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A revised method for robust optimal design of energy supply systems based on minimax regret criterion

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Abstract

A robust optimal design method of energy supply systems under uncertain energy demands is revised so that it can be applied to systems with complex configurations and large numbers of periods for variations in energy demands. First, a robust optimal design problem is described by using the minimax regret criterion and considering the hierarchical relationship among design variables, energy demands, and operation variables, which is followed by a solution method. Especially, a novel solution method is proposed for efficiently evaluating an upper bound for the optimal value of the maximum regret. Then, a method of comparing two energy supply systems under uncertain energy demands is proposed by utilizing a part of the revised robust optimal design method. Finally, through a case study on a gas turbine cogeneration system for district energy supply, the validity and effectiveness of the revised optimal design

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method and features of the robust optimal design are clarified. In addition, the gas turbine cogeneration system is compared with a conventional energy supply system using the proposed comparison method.

Keywords: Energy supply systems, Uncertainty, Robust optimal design, Minimax regret criterion, Multilevel linear programming, Mixed-integer linear programming

1. Introduction

In designing energy supply systems, the estimation of energy demands is an important work. This is because designers are requested to rationally determine what types, numbers, and capacities of equipment should be installed in consideration of equipment operational strategies corresponding to seasonal and hourly variations in the estimated energy demands, which significantly affects system economic and energy saving characteristics. Related with this issue, many optimal design methods have been proposed. With the aid of these methods, equipment types, numbers, and capacities as well as utility contract demands can be determined to minimize the annual total cost and primary energy consumption in consideration of equipment operational strategies for average energy demands estimated on several representative days, and to satisfy peak energy demands estimated in summer and winter. In most of the methods, however, it is assumed that the estimated energy demands are certain. Therefore, the methods can be useful tools for designing energy supply systems if energy demands can be estimated precisely. However, many conditions under which energy demands are estimated have some uncertainty at the design stage, which makes it impossible to estimate energy demands precisely. If the design is conducted in consideration of the estimated energy demands, the economic and energy saving characteristics expected at the design stage may not be attained and the deficit in energy supply may occur at the operation stage. This is because the energy demands which occur at the operation stage differ from those estimated at the design stage. Therefore, designers should consider that energy demands have some uncertainty, evaluate the robustness in economic and energy saving characteristics against the uncertainty, and design the systems rationally in consideration of the robustness.

Verderame et al. reviewed many papers on planning and scheduling under uncertainty in multiple sectors, and reviewed some papers on energy planning [1]. Zeng et al. also reviewed many papers on optimization of energy systems planning under uncertainty [2]. In these review papers, the approaches adopted for optimization of energy systems planning were categorized into three ones: stochastic, fuzzy, and interval programming. First, the stochastic approach includes the following papers: Goumas et al. presented computational methods for planning and evaluating geothermal projects under uncertain conditions using a stochastic analysis approach [3]; Gamou et al. proposed an optimal unit sizing method of cogeneration systems in consideration of uncertain energy demands as continuous random variables [4]; Arun et al. proposed an optimum sizing method of photovoltaic battery systems by incorporating uncertainty related with solar insolation [5]; Krukanont and Tezuka presented a near-term analysis of capacity expansion of energy systems under various uncertainties based on two-stage stochastic programming [6]; Krey et al. investigated the effects of stochastic energy prices on long-term energy-economic scenarios [7]. Second, the fuzzy approach includes the following papers: Mamlook et al. used the fuzzy sets programming to perform evaluation of different solar systems [8]; Mavrotas et al. presented a fuzzy linear programming model for energy planning in buildings under uncertainty associated fuel costs [9]; They also provided a fuzzy mathematical programming framework for energy planning in buildings under uncertainty associated with load demand [10]; Sadeghi and Hosseini demonstrated the application of the fuzzy linear programming to energy supply planning with uncertainties associated with investment costs [11]; Mazur developed an approach to thermoeconomic optimization of energy-transforming systems using a fuzzy nonlinear programming [12]; Bitar et al. examined expansion planning of isolated electrical systems under uncertainty associated with energy consumption using fuzzy multi-objective mathematical programming [13]. Finally, the interval approach includes the following papers: Lin and Huang introduced an interval-parameter linear programming approach to energy systems planning [14]; Zhu et al. developed an interval-parameter full-infinite linear programming approach to energy systems planning under multiple uncertainties with crisp and functional intervals [15]; They also proposed an interval-parameter full-infinite mixed-integer programming approach to energy systems planning under uncertainties with functional intervals [16]; Dong et al. developed an interval-parameter minimax regret programming method for power management systems planning under uncertainty [17]. However, it is difficult for designers to specify stochastic distribution and fuzzy membership functions for uncertain parameters in the first and second approaches. From the viewpoint of practical applications, it is much more meaningful for designers to specify fluctuation intervals for uncertain parameters in the third approach. However, all the methods in the third approach do not consider the difference between design and operation variables whose values are determined at the design and operation stages, respectively. In addition, most of the methods in the third approach cannot produce a unique optimal solution but an interval one, which cannot support the decision-making for design.

Yokoyama and Ito proposed a robust optimal design method of energy supply systems in consideration of the economic robustness against the uncertainty in energy demands using the minimax regret criterion [18]. This method not only treats energy demands as uncertain parameters with intervals, but also considers the hierarchical relationship among design variables, energy demands, and operation variables, and produces a unique optimal solution. They extended this method to the multiobjective robust optimal design in consideration of the economic and energy saving robustness against the uncertainty in energy demands [19]. They also extended it to the robust optimal design based on the relative robustness criterion [20]. Furthermore, they extended it to the robust optimal design for multistage expansion planning [21]. In these papers, the robust optimal design method was applied to a gas turbine cogeneration system. However, the system is composed only of a gas turbine generator, a waste heat recovery boiler, a gas-fired auxiliary boiler, and a device for receiving electricity, and its configuration is fairly simple. In addition, the numbers of periods for seasonal and hourly variations in energy demands are also fairly small. Namely, the method can be applied limitedly to energy supply systems with fairly simple configurations and fairly small numbers of periods. This is because the complexity of the robust optimal design problems depends on the numbers of inequality constraints including upper and lower limits for operation variables as well as the numbers of periods. This dependence should be removed or relaxed so that the method can be applied to energy supply systems with more complex configurations and larger numbers of periods.

In addition, it is often necessary to compare multiple energy supply systems in terms of economic and energy saving characteristics. Under certain energy demands, it is very easy to compare them by evaluating economic and energy saving characteristics of each system and their differences among all the systems certainly. Under uncertain energy demands, however, it is not so easy to compare them, because economic and energy saving characteristics of each system, and resultantly their differences among all the systems depend on energy demands. Therefore, it is necessary to establish a method of comparing multiple energy supply systems in terms of economic and energy saving characteristics under uncertain energy demands.

In this paper, the robust optimal design method proposed previously is revised so that it can be applied to energy supply systems with more complex configurations and larger numbers of periods. First, a robust optimal design problem is described by using the minimax regret criterion and considering the hierarchical relationship among design variables, energy demands, and operation variables, which is followed by a solution method. Here, a novel solution method is proposed for efficiently evaluating an upper bound for the optimal value of the maximum regret. Then, a method of comparing two energy supply systems under uncertain energy demands is proposed by utilizing a part of the revised robust optimal design method. Finally, a case study on a gas turbine cogeneration system for district energy supply is conducted to investigate the validity and effectiveness of the revised optimal design method and features of the robust optimal design. In addition, the gas turbine cogeneration system is compared with a conventional energy supply system using the proposed comparison method.

2. Robust optimal design based on minimax regret criterion

2.1. Basic concept

In designing an energy supply system under uncertain energy demands, two criteria should be considered: flexibility and robustness [22]. The former means the feasibility in energy supply for all the possible values of uncertain energy demands, and is related with constraints. The latter means the sensitivity of criteria for all the possible values of uncertain energy demands, and is related with objective functions. In this paper, an optimal design method is proposed by which the robustness is improved while the flexibility is secured for all the possible values of uncertain energy demands.

As a criterion for the robustness, the minimax regret criterion is adopted here [23]. Figure 1 shows a basic concept of the robust optimal design based on the minimax regret criterion. The regret is defined as the difference in an objective function between non-optimal and optimal designs for some values of uncertain parameters. The minimax regret criterion means that the values of decision variables are determined to minimize the maximum regret for all the possible values of uncertain parameters. Therefore, if this criterion is adopted, the difference in the objective function between the corresponding design and the optimal one can be small for all the possible values of uncertain parameters.

2.2. Summary of formulation and solution of robust optimal design problem

Following the aforementioned basic concept, a robust optimal design problem for an energy supply system is described as follows: equipment capacities and utility contract demands $\boldsymbol{x}$ as well as energy flow rates $\boldsymbol{z}$ are determined to minimize the maximum regret in the annual total cost f and to satisfy all the constraints for all the possible values of uncertain energy demands $\boldsymbol{y}$. Here, it is assumed that all the objective function and constraints are expressed by linear equations with respect to $\boldsymbol{x}$, $\boldsymbol{y}$, and $\boldsymbol{z}$. In addition, it should be noted that although the design variables $\boldsymbol{x}$ must be determined at the design stage when energy demands are uncertain, the operation variables $\boldsymbol{z}$ can be adjusted for energy demands which become certain at the operation stage. Therefore, there is a hierarchical relationship among the design variables, energy demands, and operation variables as shown in Fig. 2.

From appendix A.1, the robust optimal design problem is formulated as

$$\begin{aligned}\min_{\boldsymbol{x} \in X} \Bigg[\max_{\boldsymbol{y} \in Y} \Bigg\{ & \min_{\boldsymbol{z} \in Z} f(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \\ & - \min_{\boldsymbol{x}' \in X} \Bigg(\min_{\boldsymbol{z}' \in Z'} f(\boldsymbol{x}', \boldsymbol{y}, \boldsymbol{z}') \\ & \qquad + W \max_{\boldsymbol{y}'' \in Y} \min_{\boldsymbol{z}'' \in Z''} p(\boldsymbol{x}', \boldsymbol{y}'', \boldsymbol{z}'') \Bigg) \Bigg\} \\ & + W \max_{\boldsymbol{y}''' \in Y} \min_{\boldsymbol{z}''' \in Z'''} p(\boldsymbol{x}, \boldsymbol{y}''', \boldsymbol{z}''') \Bigg] \end{aligned} \tag{1}$$

where X is the set for all the possible values of $\boldsymbol{x}$, Y is the set for all the possible values of $\boldsymbol{y}$, Z, Z', Z'', and Z''' are the sets for all the possible values of $\boldsymbol{z}$, $\boldsymbol{z}'$, $\boldsymbol{z}''$, and $\boldsymbol{z}'''$, and depend on the combinations $(\boldsymbol{x}, \boldsymbol{y})$, $(\boldsymbol{x}', \boldsymbol{y})$, $(\boldsymbol{x}', \boldsymbol{y}'')$, and $(\boldsymbol{x}, \boldsymbol{y}''')$, respectively, p is the objective function for the infeasibility in energy supply, and W is the coefficient for penalty terms, and should be given a value large sufficiently.

The optimization problem of Eq. (1) includes the operations of minimization and maximization hierarchically, and is formulated as a kind of multilevel linear programming problem [24]. This problem is solved using the following procedure.

First, the operation of minimization with respect to $\boldsymbol{x}'$ is moved forward to reformulate Eq. (1) as

$$
\begin{aligned}
\min_{\boldsymbol{x}\in X}\Bigg[\max_{\boldsymbol{y}\in Y}\max_{\boldsymbol{x}'\in X}\Bigg\{&\min_{\boldsymbol{z}\in Z} f(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})\\
&-\Bigg(\min_{\boldsymbol{z}'\in Z'} f(\boldsymbol{x}',\boldsymbol{y},\boldsymbol{z}')\\
&\quad+W\max_{\boldsymbol{y}''\in Y}\min_{\boldsymbol{z}''\in Z''} p(\boldsymbol{x}',\boldsymbol{y}'',\boldsymbol{z}'')\Bigg)\Bigg\}\\
&+W\max_{\boldsymbol{y}'''\in Y}\min_{\boldsymbol{z}'''\in Z'''} p(\boldsymbol{x},\boldsymbol{y}''',\boldsymbol{z}''')\Bigg]
\end{aligned}
\tag{2}
$$

Next, appropriate values of $\boldsymbol{x}$ and $\boldsymbol{y}''$ are assumed in Eq. (2), and the following optimization problem is considered:

$$
\begin{aligned}
\max_{\boldsymbol{y}\in Y}\max_{\boldsymbol{x}'\in X}\Bigg\{&\min_{\boldsymbol{z}\in Z} f(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})\\
&-\Bigg(\min_{\boldsymbol{z}'\in Z'} f(\boldsymbol{x}',\boldsymbol{y},\boldsymbol{z}')\\
&\quad+W\min_{\boldsymbol{z}''\in Z''} p(\boldsymbol{x}',\boldsymbol{y}'',\boldsymbol{z}'')\Bigg)\Bigg\}\\
&+W\max_{\boldsymbol{y}'''\in Y}\min_{\boldsymbol{z}'''\in Z'''} p(\boldsymbol{x},\boldsymbol{y}''',\boldsymbol{z}''')
\end{aligned}
\tag{3}
$$

The maximum regret corresponding to the optimal solution of this problem gives an upper bound for that of the original one of Eq. (2). The problem is solved using the conventional solution method proposed previously or the novel one proposed in this paper, which are described in appendix A.2.

