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COMPLEX FLUIDS & HYDRAULIC FRACTURING

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□ **Abstract** Nearly 70 years old, hydraulic fracturing is a core technique for stimulating hydrocarbon production in a majority of oil and gas reservoirs. Complex fluids are implemented in nearly every step of the fracturing process, most significantly to generate and sustain fractures, and transport and distribute proppant particles during and following fluid injection. An extremely wide range of complex fluids are used: naturally occurring polysaccharide and synthetic polymer solutions, aqueous physical and chemical gels, organic gels, micellar surfactant solutions, emulsions, and foams. These fluids are loaded over a wide range of concentration with particles of varying size, and aspect ratio, and are subjected to a extreme mechanical and environmental conditions. We describe the settings of hydraulic fracturing (framed by geology), fracturing mechanics and physics, and the critical role that non-Newtonian fluid dynamics and complex fluids plays in the hydraulic fracturing process.

Keywords hydraulic fracturing, porous media, rheology, complex fluids, suspension mechanics, particulate transport

1. INTRODUCTION

Hydrocarbons are crucial feedstocks across all sectors of the global economy. The ease of hydrocarbon extraction translates directly into petroleum availability and lower cost for products derived from petrochemicals. Periods of increased worldwide demand, coupled with restricted supply of hydrocarbons, have

encouraged and enabled various technological advances to recover oil and gas that is increasingly difficult to remove from the ground. While a variety of reservoir stimulation techniques are currently used, we describe the process of hydraulic fracturing and the central role that particulate transport by complex fluids plays through it.

Although hydraulic fracturing of source rocks (also loosely called “shales”) - as opposed to hydrocarbon reservoirs - is a topic of contemporary (and often contentious (2)) interest, hydraulic fracturing is used in a majority of oil and gas reservoirs at some point in their lifetime. Fracturing is used in conventional hydrocarbon reservoirs to increase permeability in damaged formations or in formations which exhibit significantly lower production rates over what could be achieved after fracturing stimulation. It is also used in reservoirs where the intrinsic permeability is too low to yield economical production without it.

Reservoir stimulation by hydraulic fracturing creates reservoir surface by pumping a particle-laden fluid into the rock, generating a large pressure relative to the hydrostatic pressure downhole. Since the rock has low permeability, and the fluid is (mostly) incompressible, the growing pressure is relieved by the fracturing of reservoir rock generating a new flow path. Pumping stops when the desired volume (and fracture extensions) is attained or when the pressure required to sustain fracture growth exceeds the available pumping capacity; at this point the well is shut. Over time, the pressure in the fracture and in the formation will equilibrate, following the penetration of the injected fluid into the formation and the closure of the fracture. This process depends largely on the fluid leak-off rate inside the formation, and can take several days in low permeability formations. The fractures do not close completely, as the sand or other proppant carried into the fracture by fluid transport remains, and will prop the fractures open. Following this shut-in and leak-off period, the well is reopened and liquid or gaseous hydrocarbons flow out into the well from which they can be extracted or produced.

The hydraulic fracturing process depends critically upon complex fluids. Prior to hydraulic fracturing, a well must be drilled, cased and cemented. Drilling muds (either aqueous or oil-based) are employed as mechanical stabilizers in the construction of the wellbore to pressurize (via gravitational hydrostatic pressure) the sidewall against collapse, cool the drill bit, and carry away rock cuttings.

Cements are pumped downhole to form a reinforced casing and (along with metal pipe) isolate sections of the wellbore, providing crucial isolation (1) of the well from the surrounding environment. The first step of fracturing consists of pumping a solid-free fluid, called a pad, that initiates the fracture prior to the introduction of a particle-laden fracturing fluid. This way, risks associated with particles reaching the tip of the fracture (described in Section 4) are mitigated.

The fracturing fluids used in commercial operations are proprietary formulations specific to the geology of the formation and desired treatment, yet nearly all contain rigid proppant particles used to hold the newly-generated fracture network open after the cessation of flow. Fluids ranging from dilute polymer solutions (i.e., ‘slickwater’), polysaccharide solutions (crosslinked and linear), foams (also called energized fluids), micellar fluids (viscoelastic surfactants), and oil-based fluids are loaded with particles of varying size, aspect ratio, and density to achieve the desired combination of chemical and mechanical properties.

Fluid selection for hydraulic fracturing is a design problem. Optimality is economic: given a hydrocarbon reservoir with a specific set of geological characteristics, what set of fluid properties will produce the most hydrocarbon for the least cost? This question is straightforward but nontrivial, as it requires a thorough understanding of (i) the petrophysical properties of the hydrocarbon reservoir, (ii) the rheology and formulation costs of the particle-laden complex fluid used in fracturing, (iii) the fluid--solid interaction driving the hydraulically-induced fractures in the anisotropic hydrocarbon reservoir, and (iv) quantification of risk--reward since no process is 100% reliable. The industry has turned to complex fluids to satisfy this optimization process.

In this review, we focus on the application of complex fluids and non-Newtonian hydrodynamics which governs the hydraulic fracturing of oil and gas reservoirs. This necessarily requires an understanding of hydrocarbon reservoirs, with particular attention to the fluid--solid interactions that occur during the fracturing process, and the nature of reservoir rock (and bounding layers) in general. We first consider a brief history of hydraulic fracturing. This is followed by a discussion of hydrocarbon reservoir petrology and geomechanics. Then, we consider the mechanics of hydraulic fracturing, focusing on the role of fluid, reservoir, and flow properties in determining the fracture geometry. Finally, with

the fracturing process quantitatively described, we detail the rheology of complex fluids relevant in the oilfield and explore the wide-ranging and varied types of fracturing fluids that are employed downhole, highlighting the various processes that these complex fluids are involved in during the hydraulic fracturing process, as well as the potential knowledge gaps associated with them.

1.1. A Brief History of Hydraulic Fracturing

Hydraulic fracturing is the progeny of reservoir acidizing processes that date back to the late 1800s (3). Acidizing increases permeability and production in reservoirs through the injection of acids directly into the formation which react with, and subsequently dissolve, carbonates and some sandstones (4, 5). The Van Dyke acidizing patent (3) describes several of the features present in fracturing today: the use of a rubber packer to isolate target areas of the well, the loading of the well with a target fluid (in this case an acid), the pressurization (via an imposed hydrostatic head) of the fluid to improve penetration into the formation, and then the breaking (in this case neutralization) of the fluid with a base. This technique was later expanded upon by Grebe and Stoesser (6) who describe an “organic jellifying material” for use in wells. Grebe and Stoesser further describe a wide range of fluids and fluid-property modifiers in a 1935 article (7), and also a description of the hydraulic rock-splitting action (Figure 1) to increase the effectiveness of acidizing. All of these technologies have descendants in the modern oilfield. Hydraulically-driven reservoir deformation was further recognized in water injection and cementing operations throughout the 1940s, as summarized in the monograph by Howard and Fast (8).

<COMP: PLEASE INSERT FIGURE 1 HERE>

Figure 1 Then and now. Upper left: Schematic diagram of fracturing during an acidizing treatment ca. 1935, from product literature (7). Upper right: The first fracturing operation (1947) in Grant County, Kansas (courtesy Michael B. Smith, credit: Robert C. Fast). Lower left: schematic diagram of a contemporary fracturing job (courtesy Schlumberger) Lower right: Layout for a contemporary fracturing operation. Pump trucks are arrayed at center, and outlying trailers contain liquids. (©CustomAerialImages.com).

The first hydraulic fracturing operation occurred in the Hugoton field, Kansas in 1947, where a gelled gasoline was used as the fracturing fluid. In the decades since, hydraulic fracturing has grown in scale and importance, following innovations in fluid chemistry and pumping capacity. Coupled with the ability to

steer a drill bit horizontally over long distances, it enabled the production of source rocks that were not considered produceable previously. In the United States alone, it is estimated that 986,000 wells received fracturing treatments between 1947 and 2010 (9); 278,000 of these wells, mostly focused on the development of source rocks, have been drilled since the year 2000. One reason for this uptick are the increased lateral distances that can be drilled; contemporary wells routinely exhibit horizontal laterals (l_h) in excess of one mile (10--12). Operators are presently pursuing lengths in excess of 2.25 miles (13). The increase in length is enabled by a decrease in cost per length drilled, which has fallen 57% in the past 4 years (10), current costs (2015) being about 1000\$/ft, more (wet) or less (dry) depending on the produced fluid. Fractures are introduced along these laterals by injecting liquids at high flowrate (in excess of 90 bbl/min (14)), stimulating ever-increasing reservoir volumes from isolated wells, recently termed 'super fracking' (15).

bbl/min: Common rate of flow in the oilfield, equivalent to $0.111 \text{ m}^3 \text{ s}^{-1}$. 1 bbl is equivalent to 42 gallons

1.2. Requirements for a hydraulic fracturing fluid

The demands on a hydraulic fracturing fluid are many, and will be briefly mentioned here, from the sourcing of the material all the way to the cleanup of the fracture after the treatment has stopped. The material for the base fluid should be inexpensive, easy to source, of constant quality and harmless environmentally. It should be easy to mix on the fly and to pump, and should exhibit a low friction pressure in a pipe in turbulent regime, as it goes down the wellbore. When it hits the perforations in front of the rock to be stimulated, it should transport proppant through these perforations, which make a 90 degree angle with the direction of the flow. After going through the perforations, which behave like jets, it should recover viscosity thinned in the high-shear perforation to create fracture width. It should suspend proppant both in dynamic and static conditions, and exhibit low leakoff into the formation. After pumping has stopped, it should allow the fracture to close quickly and prevent proppant from settling. After the fracture has closed, it should flow back to the surface easily, without impairing the flow of hydrocarbons either through the matrix where it had previously leaked off or through the proppant pack. Finally, its properties

should be tunable to a wide range of downhole temperatures and chemical environments.

Let us note that, for simple Newtonian fluids, many of those requirements are contradictory in nature, e.g. the low friction pressure in the pipe and the high pressure drop in the fracture, or the proppant suspending capability and the ease of proppant pack cleanup. This is the main reason why the industry has turned to the complex fluids to satisfy these requirements.

2. HYDROCARBON RESERVOIR PETROLOGY

Sedimentary rocks form the majority of oil reservoirs. Sedimentary rocks can be formed by the successive deposition of fluids and minerals from weathering (mechanical and chemical), evaporation, or biogenic activity, which, over time, become continuous and porous structures such as sandstones, limestones, and mudstones (16, 17). We shall omit evaporites as they tend to have no porosity or permeability. This sedimentation process occurs as a set of discrete depositions, and the rock is formed by a combination of compaction and cementation (17). Sedimentary reservoirs are porous and present many distinct and identifiable layers, with the precise nature of the sedimentary rock determined by the depositional environment (e.g. a marine shale versus an aeolian dune set), along with the thermodynamic and mechanical history of the reservoir as a whole. The size, shape and topology of sedimentary structures, along with the thermo-mechanical properties exhibited locally in the lithosphere, determine many of the pertinent reservoir properties for hydrocarbon production (18).

Kerogen: A nanoporous (19) material, insoluble to both alkaline and organic solvents.

In some sediments, interspersed among clay, sand, and silt are organic materials that will degrade to form oil and gas. Organics trapped within the sedimentary rock experience progressively greater temperatures (increasing at a rate of about 2.5°C/100m (20, 21)) and stresses as the layers travel deeper into the earth (21). The gradual degradation of organics in these source rocks occurs in three stages: diagenesis, catagenesis, and metagenesis; they can be loosely thought of as the ‘heating’, ‘cooking’, and ‘burning’ of organic material.

Diagenesis occurs in the relatively low-temperature region (<50°C) up to a depth

of several hundred meters; Here, organic matter is converted to kerogen (21, 22). Kerogen is acknowledged to exist in three types, delineated by origin, structure (aliphatic versus polyaromatic), and constitution (atomic ratios of hydrogen to carbon and oxygen to carbon (21, 23, 24)). Oil and gas molecules are derived from kerogen during catagenesis, which occurs at temperatures between 50 and 150°C. Gas is generated primarily at the higher end of this window. Organic material subjected to yet greater temperatures continue to degrade into dry gas (i.e., methane), with carbon as the ultimate endpoint of thermolysis.

