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Numerical analysis on performance enhancement of a CO$_2$ heat pump water heating system by extracting tepid water

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Abstract

Water heating systems each of which is composed of an air-to-water heat pump using CO$_2$ as a natural refrigerant and a hot water storage tank have been developed, and are expected to contribute to energy saving in residential hot water supply. However, the area of tepid water in the storage tank expands because of heat conduction during long time storage, which leads to a degradation of the system performance. One of the ways to reduce the area of tepid water and enhance the system performance is to extract tepid water from the side of the storage tank. In this paper, the performance enhancement of a CO$_2$ heat pump water heating system with such a revised storage tank is analyzed by numerical simulation. The simulation model developed previously for a conventional system is extended by modifying the model for the storage tank and verifying its validity through experiments. Through a performance analysis using a standardized hot water demand, it is clarified how system performance...
values of the revised system are enhanced, and especially how the trade-off relationship between the system efficiency and the volume of unused hot water is improved, in comparison with those of the conventional system.

**Keywords:** Heat pump, Water heater, Thermal storage, Natural refrigerant, System performance enhancement, Numerical simulation

1. **Introduction**

Hot water demand occupies about one-third of the energy consumption in the residential sector in Japan, and energy saving in hot water supply has been an important issue. Under this situation, water heating systems each of which is composed of an air-to-water heat pump using CO₂ as a natural refrigerant and a hot water storage tank have been developed and commercialized widely, and are expected to contribute to energy saving in residential hot water supply [1]. In order to promote energy saving, it is necessary to enhance the performances of not only components but also systems. The performance of CO₂ heat pumps, or coefficient of performance (COP), has been enhanced dramatically through the technological development of their elements such as compressors and gas coolers for the last decade. On the other hand, importance has also been given to the performance of systems in case that they are operated under hourly and daily changes in hot water demand.

The performance of a CO₂ heat pump is affected by the air temperature as well as the inlet and outlet water temperatures. Many theoretical and experimental studies have been conducted for the performance analysis on CO₂ heat pumps. In the earliest
years, prototype CO2 heat pumps have been developed, and their performance characteristics have been investigated experimentally: Nekså et al. have developed a prototype heat pump water heater and have clarified its performance characteristics [2]; White et al. have constructed a heat pump for water heating and have created a model for performance prediction [3]; Richter et al. have compared the performance of a prototype heat pump with that of a conventional one [4]. On the other hand, simulation models for CO2 heat pump transcritical cycles have been developed for performance estimation: Hwang and Radermacher have developed a simulation model for refrigeration cycle and have compared its performance with that of a conventional cycle [5]; Skaugen et al. have developed a simulation model for vapor compression systems and have shown its verification against measurements [6]; Yokoyama et al. have created a simulation model for a heat pump included in a water heating system based on measured data and have investigated the influence of ambient and operating conditions on performance [7]; Laipvadit et al. have developed a simulation model for heat pump water heaters and have presented the effect of operating parameters on performance [8]; Yang et al. have developed a simulation model for a heat pump system with an expander and have compared simulated and experimental results [9]; Yamaguchi et al. have developed a simulation model for a heat pump water heater and have verified simulated results using an existing machine [10]; Lin et al. have developed a simulation model for a heat pump system in consideration of geometrical variations and have validated it with experimental measurements [11]. Some studies for performance enhancement of CO2 heat pump transcritical cycles have been conducted: Cavallini et al. have conducted a theoretical and experimental analysis to optimize a two-stage cycle [12]; Aprea and Maiorino have conducted an experimental
investigation to optimize heat rejection pressure for a split system [13]; Xu et al. have conducted an experimental study on performance of a system with ejector under optimum high-side pressure [14]; Qi et al. have conducted an experimental investigation to optimize rejection pressure for a water heater [15]. Some studies for applying CO2 heat pump systems to simultaneous cooling and heating have been conducted: Sarkar et al. have conducted studies on optimization and simulation of heat pump systems in this application [16, 17]; They have also investigated the performance of a system in this application by experiment [18]. The evaluation of CO2 heat pump transcritical cycles from the viewpoint of exergy has been conducted: Sarkar et al. have conducted the exergy analysis of the heat pump systems including heat transfer and fluid flow effects [19]; Fazelpour and Morozuk have conducted the exergoeconomic analysis of the heat pump refrigeration machines [20]. In addition, some review papers have been published: Nekså has given an overview of some activities in the CO2 heat pump field [21]; Ma et al. have given a review of studies on CO2 heat pump and refrigeration cycles [22].

On the other hand, the performance of a water heating system composed of a CO2 heat pump and a storage tank is affected by many conditions. The ambient conditions such as air and feed water temperatures, the hot water demand, and the operating conditions such as startup and shutdown, and outlet water temperature during operation of the CO2 heat pump affect the inlet water temperature and resultantly the heat pump COP through the temperature distribution in the storage tank. In addition to the heat pump COP, the storage and system efficiencies, and the volumes of stored and unused hot water, which are also considered as system performance values, are affected by the aforementioned various conditions through the temperature distribution in the storage
tank. Fewer theoretical and experimental studies have been conducted for the performance analysis on CO2 heat pump water heating systems. Some studies have been conducted under steady states: Cecchinato et al. have investigated the performance of a water heater by a simulation model in consideration of the influence of inlet water temperature by assuming perfect stratification and mixing in a storage tank [23]; Stene has conducted theoretical and experimental studies on the performance of a heat pump for combined space and water heating in consideration of the influence of inlet water temperature due to heat conduction in a storage tank [24]; Minetto has conducted theoretical and experimental analysis on the performance of a water heater in consideration of the influence of inlet water temperature [25]. On the other hand, some studies have been conducted under unsteady states including charging and tapping modes: Fernandez et al. have investigated the performance of a water heating system composed of a heat pump and a storage tank in the charging mode [26]; Yokoyama et al. have conducted numerical simulations on such a system and have clarified the effects of the ambient and operation temperatures on system performance values under a daily cyclic change in hot water demand [27, 28]; Yokoyama has conducted the exergy analysis of the system based on the results obtained by numerical simulation [29]; Yokoyama et al. have also investigated daily changes in system performance values under daily changes in standardized and simulated hot water demands [30, 31]. In addition, operation and design optimizations have been tried: Yokoyama et al. have investigated the optimal conditions for operating temperatures under daily change in simulated hot water demand, which is based on the estimation of system performance values in place of the numerical simulation [32]; Deng et al. have investigated performance analysis and optimization of a combination of a solar heater
and a heat pump water heating system [33]. However, all the studies have been conducted by assuming that conventional storage tanks with outlets for hot water supply on their tops are installed.

