AUTOMATICITY OF BASIC MATH FACTS: THE KEY TO MATH SUCCESS?

THESIS

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ABSTRACT

Despite a concerted effort by policy makers and educators to improve U.S. math education, student math scores remain unimpressive. In exploring possible reasons for this, this study examined the role that basic math fact automaticity plays in math success, where automaticity is defined as the ability to recall facts with speed and accuracy at an unconscious level. Information processing theory posits that automaticity frees up cognitive resources for more complex processes, and should therefore be an important part of learning math. To investigate this, basic multiplication fact fluency levels of a group of college students were determined by a timed math probe. These same students also completed a short survey assessing their attitudes and high school math achievements. The students' levels of fluency as demonstrated on the math probe were then compared to their reported math attitudes and achievements. Relationships between levels of fluency and math attitude and achievement were noted. The most interesting finding, however, was the lack of basic multiplication fact automaticity in 90% of the college students tested. The implications of this lack of automaticity present an interesting subject for future study.

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CHAPTER ONE

Introduction

College students have graduated from high school and have been accepted to college, but do they really know their multiplication tables? This is not an idle question. With alarming frequency, college professors have noted with shock and dismay how ill-prepared their incoming students are (Alsup, 2005; Latterell, 2005; National Science Foundation, 2006; Sanoff, 2006; Wilson, 2007). This is particularly vexing because many school districts have increased both the requirements and the rigor of their secondary math programs in recent years. Greater numbers of high school students are now taking more math at higher levels (Reys, Dingman, Nevels, & Teuscher, 2007). What, then, accounts for this discrepancy between the increased emphasis on math and the lack of commensurate results?

In the public realm, discussions about math education by politicians and business people have concentrated on ways to enable students graduating from American schools to be competitive in STEM (Science, Technology, Engineering, and Mathematics) fields (Heritage Foundation, 2009; National Governor's Association, 2008; National Science Foundation, 2006; U.S. Department of Education, 2008). To this end, there has been additional pressure from policy-makers and administrators to teach more math to students before they graduate from high school. But perhaps the focus should not be on how much math is taught before graduation, but rather how solid the foundations of math are.

Research Problem and Purpose

Research has shown that higher level math requires automaticity of basic math facts, where automaticity is defined as the ability to recall basic math facts with speed and accuracy at an unconscious level (Baroody, Bajwa, & Eiland, 2009; Cumming & Elkins, 1999; Poncy, Skinner, & Jaspers, 2006; Verschaffel, Luwel, Torbeyns, & VanDooren, 2009; Woodward, 2006). It is usually assumed that because students have progressed through the primary grades, they have achieved automaticity of basic math facts. However, this may not be true in many cases. There is a dearth of data on whether or not students actually achieve automaticity in American schools, and how this might affect their achievement in math.

The concept of automaticity is related but different from the concept of fluency. Fluency is the rate of accurate recall, which includes the entire spectrum of rates from high to low. Automaticity is the highest rate of fluency: immediate, accurate recall at an unconscious level.

The purpose of this study was to assess the level of basic math fact fluency of college students, to determine if they demonstrated automaticity, and to compare their level of fluency to their math attitude and achievement. The independent variable, fluency, was measured by the number of correct answers per minute on a math probe of single-digit multiplication facts. The dependent variables were (a) math achievement, as measured by self-reported high school math levels (e.g. algebra, honors geometry, Advanced Placement (AP) calculus) and self-reported high school math grades, and (b) attitude toward math, as measured by participants' responses to survey questions about whether or not they liked math, and how difficult they found math.

Research Questions

There were two research questions guiding this study: (a) Do post-secondary students have automaticity of basic math facts? And, (b) Is there a relationship between the level of fluency of basic math facts and math attitude and achievement in students?

Background

By the end of elementary school, students are expected to know basic math facts, which are the building blocks of higher math. In fact, research shows that students should not only know basic math facts, but they should have automaticity of the basic facts—the ability to recall answers with both speed and accuracy at an unconscious level (Baroody et al., 2009; Cumming & Elkins, 1999; Poncy et al., 2006; Verschaffel et al., 2009). All elementary students study basic math facts, and it is assumed that they master them—but do they? Do elementary students actually achieve the automaticity required for the more complex math in middle school, high school, and beyond?

The current favored approach to math instruction is based on constructivist principles, emphasizing understanding over memorization. The old-fashioned notion of learning math facts “by heart” with both speed and accuracy (automaticity) has been unpopular because it clashes with constructivism and brings to mind stultifying, robotic memorization drills. Indeed, a wide body of literature supports much of the constructivist approach to math instruction (Baroody et al, 2009; Caron, 2007; Kamii, Lewis, & Livingston, 1993; Van de Walle, 2007; Woodward, 2006). However, the literature also states that a conceptual understanding of basic math (as learned through a constructivist curriculum) is, by itself, insufficient for mastery of higher math; there must be automaticity of the basic math facts as well (Baroody et al., 2009; Cumming & Elkins, 1999; Poncy et al., 2006; Verschaffel et al., 2009; Woodward, 2006).

Information processing theory suggests that the basic math facts should be known so well as to be unconsciously available. In this way a student can have more cognitive resources free for understanding and performing increasingly complex mathematical chores (Caron, 2007; Poncy et al., 2006; Ramos-Christian, Schleser, & Varn, 2008; Woodward, 2006). It is difficult enough to concentrate on the process of long division when still adding and subtracting using fingers; imagine trying to factor polynomials while still counting-on to recall multiplication facts.

Although the importance of automaticity for higher math skills is supported in the literature, there is a paucity of data on whether or not students actually achieve automaticity in schools, or how their level of basic math fact fluency might affect their success and attitude in math. This data may be lacking because it is always taken for granted that students have achieved automaticity of basic math facts by virtue of having completed elementary school. However, this may not be true in many cases.

