Euler Diagram Reasoning

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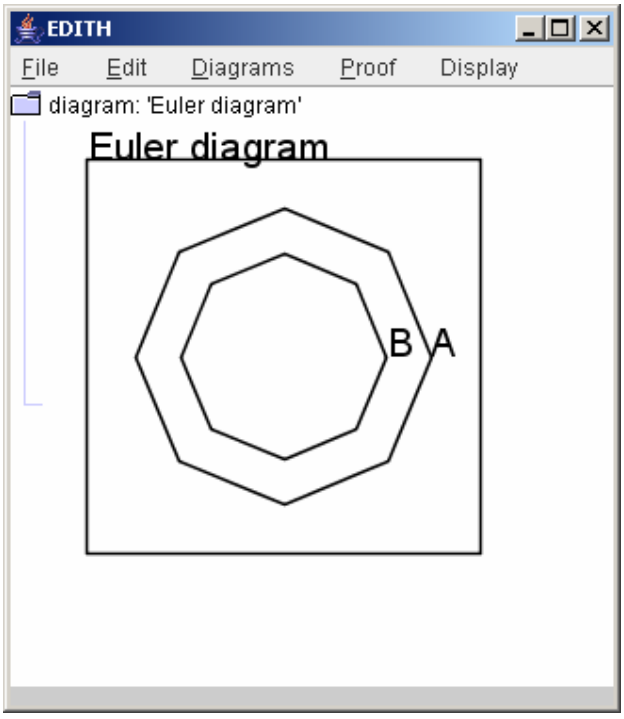
This guide has been written as an accompaniment to the How to Use Edith documentation available from http://www.cmis.brighton.ac.uk/research/vmg/HowToUseEdith.pdf. It is intended to aid users of Edith who are less familiar with Euler diagram reasoning, allowing them to gain an understanding of the terminology required in order to use Edith successfully.

What is an Euler Diagram

Before we can reason with an Euler diagram, we need to know what an Euler diagram is and what it means. The example below shows an automatically generated Euler diagram.

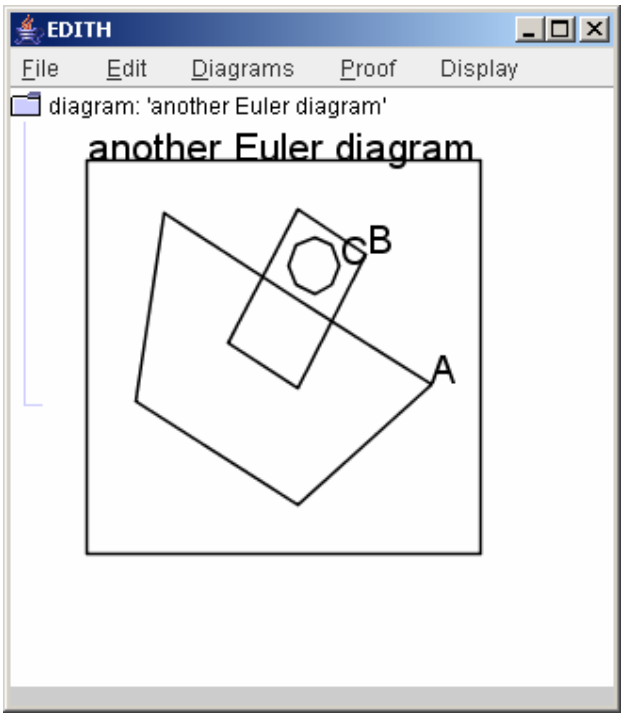


This Euler diagram has two *contours*, which are labelled A and B. B is drawn inside A, so the diagram asserts that B is a subset of A or, equivalently, everything that is B is also A.

