School Choice, Competition and Ethnic Segregation in Lancashire: Evidence from structural models of two-sided matching

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Declaration

I declare that, except where explicit attribution is made, the thesis is my own work, and has not been submitted by me in substantially the same form for the award of a higher degree elsewhere.

Abstract

This thesis uses administrative data from the National Pupil Database (NPD) to examine the phenomenon of ethnic sorting into English secondary schools. The thesis investigates the contribution of parental decision-making to observed patterns of school segregation, and also argues that sorting influences the efficacy of competition-based policies for educational improvement.

Focusing upon three towns with ethnically-mixed populations in the North West of England – Blackburn, Oldham and Preston – the thesis investigates the hypothesis that segregation can be explained by the aggregate effect of individual choices. It is proposed that parents' preferences for the ethnic characteristics of school peer groups, and socio-economic differences in parental regard for school test score performance, are important factors in contributing to uneven sorting in schools. The impact of sorting on market structure is also examined, both in relation to the amount of choice that parents enjoy in segregated school markets, and the amount of competition that schools face from neighbouring schools.

In order to study school choice in the three markets, the thesis develops a random utility model for parental preferences, incorporating proximity, test scores, and preferences for ethnic peer groups. However, data on individuals' choice of school are lacking in England, and data on pupil allocations from the NPD do not relate straightforwardly to preferences, since popular schools have limited capacity. To deal with this limitation of the data, the thesis adopts a structural game-theoretic model of two-sided matchings, and develops an innovative method to estimate the parameters of the random utility model. In developing the method the thesis considers the inferential properties – notably identification and large-sample bias – of two-sided empirical models, and provides simulation evidence to support the use of such models for estimation of preferences in large many-to-one markets.

The thesis contributes to the, hitherto primarily US-based, evidence on ethnic self-segregation in schools. It concludes that there is robust evidence that preferences for ethnic peer groups contribute to sorting in the markets studied. The thesis also presents evidence that different socio-economic groups place different weight upon academic performance when choosing a school. In relation to the effects of sorting on competition, the thesis shows that peer preferences and sorting reduce the potential for schools to compete on quality. Finally, the thesis concludes that preference-induced sorting not only hinders the establishment of self-improving school markets, but also inhibits the inter-group contact necessary for socially-cohesive, diverse communities to flourish.

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Chapter 1

Introduction

A central concern of research in education has been the separation of children from different ethnic and socio-economic backgrounds into different schools. The unbalanced sorting or segregation of children into different schools is an important regularity in education systems internationally. Sorting in schools is important for several reasons. First, and perhaps most importantly, schools are intended to be places where children are socialised, first mix with children of other families in a safe context, and learn about the diversity of society and their place in it (Green et al., 2006). In this respect, segregated schools have been blamed for perpetuating deep-rooted ethnic divisions in some communities in the North of England, for example (cf. Cantle, 2001). It is argued that, if schools do not allow the different socio-economic, ethnic and religious groups to come into contact with each other, social cohesion is weakened, and inequalities and divisions of the current generation are reproduced in a new generation.

This thesis is also concerned with another reason for studying sorting which has been overlooked – the effect of segregated school demographics on the structure and functioning of school markets, both as sites for constructive competition, innovation, and the raising of standards, and as facilitators of parental choice. Much theoretical and empirical economic literature has drawn causal links between choice-based education policies and sorting. However, usually the focus is upon policies as causes, and sorting as a consequence. For example, economic models of voucher-based market reforms in the US have shown that free markets in education can lead to cream-skimming and socio-economic stratification (Epple and Romano, 2012a). It is also possible – as we intend in this thesis

– to consider the ways in which *pre-existing* sorting affects the economic behaviour of parents, and the consequent conduct of schools.

1.1 School choice and quasi-markets

Quite apart from their relevance for explaining sorting, school choice policies have, themselves, formed a central topic of education research. In the nearly thirty years since a system of school choice was introduced in England in 1988, successive waves of reform under different governments have continued to reconfigure the structure of the English education system around a *quasi-market* model. This process has paralleled diverse choice-based reforms in other countries. Sweden and Chile have, since the 1980's, implemented systems incorporating substantial school autonomy, private providers and pupil-based funding of schools. The US has experienced a variety of small and medium scale school choice programmes. In England, reforms to the financial structure of the sector have not gone as far as in some other countries, but successive waves of grant-maintained schools, city technology colleges, foundation schools, academies and free schools have ensured that in 2016 only 35% of secondary schools are under local authority control (Gee et al., 2016).

This is not to say, however, that the dominance of school choice is uncontroversial. Researchers and policy-makers have continued to debate the consequences of choice-based reforms for welfare, standards, and social justice. Proponents of school choice tend to rely on three rationales (Gibbons et al., 2006). The first is that parents know their circumstances and their children's needs better than the state, so allowing parents to decide where their children are educated leads to improvements in allocative efficiency and family welfare. The second argument supporting school choice is that choice will impose market-like discipline, by inducing schools to compete for pupils, raising standards and therefore improving outcomes at little cost. The third rationale for school choice is that it will improve social mobility and reduce inequality by providing families an *exit* from failing schools, and disrupting *selection by mortgage* whereby less affluent families tend to be priced out of neighbourhoods containing popular schools. These arguments directly counter arguments that school choice creates sorting, by pointing out that sorting is already present in the absence of choice.

However, the actual causal mechanisms leading to sorting are not well understood. It is clear that much of the imbalance of different groups in schools is due to imbalances in the local demographics schools draw from. However, researchers disagree on whether schools are more, or less segregated than their surrounding communities. Proponents of school choice have argued that, where parents have choice, schools can be less segregated than their communities. However, this does not appear to be borne out by research (eg. Johnston et al., 2004, 2006; Burgess et al., 2005; Harris, 2010), which tends to show a small but robust element of *post-residential* sorting.

Researchers also disagree on the relative contribution of *choices* and *constraints* in explaining observed patterns of post-residential sorting. Some argue that sorting is explained by the ways that different social and ethnic groups engage differently with the choice process, whereas others argue that the different constraints different groups face, in terms of admissions policies, travel and the location of good schools, are more important for understanding patterns of sorting. The relative importance of these factors clearly matters if we wish to study the effects of sorting on school choice and quasi-markets.

1.2 Preferences and sorting

One way to approach sorting, that avoids the dichotomy between choices and constraints is to recognise that, in a quasi-market school system, all constraints facing a family can be understood as the consequences of choices made elsewhere, and that these knock-on effects can ultimately be traced back to the choices of other families. Even the strategic actions of schools with regards to admissions policies, capacity and cream-skimming must ultimately be explained as a response to the actual or expected behaviour of parents.

The various models linking parental preferences to sorting emphasise different decision processes. Whilst much of the English sociological literature focuses on asymmetries of information and social capital that disadvantage working class groups in the admissions process (eg. Ball, 1993), economic models (and the rational choice strand in sociology) tend to focus upon the influence of economic factors on the importance accorded to school performance relative to proximity and convenience (Breen and Goldthorpe, 1996). However, a largely overlooked dimension of parental choice, at least in the English quantitative vein, is the possibility that parents actually have preferences for sorting *per se*; that is, that the criteria parents use to select a school include, amongst others, the composition of each school and the likely socio-economic and ethnic characteristics of their child's future peer group.

In the US, on the other hand, there are several studies that have economically modelled the consequences of peer preferences for sorting (eg. Clotfelter, 1976; Caetano and Maheshri, 2013). In the study of ethnic segregation, for example, it is widely recognised that a significant factor in the severe ethnic segregation of some communities and schools is the reluctance of white families to remain in schools and neighbourhoods when the proportion of minority ethnic groups becomes subjectively large. The phenomenon of white families deserting schools and neighbourhoods, known as "white flight" (Clotfelter, 1976) came to prominence as a result of the de-segregation policies of the 1970's, and has been incorporated into economic models of *tipping dynamics* to explain long-term sorting processes. This thesis posits that parental decision-making is the basic unit of analysis for understanding the existence of segregation and the effects of segregation on quasi-markets.

1.3 Substantive research questions

In relation to the English secondary school sector, the thesis is concerned with two very well-studied phenomena: school segregation and the structure of educational quasi-markets. However, despite the abundance of studies approaching either or both of these topics, there is a scarcity of studies, especially in the UK, that make explicit the role of parental preferences in both explaining and linking these phenomena. One of the reasons for this scarcity is the lack of good data on preferences, as discussed below.

We are interested in the ways that realistic models of school decision-making set up intractable, self-reinforcing dynamics of segregation that problematise the posited competitive mechanisms underlying secondary school markets. Our main focus is the hypothesis that parents take into account the prospective ethnic peer groups of their children, and the role of these *peer preferences* in contributing to ethnic (self-)segregation. We also investigate preferences for "quality" which are vital to quasi-market arguments and, following the lead of other studies (eg. Hastings et al., 2009; Allen et al., 2014), we ask whether these preferences are moderated by socio-economic status.

With regards to these hypothesised choice phenomena, the thesis asks three questions:

1. What is the relative importance of heterogeneous preferences for academic performance and peer groups, in the choice of school, relative to school proximity and other school characteristics?

2. How do these heterogeneous preferences affect the short- and long-run dynamics of sorting in English secondary schools?

3. What is the long-run effect of these preferences on both the amount of school choice parents enjoy, and the competitive pressures schools face?

The ordering of these questions is important: understanding preferences sheds light on the dynamics of sorting; understanding the dynamics of sorting sheds light on the structure and efficacy of secondary school markets. To answer the first question requires a quantitative assessment of the weightings of decision-criteria in school choice. This in turn allows us to populate the parameters of economic models for the prediction of aggregate effects of individual choice.

Although socio-economic differences in school choice are considered, the focus of the thesis is on ethnic sorting and the preferences that lead to this. Whilst the literature on sorting in the US tends to focus on ethnic segregation, the focus in England is usually much more upon socio-economic or class stratification. This difference of emphasis reflects the fact that ethnic segregation in the UK is not nearly as pronounced or as deeply rooted as in the US. In England, the minority ethnic population is a much smaller proportion of the population, and it is only in some urban areas that the minority ethnic population reaches large enough numbers for sorting to even be a meaningful concept.

However, the geographical clustering of minority ethnic communities means that there are places in England where ethnic segregation in schools *is* a significant phenomenon. Where present, the separation of pupils into different schools along ethnic lines, and the interaction of this with socio-economic sorting, sets up patterns of mutual self-segregation that may arguably be more intractable than the more widespread sorting along class lines.

These research questions are motivated partly by a concern that under different behavioural assumptions, quasi-market mechanisms may not work as advertised, and may even be damaging to standards and social mobility. There is necessarily an element of harshness to quasi-market reforms; in order for them to work, schools (and therefore school staff, leaders, and pupils) must be put under pressure to perform, or face the threat of closure. If this is the best way to improve standards and align educational offerings to parents' wishes then this pressure is justified; however, if school systems are such that some schools are unfairly doomed to failure, then much disruption and suffering may be caused for no good purpose. If "failing" schools are predominantly those that serve less affluent communities, the disruption may serve to worsen, rather than improve, social mobility and equity.

1.4 Methodology

In order to obtain estimates of both socio-economically and ethnically heterogeneous preferences, a repeated cross-section of three separate urban school markets is used from the NPD. The three markets are Oldham, Preston, and Blackburn – all in the North West of England. These markets are selected for socio-economically and ethnically diverse patterns of mixing and sorting and a recent history of market-based structural changes.

The primary empirical task in this thesis is the estimation of the relative weightings of different criteria in the decision-making of parents about secondary schools. Econometric models of behaviour within the structural tradition are ideal for this task. The basic idea behind structural models is that empirically-estimable parameters in the statistical model correspond to theoretical parameters in a counterpart economic model of decision-making. These parameter values, once estimated, can be used to mechanistically predict the consequences of counterfactual scenarios.

The most commonly-used structural econometric models are discrete-choice *Random Utility Models* (RUMs), for the estimation of criteria weightings using data relating observed choices to observable characteristics of the chooser and the options to be chosen from. The name random utility model stems from the parametrisation of choice functions in terms of latent decision-utilities. Choosers are assumed to select the option from which they receive the greatest utility. This is the strategy adopted in the thesis, gathering rich data on school and pupil characteristics from the National Pupil Database (NPD).

However, one of the reasons for the scarcity of studies explicitly modelling preferences is that data on pupil preferences or choices is rarely available. With school allocations data the unconstrained choice of each family is not observed, which hinders the clean identification of utility weightings. To this end we develop a new structural model equating observed school *allocations* to the stable equilibria of two-sided matching games. This structural methodology draws on the theory of stable matchings (Gale and Shapley, 1962; Roth and Sotomayor, 1990), and derives an *incomplete* structural model for the identification of choice parameters. The incompleteness of the model is considered and conditions for the consistency of estimates obtained from it are conjectured. Through this model, using some justifiable assumptions about the allocation process and the limiting behaviour of stable matchings, we can estimate the parameters of random utility models for parental preferences that are economically equivalent to parameters estimated from straightforward revealed choice data.

To answer the second and third research questions requires prediction of aggregate patterns of demand for schools based on underlying demographics and counterfactual school characteristics. The main feature of the structural modelling approach is that the economic interpretation of estimated choice models is used to validate out-of-sample predictions made using them. Thus an estimated RUM can be used to predict aggregate demand in counterfactual scenarios in which exogenous features of the data have been manipulated. This feature is useful for both the prediction of long-run sorting dynamics, and the analysis of market structure based on cross-derivatives of aggregate demand.

1.5 Structure of the thesis

The thesis deals with both substantive and methodological subject matter in fairly equal measure, the methodological content forming a parallel and not subsidiary part of the thesis's original contribution. A slightly unusual feature of the structure of the thesis is that, in order to maintain the flow of the development of the substantive investigation, discussion of and motivation of the methodological approach is dealt with lightly during the first few chapters. After the substantive investigation is complete, we then return to more fully motivate and describe the methods that have been developed as part of the thesis.

Chapter 2 surveys the current state of the fields of school segregation and school choice policy, both in England and internationally. The literature on socio-economic and ethnic sorting is reviewed. It is argued that, rather than treating each possible dimension of sorting independently, it is useful to consider generalised causal models of sorting. A framework is posited for the analysis of the causes of sorting. The chapter also discusses the theoretical, empirical, and structural literature on the effectiveness of educational quasi-markets, arguing that structural approaches, based on models of parental choice, bridge the gap between "black-box" empirical analyses and uncalibrated theoretical models. After this, Chapters 3 and 4 describe, motivate and explore the sample of pupils from three towns in the North West of England. Chapter 4 investigates the question of whether sorting in the sample is *post-residential* and uses *Herfindahl* and *Dissimilarity* indices to provide an initial analysis of market structure. It is argued that methods that do not explicitly model underlying decision-making are limited in their ability to explain observed regularities.

Chapter 5 describes the structural model to be used, motivating and comparing this with reference to standard discrete choice models, and lists several specifications of school and pupil covariates to be estimated. Chapter 6 presents the estimated models. After estimating models of parental decision-making, in Chapter 7 these are put to use to study sorting dynamics and the influence of preferences on market structure. It is argued that models of individual decision-making uniquely offer insights into the operation and likely long-run effectiveness of education markets. This completes the substantive investigation.

Chapters 8 and 9 return to methodological considerations: Chapter 8 introduces the theory of stable matchings, and reviews the usefulness of this theory for developing structural econometric models of two-sided markets. The theory also has considerable substantive interest in relation to school choice policy and the design of markets, and these are discussed. Chapter 9 builds upon the theory of stable matchings to investigate the inferential properties of empirical models of two-sided matchings. Issues relating to bias and identification are thoroughly investigated using simulations, and it is argued that despite these issues it is still possible to obtain practically useful inferences using incomplete structural models based upon stable matching theory. We return to the stability likelihood method developed for the substantive investigation, comparing it to similar methods and explaining how it is used in computation. Finally, Chapter 10 draws conclusions from both the substantive and methodological strands of the thesis.

Chapter 2

School choice, quasi-markets and segregation

Within the field of school choice research, two overarching themes stand out. The first is concerned with the distribution of social groups within schools, and asks whether unevenness exists, making the distribution of school demographics unbalanced on socio-economic, ethnic, religious or other measures. This unevenness is usually called segregation, stratification or sorting. One strand of this research programme examines the causes of segregation, whilst a second strand examines the effects of segregation on outcomes such as test scores, attitudes and employment to determine whether sorting leads to disadvantage for certain groups of children, or all children.

The second theme is concerned with the evaluation of policies that seek to liberalise the administration of school allocations, usually accompanied by the decentralisation of school administration and funding. Work within this theme includes economic models and empirical studies that seek to link liberalisation to desirable outcomes by invoking arguments that link competition and market-like forces to performance and efficiency, and studies that criticise these arguments. The implications of liberalisation for segregation and equity are also central to arguments on both sides of this debate.

The first part of this chapter reviews the literature on sorting between schools. After enumerating the various dimensions along which segregation has been studied, a framework is developed for navigation and analysis of the literature relating to the causes of school segregation. The second part surveys the theory and practice of school choice policies in the UK and internationally. The concept of a school choice policy is dissected, before reviewing the literature assessing the effectiveness of school choice reforms, both from a theoretical and empirical approach.

2.1 School choice and segregation

A major theme within the school choice literature is the existence of uneven school populations, or segregation. Segregation in school populations may refer to ethnic segregation, such has been the focus of many studies in the US since the 1960s (see, for example Coleman, 1966), or socio-economic, or "class", segregation, as has been emphasised in the UK context since the mid-1990s (see Gorard (1999) for a review of the early work). Within the latter category, the UK sociological literature has tended to focus on explanations involving theories of class and various kinds of social capital, building upon the theoretical framework of Pierre Bourdieu (Bourdieu and Passeron, 1990; Ball, 1993, 2003), whilst within the economics literature segregation on income, wealth or occupational status has been emphasised. Gibbons and Telhaj (2007) study a different dimension of sorting, sorting by ability, arguing that the "real consideration that seems to be at the back of most parents' and pupils' minds is ability stratification"(Gibbons and Telhaj, 2007, Exec. Summary). They argue that the important aspect of segregation is the extent to which it creates hierarchical relations between schools on measurable aspects of performance, and that the most important cause of this is disparities in intake ability.

Another overlooked dimension of sorting, particularly in UK studies of ethnic segregation, is religion. While there appears to be a dearth of literature explicitly focussing on religious stratification, ethnographic studies such as Byrne and de Tona (2014) report that in parental discourse, ethnic differences are often a shorthand for, or conflated with, religious differences. Whilst this dimension of difference was absent from the original twentieth century debates on racial segregation in the US, this distinction may have increased in importance on both sides of the Atlantic as a consequence of the increasing role of religion in debates around immigration and multiculturalism.

However, a closer reading of the literature reveals that none of these dimensions of segregation can be comfortably treated in isolation. In most contexts, the particular histories of colonial and post-colonial migration dictate the form of relations between ethnicity and economic prosperity. Likewise, socio-economic status is strongly correlated with intake ability, whilst ethnic groups appear to differ in their attitudes towards academic success and consequent attainment. For example, Burgess (2014) provide evidence that minority ethnic children make greater progress at school than other groups. Part of the difficulty in separating these dimensions of segregation lies in the analytical inadequacy of the concepts of ethnicity, class and socio-economic status themselves as simplifications of a more complex set of individual affiliations.

For this reason, before focusing on ethnic segregation and socio-economic segregation in turn, we will develop a framework for the analysis of segregation in schools that is agnostic with regards to the typology used. This approach highlights the common elements running across the analysis of different kinds of segregation, whilst avoiding the reification of one way of categorising children.

In elucidating this causal framework, we will give examples of how different sub-disciplines have emphasised different parts of the causal model, and also how studies focusing upon different sorting typologies tend to focus on different causes. The framework is causal in that it attempts to comprehensively enumerate all of the most important variables that lead to segregation in school populations, and should be applicable in most contexts, although not all variables will be relevant in all contexts.

Figure 2.1 represents the framework diagrammatically. The model is constructed around two dichotomies that arise frequently, either implicitly or explicitly, in the school choice literature. The first is the dichotomy between choices and constraints, as described by Dorsett (1998) in relation to residential sorting; that is, those causes of segregation that relate to parental choices, and those causes that relate to the nature of the school system and the scarcity of supply. This is almost, but not quite, equivalent to the economic distinction of demand-side and supply-side factors. The difference is that factors, such as informational asymmetries, which act on the demand side but cannot be thought of as choices, here are categorised as constraints.

The second dichotomy is the distinction between residential causes of segregation and post-residential causes. Post-residential sorting is sorting that cannot be explained by where families live; it includes the influence of choices parents make, and constraints upon those choices, once residential abode is fixed. The residential and post-residential stage can be thought of as sequential points in time at which decisions are made. The residential decision is made first, and home location may be thought of as, in most cases, exogenous at the point the formal school choice process begins. There is considerable variation in the relative timing of these housing and schooling decisions (Allen et al., 2010). Clearly this decision has an important bearing on subsequent school choice decisions. However, the availability of good schools and school catchments also has an important bearing on residential decisions for many home-buyers. Gibbons et al. (2013) and Machin and Salvanes (2016) have shown that the quality of nearby schools is taken account of in the price of some types of family house. In the US, where neighbourhood schooling is the norm, studies have tended to focus on residential choice within so-called *Tiebout* models (Tiebout, 1956), as a proxy for direct school choice. This evidence suggests that residential decisions already incorporate school decision-making, and therefore considering only post-residential sorting as being related to "school choice" risks underestimating the important contribution of schooling to patterns of sorting.

In fact, a feature of the framework is that very few of its elements can be causally isolated from each other, and any of its elements can be modelled as endogenous with respect to other elements. Most importantly, whilst it is possible to model the factors we have labelled constraints as exogenous features of the context, more accurately they are usually created as a result of the choices of others. At the simplest level, with fixed school supply, each person's choice to express a preference for a given school will make another student less likely to obtain a place at that school. At the next level of sophistication capacities and even the entry and exit of schools from the market may be modelled as endogenous outcomes of the operation of market-like mechanisms, with schools treated as strategic agents with their own incentives. At the highest level, in so-called *political economy* models, the legal, fiscal and administrative structure of the market may be considered as the equilibrium outcome of the competing incentives of parents-as-voters.

An aspect of the relationship between choices and constraints that has perhaps been overlooked, is the extent to which parents may approve of and even prefer schools that manifest certain barriers to entry. Usually in economic models of school choice, parents are assumed to be indifferent with regards to school admissions policies (and other school-level constraints such as capacity control); they obviously prefer that their child be able to access as many schools as possible, but they do not have preferences with regards to the admissions constraints *per se*. However, what little evidence there is suggests that parents do positively value certain kinds of admissions constraints, and will vote to maintain selection policies that are threatened such as, for example, selection by ability in grammar schools (Smithers and Wainwright, 2000). Sociologist Richard Ball makes this point forcibly:

> "For some consumers the point about choice is that they 'require' exclusivity and/or performance advantage... The sort of schooling they value is that sort which is difficult to get into and which produces superior performance outcomes."(Ball, 1993, p12)

Falabella et al. (2014) provide international evidence from Finland and Chile, countries with liberalised school markets, that suggests that parents in those countries explicitly value schools that set up barriers to entry. This kind of behaviour is difficult to explain using current economic models.

With regards to parental choices, the most direct way in which the aggregate decisions of parents may cause sorting is if parents have preferences for sorting *per se*. For example, parents may prefer schools where a greater proportion of the students belong to their child's *in-group*. More controversially, parents may have negative preferences for certain other groups, or parents may even prefer schools with a smaller proportion of their own group.

Within parental choices, it is not only preferences for peer groups that can lead to segregation. If different social groups have different preferences for some school characteristic this may also lead to sorting. Therefore, any systematic between-group variation in preferences may lead to segregation of groups between schools. The main examples of preference differentials that have been studied in the literature are group-moderated preferences for quality and distance to school. Within the sociological literature, there has been an emphasis, as far as heterogeneous preferences are concerned, on the way that social class moderates the overall importance given to school choice, so that lower-SES families will be more likely to choose the neighbourhood school as a default option

(Taylor, 2002). Evidence from qualitative interviews (Taylor, 2002) also suggests that lower-SES groups are more likely to take the preferences of their children into account. A final hypothesis within the remit of parental choice is that some parents "internalise" externally-imposed limitations on their choice set, and post-rationalise decisions that have been made with these limitations in mind. In this way, even choices can be seen to be causally dependent upon constraints. To the extent that there is a group differential in constraints, there may also be differentials in constraint internalisation leading to sorting.

Under the heading of constraints, arguably the most important constraint in the UK is house prices. Parents cannot exercise choice of school if they do not live within the catchment areas of their preferred schools. As mentioned previously, Machin and Salvanes (2016) and other authors have robustly estimated a quite large capitalisation of school quality in the market prices of houses within the relevant (de facto) catchment areas. As for post-residential constraints, most of these relate to supply-side factors that may be, to a greater or lesser extent, within the control of individual schools. Some types of school, including Academies, Free Schools and Voluntary Aided schools, act as their own admissions authorities and can set admissions policies for over-subscription within the Admissions Code. Other schools may not set their own admissions policies, but their Local Authority-mandated admissions policies still constitute constraints on parental choice if the school is over-subscribed.

Apart from admissions policies, other factors within a school's control include the school's overall intake capacity and the school's marketing strategy. It has also been argued (Office of the Children's Commissioner, 2014) that some schools have subtle means of exclusion by, for example, maintaining expensive school uniform policies, or minimising their provision for Special Educational Needs and Disabilities (SEND). Capacity control is an important factor in school choice constraint, as even community-controlled schools have some discretion over the size of their intake. If a school is oversubscribed, the decision to grow involves commitments to invest in capital (extra buildings) and labour (extra teachers) that are inherently risky in the face of uncertain future demand. Allen and Burgess (2010) look at data on high-performing, low-performing and middling schools over a ten-year period, and find that high-performing schools grew by on average 10.3

per cent, whereas middling schools grew only slightly less rapidly at 7.0 per cent. This suggests that school incentives to grow are not strongly linked to performance.

What is more, it is often the case that popular schools enjoy some sort of comparative advantage in terms of intake, where – either as a result of geographical, faith, or academic selection – these schools' intakes consist to a greater degree of pupils who have better prior attainment, more supportive family envionments and/or better behaviour than surrounding schools. For such schools, any increase in capacity would risk diluting this comparative advantage.

Given the riskiness of increasing supply to match demand, it would be unsurprising if some popular schools chose not to grow, but instead to use their popularity to "cream-skim" their applicants to obtain an intake that is both easy to teach and likely to cast the school in a good light. Although cream-skimming is widely discussed in the literature on school choice, there is very little quantitative evidence that schools systematically strategically select students (Office of the Children's Commissioner, 2014; Coldron et al., 2008). On the other hand, there is also little evidence that popular schools make an effort to match supply to demand. Finally, the limited amount and quality of information parents receive to inform school choice constitutes a constraint that affects all parents to a greater or lesser degree.

2.1.1 Horizontal vs. vertical sorting

The preceding section articulated an analytical framework for school sorting anchored by two dichotomies: choice vs constraints; and residential vs. post-residential. A final dichotomy, which will be very useful in analysing sorting and segregation is one which bases its typology on the desires and intentions of members of two groups who are sorted. Assuming, for the moment, that agents have conscious preferences and intentions with regards to sorting (i.e. peer preferences), we can ask whether sorting is driven primarily by one group's preferences for sorting, or both groups' preferences. If it is the former, we can ask whether the "passive" group is ambivalent, or actually has preferences for integration.

Clearly these two scenarios have very different implications for those concerned and for policy-makers. In a scenario where one group prefers separation, but the second group

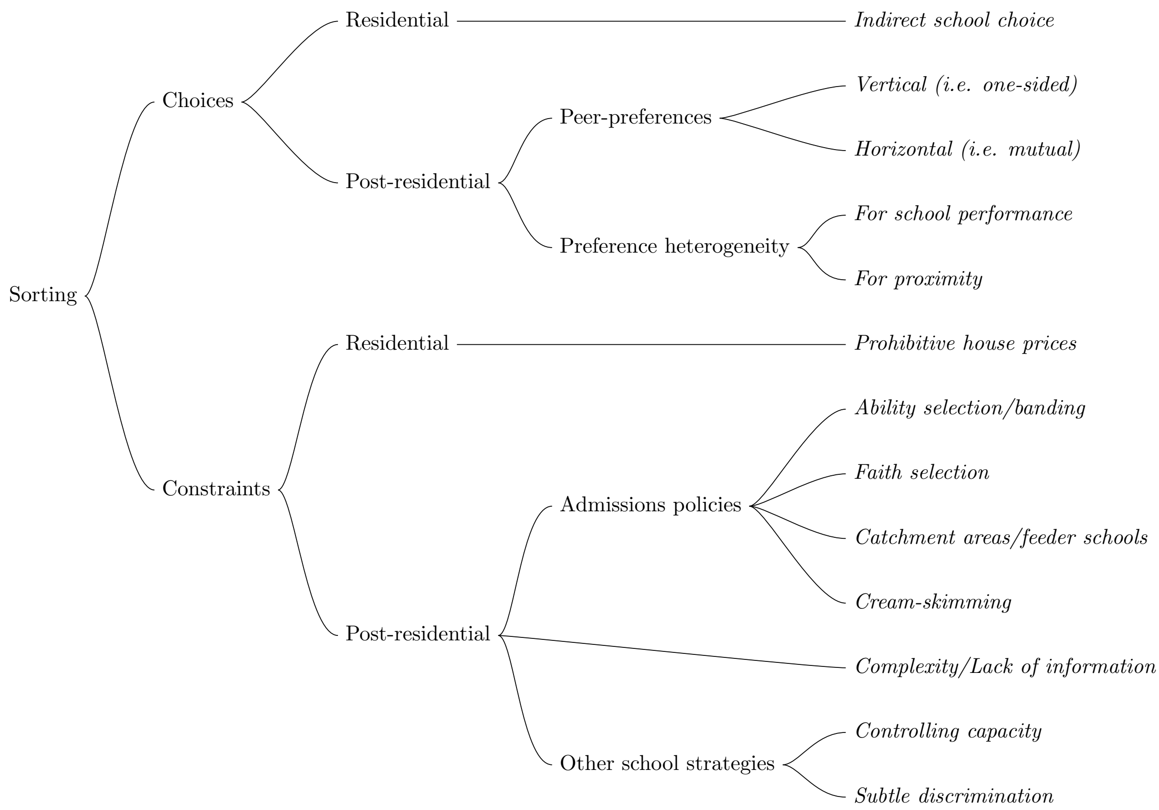


Figure 2.1: *Diagram of the causal factors associated with sorting in schools.* Choices *are those factors where sorting arises as a consequence of the aggregate decisions of parents, whereas* constraints *describe barriers to choice that unequally affect different groups, leading to sorting. The further distinction between* residential *and* post-residential *causes acknowledges that much sorting in schools is caused by underlying sorting in communities, but that schools are themselves a contributory factor in residential sorting.*

wants to integrate, sorting will only arise if the first group has some means to exclude the second group from its favoured schools. Such means may include house prices or admissions policies. In contrast, if both groups prefer sorting, then sorting may arise without constraints, and may be difficult to remedy without somehow changing people's preferences. To distinguish between these two situations we will call sorting arising from mutual peer-preferences "horizontal" sorting, and sorting arising from one-sided preferences "vertical" sorting. Rather than being a dichotomy, it is more accurate to describe this dimension as a continuum, since the extent to which each group prefers separation or integration is continuous.

The next two sections apply the framework to the literature on socio-economic sorting, and then ethnic sorting, highlighting the way that focusing on different dimensions of segregation invokes different emphasis on causes, particularly the relative importance accorded to horizontal and vertical sorting.

2.1.2 Socio-economic or class segregation

There is a large literature on class segregation in England's schools. A central concern of this literature has been the evaluation of the effect of school choice on class segregation, since the introduction of school choice in the late 1980's (e.g. Ball, 1993; Gorard, 1999). A feature of the sociological tradition at least since Ball (1993) has been the de-emphasis of explicit preferences, in favour of a focus on class differentials in capability to make effective choices. Although preferences are implicit in much of the literature (especially the preference of higher-SES groups not to mix with lower-SES groups), the central argument in the sociological tradition seems to have been of school choice as a "game" at which more privileged groups have an unfair advantage. Whilst higher-SES groups are sophisticated choosers, lower-SES groups are forced to settle, and post-rationalise their constrained choices, leading to unreliable reports of satisfaction.

This research inherently imposes a vertical model of peer preferences, where higher-SES groups seek to exclude lower-SES groups, but lower-SES groups are implicitly assumed to be either ambivalent or have an underlying wish to assimilate. In contrast, the rational choice strand championed by Breen and Goldthorpe (1996) suggested that parents have similar underlying preferences, but face very different risks and uncertainty and, beginning from different reference levels, frame the same outcomes in very different ways.

2.1.3 Ethnicity and segregation

The body of literature examining ethnically segregated schools in England is smaller than that for class segregation. Within the qualitative sociological tradition there is a considerable overlap with the work on class segregation in school. During the last ten years Diane Reay, especially, has focused on the intersection of class and ethnicity in school choice (Reay et al., 2007; Reay, 2008) (cf. Weekes-Bernard, 2007). Bridget Byrne and Carla de Tona, on the other hand, specifically investigate the role of ethnicity, and explicit preferences for ethnic mix, in school choice (Byrne and de Tona, 2014).

More demographically-oriented literature such as Johnston et al. (2004), Burgess et al. (2005), Johnston et al. (2006) and Harris (2010) has attempted to quantify the extent of ethnic segregation in England, and answer the question: how much of ethnic segregation in schools is additional to that which can be explained by residential segregation?

Johnston et al. (2004) plot "concentration profiles" that show different levels of segregation for different ethnic groups, and substantial regional variation. In general, Asian students (predominantly south Asian – Indian, Pakistani and Bangladeshi) are more concentrated into segregated schools than black students. Nationally, about 35% of Asian students are enrolled in schools where at least 50% of the students are Asian, whilst only about 10% of black students are in schools where black students are a majority (Johnston et al., 2004, Figure 5). Whilst the novel use of concentration profiles displays a lot of information, Johnston et al. have not compared each ethnicity's concentration profile to a reference concentration profile that it might have if students were allocated randomly. These reference profiles would depend upon each ethnicity's proportion of the total population, and their absence means that it is not easy to compare the levels of segregation of different ethnicities. Nor have they attempted to separate the segregation resulting from uneven residential distribution from post-residential segregation. The latter is arguably of more importance in studying school segregation.

Burgess et al. (2005) calculate Indices of Dissimilarity (ID) and Isolation (II) for both secondary schools and census wards (a geographical unit of analysis), and compare these nationally and by LEA. They find marginally higher levels of segregation, as measured by these indices, in schools than wards. In common with Johnston et al., they find that post-residential segregation is more associated with communities where south Asians are numerous, than communities where blacks are numerous. The study highlights the difficulty of comparing residential segregation to school segregation, since the two are based upon very different areal units. School segregation is implicitly based upon catchment areas as areal units, but these are not directly measurable, usually overlap each other, and are endogeneous with respect to preferences. In addition, they point out that there are on average twice as many wards per LEA as schools, and cite Massey and Denton (1988, p299), advising that "smaller areal units generally produce higher indices of segregation because they are more homogeneous." This fact is used to argue that, if anything, their method underestimates additional post-residential segregation.

Acknowledging this difficulty, Johnston et al. (2006) compare two levels of schooling (Primary and Secondary) to several layers of geographical aggregation (output areas, and lower- and middle-layer super output areas). Rather than using indices to measure segregation, they classify both schools and areas into types depending upon the proportions of whites and ethnic minorities. Types I and II correspond to units where whites form a majority of more than 80% and 50-80% respectively. In type III and IV units, whites form a minority of 30-50% and less than 30% of the population, but no other ethnic group dominates. In type V units, whites form less than 30% of the population *and* some other single ethnic group forms a majority. In the tables they produce by each unit, it is clear that ethnic minorities are much more likely to be enrolled in a type IV or V school than they are to be living in a type IV or V neighbourhood. However, although Johnston et al. provide more options than Burgess et al., they do not get around the problem of incommensurate units of analysis. This pitfall will affect any measure of segregation that is used to compare schools to residential areas on different units, whether it be an index, a graph or a set of categories. In order to overcome this problem, it is necessary to either compare school to residential segregation on the same units (which would have to be school catchment areas) or to model the process of education decision-making itself.

Allen (2007) overcomes the incomparability of units by comparing observed Indices of Dissimilarity at schools, to those computed under a counterfactual where all pupils are allocated using a proximity rule. Allen finds evidence of post-residential segregation, especially in those schools designated own-admissions authorities.

A common feature of these studies is the finding that ethnic segregation is not a nationally homogeneous phenomenon, but depends strongly on local contexts, and manifests itself differently in different areas. A primary observation is that, in most rural and suburban areas – in fact the majority of areas – ethnic minorities do not reach large enough concentrations for ethnic segregation to be apparent in schools. This is because on a larger geographical scale the UK's population is highly segregated, and most of the ethnic minority population is concentrated into a few towns and cities. In localities with smaller concentrations of ethnic minorities, it is not possible for ethnic minorities to reach a critical mass to form a majority in any school. It is not clear whether ethnic segregation still tends to occur in the sense that ethnic minorities form a larger minority in some schools than others, but given that in the country as a whole ethnic minorities form about

20% of the population (Office for National Statistics, 2016) this micro-clustering would be difficult to measure using indices and would have negligible impact on school populations. In spite of this, there appears to be a generalised perception that ethnic segregation is ubiquitous, even (or especially) in areas with very low ethnic minority populations. Johnston et al. (2008, p.88) observe that "the perception of increased segregation appears to be based on more extreme (or localised) cases than on typical experience."

In areas with substantial minority populations Burgess et al. (2005) find that the specifics of segregation, and post-residential school segregation in particular, depend not only on the ethnic mix of the locality but also on the details of each specific situation. For example, they found that towns in the North of England – Oldham, Blackburn and Bradford – tended to have higher levels of segregation (both residential and school) than other areas with comparable ethnic minority populations such as Birmingham and Slough. They also found that London Boroughs tended to have higher levels of post-residential segregation. This perhaps reflects the combination of greater density of schools in the capital, and the greater residential costs, lowering the cost of travelling to school relative to moving house, so that ethnic peer preferences for schools are less likely to be already incorporated into residential choice. Johnston et al. (2007) study ethnic segregation in two towns with substantial minority populations, Bradford and Leicester, with very different outcomes in terms of attitudes and performance.

The effects of ethnically segregated schools are also geographically differentiated. In 2001 Oldham, Burnley and Bradford suffered riots motivated by grievances related to ethnic divisions. In the wake of the Bradford riots, both the Ouseley Report (Ouseley, 2001) and the Cantle Report (Cantle, 2001) stressed the importance of schools in perpetuating cultural isolation. Ouseley (2001) strongly criticised schools for their role in this, advocating "confronting all-white and/or all-Muslim schools about their contribution, or rather lack of contribution, to social and racial integration" (Ouseley, 2001, p.1). However, they also acknowledged that parental choices had a role in school segregation and that many schools were "racially self-segregated" (Ouseley, 2001, p.16). Cantle (2001) emphasised the role of mono-cultural schools in failing to promote ethnic integration, and focussed on faith-based admissions criteria as an example of policies that exacerbate polarisation of intakes.

The case of ethnic tension leading to civil unrest in northern towns, notably Bradford, Burnley, and Oldham begs an explanation that goes beyond ethnicity, since towns in other areas with similar ethnic demographics do not face all of the same problems. An explanation must take account of the economic history of the region during the twentieth century. This will be discussed in more detail in Chapter 3.

In other areas with large ethnic minority populations, the relationship between ethnicity and school choices is more complex. Byrne and de Tona (2014) explore the complexities of preferences for multi-culturalism and ethnic mix of parents of different ethnic backgrounds three areas of Manchester. In face-to-face interviews, they obtain candid reflections on parents' incorporation of "the ethnic mix" into school decision-making. They find that considering ethnicity alongside other demographics such as economic and occupational background is ubiquitous, but that variations emerge depending on the outlook of the parents. Some white parents express concerns about the dilution of what they consider to be traditional British values such as teaching Christian festivals (although Byrne and de Tona note that there is no evidence that this belief is justified). In contrast, more liberal white parents actively seek out ethnically-mixed schools as they perceive mixing with other cultures to be beneficial for their children. Byrne and de Tona find that ethnic minority parents also tend to prefer ethnically-mixed schools where their children are not in a very small minority. They similarly perceive that ethnic mixing is good for their own children, but also express the opinion that it is good for other ethnicities (especially white children, who are nationally the most isolated ethnic group) to mix with their children. What is striking is that in general parents interviewed by Byrne and de Tona do not profess to be "colour-blind" in choosing schools, but rather to take race and ethnicity (and class) into account in finely differentiated ways.

Studies of the extent of, and the effects of ethnic segregation on school populations have a much longer pedigree in the US than in the UK. Until the 1950's several US states operated *de jure* racially segregated school systems in which black children were not allowed to attend schools reserved for white children. When this state of affairs was successfully challenged in *Brown vs. Board of Education* in 1954 (Clotfelter, 2001), many studies asked whether the end of official segregation would lead naturally to desegregation, and what the effects of (de-)segregation on outcomes for both black and white children might be (Coleman, 1966; Rivkin, 1994).

In the years following *Brown*, and in the light of evidence (Coleman, 1966) that segregated schools would lead to inherently unequal outcomes, many states implemented mandatory de-segregation policies, which usually included "busing" black students many miles to attend schools in predominantly white areas. In addition to the econometric literature on the effects of segregation, other studies (Clotfelter, 1976; Rivkin, 1994) examined the long-term trends in segregation to determine if court-mandated de-segregation was working. It was noticed that de-segregation policies appeared to be accompanied by an increase in enrolments of white pupils into the private sector, and also patterns of net migration of white families away from districts with integrated schools, a phenomenon which became known popularly as "white flight" (Clotfelter, 1976).

2.2 Markets, quasi-markets and performance

The second major strand in school choice literature is the analysis and evaluation of school choice from a policy perspective, with a particular focus on the potential for school choice deregulation to create benefits similar to those attributed to the operation of markets: greater choice for the consumer; more efficient allocation of resources; improved school conduct and greater productivity.

Obtaining an overview of academic work in this field is challenging. Much of the difficulty stems from the diversity of geographic and political contexts under which school choice deregulation has been debated, and the many policy variables (not to mention cultural and economic differences) that make comparison between any two contexts problematic. For example, empirical estimates of the effect of greater choice on school performance in England, which operates a constrained hybrid quasi-market, will not be comparable to estimates obtained using similar methods in Chile, which operates a fully deregulated voucher system.

In very broad terms, school choice systems can be split into three categories: fixed allocations; quasi-markets (Le Grand et al., 2008); and full markets. In a fixed-allocation system, which is usually, but not always, a neighbourhood-schooling system, parents have no choice and children are allocated to schools by local authorities. An example of a fixed-allocations system which was not a neighbourhood-schooling system was the bused desegregation system of the 1960's and 1970's in the US.

In a quasi-market, parents are allowed to express a preference for any school, and this is usually coupled with supply-side reforms to allow schools to expand and diversify to meet demand. School funding is dependent to some extent on enrolments, so that schools have incentives to attract pupils. Quasi-markets are differentiated from markets by the fact that school funding is set by the government rather than being determined by the market. In full markets, in contrast, each school has some degree of freedom to set its own prices. The prices schools charge will then, at least under ideal conditions, be determined by a competitive equilibrium. This does not preclude public subsidy of the cost of education, but usually in full markets parents may bear at least part of the cost of their children's education. Note that the term "voucher" system, which is often used in discussion of school choice policy, may denote either a full or quasi-market system. The important aspect is whether or not schools are allowed to charge top-up fees.

Although these categories are broad, many countries operate systems that combine elements from each of them. For example, the allocation system in the UK is a quasi-market, since parents have a right to express preferences for any school, and schools have some freedom to expand to meet demand. However, the quasi-market co-exists with a small fee-paying private sector. In other ways, the UK system operates in practice more like a fixed-allocation system, as supply-side constraints mean that pupils often end up in a local school regardless of their preferences. Fixed-cost funding and smoothed income streams also dilute the effect of per-pupil funding on schools' incentives. Likewise, the US system is differentiated at a state, and even at a county level, with several small-scale voucher programmes (around 65 according to Epple and Romano (2012a)). However, the predominant system throughout the US is a fixed-allocation system, with a large private sector and some open-enrolment "magnet schools".

The main research question in this strand of the literature is whether moving from fixed-allocation to market-like systems (either quasi-markets or markets) improves school conduct and performance. Debates around the level of state intervention in school choice are as old as state-funded education itself. As early as 1859 John Stuart Mill advocated that the government should "leave to parents to obtain the education where and how they pleased, and content itself with helping to pay the school fees."(Mill, 1859; Scarre, 2007). In many countries the introduction of universal free education was accompanied by centralisation of allocative decisions based on local districts. In common with other aspects of the welfare state, such as healthcare, in Western countries the idea of choice as a driver for improvement in standards appears to have gained prominence only after the fixed-allocations system was well-established.

2.2.1 Theoretical economic arguments for choice

An early proponent of the marketisation of school allocations was Milton Friedman (Friedman, 1955, 1962), who argued for a voucher-based full market system. Economic arguments in favour of choice, in the form of full or quasi-markets for education, gained popularity in the US during the next thirty years (Jencks, 1966; Levin, 1968; Coons and Sugarman, 1978). Since the early 1990's the field of school choice has developed alongside, and usually lagging slightly behind, developing political rhetoric. Perhaps the first big development in the 1990's was a cluster of quantitative theoretical studies, building increasingly sophisticated economic models of school choice. Manski (1992) provided a forceful argument for this academic effort, frustrated that, as he saw it "the policy debate has been long on advocacy and short on analysis." (Manski, 1992, p353). As an aside, it is amusing to note that in the UK at about the same time sociologist Stephen Ball was articulating a similar sentiment, characterising the political discourse on school choice as "Adam Smith meets Walt Disney"(Ball, 1993, p5). Manski criticised the existing literature for its one-sidedness and economic simplicity, stating:

> "During the past 30 years, the basic intellectual argument for systemic choice has not notably advanced beyond the classical economic ideas sketched by Friedman (1955, 1962). If anything, recent writing advocating choice views the matter more simplistically than did Friedman. It seems enough today to declare that choice promotes consumer sovereignty and competition." (Manski, 1992, p355)

In response to this perceived hyperbole and lack of rigour, he constructed an economic model of a mixed school system with a large public sector and a voucher-subsidised fee-charging private sector. The parameters of his model were calibrated to plausible values with reference to aggregate empirical properties of the US market. Under Manski's model, voucher systems did not uniformly improve attainment for low-income children, and he concluded by urging caution in the drive to implement market-like educational systems.

