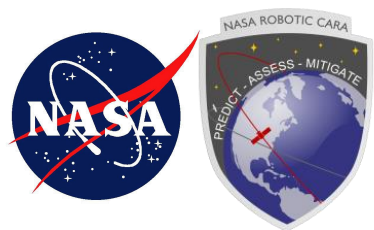


Conjunction Assessment Risk Analysis



3D Pc Operational Issues and Ways Forward

M.D. Hejduk
23 MAY 2017



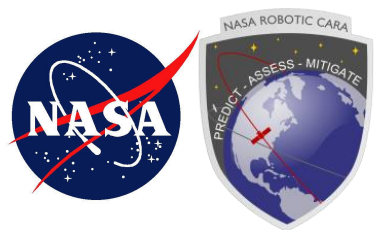
2D and 3D Pc: Abbreviated Technical Background

- **Two-dimensional (2D) probability of collision (P_c)**

- Developed for Shuttle program in early 1990's
- Presumes hyperkinetic encounter—rectilinear motion, position covariance only, and static position throughout encounter
- Applicable to great majority of conjunctions

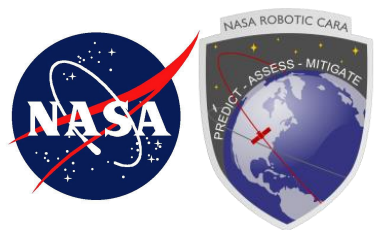
- **3D P_c formulated to operate when these restrictions relaxed**

- Theory developed by V.T. Coppola of AGI; integrates time-series of instantaneous penetrations of HBR sphere by uncertainty volume
- Allows curved rather than straight trajectories, uses full 6 x 6 covariance, and allows covariance to evolve over conjunction duration
- Attractive methodology to expand domain of P_c analytical calculation
 - Persistent conjunctions and others that respond poorly to 2D P_c
- Also introduces/frames concept of first derivative of P_c ; useful for understanding conjunction dynamics and determining background risks
- Operates only in reference frames in which position and velocity components can be separated (e.g., Cartesian orthogonal frame)



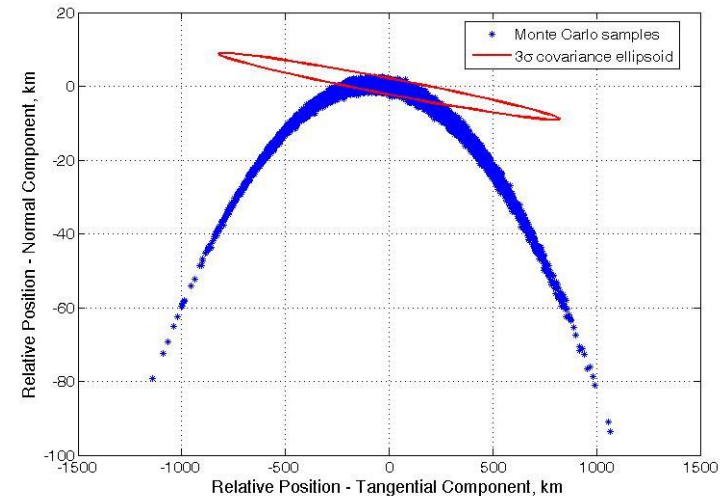
3D Development/Validation at CARA

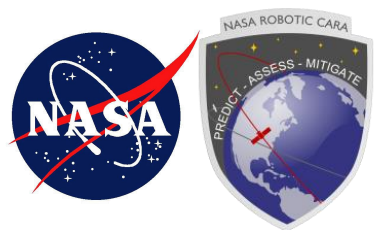
- **Developed full operational implementation/prototype**
- **Developed non-rectilinear TCA Monte Carlo for 3D Pc validation**
- **Executed and profiled 2D/3D comparison against ~80,000 conjunctions from latter part of 2016; results grouped as follows:**
 - Group 0: 2D and 3D Pc match to within operational tolerances (most events)
 - Group 1: Persistent Conjunctions—3D Pc substantially larger
 - Group 2: Modest improvements—3D Pc somewhat different
 - Group 3: “Distended Covariances”—3D Pc substantially larger or smaller
 - Group 3 most prevalent (~6% of significant events) and most surprising
- **All four groups validated by Monte Carlo (~40 events run)**
 - Matches for Groups 2 and 3 nearly exact; Group 1 confirmatory
- **Implemented in operations December 2016**
 - Large 2D/3D Pc differences (Group 3) believed to pose safety-of-flight risk



