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Robust Trajectory Planning for Robotic Communications under Fading Channels.

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Abstract. We consider a new problem of robust trajectory planning for robots that have a physical destination and a communication constraint. Specifically, the robot or automatic vehicle must move from a given starting point to a target point while uploading/downloading a given amount of data within a given time, while accounting for the energy cost and the time taken to download. However, this trajectory is assumed to be planned in advance (e.g., because online computation cannot be performed). Due to wireless channel fluctuations, it is essential for the planned trajectory to be robust to packet losses and meet the communication target with a sufficiently high probability. This optimization problem contrasts with the classical mobile communications paradigm in which communication aspects are assumed to be independent from the motion aspects. This setup is formalized here and leads us to determining non-trivial trajectories for the mobile, which are highlighted in the numerical result.

1 Introduction

Traditionally in wireless literature, the trajectory of the mobile node is assumed to be an exogenous variable and the communication resources are optimized based only on the wireless parameters. However, we have seen an emergence of new technology like unmanned aerial or ground vehicles, drones and mobile robots which have communication objectives in addition to their destination or motion based objectives [1]. Several works have studied trajectory optimization problems when the communication constraint is that of having a target SNR. However, we are interested in the case where the communication requirement is downloading a certain number of bits within a given time.

Previously, we have studied the problem where a mobile robot (MR) must download (or upload) a given amount of data from an access point and also reach a certain destination within a given time period in [1]. However, in [1], we did not account for wireless channel fading and in fact assumed that the wireless signal strength is determined purely based on the path loss. In this work, we want to relax this strong assumption and account for small-scale fading and shadowing effects. In this article we will show how to design offline a robust reference trajectory under limited amount of information and high uncertainty about the wireless channel. This trajectory will allow the MR to reach the goal point and completely transmit the content of its buffer to the access point (AP) with a sufficiently high probability.

In practice, this reference trajectory will be preloaded on the MR prior to the execution on the task and it will serve the MR as guide which may need to be slightly modified according to the wireless channel measurements collected by the MR while executing its task. This adaptation mechanism is outside the scope of this article and we will only focus on the design of the reference trajectory. Future works will address the online adaptation mechanism. The main contributions of this paper are as follows.

- Trajectory planning of a MR starting from an arbitrary point, which must reach a certain target point and download a certain number of bits from a nearby access point.
- Optimization of the trajectory to minimize a cost function which depends on the amount of data left in the buffer to be downloaded and the energy consumed.
- Considering a robust cost function which accounts for the random fluctuations of the wireless channel due to small-scale fading and shadowing effects.

Note that the first two contributions were also provided in [1] for the much more simpler case in which only path-loss is assumed to determine the wireless signal. The rest of the paper is structured in the following manner. We provide the model for the wireless communication system and the robot motion in Section 2. We then provide the problem statement in Section 3 and provide a solution concept in Section 4. Finally, we provide numerical simulations in Section 5.

2 System Model

The position of the MR is given by $\mathbf{p}(t) \in \mathbb{R}^2$, at any time $t \in \mathbb{R}_{\geq 0}$. We assume that the robot starts at position $\mathbf{s}$, i.e. $\mathbf{p}(0) = \mathbf{s}$. The MR and the AP communicate with a frame duration T during which the channel fading is assumed to be a constant, i.e we assume a block fading model. The robot has a buffer with state $b(k) \in \mathbb{Z}_{\geq 0}$ denoting the number of bits it must transmit at the discretized time $k = \lfloor \frac{t}{T} \rfloor$. The initial buffer size is the total file size and is assume to be given by N, i.e., $b(0) = N$. The robot is equipped with a wireless system to communicate with an access point at $\mathbf{p}_{\text{AP}}$ satisfying the following properties.