The theoretical economic models developed since have built upon this framework, adding various economic features and usually using parameters that have been "calibrated" but not statistically estimated. Epple and Romano (1998, 2008) allow the private sector to charge variable fees, "cream-skim" students with high prior attainment, and earn profits. In the former paper, the quality of schooling depends only on peer effects, whilst the latter paper allows schools to choose how much to spend on teaching, and allows quality to vary with both peers and expenditure. Further developments incorporate residential inequality (Nechyba, 1999, 2000, 2003), religious schools (Ferreyra, 2007), information asymmetries (Macleod and Urquiola, 2009) and the political economy of vouchers[1] (Epple and Romano, 2012b). Epple and Romano (2012a) provide a comprehensive review of the theoretical school choice literature. The conclusions from these studies are varied, but have several elements in common: first, private schools in voucher systems are motivated to "cream-skim" students if they are not prohibited from doing so; second, outcomes for low-income groups depend on the extent of cream-skimming and selective admissions.

2.2.2 Empirical evidence for quasi-market effects

In the mid to late 1990's the theoretical economic work on markets in education began to be supplemented by a wave of empirical studies with the aim of quantifying the "effect" of competition on school performance and efficiency. Influential, and later controversial, contributions by Caroline Hoxby (Hoxby, 2000, 2003) set a template for studies which operationalised competition in various ways, and used instrumental variables or other quasi-experimental methods to estimate a causal effect on test scores or other measures of success. As an example, Hoxby (2000) used geographical data on waterways as natural boundaries between districts, as an instrument for the amount of Tiebout choice families face, and regressed this on school aggregate test scores. She found large positive effects

[1] So-called "political economy" models go further than models where the researcher manipulates policy, in that they incorporate the political process that determines levels of taxation and funding, and the details of school choice policies.

of choice on outcomes. Although influential, the validity of the results were called into question by Rothstein (2005) after a failed replication revealed the sensitivity of the instrumental variable to seemingly arbitrary coding decisions.

This early work has been followed by a huge international literature following the same template, focussing especially on large-scale voucher programmes in the US, Chile and Sweden, but also including studies of choice in the Netherlands, Denmark, Finland, India and the UK. Notable reviews of this literature have been provided by Belfield and Levin (2002), Allen and Burgess (2010), Sahlgren (2013) and Epple et al. (2015). Belfield and Levin summarised their findings by concluding that the majority of studies showed a substantively modest positive effect of competition on a range of outcomes, but that more than half of estimated effects were not statistically significant. Allen and Burgess find positive effects of competition in Sweden, but ambivalent findings in the US and Chile. Sahlgren provides a thorough review of the literature, concluding that there is enough evidence to suggest that competition between schools can improve performance, yet commenting that “it is evident that the design of choice programmes matters quite a lot for outcomes,” (Sahlgren, 2013, p97). Finally, Epple and Romano conclude:

> “To summarize, the evidence thus far, while uneven, is sufficiently encouraging to make a strong case for continued experimentation and evaluation. the evidence also suggests that work originating in a single country or in a single research approach is unlikely to completely answer questions regarding vouchers ... the ongoing tasks include continuing refinement of identification strategies, investigating longer-term impacts, providing a better understanding of why effects emerge or fail to emerge, and marshalling theory and evidence to improve voucher design.” (Epple et al., 2015, p56)

Reduced-form evidence from England is less numerous than some other countries, but given that quasi-experimental studies make no claims of external validity (that is, validity beyond their political and geographic context) it is worth paying special attention to the English evidence. Taylor et al. (2000) test the effect of increased competitiveness in a local area on four school-level variables: exam performance; new admissions; change in school size and stratification. The local level of competition is represented by the mean lagged exam performance of other local schools. Although plausible as a measure of competition, this suffers from endogeneity issues, since it is not clear whether mean local exam performance "causes" variation in school performance, or whether both are affected by unobserved local variables. Taylor et al. find modest and unstable effects of local competition on exam performance, and this effect, where significant, is dwarfed by the strong effect of family background on exam performance. Otherwise, they find that a school's own lagged exam performance is positively correlated with both new admissions and overall size, whereas the performance of its competitors is negatively correlated with admissions and size. They take this as evidence that, although within-school efficiency gains may be weak, systemic efficiency gains (i.e. gains from the growth of successful schools relative to unsuccessful schools) may be larger. Finally, Taylor et al. regress the change in the % of pupils claiming Free School Meals (FSM) at each school against the school's own lagged exam performance, and find a significant negative correlation, suggesting that high-performing schools become more selective.

In a follow-up study, Bradley et al. (2001) use Data Envelopment Analysis (DEA) to measure the efficiency of each school in converting multiple inputs (% FSM-eligible pupils and % of qualified teachers) into multiple outputs (% GCSE grades A*-C and attendance rates). DEA accounts for inputs in a way analogous to (although methodologically very different from) multilevel value-added (VA) models. However, Bradley et al. choose only to account for % FSM, which leaves out a lot of prior variation on gender and test scores. Unlike VA models, DEA allows schools to have heterogeneous goals, as each school maximises a unique weighted sum of outputs given inputs. DEA efficiency estimates were then regressed on measures of competition, represented as counts of the number of other schools within various radii. The results show that an additional school within 1km increases efficiency by 0.51% (±0.12%) in 1998, and an additional school within 1-2km increases efficiency by 0.26% (±0.06%).

Other authors assess the effect on school performance of obtaining greater independence from local authority control (Clark, 2009; Allen, 2013). Both authors use data on voting outcomes from ballots to determine whether schools will become grant-maintained schools, within a regression discontinuity design. The aim of a regression discontinuity design is to eliminate the endogenous selection mechanism, by including in the sample only those schools close enough to the voting threshold to be considered *as good as* *randomly selected*. Any confounding variables correlated with the vote should have a negligible effect if schools with very similar vote outcomes (except that they fall either side of the threshold) are compared. Clark (2009) evaluates the short-run effect of obtaining grant-maintained status on school performance and finds positive effects, while Allen (2013) assesses performance more than 10 years after the vote, and finds no significant association between grant-maintained status and performance. Allen argues that in the short-run, "euphoria" effects may dominate and boost staff morale, whereas in the longer term these effects will no longer be important.

Given the sheer number, diversity and international scope of empirical studies of competition, it is perhaps surprising that a consensus has not emerged. However, the empirical "programme evaluation" approach to studying school choice is hampered by two weaknesses. First, there is a lack of methodological consensus, hampering comparability of findings, on such questions as a) how to operationalise choice and competition and whether or not to differentiate choice from competition; b) how to measure school outcomes; and c) how to achieve causal identification of effects. Second, as the diversity of international results makes clear, it cannot be assumed that a study of competition in one voucher programme is applicable to other voucher programmes, with different technical features, implemented in a different political, economic and cultural context. By minimising structural assumptions, the reduced form approach aims to finesse the whole question of causal mechanisms. However, as Manski (1992) and others have shown, the design of voucher systems, and particularly the design of details that affect the decisions of families and schools, cannot be glossed over.

2.2.3 Evidence of "quality-seeking" behaviour

The most important underlying processes that reduced form approaches leave unmodelled involve the decisions of families when faced with school choice. A necessary condition for competition between schools to improve academic outcomes is that families are conscious of, and value academic outcomes, such that they are more likely to choose a school where expected outcomes are better. A handful of studies have attempted to explicitly model educational decision-making to assess the extent to which parents value academic performance. This structural approach can be thought of as a bridge between reduced form empirical methods and theoretical economic models, in that they explicitly aim to estimate structural parameters that appear in theoretical models of decision-making.

Many of these studies model the importance attached to different aspects of school choice indirectly, via residential choice and house prices. These studies exploit spatial *Tiebout* variation in public services (i.e. school quality) to extract families' willingness-to-pay for educational quality. Models that use regression and quasi-experimental methods to disaggregate the components of house price are known as *hedonic* pricing models (Rosen, 1974). Advantages of such models are that they are applicable in contexts without school (quasi-)markets, and they allow researchers to attach a monetary value to parents' *willingness-to-pay* for improvements in quality. Gibbons and Machin (2008) provide a review of recent international literature on the contribution of school quality to house prices. They find that, despite diverse contexts, studies fairly consistently find that parents are willing to pay an additional 4% of the value for a one-standard-deviation improvement in test scores (with an inter-quartile range of 4%).

Other studies do not use the hedonic pricing approach. Rothstein (2006) analyses the correlations between house prices, family characteristics and school effectiveness, and finds no evidence that families value school effectiveness. Instead of attempting to estimate parents' weightings for quality, Buckley and Schneider (2003) investigate the decision-making processes of highly-motivated parents, to examine the claim that a "marginal" group of well-informed consumers drives competition. They apply an experimental method, in which parents use a specially-designed website to search for school information, and find evidence that a subgroup of parents does take into account performance information. Allen et al. (2014) use information on school capacities to infer choice sets available to families in England. They find that parents of children on Free School Meals are less likely to choose the highest-quality school available to them, compared to other families.

In contexts with school choice – whether quasi-markets or full markets – it is possible to study decision-making directly, although access to sensitive data about preferences is often difficult. An increasing number of studies use choice data to examine parental weighting of school academic performance against other criteria. Hastings et al. (2009) use data on preferences from a school choice programme in North Carolina. They find that higher-SES parents place greater weight on performance than lower-SES parents, and also that parents of minority ethnic groups place some weight on ethnic peer groups. Using a very large ranked preferences dataset from the New York City school market, Abdulkadiroğlu et al. (2015) also find evidence of the moderation of preferences for test scores by SES, and also find evidence of preferences for ethnic peer groups. In general the studies that have used data on choices have found an interaction between socio-economic status and preferences for academic performance measured by test scores.

Throughout the school choice literature the importance of context, and the details of policy design, have been stressed. The next section reviews the policy background of school choice in England, elaborating on aspects of the recent history of education policy in England that shed light on the development of the current system.

2.3 School choice in England: policy background

As described above, the UK public school system can be characterised as a quasi-market in which tuition is fully subsidised by the state, but parents have the right to express a preference for any school, and schools are funded based on the number of pupils they attract. The current system has a number of interacting elements which affect the structure of the market, the conduct of schools and parents and the efficiency of allocations. The development of each of these elements will be described for England. They are:

1. The right to express preferences
2. The allocation mechanism
3. Availability of relevant information for parents
4. School governance and autonomy

The first criterion is self-explanatory. A prerequisite for any school choice policy is that parents are enabled to express a preference. However, this alone is not sufficient for the establishment of an institution of school choice. For example in England, this right was first formalised in the 1980 Education Act: "Every local education authority shall make arrangements for enabling the parent of a child in the area of the authority to express a preference as to the school at which he wishes education to be provided for his child ..." (Education Act 1980, 6.1 [orig.]). However, the Act did not contain details of how this should be implemented, and included a clause obviating Local Authorities from their duty to respect such preferences if compliance "would prejudice the provision of efficient education or the efficient use of resources" (Education Act 1980, 6.3 [orig.]). Thus the Act seemingly had little impact on school choice until supplemented by the Education Reform Act (1988). The latter piece of legislation, which also introduced the National Curriculum and grant maintained schools, is commonly taken to be the one that introduced school choice into England.

The second element, the allocation mechanism, is perhaps the most technically esoteric topic in school choice research. The link between school choice and mechanism design was first made in a seminal paper by Abdulkadiroğlu and Sönmez (2003), who exposed flaws in the algorithms used at the time in Boston and other US cities. They showed that school allocation mechanisms are related to other matching mechanisms studied under the economics sub-discipline of *mechanism design*. Work by Abdulkadiroğlu and others showed that these algorithms could be designed in such a way as to guarantee axiomatic properties such as pareto efficiency and strategy-proofness. The first property, pareto efficiency, means that the allocation selected by the mechanism is optimal in the sense that no family can be made better off (according to their preferences) without making some other family worse off.

The second property ensures that parents have no incentives to misrepresent their true preferences in order to improve their allocation. This point is important: if parents have a right to express preferences, but they are penalised for expressing their true preferences, the value of school choice is diminished. In England, local authorities manage their own allocation clearinghouses. Until 2007 a variety of methods were in use, many of which were variants on the Boston Mechanism that is known to be vulnerable to manipulation(Pathak and Sönmez, 2011). The School Admissions Code published in that year made first-preferences-first allocation methods illegal (Department for Education and Skills, 2007) and since then most local authorities have switched to stable matching mechanisms.

Parents may be able to express preferences between schools, but in order for parental choice to be effective parents must have access to relevant information about aspects of school performance. Relevant in this context means information that is likely to make a difference to parental choices, and also information that is likely to accurately predict some aspect of parents' and students' future satisfaction with the school. Hence, the third aspect of effective school choice – performance information – can be evaluated in two ways: by its perceived usefulness to parents at the time of decision-making; and also by the long-term effectiveness of the decisions it supports.

In the UK, measures of performance have been reinvented several times in the last two decades as a result of continuing methodological advances as well as changing political priorities. Beginning in 1992 the government began to publish league tables, which are rankings of schools based on some measure or combination of measures. However, league tables often rely on measures of "raw" performance that have been shown to largely reflect the backgrounds and previous education of a school's intake, rather than the quality of the education offered by the school. An important concept in educational performance measurement has been the evolution of what are called "value-added" measures, which are measures designed to control for pupils' different starting points when they enter a given stage of education. In addition to quantitative measures, and perhaps more high-profile, the independent schools inspectorate, the Office for Standards in Education (Ofsted), established by the Education (Schools) Act (1992), publishes regular detailed reports on all state schools. These reports include an overall grade reflecting the inspectors' opinion of the school, and it is often these grades alone that are publicised and well-known by parents.

The fourth element of school choice policies is the creation of diverse educational institutions, and the provision of sufficient school places, to increase the likelihood that parents will be able to select a school that suits the needs of their children. The issue of sufficient capacity has been largely ignored in the school choice literature, but the amount of capacity in a school market compared to the number of students can have a large effect on the real amount of choice that parents have (Calsamiglia and Miralles, 2012). Much recent political effort in the UK has focused on the proliferation of different types of school. These new school types such as specialist schools, academies and free schools are characterised by greater autonomy in respect of both pedagogy and governance, and direct financing from central government. Whilst a rationale for these developments has usually been the promotion of choice by improving diversity and providing incentives for schools to maximise enrolments, a side-effect has been a reduction in the role of the local authority, and an increasingly individualistic role for schools as enterprises in competition with each other for resources and pupils.

Chapter 3

The sample

Chapter 2 introduced two topics of concern in studying school choice: the existence and effects of segregation within school populations; and the degree to which the system provides real choice for parents, and promotes competition between schools to attract students. This chapter introduces and briefly describes the sample of students entering secondary school in the North West of England between 2009 and 2012, and describes and justifies the selection of a sub-sample of urban school markets from the regional data for more detailed analysis. The three towns that are the focus of the main substantive study are introduced and their socio-economic and ethnic demographics and recent history are briefly explained.

3.1 The regional sample

All analyses are based on school and pupil data from the National Pupil Database. The central dataset consists of four cohorts of year 7 secondary school entrants, corresponding to the 2009/10 to 2012/13 school years. This is supplemented by school-level census data for the years 2007/08 to 2012/13. The first two cohorts of school-level data were used to construct lagged measures. Although the main analysis is conducted on medium-sized urban school markets, data was initially obtained for Local Authorities covering a large area of the North West of England. This includes school and year 7 pupil data for the following local authorities: Lancashire; Blackpool; Blackburn with Darwen; Wigan; Bury; Bolton; Rochdale; and Oldham.

The full sample contains 92,110 pupils entering into year 7, the first year of secondary school[1]. Table 3.1 shows the number of pupils in each cohort, and the ethnic and socio-economic characteristics of the cohort.

Table 3.1: Regional Sample Descriptive Statistics

			Ethnicity %			FSM %*	
Year	N	N schools	White	Asian	Other	No	Yes
2009/10	23,385	176	82.5	13.1	4.4	84.7	15.3
2010/11	23,442	169	80.9	13.9	5.1	84.5	15.5
2011/12	22,474	168	79.7	14.8	5.5	85.0	15.0
2012/13	22,809	171	78.2	16.2	5.6	84.4	15.6
All	92,110	189	80.3	14.5	5.2	84.7	15.3

*Eligible for Free School Meals (FSM)

With this larger dataset we were able to visually represent our urban school markets in their wider context and assess whether schools at the boundaries of each market should be included. In this way the selection of the urban samples was fine-tuned. The next section describes the selection of three urban school markets for the main substantive analysis.

3.2 The urban samples

From the regional sample, three smaller urban school markets were selected for more detailed analysis and choice modelling. It was necessary to select medium-sized sub-samples, rather than using the regional data, for computational feasibility. The innovative and computationally intensive nature of the structural estimation method used, involving numerical integration and loops over all possible choice possibilities, limited the size of the samples that could be processed. In particular the computational complexity of the method is sensitive to the number of schools in the sample, as it implies a separate latent variable for each school.

The integrated nature of the estimation method means that each sample is necessarily localised; that is, we must sample schools with sufficiently overlapping catchment areas

[1] In England there are a small number of local authorities that operate middle schools (schools serving children from 9 to 13/14 years of age). In these systems year 7 pupils are not necessarily secondary school entrants. However, there are only a small number of these schools remaining in England, and none of the local authorities in the sample operates middle schools.

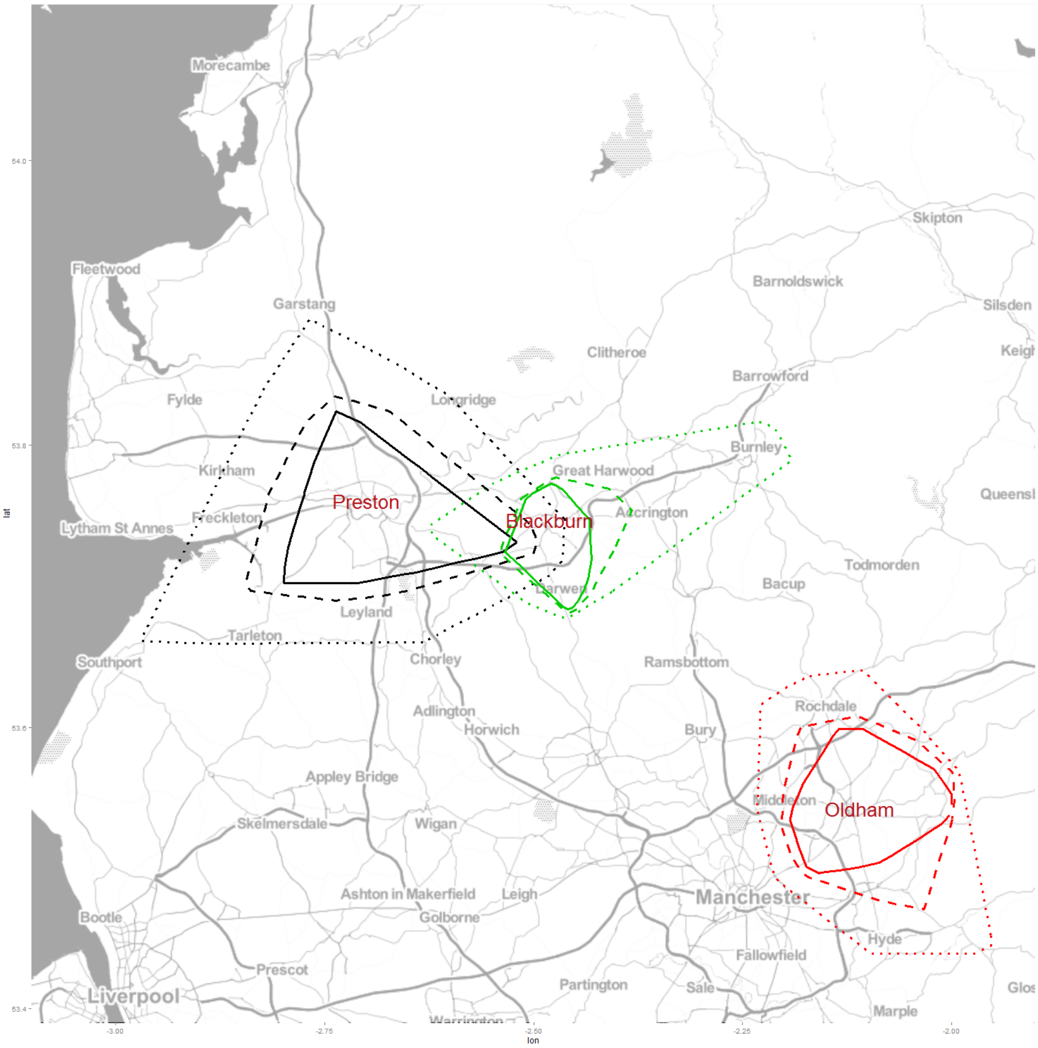


Figure 3.1: *Empirical quantile contours containing* (90%, 95%, 99%) *of the 2009–2012 intake for each study area. Contours are calculated using convex hull peeling (Wolf and Bielefeld, 2014). It is clear that the catchment areas of Preston schools and Blackburn schools are not entirely isolated.*

and probabilities of allocation. Failure to do this would lead to statistical separation and loss of information. In this respect, the two-sided method is no different from normal discrete choice methods. In addition, for our structural model we must sample all of the entrants to a given school, as failure to sample a pupil who is on the borderline of acceptance could materially bias results.

There are also substantive benefits to selecting medium-sized school markets. First, as discussed in Chapter 2, there is no reason to expect parental preferences, and peer preferences in particular, to be homogeneous throughout England. By comparing several self-contained and integrated school markets this allows us to assess the between-market heterogeneity of preference parameters. By not choosing communities that are too large we increase the probability that the parameters we estimate are reasonably internally homogeneous. Second, by sacrificing scope we gain the advantage of methodological intimacy in that it is possible to gain a more qualitative understanding of the sample. Of course, we do not gain much detail at the level of individual pupils, since sample sizes are still in the thousands, but at the level of the medium-sized town we can become familiar with individual schools, neighbourhoods and the meso-level geography of a single school market. As is shown in Chapter 4, school markets tend to have quite well-defined boundaries.

Another reason for focusing on urban areas is that, at a regional level, the proportion of minority ethnic groups in the pupil population is too low to discern sorting patterns. The distribution of minority ethnic groups is strongly clustered in urban centres. Similarly, patterns of socio-economic sorting are dominated by the urban-rural divide. As a consequence, it is only in urban areas that ethnic and socio-economic groups are mixed enough to discern post-residential sorting and schools with large proportions of minority ethnic pupils.

It may be that preferences depend upon structure of the school market as well as aspects of each school and family. To this end, three study areas have been chosen that are similar, yet sufficiently different to reflect structural variation in the composition of school markets. The three study areas are all in the North West of England, but despite this they manifest considerable variation in such factors as the presence of private school options, selective schools, faith schools, local demographics and school governance. The three areas are located in different Local Education Authorities (LEAs); they are Preston (part of Lancashire LEA), Blackburn with Darwen, and Oldham.

The study period spans four cohorts of secondary school entrants from 2009/10 to 2012/13. This period covers rapid structural reform, with the "Building Schools for the Future" programme, and the second-wave academisation of many schools. In Oldham, in particular, these changes have been particularly drastic. The study period allows us to analyse preferences before and after these changes, and may potentially provide a natural experiment for the study of preferences under structural reform.

It is worth noting that Preston and Blackburn are not quite far apart enough for their intakes to be entirely non-overlapping, as can be seen in Figure 3.1. There are a few families in Blackburn who send their children to a school in Preston, and very small number who send their children from Preston to Blackburn. An aspect that all of the study areas have in common is that none of them have any ability-selecting state schools (Grammar schools).

3.2.1 Preston

Table 3.2: Preston Descriptive Statistics

			Ethnicity %			FSM %*	
Year	N	N schools	White	Asian	Other	No	Yes
2009/10	1,701	15	79.3	15.6	5.1	88.5	11.5
2010/11	1,716	15	76.6	16.6	6.8	87.4	12.6
2011/12	1,659	16	75.2	13.9	10.9	87.9	12.1
2012/13	1,800	16	75.3	17.9	6.7	86.8	13.2
All	6,876	16	76.6	16.0	7.4	87.6	12.4

*Eligible for Free School Meals (FSM)

A medium-sized city of around 240,000 inhabitants, Preston is the administrative capital of Lancashire. The study area comprises two districts, separated by the River Ribble. These are the non-metropolitan districts of City of Preston and South Ribble. City of Preston has a more ethnically-diverse population than South Ribble. Major religions in Preston (not including South Ribble) are Christianity 61% and Islam 11% (Office for National Statistics, 2016). Preston's main ethnic groups are: white British, 76%; Indian, 10%; and Pakistani, 3%. Preston's ward with the highest proportion of Asian residents is Deepdale, near the city centre.

Together, the districts encompassed 16 secondary schools in 2013. The city is perhaps unusual for a city of its size in that it has no independent secondary schools within the two districts studied. This means that it is possible to include the entire local secondary education market within the sample. However, families in the area do send their children to private schools further away. Eight of the 16 schools have a religious denomination, including two Church of England schools, one Muslim school and five Roman Catholic schools. The large number of RC schools reflects Preston's historical association with Catholicism and continuing large Catholic population (Hunt, 2009).

The secondary school offering in Preston has been relatively stable during the study period. One new school was created (Preston Muslim Girls' School), and a couple of schools gained academy status. However, a long-established school, Tulketh High School, had closed and merged with Fulwood High School in 2007, two years before the first cohort we study.

3.2.2 Blackburn with Darwen

Table 3.3: Blackburn Descriptive Statistics

			Ethnicity %			FSM %*	
Year	N	N schools	White	Asian	Other	No	Yes
2009/10	1,290	10	57.8	38.7	3.5	80.1	19.9
2010/11	1,229	10	54.7	41.1	4.2	81.3	18.7
2011/12	1,239	9	52.8	44.1	3.1	84.3	15.7
2012/13	1,286	10	48.8	46.4	4.8	83.0	17.0
All	5,044	11	53.5	42.6	3.9	82.1	17.9

*Eligible for Free School Meals (FSM)

Blackburn with Darwen is a unitary authority in East Lancashire, encompassing Blackburn and the smaller town of Darwen to the south, with a combined population of around 140,000. The area has a large Muslim population; at 27% of the population it is the third largest proportion of any local authority in the UK, and the largest outside London. 53% of the population are Christian. The major ethnic groups in the area are: white British 67%; Indian 13%; and Pakistani 12%. Blackburn's ward with the highest proportion of Asian residents is Whalley Range, pictured in Figure 3.4

Blackburn had six independent schools[2] (not included in the sample), and ten state secondary schools in 2013. Of the state schools, five are faith schools, with two Roman Catholic, two Muslim schools and one Church of England school. During the study period two state schools merged, and a Muslim boys' state school opened.

3.2.3 Oldham

The metropolitan borough of Oldham has a population of around 230,000, with the town of Oldham at its centre. In the early 2000's Oldham became infamous for ethnic

[2]Four of the six independent schools are Muslim faith schools.

Table 3.4: Oldham Descriptive Statistics

			Ethnicity %			FSM %*	
Year	N	N schools	White	Asian	Other	No	Yes
2009/10	2,359	15	67.2	27.0	5.8	80.6	19.4
2010/11	2,364	13	64.2	27.7	8.1	79.5	20.5
2011/12	2,308	12	66.5	27.9	5.6	78.7	21.3
2012/13	2,362	12	62.5	30.6	7.0	79.5	20.5
All	9,394	19	65.1	28.3	6.6	79.6	20.4

*Eligible for Free School Meals (FSM)

tension and segregation as a result of the 2001 Oldham race riots (Amin, 2003). The major ethnic groups in the area are: white British 76%; Pakistani 10% and Bangladeshi 7%. The district has the third largest proportion of Bangladeshi residents in the country, and the largest Bangladeshi community outside London. Major religions are Christianity (60%) and Islam (18%) (Office for National Statistics, 2016). In terms of the geographical distribution of Oldham's minority ethnic groups, many Asian families choose to live in the Glodwick area to the east of Oldham town centre (see Figure 3.6). The ward of Westwood has a concentration of Bangladeshi families.

At the end of the study period Oldham had 12 schools, of which two had Church of England denominations and one Roman Catholic. In addition, one school was designated as a Christian Academy, but without faith criteria for selection. This is the smallest proportion of schools with faith admissions criteria of the three study areas. There were four independent schools, not included in the sample.

Of the three study areas, Oldham has experienced the most drastic changes to its secondary school offering during the study period. Seven schools were closed, and merged into four new "super-academies". These changes were, at least in part, in response to the extreme segregation in Oldham's secondary schools. The most high-profile of these mergers involved the closing of a school that was more than 90% white British – Counthill School – and a school that was 90% of Pakistani origin – Breeze Hill School. The academy that was opened to replace them – Waterhead Academy – was discussed in the national news using such terminology as "forced integration" and "social experiment" (Nye, 2011). The dataset encompasses these changes and this gives us a valuable opportunity to observe the evolution (or stability) of preferences during a time of structural reform.

Figure 3.6 shows that in 2009 Oldham's schools were not only segregated along white/Asian lines, but also between Pakistani and Bangladeshi pupils. The Grange school predominantly served Bangladeshi pupils, whereas Counthill School served almost exclusively Pakistani pupils. As Figure 3.7 shows, by 2012 Counthill School had been merged into Waterhead Academy, and The Grange School had been converted into Oldham Academy North at a new site.

3.3 A comparative recent history

Although they have developed differently, the economic and demographic history of the three towns in the twentieth century shares common elements. During the nineteenth and into the first decades of the twentieth century, the main industry in all three towns was provided by the manufacture of cotton fabrics. Although this industry began to decline after the First World War, favourable economic conditions in the aftermath of the Second World War brought a temporary boost to the cotton industry (Singleton, 1991). Faced with labour shortages in this and other manufacturing sectors (for example, the aerospace industry in Preston and Blackburn), industry leaders actively sought immigration from Asian and Caribbean countries (Hunt, 2009). Immigrants from India, Bangladesh and Pakistan settled in the three towns in the 1950's and 1960's, and other textile-manufacturing towns in the North of England. As these workers usually arrived with very little wealth, and were not in well-paid jobs, they tended to settle in inner-city areas in each town, and create urban communities from a single country of origin.

The eventual terminal decline of the cotton industry brought unemployment and hardship for those whose livelihoods depended on the industry, and this affected both white and minority ethnic groups. First-generation immigrants, often having poorly-paid work and less diverse support networks, arguably fared worse than white workers. In the latter part of the twentieth, and the first decades of the twenty-first century the economic prosperity of the three towns has diverged. Preston, having access to important aerospace, nuclear and other industries, and being the administrative and commercial centre of Lancashire, has been least vulnerable to the loss of cotton. Nearby Blackburn has arguably shared the benefit of aerospace and other manufacturing industries.

Oldham and similar towns such as Burnley, however, seem never to have fully recovered from the economic effects of de-industrialisation (Singleton, 1991). At the turn of the new century, unemployment among Asians in Oldham was around 50% according to Amin (2003). The economic decline and lack of industry as a force for cohesion in Oldham and other towns around the Pennines[3] had negative effects upon inter-ethnic relations, summarised by Arun Kundnani: “the textile industry was the common thread binding the white and Asian working class into a single social fabric. But with its collapse, each community was forced to turn inwards on to itself” (Kundnani, 2001, p106).

This short history is only a sketch, but it is not unreasonable to suggest that a basic understanding of the economic and demographic history of Oldham, in particular, sheds some light on the tensions that resulted in three days of race riots in Oldham in May 2001. The riots centred upon the predominantly-Asian neighbourhood of Glodwick, just to the east of the town centre. The impact of the riots was summarised in The Ritchie Report (2001), which called it “the worst racially motivated riots in the UK for fifteen years” (Ritchie, 2001, p2). Describing the aftermath of the riots, Ritchie went on to describe a string of worrying events:

> “On the 1st June the house of the Asian Deputy Mayor of Oldham was fire-bombed, he and his family only narrowly escaping. A week later the town was again in the news as the British National Party scored its biggest success ever in a UK General Election, taking over 6,500 votes in Oldham West and Royton, 16% of the vote, and over 5,000 votes in Oldham East and Saddleworth, or 11%. This confirmed the view of many that something was seriously wrong in Oldham, that there was a community more polarised on racial lines than anything seen before in the UK.” (Ritchie, 2001, p2)

The disturbances in Oldham were followed by riots in Burnley, Bradford and Leeds.

As the ramifications of demographic change and economic decline have been felt most acutely in Oldham, it is plausible to expect the effect of these trends on schools and preferences for schools to be felt most strongly in Oldham. In relation to the interaction of ethnic and socio-economic factors at least, Oldham can be considered to be the primary “treatment” sample, with Preston and Blackburn acting in this sense as “controls”

[3] An upland region encompassing parts of Lancashire and Yorkshire in England

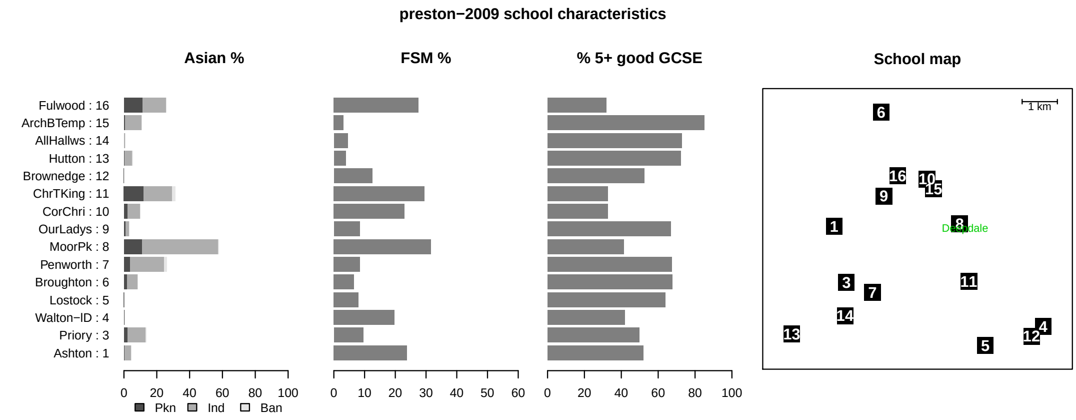


Figure 3.2: *Characteristics of schools in Preston in 2009*

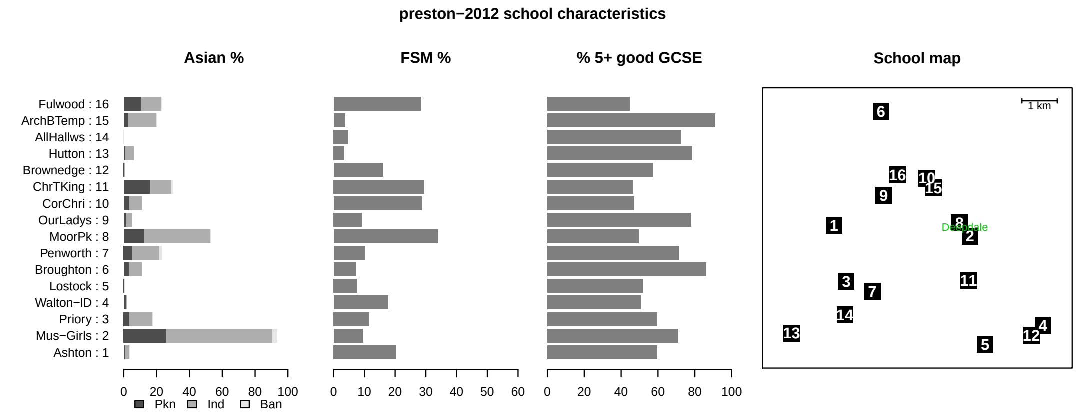


Figure 3.3: *Characteristics of schools in Preston in 2012*

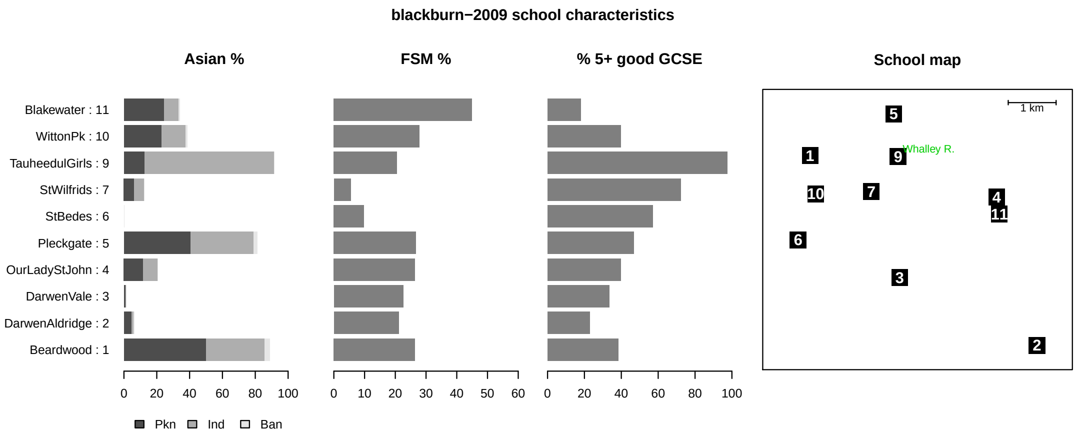


Figure 3.4: *Characteristics of schools in Blackburn in 2009*

which bracket Oldham in terms of ethnic demographics; schools in Preston have a similar proportion of minority ethnic pupils, but less apparent segregation, and schools in Blackburn have a higher proportion of minority ethnic pupils than Oldham.

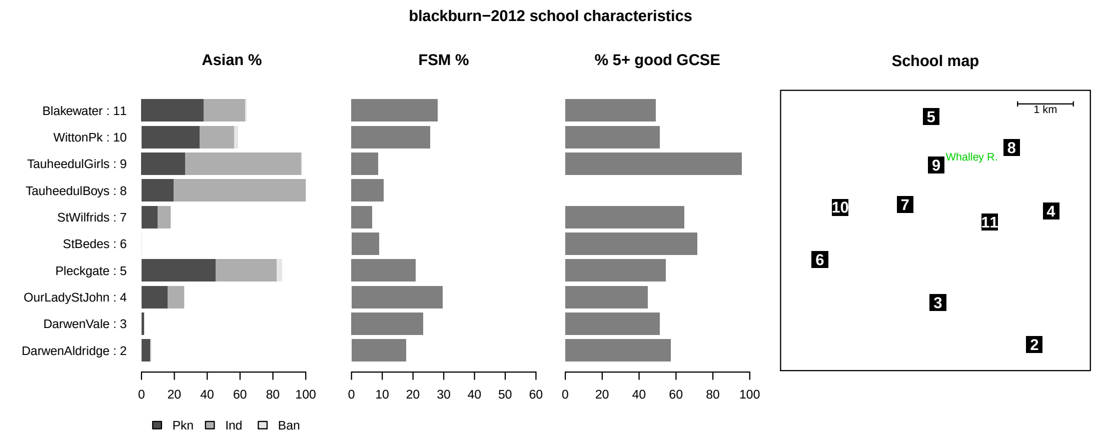


Figure 3.5: *Characteristics of schools in Blackburn in 2012. As a new school, GCSE attainment is missing for Tauheedul Boys' school.*

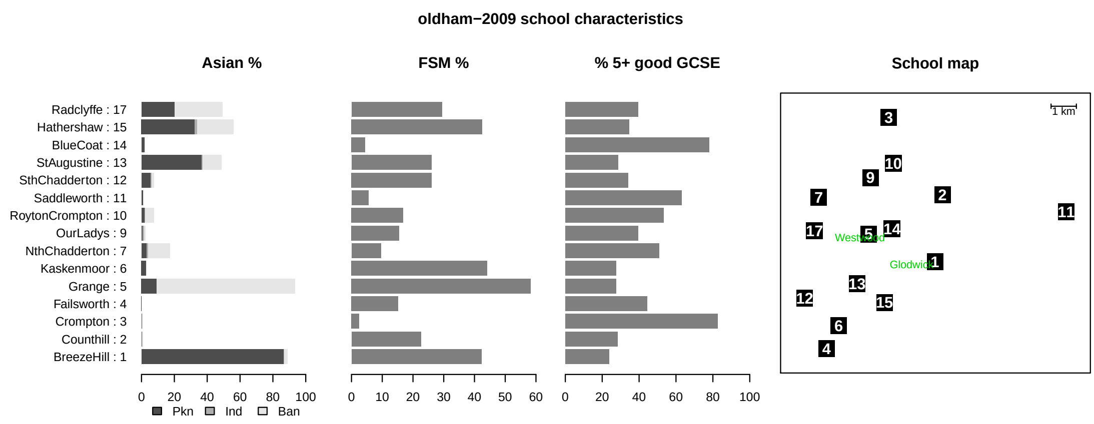


Figure 3.6: *Characteristics of schools in Oldham in 2009*

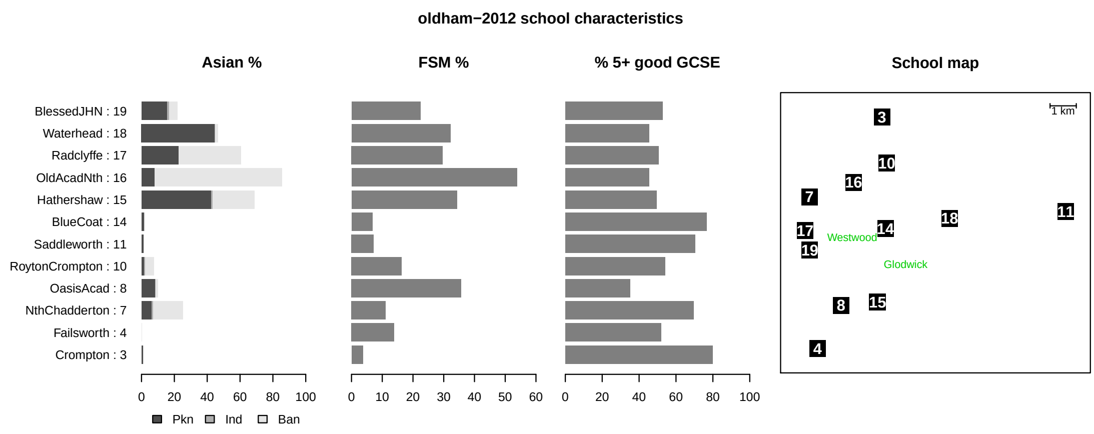


Figure 3.7: *Characteristics of schools in Oldham in 2012*

Table 3.5: Preston Schools

ID	Full name	Denom.	Type	Gender	Active
1	Ashton Community Science College	–	CC	–	–
2	Preston Muslim Girls' School	Mus	VA	G	2011–
3	Priory Sports & Technology College	–	CC*	–	–
4	Walton-le-Dale Arts College	–	CC	–	–
5	Lostock Hall Community High Sch.	–	CC*	–	–
6	Broughton High School	–	CC	–	–
7	Penwortham Girls' High School	–	CC	G	–
8	Moor Park Business & Enterprise Coll.	–	CC	–	–
9	Our Lady's Catholic High School	RC	VA	–	–
10	Corpus Christi Cath. Sports Coll.	RC	VA	–	–
11	Christ The King Cath. Mathematics Coll.	RC	VA	–	–
12	Brownedge St. Mary's Cath. High Sch.	RC	VA	–	–
13	Hutton C. of E. Grammar School	CofE	VA	B	–
14	All Hallows Catholic High School	RC	VA	–	–
15	Archbishop Temple School	CofE	VA	–	–
16	Fulwood Academy	–	AC	–	–

Key: Mus-Muslim; RC-Roman Catholic; CofE-Church of England; CC-Community Controlled; VA-Voluntary Aided; G-Girls; B-Boys; *School converted to academy during the study period.

Table 3.6: Blackburn Schools

ID	Full name	Denom.	Type	Gender	Active	Successor
1	Beardwood Humanities College	–	CC	–	–2010	11
2	Darwen Aldridge Community Academy	–	AC	–	–	
3	Darwen Vale High School	–	CC	–	–	
4	Our Lady & St. John Cath. High Sch.	RC	VA	–	–	
5	Pleckgate High School	–	CC	–	–	
6	St Bede's RC High School	RC	VA	–	–	
7	St Wilfrid's C. of E. High School	CofE	VA*	–	–	
8	Tauheedul Islam Boys' High School	Mus.	Free	B	2012–	
9	Tauheedul Islam Girls' High School	Mus.	AC	G	–	
10	Witton Park High School	–	CC	–	–	
11	Blakewater Coll./Blackburn Ctrl.	–	CC†	–	–	

Key: Mus-Muslim; RC-Roman Catholic; CofE-Church of England; CC-Community Controlled; VA-Voluntary Aided; Free-Free School; AC-Academy Converter; G-Girls; B-Boys; *School converted to academy during the study period; †Converted to Foundation School and merged with Beardwood in 2010.

Table 3.7: Oldham Schools

ID	Full name	Denom.	Type	Active	Successor
Closing schools					
1	Breeze Hill School	–	CC	–2009	18
2	Counthill School	–	CC	–2009	18
5	Grange School	–	CC	–2009	16
6	Kaskenmoor School	–	CC	–2009	8
12	South Chadderton School	–	CC	–2009	8
9	Our Lady's RC High School	RC	VA	–2010	19
13	St Augustine of Canterbury RC	RC	VA	–2010	19
Continuing schools					
3	Crompton House CofE School	CofE	VA*	–	
4	Failsworth School	–	CC	–	
7	North Chadderton School	–	CC	–	
10	Royton and Crompton School	–	CC	–	
11	Saddleworth School	–	CC	–	
14	The Blue Coat CofE School	CofE	VA*	–	
15	The Hathershaw College of Tec	–	CC*	–	
17	The Radclyffe School	–	CC	–	
New schools					
8	Oasis Academy Oldham	–	SA	2010–	
16	Oldham Academy North	–	SA	2010–	
18	Waterhead Academy	–	SA	2010–	
19	Blessed J.H. Newman RC Coll.	RC	VA	2011–	

Key: RC-Roman Catholic; CofE-Church of England; CC-Community; VA-Voluntary Aided; SA-Sponsored Academy; *Converted to Academy during study period.

