Mathematical support for automated geometry analysis of lathe machining of oblique peakless round–nose tools

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Abstract. Automatization of engineering processes requires developing relevant mathematical support and a computer software. Analysis of metal cutting kinematics and tool geometry is a necessary key task at the preproduction stage. This paper is focused on developing a procedure for determining the geometry of oblique peakless round-nose tool lathe machining with the use of vector/matrix transformations. Such an approach allows integration into modern mathematical software packages in distinction to the traditional analytic description. Such an advantage is very promising for developing automated control of the preproduction process. A kinematic criterion for the applicable tool geometry has been developed from the results of this study. The effect of tool blade inclination and curvature on the geometry-dependent process parameters was evaluated.

1. Introduction

To develop the mechanical processing automation and condition monitoring, one needs a detailed understanding of both mechanics and kinematics of the process. The mistakes in kinematics description will result in the tool trajectory distortion, improper performance of the process system, premature tool/equipment failure, and degradation of both machining quality and accuracy [1-5]. For condition monitoring, various methods of non-destructive control such as acoustic emission, vibrometry, eddy current, inductive and etc can be applied [6-11]. In connection with it, the first thing to do is to establish true kinematics specific to the process described. To achieve this goal, one can use several approaches as follows. The first approach is to describe the process kinematics by analytic expressions, however, this traditional approach implies tediousness and high risk of error due to using complicated trigonometric functions. Therefore, it is not of common use for automated design and control applications. The second approach is to analyze the process kinematics using the 3D computer-aided design (CAD) modeling. It is proved to be good for analyzing complicated kinematics cases, such as a complicated form of a workpiece or/and a tool as well as a complex trajectory of the tool/workpiece relative motion. The use of this approach is limited due to its increased working time required for analysing a large number of the nodes modeling the contacting surfaces. The third, relatively new aproach is to describe the kinematics using the vector/matrix functions. It works well and fast to describe the process kinematics even for complex tool/workpiece trajectories and geometry. Another advantage of such a description is its potential for automated computations.

The peakless turning differs from the traditional one by more complex contacting conditions.
between the tool and the workpiece and, therefore, by complicated kinematics of the metal cutting process [12-18]. The peakless cutting tools having either a straight or round-nose blade are most commonly used in practice. The kinematics of cutting by the round-nose blade tools is not very well understood now. The basic versions of such tools possess the curved rake and clearance faces (Figure 1). The advantages of the peakless cutting tools in lathe operations are determined by their high wear resistance, high machining accuracy and low roughness combined with the high performance rate.

Summing up the above mentioned, the aim of this study is to describe the peakless turning by round-nose tools using vector/matrix transformations, to estimate the geometry changes and optimize this geometry.

2. Materials and methods
This study was carried out by developing a 3D CAD model of oblique turning using a peakless round-nose tool with the specified rake and clearance surfaces curvatures (Figure 1). Theoretical analysis of the process geometry and kinematics has been conducted using the Mathcad software package.

![Figure 1](image-url) The 3D CAD models of peakless round-nose tool turning by rake (a, d) and clearance (b, c) faces

3. Results and discussions
To computerize the process, it is necessary to set the initial position, shape and geometry of the tool, i.e a tool-in-hand system. The orientation of the system axes is defined by unit vectors $\vec{e}_x$, $\vec{e}_y$, $\vec{e}_z$. The $x_u$ axis orientation coincides with that of the cutting blade.

$$
\begin{align*}
\vec{e}_x &= \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, & \vec{e}_y &= \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, & \vec{e}_z &= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, & \vec{s} &= \begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}.
\end{align*}
$$

The cutting blade geometry is given by the rake ($\alpha_u$) and clearance ($\gamma_u$) angle rotation matrices as follows:
The blade curvature is given by the parametric equations below:

\[
\begin{align*}
    x &= R \cdot \cos(\chi); \\
    y &= R \cdot \sin(\chi).
\end{align*}
\]

Here, \( \chi \) is the parametric angle determining a point position on the blade; \( R \) is the blade curvature radius.