2.1 Communications system

The MR will move among dynamic scatterers and the bandwidth used for the communication will be lower than the coherence bandwidth. As a consequence the wireless channel between the MR and the access point (AP) will experience time-varying and flat multipath (small scale) fading as well as shadowing (large-scale fading). With loss of generality, we assume, from now on, that the communication problem consists in uploading data from the MR to the AP. The signal received by the AP at time t can be written as

$$y_{\mathrm{AP}}(t, \mathbf{p}(t)) = \left(\frac{h(\mathbf{p}(t), t) s(\mathbf{p}(t))}{\|\mathbf{p}(t) - \mathbf{p}_{\mathrm{AP}}\|_2^{\alpha/2}} \right) x(t) + n_{\mathrm{AP}}(t), \tag{1}$$

where $\mathbf{p}_{\mathrm{AP}}$ is the location of the AP, $h(\mathbf{p}(t), t)$ represents the time-varying small-scale fading which we assume to be Nakagami distributed and $s(\mathbf{p}(t))$ represents the shadowing term which we assume to be lognormal distributed [2]. Nakagami fading is well suited to model the behavior of the multipath fading in many practical scenarios [3]. Without loss of generality we assume $\mathbb{E}[|h(\mathbf{p}(t))|^2] = 1$ and so the p.d.f. of $|h(\mathbf{p}(t))|$ becomes

$$f_h(z, m) = \frac{2m^m}{\Gamma(m)} z^{2m-1} \exp\left(-mz^2\right), \tag{2}$$

where m is the shape factor of the Nakagami distribution. As mentioned before, the shadowing term $s(\mathbf{p}(t))$ is lognormal distributed and so we have $\log\left(s(\mathbf{p}(t))\right) \sim \mathcal{N}\left(0, \sigma_s^2\right)$ with σ_s^2 being the its variance. Also, the normalized spatial correlation of the shadowing is

$$r(\mathbf{p}, \mathbf{q}) = \exp\left(- \frac{\|\mathbf{p} - \mathbf{q}\|_2}{\beta} \right), \tag{3}$$

where β is the decorrelation distance which will be unknown to the MR prior to the execution of the trajectory. Now, the coefficient α in (1) is the power path loss coefficient which usually takes values between 2 and 6 depending on the environment; $x(t)$ is the signal transmitted by the robot with average power $\mathbb{E}[|x(t)|^2] = P$ and $n_{\mathrm{AP}}(t) \sim \mathcal{CN}(0, \sigma_n^2)$ is the zero mean additive white Gaussian (AWGN) noise at the AP's receiver. From (1) we have that the signal-to-noise ratio (SNR) at the AP (in dB) is:

$$\begin{aligned} \Gamma_{\mathrm{dB}}(\mathbf{p}(t)) = {} & 10 \log_{10}\left(\frac{P}{\sigma_n^2}\right) + 20 \log_{10}\left(s(\mathbf{p}(t))\right) + 20 \log_{10}\left(|h(\mathbf{p}(t), t)|\right) \\ & - 10\alpha \log_{10}\left(\|\mathbf{p}(t) - \mathbf{p}_{\mathrm{AP}}\|_2\right). \end{aligned} \tag{4}$$

As a result, the number of bits in the MR's buffer is given by:

$$b(k) = \left\lceil N - \sum_{j=0}^{k} R\left(\widehat{\Gamma}(\mathbf{p}(jT))\right) \right\rceil^{+} \tag{5}$$

where and $\lceil a \rceil^+ = a$ for $a > 0$ and $\lceil a \rceil^+ = 0$ for $a \leq 0$; $\widehat{\Gamma}(\mathbf{p}(jT))$ is the estimate of $\Gamma(\mathbf{p}(jT))$ which is $\Gamma_{dB}(\mathbf{p}(jT))$ in linear scale, N is the initial number of bits in the buffer and $R\left(\widehat{\Gamma}(\mathbf{p}(kT))\right)$ is the number of bits in the payload of the packet transmitted during the duplexing period k. As mentioned above, the number $R\left(\widehat{\Gamma}(\mathbf{p}(kT))\right)$ of bits transmitted in the payload is computed by the MR according to its most recent SNR estimate. So we have (for $b(k) \neq 0$):

$$R\left(\widehat{\Gamma}\right) = R_j, \quad \forall\, \widehat{\Gamma} \in [\eta_j, \eta_{j+1}), \quad j = 0, 1, \cdots, J \tag{6}$$

with $R_j < R_{j+1}$, $\eta_j < \eta_{j+1}$, $R_0 = 0$, $\eta_0 = 0$ and η_1 must be above the sensitivity of the AP's receiver.