Setting

Participants in this study were college students enrolled in a medium-sized, Midwestern U.S. public university. The math probe and survey were administered on campus in the students' usual classroom setting during the usual class meeting time.

Assumptions

Concerns about the general decline in U.S. math scores, and noticing the concurrent changes in math instruction deemphasizing basic math fact memorization, prompted this study. The researcher assumed that most elementary teachers today use a predominantly constructivist approach to mathematics instruction. Although cognizant of the importance of basic math fact mastery, teachers do not necessarily emphasize automaticity, nor do they determine definitively if it has been achieved before students move on to higher math. Automaticity in this study was considered to be greater than or equal to 40 correct math fact answers per minute (Shapiro, 1989).

Scope of the Study

The levels of fluency of the students in the study were measured by a timed probe of single-digit multiplication problems. (Although basic math facts include addition, subtraction, multiplication, and division, this study tested multiplication only.) The students' fluency was then related to their math achievement (determined by self-reported high school math classes and grades), and their attitudes toward math (determined by how much they reported liking math and how difficult math was for them). Fluency was the independent variable. Math attitude and achievement were the dependent variables. There were no researcher-manipulated variables.

This study's intent was to assess the level of fluency in college students in order to determine whether they demonstrated automaticity of basic math facts, and whether their level of fluency was related to their math attitude and achievement. It did not take into account students' genders, socioeconomic statuses, cultural differences, learning styles, aptitudes for math, attitudes toward tests, or their adherence to the parameters of the test. No special accommodations were made for students' learning styles or difficulties (including possible issues with memorization, test environments, anxiety, and difficulties with reading or physically writing numbers)—except as were already in place in their current classroom situation. Therefore, the math probe employed in this study was an imperfect indicator of fluency in general, and automaticity specifically, for reasons listed above. Also, subjective classroom grades, as well as instruction and class levels that vary from school to school, do not necessarily define math achievement for each student. Nor do two simple questions (about liking math and how difficult it is) provide a complete picture of attitude. Furthermore, self-reported data is always subject to inaccuracies. The sample of students was not necessarily representative of all students across the United States. This study did not attempt to discern reasons for varying levels of automaticity, or to suggest methods for improving automaticity.

Definitions

In this study, *fluency* was defined as the speed of accurate recall of basic math facts, given in problems per minute (ppm); the level of fluency can be high or low. *Automaticity,* on the other hand, is the highest level of fluency—the ability to recall basic math facts with speed and accuracy at an unconscious level (Baroody et al., 2009; Campbell (2005); Cumming & Elkins, 1999; Poncy et al., 2006; Shapiro, 1989); Verschaffel et al., 2009; Woodward, 2006). Automaticity is an immediate and unconscious retrieval of answers, which suggests a rate faster than one answer per second. For this study, allowing for time for writing, automaticity was considered anything equal to or greater than 40 correct problems per minute.

Summary

Determining the level of basic math fluency of current U.S. students may shed light on the disappointing performance of students in math despite recent efforts by teachers and administrators to improve curriculum and instruction. Have schools been pushing higher math before their students have secured the most basic foundations of math—automaticity? To address this question, this study measured basic math fact fluency of students, and sought to determine if there was a relationship between levels of fluency, including automaticity, and math attitude and achievement.

CHAPTER TWO

Literature Review

Knowing basic math facts is essential. This truism can be interpreted in different ways, however. Is it enough to understand the facts conceptually? Does it count to know how to find them (in this age of the ubiquitous calculator)? Or does truly knowing basic math facts mean being able to rattle them off without thinking? Most importantly, where does mastery of basic math facts, or automaticity, fit into understanding and succeeding at math? Although math facts are supposed to be learned in elementary school, it may not be the case that automaticity—the unconscious recall of facts with speed and accuracy—is ever achieved or retained (Cumming & Elkins, 1999).

To place automaticity in context, this literature review begins with an overview of the current math education situation in the U.S., followed by a brief explanation of how children learn math facts, and why they do not learn math facts well. The topic of automaticity will then be reviewed and tied into the process of learning math in the current education climate.

U.S. Math Education Today

The current national focus on improving U.S. competitiveness has resulted in much literature regarding the improvement of math education for STEM (science, technology, engineering, and math) fields. There have been exhortations and recommendations from government agencies, education communities, and businesses (Heritage Foundation, 2009; National Governor's Association, 2008; National Science Foundation, 2006; U.S. Department of Education, 2008). Although there has been general agreement on the need for improved education in mathematics, there remains much disagreement as to how this should be accomplished.

Recent history of math education reflects this disagreement. In 1983, the Secretary of Education, T.H. Bell, published a scathing report, *A Nation At Risk,* on the state of education in America, with particular criticism of math education. The public was alarmed, and in 1989, the influential National Council of Teachers of Mathematics (NCTM) published new standards that advocated a completely different approach to math education. The wide implementation of this new approach, often called *standards-based math* or *reform math,* was the beginning of the so-called *math wars,* a heated philosophical debate about how to best teach math. The math wars pitted the reform proponents against the traditionalists. Reform math (backed by most teachers) favored more conceptual math instruction, with open-ended questions and less rote memorization; traditional math (backed by most mathematicians) favored an emphasis on basic facts and skills (Latterell, 2005).

The literature on reform math is abundant and somewhat mixed. Early on, the literature was generally positive, and reform math was hailed as progressive and proactive. Teachers and students liked that it lessened math anxiety, took away the dullness of rote memorization and drilling, and focused on concepts and real-life problem solving. Because of its constructivist foundation, it was more hands-on and more engaging than traditional math (Dempsey, 2002; Hiele, 1999).

Over time, however, policy makers began to take note that math test scores were not improving. College math professors were becoming alarmed by the poorly prepared students entering their classes (Alsup, 2005; Latterell, 2005; National Science

Foundation, 2006; Sanoff, 2006; Wilson, 2007). It appeared that reform math might not be the best approach to math after all.

