be applied, and
\[ \delta t = - \int_{\text{path}} \frac{\delta v}{v^2} ds + \frac{K_{PcP}}{c} \Delta \rho_{\text{sub}} \sum_{l,m} \int_c B_l(r) \delta \rho_{lm}(r) dr Y_{lm}(\theta_b, \phi_b). \] (12)

As discussed above, if we assume that compositional heterogeneity be relatively small (Schubert et al. 2009; Simmons et al. 2010; Della Mora et al. 2011), a constant or spherically symmetric scaling factor \( \beta(r) \) can be introduced so that \( \delta \rho_{lm}/\rho(r) = \beta(r) x_{lm}(r) \), with \( x_{lm}(r) \) the \( r \)-dependent harmonic coefficients of relative \( P \)-wave
velocity heterogeneity $\delta v(r, \theta, \phi)$. Then

$$\delta t = - \int_{\text{path}} \frac{\delta v}{v^2} \, ds + \frac{K_{\text{pkp}}}{c \Delta \rho_{\text{cmb}}} \sum_{l,m} \int_{r}^{\infty} \frac{1}{v} \left( \frac{1}{v} \right) \beta(r) \rho(r) x_{lm}(r) \, dr \, Y_{lm}(\theta, \phi).$$

We now make use of the results of Section 2.2. Specifically, we replace $x_{lm}(r)$ with its expression (9). Denoting $R_i(r)$ the basis functions used to describe the $r$-dependence of $\delta v$, so that $x_i(r) = \sum x_{ik} R_i(r)$, eq. (9) takes the form

$$x_{lm}(r) = \sum_{i,k} x_{ik} p_{i,lm} R_i(r),$$

and substituting into (13),

$$\delta t = - \int_{\text{path}} \frac{\delta v}{v^2} \, ds + \frac{K_{\text{pkp}}}{c \Delta \rho_{\text{cmb}}} \sum_{l,m} \int_{r}^{\infty} \frac{1}{v} \left( \frac{1}{v} \right) \beta(r) \rho(r) \sum_{i,k} x_{ik} p_{i,lm} R_i(r) \, dr \, Y_{lm}(\theta, \phi).$$

In practice, we choose the $R_i(r)$ to be a set of non-overlapping adjacent layers spanning the entire mantle; in analogy with Boschi & Dziewonski (1999), Boschi & Dziewonski (2000) and Soldati et al. (2003), the product $R_i(r) \rho(\theta, \phi)$ then defines a voxel. $\delta v/v$ can be written as a linear combination of voxels,

$$\delta v/v = \sum_{i,k} x_{ik} p_{i}(\theta, \phi) R_i(r).$$

Replacing $\delta v/v$ in the first term at the right-hand side of (15) with its expression (16),

$$\delta t = - \int_{\text{path}} \frac{1}{v^2} \sum_{i,k} x_{ik} p_{i}(\theta, \phi) R_i(r) \, ds + \frac{K_{\text{pkp}}}{c \Delta \rho_{\text{cmb}}} \sum_{l,m} \int_{r}^{\infty} \frac{1}{v} \left( \frac{1}{v} \right) \beta(r) \rho(r) \sum_{i,k} x_{ik} p_{i,lm} R_i(r) \, dr \, Y_{lm}(\theta, \phi).$$

After changing the order of summation,

$$\delta t = \sum_{i,k} x_{ik} \left[ - \int_{\text{path}} \frac{1}{v} p_{i}(\theta, \phi) R_i(r) \, ds + \frac{K_{\text{pkp}}}{c \Delta \rho_{\text{cmb}}} \sum_{l,m} p_{i,lm} \int_{r}^{\infty} \frac{1}{v} \left( \frac{1}{v} \right) \beta(r) \rho(r) R_i(r) \, dr \, Y_{lm}(\theta, \phi) \right].$$

The expression in square brackets at the right-hand side of eq. (18) can be computed for any $PcP$ ray path once a reference Earth model ($\rho(r), v(r), \beta(r), \Delta \rho_{\text{cmb}}$) and a voxel grid ($p_{i}(\theta, \phi)$, $R_i(r)$) are defined. If eq. (18) is implemented for each observation of $PcP$ traveltime in our database, and an index $j$ is assigned to the resulting equations, one can denote $A_{j,ik}$ the term within square brackets in (18), and

$$\delta t_j = \sum_{i,k} A_{j,ik} x_{ik},$$

which is typically a large, mixed-determined linear system (Menke 1989). Eq. (19) is analogous to eq. (7) of Boschi & Dziewonski (1999), and can be solved by least-squares methods in exactly the same way. The coefficients $x_{ik}$ describing mantle $\delta v/v$, and CMB topography are not free parameters in the inversion.