In order to enhance the performance of a water heating system composed of a CO\textsubscript{2} heat pump and a storage tank, it is necessary to pay attention to not only each component but also the combination of the components because the performance of CO\textsubscript{2} heat pump depends on that of the storage tank. In the system with a conventional storage tank, the gradient of the vertical temperature distribution in the storage tank with water temperature stratified becomes small because of heat conduction in the vertical direction during long time storage. Namely, the temperature at the lower part rises while that at the upper part drops, and the area of tepid water at the middle part expands. Here, tepid water is used for the water with middle temperatures except the highest and lowest ones. As a result, the temperature of the inlet water entering CO\textsubscript{2} heat pump rises, which leads to a decrease in the heat pump COP, and resultantly system efficiency also decreases. On the other hand, the volumes of hot water stored after heat pump operation and unused after hot water supply decrease, which may results in a shortage in hot water supply. In order to overcome these defects of the system with the conventional storage tank simultaneously, it is necessary to restore the large gradient of the temperature distribution in the storage tank. For this purpose, it is considered to be effective to extract tepid water from the side of the storage tank and reduce the area of tepid water in the storage tank. In fact, water heating systems with such revised storage tanks have been developed and commercialized. However, it is never clarified how the performance enhancement of a water heating system can be attained by replacing a conventional storage tank with a revised one.
In this paper, the performance of a CO₂ heat pump water heating system with extracting tepid water from the side of the storage tank is analyzed by numerical simulation. The simulation model developed previously for a conventional CO₂ heat pump water heating system is extended for this purpose. The model for the storage tank is modified, and its validity is investigated through experiments using a storage tank model. A performance analysis for the conventional and revised systems is conducted using a standardized hot water demand, and their performances are compared in terms of the temperature distribution in the storage tank, heat pump COP, storage and system efficiencies, and volumes of stored and unused hot water. Through this analysis, the effect of extracting tepid water from the side of the storage tank on the performance enhancement is investigated.

2. CO₂ heat pump water heating system

Figure 1 shows the configuration of the CO₂ heat pump water heating system investigated in this paper. This system is composed of a CO₂ heat pump and a hot water storage tank. The CO₂ heat pump is composed of a compressor, a gas cooler, an expansion valve, and an evaporator. The system is equipped with a fan, a pump, and motors M1 to M3 as auxiliary machinery. In the charging mode, the system heats water using the refrigeration cycle of the CO₂ heat pump and stores hot water in the storage tank. In the tapping mode, hot water stored in the storage tank is retrieved and supplied to a tapping site. The storage tank has the outlets for hot water supply on its top and side. If water extracted from the side can be used for hot water supply by mixing it with feed water or water extracted from the top, it has priority over water
It is preferable to determine the position for extracting water from the side so that the system performance is maximized. However, the hourly change in hot water demand depends on users significantly, and the area of tepid water changes with time. Therefore, it is impossible to determine the position for water extraction optimally. In this paper, the position for water extraction is assumed, and it is investigated how water extraction affects the enhancement of the system performance under a certain hourly change in hot water demand.

3. Numerical simulation

3.1. Component models

3.1.1. Conventional models for CO₂ heat pump and mixing valves

Conventional component models proposed previously are used for the CO₂ heat pump and mixing valves [30, 34]. Since the capacity of the CO₂ heat pump is much smaller than that of the storage tank, a static model is adopted for it. Namely, it is assumed that its states become steady instantaneously at each time. In addition, although the heat pump includes several elements, they are not taken into account explicitly, and it is expressed by a simplified component model. A static model is also adopted for the mixing valves.

The model of the CO₂ heat pump is shown in Fig. 2. The mass flow rates and temperatures of water at the inlet and outlet, heat pump COP, heat output, power consumption, and air temperature are adopted as basic variables whose values are to be
determined. The mass and energy balance relationships to be satisfied as basic equations are expressed by

\[
\begin{align*}
\dot{m}_{\text{HP}i} &= \dot{m}_{\text{HP}o} \\
\dot{m}_{\text{HP}i} cT_{\text{HP}i} + \dot{Q}_{\text{HP}} &= \dot{m}_{\text{HP}o} cT_{\text{HP}o}
\end{align*}
\]

(1)

where \( \dot{m}_{\text{HP}} \) and \( T_{\text{HP}} \) are the mass flow rate and temperature of water, respectively, and these variables at the inlet and outlet are denoted by the subscripts \( i \) and \( o \), respectively. In addition, \( \dot{Q}_{\text{HP}} \) is the heat output, and \( c \) is the specific heat of water, which is assumed to be constant. The energy input and output relationship is expressed by

\[
\dot{Q}_{\text{HP}} = \eta_{\text{HP}} \dot{W}_{\text{HP}}
\]

(2)

where \( \dot{W}_{\text{HP}} \) is the power consumption, and \( \eta_{\text{HP}} \) is the heat pump COP. The remaining equations to be considered are approximate functions of the power consumption and heat pump COP, and they are expressed by quadratic functions with respect to the air and inlet/outlet water temperatures as follows:

\[
\begin{align*}
\dot{W}_{\text{HP}} &= (\alpha_1 + \beta_1 T^a + \gamma_1 T^{a2}) (\alpha_2 + \beta_2 T_{\text{HP}i} + \gamma_2 T_{\text{HP}i}^2) (\alpha_3 + \beta_3 T_{\text{HP}o} + \gamma_3 T_{\text{HP}o}^2) \\
\eta_{\text{HP}} &= (\alpha_4 + \beta_4 T^a + \gamma_4 T^{a2}) (\alpha_5 + \beta_5 T_{\text{HP}i} + \gamma_5 T_{\text{HP}i}^2) (\alpha_6 + \beta_6 T_{\text{HP}o} + \gamma_6 T_{\text{HP}o}^2)
\end{align*}
\]

(3)

where \( T^a \) is the air temperature, and \( \alpha_i \) to \( \alpha_6 \), \( \beta_i \) to \( \beta_6 \), and \( \gamma_i \) to \( \gamma_6 \) are the coefficients of the quadratic functions, whose values are to be identified based on measured data for an existing device.