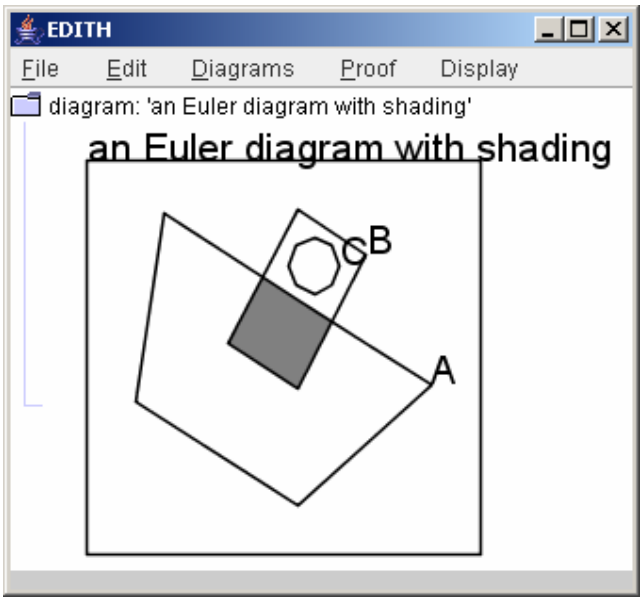
Unfortunately, Edith does not yet have facilities that allow users to draw diagrams themselves (i.e. Edith is not an editing tool) so the diagrams have to be entered via a diagram description. Details of how to enter diagrams can be found in the previously mentioned How to Use Edith guide. Here, we will explain how to describe a diagram.

An Euler diagram can be described by the minimal regions in the plane that the contours give rise to. In the Euler diagram above, the curves give rise to three pieces of the plane: that outside both contours, that inside A but outside B, and that inside both A and B. These three pieces are called *zones* and are be described in the following way: (, AB), (A, B) and (AB,). The diagram also has what is known as a missing zone: there is no zone inside B but outside A; this missing zone is described by (B,A).

The Euler diagram below has 5 zones which are described by (, ABC), (A, BC), (AB, C), (B, AC) and (BC, A). It has 3 missing zones: (AC, B), (ABC,) and (C, AB).



Shading allows us to assert the emptiness of a set. The diagram below has the same zone set as that above, but also has a shaded zone, namely (AB, C). The diagram asserts that everything that is C is also B but not A (because C is inside B band completely outside A) and nothing is both B and A but not C (because there is shading inside A and B but outside C).