On the other hand, the values of $\boldsymbol{y}$ and $\boldsymbol{x}'$ are assumed to be selected only from their combinations obtained by solving Eq. (3), and the following optimization problem is considered in place of Eq. (2):

$$
\begin{aligned}
\min_{\boldsymbol{x}\in X}\Bigg[\max_{(\boldsymbol{y},\boldsymbol{x}')\in A}\Bigg\{&\min_{\boldsymbol{z}\in Z} f(\boldsymbol{x},\boldsymbol{y},\boldsymbol{z})\\
&-\Bigg(\min_{\boldsymbol{z}'\in Z'} f(\boldsymbol{x}',\boldsymbol{y},\boldsymbol{z}')\\
&\quad+W\max_{\boldsymbol{y}''\in Y}\min_{\boldsymbol{z}''\in Z''} p(\boldsymbol{x}',\boldsymbol{y}'',\boldsymbol{z}'')\Bigg)\Bigg\}\\
&+W\max_{\boldsymbol{y}'''\in Y}\min_{\boldsymbol{z}'''\in Z'''} p(\boldsymbol{x},\boldsymbol{y}''',\boldsymbol{z}''')\Bigg]
\end{aligned}
\tag{4}
$$

where A is the set for combinations of values of $\boldsymbol{y}$ and $\boldsymbol{x}'$. The maximum regret corresponding to the optimal solution of this problem gives a lower bound for that of the original one of Eq. (2). The problem is solved using the solution method proposed previously, which is described in Appendix A.3.

Following the aforementioned procedure, the evaluation of upper and lower bounds for the optimal value of the maximum regret in Eq. (2) is repeated. If upper and lower bounds coincide with each other, it is judged that the optimal solution of Eq. (2) is obtained, and the repeat is stopped.

3. Comparison of energy supply systems

The robust optimal design method presented previously can determine the equipment capacities and utility contract demands, and can assess the maximum regret in the annual total cost for an energy supply system under uncertain energy demands. However, it cannot compare multiple energy supply systems under uncertain energy demands. A method of comparing two energy supply systems under uncertain energy demands is proposed here.

Equipment capacities and utility contract demands for two energy supply systems are assumed to be determined by the robust optimal design method, although they may be given a priori. The annual total cost is adopted as the criterion for the comparison, although any criterion may be adopted. The difference in the annual total cost between the two systems is evaluated. However, it varies under uncertain energy demands. Therefore, its maximum and minimum are evaluated for all the possible values of uncertain energy demands, and resultantly their interval is obtained.

The maximum and minimum of the difference in the annual total cost between systems A and B are expressed as follows:

$$\overline{\Delta f}_{\mathrm{AB}} = \max_{\boldsymbol{y} \in Y} \left(\min_{\boldsymbol{z}_{\mathrm{A}} \in Z_{\mathrm{A}}} f_{\mathrm{A}}(\boldsymbol{x}_{\mathrm{A}}, \boldsymbol{y}, \boldsymbol{z}_{\mathrm{A}}) - \min_{\boldsymbol{z}_{\mathrm{B}} \in Z_{\mathrm{B}}} f_{\mathrm{B}}(\boldsymbol{x}_{\mathrm{B}}, \boldsymbol{y}, \boldsymbol{z}_{\mathrm{B}}) \right) \tag{5}$$

and

$$\underline{\Delta f}_{\mathrm{AB}} = \min_{\boldsymbol{y} \in Y} \left(\min_{\boldsymbol{z}_{\mathrm{A}} \in Z_{\mathrm{A}}} f_{\mathrm{A}}(\boldsymbol{x}_{\mathrm{A}}, \boldsymbol{y}, \boldsymbol{z}_{\mathrm{A}}) - \min_{\boldsymbol{z}_{\mathrm{B}} \in Z_{\mathrm{B}}} f_{\mathrm{B}}(\boldsymbol{x}_{\mathrm{B}}, \boldsymbol{y}, \boldsymbol{z}_{\mathrm{B}}) \right) \tag{6}$$

respectively, where subscripts A and B are attached to the symbols for systems A and B, respectively. The penalty terms to secure the flexibility, or the feasibility in energy supply are not included in Eqs. (5) and (6). This is because equipment capacities and utility contract demands satisfy the flexibility when they are determined by the robust optimal design method, and they are assumed to satisfy the flexibility when they are given a priori. The interval between the minimum and maximum is obtained as [$\underline{\Delta f}_{\mathrm{AB}}$, $\overline{\Delta f}_{\mathrm{AB}}$].

The maximum and minimum can be evaluated by methods similar to that of evaluating an upper bound for the optimal value of the maximum regret presented previously. Equations (5) and (6) show that the corresponding optimization problems are bilevel linear programming ones. Therefore, based on appendix A.2, they are converted into one-level bilinear programming problems by considering dual maximization problems in place of primal minimization ones as follows:

$$\begin{aligned} \overline{\Delta f}_{\mathrm{AB}} &= \max_{\boldsymbol{y} \in Y} \left(\max_{\boldsymbol{\mu}_{\mathrm{A}} \in M_{\mathrm{A}}} g_{\mathrm{A}}(\boldsymbol{x}_{\mathrm{A}}, \boldsymbol{y}, \boldsymbol{\mu}_{\mathrm{A}}) - \min_{\boldsymbol{z}_{\mathrm{B}} \in Z_{\mathrm{B}}} f_{\mathrm{B}}(\boldsymbol{x}_{\mathrm{B}}, \boldsymbol{y}, \boldsymbol{z}_{\mathrm{B}}) \right) \\ &= \max_{\boldsymbol{y} \in Y,\, \boldsymbol{\mu}_{\mathrm{A}} \in M_{\mathrm{A}},\, \boldsymbol{z}_{\mathrm{B}} \in Z_{\mathrm{B}}} \left(g_{\mathrm{A}}(\boldsymbol{x}_{\mathrm{A}}, \boldsymbol{y}, \boldsymbol{\mu}_{\mathrm{A}}) - f_{\mathrm{B}}(\boldsymbol{x}_{\mathrm{B}}, \boldsymbol{y}, \boldsymbol{z}_{\mathrm{B}}) \right) \end{aligned} \tag{7}$$

$$\begin{aligned} \underline{\Delta f}_{\mathrm{AB}} &= \min_{\boldsymbol{y} \in Y} \left(\min_{\boldsymbol{z}_{\mathrm{A}} \in Z_{\mathrm{A}}} f_{\mathrm{A}}(\boldsymbol{x}_{\mathrm{A}}, \boldsymbol{y}, \boldsymbol{z}_{\mathrm{A}}) - \max_{\boldsymbol{\mu}_{\mathrm{B}} \in M_{\mathrm{B}}} g_{\mathrm{B}}(\boldsymbol{x}_{\mathrm{B}}, \boldsymbol{y}, \boldsymbol{\mu}_{\mathrm{B}}) \right) \\ &= \min_{\boldsymbol{y} \in Y,\, \boldsymbol{z}_{\mathrm{A}} \in Z_{\mathrm{A}},\, \boldsymbol{\mu}_{\mathrm{B}} \in M_{\mathrm{B}}} \left(f_{\mathrm{A}}(\boldsymbol{x}_{\mathrm{A}}, \boldsymbol{y}, \boldsymbol{z}_{\mathrm{A}}) - g_{\mathrm{B}}(\boldsymbol{x}_{\mathrm{B}}, \boldsymbol{y}, \boldsymbol{\mu}_{\mathrm{B}}) \right) \end{aligned} \tag{8}$$

where g is the objective function of the dual maximization problem converted from the primal minimization one with the objective function f, $\boldsymbol{\mu}$ is the Lagrange multiplier vector corresponding to the constraints of the primal minimization one, and M is the set for all the possible values of μ, as used in Eq. (A6) of appendix A.2. They are further converted into mixed-integer linear programming problems by applying the theorem on bilinear programming problems.

4. Case study

To investigate the validity and effectiveness of the revised robust optimal design method and features of the robust optimal design, a case study is conducted on a gas turbine cogeneration plant for district energy supply. Since there are relationships between supplies and demands of electricity and heat, it is impossible to determine equipment capacities simply using maxima of electricity and heat demands. Here, the design is conducted to minimize the maximum regret in the annual total cost, and features of the economic robust optimal design is investigated.

4.1. System configuration

A gas turbine cogeneration system for district energy supply shown in Fig. 3 is considered in this paper. This system is composed of a gas turbine generator (GT), a waste heat recovery boiler (BW), a gas-fired auxiliary boiler (BG), an electric compression refrigerator (RE), a steam absorption refrigerator (RS), a device for receiving electricity (EP), and a pump for supplying cold water (PC). Electricity is supplied to users by operating the gas turbine generator and purchasing electricity from an outside electric power company. Electricity is also used to drive the electric compression refrigerator, pump, and other auxiliary machinery in the system. Exhaust heat generated from the gas turbine is recovered in the form of steam by the waste heat recovery boiler, and is used for heat supply. An excess of exhaust heat is disposed of through an exhaust gas dumper. A shortage of steam is supplemented by the gas-fired auxiliary boiler. Cold water for space cooling is supplied by the electric compression and steam absorption refrigerators. Steam is used for space heating and hot water supply.

The symbols which are used as energy flow rates in the following formulation are included in Fig. 3.

4.2. Formulation of optimal design problem

Symbols used for the design are defined, and constraints and an objective function are formulated. To take account of seasonal and hourly variations in energy demands, a typical year is divided into multiple representative days, and further each day into several periods. A quantity corresponding to each period is identified by the subscript t. The set for all values of t is designated by T. The duration per year of each period is denoted by d_t.