While kerogen is converted to oil and gas in such formations called “source rocks”, traditional drilling operations do not extract hydrocarbons directly from them, as the volume concentration of hydrocarbon is low. Hydrocarbons may undergo primary migration, exiting the source rocks where they originated and then undergo secondary, buoyancy-driven, migration to and within distal porous layers. Eventually, further upward motion of the hydrocarbons is arrested by an impermeable cap (16) and oil and gas sit bounded. Conventional extraction targets oil and gas away from source rocks, in these secondary porous structures where hydrocarbons are concentrated.

The most economically favorable reservoirs exhibit large porosities and permeabilities. Porosity, hydrocarbon saturation and reservoir volume set the available amount of hydrocarbons to be extracted, while reservoir permeability sets the relative ease by which the hydrocarbons may be removed. Early oil and gas drilling relied upon the pressure in the earth to expel hydrocarbons directly or with minimal pumping from the wells (primary recovery). If the hydrocarbons are liquid, this production technique produces only a minimal amount of the hydrocarbons in place and other methods have been developed to recover more of them. For example, as the pressure in the reservoir will decrease as hydrocarbons are produced at the surface, positive displacement of water can then be used to drive hydrocarbons to the surface, known as secondary recovery. Finally, enhanced oil recovery (or tertiary recovery) makes use of chemical additives pumped downhole to coax and recover oil that could not be removed in earlier stages. In all cases, the techniques used are determined by the hydrocarbon content and flow properties of the reservoir --- multiple techniques may be used on the same reservoir over the course of its lifetime.

2.1. Fluid Flows in Reservoir Rocks

The sedimentary process endows reservoirs with widely varying porosity and permeability (18, 25--27). The initial sedimentation of material forms a porous structure which, through time, is acted upon by chemical and mechanical forces (28--30) to provide (i) pores in organic material, (ii) pores in inorganic material, and (iii) faults and fractures (31--33); all processes and features yield widely variable porosity and permeability within the reservoir (18, 25--27). Additionally, the sediment settling process results in anisotropic permeabilities such that transport is favored along the sedimentary beds rather than the direction of deposition (18).

Inorganic (i.e., mineral-based) and organic (i.e., kerogen-based) pores in mature source rocks are discussed in detail by Bernard and Horsfield (34), highlighting the change in morphology of the pore network as the source rocks age. Source rocks are distinguished from conventional oil reservoirs by reduced pore size (35) and the role of nanoscale pores within kerogen trapped in the rock. Nanoscale pores in kerogen may not form a formation-spanning connected network, but do form a local network on the scale of the organic deposit and are storage sites for gas (19, 30).

Many porous media flows are well-described by Darcy's law,

$$\langle u_i \rangle = -\frac{k_{ij}}{\eta} \frac{\partial p}{\partial x_j} \quad 1.$$

Here, u is the local velocity of the pore fluid, and the brackets, $\langle \rangle$, indicate that this velocity is an averaged quantity. This velocity is proportional to the pressure gradient across the porous media via a permeability tensor, k_{ij} , and a fluid viscosity η . The quality or 'goodness' of reservoirs is often quantified using permeability values, along with the porosity and total amount of organics available for extraction in the reservoir. Bear (36) (his Table 5.5.1) tabulates permeabilities for soils and rocks. Notably, $10 \leq k \leq 10^4$ mD for 'oil rocks', $10^{-1} \leq k \leq 10$ mD for sandstone, and $k \leq 1$ mD is listed as 'impermeable', or 'tight' in contemporary parlance (18). Modern production technology has significantly modified what we consider as impermeable, see Table 1 for more contemporary values and classification.

<COMP: PLEASE INSERT TABLE 1 HERE>

Permeability: A geometric constant of a porous material indicating resistance to flow. It incorporates both the porosity and the tortuosity of the porous media.

Porosity: A measure of the open space in a material, reported as percent of void space.

Darcy: Permeability carries units of length squared, and is typically cited in Darcies, $1 \text{ D} = 9.867 \times 10^{-13} \text{ m}^2$; milli- and nanodarcy permeabilities are commonly reported.

The linear Darcy relation generally holds for creeping flows of Newtonian fluids. Corrections exist to accommodate larger Reynolds number flows (37), large-scale velocity gradients (38), non-Newtonian effects (39--41), and pressure-dependent permeabilities (25, 37). Despite these corrections, departures from strictly Darcian flow conditions remain an open problem. Complications arise in time-dependent processes, like hydraulic fracturing, where the permeability may vary owing to components adsorbed onto the porous matrix from the fluid, and/or variations in the pressure difference between the matrix and fluid modulating the pore structure.

The Darcy law (Equation 1) is invalid for gas flows in nanopores such as those found in kerogen. For these flows, the mean free path (λ) of gas within the pores is large as compared to a characteristic diameter of the pores, d_p , and the continuum Darcy law is strictly inapplicable. The ratio of these lengths form the Knudsen number, $\text{Kn} = \frac{\lambda}{d_p}$, revealing if the flow may be described using classical continuum descriptions of transport ($\text{Kn} = 1$), if slip effects must be included ($10^{-3} \leq \text{Kn} \leq 10^{-1}$), or if free-molecule flow ($\text{Kn} \geq 10^1$) must be used to describe gas transport. Several models have been proposed (42--44), starting with the Klinkenberg relation (42) which introduced an empirical permeability, k_s , slip 'corrected' to augment the permeability in the Darcy law, $k_s = k_\infty(1 + K_b / p)$, so that decreases in thermodynamic pressure (p) will augment the permeability; here, k_∞ is a liquid permeability in the same medium, p is the pressure in the pores, and K_b is the Klinkenberg constant.

There is no universal relation to connect the porosity, ϕ , to the permeability k_{ij} in rocks or porous media in general. This failure arises from the inherent dependence of the permeability on the detailed geometry of the pores, and the distribution of pore sizes, whereas the porosity is a scalar measure that does not

include pore-scale information beyond the volume fraction of solids. Strikingly, the permeability in mudstones can vary over three orders of magnitude for the same value of porosity (27)! Despite this limitation, several empirical correlations and analytical models exist (27, 37); The success of the various models depends upon how well the underlying assumptions match physical characteristics of the targeted reservoirs. Katz and Thompson, recognizing the self-similarity of pore spaces for several sandstones (45), developed a predictive permeability model (46) $k = c l_c \sigma / \sigma_0$, where c is a numeric constant, l_c is a length scale from mercury injection measurements, and G and G_c are electrical conductivities of the brine saturated porous media and brine, respectively. The Katz and Thompson model includes two important components: (i) a single characteristic length, l_c , which dominates momentum transport, determined by the inflection point of pressure during a mercury injection test (37), and (ii) a measure of the pore connectivity through the independent electrical conductivity measurements of brine.

Brine: A salt solution typically at a concentration greater than seawater.

Concentrated salt solutions suppress surface conductance effects and encourage measurement of solution conductance.

Natural fractures can also contribute to the porosity and permeability.

Fractures in rocks form from stresses and natural fluid motion. These fractures and associated fracture networks have been identified as major contributors to shale gas reservoirs (47--49); it is therefore desirable in these systems to intersect the natural fracture network during the drilling and hydraulic fracturing process (50). Natural fractures are characterized by orientation, length, aperture, and roughness (33, 49, 51, 52). Similar to the porous media described above, the natural fracture network can exist in a percolated or unpercolated state, and can also be both anisotropic and heterogeneous (51). Percolated fractures within hydrocarbon reservoirs do not necessarily conduct liquids or gases; the fractures may be filled with calcite, or are otherwise sealed (48, 50). Transport models for fractures and fracture networks suffer similar limitations to the porous media discussed previously -- the geometry of the networks and individual fractures is strongly variable and typically unknown. Despite these uncertainties, individual

fractures (53) and fracture networks in porous matrices (52, 54--56) have been analyzed to elucidate estimated flows and permeabilities for Darcy flows.

Despite all complications associated with the description of multiphase flow in rocks (brine, liquid and gaseous hydrocarbons), one can infer that the total flux that can be drained from a reservoir into a well is a function of the rock permeability, viscosity of the reservoir fluids, pressure gradient and surface area over which that pressure gradient is applied. Rock permeability is provided by geology and can only be altered locally (e.g. by the injection of reactive fluids). Mobility of the flowing fluid(s) is also given by the nature of the fluids in place and the temperature and pressure conditions in the reservoir– even if it can be modified in some cases by heat (e.g. injection of steam) or injection of surfactants. The amount of pressure gradient that can be applied is also limited, roughly by the difference between the initial fluid pressure of the reservoir and the weight of the hydrostatic column in the wellbore. It is therefore natural to look at the surface area over which a pressure gradient can be applied from the well to the reservoir.

A vertical well drilled in a hydrocarbon reservoir drains the reservoir fluids through a small contact area with the producing layers: the intersection of a 0.2m diameter well with a 15m thick producing layer results in about 20m² of contact area. The magnitude of this contact area can be increased if the well is drilled with a long portion following the producing layer: a 1000m lateral drain placed in the same producing layer would have a contact area with the reservoir of about 1,250m². Another means is hydraulic fracturing, whose goal is to create a large surface area in contact with the reservoir. For example, the creation of a 100m long bi-wing fracture in contact with the same reservoir would result in a contact area of 3,000m². If the permeability of the fracture is large enough so that the pressure in the fracture is close to that in the wellbore, hydraulic fracturing is extremely effective at promoting flow from low permeability reservoirs. One of the goals of hydraulic fracturing is thus to create a fracture whose permeability is infinitely large compared to that of the reservoir that it is draining. Let us remark, finally, that for reservoirs with extremely poor flow characteristics (e.g. with permeabilities of the order of 100nD or less), lateral drains can be hydraulically fractured to create enough drainage area to concentrate enough flow into the wellbore.

2.2. Mechanical Attributes of Rocks

Hydraulic fractures arise from fluid--solid interaction resulting in mechanical failure of the formation. The orientation, geometry and extent of these fractures depends strongly on the intrinsic mechanical properties and state of stress of the reservoir rock. As one wishes to limit the extension of the created hydraulic fracture to the producing layers of interest as well as extending the fracture as long as required to obtain the desired producing area, a brief description of the mechanical system formed by both reservoir rocks and the layers adjacent to them follows. Reservoir rocks are usually modeled as linear elastic materials. This is not strictly true for rocks in general (18); elevated temperatures and the presence of pore fluid can lead to plastic rather than elastic failure (57, 58). However, for the hydraulic fracturing process, this has proved extremely effective, mostly because of the loading path followed by the material during the fracturing process.

Within the linear elastic framework, reservoir rocks are characterized by a modulus of elasticity, E , and Poisson's ratio, ν . Note that anisotropy of the rock can also be taken into account. Measurements of these properties can be performed on drill cores removed from the earth, although the cost can be prohibitive and care must be taken to preserve the in situ rock conditions (water/fluid content, overburden pressure) when tests are performed (58). Sonic logging tools permit direct interrogation of rocks downhole (59), accomplished by measuring the propagation speed of waves through rock and using the density to recover elastic moduli and Poisson's ratio. These and other dynamic methods typically yield larger values of both the Poisson ratio (60) and elastic moduli (60, 61) as compared to measurements using static methods, as would be done in the lab on core samples in gradual compression. Correlations have been developed to allow the transformation of the parameters obtained under small strain / large frequency (dynamic) conditions to parameters corresponding to large strain / low frequency (static) conditions required for modeling the hydraulic fracturing process. Beyond variations in these values due to test type and configuration, mechanical properties also vary substantially from reservoir to reservoir and

within rock layers forming the reservoirs, as presented in Table 1, and in the references cited therein.

Tensile failure in rocks can be characterized by a fracture toughness K_{IC} . Contrary to the variation in the elastic parameters, the value of the fracture toughness does not vary strongly between rocks and is of the order of 1 MPa $\sqrt{\text{m}}$.

The lithostatic stress determines the orientation and, along with rock properties and the injection pressure, the extent to which the reservoir may be fractured. Lithostatic stresses can be highly anisotropic. These stresses arise chiefly from the weight of rock atop any given point in the reservoir. Typically, the lithostatic stress is highest in the vertical direction (aligned with gravity), with a horizontal stress that varies as a function of depth. Brown and Hoek (64) tabulate the ratio $r = \frac{\sigma_v}{\sigma_h}$ and write the empirical bounds $\frac{100}{z} + 0.3 \leq r \leq \frac{1500}{z} + 0.5$, where z is the vertical depth in meters. Because the density of rocks is not a strongly varying quantity, the vertical stress at the same vertical depth z correlates with $\sigma_v = 27 \times 10^3 z$, with σ_v in Pascals. Complexities arise due to the superposition to gravity of both tectonic activity, thermal and fluid pressure effects on the potentially complex geological structure hosting the reservoir of interest.