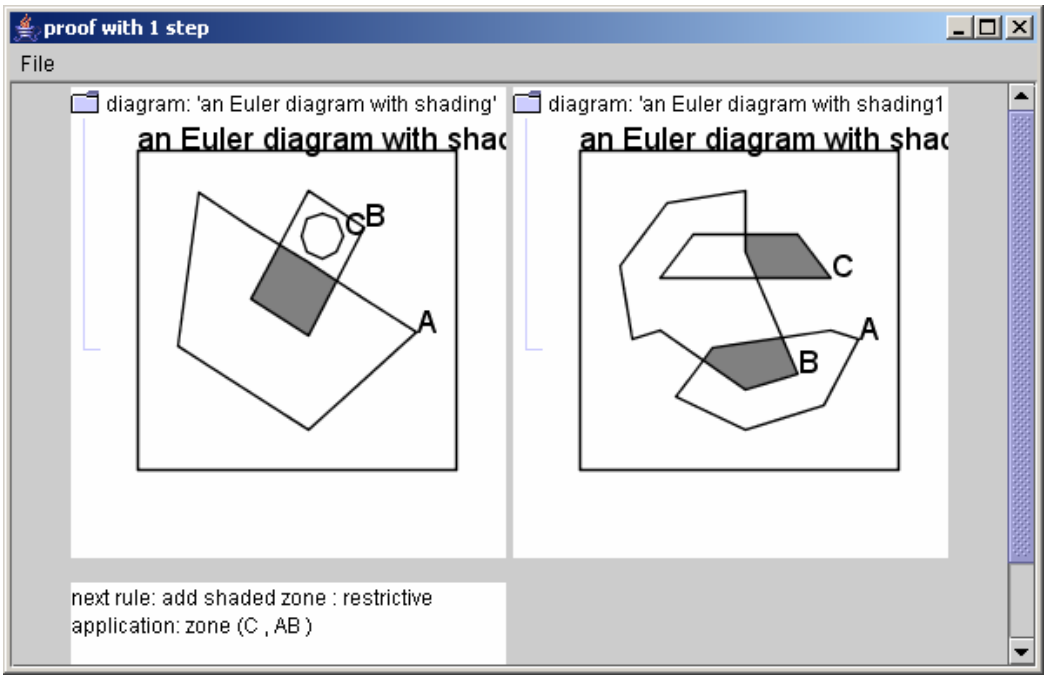


Reasoning with Euler Diagrams

Suppose we have two diagrams, d_1 and d_2, such that we can deduce d_2 from d_1. We would like to be able to *prove* that d_2 does, indeed, follow from d_1. Roughly speaking, a proof is a sequences of syntactic transformations (reasoning rules) applied to d_1 that result in d_2. Formal proofs (or *deductions*) are written using specified reasoning steps. Here, we give details on the so-called restrictive reasoning rules that Edith incorporates. The other rules that Edith has access to are generalizations of these five. However, the five rules presented here form what is known as a *complete set* which means that any time we want to write a proof, these rules are sufficient for us to be able to find a proof provided the meaning of d_2 can be deduced from the meaning of d_1. For each of the five rules, an illustration of its application can be seen in the screenshot immediately beneath it.

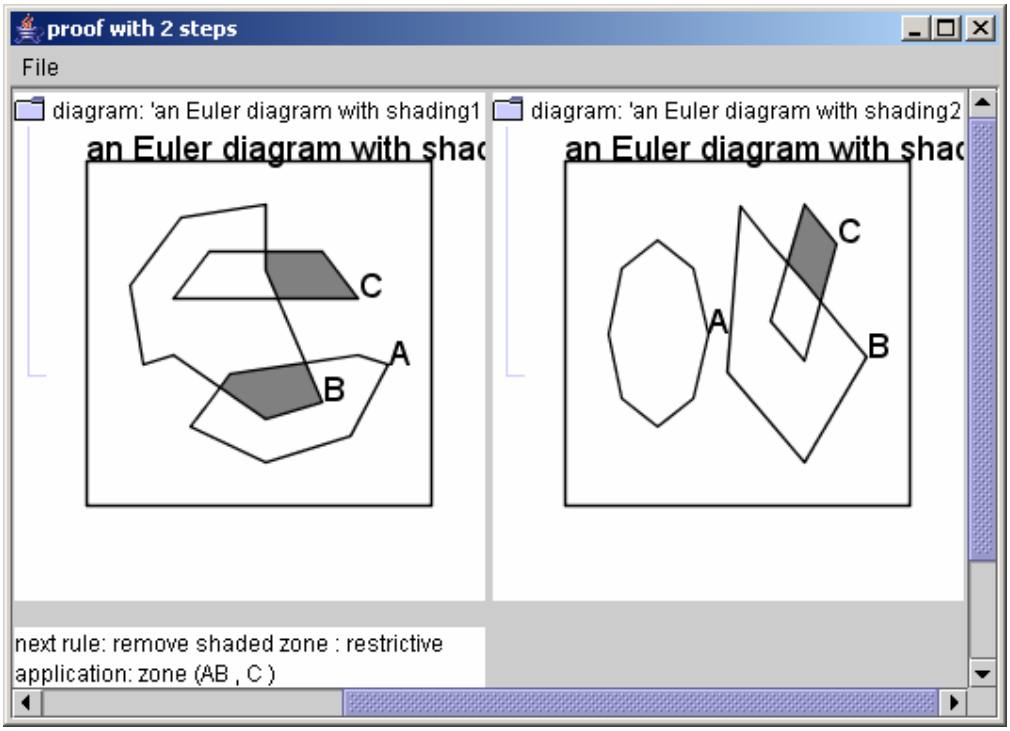
Rule 1: Add shaded zone : restrictive

If a diagram, *d*, has a missing zone then we can add that missing zone to *d* and shade it in.



