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Cheap Talk with Strategic Substitutability

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Cheap Talk with Strategic Substitutability

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Abstract

In the classic Crawford-Sobel (CS) model of strategic communication between an informed Sender and uninformed Receiver, perfect information transmission is never achieved as an equilibrium outcome. I present a modified version of the CS cheap talk game with the following two innovations: *(i)* both players take actions, and *(ii)* actions are strategic substitutes. In contrast to the CS setup, the modified game can facilitate perfect information revelation. I characterize the conditions under which a full information revelation equilibrium exists. When these conditions are violated, only partial revelation equilibria exist. Under partial revelation, the Sender reveals information up to a threshold state and pools beyond this threshold, resulting in some loss of information. Welfare analysis suggests that partial revelation equilibria with a higher threshold pareto dominate those with lower thresholds. Crucially, a higher threshold equilibrium is *also* interim efficient – every Sender type at least weakly prefers this over a lower threshold equilibrium.

Keywords: Cheap talk, interdependent action games, full information revelation

1 Introduction

Crawford and Sobel (1982) (hereafter CS), in their seminal paper, consider the problem of strategic communication between an informed Sender and uninformed Receiver. The main result of their work is that when the Sender is biased, cheap talk messages -costless, unverifiable and non-binding- never perfectly reveal information and communication is always noisy. Equilibrium communication takes the form of (noisy) state partitions. The Sender reveals that the state is within a certain partition, but never truthfully reports the true state, as long as there is some degree of conflict of interest. This fundamental analysis of cheap talk has spawned a large literature on both the theoretical[1] and applied[2] front.

An important feature of the CS framework is that the Receiver is the only decision maker. In this paper, I focus on a variation of the standard set-up. Instead of a single decision maker, I consider a scenario in which both the Sender and Receiver take an action after communication, and further, these actions are *substitutable*.

Almost all of the work on cheap talk models so far have neglected the possibility of information transmission with action substitutability.[3] Yet, a number of real world situations involve this kind of interdependence. For example, leaders of countries work cooperatively to achieve a common foreign policy objective. Alternatively, organizations (private or governmental) with multiple departments work together on a common project by sharing information and taking actions that jointly affect the success of a project.

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[1]Farrell and Gibbons (1989) study the case of a single sender and two receivers; Krishna and Morgan (1999), and Li, Rantakari, and Yang (2016) analyze information transmission between multiple senders and a single receiver; Galeotti, Ghiglino, and Squintani (2013) look into cheap talk in networks; Battaglini (2002) models the case of many senders with a multidimensional state space; Sobel (1985), and Morris (2001) investigate the role of reputation building in a repeated cheap talk setting; and Baliga and Morris (2002) throw light on coordination incentives and cheap talk communication.

[2]Cheap talk communication has been used extensively to study pertinent questions in a wide array of fields including political science (see Austen-Smith (1990), Austen-Smith (1993), Gilligan and Krehbiel (1989), and Morgan and Stocken (2008)), organizational theory (see Melumad and Shibano (1991), Rantakari (2008), and Alonso, Dessein, and Matouschek (2008)), finance (see Morgan and Stocken (2003)) and macroeconomics (see Stein (1989) and Moscarini (2007)), among others.

[3]Barring Alonso (2007), who considers a principal-agent setting in which an uninformed principal controls decision rights and the informed agent communicates information strategically, and actions of the two players are either strategic complements or substitutes.

In both instances, there is an element of credible information transmission and substitutability in actions of players. This paper provides a useful starting point to study the nature of strategic information transmission in such interdependent environments.

To understand the nature of trade-offs with action substitutability, consider the augmented CS model. Suppose the Sender is also allowed to take an action after the communication round. The Sender can anticipate the (posterior) beliefs induced by her message to the Receiver and precisely predict the Receiver's action. This way, the Sender can best respond to this action to maximize her own payoff. Therefore, what matters for truthful communication is whether the Sender, given the permissible set of actions, is able to compensate sufficiently for the Receiver's action, that in itself is induced by the message.

The addition of action substitutability, therefore, alters the insights of the standard CS model by allowing the Sender to reveal the state truthfully. Specifically, contrary to the standard CS prediction, I find that when actions are strategic substitutes, there can be *full information revelation* by the Sender.

Perfect information revelation crucially depends on whether the Sender is able to communicate the lowest state and the highest state truthfully.[4] Specifically, irrespective of the conflict of interest between the players, if the domain of the Sender's action set is sufficiently large enough, the Sender can credibly reveal the state and subsequently take an action that precisely compensates for the Receiver's action. Therefore the Sender can credibly reveal the highest state[5] and there is full information revelation in equilibrium.

When the Sender is able to reveal the lowest state but is unable to do so for the highest state, communication deteriorates and there is only partial revelation of information. A partial revelation equilibrium takes the form of a cut-off equilibrium in which the Sender reveals truthfully up to a threshold state, and beyond the threshold, pools information. I precisely characterize the conditions under which there are only partial revelation equilibria.

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[4]When the former is violated, the Sender does not have any incentive to reveal her private information, and there is communication breakdown in the sense that only a '*babbling*' equilibrium of the cheap-talk game exists.

[5]The assumption of single crossing ensures that any other lower state, ipso facto, can also be revealed truthfully. See Aumann (1990) and Baliga and Morris (2002) for more on credibility of cheap talk messages.

Further, in the *absence* of full revelation, the most informative equilibrium is the one with the highest threshold. Comparative statics reveal that this threshold is higher when biases are closer to each other, and when the upper bound on actions is greater. The former result is similar to the original CS model argument, while the latter is a direct consequence of action substitutability. A greater upper bound on the action set implies the (upward biased) Sender is able to compensate efficiently for a greater measure of types. This enables more truthful communication, leading to a higher threshold. Hence, either a decrease in conflict of interest between players, fixing the bound, or an increase in the domain of available actions, keeping the bias constant, leads to more information transmission.

Finally, I study some welfare properties of partial revelation equilibria. Intuitively, I find that the most informative equilibrium (ex ante) Pareto dominates every other partial revelation equilibria. Further, contrary to the nature of CS equilibria, I establish that the most informative equilibrium also achieves interim efficiency.[6] That is, for every type of Sender, the most informational equilibrium is at least weakly preferred. There are two reasons for this finding. First, a higher threshold reduces the measure of states for which the Sender is unable to compensate sufficiently. Second, by providing more information, the Sender raises the expected action of the Receiver over the states that are not revealed truthfully. These two effects reinforce each other making every Sender type at least weakly better off from revealing more information.

The papers closely related to my work are those by Kartik, Ottaviani, and Squintani (2007), Kartik (2009), Morgan and Stocken (2003), and Ottaviani and Squintani (2006). Kartik et al. (2007) derive a fully separating equilibrium with lying costs and the possibility of a naive Receiver. The key condition driving their result is the unboundedness of the domain of private information. In Kartik (2009), truthful communication is restricted by the presence of a bound on the state space, leading to incomplete separation. In Morgan and Stocken (2003), threshold equilibria result is driven by uncertainty in the extent of bias of the informed party. Finally, Ottaviani and Squintani (2006) construct cheap talk equilibria with naive receivers and a bounded state space in which communication is

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[6]In the most informational partition of the standard CS setup, there is always a low type Sender that prefers the babbling equilibrium over the more informational partition equilibrium. Hence, even though partitional equilibria could be ordered in the Pareto sense, they never achieve never interim efficiency. See Chen, Kartik, and Sobel (2008) for more on this point.

truthful up to a threshold, and partitional beyond.

The key difference in my results is that it is driven by substitutability and restricted domain of actions that act as a form of resource constraint on the Sender. Resource constraints indirectly affect the capacity of the Sender to compensate, and this in turn affects truthful communication in equilibrium.

The paper proceeds as follows. In Section 2, I present a simple example to show the main intuition driving my results. Section 3 builds the basic model and develops the necessary condition for full information revelation equilibrium. Section 4 contains the results pertaining to partial information revelation equilibria. In Section 5, I present some comparative statics results and welfare analysis of the partial revelation equilibrium follows in Section 6. Finally, Section 7 contains concluding remarks.

