

The improvement of strain estimation using universal kriging

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Abstract In this paper, universal kriging with linear trend is used to interpolate the strain tensor elements over a region along San Andreas Fault in California. The main goal of this paper is to improve the ordinary kriging interpolation results. A 7-year time series (2006–2012) of 12 permanent stations is utilized to obtain the coordinate changes in UTM coordinates system and calculate the strain tensor elements by means of finite difference method. Comparing the results we can find an improvement about 40 % for universal kriging at critical points in which ordinary kriging can't be appropriate method of interpolation.

Keywords Strain · Ordinary kriging · Universal kriging · Linear model

1 Introduction

By advancement of satellite geodesy, the strain tensor can be estimated by the assistance of coordinates obtained from GPS stations (Burbank and Anderson 2008; Rontogianni 2010; Palano et al. 2013). Although triangulation is one of the most familiar methods of spreading the strain tensor over the desired area, but one cannot decide clearly to which point the strain tensors should be referred. This problem arises from large extent of projections of these elements on the Earth. This fact shows the important role of other methods like *finite difference* which can calculate the strain tensor point-to-point (Becker and Kaus 2010; Ismail-Zadeh and Tackley 2008). In addition, it's shown that the standard deviation of point-wise methods, in particular finite difference, is less than those of other methods (Bagherifam 2014).

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Therefore having a suitable surface is an important requirement for determination of the strain tensor of arbitrary points. Since the tensor of strain is assumed to be homogenous-like in a desired region, therefore it's common to use deterministic and well-known methods of interpolation such as nearest neighborhood, linear, and sometimes spline or polynomial surfaces (Ardalan and Raoofian 2008; Mashhadi Hossainali et al. 2010; Palano et al. 2013; Tong et al. 2012). But geostatistical methods of interpolation such as ordinary kriging can produce more accurate results.

Combination of various constraints with the specific mathematical equations can be considered as a method to improve the strain estimation results. This technique has been widely studied during past two decades. Kreemer et al. (2003) constructed a global model for horizontal velocity and horizontal strain rate over major plate boundaries. Within their research the velocity field was derived through a least-squares interpolation method using bicubic Bessel spline. Freed et al. (2007) found the relation between the earthquakes and the stress changes and accordingly, other effective factors were interfered. One of the simplest kinds of these constraints is the trend of the strain tensor that can be considered linear or of higher degree polynomials. For strain estimation in a dense network of stations, the linear model is the best model which is always valid (Ardalan and Raoofian 2008).

The kriging as more reliable interpolation method could be categorized into subdivisions such as simple kriging, ordinary kriging and universal kriging. Ordinary kriging is the simplest and the most commonly used, because no trend is defined to comply with. The universal kriging includes modeling of the local trend through the surrounding stations. This modeling as a smooth function of the coordinates is the most important characteristics of the universal kriging (Goovaerts 1997). Geostatistical methods, especially universal kriging have been increasingly utilized in many disciplines such as mining, meteorology, hydrology, geology, remote sensing, soil science, ecology and environmental science (Chirlin and Dagan 1980; Bastin et al. 1984; Hill and Alexandar 1989; White et al. 1997; Duc et al. 2000). For example, Gundugdu and Guney (2007) applied universal kriging with linear trend for spatial analysis of groundwater level. As mentioned before, strain has a linear trend, therefore in this paper we show the improvement of ordinary kriging by assuming a linear trend which converts it to universal kriging.

2 The tensor of strain and its linear model

The strain tensor in three dimensions is a symmetric matrix including nine elements which reduce to four elements in two-dimensional case as follow (Turcotte and Schubert 2002):

$$\varepsilon = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} \\ \varepsilon_{yx} & \varepsilon_{yy} \end{bmatrix}, \quad (1)$$

where ε_{xx} and ε_{yy} are the normal strain or strain in x and y directions respectively, and $\varepsilon_{xy} = \varepsilon_{yx}$ indicate shear strain. It's common to define other parameters based on eigenvalues of this tensor which are obviously invariant with respect to changes in coordinate system, for example (Turcotte and Schubert 2002):

$$\text{Dilation} = \varepsilon_x + \varepsilon_y \quad (2)$$

$$\text{MaxShear} = \max\{\text{EigenValues}\} - \min\{\text{EigenValues}\} \quad (3)$$