2.3. Designing around geological constraints

In order for a hydraulic fracturing treatment to be effective, one wants to create a hydraulic path that is localized to the producing layer targeted, perpendicular to the bedding of that layer (to take advantage of the permeability anisotropy of reservoir rocks). Note that, if the minimum principal stress is vertical, the reservoir is unfit for hydraulic fracturing as horizontal fractures will be favored, which will not drain the reservoir effectively. That hydraulic path should also extend sufficiently away from the wellbore and be of an infinite conductivity compared to that of the targeted layer.

Creating a hydraulic fracture of a desired geometry by designing around the geological constraints is only the first step to attain these objectives. One also needs to ensure that there is enough residual permeability of the hydraulic

fracture once pressure is decreased to start flow from the reservoir. This is achieved by placing solid particles that form a highly permeable layer inside the fracture (proppant pack). Transport of these particles from surface to the fracture, so that a permeable enough pack is formed in front of the target layer, is therefore a critical aspect of hydraulic fracturing. Using fluids that do not damage the flow capacity of the reservoir or that of the proppant pack is also of high importance.

Finally, the - possibly large - variations of both mechanical and flow properties from one layer to the next, along with often poorly constrained variations of these properties along a layer, 100m away or more from the wellbore, put a very strong requirement of robustness on the design of a hydraulic fracture to such variations, and thus on the fluid systems that will be used to create them.

3. MECHANICS OF HYDRAULIC FRACTURING

The mechanics of fracture generation and arrest have been extensively studied in the decades following the first field tests. Since the choice of fluid and pumping rate are the main design levers to achieve the desired fracture geometry in a given geological setting, these studies have highlighted the - often conflicting - requirements placed on hydraulic fracturing fluids. In the following section, we review the phenomena specific to the fluid-solid interactions in hydraulic fracturing.

3.1. Fracture Orientation and Geometry

The fracture orientation conspires with the flow properties of the reservoir rock to determine the efficacy of the fracturing operation. Reservoirs have anisotropic permeability. The permeability is generally smallest through the plane of bedding, thus the desired flow path is perpendicular to the formation bedding. The orientation of hydraulic fractures is predominantly controlled by the *in situ* stresses of the reservoir (61, 89), and fractures tend to open in the direction of the minimum compressive stress. This was clearly demonstrated in a set of laboratory experiments by Hubbert and Willis (90), using a gelatin mold as the reservoir and liquid plaster as the fracturing fluid; after curing of the plaster, fractures of vertical and horizontal orientation were observed consistent with

compressive stresses applied to the mold. A study of the rank ordering of the stresses in the reservoir is thus a pre-requisite to decide if hydraulic fracturing can be applied or not. If the minimum stress is near perpendicular to the reservoir bedding, a hydraulic fracture parallel to bedding will be created regardless of the fluids being used. Thus, flow on the fracture face proceeds through the direction of minimal permeability, limiting the benefit of the fracturing process to increase production.

Pre-existing fractures, joints, and faults can, however, redirect or modify the propagation direction of induced fractures (89, 91--94). In conventional reservoirs, this reorienting effect is not generally strong enough to override the global fracture orientation that is dictated by lithostatic stresses (95). In unconventional reservoirs (often fractured source rocks), the interaction between the hydraulic fracture, the bedding planes, and the pre-existing fractures and faults has tremendous importance, directly impacting the amount of stimulated reservoir area. Whether a hydraulic fracture ignores a pre-existing fracture / bedding plane is not only a function of the geomechanical setting but also of the fracturing fluid (Yew2014), which adds another dimension to the selection of fracturing fluids.

Measurement of hydraulic fracture geometry is challenging, requiring remote downhole, or direct measurement from mineback experiments (95). A wide range of fracture lengths, widths, and heights are reported in the petroleum literature, and, along with treatment and reservoir parameters (50), are highly variable (96, 97); see Tables 1 and 4 for ranges of natural and engineered parameters.

Mineback: Excavation of a fractured formation to measure fracture geometry and extent.

<COMP: PLEASE INSERT TABLE 4 HERE>

The width of the fracture is the smallest lengthscale in the system; It is largest at the wellbore, where the pressure is greatest, and decreases towards the fracture tip. Cipolla et al. (97), in a discussion on the relation between fracture complexity and dimension, provide fracture widths varying from 2.5 mm to 185 mm when fluids ranging from waterfrac/slickwater (basically dilute polymer

solutions with $\eta \sim O(1)$ mPa-s) to gel-like fluids ($\eta \sim O(100 - 1000)$ mPa-s) are used. The fracture length is typically hundreds to more than a thousand meters, and the fracture height may be 50--100 m. The flow may branch during stimulation, forming a tortuous network of fractures. Treatments of the same reservoir with different viscosity fluids have been observed to follow the general expectation that lower viscosity fluids produce longer (and thinner) fractures than thicker gel-based systems (97). The impact of multiple fracture treatments staged along a horizontal wellbore can be inferred from microseismic measurements (98), where the seismic activity is assumed to coincide with fracture progress.

3.2. SIMPLE MODELS FOR HYDRAULIC FRACTURE GEOMETRY

One of the strong peculiarities of hydraulic fracturing is the robust coupling between fluid flow, reservoir deformation, and fracture growth. We discuss here insights provided by the two-dimensional Perkins-Kern-Nordgren (PKN) and radial and Kristianovic-Geertsma-de Klerk (KGD) models. These models are physical and geometrical simplifications of the full transient and three-dimensional fluid--structure interaction problem. These approaches, following a variety of assumptions, provide analytical solutions to the fracture problem illustrating the dominance of dissipation mechanisms and how these mechanisms translate to fracture geometry and fluid design. Regardless of their simplifying assumptions, these models are used to validate hydraulic fracture simulations (65) and inform fracturing job design where applicable.

3.2.1. PK, PKN, KGD, AND RADIAL FRACTURE MODELS

Models by Perkins and Kern (66) (PK), later modified by Nordgren (67) (PKN), a plane-strain model by Kristianovic (68), Geertsma and de Klerk (69) (KGD), along with a radial model (69, 70) all present simplifying assumptions in the state of strain and dissipation, many of which been revisited in later refinements.

In the PKN model, a bi-wing fracture with elliptic cross-section emanates from the wellbore. The height (ellipse major axis) of the fracture is set *a priori* by the height, h , of the formation. bounded by layers that are known to (by experience)

arrest vertical fracture growth. The fracture width, w , (ellipse minor axis) is determined via a plane strain elasticity relation (71):

$$w(x,t) = 2h \frac{(1-\nu^2)}{E} (p(x,t) - \sigma_{\perp}) \quad 2.$$

Here, the pressure in the invading fluid, $p(x, t)$, balances against the far--field solid stress, σ_{\perp} , in the formation and compression of the linear elastic reservoir with Poisson's ratio ν and Young's modulus E . Note that only the difference is significant, $\Delta p = p(x,t) - \sigma_{\perp}$. The plane strain condition in each vertical plane constitutes a crucial simplifying assumption --- the elliptical deformation profiles in the direction of the fracture propagation become uncoupled. Whence, the fracture width (minor ellipse axis) is determined uniquely by the local fluid pressure, reservoir stress, Young's modulus, and Poisson's ratio at each location along the fracture. Furthermore, in this model, fracture behavior at the propagating tip is completely neglected; the fracture toughness does not appear in the solution.

The pressure in the fluid is given by a lubrication approximation to the Navier-Stokes equations, inherently assuming laminar flow prevails. Here, the pressure drop along the fracture depends linearly on the viscosity, η , and flow rate, while depending more strongly on the fracture width, so that,

$$\frac{\partial \Delta p}{\partial x} = - \frac{64\eta}{\pi} \frac{Q}{w^3 h} \quad 3.$$

Note that the flowrate in the fracture, Q , and the width, w , are functions of position and time. The flowrate varies in time and space along the fracture by any combination of (i) a change in pumping rate (or 'flow schedule') at the surface, (ii) leak-off of fluid through the porous fracture walls into the formation, and (iii) accumulation in the fracture as the width varies in space and time.

A statement of mass conservation connects the fracture width (Equation 2), pressure drop (Equation 3), flow schedule, and other system properties to enable prediction of fracture geometry as a function of time:

$$\frac{\partial Q}{\partial x} + q_L + \frac{\pi}{4} h \frac{\partial w}{\partial t} = 0 \quad 4.$$

The first term accounts for the change in volumetric flow as a function of position along the fracture, the second term, q_L is the volumetric flowrate of leak-off per unit fracture length from the fracture into the formation, and the final term accounts for the expansion or contraction of the fracture width as a function of time.

The early work by Perkins and Kern (PK) (citation) assumed $\frac{\partial Q}{\partial x} = 0$, neglecting both changes in fracture area and leak-off. Nordgren (citation) recognized that both leak-off and evolution of the fracture aperture must be included, resulting in a nonlinear partial differential equation for the fracture width:

$$-\frac{1}{128\eta} \frac{E}{1-\nu^2} \frac{1}{h} \frac{\partial^2}{\partial x^2} (w^4) + \frac{4}{\pi} \frac{1}{h} q_L + \frac{\partial w}{\partial x} = 0 \quad 5.$$

This equation is subject to the initial condition $w(x, 0) = 0$, and the distal boundary condition $w(x, t) = 0$ when the coordinate $x \geq l(t)$, where $l(t)$ is the fracture length at arbitrary time. The problem is closed by an additional boundary condition on the flow into the fracture as a function of time: $Q(0, t) = \Gamma(t)$. In the absence of leak-off, the fracture length, width, and pressure difference in the fracture have all been determined analytically for a constant injection rate at the wellbore (72). These results are shown in Table 2. Note that this model corresponds to the limiting case of a height-constrained fracture with a length much larger than the height.

<COMP: PLEASE INSERT TABLE 2 HERE>

Models for other limiting cases include that of Geertsma and de Klerk (69), incorporating work from Kristianovich (68), who developed models that include the influence of the fracture tip to describe the fracture geometry. As before, laminar flow is assumed within a fracture embedded in an isotropic linear elastic solid. Models were developed for both linear (KGD) and radial fractures. The KGD case considers plane strain along the propagation direction and corresponds to the limiting case where the height of the fracture is much larger than its length, whereas the radial case corresponds to the propagation of a hydraulic fracture in an infinite isotropic medium. In each, the fracture width is determined nonlocally

by the fluid pressure and far-field stress. Both are constrained by a smooth fracture closure condition (68, 73), for the radial case: $\frac{\partial w}{\partial r}\Big|_{r=R(r)} = 0$. We present results for both (Table 2), but consider only the radial fracture in detail, as it is the most simple, yet complete, model for practical applications.

The radial model assumes a penny-- or disc--shaped fracture of radius R , emanating from a well of radius r_w , with a local width, $w(r)$. The closure condition places a constraint on the pressure,

$$\int_{r_w/R}^R \frac{sp(s)}{\sqrt{1-s^2}} ds = \sigma_{\perp} \quad 6.$$

This mechanical condition is combined with a description of the hydrodynamics in the fracture. A piecewise pressure profile is assumed in the fracture to satisfy Equation 6. It consists of two regions: a section with logarithmic pressure decay due to viscous dissipation and expanding fracture area along the radius, and a zero pressure region near the fracture tip:

$$\begin{aligned} p(r) &= p(r_w) - \frac{6\eta Q}{\pi \bar{w}^3} \ln \frac{r}{r_w}, \quad r_w \leq r < r_c \\ p(r) &= 0, \quad r_c \leq r \leq R \end{aligned} \quad 7.$$

Here, \bar{w} is the average fracture width. Equation 7 implies a gradual decrease in pressure outward along the fracture followed by a severely dissipative region near the tip, and a zero pressure region to satisfy the tip closure condition. The length scale corresponding to this dissipation region, r_c , is found through the tip closure condition (72), $r_c / R : 1 - 0.368 \left[\frac{E^3 \eta Q}{\sigma_{\perp}^4 (1-\nu^2)^3 R^3} \right]^{1/3}$.

The pressure, fracture width, and radius are determined via an analysis of the strain induced by the pressure, and the hydrodynamic model of laminar flow in a radial fracture (69, 72). These results are shown in Table 2, where both storage and leak-off are neglected. The radial results are semi-analytical (69). The pressure varies weakly with radius, except near the tip through the step function in equation 7.