In 2008, the U.S. Department of Education released *The Foundations of Success: The Final Report of the National Mathematics Advisory Panel*. This report sought to end the conflict over how to best teach math to improve student math competency, stating that "the mutually reinforcing benefits of conceptual understanding, procedural fluency, and automatic (i.e., quick and effortless) recall of facts" (p. 11) are all critical aspects of learning math (U.S. Department of Education, 2008). Among its recommendations were to prepare students for algebra by eighth grade by teaching fewer math topics, but in more depth, and requiring proficiency with math facts.

Although the importance of proficiency with math facts, procedural fluency, and automatic recall are concepts that almost everyone involved in math would agree on in principle, their exact meanings remain somewhat ambiguous. For instance, how fast, exactly, is the "automatic recall" spoken of by the U.S. Department of Education (2008)? Many researchers don't put an actual number to this definition. A few educators, such as Van de Walle (2007), posit that automaticity is demonstrated if an answer is produced in under three seconds. Other educators, cognitive psychologists, and mathematicians point out that three seconds is too long to demonstrate true automaticity because it gives time to consciously construct a response—clearly not the immediate, unconscious response defined by automaticity (Isaacs & Carroll, 1999; Shapiro, 1989).

Mathematical Learning in Children

The national push for students to be better educated in math is reflected in literature that proposes teaching more math to more students in earlier grades. Some articles even suggest algebraic reasoning be taught as early as kindergarten (Ketterlin-Geller, Jungjohann, Chard, & Baker, 2007). Teaching younger students higher math concepts is intriguing, but is at odds with most of the research on children's brain development.

Humans have an inherent number sense from infancy—an implicit understanding of basic quantitative concepts (Krajewski & Schneider, 2009; Sarnecka & Carey, 2008; Stock, Desoete, & Roeyers, 2009). Many of the secondary quantitative skills that children are expected to learn in school, however, are not inherent, but are built on primary quantitative competencies, and require organized and repetitive instruction (Geary, 2000). Successful arithmetic abilities in elementary school are built upon early experiences with number sense as well as mastering essential counting principles (Krajewski & Schneider, 2009; Stock et al., 2009).

Theories of arithmetic development assert that children progress from concrete to more abstract thinking, that context is important, and that the readiness of the individual to move to a higher level of thought cannot be rushed (Kamii et al., 1993; Prather & Alibali, 2009; Vilette, 2002). Much of the literature warns against teaching arithmetic algorithms and abstract concepts before children have developed a good number sense and an understanding of patterns and relations because they will be developmentally unready to master them. Children are not ready to memorize math facts until they have an understanding of the meaning of addition and subtraction equations as well as their inverse relationship (Vilette, 2002; Waite-Stupiansky & Stupiansky, 1998).

Individual differences in students, learning styles, and experiences must also be taken into consideration. Instruction appropriate to development and type of mind may prevent or remediate possible problems, and will allow students to move forward with the fluency needed for automaticity and more advanced math (Levine, 2002; Ramos-Christian et al., 2008; Wadlington & Wadlington, 2008).

In learning basic math fact combinations, children progress through three phases: counting strategies, reasoning strategies, and finally, mastery. The first two phases—counting and reasoning—require deliberate, conscious work to determine and understand the math combinations; these are relatively slow cognitive processes. The third and final phase of learning math facts occurs when the retrieval of answers from memory becomes automatic (Baroody et al., 2009).

There are two main ways to accomplish this third phase of mastery of basic fact combinations: through rote memorization or through meaningful memorization. Rote memorization produces *routine expertise*: The knowledge can be applied to familiar tasks, but not necessarily to new, unfamiliar tasks, and so is considered a *mastery with limited fluency*. On the other hand, meaningful memorization combines factual, strategic, and conceptual knowledge resulting in an *adaptive expertise* that can be applied to familiar as well as new tasks, and becomes *mastery with fluency* (Baroody et al., 2009). In mastery with fluency, math facts are immediately retrieved from memory with speed and accuracy on an unconscious level—automaticity.

Automaticity

Students who have not mastered addition combinations will be handicapped in their efforts to master subtraction, multiplication, and division. Furthermore, because all of the basic facts and processes of computation undergird problem solving, estimation, mental computation, and more complex skills, information processing theory suggests that it is essential that these facts be not only learned, but that they become automatic (Baroody et al., 2009; Cumming & Elkins, 1999; Gagne, 1983; Poncy et al., 2006; Verschaffel et al., 2009). Information processing theory holds that people have limited cognitive capacity, which makes accomplishing a set of tasks very difficult unless some of the tasks require less time, less cognitive effort, less working memory, and/or less attention. Developing automaticity frees up cognitive resources for complex tasks (Ashcraft, 1992; Campbell, 2005; Caron, 2007; Gagne, 1983; Poncy et al., 2006; Ramos-Christian et al., 2008; Woodward, 2006). The added cognitive load from inefficient methods of computations (such as counting on fingers) that arise from a lack of automaticity often leads to procedural errors and difficulties in learning new procedures (Ashcraft & Krause, 2007; Cumming & Elkins, 1999; Woodward, 2006).

Although depth of understanding (as promoted in reform math) is important, without automatic recall of the basic facts, students will only be able to make minimal progress in math and related subjects (Caron, 2007). The additional load on working memory and processing speed when students do not have automaticity of basic math also contributes to math anxiety, further reducing progress in math (Ashcraft & Krause, 2007). Furthermore, research has shown that the best predictor of maintained mathematical skills (including basic computational and algebraic skills) into adulthood is the frequency and distribution of practice of these skills in secondary school—even when individual mathematical abilities were taken into account (Geary, 2000). Automaticity, then, appears to be important not only for learning complex math, but to help increase math retention as well as to decrease math anxiety.