As mentioned before, it's preferred to utilize linear model for strain estimation if the network of node points is assumed to be dense. It means that displacements can be modeled as follow:

$$\Delta x = a_0 + a_1 x + a_2 y \quad (4)$$

$$\Delta y = b_0 + b_1 x + b_2 y, \quad (5)$$

where Δx and Δy are displacements in x and y directions and the coefficients form a matrix as follow:

$$\mathbf{E} = \begin{bmatrix} a_1 & a_2 \\ b_1 & b_2 \end{bmatrix} \quad (6)$$

Considering basic definition of strain as spatial variation or gradient of displacement, one can find:

$$\varepsilon = \frac{1}{2} (E + E^T) \quad (7)$$

This is the desired linear function of coordinates which should be considered in universal kriging.

3 Models

Since weighting procedure in geostatistical methods, especially kriging, depends on both distance and statistical distribution, therefore the method of *variography* is used to estimate the weight. Variography is a term which refers to fitting a spatial model to data and its implementation leads to *semivariance* as follow (Borrough 1986):

$$\gamma(h) = \frac{1}{2N} \sum_{i=1}^N [Z(x_i + h) - Z(x_i)]^2, \quad (8)$$

where γ is the variogram, h is the distance between two desired points x_i and $x_j (= x_i + h)$, N is the number of all points, Z is the value at each point. If the mean is assumed to be constant, for example in ordinary kriging which will be discussed later, it can be concluded that:

$$Var[Z(S_i) - Z(S_j)] = 2\gamma_{ij} \quad (9)$$

The semivariance is half of the variance of the differences between all possible points with a constant distance apart. For semivariance we can define different models i.e. spherical, exponential and Gaussian. For example exponential model which is used in this paper is formulated as follow (Borrough 1986):

$$\gamma(h) = c \left(1 - \exp \left(-\frac{h}{a} \right) \right), \quad (10)$$

where c and a are sill and range of the semivariogram, respectively.

3.1 Ordinary kriging

The ordinary kriging model assumption is (Goovaerts 1997):

$$Z(S_i) = \mu + \delta(S_i), \quad (11)$$

where μ is the constant mean of the model and δ is a zero mean stochastic term with variogram 2γ . Also the predictor assumption is (Goovaerts 1997):

$$\widehat{Z}(S_0) = \sum_{i=1}^N \omega_i Z(S_i), \quad (12)$$

where S_0 is a point which its value $\widehat{Z}(S_0)$ is going to be predicted, S_i is the node point which its value $Z(S_i)$ is used to predict the value of S_0 and ω_i is the weight of the related point, subject to the constraint $\sum_{i=1}^N \omega_i = 1$ to ensure unbiasedness.

Ordinary kriging minimizes the mean squared error of prediction. It means that:

$$\min \sigma_p^2 = E \left[Z(S_0) - \widehat{Z}(S_0) \right]^2 = E \left[Z(S_0) - \sum_{i=1}^N \omega_i Z(S_i) \right]^2 \quad (13)$$

By expanding σ_p^2 and differentiating it with respect to ω_i , it would be possible to obtain ω_i for all points of the model:

$$\begin{aligned} & \left[Z(S_0) - \sum_{i=1}^N \omega_i Z(S_i) \right]^2 = Z^2(S_0) - 2Z(S_0) \sum_{i=1}^N \omega_i Z(S_i) + \sum_{i=1}^N \sum_{j=1}^N \omega_i \omega_j Z(S_i) Z(S_j) \\ &= \sum_{i=1}^N \omega_i Z^2(S_0) - 2 \sum_{i=1}^N \omega_i Z(S_0) Z(S_i) + \sum_{i=1}^N \sum_{j=1}^N \omega_i \omega_j Z(S_i) Z(S_j) \\ & - \frac{1}{2} \sum_{i=1}^N \omega_i Z^2(S_i) - \frac{1}{2} \sum_{j=1}^N \omega_j Z^2(S_j) + \sum_{i=1}^N \omega_i Z^2(S_i) \\ &= -\frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N \omega_i \omega_j [Z(S_i) - Z(S_j)]^2 + \sum_{i=1}^N \omega_i [Z(S_0) - Z(S_i)]^2 \end{aligned} \quad (14)$$