Chapter 4

Indices of segregation and market structure

This chapter presents exploratory empirical analyses of allocations data that can be performed without imposing a structural model of parental preferences. Each of these analyses makes use of some notion of schools' spatial catchment areas. In the first analysis the relevant notion is the Voronoi neighbourhood of each school – that is, the neighbourhood of geographical points for which that school is the nearest school. This catchment area is used to construct a counterfactual experiment to disaggregate residential sorting from post-residential sorting and evaluate the strength of evidence for the latter. The second and third analyses construct a catchment area for each school by using kernel-smoothed counts of pupils in each location. This catchment area is used to analyse the structure of school markets in terms of choice and competition.

4.1 Post-residential segregation: Nearest-school counterfactual analysis

One of the main processes to be understood in this study is the uneven sorting of pupils into schools along ethnic and socio-economic lines. Chapter 2 summarised the extensive quantitative literature measuring and analysing socio-economic and ethnic segregation, both in the UK and internationally. Two substantive questions stand out in this literature: the question of trends – whether segregation tends to be increasing or decreasing over time; and the question of causes – whether in-school segregation is primarily a reflection of segregated neighbourhoods, or whether there are additional causes in addition to residential segregation.

Many of the studies addressing these questions have used various indices to measure segregation (Duncan and Duncan, 1955; Massey and Denton, 1988; Gorard, 1999; Burgess et al., 2005). When addressing the second question – the existence of post-residential segregation – the main strategy has been to compare indices of segregation for schools to indices of segregation on similarly-sized geographical units. However, these studies have had to contend with the Modifiable Areal Unit problem. This means that indices are highly sensitive to the choice of geographical units. The Index of Dissimilarity is sensitive to the size of units, since smaller units tend to be more homogenous and hence more segregated. In order to deal with this problem Allen (2008) compares segregation indices for schools, to a counterfactual in which all students are re-allocated strictly by distance. In the following section a similar, although not identical, strategy is used to assess segregation in schools.

The main index used to measure segregation since Duncan and Duncan (1955) is the Index of Dissimilarity (D). For two disjoint sub-groups of the population indexed by $t \in \{0, 1\}$ representing, for example, white and minority pupils, and G non-overlapping geographical units, the index is defined as

$$D \equiv \frac{1}{2} \sum_{g=1}^{G} \left| \frac{n_{g,0}}{N_0} - \frac{n_{g,1}}{N_1} \right|$$

where $n_{g,t}$ is the number of group t in unit g, and N_t is the total population of group t across all units.

The Index of Dissimilarity takes the form of a scaled absolute (or L_1) distance metric between two discrete distributions[1]. If D takes its maximum value of 1, this implies that no two members of different sub-groups share the same geographical unit; that is, the support of the two distributions is entirely disjoint. At its minimum value of 0, D implies that the empirical distribution of each sub-group is identical to that of the other. In the context of school market analysis, the measure can also be characterised as comparing the group-specific market share distributions of schools, and asking how similar the market

[1] D is similar to the Euclidean Hellinger distance for discrete distributions.

structure is for the two groups. The index has an intuitive interpretation as the proportion of either of the groups who would have to move between geographical units, to equalise the spatial distributions of the two groups.

Although popular in the segregation literature, D, when used as an estimate of underlying unevenness, is known to be upward-biased for finite samples. The reason for this is that random variation in proportions does not cancel out in the sum of absolute differences, and so the observed D is the sum of unevenness caused by segregation, and unevenness caused by random variation. This problem is especially acute when numbers of one or both groups are small in some geographical units. Several bias-corrections have been proposed to deal with this. Bootstrap bias-correction was compared to other methods by Allen et al. (2015) and found to be insufficient. Both Allen et al. (2015) and Mazza and Punzo (2015) proposed corrections based on a large sample approximation of the absolute difference between proportions to a folded normal distribution.

The correction in Allen et al. (2015) begins by calculating, for each geographical unit, the observed proportions $p_{g,t}^{\text{obs}} = n_{g,t}/N_t$ as estimates of the population parameters $\pi_{g,t}$. Since the proportions are modelled as arising from a multinomial distribution, in large samples the random variable $Z_g = |p_{g,0}^{\text{obs}} - p_{g,1}^{\text{obs}}|/\hat{\sigma}_g$ has approximately a folded normal distribution with mean μ_g and variance σ_g^2. The variance is estimated by plugging in

$$\hat{\sigma}_g^2 = \frac{p_{g,0}^{\text{obs}}(1 - p_{g,0}^{\text{obs}})}{N_0} + \frac{p_{g,1}^{\text{obs}}(1 - p_{g,1}^{\text{obs}})}{N_1},$$

and the mean by maximum likelihood:

$$\hat{\mu}_g = \underset{\mu_g \in \mathbb{R}^+}{\arg\max} \left\{ \phi(Z_g - \mu_g) + \phi(Z_g + \mu_g) \right\},$$

where $\phi(\cdot)$ is a standard normal density function. Since $\mu_g = |\pi_{g,0} - \pi_{g,1}|/\sigma_g$ then the density-corrected estimate of the Index of Dissimilarity is

$$D_{\text{dc}} = \frac{1}{2} \sum_{g=1}^{G} \hat{\sigma}_g \hat{\mu}_g.$$

For any $Z_g \leq 1$, that is, whenever the observed absolute difference is smaller than the standard deviation of the estimate, the density corrected estimate of unevenness at that site is zero, and as Z_g grows, μ_g approaches Z_g. All results below use this method of calculating D.

In this analysis the geographical units g are schools, and the groups are drawn from any two disjoint ethnic or socio-economic groups defined by indicators in the NPD. For the purpose of this analysis, "White" includes white British, but excludes other white. "Asian" includes Indian, Bangladeshi, Pakistani and any mixtures of these with white, but excludes Chinese and other East Asian. "Black" includes pupils of African and Caribbean origin, and any mixed ethnicity including black. "FSM" includes children who were eligible for Free School Meals.

In order to assess the degree to which D reflects post-residential, rather than underlying residential segregation, the observed values of D will be compared to those calculated under a simulated scenario in which all children attend their nearest school. In this scenario, *all* segregation is residential, so the difference between the actual measured levels of segregation, and the levels measured under this counterfactual, should indicate the extent of post-residential segregation. This method is very similar to that described by Allen (2008); the main difference lies in the way that the counterfactual is operationalised with respect to school capacities.

In order to treat school capacities as exogenously fixed, Allen uses a first-preferences-first matching mechanism (as described in Chapter 8) in which all schools' priorities and all pupils' preferences correspond strictly to distance. This means that if a school has a small capacity, it may not admit all those pupils for whom it is the closest school. A consequence of this is that under this mechanism, and with fixed capacity, there may be some pupils who are allocated to schools which are much more distant than their nearest school. In fact there is no guarantee that some pupil will not be allocated to his or her furthest school.

What is more, pupils who tend to miss out on nearest school allocation under this model may tend to come from some groups rather than others. Although in general the residential location of pupils is not entirely exogenous, it is plausible that residential locations with respect to the *de facto* catchment boundaries induced by school capacities are even more sensitive to fine sorting, in that house prices may respond to probability of admission to a popular school. Any counterfactual that fixes school capacities may incorporate this sub-neighbourhood sorting.

Nor is this mechanism necessarily the optimal mechanism for respecting capacities whilst allocating pupils as close as possible to home. Assuming that pupils preferences are simply proximity ranks, First-Preferences-First is weakly Pareto optimal, meaning that no pupil could be allocated closer to home without allocating some other pupil further away. However, there is no reason why distances must be converted to ranks, and the cardinal information within distances could not be used, such as in a combinatorial optimisation algorithm (see, for example Aksoy et al. (2013)).

By removing the requirement of fixed capacities, the matching becomes much simpler, and each pupil is simply allocated to his or her nearest school. De facto catchment areas are simple to visualise as the Voronoi neighbourhood of each school. This does mean that the sizes of some schools may be very different under the counterfactual. However, because the total number of students in the area, and the number of units remains the same, the average size of geographical units is the same in the counterfactual as in the actual allocation.

An advantage of D as a measure of segregation is that it is additive; for a given set of sites the calculated index can be decomposed into elements of unevenness arising from different causes. Therefore, the difference between the observed D and the counterfactual estimates the value of post-residential segregation in the same units as D. The interpretation is that this is the proportion of either group who would have to relocate to remove post-residential segregation. Since it is a signed difference, it is also possible that post-residential segregation is negative; that is, schools are less segregated than their catchment areas.

4.1.1 Results

Table 4.1 shows the calculated density-corrected indices of dissimilarity for both the actual intakes and the simulated counterfactual. In most cases D is lower under the counterfactual, implying that actual school intakes are more segregated than they would be if all pupils were allocated to their nearest school. At a regional level the difference is very small, reflecting the fact that at a larger scale patterns of segregation are dominated by the clustering of ethnic minorities within some towns and cities. Reallocating pupils to schools cannot make much difference to this larger scale segregation. However, it appears that within Oldham and Blackburn at least, segregation is at least partly post-residential. In Preston, on the other hand, the effect of school choice appears to be to reduce segregation for most comparisons.

The total segregation of Asian pupils is about twice that of black pupils, and also about twice that of FSM pupils. This pattern broadly holds when each town is considered. Asian pupils are most segregated in Blackburn schools, where segregation is 0.712. However, when only residential segregation is considered this reduces to 0.531. On the counterfactual measure of residential segregation, Oldham is slightly more segregated (0.566). We can calculate the proportional contribution of post-residential segregation to total segregation of Asian pupils: in Blackburn post-residential segregation explains 25% of total segregation, whereas in Oldham it is only 11%. In the region as a whole it is 1.2%.

Table 4.2 presents year-on-year Indices for Asian segregation. In Blackburn and Oldham post-residential segregation appears to be increasing each year, whereas the residential part of segregation is not increasing. The effect of structural changes in Oldham appears to have been to reduce the residential segregation of school catchments by a quite large amount, from 0.665 in 2009 to 0.496 in 2012. The post-residential element of segregation has gone up from 0.036 in 2009 to 0.091 in 2012. This may have been caused by families continuing to choose the "local" school, even though it has changed location. In Preston, in contrast, there is no clear trend in post-residential *desegregation*. Table 4.3 provides the same comparisons for FSM segregation, but no clear patterns emerge.

Table 4.4 sub-divides Asian pupils into individual ethnicities: Pakistani; Bangladeshi; and Indian. In Preston, the pattern of post-residential *desegregation* appears to hold for each individual group. Of the three ethnicities, Pakistani students have the least post-residential segregation overall, but in terms of overall segregation the three towns differ in terms of which group is the most segregated. There are some small counts in this table, so some of the calculated Indices cannot be relied upon.

Each town has two substantial Asian communities: Preston and Blackburn have Pakistani and Indian communities; whereas Oldham has Pakistani and Bangladeshi communities. Total school segregation between Indian and Pakistani pupils in Blackburn and Preston is quite low (0.204 and 0.226, respectively) with low post-residential segregation (0.036

and 0.045). In contrast, both total segregation (0.536) and post-residential segregation (0.106 – 20% of total) between Pakistani and Bangladeshi pupils in Oldham is higher. Segregation between Pakistani and Bangladeshi pupils in Oldham is almost as high as that between white and Asian pupils.

Finally, the post-residential segregation between black and Asian pupils in Preston (0.153 – 36% of total) and Oldham (0.213 – 39% of total) is very high, especially as the pairwise *residential* segregation of these two groups is not very high in Preston and Oldham – 0.262 and 0.335, respectively.

4.1.2 Discussion

Although the calculation and analysis of segregation indices appears to be the dominant mode of segregation analysis in the literature, this kind of modelling is limited in several ways. First, it is not easy to embed segregation indices within a statistical framework, to perform statistical inference such as producing interval estimates and calculating probabilities. The work of Allen et al. (2015) ameliorates this concern considerably, providing confidence intervals and significance tests for the index, but it is not clear whether the multinomial model they describe is entirely compatible with a capacity-constrained school system in which allocations are realisations of the equilibria of complex matching mechanisms. The model could possibly be extended to accommodate this, but that may be conceptually and computationally difficult.

Second, reducing a complex phenomenon such as segregation to a single metric, whilst useful for exploratory purposes, hides a lot of the detail that would be useful in understanding the geographically and temporally specific patterns of sorting that lie behind the index. D is a dissimilarity measure between spatial distributions, but it does not encode any other information about the distributions themselves. Nor does it illuminate, for example, whether a given value is the result of small discrepancies in market share at many schools, or large discrepancies at a couple of schools.

Third, and most importantly, assuming that the causes of segregation operate at the level of individuals – either as decision-makers or objects of the strategic decision-making of others – an aggregate metric such as the Index of Dissimilarity cannot model mechanisms. In order to dissaggregate decision-making, structure and supply-side factors, it is necessary to find a way to use data in such a way that preferences are revealed, whereas D only measures the aggregate effects of preferences, constraints, and market structure.

Table 4.1: Indices of Dissimilarity: comparison of actual intakes (2009–2012) to nearest-school counterfactual

Comparison	Actual	Nearest School	Difference	N_1	N_2	Group 1 Proportion
White vs non-White						
Region	0.579	0.574	-0.005	75285	16825	0.82
3towns	0.554	0.509	-0.045	14361	6953	0.67
Preston	0.401	0.450	0.049	5379	1497	0.78
Oldham	0.549	0.508	-0.041	6225	3169	0.66
Blackburn	0.686	0.530	-0.156	2757	2287	0.55
Asian vs non-Asian						
Region	0.659	0.651	-0.008	13345	78765	0.14
3towns	0.611	0.552	-0.060	5909	15405	0.28
Preston	0.424	0.493	0.069	1103	5773	0.16
Oldham	0.636	0.566	-0.070	2659	6735	0.28
Blackburn	0.712	0.531	-0.181	2147	2897	0.43
Black vs non-Black						
Region	0.352	0.332	-0.020	1637	90473	0.02
3towns	0.269	0.234	-0.035	480	20834	0.02
Preston	0.278	0.219	-0.059	163	6713	0.02
Oldham	0.202	0.101	-0.101	267	9127	0.03
Blackburn	0.415	0.216	-0.199	50	4994	0.01
FSM vs non-FSM						
Region	0.306	0.250	-0.056	14130	77980	0.15
3towns	0.319	0.272	-0.046	3670	17644	0.17
Preston	0.321	0.297	-0.024	850	6026	0.12
Oldham	0.342	0.269	-0.073	1919	7475	0.20
Blackburn	0.221	0.240	0.019	901	4143	0.18

Table 4.2: Indices of Dissimilarity: Year-on-year comparison of D for Asian vs non-Asian intake.

Comparison	Actual	Nearest School	Difference	N_1	N_2	Group 1 Proportion
Preston						
2009	0.432	0.489	0.057	265	1436	0.16
2010	0.502	0.518	0.016	285	1431	0.17
2011	0.399	0.443	0.045	230	1429	0.14
2012	0.429	0.505	0.077	323	1477	0.18
Oldham						
2009	0.700	0.665	-0.036	637	1722	0.27
2010	0.661	0.615	-0.046	656	1708	0.28
2011	0.606	0.493	-0.113	644	1664	0.28
2012	0.587	0.496	-0.091	722	1641	0.31
Blackburn						
2009	0.697	0.543	-0.154	499	791	0.39
2010	0.698	0.538	-0.160	505	724	0.41
2011	0.720	0.535	-0.184	546	693	0.44
2012	0.727	0.543	-0.184	597	689	0.46

Table 4.3: Indices of Dissimilarity: Year-on-year comparison of D for FSM vs non-FSM intake.

Comparison	Actual	Nearest School	Difference	N_1	N_2	Group 1 Proportion
Preston						
2009	0.388	0.284	-0.104	196	1505	0.12
2010	0.295	0.324	0.028	217	1499	0.13
2011	0.281	0.314	0.033	200	1459	0.12
2012	0.290	0.290	0.000	237	1563	0.13
Oldham						
2009	0.416	0.273	-0.143	457	1902	0.19
2010	0.308	0.316	0.008	485	1879	0.21
2011	0.322	0.233	-0.089	492	1816	0.21
2012	0.316	0.237	-0.080	485	1878	0.21
Blackburn						
2009	0.215	0.276	0.061	257	1033	0.20
2010	0.261	0.273	0.012	230	999	0.19
2011	0.257	0.198	-0.059	195	1044	0.16
2012	0.157	0.220	0.062	219	1067	0.17

Table 4.4: Indices of dissimilarity: Detailed ethnicity and inter-ethnic comparisons

Comparison	Actual	Nearest School	Difference	N_1	N_2	Group 1 Proportion
Pakistani						
Region	0.686	0.681	-0.006	6817	85293	0.07
3towns	0.585	0.571	-0.014	2471	18843	0.12
Preston	0.416	0.488	0.071	229	6647	0.03
Oldham	0.594	0.587	-0.007	1274	8120	0.14
Blackburn	0.448	0.382	-0.066	968	4076	0.19
Bangladeshi						
Region	0.810	0.789	-0.021	1849	90261	0.02
3towns	0.780	0.749	-0.031	1283	20031	0.06
Preston	0.455	0.529	0.074	15	6861	0.00
Oldham	0.683	0.585	-0.098	1223	8171	0.13
Blackburn	0.552	0.395	-0.157	45	4999	0.01
Indian						
Region	0.758	0.770	0.012	3033	89077	0.03
3towns	0.667	0.665	-0.003	1697	19617	0.08
Preston	0.431	0.508	0.077	648	6228	0.09
Oldham	0.379	0.272	-0.107	37	9357	0.00
Blackburn	0.579	0.473	-0.106	1012	4032	0.20
Indian vs Bangladeshi						
Region	0.906	0.900	-0.006	3033	1849	0.62
3towns	0.929	0.933	0.004	1697	1283	0.57
Preston	0.460	0.555	0.095	648	15	0.98
Oldham	0.525	0.383	-0.142	37	1223	0.03
Blackburn	0.389	0.191	-0.198	1012	45	0.96
Indian vs Pakistani						
Region	0.667	0.663	-0.004	3033	6817	0.31
3towns	0.504	0.507	0.003	1697	2471	0.41
Preston	0.226	0.181	-0.045	648	229	0.74
Oldham	0.426	0.331	-0.095	37	1274	0.03
Blackburn	0.204	0.169	-0.036	1012	968	0.51
Pakistani vs Bangladeshi						
Region	0.640	0.585	-0.055	6817	1849	0.79
3towns	0.637	0.570	-0.067	2471	1283	0.66
Preston	0.269	0.551	0.281	229	15	0.94
Oldham	0.536	0.430	-0.106	1274	1223	0.51
Blackburn	0.241	0.239	-0.002	968	45	0.96
Asian vs Black						
Region	0.541	0.450	-0.091	13345	1637	0.89
3towns	0.554	0.393	-0.161	5909	480	0.92
Preston	0.415	0.262	-0.153	1103	163	0.87
Oldham	0.548	0.335	-0.213	2659	267	0.91
Blackburn	0.606	0.394	-0.212	2147	50	0.98

4.2 Analysis of spatial market structure: choice and competition

The thesis' third research question asks how sorting affects the ability of the market to offer choice to parents and to impose competition upon schools. When asking such questions about the character and effects of choice and competition in education, it is necessary to take into account the varied nature of choice and competition as they are actually experienced by families and schools. Choice and competition are not homogenous forces that operate in the same way in all places at all times – they are rather a convenient shorthand for all of the interactions between families and schools, and between schools and other schools, that arise as part of the allocation process. For families, ideally these interactions are experienced as choice, and for schools, as competition.

The geographical specificity of school markets has been recognised in much of the empirical economics literature on quasi-markets. Bradley et al. (2001) evaluate the effect of the number of local competitors on secondary school efficiency, estimated using Data Envelopment Analysis. Gibbons et al. (2008) and Bayer and McMillan (2010) provide a more nuanced empirical definition of competition, defining it as the local elasticity of demand with respect to quality, distinguishing it from choice as that which makes schools compete on quality. However, in the economics literature there has been little attention given to the question of how to visually represent market structure.

A handful of papers in geographical economics have used methods to visually represent the geographical distribution of choice (Gibbons et al., 2006; Singleton et al., 2011; Harris et al., 2016). Visual representations of market structure are worthwhile because, despite the existence of choice, school markets are very localised – distance to school matters much more than it does for, say, choice of hospital, because the choice that is made dictates the daily commute for the next five to seven years. Furthermore, visualisations have intuitive exploratory power, and aid in hypothesis generation. They allow the analyst to pinpoint areas for further investigation. Visualisation techniques also have applications in policy; a matter of some interest to policy-makers currently is the existence of "cold spots" – locations where there is less choice of good schools[2].

[2] Based on conversations with civil servants at the Department for Education.

4.2.1 Spatial variation in effective choice

A key concept in market structure research is market concentration, the extent to which a local market is dominated by one or a small number of providers. A commonly-used measure of market concentration is the Herfindahl (or Herfindahl-Hirschman) Index, H. The Herfindahl index at a particular location, g is defined as

$$H_g \equiv \sum_{s=1}^{N_s} \left(\frac{n_{s,g}}{N_g} \right)^2$$

where $\frac{n_{s,g}}{N_g}$ is the market share of school s at site g. In contrast to the notation for the Index of Dissimilarity, in this context g is not a school, but a geographical area. The Herfindahl Index takes values in $[\frac{1}{N_s}, 1]$, where N_s is the total number of schools. The closer the index to one, the more concentrated is the market. A related measure which we will call the Effective Choice Index, ES, defined as the reciprocal of the Herfindahl index, is used to represent the effective amount of choice in the market; it takes values between one and N_s. If all schools' catchments were distributed uniformly over the entire space the effective number of schools operating in each location would be equal to N_s. The more one school dominates in a particular location, the closer ES is to one at that location.

In order to analyse local variations in the amount of choice available to families, the geo-coded region-wide intake data has been counted into a regular 500m grid, and the Effective Choice Index, ES, calculated for each grid square. Although this level of granularity allows detailed spatial analysis, the gridded counts become very sparse, with many zeroes; this means that the Herfindahl Index and the Effective Choice Index are only defined on a handful of grid squares. For this reason the gridded counts were smoothed using a Kernel Density smoother. The function `image.smooth` in the R package `fields` was used (Nychka et al., 2015). The smoother treats the count in each square as a point mass in the centre of the square, and replaces this point mass with a density shared between the original square and other squares, based upon the distance between grid centroids. The density, and the bandwidth of the density, are chosen by the researcher.

The density and the bandwidth were chosen by eye to obtain coverage of the map, whilst avoiding oversmoothing. Densities with bounded support did not provide coverage of the map. A relatively heavy-tailed sigmoid kernel was used, as this appeared to cover the map whilst maintaining detail in urban areas. The sigmoid kernel has unnormalised density $K(x) = 1/[\exp(x) + \exp(-x)]$ (where x is the squared distance between grid centroids). A bandwidth of 500m was considered to provide a minimum feasible amount of smoothing. Gaussian, Epanechnikov and Logistic kernels were tested with several different bandwidths, with very similar results. For each school, these gridded counts represent a smoothed estimate of the *de facto* catchment area of that school.

Figure 4.1 presents the local Effective Choice Indices for the whole region in 2012. Although much of the pattern in the figure is caused by the underlying distributions of families and schools, there are a few points of interest. Firstly, Preston stands out clearly as a hotspot, and this does not appear to be simply a function of population and the number of schools since Blackpool and areas of Greater Manchester have similar populations but do not appear to have as much local mixing. Second, it is possible to compare schools that are similarly remote but have very different local market structures. For example, Garstang High School, halfway between Preston and Lancaster, appears to have a slightly more mixed local market than Carr Hill High School, halfway between Preston and Blackpool.

There appear to be some schools, even in urban areas, that act as local monopolies, repelling competition from other schools. There is an example in Blackpool of a school with a ring of blue around it. It is not clear from the figure whether this is caused by the popularity of the school, or some unseen barriers to accessing other schools in the locality. Unfortunately the market structure of Oldham cannot be read at face value from the plot, because Oldham is at the boundary of the dataset, and the smoothed counts do not adjust for boundary effects. It is probable that there are schools in the areas to the south and east of Oldham that provide additional choice to families in Oldham, but are not observed in the data.

4.2.2 Analysis of pairwise competition between schools

The second analysis visually represents the competitiveness of the market in terms of the pairwise competition between any pair of schools. To represent the pairwise competition, the smoothed, gridded counts of pupils from the previous analysis have been used to represent each school's *de facto* catchment area. The catchment areas each pair of schools have been compared using the index of Dissimilarity D to measure the separateness of the two spatial distributions. In this context g is a grid square, and t indexes the two schools. Pairs of schools whose catchments overlap to a large degree will have a small pairwise D. It is expected that D between two schools will correlate closely, but not exactly, with distance, and it is the exceptions to this rule that are of most interest. These pairwise dissimilarity indices can be presented in a symmetric matrix. To aid visual inspection the rows and columns of the matrix can be permuted so that small dissimilarities are closer to the main diagonal. This reveals cliques – sets of schools with dense pairwise interactions defined by overlapping catchment areas. The permutation was accomplished using anti-Robinson optimisation by simulated annealing, using the R package `seriation` (Hahsler et al., 2008).

Figure 4.2 shows the dissimilarity matrix for North and East Lancashire, and Figure 4.3 shows the matrix for South Lancashire and Greater Manchester. Figure 4.2 reveals clear clique structure, but also some overlap between Preston and schools in Leyland, Longridge, Garstang and Lancaster. It can be seen that pupils travel from schools in Preston to attend Lancaster Royal Grammar School and to a lesser extent Lancaster Girls' Grammar. This is probably explained by the lack of state grammar schools in Preston.

In the previous chapter it was shown that the catchment area of schools in Preston extends into Blackburn. Figure 4.2 reveals that almost all of this overlap is explained by Walton-le-Dale Arts College, in the south east of Preston, which has a catchment that extends to Blackburn. Schools in Blackburn have catchments that overlap with schools up the Calder Valley towards Burnley. There is some evidence that faith schools, in particular Church of England schools (for example St Wilfrid's CofE School in Blackburn), have larger catchment areas than community schools.

Figure 4.3 suggests that the school markets in the more densely-populated south of the region are less self-contained. Nevertheless schools in Oldham form a clear clique. Two schools in Oldham, The Blue Coat CofE School and The Crompton House CofE School, have larger catchment areas than the other schools, which extend north into the neighbouring authorities Rochdale and Bury.

4.2.3 Discussion

The analyses of choice and competition in this section are intended to provide *model-free* visual insights into market structure. Whilst they have provided some tentative insights into choice and competition, an evaluation of the usefulness of these methods must contend with several weaknesses. First, although the analyses were intended to be model-free and hence, in some sense, objective, several design decisions were necessary – for example, in the choice of kernels and bandwidths – to operationalise the analysis. Second, the resulting visualisations are sensitive to the underlying distribution of the population. It is not clear, for example, how to read areas of relatively abundant school choice that fall in rural areas in between two or more urban areas. These rural areas clearly benefit from the choice of more than one urban school market, but they also suffer from the increased distance to schools and lack of public transport options.

In general it is difficult to disaggregate the effects of sorting from purely spatial structure. This could perhaps be remedied by comparing visualisations of observed data with equivalent visualisations calculated using a nearest-school counterfactual.

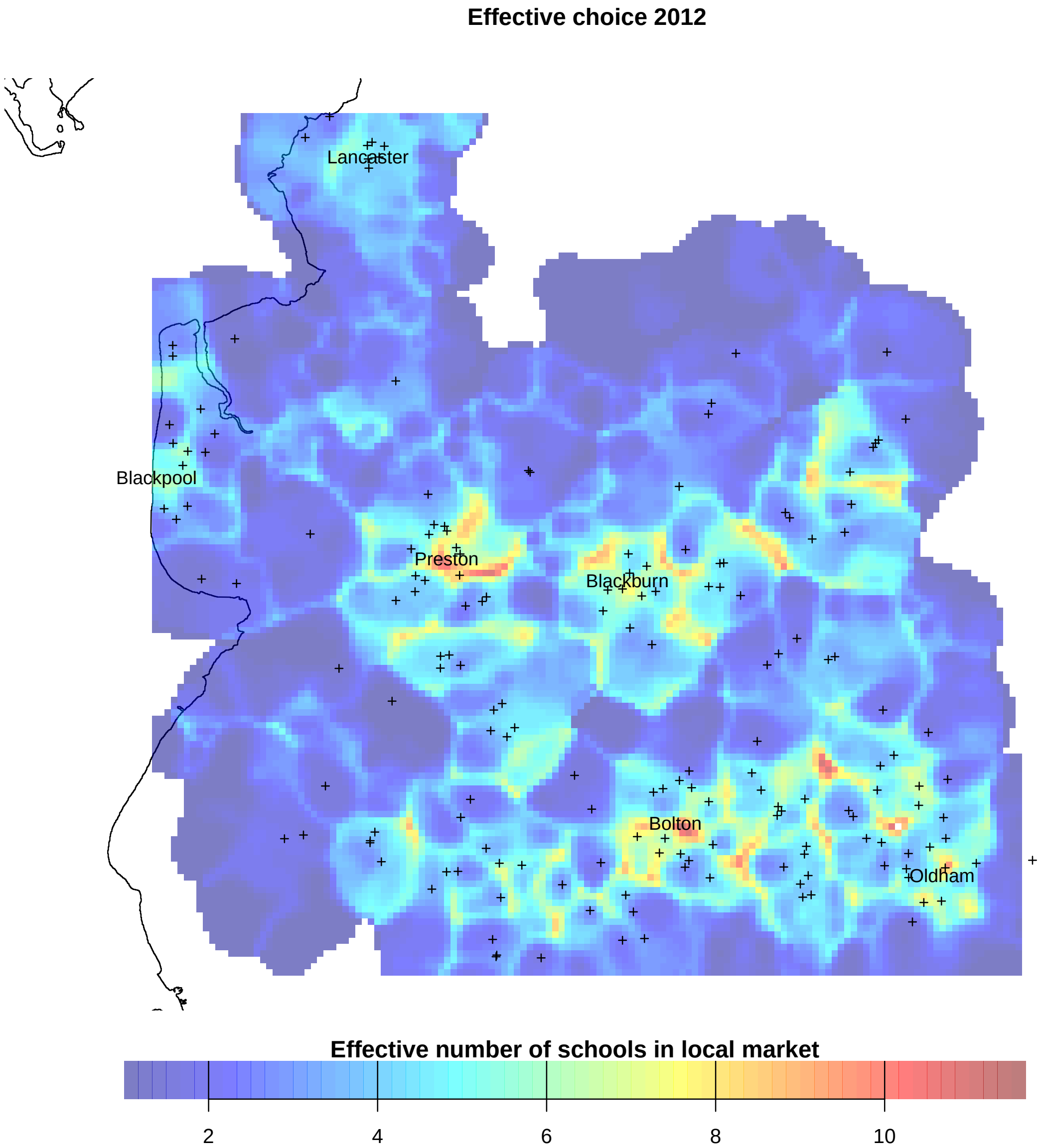


Figure 4.1: *The heat map shows the "effective number of schools" available in each grid square, calculated as the reciprocal of the Herfindahl Index. Underlying pupil counts were smoothed using a kernel smoother, before calculating the plotted values. Crosses are locations of schools.*

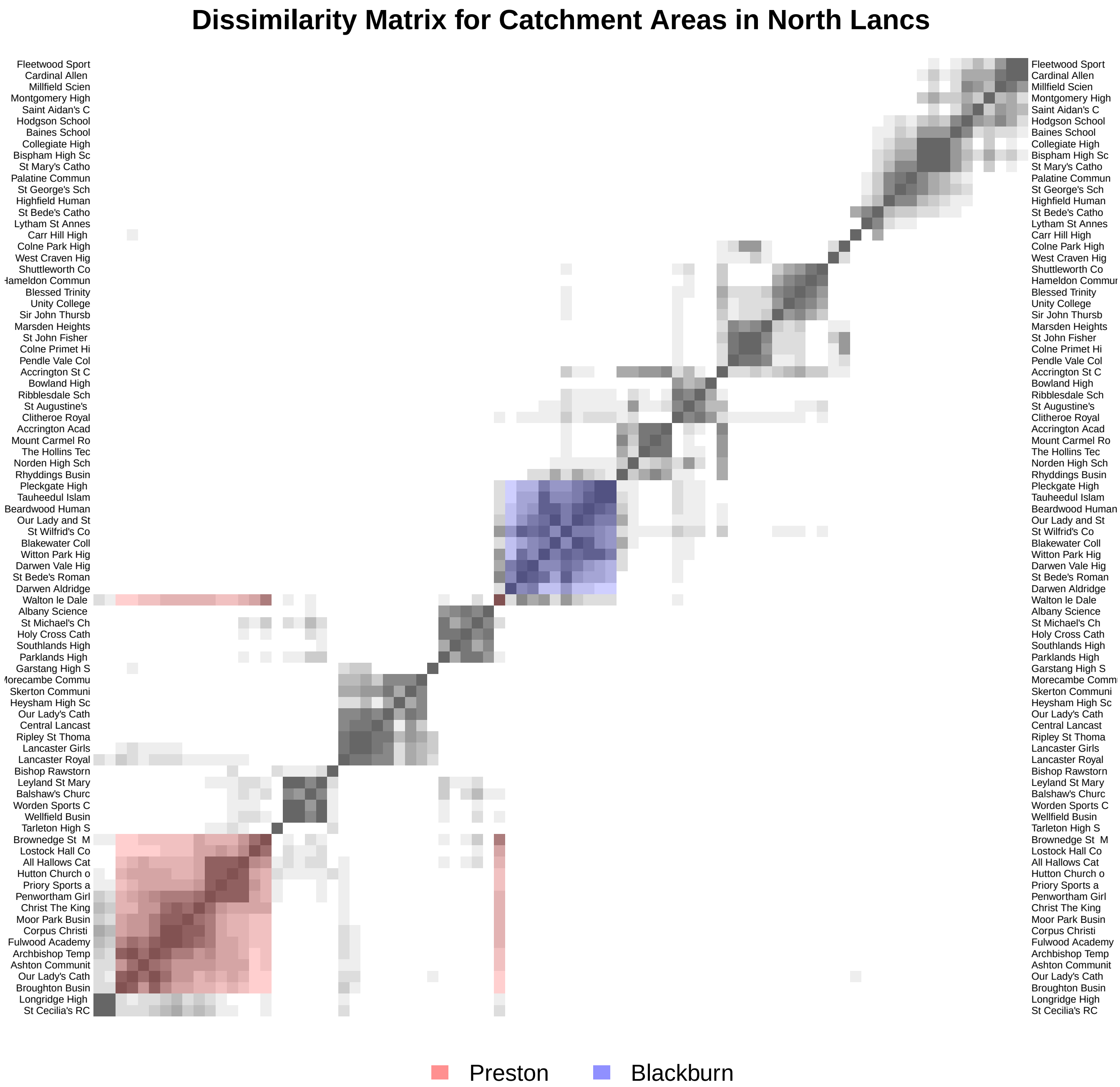


Figure 4.2: *Dissimilarity indices between the gridded, KDE-smoothed catchment distributions of pairs of schools in North and East Lancashire. Matrix indices have been permuted using anti-Robinson optimisation to reveal clusters.*

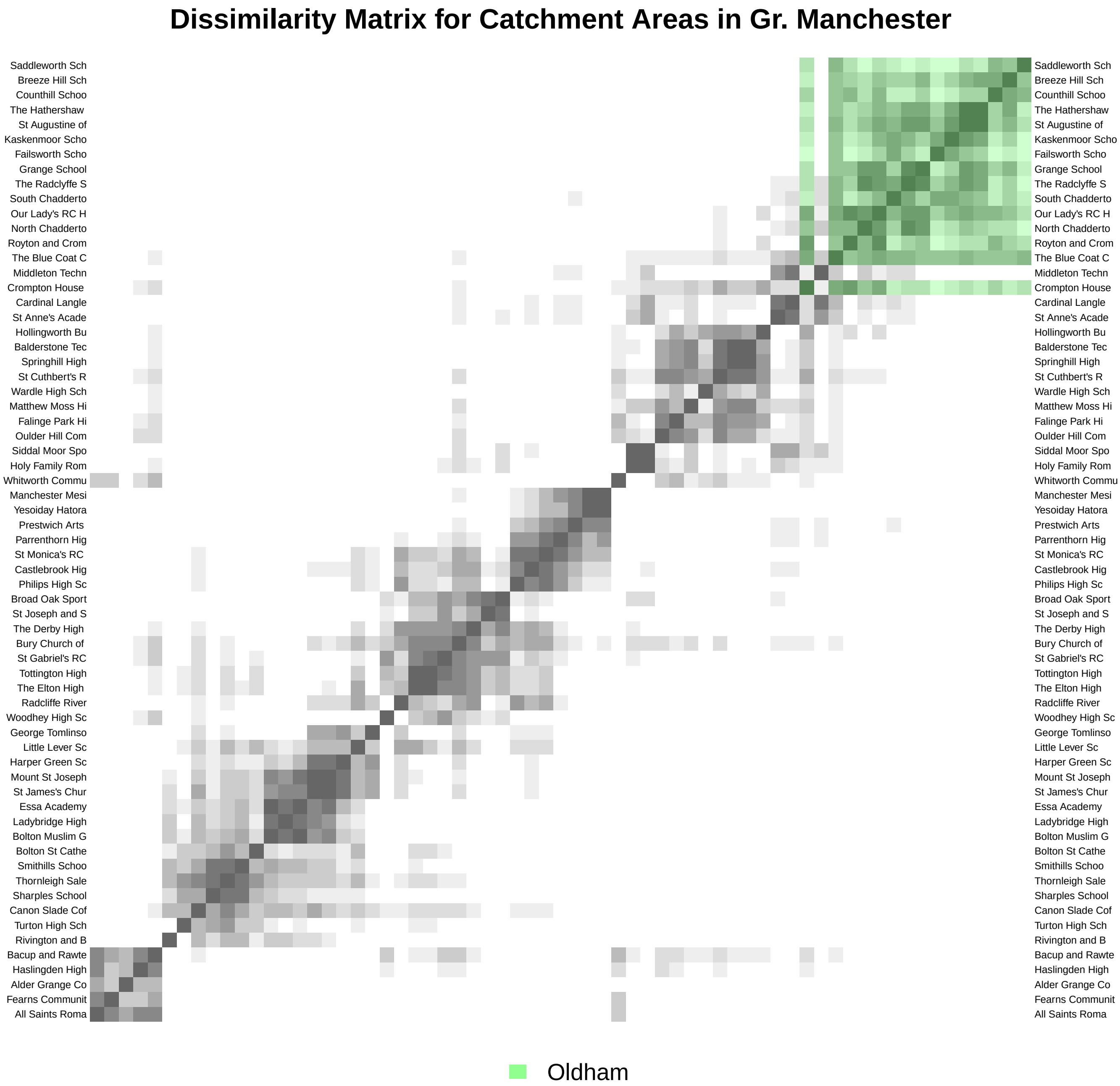


Figure 4.3: *Dissimilarity indices between the gridded, KDE-smoothed catchment distributions of pairs of schools in South Lancashire and Greater Manchester. Matrix indices have been permuted using anti-Robinson optimisation to reveal clusters.*

Chapter 5

Studying parental choices

The analyses presented in Chapter 4 – segregation indices, effective choice sets and competition networks – provided useful insights into associations between geography, choice, sorting and competition. However, the implicit aim of all three analyses was to illuminate the causal factors underlying these themes. To this end, the analyses were suggestive of causal effects. For example, the segregation indices gave evidence for post-residential sorting in urban neighbourhoods. However, according to the framework for sorting laid out in Chapter 2 there are several causal factors that could lead to post-residential sorting. Similarly, with the other analyses it was difficult to disentangle geography from other factors underlying patterns of choice and competition. The analyses fell short of illuminating causal explanations because they were not based on a model of the underlying causal processes, of which arguably the most important is the decision-making of families.

Given that the field is called School Choice, there have been surprisingly few attempts to model family choices of school explicitly, as opposed to modelling the effects of choice. The main barrier has been the lack of good quality choice data. Of the studies reviewed in Chapter 2, only a couple (Hastings et al., 2005; Abdulkadiroğlu et al., 2015) have managed to obtain stated preferences data from the allocation process. Burgess et al. (2015) use a large longitudinal survey, linked to data from the National Pupil Database. Bayer and McMillan (2010) and Gibbons et al. (2013) use house price data to analyse Tiebout choice. Having only observed allocations data, Caetano and Maheshri (2013) and Gallego and Hernando (2009) assume that supply is perfectly elastic, and allocations therefore represent choices (which may be a reasonable assumption in their contexts).

But school preferences cannot be ignored, however difficult it may be to obtain revealing data. Choices underlie sorting, directly via peer-preferences, and indirectly through the way that some families' choices constrain other families choices. Choices also underlie quasi-market competition. School markets are defined by schools competing against other schools to achieve and maintain market share. Market share is simply the aggregate of family decision-making. This implies that those qualities of schools, families and the interaction between the two, that affect the choices families make, may help to explain both sorting and competition. What is more, understanding school choices may also lead to insights into how competition affects sorting (which has been the aim of studies such as Epple and Romano (1998)), and also – which has been less well studied – how sorting may affect competition. In short, family decision-making is the glue that links competition and sorting.

There are two main post-residential choice processes that may contribute to sorting and competition effects. The first process is the differentiation of preferences by socio-economic circumstances. There is increasing econometric evidence from several countries (eg. Hastings et al., 2005; Gallego and Hernando, 2009; Burgess et al., 2015) that, although in general parents from all backgrounds value school performance as measured by test scores, middle-class parents tend to place a greater weight on test scores, and less weight on proximity when choosing a school. Burgess et al. (2015) use the Millenium Cohort Study to analyse parental preferences for primary schools, finding that parents value proximity and educational performance as measured by test scores. They also find that the relative weight of these two factors is differentiated by the socio-economic background of the family, as measured by SES variables included in the survey. These elements of preferences are important in understanding competition, as "quality-seeking behaviour" plays a central role in creating the necessary conditions for raising standards in market-like systems. Moderated preferences for quality may also play a role in sorting; in a school market with moderated preferences for test scores, sorting on SES would arise simply because higher-SES families are more likely to travel further to attend a school with good academic performance.

The second process concerns the importance of the ethnic demographics of the school in the formation of school preferences. Alongside proximity, quality and other factors, parents may consciously or unconsciously evaluate the prevalence of different ethnic groups at a school, and weigh this in choosing a school for their child. We can evaluate the relative strength of preferences of each group; this would shed light on the extent to which preference-induced sorting is one-sided ("vertical") or mutual ("horizontal"). It is also important to allow for the possibility that peer preferences are not *linear*. Perhaps parents would prefer a mixed school, rather than either a white or minority "monoculture". It is necessary to specify peer preferences in such a way that various hypotheses can be accommodated.

A handful of studies of school choice have allowed for preferences for peer groups (Elacqua et al., 2006; Hastings et al., 2009; Abdulkadiroğlu et al., 2015; Burgess et al., 2015). Of the econometric papers, Caetano and Maheshri (2013) is worthy of note, as the authors not only estimate a model for ethnic in-group preferences (homophily) but also empirically estimate a model of *tipping points* leading to segregation.

Although we have chosen to focus on socio-economic groups for the first question, and ethnic groups for the second, this is primarily for practical rather than substantive reasons. There is no reason *a priori* why we shouldn't expect peer preferences for SES, and moderation of preferences for quality by ethnicity. However, with the data we have it would be difficult to simultaneously identify the moderation of preferences for quality and peer preferences for the same set of groups. This is because the correlation between, say, the socio-economic composition of, and the academic performance of schools is so great that it is difficult to distinguish between the two types of preference. By studying only one type of preference phenomenon in relation to each category, we in effect set up an exclusion restriction that enables stable estimation. For this reason we follow previous studies in focussing on SES in relation to the question of preferences for quality, and ethnicity in relation to peer preferences.

These two processes – peer preferences and moderated preferences for quality – are our primary focus in modelling school preferences. We will also account for preferences for proximity, and preferences for religious schools.

5.1 Discrete choice models

The statistical modelling of choice for a person faced with several discrete alternatives was studied in the field of psychology by Thurstone (1927), axiomatised by Luce (1959), and further refined and popularised in the field of Economics by McFadden (1974, 1981). Discrete choice models are based on a representation of decision-making agents as aiming to maximise the utility of their decision. Each agent has a utility function, that maps relevant choice criteria and idiosyncratic preferences to a latent quantity called utility, quantifying the value of each choice to the agent. The agent chooses the option that maximises his or her utility. A stochastic element in the utility function converts a deterministic decision problem into a statistical model. For this reason, discrete choice models are also called Random Utility Models (RUMs).

In the terminology of econometrics, RUMs are *structural models*, meaning that the model parameters are equated to parameters of a theoretical economic model. In the case of RUMs, the corresponding economic models are models of human behaviour under uncertainty. It is possible to elaborate on RUMs by drawing on economic theories that aim to predict the decision-making behaviour of agents, or normatively specify optimal behaviour, based on rationality (expected utility theory) intertemporal optimality (dynamic discrete choice; see Aguirregabiria and Mirak (2010) for a recent review), bounded rationality (Kahneman and Tversky, 1979), or other elaborations. However, discrete choice models can also be used agnostically as descriptive models of behaviour, without adopting utility-based behavioural theories of choice.

In the context of school choice, a typical utility function for a family might look like

$$\begin{aligned} U_{as} &= f(d_{as}, \boldsymbol{x}_s, \boldsymbol{x}_{as}; \boldsymbol{\beta}) + \epsilon_{as} \\ &= \beta_1 d_{as} + \sum_{k=2}^{K_1} \beta_k x_{ks} + \sum_{k=K_1+1}^{K_2} \beta_k x_{ask} + \epsilon_{as}, \end{aligned}$$

where d_{as} is home-school distance, x_s are observable school characteristics, and x_{as} are interactions between pupil and school characteristics. ϵ_{as} are terms relating to unobserved preference heterogeneity assumed to follow some parametric distribution, say $\mathrm{N}(0, 1)$. The model is parametrised by $\boldsymbol{\beta}$. For compactness we can write this as $U_{as} = f_{as} + \epsilon_{as} = \boldsymbol{x}_{as}^{\mathrm{T}}\boldsymbol{\beta} + \epsilon_{as}$, bearing in mind that $\boldsymbol{x}_{as}$ subsumes d_{as} and contains a mixture of covariates that vary by school only and by pupil and school. Note that neither pupil characteristics, nor an intercept term, enter into pupils' utility functions on their own.