The model of the mixing valve is shown in Fig. 3. The mass flow rates and temperatures of water at the inlets and outlet are considered as basic variables. The mass and energy balance relationships to be satisfied as basic equations are expressed
by

\[
\begin{align*}
\dot{m}_{MV1} + \dot{m}_{MV2} &= \dot{m}_{MV0} \\
\dot{m}_{MV1} c_{TMV1} + \dot{m}_{MV2} c_{TMV2} &= \dot{m}_{MV0} c_{TMV0}
\end{align*}
\]

where \( \dot{m}_{MV} \) and \( T_{MV} \) are the mass flow rate and temperature of water, respectively, and these variables at the inlets and outlet are denoted by the subscripts \( i \) and \( o \), respectively. In addition, the two inlets are distinguished by the subscripts 1 and 2.

The validity of the model for the CO₂ heat pump should be investigated by comparing simulated and experimental results. An extensive experiment has already been conducted in a room where the air and inlet water temperature can be controlled [34]. An existing CO₂ heat pump has been operated by changing the air and inlet water temperatures, and the power consumption, mass flow rate of water, and outlet water temperature have been measured. The heat pump COP has been calculated from the measured data obtained by the experiment. The values of the coefficients of the quadratic functions in Eq. (3) have been identified so that calculated power consumption and heat pump COP coincide well with measured ones.

### 3.1.2. Revised model for storage tank

A conventional component model proposed previously is revised for the storage tank, especially for the control volume with water extraction. Since the capacity of the storage tank is large, a dynamic model is adopted for it [27, 30, 34]. The temperature of water in the storage tank is assumed to be stratified. To consider the one-dimensional vertical temperature distribution in the storage tank, it is vertically divided into many control volumes with the same volume, in each of which the water temperature is assumed to be uniform. It is also assumed that the heat transfer occurs
by one-dimensional water flow and heat conduction as well as heat loss from the tank surface.

The model of the storage tank is shown in Fig. 4. The storage tank is divided into \( J \) control volumes. The mass flow rates and temperatures of water for the control volumes are adopted as basic variables whose values are to be determined. The mass balance relationships for the control volumes to be satisfied as basic equations are expressed by

\[
\begin{align*}
\dot{m}_{STi}^l &= \dot{m}_{ST1}^l + \dot{m}_{STo}^l \\
\dot{m}_{STj} &= \dot{m}_{STj}^l \quad (j = 2, 3, \ldots, n - 1, n + 1, n + 2, \ldots, J - 1) \\
\dot{m}_{STn-1} &= \dot{m}_{STn}^b + \dot{m}_{STo}^e \\
\dot{m}_{STi}^b + \dot{m}_{STJ-1} &= \dot{m}_{STo}^b
\end{align*}
\]

where the mass flow rate of water which flows downward from the \( j \)th to \((j+1)\)th control volumes is denoted by \( \dot{m}_{STj}^l \), and \( \dot{m}_{STj} \geq 0 \) and \( \dot{m}_{STj} < 0 \) when water flows downward and upward, respectively. The variables at the top and bottom inlet/outlet are denoted by the superscripts \( t \) and \( b \), respectively, and at the inlet and outlet by the subscripts \( i \) and \( o \), respectively. It is assumed that water extraction is conducted from the \( n \)th control volume, and the variables for water extraction are denoted by the superscript \( e \) and the subscript \( o \). The energy balance relationships for the control volumes to be satisfied as basic equations are expressed by
\[ \frac{\rho V_{ST}}{J} \frac{dT_{ST1}}{dt} = m_{ST1}^t c T_{ST1}^t - m_{STo}^t c T_{STo}^t - \dot{Q}_{ST1} \]
\[ \quad - \lambda_{ST1}^t J / H_{ST} (T_{ST1} - T_{ST2}) - U_{ST} (A_{ST} + 2S_{ST}) / J (T_{ST1} - T^a) \]
\[ \frac{\rho V_{ST}}{J} \frac{dT_{STj}}{dt} = -\dot{Q}_{STj} + \lambda_{j-1}^t S_{STj}^t / H_{ST} (T_{STj-1} - T_{STj}) \]
\[ \quad - \lambda_{STj}^t J / H_{ST} (T_{STj} - T_{STj+1}) - U_{ST} (A_{ST} + 2S_{ST}) / J (T_{STj} - T^a) \]
\[ \left( j = 2, 3, \ldots, J \right) \]
\[ \frac{\rho V_{ST}}{J} \frac{dT_{STj}}{dt} = m_{STj}^b c T_{STj}^b - m_{STo}^b c T_{STo}^b - \dot{Q}_{STj} \]
\[ \quad + \lambda_{j-1}^t S_{STj}^t / H_{ST} (T_{STj-1} - T_{STj}) - U_{ST} (A_{ST} + 2S_{ST}) / J (T_{STj} - T^a) \]

where the water temperature in the \( j \)th control volume is denoted by \( T_{STj} \), and \( t \) is time. \( \rho \) and \( \lambda \) are the density and heat conductivity of water, respectively, \( \rho \) is assumed to be constant, and \( \lambda \) is assumed to depend on temperature, which is denoted by the subscript \( j \). \( H_{ST} \), \( V_{ST} \), \( S_{ST} \), and \( A_{ST} \) are the height, volume, horizontal sectional area, and cylindrical surface area of the storage tank, respectively. In addition, \( U_{ST} \) is the overall heat transfer coefficient, and \( \dot{Q}_{STj} \) is the rate of heat transfer by water flow, which is expressed by

\[ \dot{Q}_{ST1} = \begin{cases} 
\dot{m}_{ST1}^t c T_{ST1}^t & (\dot{m}_{ST1} \geq 0) \\
\dot{m}_{ST1}^t c T_{ST2}^t & (\dot{m}_{ST1} < 0) 
\end{cases} \]

\[ \dot{Q}_{STj} = \begin{cases} 
-\dot{m}_{STj-1}^t c T_{STj-1}^t + \dot{m}_{STj}^t c T_{STj}^t & (\dot{m}_{STj-1} = \dot{m}_{STj} \geq 0) \\
-\dot{m}_{STj-1}^t c T_{STj}^t + \dot{m}_{STj}^t c T_{STj+1}^t & (\dot{m}_{STj-1} = \dot{m}_{STj} < 0) 
\end{cases} 
\]