By applying expectation on the last expression, it will be:

$$\begin{aligned} & -\frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N \omega_i \omega_j E[Z(S_i) - Z(S_j)]^2 + \sum_{i=1}^N \omega_i E[Z(S_0) - Z(S_i)]^2 \\ & - \frac{1}{2} \sum_{i=1}^N \sum_{j=1}^N \omega_i \omega_j Var[Z(S_i) - Z(S_j)] + \sum_{i=1}^N \omega_i Var[Z(S_0) - Z(S_i)] \end{aligned} \quad (15)$$

But $Var[Z(S_i) - Z(S_j)] = 2\gamma_{ij}$ is the definition of the variogram. Therefore ordinary kriging minimizes

$$\sigma_p^2 = E \left[Z(S_0) - \sum_{i=1}^N \omega_i Z(S_i) \right]^2 = - \sum_{i=1}^N \sum_{j=1}^N \omega_i \omega_j \gamma_{ij} + 2 \sum_{i=1}^N \omega_i \gamma_{i0} \quad (16)$$

Subject to $\sum_{i=1}^N \omega_i = 1$

The minimization is carried out over ω_i subject to non-biased constraint. Therefore this minimization calls for the definition of a Lagrangian $L(u)$ which is a function of the data weights ω_i and Lagrange parameter 2λ . Therefore the minimization problem can be written as (Edwards and Penney 1982):

$$\min 2 \sum_{i=1}^N \omega_i \gamma_{i0} - \sum_{i=1}^N \sum_{j=1}^N \omega_i \omega_j \gamma_{ij} - 2\lambda \left(\sum_{i=1}^N \omega_i - 1 \right) \quad (17)$$

After differentiating with respect to ω_i and λ and setting the derivatives equal to zero, it yields:

$$\begin{cases} \sum_{i=1}^N \omega_i \gamma_{ij} - \lambda = \gamma_{i0} \\ \sum_{i=1}^N \omega_i = 1 \end{cases} \quad (18)$$

and in matrix form:

$$\Gamma W = C, \quad (19)$$

where

$$W = [\omega_1 \omega_2 \dots \omega_n \lambda]^T$$

$$C = [\gamma_{10} \gamma_{20} \dots \gamma_{n0} 1]^T$$

$$\Gamma = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} & \cdots & \gamma_{1n} & 1 \\ \gamma_{21} & \gamma_{22} & \gamma_{23} & \cdots & \gamma_{2n} & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ \gamma_{n1} & \gamma_{n2} & \gamma_{n3} & \cdots & \gamma_{nn} & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{bmatrix} \quad (20)$$

Therefore the weights ω_i and Lagrange multiplier can be obtained by $W = \Gamma^{-1} C$.

3.2 Universal kriging

The ordinary kriging algorithm implicitly considers a non-stationary random function, but in some situations it is not reasonable to consider the local mean μ as a constant value. In other words, the mean can be a function of the coordinates X, Y, in linear, quadratic, or higher form. Considering a linear trend for strain as follow:

$$\hat{Z}(S_i) = \beta_0 + \beta_1 X_i + \beta_2 Y_i + \delta(S_i) \quad (21)$$

The value of $\widehat{Z}(S_i)$ can be expressed as:

$$\begin{aligned}\widehat{Z}(S_i) &= \sum_{i=1}^N \omega_i Z(S_i) = \sum_{i=1}^N \omega_i \beta_0 + \sum_{i=1}^N \omega_i \beta_1 X_i + \sum_{i=1}^N \omega_i \beta_2 Y_i + \sum_{i=1}^N \omega_i \delta(S_i) \\ &= \beta_0 + \beta_1 \sum_{i=1}^N \omega_i X_i + \beta_2 \sum_{i=1}^N \omega_i Y_i + \sum_{i=1}^N \omega_i \delta(S_i)\end{aligned}\quad (22)$$

which means three constraints of unbiasedness versus one constraint of ordinary kriging:

$$\sum_{i=1}^N \omega_i X_i = X_0; \sum_{i=1}^N \omega_i Y_i = Y_0; \sum_{i=1}^N \omega_i = 1 \quad (23)$$