2.2 Mobile robot

We assume the MR to be omnidirectional and its velocity is assumed to be controlled directly. This results in its motion described by

$$\dot{\mathbf{p}}(t) = \mathbf{u}(t), \tag{7}$$

where $\mathbf{p}(t)$ is the MR position at time t and $\mathbf{u}(t)$ is the control input which is bounded by:

$$\|\mathbf{u}(t)\|_2 \leq u_{\max}, \tag{8}$$

Finally, the mechanical energy spent by the MR between t_0 and t_1 while using the control signal $\mathbf{u}(t)$ is:

$$E_{\text{mechanical}}(t_0, t_1, \mathbf{u}) = m \int_{t_0}^{t_1} \|\mathbf{u}(t)\|^2 \mathrm{d}t. \tag{9}$$

where m is the mass of the MR.

3 Problem statement

The objective of the robot is to depart from a starting point $\mathbf{s}$ to a goal point $\mathbf{g}$ within a time t_f and transmit the all the content from its buffer to the AP. The desired trajectory is such that it consumes little mechanical energy from the robot and also allows the robot the transmit all the content of the buffer quickly. In addition we want that when the MR follows this trajectory it succeeds in emptying its buffer with a high probability.

We assume that the only knowledge available to the MR (and the designer) about the environment (prior to the execution of the trajectory) is the position of the starting and goal points (i.e., $\mathbf{s}$ and $\mathbf{g}$); an estimate of the path loss coefficient α, but we assume no knowledge about the severity of the small-scale fading (i.e., about the shaping factor m in (2)). Solving the general problem with no approximation is very hard due to the large amount of stochastic perturbations, the shadowing correlation and the large number of terms in the sum of (5). This results in a very complicated expression for the probability of the buffer to be empty at t_f. Therefore, we look at the *most likely* buffer state given by

$$\widetilde{b}(k) = \left[N - \sum_{j=0}^{k} \widetilde{R}\left(\Gamma(\mathbf{p}(jT))\right)\right]^{+} \tag{10}$$

where $\widetilde{R}\left(\Gamma(\mathbf{p}(jT))\right)$ is the statistical mode of $R\left(\Gamma(\mathbf{p}(jT))\right)$, i.e.,

$$\widetilde{R}\left(\Gamma(\mathbf{p}(kT))\right) = \max\left(\underset{R\in\{R_j\}_{j=0}^{J}}{\operatorname{argmax}} \Pr\left(R\left(\Gamma(\mathbf{p}(kT))\right) = R\right)\right). \tag{11}$$

This results in the following optimization problem

$$\begin{aligned}
&\underset{\mathbf{u}}{\text{minimize}} \qquad \theta_1 \int_0^{tf} \frac{\|\mathbf{u}(t)\|_2^2}{u_{\max}^2}\mathrm{d}t + \theta_2 \sum_{k=0}^{\lfloor \frac{t_f}{T} \rfloor} \frac{T\widetilde{b}(k)}{N}\\
&\text{s.t.}\\
&\dot{\mathbf{p}}(t) = \mathbf{u}(t)\\
&\|\mathbf{u}(t)\|_2 \leq u_{\max},\\
&\mathbf{p}(0) = \mathbf{s}, \quad \mathbf{p}(t_f) = \mathbf{g},\\
&\sum_{k=0}^{\lfloor \frac{t_f}{T} \rfloor} \widetilde{R}\left(\Gamma(\mathbf{p}(kT))\right) \geq r_R N.
\end{aligned} \tag{12}$$

The optimization target is a convex combination of the energy spent in motion by the robot (9) and of a second term which estimates how quickly the buffer is emptied. This second term is a sum over the most likely number of bits left in the buffer at time instant $t = kT$ (i.e., $\mathbb{E}\left[b(k)\right]$). The coefficients $\{\theta_k\}_{k=1}^{2}$ of the convex combination determine the relative importance of each optimization criterion.