Why Students Do Not Develop Automaticity

General difficulties with math account for many math deficiencies and are often partly to blame for the lack of automaticity. There is considerable overlap in the literature on math learning with the literature on learning difficulties and disabilities. Problems with memorizing basic facts are more severe for children who have language difficulties, math anxiety, and other mathematics difficulties (MD), as well as for children who are at-risk for mathematical difficulties. The at-risk indicators include low family income; a single, poorly-educated, or teenage parent; minority status; physical disabilities; or emotional difficulties. Children with MD may or may not have a cognitive impairment associated with mathematical learning disabilities (MLD) (Baroody et al., 2009). Although some children do have troubles with math because of MLD, many, and perhaps most, children have difficulties with math because they haven't had the chance to develop a good number sense (Baroody et al., 2009; Krajewski & Schneider, 2009; Wadlington & Wadlington, 2008). Regardless of the nature of the learning difficulty, all learners must have foundational, basic skills on which to build their math understanding, and should not be pushed into math of higher complexity before they have mastered the basics (Miller & Mercer, 1997; Zentall & Ferkis, 1993). In fact, pushing students into more complex math before they have established a strong foundation in the basics can overburden the working memory and processing speed of students, leading to poor performance and anxiety about math. This anxiety, in turn, further burdens the working memory, and a cycle of math anxiety and poor performance can ensue (Ashcraft & Krause, 2007; Campbell, 2005).

Another reason students do not develop automaticity is that teachers are sometimes uncertain as to how to help students achieve automaticity without resorting to rote drilling, which is generally frowned upon by the education community. Indeed, there are reasons for educators' avoidance of rote drilling. For instance, there is evidence that memorizing facts is less effective if not supported by memory gained through actual experience and understanding (Baroody et al., 2009; Caron, 2007). Some studies also show that superficial rote memory alone is an ineffective way to memorize, and takes up much of a student's time and effort. Furthermore, memorizing by rote does not guarantee automatic recall, does not necessarily change the slow procedures students use to arrive at answers, and does not improve quantitative thinking (Caron, 2007). In addition, forced drilling of math facts can actually reinforce students' use of immature methods for answering fact problems, can create math anxiety, and can make math both uninteresting and uninviting. In an effort to avoid all of these concerns, many educators have emphasized using explicit strategy instruction (as in reform math) over traditional rote instruction (Kamii et al., 1993; Van de Walle, 2007; Woodward, 2006). Although this has helped students organize and understand facts, these strategies alone do not usually lead to automaticity (Woodward, 2006).

Ultimately, however, the key factor in automaticity may not be the type of instruction used as much as when the instruction takes place in the student's numerical cognitive development. Some literature points out that working memory and processing speed increase as children get older (Campbell, 2005). This suggests that some children might be able to master math facts more effectively at a later time than is usually expected in the primary grades.

Summary

Automaticity has traditionally been an expected basic outcome of elementary school math education. In recent years, however, reform math programs have deemed automaticity old-fashioned, dull, and somewhat unnecessary; therefore, automaticity has not been emphasized. Unfortunately, during this same time, student math scores have fallen (Institute of Education Sciences, 2009).

Teaching math is a complex undertaking, and experts disagree on the best way to teach it. There is a growing body of literature, however, that shows a consensus on the need for learning basic math facts completely—including both conceptual understanding as well as speed and accuracy—for true automaticity as a foundation of all math, and especially for the more complex tasks of higher math. Although the benefits of automaticity have been noted, there remain gaps in the literature: whether automaticity is being pushed too early for some students; whether automaticity is attained gradually throughout the years of elementary and secondary schooling; whether students actually achieve automaticity at all; whether students retain automaticity once they have attained it; and the exact extent to which automaticity benefits higher math thinking.

The literature indicates that automaticity of basic math facts is the foundation for higher math skills. This study seeks to determine whether college students do in fact demonstrate basic math fact automaticity (assumed to have been acquired years before), and to explore whether there is a relationship between automaticity, levels of fluency, and math attitude and achievement.

CHAPTER THREE

Methodology

To determine the level of fluency of basic math facts in college students and how that level of fluency related to their math attitude and achievement, a survey and a probe of basic math facts were administered to participants. This chapter describes the research questions guiding this study, the participants and the research design, and then explains the data gathering and analysis.

Research Questions

There were two research questions guiding this study: Do post-secondary students have automaticity of basic math facts? And, is there a relationship between the level of fluency of basic math facts and math attitude and achievement in students?

Participants

In addressing this study's research questions, the population of interest was post-secondary students from U.S. high schools. College students were used in this study because it was assumed that students who had graduated from high school and were enrolled in college would know their multiplication facts. The math fluency of post-secondary students enrolled in college would be a reflection of math education in U.S. secondary schools.

The sample for this study was a class of college students in a general education course in a medium-sized, Midwestern public university. These participants were chosen for the following reasons: a) college students are assumed to already have automaticity of basic math facts, b) the large number of students in this general education class would include varying math skills as well as differing majors, and c) the instructor was amenable to the interruption for investigation.

Research Design

The instrument used in this cross-sectional study consisted of two parts. The first part was a ten-question survey created by the researcher to collect data on math experience and attitude, including high school math classes, high school math grades, attitude toward math, and intended college major (see Appendix A). The second part of the research instrument, on the reverse side of the survey, was a single-skill math probe created by the researcher based on current guidelines for Curriculum-Based Assessment (Shapiro, 1989). The researcher created this probe because no suitable existing probes were found for this particular situation. Most standardized tests assess multiple skills (instead of the single skill of interest in this study), and other probes were computerized assessments (not applicable to this study), or computer-generated worksheets from online sources (insufficient in quantity of problems presented per page). The probe created for this study was a one-minute timed test consisting of 120 single-digit vertical multiplication problems using factors between two and nine, inclusive (see Appendix B). The problems were presented on one side of a sheet of paper in 12 rows and 10 columns.

For this study, the combination of math probe and survey was an expedient way to assess automaticity, other levels of fluency, math achievement, and math attitude. Because timed math fact tests are common classroom diagnostic tools used in elementary and middle schools, the format should have been familiar to the students. Furthermore, because the probe and survey were both short, they provided a convenient and efficient means of assessing the fluency, attitude, and achievement of many participants at once in a short amount of time.