In the case of seismic phases refracted through the CMB, for example, $PKP$, the above treatment can be repeated after replacing eq. (10) with

$$\delta t = - \int_{\text{path}} \frac{\delta v(r, s), (\theta, \phi)(s)}{v^2(r)} \, ds + K_{\text{pkp}} \frac{\delta c(r, s), (\theta_{\text{in}}, \phi_{\text{in}})(s)}{c}$$

$$+ K_{\text{out}} \frac{\delta c(r, s), (\theta_{\text{out}}, \phi_{\text{out}})}{c},$$

where $(\theta_{\text{in}}, \phi_{\text{in}})$ and $(\theta_{\text{out}}, \phi_{\text{out}})$ denote the coordinates of the ray path’s entry and exit points at the CMB. The sensitivity $K_{\text{pkp}}$ and $K_{\text{out}}$ of the $PKP$ traveltime to CMB topography depends, again, on the ray path incidence angle at the CMB (e.g. Dziewonski & Gilbert 1976).
2.4 Tomographic and geodynamically constrained inversions

The treatment of Section 2.3 should be compared with that of Boschi & Dziewonski (2000), who simply replace \( \delta v/c \) in eq. (10) with its voxel expansion (16) and \( \delta c/c \) with an analogous, pixel expansion \( \delta c/c = \sum_{\text{c,p} \theta, \phi} \). From the resulting equation they derive a linear system of the form (19), where the unknown values of CMB above (‘dynamics).

described in Section 2.3, there is no guarantee that the CMB to-
unknown vector at the right-hand side. As opposed to the method
described in Section 2.3, there is no guarantee that the CMB to-
unknown vector at the right-hand side. As opposed to the method
of Boschi & Dziewonski (2000) (resulting in ‘T models) and the joint tomography–geodynamics approach of Section 2.3
above (‘TG models).

In the following, we shall alternatively apply the tomography
approach of Boschi & Dziewonski (2000) (resulting in ‘T models)
and the joint tomography–geodynamics approach of Section 2.3
above (‘TG models).

3 SEISMIC DATA, TOMOGRAPHIC PARAMETERIZATION AND REGULARIZATION

Our tomographic inversions are all derived from the database of An-
tolik et al. (2001), including P, PKP, PKPdf and PcP traveltimes
corrected from the International Seismological Centre (ISC) bul-
letin. Exactly the same database was employed by Soldati et al.
(2003). This database is preferable to the updated bulletin of
Engdahl et al. (1998), in that hypocentres were further relocated
(and traveltimes accordingly corrected) to account for 3-D mantle
structure and the crustal model Crust5.1 of Mooney et al. (1998).
In recent years the global earthquake and station distribution remained
essentially unchanged.) The contribution of crustal structure to trav-
eltimes is also estimated on the basis of Crust5.1, and subtracted
from the data, such that the residual traveltimes should then be ap-
proximately sensitive only to the mantle. The observations (several
millions) of Antolik et al. (2001) are combined to form sum-
mary-traveltime catalogues: \( \sim 70 \times 10^3 \) PcP, \( \sim 30 \times 10^3 \) PKP, \( \sim 15 \times 10^3 \) PKPdf,
\( \sim 630 \times 10^3 \) P. Following Boschi & Dziewonski (2000) and Soldati
et al. (2003), we neglect the more noisy ab branch of PKP. In
the following, we shall jointly invert various combinations of the
P, PcP and PKP databases, investigating whether different phases pro-
vide consistent maps of the CMB. P data serve to constrain mantle
structure as robustly as possible, limiting the amount of smearing
of unresolved mantle heterogeneity into the CMB maps (e.g. Boschi &
Dziewonski 2000).

We follow Soldati et al. (2003) also in their choice of parametriza-
tion, consisting of 15 equal-thickness (\( \sim 200 \) km) layers each sub-
divided into 1656 voxels of approximately equal horizontal extent
(\( 5^\circ \times 5^\circ \) at the equator). Our TG models then consist of 24840
free parameters. T models include 1656 additional pixels describ-
ing the CMB topography, resulting in 26496 free parameters to-
total. All inversions are regularized, requiring that the roughness of both \( \delta v_P \) and \( \delta c \), as implemented by Boschi & Dziewonski (1999)
for voxel/pixel parametrizations, be minimum. Consistently with
Soldati et al. (2003), the norm of \( \delta c \) (not of \( \delta v_P \)) is also damped.
Independent regularization weights (‘damping parameters’) for \( \delta v_P \)
and \( \delta c \) are selected based on the synthetic tests described in Sec-

4 SYNTHEtiC TESTS

A synthetic test consists of (i) selecting an arbitrary, ‘input’ model of
mantle and CMB structure; (ii) computing input-model theoretical
traveltimes for all summary source-station couples in our database,
and for all observed phases (P, PcP, PKP); (iii) substituting \( \delta t_i \) with
those theoretical traveltimes in eq. (19), and least-squares solving.
The similarity of the resulting ‘output’ model to the input is a
measure of data and algorithm resolution (e.g. Boschi 2003).