3.2.2. LEAK-OFF IN THE PKN AND RADIAL MODELS. Substantial complications in the mass conservation statement for both the PKN and radial models are

introduced by fluid leak-off into the formation. The leak-off term q_L is often given by the *Carter law* (originating in an appendix to a conference proceeding by Howard and Fast (74)), and is typically of the form $q_L = \frac{2hC_L}{\sqrt{t-t_{\text{exp}}}}$ when the height is constant. The leak-off rate varies as a function of the elapsed exposure time at the fracture face, $t-t_{\text{exp}}$; this decrease in leak-off is due to the progressive build-up of a resistance in the reservoir to the invasion of the fracturing fluid, be it by simple diffusion gradient or by the formation of a filter-cake, either at the fracture wall or in the near fracture region of the reservoir. In the case of filter cake formation, material is deposited on the wall and/or within the pores (colmatage (37) or clogging may also occur). The leak-off coefficient, C_L , typically depends on the fluid viscosity, the pressure difference between the fluid in the fracture and fluid in the pores, and the permeabilities of the filter cake and reservoir. Models using Darcy's law can be used to construct the leak-off coefficient (37). However, because the process relies on the detailed pore-scale structure of the rock averaged over large surfaces where the variation of such details is unknown, the leak-off rate is typically determined experimentally, both in the field and in the laboratory (75, 76).

The fracture problem with leak-off can be solved numerically in the general case, as originally done by Nordgren (67) for the PK model. These results indicate a matching at long and short times with restricted versions of the full mass transport equations. Nordgren's characteristic time scale, $t^* = \pi^2 \left(\frac{(1-\nu)\eta Q^2}{32C_L^2 hG} \right)^{2/3}$, developed by making the PKN equation set dimensionless, indicates early times $t = t^*$ where leak-off is minimal and late times, $t \gg t^*$, where fluid loss to the formation is significant. Limiting results for the fracture length, width, and pressure following this early/late time approach appear in Nordgren (67) and Geertsma (72) for the PKN fracture.

The rapidly increasing complexity of solving the coupled equations for this non-linear moving boundary problem leads very quickly to fully numerical solutions, be it for the radial geometry (citation), or more complex planar or even fully 3-dimensional geometries (citation). Another approach to the problem is to focus on what is happening at the tip of the fracture. Here, the coupling of the various processes at play yields a series of multi-scale solutions

highlighting mechanisms that control energy dissipation as the fracture propagates.

3.3. Asymptotic Solutions

Following the KGD and radial results outlined above, the fracturing fluid-structure interaction problem has been approached more carefully by Spence and Sharpe (78) through a similarity solution for flow into a penny- (or lens-)shaped cavity. Related approaches have been adopted to explore the formation of dykes (magma-driven fractures) by Spence and Turcotte (79), and also for studying buoyancy-driven propagation (Spence, Sharp, and Turcotte (80) and Lister (81)). These analyses highlight the importance of various physical processes in determining the final geometry of the fracture, and, crucially, the localization of processes at the fracture tip.

Lister (81) notes that when the crack tip is saturated with liquid, large pressures are required to fill the fracture; Instead, it can be supposed that there is a region of fluid lag with inviscid material filling the remainder of the fracture between the liquid front and fracture tip. This distinction between fluid filling and fluid lagging the fracture tip is discussed further by Desroches et al. (82), indicating that fluid approaching the fracture tip implies a solution of zero fracture toughness ($K_{IC} = 0$) in the leading order term, with a stress singularity weaker than predicted by linear elastic fracture mechanics. The dominant dissipation mechanism is due to the pumping of viscous liquid into the fracture tip (Poiseuille flow) rather than the fracture process itself. Solutions with non-negligible fracture toughness, then, require a fluid lag between the advancing fracture tip and the driving fluid.

The strength of the singularity at the crack tip indicates the dominant physical process (83--86). In 2004, Detournay (84) summarized this work for fracturing in impermeable rock, $q_L = 0$, considering both the radial and plane strain KGD fractures. Absent of leak-off, a dimensionless toughness $K = \left(\frac{L_m}{L_k}\right)^p$ identifies if the fracture process is dominated by viscosity ($K \leq 1$) or fracture toughness ($K \geq 4$). The exponent p is 5/2 and 3/2 for the radial and KGD fracture, respectively. Expressions for L_m and L_k , along with the controlling ratio K are presented in Table 3. These scalings show that the KGD fracture is dominated by

the same physics for all time, as determined by the temporal invariance of the parameter K . The radial fracture exhibits a $t^{\frac{1}{9}}$ time dependence, indicating that the dominant dissipation mechanism changes from viscous dissipation to fracture toughness dominated at long times.

<COMP: PLEASE INSERT TABLE 3 HERE>

3.3.1. ASYMPTOTICS WITH LEAK-OFF. Inclusion of leak-off for permeable formations adds an additional dissipation mechanism; relaxation of the impermeability condition permits fluid accumulation in the formation, in addition to the fracture. Lenoach (87) considered leak-off with a viscosity-dominated fracture and Bunger et al. (88) consider leak-off in the toughness-dominated case. Subsequently, Garagash et al. (86) has presented a generalized approach where leak-off, storage in the fracture, along with viscous and toughness losses are all active within the fracture and at the tip. This unifying approach verifies that previous analyses (as detailed in (86)) are obtained as limiting cases of the general problem. Tip asymptotics can be used to indicate the dominant mechanisms in planar fracture propagation. A recent review, highlighting the multi-scale and time dependence of the leading phenomena at play during creation of a hydraulic fracture, is provided by Detournay(citation).

Verifying the regime in which a hydraulic fracture will be propagating is of crucial importance in the design of laboratory tests and small scale experiments to understand the behavior of a hydraulic fracture. Improper scaling of the dominant physics yields results that are purely of academic interest and cannot be practically exploited.

3.4. Transport of solids in hydraulic fracturing

Solid particles are injected to sustain fractures of the desired geometry against closure stresses imposed by the reservoir. Fractures which close before, or during, hydrocarbon production severely limit the efficacy of the fracturing process. Injected particles are transported by the fracturing fluid to minimize the as designed and final fracture dimensions. A straightforward dimensional analysis provides insight to the transport phenomena at play during creation and

propagation of the hydraulic fracture. The geometry of the fracture (w, h, l), the injection rate (Q , or shear rate $\dot{\gamma}$), fluid rheology (chiefly: viscosity η , relaxation time λ , fluid density ρ_f), and proppant properties (solids volume fraction ϕ , radius a , density ρ_s) frame the geometric and dynamic scales, and appropriate dimensionless parameters in the fracturing process. Application of the Buckingham Pi theorem to the variables ($w, h, a, \phi, \rho_s, \rho_f, \eta, \lambda, \dot{\gamma}$) forms a set of six dimensionless groups, two purely geometric ($\Pi_1 = w/h, \Pi_2 = w/a$), three dynamic ($Re_w = \rho_f w^2 \dot{\gamma} / \eta, Wi = \lambda \dot{\gamma}, Sk = \Delta \rho a w \dot{\gamma} / \eta$), and the volume fraction, ϕ . Here, $\Delta \rho = |\rho_s - \rho_f|$ is the density difference. The geometric parameters define the aspect ratio of the fracture (w/h), and the aperture as measured by the particle size (w/a). The relevant dynamic conditions of the flow are indicated by the dominance of viscosity versus inertia through the Reynolds number (Re), the flow timescale versus the fluid relaxation time through the Weissenberg number (Wi), and the importance of particle inertia in unsteady flows via the Stokes number (Sk). The shear rate, $\dot{\gamma}$, varies throughout the fracture and can be computed from the flowrate and the fracture geometry. Here, we choose the simple relation, $\dot{\gamma} = Q w^{-2} h^{-1}$, assuming flow in a slit. Note also that a particle-based Reynolds number can also be defined, $Re_p = \Pi_2^{-1} Re_w$.

Additional physical variables give rise to additional dimensionless groups. These groups will reflect the important physics of the transport process, and the rheology of the complex fluid in the fracture. A *Shields parameter*, for example, captures the ability of a slickwater treatment to transport proppant particles settled along the bottom of a fracture. Further, we could quite reasonably specify a particle settling velocity to compare against the flow velocity, a wall roughness or channel bend radius to compare against the channel width or particle size (53, 99), a thermodynamic energy scale to compare against viscous dissipation around the particle (100), or additional timescales for the fluid rheology or pumping schedule.

3.5. Implications for fracturing fluids

These models highlight expected geometric and dynamical scalings associated with fluid selection. Parameterization of the particle-laden fracturing fluid is often relegated to the magnitude of the viscosity only, occasionally also capturing shear rate dependence (82). The fracture geometry depends weakly on the viscosity: For a PKN fracture geometry, the length scales with the viscosity as $l : \eta^{-1/4}$, width as $w : \eta^{1/5}$, pressure as $\Delta p : \eta^{1/4}$. Fluid viscosity must thus be varied by orders of magnitude to significantly alter length and width. However, assuming simple Darcy flow into the formation, $C_L : \frac{1}{\sqrt{\eta}}$, and Nordgren's crossover time scales as $t^* : \eta^{7/3}$, indicating a strong viscosity dependence for leak-off. Settling also depends strongly on the viscosity; the Stokes settling velocity of a suspension with volume fraction ϕ scales as $u : \eta^{-1} (1 - \frac{\phi}{\phi_{\max}})^5$ (101).

These scaling considerations highlight contradictory fluid requirements, and the need for complex fluids in hydraulic fracturing. The implications are significant: a fluid designed to create large fracture widths (high viscosity) will also exhibit low leak-off resulting in a long fracture closure time. This long closure time will result in proppant settling to the bottom of the fracture, leaving the top part mostly un-propped and it will re-seal during leak-off reducing the net flow of reservoir fluids. Thus, a large viscosity is required during pumping and a small viscosity is required to aid leak-off before proppant can settle. Further examples include: (1) low pressure drop in the pipe and the simultaneous creation of fracture width, (2) low leak-off during fracture creation but fast fracture closure after pumping has stopped with low damage of the rock permeability (102), and (3) transport of solid particles whilst providing maximum permeability of the placed proppant pack to hydrocarbon fluids.

Three main routes exist to optimize between these conflicting requirements. A 'train of fluids' is often used: For example, creation of fracture complexity requires a thin, solids-free fluid. A slurry created by adding solids to the base fluid will not create enough fracture width to admit large concentrations of proppant particles. Therefore, a sequence of at least two different fluids is preferred: a low-viscosity fluid to propagate the leading edge of the fracture, followed by a particle-laden fluid with a larger viscosity. A second route to

address this fluids design challenge is the use of additives to decouple fluid properties: for example, starch particles can be added to control leak-off. A third route is to use fluids whose properties can be altered with time and temperature.

4. FRACTURING FLUIDS

The formulation and properties of hydraulic fracturing fluids vary greatly in response to performance requirements that are set by constraints on the surface and downhole (citation). The choice of fracturing fluid is largely set by reservoir properties, which are strongly variable across different reservoirs (if not within a particular reservoir). Thus, a complex design of the base fluid must be performed for each job. Today, this design is largely empirical and is guided by return on experience.

Fracturing fluid design is constrained. All fluids should: (1) have minimal environmental impact, (2) be easy to mix/hydrate in water with varying ion content, (3) be easy to pump with low pressure drop in the wellbore, (4) be able to travel 1--5 miles through the pipe (typical pipe diameter is 4.5") and transit the perforation (aperture of about 1" diameter connecting the wellbore to the formation) without significant pressure loss and material degradation, (5) be compatible with the formation, (6) generate a high pressure drop in the fracture to create fracture width, (7) transport the proppant, (8) minimize leak-off during fracture creation, (9) (but) maximize leak-off after proppant placement to hasten fracture closure, (10) have low residue content to minimally alter fracture/proppant pack conductivity, and (11) limit proppant flowback during hydrocarbon production. In spite of these (often contradictory) requirements, typical fluids cost less than \$1/liter (citation).

Fluids designed to meet these requirements are complex. Additives, both chemical (such as polymers) and physical (e.g. spherical particles and fibers) endow the fluid with a non-Newtonian response in which the stress varies nonlinearly with the shear rate, and changes also as a function of mechanical (and thermodynamic) variables. Other important non-Newtonian characteristics include strain-dependent stresses, viscoelasticity, thixotropy, and a finite yield stress during shearing, amongst others (105, 106). Importantly, non-Newtonian

fluids can also exhibit normal stresses which are not observed in Newtonian fluids; normal stresses give rise to several important phenomena, including elastic tension along streamlines (105, 107), particle migration (108, 109), and elastic instabilities (110, 111). The various processes described here, and how they manifest in the fracturing process, are shown schematically in Figure 2.