Data Collection

The researcher brought all materials to the test setting: copies of the survey and probe for each student, extra pencils, and a timer. The study was explained to the students orally, and information sheets (stapled to the survey and math probe) with additional information (including contact names and numbers) were passed out to the students (see Appendix C). Students were given time to look over the information and ask questions. Those interested in participating were then asked for their oral assent.

The participating students were asked to complete the survey on the front side of the probe, and were asked not to turn their papers over when finished with the survey. When these were completed, the students were given instructions for the timed test. They were told that automaticity of math facts was being studied, so they should work quickly, completing each problem in order from left to right and from top to bottom, but that the test was designed so that they were not expected to finish all the problems within the allotted time period. Students were assured that their performance on the test would have absolutely no bearing on their class grade, and that their results would be anonymous. After time was allowed for questions, the participants were told to turn their papers over and begin; they were given one minute to complete as many of the problems on the probe as they could.

The numbers of correct problems per minute (ppm) from the completed timed tests were recorded for each student. These data were the independent variables, indicating the math fluency (in ppm) of each student.

The dependent variables, math attitude and achievement, were measured by the self-reported high school math grades, the level of last high school math class (e.g., algebra I, honors geometry, AP calculus), and the attitude information gathered from the survey.

Treatment of the Data

Each completed math probe was given a reference number. Then, the participants' answers on the probes were checked against the correct answers. The number of problems attempted, the number of incorrect answers, and the number of correct answers were recorded for each math probe. These numbers of correct answers on each probe were labeled *correct problems per minute (ppm)*. The math probes that were not completed according to the directions were marked as such, and those data were eliminated from the results.

The data gathered from the survey and math probe were compiled in a spreadsheet with the following column headings: Test Number, Missed Directions, Attempted, Incorrect, Correct, City, State, Math Effort, Math Enjoyment, Years of High School Math, Highest High School Math Class, Grades in High School, Year in College, Math in College, Intended Major, and Future Use of Math. The data from those participants who misunderstood the directions were separated, and the rest of the data were then sorted by number of correct answers, from highest to lowest. These data were further separated into tiers of fluency. The highest level of fluency, automaticity, was greater than or equal to 40 correct problems per minute, and was labeled Tier 1. Tier 2 was between 30 and 39ppm, Tier 3 was between 20 and 29ppm, and Tier 4 was below 20ppm.

The data were then broken down into the numbers of mistakes, intended college majors, those who reported liking math, those who reported math as being difficult, high school grades, and highest level of high school math courses. These numbers were further separated within each tier.

Summary

College student participants filled out a short survey and a one-minute timed math probe of single-digit multiplication problems. From these, levels of fluency, attitudes toward math, and math achievement in high school, were recorded. The results and descriptive analysis of the data are discussed in the following section.

CHAPTER FOUR

Results and Discussion

This study was undertaken to determine the prevalence of automaticity of basic math facts in college students and to determine if the level of basic math fact fluency was related to math attitude and high school math achievement. To this end, a short survey in which students reported their attitudes toward math and their high school math achievement (see Appendix A), as well as a one-minute timed math probe of single-digit multiplication facts (see Appendix B), were given to college students in an undergraduate-level general education class. This particular lecture class was chosen because it was readily accessible and consisted of a large cross-section of college students who were in different years in their undergraduate college careers, and who had a wide variety of intended majors.

The results of the survey and math probe are presented in this chapter, followed by a discussion of these results.

Results

Of the 225 students enrolled in the class, 137 chose to participate. Of these 137, 12 did not follow directions (skipping around on the timed probe); the data from these 12 were eliminated from the study, leaving 125 as the total number of useable respondents. The following results were noted about fluency, math achievement, and math attitude.

Fluency. To assess basic math fact fluency, participants were given one minute to complete a math probe of single-digit multiplication problems. The math facts on the probe were to be done in order (so as to eliminate an unfair advantage of skipping difficult problems), and only correct answers were counted toward their score. The number of correct problems per minute (ppm) indicated each student's level of fluency. To organize and examine the data more easily, the scores from the math probes were divided into four tiers of fluency: Tier 1 was greater than or equal to 40ppm; Tier 2 was 30-39ppm; Tier 3 was 20-29ppm; and Tier 4 was less than 20ppm.

Thirteen of the 125 participants (10%) were in the top tier, Tier 1, answering 40 or more problems correctly on the math probe; 22% were in Tier 2 (30-39ppm); 31% were in Tier 3 (20-29ppm); and 36% were in Tier 4 (less than 20ppm) (see Appendix D). The highest number of correct answers was 53ppm. Eight students answered 10 or fewer questions correctly in the allotted time; two of these students answered only four questions correctly (see Appendix E). Two-thirds (66%) of all the students answered at least one problem incorrectly.

Achievement. The survey asked questions about students' math achievements. Self-reported data from the participants indicated that 98% of them had completed three or more years of high school math. Overall, 75% of the participants reported taking higher-level math in high school (completing algebra II, pre-calculus, or higher); 36% had completed the highest-level high school math classes (calculus, AP calculus, or AP statistics). All of the students who scored with automaticity (>40ppm) had taken higher-level math courses. Overall, 77% of the students reported receiving all As and Bs in high school math; 2% reported receiving Cs and lower (see Appendix F).

Attitude. Of the 125 participants, well over one-third (36%) reported that math was *difficult* or *very difficult* for them; over half (56%) of these students were in Tier 4, the lowest fluency tier (<20ppm). Only two students (less than 2%) in the highest fluency tier, Tier 1, reported finding math *difficult*, and none in this tier reported finding it *very difficult* (see Appendix G).