“Non-Gaussian” Covariance Behavior: Brief Introduction

- **Two issues with covariance for CA analytic calculation (2D or 3D)**
 - Actual error volume must be Gaussian
 - Covariance as formulated can describe only multivariate Gaussian error distributions
 - Covariance must be represented in coordinates in which position and velocity can be separated
- **Concern that, with long distended covariances, error volume not properly representable in Cartesian coordinates**
 - Actual in-track error follows curved orbit path
 - In-track component of Cartesian covariance remains tangent to orbit path
 - Disjunction possible between Cartesian error representation (used in Pc calculations) and actual error distribution

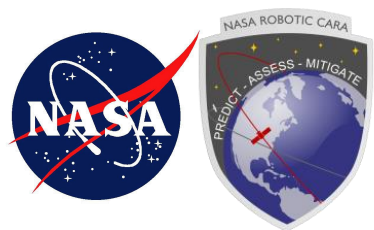




Non-Gaussian Covariance Behavior

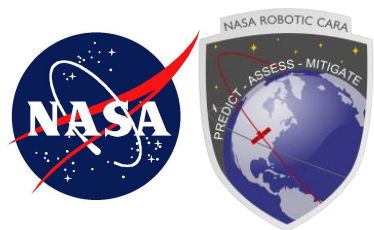
- **In 2012, CARA performed study on the effect of potential non-Gaussian behavior on CA calculations***
 - Examined 248 high-Pc conjunctions with covariances of various levels of “in-track distention”
 - Only one of the 248 showed appreciable difference in Pc between 2D value and that with methods to correct for non-Gaussian behavior
 - And for that one case, covariance was not all that seriously distended
- **Conclusion was that non-Gaussian behavior, manifested by covariance distention, is not problematic for Pc calculation**
- **Based on 2012 study, did not suspect any non-Gaussian problem with 3D Pc**
- **However, at March 2017 Users Forum committed to investigating conjunctions with large 2D/3D Pc differences for any evidence of non-Gaussian behavior**
 - Enhanced Monte Carlo capability under development to check for this

*Ghrist, R. and Plakalovic, D. “Impact of Non-Gaussian Error Volumes on Conjunction Assessment Risk Analysis.” AIAA/AAS 2012 Astrodynamics Specialist Conference, Minneapolis MN, August 2012.



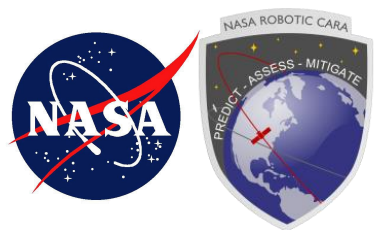
3D Pc Problem Discovery

- **Enhanced Monte Carlo (MC) capabilities (two separate lineages) became available last week of April 2017**
 - Performs MC in element space (equinoctial elements); reference frame insulated from non-Gaussian behavior due to orbit curvilinearity
- **First application of new Monte Carlo was to check cases with large 2D/3D Pc differences**
 - Initial two cases disturbing—Monte Carlo Pc much closer to 2D value
- **Immediately launched high-priority study effort**
 - 1) Is the 3D Pc calculation miscarrying in cases of large 2D/3D Pc differences?
 - 2) What can we do operationally to respond while problem is studied enough to understand it (and if necessary propose remediations)?
 - 3) If 3D Pc calculation is in fact erring in some circumstances, why is this so?
 - 4) What can be done to repair/enhance the 3D Pc calculation?
- **Purpose of today's Users' Forum is to report on the four questions above**



Question 1: Is the 3D Pc Calculation Miscarrying?