Note that due to the stochastic nature of the channel we can not ensure that when the MR follows the reference trajectory it will always be able to empty its buffer but we can ensure that this happens with a certain probability. As calculating the actual probability of failing to meet the communication requirement constitutes a very hard task as explained above, we introduce $r_R \geq 1$ which is an overestimation parameter selected by the designer. The final constraint in (12) ensures that the sum of the statistical mode of the bits transmitted in the payload over all the trajectory is equal to an overestimation of the initial number of bits in the buffer, i.e., $r_R N$. So when the trajectory is actually executed, the probability that the buffer will be emptied will be high and by increasing the overestimation parameter r_R we can reduce the probability of the MR failing to empty its buffer when it reaches the goal point $\mathbf{g}$. The term $\widetilde{b}(t)$ is a discreet and deterministic function of the MR's position. This difference makes the problem much more feasible to solve.

4 Proposed Solution

Now, to solve the optimization problem (12) we first define the region $\mathcal{A}_j$ as:

$$\mathcal{A}_j = \{\mathbf{p} \mid \widetilde{R}\left(\Gamma(\mathbf{p})\right) = R_j\}. \tag{13}$$

Due to the wireless channel model the region $\mathcal{A}_J$ is circular while the shape of region $\mathcal{A}_j$, for $j = 1, 2, \cdots, J-1$, is a ring with inner and outer radii of r_{j+1} and r_j respectively. And r_j is given by:

$$r_j = \min\left(\left\{r \mid \widetilde{R}\left(\Gamma\left(r[\cos(\theta) \quad \sin(\theta)] - \mathbf{p}_{\mathrm{AP}}\right)\right) = R_j\right\}\right) \tag{14}$$

The radii r_j are computed from the channel statistics which can be estimated using the techniques presented in [4]. Nevertheless, for lack of space we do not provide here the details on how to compute it.

We also define $\mathbf{u}_j(t)$ as any control law that takes the vehicle through the regions $\{\mathcal{A}_k\}_{k=0}^{j}$. The set of all control laws $\mathbf{u}_j(t)$ will be denoted as $\mathcal{U}_j$ and $\mathcal{U} = \cup_{j=0}^{J}\mathcal{U}_j$ is the set of all control laws.

One simple way to solve (12) is to first solve it with the additional constraint $\mathbf{u} \in \mathcal{U}_j$, once for each different value of $j = 1, 2, \cdots, J$. We will denote as $\mathbf{u}_j^*(t)$ the optimum control law that solves (12) under the additional constraint $\mathbf{u} \in \mathcal{U}_j$ and $\mathbf{u}^*(t)$ as control law that solves (12) under the constraint $\mathbf{u} \in \{\mathbf{u}_j^*(t)\}_{j=1}^{J}$. Therefore to solve (12) we will calculate all the optimum control signals $\mathbf{u}_j^*$.

In order to minimize the mechanical energy term in the optimization target of (12) the optimum control law $\mathbf{u}_j^*(t)$ must make the robot enter and exit the convex hull of each region $\{\mathcal{A}_n\}_{n=0}^{j}$ at most once. These input and output points to the convex hull of the area $\mathcal{A}_j$ are denoted by $\mathbf{i}_j$ and $\mathbf{o}_j$ respectively. We regroup these points in the following set $\mathcal{C}^j = \{\mathbf{s}, \mathbf{i}_1, \mathbf{i}_2, \cdots, \mathbf{i}_j, \mathbf{o}_j, \mathbf{o}_{j-1}, \cdots, \mathbf{o}_1, \mathbf{g}\}$ and index them as follows:

$$\begin{array}{l} \mathbf{c}_0^j = \mathbf{s}, \\ \mathbf{c}_n^j = \mathbf{i}_n, \quad \text{for} \quad n = 1, 2, \cdots, j, \\ \mathbf{c}_n^j = \mathbf{o}_{2j+1-n}, \quad \text{for} \quad n = j+1, j+2, \cdots, 2j, \\ \mathbf{c}_{2j+1}^j = \mathbf{g}. \end{array} \tag{15}$$

where $\mathbf{s}$ and $\mathbf{g}$ are the starting and goal points for the robot. In addition, t_n is the time instant in which the robot is at $\mathbf{p}_n^j$ and:

$$\tau_n t_f = (t_{n+1} - t_n), \quad n = 0, 1, \cdots, 2j \tag{16}$$

where:

$$\sum_{n=0}^{2j} \tau_n = 1, \quad \tau_n > 0, \tag{17}$$

Note that the coefficients $\{\tau_n\}_{n=0}^{2j}$ determine the portion of time t_f that the robot takes to go from $\mathbf{c}_{n-1}^j$ to $\mathbf{c}_n^j$. Let us also write the points belonging to $\mathcal{C}^j$ in polar coordinates as:

$$\mathbf{c}_n^j = r_n^j[\cos(\phi_n^j) \quad \sin(\phi_n^j)]^T. \tag{18}$$

From the definition of $\mathbf{i}_n$ and $\mathbf{o}_n$ we know that they lie in a circle of radius r_n which can be computed from the p.m.f. of $R\left(\Gamma(\mathbf{p}(kT))\right)$. Therefore we know $\{r_n^j\}_{n=1}^{2j}$ and as a consequence the only unknowns to uniquely determine $\mathcal{C}^j$ are the angles[5] $\{\phi_n^j\}_{n=1}^{2j}$, where the ϕ_n^j is the angle of $\mathbf{c}_n^j$ respect to the AP.

[5] Since ϕ_0 and ϕ_{2j+1} are the angles of $\mathbf{s}$ and $\mathbf{g}$ they are also known.

Now, the optimum control law $\mathbf{u}_j^*(t)$ takes the robot from $\mathbf{c}_0^j$ up to $\mathbf{c}_{2j+1}^j$ in ascending order through each point in $\mathcal{C}^j$. We can also see that the second term in the optimization target of (12) depends only the time spent in each region $\mathcal{A}_j$ (i.e., on the durations $\tau_k t_f$) and not on the shape of the particular path followed by the robot nor by its velocity profile. So, the velocity profile and the path must be selected to minimize the mechanical energy (i.e., the first term in the optimization target (12)). To do so the vehicle must go from $\mathbf{c}_{n-1}^j$ to $\mathbf{c}_n^j$ in a time $\tau_n t_f$ (to be determined) using minimum energy. Using calculus of variations [5] we can show that this is achieved by:

$$\mathbf{u}_j(t) = \frac{\mathbf{c}_n^j - \mathbf{c}_{n-1}^j}{\tau_{n-1} t_f} \quad \forall \quad t \in [t_{n-1}, t_n). \tag{19}$$

Therefore if we add the constraint $\mathbf{u} \in \mathcal{U}_j$ and then we optimize $\{\tau_n\}_{n=0}^{2j}$ and the angles $\{\phi_n^j\}_{n=1}^{2j}$ we obtain $\mathbf{u}_j^*(t)$. Now, if we use the constraint $\mathbf{u} \in \mathcal{U}_j$ and select $\mathbf{u}_j(t)$ to take the form (19) then the optimization target of problem (12) becomes:

$$\mathcal{J}\left(\{\tau_n\}_{n=0}^{2j}, \{\phi_n^j\}_{n=1}^{2j}\right) = \theta_1 \sum_{n=1}^{2j+1} \frac{\|\mathbf{c}_n^j - \mathbf{c}_{n-1}^j\|^2}{u_{max}^2 \tau_{n-1} t_f} + \theta_2 \sum_{k=0}^{\lfloor \frac{t_f}{T} \rfloor} \frac{T\widetilde{b}(k)}{N}. \tag{20}$$