2 Leading Example

Consider a variant of the basic Crawford-Sobel set-up with strategic interdependency in actions. An informed player, S, receives a perfectly observable signal about the state of the world θ, drawn from an uniform distribution $[0, 1]$ and communicates this information through a cheap talk message m to an uninformed player, R. Upon communication, both R and S take actions in a way that affects both their payoffs. Let the modified utility function be the following:

$$U^R = -\left[\left(\frac{x_R + \eta x_S}{1 + \eta}\right) - \theta\right]^2$$

$$U^S = -\left[\left(\frac{x_S + \eta x_R}{1 + \eta}\right) - \theta - b\right]^2$$

Observe the small departure from the CS set-up. Both players now are allowed to take actions after communication, and actions are substitutes in that $\frac{\partial^2 U^i}{\partial x_R \partial x_S} < 0$, where $\eta \in (0, 1)$ captures the degree of substitutability[7]. Further, let the actions of players x_i have a domain $[-a, a]$. Given this structure, when S truthfully reveals the true state of the world through her message, $m = \theta$, the two players solve the

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[7]The case of $\eta = 1$ is one where actions are perfect substitutes. Under perfect substitutes, no information can be credibly revealed and there is only a babbling equilibrium.

following best responses:

$$R : x_R = (1+\eta)m - \eta x_S$$

$$S : x_S = (1+\eta)(m+b) - \eta x_R$$

To simplify the exposition, let $b = \frac{2}{5}$ and $\eta = \frac{1}{2}$. Equilibrium actions after (truthful) messaging are given by: $x_R^* = m - \frac{2}{5}$, $x_S^* = m + \frac{4}{5}$. Notice immediately that full information revelation is possible if $a \geqslant \frac{9}{5}$. This is so because S is able to compensate precisely even for the highest type, $\theta = 1$. At the other extreme, if $a < \frac{4}{5}$, no information can be credibly revealed by S, since irrespective of what the true state is, reporting the truth is never optimal for S. This stems from her inability to sufficiently compensate even for the lowest type signal.

For example, when $a = \frac{2}{5}$, the equilibrium action of the sender under truthful communication is $x_S^* = \frac{2}{5}$, irrespective of the state. However, S can inflate her signal in order to make R play a higher action. To see this, suppose instead of $m = 0$, S inflates and sends a message $m = \frac{2}{5}$. Then, R best responds by taking an action $x_R^* = \frac{2}{5}$. But notice that S can fully anticipate this response by R and suitably adjust her optimal action. In particular, S takes an action $x_S^* = \frac{3}{5} - \frac{1}{5} = \frac{2}{5}$. Though S's action has not changed, she has managed to push R's action upwards, and thereby achieves a payoff of 0. But this incentive to misrepresent means that R would never believe any message from S, and therefore communication is rendered ineffective in equilibrium.

Finally, when $\frac{4}{5} < a < \frac{9}{5}$, S has an incentive to reveal some information. To see this, let $a = 1$. Then, for any $\theta \in [0, \frac{1}{5}]$, S reveals the state truthfully since her optimal action is within the domain of available actions (in this case $x_S^*(\frac{1}{5}) = 1$). But, for any $\theta > \frac{1}{5}$, S cannot sustain a truthful messaging strategy. To see this, suppose $\theta > \frac{1}{5}$, and S reports truthfully. Then the optimal action for S is bounded by $x_S = 1$, while R provides the residual as demanded by her best response function, which is $x_R = \frac{3}{2}\theta - \frac{1}{2}$. This cannot be an equilibrium since there is under-provision as far as S is concerned: S gets a payoff of $U_S = -\left(\frac{1 + \frac{1}{2}(\frac{3}{2} \cdot \theta - \frac{1}{2})}{\frac{3}{2}} - \theta - \frac{2}{5}\right)^2 = -(\frac{1}{10} - \frac{\theta}{2})^2 < 0$ for $\theta > \frac{1}{5}$. Therefore, S has an incentive to exaggerate her information in order to induce R to contribute more. This precludes separation beyond $\theta = \frac{1}{5}$.
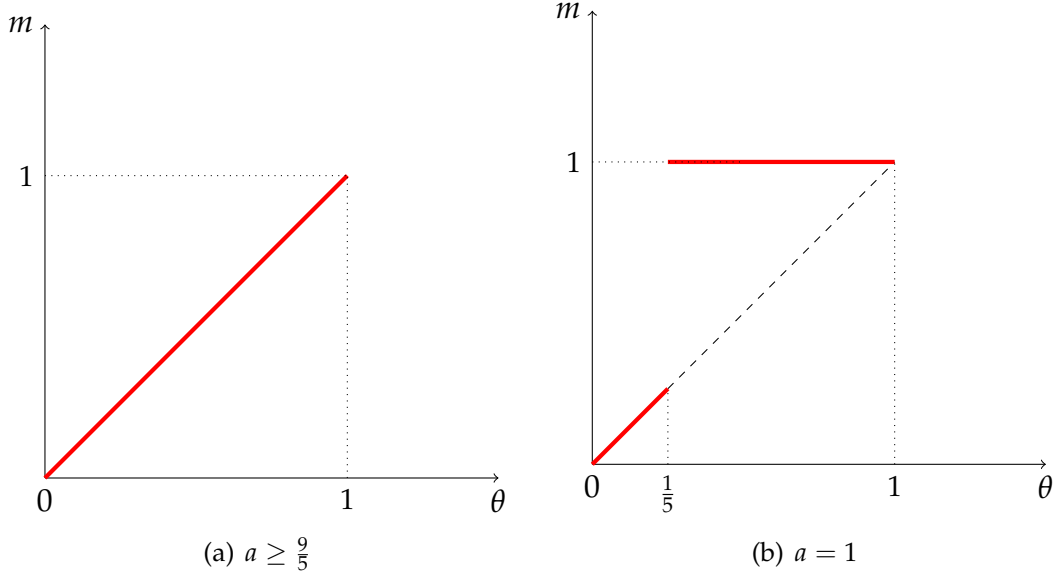
6

(a) $a \geq \frac{9}{5}$ (b) $a = 1$

Figure 1: When $a \geq \frac{9}{5}$, there are no resource constraints for the Sender, resulting in full information revelation. On the other hand, when $a \in (\frac{4}{5}, \frac{9}{5})$ there is only partial revelation of information. Specifically, for all states above $\frac{1}{5}$, the Sender pools and sends a message $m = 1$.

In fact, all types above this cutoff must pool and send the highest message, $m = 1$. This is primarily because the signals are (imperfectly) invertible in this environment. Any pure message $m < 1$ could be interpreted as coming from one of the many possible (weakly lower) types. For instance, when $\theta = \frac{2}{5}$, S would want to exaggerate and send a message of at least $m \geq \frac{3}{5}$, since this would ensure that S's action is within the bound $a = 1$. Say S sends $m = \frac{3}{5}$. But this message could possibly come from any of the types $\theta \in (\frac{1}{5}, \frac{2}{5}]$, each of whom have incentives to deviate and send $m = \frac{3}{5}$. Hence, R could invert the message and form beliefs accordingly[8]. But if this is the case, every type in the interval $(\frac{1}{5}, 1]$ would find it profitable to send the highest pooling message possible, $m = 1$. At most, there is a partial revelation equilibrium, in which S reveals truthfully (or separates) for $\theta \in [0, \frac{1}{5}]$ and pools her messages for $\theta \in (\frac{1}{5}, 1]$.

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[8]For precisely a similar argument, partition equilibria of the kind developed by CS are also ruled out on the interval $(\frac{1}{5}, 1]$. Again, this is because there would not exist an indifference type in this interval, since there is a natural propensity to inflate information. This incentive to exaggerate ensures that if there are two partitions, the high types in the lower partition would find it profitable to deviate to the higher partition, precluding the existence of an indifference type in the first place.

The example suggests a novel trade-off for information transmission with action substitutability. That is, the ability to truthfully reveal information depends on how large the bounds on actions are, namely the size of a. Crucially, as a increases (on the interval $(\frac{4}{5}, \frac{9}{5})$ in the example above), there is more information revealed by S. The informed Sender is able to provide more information regardless of the extent of the biases between the two players.

3   The Model

Consider two players, a receiver R and sender S, who decide on contributions to a joint project. The payoff from the project is contingent on an unknown state $\theta \in \Theta \equiv [0,1]$, distributed according to the density function $f(.)$. The sender receives a perfectly observable private signal about the state θ, while the receiver has no information.