The parametric random utility model model allows the estimation of the relative importance of different factors in the decision-making process, as well as the probabilities of a family choosing each school from a known choice set, and the aggregate demand that each school would enjoy under counterfactual scenarios. It is therefore a very useful model for our purposes.

The Maximum Likelihood estimator for the random utility model when choices are observed is based upon the joint probability of observing each choice, given the underlying utilities:

$$\begin{aligned} p(y|\theta) &= \prod_{a=1}^{N_a} \Pr\left(\bigcap_{s \neq \mathrm{m}(a)} U_{a\,\mathrm{m}(a)} > U_{as} \right) \\ &= \prod_{a=1}^{N_a} \int_{\mathbb{R}} \left\{ \prod_{s \neq \mathrm{m}(a)} \Pr\bigl(\epsilon_{as} < f_{a\,\mathrm{m}(a)} + e - f_{as}\bigr) \right\} p_{\epsilon_{a\,\mathrm{m}(a)}}(e)\,\mathrm{d}e. \end{aligned}$$

In this notation N_a is the number of students in the sample, and $\mathrm{m}(a)$ is the index of the alternative chosen by student a – in the context of school choice, the "match" of a. Schools are indexed by s, and the set of all schools (students) is $\mathcal{S}$ ($\mathcal{A}$). When $\epsilon \sim \mathrm{N}(0,1)$, the model is

$$p(y|\theta) = \prod_{a=1}^{N_a} \int_{\mathbb{R}} \left\{ \prod_{s \neq \mathrm{m}(a)} \Phi\bigl(\beta^{\mathrm{T}} \left[\boldsymbol{x}_{a\,\mathrm{m}(a)} - \boldsymbol{x}_{as}\right] + e\bigr) \right\} \phi(e)\,\mathrm{d}e,$$

where $\phi(\cdot)$ is the standard normal density, and $\Phi(\cdot)$ is the standard normal CDF. This is known as the multinomial probit discrete choice model.

Another important variant of the model occurs when the stochastic part of utility is distributed Extreme Value Type-II (Gumbel). Due to the properties of the Gumbel distribution, the integral in the likelihood has a tractable form:

$$p(y|\theta) = \prod_{a=1}^{N_a} \frac{\exp f_{a\,\mathrm{m}(a)}}{\sum_{s \in \mathcal{S}} \exp f_{as}}.$$

This is known as the multinomial logit (MN-logit) discrete choice model.

There are three main conditions for identification of a discrete choice model:

1. Only differences matter (Train, 2009). It is clear from the model above, that the likelihood contribution of each observation depends only upon the differences in observable characteristics, captured by $\left[x_{a\,\mathrm{m}(a)} - x_{as}\right]$. Adding a term that is constant for all options for each pupil, such as an intercept term, or a term depending only on pupil characteristics, would not affect this difference, and therefore such terms are not identifiable.

2. Utilities are scale-invariant (Train, 2009). Scaling the utilities by a constant does not affect the choice probabilities. This means that σ_ϵ^2 must be fixed, to allow the estimation of β. In the MN-probit model σ_ϵ^2 is usually fixed at one, whereas the MN-logit model usually has $\sigma_\epsilon^2 = \pi^2/6 \approx 1.6$.

3. Non-separation of differences. This condition is not usually discussed in relation to discrete choice models specifically, but *separation* is a well-known phenomenon in modelling dichotomous outcomes more generally. In the context of the discrete choice model, separation would occur when, for some covariate, the distribution of covariate differences $\Delta_a^{(x)} \equiv \left[x_{a\,\mathrm{m}(a)} - x_{as}\right]$ does not have support on both sides of zero. If such differences are all positive or all negative, the likelihood function for the corresponding parameter is not bounded and the MLE for that parameter is infinite. This condition is mentioned now as it will be relevant in discussing identification for two-sided choice models.

5.1.1 Aggregate demand and substitution

A common use of discrete choice models is to predict the aggregate demand for or *market share* of alternatives under different counterfactual scenarios. Such scenarios involve the exogenous alteration of observable characteristics of the alternatives, or the distribution of observable characteristics in the population, or both. Given an estimated model, the aggregate demand for alternative s is defined as the sum over all predicted choice probabilities for that alternative,

$$D_s = \sum_{a \in \mathcal{A}} \Pr\Big(U_{as} = \max_{s' \in \mathcal{S}} (U_{as'}) | \boldsymbol{X}, \hat{\beta} \Big) .$$

For the evaluation of counterfactuals involving characteristic x_s, either the derivative of aggregate demand $\frac{\partial D_s}{\partial x_s}$ or the elasticity of demand $\frac{\partial D_s}{\partial x_s}\frac{x_s}{D_s} = \frac{\partial \log D_s}{\partial \log x_s}$ may be of interest. It is also possible to define, for alternative t, the cross-derivative $\frac{\partial D_t}{\partial x_s}$ and cross-elasticity $\frac{\partial D_t}{\partial x_s}\frac{x_s}{D_t}$. These estimable quantities describe how strongly the market demand for t responds to changes in observable characteristic x_s. The cross-derivatives and -elasticities describe the pairwise dependence of each alternative on the other alternatives for demand.

If x_s changes, such that demand for alternative s goes down, the cross-derivatives $\frac{\partial D_{s'}}{\partial x_s}$ describe how families *substitute* other schools for school s. These *patterns of substitution* are often an important output of market analysis, so it is important that they are accurate. However, discrete choice models as described above are limited in their ability to model complex patterns of substitution, as they do not allow for unobserved systematic variation in tastes leading to correlated errors ϵ_{as} across alternatives for a given individual. For example, there may be an unobservable (to the researcher) characteristic of schools, say the smartness of their uniforms, and a certain sub-group of parents who value smart school uniforms highly. If they move away from a school with smart uniforms they are more likely to substitute another school with smart uniforms. This shared unobservable variation in tastes would cause the error terms for the two schools to be correlated, and this correlation would not be captured by the model, leading to inaccurate patterns of substitution. There are two ways to deal with this problem: either ensure that all relevant variation in tastes is captured by observable characteristics within the model; or allow the error terms to be correlated. The latter option would usually require computationally difficult numerical integration.

5.2 The two-sided matching model

By modelling school choices we can uncover the determinants of family decision-making to explain processes of sorting and competition. The problem with this approach is that discrete choice estimation requires data relating to choices that relate simply to underlying preferences. In the context of school choice, the allocation of a child to a school is not always a direct reflection of the family's preferences, as some proportion of children do not get into their first choice school. The more supply of school places is constrained, the greater the proportion of parents who will not receive their first choice of school, and the greater the potential for bias from treating school allocations as if they revealed first preferences. In previous international literature (Caetano and Maheshri, 2013; Gallego and Hernando, 2009) this issue has been ignored on the rationale that supply of school places is reasonably elastic and the majority of parents achieve their first preference school. However, in the UK the supply of school places is highly constrained and the strategy of using a naive choice model risks introducing an unacceptable amount of bias.

In the terminology of the discrete choice literature, the constrained matching problem corresponds to each family having a choice set that is both endogenous and unobserved. Each party's choice set is only defined in equilibrium and is not visible to the researcher[1]. This is in contrast to traditional discrete choice models, which rely on the assumption that the choice set, and all relevant characteristics of all available options, can be observed. Any solution to this problem must therefore deal with unobserved variables and endogeneity.

To deal with this problem, we have developed a structural model based on the game-theoretic properties of school allocations as two-sided matchings. The model and associated estimation method draw on a branch of game theory and mechanism design known as *Stable Matching Theory*, introduced by Gale and Shapley (1962). The theory analyses two-sided matching problems known as the *Marriage Problem* and the *College Admissions Problem*, and develops a relevant solution concept known as *stability.* Chapter 8 discusses the background and relevance of stable matching theory in greater detail. The particular relevance of stable matching theory for school choice stems from the widespread use of concepts from stable matching theory in designing centralised school allocation mechanisms.

Chapter 9 motivates the "stability" partial likelihood model in greater depth; in this section we briefly describe the model and method. The model begins with the assumption that an observed school matching (a set of data on pupils allocated to schools, with pupil and school characteristics) is a *stable matching.* This assumption is justified by the predominant use by LEAs, of centralised allocation algorithms designed to return a stable matching[2]. Given a stable matching, it is possible to derive a set of preference relations or

[1]See Section 8.3 in Chapter 8 for a full discussion of the concept of "stable choice sets."

[2]Existing legislation does not mandate the use of stable-matching mechanisms by admissions clearing-

inequalities on underlying utilities. These revealed inequalities provide information about the parameters of underlying Random Utility Models in a way similar to the inequalities that allow identification in standard discrete choice models.

In order to implement the model, it is necessary to specify random utility models for both sides of the matching – that is, for both pupils *and* schools. This is problematised by the fact that in educational quasi-markets such as in England, schools do not express preferences for pupils in the strictest sense. However, schools do use over-subscription criteria, codified in each school's admissions policy, to rank pupils when the school is oversubscribed. It is these criteria that we model as "pseudo-utilities". We can do this by setting up a random utility function for schools, such as $V_{sa} = g(d_{sa}, \boldsymbol{z}_a, \boldsymbol{z}_{sa}; \boldsymbol{\delta}) + \eta_{sa}$, where $g(\cdot)$ is a linear model of observable characteristics, and η_{sa} is an unobservable random term distributed according to a known parametric distribution, say $\mathrm{N}(0, 1)$. $\boldsymbol{\delta}$ denotes a set of parameters, and these are combined with the pupil-side parameters in $\theta = (\beta, \delta)$. The pseudo-utility function does not contain terms relating only to characteristics of the school, as these would only shift each school's utility for all options, and would therefore not affect preference formation. There are no other restrictions on the inclusion of observables in either the pupils' or schools' utility functions. In particular the two utility functions may contain some or all of the same covariates, as long as the identification restrictions already mentioned are met. Identification is further discussed in Chapter 9.

A market is an ordered triple $(\mathcal{A}, \mathcal{S}, \pi)$, where $\mathcal{A}$ is a set of pupils, $\mathcal{S}$ is a set of schools, and π is a full set of (pseudo-)preferences on both sides of the market – a *preference profile*. The function $\mathrm{m}(\cdot)$ maps each pupil or school to his/her/its set-valued "match" on the other side of the market[3]. Let $\mathbf{m}$ denote a matching – that is, a full set of pupil-school matched pairs. The set of all possible matchings for a given market is denoted by $\mathcal{M}$ and the subset of matchings that are stable with respect to a given market are denoted by $\mathcal{M}^*$. Dependence of $\mathcal{M}^*$ upon the market, and especially upon the preference profile, is implicit.

Informally, stability of a matching $\mathbf{m}$ is equivalent to the condition that each school-

houses. However, the Admissions Code 2007 did outlaw a popular form of non-stable-matching algorithm, which has led most LEAs to adopt stable-matching algorithms (Pathak and Sönmez, 2011)

[3]We abuse notation slightly by allowing $s = \mathrm{m}(a)$ rather than $\{s\} = \mathrm{m}(a)$, which enables the use of the function in indices.

student pair who are *not* matched would not mutually prefer to match with each other than their current matches. Let $\overline{U}_a = U_{as} : \{s\} = \mathrm{m}(a)$. This is the student's utility for her allocated school. Similarly, let $\overline{V}_s = \min_{a \in \mathrm{m}(s)}\{V_{sa}\}$ be the utility of the worst student matched with school s. For each unmatched pair (a, s), the pair blocks the matching if and only if $U_{as} > \overline{U}_a$ and $V_{as} > \overline{V}_s$. If there are no such pairs, the matching is stable.

For a given matching $\mathbf{m}$, the probability of pairwise stability is equivalent to the probability that no unmatched pair (a, s) blocks the matching. The conditional likelihood of stability, given the matching, the parameters and the matched utilities is defined as

$$\Pr\bigl(\mathbf{m} \in \mathcal{M}^* | \theta, \overline{\mathbf{V}}, \overline{\mathbf{U}}\bigr) = \prod_{(a,s) \notin \mathbf{m}} \left\{1 - \Pr\bigl(U_{as} > \overline{U}_a\bigr) \Pr\bigl(V_{sa} > \overline{V}_s\bigr)\right\},$$

and marginalising over $\overline{\mathbf{U}}$ gives

$$\Pr\bigl(\mathbf{m} \in \mathcal{M}^* | \theta, \overline{\mathbf{V}}\bigr) = \prod_{a \in \mathcal{A}} \int_{\mathbb{R}} \left\{ \prod_{\{s \notin \mathrm{m}(a)\}} 1 - \Pr(U_{as} > u) \Pr\bigl(V_{sa} > \overline{V}_s\bigr) \right\} p_{\overline{U}_a}(u)\ \mathrm{d}u.$$

The integrals over $p_{\overline{U}_a}(u)$ (a standard normal density given the Probit model) are easily approximated using gaussian quadrature rules. The joint probability distribution of stability and the unobserved $\overline{\mathbf{V}}$ is

$$\Pr(\mathbf{m} \in \mathcal{M}^*, \mathbf{v} | \theta) = \Pr\bigl(\mathbf{m} \in \mathcal{M}^* | \theta, \overline{\mathbf{V}}\bigr) \prod_{s \in \mathcal{S}} p_{\overline{V}_s}(v_s | \mathbf{m}, \delta), \tag{5.1}$$

where

$$\begin{aligned} p_{\overline{V}_s}(v_s | \mathbf{m}, \delta) &= \frac{\partial}{\partial v_s} \left[1 - \prod_{a \in \mathrm{m}(s)} \Pr(V_{sa} > v_s | \delta) \right] \\ &= \left[\prod_{a \in \mathrm{m}(s)} \Pr(V_{sa} > v_s | \delta) \right] \left[\sum_{a \in \mathrm{m}(s)} \frac{p_{V_{sa}}(v_s)}{\Pr(V_{sa} > v_s | \delta)} \right]. \end{aligned}$$

The estimation method we adopt to estimate the parameters θ using this likelihood function is described below.

5.2.1 Specification of the pupil utility model

The pupil utility model must take into account the factors potentially influencing school choice that were discussed earlier – peer preferences and moderated preferences for academic performance. It must also take account of proximity, as parents have obvious incentives to prefer closer schools over schools further away, all things being equal. The NPD data contains the postcodes of both schools and pupils as well as the Ordnance Survey northings and eastings of the postcode centroids. The OS co-ordinates are used to calculate the linear distance between each pupil's postcode and each school in kilometres. Since postcode is the lowest-level geographical identifier, the measurement error in using postcode centroid to approximate location is considered negligible and is not accounted for in the model.

Previous discrete choice studies of school choice have assumed that utility is linear in distance from school. Abdulkadiroğlu et al. (2015) stated that this assumption allowed distance to be used as a "numeraire" to set a scale for utility comparisons in the absence of money. We wish to accommodate the possibility that utility for proximity is non-linear. However, a polynomial specification for distance, such as a quadratic or cubic, is too constraining. A quadratic specification, for example, may be influenced too much by the extremes of distance where choice probabilities are very low. To allow for non-linear preferences for proximity we have specified a model in which utility for proximity is piecewise linear, with the "knots" chosen so that they fall roughly at the quartiles of allocated distances across all samples; the knots (including boundary knots) are at $\{0, 2.5, 4, 6, 22\}$ kilometres. This allows the gradient of the proximity utility function to change at each quartile. The piecewise linear terms were parametrised as a B-spline to minimise correlation between terms for computational reasons. The R functions `splineDesign` and `bs` were used to create the B-spline design matrices (R Core Team, 2016). In addition, a dummy for the closest school to the pupil is included to allow for the possibility that families give additional weight to their nearest school, which is not accounted for by proximity alone.

To operationalise peer effects pupil ethnicities and data on the ethnic mix at schools are used. In the urban samples of pupils the predominant ethnic groups are: white British; Indian; Pakistani; and Bangladeshi; with much smaller proportions of other ethnic groups. Because the estimation of peer preferences requires reasonably large group sizes, ethnicity is re-categorised into three groups: white British; Asian (including Indian, Pakistani and Bangladeshi but excluding 'other asian'); and Other (including 'white other' and 'asian other'). In the context of the three urban school markets, this categorisation is appropriate as south asian pupils tend to form the main minority ethnic group in all three markets.

School ethnic mix is represented by two-year lagged mean proportions of White and Asian categories at each school (whole-school population). Proportions are calculated as the mean of proportions at $y - 2$ and $y - 1$, where y is the year of entry. Lagged mean percentages have been divided by 10 so that they take values between one and 10 and parameter estimates represent the change in utility per 10 point change in ethnic composition. The lagged means are designed to represent parents' qualitative recognition of the relative ethnic compositions of schools, gleaned from direct observation and social networks, as opposed to a precise knowledge of ethnic proportions, which are not published. Lagged mean proportions are interacted with pupil ethnicity dummies, such that only white pupils have a term for the white composition of schools, and likewise for asian pupils. In this way homophily (preference for one's in-group) is modelled explicitly, but xenophobia (dis-preference for another group) is not modelled. Linear and quadratic terms on these interactions are included in the pupil utility function to allow for non-linear peer preferences. Peer preferences for the Other category are not modelled as the group size is too small.

Academic performance is operationalised as the % of children in each year achieving five or more A*–C at GCSE (5AC). During the sampled period this measure was the main raw performance measure used in performance tables published by the government and in league tables. This raw performance measure was used rather than a value-added measure such as Contextualised Value-Added (CVA) for two reasons. First, there is evidence from empirical studies that parents focus on raw performance when choosing schools, rather than value-added measures (Wilson et al., 2006). Second, value-added measures used in performance tables changed during the study period, whereas the 5AC measure remained unaltered[4]. Using a measure that has not been altered during the sampled period facilitated the combination of estimates across years. As with ethnic

[4]The 5AC measure has recently been superceded by Progress 8.

preferences, the two-year lagged mean of 5AC was used to represent parents' weighing several information sources (including out-of-date word of mouth as well as up-to-date statistics). The measure was standardised in each sample, so that parameter estimates could be interpreted per standard deviation change in 5AC.

Measuring socio-economic status (SES) using data from the NPD is problematic. The two variables that are widely used as proxies for SES are Free School Meals eligibility (FSM) and the Income Deprivation Affecting Children Index (IDACI). Each has been criticised for different reasons. FSM indicators denote that a child is, or has been at some point in the past six years, eligible for a free school meal because the family receives certain state benefits. However, this is not an indicator of poverty *per se* and it has been criticised on the grounds that it contains an element of arbitrariness and fails to measure the working poor (Hobbs and Vignoles, 2007). IDACI, on the other hand, is a geographically-imputed variable – that is, a composite of the prevalence of certain indicators of deprivation in census tracts, that has been assigned to each pupil based on the pupil's postcode. This means that, rather than measuring the pupil's income deprivation, it assumes that the pupil's circumstances are explained by the pupil's home location. As a geographically-imputed variable, IDACI is only valid as a proxy for pupil SES to the extent that residential sorting on SES exists.

Each of these proxies for SES is problematic. Crawford and Greaves (2013) compared several proxies for SES and concluded that at a *school level*, the proportion receiving FSM was a better predictor of "educational disadvantage" than IDACI. However, we require a proxy for SES that has good properties at the level of an *individual pupil*, and for this purpose there are several reasons to prefer IDACI. First, FSM codifies a single administrative rule based on somewhat arbitrary criteria, whereas IDACI at least averages over several rule-based classifications, and many census respondents, leading to an estimate of deprivation that should be expected to be more stable and less sensitive to arbitrary rules. Second, IDACI allows more finely-grained classification into five (or any number of) levels, whereas FSM only allows binary classification. A single 'outlier' may be more influential when using FSM than IDACI quintiles, since the influence of a single case on the estimate for one quintile will not affect neighbouring quintiles.

The IDACI variable is a proportion of households in each census tract experiencing income deprivation. These were transformed into a national rank, and thence into national quintiles. The reason for this transformation was that transforming IDACI into an ordered factor variable avoids having to assume linearity in the effect of IDACI on utilities. Finally, to allow for the moderation of preferences for academic performance by SES, the standardised 5AC measure was interacted with IDACI quintiles. In one model IDACI quintiles were replaced by an indicator of FSM eligibility.

The final part of the pupil utility model was a term interacting the Christian religious denomination of the school with the religious denomination of the pupil's primary school. Separate terms are estimated for Church of England (CofE) and Roman Catholic (RC) schools. The CofE term takes a value of one if the secondary school has a CofE denomination and the pupil's primary school also has a CofE denomination, zero otherwise. The RC term is defined similarly. The substantive interpretation of this measure is the extent to which families who send their children to faith primary schools prefer faith secondary schools, or if primary school denomination is viewed as a proxy for faith, the extent to which people of faith prefer schools of that faith. The practical reason for the inclusion of the terms is that they appear in the school utility model; we therefore avoid adding an additional exclusion restriction to the set of assumptions. By including this term in both the pupil and the school models we allow for the possibility that sorting based on religious denomination is by choice as well as constraint.

The existence of all-boys schools and all-girls schools means that for both boys and girls there are some schools that are not available, and should not be considered to be part of the choice set. To account for this, cases corresponding to a girl's probability of forming a blocking pair with a boys' school, or a boy with a girls' school, are simply excluded from the log-likelihood function.

The following pupil utility functions have been estimated. The first, model A, specifies a linear function for distance and ethnic mix, and uses IDACI:

$$
\begin{aligned}
U_{as} = {} & \beta_1 \texttt{distance}_{as} + \beta_2 \texttt{closest}_{as} + \beta_3(\texttt{Asian}_a \times \texttt{\%Asian}_s) + \beta_4(\texttt{WhiteBr}_a \times \texttt{\%WhiteBr}_s) \\
& + \sum_{j=1}^{5} \beta_{5j}(\texttt{5AC}_s \times \texttt{IDACI.Q}_{aj}) + \beta_6(\texttt{CofE}_s \times \texttt{CofEPrimary}_a) + \beta_7(\texttt{RC}_s \times \texttt{RCPrimary}_a) + \epsilon_{as},
\end{aligned}
$$

where $\texttt{IDACI.Q}_a$ are a set of dummy variables indicating which IDACI quintile the pupil belongs to. Model B allows quadratic peer preferences:

$$
\begin{aligned}
U_{as} = {}& \beta_1 \texttt{distance}_{as} + \beta_2 \texttt{closest}_{as} + \beta_3(\texttt{Asian}_a \times \texttt{\%Asian}_s) + \beta_8(\texttt{Asian}_a \times \texttt{\%Asian}_s)^2 \\
& + \beta_4(\texttt{WhiteBr}_a \times \texttt{\%WhiteBr}_s) + \beta_9(\texttt{WhiteBr}_a \times \texttt{\%WhiteBr}_s)^2 + \\
& \sum_{j=1}^{5} \beta_{5j}(\texttt{5AC}_s \times \texttt{IDACI.Q}_{aj}) + \beta_6(\texttt{CofE}_s \times \texttt{CofEPrimary}_a) + \beta_7(\texttt{RC}_s \times \texttt{RCPrimary}_a) + \epsilon_{as}.
\end{aligned}
$$

Model C adds a piecewise linear specification of distance:

$$
\begin{aligned}
U_{as} = {}& \sum_{i=1}^{I} \beta_{1i} B_i(\texttt{distance}_{as}) + \beta_2 \texttt{closest}_{as} + \beta_3(\texttt{Asian}_a \times \texttt{\%Asian}_s) + \beta_8(\texttt{Asian}_a \times \texttt{\%Asian}_s)^2 \\
& + \beta_4(\texttt{WhiteBr}_a \times \texttt{\%WhiteBr}_s) + \beta_9(\texttt{WhiteBr}_a \times \texttt{\%WhiteBr}_s)^2 \\
& + \sum_{j=1}^{5} \beta_{5j}(\texttt{5AC}_s \times \texttt{IDACI.Q}_{aj}) + \beta_6(\texttt{CofE}_s \times \texttt{CofEPrimary}_a) + \beta_7(\texttt{RC}_s \times \texttt{RCPrimary}_a) + \epsilon_{as},
\end{aligned}
$$

where $B_i()$ is a linear B-spline basis function. The final specification, model D, features linear distance and quadratic peer effects, and uses FSM in place of IDACI:

$$
\begin{aligned}
U_{as} = {}& \beta_1 \texttt{distance}_{as} + \beta_2 \texttt{closest}_{as} + \beta_3(\texttt{Asian}_a \times \texttt{\%Asian}_s) + \beta_8(\texttt{Asian}_a \times \texttt{\%Asian}_s)^2 \\
& + \beta_4(\texttt{WhiteBr}_a \times \texttt{\%WhiteBr}_s) + \beta_9(\texttt{WhiteBr}_a \times \texttt{\%WhiteBr}_s)^2 \\
& + \sum_{j=1}^{2} \beta_{5j}(\texttt{5AC}_s \times \texttt{FSM}_{aj}) + \beta_6(\texttt{CofE}_s \times \texttt{CofEPrimary}_a) + \beta_7(\texttt{RC}_s \times \texttt{RCPrimary}_a) + \epsilon_{as}.
\end{aligned}
$$

In addition to the specifications above, we also undertake a sensitivity analysis using model B (quadratic ethnic preferences) whereby all Islamic schools are removed and the model is re-estimated. Islamic schools, of which there is one in Preston and two in Blackburn, tend to have large proportions of Asian students. They also have faith-based admissions criteria for at least part of their intake. The sample does not contain any information about feeder schools, regular worship, or other criteria that constitute the admissions criteria, so it is possible that religious preferences, and religious constraints, may be confounded with ethnic preferences. For this reason we exclude Islamic schools to test the influence of these schools on results.

One aspect that is missing from the pupil utility model, that is often included in discrete choice models, is a school fixed effect (a dummy variable for each school) whose purpose would be to mop up any unobserved systematic variation in school popularity, not explained by school-level covariates. Although theoretically identifiable in many-to-one two-sided structural models, we found that in practice adding fixed effects to the model led to correlation and local optima in the likelihood function, hindering stable estimation. This appears to be due to the fact that the two-sided method already includes a latent variable for each school that mops up some unobserved variation in the popularity of schools, since less popular schools tend to have lower threshold utilities $\overline{\mathbf{V}}$. Fixed effects interact with these latent variables to produce local optima in the likelihood function, which makes estimation problematic. Excluding fixed effects should not affect the unbiased estimation of other pupil utility parameters, but is relevant to the accuracy of aggregate demand estimation for policy analysis.

Assuming that ϵ_{as} are uncorrelated across schools implies that the model does not allow for substitution in the sense defined by Train (2009) – that is, non-proportional substitution in the individual choice probabilities, conditioning on observables. In other words, the model does not allow for the possibility that there is some unobservable characteristic shared by schools s and s' such that people who like school s would be more likely to like school s', and vice-versa. However, a more important question for policy analysis is whether the model allows for realistic patterns of substitution in aggregate demand for schools, averaging over observables. This is indeed the case; by specifying a model that richly interacts school characteristics with pupil characteristics (location, ethnicity, SES and primary school denomination), rich patterns of substitution are possible. The presence of non-proportional substitution can be confirmed by analysing cross-elasticities of demand for a school with all other schools, and checking that they are not all equal.

5.2.2 Specification of the school pseudo-utility model

Admissions policies are differentiated by school, and generally rank pupils on the basis of faith, siblings, catchment areas and distance. Looked after children, or children who have been adopted from care, receive the highest priority. In addition, pupils with Special Educational Needs (SEN) are allocated on the basis of an assessment of their needs, independently of capacity. However, the data does not contain indicators of looked after children or special educational needs as these data are too sensitive. This raises the issue that there might be pupils in schools who appear to live, for example, too far away to gain acceptance, but are higher priority for those schools than they appear to be, because they are either looked-after children or have SEN. The model is sensitive to outliers who influence a school's apparent threshold of acceptance. However, the model is not as sensitive to missing pupils when those pupils are high priority and therefore not at or near the threshold. For this reason, the NPD team were requested to identify and exclude SEN and looked after children from the sample.

The sibling rule is usually the most important element of rankings in terms of the number of children admitted under that rule, but siblings are not observed at all in the sample. We do not have data on catchment areas. This means that the observed pupil characteristics available to model school pseudo-utilities are necessarily limited. Distance appears as a tie-breaker in almost all admissions policies. Similarly to the pupil utility model, we use the straight-line distance between the pupil's address and the school's address. A separate term is allowed for community schools and faith schools, to allow for the fact that faith schools use faith criteria in addition to distance, and therefore accept pupils from a wider area.

We do not directly observe the faith of pupils, but we do observe the primary school that was attended. Use of this information as a proxy for faith is justified on the grounds that gaining admission to a faith primary school often requires the same kind of evidence of religious practice as that required for admission to a faith secondary school. In addition, faith secondary schools, particularly Catholic schools, often include attendance at feeder primary schools in their admissions criteria.

The school utility function is specified as follows:

$$\begin{aligned} V_{sa} = {} & \delta_1(\texttt{Distance}_{sa} \times \texttt{FaithSch}_s) + \delta_2(\texttt{Distance}_{as} \times [1 - \texttt{FaithSch}_s]) \\ & + \delta_3(\texttt{CofE}_s \times \texttt{CofEPrimary}_a) + \delta_4(\texttt{RC}_s \times \texttt{RCPrimary}_a) + \eta_{sa}, \end{aligned}$$

where $\texttt{FaithSch}_s$ is an indicator of whether the school is a faith school.

The school pseudo-utility function represents all supply-side constraints that limit the ability of families to choose the school they would prefer. The school model incorporates distance and faith feeder schools, but not ethnicity or SES; this exclusion restriction implies that ethnicity and SES do not enter directly into the constraints that families face in accessing schools. However, ethnicity and SES may affect admission probabilities indirectly to the extent that they are correlated with residential location and the primary schools attended by pupils. Although it would be possible to add ethnicity and SES into the supply-side model, it was decided to only account for those pupil characteristics that may reasonably be expected to be visible to schools, as schools are prohibited from interviewing pupils or asking for information about their ethnicity or socio-economic background.

An implicit assumption in the model is that each school's intake is equal to its capacity. This is a strong assumption, since in a given market there may be one or more schools that have spare capacity. However, the assumption is unavoidable as we do not have data on school capacities. If capacity data were available, knowing that a school is not at capacity would be equivalent to knowing that its threshold utility $\overline{V}_s$ is equal to the utility of the *outside option* – V_{s0}. Since in England schools are not permitted to keep spare capacity voluntarily if there are students who wish to go to the school, this implies that the utility of the outside option must be lower than the utility of any child – therefore $\overline{V}_s = V_{s0} = -\infty$. It is not clear how sensitive estimation would be to the exact value of $\overline{V}_s$ if it were fixed at some large negative value, however simulation evidence (not presented) suggests that estimates of θ are somewhat robust to misspecifications of threshold utilities or capacities. In practice, setting $\overline{V}_s$ for some s to a low value would imply that some preference inequalities on the pupil-side would be certain, rather than contingent on the school-side.

5.2.3 Estimation

The stability likelihood model contains two sets of latent variables: $\overline{\mathbf{U}}$ and $\overline{\mathbf{V}}$. The model marginalises over the former, using numerical integration (Gaussian quadrature), leaving the latter as nuisance variables. There are several possible strategies for eliminating these nuisance variables and obtaining estimates of the parameters of interest, all of which are computationally demanding. The strategy we have adopted produces accurate point estimates with a moderate computational cost, at the expense of assuming normality, which may impact upon interval estimation. The Maximum h-Likelihood Estimator

(MhLE) jointly maximises the log-likelihood over both $\theta = (\beta, \delta)$ and $\overline{\mathbf{V}}$. The properties of this estimator are discussed in Chapter 9.

Estimation of parameters on combined datasets was achieved using the Expectation Propagation (EP) algorithm described in Chapter 9. The EP algorithm permits the use of a prior on θ, to regularise estimation – i.e. to reduce collinearity and improve the stability of optimisation. An appealing property of the EP algorithm is that priors can be used that are informative for each individual dataset but not informative enough to affect the combined approximate posterior. Priors were used liberally during the development of the model to stabilise estimation. However, the only informative prior used in the final models was a weakly informative ridge prior on the feeder school parameters, with diagonal elements equivalent to $\sigma_0^{\text{Feeder}} = 0.5$. The purpose of this prior was to reduce the collinearity of these parameters in the pupil-side and school-side of the model. A diffuse ridge prior with mean 0 and precision matrix $0.01\mathbb{I}$ was used for all of the other parameters.

Finally we briefly describe the method for numerically integrating out the latent variables $\overline{\mathbf{U}}$. We wish to approximate an integral such as

$$p(y|\theta) = \int_{\mathbb{R}} p(y|\theta, z)\, \phi(z)\, \mathrm{d}z$$

where z is a latent variable and $\phi()$ is the distribution of z, which is assumed to be a standard normal distribution. Gaussian quadrature replaces the integral with a weighted finite sum of N terms,

$$p(y|\theta) \approx \sum_{i=1}^{N} w_i p(y|\theta, h_i)$$

where w_i are the weights, and each term is evaluated setting the variable of integration to a particular value h_i, known as the node. The weights and nodes are chosen to optimise the accuracy of the approximation, and the optimal choice of weights and nodes depends upon the analytical form of the integrand. For integrands weighted by a normal distribution, the method known as Gauss-Hermite quadrature provides the correct nodes and weights. The function `gauss.quad.prob` in the R package `statmod` (Smyth et al., 2015) calculates the weights and nodes for Gauss-Hermite quadrature.

Chapter 6

Results of the two-sided model

This chapter presents the empirical results from estimating the four model specifications described in the previous chapter (plus the estimation of model B with Islamic schools removed). The estimated weightings are described and compared. These weightings shed light on the relative importance of different decision-criteria for families, and allow decision-weightings to depend on family membership of socio-economic and ethnic groups. Lastly, simulation evidence is presented to assess the fidelity with which the estimated model parameters predict observed joint distributions of matched pupil and school characteristics.

6.1 Empirical findings

Table 6.1 shows the estimated co-efficients from the model with linear peer preference effects. The first column of estimates are the combined estimates from estimating the model on all 12 samples, and the other three columns present the results from estimating the model on the four cohorts of each town separately. All results are estimated using the Expectation Propagation (EP) algorithm. Distance co-efficients are of similar magnitude in the three towns; the co-efficient for Oldham is the largest, at -0.754, and the combined co-efficient is the smallest, at -0.613. The similarity of linear estimates of the proximity effect, robust across all the estimated models, is encouraging since the distance effect serves as a *numeraire*, setting the scale for the other parameters of systematic utility. The similarity suggests that the overall scale of variation in systematic preferences, relative

Table 6.1: Estimates from model A with linear preferences.

	All		Blackburn		Oldham		Preston	
Demand-side:	Est	SD	Est	SD	Est	SD	Est	SD
Distance (km)	-0.613	(0.00998)	-0.652	(0.0269)	-0.754	(0.0209)	-0.639	(0.015)
Closest school	0.213	(0.0256)	0.435	(0.0551)	0.117 *	(0.0413)	0.278	(0.0435)
Ethnic (10% increase):								
Asian	0.254	(0.0122)	0.208	(0.0129)	0.296	(0.0174)	0.469	(0.0372)
White Brit.	0.189	(0.00759)	0.318	(0.0157)	0.112	(0.0101)	0.0916	(0.013)
% 5 or more A*–C (SDs)								
× IDACI q. 1	1.01	(0.0283)	0.761	(0.0758)	1.88	(0.0913)	1.04	(0.0380)
× IDACI q. 2	0.867	(0.0290)	0.546	(0.0775)	1.27	(0.0605)	0.956	(0.0414)
× IDACI q. 3	0.547	(0.0282)	0.106 *	(0.0584)	1.09	(0.0547)	0.533	(0.0421)
× IDACI q. 4	0.213	(0.0225)	-0.342	(0.0400)	0.806	(0.0437)	0.204	(0.0306)
× IDACI q. 5	-0.0700	(0.0197)	-0.506	(0.0394)	0.348	(0.0330)	0.0255 *	(0.0267)
Feeder schools:								
Church of Eng.	0.182	(0.0575)	0.361	(0.0688)	-0.208 *	(0.116)	0.666	(0.107)
Roman Catholic	1.69	(0.0441)	2.09	(0.101)	2.73	(0.136)	1.71	(0.0628)
Supply-side:								
Distance (km):								
Comprehensives	-0.512	(0.0139)	-0.223	(0.0229)	-0.597	(0.0190)	-0.453	(0.0311)
Faith Schools	0.00813 *	(0.00636)	-0.153	(0.0177)	0.160	(0.0105)	-0.0943	(0.00794)
Feeder schools:								
Church of Eng.	0.926	(0.0609)	0.0449 *	(0.481)	1.06	(0.0665)	0.280	(0.0895)
Roman Catholic	2.10	(0.0820)	2.60	(0.215)	1.02	(0.101)	1.60	(0.0730)
# pupils	21,313		5,044		9,393		6,876	
# school-years	153		39		52		62	

* $|\hat{\theta}| < 3\hat{\sigma}$

to the random term, is similar across markets. Estimates for the closest-school dummy are more variable.

When peer preferences are linear, in Preston and Oldham they are larger for Asian families than for white families. However in Blackburn peer preferences are stronger for Asian families. In general, peer preferences exist on both sides, suggesting *horizontal* sorting along ethnic lines. The linear model for peer preferences, estimated on all three markets, implies that an Asian family will be willing to send their child 0.41 km further for a 10% increase in Asian children at school, whereas a white family will send their child 0.31 km further for a 10% increase in white children at school.

There is a clear pattern of socio-economic status (represented by quintiles of IDACI) moderating preferences for raw academic performance (represented by "% 5+ A*–C at GCSE"). In model A, preferences for academic performance are strongest in Oldham, and weakest in Blackburn. For the highest 40% of IDACI, estimated preferences for academic performance are *negative* in Blackburn, implying that those families avoid schools with better GCSE grades. Estimates from the combined model imply that those living in the most affluent areas (IDACI quintile 1) will be willing for their child to travel 1.65 km further for a one-standard-deviation improvement in test scores, whereas those living in the least affluent areas (IDACI quintile 5) will send their child 0.11 km further for a one-standard-deviation *reduction* in test scores!

The final group of parameters in the pupil-side utility model is a set of parameters interacting probability of choosing faith secondary schools, with faith primary school attendance. In general the estimates for the interaction of Church of England (CofE) school choice with CofE primary school attendance are smaller and more variable than the Roman Catholic (RC) co-efficients. The latter are large, and similar in magnitude across the three markets. The combined model implies that a family attending a Catholic primary school will send their child 2.76 km further to attend a Catholic secondary school.

On the school side, coefficients for proximity are more variable, possibly reflecting the fact that in each market there is only a small number of schools and hence the effect of the spatial distribution of individual schools has a greater influence on estimates. The distance co-efficient for faith schools is smaller than the distance co-efficient for comprehensives, and is negligible in the combined model, suggesting that selection in faith schools is dominated by faith criteria and distance does not play an important role as a tie-breaker.

It is not as informative to interpret the parameters of the school-side model, except perhaps to calculate the probability of a student being ranked higher than another student given a fixed difference in proximity. For example, with the combined model estimate of -0.512 (s.d. 0.0139), if one pupil is 1 km closer to a community school than the other, the model implies that the probability of the school ranking the closer student higher than the further student is 0.641 [0.634,0.649]. Given that the actual admissions rule is deterministic, this implies that only a small proportion of the variation in admissions probabilities is explained by distance. However, attending a Catholic primary school confers a probability of 0.931 [0.915,0.945] of being higher-ranked at a Catholic secondary school than someone who did not.

Model B, presented in Table 6.2, presents the estimates from the model with quadratic

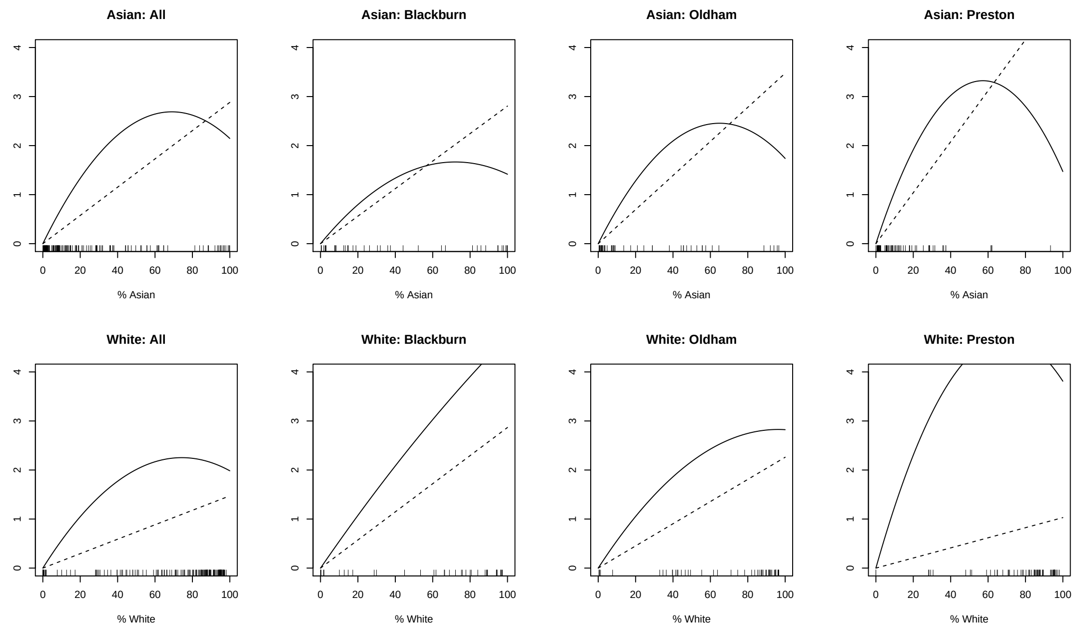


Figure 6.1: *Pupil utility functions for lagged mean ethnic composition. Dashed lines denote linear specification, while solid lines denote quadratic specification. Tick marks on x-axis show the distribution of observed lagged mean ethnic proportions.*

peer preferences for ethnicity. The model is otherwise identical to model A. Figure 6.1 plots estimates from model B (quadratic) against estimates from model A (linear). In general, model B indicates slightly inverse-U shaped utility functions for the ethnic in-group. This is consistent with a hypothesis that families may be strongly averse to schools where their child is an isolated minority, but at the same time be more indifferent to in-group dominance, or have weak preferences for integration. In the combined results, the Asian utility function is slightly steeper, but also more non-linear, than the white utility function suggesting that Asian families may have stronger preferences for integration. However, the estimates seem to support a hypothesis of broadly similar preferences for in-groups for both white and Asian families.

Apart from the peer preference estimates, the main difference between Tables 6.1 and 6.2 is that IDACI-moderated estimates of preferences for academic performance in Blackburn and Preston are larger, and are no longer negative for high-IDACI quintiles in Blackburn. The estimated test score co-efficients are now more similar across markets.

When a piecewise linear utility function for distance is introduced in model C (Table 6.3) the estimates of other parameters are not substantively altered. The only material change is in the closest-school dummy effects, which are all attenuated. Figure 6.2 plots

Table 6.2: Estimates from model B with quadratic peer preferences for ethnicity.

	All		Blackburn		Oldham		Preston	
Demand-side:	Est	SD	Est	SD	Est	SD	Est	SD
Distance (km)	-0.636	(0.0104)	-0.657	(0.0305)	-0.751	(0.0210)	-0.738	(0.0192)
Closest school	0.252	(0.0251)	0.318	(0.0568)	0.124	(0.0409)	0.404	(0.0509)
Ethnic (10% increase):								
Asian	0.803	(0.0289)	1.03	(0.0797)	0.784	(0.0467)	1.11	(0.0759)
Asian Sq.	-0.0651	(0.00265)	-0.0818	(0.00695)	-0.0618	(0.00434)	-0.0962	(0.00828)
White Brit.	0.744	(0.0345)	0.793	(0.0773)	0.601	(0.0596)	1.13	(0.0904)
White B. Sq.	-0.0462	(0.00264)	-0.0415	(0.00635)	-0.0413	(0.00485)	-0.082	(0.00667)
% 5 or more A*–C (SDs)								
× IDACI q. 1	1.08	(0.0289)	1.31	(0.105)	1.94	(0.0902)	1.31	(0.0528)
× IDACI q. 2	0.942	(0.0294)	1.25	(0.108)	1.33	(0.0620)	1.20	(0.0535)
× IDACI q. 3	0.629	(0.0288)	0.760	(0.0900)	1.15	(0.0564)	0.679	(0.0508)
× IDACI q. 4	0.298	(0.0223)	0.308	(0.0585)	0.837	(0.0444)	0.312	(0.0374)
× IDACI q. 5	0.0221 *	(0.0188)	0.101 *	(0.0499)	0.339	(0.0339)	0.122	(0.0336)
Feeder schools:								
Church of Eng.	0.195	(0.0598)	0.594	(0.137)	-0.185 *	(0.117)	0.733	(0.116)
Roman Catholic	1.94	(0.0482)	2.14	(0.0912)	2.90	(0.146)	2.04	(0.0790)
Supply-side:								
Distance (km):								
Comprehensives	-0.569	(0.0155)	-0.456	(0.0473)	-0.611	(0.0187)	-0.203	(0.0147)
Faith Schools	0.0124 *	(0.00612)	0.0266 *	(0.0123)	0.149	(0.0105)	-0.0946	(0.00766)
Feeder schools:								
Church of Eng.	0.863	(0.0586)	0.546	(0.129)	1.04	(0.0681)	0.273	(0.0775)
Roman Catholic	1.70	(0.0586)	2.79	(0.178)	0.953	(0.0862)	1.41	(0.0648)
# pupils	21,313		5,044		9,393		6,876	
# school-years	153		39		52		62	

* $|\hat{\theta}| < 3\hat{\sigma}$

the linear distance utility functions from model B against the piecewise linear utility functions of model C. Of the three towns, the piecewise utility function for Oldham appears to be the most similar to the linear utility function, and the piecewise utility function for Blackburn appears to be the most different, although this may be misleading as the actual substantive difference between utility functions depends upon the underlying distribution of pupil-school distances. For the combined result, the maximum divergence is at 2.5 km, where the difference between the piecewise utility and the linear utility is $2.5(-0.93) - 2.5(-0.64) = -0.73$. The main regularity from the piecewise estimates is that the utility function for distance is decreasing more quickly in the first 2.5 km, and thereafter is less steep.