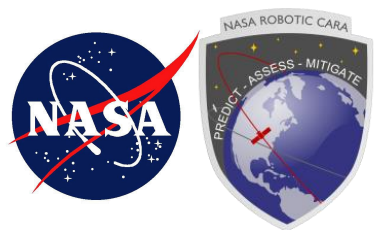
- **Profiled set of ~500,000 conjunctions (CDMs) over past year**
 - 0.4% have 2D Pc > 1E-07 and > 1 order of magnitude (OoM) difference between 2D and 3D Pc
- **33 high-Pc cases examined with 2D, 3D, and MC calculation**
 - Used both “old” MC and both lineages of enhanced (“new”) MC
 - In all 33 cases, new MC and 2D calculations very close
 - Old MC and 3D Pc also very close, but different from above, and presumably wrong
- **Conclusion is that non-Gaussian effect of some type seems to be corrupting 3D Pc (and Cartesian Monte Carlo) in certain cases**



Question 2:

What Immediate Operational Response is Possible?

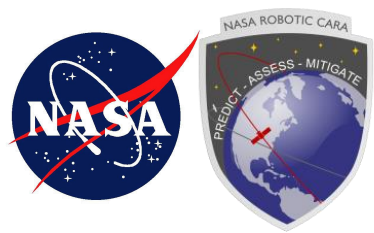
- **Once first few examples of 3D Pc miscarriage were verified by Monte Carlo, the following operational procedure was instantiated:**
 - Script written to identify all cases in which 2D and 3D Pc differed by at least an order of magnitude. This script is run daily by on-call analyst.
 - If either 2D or 3D value exceeded $1E-05$, enhanced Monte Carlo is run to validate Pc and increased tasking is requested on the secondary object.
 - If “true” Pc as established by Monte Carlo exceeded worrisome level ($\sim 5E-05$) and less than five days to TCA, mission is to be notified
 - No cases since procedure development have met these criteria
 - Usually natural event evolution and increased tasking shrinks covariance, and 2D and 3D Pc calculations come into alignment
- **Procedure is to be followed until fix can be put into CARA software**



Question 3:

Why does the 3D Pc Occasionally Err?

- **Initial supposition: long, thin covariances introduce non-Gaussian covariance behavior that somehow trips up 3D Pc**
 - Further analysis revealed that 2D/3D Pc mismatches are not all that strongly correlated with covariance distension
- **Current thinking: issues with the velocity portion of the covariance introduce problems in 3D Pc calculation**
 - Discovery: zeroing out the velocity portion of the covariance makes 3D Pc and old MC match 2D Pc and new MC
 - Cartesian rendering of covariance appears to overstate velocity uncertainties—causes additional dispersion that erroneously raises or lowers Pc, depending on circumstances
 - Significant finding that came as surprise to major researchers in CA discipline
 - Velocity portion of JSpOC covariances has never been studied in depth
 - Non-Gaussian behavior manifests itself through velocity uncertainties, not positional issues
 - Reason why behavior not correlated with covariance distention

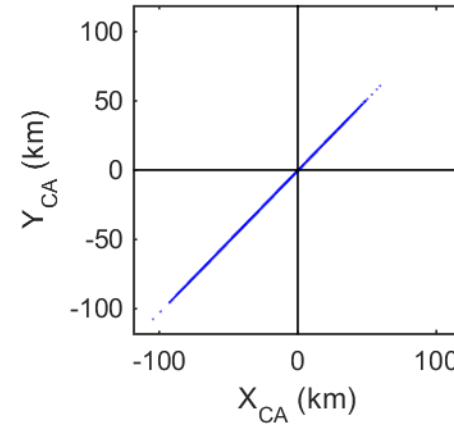
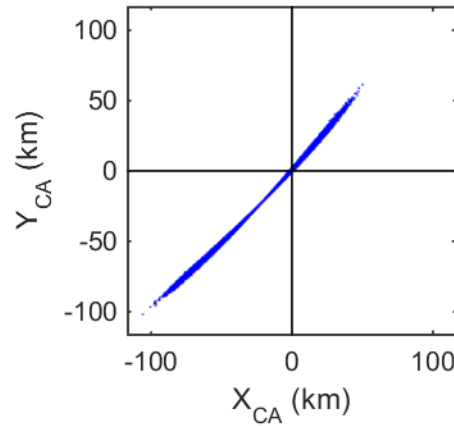
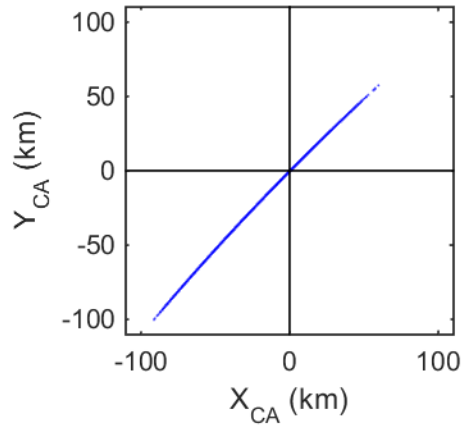