And using (13), (19) and the constraint in (12) we have the following approximation:

$$\frac{\tau_j t_f}{T} R_j + \sum_{n=0}^{j-1} \left(\frac{(\tau_{2j-n} + \tau_n) t_f}{T} \right) R_n \geq r_R N \tag{21}$$

So, taking into account (19)-(21) the optimization problem (12) becomes:

$$\begin{aligned}
&\underset{\{\tau_n\}_{n=0}^{2j}, \{\phi_n^j\}_{n=1}^{2j}}{\text{minimize}} \quad \mathcal{J}\left(\{\tau_n\}_{n=0}^{2j}, \{\phi_n^j\}_{n=1}^{2j}\right) \\
&\text{s.t.} \\
&\sum_{n=1}^{2j+1} \tau_n = 1, \quad \tau_n > 0, \\
&\frac{r_n^2 + r_{n-1}^2 - 2 r_n r_{n-1} \cos\left(\phi_n^j - \phi_{n-1}^j\right)}{\tau_n^2 t_f^2} \leq u_{max}^2, \; n = 0, 1, \cdots, 2j \\
&\mathbf{c}_n^j = r_n^j [\cos(\phi_n^j) \quad \sin(\phi_n^j)]^T, \\
&\mathbf{c}_0^j = \mathbf{s} \;\; \mathbf{c}_{2j+1}^j = \mathbf{g}, \\
&\tfrac{\tau_j t_f}{T} R_j + \sum_{n=0}^{j-1} \left(\frac{(\tau_{2j-n} + \tau_n) t_f}{T} \right) R_n \geq r_R N
\end{aligned} \tag{22}$$

where the first line of constraints ensures that the coefficients $\{\tau_k\}_{k=0}^{2j}$ determine the portion of the total time t_f taken to go from one point in $\mathcal{C}^j$ to the next one. The next line of constraints establishes the maximum velocity of the robot. The final constraint is the robust constraint which will allow the designer to obtain a high probability of the MR emptying completely its buffer.

Table 1. Performance of different trajectories

trajectory	$\mathbb{E}[t_{empt}]$ (s)	P_S	$Energy/(m\lambda^2)$ (J)
$\mathcal{T}_0$	14.46	0.0868	0.1859
$\mathcal{T}_1$	10.41	0.7206	0.8139
$\mathcal{T}_2$	7.72	0.9347	4.8924
$\mathcal{T}_3$	7.68	0.8652	3.8928
$\mathcal{T}_4$	7.59	0.9262	4.7909

To solve the optimization problem (22) we first express the angles $\{\phi_n^j\}_{n=1}^{2j}$ as function of the durations $\{\tau_n\}_{n=0}^{2j}$. This is achieved by deriving the optimization target of (22), see more details in [1]. Then we use simulated annealing algorithm (SAA) [6] to optimize the durations $\{\tau_n\}_{n=0}^{2j}$. This concludes the discussion about the optimization of the trajectory and in the next section we present some simulations to better understand its behaviour and observe its performance.

5 Simulations

In this section we present some simulations to gain some insight about the trajectories obtained by the method presented in this paper. We select $10\log_{10}\left(\frac{P}{\sigma_n^2}\right) = 33dB$. Now, the initial number of bits in the buffer $b(0) = 600N_s$ while the possible amount of bits transmitted in one packet can be $R_0 = 0$, $R_1 = 4N_s$, $R_2 = 16N_s$, $R_3 = 64N_s$ where N_s is the number of symbols transmitted in one packet. Note that such values for the number of bits transmitted in the payload can be obtained using a rectangular M-QAM modulation. Now, regarding the thresholds $\{\eta_j\}_{j=0}^{J}$ we fix them so that the bit error rate is at least 10^{-3}.

Now regarding the channel we select the path loss coefficient as $\alpha = 2$, shadowing variance $\sigma_s^2 = 2.5$ and then for the decorrelation distance we select $\beta = 10\lambda$, where λ is the wavelength of the RF carrier used for communications.