<COMP: PLEASE INSERT FIGURE 2 HERE>

Figure 2 Schematic diagram of the fracturing process (above), and various transport phenomena involved in hydraulic fracturing.

The majority of fracturing fluids used today are aqueous (9). Diesel, alcohol and other organic-based fluids, while desirable since they tend not to cause formation damage and are easily viscosified, tend not to be used as they are hazardous to pump at high pressures and in large volumes (104). Both aqueous and organic fluids can be foamed (or ‘energized’), and all are eventually filled with proppant. Due to the proprietary nature of fracturing fluids, definitive information on components and concentrations are not generally available. Also, the chemical structure of components can be ambiguous. Reviews by Barati and Liang (197), Montgomery (104, 198), Gulbis and Hodge (192), and Ely (103) describe many chemical and physical properties of commonly used fracturing fluids. We summarize material properties and parameters within the supplementary material; follow the Supplemental Material link in the online version of this article or at <http://www.annualreviews.org/>.”

4.1 POLYSACCHARIDE-BASED FLUIDS.

Polysaccharide-based fluids are inexpensive and effective viscosifiers, achieving desired fracture widths and reduced proppant settling. Most of these materials are well-studied (200). Guar is the most common polysaccharide used in hydraulic fracturing (198). It is mostly soluble in water (the residue varies depending on the guar), biodegradable, presents low health concern (it is commonly used as food additive), and can be readily broken. Xanthan gum and cellulosic materials are also used. Xanthan is less-common owing to its higher cost, and materials modified from cellulose (which is itself not soluble in water)

are also used since they have fewer impurities than guar (192) and easily form gels when derivitized (e.g., hydroxyethyl cellulose or HEC) (200). Here, we focus on guar exclusively, as guar-based fluids can answer all of the requirements enumerated above. Consequently, guar based fluids have been the fracturing fluid of choice for decades.

Uncrosslinked, or linear, guar consists of a mannose backbone substituted galactose with an average ratio 1.8 mannose (201). Uncrosslinked guar behaves as a viscoelastic shear thinning fluid (202). The galactose units are commonly modified (192) to change (i) the solvation properties of the guar, (ii) the availability of crosslinking sites and chemistries, and (iii) the performance of the material at elevated temperature. Typical guar concentration is 0.5 wt%, providing viscosity of the order of 0.1 Pa.s at 170s⁻¹ and room temperature (citation). Borate and various metal ions crosslink the linear material to a physical or chemical gel. Crosslinking with borate requires elevated pH (\hat{a} 7.5) (190, 197), resulting in a dynamic ionic bond in which the borate ion connects *cis*-diols on the galactose. The dynamics of these crosslinks have been extensively studied (191, 203), and they give rise to a classical Maxwell-like linear viscoelastic response of the material (190) and a more viscous solution (as compared to the uncrosslinked case) in steady shear that can also display shear thickening above a critical shear rate (204). Crosslinking with metal ions is performed over a larger range of pH (depending on the ion) (192), and gels crosslinked with zirconate can tolerate a higher downhole temperature than with borate (197). Since the metal crosslinked guar forms a chemical gel, shearing disrupts the bonding network and the gels do not completely heal (193, 197). Temperature and pH accelerate the crosslink reaction, while the addition of organic molecules (ligands) can delay it. Pressure also affects the rheology: a decrease of 70% was reported in the viscosity of 0.3wt% hydroxypropyl guar crosslinked with diboronic acid when pressure was increased from atmospheric to 10,000psi (citation). Water chemistry (pH and ion content) must also be monitored, as species like bicarbonate, phosphate, magnesium, iron, and silicate can affect the crosslink density and hydration state of the guar (citation).

Guar is often injected with delayed crosslinker and a breaker to modify the fluid rheology away from surface equipment. Ideally, crosslinking would occur

immediately before the fluid transits the perforation to the fracture: (1) Crosslinking in the wellbore subjects the fluid to high rates of shear (typically $500\text{--}1,500\text{ s}^{-1}$), and may irreversibly damage the crosslinked network. (2) Crosslinked guar is more viscous and more difficult to pump, limiting the pumping rate (citation). (3) Crosslinked guar more effectively carries proppant into the fracture preventing sand from settling to the bottom of the pipe instead of entering the fracture. Even when crosslinking is delayed, the gel is exposed to extremely high shear rates in the jet-like flow through the perforation. Thus, the ability of the gel to heal and recover must be quantified to ensure that the desired rheological properties of the fluid are retained in the fracture. The breaker is added to degrade the guar (crosslinked or uncrosslinked) once the fracture has closed (192). Degradation is necessary to ensure that the ultimate conductivity of the fracture is not hindered by the presence of a viscous fluid, or residual polymer. Oxidizers and enzymes can be used (such as β -mannanase) to cleave the acetyl linkages connecting mannose--mannose and mannose--galactose groups (citation, 205).

Breaker: Additive to reduce fluid structure and viscosity.

4.2 VISCOELASTIC SURFACTANT-BASED FLUIDS.

Micellar surfactants in water constitute the so-called viscoelastic surfactant (VES) class of fluids. Introduced because of simplicity in design, preparation, and breaking requirements, the rheology of VES can be tuned by varying the surfactant concentration and also the amount and type of salt or co-surfactant added to water. Since the surfactant molecules are amphiphilic and relatively short-chained (as compared to polysaccharides), these fluids do not require time to hydrate and generally build structure rapidly upon addition to water. Above the critical micelle concentration, the VES forms long worm--like structures that continuously break and reform, endowing the fluid with a Maxwell--like viscoelastic response (206). This microstructure is perturbed and the fluid thins when flowing through the high shear perforation (like guar). Since the proppant-carrying ability depends critically upon the viscosity, the crucial design

parameter is the fluid rehealing time versus the proppant settling time in the near-perforation region. If the fluid does not recover sufficiently fast, the proppant will settle, accumulate, and occlude the perforation resulting in screenout. The micellar fluid rheology is also strongly temperature sensitive; VES are only typically used in formations below 115°C.

VES has additional benefits: A breaker is not required when the formation contains mobile water. In other cases, VES can be broken by dilution with injected water (citation). Permeability damage is reduced in the formation and proppant pack as the surfactant micelles can dissolve in both aqueous and organic liquids. The strain-hardening extensional viscosity of VES reduces leak-off in formations with small pores. Furthermore, the rheology of such systems can be tuned such that the very high shear rate rheology translates into low frictional pressure losses in the pipe (in turbulent regime), eliminating the need for a cross-linker. The main disadvantage, however, is the low pressure drop they create in the fracture (related to their shear banding). To remediate this issue and ensure enough fracture aperture for proppant placement, VES are generally foamed. While much is known about unladen VES, the current body of knowledge has not enabled a clear link between the structure of the surfactant and the resulting properties of the slurry, limiting the development of this family of fluids.

4.3 ENERGIZED FLUIDS, FOAMS, AND EMULSIONS.

All of the fluids described above can be energized (foamed) with nitrogen and/or carbon dioxide, and can also be added as inclusions in an immiscible organic fluid to build an emulsion (197, 198). Energized fluids and foams have the advantage of reduced water usage, while maintaining proppant transport ability. Furthermore, energized fluids require little or no breaker (the foam life can be controlled by surfactant chemistry), and tend not to damage the formation since a majority of the material (by volume) pumped downhole is gas and not water or oil (192). The rheology of foams and emulsions can be tuned through the fraction of the various phases, distribution of the size of the dispersed phase, and the interfacial tensions of the phases (106). Foams can exhibit viscosities far greater than the liquid phase and can exhibit a yield stress (207): they thus

suspend proppant adequately, provide enough viscosity to create fracture width and can provide adequate leakoff control.

Temperature and pressure variations from the surface to the fracture tip yield changes in the rheology of the foam, and changes in the overall gas volume fraction. Foams must be designed to accommodate these changes, requiring interrogation at variable temperature, pressure, and mechanical stimuli. These materials are difficult to characterize in the lab, chiefly due to slip and related artifacts (120, 207). Furthermore, the structure of foams, although shown to strongly control flow properties in both straight channels and porous media, is difficult to replicate in the laboratory.

Foams have been historically used to decrease the hydrostatic pressure in the wellbore, enabling hydraulic fracturing treatments in depleted reservoirs (with low pore pressure and low reservoir stress). Despite strong potential benefits in reservoirs generally, the use of foams remains restricted to lower pressure regions and low volume treatments. Fluid selection is (again) mainly based on empirical experience, largely because of the lack of systematic understanding noted above. More recently, liquefied gas has been used as the liquid phase (to decrease formation damage) (citation), but this approach has not yet been comprehensively developed.

4.4 SLICKWATER AND WATERFRACS.

The simplest materials used in hydraulic fracturing are the slickwater/waterfrac fluids. These materials target very low permeability reservoirs, where leak-off is minimal and generation of fracture surface area is paramount (citation). These fluids are dilute polymer solutions (usually polyacrylamide (198) or propylene oxide) where the polymer acts as a friction reducer (199) in the pipe, and not a viscosifier. The concentration of these additives is of the order of 0.01-0.1w% (Montgomery2013,Gulbis1999). Salts, clay control additives (to mitigate clay swelling and permeability reduction), and other components may be added in small amounts (104). Mitigating pumping losses are a key issue, as these fluids are pumped at high rates to treat multiple fractures at once (upwards of 120

bbl/min for 4 to 6 concurrent hydraulic fractures). Of all the fracturing fluids, proppant settling occurs most rapidly in slickwater. Since the carrying ability of these fluids is minimal, proppant concentrations $0 \leq \phi \leq 0.07$ are typically used.

The main advantage of slickwater fluids is their low cost, which has enabled economical production of unconventional reservoirs in North America. The major drawbacks are the minimal proppant-carrying ability, minimal fracture width generated, and challenges in fracture containment. Slickwater has been used recently in hybrid jobs, where slickwater is first pumped to create narrow fractures in a complex network, and is followed by a more viscous fluid to widen the fracture and carry higher concentrations of proppant (**citation**). As with other fluids, the design of slickwater and hybrid jobs is largely empirical.

4.5 FIBER-BASED FLUIDS

Fiber-based fracturing fluids are a relatively recent innovation, developed to improve proppant transport (**citation**). Benefits of fiber-based fluids lie in their ability to partially decouple proppant transport ability from the base fluid rheology. This enables independent control of viscosity to achieve fracture width, pumping rate, and invasion of proppant into the fracture. The addition of fibers is particularly useful for high-temperature application, where maintaining the fluid viscosity with chemical modifiers can prove difficult. Furthermore, fibers strongly reduce the effect of proppant diffusion during transport, both in the pipe and in the fracture. This ensures that the schedule of proppant concentration imposed at the surface is preserved along the wellbore and into the fracture. Specifically, this feature permits the pulsing of proppant-laden slugs with particle-free fluid, enhancing the post-treatment fracture conductivity.

Despite the success and wide use of fiber-based fluids, fundamental understanding of these systems are scant. This knowledge gap is likely the largest among all commonly-used fracturing fluids. Design empiricisms rule: systems adjudged to be adequate typically consist of a polysaccharide (guar) base fluid, flexible fibers in the semi-dilute to dense regime, and proppant. The properties of these systems have not been comprehensively studied in the literature. Typically, simpler canonical systems have been studied, focusing on

rigid fibers in Newtonian matrix fluids, in the absence of other solid particles (i.e., proppant). Despite their relative simplicity, studies demonstrate that fiber based suspensions have unique properties.

4.6 THE NEED FOR MEASUREMENTS

Fracturing fluids operate at high shear, high temperature, high pressure, are laden with dense spherical particles, long and flexible fibers, and can be foamed. As experimental requirements, any one of these is a challenge. In conjunction the experimental task is daunting. Regardless, measurement and theory are required to enable model-based fluid selection and design, rather than Edisonian approaches guided by empirical experience. Parametric exploration of the rheology of these materials, and measurements at high pressure and high temperature under both steady and dynamic forcing would advance our understanding substantially.