We compute two different sets of synthetic data, each associated
with a different input model, and in both cases invert them with
both the T and TG approaches: this allows us to evaluate the rel-
ative performance of our new, joint seismic–geodynamic method
with respect to classical tomography. Both input models are by con-
struction geodynamically self-consistent, that is, the input CMB
topography is computed, through eq. (3), from the assumed mantle
\( v_P \) model, based on the viscosity profile of Mitrovica & Forte (1997)
and on our assumed scaling factor \( \delta \nu = \delta \nu_P \).

Gaussian random noise is added to the synthetic data, with vari-
ance selected as half the variance of each data set, namely 0.8 s
for direct P traveltimes, 0.7 s for PKP, 1.3 s for PKPdf and 1.9 s
for PcP.

4.1 Checkerboard test

We first compute traveltime synthetics based on the ‘checkerboard’
input model shown in Fig. 4 (left column), coinciding with the
\( l = 8, m = 4 \) spherical harmonic function with amplitude \( \delta v_P/v_P =
1.5 \) per cent and a change of sign (not of pattern) every 400 km rad-
ially (two layers). As anticipated, the input CMB model is not defined
arbitrarily, but derived from the input \( \delta v_P/v_P \) through eq. (3); this
results in a checkerboard-like topography perfectly anticorrelated
(negative topography under fast, cold, dense mantle) with the veloc-
ity heterogeneity of the immediately overlying, lowermost-mantle
layers (Section 1).

From this model, we compute synthetic traveltimes associated with
recorded P and PcP arrivals. We then invert them jointly, first
with the T and then with the TG approach. In the TG case, the
output CMB is of course not part of the least squares solution, but
computed from the output mantle via eq. (3).

The results of both experiments are illustrated in the middle
and right columns of Fig. 4. The T and TG methods are equally
successful (or unsuccessful) in reproducing the input pattern within
the mantle. In both cases, good tomography resolution is limited to
areas that are covered well by the data (compare, e.g. with fig. 2
of Soldati & Boschi (2005)). Both output models are characterized
by an evident loss in amplitude of \( \delta v_P/v_P \) with respect to the input.
More interestingly, the CMB topography is only recovered by the
TG inversion. For such a complex input model (many changes of
sign in the vertical direction within the mantle, and relatively short-

 wavelength CMB topography), the geographic coverage of PcP is
apparently too poor, and at many locations the output model is
actually anticorrelated with the input.
Figure 4. Result of a checkerboard test: input model (left hand side), output $P + P_C P$-derived seismic model (middle panel), output $P + P_C P$-derived seismic-geodynamic model (right hand side). Relative velocity heterogeneities (top panel) range between $-1.5$ per cent and $1.5$ per cent, CMB topography (bottom panel) between $-3$ and $+3$ km.

We repeat this exercise with the same input model for the combined $P$ and $PKPdf$ data set, and the results are shown in Fig. 5. Similar conclusions as above can be drawn, with the important difference that $PKPdf$ data can resolve both mantle anomalies and CMB topography also via the $T$ approach. $TG$ yet clearly outperforms $T$: the amplitude of output CMB undulations is comparable with the input, and a consistent pattern of undulations is reproduced over a much wider area.

The rms of the difference between input and output model at each model voxel/pixel (Table 1), and the correlation between input and output model (Table 2) confirms that $TG$ output models are closer to the input than $T$ ones. Inversions of $P_C P$ or $PKPdf$ (along with $P$) synthetic traveltimes are equally successful in reproducing the input mantle, whether the $TG$ or $T$ approach is used; but, in the assumption that heterogeneity be of purely thermal origin, the $TG$ approach is systematically better at resolving CMB topography.

4.2 Recovery test

We next select as input a ‘realistic’ model of mantle $v_p$ heterogeneity (obtained by inverting the joint direct $P$ and $P_C P$ data set for mantle $\delta v_p/v_p$ and CMB topography, and shown in...
Tomography of core–mantle boundary

Figure 5. Same as previous figure, with output models obtained inverting $P+PKPdf$ data. All models include mantle velocity heterogeneities (per cent) and CMB topography (km).