Each player's utility is given by $U(\phi^i(x_i, x_{-i}), \theta, b_i)$, where $\phi^i(.)$ is the player-specific (symmetric) joint contribution function.[9] The contribution function $\phi^i(.)$ depends on player i's action x_i, as well as the contribution of the other player, x_{-i}. Actions of players are such that $x_i \in V \subseteq \mathbb{R}$, where the set V is closed and compact. The bias parameter b_i measures the conflict of interest between the two players.

The standard Crawford-Sobel assumptions on the utility function of players hold. Specifically, $U(.)$ is twice continuously differentiable, $U_1(.) = 0$ for some ϕ^i, $U_{12}(.) > 0$, $U_{13}(.) > 0$ and $U_{11}(.) < 0$ so that U has a unique maxima for any given pair (θ, b_i). This implies that there is an unique joint contribution function ϕ^i for each player that satisfies their maximization problem.

The utility functions of the players satisfy the condition $\frac{\partial^2 U}{\partial x_i x_j} < 0$, implying that actions of the two players are strategic substitutes. For sake of exposition, I normalize the bias of receiver to 0 and that of sender to $b > 0$[10]. The two players therefore maximize their payoff functions given by $U(\phi^R(x_R, x_S), \theta)$ and

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[9]The assumption of symmetry is not important in order to generate the main findings. However, it aides comprehension of the same.

[10]The biases could be a function of θ in that $b(\theta)$ may be the extent of conflict of interest, instead of a constant b. This, however, does not change the main results of the paper as long as single-crossing property holds, meaning $U_{13} > 0$.

$U(\phi^S(x_S, x_R), \theta, b)$.

Players have action interdependence in the sense that each players' action x_i affects the contribution function of the other player $\phi^{-i}(.)$. Since $b > 0$ and $U_{13}(.) > 0$, it implies that $\phi^S(.) > \phi^R(.)$ for every θ. I make the following further assumptions on the functional form of $\phi^i(.)$ to ensure an interior solution to the contribution decision of the players:

Assumption 1 *Non-decreasing marginal contribution:* $\frac{\partial \phi^i(.)}{\partial x_i} \geq 0$

Assumption 2 *Non-negative spillover:* $\forall i, j \neq i : \frac{\partial \phi^i(.)}{\partial x_j} \geq 0$

Assumption 3 *Imperfect substitutability:* $\forall i, j \neq i : \frac{\partial \phi^i(.)}{\partial x_i} > \frac{\partial \phi^i(.)}{\partial x_j}$ *and* $\frac{\partial \phi^i(.)}{\partial x_j} < 1$

Assumption 1 ensures that the contribution function is non-decreasing in the player's own action, while the second assumption implies that a player's contribution function is non-decreasing in the other player's action. Assumption 3 implies that the *marginal contribution effect* dominates the *non-negative spillover effect*. Further, it rules out perfectly substitutable actions.[11]

The game proceeds in two stages. In the first stage, the sender observes the true state θ and sends a message (or report) $m \in M$ to the receiver. Let this messaging strategy be defined by a mapping from the private signal of the sender into a message $m = \mu(\theta)$. Let $p(\theta \mid m)$ denote receiver's posterior beliefs on θ after receiving the message m. In the next stage, both players simultaneously decide on contributions.

A communication equilibrium of this game is a (pure strategy) perfect Bayesian Equilibrium (PBE) which satisfies the following properties:

- given the sender's message m and the posterior beliefs $p(. \mid m)$ over the state, both players simultaneously choose actions that maximize their respective payoffs

- the posterior beliefs are updated using Bayes' rule where possible

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[11]When actions are perfect substitutes, notice that there is no guarantee of an interior equilibrium. Take the example presented in Section 2 and substitute $\eta = 1$. The best responses are such that the equations do not have a solution. For this reason, I focus on imperfect substitutability of actions in this paper.

- given the beliefs and second stage contributions $x_R(m)$ and $x_S(\theta, m)$, the sender's reporting strategy $\mu(.)$ maximizes the expected payoff in the first stage

A PBE always exists in games with cheap talk. This is a babbling equilibrium in which the sender's message is ignored and the receiver acts based on her prior information, while the sender anticipates this and acts accordingly. In this paper, I try to identify conditions under which more informative equilibria emerge.

3.1 Full Revelation

In a full revelation equilibrium, the private information of the sender is completely revealed to the receiver, meaning $\mu(\theta) = \theta$ for all $\theta \in [0, 1]$. To see if a full revelation equilibrium[12] exists, it is important to understand the trade-offs for the sender. Since actions are substitutable, players compensate for each others' action by contributing the residual action required. The players are constrained by the lower and upper bound on permissible actions, given by $\inf V$ and $\sup V$ respectively. That is, the size of the bound directly affects the ability of either player to contribute.

Revealing information truthfully then becomes a question of whether the sender can follow up a truthful message with an appropriate action that is within the available domain of the actions. If the sender's action upon truthful communication is within the bound, then it precludes her incentive to misrepresent. Therefore, in some sense, the action set V acts as an *incentive compatibility constraint* for truth-telling.

However, since $b > 0$ and $U_{13} > 0$, we know that $\phi^S(.) > \phi^R(.)$. Further, given the assumption of imperfect substitutability ($\phi_1^{S'}(.) > \phi_2^{S'}(.)$), the sender's only concern is whether the optimal best response is within the upper bound of the action set. Because single crossing property $U_{12} > 0$, the only incentive compatibility constraint of interest is the one where $\theta = 1$. That is, if the optimal action for the sender upon revealing the highest state $\theta = 1$ is within the domain of available actions, then it must be so for every $\theta < 1$. This property is made clear in the following definition.

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[12]Any messaging function $\mu : [0,1] \to [0,1]$ that is one-to-one and onto is a fully revealing messaging strategy. I will, however, concentrate on the most intuitive one in which if the state is θ, the sender sends a message that is equivalent to the statement - "The state is θ".

Definition 1 *Let $\tilde{x}_S(\theta, m)$ be the optimal actions of the sender when, i) unrestricted domain is satisfied ($x_S \in \mathbb{R}$); and ii) the sender's message m (truthful or otherwise) is believed by the receiver to be the true state. That is, the action $\tilde{x}_S(\theta, m)$ is the solution to the unconstrained optimization problem of the sender when her message is believed. Stated formally:*

$$R\text{'s action: } \tilde{x}_R(m) \text{ solves } \max_{x_R \in V} U(\phi^R(x_R, \tilde{x}_S(m)), m) \text{ subject to } \tilde{x}_S(m)$$

$$\equiv \arg\max_{x_S \in \mathbb{R}} U(\phi^S(x_S, \tilde{x}_R(m)), m, b)$$

$$S\text{'s action: } \tilde{x}_S(\theta, m) \text{ solves } \max_{x_S \in \mathbb{R}} U(\phi^S(x_S, \tilde{x}_R(m)), \theta, b)] \text{ subject to } \tilde{x}_R(m)$$

$$\equiv \arg\max_{x_R \in V} U(\phi^R(x_R, \tilde{x}_S(m)), m)$$

Further, when communication is truthful, let the optimal action of players under the unconstrained optimization problem be $\tilde{x}_R(\theta)$ and $\tilde{x}_S(\theta) = \tilde{x}_S(\theta, \theta)$.

Assumption 4 $\tilde{x}_S(0) \geq \inf V$

Note that Definition 1 does not necessarily prescribe the action of the sender in equilibrium, $x_S^*(.)$. Instead, $\tilde{x}_S(\theta, m)$ allows us to intuitively characterize the optimal response of the sender when her message is believed to be true[13], and her actions have an unrestricted domain $\mathbb{R}$. $\tilde{x}_S(\theta, m)$ takes into account the fact that the receiver's action is constrained by the bounds imposed by V. Assumption 4 ensures that the optimal action of the sender is above the lower bound of permissible actions, when she reveals the lowest state. Given the above formulation, let $c = \sup V$ be the upper bound of the domain of action set V. The following conditions are then useful to characterize the communication breakdown and full information revelation equilibrium.