\( (j = 2, 3, \ldots, n-1, n+1, n+2, \ldots, J-1) \)

\[ \dot{Q}_{STn} = \begin{cases} 
-\dot{m}_{STn-1}^t c T_{STn-1}^t + \dot{m}_{STn}^t c T_{STn}^t + \dot{m}_{STo}^e c T_{STo}^e & (\dot{m}_{STn-1} \geq \dot{m}_{STn} > 0) \\
-\dot{m}_{STn-1}^t c T_{STn-1}^t + \dot{m}_{STn}^t c T_{STn}^t + \dot{m}_{STo}^e c T_{STo}^e & (\dot{m}_{STn-1} \geq 0, \dot{m}_{STn} \leq 0) \\
-\dot{m}_{STn-1}^t c T_{STn}^t + \dot{m}_{STn}^t c T_{STn+1}^t + \dot{m}_{STo}^e c T_{STo}^e & (\dot{m}_{STn} \leq \dot{m}_{STn-1} < 0) 
\end{cases} \]

\[ \dot{Q}_{STj} = \begin{cases} 
-\dot{m}_{STj-1}^t c T_{STj-1}^t & (\dot{m}_{STj-1} \geq 0) \\
-\dot{m}_{STj-1}^t c T_{STj}^t & (\dot{m}_{STj-1} < 0) 
\end{cases} \]
where the temperature for heat transfer by water flow is set at its value in the upstream control volume. The water temperatures at the top, side, and bottom outlets of the storage tank are the same with those in the corresponding control volumes as follows:

\[
\begin{align*}
T^a_{STo} &= T_{ST1} \\
T^e_{STo} &= T_{STn} \\
T^b_{STo} &= T_{STJ}
\end{align*}
\]

The validity of the model for the storage tank should also be investigated by comparing simulated and experimental results. An experiment has already been conducted in a room where the air and feed water temperature can be controlled [30, 34]. An existing CO2 heat pump water heating system has been operated in the charging and tapping modes, and the temperatures at 16 representative points located vertically on the side of the storage tank have been measured. In the charging mode, the performance of the CO2 heat pump identified by the aforementioned experiment has been used in the numerical simulation, and the same outlet water temperature has been set in both the experiment and numerical simulation. The changes in the temperature distribution in the storage tank for about three hours have been compared. In the tapping mode, the same hourly change in the hot water demand, i.e., flow rate, temperature, and duration has been used in both the experiment and numerical simulation. The changes in the temperature distribution in the storage tank for about fifteen hours have been compared. It has turned out that calculated and measured temperatures at some selected times are in good agreement. However, this experiment has been conducted without water extraction in the tapping mode. In this paper, therefore, experiments are conducted newly with water extraction in the tapping mode, and simulated and experimental results are compared. These experiments are described in the next chapter.
3.2. Modes for hot water supply

As described previously, the storage tank is divided into $J$ control volumes in the vertical direction, and the position for water extraction is set at the $n$th control volume CV$n$. On the assumption that the temperature of hot water supplied to the tapping site is prescribed exactly, the following strategy for hot water supply is adopted: If the water in CV$n$ can be used for hot water supply by mixing it with feed water or water extracted from the top, it has priority over water extracted from the top. Based on this strategy, the following three modes for hot water supply shown in Fig. 5 are set using the temperature $T_{STn}$ of water in CV$n$, and thresholds $\bar{T}$ and $T$, which are higher and lower slightly than the temperature for hot water supply, respectively, in consideration of the error in measuring temperature:

Mode A: In case that $T_{STn} > \bar{T}$, the heat of water in CV$n$ can be used for hot water supply by mixing it with water with a lower temperature, and water in CV$n$ mixed with feed water is supplied to the tapping site.

Mode B: In case that $\bar{T} < T_{STn} \leq T$, the temperature of water in CV$n$ may not be suitable for hot water supply in modes A and C in consideration of the error in measuring temperature, and water in CV1 mixed with feed water is supplied to the tapping site. This mode denotes a conventional one without water extraction.

Mode C: Even in case that $T_{STn} \leq \bar{T}$, the heat of water in CV$n$ can be used for hot water supply by mixing it with water with a higher temperature, and water in CV$n$ mixed with water in CV1 is supplied to the tapping site.
Modes A to C are switched to another one using the mixing valves MV1 and MV2 based on the temperature $T_{STn}$. When the system is operated in the charging mode without hot water demand during the nighttime, the temperature $T_{STn}$ rises with time, and the mode changes from C to A. On the other hand, when the system is operated in the tapping mode without additional heat pump operation during the daytime, the temperature $T_{STn}$ drops with time, and the mode changes from A to C.

### 3.3. Various conditions

Connection, boundary, control, ambient, and initial conditions are considered as the other conditions to be satisfied. At the connection points among the CO$_2$ heat pump, storage tank, and mixing valves, connection conditions are taken into account to equalize the values of the corresponding variables. The feed water temperature as well as the mass flow rate and temperature of hot water supplied to the tapping site are given as boundary conditions. The outlet water temperature during heat pump operation is given as a control condition. The air temperature is given as an ambient condition. The temperature of water in the storage tank is given as an initial condition.

### 3.4. Numerical solution

The aforementioned system modeling for the performance analysis by numerical simulation is conducted by a building block approach: The component models for the CO$_2$ heat pump, storage tank, and mixing valves as well as the substance model for water are defined independently; The system model is composed of the component and substance models as well as the connection, boundary, control, ambient, and initial conditions. The equations for the CO$_2$ heat pump and mixing valves are static, while
Those for the storage tank are dynamic. Therefore, the modeling results in a set of nonlinear differential algebraic equations as follows:

\[
\begin{align*}
  f(x(t), \dot{x}(t), y(t), t) &= 0 \\
  x(t_0) &= x_0
\end{align*}
\]

where \( f \) is the vector for equations, \( x \) is the vector for variables with their derivatives, \( \dot{x} \) is the derivative of \( x \), \( y \) is the vector for variables without their derivatives, \( t_0 \) is the initial time, and \( x_0 \) is the initial values of \( x \). This set of equations is solved numerically by a hierarchical combination of the Runge-Kutta and Newton-Raphson methods.