Fig. 6, left column), use, again, eq. (3) to compute the associated CMB topography, and compute, noise and invert the associated synthetic data as described in Section 4.1. We summarize the results in Figs 6 and 7, corresponding to inversions of the combined $P$ and $PcP$, and combined $P$ and $PKPdf$ data sets, respectively.

Velocity anomalies in the mantle are reproduced well in both $T$ and $TG$ output models, with amplitudes only slightly weaker than the input ones in the lowermost mantle of the $T$ model. The CMB is recovered almost perfectly by our $TG$ inversions; $T$ inversions of both $PcP$ and $PKPdf$ data lead to quite different output CMB maps, although both at least partially consistent with the input.

Again, we compute difference rms and correlation between output and input models in the various cases, and summarize them in Tables 3 and 4, respectively. The performance of the two methods is comparable as far as only mantle structure is concerned, but the $TG$ method outperforms the $T$ one in resolving CMB topography.

5 APPLICATION TO THE ISC DATABASE

5.1 Geodynamically self-consistent tomographic images

To correctly evaluate the difference between classic tomographic inversions ($T$) and seismic/geodynamic results ($TG$), one must make
Table 1. Checkerboard test: rms of the difference between input and output models, computed from the entire mantle (MAN), the two lowermost mantle layers (LMM), and the CMB topography (CMB). These values are obtained from independent T and TG inversions of either the combined P and PKPdf, or P and PcP data sets, as indicated in the first column.

<table>
<thead>
<tr>
<th>Inverted data</th>
<th>MAN</th>
<th>LMM</th>
<th>CMB</th>
</tr>
</thead>
<tbody>
<tr>
<td>P + PcP (T)</td>
<td>0.479</td>
<td>0.245</td>
<td>1.192</td>
</tr>
<tr>
<td>P + PcP (TG)</td>
<td>0.478</td>
<td>0.234</td>
<td>0.418</td>
</tr>
<tr>
<td>P + PKPdf (T)</td>
<td>0.477</td>
<td>0.200</td>
<td>0.813</td>
</tr>
<tr>
<td>P + PKPdf (TG)</td>
<td>0.477</td>
<td>0.207</td>
<td>0.279</td>
</tr>
</tbody>
</table>

Table 2. Checkerboard test: correlation between input and output models (acronyms same as Table 1).

<table>
<thead>
<tr>
<th>Inverted data</th>
<th>MAN</th>
<th>LMM</th>
<th>CMB</th>
</tr>
</thead>
<tbody>
<tr>
<td>P + PcP (T)</td>
<td>0.680</td>
<td>0.943</td>
<td>−0.168</td>
</tr>
<tr>
<td>P + PcP (TG)</td>
<td>0.683</td>
<td>0.944</td>
<td>0.919</td>
</tr>
<tr>
<td>P + PKPdf (T)</td>
<td>0.685</td>
<td>0.960</td>
<td>0.627</td>
</tr>
<tr>
<td>P + PKPdf (TG)</td>
<td>0.683</td>
<td>0.961</td>
<td>0.966</td>
</tr>
</tbody>
</table>

sure to follow the same procedure in the two cases, and that the mantle models be equivalently regularized. As for the CMB topography, the damping applied to the T inversions has no counterpart in the TG method, where the CMB is obtained by integration of mantle velocity heterogeneity rather than by inversion of traveltimes.

Similar to Section 4 and to Soldati et al. (2003), we first conduct independent T inversions of different combinations of the P, PcP, PKPdf and PKPdf,bc traveltimes data sets. The results, obtained with the same regularization scheme and weights as the synthetic tests in Section 4, are shown in Fig. 8. Direct T traveltimes are always included in the inverted database to guarantee that mantle structure is constrained as robustly as possible: trade-off between mantle and CMB structure is thus minimized. Following Boschi & Dziewonski (1999) and Boschi & Dziewonski (2000), data are weighted before inversion according to the extent to which each traveltimes deviates from PREM predictions; the weight w_d of the datum τ_d is an exponential function whose argument is proportional to the difference between observed and reference traveltimes:

\[
\frac{\delta t}{h_{\text{ref}} - |\delta t|}.
\]  

(21)