Definition 2 *Lowest type incentive compatibility (**LTIC**) : $\tilde{x}_S(0) \leq c$*

Definition 3 *Highest type incentive compatibility (**HTIC**) : $\tilde{x}_S(1) \leq c$*

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[13]One way to interpret this is to think of a naive receiver, as studied in Kartik et al. (2007) and Ottaviani and Squintani (2006). A naive receiver is not rational, and believes any message sent by the sender.

LTIC provides an intuitive criteria for any information transmission with action substitutability. When *LTIC* fails, no information can be credibly revealed by the sender, since the receiver always believes that the sender is exaggerating her information. To put it differently, the sender would always find it profitable to inflate her message so that the receiver contributes more. Instead, if the sender does reveal truthfully, then the actions from the constrained optimization problem (given by equations 1 and 2 in Appendix A.1) and unconstrained optimization problem are such that $x_S^*(\theta) = c \leq \tilde{x}_S(\theta)$. The sender, therefore, does not maximize her payoffs from truthful revelation for any $\theta \in [0, 1]$.

On the contrary, the *HTIC* condition provides a sort of *IC* constraint for full revelation. As long as HTIC is satisfied, the sender can never do better than reveal the truth. This is because the solution to her constrained optimization problem coincides with that of the unconstrained optimization problem, implying that $x_S^*(\theta) = \tilde{x}_S(\theta)$ for all $\theta \in [0, 1]$. This ensures that there is no incentive for *S* to lie, and full revelation is achieved as an equilibrium.

Proposition 1 *Under Assumptions (1-3 and 4), given a bias* **b** *of the sender,*
1. No information is credibly revealed in equilibrium if the LTIC condition is violated.
2. There is full information revelation if the HTIC condition is satisfied.

Proof. See Appendix B.1 ∎

Notice that the condition for truth-telling with one-sided incomplete information and strategic substitutability in actions resembles the credibility notion of *self-signaling*[14], identified by Aumann (1990), and Farrell and Rabin (1996). When the unconstrained action is above the bound under truthful revelation, it implies that the sender faces a positive spillover from the receiver's action, implying that $U_1(\phi^{S^*}(c, x_R^*(\theta)), \theta, b) > 0$ when $\tilde{x}_S(\theta) > c$. This '*positive spillover effect*' implies that communication ceases to be credible at the bound, for the sender (weakly) prefers to induce a higher action from the receiver, by inflating her private information.[15]

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[14]A cheap talk message is self-signaling if the sender intends it to be believed only if it is true.

[15]Baliga and Morris (2002) study a game of strategic complementarities in actions, in which the presence of positive spillovers precludes cheap talk communication. In this sense, the first part of Proposition 1 illustrates how the communication breakdown result holds true even when actions are strategic substitutes.

4   Partial Revelation

So far, I have identified the sufficient conditions for both extremes -communication breakdown and fully revealing equilibria- to emerge. Consequently, as long as *LTIC* holds, there is always some information transmission. However, when the HTIC condition fails, the sender may only be able to reveal information up to a cutoff state, and not beyond. This section will focus on the nature of such *'partial revelation equilibria'* (henceforth PRE)[16].

In a PRE, the sender sends a truthful message $m = \theta$ up to a threshold cutoff θ^* and for all states above this cutoff, pools her message ($m = 1$ if $\theta > \theta^*$). Before stating the result on PRE, one needs to identify the states for which truthful messages can never be credible. As pointed out earlier, these are states for which the IC constraint, given by the upper bound, is violated. Let $G = \{\theta : \tilde{x}_S(\theta) > c\}$ be the set of states for which truthful revelation is not possible for the sender. The following lemma establishes the condition for G to be non-empty.

Lemma 1 *If LTIC holds and HTIC is violated, then the set G is non-empty.*
Proof. *When $\tilde{x}_S(0) \geq \inf V$ and $\tilde{x}_S(1) > c$, by continuity of $U(.)$ and $\phi^i(.)$, and single-crossing property $U_{12}(.) > 0$, the set G must be non-empty.* ∎

Given this property, observe that there must then exist a cutoff $\bar{\theta}$ such that $\tilde{x}_S(\bar{\theta}) = c$ and $\bar{\theta} = \sup\{[0,1]\backslash G\}$. The set $G = (\bar{\theta}, 1]$ represents the signal types for which there are incentives to lie for the sender. This is because for all signals in the set G, reporting the truth implies that the sender's optimal action is outside the domain of permissible actions. Therefore, by misreporting her private information θ, say by reporting $m > \theta$, the sender can induce the receiver to take a higher action. As a result, none of the messages in this interval are credible and will never be believed in equilibrium. That is, the sender must pool her messages for all signals in G by sending the highest message[17], $m = 1$. I will now state the result formally.

Proposition 2 *Under assumptions 1-4, if LTIC holds and HTIC is violated, then there exists a PRE with threshold $\theta^* = \bar{\theta}$.*

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[16]See Appendix A.1 for a more detailed and formal definition.

[17]R assigns the off-equilibrium path beliefs that would discourage deviations on-the-equilibrium path. For example, for any off-equilibrium path message $m \in (\bar{\theta}, 1)$, R could assign the state to be the cutoff θ^*.

Proof. See Appendix B.2 ∎

Proposition 2 suggests that inflated messaging occurs above the cutoff state, while every message within the cutoff is truthful.[18] Moreover, the incentive to exaggerate above the cutoff is exacerbated by the fact that any inflated message could now possibly come from a continuum of types, instead of a one-to-one mapping (see proof of Proposition 2 in Appendix). Specifically, any message above the cutoff that is not $m = 1$ is inverted as being from a set of types, rather than a single type. This ensures that the sender sends the highest possible message for all types of θ above the cutoff, in order to avoid imitation by lower types.[19]

4.1 Multiplicity of threshold equilibria

The cutoff $\theta^* = \bar{\theta}$ is only one of many possible thresholds that could be supported as an equilibrium. To characterize all the PRE and simplify analysis of the same, I make the following assumption:

Assumption 5 *Feasible Low type deviation:* $\tilde{x}_S(0,1) \in V$

This assumption ensures that when the lowest type sender ($\theta = 0$) misrepresents her signal and sends the most inflated message ($m = 1$) that is further believed to be true by the receiver, the optimal (unconstrained) action of the sender is within the domain of permissible actions.[20]

Proposition 3 *Under Assumptions 1-5, if LTIC holds and HTIC is violated, then every threshold $\theta^* \in (0, \bar{\theta})$ is a PRE.*

Proof. See Appendix B.3 ∎

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[18]The PRE expressed above is similar to the cut-off equilibria obtained by Kartik (2009). The exaggeration in communication is driven by lying costs. In Kartik's work, however, the sender (almost) always uses inflated communication even though the rational receiver is able to invert, and thereby decode, the inflated message. See also Morgan and Stocken (2003).

[19]This is in contrast with the result of Ottaviani and Squintani (2006), who consider the presence of naive receivers. They construct a cutoff equilibrium in which messages are revealing (albeit inflated) below the threshold, and for states above the cutoff, information transmission is partitional in nature.

[20]Note that this is a stronger version of assumption 4, which concerns the feasibility of truthful communication for the lowest type information. Assumption 5 makes upward deviations by the sender possible in the sense that they would never induce an action that goes below the lower bound of the feasible action set, meaning $\tilde{x}_S(0, m) \in V$ for all $m < 1$.

One implication of the above result is that the sender could possibly choose how much information to reveal in equilibrium. Though the PRE $\theta^* = \bar{\theta}$ is the most informative one, it does not necessarily restrict the sender from providing lesser information. In section 6, I address this multiplicity problem pertaining to PREs, by looking at the welfare properties of the different threshold equilibria.

5 Resource constraints

As pointed above, the PRE relies on two crucial parameters - the bias of the sender and the size of the upper bound c. In both cases, what is relevant is to check how the most informative PRE reacts to changes in these parameters. With bias b, the results from the standard literature on cheap talk hold in my setting as well. Specifically, if b decreases, then since $U_{13} > 0$, it implies that the sender is able to to compensate sufficiently for more types in the state space.