4. Experiments for storage tank

4.1. Experimental setup

The experiments using a storage tank model in modes A and C for hot water supply are conducted to investigate the validity of the one-dimensional model for the storage tank proposed in subsection 3.1.2. The schematic of the experimental setup is shown in Fig. 6. Tank 1 is the storage tank model made of acryl, and its inside diameter and height are 0.38 and 1.05 m, respectively. The overall heat transfer coefficient of the tank wall without insulation is estimated at 5.7 W/(m\(^2\)·°C) based on an experiment conducted in advance of the ones in modes A and C in this paper, where tank 1 full of hot water with an initial uniform temperature distribution is left without water flow for several hours, and temperature drop with time is measured.

Tank 1 has inlet and outlet ports on both the top and bottom, and it also has 10 outlet ports on the side with a span of 0.10 m. The inlet port on the bottom, and the 1st
and 5th outlet ports from the top on the side are used for the experiments. City water is supplied to the inlet port on the bottom through tank 2, although the electric heater installed for other purposes is not used in the experiments. Tank 3 is used for water exhaust in place of hot water supply.

A series of type-T thermocouples with a measurement accuracy of ±1.0 °C is placed vertically in tank 1 to measure water temperatures at 10 points. It is close to the inside wall of the tank apart from the outlet ports on the side by a circumferential angle of 90 °. The thermocouples are placed along the series with a span of 0.10 m. A type-K thermocouple with a measurement accuracy of ±2.5 °C is inserted into each port to measure water temperature there. The pump has a constant rotational speed. Thus, the flow rates of water extracted are determined by the performance of the pump and the pressure losses of pipes and valves through which water flows. The valve and mass flow sensor are installed to set the flow rate of water which flows from tank 2 to tank 1. The mass flow sensor has a measurement accuracy within ±3.0 % of the full scale volumetric flow rate 0.067 L/s, or within ±0.0020 L/s, while it has a reproducibility accuracy within ±0.5 % of the same full scale volumetric flow rate, or within ±0.00033 L/s. Although the measurement accuracy is not necessarily high, the reproducibility accuracy is high. As stated later, simulated results coincide very well with experimental ones, which indicates that the measurement accuracy seems to be also high. The ambient air temperature is measured to assess the heat loss from the tank surface.

The initial temperature distribution with the area of tepid water in tank 1 is realized by pouring water into tank 1 with its temperature elevated stepwise from the level for city water to that for hot water supply.
4.2. Experiment for mode A

In the experiment for mode A, water is extracted from only the 5th outlet port on the side, and water is supplied from the inlet port on the bottom. The pump is operated and water is extracted for 1200 s from the initial time. The volumetric flow rate of water during water extraction is adjusted by the valve and mass flow sensor to be set at 0.013 L/s. Tank 1 is left without water flow for 8800 s after that.

A numerical simulation is also conducted under the same conditions with those for the experiment. The number of control volumes for the storage tank model is set at 200, and the sampling time interval for the Runge-Kutta method is set at 10 and 180 s for the cases with and without water flow, respectively.

Figure 7 compares experimental and simulated results in terms of the hourly change in the temperature distribution in tank 1. Figures (a) and (b) correspond to the times with and without water flow during and after water extraction, respectively. During the time with water flow, the temperature distribution above the 5th outlet port hardly changes, while that below the port changes due to water flow, and the temperature gradient near the port increases drastically. The simulated results coincide very well with the experimental ones. The three-dimensional effect due to convection near the port seems to be very small. During the time without water flow, the temperature above the port drops gradually due to heat loss, and the temperature gradient near the port decreases drastically due to heat conduction. On the other hand, the temperature distribution below the port hardly changes. This is because both heat conduction and loss are small. The temperatures near the top by the experiment become slightly lower than those by the numerical simulation. This difference seems
to arise because of a three-dimensional effect by heat loss from the top. Except near
the top, the simulated results coincide well with the experimental ones. In an actual
storage tank with insulation, the three-dimensional effect by heat loss from the top
seems to be much smaller.

4.3. Experiment for mode C

In the experiment for mode C, water is extracted from both the 1st and 5th outlet
ports on the side, and water is supplied from the inlet port on the bottom. The pump is
operated and water is extracted for 3000 s from the initial time. The total volumetric
flow rate of water during water extraction is adjusted by the valve and mass flow sensor
to be set at 0.010 L/s. In addition to the total volumetric flow rate, the temperatures at
the outlet ports and their juncture are used to estimate the volumetric flow rates of water
extracted from the 1st and 5th outlet ports on the side. They are estimated at 0.0026
and 0.0074 L/s, respectively. Water is extracted only from the side in mode A, while
water is extracted from the top and side in mode C. Thus, the pressure loss in mode C
is larger than that in mode A, and the water flow rate in mode C is smaller than that in
Mode A. Since the water flow rate from side in mode C is about a half of that in mode
A, the duration in mode C is longer than double of that in mode A. Tank 1 is left
without water flow for 7000 s after that.

A numerical simulation is also conducted under the same conditions with those for
the experiment. The number of control volumes for the storage tank model and the
sampling time interval for the Runge-Kutta method are set as aforementioned in the
numerical simulation for mode A.