By knowing the trend, now the weighting procedure in universal kriging would be similar to ordinary kriging which is done by minimization of the mean squared error of prediction, σ_p^2 . Using matrix notation, the system of equations will be:

$$\begin{aligned}W &= [\omega_1 \omega_2 \dots \omega_n \lambda_1 \lambda_2 \lambda_3]^T \\ C &= [\gamma_{10} \gamma_{20} \dots \gamma_{n0} X_0 Y_0 \quad 1]^T \\ \Gamma &= \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} & \dots & \dots & \gamma_{1n} & X_1 & Y_1 & 1 \\ \gamma_{21} & \gamma_{22} & \gamma_{23} & \dots & \dots & \gamma_{2n} & X_2 & Y_2 & 1 \\ & & & & & \vdots & & & \\ \gamma_{n1} & \gamma_{n2} & \gamma_{n3} & \dots & \dots & \gamma_{nn} & X_n & Y_n & 1 \\ X_1 & X_2 & X_3 & \dots & \dots & X_n & 0 & 0 & 0 \\ Y_1 & Y_2 & Y_3 & \dots & \dots & Y_n & 0 & 0 & 0 \\ 1 & 1 & 1 & \dots & \dots & 1 & 0 & 0 & 0 \end{bmatrix}\end{aligned}\quad (24)$$

4 Numerical results

A numerical comparison between ordinary kriging and universal kriging interpolations was carried out by downloading the time series of 12 permanent GPS stations from <http://sideshow.jpl.nasa.gov/post/series.html> which are located along southern part of San Andreas Fault. This time series is collected in a 7-year interval from 2005 to 2012 (2183 observational day) and its stations are named P210, P217, P212, P233, P234, P236, P238, P240, P242, P244, P251 and P239. The coordinates and the strain tensor of network points are tabulated in Table 1 and the map of these stations is shown in Fig. 1. Strain tensor is calculated for each point through finite difference method. Then, the strain tensor is interpolated through four methods of linear, spline, ordinary kriging and universal kriging. The results are tabulated in Table 2. It can be noticed that linear and spline interpolations are able to interpolate only points located inside the desired region and for marginal points it is not possible to calculate the values.

Talking about the details of kriging interpolation, it can be paid attention that the semivariogram is assumed to be exponential; because these methods are very similar to each other and there is no significant difference between various models of semivariogram in strain interpolation, e.g. spherical, exponential, Gaussian. Also, as mentioned before, there is a constant value in kriging, which is named as sill of the semivariogram. Estimating this constant value might be done by difference methods e.g. ordinary least squares, maximum likelihood, by-eye methods, etc. In this paper, the value of c is tested by-eye, so

Table 1 Coordinates and strain tensors of network points. X1, Y1 are the initial coordinates (before the deformation), and X2, Y2 are the final coordinates (after the deformation)

Station no.	X1 (m)	Y1 (m)	X2 (m)	Y2 (m)	ε_{11}	ε_{12}	ε_{22}
1(P210)	613,109,615	4,075,226,544	613,109,636	4,075,226,504	-4,67E-07	1,38E-07	-4,43E-07
2(P212)	601,241,847	4,091,262,413	601,241,868	4,091,262,375	-5,31E-07	1,62E-07	5,95E-08
3(P240)	619,900,128	4,107,316,674	619,900,14	4,107,316,643	-4,43E-07	1,39E-07	4,09E-08
4(P217)	614,123,844	4,118,223,939	614,123,856	4,118,223,907	-4,39E-07	1,31E-07	3,05E-08
5(P233)	640,928,664	4,073,897,468	640,928,678	4,073,897,438	-4,56E-07	1,48E-07	5,16E-08
6(P234)	625,582,809	4,080,104,727	625,582,829	4,080,104,689	-4,27E-07	1,16E-07	4,75E-08
7(P236)	628,784,699	4,085,148,163	628,784,713	4,085,148,13	-4,43E-07	1,42E-07	4,14E-08
8(P238)	637,943,101	4,079,247,783	637,943,113	4,079,247,754	-4,47E-07	1,48E-07	4,22E-08
9(P239)	629,714,952	4,096,732,087	629,714,963	4,096,732,055	-4,46E-07	1,49E-07	3,98E-08
10(P242)	636,827,123	4,090,865,357	636,827,134	4,090,865,327	-4,59E-07	1,48E-07	4,14E-08
11(P244)	646,393,567	4,097,338,867	646,393,566	4,097,338,842	-3,82E-07	1,44E-07	6,33E-08
12(P251)	647,361,026	4,075,229,526	647,361,028	4,075,229,502	-3,91E-07	1,63E-07	9,70E-09