Claim 1 *Take two biases b_1 and b_2 such that $b_1 < b_2$. Then the most informative PRE under the two biases are such that $\bar{\theta}_1 > \bar{\theta}_2$.*

The more interesting comparative static finding comes from varying the extent of the bound c, or in other words, expanding the domain of actions imposed on the sender. Remember that c affects truth telling by enabling the sender to compensate for types up to a certain threshold. Increasing this bound leads to more communication since the sender can now reveal truthful information for more types, pushing the threshold to the right. I summarize these two intuitive findings in the following claim.

Claim 2 *Take two bounds $c_1 = \sup V_1$ and $c_2 = \sup V_2$ such that $c_1 < c_2$. Then the most informative PRE under the two bounds are such that $\bar{\theta}_1 < \bar{\theta}_2$.*

In the context of the examples put forth in the introduction, c could be interpreted as a form of resource or capacity constraint imposed on the sender that prevents her from revealing information. Propositions 1 and 2, therefore, highlight the importance of the interaction between resource constraints and the bias of the sender when there is action substitutability.

Resource constraints are frequently observed in the real world. As a motivation, think of the following scenario. Suppose two departments in an organization decide to implement a project that involves contributions from both entities. Further, let us assume that only one department holds information relevant to the implementation of the joint project, and contributions are substitutable. In this situation, an absence of any constraints (shortage of manpower or financial burdens, say) would enable the two departments to perfectly cooperate their activities, and all private information regarding the project may be credibly conveyed.

However, in the presence of resource constraints, the informed department could misrepresent its information for higher states of the world, in order to induce the other to spend more resources on the project. Any information loss could then be viewed as a source of inefficiency in the project. Above results show that in order to improve informational efficiency, it may be in the interest of an informed party to either choose a partner with more aligned interests, or mitigate the burden of constraints imposed upon it by the organization.[21]

6 Welfare - Ex-ante and Interim efficiency

The previous section establishes that more information can be revealed when the upper bound of the domain of actions available to the sender is increased. However, the sender may also choose to reveal any threshold of information, starting from a cutoff $\theta^* = 0$, up to a $\theta^* = \bar{\theta}$. An important question that arises is whether the sender would find it in her interest to convey less information. Given a cutoff θ^*, the ex ante utility of receiver R can be expressed as,

$$V_R(\theta^*) = \int_0^{\theta^*} U(\phi^{R^*}(x_R^*(t), x_S^*(t)), t) f(t) dt +$$

$$\int_{\theta^*}^1 U(\phi^{R^*}(x_R^*((\theta^*, 1]), x_S^*(t, (\theta^*, 1])), t) f(t) dt$$

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[21]In a typical organization, this could involve hiring more staff or increasing the budget allocated for the project within the department.

That of sender S can be similarly written as,

$$V_S(\theta^*) = \int_0^{\theta^*} U(\phi^{S^*}(x_S^*(t), x_R^*(t)), t, b) f(t) dt +$$

$$\int_{\theta^*}^1 U(\phi^{S^*}(x_S^*(t, (\theta^*, 1]), x_R^*((\theta^*, 1])), t, b) f(t) dt$$

where $x_S^*(t, (\theta^*, 1])$ and $x_R^*((\theta^*, 1])$ are the equilibrium actions of the sender and receiver respectively, given the receiver's beliefs that the state is in the interval $(\theta^*, 1]$, given a cutoff θ^*.

Notice that a higher θ^* may benefit the receiver since providing more accurate information over a larger domain of type space makes it possible for the receiver to compensate more precisely for these (truthfully) reported states. But does this hold from the perspective of the sender?

Proposition 4 *Ex-ante efficiency: The most informative PRE where $\theta^* = \bar{\theta}$ Pareto dominates every other PRE.*

Proof. See Appendix B.4 ∎

The intuition for this Pareto ranking of equilibria is the following. Think of a sender providing information up to some threshold, say $\varepsilon < \bar{\theta}$. Then, for all other states $\theta > \varepsilon$, the sender, by pooling her message on $m = 1$, induces an expectation over the possible states for the receiver given by beliefs that $\theta \in (\varepsilon, 1]$. The action of the receiver takes into account this posterior belief and induces an action $x_R^*((\varepsilon, 1])$. Notice that for any such cutoff ε, there must exist a θ_ε such that $x_S^*(\theta_\varepsilon, (\varepsilon, 1]) = c$, by assumption 5 and single crossing. But, for every $\theta > \theta_\varepsilon$, the sender suffers inefficiency since she is unable to compensate sufficiently.

Pareto ranking the equilibria then becomes possible by observing two sources of inefficiency that arise with pooling of information. First, the greater the cutoff ε, the smaller are the measure of types $(\theta_\varepsilon, 1]$ for which the sender is unable to compensate efficiently. Second, the severity of this inefficiency for states $\theta > \theta_\varepsilon$, given that $x_S^*(\theta, (\varepsilon, 1]) = c$, is greater when ε is smaller. Both these sources of inefficiencies are reduced when more information is communicated in equilibrium. As a result, it is always in the sender's interest to provide more information in equilibrium.

Usually, however, ex ante Pareto dominance is not a sufficient criterion to select equilibria since it only provides an aggregate welfare measure. In particular, it may be that different types of sender may have varying preferences over the equilibria, making it harder to tackle the multiplicity problem prevalent in cheap talk models.[22] However, when actions are strategic substitutes, I find that a higher cutoff PRE is not only ex-ante efficient, but also guarantees interim efficiency. That is, every sender type weakly prefers a higher cutoff PRE to a lower one.

Proposition 5 *Interim efficiency: Every sender type weakly prefers a PRE with $\theta^* = \bar{\theta}$.*

Proof. See Appendix B.5 ■

The intuition is an extension of the arguments made for Proposition 4. Specifically, sender types that are at the higher end of the spectrum tend to prefer $\bar{\theta}$ since there is a positive spillover effect at the bound. Hence, for these high types, inducing a higher (expected) action from the receiver increases payoffs. Since the cutoff $\bar{\theta}$ induces a greater action from the receiver, meaning $x_R^*((\bar{\theta}, 1]) > x_R^*((\theta^*, 1])$ for all $\theta^* < \bar{\theta}$, the highest types strictly prefer the PRE with cutoff $\bar{\theta}$. For all other types the optimal response is within the bound, making them indifferent between $\bar{\theta}$ and $\theta^*(< \bar{\theta})$. Therefore, either every type of sender is indifferent to or strictly prefers a PRE with threshold $\bar{\theta}$.

7 Conclusion

This paper investigates the nature of cheap talk communication with (one sided) incomplete information when actions of players are strategic substitutes. With pure messaging strategies, I show that cheap talk communication fully reveals information when the informed sender is able to compensate for the actions of the uninformed receiver for every possible state, once that private information is revealed. Conversely, when the sender is unable to reveal even the lowest state truthfully, there is complete communication breakdown and no information is conveyed.

Consequently, when the domain of the action set constrains the sender from taking an efficient action for some states, I find that communication deteriorates.

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[22]In view of this, Chen et al. (2008) develop a selection criterion to deal with multiplicity problem in the classic CS set up - a notion they call the '*No incentive to separate*' condition.

Specifically, there is only partial information revelation in equilibrium. The sender reveals the state up to a threshold, and for states higher than this threshold, no information is conveyed. An interesting property of the cutoff equilibria is that they are dependent on the boundedness of action sets. Specifically, the bounds on actions act as an incentive constraint for truthful revelation.

The framework I have presented could be extended to applications with multiple senders and (or) receivers (contribution games in networks and team theory, for example). With multiple sources of information, the nature of truth-telling would be dependent on the distribution of the biases and the size of the constraints on actions. In case of multiple receivers, the sender's incentive to reveal information would be similar to the conditions I developed in this paper. This means that full revelation ensues as long as the highest type signal can be credibly revealed by the sender. Such scenarios require a more detailed analysis, and are left for future work.

Another interesting avenue to explore is the role of commitment on part of the receiver (see, e.g., Alonso and Matouschek (2008), and Melumad and Shibano (1991)). Can the receiver, by committing to a message contingent decision rule (a *deterministic mechanism*), do better than the cheap talk equilibrium? Of course, when there is full information revelation, the receiver can do no better from commitment. However, in the case of partial revelation, whether a mechanism of the kind described above is better for the receiver remains an open question.