Velocity Uncertainties Push 3D Pc too High

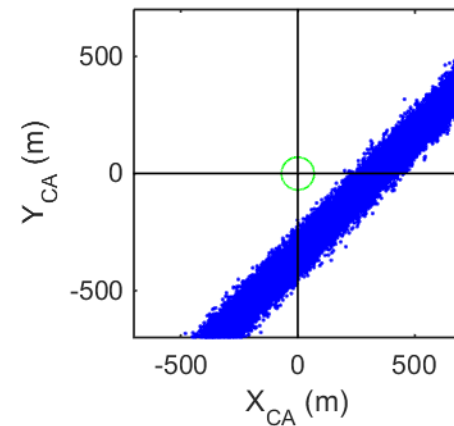
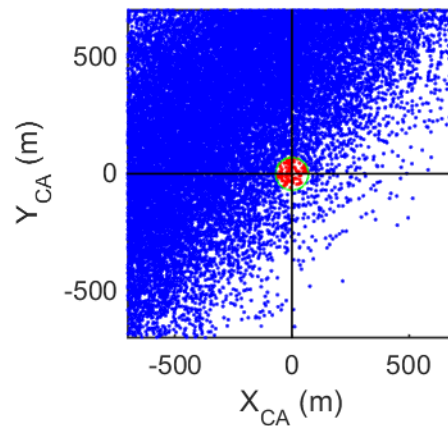
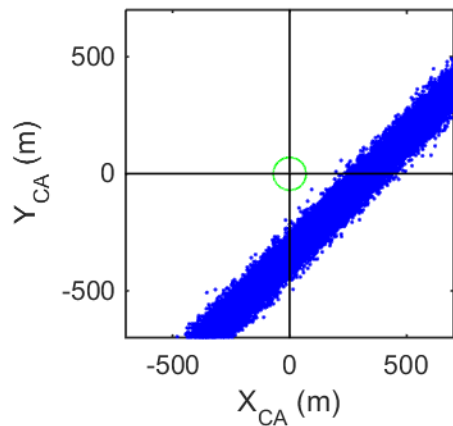
**“New” MC
Equinoctial Sampling**