Model D replaces IDACI quintiles with Free School Meals (FSM) indicators. Preferences for academic performance of families where the child is in receipt of Free School Meals are much lower than those for the rest of the population. In the combined model the absolute

Table 6.3: Estimates from model C with piecewise linear preferences for proximity.

	All		Blackburn		Oldham		Preston	
Demand-side:	Est	SD	Est	SD	Est	SD	Est	SD
Closest school	0.0235 *	(0.0292)	-0.0241 *	(0.0575)	-0.138	(0.0459)	0.26	(0.059)
Distance B-spline components:								
$B_1(d_{as})$	-2.32	(0.057)	-2.67	(0.121)	-2.68	(0.0969)	-2.15	(0.11)
$B_2(d_{as})$	-3.3	(0.0594)	-3.19	(0.126)	-3.68	(0.103)	-3.47	(0.113)
$B_3(d_{as})$	-4.07	(0.0732)	-3.94	(0.199)	-4.55	(0.143)	-4.57	(0.135)
$B_4(d_{as})$	-14.5	(0.437)	-8.44	(1.34)	-17.0	(0.914)	-13.8	(0.648)
Distance gradients by segment:								
0 – 2.5 km	-0.927	(0.0228)	-1.07	(0.0485)	-1.07	(0.0388)	-0.86	(0.0438)
2.5 – 4 km	-0.656	(0.0248)	-0.347	(0.0538)	-0.67	(0.0426)	-0.882	(0.0434)
4 – 6 km	-0.385	(0.0248)	-0.376	(0.0714)	-0.434	(0.0484)	-0.551	(0.0399)
6 – 22 km	-0.654	(0.0268)	-0.281	(0.0847)	-0.777	(0.0578)	-0.578	(0.0383)
Ethnic (10% increase):								
Asian	0.788	(0.0288)	0.868	(0.0679)	0.753	(0.0452)	1.11	(0.0758)
Asian Sq.	-0.065	(0.00264)	-0.0692	(0.00595)	-0.0601	(0.00419)	-0.0967	(0.00843)
White Brit.	0.783	(0.0367)	0.647	(0.0641)	0.579	(0.055)	1.13	(0.0914)
White B. Sq.	-0.0484	(0.00278)	-0.0303	(0.00531)	-0.0388	(0.00448)	-0.0816	(0.00673)
% 5 or more A*–C (SDs)								
× IDACI q. 1	1.12	(0.0308)	1.13	(0.0914)	1.82	(0.0806)	1.32	(0.0531)
× IDACI q. 2	0.955	(0.0305)	0.869	(0.0895)	1.25	(0.0588)	1.22	(0.0542)
× IDACI q. 3	0.617	(0.0286)	0.611	(0.0769)	1.08	(0.0523)	0.68	(0.0508)
× IDACI q. 4	0.296	(0.0226)	0.192	(0.052)	0.818	(0.0424)	0.316	(0.0381)
× IDACI q. 5	-0.0000476 *	(0.019)	0.00963 *	(0.0454)	0.322	(0.0327)	0.109	(0.0336)
Feeder schools:								
Church of Eng.	0.101 *	(0.0593)	0.486	(0.14)	-0.473	(0.108)	0.733	(0.115)
Roman Catholic	1.97	(0.0482)	2.04	(0.0844)	2.78	(0.131)	2.01	(0.0778)
Supply-side:								
Distance (km):								
Comprehensives	-0.497	(0.015)	-0.459	(0.0572)	-0.59	(0.0194)	-0.214	(0.0141)
Faith Schools	0.0055 *	(0.0059)	0.0287 *	(0.0156)	0.131	(0.0113)	-0.102	(0.00763)
Feeder schools:								
Church of Eng.	0.92	(0.0613)	0.606	(0.168)	1.21	(0.0769)	0.288	(0.0767)
Roman Catholic	1.72	(0.0565)	3.04	(0.198)	1.01	(0.0893)	1.46	(0.0665)
# pupils	21,313		5,044		9,393		6,876	
# school-years	153		39		52		62	

* $|\hat{\theta}| < 3\hat{\sigma}$

value of the effect is smaller than one standard deviation. For those not in receipt of FSM, families are willing to travel 0.98 km further for a one-standard-deviation improvement in test scores. The estimates of other parameters are not materially different from model B.

Finally, a sample has been constructed in which the Islamic faith schools have been removed. This is intended to check sensitivity of estimates to the presence of Islamic schools, which tend to have faith-based admissions criteria for at least part of their intake, and also might conflate ethnic peer preferences with preferences for religious education. Table 6.5 presents the estimates from this model. Although the results are qualitatively similar, in general, the removal of Islamic schools alters the coefficients relating to ethnic peer preferences in Blackburn and Preston (Oldham has no Islamic state schools). In

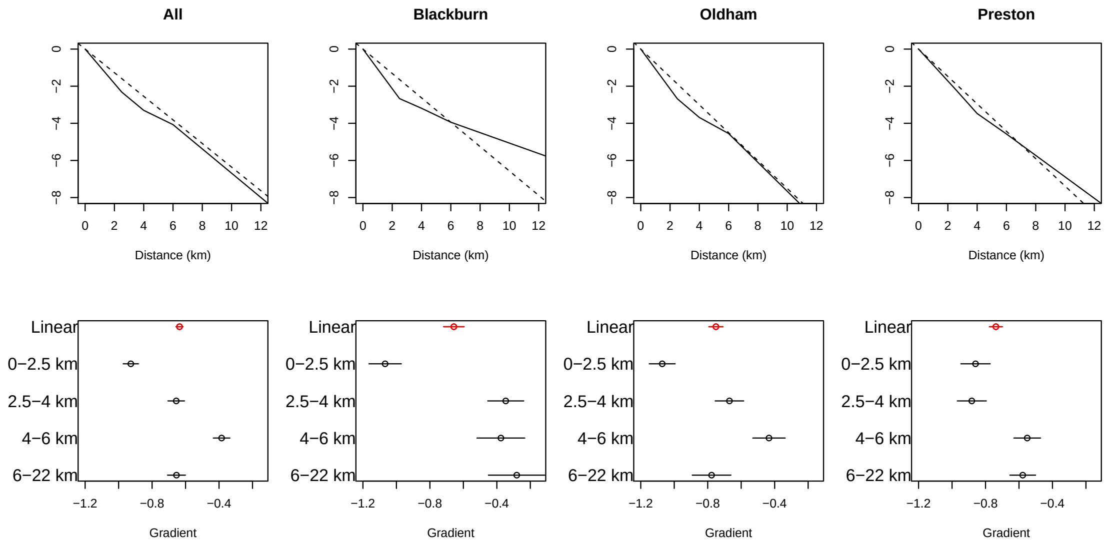


Figure 6.2: *The top figures show pupil utility functions for distance from school (km). Dashed lines denote the linear specification, while solid lines denote the piecewise linear specification. The bottom figures show the estimated gradients and confidence intervals for each line segment, compared to the linear specification.*

Blackburn, the linear part of white families' peer preferences is attenuated from 0.793 to 0.374, and the linear part of Asian peer preferences is increased slightly, making peer preferences slightly more unbalanced. In Preston, peer preferences on both sides are increased. In Blackburn, the pupil-side distance co-efficient is decreased from -0.657 to -0.519, while the school-side distance coefficient for comprehensives is increased from -0.456 to -0.863. However, apart from these changes the estimates are reasonably stable.

6.2 Model comparison and checking

The purpose of this section is to comparatively evaluate the predictive fidelity of the models with respect to the observed data. There are three parts to this. First, we wish to know whether one model specification from models A–D fits the data more parsimoniously than the others. Second, for a given model specification, it would be convenient if a single set of estimated parameter values were sufficient to summarise the analysis, without having separate estimates for each market. The apparent stability and similarity of estimates across markets suggests that using a single combined estimate, rather than market-specific estimates, might be reasonable. The third task in comparatively evaluating the estimated models is to evaluate whether *any* of the models captures the essential features of the data well enough to be of use in counterfactual analysis.

Table 6.4: Estimates from model D (Free School Meals (FSM) status).

	All		Blackburn		Oldham		Preston	
Demand-side:	Est	SD	Est	SD	Est	SD	Est	SD
Distance (km)	-0.622	(0.00984)	-0.573	(0.0248)	-0.688	(0.0196)	-0.744	(0.0203)
Closest school	0.193	(0.0244)	0.225	(0.0539)	0.133	(0.0395)	0.216	(0.0460)
Ethnic (10% increase):								
Asian	0.924	(0.0308)	1.28	(0.0736)	0.874	(0.0526)	1.16	(0.0769)
Asian Sq.	-0.0741	(0.00282)	-0.104	(0.00654)	-0.0656	(0.00482)	-0.101	(0.00853)
White Brit.	0.692	(0.0346)	0.695	(0.0649)	0.504	(0.0527)	1.16	(0.0939)
White B. Sq.	-0.0429	(0.00264)	-0.0386	(0.00506)	-0.0311	(0.00428)	-0.0862	(0.00710)
% 5 or more A*–C (SDs)								
× FSM	0.0140 *	(0.0231)	0.107 *	(0.0597)	0.216	(0.0404)	0.114 *	(0.0428)
× No FSM	0.609	(0.0155)	0.662	(0.0604)	0.871	(0.0358)	0.750	(0.0288)
Feeder schools:								
Church of Eng.	0.363	(0.0597)	0.686	(0.130)	-0.000267 *	(0.107)	0.885	(0.104)
Roman Catholic	1.91	(0.0468)	2.14	(0.0823)	2.42	(0.124)	2.19	(0.0871)
Supply-side:								
Distance (km):								
Comprehensives	-0.630	(0.0152)	-0.758	(0.0603)	-0.651	(0.0190)	-0.203	(0.0157)
Faith Schools	0.0290	(0.00636)	0.0322 *	(0.0126)	0.165	(0.0112)	-0.0985	(0.00787)
Feeder schools:								
Church of Eng.	0.734	(0.0528)	0.426	(0.123)	0.968	(0.0688)	0.292	(0.0754)
Roman Catholic	1.69	(0.0592)	2.95	(0.190)	1.15	(0.099)	1.19	(0.0625)
# pupils	21,313		5,044		9,393		6,876	
# school-years	153		39		52		62	

* $|\hat{\theta}| < 3\hat{\sigma}$

Table 6.6 presents the AIC statistics for each set of model estimates. The AIC for market i and year y is defined as

$$AIC = -2 \log \Pr\Big(\mathbf{m}_{iy} \in \mathcal{M}^*, \hat{\mathbf{V}}(\hat{\theta})|\hat{\theta}\Big) + 2p$$

where p is the number of parameters used in the model. Model C clearly dominates for every sample, with the ranking – $AIC_C < AIC_B < AIC_A < AIC_D$ – consistent throughout. Since the markets are independent, it is also possible to use the sum of the AIC statistics for the three markets, estimated individually, to give a rough comparison of the performance of estimating market-specific models compared to estimating a single model for all markets. Since each AIC statistic contains a penalty term for the number of parameters, the sum of the unpooled models contains a penalty that is equal to $6p$, whereas the AIC for the pooled model is only penalised by $2p$. Even so, estimating a model for each market dominates estimating a single pooled model.

A more detailed analysis of the predictive performance of the estimated models requires simulating data according to the models. This kind of model checking is called *posterior*

Table 6.5: Estimates from model B with quadratic peer preferences for ethnicity (Islamic faith schools removed).

	All		Blackburn		Oldham		Preston	
Demand-side:	Est	SD	Est	SD	Est	SD	Est	SD
Distance (km)	-0.614	(0.0101)	-0.519	(0.0243)	-0.753	(0.021)	-0.727	(0.0194)
Closest school	0.296	(0.025)	0.42	(0.0556)	0.122	(0.0409)	0.458	(0.0515)
Ethnic (10% increase):								
Asian	0.838	(0.0304)	1.18	(0.0739)	0.785	(0.0467)	1.47	(0.1)
Asian Sq.	-0.0693	(0.00285)	-0.0969	(0.00664)	-0.0619	(0.00435)	-0.2	(0.0149)
White Brit.	0.552	(0.0366)	0.374	(0.0589)	0.6	(0.0596)	1.35	(0.116)
White B. Sq.	-0.0331	(0.00275)	-0.017	(0.00463)	-0.0412	(0.00485)	-0.0955	(0.00828)
% 5 or more A*–C (SDs)								
× IDACI q. 1	1.08	(0.0283)	1.08	(0.0823)	1.95	(0.0902)	1.31	(0.0537)
× IDACI q. 2	0.944	(0.0283)	0.998	(0.0686)	1.33	(0.062)	1.19	(0.0544)
× IDACI q. 3	0.637	(0.0282)	0.687	(0.0714)	1.15	(0.0564)	0.671	(0.051)
× IDACI q. 4	0.317	(0.0219)	0.332	(0.0489)	0.837	(0.0444)	0.294	(0.0379)
× IDACI q. 5	0.0358 *	(0.0196)	0.129 *	(0.0482)	0.339	(0.0339)	0.112 *	(0.0342)
Feeder schools:								
Church of Eng.	0.0358	(0.0611)	0.408	(0.138)	-0.181 *	(0.117)	0.749	(0.117)
Roman Catholic	1.9	(0.0462)	2.1	(0.0849)	2.91	(0.146)	2.04	(0.0806)
Supply-side:								
Distance (km):								
Comprehensives	-0.588	(0.016)	-0.863	(0.0686)	-0.611	(0.0187)	-0.195	(0.0148)
Faith Schools	0.0193 *	(0.00643)	0.0466	(0.0151)	0.149	(0.0105)	-0.0942	(0.00778)
Feeder schools:								
Church of Eng.	0.966	(0.0626)	0.632	(0.137)	1.03	(0.0681)	0.267	(0.0778)
Roman Catholic	1.8	(0.0596)	2.96	(0.171)	0.95	(0.0862)	1.42	(0.0669)
# pupils	20,721		4,595		9,393		6,733	
# school-years	146		34		52		60	

* $|\hat{\theta}| < 3\hat{\sigma}$

predictive checking (Gelman et al., 2015), because it involves simulation of matchings from the posterior predictive distribution of the matching, defined as

$$\Pr\big(\mathbf{m}^{\text{Rep}}|\mathbf{m} \in \mathcal{M}^*\big) = \int \Pr\big(\mathbf{m}^{\text{Rep}}|\theta\big)\, p(\theta|\mathbf{m} \in \mathcal{M}^*)\, \mathrm{d}\theta.$$

where $\mathbf{m}^{\text{Rep}}$ is a hypothetical replication of a matching from the same sample (i.e. with the same observable characteristics). In the above distribution, $p(\theta|\mathbf{m} \in \mathcal{M}^*)$ is the posterior distribution of the parameters of interest, conditioning on the stability of the observed matching. We approximate the posterior with the multivariate normal approximation $g(\theta)$ produced by the EP algorithm. Once simulated, the joint distribution of matched pupil and school characteristics can be plotted to assess whether the estimated model faithfully recreates the main features of the data.

In the case of the stability model this is made more complicated by the fact that the model does not describe the full data generating process of the data, as the method

Table 6.6: Comparison of Models on Akaike Information Criterion (AIC)

Model	A	B	C	D
All	54,564	53,782	$\underline{53{,}488}$	55,077
Blackburn	11,520	11,496	$\underline{11{,}305}$	11,692
Oldham	23,674	23,432	$\underline{23{,}279}$	23,967
Preston	18,088	17,869	$\underline{17{,}793}$	18,599
Sum of unpooled models	53,282	52,797	$\underline{52{,}377}$	54,258

for selecting a matching from $\mathcal{M}^*$ is not specified. It is necessary to choose an explicit matching algorithm, and the choice may affect the posterior predictive distribution of matched characteristics. For this reason in simulating from the posterior predictive distribution both the *Pupil-Optimal Stable Matching* (POSM) algorithm and the *School-Optimal Stable Matching* (SOSM) algorithm were used to select a stable matching (see Chapter 8 for a description of these algorithms). A well-known result in stable matching theory is that the two algorithms produce matchings that are at opposite extremes of $\mathcal{M}^*$ in terms of pupil and school welfare, so this choice of equilibrium-selection rules is likely to reveal any sensitivity of the model to stable-matching selection.

Simulating a matching from the posterior predictive distribution entails sampling a parameter vector θ' from the multivariate normal approximate posterior, sampling a set of utilities on both sides of the matching conditional upon θ', which induces a full preference profile π^{Rep}, and then using one of the stable matching algorithms to compute a matching that is stable for those preferences. Matchings simulated thus are used to examine the joint distribution of matched pupil and school characteristics. We have chosen three comparisons, to capture the main dimensions of sorting: the joint distribution of pupil ethnicity and school ethnic composition; the joint distribution of pupil IDACI quintile and school academic performance; and the distributions of matched distance in each school. Gelman et al. (2015) suggests using the data to calculate a simulated significance test. However, the data can also just be used to provide a visual indication of the performance of the model.

One possible criticism of the use of posterior predictive visual checks is that they are not *out-of-sample* checks, which means that the same data are used to estimate the model and to check its predictive fidelity. This may lead to optimistic evaluation of the predictive accuracy of the model when applied to new datasets. However, in this exercise the intention is only to assess the fit of the model to the data it was estimated on, for which these graphical checks are appropriate.

Results for Oldham in 2012 and Preston in 2012 are presented here. Figure 6.3 shows the distribution of school and pupil ethnic characteristics from 100 simulations, overlayed with the observed joint distribution in Oldham in 2012. The three rows of the figure correspond to models A–C. The simulations in the left-hand column use the named model, estimated on all datasets, whereas the simulations on the right-hand side use the named model, estimated using only the four cohorts of Oldham data. All of the models produce simulated data that is reasonably similar to the observed data, with no single set of estimates outperforming the others in all circumstances. The simulations based on estimates from all samples tend not to be able to reproduce the Asian proportion at North Chadderton school (no. 7), Oasis Academy (no. 8) and The Radclyffe School (no. 17), whereas simulations based on market-specific estimates are more faithful for these schools, but tend to not to be able to reproduce the ethnic distribution at The Hathershaw school (no. 15).

Figure 6.4 again presents the results of 100 simulations, focussing on the proportions of high-IDACI pupils (the top two quintiles) matched to schools by academic performance. Again patterns of sorting in North Chadderton School (no.7) are not as faithfully reproduced. The models also fail to exactly match up Oldham Academy North (no.16) and Waterhead Academy (no.18). The observed and simulated distributions are close in general. Differences between the simulation and the model may be explained by some source of heterogeneity that is not modelled. Probably the inclusion of fixed effects for schools, if feasible, would improve the fit.

The final visual comparison plots the distribution of matched distance by school. Figure 6.5 presents this comparison for Oldham in 2012. Five simulated datasets are compared to the actual distribution, for models B and C (model A is not compared). Although all models perform well, the combined-market estimates fail to capture the distance distribution of The Bluecoat School (no.14), which has a wider catchment than other schools. This is probably because the school is popular, but without fixed effects the estimated models cannot capture popularity that is not correlated with observable school

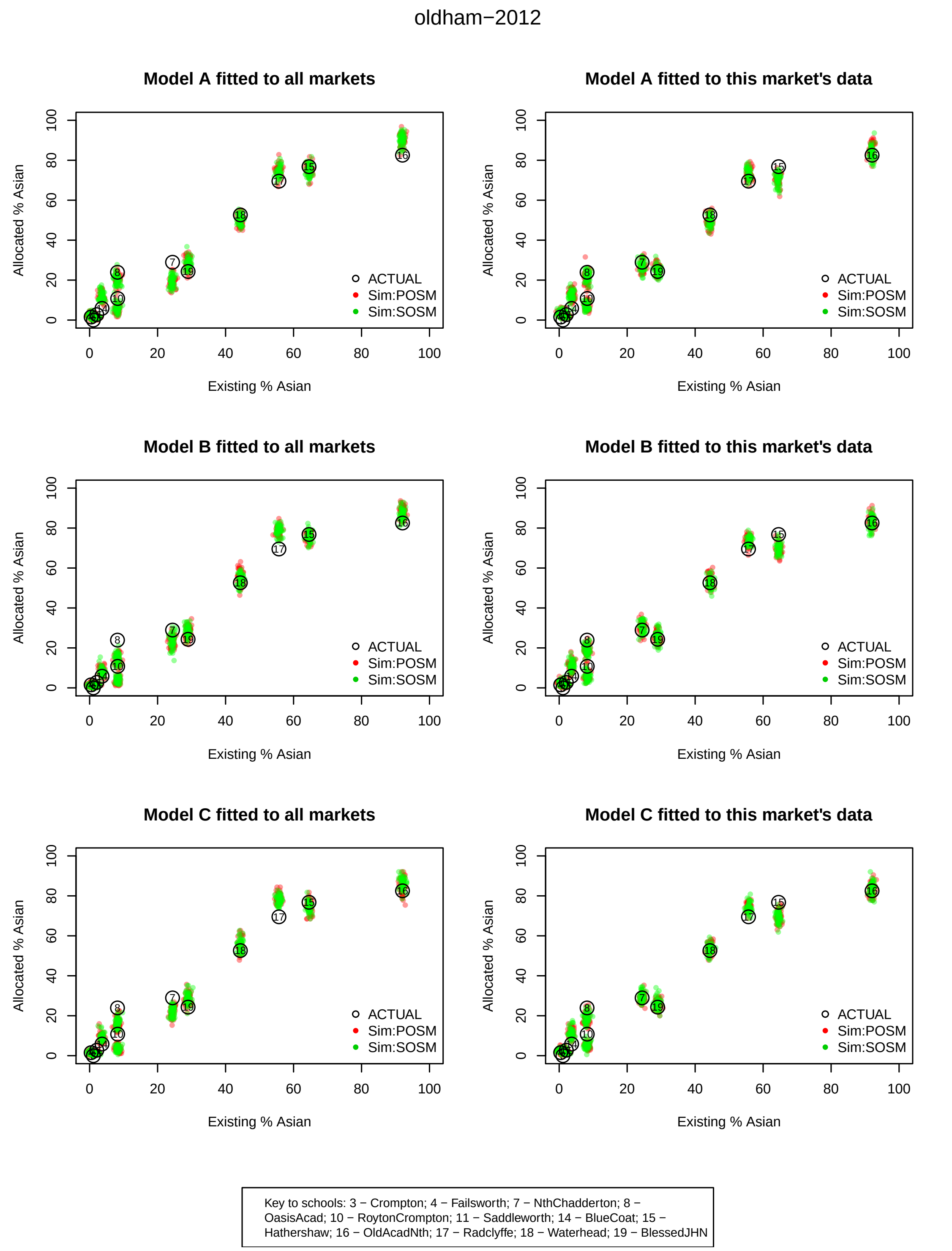


Figure 6.3: *100 Simulations from the joint posterior predictive distribution of school Asian proportion in the full population, and the proportion of Asian pupils in the intake, at schools in Oldham in 2012.*

characteristics. Interestingly, model B appeared to capture the distribution of no.14 more faithfully than model C.

Two of the posterior predictive plots for Preston in 2012 are presented because they

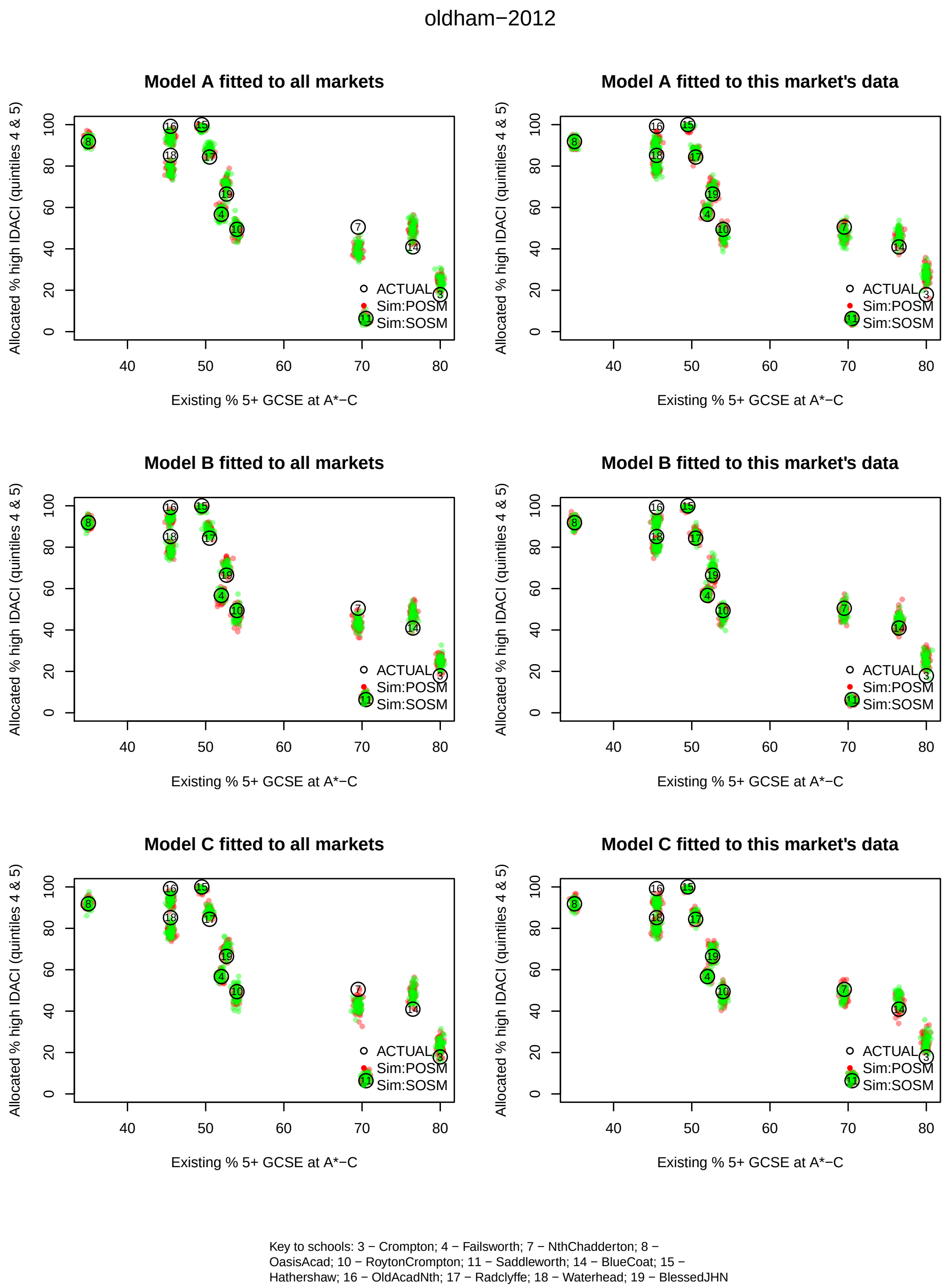


Figure 6.4: *100 Simulations from the joint posterior predictive distribution of school academic performance and the proportion of high-IDACI pupils in the intake, at schools in Oldham in 2012.*

reveal a generally poor fit of the models to the observed matching in Preston in 2012. Figure 6.6 shows that models A–C fail to capture the matched distribution of ethnicity for Preston Muslim Girls' School (no.2), Moor Park School (no. 8), Hutton Grammar CofE Boys' School (no. 13) and Archbishop Temple CofE School (no.15). The predicted

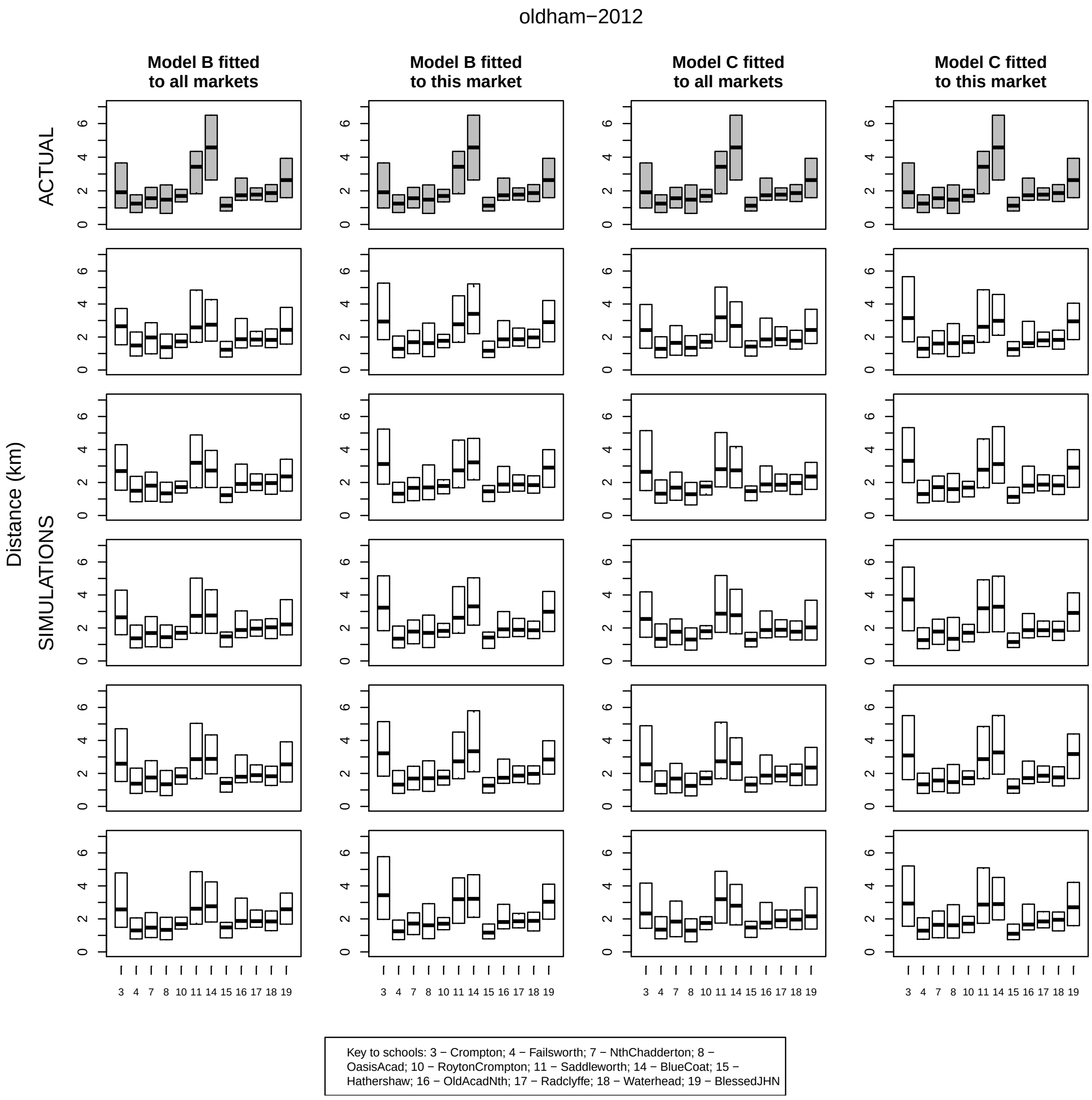


Figure 6.5: *The distribution of each school's catchment in Oldham in 2012, compared to five simulations for each set of estimates.*

ethnic proportion for no.2 is smaller than the actual proportion, whereas the predicted ethnic proportion for Moor Park is higher. The predicted ethnic proportions for no.13 and no.15 (both high-performing CofE schools) are lower than the actual proportions. It may be the case that the models do not capture a positive interaction between belonging to an Asian ethnic group and preferences for test scores that would explain the lack of fit to these two schools.

Figure 6.7 reveals that the models do not capture the fact that almost all of the, predominantly white British, pupils who travel from Blackburn to Preston attend Walton-le-Dale Arts College (no.4). The models distribute these long-distance pupils over several schools, particularly Lostock Hall Academy (no.5), biasing the distributions of these other schools.

The models appear not to be able to capture the joint distributions of matched characteristics in Preston. This indicates that caution is required in interpreting counterfactual analyses undertaken on data for Preston. Results for Blackburn are more encouraging, although Oldham appears to be the market for which the models have achieved the best fit.

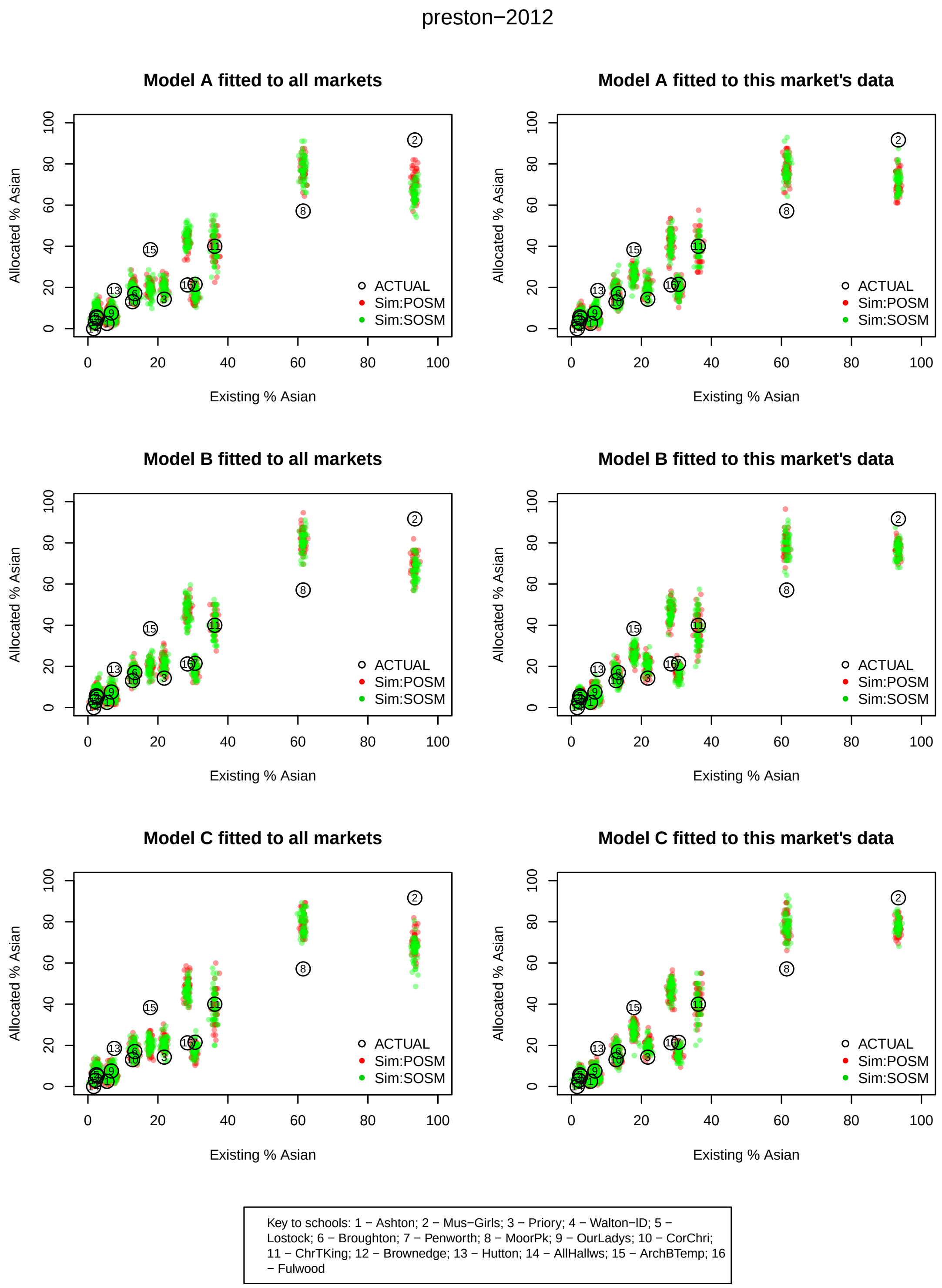


Figure 6.6: *100 Simulations from the joint posterior predictive distribution of school Asian proportion in the full population, and the proportion of Asian pupils in the intake, at schools in Preston in 2012.*

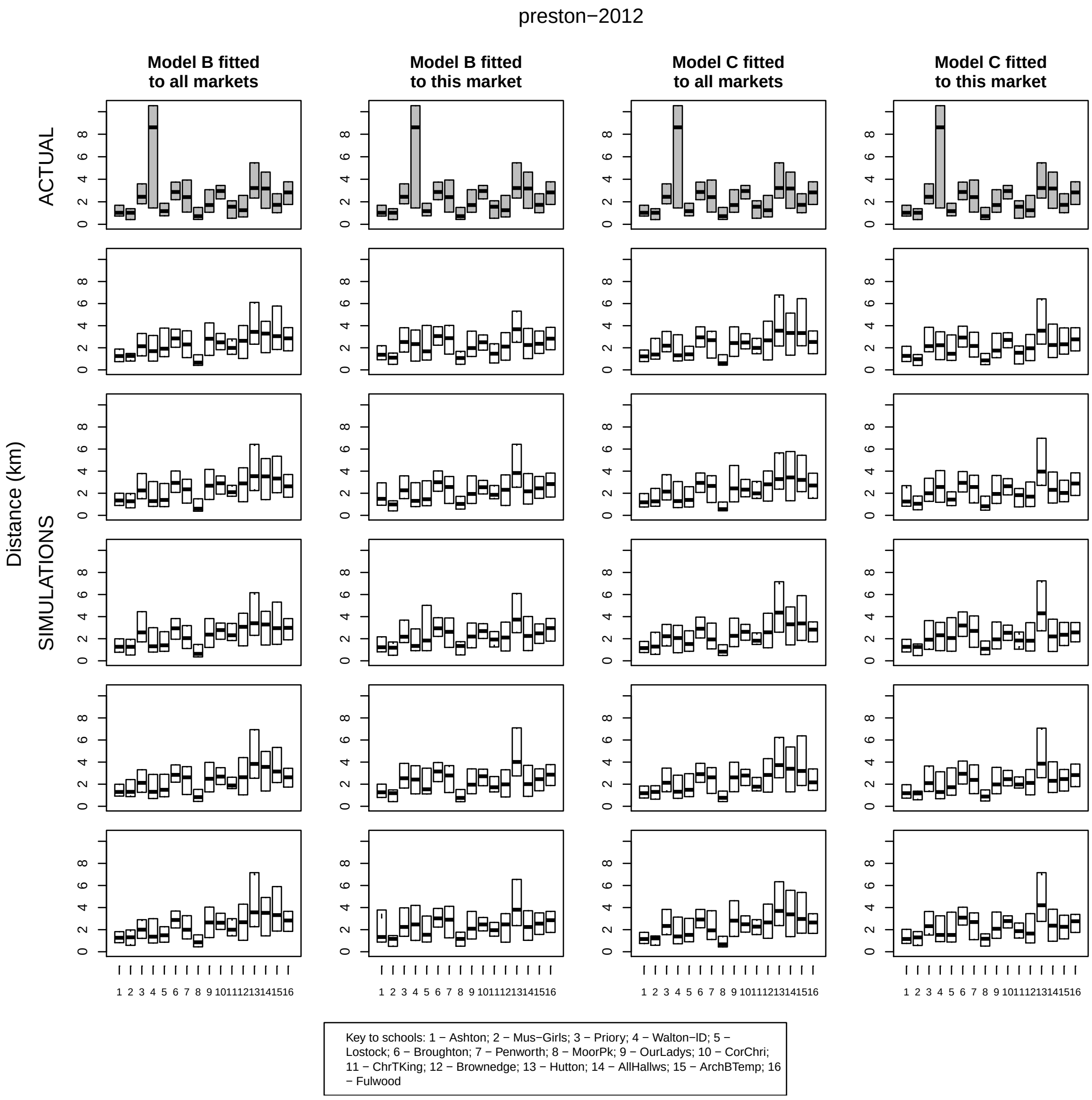


Figure 6.7: *The distribution of each school's catchment in Preston in 2012, compared to five simulations for each set of estimates.*

6.3 Discussion

The general similarity of estimates from the three markets, estimated independently, and from the four model specifications is encouraging as it suggests that the structural model has successfully identified so-called “deep” parameters (i.e. behavioural parameters that are not contingent on details of the market). Identification of deep or *policy-invariant* parameters is necessary if we intend to use estimated parameters to simulate counterfactuals. The most important regularities are:

1. Peer preference coefficients are positive in all models. Coefficients for both Asian and white families are similar in magnitude, suggesting that homophily is not confined to one group, and that “horizontal” sorting may be present. Both sides exhibit steep preferences for ethnic peer groups when the proportion of their own group is small, and no increase in utility after the proportion of their own group passes about 60%. However, Asian families tend to have slightly more non-linear, inverse-U shaped utility for ethnic peers, suggesting that Asian families prefer for their child not to be part of an extreme majority.

2. There is a strong socio-economic gradient in preferences for raw academic performance, measured as standardised % achieving five or more GCSEs at A*–C. This is robust across all specifications and samples. Parents from low-IDACI (i.e. high-SES) neighbourhoods would typically be willing to send their children 1–3km further for a one-standard-deviation improvement in test scores, whereas parents from neighbourhoods with the highest quintile of IDACI have muted or negligible preferences for test scores.

3. All models imply that those attending Catholic primary schools strongly prefer Catholic secondary schools, and are also much more likely to be accepted if they apply. Preferences are typically equivalent to about 2.5-3km distance. For those who attended a Church of England primary school, preferences are less strong and more variable, but the effect on probability of acceptance is more stable and about half as strong as that for Catholic schools.

4. Distance is an important term in explaining admissions probabilities to non-faith schools, but is not an important term in explaining admissions probabilities for

faith schools. This suggests that faith schools are able to select all or almost all of their pupils on the basis of faith, and in relation to Muslim schools means that there is effectively no systematic component in the model for acceptance.

All models imply both "horizontal" ethnic peer-preferences and socio-economic moderation of preferences for academic performance. This is consistent with all of the existing discrete choice evidence on school choice (Hastings et al., 2009; Abdulkadiroğlu et al., 2015; Caetano and Maheshri, 2013), and constitutes an answer to our first research question. The final point implies that caution must be used in inferences about preferences for Muslim schools. Refitting model B to the data with Muslim schools removed did not reveal any major sensitivities, but ethnic peer preferences were affected.

It is not clear why the models we have estimated have failed to achieve a consistently good fit to data from Preston. It is possible that the market is just too large and dispersed for the distance parameters to be able to capture the spatial arrangement of pupil allocations. However, it is also possible that underlying ethnic preferences are weaker in Preston, leading to uncertainty in the model for preferences. However, whatever the reason for the poor fit, it implies that caution must be used in applying substantive conclusions from Preston. In general the models appeared to achieve the best fit to data from Oldham.

Chapter 7

The implications of preferences for sorting and competition

Having estimated random utility models of parental preferences for schools in the last chapter, thus answering the first research question, the purpose of this chapter is to put those models to work to help answer the second and third research questions. For the second research question we wish to investigate the potential for heterogeneous preferences for ethnic peers to contribute to cumulative sorting processes. The third research question asks whether, and to what extent, heterogeneous preferences affect the amount of choice parents enjoy, and the amount of competitive pressure schools face. Since investigating each of these questions requires the use of a concept of *aggregate demand*, and since it is more convenient to define the aggregate demand in relation to competition on quality, the chapter deals with these questions in reverse order, analysing preferences and competition first, and then preferences and sorting.

7.1 Preferences and competition

A central concern in the study of school choice is the extent to which choice induces competition between schools to raise standards. As we have seen, the idea of introducing competitive discipline into the operation of schools is often cited as one of the most important justifications for school choice policies. The literature attempting to quantify the effect of school choice on school performance is huge, much of it adopting experimental and quasi-experimental techniques (eg. Epple and Romano, 2012b; Sahlgren, 2013).

However, within this literature, with its focus on arguments for or against quasi-markets as a whole, there has been little attempt to enumerate or quantify the structural conditions under which quasi-markets can be expected to function optimally. At the same time, it has been argued that the local structure of school markets, and particularly demand-side variation caused by local demographics, has a significant impact on local competitiveness. Allen and Burgess (2010) argue that "competitive markets work best where the social mix at schools is reasonably similar" (Allen and Burgess, 2010, p10). Among quasi-market studies, Bayer and McMillan (2010) go further, explicitly operationalising competition in terms of a structural model of aggregate demand for quality. They characterise functioning school quasi-markets as depending upon: (i) responsiveness of parental demand to observable aspects of school performance, which they signal through their choices; and (ii) responsiveness of schools to these parental demand signals. Their model extends the previous literature by explicitly allowing competition to depend on demand at the local level.

In contrast, previous work such as Hoxby (2000) and Bradley et al. (2001) operationalised choice/competition by measuring the density of local schools. Bayer and McMillan explicitly distinguish between choice and competition, and claim it is the latter that drives the quasi-market pressure that incentivises school improvement. Gibbons et al. (2008) also distinguish between choice and competition in primary school markets. Studies in UK school choice that have calculated elasticities of demand, for example Burgess et al. (2015) have not modelled spatial variation in these elasticities at a local level.

In Chapter 4, we used spatial overlap of *de facto* catchment areas as a proxy for pairwise competition between schools. Using this method, schools were clearly arranged in cliques geographically, although there was some evidence of popular schools with very broad catchments (e.g. Lancaster grammar schools). Now, having estimated a random utility model for school choice, we are in a position to operationalise competition between schools in terms of demand.

An advantage of this approach is that, in defining competition in terms of demand, we can be explicit about what aspects of "quality" schools are competing on, and can isolate the level of competition on quality from competition on any other aspect such as, say, ethnic or socio-economic composition of intakes. Another advantage is that we can evaluate counterfactual demand functions by altering model parameters, and counterfactual market structures by setting observables to different levels.

7.1.1 Conversion to logit co-efficients

Since normally-distributed errors have been used in the stability likelihood, the estimated parameters are structurally equivalent to those from a multinomial probit discrete choice model. This means that it is possible to calculate choice probabilities using the probit probability formula. However, the probit model requires integration to calculate choice probabilities and derivatives.

The logit model, on the other hand, has a simple analytical expression for both the probability and the derivatives. It is well known that the two models are virtually empirically interchangeable, in that there are few substantive differences between inferences derived from each one. This means that parameters estimated using either model can be converted to the other model by rescaling to account for the fixed standard deviation of errors, σ_ϵ.

The probit model has $\sigma_\epsilon = 1$, whereas the EV1 distribution usually has $\sigma_\epsilon = \sqrt{\pi^2/6} \approx 1.3$ (Train, 2009). This means that by multiplying the probit parameters by about 1.3 we can use the logit model and benefit from the tractability of results. In effect, we have used the probit model for its convenience with respect to gaussian integration, and so converting the estimates to make use of the convenience of the logit model is reasonable. In the analyses that follow the estimated parameters have been converted into logit parameters, and the logit model is used.

