# Quench-Induced Floquet Topological $p$-Wave Superfluids

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Ultracold atomic gases in two dimensions tuned close to a $p$-wave Feshbach resonance were expected to exhibit topological superfluidity, but these were found to be experimentally unstable. We show that one can induce a topological Floquet superfluid if weakly interacting atoms are brought suddenly close ("quenched") to such a resonance, in the time before the instability kicks in. The resulting superfluid possesses Majorana edge modes, yet differs from a conventional Floquet system as it is not driven externally. Instead, the periodic modulation is self-generated by the dynamics.

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An intense search is underway to identify experimental realizations of topological superconductors, exotic quantum states of matter that carry robust energy currents along their boundaries. Topological superconductors can host Majorana fermions, fractional particles whose discovery could enable fault-tolerant quantum computation.

A system of identical fermionic atoms confined to two dimensions and interacting attractively through a $p$-wave Feshbach resonance was predicted to form a $p_x + ip_y$ superfluid [1]. Here, $p_x + ip_y$ refers to a particular symmetry of the superfluid order parameter; phases of other symmetries are also possible, but are not as energetically favorable [2–4]. The excitation spectrum [5] of such a $p_x + ip_y$ state is fully gapped, as long as the chemical potential $\mu$ is not zero. However, if the sample possesses a boundary and if $\mu > 0$, then gapless excitations appear at the superfluid edge [5,6], propagating in a particular direction. The edge excitations can be thought of as a one-dimensional band of Majorana fermions. When the $p_x + ip_y$ superfluid is deformed by a perturbation that does not close the bulk energy gap, the gapless boundary excitations remain and retain their properties. These are said to be "topologically protected," and the phase of the superfluid with $\mu > 0$ is a two-dimensional topological superconductor. This picture applies to the weak coupling BCS regime; the strong pairing Bose-Einstein condensate (BEC) phase [7,8] has $\mu < 0$ and is topologically trivial. These are separated by a quantum critical point at $\mu = 0$ [3,5].

Several attempts were made to create the $p$-wave superfluid experimentally in a gas of fermionic $^{40}$K or $^{6}$Li atoms. Unfortunately these gases were found to be unstable due to losses involving three-body processes [9–11], with the lifetime $t_3$ ranging from a few milliseconds in $^{40}$K [12] to about 20 ms in $^{6}$Li [13,14] at a particle density corresponding to a Fermi energy of about 10 kHz. This instability prevents the gas from reaching its ground state; instead, it decays with atoms simply leaving the trap where the gas is held. Interestingly, a very *weakly interacting* $p$-wave gas is predicted to be significantly more stable [10], and yet such a gas would also have a very small gap, potentially preventing direct observation of its topological properties.

In this Letter, we show that one can induce a topological Floquet superfluid [15,16] if weakly interacting atoms are brought suddenly close to a Feshbach resonance, in the time before the instability kicks in. We build off of our recent work [17], in which we determined the exact asymptotic dynamics of a BCS $p$-wave superfluid following a quantum quench. Specifically, we propose to start with a weakly interacting gas of $^{40}$K or $^{6}$Li and then suddenly tune the interaction strength to the desired value by means of a Feshbach resonance. The gas would evolve in an out-of-equilibrium fashion from its initial state. Our results determine the evolution over the time scale before the instability destroys the gas. Note that the ratio of the lifetime $t_3$ to the inverse Fermi energy $t_F \equiv 2m/p_F^2$ can be as high as 200, which gives plenty of room for the gas to evolve and reach a quasistationary state before decaying, as will be elaborated below. The types of superfluid states that we describe have topologically trivial $s$-wave analogs [18–25]. An exciting recent development is the observation of nonequilibrium order parameter dynamics in superconducting thin films [26].

Depending on the initial state and the strength of the quench, the resulting out-of-equilibrium superfluid may find itself in one of three regimes [17]: a steady state with a vanishing order parameter $\Delta(t) \to 0$ as $t \to \infty$ (region I), a state with $\Delta(t) \to \Delta_\infty$, a nonzero constant (region II), or a quasisteady state with an oscillating $\Delta(t)$ (region III). The phase diagram of all possible quenches of the superfluid is shown in Fig. 1.