**“Old” MC
With Vel. Covariance**

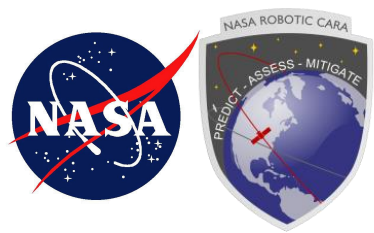
**“Old” MC
Zero Vel. Covariance**



**Neglecting TCA
vel. covariances
eliminates the
erroneous
curvature (mostly)
and scatter**



**Zoom shows no
close approaches
occur **inside** the
combined HBR
when velocity
covariances are
set to zero**



Velocity Uncertainties Push 3D Pc too Low

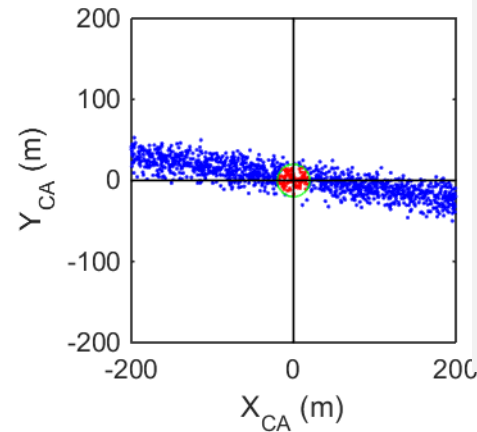
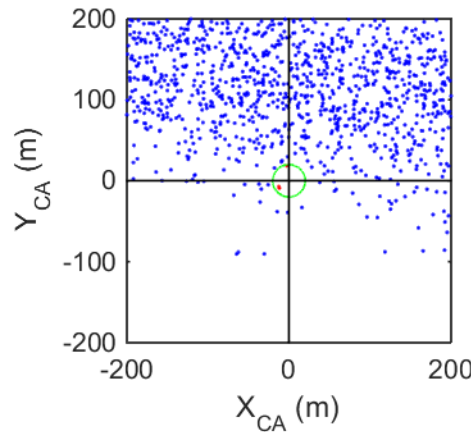
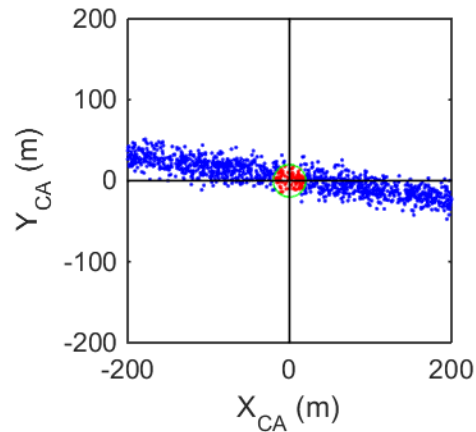
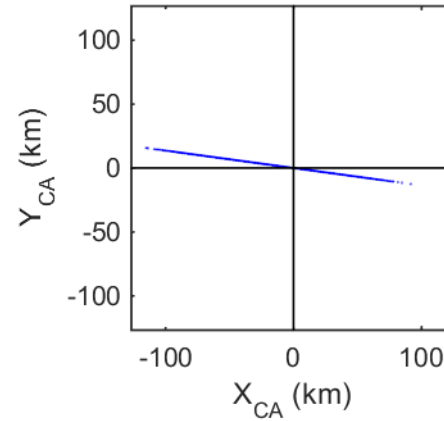
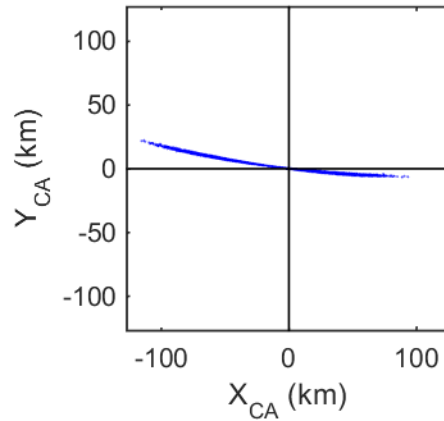
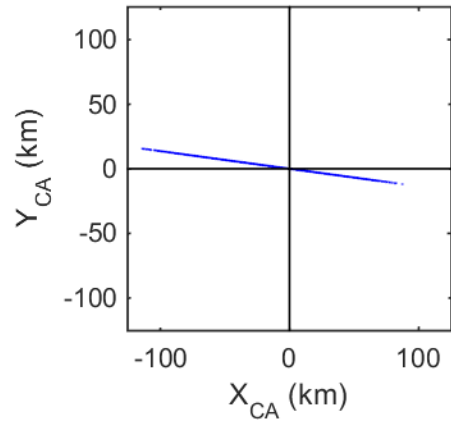
**“New” MC
Equinoctial Sampling**

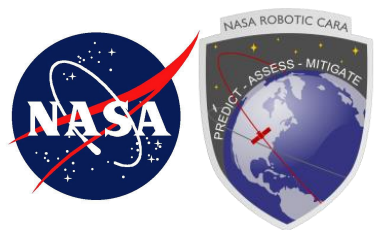
**“Old” MC
With Vel. Covariance**

**“Old” MC
Zero Vel. Covariance**

**Neglecting TCA
vel. covariances
eliminates the
erroneous
curvature and
scatter**

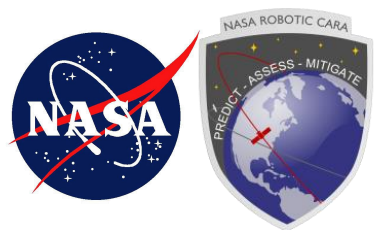
**Zoom shows that
setting the vel.
covariances to
zero makes the
Level II and Level
III distributions
match closely**





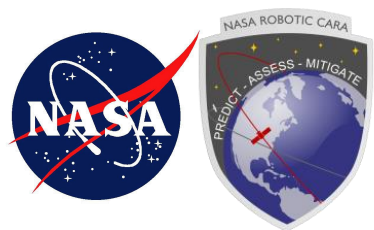
Question 4: Repair to 3D Pc Algorithm (Near-Term Fix)