7.1.2 The model for quality-responsiveness

We define the notional aggregate demand that each school faces, as the expected number of pupils who would choose that school if able to do so (i.e. absent capacity constraints):

$$D_s(\boldsymbol{Q};\hat{\theta}) = \sum_{a\in\mathcal{A}} \Pr\Big(U_{as} = \max_{s'\in\mathcal{S}}(U_{as'})|\boldsymbol{Q},\boldsymbol{X},\hat{\theta}\Big)\,.$$

The aggregate demand is defined as a function of some vector of school quality measures, $\boldsymbol{Q}$ and estimated parameters, with a parametric form determined by the random utility model adopted.

Note that the notional aggregate demand is not the only way of conceptualising demand in constrained markets. The dependence of the allocation on fixed school capacities means that demand can be defined in at least three ways. In addition to notional demand, the *realised* demand is defined as the number of pupils allocated to school s in equilibrium, and the *effective* demand is defined as the size of the school's "stable choice set" – that is, the number of pupils allocated to school s, plus the number of pupils who would prefer s to their allocated school[1] (see Chapter 8 for a full discussion of the stable choice set).

The question of whether schools respond to aggregate demand, and if so whether they respond to notional demand or some other type of demand, is an important empirical question, albeit one outside the scope of this work. Bayer and McMillan (2010) find a quite strong response to local elasticities of demand. We assume that schools respond to notional demand. This is not an unreasonable assumption, since schools have access to, and may be expected to monitor and evaluate, the number of first preferences they receive. Notional demand may be thought of as the "safest" measure of demand to monitor from a school's perspective, since it is the only demand that is not contingent on other schools maintaining capacity constraints.

In defining the notional demand in terms of $\boldsymbol{Q}$, all other school and pupil characteristics that affect choices are implicitly fixed at their observed values in the sample. Note that for each s, $D_s(\boldsymbol{Q};\hat{\theta})$ is a function of the full vector $\boldsymbol{Q}$, since the quality measures of all schools enter into the demand for each school. We can therefore define the marginal change in demand for school s_1, given a change in quality at s_2 (where s_1 may equal s_2) as the partial derivative

$$\frac{\partial D_{s_1}}{\partial Q_{s_2}} = \sum_{a\in\mathcal{A}} \frac{\partial}{\partial Q_{s_2}} \Pr\left(U_{as_1} = \max_{s'\in\mathcal{S}}(U_{as'}) | Q_{s_2}, \boldsymbol{Q}_{-s_2}, \boldsymbol{X}, \hat{\theta}\right).$$

[1] As well as preferences, realised and effective demand also depend to different extents on the details of the market and mechanism. The realised demand depends on the capacities of all schools, but does not depend on the specific stable matching algorithm used, due to a result in stable matching theory known as the Rural Hospitals Theorem (Roth, 1986). The effective demand, on the other hand, does depend on the stable matching algorithm if the size of the set of stable matchings is greater than one.

When the random utility model is logit, the individual derivatives have the simple form

$$\frac{\partial P_{as_1}}{\partial Q_{s_2}} = \begin{cases} \frac{\partial f_{as_1}}{\partial Q_{s_1}} P_{as_1}(1 - P_{as_1}) & \text{if } s_1 = s_2, \\ -\frac{\partial f_{as_2}}{\partial Q_{s_2}} P_{as_1} P_{as_2} & \text{otherwise.} \end{cases}$$

If the utility model is linear in Q_s, then $\frac{\partial f_{as}}{\partial Q_s}$ is simply β_Q.

Combining the demand functions for each school produces the vector-valued total demand function $\boldsymbol{D}(\boldsymbol{Q}; \hat{\theta})$. We can then define the Jacobian matrix

$$\boldsymbol{J}_D = \begin{bmatrix} \frac{\partial D_1}{\partial Q_1} & \frac{\partial D_1}{\partial Q_2} & \cdots & \frac{\partial D_1}{\partial Q_n} \\ \frac{\partial D_2}{\partial Q_1} & \frac{\partial D_2}{\partial Q_2} & \cdots & \frac{\partial D_2}{\partial Q_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial D_n}{\partial Q_1} & \frac{\partial D_n}{\partial Q_2} & \cdots & \frac{\partial D_n}{\partial Q_n} \end{bmatrix}$$

The diagonal elements of the matrix are the marginal responses of each school's demand to a change in its own quality, and the off-diagonal elements are the marginal responses of each school's demand to a change in another school's quality. The matrix can be thought of as describing the flow of demand from one school to another as observable measures of quality change. This characterisation explains a couple of properties of the matrix that are not common to all Jacobian matrices. First, the sum of each column is zero; since each flow *to* a school must be a flow *from* another school, therefore $\partial D_s / \partial Q_s = -\sum_{s' \neq s} \partial D_{s'} / \partial Q_s$. Second, and less obviously, the matrix is symmetric – that is, $\partial D_s / \partial Q_t = \partial D_t / \partial Q_s \quad \forall (s, t)$. This property follows from the fact that "only differences in utility matter" (Train, 2009). Due to this property of discrete choice models, we can restate the partial derivative $\partial D_s / \partial Q_t$ as $\partial D_s / \partial (Q_t - Q_s)$. But since changes to $(Q_t - Q_s)$ only affect s and t, using the column-sum rule we can see that

$$\begin{aligned} \frac{\partial D_s}{\partial (Q_t - Q_s)} &= -\sum_{s' \neq s} \frac{\partial D_{s'}}{\partial (Q_t - Q_s)} \\ &= -\frac{\partial D_t}{\partial (Q_t - Q_s)} = \frac{\partial D_t}{\partial (Q_s - Q_t)} \\ &= \frac{\partial D_t}{\partial Q_s}. \end{aligned}$$

This symmetricity is true of the matrix of partial derivatives. However, if we were to use instead *arc derivatives* – that is, responses of demand to a non-infinitesimal change in quality – the resulting matrix would not be symmetric, since the aggregate demand function is non-linear.

The Jacobian matrix can be used in analysis as a model-based alternative to the catchment overlap matrices calculated in Chapter 4. The off-diagonal elements capture the amount that each pair of schools competes on quality. Similarly to the catchment overlap matrix, geographical distance causes the cross-derivative to shrink. However the model-based Jacobian matrix also captures variation in levels of competition not correlated with proximity.

Studying the marginal response of demand to quality allows us to investigate the effect of sorting and social factors on competitiveness between schools. This is the case because aggregate demand, and the rate at which aggregate demand changes in response to changes in school performance, can be affected by existing sorting patterns.

In the two-school example in Figure 7.1 the marginal response of demand to quality (the slope of the curve) depends on the existing pairwise demand between the two schools. It is greatest when demand for the two schools is already fairly evenly matched (i.e. when p is close to 0.5). If demand for two schools is very unbalanced, due to large differences in test scores or any other reason, the flow of demand resulting from a change in test scores may be small. Intuitively, the return to expending effort to improve test scores may be small for either popular or unpopular schools, or both, in very unbalanced markets, incentivising "coasting" strategies at both ends of the performance spectrum.

Furthermore, since the aggregate demand is defined by the sum of heterogenous individual choice functions, the response of the aggregate demand to quality may be lower than the overall market shares suggest. If overall pairwise demand is balanced, but the population can be split into subgroups, within each of which the demand is very unbalanced towards one school, then a small change in test scores may not be enough to overcome sorting. Such a situation may arise when neighbouring schools have very segregated intakes, for example, or when neighbouring schools serve different faiths. The demand model can therefore predict the effect on competition of both vertical sorting (stratification) and horizontal sorting (segregation).

In order to operationalise the model, we used "% achieving 5 or more A*-C" as the quality measure $\boldsymbol{Q}$. The estimated parameters from model C estimated on each market,

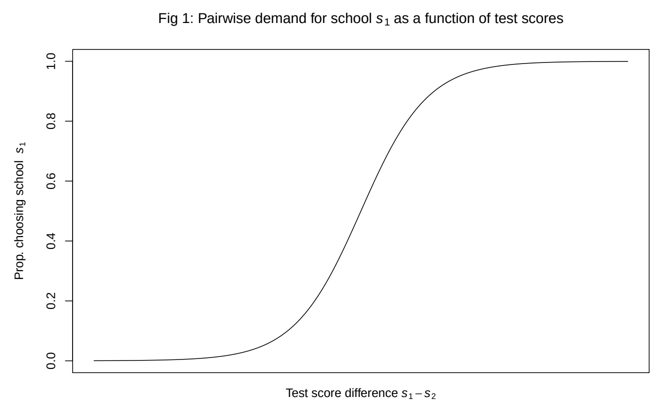


Figure 7.1: *At an individual level, the marginal response of demand to quality (the slope of the pictured function) depends on the existing balance of choice probabilities between alternatives.*

as reported in Chapter 6, were logit-converted by multiplying by 1.34. Although the logit derivative is tractable, we choose to calculate the Jacobian numerically using the R package *numDeriv* (Gilbert and Varadhan, 2015). The reason for this is that it is then much easier to adapt and reuse the code for the analysis in the next section. The quality of the numerically-estimated Jacobian can be checked by confirming that the row and column sums are all zero, and that the matrix is symmetric.

To aid interpretability of results and comparison across markets, the demand function has not been normalised, so that the demand estimates are reported in terms of pupil headcount, and the derivatives are in units of pupil flow per unit change in quality. The original units of quality are percentage points, and the derivatives have been multiplied by five to provide a linearised approximate demand response for a five percentage point increase in Q_s.

Two counterfactual scenarios are considered:

1. To analyse the effect on competition of ethnic peer preferences, the first counterfactual sets the corresponding parameters in the random utility model to zero, and re-estimates cross-derivatives of demand.

2. In order to evaluate the impact on competition of large discrepancies in test scores caused by sorting, that is correlated with sorting on SES, the second counterfactual equalises all test scores and re-estimates cross-derivatives of demand.

Neither of these counterfactuals are strictly *policy* counterfactuals, as they are not aspects of the market that can be exogenously altered by policy-makers. Nevertheless, they capture the main elements of sorted markets that may impact upon market competitiveness.

7.1.3 Results

Table 7.1: Mean response of notional aggregate demand at school s to a 5 point increase in quality at s.

	Mean demand response (5 point change)	Counterfactual % change from actual
Blackburn		
2009		
Actual	8.8	
Cf 1	9.3	5.8
Cf 2	8.9	1.6
2012		
Actual	11.3	
Cf 1	11.8	4.3
Cf 2	13.0	14.5
Oldham		
2009		
Actual	14.7	
Cf 1	15.2	3.7
Cf 2	16.3	11.4
2012		
Actual	19.4	
Cf 1	21.0	8.0
Cf 2	22.4	15.2
Preston		
2009		
Actual	16.6	
Cf 1	16.7	0.6
Cf 2	19.8	19.3
2012		
Actual	18.7	
Cf 1	19.2	2.8
Cf 2	20.7	10.6

"Cf 1" – counterfactual experiment with ethnic preference parameters switched off.
"Cf 2" – counterfactual experiment with all quality measures set equal.

Table 7.1 shows the mean response of demand corresponding to a five percentage point improvement in test scores in each market. In each market, the competitiveness appears to increase over the period 2009–2012. For example, in Oldham in 2009 the average school, in response to a 5 point improvement in test scores, gains notional demand equivalent to 14.7 pupils. However, by 2012 this has risen to 19.4 pupils. However, this is probably explained by the larger average size of schools in 2012 than in 2009.

In most cases, equalising test scores has a greater effect upon the competitiveness of markets as a whole than removing peer preferences. A closer look at the Jacobian of cross-derivatives of demand in Oldham in 2012 (see Tables 7.2) reveals that peer preferences have a large affect on competition for a small number of schools with large minority ethnic populations. For example, the demand response for Oldham Academy North increases from 3 to 8 if peer preferences are removed, and the demand response for Waterhead increases from 8 to 14. In contrast, equalising test scores affects all schools to a certain degree, and the most popular schools experience a reduction in the competitive response under the counterfactual.

Table 7.2 also illuminates the effect of sorting upon the overall integration of the market. The top panel has one school, The Blue Coat School, that spans the market, but otherwise the market appears to be split into two cliques containing the schools in the south and west of Oldham (North Chadderton, Oasis Academy, etc.) and the schools in the north and east (Royton Crompton, Waterhead, etc.). Oldham Academy North does not appear to compete effectively with any other schools. This is interesting because Oldham Academy North is not geographically-isolated, but has the largest Asian population (at 92.5%) in 2012. In the third panel the split between the north-east and south-west of Oldham is not apparent, as schools compete across Oldham.

Tables 7.3, 7.4 and 7.5 present the estimated aggregate demand and estimated demand response in 2012 for each school in Blackburn, Oldham and Preston respectively. In Table 7.3, the first counterfactual has a large effect on demand for high-performing Tauheedul Islam Girls' School, whose demand almost doubles from 126 to 220. In contrast demand for Witton Park school goes down from 112 to 67.

Table 7.2: Oldham 2012 demand response Jacobian matrix for 5-point increase in test scores

Actual

OldAcadNth	3											
RoytonCrompton		15	-1	-1	-7	-4	-2					
Waterhead		-1	8	-1	-1	-4						-1
Saddleworth		-1	-1	12	-3	-7						
Crompton		-7	-1	-3	36	-15	-7				-1	
BlueCoat		-4	-4	-7	-15	53	-9	-1	-1	-6	-4	-2
NthChadderton		-2			-7	-9	32	-1	-3	-3	-6	-1
OasisAcad						-1	-1	8		-3	-1	-1
Radclyffe						-1	-3		11	-1	-3	-2
Failsworth						-6	-3	-3	-1	20	-5	-1
BlessedJHN					-1	-4	-6	-1	-3	-5	24	-2
Hathershaw			-1			-2	-1	-1	-2	-1	-2	10

"colour-blind" counterfactual

OldAcadNth	8	-1			-2	-2	-1				-1	
RoytonCrompton	-1	14	-1		-6	-3	-1					
Waterhead		-1	14	-2	-2	-7						-1
Saddleworth			-2	12	-3	-6						
Crompton	-2	-6	-2	-3	36	-13	-7		-1		-1	
BlueCoat	-2	-3	-7	-6	-13	56	-8	-2	-2	-5	-5	-4
NthChadderton	-1	-1			-7	-8	31	-1	-4	-2	-5	-1
OasisAcad						-2	-1	9	-1	-3	-2	-2
Radclyffe					-1	-2	-4	-1	14	-2	-3	-1
Failsworth						-5	-2	-3	-2	21	-5	-3
BlessedJHN	-1				-1	-5	-5	-2	-3	-5	24	-2
Hathershaw			-1			-4	-1	-2	-1	-3	-2	14

Equal test scores counterfactual

OldAcadNth	4	-1	-1				-1		-1		-1	-1
RoytonCrompton	-1	36	-5	-2	-14	-6	-4	-1	-1		-2	
Waterhead	-1	-5	24	-4	-2	-7	-1	-1	-1		-1	-2
Saddleworth		-2	-4	11	-1	-2		-1				-1
Crompton		-14	-2	-1	28	-3	-4		-1		-2	
BlueCoat		-6	-7	-2	-3	29	-3	-2	-1	-1	-2	-1
NthChadderton	-1	-4	-1		-4	-3	26	-2	-4	-1	-6	-1
OasisAcad		-1	-1	-1		-2	-2	24	-1	-10	-5	-2
Radclyffe	-1	-1	-1		-1	-1	-4	-1	18	-1	-5	-2
Failsworth						-1	-1	-10	-1	22	-6	-2
BlessedJHN	-1	-2	-1		-2	-2	-6	-5	-5	-6	32	-2
Hathershaw	-1		-2	-1		-1	-1	-2	-2	-2	-2	14

Table 7.3: Market Response results: Blackburn, 2012

		Actual		CF 1		CF 2*	
	Q	D	$5\frac{\partial D}{\partial Q}$	D	$5\frac{\partial D}{\partial Q}$	D	$5\frac{\partial D}{\partial Q}$
TauheedulBoys	60	50	4	83	7	77	3
DarwenVale	51	57	10	58	9	88	16
Pleckgate	54	105	9	95	10	128	12
WittonPk	51	112	11	67	7	101	17
TauheedulGirls	96	126	9	220	19	106	5
OurLadyStJohn	44	150	9	140	7	183	17
Blakewater	49	156	8	103	7	129	12
StWilfrids	64	161	21	167	18	162	18
DarwenAldridge	57	165	11	154	11	159	12
StBedes	72	204	21	199	23	152	17

* $Q = 50$ for all schools.

Table 7.4: Market Response results: Oldham, 2012

		Actual		CF 1		CF 2*	
	Q	D	$5\frac{\partial D}{\partial Q}$	D	$5\frac{\partial D}{\partial Q}$	D	$5\frac{\partial D}{\partial Q}$
OldAcadNth	46	73	3	100	8	159	4
RoytonCrompton	54	92	15	96	14	154	36
Waterhead	46	119	8	121	14	224	24
Saddleworth	70	142	12	140	12	118	11
OasisAcad	35	145	8	169	9	230	24
Radclyffe	50	176	11	119	14	152	18
NthChadderton	70	194	32	181	31	124	26
Failsworth	52	231	20	230	21	232	22
BlessedJHN	53	249	24	242	24	266	32
Crompton	80	282	36	287	36	160	28
Hathershaw	50	302	10	225	14	246	14
BlueCoat	76	356	53	453	56	298	29

* $Q = 50$ for all schools.

Table 7.5: Market Response results: Preston, 2012

		Actual		CF 1		CF 2*	
	Q	D	$5\frac{\partial D}{\partial Q}$	D	$5\frac{\partial D}{\partial Q}$	D	$5\frac{\partial D}{\partial Q}$
Fulwood	44	24	3	24	3	75	19
Mus-Girls	71	44	4	83	10	73	3
Priory	60	53	13	49	12	87	26
Lostock	52	63	16	66	16	102	23
Penworth	72	65	17	61	17	47	14
Ashton	60	66	9	66	9	104	19
Walton-lD	50	73	14	74	15	105	21
CorChri	47	74	7	72	7	163	30
Hutton	78	74	14	73	13	48	10
Broughton	86	86	19	86	19	50	11
MoorPk	50	117	7	89	6	132	14
ChrTKing	46	135	11	132	12	198	24
Brownedge	57	184	27	184	28	185	30
AllHallws	72	212	42	219	43	169	35
OurLadys	78	234	40	239	42	170	30
ArchBTemp	91	296	57	283	56	93	21

* $Q = 50$ for all schools.

7.2 Preferences and sorting models

One of the central motivations of the thesis has been to assess the extent of, and understand the causes of, post-residential ethnic sorting in schools. Chapter 2 examined the "ingredients" of sorting, without elaborating on the mechanisms that might lead to the concentration of different ethnic groups in different schools. It is clear that residential sorting plays a part in school sorting, but the analysis of indices of dissimilarity in Chapter 4 suggested that residential sorting was not the whole story. Having estimated our choice model, we have evidence that preferences for peers play a role as well.

An important aspect of sorting that the causal framework in Chapter 2 did not discuss, especially in the light of peer preferences, is the potential for cumulative sorting dynamics due to the endogeneity of school composition with respect to choice. If, as our estimates suggest, it is true that the demographics of a school enter into a family's choice process, then this affects the group-level aggregate demand function for that school, which in turn determines the demographics upon which future cohorts base their choices. In this way preferences for peer groups may have multiplier effects that lead to greater sorting than a single-period model would predict.

Models predicting such sorting dynamics or *tipping* dynamics have a long history in economics, beginning with the work of Schelling (1972). More recent approaches to so-called *social interaction models* include Becker and Murphy (2003) and Card et al. (2008). Clotfelter (1976) studied models of sorting dynamics in relation to schooling. More recently Caetano and Maheshri (2013) used a similar model with structural estimation of parameters to estimate tipping points and segregated equilibria in US schools. The model adopted in this section is similar to both of these models.

7.2.1 The tipping model

The tipping model again uses the definition of notional aggregate demand as described in the previous section, except that demand is now defined as a function of the proportion of minority ethnic pupils in the school's population, rather than quality. In addition, the notional aggregate demand is disaggregated by ethnic group indicator g.

$$D_s^g(p_s^g) = \sum_{a \in \mathcal{A}_g} \Pr\biggl(U_{as} = \max_{s' \in \mathcal{S}}(U_{as'})|p_s^g, \boldsymbol{p}_{-s}^g, \boldsymbol{X}, \hat{\theta}\biggr),$$

where $\mathcal{A}_g$ is the set of pupils in ethnic group g, p_s^g is the proportion of the population of school s who are of group g, and $\boldsymbol{p}^g$ is the vector of group g proportions at all schools. The parametric form of each term in the aggregate demand is determined by the random utility model adopted. We can then define the expected proportion of group g in the school's population

$$\pi_s^g = \frac{D_s^g(p_s^g)}{\sum_{g'=1}^{N_g} D_s^{g'}(p_s^g)}.$$

This definition implies a number of simplifying assumptions. First, it conflates the *intake* with the *whole-school* population. In reality a single year's intake is only about one fifth (excluding sixth form) of the entire school population. Second, it treats the elements of the vector $\boldsymbol{p}^g$ as independent, as though we can exogenously adjust one p_s^g whilst holding the others constant. If there are a large number of schools the impact of this assumption will be less serious. Third, the model assumes that decision makers at time t observe the ethnic proportions in each school at t with no lag or error. Finally, the model ignores capacity constraints and admissions policies, and assumes that realised demand is equal to notional demand.

Accepting these simplifications, for the moment, the model implies that in a given admissions year t, $p_{s,t}^g$ is equal to $\pi_{s,t-1}^g$. As long as $\pi_{s,t}^g \neq p_{s,t}^g$, demand for school s will not be stable across years, and will tend to shift towards a point such that $\pi_{s,t}^g = p_{s,t}^g$. These fixed points constitute the equilibria of the model, and come in two varieties: stable equilibria; and unstable equilibria or "tipping points". The tipping points separate the regions of attraction of multiple equilibria (if multiple equilibria exist). In plotted curves such as Figure 7.2 stable equilibria can be distinguished from tipping points as the former cross the 45 degree line from above, whereas the latter cross from below. As Figure 7.2 shows, the model can predict multiple segregated equilibria and tipping points in the presence of linear preferences for school composition on ethnic or socio-economic groupings. With non-linear peer preferences more complex equilibria and dynamics may arise.

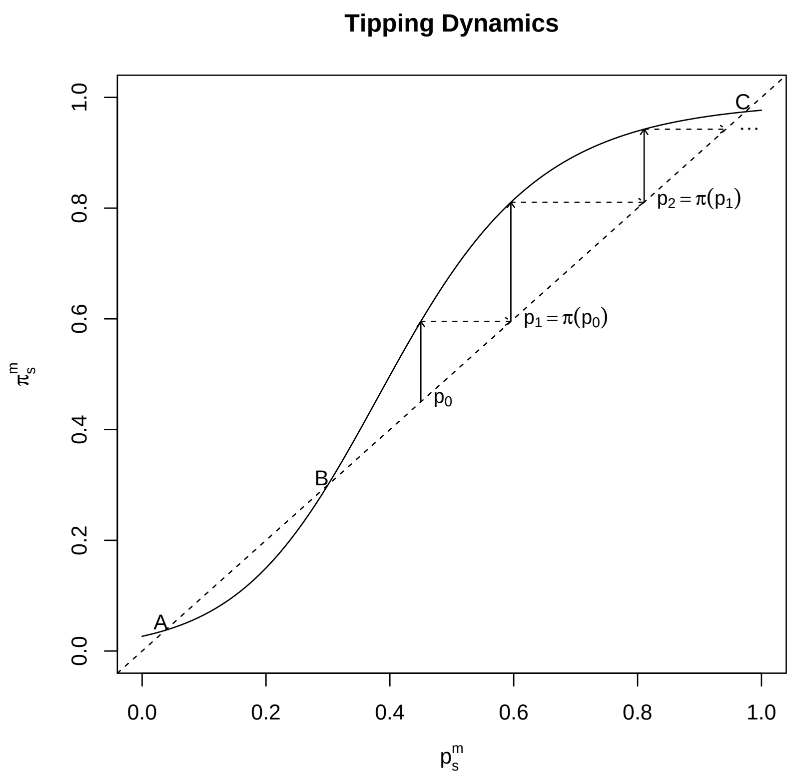


Figure 7.2: *The curve plots p_s^m against π_s^m for school s. The curve is induced by linear utilities for composition by both white and minority families. Of the three fixed points, A and C are stable equilibria, and B is a tipping point. The arrows show how the school's composition, starting from a point out of equilibrium (p_0), may move towards a stable equilibrium over a period of several admissions cycles.*

7.2.2 Results

The same datasets and estimated parameters were used as in the previous analysis. Applying the model to the data was made slightly more complex by the fact that we have not used a dichotomous coding for ethnicity, instead using the coding (white British, South Asian, Other). However, the proportion of 'Other' ethnic groups in the population, and in all schools, was small, so we treated this proportion as fixed, and allowed the 'White British' proportion to be $1 - \%\text{other} - \%\text{Asian}$. Both D_s and π_s^g were treated as functions of p_s^g only, with all other schools' compositions fixed.

The predicted school composition was also calculated using a counterfactual choice model, with the parameters relating to ethnic peer preferences set to zero. This prediction is not a function of the composition and is plotted alongside the estimated tipping curves as a single point. This prediction provides an indication of the effect of residential segregation and other factors on the expected composition of the school.

Figure 7.3 displays the estimated tipping curves for Oldham schools in 2012. These results suggest that for some schools, such as Royton Crompton and Oldham Academy North, tipping dynamics induced by peer preferences have explanatory power beyond that of the baseline model. However, other schools such as The Radclyffe School and Waterhead Academy appear far from the equilibrium predicted by the tipping model. There are several possible explanations for this. First, the compositions might simply be transitioning to a new equilibrium after some shock to the composition of the market, or to demand, or both. It is worth noting that the populations of at least some of Oldham's schools have been radically altered in the period 2010–2012, with the express purpose of disrupting a segregated equilibrium. The case of Waterhead Academy is the most high-profile (Nye, 2011; Edmonds, 2015), but other new academies in Oldham have intakes drawn from two or more parent schools. Given that in reality only a fifth of each school's population is renewed at a time, any re-equilibration might be expected to take a number of years.

Second, supply-side constraints might make some equilibria more difficult or impossible to attain; for example, where admissions criteria related to faith may have implications for the ethnic composition of the school; or where a school's capacity cannot grow enough to accommodate the predicted demand.

The final reason why school compositions may not correspond to the equilibria of the tipping model is that the difference may reflect an inadequacy of the underlying choice model to accurately predict demand. The tipping model is quite sensitive to the specification of the choice model. See, for example, the estimated tipping model for the Blue Coat School, whose curve only just fails to intersect with the 45 degree line near $p_s \approx 0.2$. Given that the choice model is quite simple, and does not account for unobservable variation in school popularity, it is unlikely that the true tipping points and stable equilibria are all accounted for.

It is worth noting that, because each tipping curve is created independently of the others, they may predict segregated equilibria for all schools, whereas in reality the schools interact so that if, say, one school's Asian proportion becomes large, another school's Asian proportion must become smaller.

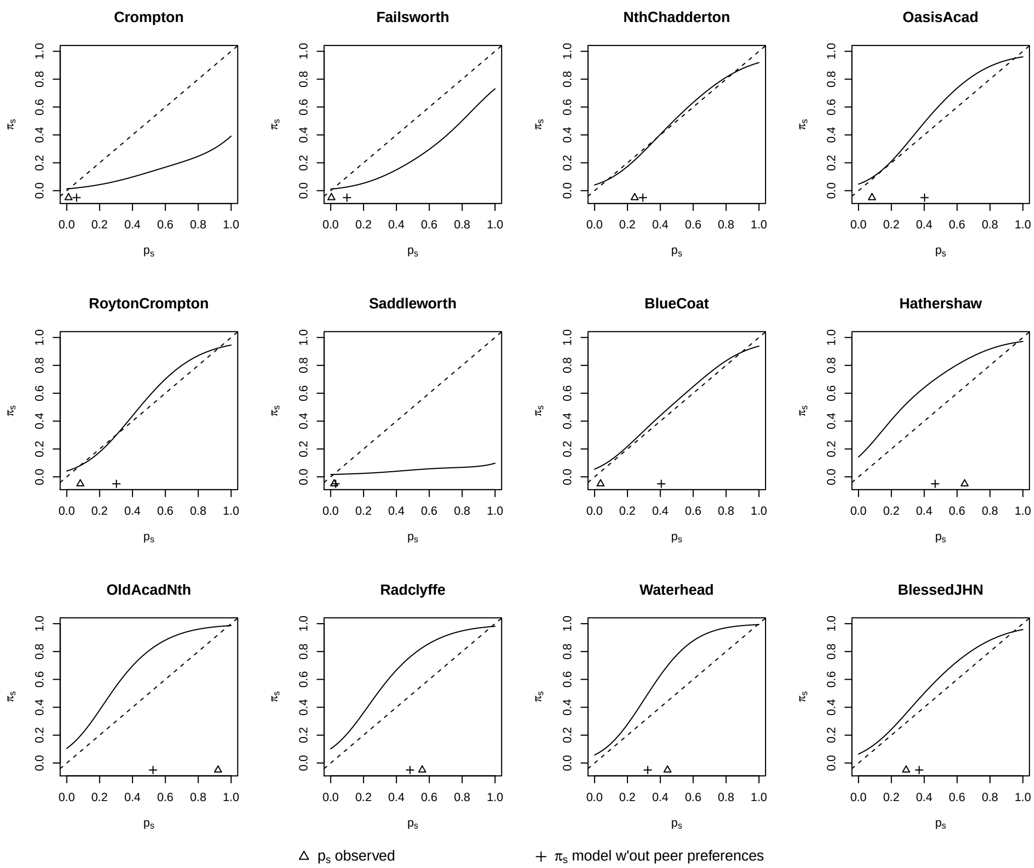


Figure 7.3: *Estimated tipping curves for Oldham schools in 2012. The actual proportion of asian pupils in the school, and the proportion predicted from the non-ethnic "null" model, are plotted on the x-axis.*

7.2.3 Simulation evidence

The tipping model results suggest that the estimated ethnic peer preferences may create sorting dynamics leading to complete segregation in some schools. However, a more realistic model might account for the constrained nature of the market, which may impact upon sorting dynamics. Using both the demand and supply sides of the two-sided model, and Deferred Acceptance matching algorithms, we can computationally simulate intakes and the effect of these on ethnic composition and hence demand.

For the purposes of the simulation, the capacity of each school was assumed equal to its current intake, and each schools' population "rolled through" a five-year cycle. It was assumed that families could observe the composition of a school perfectly, without any lag[2].

Table 7.6 presents the results of this model, while Figure 7.4 shows the trace of the

[2]Note that the parameters of the choice model were estimated on 2-year lagged data.

Table 7.6: Simulation Results

School	% Asian 2012	Predicted % Asian 2092
Crompton	1.1	0.7
Failsworth	0.4	0.8
NthChadderton	24.4	3.2
OasisAcad	8.2	66.2
RoytonCrompton	8.2	3.6
Saddleworth	2.1	1.6
BlueCoat	3.6	19.8
Hathershaw	64.5	72.3
OldAcadNth	92.0	87.8
Radclyffe	55.7	81.0
Waterhead	44.4	28.9
BlessedJHN	29.0	9.2

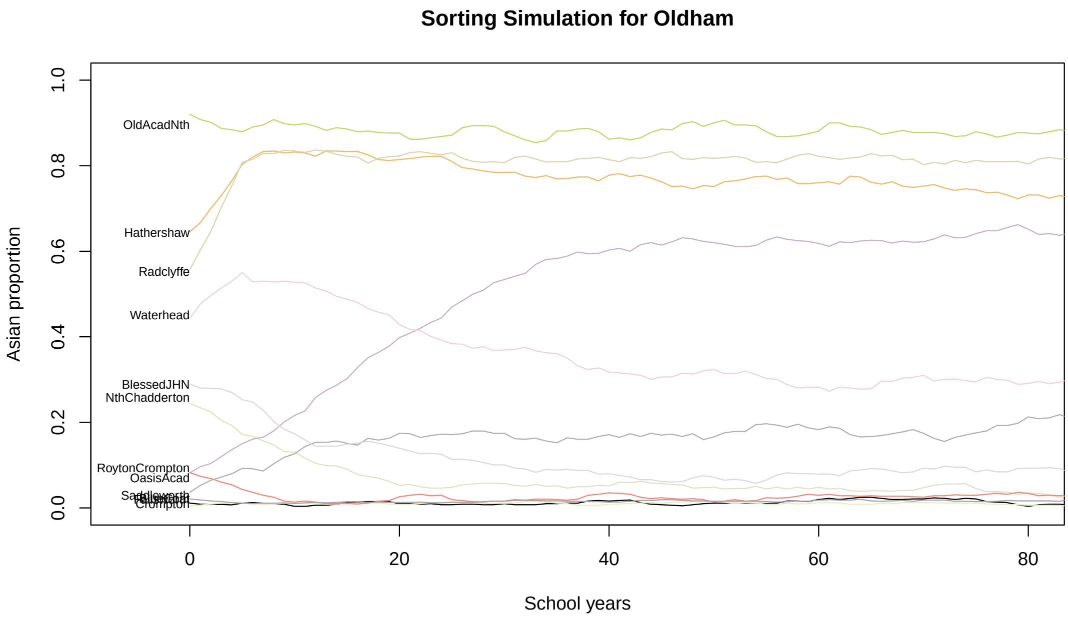


Figure 7.4: *Simulated sorting dynamics for Oldham schools, initialised with the 2012 intake.*

Asian proportions. As Figure 7.4 makes clear, the school compositions do not necessarily converge quickly to a segregated equilibrium; school ethnic compositions appear to settle down after 30–40 years! The model predicts that, without any shocks or demographic changes, Oldham schools will gradually become more segregated. In 2092 the difference between the Asian proportions of Oasis Academy and Waterhead is about 40 points. However, the model does not predict complete segregation. This is possibly due to capacity constraints and location.

7.3 Discussion

This chapter has presented evidence based upon structural models of aggregate demand, the parameters of which were provided by the estimates of the random utility model. Models of parental decision-making enable investigations of market structure that are analogous to the investigations based on various indices – notably the Herfindahl Index and the Index of Dissimilarity. Where the structural approach excels, however, is in allowing the researcher to undertake investigations of counterfactual hypotheses. This approach offers considerable flexibility in the design of computational experiments to test theories of sorting and competition.

However, the validity of a structural counterfactual analysis is limited by the predictive accuracy of the random utility model it is based upon. In the case of the models estimated in this thesis the lack of choice data, and the lack of school fixed effects, means that there is no guarantee that the predicted aggregate demand for a school is accurate even in the baseline scenario, let alone the counterfactuals. For this reason the potential for detailed quantitative predictions from these models is limited.

What the models offer is convincing evidence of *qualitative* regularities, such as the potential for cumulative sorting dynamics, or the association between sorting and competition. These "stylised facts" provide general answers to the second and third research questions, without justifying firm predictions about the size of effects. However, there is every reason to believe that methods based upon choice models, with a richer random utility model and better data, are capable of providing accurate predictions of not only the existence, but also the extent, of the influence of preferences on sorting and competition.

A final avenue for extension of the structural approach is in the quantification of parental choice, and especially spatial variations in parental choice. This part of the final research question has not been addressed in this chapter. Chapter 4 presented a visual analysis of parental choice based on Herfindahl Indices. Although not implemented here, it would also be possible to estimate spatial variation in choice using the *expected value of the choice set*, otherwise known as the *inclusive value*, for each location (Train, 2009). Although computationally-difficult in general, the logit model provides a tractable expression for the expected value of choice for a hypothetical family at location a:

$$\mathbb{E}\left[\max_{s\in\mathcal{S}}\left(U_{as}\right)\middle|\hat{\theta}\right] = \log\sum_{s\in\mathcal{S}}\exp\left(\hat{f}_{as}\right).$$

This quantity could be calculated at a grid of locations, to produce a visualisation revealing "cold spots" of effective choice. An advantage of this method would be that it takes account not only of the quantity of schools, but also the quality of schools. A difficulty arising with such a method would be the value judgements that are implicit in selecting a utility function to represent the *rational* or *normative* utility of school choice, given that the model would produce different utilities for members of different groups. However, it would be a useful method to illustrate how members of different groups perceive the choice available to them. As with all methods based on structural random utility models, it would enable the analysis of counterfactuals in which either details of the school market, or parameters of individual preferences have been changed.

Chapter 8

Stable matching theory

In previous chapters theoretical considerations relating to mechanism design and stable matchings have been treated lightly, so as not to interrupt the development of the substantive investigation. In this chapter and the following chapter we return to theoretical and methodological issues with two main purposes: first, to more thoroughly motivate and justify the structural econometric methodology adopted in this thesis; and second, to provide an informal review of a topic that is intrinsically interesting, and directly relevant to school choice.

The rest of the chapter is organised as follows. The next section introduces the branch of game theory and mechanism design known as stable matching theory. The development of the field is traced alongside its relevance to real situations in school choice. The following section enumerates a handful of results from stable matching theory and mechanism design that are either key to our econometric strategy, or especially relevant to our study of school markets. The final section provides a motivation to econometric uses of stable matching theory by investigating the empirical content of one-sided matching models, that use knowledge of ranked preferences on one side of a matching market to infer preferences on the other side of the market.

8.1 Theory of Stable Matchings

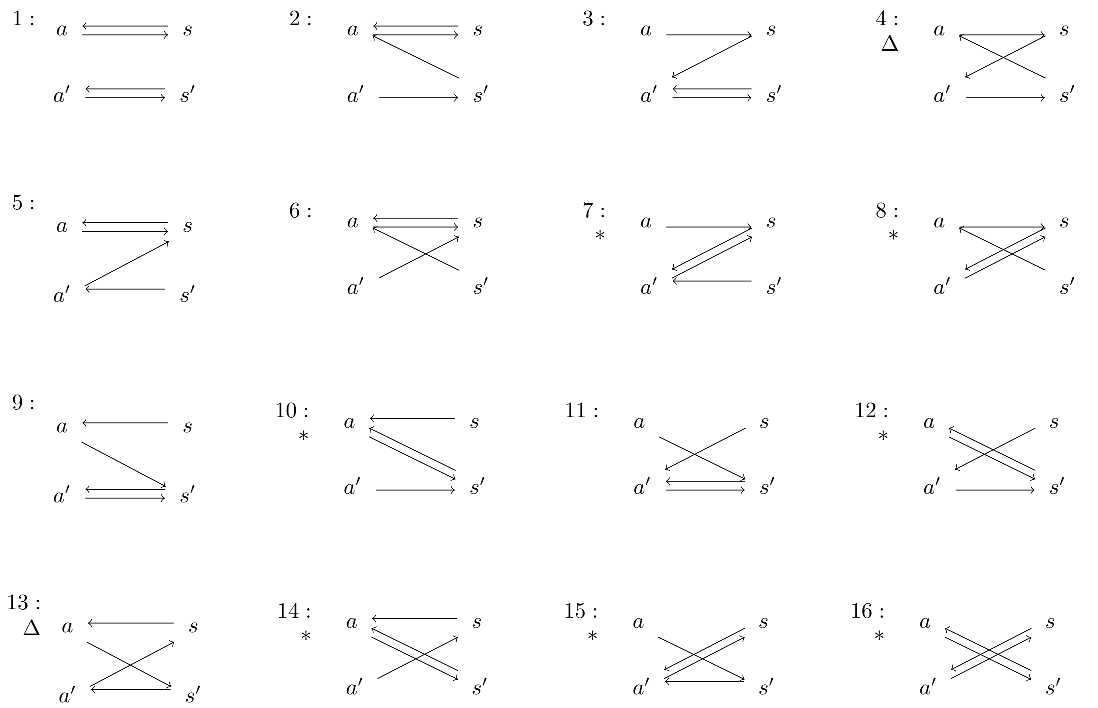
The problem of matching agents with heterogeneous preferences for institutions, to institutions with heterogeneous preferences for agents, has been studied formally in economics as the game theoretical "college admissions problem". An important aspect of many such situations is that, although they are often described as markets in an economic sense, money does not change hands between agents and institutions, which deprives these markets of a price mechanism for obtaining an allocation in equilibrium[1]. Gale and Shapley (1962) were the first to define this class of problems formally and derive a solution concept, called stability, to characterise feasible matchings. They proved that, under certain not-too-restrictive conditions on preferences, a stable matching always exists, and that a given preference profile may admit several different stable matchings. Furthermore, they described a matching algorithm, called "deferred acceptance" (often referred to as the "Gale-Shapley" algorithm) that is guaranteed to produce a stable matching.

Part of the appeal of stable matching theory comes from the fact that, whilst the motivating solution concept, stability, is conceptually simple and intuitive, the theoretical insights it enables are descriptively rich and practically useful in many real-world contexts. Roth and Sotomayor (1990) is credited with popularising the theory and demonstrating its applicability to improving the operation of real world matching mechanisms, leading to a significant re-invigoration of the field of mechanism design.

8.1.1 The College Admissions problem

In order to motivate the school choice problem we will first consider the original college admissions problem. Let $\mathcal{A} = \{a_1, a_2, \ldots, a_A\}$ be a set of students, and $\mathcal{C} = \{c_1, c_2, \ldots, c_C\}$ be a set of colleges. Each student wishes to attend a college, and each college wishes to accept its quota q_c of students. Each college is assumed to act as a single agent with a unique preference ordering over students. Each student ranks the colleges in order of preference, excluding only those colleges s/he would not attend under any circumstances, even if it meant not attending college at all. Likewise each college ranks all potential students in order of preference, excluding only those it would never accept, even if it meant not achieving its quota. For students, not attending college is always an option, and for colleges there is always an option to leave seats unfilled, rather than accept "unacceptable" students.

[1] Although originally studied in relation to money-less matching markets, stable matching theory is easily extended to allow endogenous prices, such as for example in job markets, although this branch of the theory is not discussed here.



$*$: Matching is not stable for these preferences. Δ: stable matching is not unique.

Figure 8.1: *All possible preference profiles in a* 2×2 *matching* $\{(a,s),(a',s')\}$. *Arrows represent preferences. There are nine preference profiles for which the matching is stable, and seven for which it is unstable. Unstable matchings are characterised by the presence of blocking pairs, represented here by diagonal arrows that are "reciprocated".*

Assuming for the moment that the number of students is equal to the number of available places at colleges, such a situation necessarily introduces conflicts of interest, not only between students and colleges, but also amongst colleges as competitors for students, and amongst students as competitors for college places. Without imposing hierarchical priority relations over students or colleges, it is not clear how these conflicts should be resolved to come up with an allocation that represents a good compromise.

Gale and Shapley (1962) proposed a solution concept based on a hypothetical situation that should be avoided by any feasible matching. In this hypothetical situation, there exists some pair of college and student who are *not* matched, who would both benefit from dissolving their existing match to match with each other. In the case of colleges this would mean expelling some student to create a spare place, and in the case of students this would mean leaving their college to take up the new place. Since such a coalition could destabilise an allocation, a matching in which there are no such "blocking" pairs is called a stable matching.

Figure 8.1 demonstrates stable and unstable matchings in a hypothetical school system with two schools and two pupils. It can be seen that, although stability is a property of the matching as a whole, because it is defined by the absence of a particular preference relation between pairs, it is easy to verify whether a given matching is stable by checking for blocking pairs.

Note that stability is a feasibility condition rather than an an optimality condition. Stability does not imply that the matching is optimal in relation to the preferences of either the colleges or students. Stability only implies that that there is no match that any college or student would prefer, that is available to them given the preferences of the other colleges and students.

Under certain regularity conditions Gale and Shapley proved that a stable matching always exists. It is only assumed that the preferences are strict (i.e. no college is indifferent between two or more students, or vice-versa), responsive (i.e. institutions only consider preferences over individuals and not sets of students) and transitive (i.e. no preference cycles). In fact, in balanced matching markets such that the number of places equals the number of students there will often be several distinct stable matchings.

As part of a constructive proof of the existence of a stable matching, Gale and Shapley described an algorithm that was guaranteed to produce a stable matching: the deferred-acceptance (DA) algorithm. In the algorithm one side of the market takes the active role of “proposer” and the other side takes the passive role of “accepter”. There are thus two variants of the algorithm: student-proposing DA; and college-proposing DA.

The student-proposing algorithm works as follows. In the first round, each student applies to his or her favourite college. Then, each college considers all applications, and *provisionally* accepts its preferred students up to its quota. In each subsequent round, all students who are not provisionally matched apply to the next college on their preference list, crossing colleges off as they apply to them. After each round of proposals, colleges consider all new proposals in combination with their current provisional list, and update their provisional list, rejecting previously-accepted students as necessary to make room for preferred students. Thus each student may be on a college’s list and then rejected in later rounds. This process continues until there are no students who are not on a college’s list, who have colleges left to apply to.

The college-proposing algorithm is essentially the same in reverse. In the first round, each college offers places to preferred students up to its quota. Each student considers all offers s/he has received and *provisionally* accepts the preferred offer. In each subsequent round, each college that has not filled its quota makes offers to students, working down its preference list to make up the difference between current matches and its quota. Each student considers new offers in combination with its current provisionally accepted offer, and accepts the best of these, rejecting the currently accepted offer if necessary. The process ends when all colleges have either filled their quotas or exhausted their preference lists.

Both variants of the algorithm are guaranteed to produce a stable matching, but these matchings are not usually identical unless there is only one possible stable matching. In fact, when there is more than one possible stable matching the two algorithms will produce matchings that have very different implications for the two sides. Gale and Shapley showed that for a given preference profile, student-proposing DA will produce the best possible stable allocation for the students, in that no student can improve on the matching without making some other student worse off or violating stability. This kind of optimality condition is called Pareto optimality[2]. At the same time, this student-optimal stable matching will be the worst possible stable matching for the colleges, in that *any* other stable matching can improve the result for at least one college without making any other colleges worse off.

The same result is true in reverse when a stable matching is found using college-proposing DA, which turns out to produce the college-optimal and student-pessimal stable matching. In fact, the set of possible stable matchings for a given preference profile follows a highly ordered lattice-like structure. Within this structure there is a unique college-optimal matching, and a unique student-optimal matching, which only coincide if there is a unique stable matching; in between these two extremes every Pareto improvement for one side is to the detriment of the other side. Thus the interests of colleges and students are always in opposition.