 

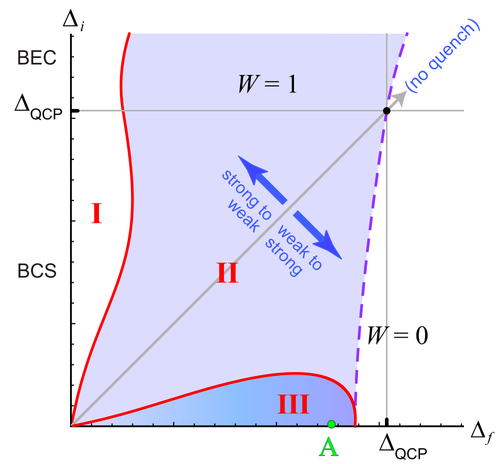


FIG. 1 (color online). Phase diagram showing the three regimes (I–III) of nonequilibrium superfluidity reached after a quantum quench in a $p$-wave gas [17]. Each point in this phase diagram represents a particular quench, wherein one takes an initial state with order parameter amplitude $\Delta_i$ and subsequently ramps the strength of attractive atom-atom interactions to weaker or stronger pairing. The initial state is specified via the vertical axis. The horizontal axis measures $\Delta_f$, which is the amplitude one would find in the *ground state* of the postquench Hamiltonian. The diagonal line $\Delta_i = \Delta_f$ is the case of no quench; $\Delta_{QCP}$ locates the BCS-BEC ground-state transition (see the Supplemental Material [33]). Each off-diagonal point to the left (right) of this line denotes a particular quench from stronger-to-weaker (weaker-to-stronger) pairing. The regions labeled I, II, III denote three different regimes of nonequilibrium superfluid dynamics. For a strong-to-weak quench in I, the order parameter $\Delta(t)$ decays to zero. A quench in II leads to a nonzero steady-state order parameter amplitude. A weak-to-strong quench in III induces persistent oscillations in $|\Delta(t)|$. $W$ denotes the winding number described in the text. Quenches in II with $W = 1$ and in III produce topological states. Point A specifies a quench from very weak initial pairing that produces a Floquet topological state, which could be accessible experimentally.

To realize a topological superfluid in an ultracold gas, the most relevant quenches are those in region III. An initial state with weak pairing is prepared far from the Feshbach resonance (i.e., $\Delta_i$ in Fig. 1 is close to zero), where three-body losses can be neglected [10]. Then the coupling in the Hamiltonian is quenched close to the resonance, and the system evolves coherently. Here, we show that region III is topologically nontrivial, and the oscillating order parameter induces Majorana edge modes. Region III, therefore, realizes a Floquet topological superfluid, yet this differs from a conventional Floquet system [15,16,27,28] as it is not driven externally. Instead, the periodic modulation is self-generated by the dynamics.

We briefly recount the setup of the problem from Ref. [17]. Neglecting the terms responsible for the losses, the gas can be described by the Hamiltonian [1]

$$\hat{H} = \sum_{\mathbf{p}} \frac{p^2}{2m} \hat{a}^\dagger_{\mathbf{p}} \hat{a}_{\mathbf{p}} - \frac{\lambda}{V} \sum_{\mathbf{p},\mathbf{q},\mathbf{k}} \mathbf{q}\cdot\mathbf{k}\, \hat{a}^\dagger_{(\mathbf{p}/2)+\mathbf{q}} \hat{a}^\dagger_{(\mathbf{p}/2)-\mathbf{q}} \hat{a}_{(\mathbf{p}/2)-\mathbf{k}} \hat{a}_{(\mathbf{p}/2)+\mathbf{k}}. \quad (1)$$

Here, $\hat{a}^\dagger_{\mathbf{p}}$ and $\hat{a}_{\mathbf{p}}$ create and annihilate fermions of mass $m$ with momentum $\mathbf{p}$, $\lambda > 0$ denotes their interaction strength, and $V$ is the volume of the system. In the following, we imagine fixing the coupling strength to some initial value $\lambda = \lambda_i$ and preparing the system of atoms in the corresponding ground state. Then we suddenly change (quench) the coupling to a different value $\lambda_f$. We then evaluate how the state of the fermions evolves in time after this quench.