As constraints, performance characteristics of equipment are formulated as linear equations. For example, for the gas turbine generator, relationships among natural gas consumption $F_{\mathrm{GT}t}$, electric power generated $E_{\mathrm{GT}t}$, heat flow rate of exhaust gas $Q^{\mathrm{e}}_{\mathrm{GT}t}$, electric power consumption for auxiliary machinery $E^{\mathrm{a}}_{\mathrm{GT}t}$, and power generating capacity $\bar{E}_{\mathrm{GT}}$ are expressed as follows:

$$\left.\begin{array}{l} E_{\mathrm{GT}t} = a_{\mathrm{GT}} F_{\mathrm{GT}t} \\ Q^{\mathrm{e}}_{\mathrm{GT}t} = b_{\mathrm{GT}} F_{\mathrm{GT}t} \\ E^{\mathrm{a}}_{\mathrm{GT}t} = a^{\mathrm{a}}_{\mathrm{GT}} F_{\mathrm{GT}t} \\ 0 \leq E_{\mathrm{GT}t} \leq \bar{E}_{\mathrm{GT}} \end{array}\right\} (t \in T) \tag{9}$$

where a and b are performance characteristic values. Similarly, performance characteristics of the waste heat recovery and gas-fired auxiliary boilers, electric compression and steam absorption refrigerators, and pump are expressed by

$$\left.\begin{aligned} &Q^{\mathrm{s}}_{\mathrm{BW}t} = a_{\mathrm{BW}} Q^{\mathrm{e}}_{\mathrm{BW}t} \\ &E^{\mathrm{a}}_{\mathrm{BW}t} = a^{\mathrm{a}}_{\mathrm{BW}} Q^{\mathrm{e}}_{\mathrm{BW}t} \\ &0 \le Q^{\mathrm{s}}_{\mathrm{BW}t} \le \overline{Q}^{\mathrm{s}}_{\mathrm{BW}} \end{aligned}\right\} (t \in T) \tag{10}$$

$$\left.\begin{aligned} &Q^{\mathrm{s}}_{\mathrm{BG}t} = a_{\mathrm{BG}} F_{\mathrm{BG}t} \\ &E^{\mathrm{a}}_{\mathrm{BG}t} = a^{\mathrm{a}}_{\mathrm{BG}} F_{\mathrm{BG}t} \\ &0 \le Q^{\mathrm{s}}_{\mathrm{BG}t} \le \overline{Q}^{\mathrm{s}}_{\mathrm{BG}} \end{aligned}\right\} (t \in T) \tag{11}$$

$$\left.\begin{aligned} &Q^{\mathrm{c}}_{\mathrm{RE}t} = a_{\mathrm{RE}} E_{\mathrm{RE}t} \\ &E^{\mathrm{a}}_{\mathrm{RE}t} = a^{\mathrm{a}}_{\mathrm{RE}} E_{\mathrm{RE}t} \\ &0 \le Q^{\mathrm{c}}_{\mathrm{RE}t} \le \overline{Q}^{\mathrm{c}}_{\mathrm{RE}} \end{aligned}\right\} (t \in T) \tag{12}$$

$$\left.\begin{aligned} &Q^{\mathrm{c}}_{\mathrm{RS}t} = a_{\mathrm{RS}} Q^{\mathrm{s}}_{\mathrm{RS}t} \\ &E^{\mathrm{a}}_{\mathrm{RS}t} = a^{\mathrm{a}}_{\mathrm{RS}} Q^{\mathrm{s}}_{\mathrm{RS}t} \\ &0 \le Q^{\mathrm{c}}_{\mathrm{RS}t} \le \overline{Q}^{\mathrm{c}}_{\mathrm{RS}} \end{aligned}\right\} (t \in T) \tag{13}$$

and

$$E^{\mathrm{a}}_{\mathrm{PC}t} = a^{\mathrm{a}}_{\mathrm{PC}} (Q^{\mathrm{c}}_{\mathrm{RE}t} + Q^{\mathrm{c}}_{\mathrm{RS}t}) \ \ (t \in T) \tag{14}$$

respectively. In addition, the electricity and natural gas consumptions, $E_{\mathrm{elec}t}$ and $F_{\mathrm{gas}t}$, and their contract demands, $\overline{E}_{\mathrm{elec}}$ and $\overline{F}_{\mathrm{gas}}$, are related by

$$\left.\begin{aligned} &0 \le E_{\mathrm{elec}t} \le \overline{E}_{\mathrm{elec}} \\ &0 \le F_{\mathrm{gas}t} \le \overline{F}_{\mathrm{gas}} \end{aligned}\right\} (t \in T) \tag{15}$$

Here, the capacity of device for receiving electricity $\overline{E}_{\mathrm{EP}}$ is assumed to be equal to the contract demand of purchased electricity $\overline{E}_{\mathrm{elec}}$ as follows:

$$\overline{E}_{\mathrm{EP}} = \overline{E}_{\mathrm{elec}} \tag{16}$$

Furthermore, energy balance and supply-demand relationships are expressed by

$$\left.\begin{aligned}
&E_{\text{elec}t} + E_{\text{GT}t} = E_{\text{RE}t} + E^{\text{a}}_{\text{GT}t} + E^{\text{a}}_{\text{BW}t} + E^{\text{a}}_{\text{BG}t} \\
&\qquad\qquad + E^{\text{a}}_{\text{RE}t} + E^{\text{a}}_{\text{RS}t} + E^{\text{a}}_{\text{PC}t} + E_{\text{dem}t} \\
&F_{\text{gas}t} = F_{\text{GT}t} + F_{\text{BG}t} \\
&Q^{\text{e}}_{\text{GT}t} = Q^{\text{e}}_{\text{BW}t} + Q^{\text{e}}_{\text{disp}t} \\
&Q^{\text{s}}_{\text{BW}t} + Q^{\text{s}}_{\text{BG}t} = Q^{\text{s}}_{\text{RS}t} + Q^{\text{s}}_{\text{dem}t} \\
&Q^{\text{c}}_{\text{RE}t} + Q^{\text{c}}_{\text{RS}t} = Q^{\text{c}}_{\text{dem}t}
\end{aligned}\right\} (t \in T) \tag{17}$$

From the economic viewpoint, the annual total cost is adopted as the objective function to be minimized, and is evaluated by the annualized costs method as follows:

$$\begin{aligned}
f = {} & \{R(1-s) + is\}(C_{\text{GT}}\overline{E}_{\text{GT}} + C_{\text{BW}}\overline{Q}^{\text{s}}_{\text{BW}} + C_{\text{BG}}\overline{Q}^{\text{s}}_{\text{BG}} + C_{\text{RE}}\overline{Q}^{\text{c}}_{\text{RE}} \\
& + C_{\text{RS}}\overline{Q}^{\text{c}}_{\text{RS}} + C_{\text{EP}}\overline{E}_{\text{EP}}) + 12(\psi_{\text{elec}}\overline{E}_{\text{elec}} + \psi_{\text{gas}}\overline{F}_{\text{gas}}) \\
& + \sum_{t \in T} (\varphi_{\text{elec}} E_{\text{elec}t} + \varphi_{\text{gas}} F_{\text{gas}t}) d_t
\end{aligned} \tag{18}$$

where C is the capital unit cost of each piece of equipment, R is the capital recovery factor, s is the ratio of salvage value to capital cost of equipment, i is the interest rate, ψ is the monthly unit cost for demand charge of each utility, and φ is the unit cost for energy charge of each utility.

The design variable vector composed of equipment capacities and utility contract demands $\boldsymbol{x}$, the uncertain parameter vector composed of energy demands $\boldsymbol{y}$, and the operation variable vector composed of energy flow rates $\boldsymbol{z}$ are defined as

$$\boldsymbol{x} = (\overline{E}_{\text{GT}}, \overline{Q}^{\text{s}}_{\text{BW}}, \overline{Q}^{\text{s}}_{\text{BG}}, \overline{Q}^{\text{c}}_{\text{RE}}, \overline{Q}^{\text{c}}_{\text{RS}}, \overline{E}_{\text{EP}}, \overline{E}_{\text{elec}}, \overline{F}_{\text{gas}})^{\text{T}} \tag{19}$$

$$\left.\begin{aligned}
&\boldsymbol{y}_t = (E_{\text{dem}t}, Q^{\text{s}}_{\text{dem}t}, Q^{\text{c}}_{\text{dem}t}) \\
&\boldsymbol{y} = (\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots,)^{\text{T}}
\end{aligned}\right\} \tag{20}$$

and

$$\left.\begin{aligned}
\boldsymbol{z}_t = {} & (E_{\text{GT}t}, Q^{\text{e}}_{\text{GT}t}, E^{\text{a}}_{\text{GT}t}, F_{\text{GT}t}, Q^{\text{s}}_{\text{BW}t}, E^{\text{a}}_{\text{BW}t}, Q^{\text{e}}_{\text{BW}t}, Q^{\text{s}}_{\text{BG}t}, E^{\text{a}}_{\text{BG}t}, F_{\text{BG}t}, \\
& Q^{\text{c}}_{\text{RE}t}, E^{\text{a}}_{\text{RE}t}, E_{\text{RE}t}, Q^{\text{c}}_{\text{RS}t}, E^{\text{a}}_{\text{RS}t}, Q^{\text{s}}_{\text{RS}t}, E^{\text{a}}_{\text{PC}t}, E_{\text{elec}t}, E_{\text{gas}t}, Q^{\text{e}}_{\text{disp}t}) \\
\boldsymbol{z} = {} & (\boldsymbol{z}_1, \boldsymbol{z}_2, \ldots,)^{\text{T}}
\end{aligned}\right\} \tag{21}$$

respectively. As the feasible region X for $\boldsymbol{x}$, Eq. (16) and the non-negative condition

$$\boldsymbol{x} \geq \mathbf{0} \tag{22}$$

are considered. As the possible region Y for $\boldsymbol{y}$, the following equation is considered:

$$\underline{\boldsymbol{y}} \leq \boldsymbol{y} \leq \bar{\boldsymbol{y}} \tag{23}$$

where $\bar{\boldsymbol{y}}$ and $\underline{\boldsymbol{y}}$ are upper and lower limits, respectively, of $\boldsymbol{y}$. As the feasible region Z for $\boldsymbol{z}$, Eqs. (9) through (15) and Eq. (17) are considered. It should be noted that Z depends on the values of $\boldsymbol{x}$ and $\boldsymbol{y}$, which means that energy flow rates must be determined to satisfy equipment capacities and utility contract demands as well as performance characteristics of equipment, and energy balance and supply-demand relationships.

The optimal design problem is described as follows:

Find	equipment capacities and utility contract demands $\boldsymbol{x}$ of Eq. (19) as well as energy flow rates $\boldsymbol{z}$ of Eq. (21)
which minimize	the annual total cost f of Eq. (18)
subject to	the constraints of Eqs. (9) through (17) and Eq. (22)
under	uncertain energy demands $\boldsymbol{y}$ of Eq. (20) limited by Eq. (23).

4.3. Input Data

A typical year is divided into three representative days, i.e., summer, winter, and mid-season each of which has 122, 121, and 122 days per year. Furthermore, each day is divided into 6, 12, and 24 periods each of which has 4, 2, and 1 hour(s) per day in cases I, II, and III, respectively, and correspondingly the duration per year of each period d_t is given. Averages of electricity, steam, and cold water demands for each period are estimated. Electricity, steam, and cold water demands for each period are assumed to vary within $\pm\alpha$ times of their averages, and correspondingly their upper and lower limits, $\bar{y}$ and $\underline{y}$, are given. As an example, Fig. 4 shows the averages of electricity, steam, and cold water demands on the representative day in summer in case III. Other input data are given in Table 1. All values for performance characteristic values of equipment as well as unit costs of equipment and utilities are set based on their real data. In addition, all values for costs are stated in yen, which is equivalent to about 9.8×10^{-3} dollars and 7.2×10^{-3} euros on the recent exchange rate.

All the optimization calculations are conducted on a MacBook Air using the commercial solver CPLEX Ver. 12.4 for the mixed-integer linear and linear programming problems through the modeling system for mathematical programming GAMS Ver. 23.9 [25].

4.4. Results and Discussion

4.4.1. Effectiveness of revised robust optimal design method

The effectiveness of the revised robust optimal design method proposed in this paper can be investigated through the evaluation of an upper bound for the optimal value of the maximum regret. The number of binary variables affects the efficiency of this evaluation significantly. In the conventional solution method, the number of binary variables is equal to the product of the number of inequality constraints including upper and lower limits for the operation variables for energy flow rates for each period and the number of periods, and is 270, 540, and 1080 in cases I, II, and III, respectively. In the novel solution method, on the other hand, the number of binary variables is equal to the product of the number of energy demand types and the number of periods, and is 54, 108, and 216 in cases I, II, and III, respectively. Therefore, the novel solution method can reduce the number of binary variables drastically.

To confirm this effectiveness, the optimization calculations are conducted using both the methods in cases I to III when uncertainty in energy demands α = 0.2 and 0.4. Table 2 shows the comparison of computation times in evaluating upper bounds for the optimal value of the maximum regret by these methods. When α = 0.2, in cases I and II, both the methods can obtain the optimal solutions in short computation times, and in many cases the novel solution method needs slightly longer computation times than the conventional one. In case III, however, the conventional solution method needs much longer computation times than the novel one. When α = 0.4, in case I, both the methods can obtain the optimal solutions in short computation times, and the novel solution method tends to need slightly longer computation times than the conventional one. In case II, however, the conventional solution method needs much longer computation times than the novel one. Additionally, in case III, the novel solution method also needs longer computation times, but the conventional one cannot obtain the optimal solution in a reasonable computation time at the third repeat, and it still has a large gap in an upper bound for the optimal value of the maximum regret between the current and possible best solutions even in a long computation time of 3600 s. This is because the bilevel linear programming problem is converted into the mixed-integer linear programming one with many binary variables, which is too complex to solve. These results show that the revised robust optimal design method is very effective in terms of computation time.