- **Fix to CAS Pc calculation/rendering involves two items:**
- **#1: Modify 3D Pc calculation to zero out velocity uncertainties**
 - Eliminates large differences between 2D and 3D Pc calculations
 - Smaller differences that remain have so far been shown to be correct
 - Most of these factor of 2 or smaller
- **#2: Calculate both 2D and 3D Pc for each CDM, and make the reportable Pc the larger of the two values**
 - #1 above probably sufficient; but given “discoveries” to date with velocity covariances, best to be conservative
 - Allows future enhancements to 3D Pc in a conservative framework
 - Because seeds risk assessment process with larger value, will ensure sufficient attention to perform supporting functions, such as running MC



Question 4: Fix Details

- **Summary section of report will contain “high watermark” Pc**
 - Basis for color assignment, tasking increases, and other risk assessment tasks
- **Details section of report will contain both 2D and 3D Pc values**
 - Placed together, in easy-to-access area
- **“Why not put both 2D and 3D Pc in summary section?”**
 - Wanted to get fix into CAS as quickly as possible; this is easiest change
 - As situation is studied further, will probably refine what should be shown in summary section
 - e.g., may be best to show 2D, 3D, and a MC result, the latter of which would be automatically populated when 2D/3D difference exceeds given threshold
 - Don't want to jump the gun on changing summary section until understand precisely what decision information is necessary



Question 4: Fix Status and Schedule

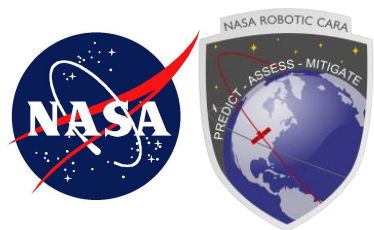
- **Development**

- 3D Pc Code Correction: Complete
- Data Integrity Correction: Complete
- Report Corrections: Complete
- MSA Updates: In progress

- **Testing**

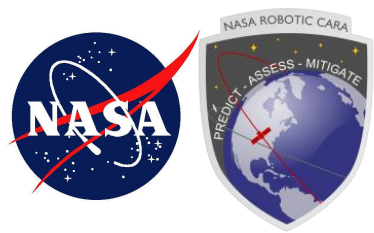
- Current completed code soaking on integration string
- Testing Prep: Complete
- Currently in Testing

- **Expected Delivery: week of 5 JUN 2017**



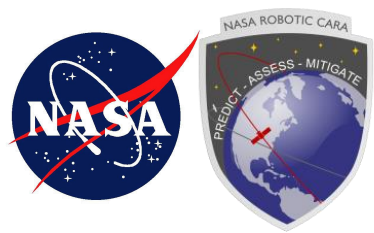
Question 4: Longer-Term Repair

- **Technical consultation held yesterday with external reviewers**
 - R. Carpenter, SSMO deputy and distinguished CA researcher
 - J. Frisbee, JSC CA senior SME
 - S. Casali, JSpOC OD and CA algorithm architect
- **Summary of findings/direction**
 - 3D Pc algorithm is technically sound and should be retained as CARA's principal analytic Pc calculation method
 - Repair used in near-term fix is acceptable for the present, but it is heavy-handed and should be replaced with a more nuanced approach
 - Transformation of 3D Pc reference frame to satellite-centered spherical coordinates may be an effective long-term solution to the non-Gaussian problem, at least for near-circular orbits
 - HEO satellites may require MC approaches as only reliable Pc calculation method

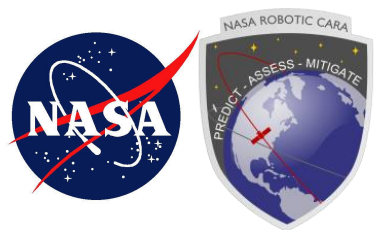


3D Pc Way Forward

- **Current operational procedure to be used until fix deployed**
- **After fix deployed, for events of significance with 2D/3D Pc discrepancy, MC will be run as a matter of course**
- **Analysis effort will continue on Pc calculation**
 - Development of a re-framed or otherwise enhanced 3D Pc approach to replace current fixed version
 - More definitive determination of when MC should be run as principal Pc calculation
 - Enhanced CAS software and reports to incorporate this expanded functionality and communicate results to users