Similarly, 22% of the students in the lowest tier, Tier 4 (<20ppm), reported *not liking math at all,* whereas none of the students in the highest tier, Tier 1 (>40ppm), reported the same. In fact, 92% of the students in the top tier, Tier 1 (>40ppm), reported liking math *a lot, a little,* or *neutral,* as opposed to 44% in the bottom tier, Tier 4 (<20ppm), reporting the same (see Appendix G).

All the different categories of majors were represented in each tier of fluency, with the exception of one, as shown in Appendix H.

Discussion

To determine the prevalence of automaticity among college students, a timed math probe (see Appendix B) tested participants' levels of fluency of basic math facts. Interestingly, only 10% of the students demonstrated automaticity. Because basic math facts are assumed to have been mastered in elementary school, and because they are part of the foundation of most mathematics, it was surprising that 90% of the college students tested did not have basic multiplication facts mastered to the point of automaticity.

Perhaps even more remarkable was the number of incorrect answers by the participants. Some mistakes were to be expected, of course, especially because speed was demanded and some participants might have felt stressed. Still, it was surprising how many mistakes were made, even among those who took considerable time with their answers (with fluency in the lower tiers). This argued against the proposition that students "know" their math facts even when their fluency is low. Further evidence of this was the fact that some students made the same mistakes on subsequent problems involving the same factors, suggesting their mistakes were due to not knowing those particular facts rather than just random errors. The frequency and types of mistakes showed that many students not only lacked automaticity, but were in fact unable to accurately construct the answers. Clearly these facts were not mastered by most of the students.

Fluency and achievement. Although the level of math achievement did, in general, increase with the level of fluency, the relationship between the two was not as pronounced as expected. There were students at all levels of fluency who reported having taken the highest math classes (calculus or AP classes). In other words, even those students with very low fluency levels reported having completed the highest math classes in high school. However, students in the top tier of fluency (automaticity) reported the highest percentage of higher math classes taken. The relationship between fluency and grades was even less clear: A large number of all students (77%) reported receiving all As and Bs in high school math classes. Although the greatest number of students who reported all As and Bs were in the top two fluency levels (Tiers 1 and 2), the differences in grades between all of the tiers was not overwhelming.

Several factors may have accounted for the fact that the great differences in math fluency were not matched by the differences in math achievement. To begin with, self-reported high school math levels and grades were not necessarily reliable indicators of achievement—not only because students might not recall correctly, but because the title and rigor of classes as well as the consistency of grading vary widely between schools and teachers. Without accessing standardized test scores or evaluating the teachers and instruction, the comparisons of grades and class levels between students who had different teachers at different schools cannot be highly accurate. With this caveat in mind, general trends were noted. Overall, self-reported achievement appeared fairly high: 98% of students finished three or more years of high school math; 75% of students completed courses through algebra II, pre-calculus, or higher; and 77% of students reported math grades of all As and Bs.

Reviewing these numbers in context of fluency tiers put a finer point on them: 69% of students in Tier 1 (the students with automaticity) reported having taken the highest possible levels of high school math (calculus or AP math courses), and 85% of this tier reported receiving all As and Bs. Comparing by tier, the percentages of students who took the highest possible math (calculus or AP) courses in high school were 69%, 46%, 28%, and 27% (corresponding to Tiers 1, 2, 3, and 4, respectively). Similarly, the percentages of students who reported all As and Bs in high school math were 85%, 93%, 74%, and 67% (again, corresponding to Tiers 1, 2, 3, and 4, respectively) (see Appendix F).

Fluency and attitudes. The attitudes participants had toward math were determined by their answers to several survey questions (see Appendix A). Based on that survey, it was found that there was a relationship between the number of students who had low fluency scores and those who found math *difficult* or *very difficult:* In Tier 4 (the lowest tier), 56% found math *difficult* or *very difficult,* whereas only 15% of students in Tier 1 (with automaticity) found math *difficult* (see Appendix G). Similarly, there was relationship between levels of fluency and the percentage of students reporting to like math *a lot, a little,* or were *neutral* to it: 92% of students with automaticity liked math (or were at least neutral to it), as opposed to only 44% of students in the lowest tier (see Appendix G).

The data also showed that students of all interests, attitudes, and abilities were represented at almost all levels of fluency (Tiers 1-4) and accuracy (numbers of mistakes made). In other words, there were science majors as well as liberal arts majors in all four tiers of fluency, there were those who liked math who were represented in the bottom tier as well as in the top tier, and there were those who found math difficult in the top tier as well as in the bottom tier (see Appendices F, G, and H).

Analysis. Although levels of fluency (and automaticity in particular) were associated with higher math achievement and attitudes, the relationships were not as pronounced as might have been expected, given the lack of automaticity of 90% of the students. There were students at all levels of fluency who claimed high math achievement or positive math attitudes. It was interesting to note, for instance, that there were some students who had completed four years of high school math (including calculus and AP calculus), had received grades of all As and Bs, had reported that math was *easy* for them, and yet they answered fewer than 10 math facts correctly. This, however, may speak more to the fact that the criteria for achievement were not definitive rather than to the possibility that fluency did not have a great impact on achievement.

In looking at the data, several questions were immediately raised: Was it reasonable that those students who demonstrated such poor automaticity could have done so well in high school math? Did it make sense that students who could only answer fewer than 15 single-digit multiplication facts in a minute, and then got several of those incorrect, really have completed AP calculus and have received all As and Bs? Might this mean that the self-reported skills, grades, and high school class levels don't accurately reflect math achievement (or understanding)? Or could it be that these students might have done even better had they had automaticity? Did many of the lowest-fluency (Tier 4) students have special circumstances (such as a learning disability) that may have affected their performance (a possibility this study did not take into account)? Or might these data suggest something else entirely, that automaticity is, in fact, not necessary for math success?