Table 3 shows the comparison of convergence characteristics, or upper and lower bounds for the optimal value of the maximum regret, by both the methods in cases I to III when α = 0.2 and 0.4. Both the methods have the tendency that the optimal solutions are obtained by several repeats. At the first and second repeats, large differences between the upper and lower bounds are found. However, the upper and lower bounds coincide well with each other after three to five repeats, and the optimal solutions are obtained. Generally, convergence processes may be different between the two methods, because the energy demands obtained in evaluating an upper bound for the optimal value of the maximum regret may be different. This is because the energy demands which give the maximum regret are not necessarily unique, as described below, which affects the results obtained in evaluating a lower bound for the optimal value of the maximum regret. Thus, the number of repeats may be different between the two methods, as shown in case II when α = 0.2. However, the optimal solutions obtained finally by the two methods coincide well with each other. These results show that the revised robust optimal design method is valid in terms of solution optimality.

4.4.2. Features of robust optimal design

The optimization calculations are conducted using the revised robust optimal design method in case III by changing the uncertainty in energy demands α within 0 to 0.4. Figure 5 shows the relationships between α and the optimal values of equipment capacities and utility contract demands. With an increase in α, the capacities of gas turbine generator and waste heat recovery boiler decrease slightly, and those of device for receiving electricity and gas-fired auxiliary boiler increase dramatically. This is because the robust optimal design avoids an increase in the economic regret due to unbalanced electricity and heat demands which tend to occur with an increase in α. The capacity of steam absorption refrigerator is much larger than that of electric compression refrigerator, and the former and latter increase drastically and slightly, respectively, with an increase in α.

Figure 6 shows the relationship between α and the optimal value of the maximum regret in the annual total cost. The maximum regret and its increasing rate increase with an increase in α. This shows that the cogeneration system becomes disadvantageous for unbalanced electricity and heat demands with an increase in α. However, the maximum regret remains much small as compared with the annual total cost. For example, when α = 0.2, the annual total costs of the robust optimal design for the largest and smallest energy demands mentioned in the following are 1845.34×10^6 and 1591.45×10^6 yen/y, respectively. Therefore, the maximum regret 4.95×10^6 yen/y as shown in Fig. 6 is only 0.27 and 0.31 % of these annual total costs. This result shows the robust optimal design gives a small ratio of the maximum regret to the annual total cost.

Since the energy demands which give the maximum regret are not necessarily unique, two featured ones are derived newly using the following equations after the optimization calculation:

$$\max_{\boldsymbol{y} \in Y,\, \boldsymbol{\mu} \in M,\, \boldsymbol{z}' \in Z'} \left\{ \Omega\big(g(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{\mu}) - f(\boldsymbol{x}', \boldsymbol{y}, \boldsymbol{z}')\big) \pm \sum_i \delta_i \right\} \tag{24}$$

where δ_i is the binary variable introduced into Eq. (A11) of appendix A.2 to select the upper or lower limit of an uncertain energy demand, and Ω is the coefficient for the penalty term, and should be given a value large sufficiently. These equations with the plus and minus signs mean that the energy demands are maximized and minimized, respectively, subject to the maximum regret obtained by the optimization calculation. As an example, Figure 7 shows the energy demands in summer which give the maximum regret when α = 0.2. In this figure, dot-dash lines show the average energy demands, and solid lines and marks show the energy demands which give the maximum regret, while (a) and (b) show the largest and smallest energy demands, respectively, obtained using Eq. (24). During some periods, both the energy demands coincide with each other at their upper or lower limits. This means that if energy demands are selected differently, the maximum regret decreases. It should be noted that there may be alternative energy demands which give the maximum regret between their upper and lower limits. During the other periods, on the other hand, the largest and smallest energy demands are selected differently at their upper and lower limits, respectively. This means that even if energy demands are selected differently, the maximum regret remains constant. It is not so easy to perfectly understand the validity of these energy demands, because the energy demands depend on the optimal operational strategies of the robust optimal and optimal designs. However, it is relatively easy to understand the validity of the energy demands for limited periods as shown in the following.

Figures 8 and 9 show one of the optimal operational strategies or the allocation of electricity supply of the robust optimal and optimal designs, respectively, corresponding to the largest energy demands in summer which give the maximum regret. For example, during 0:00 to 6:00 and 22:00 to 24:00, the electricity demands are selected differently at their upper and lower limits in Figs. 7 (a) and (b), respectively. This is because the optimal operational strategies for electricity supply do not change qualitatively in both the designs, even if the energy demands change between their upper and lower limits, which does not change the maximum regret. However, during 6:00 to 7:00 and 19:00 to 22:00, the electricity demands are selected at their lower limits in Figs. 7 (a) and (b). This is because the optimal operational strategy for electricity supply of the optimal design changes qualitatively if the energy demands change from the lower limits to the upper ones, which decreases the maximum regret. This result shows that the robust optimal design method determines the energy demands which give the maximum regret rationally in consideration of the optimal operational strategies of the robust optimal and optimal designs.

4.4.3. Comparison of cogeneration and conventional energy supply systems

Finally, the gas turbine cogeneration system obtained by the robust optimal design method is compared with a conventional energy supply system without cogeneration using the proposed comparison method. For this purpose, equipment capacities and utility contract demands of the conventional energy supply system are also obtained by the robust optimal design method, where the capacities of gas turbine generator and waste heat recovery boiler are set at zeros. The optimization calculations are conducted using the revised robust optimal design method in case III by changing the uncertainty in energy demands α within 0 to 0.4. Figure 10 shows the relationships between α and the optimal values of equipment capacities and utility contract demands. The capacities of the other pieces of equipment as well as the utility contract demands increase with an increase in α. However, there are hardly alternatives for the operational strategy, because electricity is purchased, steam is generated by the gas-fired auxiliary boiler, and cold water is mainly generated by the steam absorption refrigerator. As a result, the optimal value of the maximum regret in the annual total cost is almost zero, as shown in Fig. 6.

The maximum and minimum differences in the annual total cost between the conventional energy supply and cogeneration systems are evaluated by the comparison method. Figure 11 shows the relationship between α, and the maximum and minimum differences in the annual total cost. With an increase in α, the maximum difference increases, while the minimum one decreases, and resultantly their interval increases. However, the increasing rate of the maximum difference decreases, while the decreasing rate of the minimum difference increases. As a result, the averaged difference decreases. This is also because the cogeneration system becomes disadvantageous for unbalanced electricity and heat demands with an increase in α.

Since the energy demands which give the maximum and minimum differences are also not necessarily unique, two featured ones are derived alternatively using the equations

$$\max_{\boldsymbol{y} \in Y,\, \boldsymbol{\mu}_{\mathrm{A}} \in M_{\mathrm{A}},\, \boldsymbol{z}_{\mathrm{B}} \in Z_{\mathrm{B}}} \left\{ \Omega\Big(g_{\mathrm{A}}(\boldsymbol{x}_{\mathrm{A}}, \boldsymbol{y}, \boldsymbol{\mu}_{\mathrm{A}}) - f_{\mathrm{B}}(\boldsymbol{x}_{\mathrm{B}}, \boldsymbol{y}, \boldsymbol{z}_{\mathrm{B}}) \Big) \pm \sum_i \delta_i \right\} \tag{25}$$

$$\min_{\boldsymbol{y} \in Y,\, \boldsymbol{z}_{\mathrm{A}} \in Z_{\mathrm{A}},\, \boldsymbol{\mu}_{\mathrm{B}} \in M_{\mathrm{B}}} \left\{ \Omega\Big(f_{\mathrm{A}}(\boldsymbol{x}_{\mathrm{A}}, \boldsymbol{y}, \boldsymbol{z}_{\mathrm{A}}) - g_{\mathrm{B}}(\boldsymbol{x}_{\mathrm{B}}, \boldsymbol{y}, \boldsymbol{\mu}_{\mathrm{B}}) \Big) \pm \sum_i \delta_i \right\} \tag{26}$$

in place of Eqs. (7) and (8), respectively. The equations with the plus and minus signs of Eq. (25) mean that the energy demands are maximized and minimized, respectively, subject to the maximum difference. On the other hand, the equations with the plus and minus signs of Eq. (26) mean that the energy demands are minimized and maximized, respectively, subject to the minimum difference. As examples, Figures 12 and 13 show the energy demands in summer which give the maximum and minimum differences, respectively, when α = 0.2. In these figures, dot-dash lines show the average energy demands, and solid lines and marks show the energy demands which give the maximum and minimum differences, while (a) and (b) show the largest and smallest energy demands obtained using Eqs. (25) and (26), respectively. During some periods, both the energy demands coincide with each other at their upper or lower limits. This means that if energy demands are selected differently, the maximum and minimum differences decreases and increases. It should be noted that there may be alternative energy demands which give the maximum and minimum differences between their upper and lower limits. During the other periods, on the other hand, the largest and smallest energy demands are selected differently at their upper and lower limits, respectively. This means that even if energy demands are selected differently, the maximum and minimum differences remain constant. It is also not so easy to perfectly understand the validity of these energy demands, because the energy demands depend on the optimal operational strategies of the conventional energy supply and cogeneration systems. However, it is relatively easy to understand the validity of the energy demands for limited periods as shown in the following.

Figures 14 and 15 show one of the optimal operational strategies or the allocation of electricity supply of the cogeneration system corresponding to the largest energy demands in summer which give the maximum and minimum differences, respectively. For example, during 0:00 to 7:00 and 18:00 to 24:00, although the electricity demands are selected similarly at their upper and lower limits for the maximum and minimum differences in Figs. 12 (a) and (b), and Figs. 13 (a) and (b), respectively. This is because the optimal operational strategies for electricity supply of the conventional energy supply and cogeneration systems differ qualitatively, and resultantly the electricity demands are selected similarly at their upper and lower limits so that the difference in the annual total cost between the conventional energy supply and cogeneration systems is maximized and minimized, respectively. This result shows that the comparison method also determines the energy demands which give the maximum and minimum differences rationally in consideration of the optimal operational strategies of the conventional energy supply and cogeneration systems.

5. Conclusions

A robust optimal design method of energy supply systems under uncertain energy demands proposed previously has been revised so that it can be applied to systems with complex configurations and large numbers of periods for variations in energy demands. This revised method inherits the features of the conventional method that it is based on the minimax regret criterion, it considers the hierarchical relationship among design variables, energy demands, and operation variables, and it can produce a unique optimal solution. Equipment capacities and utility contract demands as well as energy flow rates have been determined to minimize the maximum regret in the annual total cost and to satisfy all the possible energy demands. This robust optimal design problem has been formulated as a kind of multilevel linear programming one, and its solution has been derived by repeatedly evaluating lower and upper bounds for the optimal value of the maximum regret. Especially, a novel solution method has been proposed for efficiently evaluating an upper bound for the optimal value of the maximum regret. A method of comparing two energy supply systems under uncertain energy demands has also been proposed by utilizing a part of the revised robust optimal design method. A case study on a gas turbine cogeneration system for district energy supply has been conducted to investigate the validity and effectiveness of the revised optimal design method and features of the robust optimal design. In addition, the gas turbine cogeneration system has been compared with a conventional energy supply system using the proposed comparison method. The following are the main results obtained here:

1) The proposed novel solution method can evaluate an upper bound for the optimal value of the maximum regret in a much shorter computation time than the conventional

one, and it can significantly contribute to efficiently deriving the optimal solution of the robust optimal design problem. As a result, the revised robust optimal design method can be applied to energy supply systems with complex configurations and large numbers of periods.

2) With an increase in the uncertainty in energy demands, the optimal capacity of cogeneration unit decreases slightly, while those of auxiliary equipment increase dramatically, to avoid the economic regret. This is because the cogeneration system becomes disadvantageous for unbalanced electricity and heat demands.

3) With an increase in the uncertainty in energy demands, the maximum regret and its increasing rate increase. However, the maximum regret remains much small as compared with the annual total cost. This means that the robust optimal design is very effective.

4) With an increase in the uncertainty in energy demands, the maximum and minimum differences in the annual total cost between the conventional energy supply and cogeneration systems increases and decreases, respectively, and resultantly their interval increases. However, the increasing rate of the maximum difference decreases, while the decreasing rate of the minimum difference increases. As a result, the averaged difference decreases. This is also because the cogeneration system becomes disadvantageous for unbalanced electricity and heat demands.