A Formal definitions of PBE and PRE

A.1 PBE:

A communication equilibrium of the game is given by:

1. given the message m and posterior beliefs $p(\theta \mid m)$, R and S simultaneously choose actions $\left(x_R^*(m), x_S^*(\theta, m)\right)$ that maximizes their expected utility according to the following dual (constrained) optimization problem:

$$\max_{x_R \in V} E_\theta[U(\phi^R(x_R, x_S^*(\theta, m)), \theta)] \text{ subject to } x_S^*(\theta, m) \equiv$$
$$\max_{x_S \in V} U(\phi^S(x_S, x_R^*(m)), \theta, b), x_S \in V \tag{1}$$

$$\max_{x_S \in V} E_\theta[U(\phi^S(x_S, x_R^*(m)), \theta, b)] \text{ subject to } x_R^*(m) \equiv$$
$$\arg\max_{x_R \in V} U(\phi^R(x_R, x_S^*(\theta, m)), \theta), x_R \in V \tag{2}$$

2. the optimal actions by the two players in equilibrium, $x_R^*(m)$ and $x_S^*(\theta, m)$, together, must ensure that the contribution function maximizes each players' expected utility conditional on their information, ie, $\phi^{R^*}(x_R^*(m), x_S^*(\theta, m)) \equiv$ $\arg\max_{\phi^R} U(\phi^R(x_R, x_S), \theta)$ and $\phi^{S^*}(x_S^*(\theta, m), x_R^*(m)) \equiv$ $\arg\max_{\phi^S} U(\phi^S(x_S, x_R), \theta, b)$

3. the posterior beliefs, $p(. \mid m)$, are updated using Bayes' rule whenever possible, given the messaging rule $\mu(.)$

4. given the beliefs and second stage contributions $x_R(m)$ and $x_S(\theta, m)$, S chooses a reporting strategy that maximises expected payoff in the first stage,

$$\mu(\theta) \in \arg\max_{m \in M} U(\phi^S(x_S(\theta, m), x_R(m)), \theta, b)$$

A.2 PRE:

The PRE of the game consists of a θ^* such that,

- for all $\theta \leq \theta^*$, S sends a separating (truthful) message $m = \theta$; for every $\theta > \theta^*$, S sends a pooling message $m = 1$

- When $m \leq \theta^*$, the posterior beliefs are $p(\theta \mid m = \theta) = 1$; when $m = 1$, $p(\theta \mid m) = f(\theta \mid \theta > \theta^*)$

- Upon receiving message $m \leq \theta^*$, players' optimal actions are $x_S^*(m) = \tilde{x}_S(m)$ and $x_R^*(m) = \tilde{x}_R(m)$

- Upon receiving message $m = 1$, players' optimal actions are,

$$\tilde{x}_S(\theta, (\theta^*1]) \equiv \arg\max_{x_S \in \mathbb{R}} \int_{\theta^*}^{1} U(\phi^S(x_S, x_R^*((\theta^*, 1])), \theta, b) f(\theta \mid \theta > \theta^*) d\theta \text{ and}$$

$$x_R^*((\theta^*, 1]) \equiv \arg\max_{x_R \in V} \int_{\theta^*}^{1} U(\phi^R(x_R, \tilde{x}_S(\theta, (\theta^*1])), \theta) f(\theta \mid \theta > \theta^*) d\theta$$

  - If $\tilde{x}_S(\theta, (\theta^*1]) \leq c$, then $x_S^*(\theta, (\theta^*1]) = \tilde{x}_S(\theta, (\theta^*1])$ and $x_R^*((\theta^*, 1])$ is as above

  - If $\tilde{x}_S(\theta, (\theta^*1]) > c$, then $x_S^*(\theta, (\theta^*1]) = c$ and $x_R^*((\theta^*, 1]) \equiv$
  $\arg\max \int_{\theta^*}^{1} U(\phi^R(x_R, c), \theta) f(\theta \mid \theta > \theta^*) d\theta$

The first condition says that for all states in $[0, \theta^*]$, S communicates truthfully, and for any state above, pools by sending an exaggerated message $m = 1$. The second condition describes the formation of posterior beliefs. For any message on $[0, \theta^*]$, R believes it to be truthful and for messages $m = 1$, the posterior is just the conditional prior on the state space.

The third and fourth statements indicate the equilibrium actions conditional on the message and the posterior beliefs of R. The important departure to note is when the sender's unconstrained action goes above the bound. Specifically, R best responds by taking into account the possibility that S's action is constrained by the upper bound - $\hat{x}_S(\theta, m) > c$ implies $x_S^*(\theta, m) = c$- and adjusts her actions accordingly. This revised best response is indicated by the last sub-condition.

B Proofs

B.1 Proof of Proposition 1

B.1.1 Communication Breakdown:

When LTIC is violated, $\tilde{x}_S(0) > c$, meaning that the unconstrained actions does not coincide with the constrained optimization action $x_S^*(0) = c$. Moreover, since $U_{12} > 0$, it must hold for every $\theta \in [0,1]$ that $\tilde{x}_S(\theta) > c$. But if this is so, truth-telling can never be optimal since the sender can always inflate her message and ensure R contributes more.

To see this, think of a generic θ'. If S reports θ', the optimal actions are $x_S^*(\theta') = c$ and $x_R^*(\theta')$ solves $\max_{x_R \in V} U(\phi^R(x_R, c), \theta')$. However, this $x_R^*(\theta')$ is inefficient since $\phi^{R^*}(x_R^*(\theta'), c) < \phi^{S^*}(c, x_R^*(\theta')) < \tilde{\phi}^S(\tilde{x}_S(\theta'), \tilde{x}_R(\theta'))$. The term $\tilde{\phi}^S(\tilde{x}_S(\theta'), \tilde{x}_R(\theta'))$ is the optimal contribution function that maximizes S's payoff. But, because LTIC is violated, the contribution under truth-telling is $\phi^{S^*}(c, x_R^*(\theta'))$ which is clearly sub-optimal in the sense that $U_1(\phi^{S^*}(c, x_R^*(\theta')), \theta', b) > 0$. This condition corresponds with the *'positive spillover effect at the bound'*.

Finally, note that $\phi_2^{S^*}(c, x_R^*(\theta')) \geq 0$ by assumption 2. This implies that if S instead sends a higher message $\theta'' > \theta'$, R increases her optimal action thereby improving S's payoff. Of course, anticipating this, R must never believe any message m as being truthful. The same argument can be applied for any $\theta \in [0,1)$. Hence, this leads to a communication breakdown with all types pooling on the message $m = 1$.

B.1.2 Full revelation result:

When HTIC condition is satisfied, it implies that for every $\theta \in [0,1)$, $\tilde{x}_S(\theta) < c$ by single crossing property of the utility function ($U_{12} > 0$). But if this is the case, when the sender sends a truthful message $m = \theta$, the optimal action under both constrained optimization and unconstrained optimization coincide for the sender. This means that for every $\theta \in [0,1]$, $x_S^*(\theta) = \tilde{x}_S(\theta)$. This ensures there is no inefficiency in terms of contributions. Hence, there always exists a full reve-lation equilibrium in which the sender has an incentive to reveal her information truthfully.

B.2 Proof of Proposition 2

For $\theta^* = \bar{\theta}$ to be supported as an equilibrium, I will construct the following beliefs: for any $m \leq \bar{\theta}$, R believes it to be the true type; for the message $m = 1$, R believes it to be the types $(\bar{\theta}, 1]$. For off-equilibrium path messages $m \in (\bar{\theta}, 1)$, R assigns the belief $\theta = \bar{\theta}$, that is the deviation comes from the lowest possible type in the set of possible off-equilibrium path messages. Then, for an equilibrium with cutoff $\bar{\theta}$ to exist, there should be no profitable deviations for any of the types of players. To check this, consider the types of in $(0, \bar{\theta}]$ and $(\bar{\theta}, 1]$.

For any $\theta \in (0, \theta^*)$, S does not have an incentive to deviate from truth telling since the optimal action under unconstrained optimization coincides with the constrained optimization problem, implying that $\tilde{x}_S(\theta) = x_S^*(\theta) \leq c$.