5. COMPLEX FLUIDS IN HYDRAULIC FRACTURING

Fracturing fluid characterization is essential for both the design of new fracturing fluid systems and for the optimization of hydraulic fracturing treatments. It can be decomposed into two parts: Firstly to provide an understanding of how the fluid answers the tremendously complex and often contradictory engineering requirements described in sections 3.5 and 4.1, and result in a knowledge base that forms the basis for design and optimization. The second goal is to inform fracturing models that are used to design and optimize hydraulic fracturing treatments. Tests associated with these goals should (ideally) be standard protocols as they be repeated for any new fluid formulation and, to some degree, for any new fracture design. Such standard tests should be guided by the knowledge developed in the former, more detailed characterization. In this section, we consider the rheology of the base fluids (that also constitute the pad) and that of the particle laden fluids (slurries).

Propping the fracture is an essential outcome of the hydraulic fracturing treatment. In addition to initiating and propagating the fracture, the fracturing fluid must transport and enable an even distribution of particles to the instant of fracture closure. Avoidance of particle settling is thus a major consideration in particle transport – settling results in inhomogeneous placement along the length and height of the fracture, and can cause the fracture to close in a wedge-like shape (124). Transported particles must navigate the vertical and horizontal wellbore, the perforation, the fracture width and height, as well as the bifurcations and tortuosity encountered along the fracture length; Nearly every point along the flow path is an opportunity for occlusion by the proppant.

5.1. Rheology and Rheometry of Fracturing Fluids

The response of the fluid stress as a function of shear rate and temperature is an important input to fracturing models (65) both to determine the geometry of the fractures and the power required to pump fluids downhole. As described in section 1, most codes model the slurry as a continuous fluid using a shear-thinning viscosity model (citation). Oilfield operators argue that determining a precise fluid viscosity is unnecessary owing to the weak dependence of fracture dimensions on the viscosity, as described in section 3.5. Even as the viscosity strongly influences leak-off, knowledge of the exact value may not be critical, as the necessary rock properties are often *a priori* unknown and independent tests must be carried out to characterize leak-off. The flow behavior of suspensions, on the other hand, is critical to optimize proppant transport, as described in the next section.

Complex fluids exhibiting a dependence of stress on shear rate only (absent material memory and strain dependence) are classified as generalized Newtonian fluids (GNFs). We provide a set of commonly used GNF constitutive laws in Table 5. Common Fracturing fluids are also often viscoelastic in character exhibiting normal stress differences, memory, and other effects. More accurate descriptions and fits can be obtained by viscoelastic equations of state like the Giesekus or K-BKZ models accounting for memory and strain effects (105, 107, 112), but these models are more challenging to implement analytically and computationally.

<COMP: PLEASE INSERT TABLE 5 HERE>

5.1.1. FRACTURING FLUID RHEOMETRY. The bespoke design of fracturing fluids, coupled with the absence of universally predictive structure–function relationships, necessitates the direct measurement of complex fluid properties. Rheometers are used to measure the stress response to well-defined shearing motions generating (i) steady applied rate or stress, (ii) steady-state and transient response to oscillations in amplitude and frequency (capturing both linear and nonlinear viscoelastic material response), and (iii) startup and cessation of stress or strain. Pressure-driven flows through slits, pipes, and constrictions can also be used to measure properties of pure fluids and suspensions under kinematics conditions closer to those downhole. Fewer commercial instruments exist to probe the extensional properties of complex fluids (useful for flow through perforations and porous media), with most being custom-built (114). This is mainly due to difficulties in eliminating no-slip boundary conditions and in controlling the kinematics and flow strength (115).

Rheometric tests quantify several material functions: the shear viscosity $\eta(\dot{\gamma})$, normal stress differences $N_1(\dot{\gamma})$ and $N_2(\dot{\gamma})$, viscoelastic storage and loss moduli in response to imposed oscillations (at frequency ω and amplitude γ_0) $G'(\gamma_0, \omega)$ and $G''(\gamma_0, \omega)$, compliance $J(t)$, relaxation modulus $G(t)$, extensional viscosity $\eta_E(\dot{\epsilon})$, or CaBER relaxation time (116), among others (105, 117). Measures of linear viscoelasticity are relatively well-defined and can be reproduced across different techniques, whereas numerous alternate measures of nonlinear viscoelasticity can be made and are often chosen to closely mimic the process that the data will inform.

Despite the availability of commercial fluids-characterization tools, care must be taken to properly execute experiments and interpret results. Several effects, generic to all complex fluids, are present and should be anticipated when performing experiments: **(1) Slip and shear banding;** Slip, adhesive failure, and shear banding on the measuring instrument will compromise results.

Unavoidable in many cases, they can be quantified with direct measurement (119), or inferred by repeated measurements varying the gap between parallel

plates (120). **(2) Instabilities;** Strongly viscoelastic fluids can exhibit spurious, or apparent, shear thickening owing to elastic (117, 121) or inertial instabilities (117). Predictive theories for the onset of instabilities are available in most geometries (111, 117) as a function of material properties and experiment parameters. **(3) Particle migration;** Gradients in shear rate and elastic stress, and nonzero streamline curvature promote particle migration; Most rheometric shear flows have circular streamlines. The measurement time should be short as compared to the time scale for particle migration. **(4) Boundary effects;** In addition to slip, rheometer boundaries can induce ordering and disturb the orientation of nonspherical particles (122). The geometry should be large as compared to the particle size, typically greater than 10 particle diameters (117). **(5) Measurement at elevated temperature and pressure;** Reproducing downhole temperature and pressure introduces problems in evaporation and sample containment, and is generally available only for steady measurements above 100°C. **(6) Repeatability and mixing;** Care must be taken to properly hydrate and mix additives in fluids (123), and also that the materials have not degraded or biologically decomposed when used over several days. In all cases the material should reflect the state of hydration/homogeneity used in the actual process.

In addition to shear, pressure-driven, and extensional rheometry, simpler index-based measures of rheological properties are also used (117). Common outside of academic laboratories, indicial measurements provide a simple diagnostic metric corresponding to pressure drop or torque in response to a poorly-defined flow field. They are difficult to connect to models and microstructural material theories. Such indices are useful, however, in that they can provide rough quality-control checks that the target fluid will answer all requirements adequately.

5.1.2. RHEOLOGY OF PARTICLE LADEN FLUIDS. Proppants are transported to the fracture to maintain conductivity after cessation of the flow/pressure from surface. Proppants are classified by their strength (i.e. how much fracture closure stress the proppant pack can sustain), average grain size and statistical distribution, the quantity of fines (that may reduce the pack permeability),

roundness and sphericity, and density. Sand is commonly used, as are other materials like resin or polymer-based particles, ceramics or carbides, and hollow glass spheres may be added as proppants or as components in a mixture. As mentioned in Section 4, fibers are also added, not as proppant per se, but to modify fluid rheology and decrease settling rates (132) although fibers can also mechanically bridge the fracture. At the pump, the volume fraction (vol/vol) of the flowing suspension can range from 0% to 5% for waterfrac/slickwater applications (133), and upwards of 20% in treatments using more viscous fluids (134); these values increase along the fracture due to leak-off. The size of the proppant particles used is guided by the expected width of the fracture during pumping, and proppant concentration by the desired width during production which is directly linked to the final amount of proppant per fracture area after closure. Particle size is specified using the mesh size of a sieve, larger mesh values correspond to smaller particles. 40/70 mesh sand is commonly used, corresponding to particles with approximate diameters between 210 and 420 μm .

Volume fraction: Typically, $\phi = \frac{V_{\text{solids}}}{V_{\text{total}}}$. Industrial specifications of solids concentration are typically in pounds added per fluid volume; pounds per gallon, ppg, is common.

Addition of particles will ‘thicken’ the fluid due to the additional viscous dissipation that arises at the no slip boundary of each particle. This effect is well-characterized in Newtonian fluids, and has been worked out to first order by Einstein and to second order by Batchelor: $\eta / \eta_s = 1 + \frac{5}{2} \phi + C_2 \phi^2$. The constant C_2 depends on the flow type and the Péclet number (101, 135). At larger concentrations no analytical solution is known; several empirical fits to data for monodisperse suspensions of spheres exist (100, 135), typically of the form given by the Krieger-Dougherty equation (Table 5), Where ϕ_{max} is the maximum packing fraction for spheres in the suspension, and $[\eta]$ is the specific viscosity (a function of particle shape); with $[\eta] \phi_{\text{max}} \approx 1.6$ for spheres. The viscosity diverges at ϕ_{max} , and is very sensitive to small changes in particle concentration except for small values of ϕ / ϕ_{max} .

Similar results follow for multimodal suspensions (136, 137). Chang and Powell (137) review and study the rheology of bimodal suspensions of spherical particles; At an identical total volume fraction, the viscosity of a bimodal suspension is lower than a monodisperse suspension, and this difference grows as the mismatch between particle sizes becomes larger. The same result follows for multimodal suspensions, as described by Farris (136), for whom the effect is named. This result is significant for fracturing – solids loading can be increased without sacrificing pumping power by selectively controlling the size distribution (citation).

The dynamics of a fibers are drastically different than spheres, even in the dilute limit. Jeffery (138) solved for the motion of an isolated, axisymmetric ellipsoid in a shear flow, demonstrating that particles orbit and sweep out a volume dependent upon the initial orientation of the ellipsoid. Rigid fibers fit well into this framework --- they can be modeled as ellipsoids where the aspect ratio $r_p = \frac{\text{major axis}}{\text{minor axis}} = \frac{L}{d} ? 1$. Suspensions of fibers may be classified according to the nature of interactions among the fibers (122). Suspensions are considered dilute when the distance between fiber centers avoids contact and hydrodynamic interactions are negligible ($>L$), satisfied when $\phi \propto r_p^{-2}$. Incorporating hydrodynamic interactions with minimal particle--particle contacts establishes the semidilute window, $r_p^{-2} \propto \phi \propto r_p^{-1}$; everything above this window ($\phi > r_p^{-1}$) is considered the concentrated regime, in which interparticle interactions between fibers contributes strongly to the fluid rheology.

In the dilute regime, fibers viscosify the fluid *less* than spheres at the same volume fraction, as they spend most of their existence aligned with the flow during their Jeffrey orbit. This is communicated quantitatively through the specific viscosity of fiber suspensions (ellipsoids with $r_p \rightarrow \infty$), found by Jeffery to be $[\eta] = 2$; similar results have been computed by many others (see Table 1 in Petrie's review on fiber suspension rheology (139)). In the semidilute regime, for the same solids concentration, suspensions with larger r_p will exhibit a *larger* viscosity (140). The aspect ratio and concentration are not the only controlling parameters, however, as the fiber length has been shown to influence the viscosity for fixed r_p and ϕ (140); this length effect is presumably due to the

presence of flocs formed by adhesive contacts and between the particles, and is mitigated at large stresses (141). Other effects, like the orientation and buckling of fibers contribute strongly to suspension rheology and stress growth in the start-up of shear flows.

The volume fraction required to form a mechanical contact network (ϕ_c) decreases strongly as the particle becomes less spherical (142), scaling as $\phi_c : 0.6r_p^{-1}$ to lowest order in the aspect ratio for prolate ellipsoids. This suggests that mechanically--connected networks of fibers can be formed at smaller volume fractions as compared to spheres, where $\phi_c : 0.28$ (142). This result has been exploited to fiber-based fracturing fluids (132).

Shear thickening is also observed in some fracturing fluids. Particle-laden flows will exhibit an increase of viscosity with increasing shear rate provided the suspension is above about 40% solid particles; the magnitude of shear thickening, becomes increasingly severe as the concentration of particles increases (100, 113). The shear thickening described here is dynamic and distinct from the viscosity enhancement that arises from simply adding particles to a fluid (where no-slip on the particle surfaces enhances the local viscous dissipation rate as the volume fraction of the solid phase increases). Recent reviews by Denn and Morris (143), Stickel and Powell (135), and the monographs by Guazzelli and Morris (101) and Mewis and Wagner (100), provide extensive reviews of suspension rheology and mechanics for non-Brownian particles.

Describing the rheology of filled non-Newtonian liquids is significantly more complex due to the variety of fluid behaviors and constitutive laws available --- experiments and theory required to classify and describe each are extensive and difficult. Barnes (144) provides a recent review of particulate suspensions in non-Newtonian fluids. While generalized statements are difficult to justify for particles in non-Newtonian matrix fluids, many viscoelastic particle-laden systems display a magnification of viscoelastic effects in the fluid phase owing to the increased local shear rate resulting from the presence of solids. This effect has been explored by Gleissle and coworkers, showing that a shift factor dependent on volume fraction can accurately predict material functions of many particle--fluid systems (145, 146). The non-Newtonian rheology of a suspension

in a yield-stress fluid has recently been explored by Dagois-Bohy et al. (citation).