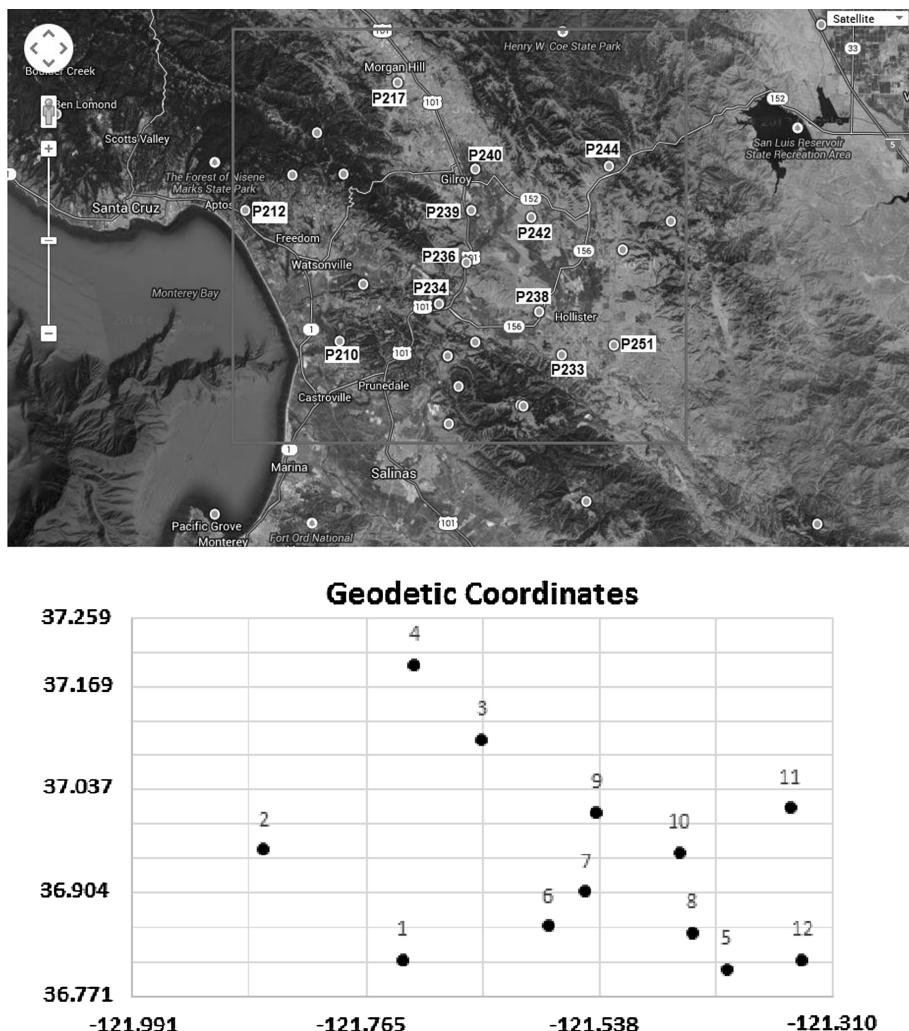


Fig. 1 Up location of 12 stations of study on the Earth map. Down geodetic coordinates of 12 station