We compute the dynamics of Eq. (1) within self-consistent BCS mean field theory, as governed by the Hamiltonian

$$\hat{H}_{\rm eff} = \sum_{\mathbf{p}} \frac{p^2}{2m} \hat{a}^\dagger_{\mathbf{p}} \hat{a}_{\mathbf{p}} + \left[\Delta(t) \sum_{\mathbf{p}}{}' p \hat{a}^\dagger_{\mathbf{p}} \hat{a}^\dagger_{-\mathbf{p}} + \text{H.c.}\right]. \quad (2)$$

Here, the symbol $\sum'$ signifies that the summation is restricted to $\mathbf{p}$ values that satisfy $p_x > 0$, and $\Delta(t)$ is the amplitude of the gap function, defined as

$$\Delta(t) = -\frac{2\lambda}{V} \sum_{\mathbf{p}}{}' p \langle \hat{a}_{-\mathbf{p}} \hat{a}_{\mathbf{p}} \rangle. \quad (3)$$

The time-dependent state of the fermions is of the BCS form [18]

$$|\Omega(t)\rangle = \prod_{\mathbf{p}}{}' [u_p(t) + v_p(t) \hat{a}^\dagger_{\mathbf{p}} \hat{a}^\dagger_{-\mathbf{p}}] |0\rangle, \quad (4)$$

where $|0\rangle$ is the vacuum, and Eqs. (2) and (3) reduce to nonlinear differential equations satisfied by $u_p(t)$ and $v_p(t)$. We solve these equations exactly, exploiting the integrability of the equations of motion [21–24,29]. The solution employs a Lax spectral method. The analysis closely parallels work done for the corresponding $s$-wave problem in three-dimensional space [18–25]. This was carried out in Ref. [17], and the results are shown in Fig. 1.

The mean field approach differs from Eq. (1) in three ways. First, the interaction terms in Eq. (1) with $\mathbf{p} \neq 0$ have been removed. This is a standard approximation in the theory of superconductivity: the only terms retained in the interaction are those responsible for the pairing of the fermions into Cooper pairs or strongly bound molecules that then Bose condense. Since our goal is to predict dynamics from a given initial state, our results will hold over a time interval in which the effects of neglected terms remain small. This is the minimum of $t_3$ and $t_{\rm pb}$, where $t_{\rm pb}$ is the pair-breaking lifetime induced by $\mathbf{p} \neq 0$ terms [18,30].

Second, the mean field approach neglects quantum fluctuations in $\Delta(t)$. Without pair-breaking terms, this is exact in the thermodynamic limit. It is well known [31,32] that fluctuations induce only finite-size corrections. The reason is that $\Delta(t)$ is a *global*, not merely a local mean field. It becomes macroscopic and classical if the number of fermions is sufficiently large and the system exhibits superconducting order (irrespective of equilibrium). Finally, we assume an initial state with $p_x + ip_y$ symmetry. In mean field theory this "projects out" the $p_x - ip_y$ channel so that it does not participate in the dynamics. The change of variables $\hat{a}_{-\mathbf{k}}\hat{a}_{\mathbf{k}} \to e^{i\phi_{\mathbf{k}}}\hat{a}_{-\mathbf{k}}\hat{a}_{\mathbf{k}}$ leads to Eqs. (2)–(4) [17]; $\phi_{\mathbf{k}}$ is the polar angle.

Let us now examine region III, of particular interest here. In this case, the order parameter asymptotes to

$$\Delta(t) = \Delta_\infty(t)e^{-2i\mu_\infty t}, \tag{5}$$

where $\Delta_\infty(t)$ is a complex-valued periodic function of time with some period $T$, and $\mu_\infty$ is a real constant. These are completely determined by the particular quench specified by $\{\Delta_i, \Delta_f\}$ [17] (see the Supplemental Material [33]). In general, $T$ and $\pi/\mu_\infty$ are incommensurate periods. By absorbing $\mu_\infty$ into the phase of the operators $\hat{a}_{\mathbf{p}}$ and $\hat{a}^\dagger_{\mathbf{p}}$, we can map our effective Hamiltonian in Eq. (2) to a superconductor with an oscillatory complex-valued order parameter $\Delta_\infty(t)$ and chemical potential $\mu_\infty$. A useful quantity to characterize such a superconductor is its retarded Green's function $\mathcal{G}(t,t')$, defined as the solution of the matrix Bogoliubov–de Gennes (BdG) equation (see the Supplemental Material [33])