Several factors argue against this last possibility. First, although students reported a fair amount of success in high school math at all levels of fluency, there was still a marked increase in reported success at higher levels of fluency (see Appendix F). Second, college math professors claim their students are not adequately prepared for college math classes (in spite of the fact that their students are a self-selected group who have chosen to study math at college) (Alsup, 2005; Latterell, 2005; Sanoff, 2006; Wilson, 2007). Third, this contradicts cognitive research, which has demonstrated that brains can accomplish more when there is automaticity of component pieces (Ashcraft, 1992; Campbell, 2005; Caron, 2007; Gagne, 1983; Poncy et al., 2006; Ramos-Christian et al., 2008; Woodward, 2006). It is more likely that some of these students had managed to get through their high school math classes without automaticity, but did not excel as they might have had they had automaticity to free up more of their working memory. If they had had automaticity (instead of low fluency) starting in the primary grades (and hence a stronger foundation of math), it could be that their accomplishments in math would have been greater, their interest in it stronger, and their difficulties fewer. Another possibility is that secondary math classes and grades do not accurately reflect actual math achievement: what looks like high achievement on transcripts does not translate into proof of such beyond high school.

In any case, both the large numbers of students with low fluency as well as the number of mistakes made, point to a lack of automaticity in these college participants. If the automaticity of most college students is lacking, what does that imply about the automaticity of secondary students in general?

Summary

These results revealed a marked lack of automaticity in U.S. college students. This supports literature in which college math professors noted how unprepared their incoming students were for college math. However, the affect this lack of automaticity has on math achievement and attitudes (as defined in this study) is not overwhelmingly obvious from the data and further investigation is needed, as detailed in the next chapter.

CHAPTER FIVE

Summary and Conclusions

In addressing the question of whether or not college students have actually learned their multiplication tables, this study set out to determine four things: (a) the level of basic math fluency of students in college, who had presumably mastered their multiplication tables some 10 years previously; (b) their achievement in math through high school; (c) the attitude of these same students toward math; and (d) the relationship between the levels of fluency and the attitudes and achievements of these students.

Significant Findings

Of the 125 students tested for math fluency, only 10% demonstrated automaticity of basic multiplication facts, where automaticity was considered accurate recall of math facts at a rate of 40ppm or greater. This means that 90% of the college students tested did not know their multiplication tables to the point of automaticity. Although the review of literature for this study suggested that there was a general lack of automaticity in U.S. students, no articles were found delineating prevailing fluency levels or the extent of automaticity of current students. Therefore, the high percentage of students lacking automaticity in this study was surprising. Furthermore, the high number of students making mistakes on the math probe (66% of the students answered at least one of the math problems incorrectly, and some students had up to 45% of their answers wrong) suggests a lack of mastery of the facts as well as a lack of automaticity.

The self-reported math achievement of the students was higher than expected considering the low levels of fluency demonstrated on the probe. Most of the students (77%) reported receiving all As and Bs in high school math, and most (75%) also reported completing higher-level math classes, such as algebra II, pre-calculus, or above, regardless of their level of basic math fact fluency. The self-reported attitudes of the students generally reflected their fluency levels. Students in the higher tiers of fluency reported liking math more, and finding it easier as well. Overall, the data showed that students in the higher tiers of fluency generally reported higher achievement and attitude.

Educational Implications

The educational implications of these results are open for debate. To begin with, it must be decided if automaticity is truly necessary, or if a certain low level of fluency is acceptable. There are some educators who claim that understanding the concepts of multiplication and being able to figure out math facts is the important part of learning math, and that automaticity as defined in this paper is not necessary (Dempsey, 2002; Hiele, 1999; Kamii et al., 1993). But there are others—most notably some cognitive psychologists and mathematicians—who claim that automaticity is indeed necessary, that relying on strategies or rules to figure out math facts when needed (as is done with low levels of fluency) is insufficient for higher-level math (Ashcraft, 1992; Campbell, 2005; Caron, 2007; Gagne, 1983; Poncy et al., 2006; Ramos-Christian et al., 2008; Woodward, 2006).

If lack of automaticity is taxing students' cognitive functions in math, then a further educational question might be: Is this where some math anxiety starts? If students must devote cognitive resources to math fact basics in addition to whatever lesson is being learned, then this puts an additional burden on their working memories and processing speeds. This can cause them to have more difficulty learning new material, which often causes anxiety. Anxiety itself, in turn, creates an additional cognitive burden

(Ashcraft & Krause, 2007). Thus, a self-perpetuating cycle of poor performance and anxiety might begin due to the lack of automaticity.

A related educational observation is that pushing students to take high levels of math in high school (as is currently done in secondary schools) does not in itself make them more proficient in math or prepare them for college math. Furthermore, this policy has not had the intended result of increasing math scores and student interest in STEM careers (Alsup, 2005; Latterell, 2005; National Science Foundation, 2006; Sanoff, 2006; Wilson, 2007). Perhaps instead, the opposite should be done: Math education should slow down and allow young students to become comfortable with numbers before mastery of facts is required. Then automaticity of basic math should be required before pushing on into more complex aspects of math education. This would give students a solid foundation for any future math (and for real life), without the discouragement or tenuous understanding that often accompanies an accelerated math education for those who are not ready for it.

Recommendations for Future Research

This study was a brief look at basic math fluency. A clearer picture should be obtained using a broader sample—students in different grade levels and various circumstances. The results of probes and surveys should then be examined against standardized test scores, actual classroom data, and teacher, as well as student, interviews. Also, a better instrument could be developed and tested, and the incorrect answers given on probes could be analyzed to illuminate possible problem areas for fluency. In these ways, a more accurate and complete picture of achievement and attitude, and the role fluency and automaticity play in them, could be obtained.

Following the trajectory of fluency would be also be instructive, to determine how and when fluency (and then automaticity) is developed and retained. Another interesting aspect would be to test the fluency of basic addition and subtraction facts to see if students have automaticity with those, as those are even more fundamental to math than multiplication.