The blade position in the kinematical coordinate system, i.e. when installing the tool in the corresponding equipment, is given by the rotation angle (\( \omega \)) matrices, whereas the current point position on the blade with respect to the workpiece is determined by rotation angle \( \psi \).

\[
\begin{align*}
    \text{rot}\alpha &= \begin{pmatrix}
        1 & 0 & 0 & 0 \\
        0 & \cos(\alpha) & -\sin(\alpha) & 0 \\
        0 & \sin(\alpha) & \cos(\alpha) & 0 \\
        0 & 0 & 0 & 1
    \end{pmatrix}, \\
    \text{rot}\gamma &= \begin{pmatrix}
        1 & 0 & 0 & 0 \\
        0 & \cos(\gamma) & -\sin(\gamma) & 0 \\
        0 & \sin(\gamma) & \cos(\gamma) & 0 \\
        0 & 0 & 0 & 1
    \end{pmatrix}.
\end{align*}
\]

As soon as the cutting speed vector direction is changed with the current point position and is determined by basic plane inclination angle \( \psi_c \), it becomes necessary to find its value with allowance for angle \( \chi \). For this purpose, the relationship equations have been derived for these angles as shown below.

- **Rake face curvature blade:**

\[
\begin{align*}
    \sin(\chi) &= \pm \left( \frac{R \cdot \cos(\omega) + r \cdot \sin(\psi_c) - a \sin(\cos(\omega))}{R} \right), \\
    \sin(\psi_c) &= \frac{\sin(\chi) + a \sin(\cos(\omega)) - R \cdot \cos(\omega)}{R}.
\end{align*}
\]

- **Clearance face curvature blade:**

\[
\begin{align*}
    \sin(\chi) &= \pm \frac{r \cdot \sin(\psi_c)}{R \cdot \sin(\omega)}, \\
    \sin(\psi_c) &= \frac{R \cdot \sin(\chi) \cdot \sin(\omega)}{r}.
\end{align*}
\]

The sign of the \( \chi \)-angle depends on that of the \( \omega \)-angle, so that for \( \omega+\omega \) and \( \omega-\omega \) it will be \( \pm \chi \) and \( -\pm \chi \), respectively. The sign of the blade curvature radius is determined by the blade curvature type as follows: for convex and concave blades it will be \( \pm \psi_c \) and \( \mp \psi_c \), respectively.

The tool shape and geometry in each point are determined by vectors \( \vec{a}, \vec{n}\alpha \) and \( \vec{n}\gamma \), so that

\[
\vec{a} = \text{rot}\omega \cdot \text{rot}\vec{R} \cdot \vec{x}_u,
\]

where \( \text{rot}\vec{R} \) is to set the coordinate system rotation along the cutting blade length as follows.

For the rake face curvature blade:

\[
\text{rot}\vec{R} = \begin{pmatrix}
    \cos(\chi) & 0 & \sin(\chi) & 0 \\
    0 & 1 & 0 & 0 \\
    -\sin(\chi) & 0 & \cos(\chi) & 0 \\
    0 & 0 & 0 & 1
\end{pmatrix}.
\]
For the clearance face curvature blade:

\[
\mathbf{rotR} = \begin{bmatrix}
\cos(\chi) & -\sin(\chi) & 0 & 0 \\
\sin(\chi) & \cos(\chi) & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}.
\]

The normal curvature rake and clearance face vectors for the curvature blade are defined as follows:

- The normal curvature rake vector for a blade curved with respect to the rake face:

\[
n_{\gamma} = \mathbf{rot\omega} \cdot \mathbf{rot\gamma} \cdot \mathbf{rotR} \cdot \mathbf{z}.
\]

- The normal curvature clearance vector for a blade curved with respect to the rake face:

\[
n_{\alpha} = \mathbf{rot\omega} \cdot \mathbf{rot\alpha} \cdot \mathbf{y}.
\]