$$i\frac{\partial\mathcal{G}}{\partial t} - \mathcal{H}\mathcal{G} = \delta(t-t'),$$

$$\mathcal{H} = \begin{pmatrix} \frac{p^2}{2m} - \mu_\infty & \Delta_\infty p e^{i\phi_{\mathbf{p}}} \\ \Delta^*_\infty p e^{-i\phi_{\mathbf{p}}} & -\frac{p^2}{2m} + \mu_\infty \end{pmatrix}, \tag{6}$$

where $\phi_{\mathbf{p}}$ is the angle $\mathbf{p}$ forms with the positive $x$ direction. This equation is identical to that for a driven superconductor with a gap function imposed to be a given function of time. We must still keep in mind that we are describing a strongly out-of-equilibrium state, with $\Delta_\infty$ determined by the contributions of many fluctuating Cooper pair amplitudes such as $\langle \hat{a}_{-\mathbf{p}}\hat{a}_{\mathbf{p}}\rangle(t)$ in Eq. (3).

Interestingly, in region II where $\Delta_\infty$ is a constant, the corresponding BdG equations formally match those of an equilibrium superconductor. The equilibrium superconductor is characterized by a topological number $W$, which depends solely on the sign of the chemical potential [5]. If the chemical potential is positive, $W = 1$ and the system, while gapful in the bulk, is known to have gapless edge states. The authors of Ref. [34] argued that any retarded Green's function with a topological number $W = 1$ when computed in a geometry with a boundary will have poles corresponding to gapless excitations in the boundary. Therefore, even the far-from-equilibrium superconductor discussed here will have topologically protected edge states as long as $\mu_\infty$ is positive (see the Supplemental Material [33]). The range of positive $\mu_\infty$ is shown in Fig. 1 as a subregion of region II with $W = 1$.

Unitary time evolution is a smooth rotation of the initial state. It may, therefore, appear surprising that a quench can induce a change in a winding number within a finite time interval. In fact, one must distinguish two different notions of topology here. The topology of the state (pseudospin winding) does not change but that of the effective single particle Hamiltonian can ($W$, as defined via the retarded Green's function in the Supplemental Material [33]). The Green's function determines the frequency spectrum that appears when transitions are driven by an external probe, while the state encodes the occupation of the modes [17].

In region III, $\Delta_\infty(t)$ is a complex-valued periodic function of time that can be determined analytically [18,21,22]. The parameters of this function including its turning points and the period $T$ are computed for a particular quench by solving a certain transcendental equation [17] (Supplemental Material [33]). A periodically driven system can be topological in the Floquet sense, as was recently discussed in the literature [15,16,27]. What this implies is that one needs to construct its Green's function $U(T) = \mathcal{G}(t+T, t)$ with $t$ being arbitrary [but sufficiently large so that the large-time asymptotic for $\Delta(t)$ applies]. The edge states of this system are then the eigenstates of $U(T)$, with their energy related to the eigenvalues of $U(T)$. More precisely, the eigenvalues of $U(T)$, a unitary operator, assume the form of $\exp(-i\epsilon T)$, where $\epsilon$ is such an energy level, taken to reside in the compact interval $[-\pi/T, \pi/T]$. These "quasienergies" are similar to the crystalline quasimomentum in systems periodic in space (while here the Hamiltonian is periodic in time).

It is possible to extract whether this system is topological by analyzing [28] the winding of $\mathcal{G}(t,t')$. In practice, this may not be easy to do. Instead, given the analytic expression for the time-dependent $\Delta_\infty(t)$ associated to a particular quench [17], we solved the Bogoliubov–de Gennes equation numerically in the cylinder geometry (periodic in one direction, with a hard wall boundary in the other) (Supplemental Material [33]). After computing $U(T)$ in this geometry, we extracted its eigenvalues and checked whether the edge states appear. By doing this at various points in region III, we can map out the topological character of this dynamical phase. At the boundary of regions III and II where $\Delta_\infty$ becomes a constant, we know from Fig. 1 that the system is topological. Thus, we expect that within the region III close to the boundary with region II the topological aspects of the phase (the boundary states) remain, even though the winding number $W$ may change, as was recently pointed out [28], while deep within region III there might, in principle, be nontopological domains or domains with a different topology from that in II.