The mantle v_P heterogeneity models derived with different subsets of data essentially agree with each other and with previously published ones (e.g. Boschi & Dziewonski 1999, 2000), except for minor differences in the lowermost mantle, likely reflecting the non-uniformity of data coverage: fictitious structure may be mapped in regions that are sampled relatively sparsely by seismic rays. Most importantly, the T inversion of the combined P and PcP data set results in a CMB map that is inconsistent with the P and PKPdf,bc ones: PcP-derived CMB topography is characterized by a C-shaped depression under Eastern Asia, Kamchatka peninsula and the Americas, while the PKPdf maps are dominated by a major topographic high under Southeastern Asia. A similar discrepancy has been observed in previous studies (e.g. Rodgers & Wahr 1993); since all the regions in question are sampled well by the data, lack of tomography resolution cannot, alone, account for it. The discrepancy has been shown to be reduced when assuming a laterally heterogeneous outer core, consistent with the results of some independent studies (e.g. Ritzwoller et al. 1986), but the latter scenario still lacks a physical explanation (Boschi & Dziewonski 2000; Piersanti et al. 2001; Soldati et al. 2003).

We repeat all inversions with our new TG method described in Section 2.3, to find the solution models shown in Fig. 9. CMB topography is computed a posteriori from the mantle solution. The significant discrepancy between PcP- and PKPdf-based models of Fig. 8 disappears in the new maps of CMB topography, characterized by (i) depression along the Pacific rim, possibly associated with deep subducting plates, and (ii) elevation under the large, lowermost-mantle low shear velocity provinces under central Pacific and south/west Africa. This pattern is consistent with the discussion of Section 2.1.

An important metric of model quality is variance reduction. After each T or TG inversion, we calculate how much the solution model reduces the variance of each individual data set (P, PcP, PKPdf,bc). We first calculate variance reduction of data weighted as described earlier. The results are shown in Fig. 10. There is a slight but systematic decrease in the variance reduction achieved with TG with respect to T inversions. This is not surprising, as the TG method involves a decrease in the number of free parameters (model coefficients) available to fit the data: the 1656 CMB tomography pixels that are now coupled with mantle v_P voxels. We next calculate variance reduction of raw, non-weighted data, and find that the value achieved by TG models is higher for all phases (Fig. 11): the effect seen in Fig. 10 is reversed. Our TG models explain better than T models data that were not, or not entirely accounted for in both T and TG inversions. This improvement in data fit cannot be explained by tomographic artefacts as it corresponds to a decrease in the number of free parameters, and we infer that it might reflect an effective enhancement of model quality (Menke 1989; Tarantola 2005).

We compare the T and TG models quantitatively using the same metrics of Section 4, that is, model correlation and rms difference. We compute their values over the entire mantle, over the bottom two lowermost-mantle layers, and over the CMB topography only. The results are shown in Fig. 12. T and TG images of the mantle as a whole are very similar; maps of the lowermost mantle alone are slightly less well correlated than average (rms difference is also slightly stronger); discrepancies in T and TG maps of CMB topography are comparably large.

5.2 Comparison with independent v_P and v_S tomography models

To understand whether our TG models of v_P velocity introduce some change in the ratio v_S/v_P compared to the purely tomographic models T, and consequently a different interpretation of mantle structure/composition, we calculate the correlation of the T and TG models derived here from the entire P, PcP, PKPdf, database, with the PMEAN and SMEAN models of Becker & Boschi (2002). The results are shown in Fig. 13 in the same style as analogous figures in Becker & Boschi (2002). Being obtained from the combination of different earlier models, each particularly robust over a different depth range, PMEAN and SMEAN are a reasonable choice of ‘reference’ models. SMEAN in particular has been shown to fit broadband seismic data at least as well (Qin et al. 2008), and geoid observations better (Steinberger & Anttetre 2006) than other recent tomographic models.

Figs 13(a) and (c) show that the correlation between PMEAN and both our T and TG models is very high over most of the mantle. This is not surprising given that PMEAN was derived from essentially the same data as our models. In the lowermost mantle, where
Figure 6. Result of a recovery test: input model (left-hand side), output $P + PcP$-derived seismic model (middle panel), output $P + PcP$-derived seismic-geodynamic model (right-hand side). All models include mantle velocity heterogeneities (per cent) and CMB topography (km).

the latter are also based on PKP and $PcP$ data, not accounted for by PMEAN, the correlation is lower. Correlation between SMEAN and our models (Figs 13b, d) is much lower than that of PMEAN, confirming the differences between $v_S$ and $v_P$ patterns at relatively high harmonic degrees throughout the mantle, and particularly in the lowermost mantle (Della Mora et al. 2011). Together with the anomalously high ratio of $v_S$ to $v_P$ anomaly, this decorrelation has been interpreted as being caused by chemical, rather than purely thermal heterogeneity (e.g. Ishii & Tromp 1999; Forte & Mitrovica 2001; Trampert et al. 2004). In this context, it is interesting to note that $v_P$ anomaly amplitudes from $TG$ models are systematically, albeit only slightly higher than those of $T$ models, although the correlation between $T$ and PMEAN/SMEAN models is essentially as high as that between $TG$ and PMEAN/SMEAN.