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Appendices

A.1. Formulation of robust optimal design problem

First, the ordinary optimal design problem in which equipment capacities and utility contract demands $\boldsymbol{x}'$ as well as energy flow rates $\boldsymbol{z}'$ are determined to minimize f under certain energy demands $\boldsymbol{y}$ is expressed by

$$\min_{\boldsymbol{x}' \in X} \min_{\boldsymbol{z}' \in Z'} f(\boldsymbol{x}', \boldsymbol{y}, \boldsymbol{z}') \tag{A1}$$

where X is the set for all the possible values of $\boldsymbol{x}$, and Z' is the set for all the possible values of $\boldsymbol{z}'$, and depends on the combinations $(\boldsymbol{x}', \boldsymbol{y})$. Therefore, the robust optimal design problem in which equipment capacities and utility contract demands $\boldsymbol{x}$ as well as energy flow rates $\boldsymbol{z}$ are determined to minimize the maximum regret in f under uncertain energy demands $\boldsymbol{y}$ is expressed by

$$\min_{\boldsymbol{x} \in X} \max_{\boldsymbol{y} \in Y} \left(\min_{\boldsymbol{z} \in Z} f(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) - \min_{\boldsymbol{x}' \in X} \min_{\boldsymbol{z}' \in Z'} f(\boldsymbol{x}', \boldsymbol{y}, \boldsymbol{z}') \right) \tag{A2}$$

where Y is the set for all the possible values of $\boldsymbol{y}$, and Z is the set for all the possible values of $\boldsymbol{z}$, and depends on the combinations $(\boldsymbol{x}, \boldsymbol{y})$.

Next, the flexibility, or the feasibility in energy supply is incorporated into Eq. (A2). To secure the flexibility for all the possible values of uncertain energy demands $\boldsymbol{y}$, an objective function which expresses the infeasibility in energy supply is introduced [26], and equipment capacities and utility contract demands $\boldsymbol{x}$ are determined to minimize (make zero) the maximum of this objective function for all the possible values of $\boldsymbol{y}$. This idea is applied to the ordinary and robust optimal designs, and the corresponding optimization problems are expressed by

$$\min_{\boldsymbol{x}' \in X} \max_{\boldsymbol{y}'' \in Y} \min_{\boldsymbol{z}'' \in Z''} p(\boldsymbol{x}', \boldsymbol{y}'', \boldsymbol{z}'') \tag{A3}$$

and

$$\min_{\boldsymbol{x} \in X} \max_{\boldsymbol{y}''' \in Y} \min_{\boldsymbol{z}''' \in Z'''} p(\boldsymbol{x}, \boldsymbol{y}''', \boldsymbol{z}''') \tag{A4}$$

respectively, where p is the objective function for the infeasibility in energy supply. In addition, Z'' and Z''' are the sets for all the possible values of $\boldsymbol{z}''$ and $\boldsymbol{z}'''$, and depend on the combinations $(\boldsymbol{x}', \boldsymbol{y}'')$ and $(\boldsymbol{x}, \boldsymbol{y}''')$, respectively.

To take account of Eqs. (A3) and (A4) prior to Eq. (A2), they are added to Eq. (A2) as penalty terms. As a result, the robust optimal design problem is formulated as

$$\begin{aligned}\min_{\boldsymbol{x} \in X} \Bigg[\max_{\boldsymbol{y} \in Y} \Bigg\{ & \min_{\boldsymbol{z} \in Z} f(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) \\ & - \min_{\boldsymbol{x}' \in X} \Bigg(\min_{\boldsymbol{z}' \in Z'} f(\boldsymbol{x}', \boldsymbol{y}, \boldsymbol{z}') \\ & \qquad + W \max_{\boldsymbol{y}'' \in Y} \min_{\boldsymbol{z}'' \in Z''} p(\boldsymbol{x}', \boldsymbol{y}'', \boldsymbol{z}'') \Bigg) \Bigg\} \\ & + W \max_{\boldsymbol{y}''' \in Y} \min_{\boldsymbol{z}''' \in Z'''} p(\boldsymbol{x}, \boldsymbol{y}''', \boldsymbol{z}''') \Bigg]\end{aligned} \tag{A5}$$

where W is the coefficient for penalty terms, and should be given a value large sufficiently.

A.2. Evaluation of upper bound

A.2.1 Conventional solution method

The optimal solution of Eq. (3) is obtained by solving two optimization problems corresponding to the first through third lines and the fourth one independently. These problems are formulated as bilevel linear programming ones which include the operations of maximization and minimization hierarchically. In the conventional solution method adopted in the previous papers [18–21], these problems can be reformulated as ordinary one-level optimization ones by applying the Kuhn-Tucker optimality condition to their lower-level minimization problems. This reformulation produces complementarity conditions which are inner products of the inequality constraint vectors and the corresponding Lagrange multiplier vectors. To avoid the nonlinearity due to these complementarity conditions, binary variables are introduced to linearize the nonlinear terms exactly, and the problems are reduced to mixed-integer linear programming ones [27].

However, these problems become large scale with increases in the complexity of system configuration and number of periods for seasonal and hourly variations in energy demands. Especially, the binary variables introduced to linearize the nonlinear complementarity conditions are necessary corresponding to inequality constraints including upper and lower limits for the operation variables for energy flow rates. Therefore, the solution method takes a long computation time when the number of equipment increases with the complexity of system configuration.

A.2.2 Novel solution method

In this paper, a novel solution method for efficiently evaluating an upper bound for the optimal value of the maximum regret is proposed. In this method, the lower-level minimization problems are converted into their dual maximization ones, and resultantly the bilevel linear programming problems are converted into ordinary one-level bilinear programming ones. Then, the value of $\boldsymbol{y}$ is limited by applying a theorem on bilinear programming problems, and is expressed by binary variables. Finally, the products of binary and continuous variables are linearized exactly, and the bilinear programming problems are converted into mixed-integer linear programming ones. A detailed procedure is described in the following.

By considering the dual maximization problems in place of the primal minimization ones, the optimization problem of Eq. (3) can be converted into the following one:

$$\begin{aligned}&\max_{\boldsymbol{y} \in Y} \max_{\boldsymbol{x}' \in X} \left\{ \max_{\mu \in M} g(\boldsymbol{x}, \boldsymbol{y}, \mu) \right.\\ &\qquad - \left(\min_{\boldsymbol{z}' \in Z'} f(\boldsymbol{x}', \boldsymbol{y}, \boldsymbol{z}') \right.\\ &\qquad\quad \left.\left. + \; W \min_{\boldsymbol{z}'' \in Z''} p(\boldsymbol{x}', \boldsymbol{y}'', \boldsymbol{z}'') \right) \right\}\\ &+ W \max_{\boldsymbol{y}''' \in Y} \max_{\mu''' \in M'''} q(\boldsymbol{x}, \boldsymbol{y}''', \mu''')\end{aligned} \tag{A6}$$

where g and q are the objective functions of the dual maximization problems converted from the primal minimization ones with the objective function f and p, respectively, μ and μ''' are the Lagrange multiplier vectors corresponding to the constraints of the primal minimization ones, and M and M''' are the sets for all the possible values of μ and μ''', respectively. The operations of maximization with respect to μ and minimization with respect to $\boldsymbol{z}'$ and $\boldsymbol{z}''$ are moved forward to reformulate Eq. (A6) as

$$\begin{aligned}&\max_{\boldsymbol{y} \in Y,\, \boldsymbol{x}' \in X,\, \mu \in M,\, \boldsymbol{z}' \in Z',\, \boldsymbol{z}'' \in Z''} \left\{ g(\boldsymbol{x}, \boldsymbol{y}, \mu) \right.\\ &\qquad \left. - \left(f(\boldsymbol{x}', \boldsymbol{y}, \boldsymbol{z}') + Wp(\boldsymbol{x}', \boldsymbol{y}'', \boldsymbol{z}'') \right) \right\}\\ &+ W \max_{\boldsymbol{y}''' \in Y,\, \mu''' \in M'''} q(\boldsymbol{x}, \boldsymbol{y}''', \mu''')\end{aligned} \tag{A7}$$

The optimization problem corresponding to the first and second lines in Eq. (A7) is examined here. To consider $\min_{\boldsymbol{z} \in Z} f(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z})$ in Eq. (3) and $\max_{\mu \in M} g(\boldsymbol{x}, \boldsymbol{y}, \mu)$ in (A6) concretely, the primal and dual problems are described as follows:

$$
\left.\begin{array}{ll}
\min\limits_{\boldsymbol{z}} & \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x} + \boldsymbol{d}^{\mathrm{T}}\boldsymbol{z} \\
\text{sub. to} & \boldsymbol{B}_1\boldsymbol{z} = \boldsymbol{y} \\
& \boldsymbol{B}_2\boldsymbol{z} = \boldsymbol{b} \\
& \boldsymbol{B}_3\boldsymbol{z} \leq \boldsymbol{x} \\
& \boldsymbol{z} \geq \boldsymbol{0}
\end{array}\right\} \tag{A8}
$$

and

$$
\left.\begin{array}{ll}
\max\limits_{\boldsymbol{\mu}_1,\, \boldsymbol{\mu}_2,\, \boldsymbol{\mu}_3} & \boldsymbol{c}^{\mathrm{T}}\boldsymbol{x} + \boldsymbol{y}^{\mathrm{T}}\boldsymbol{\mu}_1 + \boldsymbol{b}^{\mathrm{T}}\boldsymbol{\mu}_2 - \boldsymbol{x}^{\mathrm{T}}\boldsymbol{\mu}_3 \\
\text{sub. to} & \boldsymbol{B}_1^{\mathrm{T}}\boldsymbol{\mu}_1 + \boldsymbol{B}_2^{\mathrm{T}}\boldsymbol{\mu}_2 - \boldsymbol{B}_3^{\mathrm{T}}\boldsymbol{\mu}_3 \leq \boldsymbol{d} \\
& \boldsymbol{\mu}_3 \geq \boldsymbol{0}
\end{array}\right\} \tag{A9}
$$

respectively, where $\boldsymbol{c}$ and $\boldsymbol{d}$ are the coefficient vectors for equipment capacities and utility contract demands, and energy flow rates, respectively, in the objective function of the primal problem, $\boldsymbol{B}_1$ to $\boldsymbol{B}_3$ and $\boldsymbol{b}$ are the coefficient matrices and constant term vectors, respectively, in the constraints of the primal problem, $\boldsymbol{\mu}_1$ to $\boldsymbol{\mu}_3$ are the dual variables corresponding to the constraints of the primal problem, and the superscript T means the transposition of a vector or matrix. Since the value of $\boldsymbol{x}$ is fixed in evaluating an upper bound, the objective function of the dual problem becomes a nonlinear function with the bilinear term as the inner product of $\boldsymbol{y}$ and $\boldsymbol{\mu}_1$. The optimization problem corresponding to the third line in Eq. (A7) is examined similarly. As a result, Eq. (A7) results in one-level bilinear programming problems with bilinear terms only in the objective functions.

The optimization problem corresponding to the first and second lines in Eq. (A7) is examined again. The set M for all the possible values of $\boldsymbol{\mu}$ is independent from the set Y, X, Z', and Z'' for all the possible values of $\boldsymbol{y}$, $\boldsymbol{x}'$, $\boldsymbol{z}'$, and $\boldsymbol{z}''$, respectively. A theorem on bilinear programming problems states that the optimal solution exists at one of the basic feasible solutions [28]. Here, the uncertain energy demands $\boldsymbol{y}$ are assumed to be between their upper and lower limits, $\bar{\boldsymbol{y}}$ and $\underline{\boldsymbol{y}}$, respectively, as follows:

$$\underline{\boldsymbol{y}} \le \boldsymbol{y} \le \bar{\boldsymbol{y}} \tag{A10}$$

Then, the set Y for all the possible values of $\boldsymbol{y}$ is independent from the sets X, Z', and Z'' for all the possible values of $\boldsymbol{x}'$, $\boldsymbol{z}'$, and $\boldsymbol{z}''$, respectively, and the value of $\boldsymbol{y}$ becomes equal to its upper or lower limit, which means a basic feasible solution. Therefore, the ith element y_i of $\boldsymbol{y}$ can be expressed as

$$\left.\begin{array}{l} y_i = \bar{y}_i \delta_i + \underline{y}_i (1 - \delta_i) \\ \delta_i \in \{0, 1\} \end{array}\right\} \tag{A11}$$

using a binary variable δ_i. This conversion produces the bilinear term composed of δ_i and the ith element μ_{1i} of $\boldsymbol{\mu}_1$. However, it is a product of binary and continuous variables, which can be linearized exactly by replacing it with a continuous variable ξ_i and adding the following linear constraints [29]:

$$\left.\begin{array}{l} \underset{\sim}{\mu}_{1i} \delta_i \le \xi_i \le \tilde{\mu}_{1i} \delta_i \\ \mu_{1i} - (\tilde{\mu}_{1i} - \underset{\sim}{\mu}_{1i})(1 - \delta_i) \le \xi_i \le \mu_{1i} + (\tilde{\mu}_{1i} - \underset{\sim}{\mu}_{1i})(1 - \delta_i) \end{array}\right\} \tag{A12}$$

where $\tilde{\mu}_{1i}$ and $\underset{\sim}{\mu}_{1i}$ are upper and lower bounds for μ_{1i}, respectively. The optimization problem corresponding to the third line in Eq. (A7) is examined similarly. As a result, Eq. (A7) is reduced to mixed-integer linear programming problems.