5.2. PARTICLE MIGRATION

The oilfield community started to investigate particle migration under fluid flow in the late 1980s (citation). There was a concern that settling would be accelerated if particles migrated to the center of the fracture width, both under flow and at rest. Another concern is the increased velocity of the particles in the fracture midplane, which may reach the tip of the fracture and cause bridging [cite?]. Particle migration arises from several forces, present in both Newtonian and non-Newtonian fluids.

The migration of a dilute suspension of particles in low Reynolds number flow was studied by Segré and Silberberg (147, 148), observing particle concentration at ≈ 0.6 of the tube radius. Theory applied to migration in this dilute limit generally agrees with experiment (149, 150), revealing that a balance between an inertial lift force and a repulsion near the wall is responsible for the equilibrium distribution. At higher Reynolds numbers the concentrate annulus moves towards the wall. Matas et al. (151) characterize this behavior for $67 \leq Re \leq 2400$, and observe a second equilibrium position at the tube center when $Re \geq 600$, which was also a function of particle size. Asmolov (152) provides a theoretical description of particle migration in this limit, but does not predict the second equilibrium region.

Non-dilute suspensions under flow exhibit particle migration owing to gradients in viscosity, shear rate, and particle concentration. These effects were observed by Acrivos and coworkers (162, 163) in the rheometry of suspensions, where particles migrated to regions of low shear rate in a Couette cell. Similar effects have been observed in duct flows of suspensions (citation), showing the transport of particles toward the channel centerline and a blunting of the velocity profile with increasing volume fraction. Oh et al. (citation) measured that the center region could reach $\Phi=0.64$, the random close packing limit of monodisperse spheres, demonstrating that these domains progressively compact.

The driving mechanisms are well-described in constitutive models by Phillips et al. (164) and Mills and Snabre (165). Phillips et al. use a diffusive flux approach, in which particle concentration plays the role of an active scalar which diffuses as a result of several gradient terms. Particles migrate away from regions of high collision frequency, corresponding to regions of large shear and/or particle concentration, $J_{p-p} : -\nabla(\phi)$. Further, particles migrate away from regions of large viscosity; a colliding particle pair in a region of inhomogeneous ϕ (and therefore $\eta(\phi)$) will exhibit a net displacement toward the region of lower viscosity, $J_{\eta} : -\nabla(\eta(\phi))$. Mills and Snabre (165) predict migration based upon gradients in the stress (rather than upon local particle collision rates as considered by Phillips et al. (164)). Recent analyses (166, 167) employ the frictional rheology model of Boyer et al. (168). Both capture the migration of particles toward the center and resultant blunting of the parabolic flow profile. The distribution of particles has been measured (154) and demonstrates the applicability of these models (166, 168).

In viscoelastic fluids, migration can also occur due to an elastic force orthogonal to the direction of shearing. Elastic (normal) forces are largest in regions of large shear rate; gradients in the shear rate across a particle will result in a net elastic force pushing particles to regions where the gradient in shear rate--and resulting elastic stress gradient--is small. In a Poiseuille flow, particles therefore migrate toward the apex of the velocity profile. This effect has been described in a Second Order Fluid by Ho and Leal (108), and experimentally verified by Tehrani (155). Leshansky et al. (109) perform experiments showing focusing in a microdevice, and derive an identical scaling argument as Tehrani for the migration speed, $v \sim -\frac{a^2}{6\pi\eta} \frac{\partial N_1}{\partial x} \frac{\partial \dot{\gamma}}{\partial y}$ where N_1 is the first normal stress difference, and y is the cross-channel dimension. Tehrani also illustrates the importance of the shear rate, observing the absence of particle migration in the plug-flow region of an elastic solution.

Elastic forces also give rise to subtle and surprising particle--particle interactions. Feng and Joseph (156) studied these interaction effects in torsional shearing flow between parallel plates, observing the formation of annular rings using both spheres and fibers in a polyox solution; interestingly, fibers can rotate inward or outward depending if the fiber is free to rotate about the vorticity axis

or if it is stuck in a state aligned with the flow. Lim et al. (157) explored these effects in rigid microfluidic devices at high Reynolds number. Iso, Koch, and Cohen (158, 159) study semidilute concentrations of fibers in fluids of varying elasticity in a Couette flow and highlight the competition between particle--particle hydrodynamic interactions and elasticity-driven fluid effects. Fibers tend to align toward the vorticity axis in weakly elastic fluids ($Wi = 1$, 'log rolling'), but become more random when the fiber concentration increases, whereas fibers tend to align in fixed orientations (along and slightly offset from flow) in strongly elastic fluids ($Wi \hat{=} O(1)$).

The nature of the non-Newtonian fluid determines the nature of the particle-particle interactions. In a study of particle--particle interactions in a shear flow, Snijkers et al. (160) detail these differences in both the rheology of the fluid and resulting particle trajectories. A unifying theme of elastic fluids, however, is the tendency to exert an inward compressive force normal to curved streamlines, which can explain particle--particle kissing, streamwise orientation of elongated particles, and particle motion toward walls (161). Understanding these effects better may provide insight into the trajectory of proppants in a complex network of fractures, e.g. which components of a proppant will effectively move from the main fracture into an adjacent fracture making a sharp angle with it.

A myriad of these migration mechanisms have been implicated, along with fluid phase leak-off, in suspension flow and tip screenout in the fracturing process. Chekhonin and Levonyan (169) consider suspension flow in a KGD fracture to establish parameters for tip screenout, computationally modeling the suspension concentration along the fracture length only and dividing the flow into regions that are Poiseuille--like ($\phi < \phi_{\max}$) or Darcy--like ($\phi = \phi_{\max}$). Dontsov and Peirce (170, 171) consider both KGD and pseudo 3D fracture models using a frictional rheology model incorporating slip and settling within a fracture. These results establish that proppant screenout (owing to leak-off driven concentration and settling) can drive strong changes in fracture geometry.

5.3. PARTICLE SETTLING

Proppant settling occurs under both dynamic (during flow) and static conditions. Elimination and mitigation of settling is important to ensure a uniform distribution of proppant particles along a fracture; settling will occur unless the proppant and fluid are density-matched. The presence of boundaries, non-Newtonian base fluids, and other particles all modify the settling of proppants within a fracture.

Two distinct regimes of settling behavior are recognized to exist during the fracturing process: (i) rapid settling coupled with successive rolling, saltation, and resuspension of the sediment bed, and (ii) gradual settling absent a moving sediment bed. In all cases descriptions of particle transport are dominated by empirical correlations except where exact zero Reynolds number results can be applied (172). The sediment transport described in (i) was recognized in 1959 by Kern et al. (173). The relevant physical parameters are the shear stress acting on the grain compared to a gravitational stress, $\theta = \frac{\tau_p}{2ag(\rho_s - \rho_f)}$ (the Shields parameter), and the particle Reynolds number Re_p . Sediment transport is anticipated in waterfrac/slickwater operations (174), the onset of which can be predicted using a Shields plot (175). This process, specific to the fracturing configuration, was studied in more detail by Patankar et al. (176) and Wang et al. (177) who develop power law correlations to describe static and fluidized bed heights in a fracture.

Proppant particles in more concentrated polymer and gel--like fluids will experience reduced settling and flow rates, owing to the enhanced viscosity of the fluid. Particles sedimenting experience frictional drag and a variety of particle-- and wall--driven retarding phenomena prior to deposition in a pack on the bottom of the fracture. The flowing particles experience all of the migration forces described above, and additionally feel a body force due to gravity. This body force drives particles downward to the fracture floor; conservation of mass and the migration of neighboring particles drives a recirculation (101), which retards the settling speed. A result by Richardson and Zaki (101, 178), $u = u_s(1 - \phi)^{-n}$, predicts the decrease in the Stokes settling speed, u_s , as a function of volume fraction, ϕ . The power-law exponent $n \approx 5$ is determined empirically (101, 179). In this expression the volume fraction is uncorrected for maximum packing and will lose predictive capability at higher particle

concentrations; Chang and Powell (180) introduce a modified version of this law to correct for the maximum packing limit, $u = u_s (1 - 0.05\phi / \phi_{\max})^{5.3 \pm 0.2}$.

Particles sediment differently under both static and flowing conditions when embedded in non-Newtonian fluids (Figure 3). This aspect of fracturing fluids has been studied in the petroleum industry since at least the 1970s. Some efforts have focused on the development of empirical correlations to predict settling of particles in shear-thinning or Carreau-like fluids (172, 181, 182), while others have tackled the static versus dynamic settling problem directly. Novotny (183) performed experiments in a Couette cell using a polyacrylamide fluid and observed enhanced settling when the suspension was sheared, even remarking that particles which appeared to be completely stagnant in a ‘highly non-Newtonian fluid’ will sediment once sheared. McMechan and Shah (184) studied the static settling of particles in a 14-ft tall cell using a variety of metal and borate crosslinked guar fluids, and remarked on the suspending ability of the various crosslinkers. In particular, they noted variability in the time for the proppant-carrying capacity of the materials to recover following pumping and shearing to fill the cell.

<COMP: PLEASE INSERT FIGURE 3 HERE>

Figure 3 Single and many particle effects under settling and shear. (a) Trace of the conformation tensor surrounding a settling particle under shear (128) illustrating regions of pronounced molecular stretching length and regions of elastic stress (Reprinted from Journal of non-Newtonian Fluid Mechanics, Vol 197, Padhy et al., Simulations of a sphere sedimenting in a viscoelastic fluid with cross shear flow, 48--60, Copyright 2013, with permission from Elsevier). (b) Settling of particles with stationary ($\dot{\gamma} = 0$) and sheared fluid (Reprinted with permission from Tonmukayakul et al. (129) Copyright 2008, AIP Publishing LLC) illustrating increased drag on a settling suspension when $\dot{\gamma} > 0$. Many particle interactions induce chaining in a periodic triangle wave Couette flow (130) (Reprinted from Journal of non-Newtonian Fluid Mechanics, Vol 117, Won et al., Alignment and aggregation of spherical particles in viscoelastic fluid under shear flow, 141-146, Copyright 2004, with permission from Elsevier) (c) and under settling (131) in (d) (Reprinted figure with permission from Mora et al. Physical Review Letters 95:088301 2005 Copyright 2015 by the American Physical Society).

Extensive experiments using Couette and slit-flow geometries indicate that fluid elasticity is required to retard settling in dynamic conditions, and that the microstructure of the non-Newtonian fluid and shear conditions are essential to determine the ability of the fluid to retard (or enhance) settling under dynamic

conditions. Gheissary and van den Brule (185, 186) study Newtonian, constant viscosity elastic fluids (i.e., Boger fluids), and a viscoelastic shear-thinning fluid. They find that settling is retarded in the Boger and viscoelastic shear-thinning fluids, with the highest elasticity fluid (as measured by the first normal stress difference) giving slowest settling rates, even if the fluid shear-thins. They also comment on flow-induced anisotropy: flow of fluids that shear-thin due to alignment of polymer molecules (HEC for example) will exhibit less settling than shear-thinning fluids that shear-thin due to breakdown of the internal microstructure (such as carbopol). Tonmukayakul et al. (129) tune the amount of borate crosslinker in a guar gum solution, and show that at lower concentration of crosslinker (corresponding to a weakly elastic, shear-thinning fluid), the settling rate increases with shear rate, whereas at higher concentration of crosslinker (a viscoelastic gel-like fluid), the settling rate markedly decreases with shear rate. The experiments by Hu et al. (187--189) focus on differences between borate and metal crosslinked guar, with high concentration crosslinkers. Microstructurally, these materials differ in the nature of the crosslinks: Addition of borate introduces dynamic crosslinks, enabling the material to heal following breaking (190, 191), while the metal-based crosslinks are irreversible and do not reform once broken (192, 193). Settling in each gel under shear then proceeds differently. Settling in the borate crosslinked guar is retarded by imposed crossflow, and both N_1 and η increase as a result of shear. Conversely, minimal settling is observed in metal crosslinked gels at low or zero shear rates, but, above a critical shear rate, the proppant carrying ability of the material is lost and substantial settling is observed.