For $\theta \in (\bar{\theta}, 1]$, however, $\tilde{x}_S(\theta, (\bar{\theta}, 1]) \leq c$ or $\tilde{x}_S(\theta, (\bar{\theta}, 1]) > c$. Since $\tilde{x}_S(\bar{\theta}, (\bar{\theta}, 1]) < c$ and $\tilde{x}_S(1, (\bar{\theta}, 1]) > c$, from single crossing, continuity of $U(.)$ and $\phi^S(.)$ and assumption 5, there must exist a $\theta' \in (\bar{\theta}, 1)$ such that $\tilde{x}_S(\theta', (\bar{\theta}, 1]) = c$. Then, for every θ in $(\bar{\theta}, \theta']$, it must hold true that $\tilde{x}_S(\theta, (\bar{\theta}, 1]) \leq c$. Therefore, sending the message $m = 1$ maximizes payoff for the same reasons put forth above.

For types $\theta \in (\theta', 1]$, the constrained optimization solution suggests that S's action is constrained by the bound, meaning $x_S^*(\theta, (\bar{\theta}, 1]) = c$. But, the payoff to S from sending $m = 1$ is still higher than sending any other off-equilibrium path message. To see this, notice that $x_R^*((\bar{\theta}, 1]) \geq x_R^*(\bar{\theta})$ and $\phi_2^{S^*}(c, x_R^*(.)) > 0$ at the bound. Therefore, $U(\phi^{S^*}(c, x_R^*((\bar{\theta}, 1])), \theta, b) \geq U(\phi^{S^*}(c, x_R^*(\bar{\theta})), \theta, b)$ for all $\theta \in (\theta', 1]$ such that $x_S^*(\theta, (\bar{\theta}, 1]) = c$. This concludes the proof.

B.3 Proof of Proposition 3

I will start by defining the off-equilibrium path messages that would be sufficient to support any PRE with a threshold θ^*. As in Proposition 2, for any $m \in (\theta^*, 1)$, R assigns the belief $\theta = \theta^*$, that is the deviation comes from the lowest possible type in set of possible off-equilibrium path messages. Then, for an equilibrium with θ^* to exist, there should be no profitable deviations for any of the types of players.

To check this, consider the types of in $(0, \theta^*]$ and $(\theta^*, 1]$, in that order. For any $\theta \in (0, \theta^*]$, S does not have an incentive to deviate from truth telling since the optimal action upon truthful messaging is strictly within the bound, $\tilde{x}_S(\theta) \leq c$.

This implies that the unconstrained solution also coincides with the constrained optimization problem, $\tilde{x}_S(\theta) = x_S^*(\theta)$.

For $\theta \in (\theta^*, 1]$, $\tilde{x}_S(\theta, (\theta^*, 1]) \le c$ or $\tilde{x}_S(\theta, (\bar{\theta}, 1]) > c$. Since $\tilde{x}_S(\theta^*, (\theta^*, 1]) < c$ and $\tilde{x}_S(1, (\theta^*, 1]) > c$, as before, from single crossing, continuity of $U(.)$ and $\phi^S(.)$ and assumption 5, there must exist a $\theta' \in (\theta^*, 1)$ such that $\tilde{x}_S(\theta', (\theta^*, 1]) = c$. Then, for every θ in $(\theta^*, \theta']$, it must hold true that $\tilde{x}_S(\theta, (\theta^*, 1]) \le c$.

For the types $\theta \in (\theta', 1]$, the constrained optimization solution suggests that $x_S^*(\theta, (\theta^*, 1]) = c$. But, the payoff to S from sending $m = 1$ is still higher than sending any other off-equilibrium path message. To see this, notice that $x_R^*((\theta^*, 1]) > x_R^*(\theta^*)$ and $\phi_2^{S^*}(c, x_R^*(.)) > 0$. Therefore, $U(\phi^{S^*}c, x_R^*((\theta^*, 1])), \theta, b) \ge U(\phi^{S^*}(c, x_R^*(\theta^*)), \theta, b)$ for all $\theta \in (\theta', 1]$. This completes the proof.

B.4 Proof of Proposition 4

Receiver's ex-ante utility:

$$
V_R(\theta^*) = \int_0^{\theta^*} U(\phi^{R^*}(x_R^*(t), x_S^*(t)), t) f(t) dt +
$$

$$
\int_{\theta^*}^1 U(\phi^{R^*}(x_R^*((\theta^*, 1])), x_S^*(t, (\theta^*, 1]))), t) f(t) dt
$$

Taking the derivative of receiver's welfare with respect to θ^*,

$$
\frac{dV_R(\theta^*)}{d\theta^*} = U(\phi^{R^*}(x_R^*(\theta^*), x_S^*(\theta^*)), \theta^*) f(\theta^*) -
$$

$$
U(\phi^{R^*}(x_R^*((\theta^*, 1]), x_S^*(\theta^*, (\theta^*, 1]))), \theta^*) f(\theta^*) > 0
$$

for any $\theta^* \le \bar{\theta}$

Sender's ex-ante utility:

Take any two feasible cutoffs of a PRE, say θ' and θ'', such that $\theta' < \theta'' \le \bar{\theta}$. I will try to establish that sender is better off with the more informative equilibrium θ''. Similar to arguments made in earlier Propositions 2 and 3, for any cutoff equilibrium θ^* there must exist a $\theta \in (\theta^*, 1]$ such that $\tilde{x}_S(\theta, (\theta^*, 1]) = c$.

Let θ_c' and θ_c'' be such deviation types for the two cutoffs θ' and θ'', respectively.

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First, I claim that $\theta_c' < \theta_c''$. Suppose not, and $\theta_c' > \theta_c''$. Then, $\tilde{x}_S(\theta_c'', (\theta', 1]) < \tilde{x}_S(\theta_c', (\theta', 1]) = c$. But by single-crossing property, $\tilde{x}_S(\theta_c'', (\theta', 1]) \geq \tilde{x}_S(\theta_c'', (\theta'', 1]) = c$. This is a contradiction. Therefore the claim holds. In order to prove the result for the sender, I will have to consider two possible scenarios.

Scenario (a): *When $\theta_c' < \theta''$. That is, $\theta' < \theta_c' < \theta'' < \theta_c''$.* The sender's utility under the two PRE's is given by,

$$PRE' : V_S(\theta') = \int_0^{\theta'} U(\phi^S(x_S^*(t), x_R^*(t)), t, b) f(t) dt +$$

$$\int_{\theta'}^1 U(\phi^S(x_S^*(t, (\theta', 1]), x_R^*((\theta', 1])), t, b) f(t) dt$$

$$PRE'' : V_S(\theta'') = \int_0^{\theta''} U(\phi^S(x_S^*(t), x_R^*(t)), t, b) f(t) dt +$$

$$\int_{\theta''}^1 U(\phi^S(x_S^*(t, (\theta'', 1]), x_R^*((\theta'', 1])), t, b) f(t) dt$$

Under PRE' the sender's optimal action is within the bound for the interval $(0, \theta_c']$. Since $\theta_c' < \theta_c''$, the sender's optimal action is also within the bound over the interval $(0, \theta_c']$ under PRE''. Therefore, what is left to be checked are those states in which there is inefficiency because of the bounds imposed on actions of the sender. In PRE' this corresponds to the interval $(\theta_c', 1]$. On the same interval, I compare the utility (ex-ante) achieved under PRE''. I will refer to this utility as the residual welfare that results from inefficiency, $V_S^{RES}(\theta)$.

$$V_S^{RES}(\theta') = \int_{\theta_c'}^{\theta_c''} U(\phi^S(c, x_R^*((\theta', 1])), t, b) f(t) dt + \int_{\theta_c''}^1 U(\phi^S(c, x_R^*((\theta', 1])), t, b)) f(t) dt$$