The numbers of binary variables in these mixed-integer linear programming problems depend on the numbers of energy demand types and periods. The number of energy demand types is usually much smaller than that of inequality constraints including upper and lower limits for the operation variables for energy flow rates. Therefore, the novel solution method proposed here is expected to find the optimal solution much more efficiently than the conventional one.

A.3. Evaluation of lower bound

In Eq. (4), the values of

$$\Phi(\boldsymbol{x}', \boldsymbol{y}) = \min_{\boldsymbol{z}' \in Z'} f(\boldsymbol{x}', \boldsymbol{y}, \boldsymbol{z}') \tag{A13}$$

and

$$\Pi(\boldsymbol{x}') = W \max_{\boldsymbol{y}'' \in Y} \min_{\boldsymbol{z}'' \in Z''} p(\boldsymbol{x}', \boldsymbol{y}'', \boldsymbol{z}'') \tag{A14}$$

can be evaluated for each candidate of combinations of values of $\boldsymbol{y}$ and $\boldsymbol{x}'$ independently. The following procedure is used to solve the problem of Eq. (4).

First, B is defined as the set for values of $\boldsymbol{y}'''$. An appropriate value of $\boldsymbol{y}'''$ is assumed, and is made an element of B. The value of $\boldsymbol{y}'''$ is assumed to be selected only from the elements of B, and the following optimization problem is solved in place of Eq. (4):

$$\begin{aligned}\min_{\boldsymbol{x} \in X} \Bigg[\max_{(\boldsymbol{y}, \boldsymbol{x}') \in A} \Bigg\{ & \min_{\boldsymbol{z} \in Z} f(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z}) - \Big(\Phi(\boldsymbol{x}', \boldsymbol{y}) + \Pi(\boldsymbol{x}') \Big) \\ & + W \max_{\boldsymbol{y}''' \in B} \min_{\boldsymbol{z}''' \in Z'''} p(\boldsymbol{x}, \boldsymbol{y}''', \boldsymbol{z}''') \Bigg] \end{aligned} \tag{A15}$$

This problem is a three-level linear programming one which includes the operations of minimization and maximization hierarchically, and seems to be difficult to solve. However, the operations of maximization are only with respect to ($\boldsymbol{y}$, $\boldsymbol{x}'$) and $\boldsymbol{y}'''$, and are conducted by selecting the values of ($\boldsymbol{y}$, $\boldsymbol{x}'$) and $\boldsymbol{y}'''$ from their finite numbers of candidates in the sets A and B, respectively. Therefore, the introduction of variables for maxima with respect to ($\boldsymbol{y}$, $\boldsymbol{x}'$) and $\boldsymbol{y}'''$, and inequality constraints changes the problem of Eq. (A15) into an ordinary linear programming one, which can be solved easily by the simplex method.

Next, using the value of $\boldsymbol{x}$ obtained by solving Eq. (A15), the problem

$$\max_{\boldsymbol{y}''' \in Y} \min_{\boldsymbol{z}''' \in Z'''} p(\boldsymbol{x}, \boldsymbol{y}''', \boldsymbol{z}''') \tag{A16}$$

is solved by the novel solution method proposed previously, and it is tested whether the value of the objective function p for the optimal solution is zero or not. If the value of p is zero, it is judged that the optimal solution of Eq. (4) is obtained by solving Eq. (A15); otherwise the value of $\boldsymbol{y}'''$ for the optimal solution of Eq. (A16) is added to the set B, and the problems of Eqs. (A15) and (A16) are solved repeatedly until the value of p becomes zero.

Nomenclature

A, B : sets for values of ($\boldsymbol{y}$, $\boldsymbol{x}'$) and $\boldsymbol{y}'''$, respectively

a, b : performance characteristic values [kW/(m^3/h), kW/kW]

$\boldsymbol{B}_1$, $\boldsymbol{B}_2$, $\boldsymbol{B}_3$: coefficient matrices

$\boldsymbol{b}$: constant term vector

C : capital unit cost of equipment [yen/kW]

$\boldsymbol{c}$, $\boldsymbol{d}$: coefficient vectors

d : duration of period [h/y]

E : electric power [kWh/h]

F : natural gas consumption [m^3/h]

f : objective function of primal problem (annual total cost) [yen/y]

Δf : difference in annual total cost [yen/y]

g : objective function of dual problem (annual total cost) [yen/y]

i : interest rate

p : objective function of primal problem (infeasibility in energy supply) [kWh/y]

Q : heat flow rate [kWh/h]

q : objective function of dual problem (infeasibility in energy supply) [kWh/y]

R : capital recovery factor

s : ratio of salvage value to capital cost of equipment

T : set for all values of t

W : coefficient for penalty terms

X : feasible region for $\boldsymbol{x}$

$\boldsymbol{x}$: design variables (equipment capacities and utility contract demands) [kW, m^3/h]

Y : set for all possible values of $\boldsymbol{y}$

$\boldsymbol{y}$: uncertain parameters (energy demands) [kWh/h]

y_i : ith element of $\boldsymbol{y}$

Z : feasible region for $\boldsymbol{z}$

$\boldsymbol{z}$: operation variables (energy flow rates) [kWh/h, m^3/h]

δ_i : ith binary variable

M : feasible region for $\boldsymbol{\mu}$

$\boldsymbol{\mu}, \boldsymbol{\mu}_1, \boldsymbol{\mu}_2, \boldsymbol{\mu}_3$: dual variables of dual problem

μ_{1i} : ith element of $\boldsymbol{\mu}_1$

ξ_i : product of δ_i and μ_{1i}

Π : function with respect to $\boldsymbol{x}'$

Φ : function with respect to $\boldsymbol{y}$ and $\boldsymbol{x}'$

φ : unit cost for energy charge of utility [yen/kWh, yen/m^3]

ψ : monthly unit cost for demand charge of utility [yen/(kW·month), yen/(m^3/h·month)]

Ω : coefficient for penalty term

$\overline{(\ )}$, $\underline{(\ )}$: upper and lower limits, respectively

$\tilde{(\ )}$, $\underset{\sim}{(\ )}$: upper and lower bounds, respectively

$(\ )'$, $(\ )''$, $(\ )'''$: different values

Equipment symbols (subscripts)

BG : gas-fired auxiliary boiler

BW : waste heat recovery boiler

EP : device for receiving electricity

GT : gas turbine generator

PC : pump

RE : electric compression refrigerator

RS : steam absorption refrigerator

Subscripts

A, B : systems A and B, respectively

AB : system A to B

dem : energy demand

disp : heat disposal

elec : purchased electricity

gas : purchased natural gas

t : index for periods

Superscripts

a : electricity for auxiliary machinery

c : cold water

e : exhaust gas

s : steam

T : transposition of vector

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Captions for tables and figures

Table 1 Input data

Table 2 Computation times in evaluating upper bounds for optimal value of maximum regret

(a) α = 0.2, (b) α = 0.4

Table 3 Upper and lower bounds for optimal value of maximum regret

(a) α = 0.2, (b) α = 0.4

Fig. 1 Basic concept of robust optimal design based on minimax regret criterion

Fig. 2 Hierarchical relationship among design variables, energy demands, and operation variables

Fig. 3 Configuration of gas turbine cogeneration system

Fig. 4 Average energy demands in summer

Fig. 5 Relationships between uncertainty in energy demands and optimal values of equipment capacities and utility contract demands of cogeneration system

Fig. 6 Relationships between uncertainty in energy demands and optimal values of maximum regret in annual total cost of cogeneration and conventional energy supply systems

Fig. 7 Energy demands in summer which give maximum regret

(a) Largest demands, (b) Smallest demands

Fig. 8 Optimal operational strategy of robust optimal design for largest energy demands in summer which give maximum regret

Fig. 9 Optimal operational strategy of optimal design for largest energy demands in summer which give maximum regret

Fig. 10 Relationships between uncertainty in energy demands and optimal values of

Table 1 Input data

Item	Value		
Performance characteristic values of equipment	a_{GT} =	3.23	kW/(m^3/h)
	b_{GT} =	6.71	kW/(m^3/h)
	$a^{\mathrm{a}}_{\mathrm{GT}}$ =	0.121	kW/(m^3/h)
	a_{BW} =	0.78	kW/kW
	$a^{\mathrm{a}}_{\mathrm{BW}}$ =	0.005	kW/kW
	a_{BG} =	10.40	kW/(m^3/h)
	$a^{\mathrm{a}}_{\mathrm{BG}}$ =	0.051	kW/(m^3/h)
	a_{RE} =	5.04	kW/kW
	$a^{\mathrm{a}}_{\mathrm{RE}}$ =	0.215	kW/kW
	a_{RS} =	1.25	kW/kW
	$a^{\mathrm{a}}_{\mathrm{RS}}$ =	0.079	kW/kW
	$a^{\mathrm{a}}_{\mathrm{PC}}$ =	0.025	kW/kW
Capital unit costs of equipment	C_{GT} =	230.0×10^3	yen/kW
	C_{BW} =	9.6×10^3	yen/kW
	C_{BG} =	6.9×10^3	yen/kW
	C_{RE} =	46.7×10^3	yen/kW
	C_{RS} =	43.7×10^3	yen/kW
	C_{EP} =	56.0×10^3	yen/kW
Unit costs of utilities	ψ_{elec} =	1.74×10^3	yen/(kW·month)
	ψ_{gas} =	2.37×10^3	yen/(m^3/h·month)
	φ_{elec} =	11.0	yen/kWh
	φ_{gas} =	31.0	yen/m^3
Parameters for annual total cost	R =	0.132	
	i =	0.10	
	s =	0.0	
Coefficient for penalty terms	W =	1.0×10^3	yen/kWh

Table 2 Computation times in evaluating upper bounds for optimal value of maximum regret

(a) α = 0.2

Case	Repeat	Computation time s	
		Conventional	Novel
I	1st	0.13	0.64
	2nd	0.13	0.10
	3rd	0.12	0.87
II	1st	1.26	3.93
	2nd	3.09	0.96
	3rd	1.70	2.79
	4th	2.26	3.27
III	1st	29.14	5.36
	2nd	3.37	6.62
	3rd	795.79	7.35
	4th	606.81	7.09

(b) α = 0.4

Case	Repeat	Computation time s	
		Conventional	Novel
I	1st	0.26	1.50
	2nd	0.18	0.16
	3rd	0.83	1.48
II	1st	228.56	5.38
	2nd	0.86	0.97
	3rd	562.49	6.75
	4th	304.02	9.13
	5th	494.50	—
III	1st	660.84	81.97
	2nd	33.63	5.42
	3rd	3600.00*	33.67
	4th	—	118.36

* Limit for computation time attained

Table 3 Upper and lower bounds for optimal value of maximum regret

(a) α = 0.2

Case	Repeat	Conventional $\times 10^6$ yen/y		Novel $\times 10^6$ yen/y	
		Upper bound	Lower bound	Upper bound	Lower bound
I	1st	5.85	0.00	5.85	0.00
	2nd	14.55	5.37	14.55	5.37
	3rd	5.37	5.37	5.38	5.37
II	1st	4.25	0.00	4.25	0.00
	2nd	16.45	3.87	16.45	3.87
	3rd	3.92	3.89	3.93	3.89
	4th	3.89	3.89	3.93	3.89
III	1st	6.79	0.00	6.80	0.00
	2nd	14.01	4.93	14.13	4.94
	3rd	4.98	4.94	4.99	4.95
	4th	4.94	4.94	4.95	4.95

(b) α = 0.4

Case	Repeat	Conventional $\times 10^6$ yen/y		Novel $\times 10^6$ yen/y	
		Upper bound	Lower bound	Upper bound	Lower bound
I	1st	20.13	0.00	20.13	0.00
	2nd	59.85	16.19	59.85	16.19
	3rd	16.19	16.19	16.19	16.19
II	1st	20.52	0.00	20.52	0.00
	2nd	60.37	18.37	60.37	18.37
	3rd	18.41	18.37	18.41	18.37
	4th	18.40	18.38	18.39	18.38
	5th	18.38	18.38	—	—
III	1st	22.53	0.00	22.53	0.00
	2nd	52.95	16.99	53.08	17.00
	3rd	17.03~24.63*	—	17.04	17.01
	4th	—	—	17.01	17.01