We select the starting and the goal points to be $\mathbf{s} = [8\lambda \;\; 0]$ and $\mathbf{g} = [9 \;\; -6]\lambda$ while we locate the access point at the origin. Then the time to reach the goal point is $t_f = 20$s, the period between packets $T = 100$ms and the maximum velocity of the MR is 10λ per second.

First of all we consider for references a trajectory that goes from $\mathbf{s}$ to $\mathbf{g}$ using minimum energy. This is achieved by a linear path between both points and a constant velocity profile. We will denote such trajectory as $\mathcal{T}_0$. Then we consider a trajectory $\mathcal{T}_1$ optimized according to (22) with $\theta_1 = 1$, $\theta_2 = 0$ and $r_R = 1$. This trajectory is optimized to use minimum energy while satisfying constraint (21). Then we also consider another trajectory $\mathcal{T}_1$ optimized according to (22) with $\theta_1 = 0$, $\theta_2 = 1$ and $r_R = 1$. This trajectory is optimized to empty the buffer as quick as possible.

In Fig. 1 we can observe the paths corresponding to the trajectories $\mathcal{T}_0$, $\mathcal{T}_1$ and $\mathcal{T}_2$. We first note that the path corresponding to $\mathcal{T}_1$ is shorter than the path corresponding to $\mathcal{T}_2$ which agrees with the fact that the trajectory $\mathcal{T}_1$ is optimized to minimize the energy consumed (while satisfying constraint (21)). Then regarding the shape of the paths we see that the path of $\mathcal{T}_2$ reaches $\mathcal{A}_2$ through the shortest path, this is done in order improve as quick as possible the transmission rate in order to empty the buffer as soon as possible. Now, regarding the path for $\mathcal{T}_1$ the robot reaches $\mathcal{A}_2$ by moving in an orthogonal direction with respect to the vector $\mathbf{g} - \mathbf{s}$, by doing so the robot minimizes the amount of deviation from $\mathbf{g}$ which reduces then the distance total distance travelled and consequently the energy spent.

When we observe the velocity profiles of both trajectories in Fig. 2 we first note that the period with highest velocity takes place from $t = 0$ until $t = \tau_1$ this is because the robot is rushing to get out from $\mathcal{A}_0$ to start transmitting as many bits as possible. Then we also observe that the minimum velocity occurs when the robot reaches the inner most area of the trajectory (in this case $\mathcal{A}_2$). This is in order to spend as much time as possible in that area with the best channel conditions in the trajectory.

Then, in table 1 we observe the average time in which the buffer is emptied $\mathbb{E}[t_{empt}]$, the probability of success P_S (i.e., the probability of emptying the buffer when reaching $\mathbf{g}$) and the amount of mechanical energy used normalized by $m\lambda^2$. As it is expected the trajectory $\mathcal{T}_0$ uses minimum energy but its probability of success is very low (0.0868). On the other hand the probability of success for the optimized trajectories $\mathcal{T}_1$ and $\mathcal{T}_2$ is much higher, 0.7206 and 0.9347 respectively, but due to the larger paths and velocities their energy consumption is higher.

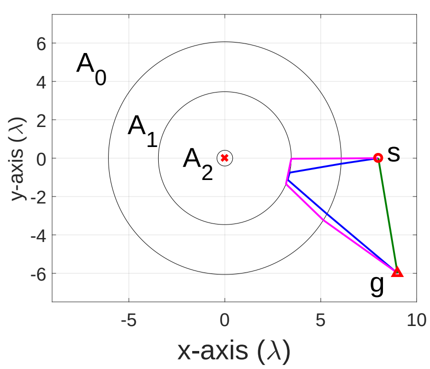

Fig. 1. Paths corresponding to trajectories $\mathcal{T}_0$ (green), $\mathcal{T}_1$ (blue) and $\mathcal{T}_2$ (magenta). Starting point $\mathbf{s}$ represented by a circle, goal point $\mathbf{s}$ represented by a triangle and AP location at the origin. We observe as well the delimitation of the areas $\{\mathcal{A}_j\}_{j=0}^{3}$.