The retarded settling of particles within non-Newtonian fluids subjected to shear flow has also been examined computationally and analytically. Tanner, Housiadas, and coworkers (194, 195) have studied the drag on a sphere, pointing to an added elastic stress contributed by the distortion of streamlines induced by the settling particle. Padhy et al. (128, 196) perform computations using FENE-P and Giesekus models to mimic a Boger fluid and a guar solution lightly crosslinked with borate (which shows enhanced settling with increasing shear rate in experiments (citation)). Their computations capture the retarded settling observed in experiments, and also capture the result of Housiadas and Tanner

(194) when walls bounding the flow are removed (infinite domain). Padhy et al. (128, 196) examine individual contributions to the total drag and highlight the competition between elasticity (which increases frictional drag) and shear thinning (decreasing drag). They further note that these phenomena scale differently with polymer concentration, and a optimum concentration exists to minimize the sedimentation speed (196).

The complexity of the nonlinear transport phenomena presented here, compounded with complex temperature, pressure, and shear rate history experienced by a slurry during a hydraulic fracturing treatment, in very conservative assumptions in the design and optimization of hydraulic fracturing treatments.

5.4. BRIDGING AND JAMMING EVENTS

Proppant occasionally becomes stuck, or jammed, during fluid injection. This event is known to occur in both the immediate vicinity of the wellbore and close to the fracture. In narrow apertures, these events may start after the formation of a localized force chain or bridge of particles between the fracture walls. Bridged proppant will redirect the local flow around it, and may either become destabilized (unbridged) or grow. Jammed proppant creates a region of locally magnified hydraulic resistance, growing until the pressure drop through the jammed pack exceeds the pumping pressure at the well surface, termed screenout. Screenout events constitute failure if they happen in the near wellbore, as they halt the fracturing job, and require well cleaning. On the contrary, these events can be planned to happen further down the fracture, ideally with limited pad left if the fracture design is perfect, stopping the fracture from growing and allowing more packing of the fracture with proppant than would be possible without this event taking place (“tip-screenout” technique).

Bridge: A localized group of stuck particles, around which other particles and fluid may flow.

Jam: A particle pack in the fracture with enhanced hydraulic resistance accumulating proppant but permitting fluid to filter through.

Screenout: Pumping failure caused by massive hydraulic resistance due proppant obstructing the flow in the well or fracture.

Control and prediction of bridging and jamming are critical to avoid and program screenout. Jamming is predicted sufficiently well when solids reach a maximum volume fraction. As for bridging, fracture models typically predict bridging events when the ratio of slot width/particle size is below 3 to 10 – a number that is tuned by experience, but consistent with bridging studies in granular (125) and non-Brownian (127) systems, which reveal the stochastic nature of the process. The difficulty partially comes from the physics controlling the phenomena: bridging is a discrete event governed by probabilities of occurrence that are not resolved by continuum models nor accurately predicted by discrete models. The sphericity and frictional properties of the particle certainly play a role and the pronounced effects such properties have on suspension rheology are now beginning to be considered (citation). In addition to tip-screenout designs, the dramatic consequences of bridging/jamming have been advantageously applied in ‘diversion techniques’, where a fracture is locally jammed and the particle-laden fluid is redirected to stimulate other parts of the oilfield reservoir (citation), or in the form of a fluid loss control additive where operators want to locally eliminate leak-off to the formation (citation).

Volume fraction: Typically, $\phi = \frac{V_{\text{solids}}}{V_{\text{total}}}$. Industrial specifications of solids concentration are typically in pounds added per fluid volume; pounds per gallon, ppg, is common.

6. CONCLUSIONS

Hydraulic fracturing, and hydrocarbon recovery operations in general, present a grand challenge for non-Newtonian fluid dynamics. Complex fluids used in fracturing operations encompass essentially all aspects of rheology and complex fluids --- linear and nonlinear viscoelasticity, physicochemical gelation, transport and orientation of spherical and fibrous particles, control of slip/shear banding, migratory and many-body particle interactions are present in abundance and intimately affect the ultimate hydrocarbon recovery that can be achieved. These

manifold effects match the spectrum of specific fluids employed to target rock formations of widely varying permeability, porosity, mechanical properties, depth, and temperature. Understanding and integrating non-Newtonian fluid mechanics with reservoir attributes and statistical variability is a key contributor to improved resource recovery, and constitutes an intellectually rich and economically relevant research problem for years to come.

SUMMARY POINTS

1. Hydraulic fracturing is a key stimulation technique in hydrocarbon recovery, primarily targeting reservoirs of low intrinsic permeability.
2. Hydrocarbon reservoirs exhibit widely variable mechanical, chemical, and environmental properties.
3. Particle-laden complex fluids with a wide range of rheological characteristics are used to open and sustain hydraulic fractures.
4. Quantitative descriptions of non-Newtonian fluid mechanics are essential to predict complex fluid (and hence fracturing) performance.

FUTURE ISSUES

1. Improvement and development of constitutive models (particularly nonlinear) for physical gels and supramolecular systems to model crosslinked and associating complex fluids.
2. Mechanistic descriptions of particle--particle and particle--fluid interactions under pressure (or shear) driven flow with superposed gravitational settling.
3. Material property measurements under representative temperatures and pressures and shear rates to match downhole conditions.
4. Development of laboratory (milli- and microfluidic) scale experiments illustrating reservoir--fluid--proppant interactions in flow regimes representative of hydraulic fracturing dynamics, geometry, and materials.
5. Creation of coupled, multiscale, time-dependent simulations capturing complex fluid rheology and particle transport during pumping, fracturing, leak-off, and fracture closure.

DISCLOSURE STATEMENT

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Table 1 Reservoir properties for shales, sandstones, and mudstones

Quantity [units]	Range ⟨representative value⟩
Poisson's ratio, ν [-] (18, 61, 62)	0.1--0.5 ⟨0.25⟩
Young's modulus, E [GPa] (18, 61)	5--95 ⟨30⟩
Toughness, K_{IC} [MPa- \sqrt{m}] (61, 63)	0.17--1.73 ⟨1.0⟩
Permeability, k [D] ^{†)} (14, 18)	
Shale gas	: 1×10^{-9}
Tight gas	$< 0.1 \times 10^{-3}$
Conventional	$< 500 \times 10^{-3}$
Porosity [%] (14)	6-- > 25 ^{‡)}
Formation depth, l_v [m] (14)	1200 ^{M)} --4100 ^{EF)}
Formation thickness, h [m] (14)	6 ^{F)} --180 ^{B)}

^{†)} $1D = 9.86 \times 10^{-13} \text{ m}^2$; ^{‡)} Source rock and tight gas formations are at the low end of this range, conventional formations at upper; Depth and thickness range extremes from: ^{B)} Barnett Shale, ^{F)} Fayetteville Shale, ^{EF)} Eagle Ford Shale, ^{M)} Marcellus Shale.

Table 2 PK, PKN, KGD, and radial model predictions for an impermeable formation ($q_L = 0$), following (69, 72)

Model	$l(t), R(t)$	$w(0,t)$	Δp
PK ^{a)} , PKN ^{b)}	$C_1 \left(\frac{EQ^3}{2(1-\nu^2)\eta h^4} \right)^{1/5} t^{4/5}$	$C_2 \left(\frac{2(1-\nu^2)\eta Q^2}{Eh} \right)^{1/5} t^{1/5}$	$C_3 \frac{1}{h} \left(\frac{E^3 Q \eta l}{8(1-\nu^2)^3} \right)^{1/4}$
KGD ^{c)}	$C_1 \left(\frac{EQ^3}{2(1-\nu^2)\eta h^3} \right)^{1/6} t^{2/3}$	$C_2 \left(\frac{2(1-\nu^2)Q^3 \eta}{Eh^3} \right)^{1/6} t^{1/3}$	$C_3 \frac{1}{2h} \left(\frac{E^3 Q \eta h^3}{8(1-\nu^2)^3 l^2} \right)^{1/4}$
Radial ^{d)}	$1.94 \left(\frac{Q}{2\pi w(0,t)} \right)^{1/2} t^{1/2}$	$2.15 \left(\frac{2(1-\nu^2)\eta QR(t)}{E} \right)^{1/4}$	$\frac{2.03}{\pi} \left(\frac{E^3 \eta Q}{8(1-\nu^2)^3 R(t)^3} \right)^{1/4} \ln \frac{R(t)}{r_w}$

Coefficients for single and bi-wing fractures, (single wing, bi-wing) ^{a)} $C_1 = (0.60, 0.395)$, $C_2 = (2.64, 2.00)$, $C_3 = (3.00, 2.52)$; ^{b)} $C_1 = (0.68, 0.45)$, $C_2 = (2.50, 1.89)$, $C_3 = (2.75, 2.31)$; ^{c)} $C_1 = (0.68, 0.48)$, $C_2 = (1.87, 1.32)$, $C_3 = (2.27, 1.19)$. ^{d)} Results are semianalytical (69).

Table 3 Length scales for viscosity and toughness in KGD and radial models under constant injection rate Q , from (84)

	L_m	L_k	K
KGD	$\left(\frac{E'Q}{\eta'}\right)^{1/6} t^{2/3}$	$\left(\frac{E'Q}{K'}\right)^{2/3} t^{2/3}$	$\frac{K'}{(E'^3 \eta' Q)^{1/4}}$
Radial	$\left(\frac{E'Q^3}{\eta'}\right)^{1/9} t^{4/9}$	$\left(\frac{E Q}{K}\right)^{2/5} t^{2/5}$	$\frac{K'}{(\eta'^5 Q^3 E'^{13})^{1/18}} t^{1/9}$

Here, $\eta' = 12\eta$, $E' = E/(1-\nu^2)$, $K' = 4(2/\pi)^{1/2} K_{IC}$. K_{IC} is the fracture toughness.

Table 4 Operational scales in hydraulic fracturing

Quantity [units]	Range	Remarks
Fracture height [m], h	10--100	Set by the rock formation and local stresses
Fracture width [mm], w	2--185	2--10 is typical
Fracture length [m], l	100--1000	Can form complex network
Fluid viscosity [mPa-s], η	1--1000	Impacts fracture width, length, network complexity
Proppant diam. [μm], a	100--850	Highly variable and polydisperse
Proppant conc. [-], ϕ	0--20	Variable during injection, correlated with η
Fluid Injection rate [$\text{m}^3 \text{s}^{-1}$], Q	0.05--0.27	20--100 bbl/min
Shear rate [s^{-1}], $\dot{\gamma}$	0--1000	The magnitude of $\dot{\gamma}$ is important for controlling settling, and the local fluid rheology
$Re_w = \rho_f w^2 \dot{\gamma} / \eta$	0--5000	Flow can be turbulent in wellbore, where $Re_r : 10,000$
$Wi = \lambda \dot{\gamma}$	0--1000	Difficult to quantify at downhole temperature; λ undetermined
$Re_p = Re_w \frac{a}{w}$	≤ 100	From upper limit on Re_w
$Pe_a = 6\pi\eta\dot{\gamma}a^3 / (k_B T)$	$> 10^3$	Proppant is noncolloidal and viscous stresses dominate

Table 5 Expressions for viscosity as a function of shear rate and particle volume fraction in generalized Newtonian fluids

Model	Expression	Remarks
Newtonian	$\eta(\dot{\gamma}) \rightarrow \mu$	Constant viscosity
Power-law	$\eta(\dot{\gamma}) = k\dot{\gamma}^{n-1}$	Easiest to implement
Carreau $\eta(\dot{\gamma} \rightarrow \infty) = \eta_\infty$, $\eta(\dot{\gamma} \rightarrow 0) = \eta_0$	$\frac{\eta(\dot{\gamma}) - \eta_\infty}{\eta_0 - \eta_\infty} = (1 + (\lambda\dot{\gamma})^2)^{\frac{n-1}{2}}$	Captures low/high shear plateau and power law thinning at intermediate rates
Bingham ($n, k = \mu$), Hershel-Bulkley ($n \neq 1, k$ consistency)	$\frac{\eta}{\eta_0} = \begin{cases} \infty & \tau \leq \tau_y \\ k\dot{\gamma}^{n-1} + \frac{\tau_y}{\eta_0\dot{\gamma}} & \tau > \tau_y \end{cases}$	Thinning above τ_y ; Finite yield stress and shear thinning
Krieger-Dougherty	$\eta(\phi) = \eta_s \left(1 - \frac{\phi}{\phi_{\max}}\right)^{-[\eta]\phi_{\max}}$	η_s is solvent phase viscosity ($\phi \rightarrow 0$)