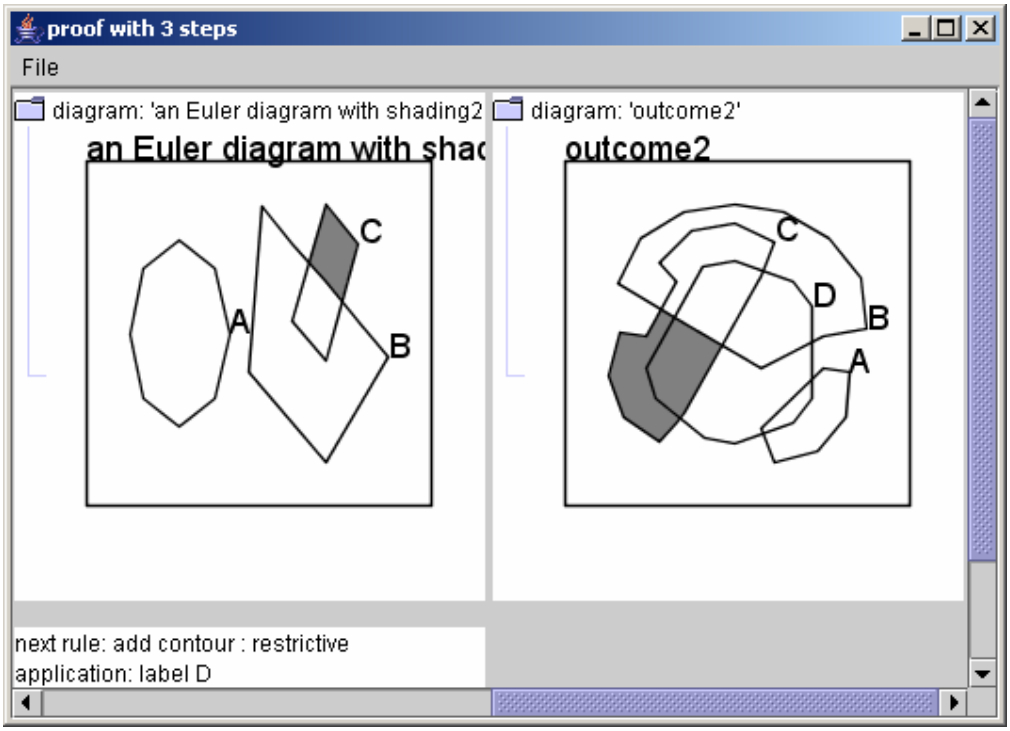
Rule 2: Remove shaded zone : restrictive

If a diagram, *d*, has a shaded zone, *z*, such that for all contour labels, *l*, in *d*, there is some zone inside *l* distinct from *z* then *z* can be removed from *d*.