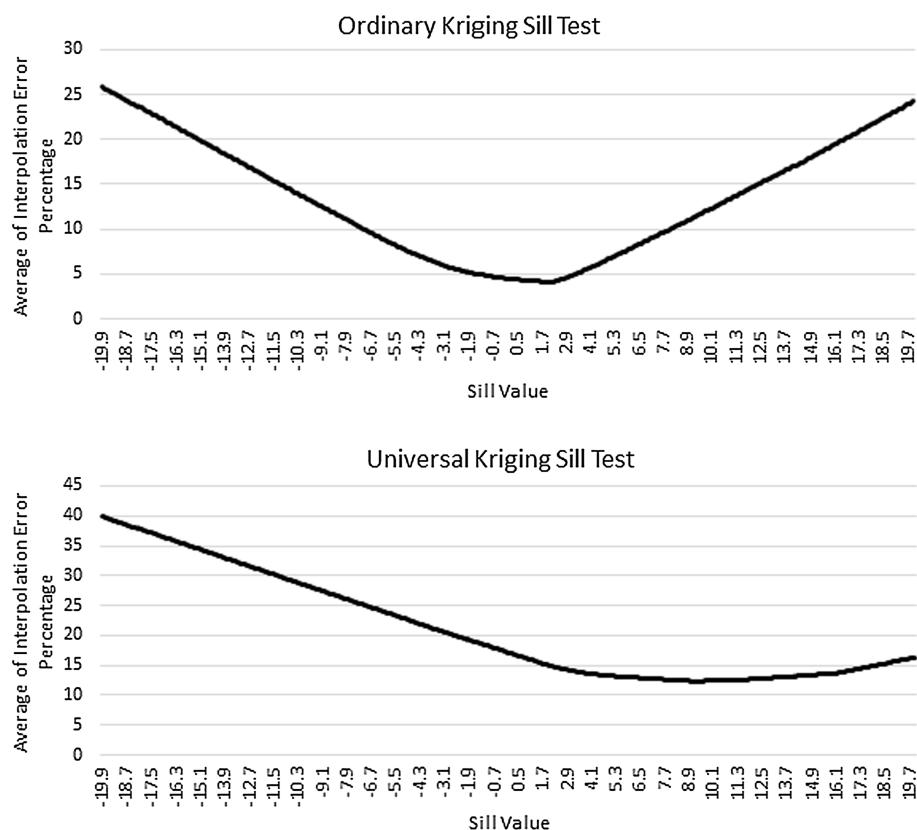
that the average of the error percentages of interpolated points are tested for a huge number of sill values and the result of this comparison is graphed in Fig. 2. As shown in this graph the value of the variogram sill, c is preferred to be one for ordinary kriging and 8.9 for universal kriging to reduce the error percentage average for all interpolated points.

Then validation procedure is implemented by excluding each point respectively and recalculating its strain tensor through the other strain points by means of ordinary kriging and universal kriging interpolation and comparing the discrepancies between calculated and predicted strain tensors. It should be noticed that by excluding each point, variogram is estimated again for remaining points. Results are shown in Table 2 and Fig. 3. All of the calculations are done and programmed in MATLAB.

As it can be seen, kriging interpolation methods, either ordinary or universal, provide acceptable results for ε_{11} and ε_{22} and on average, we can find improvements more than

Table 2 Error percentage of kriging, linear and spline for network points

Station no.	Ordinary kriging			Universal kriging			Linear			Spline		
	ε_{11}	ε_{12}	ε_{22}									
1	—	—	—	—	—	—	—	—	—	—	—	—
2	—	—	—	—	—	—	—	—	—	—	—	—
3	0.14	7.64	3.24	1.63	3.05	8.60	21.4	41.91	183.68	22.36	44.15	172.73
4	—	—	—	—	—	—	—	—	—	—	—	—
5	—	—	—	—	—	—	—	—	—	—	—	—
6	0.83	6.55	2.94	3.98	0.41	6.25	21.44	5.04	67.35	19.4	0.85	73.58
7	0.32	6.56	1.42	1.67	11.08	4.76	19.72	11.44	90.91	20.43	15.81	94.59
8	1.19	9.39	2.71	2.28	0.90	0.03	14.24	1.72	120.52	14.7	0.5	122.92
9	0.72	5.50	1.88	0.91	20.90	2.46	18.91	19.72	100.54	17.86	19.6	107.36
10	0.67	5.83	2.43	4.40	17.87	0.07	9.51	1.04	77	10.91	1.21	76.37
11	—	—	—	—	—	—	—	—	—	—	—	—
12	—	—	—	—	—	—	—	—	—	—	—	—