[2]Pareto optimality or efficiency is a criterion whereby no-one can be made better off without making someone else worse off. An allocation is Pareto optimal if this condition holds. If an allocation is not Pareto optimal it is possible to make improvements such that at least one person is made better off without making anyone else worse off. These improvements are called Pareto improvements. Pareto efficiency is a useful concept when dealing with preferences, where it may not be possible or desirable to trade off one person's utility against another's.

At this point it will be useful to introduce some terminology and notation. A match is a pair (a, c) where student a is matched to college c, and vice versa. However, it is also possible that students do not obtain a college place, $(a, \_)$, or colleges have unfilled places, $(\_, c)$. We use the notation $\mathrm{m}(a)$ to denote the match of student a, and $\mathrm{m}(c)$ to denote the set of students matched to college c[3]. A set of orderings describing the full preference sets of all students and all colleges is called a *preference profile*, denoted by π. A market is a triple $(\mathcal{A}, \mathcal{C}, \pi)$. A *matching* $\mathbf{m}$ is a full set of matches for a market, and $\mathcal{M}$ is the set containing all possible matchings for a given market. For each market the set of stable matchings $\mathcal{M}^*$ forms a subset of $\mathcal{M}$.

The set of stable matchings, $\mathcal{M}^*$, follows a highly ordered lattice-like structure. Within this structure there is a unique college-optimal matching, and a unique student-optimal matching, which only coincide if $\mathcal{M}^*$ is a singleton; in between these two extremes every Pareto improvement for one side is to the detriment of the other side. Thus the interests of colleges and students are always in opposition. Since the paper of Gale and Shapley, other algorithms have been proposed to produce matchings from the interior of $\mathcal{M}^*$ that compromise between the welfare of the two sides. However, in the original paper the authors advocated using the student-proposing algorithm, on the basis that colleges exist for the benefit of students rather than the other way round.

8.1.2 The school choice problem

The college admissions problem described above is a good description of many real life situations where agents with diverse preferences need to be allocated to institutions which also have preferences for agents, but which are not allowed to charge money for admission, or at least are not allowed to set prices. The problem of public school allocations is slightly different. In public school choice markets there are students who have, or at least their parents have, preferences for different schools. There are also schools with limited capacity, however it is problematic to think of them as assigning places according to their preferences. Abdulkadiroğlu and Sönmez (2003) summarised the distinction thus:

> "The central difference between college admissions and school choice is that

[3] It is common to use $s = \mathrm{m}(a)$ rather than the more correct $\{s\} = \mathrm{m}(a)$. We exploit this slight abuse of notation to use $\mathrm{m}(\cdot)$ within indices.

> in college admissions, schools themselves are agents which have preferences over students, whereas in school choice, schools are merely 'objects' to be consumed by the students." (Abdulkadiroğlu and Sönmez, 2003, p.731)

For Abdulkadiroğlu and Sönmez, this distinction arises for ethical reasons; the idea of an education market in which schools pick-and-choose students to achieve their own ends is, for them, an example of what Roth (2007) terms a "repugnant market". For this reason, Abdulkadiroğlu and Sönmez propose a model whereby institutional preferences are replaced with *priorities*. Unlike preferences, school admissions priorities may be prescribed by law or by local authorities; they are usually based on explicit objectively-verifiable criteria such as distance to school or residence within a catchment area. Also, unlike colleges, schools do not have the option of leaving places unfilled if they have spare capacity.

Given a set of preferences on the student side and a set of priorities on the school side, the problem is essentially the same as the college admissions problem. Priorities can be treated in the same way as preferences, and a stable matching can be reached using the student-proposing deferred acceptance algorithm. However, if respecting school priority rules is considered less important than respecting student preferences, it is possible to obtain *unstable* matchings that Pareto dominate the student-optimal stable matching by allowing students to "trade" priorities. This idea rests on a formulation of priority at a given school as an endowment *possessed by* a student; if the student does not require the priority (such as if the student prefers another school) s/he can trade it for priority at another school. In practice, priorities can be traded using the Top-trading Cycles mechanism, otherwise known as the Efficient Transfer Mechanism; this mechanism improves on simple bilateral priority trading by allowing complex cyclical trades involving multiple students. The TTC mechanism was implemented by the New Orleans Recovery School District in 2012.

Another potential difference between the college admissions problem and school choice is that schools often do not have strict priorities over all students. In many contexts, especially in the US, school priority rules may only specify that a group of students is within the catchment area, and another group of students is outside the catchment area. Within each priority group, the school is indifferent between students. Tied preferences are often broken using some kind of lottery, but this creates new problems since the student-proposing deferred acceptance algorithm is no longer guaranteed to be Pareto efficient for students. The reason for this is that the lottery creates pseudo-priorities that may block matches that would otherwise be stable and would be preferable from the point of view of students.

Abdulkadiroğlu (2013) acknowledges that many school choice markets contain a mixture of schools that are not in control of admissions, and schools that control their own admissions and act to a greater or lesser degree as strategic agents. In these cases hybrid mechanisms can be used that allow the trading of flexible priorities, but restrict the trading of fixed priorities. In England and Wales school choice can be said to operate within a hybrid mechanism in that schools have differing levels of control over their own admissions priorities, although successive legislation, most recently the 2007 Admissions Code, has ensured that no state-funded school has complete freedom in assigning admissions priorities.

8.1.3 Strategy-proofness and the Boston mechanism

Before mechanism designers began to take an interest in school choice, the preferred allocation mechanism in many public school markets was some variant of a first-preferences-first algorithm, which was dubbed the Boston mechanism by Abdulkadiroğlu and Sönmez (2003), after the Boston public schools district where it was first studied. In the UK, first-preferences-first was used by many local authorities until the rule was outlawed in the 2007 Admissions Code (Coldron et al., 2008).

The Boston mechanism begins by each school assigning seats to students who have ranked that school as first choice. Each school assigns seats to as many students as possible according to its admissions rules, until either there are no seats left or all first-choice students have been assigned. In the next round, each school assigns remaining seats to students who have ranked the school second, again according to its admissions rules. In general, in the i'th round, each school considers only the students who ranked the school in i'th place. This continues until all students have been assigned. This mechanism is similar to student-proposing deferred acceptance, except that school assignment in each round is firm rather than provisional.

Although intuitively appealing as it aims to respect first preferences, the Boston mechanism has been shown to have a number of unappealing qualities. First, it is not a stable matching algorithm. To see this, consider a student a who is ranked first by school s, but prefers another school, s', and ranks s second. Say the student is not allocated to s' in the first round, and school s managed to fill its quota in the first round. Now student a cannot get into school s even though a prefers s to all available schools, and school s prefers student a to all of the students it has admitted under the first-preferences-first rule. Student a and school s form a blocking pair that make the matching unstable.

Second, the Boston mechanism is not strategy-proof. This means that students can benefit from misrepresenting their true preferences. Clearly, in the preceding example student a would have been better off if she had stated school s as her first preference, even though this would have not been true. This is a particularly undesirable property for school choice, because it necessitates sophisticated school choice strategies on the part of parents. This may lead to some groups of parents being advantaged over others with undesirable consequences for equity.

Strategy-proofness is also an important property for the estimation of choice models. If stated preferences are not honest then there is little hope of inferring parameters of underlying random utility models with any kind of data. It is therefore important to verify that mechanisms in use are actually truth-revealing. Fortunately, first-preferences-first mechanisms are no longer used in the UK. The majority of local authorities use deferred acceptance algorithms.

Even then, it has been shown that implementations of deferred acceptance algorithms vary in their vulnerability to manipulation (Pathak and Sönmez, 2011). Student-optimal DA is strategy-proof under ideal conditions; however there are three features of real admissions systems that may compromise this. First, many local authorities limit the number of preferences that can be expressed. This compromises incentive compatibility because if parents are only allowed to express a handful of preferences, they may be incentivised to choose a "safe option" as the last choice, to avoid being allocated to an unlisted school. Since the 2007 Admissions Code each local authority has to allow parents to express at least three preferences. This means that while the first two preferences should be expressed truthfully, the third preference may be chosen to be a safe option.

The second reason mechanisms may not be strategy-proof in practice is that, although there is a clear advantage to using *student-proposing* DA, in fact some local authorities have implemented the *school-proposing* variant of the Gale-Shapley algorithm. According to stable matching theory, only the student-proposing variant is incentive-compatible for students. Intuitively, this is because when a mechanism is used that does not select the optimal stable matching for students, it is always possible for students to manipulate their preferences to obtain another stable matching within the core that is preferable. However, in large and diverse matching markets, and without knowledge of other preferences, it is unlikely that any family would be able to devise a strategy to improve their match *within* the set of stable matchings. This hypothetical manipulability is in contrast to the previously discussed issues, where strategic manipulations are obvious and probably widely understood.

The final reason why some families may not be expressing preferences truthfully under the current system is that the truth-telling property of the matching mechanisms has not been widely publicised. That is, some parents may still be operating under the misconception that school places are allocated according to a first-preferences-first rule, and may be inappropriately applying redundant strategies. Evidence from informal conversations with parents suggests that at least some parents still believe that they should adjust their stated preferences to accommodate first-preferences-first rules.

8.2 Results and observations from stable matching theory

Although our interest in stable matchings is primarily motivated by our econometric methodology, there are nevertheless a handful of less-well-known results motivated by stable matching theory that will prove useful in what follows, and it is therefore convenient to introduce them here. These results have applications in two contexts in the thesis. First, stable matchings provide a basis for our estimation strategy, and there are certain facts that support the validity of this strategy, and help us to predict its performance in real life situations. Second, some of these results and observations have substantive interest as they shed light on the impact of mechanism design on the welfare of students.

8.2.1 Size of $\mathcal{M}^*$

Much of the early literature in stable matchings studied hypothetical stable marriages (that is, one-to-one matchings) where the number of men equalled the number of women, and preferences were generated uniformly from the set of all possible preference lists. In these conditions, researchers found, not only that a stable matching always exists, but also that the size of $\mathcal{M}^*$ grows large as the number of participants grows (Knuth, 1976). In large, balanced, one-to-one matchings there are typically many possible stable matchings, and the implications of men-proposing and women-proposing DA for the welfare of men and women are very different. Arguably, this early work, and the continuing popularity of "toy problems" in the mechanism design literature, inspired an emphasis on optimising the mechanism itself, rather than studying the effect of other properties of the market.

However, when researchers began to study real matching markets they found that $\mathcal{M}^*$ is often not large at all. For example, when Roth and Peranson (1999) analysed the clearing house for US medical interns, and simulated matchings based on actual stated preferences, they found that the choice of DA algorithm did not make any difference to the result, indicating a unique stable matching.

Using a combination of simulations and analysis, Ashlagi et al. (2013) investigated the behaviour of unbalanced markets, where the number on one side of the market is greater than the number on the other side. In school choice markets this corresponds to a situation where there are either more available places than students, or more students than available places. Surprisingly, they found that even slight imbalances in the market, in the order of one in one thousand participants, caused $\mathcal{M}^*$ to collapse. They showed that, not only were there only a very small proportion of participants who had multiple stable matchings, but also that the difference in welfare between the school-optimal and student-optimal stable matchings was typicially small.

Furthermore, Ashlagi et al. used simulations of matchings with preferences drawn from various random utility models to demonstrate that almost any deviation from uniform random preferences also leads to a small or singleton $\mathcal{M}^*$. In a school choice setting, this includes models where students' preferences for schools are correlated, or where students tend to prefer schools that also prefer them. Both of these correlation structures are plausible in real settings. Azevedo and Leshno (2016) prove that in many-to-one matching markets, as the number of students in each school grows large there is a unique stable matching. This follows from the results for correlated preferences, since a many-to-one market can be represented as a one-to-one market where each school is represented by a set of q agents, each with identical preferences. Taken together, these results imply that we cannot expect $\mathcal{M}^*$ to be large in real school choice markets.

8.2.2 Welfare implications of capacity

As well as showing that $\mathcal{M}^*$ is small in unbalanced markets, Ashlagi et al. demonstrated that unbalanced markets benefit the short side of the market (i.e. the side with less participants) at the expense of the long side. Surprisingly, they found that even a tiny imbalance in the numbers of participants had dramatic implications for the comparative welfare of the two sides. As an example, they showed that in a hypothetical marriage market with 1,000 men and 1,001 women, men are matched on average with the 7^{th} woman on their preference lists, while women are matched with their 145^{th} most-preferred man (Ashlagi et al., 2013, p3).

Although Allen and Burgess (2010) and others have discussed the role of capacity constraints in the success of school choice policies, there do not appear to have been any studies that quantitatively analyse the welfare effects of different levels of capacity. The problem is complicated by correlated preferences, since any increase in capacity at unpopular schools is unlikely to have much effect on student welfare.

8.2.3 The effect of subsetting a matching

In both the mechanism design literature and the literature on estimation, it is often assumed that an entire matching market is observed. In real situations, including school choice, it is very difficult to observe a self-contained matching market. Therefore, for estimation strategies based on stability to be valid, it is necessary to show that stability holds for subsets of a matching market.

Claim 1: Let $\mathbf{m}$ *be a many-to-one matching between a set of students,* $\mathcal{A}$ *and a set of schools* $\mathcal{S}$*. Let* $\mathcal{S}' \subset \mathcal{S}$*, and let* $\mathbf{m}' \subset \mathbf{m}$*, such that all matched pairs corresponding to schools in* $\mathcal{S}'$ *are retained, and all other matched pairs are removed. Assume that* *the removal of schools does not affect students' rankings of the remaining schools, and symmetrically for schools' preferences. Then, if* $\mathbf{m}$ *is stable,* $\mathbf{m}'$ *is also stable.*

This follows from the definition of stability. Stability is defined by the absence of blocking pairs: pairs of student and school such that the student prefers the school to her allocated school, and the school prefers the student to at least one of its allocated students. If there are no blocking pairs in $\mathbf{m}$, there will be no blocking pairs in $\mathbf{m}'$. In order for subsetting the matched pairs to create a blocking pair there would have to be some student matched to $s \in \mathcal{S}'$ in $\mathbf{m}'$, who prefers some $s' \in \mathcal{S}'$, and s' either has an available place or prefers a to at least one of its allocated students. However, if s' has an available place in $\mathbf{m}'$ then s' also had an available place in $\mathbf{m}$, since the set of students in $\mathrm{m}(s')$ has not changed. Similarly, if s' prefers a to some allocated student in $\mathbf{m}'$, then s' also preferred a to one of its allocated students in $\mathbf{m}$. Therefore (a, s') would have been a blocking pair in $\mathbf{m}$, which is not true therefore (a, s') cannot be a blocking pair in $\mathbf{m}'$. $\square$

Subsetting the matching in other ways, such as removing a subset of students without decreasing the corresponding schools' quotas, or removing a subset of schools but not removing the students, may destabilise the matching, since there would be some schools with available places, or extra unmatched students, that were not previously available.

Although the subsetted matching is also a stable matching, subsetting the market may remove blocking pairs, and in so doing change the size and properties of $\mathcal{M}^*$. Counter-intuitively, subsetting a market may actually increase the number of possible stable matchings for the participants in the subset, by removing pairs that blocked certain matchings. Or, in the context of markets where only a subset is observed, it may create the appearance that more stable matchings were possible for the subset than were actually possible, because of unobserved blocking pairs.

This is best demonstrated with an example. Three students a, b and c are to be allocated to three schools s_1, s_2 and s_3. Assume these are the only participants in the market. Preferences are given below:

$$
\begin{array}{ll}
a:\ s_2 \succ s_1 \succ s_3 & \qquad s_1:\ a \succ c \succ b \\
b:\ s_1 \succ s_2 \succ s_3 & \qquad s_2:\ b \succ c \succ a \\
c:\ s_1 \succ s_2 \succ s_3 & \qquad s_3:\ c \succ a \succ b
\end{array}
$$

By doing student-proposing and school-proposing DA on these preferences it is easy to verify that there is only one stable matching, that is

$$\mathbf{m}_1 = \begin{pmatrix} s_1 & s_2 & s_3 \\ a & b & c \end{pmatrix}.$$

If we remove student c and school s_3, then

$$\mathbf{m}_2 = \begin{pmatrix} s_1 & s_2 \\ a & b \end{pmatrix}$$

is also stable. It is the school-optimal stable matching. However, there is also a matching:

$$\mathbf{m}_3 = \begin{pmatrix} s_1 & s_2 \\ b & a \end{pmatrix}$$

which is the student-optimal stable matching. If we had only observed $\mathbf{m}_2$, without knowing that it was a subset of $\mathbf{m}_1$ we may have concluded that welfare gains were possible by using the student-proposing DA, where in fact no welfare gains were possible.

The above result suggests that, unless we observe a self-contained matching market, it is not possible to use additional information about matchings produced by an algorithm, other than stability, for estimation. For example, if it is known that school-proposing DA was used we may wish to use this additional information to derive additional constraints on the likelihood. However, the example above indicates that any such efforts may produce misleading answers unless a self-contained market was observed, and stability is the only property of a matching that is robust to subsetting. In practice a self-contained market is almost never observed.

An additional implication is that one should be very careful when simulating from real matching markets for the purpose of examining the welfare effects of mechanism design. One may discover spurious welfare improvements from using different mechanisms, when in fact unobserved aspects of the wider market would not allow those improvements.

As a final note, although the example above is a toy problem, it exemplifies a more general phenomenon. When some students are matched to universally unpopular schools, such as

(c, s_3), or equivalently are unmatched, then pupils who have priority at those schools may block pupils who have priority at more popular schools from making mutually beneficial trades with other pupils. This phenomenon is discussed in a school choice context by Calsamiglia and Miralles (2012). In the presence of “sink” schools, whose pupils may all prefer to be at other schools, the amount of choice enjoyed by pupils at *other* schools is diminished. They point out that the Top Trading Cycles algorithm allows the swaps that would otherwise be blocked. This does not improve the situation for the unlucky pupils at “sink” schools, but benefits all other pupils who may wish to trade priorities. In the full market above, TTC would have resulted in

$$\mathbf{m}_4 = \begin{pmatrix} s_1 & s_2 & s_3 \\ b & a & c \end{pmatrix}$$

which is not stable because (s_1, c) and (s_2, c) are blocking pairs, but is Pareto optimal for students. It has also been conjectured (seminar discussion, 2013[4]) that a similar mechanism may be responsible for the large imbalance in welfare in unbalanced markets, because unmatched agents on the long side may block mutually beneficial trades between matched agents.

8.3 The empirical content of stable matchings

Although they have intrinsic interest for school choice from a mechanism design perspective, the main reason we are interested in stable matchings is the potential of the theory for performing inference about choices. As a first step, this section asks what information can be recovered about preferences on one side of a matching, when complete preferences on the other side of the matching are known, given that we know or can assume that the matching is stable.

To motivate the topic, let us introduce a simple example. Two students, a and b, are matched to schools s_1 and s_2, respectively. We know that both schools would prefer to swap, that is: $s_1 : b \succ a$ and $s_2 : a \succ b$. We also know that the matching is stable. Does this information about school priorities tell us anything about student preferences? Yes,

[4]Meeting of COST Action IC1205 on Computational Social Choice at the Universidad Autonoma de Barcelona, October 2013.

in fact, in this simple system, it tells us everything about student preferences. Because we know there are no blocking pairs, if school s_1 "longs" for student b, then we know that b does not reciprocate this longing, so b must prefer s_2. Likewise, given that s_2 "longs" for a, we know that a does not "long" for s_2. In order to make the exposition more expressive, hereafter the term "longing" will denote any preference on either side of the market, for a match other than the realised match. Therefore, "reciprocated longing" is synonymous with a blocking pair. The preferences inferred on the student side are $a : s_1 \succ s_2$ and $b : s_2 \succ s_1$ which are indeed the complete preference lists of the students. We will see that in general complete preference lists cannot usually be inferred.

The example above suggests that it is longing on the side with known preferences that provides information about preferences on the other side. In fact, the key mechanism is that when there is longing on one side of the market, we know that this longing is not reciprocated. This implies that if every school were matched with its preferred students, the matching and school preferences would provide no information about student preferences.

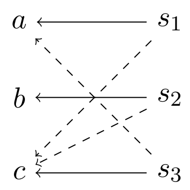
To state this result formally, if $\mathbf{m}$ is a stable matching, and schools' priorities are known, then we have that

$$[a \in \mathrm{m}(s) \;\&\; \exists\, a' : \{a' \notin \mathrm{m}(s) \;\&\; a' \succ_s a\}] \Rightarrow [\mathrm{m}\big(a'\big) \succ_{a'} s].$$

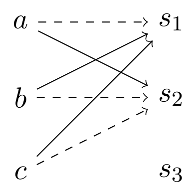
This result is symmetrical in the case where students' priorities are known, except that in this case $\mathrm{m}(s')$ is a set of students, and $[\mathrm{m}(s') \succ_{s'} a]$ means that s' prefers *each* of its allocated students to a. Equivalently, if we denote by $\overline{a}_s$ the least preferred student allocated to school s', then $[\mathrm{m}(s') \succ_{s'} a] \Leftrightarrow [\overline{a}_{s'} \succ_{s'} a]$. That is, s' prefers its least preferred student to a.

To clarify this let us return to the example in Section 8.2. The unique stable matching in the market with three students and three schools is represented diagrammatically in Figure 8.2. The two panels represent two empirical situations. In panel (a) we have observed the matching and the schools' preferences, and in panel (b) we have observed the same matching and the students' preferences.

What can be learned about students' preferences by observing schools' preferences in



(a) Schools' preferences



(b) Students' preferences

Figure 8.2: *A stable matching with three schools and three students. In panel (a), schools' preferences have been observed, whereas in panel (b) students' preferences have been observed. Solid arrows represent first preferences and dashed arrows represent second preferences.*

panel (a)? The answer is nothing, since each school is matched to its first choice, there are no longings on the school side to be used to infer preferences of the students. In panel (b), however, no student is allocated to his or her first choice, and student c is matched to his third choice. In fact there are four longings on the student side, which means that we can infer four preference relations on the school side, out of a total of nine preference relations in the schools' full preference lists. The inferred preferences are:

$$[s_1 \succ_b s_2] \Rightarrow [a \succ_{s_1} b] \qquad [s_2 \succ_a s_1] \Rightarrow [b \succ_{s_2} a]$$
$$[s_1 \succ_c s_3] \Rightarrow [a \succ_{s_1} c] \qquad [s_2 \succ_c s_3] \Rightarrow [b \succ_{s_2} c]$$

which can be written more compactly as $[s_1 : a \succ \{b, c\}]$ and $[s_2 : b \succ \{a, c\}]$.

In fact, revealed preferences for some agent i in stable matchings always take the form $[\mathrm{m}(i) \succ_i \{\dots\}]$, analogous to the revealed preferences in a classical discrete choice problem, with the realised match on the LHS and a set of other available options on the RHS. In a classical discrete choice problem, the *choice set* contains all options, whereas in a stable matching the "stable choice set" of each agent is determined by the matching mechanism. The concept of stable choice sets leads to an alternative way to characterise stable matchings, and is essential to the fixed point representation of stable matchings of Adachi (2000), although he does not use this terminology. The stable choice sets on each side of the matching correspond to a particular equilibrium. If a market has more than one stable matching, each stable matching will imply a different set of stable choice sets.

Observing the preferences on one side of the market reveals all stable choice sets on the other side of the market. In our example, we have not learned anything about the preferences of s_3, but we know that s_3's stable choice set in that particular (in this case unique) stable matching contained only student c. Denote the stable choice set of a under some stable matching $\mathbf{m}$ by $\mathrm{C}_{\mathbf{m}}(a) \equiv \{s : a \succeq_s \overline{a}_s\}$, of which $\mathrm{m}(a)$ is a subset. Observing the stable matching and all schools' preference lists reveals $\mathcal{C}_{\mathbf{m}} \equiv \{\mathrm{C}_{\mathbf{m}}(a)\}_{a \in \mathcal{A}}$, and the set of preferences revealed is denoted by $\mathcal{R}_{\mathbf{m}} \equiv \{[\mathrm{m}(a) \succ_a \mathrm{C}_{\mathbf{m}}(a) \setminus \mathrm{m}(a)]\}_{a \in \mathcal{A}}$. This applies symmetrically to revealed preferences of schools. $\mathcal{C}_{\mathbf{m}}$ can be calculated in $(A+1) \times S$ steps, by calculating $\overline{a}_s$ and then comparing the rank of $\overline{a}_s$ to each $a \in \mathcal{A}$, for each $s \in \mathcal{S}$. If $\mathrm{Rank}_s(a) \leq \mathrm{Rank}_s(\overline{a}_s)$, then s is added to $\mathrm{C}(a)$. The revealed preference set $\mathcal{R}_{\mathbf{m}}$ is used to derive inequalities for sampling from latent utilities in the LHN stability Gibbs sampler described in the next chapter (Logan et al., 2008).

Chapter 9

Inference in two-sided matching models

two

This chapter returns to the discussion of the structural estimation of two-sided matching models, introduced in Chapter 5. The purpose of this chapter is to more thoroughly motivate the model and explore its inferential and computational properties, with reference to other works focussing on similar models.

The first part of the Chapter synthesises the current state of knowledge concerning the inferential properties of structural empirical models of two-sided matchings. As incomplete econometric models, the identification and unbiasedness of estimators derived from these models cannot be taken for granted, and the existing literature contains examples of contexts in which these conditions are, and are not, met. The second part of the Chapter reviews possible methods for the estimation of two-sided matching models, and describes in detail the methods used in the thesis. Finally, simulation evidence is presented to evaluate the performance of estimators derived from our methods.

9.1 Existing literature on two-sided matching models

Econometric studies of two-sided matchings are not new; the field originated with the work on assortative matchings of Becker (1974), and is related to empirical work on discrete games (Bresnahan and Reiss, 1991; Tamer, 2003; Ciliberto and Tamer, 2009). However, models of assortative matching impose restrictive assumptions on the distribution of preferences for identification, and recently a small literature has arisen concerned with estimating models of heterogeneous preferences based on the Gale-Shapley stable marriage or college-admissions models (discussed in Chapter 8). These models seek to estimate the parameters of random utility choice models for both sides of a matching. These models can be divided into those that allow agents to maximise a joint utility or surplus of the matching (called Transferable Utility or TU models) and those that focus on situations where utilities on each side are private (called Non-Transferable Utility or NTU models). Literature on estimation of the joint surplus in TU models is somewhat more advanced[1]. However, there are many situations for which this model of matching is clearly not appropriate since no transfer, monetary compensation, or sharing of utility can take place. School allocation systems are a good example of this. In addition, TU models do not allow the separate modelling of private utilities on each side, which may be required even in cases where monetary compensation does take place.

These recent studies have addressed the fact that NTU stable matching models are *incomplete* econometric models (Tamer, 2003). In statistical terminology, this means that a probability model for stability alone does not describe the full data generating process leading to an observed matching. This is the case because, as we saw in Chapter 8, stable matching games possess multiple equilibria for a given set of preferences, and a model specifying stability does not specify how the observed stable matching has been chosen from this set. This incompleteness potentially leads to bias in estimators if the partial data generating model is substituted for the full data generating model without additional restrictions.

There are several strategies adopted in the literature to deal with the incompleteness of the model. Sørensen (2007) and Agarwal and Diamond (2013) make restrictions on the form of utility functions to ensure that the model admits a unique stable matching. Menzel and Salz (2013) treats the stability model as an upper bound on the likelihood function, and derives a lower bound on the likelihood function to allow set-identification of parameters (see Figure 9.1[2]). Whereas the upper bound on the full likelihood is the

[1] See Fox (2009) for a review

[2] Menzel and Salz (2013) derives a Bayesian method. Figure 9.1 shows how set identification of an

probability that the matching is stable, Menzel proposes as a lower bound the probability that the observed matching is the *unique* stable matching. In a later paper, Menzel (2015) adopts a different approach, arguing that two sided markets asymptotically approach a limiting approximation with a unique equilibrium but restricted identification properties.

The issue of multiple equilibria is the most important limitation in estimating two-sided stable matching models. However, at times it seems that other inferential issues are not sufficiently acknowledged, or are even conflated with, the incompleteness issue. For example, Echenique et al. (2013) claim to show partial (i.e. set) identification using a method of moments estimator. However, for the 2×2 market they study, any bias caused by multiple equilibria is negligible, because all but two of the 16 possible preference profiles admit only a single stable equilibrium (cf. Figure 9.2). The partial identification they demonstrate is caused by the use of information from only one 2×2 market. In this case, as we show in the next section, point identification is impossible, because a single 2×2 market does not contain enough likelihood components to bound the parameter space. This issue is not related to the multiplicity of equilibria, and is actually a variant of the well-understood *statistical separation* issue for discrete choice models.

Similarly, Sørensen (2007), Logan et al. (2008) and Agarwal and Diamond (2013) have pointed out that certain combinations of covariates on the two sides of the model are not identified and lead to local maxima in the likelihood function/posterior distribution. Local maxima are not necessarily a problem if, in large samples, we can expect a unique global optimum to tend towards the true parameter value. This is the case for most combinations of covariates including interactions (eg. distances). However, in the next section we show that in one-to-one markets there are combinations of covariates that lead to indistinguishable local optima and biased estimation. This non-identification is *not* caused by multiple equilibria. Agarwal and Diamond (2013) uses a model with perfectly correlated preferences and shows that many-to-one matchings contain additional information that identifies parameters in this situation. In the next section this identification issue is explored for a general random utility specification.

MLE might be achieved along similar lines.

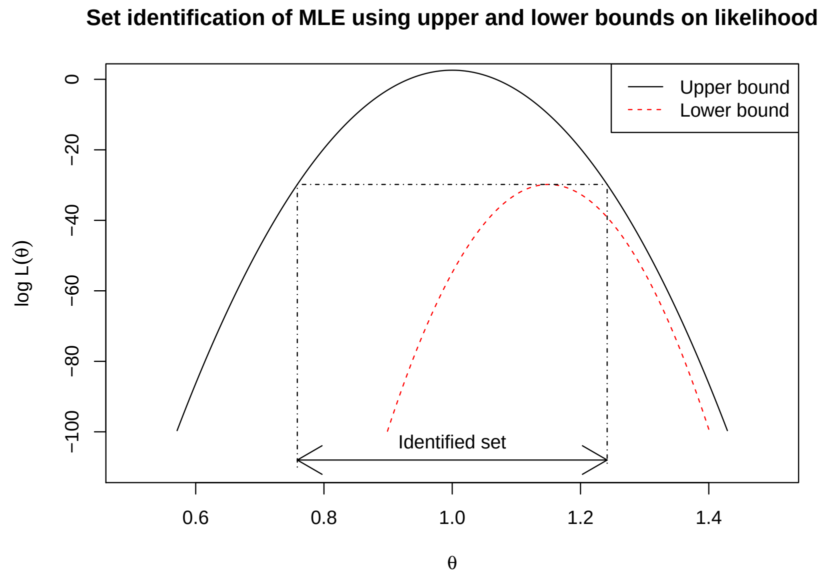


Figure 9.1: *Set identification of parameter θ by exploiting the stability part of the model* $\Pr(\mathbf{m} \in \mathcal{M}^*|\theta)$ *as an upper bound on the likelihood, and the probability of a unique equilibrium* $\Pr(\{\mathbf{m}\} = \mathcal{M}^*|\theta)$ *as a lower bound on the likelihood, as per Menzel and Salz (2013).*

9.2 Comparison of full- and partial-likelihood models

In this chapter we take a different approach to the above authors; rather than using restricted or approximate models to ensure a unique equilibrium (stable matching), we study the conditions for obtaining bounded, yet biased, point estimates in finite samples under a flexible random utility model specification. Then we ask, under which conditions do estimates derived from the stability likelihood asymptotically approach those derived from a full likelihood? At the same time we explore and clarify two inferential issues that turn out to be independent of the incompleteness issue: obtaining bounds on the parameter space; and non-identification of certain parameter combinations. In order to understand the identification problems associated with using both the full and stability models, it is useful to write down the full likelihood as an intractable sum over preference profiles.

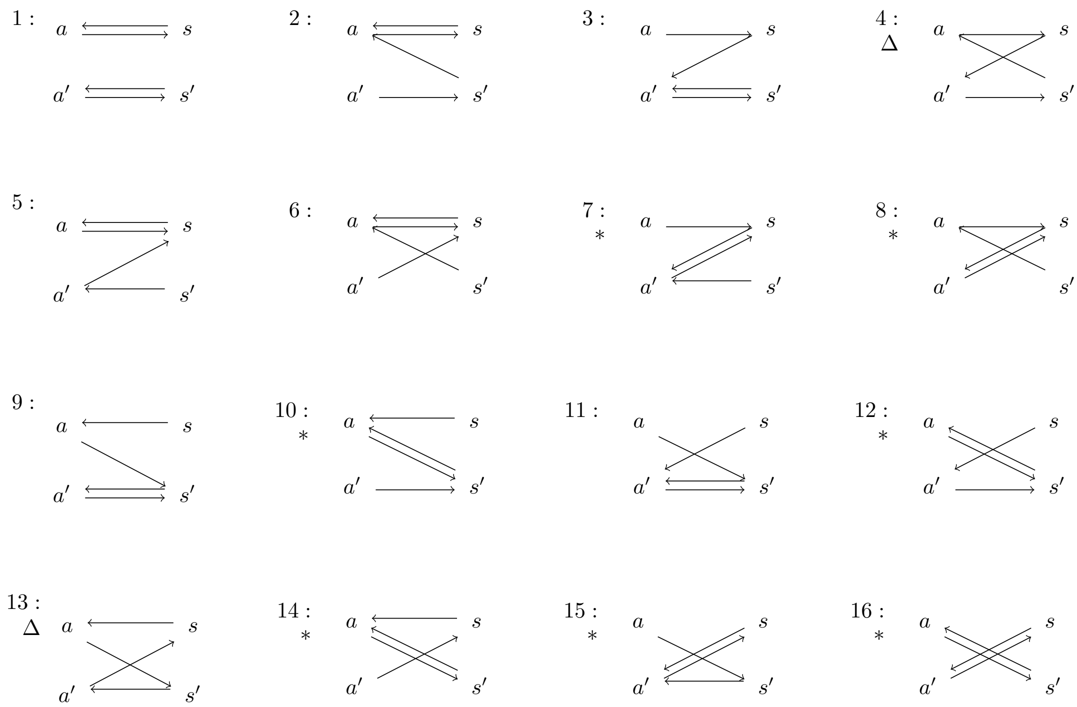
As described in Chapter 5 we set up a random utility function for pupils, such as $U_{as} = f(d_{as}, \boldsymbol{x}_a, \boldsymbol{x}_{as}; \boldsymbol{\theta}) + \epsilon_{as}$, and a random utility function for schools, such as $V_{sa} = g(d_{sa}, \boldsymbol{z}_a, \boldsymbol{z}_{sa}; \boldsymbol{\delta}) + \eta_{sa}$, where $f()$ and $g()$ are linear models of observable characteristics, and $(\epsilon_{as}, \eta_{sa})$ are unobservable random terms distributed according to a known parametric distribution, say $\mathrm{N}(0, 1)$. The usual identification restrictions of a discrete choice model, described in Section 5.1, apply. Specifically, each agent's utility function does not contain terms relating only to characteristics of the agent, as these would only shift each agent's utility for all options, and would therefore not affect preference formation. Denote by $\theta \equiv (\beta, \delta)$ the set of parameters on both sides of the market.

θ indexes a distribution over the $(N_s!^{N_a})(N_a!^{N_s})$ possible preference profiles π for a given set of pupils $\mathcal{A}$ and schools $\mathcal{S}$. There is a many-to-one mapping between each π and $\mathcal{M}^*$, which is defined as the set of all matchings $\mathbf{m}$ such that $\mathbf{m}$ is stable under $(\mathcal{A}, \mathcal{S}, \pi)$. So $\mathcal{M}^*$ can be thought of as a random variable whose distribution is completely defined by θ. The data generating process for a stable matching conditional upon θ can be characterised as follows: first draw a preference profile given θ; then select a matching from the $\mathcal{M}^*$ of that preference profile. This implies the following likelihood function:

$$\begin{aligned}
\Pr(\mathbf{m}|\theta) &= \Pr\big(\pi \in \{\pi' : \mathbf{m} \in \mathcal{M}^*(\pi')\}|\theta\big) \Pr\big(\mathbf{m}|\pi \in \{\pi' : \mathbf{m} \in \mathcal{M}^*(\pi')\}, \theta\big) \\
&= \sum_{\pi \in \mathcal{P}} \Pr(\pi|\theta)\, \mathbb{1}\{\mathbf{m} \in \mathcal{M}^*(\pi)\} \Pr(\mathbf{m}|\mathbf{m} \in \mathcal{M}^*(\pi), \theta)\,.
\end{aligned}$$

For each preference profile, the conditional probability can be written down as a discrete-choice-type product over terms for each agent, but the sum over preference profiles is already computationally difficult in the case of a 3×3 market, where there are 46,656 possible preference profiles. In addition, the *assignment probability* $\Pr(\mathbf{m}|\mathbf{m} \in \mathcal{M}^*(\pi), \theta)$ depends upon the matching mechanism used. In the case where the matching is sampled uniformly from $\mathcal{M}^*$, this probability is simply $1/|\mathcal{M}^*(\pi)|$. If the matching is the result of pupil-proposing DA then the corresponding term is $\mathbb{1}\{\mathbf{m} \text{ is pupil-optimal for } \pi\}$, which is equivalent to the absence of stable improvement cycles in the bi-partite graph representation of the matching.

In general, it is not possible to derive a tractable expression for the assignment mechanism, and even if it were we often do not know with certainty which assignment mechanism was used. In Chapter 8 it was shown that when a subset of a stable matching is used for inference, the stability property remains valid, but any properties associated with the specific assignment mechanism are lost. This means that inference based on the assignment mechanism may be sensitive to assumptions about the boundaries of the market, or small variations in the configuration of the algorithm. Therefore in practice



$*$: Matching is not stable for these preferences. Δ: stable matching is not unique.

Figure 9.2: *All possible preference profiles in a* 2×2 *matching* $\{(a, s), (a', s')\}$. *Arrows represent preferences. There are nine preference profiles for which the matching is stable, and seven for which it is unstable. Of the stable preference profiles, all but no.s 4 and 13 have a unique stable equilibrium, which is the current matching. (Copied from Figure 8.1)*

it is not only extremely difficult, but probably not desirable to treat the assignment mechanism as known.

For this reason we focus on the information that can be derived from the stability mechanism $\Pr(\pi \in \{\pi' : \mathbf{m} \in \mathcal{M}^*(\pi')\}|\theta)$. To save space, denote by $\widetilde{\mathbf{m}}$ the event $[\pi \in \{\pi' : \mathbf{m} \in \mathcal{M}^*(\pi')\}]$.

It makes sense to compare the properties of the stability and full models in the context of the only matching market for which both models are tractable, the market with two schools and two pupils. In this market there are only 16 possible preference profiles.

In order to further simplify the model, each pupil and school's utilities depend only on the distance between the pupil and the school, and unobserved random variation. The utility functions are $U_{as} = \beta d_{as} + \epsilon_{as}$ and $V_{sa} = \delta d_{sa} + \eta_{sa}$ with ϵ_{as}, η_{sa} each distributed independently. Without loss of generality we set the distribution of each as $\mathrm{N}\big(0, \frac{1}{2}\big)$ to simplify the exposition.

Say, we observe a matching $\mathbf{m} = \{(a, s), (a', s')\}$ then the stability likelihood can be written as

$$
\begin{aligned}
\Pr(\widetilde{\mathbf{m}}|\theta) =& \Pr\big(s \succ_a s' \cup a' \succ_{s'} a\big) \times \Pr\big(s' \succ_{a'} s \cup a \succ_s a'\big) \\
=& \{1 - \Phi(\beta(d_{as'} - d_{as}))\, \Phi(\delta(d_{as'} - d_{a's'}))\} \\
& \times \{1 - \Phi(\beta(d_{a's} - d_{a's'}))\, \Phi(\delta(d_{a's} - d_{as}))\}\,.
\end{aligned}
$$

Some tedious algebra confirms that this expression is equivalent to the sum over the probabilities of the nine preference profiles for which the matching is stable.

If we know that the matching was selected using the pupil-proposing deferred acceptance algorithm then we can derive the full likelihood, which turns out to be the sum over the eight preference profiles for which the matching is both stable *and* pupil-optimal. We can simply subtract the probability of the one preference profile (no. 13 in Figure 9.2) that is stable but not pupil-optimal from the expression above:

$$
\begin{aligned}
\Pr(\mathbf{m}|\theta) =& \Pr\big(s \succ_a s' \cup a' \succ_{s'} a\big) \times \Pr\big(s' \succ_{a'} s \cup a \succ_s a'\big) \\
& - \Pr\big(s' \succ_a s \cap s \succ_{a'} s' \cap a \succ_s a' \cap a' \succ_{s'} a\big) \\
=& \{1 - \Phi(\beta(d_{as'} - d_{as}))\, \Phi(\delta(d_{as'} - d_{a's'}))\} \times \{1 - \Phi(\beta(d_{a's} - d_{a's'}))\, \Phi(\delta(d_{a's} - d_{as}))\} \\
& - \Phi(\beta(d_{as'} - d_{as}))\, \Phi(\beta(d_{a's} - d_{a's'}))\, \Phi(\delta(d_{as} - d_{a's}))\, \Phi(\delta(d_{a's'} - d_{as'}))\,.
\end{aligned}
$$

It would be reasonable to ask whether the parameters (β, δ) are identified from either the stability-likelihood model or the full likelihood model. The answer to this question depends only on the signs of the covariate differences $\Delta_i^{(d)} \equiv [d_{ij} - d_{i\,\mathrm{m}(i)}]$ (where j is the agent not matched to i), and is negative for both the full and the stability-only model, as can be seen by plotting the likelihood components one at a time.

Figure 9.3 plots the first and second likelihood components of the stability model, assuming that $\Delta_a^{(d)} = 1$ and $\Delta_{s'}^{(d)} = 1$. The joint MLE for the two parameters with one component is not bounded, and the likelihood function with two components creates a *saddle point*, rather than an optimum. So it is not possible to identify parameters on both sides of a single 2×2 matching. Figure 9.4 shows that four independent likelihood components are sufficient to provide bounds on both parameters, if and only if the covariate differences $\Delta_a^{(d)}$ and $\Delta_{s'}^{(d)}$ take every combination of positive and negative

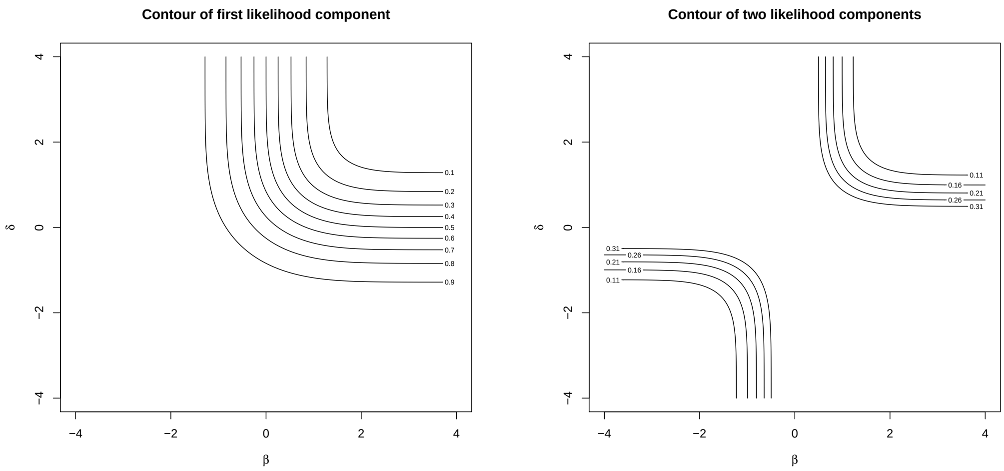


Figure 9.3: *Contours of likelihood functions formed from the first and second no-blocking-pair conditions from a single* 2×2 *matching market.*

values. Therefore the non-separation condition for identifiability in matching markets is only slightly more restrictive than the non-separation condition in normal discrete choice models. The non-separation condition holds as long as the joint distribution of $\Delta_a^{(d)}$ and $\Delta_{s'}^{(d)}$ for all unmatched pairs (a, s') has support in all four quadrants of the covariate-difference space.

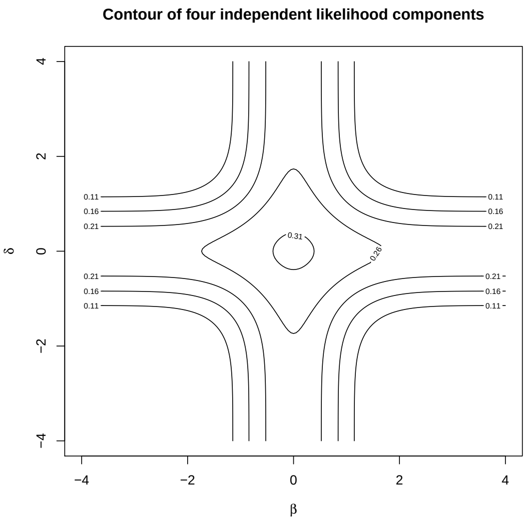


Figure 9.4: *Contours of a likelihood function formed from four independent no-blocking-pair conditions. The four components could be provided by, for example, observing two independent* 2×2 *matching markets.*

This observation sheds light on the non-identification result in Echenique et al. (2013), where the example used was an identical replication of a single 2×2 matching. However, Echenique et al. erroneously attribute the non-identification to multiple equilibria.

Table 9.1: Simulated parameter values from 2×2 markets under two utility specifications.