Figure 8 compare experimental and simulated results in terms of the hourly change
in the temperature distribution in tank 1. Figures (a) and (b) correspond to the times with and without water flow during and after water extraction, respectively. During the time with water flow, both the temperature distributions above and below the 5th outlet port change due to water flow, and the change in the former is small and that in the latter is large. This is because the flow rate above the port is smaller than that below the port. In addition, the temperature gradient near the port increases. The simulated results coincide well with the experimental ones. A three-dimensional effect due to convection near the port seems to be very small. The temperatures near the bottom by the experiment become slightly higher than those by the numerical simulation. This difference seems to arise because of a three-dimensional effect by heat invasion from the bottom, since the ambient air temperature is higher than city water temperature. During the time without water flow, the temperature above the port drops gradually due to heat loss, and the temperature gradient near the port decreases due to heat conduction. On the other hand, the temperature distribution below the port hardly changes. This is because both heat conduction and loss are small. The temperatures near the top by the experiment become slightly lower than those by the numerical simulation as aforementioned in the experiment for mode A. Except near the top and bottom, the simulated results coincide well with the experimental ones. In an actual storage tank with insulation, the three-dimensional effect by heat loss and invasion from the top and bottom, respectively, seems to be much smaller.

5. Numerical study

5.1. Conditions
The performance of the revised system is evaluated under an hourly change in a standardized hot water demand which is repeated periodically with a period of 24 hours. The performance of the conventional system is also evaluated to compare those of both the systems. The numerical simulation is conducted for eight days, because it takes at least several days until a periodic steady state appears. The following are the calculation conditions used in the numerical study:

Table 1 shows the specifications of the CO₂ heat pump water heating system. The rated heat output of the CO₂ heat pump is set at 4.5 kW. The values of the coefficients of the quadratic functions for the power consumption and heat pump COP have been estimated based on measured data for an existing system [34]. As an example, Fig. 9 shows measured values and approximate functions for the power consumption, heat pump COP, and their resultant heat output of the CO₂ heat pump in relation to the inlet water temperature for the air and outlet water temperatures of 16 and 85 °C, respectively. Here, each value is relative to its rated one for the air and inlet/outlet water temperatures of 16, 17, and 65 °C, respectively. The reason for using relative values for the performance is as follows: As aforementioned, the experiment for the CO₂ heat pump was conducted in the previous paper [34], and its performance obtained by the experiment is still used in this paper. However, the performance of CO₂ heat pumps has been enhanced dramatically during this decade because of the technological development for compressors, gas coolers, etc. Thus, absolute values for the performance obtained by the experiment are not appropriate, because they may mislead one to the performance of CO₂ heat pumps available commercially currently. The volume of the storage tank is set at 370 L. The value of the overall heat transfer coefficient has also been estimated based on measured data for an existing system [30,
Water is extracted from the 130th control volume from the top of the storage tank, i.e., \( n = 130 \). The thresholds \( T^+ \) and \( T^- \) for switching the modes for hot water supply are set at the temperature of hot water demand plus and minus 5 °C, respectively.

The mid-season is selected, and the corresponding air and feed water temperatures are set at 16 and 17 °C, respectively. The numerical simulation can treat hourly changes in the air and feed water temperatures. However, these values were prescribed as the ambient conditions in mid-season originally by the Japan Refrigeration and Air Conditioning Industry Association [35], and afterwards by the Japanese Standards Association [36]. Thus, these values are assumed to be constant throughout the days. The hourly changes in the flow rate and temperature of the standardized hot water demand are given as shown in Fig. 10, which was also prescribed by the Japan Refrigeration and Air Conditioning Industry Association [35]. Here, the height of each vertical line means the flow rate, as indicated. The temperature is shown above each vertical line. In addition, the thickness of each vertical line means the duration. The heat for the daily hot water demand is 46.15 MJ/d for the aforementioned feed water temperature in mid-season.

The system is assumed to be operated in the charging and tapping modes independently during the nighttime and daytime, respectively. In the charging mode, the outlet water temperature of the CO\(_2\) heat pump is set at 85 °C. In addition, the CO\(_2\) heat pump is started up at 2:00 and is shut down with a shutdown condition satisfied at an appropriate time before 7:00. The shutdown condition is that the inlet water temperature of the CO\(_2\) heat pump attains a specified value, which is changed from 30 to 50 °C by 5 °C to investigate its influence on the system performance. As the initial condition, the temperature of water in the storage tank is set at 17 °C at 0:00 on the 1st
The number of control volumes for the storage tank is set at 200, and the sampling
time interval for the Runge-Kutta method is set at 10 and 180 s for the cases with and
without water flow, respectively.

5.2. Results and discussion

First, temperature distributions in the storage tank obtained for the conventional
and revised systems are compared. Figures 11 (a) and (b) show the changes in the
temperature distribution in the storage tank in the charging and tapping modes,
respectively, obtained on the 8th day under the condition that the inlet water
temperature for heat pump shutdown is 50 °C, as an example, for the conventional
system, while Figs. 12 (a) and (b) show those for the revised system. In the
conventional system, since the outlet water with a temperature of 85 °C enters the top of
the storage tank and the inlet water temperature rises up to 50 °C in the charging mode,
the temperature gradient changes with the vertical position, but its change is gradual.
This is because the area of tepid water is large. The temperature gradient becomes
small gradually in the tapping mode, and the area of tepid water expands up to about
two thirds of the height of the storage tank. In the revised system, on the other hand,
the temperature gradient changes with the vertical position, and its change is marked in
the charging mode, which is different from that in the conventional system. This is
because the area of tepid water is small. The temperature distribution changes
complicatedly with time because of water extraction in the tapping mode. Before
20:00, the temperature distribution at the position higher than that for water extraction
hardly changes with time, while the temperature distribution at the position lower than
that for water extraction changes significantly. However, after 20:00, the temperature distribution at the position higher than that for water extraction also changes with time since the volume of stored hot water decreases, and the temperature gradient increases because of water extraction. As a result, the area of tepid water is small, and expands up to only about one fourth of the height of the storage tank. In addition, the temperature drop at the highest part of the storage tank is small.