**Fig. 2** Testing the value of the sill of the variogram for ordinary and universal kriging

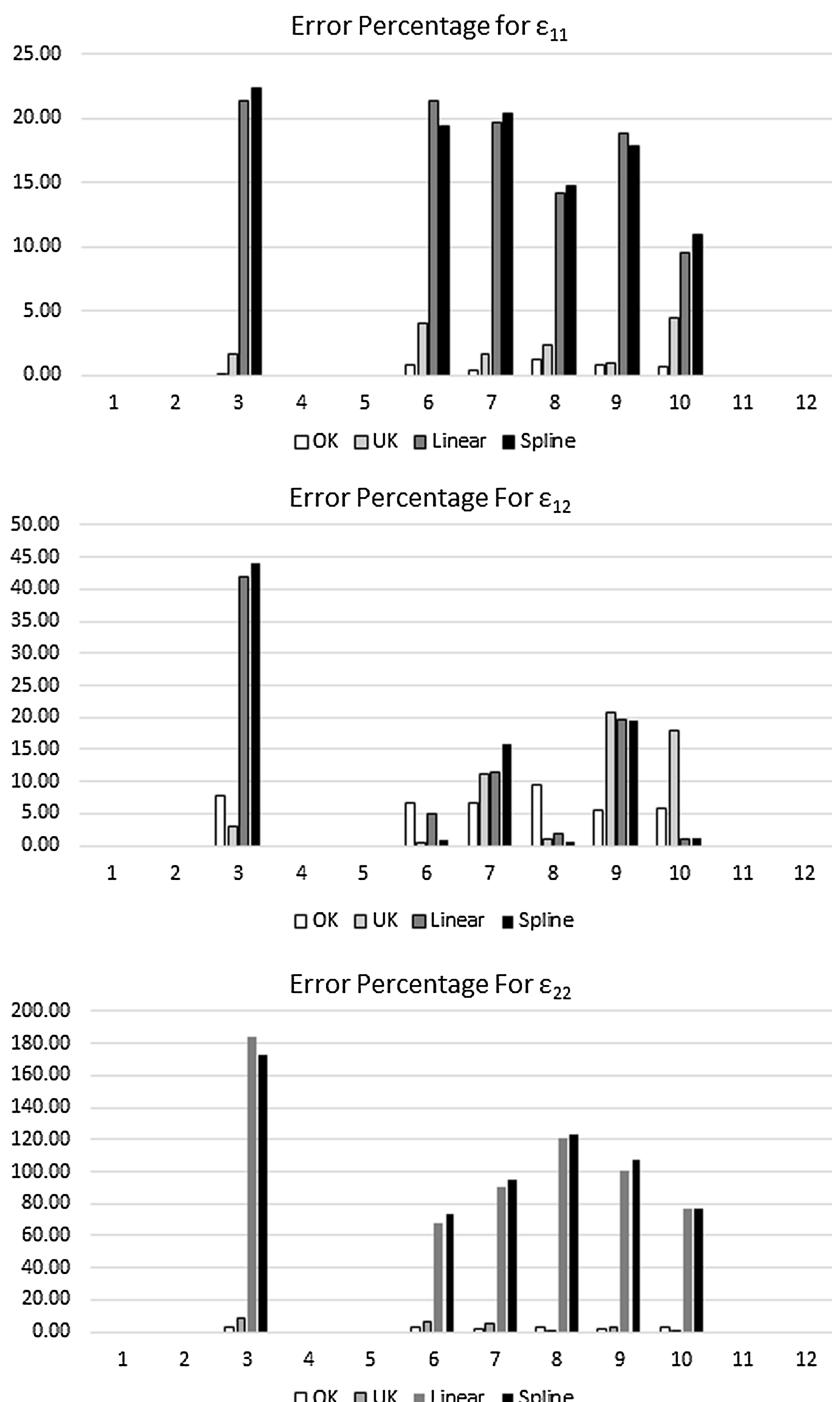


Fig. 3 The error percentage of kriging, linear and spline for each point. *Vertical axis* is the error percentage and *horizontal axis* is the number of each point

70 % with respect to results of linear and spline interpolations. But the critical case is estimation of shear strain ε_{12} ; the performance of ordinary kriging is better than linear and spline, only in 3 point, but its performance on points 6, 8 and 10 is not acceptable. Since linear interpolation is one of the most usual method for strain calculations (Ardalan and Raoofian 2008) therefore a linear trend is combined with ordinary kriging, as explained in 3.2, and estimate ε_{12} again. Results show that universal kriging error percentage for ε_{12} is usually equal or less than of linear interpolation which are acceptable. In other words, universal kriging can improve estimation of shear strain.

5 Conclusions and recommendations

The results of ordinary kriging interpolation for strain estimation show an improvement of about 70 % with respect to results of linear and spline method. But it has some deficiencies and disadvantages. For example it is not sensitive to introducing trend in the data over the study area, whereas it is known that strain has almost a linear trend. Therefore by adding linear trend to ordinary kriging and employing universal kriging for strain estimation, we can find about 40 % improvement especially for shear strain element.

It should be noticed that we can choose non-linear trend according to the condition of the case study area and also the number of unbiased constraint can differ and should be considered in the weight matrices.

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