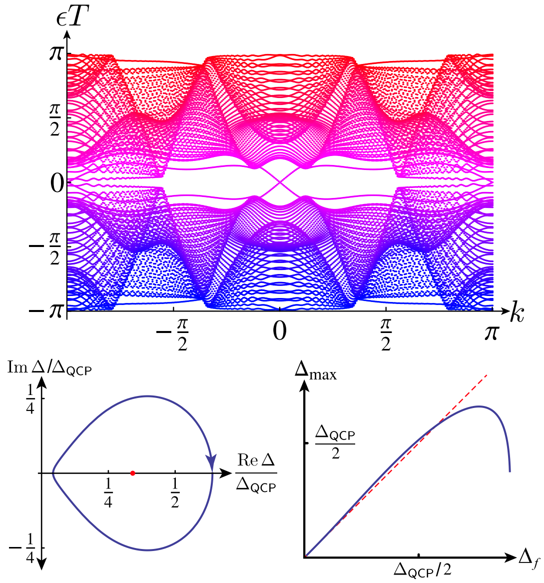


FIG. 2 (color online). Majorana edge modes for a quench-induced time-dependent state of $p$-wave superfluidity. The Floquet spectrum (top) of $\ln U(T)$ for a system on a finite cylinder is plotted in the large time asymptotic regime for a quench in region III, point A in Fig. 1. The horizontal axis represents the momentum along the boundary, and the vertical axis represents the quasienergies multiplied by the period of oscillations, both ranging from $-\pi$ to $\pi$. The edge states can clearly be seen crossing in the center of the figure. The bottom left shows the orbit swept by $\Delta_\infty(t)$ in the complex $\Delta$ plane at this point in region III. The bottom right plots the orbital maximum $\Delta_{\max} \equiv \max|\Delta(t)|$ as a function of $\Delta_f$ for quenches from very weak initial pairing ($\Delta_i \to 0$) in III. The dashed line is $\Delta_{\max} = \Delta_f$.

We find that edge states are present no matter where within region III we look. Figure 2 exhibits a typical spectrum for $U(T)$ at a point deep within region III, indicated in Fig. 1 as point A. This quench is located at $\Delta_i/\Delta_{\mathrm{QCP}} = 0.0065$, $\Delta_f/\Delta_{\mathrm{QCP}} = 0.83$. To generate the plot shown in Fig 2, we placed the superfluid on the lattice, with 50 lattice constants within the width of the cylinder (Supplemental Material [33]). The hopping amplitude on this lattice was chosen to be $J = 1/2$ so that the system would be below half filling, yet the total bandwidth was as small as possible to prevent the spectrum from folding too many times onto itself and obscuring the graph. Crossing edge states in the center of the figure prove that the time-dependent superfluid for this particular quench is topological in the Floquet sense. We conjecture that the entire region III is topological, but proving this requires further work.

A natural quench from the point of view of experiment would start from the noninteracting Fermi gas at $\Delta_i = 0$. Such a quench is much harder to describe than those studied here so far. Technically, the zero-temperature Fermi-Dirac distribution is a point of unstable equilibrium for the classical equations of motion studied above, so naively it does not evolve in time. In reality, quantum or thermal fluctuations will generate an initial condition with nonzero $p_x + ip_y$ and $p_x - ip_y$ order parameter amplitudes, and these will compete in the subsequent dynamics. The precise outcome is difficult to predict. Instead, we assume that it is possible to first switch on very weak attraction, which results in some initial very small yet nonzero $\Delta_i$ of pure $p_x + ip_y$ type. Then we quench this state into a far stronger interacting regime; as long as the quench resides within region III, we expect the resulting state to be in the topological Floquet phase (point A, where $\Delta_i \ll \Delta_f$ is a good example of such a quench). At the same time, if the interactions after the quench are stronger than the threshold depicted in Fig. 1, we will end up in the nontopological ($W = 0$) domain of region II.