* Range at limit for computation time attained

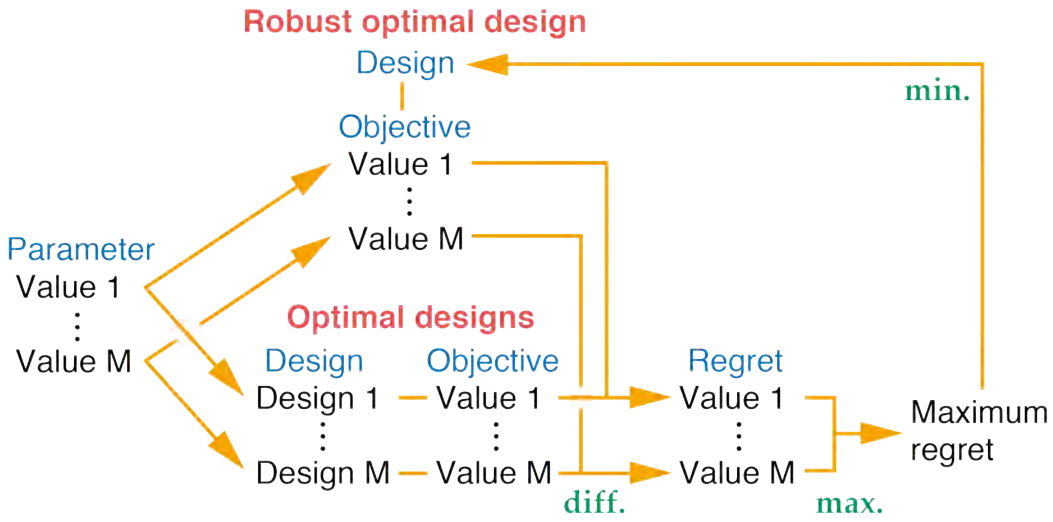


Fig. 1 Basic concept of robust optimal design based on minimax regret criterion

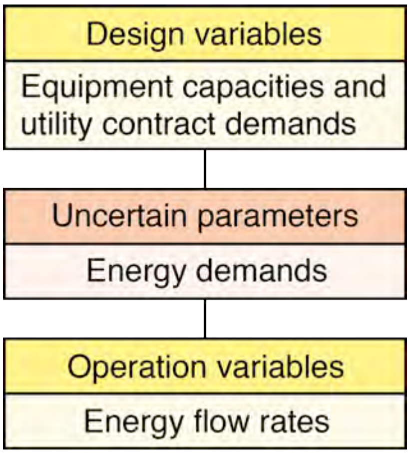


Fig. 2 Hierarchical relationship among design variables, energy demands, and operation variables

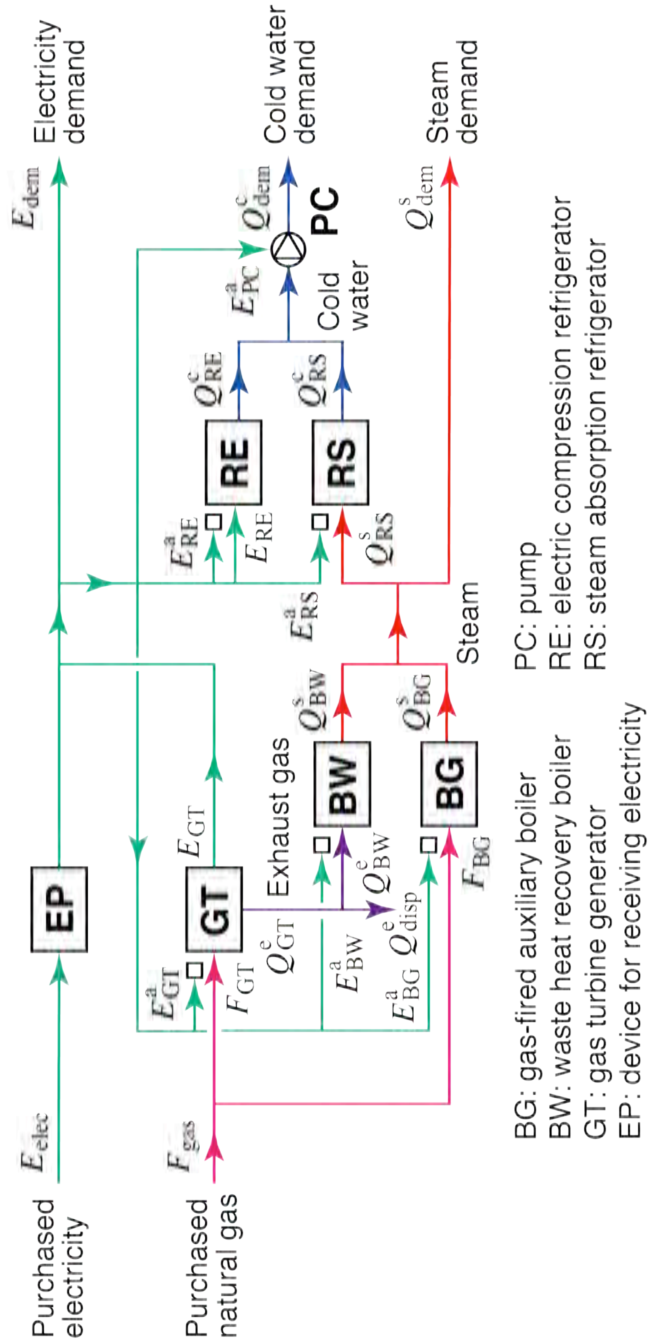


Fig. 3 Configuration of gas turbine cogeneration system

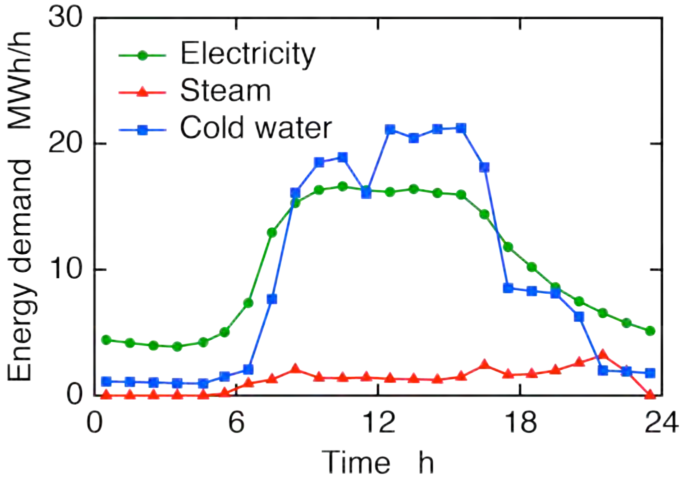


Fig. 4 Average energy demands in summer

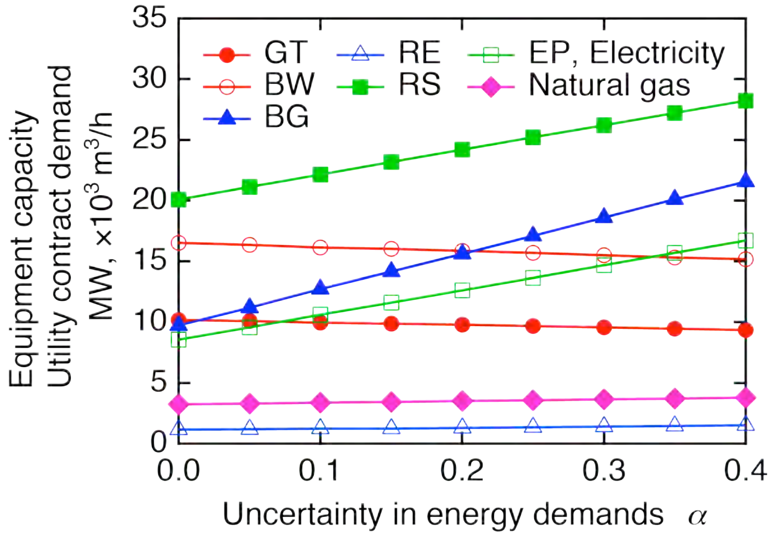


Fig. 5 Relationships between uncertainty in energy demands and optimal values of equipment capacities and utility contract demands of cogeneration system

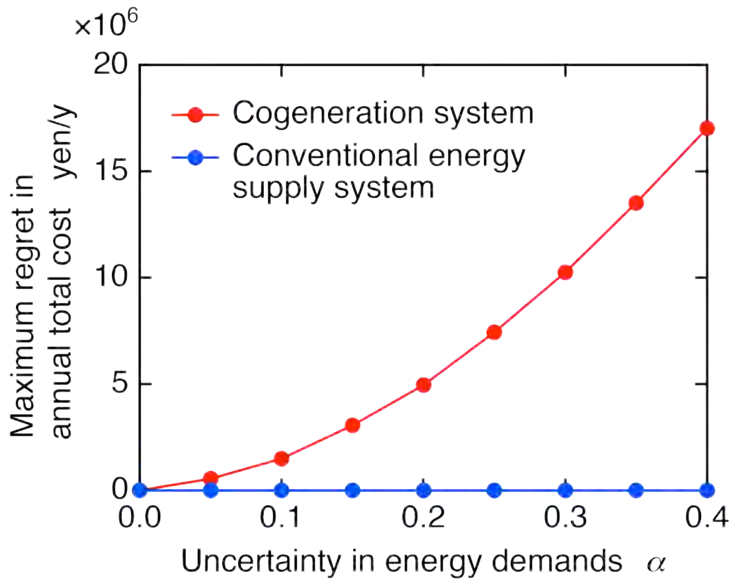
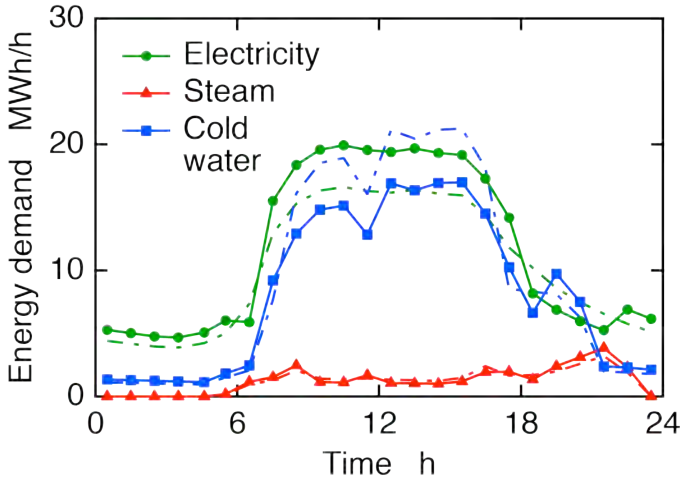
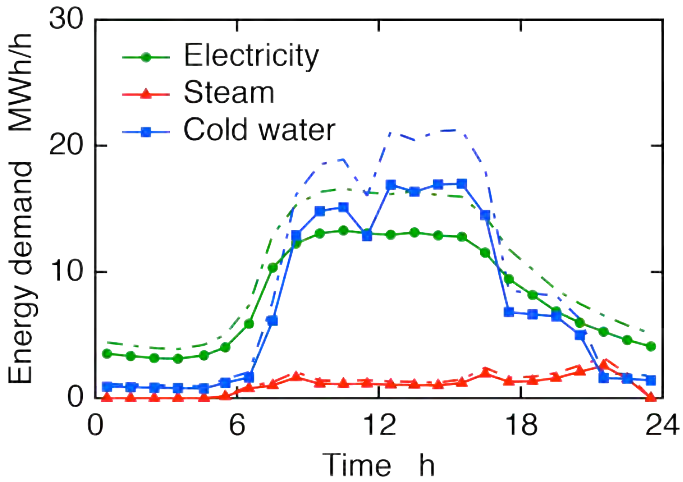


Fig. 6 Relationships between uncertainty in energy demands and optimal values of maximum regret in annual total cost of cogeneration and conventional energy supply systems