	Stability Model		Full Model	
	Mean	SD	Mean	SD
True parameters:				
(β_0, δ_0)	-0.500		-0.500	
Specification 1:				
$\beta^{(d)}$	-0.493	0.0543	-0.503	0.0547
$\delta^{(d)}$	-0.499	0.0573	-0.510	0.0576
Specification 2:				
$\beta^{(x)}$	-0.280	0.369	-0.497	0.115
$\delta^{(z)}$	-0.180	0.332	-0.055	0.419

To compare the full and stability likelihoods for the 2×2 case, it is necessary to observe multiple independent markets. Then the full (stability) likelihood is the product over the full (stability) likelihoods for each market:

$$\Pr(\mathbf{m}_1, \mathbf{m}_2, \ldots, \mathbf{m}_M) = \prod_{i=1}^{M} \Pr(\mathbf{m}_i|\theta)\,;$$

and

$$\Pr(\widetilde{\mathbf{m}}_1, \widetilde{\mathbf{m}}_2, \ldots, \widetilde{\mathbf{m}}_M) = \prod_{i=1}^{M} \Pr(\widetilde{\mathbf{m}}_i|\theta)\,.$$

In order to investigate the identification issues relating to certain combinations of covariates, as noted by Sørensen (2007), Logan et al. (2008) and others, we will estimate two different utility specifications. The first specification has utilities on both sides of the market depending only on proximity as described above. The second utility specification has $U_{as} = \beta x_s + \epsilon_{as}$ and $V_{sa} = \delta z_a + \eta_{sa}$, where (ϵ, η) are as before, x_s is a school characteristic, and z_a is a pupil characteristic. The model differs from the previous model in that covariates (x, z) do not interact schools and pupils. For each specification we simulate datasets containing 1,000 independent 2×2 matching markets.

Table 9.1 shows the means and standard deviations after estimating both the full and stability likelihoods 100 times for each specification. For the first specification, the incorporation of the additional information from the matching mechanism makes little difference; both are identified. This is unsurprising, since the only difference between the two likelihoods is that one is a sum over the probabilities of nine preference profiles for

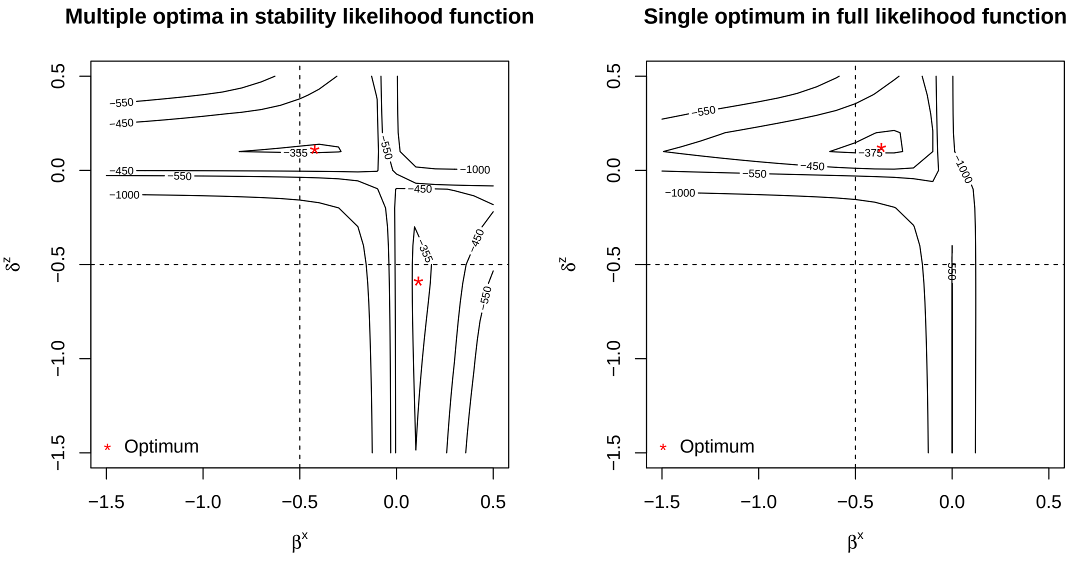


Figure 9.5: *Likelihood contours for a single simulation of the second specification (x, z) with 1,000 markets. Dashed lines show the true parameter values*

each market, and the other is the sum over the probabilities of eight preference profiles. The second specification is not fully identified. This non-identification of uninteracted parameters was as predicted by Sørensen, Agarwal and others. This non-identification affects both the full and stability likelihood inferences. The reported means in Table 9.1 do not reveal that the stability-likelihood function for the second specification has multiple optima, as shown in Figure 9.5.

The simulation of simple 2×2 markets reveals that in this context, the information provided by stability is almost equal to the information provided by knowing the full data generating process. In that case, the bias induced by multiple equilibria is negligible. However, it does not allow us to investigate the behaviour of these models in larger markets, where the sets of preference profiles satisfying stability diverge from the sets satisfying the full data generating process.

This simple example also allows us to confirm that certain specifications of utility functions on either side of the market are not identified, as noted by several authors. However, Agarwal and Diamond (2013) prove that, with a specific restriction on preferences, many-to-one markets provide identification of utility models similar to our second specification. In order to test this result on unrestricted preference specifications it is necessary to look at larger matching markets, and this will require tractable methods for estimation.

9.3 Tractable stability-based models

This section focusses on tractable methods for large markets based on the stability likelihood. The other main studies employing the stability likelihood, as opposed to method of moments estimators, are Logan et al. (2008) (hereafter LHN) and Menzel and Salz (2013). In both cases a Gibbs sampler is used to implicitly sample from the stability likelihood, by simulating draws from the latent utilities and parameters. The Gibbs algorithm proceeds as follows:

<u>The LHN Stability Gibbs Sampler:</u>

1. Initialise $\theta \equiv (\beta, \delta)$. Initialise $\boldsymbol{U}, \boldsymbol{V}|\theta$, s.t. $\mathbf{m}$ is stable.
2. Sample $\boldsymbol{U}|\beta, \boldsymbol{V}$, s.t. $\mathbf{m}$ is stable. This entails respecting inequality conditions on ϵ_{as} induced by conditioning on $\boldsymbol{V}$ (see discussion). In practice ϵ_{as} is sampled from truncated distributions.
3. Update $\beta|\boldsymbol{U}$ using a conjugate OLS update.
4. Sample $\boldsymbol{V}|\delta, \boldsymbol{U}$, s.t. $\mathbf{m}$ is stable as above.
5. Update $\delta|\boldsymbol{V}$.
6. Repeat steps 2 to 5 until convergence.

The sampler exploits the fact that, if utilities (and hence rankings) on one side of the market are known, a subset of preferences – $\mathcal{R}_{\mathbf{m}}$ – on the other side of the market is revealed. Section 8.3 discusses the revealed preferences from a one-sided matching in more detail. Given this subset of preference relations, utilities are sampled from truncated normal distributions to respect inequalities. For example, for a student a, the utility of the matched school, $U_{a\,\mathrm{m}(a)}$, is sampled such that it is greater than the maximum utility of the *revealed-dominated* schools in the previous iteration. Then the utilities of the *revealed-dominated* schools are sampled such that they are less than the current sampled value of $U_{a\,\mathrm{m}(a)}$.

This sampler is intuitive, and quick to implement. A disadvantage of the method is that it requires sampling from the full set of latent utilities on both sides of the market, of dimension $2 \times N_a \times N_s$. In simulations, we found that the sampler could be slow to converge.

To facilitate tractable estimation, we have rewritten the stability likelihood as an explicit function of parameters and *threshold* utilities. Threshold utilities are defined as utility of each agent for their marginal match – in the case of pupils, their allocated school, and in the case of schools, their lowest ranked pupil. The advantage of this approach is that there are only $N_a + N_s$ threshold utilities. The threshold utilities are defined as: $\overline{U}_a = U_{as} : \{s\} = \mathrm{m}(a)$, student a's threshold utility; and, $\overline{V}_s = \min_{a \in \mathrm{m}(s)}\{V_{sa}\}$, the threshold utility of school s.

The joint distribution of $\widetilde{\mathbf{m}}$ and $\overline{\mathbf{V}}$, introduced in Chapter 5, is

$$\Pr(\widetilde{\mathbf{m}}, \mathbf{v}|\theta) = \Pr\big(\widetilde{\mathbf{m}}|\theta, \overline{\mathbf{V}}\big) \prod_{s \in \mathcal{S}} p_{\overline{V}_s}(v_s|\mathbf{m}, \delta)\,,$$

where

$$\Pr\big(\widetilde{\mathbf{m}}|\theta, \overline{\mathbf{V}}\big) = \prod_{a \in \mathcal{A}} \int_{\mathbb{R}} \left\{ \prod_{\{s \notin \mathrm{m}(a)\}} 1 - \Pr(U_{as} > u) \Pr\big(V_{sa} > \overline{V}_s\big) \right\} p_{\overline{U}_a}(u)\ \mathrm{d}u.$$

and

$$\begin{aligned} p_{\overline{V}_s}(v_s|\mathbf{m}, \delta) &= \frac{\partial}{\partial v_s}\left[1 - \prod_{a \in \mathrm{m}(s)} \Pr(V_{sa} > v_s|\delta)\right] \\ &= \left[\prod_{a \in \mathrm{m}(s)} \Pr(V_{sa} > v_s|\delta)\right]\left[\sum_{a \in \mathrm{m}(s)} \frac{p_{V_{sa}}(v_s)}{\Pr(V_{sa} > v_s|\delta)}\right]. \end{aligned}$$

This extended likelihood is a joint distribution over the parameters of interest θ, and the random variables $\overline{\mathbf{V}}$. The random variables $\overline{\mathbf{U}}$ have been integrated out using numerical quadrature. The $\overline{\mathbf{V}}$ are considered nuisance variables. We wish to use the marginal stability likelihood

$$\Pr(\widetilde{\mathbf{m}}|\theta) = \int_{\mathbb{R}^{N_s}} \Pr(\widetilde{\mathbf{m}}, \mathbf{v}|\theta)\, \mathrm{d}\mathbf{v},$$

for inference about the parameters θ.

In order to obtain estimates of the parameters of interest we can either use MCMC methods to integrate out $\overline{\mathbf{V}}$, or approximate the integral analytically. We have found that the first method, MCMC, does not work well without a good starting point and proposal distribution, so that in practice even computational simulation of the integral requires a method for analytical approximation of the integral as a starting point.

The Laplace approximation uses the curvature of the likelihood function at the optimum to approximate the integral. Calculation of the *Laplace log-likelihood* for a given θ entails finding

$$\hat{\mathbf{V}}(\theta) = \underset{\mathbf{v} \in \mathbb{R}^{N_S}}{\arg\max} \{\log \Pr(\widetilde{\mathbf{m}}, \mathbf{v}|\theta)\}$$

and then numerically estimating the matrix of second derivatives (the Hessian matrix) H at $(\theta, \hat{\mathbf{V}}(\theta))$. The Laplace log-likelihood approximation is then

$$\log \Pr_{\text{Lap}}(\widetilde{\mathbf{m}}|\theta) = \log \Pr\Big(\widetilde{\mathbf{m}}, \hat{\mathbf{V}}(\theta)|\theta\Big) - \frac{1}{2} \log \left| -H\left(\theta, \hat{\mathbf{V}}(\theta)\right) \right|.$$

Since calculation of this approximation for a given value of θ requires numerical optimisation and numerically estimating the Hessian, optimising the Laplace log-likelihood involves optimisation-within-optimisation and is therefore computationally demanding. A cruder, but much less computationally-demanding, approximation can be derived by omitting the Hessian term from the Laplace approximation. This is essentially the *h*-likelihood approach to inference (Lee and Nelder, 1996; Lee et al., 2006). This approach treats the random variables $\overline{\mathbf{V}}$ in a similar manner to unknown population parameters, and advocates estimators that simply jointly optimise the joint likelihood for $(\theta, \overline{\mathbf{V}})$ rather than (approximately) integrating out the random variables. The *h*-likelihood approach has attracted controversy (see, for example, the responses to the discussion paper Lee and Nelder (1996), and the theoretical exploration of Meng (2009)), especially in situations where the number of units per group, and hence the amount of information available to estimate random variables, is small and does not increase with sample size. In these cases the bias arising from the approximation is not asymptotically negligible.

However, we argue that in the context of school choice, group size (i.e. the size of each school's intake) is large enough to make this approach a reasonable compromise. The *Maximum h-Likelihood Estimator* is

$$\hat{\theta}^{\text{MhLE}} = \arg\max_{\theta} \left[\max_{\mathbf{v}} \{\log \Pr(\widetilde{\mathbf{m}}, \mathbf{v}|\theta)\} \right].$$

Care must be taken to ensure that the distribution of the latent variables is reasonably unimodal and quadratic, and that they are specified on an appropriate scale, since the location of the mode is not transformation-invariant in general (Meng, 2009). When h-likelihood estimates were compared to estimates from the Laplace approximation, they were almost identical.

We also implemented a Random-walk Metropolis type MCMC sampler. The sampler converged quickly, achieving close-to-optimal acceptance rates (approx. 0.25) with a proposal distribution formed from the estimated inverse negative Hessian $\left[-H\left(\hat{\theta}, \hat{\mathbf{V}}(\hat{\theta})\right)\right]^{-1}$, with optimal scaling $2.38^2/(N_\theta + N_s)$. However, sampling from the true posterior after estimating the approximate posterior in most cases did not produce point or interval estimates that were sufficiently different to justify the additional computational cost. Additionally, the MCMC sampler could not easily be scaled to multiple datasets in combination with the EP algorithm described below.

9.3.1 Identification of parameters

With this tractable stability likelihood model we can investigate the behaviour in many-to-one markets of utility specifications containing complementary pupil and school characteristics. It is no longer possible to compare the stability likelihood to the full likelihood, but we can increase the number of pupils per school to evaluate the performance of the estimators as the sample size increases.

We investigate two specifications of utility functions on both sides of the market as before. The first specification has $U_{as} = \beta^{(d)} d_{as} + \epsilon_{as}$ and $V_{sa} = \delta^{(d)} d_{sa} + \eta_{sa}$, whilst the second specification has $U_{as} = \beta^{(x)} x_s + \epsilon_{as}$ and $V_{sa} = \delta^{(z)} z_a + \eta_{sa}$. As before, d_{as} is the home-school distance, x_s is a school characteristic, and z_a is a pupil characteristic. For each specification we simulate datasets containing a single market with $\{7, 15\}$ schools, and $\{20, 50, 100\}$ pupils per school.

Table 9.2 shows the medians and standard deviations after estimating both the full and stability likelihoods 50 times for each specification. The median has been calculated rather than the mean because estimates on smaller markets sometimes exhibit outliers. Both specifications appear to be identified in many-to-one markets. In small markets the bias is substantial, but there is clear evidence that both the bias and variance are reducing as the market size increases. Estimates of $\beta^{(x)}$ are the most biased and variable, which is to be expected since identification of the parameter depends on variation over a very small number of schools. In the next section we describe the Expectation Propagation method that has been used to allow the sample size to be increased, and especially the number of schools for the identification of school-varying parameters.

Table 9.2: Simulation evidence from the stability likelihood on many-to-one markets. 50 replications.

	Specification 1				Specification 2			
	$\beta^{(d)}$		$\delta^{(d)}$		$\beta^{(x)}$		$\delta^{(z)}$	
	Median	SD	Median	SD	Median	SD	Median	SD
True values	-1.00		-1.00		-1.00		-1.00	
7 schools and								
20	-1.20	(0.401)	-0.75	(0.350)	-1.78	(4.790)	-0.77	(0.208)
50	-1.06	(0.236)	-0.92	(0.233)	-1.45	(4.600)	-0.86	(0.181)
100	-1.05	(0.097)	-0.94	(0.144)	-1.24	(0.505)	-0.92	(0.136)
pupils per sch.								
15 schools and								
20	-1.10	(0.314)	-0.82	(0.242)	-1.72	(1.190)	-0.76	(0.213)
50	-1.05	(0.197)	-0.90	(0.182)	-1.27	(0.494)	-0.87	(0.288)
100	-1.01	(0.087)	-0.97	(0.105)	-1.17	(0.321)	-0.92	(0.306)
pupils per sch.								

The evidence in these simulations and in those in the previous section appears to support the analysis of Agarwal and Diamond (2013), by showing that specifications including uninteracted parameters are identified in many-to-one markets but not one-to-one markets. We have also presented evidence that specifications involving interactions between pupil-side and school-side parameters (such as distance) are identified in both many-to-one and one-to-one markets. Simulation evidence cannot *prove* that models are identified or consistent, as we cannot make the sample size arbitrarily large. The next section discusses informal arguments supporting the hypothesis that stability-based likelihood models have desirable large-sample properties in many-to-one markets.

9.3.2 Bias arising from multiple equilibria

As the analysis of 2×2 markets made clear, the issue of identification of complementary parameters in a two-sided market is not dependent upon the issue of bias arising from the incompleteness of the model. We now discuss the latter issue. Both Bayesian and likelihood-based approaches stress the importance of basing inference upon the full data generating process. Since we are not using the full data generating process in our model, many of the standard guarantees associated with likelihood-based inference, regarding consistency, efficiency and identification, do not automatically apply.

For this reason, those using two-sided models have frequently imposed restrictions designed to ensure that the matching equilibrium is unique and therefore that the model for equilibrium is complete (Sørensen, 2007; Agarwal and Diamond, 2013; Menzel, 2015). These models obtain unique equilibria in finite markets, at the expense of strong model restrictions. The alternative approach that we advocate is to exploit results regarding the limiting behaviour of the set of stable matchings, $\mathcal{M}^*$, in two-sided markets. The large sample properties of $\mathcal{M}^*$ in stable matching markets have been studied by several authors (Knuth, 1976; Roth and Peranson, 1999; Ashlagi et al., 2013). Using simulations, Ashlagi et al. (2013) found that in many realistic market specifications $\mathcal{M}^*$ appears to become small in large markets.

A particularly important case for our purposes is the limiting behaviour of many-to-one markets. Azevedo and Leshno (2016) studied markets in which the school-side remained fixed while the student-side increased. They provided a proof that, for a general model of two-sided many-to-one matching markets, $\mathcal{M}^*$ converges to a singleton as the number of students per school increases. This relationship is borne out by the simulation evidence in Table 9.3. If and only if the pupil-optimal and school-optimal matchings are identical for all agents, $\mathcal{M}^*$ is a singleton. Enumerating the size of $\mathcal{M}^*$, if it is not a singleton, is computationally difficult, but a good proxy is provided by the proportion of agents for whom their pupil-optimal match is not equal to their school-optimal match. Table 9.3 presents this proportion for matching markets of different sizes, showing clearly that the proportion tends towards zero as the number of pupils per school increases.

Since the stability likelihood $\Pr(\mathbf{m} \in \mathcal{M}^*|\theta)$ constitutes an upper bound on the full likelihood, and the probability that a given matching is a unique equilibrium $\Pr(\{\mathbf{m}\} = \mathcal{M}^*|\theta)$ constitutes a lower bound on the full likelihood (cf. Menzel and Salz (2013) and Figure 9.1), it is reasonable to suppose that some kind of "sandwiching" argument could be used to justify the smooth convergence of the stability likelihood to the full likelihood.

Table 9.3: The proportion of pupils for whom the pupil-optimal match is not equal to the school-optimal match. Evidence from 100 simulations of each market.

Pupils per school	5 schools	10 schools	20 schools
$\times 2$	0.0940	0.132	0.132
$\times 10$	0.0374	0.0622	0.0924
$\times 50$	0.0137	0.0218	0.0333
$\times 100$	0.00898	0.0157	0.0303

Although we do not pursue a proof of convergence here, this informal argument along with the simulation evidence is enough to satisfy us that estimators derived from stability-based likelihoods will perform reasonably well given enough data. However, a rigorous proof would probably be more generally useful, since the work of Ashlagi et al. (2013) and others suggests there may be several conditions under which the set of stable matchings converges. An approach based on large sample convergence of $\mathcal{M}^*$ may indeed be useful in empirical work on discrete games more generally.

9.4 The Expectation Propagation algorithm

Although more tractable than alternative methods, the stability h-likelihood does, like other methods, have dimensionality which grows with the number of schools. This means that it becomes prohibitively difficult to optimise the h-likelihood as datasets are combined. In practice we found that estimation became infeasible with more than about two dozen schools. In the previous section we discussed the importance of amassing enough data to overcome several sources of finite-sample bias. As there are 12 datasets (three markets $\times$ four cohorts) containing a total of 153 school-years, a method is needed to combine the estimates without combining the dimensions of the parameter space.

A simple method to combine 12 sets of substantive parameter estimates whilst ignoring nuisance parameter estimates would be by the sequential application of Bayes theorem. Such a method would require assuming that each estimated likelihood function can be summarised by its estimated mode $\hat{\theta}$ and negative Hessian matrix $-\hat{H}$, and is therefore proportional to a multivariate normal posterior distribution $N(\hat{\theta}, (-\hat{H})^{-1})$. The product of two normal distributions $N_1(\mu_1, \Sigma_1)$ and $N_2(\mu_2, \Sigma_2)$ is proportional to a normal distribution with parameters

$$\Sigma_3 = (\Sigma_1^{-1} + \Sigma_2^{-1})^{-1}$$

and

$$\mu_3 = \Sigma_3(\Sigma_1^{-1}\mu_1 + \Sigma_2^{-1}\mu_2).$$

The sequential method proceeds by treating the first estimated posterior as the prior for the estimation of the second, and the second as the prior for the estimation of the third, and so on. At each stage the *Maximum a Posteriori* estimate is used to approximate the posterior. With such a method the dimensionality of the posterior does not grow as the sequence progresses, because at each stage the nuisance variables can be analytically marginalised out by subsetting the estimated mean vector $\hat{\theta}$ and covariance matrix $\hat{\Sigma} = (-\hat{H})^{-1}$. The posterior becomes increasingly regular as the prior is increasingly informative, leading to quicker and more stable estimation. However, this method raises the problem that the order of estimation matters, especially if the individual distributions are far from normal. Only the last iteration benefits from the estimation on all of the other datasets, whereas the estimation of the first dataset does not benefit from any information to stabilise estimation and ensure normality. If the first likelihood function is irregular, the optimisation may become stuck in a local mode, which will have knock-on effects for the estimation on the other datasets, particularly if their likelihood functions are also irregular.

The Expectation Propagation method (Minka, 2001; Gelman et al., 2014) allows all partitions to provide prior information for the regularisation of estimation of all other partitions. The information from all except the k'th dataset is approximated by a *cavity* distribution $g_{-k}(\theta) = g_0(\theta)\prod_{k'\neq k} g_{k'}(\theta)$, which acts as a prior to regularise the estimation of the k'th partition. Here each $g_{k'}(\theta)$ is a multivariate normal distribution that approximates the likelihood contribution of one partition ($g_0(\theta)$ is the prior). This cavity distribution is combined with the k'th likelihood to form the *tilted* distribution $g_{\backslash k}(\theta) = \Pr(\widetilde{\mathbf{m}}_k|\theta)\, g_{-k}(\theta)$. Each of these approximations is summarised by its precision matrix $Q_k = \Sigma_k^{-1}$ and natural parameter $r_k = Q_k\mu_k$. The algorithm proceeds as follows:

Stability Expectation Propagation:

1. Initialise the approximate posterior $g(\theta) = g_0(\theta) \prod_{k=1}^{K} g_k(\theta)$ by setting the initial precision matrices $Q_k = \epsilon\mathbb{I}$ and natural parameters $r_k = 0$. Then $Q = \sum_{k=0}^{K} Q_k$ and $r = \sum_{k=0}^{K} r_k$.

2. For each k in $\{1, 2, \ldots, K\}$:

   (a) Compute the cavity distribution $g_{-k}(\theta)$ by setting $Q_{-k} = Q - Q_k$ and $r_{-k} = r - r_k$.

   (b) Form the tilted distribution $g_, \backslash k(\theta) = \Pr(\widetilde{\mathbf{m}}_k|\theta)\, g_{-k}(\theta)$ and optimise this function to obtain $Q_{\backslash k}$ and $r_{\backslash k}$. This entails approximating the full posterior with $(\hat{\theta}, \hat{\mathbf{V}})$ and $-\hat{H}$, and marginalising over the nuisance variables $\overline{\mathbf{V}}$.[3].

   (c) Divide out the cavity distribution to obtain a new approximation of the k'th likelihood, by setting $Q_k^{\text{new}} = Q_{\backslash k} - Q_{-k}$ and $r_k^{\text{new}} = r_{\backslash k} - r_{-k}$.

   (d) Update $g(\theta)$ with the new k'th approximation.

3. Repeat until convergence.

The main advantages of the EP method are, firstly, that at each iteration the dimension of the parameter space is reduced by analytically marginalising over the nuisance parameters and, secondly, that most of the operations involve simple linear algebra (Gelman et al., 2014). Additionally, the final posterior is represented by two parameters, which is easier to work with than, for example, the output of MCMC methods. The main disadvantage of the method is that convergence is not guaranteed. We have noticed that, for some specifications and datasets, parameter values may oscillate slightly rather than converging completely. This oscillation does not appear to be of a magnitude that would substantively affect estimates, but it does mean that finding a sensible stopping rule is more problematic, and more iterations may be necessary than would otherwise be the case.

[3] To obtain an estimate of the marginal precision matrix it is necessary to invert the negative Hessian, subset the covariance matrix, and then invert again. $\hat{\theta}$ approximates the marginal mean vector.

Chapter 10

Conclusions

This thesis has presented new evidence on the determinants of parental demand for schools, and the relationship between choice, competition and sorting in the English secondary education system. At the same time it has introduced a new structural method for statistical inference on two-sided matching markets, of which secondary school markets are an example. This conclusion, firstly, brings together evidence from previous chapters and discusses the relevance of the substantive findings to current debates around social cohesion, quasi-markets and choice. Secondly, it discusses and evaluates the new statistical method, and points out avenues for further research in this area.

10.1 Sorting, choice and competition

The thesis set out to answer three research questions around sorting, choice and competition. The first question concerned the different weightings parents of different ethnic and socio-economic groups placed on peers, academic performance and other criteria in choosing a school. The second question asked if heterogeneity in parents' preferences for peers and academic performance set up patterns of sorting both in the short run, and cumulatively as preferences feed back into observable school characteristics. The third question asked whether sorting has a muting effect on the amount of choice parents enjoy, and the amount of competition schools face for pupils.

The thesis first set out to confirm that *post-residential* sorting exists in the markets studied. To achieve this, a counterfactual was set up in which all pupils attend their nearest school, and an index of segregation, D was calculated for each market. These counterfactual segregation indices were compared to actual levels of school segregation to disaggregate residential from post-residential segregation. The results were consistent with substantively important levels of post-residential segregation on ethnicity in all markets except Preston, where school choice appears to act to reduce segregation. Although it is clear that residential segregation still explains the majority of sorting in schools, post-residential sorting processes play an important role. This preliminary result was important, because it validates an approach that emphasises the importance of *choices*, rather than *constraints*, in observed patterns of sorting. Post-residential sorting may be a result of choices or constraints but, as we have argued, both choices and constraints can be thought of as stemming from the decision-making of individual families; therefore studying parental preferences is vital.

On these grounds the following two chapters set up and estimated a model of parental preferences for schools based on weighing the relative importance of proximity, peers and academic performance in choosing a secondary school. We expected to find that parents value proximity, and they do. We also tentatively expected to find that parents, to some extent, are conscious of and evaluate the ethnic composition of schools. Finally, we hypothesised, based on evidence from other studies of school choice, that parents would value academic performance as measured by raw test scores, but that parents with lower socio-economic status (SES) would place less weight on academic performance, relative to other factors, than higher-SES parents. Both of these hypotheses were borne out by the estimated models, and we deal with each in turn here.

10.1.1 Tipping dynamics and self-segregation

In relation to peer preferences, we found that both white and Asian parents *usually* preferred to send their children to schools with a higher proportion of their own ethnicity on the rolls. We characterised such mutual and symmetric preferences for one's own group as *horizontal* peer preferences, in contrast to *vertical* peer preferences where the preferences of one group determine sorting. Further to this, when we allowed peer preferences to be non-linear, we found that preferences were consistent with an aversion to sending one's child to a school overwhelmingly dominated by one's own ethnic group. This pattern of revealed preferences was robust across markets, and preference weightings for white and Asian families were of a similar magnitude, although those for Asian families were slightly more non-linear. We also expected to find that the specifics of each place made a difference, especially for Oldham whose recent history suggests much more consciousness of ethnic difference than the other two towns. However, what we found was, if anything, more consistent with an underlying geographic homogeneity to ethnic peer preferences, as if we had tapped into "deep" parameters of ethnic homophily. However, the similarity of estimates does not, on its own, provide firm evidence to support this assertion. It is possible that the three towns studied were not sufficiently dissimilar to reveal geographic diversity in ethnic peer preferences.

Finding that peer preferences tend to be reversed as one's own ethnic group approaches dominance would seem to be a positive result. It suggests that parents do not want extreme ethnic segregation – that they would rather their children went to an ethnically-mixed school as long as their ethnic group constitutes the majority. This kind of ethnic preference sounds moderate, at least more moderate than an absolute aversion to ethnic mixing. However, models of tipping in sorting dynamics suggest that in the long run, even moderate preferences for ethnic in-groups can produce complete segregation.

In Chapter 7 we populated the parameters of sorting models for Oldham with our estimated random utility parameters. Oldham is of special interest both because of its recent history of ethnic unrest, and because of the structural changes in the period 2010–2012 that merged several segregated schools into fewer, larger schools with ethnically-mixed populations, thereby creating a kind of natural experiment in mixing and ethnic preferences. Waterhead Academy, in particular, received national media attention (eg. Nye, 2011) as part of a bold strategy to replace an almost-completely white school (Counthill School) and an almost-completely Asian school (Breeze Hill School) with an integrated academy. The school was used as a case study for theories of inter-group attitudes by Ramiah et al. (2015), who found that outside lessons pupils in the new school have tended to avoid contact with members of the other ethnic group. However, some (unpublished) evidence has been reported, suggesting that through daily contact, pupils' attitudes towards the other group were gradually improving (Nye, 2011).

This is a positive result of a natural experiment in integration. Yet the results in Chapter 7 suggest, unfortunately, that the integration of the school population may not be sustainable given current parental preferences; both the equilibrium model and the computational model predicted increasing (re-)segregation at Waterhead over time. Whereas the equilibrium model predicted that the school's population would become almost completely segregated over time, the computational model implied that the proportions would become more uneven over many years, but would stop short of complete segregation.

The problem, in Oldham and in the other towns we have studied, is that the preferences of each ethnic group for ethnic mix are *not realistic* with regards to the local context. Each group of parents would like a *small* proportion of the other ethnic group in their child's school; a proportion that does not reflect the actual proportion of the other group that is likely to wish to attend their local school given local demographics. This may be explained partly by people's tendency to over-estimate the proportion of ethnic groups other than their own in a given setting (Alba et al., 2005). It is this mismatch between feasible intakes and desired intakes that sets up the sorting dynamics that result in additional post-residential sorting. In this respect, our emphasis on parental choices actually seems more pessimistic than a focus on constraints, since at least where institutional constraints, such as unfair admissions practices, are present they can be removed. Admissions policies, house prices, school capacity and other constraints undoubtedly do play a role in sorting. However, where preferences for sorting are mutual, rather than one-sided, constraints are not necessary for sorting to arise.

This begs the question of whether the only way to achieve ethnically-integrated schools is to give up on choice and impose mandatory desegregation in a manner similar to the US in the 1960's and 1970's. This need not involve busing children to distant schools; it would be sufficient to build schools near the boundaries of currently segregated areas, and to "gerrymander" the catchment areas of each school to achieve ethnic balance. However, the experience of the US suggests that, apart from being undesirable from a parental sovereignty point of view, such desegregation may not be sustainable as in the medium to long run, parents would just subsume school choice (and any ethnic peer preferences) within residential choice and move to new areas. So-called "white flight" is a well-documented phenomenon as a result of desegregation in the US (Clotfelter et al., 2006).

In the light of the seeming intractability of self-segregation, in the US there has been a recent interest in policies to promote diversity within a quasi-market system. For example the US Department of Justice has published guidance establishing that schools are permitted to consider ethnicity in admissions criteria to achieve a more diverse student body (Erdil and Kumano, 2014). However, as well as moral and legal issues, the idea of two-sided markets incorporating "ethnic quotas" creates difficult theoretical problems requiring innovations in the mechanism design literature (Erdil and Kumano, 2014; Echenique and Yenmez, 2015).

It is unlikely that solutions to ethnic segregation involving the removal of choice, or even ethnic quotas on admissions, would be socially or politically acceptable in the UK. Nevertheless, the EU Referendum and other events in 2016 have brought issues around social cohesion, integration and multiculturalism to the fore again, as they were in 2001. Perhaps the only policy to tackle (self-)segregation in English schools that would be effective in the long run, would be to change parents' perceptions about the benefits of attending an ethnically-diverse school. Ted Cantle, whose report in 2001 responded to the race riots in Oldham and elsewhere, proposes what he calls "salesmanship" (Asthana and Parveen, 2016) on the part of policy-makers, to promote the benefits of integration:

> Politicians and policymakers need to encourage white British residents to remain in diverse areas; to choose, rather than avoid, diverse areas when they do relocate, encouraging similar choices with respect to placing pupils in diverse schools. (Cantle and Kaufmann, 2016)

However, changing people's perceptions is not a job for which politicians, policy-makers or civil servants have an abundance of tools. Ironically, it may be that education, the site of the problem, is also the only way in which the problem can be solved.

10.1.2 Socio-economic status and preferences for academic performance

The model of parental preferences constructed in the thesis allowed different socio-economic groups to have different preferences for academic performance, as measured by the percentage of pupils achieving five or more A*–C grades at GCSE. The pattern of preferences for test scores increasing in importance with SES was robust across all samples; at each quintile of IDACI (a proxy for SES) parent's relative weighting of test scores against other criteria decreased. These results were consistent with previous work (eg. Hastings et al., 2009; Allen et al., 2014; Burgess et al., 2015). However this feature of parental preferences, even more than ethnic peer preferences, is difficult to explain in terms of rational decision-making or well-known psychological theories. Families clearly have incentives to seek educational "quality", and measures of test scores contain information, albeit imperfect, about both the quality of teaching, and likely peer effects from other pupils. It might therefore be expected that families of all socio-economic statuses would take them into account.

There are several possible explanations for the regularity. First, it may be that higher-SES parents use various sources of information more effectively than lower-SES parents. Second, as Breen and Goldthorpe (1996) argue, parents' different commitment to educational attainment may reflect parents' different reference points, and hence different evaluations the value of a given level of educational attainment relative to the perceived risks in striving for it. Another possible explanation is that lower-SES parents believe that their children are less likely to gain admission to certain schools, and *post-rationalise* perceived constraints. It is also possible that lower-SES parents are less likely to consider test scores a good proxy for the quality of education that their child will receive. Depending on the (poorly understood) effects of peers and educational production, it may be rational for lower-SES parents to put less faith in test scores as predictors of a good quality education. Indeed, it may be the case that higher-SES parents put too much faith in test scores, given that they tend to be poor predictors of their children's satisfaction at school (Gibbons and Silva, 2010).

Finally, it may be what is actually being measured is families preferences for socio-economic peer groups, which are highly correlated with test scores. The models estimated in this thesis did not account for the possibility of peer preferences for SES. This exclusion restriction on families' utility functions was unavoidable as there was not enough information in the samples to robustly estimate both types of preference. However, with better data it should be possible to allow for the existence of socio-economic peer preferences. The more hierarchical, and arguably the more obvious, model would have higher-SES parents seeking higher-SES peer groups, but lower-SES parents indifferent.

This may interact with a more-or-less homogenous preference for test scores to produce the observed regularity. However, it is also possible that both higher and lower-SES parents seek in-group peers for their children. This possibility would produce *mutual* self-segregation similar to that discussed in relation to ethnicity.

Even without peer preferences, the endogeneity of test scores suggests the potential for a sorting dynamic, whereby the interaction of SES with preferences for academic performance would result in cumulative segregation along socio-economic lines over several years. This is because GCSE test scores depend on the intake (of five years previous) as an input, and SES is strongly correlated with ability. However, incorporating such a process would require a model for educational production – i.e. an economic model that links inputs, such as pupils' prior attainment, finances, and teaching, to outputs such as test scores. There is a long tradition of modelling the education production function in terms of so-called *value-added* models, an early and influential example of which was The Coleman Report (Coleman, 1966; Hanushek, 2008). Extending the analysis in this way would make possible a more complete investigation of sorting processes in education.

10.1.3 Choice and competition

Segregation in schools is intrinsically important, because of the influence of schools on both later life outcomes and social cohesion; however, as we have made clear, there are also instrumental reasons for studying sorting in schools, relating to the operation of education (quasi-)markets. For quasi-markets to work, parents must be able to measure educational quality, and they must value it. As we have discussed, neither of these conditions holds uniformly for all parents at all times.

Furthermore, in addition to valuing educational quality, parents must also enjoy a good choice of *attainable* and *comparable* alternatives. This choice is compromised in highly segregated markets, as school demographics dominate decision making. As articulated by Rebecca Allen and Simon Burgess, "competitive markets work best where the social mix at schools is reasonably similar"(Allen and Burgess, 2010, p10).

In Chapter 7 we showed that both sorting by ability and ethnicity attenuates the potential for demand to flow from one school to another in response to a change in relative quality. We argued that this potential flow of demand constitutes the competitive pressure that is supposed to drive improvements in school conduct and performance; therefore sorting attenuates competition. We found that schools with large proportions of minority ethnic students were most affected by reductions in competition due to ethnic segregation. Likewise, schools with poor test scores experienced the largest effects of re-balancing test scores on their potential flow of demand. The latter result is consistent with a scenario in which schools with disadvantaged intakes have muted incentives to put effort into improving academic performance, because it has a negligible effect on demand.

Schools with large minority ethnic populations also suffer from muted competition and muted demand. However, in the case of some of these schools, notably the Muslim schools in Preston and Blackburn, the muted competitive incentives for academic performance do not seem to have hindered the schools from achieving very good results. In the case of these schools at least, competitive pressure does not appear to be necessary to drive academic success.

The model of sorting and competition based upon pairwise cross-derivatives of demand was inspired by a structural model proposed by Bayer and McMillan (2010). However, the model of Bayer and McMillan had two parts: a model relating aggregate demand to competitive pressures faced by schools; and a model for schools' conduct in the face of competition. The latter model is a variant of the education production model discussed in the previous section, with an additional term for a school's response to local competition. Bayer and McMillan estimated both parts of the model, and argued that schools *do* respond to competitive pressures, but that the competitive pressures themselves were not sufficiently strong.

This thesis has not modelled the school side of the market (educational production) so the conclusions we can offer with regards to sorting and competition are incomplete. A priority for further work is to model the education production function using value-added models, and to combine these with discrete choice models to obtain a structural model of both parental decision-making and school conduct. Without the latter we cannot properly analyse the long-run influence of sorting upon competition, nor of competition upon sorting.

10.2 Two-sided structural models

A core motivation for the thesis has been the assertion that, in studying sorting and competition in education systems, it is vital to understand family preferences and decision-making with regards to school choice. Faced with a lack of data on choices, and an abundance of data on allocations, we sought a way to reveal information about the preferences underlying allocations. In order to achieve this, we investigated the branch of game theory dealing with two-sided matchings, known as *stable matching theory*, and the current state of the art regarding structural estimation of these models. We found that current structural models were either too restrictive, requiring strong restrictions on model form that could not be met in our context, or prohibitively computationally demanding. This motivated a re-writing of the incomplete likelihood model for stability, leading to an explicit functional form for the likelihood and more flexible options for estimation.

Apart from the immediate benefit of being able to implement the model to answer our research questions, the investigation led to informal insights about the properties of this class of models, which it is hoped will aid the intuitive understanding of the field for applied researchers. Using simulation evidence it was conjectured that two-sided models possess good large-sample properties for bias and identification in one-to-many matching markets (such as school markets). Such arguments, alongside the large sample sizes we have been able to achieve by combining datasets, lead to confidence in the validity of using two-sided models in the context of the thesis.

10.2.1 Assumptions impacting upon the validity of results

However, these arguments assume that the model is complete and perfectly specified. In developing the model, Chapter 5 described a number of simplifying assumptions that had to be made in the specification of both pupil and school utility functions. The most important assumptions are discussed here.

The first assumption, which motivated the entire empirical approach, was that the observed allocations constitute a *stable matching*. This assumption was justified on the grounds that each local authority's admissions system uses stable matching algorithms to compute the allocation. However, it is possible that a small number of pupils admitted during the year – i.e. outside the normal admissions process – might create blocking pairs, as a school might not usually prefer those pupils to other pupils the school has rejected. There has been little work on the sensitivity of two-sided models to small deviations from stability; this should be a priority for further work on empirical matchings. It is also assumed that parents express their true preferences on the application form. Although expressing one's true preferences is a weakly optimal strategy for stable matching markets, Chapter 8 discusses possible conditions under which this assumption might fail.

Of the two sides of the market, the school side was most limited in terms of available explanatory variables. We lacked information about siblings, religious observance, special educational needs, looked after children, feeder schools, and catchment areas, all of which have some bearing on probability of admission to a school. The sparsity of the school-side model risks introducing omitted-variables bias into estimates of preference parameters on the pupil side. Another direction for future research on empirical matching models is the extent to which each side of the model is sensitive to omission of variables on the other side of the model.

It is also not known how sensitive the model is to the capacity of each school. In the absence of data on capacities we have assumed that each school's capacity is equal to its intake, implying that all schools are full. In future work it would be advisable to obtain data about school capacities, and generalise the model to allow for schools with empty places.

On the pupil side of the model the dataset was sufficiently detailed to allow us to investigate rich interactions between pupil and school characteristics. However, there were two restrictions in the pupil utility function that may have limited the ability of the model to predict aggregate demand. First, there were no fixed effects for schools in the model. Since these terms ensure that predicted aggregate demand is equal to observed aggregate demand in the sample, their omission might be expected to affect estimates of aggregate demand.

The other main restriction of pupil utility functions is the assumption that the unobserved portion of utility ϵ_{as} is uncorrelated across alternatives for each pupil. Whilst it is not uncommon for discrete choice studies to make this assumption, it is well-known that assuming uncorrelated residuals limits the ability of the model to capture substitution patterns in aggregate demand. However, it is not clear whether a model with correlated residuals would even be identifiable in the context of a two-sided model. In Chapter 5 it is argued that the abundance of terms interacting pupil and school characteristics creates rich substitution patterns in aggregate demand, so that the omission of substitution-on-unobservables is less serious a concern.

10.2.2 Other applications of two-sided matching models

The two-sided matching paradigm has wide applicability in social science and Economics, and there are several possible areas in which two-sided empirical models may be useful. Two-sided empirical models have already been used to study topics such as the matching of teachers to schools (Boyd et al., 2013), or industrial organisation (Sørensen, 2007).

Although we have argued that the stability likelihood method works best in many-to-one markets where there are a large number of individuals for each institution, it is possible that further advances could be made in applications with one-to-one markets. This would enable the model to be used in diverse applications such as job matching, University admissions, the matching of directors to boards, or even the market for sportspeople in team sports.

It is even possible that stable matching models could be applied to one-sided matching markets. Analogous theories of stability and theoretical results exist for one-sided matching markets, in which individuals match with others of the same type (the so-called *Stable Roommates Problem*). If the empirical models we have investigated could be adapted, this may potentially provide a new avenue for the study of social networks. Although statistical models for the probability of forming an edge between two people already exist in social networks research, these models currently do not allow the participants in the match to have separate utility functions for the match; therefore they are not structural. However, there would be much work to be done to ascertain the identification and computational feasibility of such models.

10.3 Further Work

Although the thesis has answered its own research questions, it has become clear that there is ample scope to broaden the investigation of the role of parental decision making in education markets. We have found evidence of ethnic peer preferences, and the self-reinforcing sorting dynamics induced by these preferences. Likewise, we have confirmed the regularity studied by other authors, that preferences for academic performance are strongly moderated by socio-economic status. We have also demonstrated how preferences and sorting create the conditions, or fail to create the conditions, under which flows of demand between schools incentivise competition. However, the model for parental preferences was, of necessity, relatively simple, because of the computational limitations of two-sided modelling. Although there are many ways in which the two-sided structural modelling of school markets could be extended and developed, ultimately the investigation into parental preferences would benefit from new sources of data that more straightforwardly reveal parental preferences.

Such datasets exist in the US, in Sweden, and now also in England. The English Department for Education (DfE) has begun to collect rankings of primary and secondary schools submitted by parents as part of the admissions system, although it has not yet clarified the terms of access. The availability of ranked preferences data, whether from the US, from Sweden, or from England allows the modelling of parental preferences at a much larger scale, and also the specification of much richer, more detailed models.

Apart from extending and developing models of parental choice, it is also imperative that these models be augmented by combining them with models of school conduct in the production of educational outputs. These *value-added* models would complete a circuit by describing how schools react to competitive pressures, and how pupil intakes translate into observable outputs, which then influence future cohorts' decisions. This would allow the investigation of questions such as: to what extent do schools improve performance in response to competition? Do results improve for some pupils more than others in response to competition? Do heterogenous preferences for academic performance induce cumulative processes of sorting in schools? How do *these* sorting processes interact with competition and school conduct?

A final avenue for future research would be to investigate combining models of *post-residential* school choice with models of *residential* choice, incorporating indirect school choice considerations into the latter. In this thesis we have treated residential location as fixed, whereas in fact residential location is endogenous with respect to the availability of good schools. Combining these two choice processes might therefore be expected to produce new insights into the interaction of residential and post-residential sorting. Such a model might be implemented as follows: first, estimate a model of school choice with fixed residential location; then, calculate the *expected value of the choice set* at each residential location (discussed in Chapter 7); finally, estimate a model of residential choice, incorporating the expected value of the choice set at each location as a decision criterion.

10.4 Concluding Remarks

The focus upon three medium-sized towns in the North of England may seem small in scope, but there is no doubt that the themes dealt with in this thesis are anything but parochial. Neither do they appear to be transitory. Fifteen years after the Oldham riots, and fifty years after the Coleman Report into segregation in America's schools, social cohesion continues to preoccupy researchers, policy-makers, and educators. Likewise, the reform of England's schooling system around market-like principles mirrors that of other countries. That, too, continues a trend that has preoccupied researchers, policy-makers and educators for at least the last thirty years.

The thesis has treated these two topics as natural bedfellows, and has attempted to show that they are interconnected; sorting and segregation affect markets, and markets affect sorting. Although few would argue that one of the key roles of schools is to socialise children as citizens of a diverse yet cohesive society, this aspect of schooling receives perhaps less sustained focus than it deserves. In contrast to academic achievement, where national league tables and international comparative assessments such as PISA and TIMSS put schools and governments under pressure to perform, it is very hard to hold schools, local authorities and governments to account for failing to foster social cohesion. So, as always, schools and governments focus on what is measured. It is hoped that, by showing that sorting is a barrier to effective school choice and competition, this thesis has made a convincing argument for redirecting some of that focus towards tackling sorting and segregation in schools.

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