We conclude with a discussion of relevant time scales. The main effect of the $p \neq 0$ terms in Eq. (1) is to mediate pair-breaking collisions [18,30], associated to a rate $1/t_{\mathrm{pb}}$. For quenches confined to the BCS regime, the lifetime can be estimated using Fermi liquid theory [24,30,35], leading to $t_{\mathrm{pb}}/t_F \sim [\varepsilon_F/E_{\min}(\Delta_f)]^2$, where $t_F = 1/\varepsilon_F$ is the inverse Fermi energy and $E_{\min}(\Delta) = \Delta\sqrt{2\mu - \Delta^2}$ is the ground-state quasiparticle energy gap. Quenches in region III that produce topological Floquet states reside entirely within the BCS regime; for these, the ratio $t_{\mathrm{pb}}/t_F$ can easily be an order of magnitude and grows rapidly larger as $\Delta_f$ is reduced. The inverse three-body loss rate can be estimated to be $t_3/t_F \sim (\ell/b)^\alpha$, where $\ell \sim 1000$ nm ($b \sim 5$ nm) is the interparticle separation (van der Waals length) [10,11]. Near resonance, the exponent $\alpha = 1$, with $t_3 \sim 20$ ms in experiments [13,14]. However, towards the weak BCS regime $t_3$ becomes orders of magnitude larger with $\alpha = 4$ [10], making three-body losses essentially irrelevant for the creation of a weakly paired initial state.

In numerical simulations of our model [17], we find that the asymptotic behavior is reached very quickly in region III over a time $t \lesssim t_F$. For quenches from weak initial pairing with $\Delta_i \ll \Delta_f$, the period $T$ of oscillations in the order parameter magnitude can be estimated as [17] (Supplemental Material [33])

$$T \sim \frac{2}{E_{\min}(\Delta_f)} \ln\left[\frac{4\varepsilon_F}{\Lambda}\frac{E_{\min}(\Delta_f)}{E_{\min}(\Delta_i)}\right] \sim \sqrt{t_F t_{\mathrm{pb}}}, \qquad (7)$$

where $\Lambda$ is an ultraviolet energy cutoff. In the BCS regime, we always have $\varepsilon_F > 2E_{\min}$ so that

$$t_F < T < \min(t_{\mathrm{pb}}, t_3), \qquad (8)$$

where $t_3$ is associated to the (larger) postquench coupling strength. This is the window in which the topological nonequilibrium steady state can be realized. Decreasing the final pairing strength [increasing $\varepsilon_F/E_{\min}(\Delta_f)$] may increase the relative size of the window, at the cost of decreasing the detectable maximum of $|\Delta(t)|$ (see Fig. 2).

Quench-induced topological edge states could be detected using rf spectroscopy type experiments. An open question is whether these types of topological steady states support the kind of quantized thermal conductance expected in an equilibrium topological $p$-wave superconductor. Calculating energy transport and exploring possible quantized out-of-equilibrium transport phenomena remain subjects for future work.