(a) Largest demands



(b) Smallest demands

Fig. 7 Energy demands in summer which give maximum regret

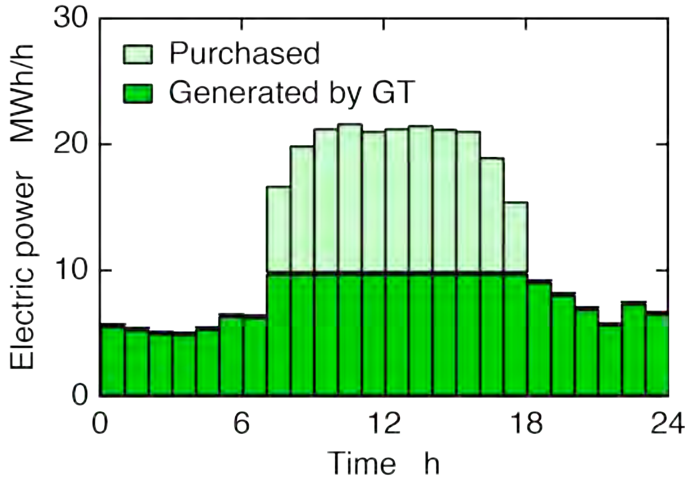


Fig. 8 Optimal operational strategy of robust optimal design for largest energy demands in summer which give maximum regret

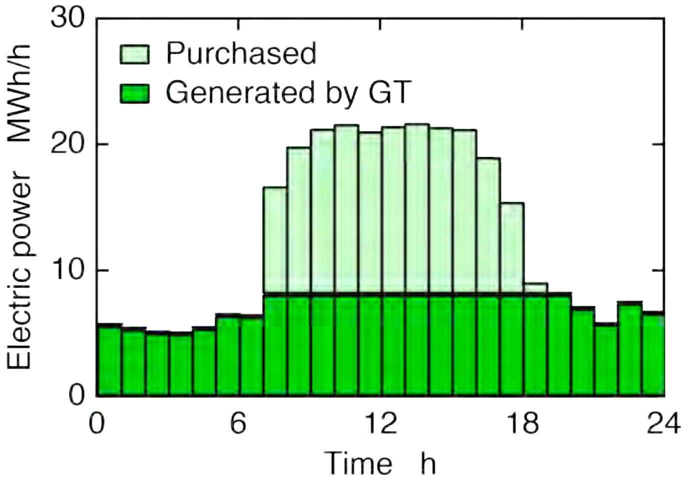


Fig. 9 Optimal operational strategy of optimal design for largest energy demands in summer which give maximum regret

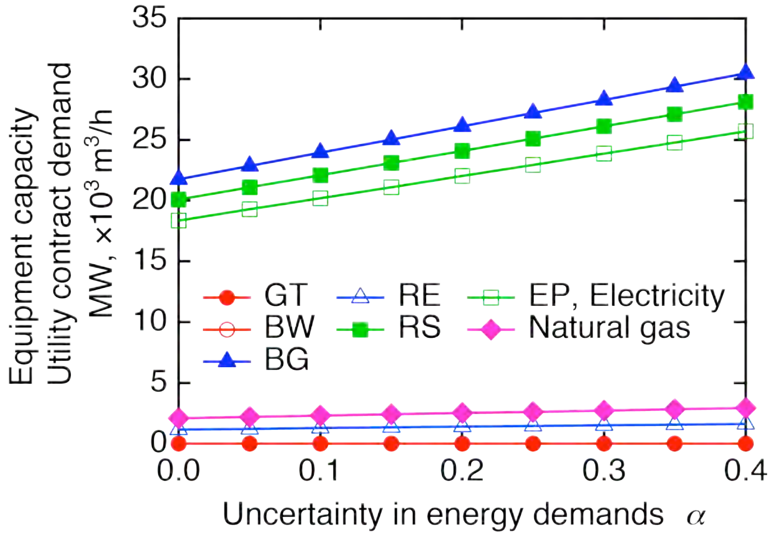


Fig. 10 Relationships between uncertainty in energy demands and optimal values of equipment capacities and utility contract demands of conventional energy supply system

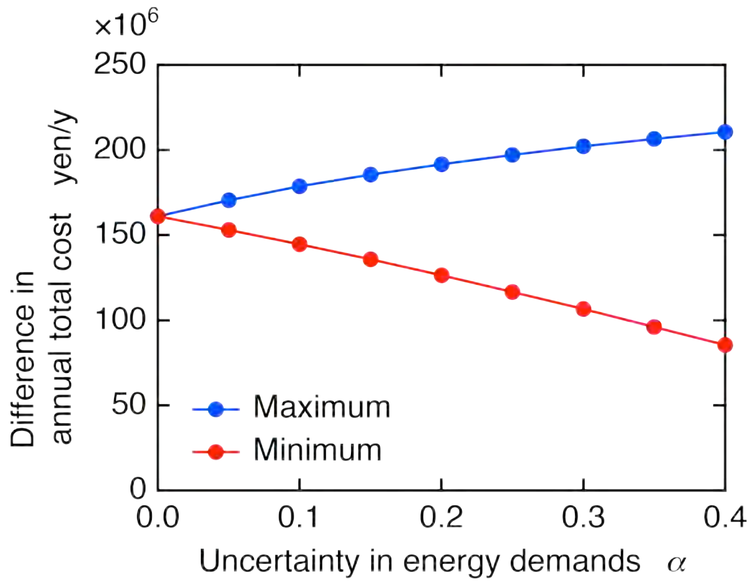
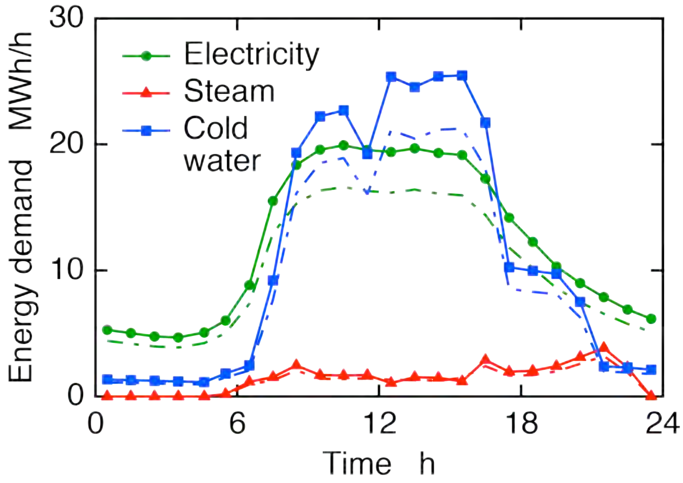
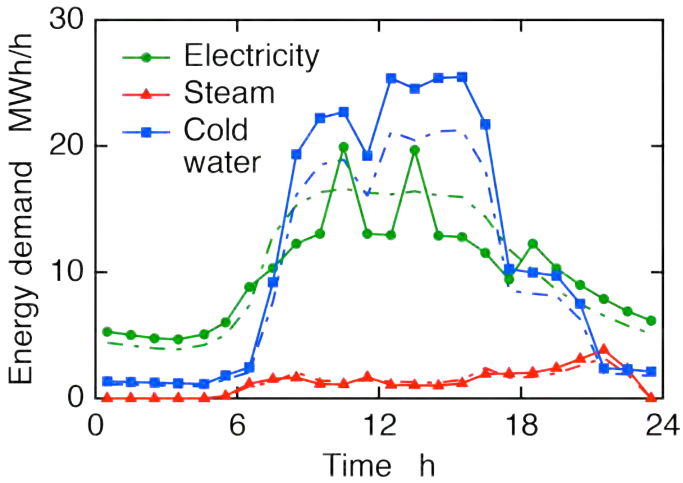


Fig. 11 Relationships between uncertainty in energy demands and both maximum and minimum differences in annual total cost of cogeneration and conventional energy supply systems

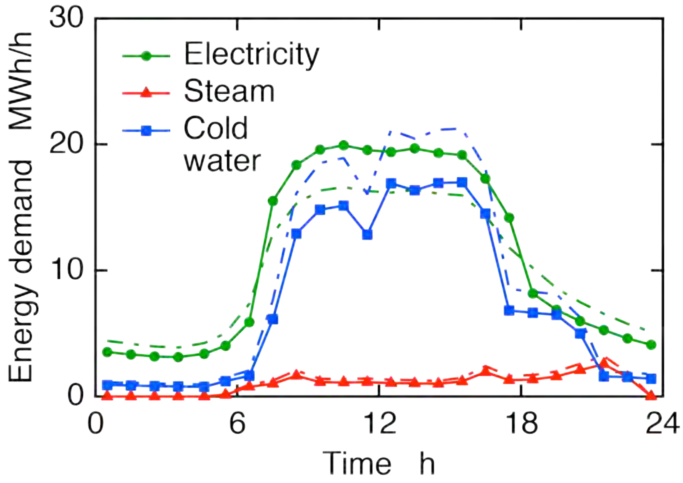


(a) Largest demands

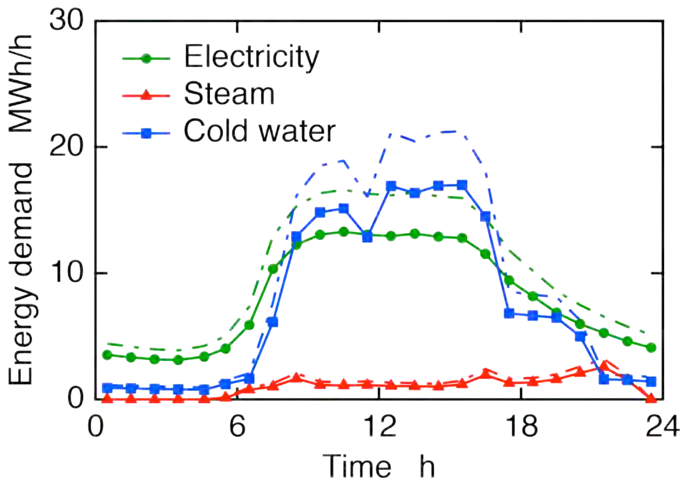


(b) Smallest demands

Fig. 12 Energy demands in summer which give maximum difference



(a) Largest demands



(b) Smallest demands

Fig. 13 Energy demands in summer which give minimum difference

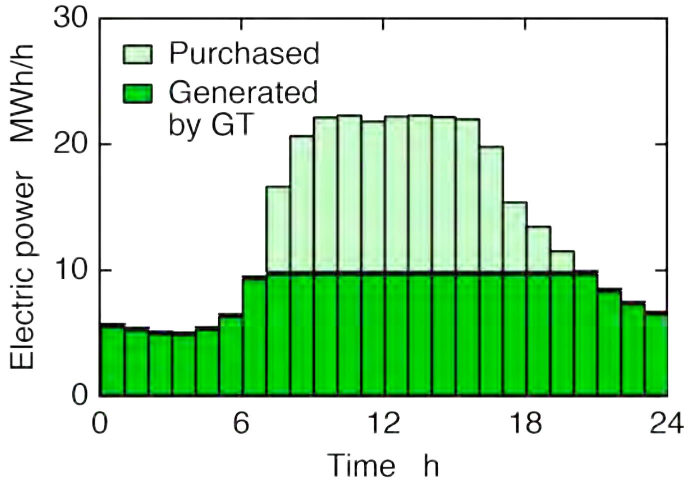


Fig. 14 Optimal operational strategy of cogeneration system for largest energy demands in summer which give maximum difference

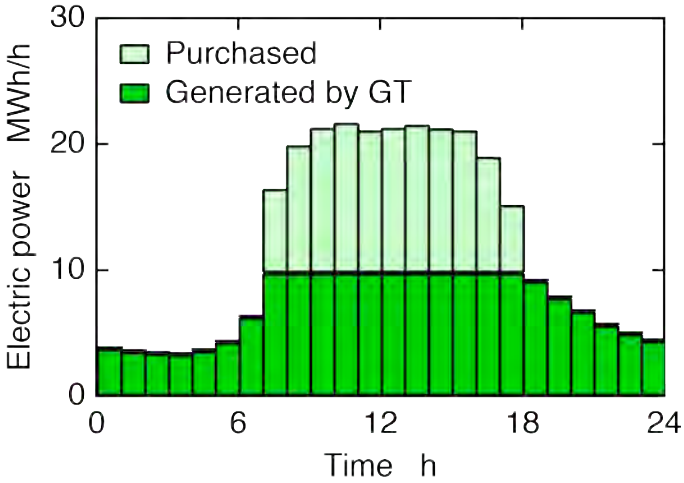


Fig. 15 Optimal operational strategy of cogeneration system for largest energy demands in summer which give minimum difference