Second, the changes in the heat pump COP obtained for the conventional and revised systems are compared. Figures 13 (a) and (b) show the changes in the ratio of heat pump COP obtained on the 8th day under the condition that the inlet water temperature for heat pump shutdown is 50 and 30 °C, respectively, as examples, for the conventional and revised systems. The ratio of heat pump COP is relative to its value under the air and inlet/outlet water temperatures of 16, 17, and 85 °C, respectively. In the charging mode, the temperature in the 200th control volume is equal to the inlet water temperature to the CO₂ heat pump. As shown in Figs. 11 (a) and 12 (a), the hourly changes in the temperature distribution are different between the conventional and revised systems. Thus, the hourly changes in the inlet water temperature are also different between the conventional and revised systems. In addition, as shown in Fig. 9, the heat pump COP decreases with an increase in the inlet water temperature. These cause the difference in the hourly changes in the heat pump COP between the conventional and revised systems, as shown in Fig. 13 (a). Under the condition that the inlet water temperature for heat pump shutdown is 50 °C, in the conventional system, since the area of tepid water is large, the time when the inlet water temperature rises and the heat pump COP decreases correspondingly is long. In the revised system, on the other hand, since the area of tepid water is small, the time when the inlet water
temperature rises and the heat pump COP decreases correspondingly is short. In addition, the time when the heat output of the CO₂ heat pump decreases is short, and resultantlly the time when the CO₂ heat pump is operated is short. Under the condition that the inlet water temperature for heat pump shutdown is 30 °C, the time when the heat pump COP decreases is short, and the decrease in the heat pump COP is small. As a result, the difference in the change in the heat pump COP between the conventional and revised systems is small. Therefore, the effect of water extraction on the increase in the heat pump COP under the condition that the inlet water temperature for heat pump shutdown is 50 °C is larger than that under the condition that the inlet water temperature for heat pump shutdown is 30 °C.

Next, the volumes of stored and unused hot water obtained for the conventional and revised systems are compared. Figures 14 (a) and (b) show the temperature distributions in the storage tank at 7:00 soon after the charging mode and 24:00 soon after the tapping mode obtained on the 8th day under the condition that the inlet water temperature for heat pump shutdown is 50 and 30 °C, respectively, as examples, for the conventional and revised systems. Under the condition that the inlet water temperature for heat pump shutdown is 30 °C, in the conventional system, since the inlet water temperature rises up to only 30 °C, the temperature gradient is small in all the temperature ranges. As a result, the difference in the temperature distributions for stored and unused hot water between the conventional and revised systems is large. Under the condition that the inlet water temperature for heat pump shutdown is 50 °C, on the other hand, in the conventional system, since the inlet water temperature rises up to 50 °C, the temperature gradient is large in lower temperature ranges and is small in higher temperature ranges. As a result, the difference in the temperature distributions
for stored and unused hot water between the conventional and revised systems is small. Therefore, the effect of water extraction on the increases in the volumes of stored and unused hot water under the condition that the inlet water temperature for heat pump shutdown is 30 °C is larger than that under the condition that the inlet water temperature for heat pump shutdown is 50 °C.

Finally, the influence of the inlet water temperature for heat pump shutdown on the following system performance values are investigated: The heat pump COP is evaluated as the ratio of the daily heat output to the daily power consumption in the charging mode; The storage efficiency is evaluated as the ratio of the daily hot water demand in the tapping mode to the daily heat output in the charging mode; The system efficiency is evaluated as the ratio of the daily hot water demand in the tapping mode to the daily power consumption in the charging mode, and thus it is equal to the product of the heat pump COP and storage efficiency; The volumes of stored and unused hot water are defined as the volumes of hot water which can be used for hot water supply at 7:00 soon after the charging mode and at 24:00 soon after the tapping mode, respectively. Here, the volume of hot water is defined as the one with a temperature of 42 °C obtained by mixing the hot water with temperatures higher than 42 °C and the feed water. In understanding the obtained results, the following should be noted: The daily hot water demand is common to the conventional and revised systems, and it is equal to the daily heat output of the storage tank; The daily heat output of the CO2 heat pump is equal to the daily heat input of the storage tank, which is equal to the sum of the daily heat output and heat loss of the storage tank; The difference in the characteristics of the storage tank between the conventional and revised systems causes the difference in the daily power consumption and heat output of
the CO₂ heat pump as well as the daily heat input and heat loss of the storage tank between the conventional and revised systems; The system efficiency is in inverse proportion to the daily power consumption of the CO₂ heat pump.

Figure 15 (a) shows the heat pump COP, and the storage and system efficiencies obtained on the 8th day for the conventional and revised systems in relation to the inlet water temperature for heat pump shutdown. All the values are relative to those for the conventional system under the condition that the inlet water temperature for heat pump shutdown is 50 °C. With an increase in the inlet water temperature for heat pump shutdown, the heat pump COP decreases in both the systems from the results shown in Fig. 13. In addition, the storage efficiency also decreases in both the systems, because the average temperature in the storage tank rises from the results shown in Fig. 14. As a result, the system efficiency also decreases in both the systems. On the other hand, the difference in the heat pump COP between both the systems becomes large from the results shown in Fig. 13. In addition, the difference in the storage efficiency between both the systems becomes small from the results shown in Fig. 14. As the result of these differences, the difference in the system efficiency between both the systems becomes large, and the effect of water extraction on the increase in the system efficiency also becomes large. It should be noted that the difference in the system efficiency becomes negative under the condition that the inlet water temperature for heat pump shutdown is 30 °C. Figure 15 (b) shows the volumes of stored and unused hot water obtained on the 8th day for the conventional and revised systems in relation to the inlet water temperature for heat pump shutdown. All the values are relative to those for the conventional system under the condition that the inlet water temperature for heat pump shutdown is 50 °C. With an increase in the inlet water temperature for
heat pump shutdown, the volumes of stored and unused hot water in both the systems increase from the results shown in Fig. 14. However, the differences in the volumes of stored and unused hot water between both the systems become small from the results shown in Fig. 14, and the effect of water extraction on the increase in the volume of unused hot water also becomes small.

Among the aforementioned system performance values, the system efficiency and the volume of unused hot water are the most important, because the former and latter are criteria for energy saving and reliable hot water supply, respectively. Figure 16 shows the relationships between these criteria in both the systems with the inlet water temperature for heat pump shutdown as a parameter. There exist trade-off relationships between the criteria in both the systems. The water extraction moves the trade-off relationship in the upper and right direction, which means the performance enhancement in terms of these criteria. For example, under the condition that the inlet water temperature for heat pump shutdown is 50 °C, the water extraction increases the system efficiency by more than 10 % with the volume of unused hot water unchanged. In addition, under the condition that the inlet water temperature for shutdown is 40 °C, the water extraction increases the volume of unused hot water by more than 20 % with the system efficiency unchanged.