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[1] V. Gurarie and L. Radzihovsky, Ann. Phys. (Amsterdam) **322**, 2 (2007).
[2] P. W. Anderson and P. Morel, Phys. Rev. **123**, 1911 (1961).
[3] V. Gurarie, L. Radzihovsky, and A. V. Andreev, Phys. Rev. Lett. **94**, 230403 (2005).
[4] C.-H. Cheng and S.-K. Yip, Phys. Rev. Lett. **95**, 070404 (2005).
[5] N. Read and D. Green, Phys. Rev. B **61**, 10267 (2000).
[6] G. E. Volovik, *The Universe in a Helium Droplet* (Oxford University Press, Oxford, 2003).
[7] A. J. Leggett, in *Modern Trends in the Theory of Condensed Matter* (Springer, Berlin, 1980).
[8] P. Nozières and S. Schmitt-Rink, J. Low Temp. Phys. **59**, 195 (1985).
[9] J. Zhang, E. G. M. van Kempen, T. Bourdel, L. Khaykovich, J. Cubizolles, F. Chevy, M. Teichmann, L. Tarruell, S. J. J. M. F. Kokkelmans, and C. Salomon, Phys. Rev. A **70**, 030702(R) (2004).
[10] M. Jona-Lasinio, L. Pricoupenko, and Y. Castin, Phys. Rev. A **77**, 043611 (2008).
[11] J. Levinsen, N. R. Cooper, and V. Gurarie, Phys. Rev. A **78**, 063616 (2008).
[12] J. P. Gaebler, J. T. Stewart, J. L. Bohn, and D. S. Jin, Phys. Rev. Lett. **98**, 200403 (2007).
[13] J. Fuchs, C. Ticknor, P. Dyke, G. Veeravalli, E. Kuhnle, W. Rowlands, P. Hannaford, and C. J. Vale, Phys. Rev. A **77**, 053616 (2008).
[14] Y. Inada, M. Horikoshi, S. Nakajima, M. Kuwata-Gonokami, M. Ueda, and T. Mukaiyama, Phys. Rev. Lett. **101**, 100401 (2008).
[15] N. H. Lindner, G. Refael, and V. Galitski, Nat. Phys. **7**, 490 (2011).
[16] T. Kitagawa, T. Oka, A. Brataas, L. Fu, and E. Demler, Phys. Rev. B **84**, 235108 (2011).
[17] M. S. Foster, M. Dzero, V. Gurarie, and E. A. Yuzbashyan, Phys. Rev. B **88**, 104511 (2013).
[18] R. A. Barankov, L. S. Levitov, and B. Z. Spivak, Phys. Rev. Lett. **93**, 160401 (2004).
[19] Yu. M. Gal'perin, V. I. Kozub, and B. Z. Spivak, Sov. Phys. JETP **54**, 1126 (1981).
[20] G. L. Warner and A. J. Leggett, Phys. Rev. B **71**, 134514 (2005).
[21] E. A. Yuzbashyan, B. L. Altshuler, V. B. Kuznetsov, and V. Z. Enolskii, Phys. Rev. B **72**, 220503(R) (2005).
[22] E. A. Yuzbashyan, B. L. Altshuler, V. B. Kuznetsov, and V. Z. Enolskii, J. Phys. A **38**, 7831 (2005).
[23] E. A. Yuzbashyan, O. Tsyplyatyev, and B. L. Altshuler, Phys. Rev. Lett. **96**, 097005 (2006).
[24] R. A. Barankov and L. S. Levitov, Phys. Rev. Lett. **96**, 230403 (2006).
[25] E. A. Yuzbashyan and M. Dzero, Phys. Rev. Lett. **96**, 230404 (2006).
[26] R. Matsunaga, Y. I. Hamada, K. Makise, Y. Uzawa, H. Terai, Z. Wang, and R. Shimano, Phys. Rev. Lett. **111**, 057002 (2013).
[27] Z. Gu, H. A. Fertig, D. P. Arovas, and A. Auerbach, Phys. Rev. Lett. **107**, 216601 (2011).
[28] M. S. Rudner, N. H. Lindner, E. Berg, and M. Levin, Phys. Rev. X **3**, 031005 (2013).
[29] M. Ibañez, J. Links, G. Sierra, and S.-Y. Zhao, Phys. Rev. B **79**, 180501 (2009).
[30] A. F. Volkov and Sh. M. Kogan, Zh. Eksp. Teor. Fiz. **65**, 2038 (1973) [Sov. Phys. JETP **38**, 1018 (1973)].
[31] R. W. Richardson and N. Sherman, Nucl. Phys. **52**, 221 (1964); **52**, 253 (1964).
[32] J. Dukelsky, S. Pittel, and G. Sierra, Rev. Mod. Phys. **76**, 643 (2004).
[33] See the Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.113.076403 for a discussion of units and scales, the definition of $\mathcal{G}$ in Eq. (6), the definition of the winding number $W$ in terms of $\mathcal{G}$, and the details of the Floquet calculation.
[34] A. M. Essin and V. Gurarie, Phys. Rev. B **84**, 125132 (2011).
[35] M. W. Zwierlein, C. H. Schunck, C. A. Stan, S. M. F. Raupach, and W. Ketterle, Phys. Rev. Lett. **94**, 180401 (2005).