5.3 A very low viscosity, post-perovskite layer in the lowermost mantle

Results discussed so far were obtained assuming the radial mantle viscosity profile of Mitrovica & Forte (1997) (Fig. 1). We have repeated our experiments with alternative viscosity profiles, for example, that of Soldati et al. (2009) (also shown in Fig. 1), and we calculated the fit to geodynamical and seismic data. The results are similar to those of Section 5.1 and we do not show them here in the interest of brevity.

Gravity and postglacial rebound data used to constrain mantle viscosity are approximately insensitive to lowermost mantle rheology (Soldati et al. 2009). However, it has been recently shown that mantle materials undergo a phase transition from perovskite to post-perovskite (Murakami et al. 2004; Oganov & Ono 2004), presumably associated with the $D'_\ast$ seismic layer in the lowermost mantle. Several studies have also shown post-perovskite to be characterized by higher density and, perhaps, orders-of-magnitude lower viscosity than post-perovskite (Yamazaki et al. 2006; Tosi et al. 2009; Ammann et al. 2010). A broad lowermost-mantle layer of anomalously low viscosity is likely to affect strongly deep mantle flow, and, subsequently, the mantle-CMB kernels of Section 2.1. As anticipated in Section 2.1, we model this effect correcting the bottom 250 km of the viscosity profile assumed so far (Mitrovica & Forte 1997), where we now impose a viscosity $10^3$
times lower than in the immediately overlying layer (Fig. 1). The resulting kernels are shown in Fig. 2 and were discussed in Section 2.1. We show in Fig. 14 TG models based on the kernels of Fig. 2. Even in the lowermost mantle, mapped \( v_P \) is only marginally perturbed with respect to the TG models of Section 5.1. CMB undulations, on the other hand, change significantly, with a similar pattern but amplitude reduced by over 75 per cent. In addition, the localized depression to the west of Cocos plate fades away almost completely.

Variance reduction of seismic data achieved by the TG models of this section is only marginally different from those illustrated in Section 5.1 (\( \sim 1 \) per cent difference depending on the phase, with the \( P\delta P \) variance reduction slightly higher for the low-viscosity models, and that of other data sets slightly lower). Interestingly, however, the low-viscosity models are characterized by mapped CMB ellipticity systematically closer to the value given by, for example, Gwinn et al. (1986) based on independent geodetic observations, as discussed in Section 5.4 later.

Table 3. Recovery test: rms of the difference between input and output models (acronyms same as Table 1) for a realistic mantle \( v_P \) structure.

<table>
<thead>
<tr>
<th>Inverted data</th>
<th>MAN</th>
<th>LMM</th>
<th>CMB</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P + P\delta P (T) )</td>
<td>0.056</td>
<td>0.0744</td>
<td>0.895</td>
</tr>
<tr>
<td>( P + P\delta P (TG) )</td>
<td>0.058</td>
<td>0.065</td>
<td>0.151</td>
</tr>
<tr>
<td>( P + PKP (T) )</td>
<td>0.0549</td>
<td>0.0737</td>
<td>0.884</td>
</tr>
<tr>
<td>( P + PKP (TG) )</td>
<td>0.057</td>
<td>0.098</td>
<td>0.218</td>
</tr>
</tbody>
</table>

Table 4. Recovery test: correlation between input and output models (acronyms same as Table 1) for a realistic mantle \( v_P \) structure.

<table>
<thead>
<tr>
<th>Inverted data</th>
<th>MAN</th>
<th>LMM</th>
<th>CMB</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P + P\delta P (T) )</td>
<td>0.980</td>
<td>0.934</td>
<td>0.677</td>
</tr>
<tr>
<td>( P + P\delta P (TG) )</td>
<td>0.979</td>
<td>0.943</td>
<td>0.991</td>
</tr>
<tr>
<td>( P + PKP (T) )</td>
<td>0.980</td>
<td>0.922</td>
<td>0.817</td>
</tr>
<tr>
<td>( P + PKP (TG) )</td>
<td>0.978</td>
<td>0.874</td>
<td>0.984</td>
</tr>
</tbody>
</table>

Figure 7. Same, with output models obtained inverting \( P+PKP \) data. All models include mantle velocity heterogeneities (per cent) and CMB topography (km).
5.4 Excess ellipticity of the CMB