6. Conclusions

The performance of a CO₂ heat pump water heating system with extracting tepid water from the side of the storage tank has been analyzed by numerical simulation. The simulation model developed previously for a conventional CO₂ heat pump water
heating system has been extended for this purpose by modifying the model for the storage tank and investigating its validity through experiments. A performance analysis for the conventional and revised systems has been conducted using a standardized hot water demand, and their performances are compared in terms of the temperature distribution in the storage tank and system performance values. Through this analysis, the effect of water extraction on the performance enhancement has been investigated. The following main results have been obtained:

1) The temperature distributions in the storage tank with water extraction obtained by the numerical simulation and experiment coincide well with each other, and the validity of the modified model for the storage tank is verified.

2) The water extraction makes the temperature gradient in the storage tank large, and resultantly makes the area of tepid water small.

3) The water extraction increases the heat pump COP but decreases the storage efficiency. As a result, the water extraction increases the system efficiency except under the condition that the inlet water temperature for heat pump shutdown is low. In addition, the water extraction increases the volumes of stored and unused hot water.

4) With an increase in the inlet water temperature for heat pump shutdown, the effect of water extraction on the heat pump COP and the system efficiency increases, but that on the volumes of stored and unused hot water decreases.

5) The water extraction moves the trade-off relationship between the system efficiency and the volume of unused hot water in the upper and right direction, and enhances the system performance significantly.
Nomenclature

\( A \): cylindrical surface area of hot water storage tank \([\text{m}^2]\)

\( c \): specific heat of water \([\text{J/(kg \cdot ^\circ \text{C})}]\)

\( f \): vector for equations

\( H \): height of hot water storage tank \([\text{m}]\)

\( J \): number of control volumes

\( \dot{m} \): mass flow rate \([\text{kg/s}]\)

\( \dot{Q} \): heat flow rate \([\text{W}]\)

\( S \): horizontal sectional area of hot water storage tank \([\text{m}^2]\)

\( T \): temperature \([^\circ \text{C}]\)

\( T \): upper threshold of \( T \)[\(^{\circ} \text{C}\)]

\( \underline{T} \): lower threshold of \( T \)[\(^{\circ} \text{C}\)]

\( t \): time \([\text{s}]\)

\( U \): overall heat transfer coefficient \([\text{W/(m}^2 \cdot ^\circ \text{C})}]\)

\( V \): volume of hot water storage tank \([\text{m}^3]\)

\( \dot{W} \): power consumption \([\text{W}]\)

\( x \): vector for variables with their derivatives

\( \dot{x} \): derivative of \( x \)

\( y \): vector for variables without their derivatives

\( \alpha, \beta, \gamma \): coefficients of quadratic functions

\( \eta \): coefficient of performance (COP)

\( \lambda \): heat conductivity of water \([\text{W/(m} \cdot ^\circ \text{C})}]\)

\( \rho \): density of water \([\text{kg/m}^3]\)
**Subscripts**

HP : CO$_2$ heat pump  
  i : inlet  
  j : index for control volumes  

MV : mixing valve  
  n : control volume for water extraction  
  o : outlet  

ST : hot water storage tank  
  0 : initial state

**Superscripts**

a : air  
  b : bottom  
  e : water extraction  
  t : top

**References**


Captions for tables and figures

Table 1 Specifications of CO₂ heat pump water heating system

Fig. 1 Configuration of CO₂ heat pump water heating system with water extraction

Fig. 2 Model for CO₂ heat pump

Fig. 3 Model for mixing valve

Fig. 4 Model for storage tank

Fig. 5 Modes for hot water supply

(a) Mode A
(b) Mode B
(c) Mode C

Fig. 6 Schematic of experimental setup

Fig. 7 Comparison between experiment and numerical simulation in terms of hourly change in temperature distribution in storage tank model in mode A

(a) During water extraction
(b) After water extraction

Fig. 8 Comparison between experiment and numerical simulation in terms of hourly change in temperature distribution in storage tank model in mode C

(a) During water extraction
(b) After water extraction

Fig. 9 Performance characteristics of CO₂ heat pump

Fig. 10 Hourly change in standardized hot water demand

Fig. 11 Changes in temperature distribution in storage tank for conventional system

(a) Charging mode
(b) Tapping mode
Fig. 12 Changes in temperature distribution in storage tank for revised system
   (a) Charging mode
   (b) Tapping mode

Fig. 13 Changes in heat pump COP
   (a) Inlet water temperature for heat pump shutdown of 50 °C
   (b) Inlet water temperature for heat pump shutdown of 30 °C

Fig. 14 Temperature distributions in storage tank for stored and unused hot water
   (a) Inlet water temperature for heat pump shutdown of 50 °C
   (b) Inlet water temperature for heat pump shutdown of 30 °C

Fig. 15 Influence of inlet water temperature for heat pump shutdown on system performance
   (a) Heat pump COP, and storage and system efficiencies
   (b) Volumes of stored and unused hot water

Fig. 16 Trade-off relationship between system efficiency and volume of unused hot water
<table>
<thead>
<tr>
<th>Equipment</th>
<th>Specification</th>
<th>Value</th>
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<td>CO₂ heat pump</td>
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<td>transfer coefficient</td>
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</table>
Fig. 1  Configuration of CO₂ heat pump water heating system with water extraction
Fig. 2 Model for CO$_2$ heat pump
Fig. 3  Model for mixing valve
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(a) Mode A  (b) Mode B  (c) Mode C
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Fig. 11 Changes in temperature distribution in storage tank for conventional system

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(a) Inlet water temperature for heat pump shutdown of 50 °C

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(a) Inlet water temperature for heat pump shutdown of 50 °C

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Fig. 14  Temperature distributions in storage tank for stored and unused hot water
(a) Heat pump COP, and storage and system efficiencies

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Fig. 15 Influence of inlet water temperature for heat pump shutdown on system performance
Fig. 16  Trade-off relationship between system efficiency and volume of unused hot water

Inlet water temperature for heat pump shutdown °C

Ratio of volume of unused hot water

Ratio of system efficiency

Δ Conventional
• Revised