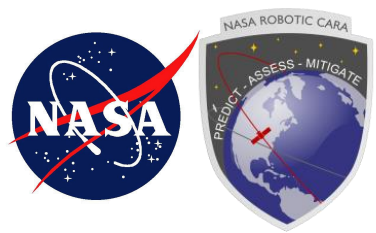


BACKUP SLIDES



Background and History: Monte Carlo Pc Estimation

- **Level I: Legacy CAS Monte Carlo not useful to validate 3D Pc**
 - Uses rectilinear motion and position covariance; just reproduces 2D Pc
- **Level II: Upgraded MC removes these limitations (“Old” MC)**
 - 2-body propagation from TCA, using position/velocity states and covariances
 - Validated against Sal Alfano’s published MC test cases
 - This version used for 3D Pc software testing and validation
- **Level III: operate in curvilinear rather than Cartesian space; better representation of the actual error volume at TCA (“New” MC)**
 - Level IIIa: Propagate covariance natively in equinoctial elements
 - Level IIIb: Use resampling technique to convert TCA covariances (Sabot 2010)
 - Both approaches pursued by CARA team in parallel
- **Level IV: Full MC from epoch (“brute force Monte Carlo”)**
 - Propagates all MC trials non-linearly from epoch
 - “Gold standard” in that no simplifying assumptions used
- **Levels III and IV became available at the end of April 2017**