The shift of the period of the Earth’s free core nutation predicted using hydrostatic theory from the one observed using VLBI geodesy has been interpreted as due to excess CMB ellipticity caused by convection (Forte et al. 1995), with a peak-to-valley deviation of 490 ± 110 m (Gwinn et al. 1986). Table 5 shows the values we find depending on data and method (T/TG); for each of our CMB topography models we compute a spherical harmonic expansion, filter out all components but the $l = 2, m = 0$ one, and measure the peak-to-valley deviation of the resulting map. T models are overall closer to the VLBI-observed CMB ellipticity than TG. If we use a more realistic rheology, including a post-perovskite layer modelled as a very low viscosity region at the base of the mantle, our TG models agree slightly better with the observations of Gwinn et al. (1986).

To rule out the possibility of trade-off between the choice of the viscosity profile and that of the density-to-velocity conversion factor, we test the effect of reducing by 1000 times the value of $\delta \ln \rho / \delta \ln v_P$ (instead of that of the radial viscosity) in the lowermost 250 km of the mantle. Such reduction in $\delta \ln \rho / \delta \ln v_P$ could be expected based on the indication, from mineral physics, that velocity-density scaling could decrease with increasing pressure. We find, again, values of CMB ellipticity too high (peak to valley anomaly of 1 km) to be consistent with the VLBI data; at this stage, a low-viscosity post-perovskite layer in the TG approach remains the only mechanism to explain better VLBI-observed CMB ellipticity.

6 DISCUSSION

We have constructed a series of tomographic models of mantle $P$-wave velocity and CMB topography accounting for the mechanical coupling between mantle heterogeneity which drives viscous flow, and fluid outer core, whose upper boundary is deformed accordingly. Instead of inverting seismic traveltimes for both mantle structure and CMB deflections, as it has been done so far, the new models are built with mantle heterogeneity as the only unknown of our inverse problem, but taking into account topography seen by the data at the CMB. We thus get mantle-CMB models that are both geodynamically sound and consistent with seismic data. The same concept can in principle be applied to other mantle discontinuities. Including this mantle-CMB geodynamic coupling is practically equivalent to a physically based regularization of the inverse problem.

Our approach rests on the assumption that the chemical contribution to mantle heterogeneity be negligible compared to the thermal one. More work (for example, combining the database used here with $S$-wave observations) will be needed to take proper account of compositional heterogeneity (e.g. Karato 2003; Deschamps & Trampert 2003; Trampert et al. 2004; Della Mora et al. 2011), though there are indications that thermal effects might still be predominant (Forte & Mitrovica 2001; Simmons et al. 2009). Our results, particularly the response of CMB topography to mantle density perturbations, also depend on the radial viscosity profile of the mantle, notoriously difficult to constrain particularly at large depths (Soldati et al. 2009); however, repeating our calculations with a set of quite different viscosity profiles (Fig. 1) we have found that at
Figure 9. Maps of mantle $v_P$ anomalies (per cent) and CMB topography (km). The maps should be read as in the previous figure. They are derived through a tomographic-geodynamic (TG) inversion.

Figure 10. Weighted variance reduction (weighting used in inversion) achieved by $T$ and $TG$ models, based on the combined $P$ and $PcP$, $P$ and $PKPdf,bc$, and complete databases: each datapoint on the horizontal axis corresponds to a different $T$ or $TG$ model, obtained from a different data set, as indicated. For each model, we compute the variance reduction of the data set associated with each phase (four values on the vertical axis), that is, $P$ (red squares), $PcP$ (green triangles), $PKPdf$ (blue circles), $PKPbc$ (purple triangles).
least the overall pattern (if not amplitude) of mapped CMB topography remains stable. In addition, since independent data are available to test the quality of our CMB maps (e.g. waveforms of diffracted seismic phases; eigenfrequency splitting of CMB-sensitive modes), in future work it might be possible to use the latter to evaluate the reliability of proposed viscosity models. A final assumption of our calculations is that the outer core be laterally homogenous. This is occasionally questioned, and there is emerging evidence of radial...
Figure 13. (a) (Left plot) correlation \( r \) up to harmonic degree 8, and \( r_{20} \) up to harmonic degree 20, as functions of depth between our \( T \) model and the \( v_P \) model \( PMEAN \) of Becker & Boschi (2002); (middle plot) correlation between the same two models as a function of depth, and harmonic degree; (right plot) rms of both models as a function of depth, with our model marked as model 1 and \( PMEAN \) as model 2. (b) same as (a), but for our \( T \) model and the \( v_S \) model \( SMEAN \) of Becker & Boschi (2002). (c) \( TG \) and \( PMEAN \). (d) \( TG \) and \( SMEAN \).