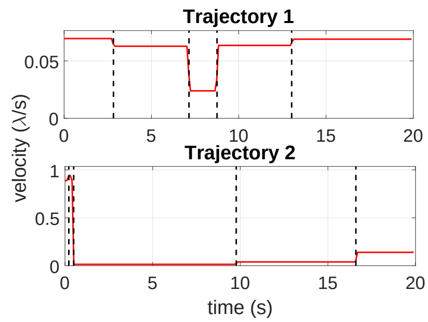

Fig. 2. Velocity profiles of trajectory $\mathcal{T}_1$ (top) and $\mathcal{T}_2$ (bottom). The vertical dashed lines separate the durations $\{\tau_n\}_{n=0}^{2j}$.

Now, we observe the effect of the robustness parameter r_R, see (21). To do so we consider two more trajectories. The first one, denoted $\mathcal{T}_3$, is optimized according to (22) with $\theta_1 = 0.3$, $\theta_2 = 0.7$ and $r_R = 1$. While the second trajectory, denoted $\mathcal{T}_4$, is optimized according to (22) with $\theta_1 = 0.3$, $\theta_2 = 0.7$ and $r_R = 1.5$. We observe in Fig. 3 that their path is really similar (the path corresponding to $\mathcal{T}_4$ is slightly larger) but their velocity profiles are clearly different as we can observe in Fig. 4. The trajectory $\mathcal{T}_4$ spends a larger time in the area $\mathcal{A}_2$ in order to increase the average data rate and therefore increase the probability of success. But by doing so the robot has to move quicker when it gets out from $\mathcal{A}_2$ in order to reach $\mathbf{g}$ in time. By comparing the probabilities of success of $\mathcal{T}_4$ with $\mathcal{T}_3$ in table 1 we observe that increasing the robustness parameter r_R indeed increases the probability of success although it also increases the energy consumption.

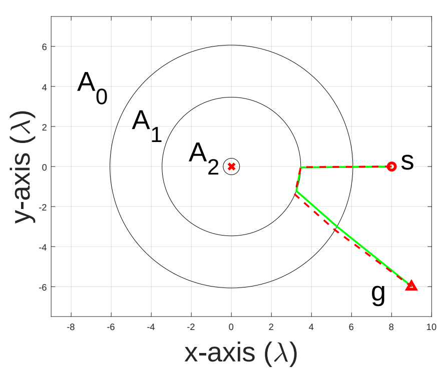

Fig. 3. Paths corresponding to trajectories $\mathcal{T}_3$ (green) and $\mathcal{T}_4$ (dashed red).

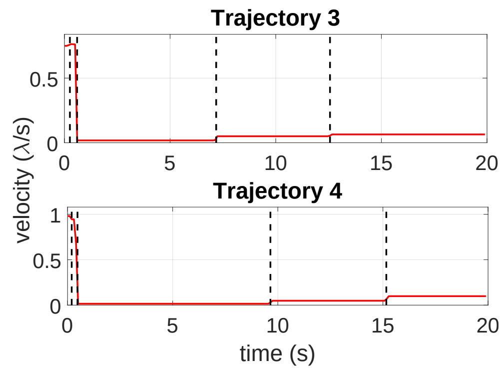

Fig. 4. Velocity profiles of trajectory $\mathcal{T}_3$ (top) and $\mathcal{T}_4$ (bottom).

Note that all the optimized predefined trajectories were able to produce a relatively large probability of success in a fading channel without the use of any kind of diversity. This large probability of success was achieved by optimizing the trajectories using only first order statistics of the wireless channel. In the future we will take into account channel measurements to develop an online mechanism which further improves the success probability while reducing the amount of mechanical energy.

6 Conclusions

We have formulated the problem of robust trajectory optimization for an MR with a target point to reach and a certain number of bits to transmit within a given time. Due to small scale fading and shadowing effects, obtaining a suitable reference trajectory offline is non-trivial. Therefore, we consider the most likely buffer state at each time determined based on the statistical mode and optimize the desired metric by introducing an overestimation parameter for robustness. This approach results in an optimization problem with a feasible solution.

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