- The normal curvature rake vector for a blade curved with respect to the clearance face:

\[
n_{\gamma} = \mathbf{rot\omega} \cdot \mathbf{rot\gamma} \cdot \mathbf{z}.
\]

- The normal curvature clearance vector for a blade curved with respect to clearance:

\[
n_{\alpha} = \mathbf{rot\omega} \cdot \mathbf{rot\alpha} \cdot \mathbf{rotR} \cdot \mathbf{y}.
\]

Static planes are:

- Working plane Ps is the plane containing both stroke speed (\(V\)) and feed (\(s\)) directions;
- Reference plane Pv is the plane drawn through the current point perpendicular to the current stroke speed direction in this point.
- Tool cutting edge plane Pt is the plane tangential to the cutting edge and perpendicular to the reference plane in the current point.
- Sectional plane Pn is the plane perpendicular to the line of intersection of reference and tool cutting edge planes.

\[
n_{v} = V = \mathbf{rotV} \cdot \mathbf{z}, \quad n_{s} = \frac{s \times V}{|s \times V|}, \quad n_{r} = \frac{a \times V}{|a \times V|}, \quad n_{m} = \frac{V \times mn}{|V \times mn|}.
\]

Static tool angles, such as cutting edge inclination angle \(\lambda\), approach angle \(\psi\), clearance angle \(\alpha\), and rake angle \(\gamma\) are introduced by equations (1) as follows:

\[
\lambda = n_{t} \cdot \frac{a \times mn}{|a \times mn|}, \quad \psi = n_{v} \cdot \frac{(n_{s} \times V) \times mn}{|(n_{s} \times V) \times mn|}, \quad \alpha = n_{m} \cdot \frac{m \times n_{\alpha} \times mn}{|m \times n_{\alpha} \times mn|}, \quad \gamma = n_{m} \cdot \frac{(n_{\alpha} \times mn) \times nt}{|(n_{\alpha} \times mn) \times nt|}.
\]

A transition from the static to kinematic coordinate system is by substituting a static speed vector for the kinematic one in the above shown formulas:

\[
\mathbf{V}_{p} = \mathbf{rotV} \cdot \begin{bmatrix}
0 \\
0 \\
V \\
0
\end{bmatrix} + \begin{bmatrix}
-s_{m} \\
0 \\
0 \\
0
\end{bmatrix}
\]

where \(s_{m} = s_{n} \cdot n\), \(s_{n}\) is the feed, mm/rev; \(n\) is the spindle rotation rate, rpm (rotation per minutes).

By the example of cutting tools with convex rake (Figure 2) and clearance (Figure 3) faces, it was shown that the static angles changed so that coloured areas corresponded to the geometry parameter ranges providing the positive clearance angle values. The latter condition is necessary for workability of the tool in lathe machining.
4. Conclusion
The cutting tool geometry parameter dependencies have been obtained in the course of this work and then verified by those obtained from 3D CAD models. The reconstruction of sections was carried out using a $15\degree$ step of the reference plane inclination for all used calculations of blade inclination angles.
The results of verification showed that the computational error is not higher than 0.5°, thus, confirming the applicability of the vector/matrix transformation approach to accurate and adequate description of the oblique peakless tool turning kinematics.

The main kinematic criterion limiting the peakless tool geometry is the necessity of using only positive clearance angle values.

The ranges of $\phi_c$, $\lambda_c$, $\gamma_c$ angles vary as dependent on the blade curvature and inclination. The higher $\omega$-angle values serve to increase both the ranges of $\phi_c$ and $\gamma_c$. The blade curvature influences mainly the static edge inclination angle. The applicability range of tool geometry parameters is greater for the round-nose clearance face tools.

It was shown by the analysis of both static and kinematic angles that the difference between them was not higher than $3^\circ$ for the same type of tool geometry behavior. Hence, it is reasonable to take into account only static angles in studying the geometry of lathe machining of the oblique peakless round-nose tool.

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