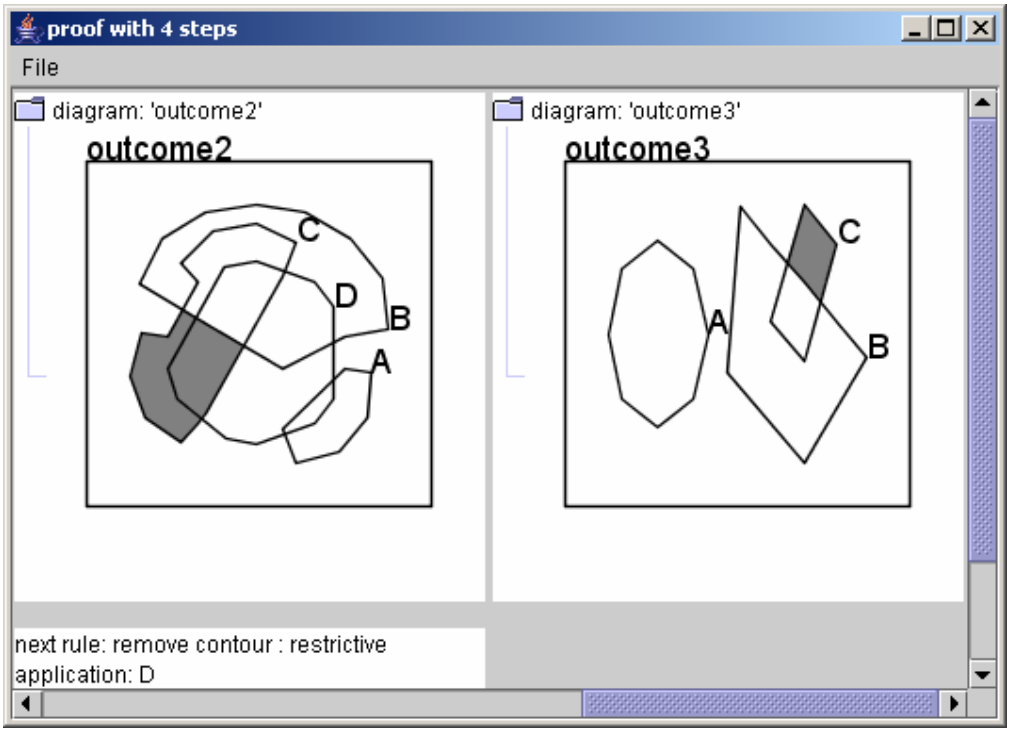
Rule 3: Add contour : restrictive

A contour with label *l* can be added to diagram, *d*, provided each zone splits into two new zones: one inside and one outside the new contour. Shading is preserved.



Rule 4: Remove contour : restrictive

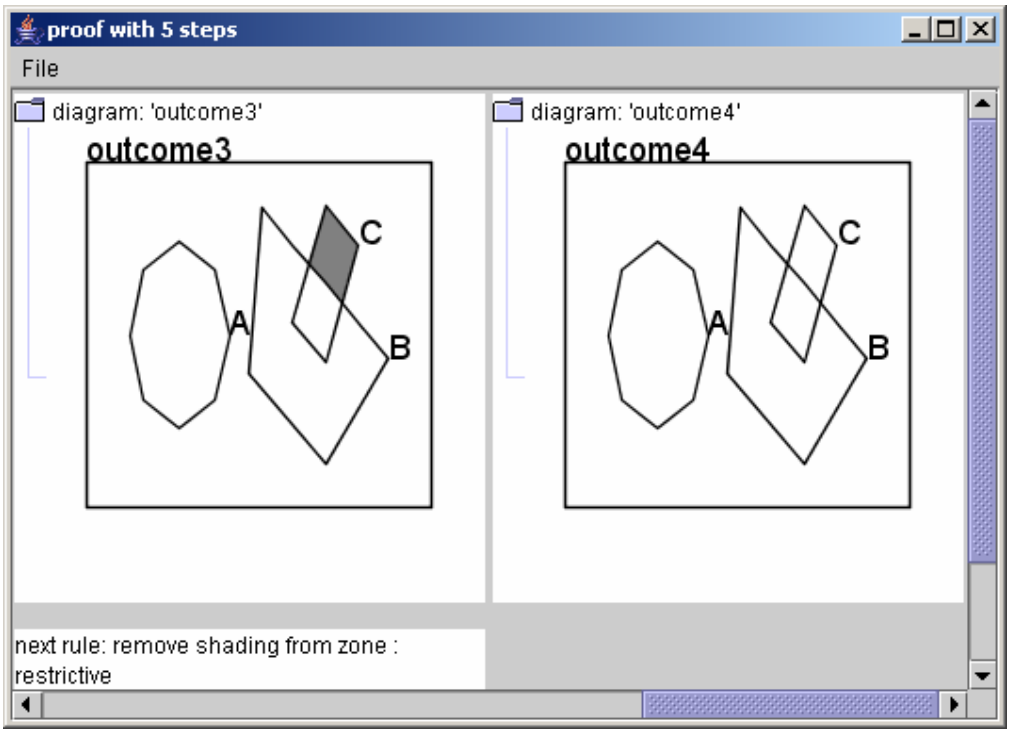
A contour with label *l* can be removed from diagram, *d*, provided each zone joins with another zone: one inside and one outside the removed contour. Shading is preserved. In other words, the removing a contour is the reverse of adding a contour.



So far, the reasoning rules have preserved information. A further reasoning rule that weakens information.

Rule 5: Remove shading from zone : restrictive

If a diagram, *d*, has a shaded zone then we can remove the shading from that zone.



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