Figure 14. \( TG \) maps of \( \delta v_P/v_P \) (per cent) in the lowermost mantle (depth = 2700–2900 km) obtained using a viscosity profile for the mantle, which mimics the presence of the perovskite–postperovskite phase transition. Bottom plots refer to the corresponding CMB topography (km) computed geodynamically from mantle \( v_P \) models. The models were obtained inverting different subsets of the data, left to right, as indicated.

Table 5. Predicted excess CMB ellipticity (km) obtained from models \( T \) (Fig. 8) and \( TG \) (Figs 9, 14), \( TG \) (ppv) with a low-viscosity post-perovskite layer (Section 5.3), and \( TG \) (\( \delta \ln \rho/\delta \ln v_P \)).

<table>
<thead>
<tr>
<th>Inverted data</th>
<th>( T ) model</th>
<th>( TG ) model (ppv)</th>
<th>( TG ) model (( \delta \ln \rho/\delta \ln v_P ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P + PcP )</td>
<td>0.948</td>
<td>1.221</td>
<td>0.360</td>
</tr>
<tr>
<td>( P + PKPdf )</td>
<td>0.490</td>
<td>0.956</td>
<td>0.295</td>
</tr>
<tr>
<td>( P + PKPbc )</td>
<td>2.172</td>
<td>1.270</td>
<td>0.390</td>
</tr>
<tr>
<td>( all \ data )</td>
<td>0.390</td>
<td>1.029</td>
<td>0.308</td>
</tr>
</tbody>
</table>

compositional variation in the outermost outer core (e.g. Helffrich & Kaneshima 2010), but preliminary experiments not shown here for brevity show that our \( TG \) maps are not strongly perturbed by the introduction of outer-core heterogeneity (S. Della Mora et al., in preparation).

The new joint tomographic–geodynamic models (labelled \( TG \)) are compared to models (\( T \)) obtained via a classic tomographic treatment (e.g. Boschi & Dziewonski 2000; Forte & Mitrovica 2001) through a series of inversions of synthetic and real data. From the checkerboard and recovery tests of Section 4 it emerges that while \( T \) and \( TG \) methods perform equally well in reproducing the input mantle model, the latter approach enhances the resolution of the CMB input topography.

Because the CMB topography is not directly inverted for, the \( TG \) approach involves a much smaller number of free parameters; nevertheless, we find that our new \( TG \) models explain the database of Antolik et al. (2001) at least as well as the \( T \) ones. This can be
interpreted as an indication that mantle flow is predominantly governed by thermal, rather than compositional, heterogeneity. On the other hand, the amplitude of the $P$-wave velocity anomalies in our $TG$ models of the lowermost mantle remains basically unchanged compared to that of $T$ models, and so does the ratio $\delta v/P_s$, confirming that a certain degree of compositional heterogeneity exists in this depth range (Simmons et al. 2010; Della Mora et al. 2011).

The CMB maps obtained on the basis of $PcP$ versus $PKP$ data are highly correlated, as opposed to the systematic discrepancy found by classic tomographic approaches (Rodgers & Wahr 1993; Boschi & Dziewonski 2000; Soldati et al. 2003). Robust structure that emerged from earlier purely tomographic inversion at least of $PcP$ data is confirmed by our $T$ and $TG$ inversions here. In particular, the spherical-harmonic degree-2 character of $TG$ maps of the CMB (bottom panels of Fig. 9), depressed under the Pacific rim (corresponding to possible sinking slabs) and relatively elevated under the Pacific and African superplumes, is very similar to what is found in classical tomography, both here (Fig. 8, bottom left and bottom right panels) and in earlier studies (e.g. Morelli & Dziewonski 1987; Rodgers & Wahr 1993; Soldati et al. 2003). In addition, our $TG$ models are found to reduce the variance of normal-mode eigenfrequency splitting observations (Koelemeijer & Deuss, personal communication, 2011). These are all strong indications that the data have at least some useful sensitivity to CMB topography.

The $TG$ approach introduced here reduces the amplitudes of CMB topography by about 30 per cent, which is consistent with geodynamic results found in the literature (e.g. Forte et al. 1995). The peak-to-valley deviation of the $\ell = 2, m = 0$ component of CMB inferred from VLBI observations is more difficult to match, but can be reproduced (although with reasonable error) provided that a more realistic rheology is employed: namely, a 250 km thick low viscosity layer above the CMB simulating the presence of post-perovskite.

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REFERENCES


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