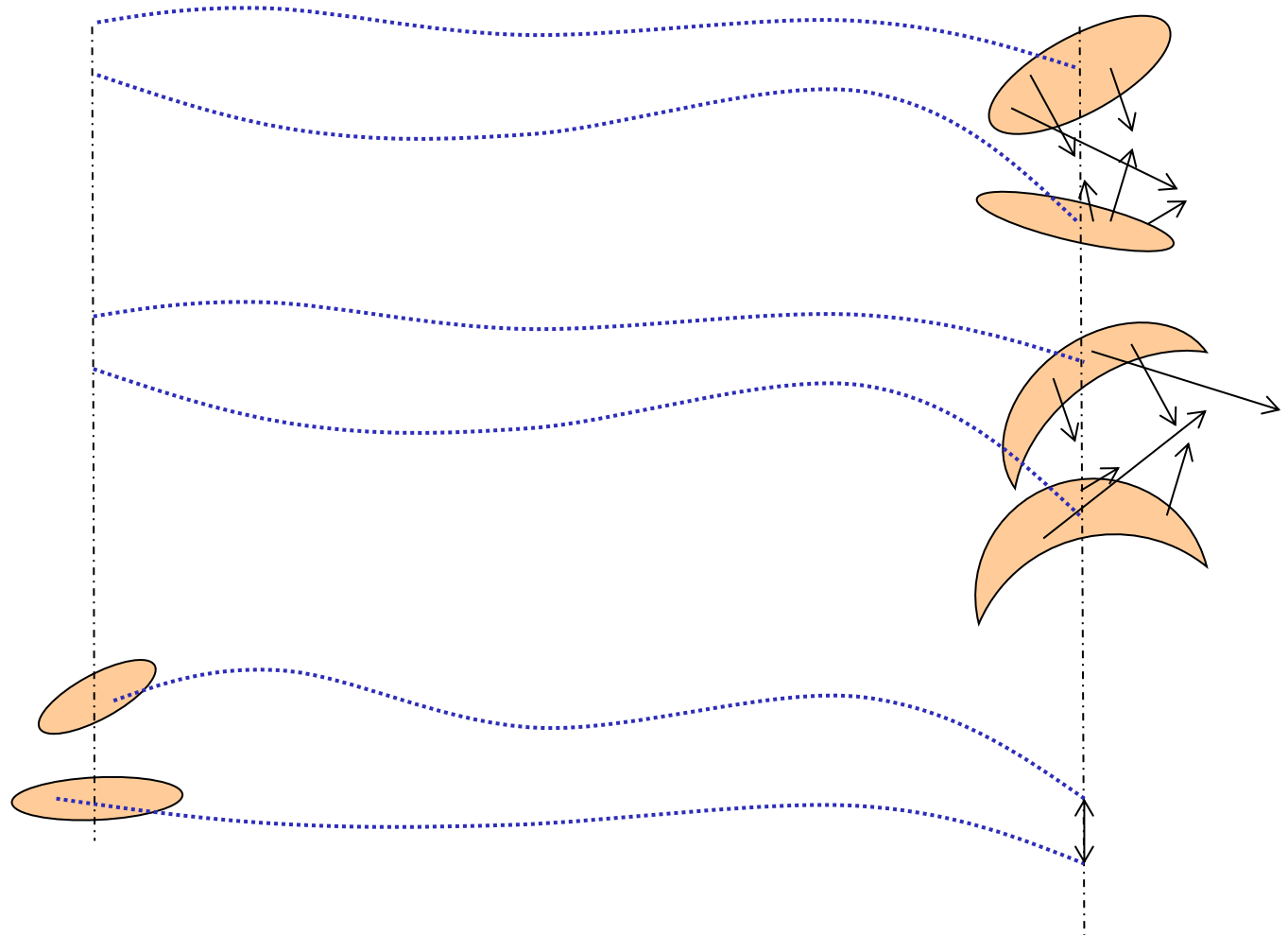


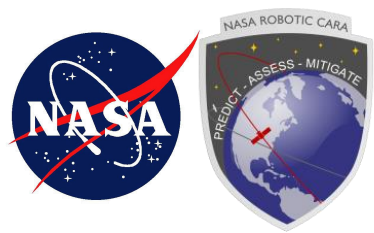
Different Monte Carlo Types: Cartoon Schematic

Level II: propagate covariances to TCA; generate MC samples in Cartesian space and find TCA between pairs

Level III: propagate covariances to TCA; generate MC samples in element space and find TCA between pairs

Level IV: Generate samples at epoch; propagate every pair of samples forward to its proper TCA





Monte Carlo Close-Approach Distributions

For each MC sample calculate :

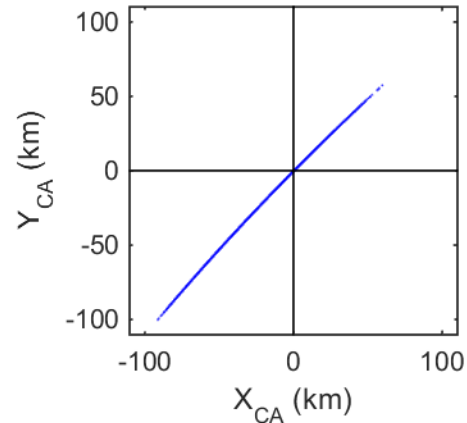
$$\hat{\mathbf{Z}}_{CA} = \frac{\mathbf{v}_s(t_{ca}) - \mathbf{v}_p(t_{ca})}{|\mathbf{v}_s(t_{ca}) - \mathbf{v}_p(t_{ca})|}$$

$$\hat{\mathbf{X}}_{CA} = \frac{\hat{\mathbf{Z}}_{CA} \times \hat{\mathbf{z}}}{|\hat{\mathbf{Z}}_{CA} \times \hat{\mathbf{z}}|} \quad \hat{\mathbf{Y}}_{CA} = \hat{\mathbf{Z}}_{CA} \times \hat{\mathbf{X}}_{CA}$$

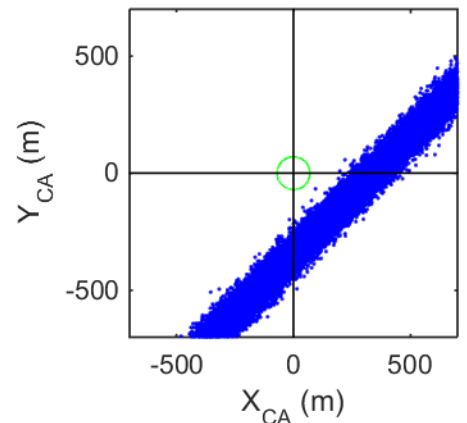
$$X_{CA} = \hat{\mathbf{X}}_{CA} \cdot [\mathbf{r}_s(t_{ca}) - \mathbf{r}_p(t_{ca})]$$

$$Y_{CA} = \hat{\mathbf{Y}}_{CA} \cdot [\mathbf{r}_s(t_{ca}) - \mathbf{r}_p(t_{ca})]$$

$$Z_{CA} = 0$$



Distribution of all close approaches for a conjunction



Zoom in to show close approaches as well as the combined HBR