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$$V_S^{RES}(\theta'') =$$

$$\int_{\theta_c'}^{\theta''} U(\phi^S(x_S^*(t), x_R^*(t)), t, b)f(t)dt \ + \int_{\theta''}^{\theta_c''} U(\phi^S(x_S^*(t, (\theta'', 1]), x_R^*((\theta'', 1])), t, b)f(t)dt$$

$$+ \int_{\theta_c''}^{1} U(\phi^S(c, x_R^*((\theta'', 1])), t, b)f(t)dt$$

Comparing the two expressions term-wise,

$$\int_{\theta_c'}^{\theta''} U(\phi^S(x_S^*(t), x_R^*(t)), t, b)f(t)dt + \int_{\theta''}^{\theta_c''} U(\phi^S(x_S^*(t, (\theta'', 1]), x_R^*((\theta'', 1])), t, b)f(t)dt >$$

$$\int_{\theta_c'}^{\theta''} U(\phi^S(c, x_R^*((\theta', 1])), t, b)f(t)dt + \int_{\theta''}^{\theta_c''} U(\phi^S(c, x_R^*((\theta', 1])), t, b)f(t)dt$$

$$= \int_{\theta_c'}^{\theta_c''} U(\phi^S(c, x_R^*((\theta', 1])), t, b)f(t)dt$$

This is because,

$$\int_{\theta_c'}^{\theta''} U(\phi^S(x_S^*(t), x_R^*(t)), t, b)f(t)dt > \int_{\theta_c'}^{\theta''} U(\phi^S(c, x_R^*((\theta', 1])), t, b)f(t)dt$$

$$\int_{\theta''}^{\theta_c''} U(\phi^S(x_S^*(t, (\theta'', 1]), x_R^*((\theta'', 1])), t, b)f(t)dt > \int_{\theta''}^{\theta_c''} U(\phi^S(c, x_R^*((\theta', 1])), t, b)f(t)dt$$

Similarly,

$$\int_{\theta_c''}^{1} U(\phi^S(c, x_R^*((\theta'', 1])), t, b)f(t)dt > \int_{\theta_c''}^{1} U(\phi^S(c, x_R^*((\theta', 1])), t, b))f(t)dt$$

The first inequality follows from the fact that on the interval (θ'_c, θ''),
$U(\phi^S(x^*_S(t), x^*_R(t)), t, b) > U(\phi^S(c, x^*_R((\theta', 1])), t, b)$ for all $t \in (\theta'_c, \theta'')$. Similarly,
for all $t \in (\theta'', \theta''_c]$, $U(\phi^S(x^*_S(t, (\theta'', 1]), x^*_R((\theta'', 1])), t, b) > U(\phi^S(c, x^*_R((\theta', 1])), t, b)$.
The last inequality follows from noting that $\phi^S(c, x^*_R((\theta', 1])) < \phi^S(c, x^*_R((\theta'', 1]))$
because $x^*_R((\theta', 1]) < x^*_R((\theta'', 1])$ and since there is positive spillover at the bound
for S, $U_1 \mid_{\hat{x}_S(.) > c} > 0$. Comparing the terms pairwise therefore yields the required
result, $V^{RES}_S(\theta'') > V^{RES}_S(\theta')$

Scenario (b): *When $\theta'_c > \theta''$. That is, $\theta' < \theta'' < \theta'_c < \theta''_c$.*
In this case, as earlier, I will look at states in which there is inefficiency generated
by information pooling and compare the residual welfare.

$$V^{RES}_S(\theta') = \int_{\theta'_c}^{\theta''_c} U(\phi^S(c, x^*_R((\theta', 1])), t, b) f(t) dt + \int_{\theta''_c}^{1} U(\phi^S(c, x^*_R((\theta', 1])), t, b)) f(t) dt$$

$$V^{RES}_S(\theta'') = \int_{\theta'_c}^{\theta''_c} U(\phi^S(x^*_S(t, (\theta'', 1]), x^*_R((\theta'', 1])), t, b) f(t) dt +$$

$$\int_{\theta''_c}^{1} U(\phi^S(c, x^*_R((\theta'', 1])), t, b) f(t) dt$$

Pairwise comparison yields,

$$\int_{\theta'_c}^{\theta''_c} U(\phi^S(x^*_S(t, (\theta'', 1]), x^*_R((\theta'', 1])), t, b) f(t) dt > \int_{\theta'_c}^{\theta''_c} U(\phi^S(c, x^*_R((\theta', 1])), t, b) f(t) dt$$

and,

$$\int_{\theta''_c}^{1} U(\phi^S(c, x^*_R((\theta'', 1])), t, b) f(t) dt > \int_{\theta''_c}^{1} U(\phi^S(c, x^*_R((\theta', 1])), t, b)) f(t) dt$$

The first inequality follows from the inefficiency of contributing c on the interval
$(\theta'_c, \theta''_c]$ when instead sender can best respond to $x^*_R((\theta'', 1])$. The second inequality
results from the positive spillover property in the interval $(\theta''_c, 1]$ and the fact that
$x^*_R((\theta'', 1]) > x^*_R((\theta', 1])$. Therefore, $V^{RES}_S(\theta'') > V^{RES}_S(\theta')$. This completes the
proof.

B.5 Proof of Proposition 5

I will prove this by making pairwise comparison between two thresholds $\bar{\theta}$ and $\theta'(< \bar{\theta})$. From Proposition 4, we know that $\theta'_c < \bar{\theta}_c$. As before, there are two scenarios to consider.

Scenario (a): *When $\theta'_c < \bar{\theta}$. That is, $\theta' < \theta'_c < \bar{\theta} < \bar{\theta}_c$.*
In this case, every type $\theta \in [0, \theta'_c]$ is indifferent between the two threshold equilibria, since the optimal actions are within the bound in both cases. Therefore, $U(\phi^S(x^*_S(\theta, (\theta', 1]), x^*_R((\theta', 1])), \theta, b) = U(\phi^S(x^*_S(\theta, (\bar{\theta}, 1]), x^*_R((\bar{\theta}, 1])), \theta, b)$ since $\tilde{x}_S(\theta, (\theta', 1]) = x^*_S(\theta, (\theta', 1]) \leq c$, and $\tilde{x}_S(\theta, (\bar{\theta}, 1]) = x^*_S(\theta, (\bar{\theta}, 1]) \leq c$.
However, every $\theta \in (\theta'_c, 1]$ strictly prefers the $\bar{\theta}$ threshold equilibrium. To see this, let us further divide the interval $(\theta'_c, 1]$ to $(\theta'_c, \bar{\theta}_c]$ and $(\bar{\theta}_c, 1]$. Now, every $\theta \in (\theta'_c, \bar{\theta}_c]$ prefers the threshold $\bar{\theta}$ since $\tilde{x}_S(\theta, (\bar{\theta}, 1]) \leq c$, whereas with threshold θ', $\tilde{x}_S(\theta, (\theta', 1]) > c$ implying that the constrained action is $x^*_S(\theta, (\theta', 1]) = c$. Therefore, for $\theta \in (\theta'_c, \bar{\theta}_c]$, $U(\phi^S(c, x^*_R((\theta', 1])), \theta, b) < U(\phi^S(x^*_S(\theta, (\bar{\theta}, 1]), x^*_R((\bar{\theta}, 1])), \theta, b)$. Lastly, for types $\theta \in (\bar{\theta}_c, 1]$, the unconstrained action under both the thresholds are above c. This means $\tilde{x}_S(\theta, (\theta', 1]), \tilde{x}_S(\theta, (\bar{\theta}, 1]) > c$. But, R's action is higher under $\bar{\theta}$ ($x^*_R((\bar{\theta}, 1]) > x^*_R((\theta', 1])$) and due to the positive spillover property, it follows that $U(\phi^S(c, x^*_R((\theta', 1])), \theta, b) < U(\phi^S(c, x^*_R((\bar{\theta}, 1])), \theta, b)$ for all $\theta \in (\bar{\theta}_c, 1]$.

Scenario (b): *When $\theta'_c < \bar{\theta}$. That is, $\theta' < \bar{\theta} < \theta'_c < \bar{\theta}_c$.*
A analogous set of arguments hold true for this case. In particular, every type $\theta \in [0, \theta'_c]$ is indifferent between the thresholds $\bar{\theta}$ and θ'. Every type $\theta \in (\theta'_c, \bar{\theta}_c]$ are strictly better off under threshold $\bar{\theta}$ because $\tilde{x}_S(\theta, (\bar{\theta}, 1]) \leq c$, whereas with threshold θ', $\tilde{x}_S(\theta, (\theta', 1]) > c$. Types $\theta \in (\bar{\theta}_c, 1]$ are also strictly better off under threshold $\bar{\theta}$ because of the positive spillover argument made earlier. This completes the proof.

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