The most pressing need for further study, however, is to determine exactly what role automaticity—or the lack thereof—plays in students' learning, understanding, attitudes, retention, and success in math. To investigate this, a study of alternative math education timelines and priorities (in comparison to current math education timelines and priorities) would be interesting. Are students ready for the math they're given? It would be useful to study students' working memory and speed of processing to see if those are being exceeded in their current math lessons. There is evidence that processing speed and working memory both increase with age (Campbell, 2005). Therefore, research is needed to see if schools require memorization too soon, before adequate number sense, processing speed, and working memory have been established. Educators might be more effective if they delayed some math instruction (such as memorization of math facts) and allowed young brains to develop and increase working memory and processing speed.

Research is needed to explore math education that would stress an extremely solid foundation for all math. This type of foundation would allow students to develop a strong number sense through increased number play, exploration, usage, and manipulation in the early years of school. Then, when students were ready, they would be helped in acquiring automaticity of basic math in meaningful and engaging ways.

Research is needed to discern if the time spent on this type of foundation would pay off. Students graduating from high school may not have calculus, then, but would they be both competent and comfortable with math through algebra, and would they be better prepared for more complex math in college?

Conclusion

There has been a concerted effort to improve math education in the U.S. in recent years, and yet math scores have not improved. In examining possible reason for this, this study examined the levels of basic math fact fluency of college students to determine what those levels were, and if there might be a connection between fluency levels and math achievement and attitude. Indeed, a general relationship was found between higher levels of basic math fact fluency, better attitudes toward math, and higher math achievement. The criteria for math achievement, however, were limited, and should be improved for future studies in order to obtain more accurate conclusions.

Because it is generally assumed that all students master their multiplication tables in elementary school, the surprising finding of this study was that 90% of the college students tested did not demonstrate automaticity (unconscious recall with speed and accuracy) of basic multiplication facts. The question now remains: How does this lack of automaticity affect math education? The role of automaticity in math education must be determined. Is it truly necessary for learning higher math, and if it is, when and how should it be achieved? Does it make sense for high school graduates to have calculus on their transcripts but not be able to multiply? To this researcher, that does not compute.

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Appendix A

Survey

University of Minnesota Duluth, M.Ed. Research

STOP

Do Not Turn Paper Over
Until Instructed To Do So

Thank you!

Please fill in the blank or circle the answer.

1. Where did you attend elementary school (city, state, country)? ____

2. Which word best completes this sentence for you? Math is______________ . Easy Fairly Easy Difficult Very Difficult

3. Which phrase best describes how much you like math? A lot A little Neutral Not much Not at all

4. How many years of math did you have in high school? 1 2 3 4

5. What was the name of your last high school math class (e.g., Algebra I, Honors Geometry, etc.)?________________________

6. What were your usual grades in high school math? A's A's and B's B's and C's C's C's and Below

7. What is your current college level? 1st Year 2nd Year 3rd Year 4th Year Above 4th Year N/A

8. Have you taken (or are you currently taking) any math classes at college? Yes No

9. What is your current/intended maior? ____

10. How much math do you anticipate using in your future career? A lot A little None

Appendix B

Math Probe

9 x 3	7 x 8	7 x 5	6 x 4	9 x 6	3 x 5	9 x 8	7 x 5	5 x 8	7 x 3
8 x 6	7 x 9	5 x 9	8 x 3	8 x 4	2 x 5	8 x 6	3 x 6	7 x 4	5 x 9
5 x 4	6 x 2	9 x 7	7 x 6	6 x 4	3 x 7	5 x 5	9 x 6	7 x 8	5 x 6
6 x 8	7 x 6	6 x 5	8 x 7	9 x 5	4 x 8	3 x 9	2 x 8	3 x 4	9 x 4
8 x 7	4 x 9	6 x 7	4 x 6	3 x 8	3 x 9	4 x 5	7 x 4	5 x 3	3 x 4
9 x 8	6 x 3	5 x 2	7 x 8	4 x 5	2 x 5	6 x 9	4 x 3	5 x 7	8 x 3
6 x 9	7 x 7	4 x 7	3 x 3	5 x 8	3 x 6	4 x 6	3 x 7	6 x 8	9 x 7
3 x 5	4 x 8	4 x 9	6 x 5	7 x 9	9 x 9	6 x 7	9 x 3	8 x 9	5 x 6
8 x 9	4 x 3	3 x 5	4 x 4	4 x 7	7 x 2	4 x 7	5 x 8	9 x 4	7 x 3
4 x 5	5 x 7	2 x 2	4 x 7	5 x 3	8 x 8	9 x 5	3 x 8	7 x 6	8 x 4
5 x 3	3 x 8	6 x 6	2 x 7	5 x 4	9 x 2	3 x 5	6 x 8	2 x 3	7 x 4
6 x 3	8 x 5	7 x 5	4 x 8	5 x 3	6 x 7	9 x 4	3 x 9	7 x 6	9 x 3

Appendix C

Information Sheet for Research

University of Minnesota Duluth, Graduate Research Study
Automaticity of Basic Math Facts: The Key to Math Success?

You are invited to be in a research study of the automaticity of basic math facts and its effects on math achievement. You were selected as a possible participant because you are a college student who has successfully earned a high school diploma or equivalent. Please read this form and ask any questions you may have before agreeing to be in the study.

This study is being conducted by Carmel DeMaioribus, a graduate student in the Education Department, University of Minnesota Duluth.

Procedures:

If you agree to be in this study, you will be asked to do the following:

Fill out a brief survey about past math experiences, and take a one-minute basic multiplication fact quiz. All results will be anonymous and your answers will in no way impact your grades in this or any other class.

Confidentiality:

The records of this study will be kept private. Any sort of report that might be published will not include any information that will make it possible to identify any participants. Research records will be stored securely and only the researcher will have access to the records.

Voluntary Nature of the Study:

Participation in this study is voluntary. Your decision whether or not to participate will not affect your current or future relations with the University of Minnesota or any other institution. If you decide to participate, you are free to not answer any question or withdraw at any time without affecting those relationships.

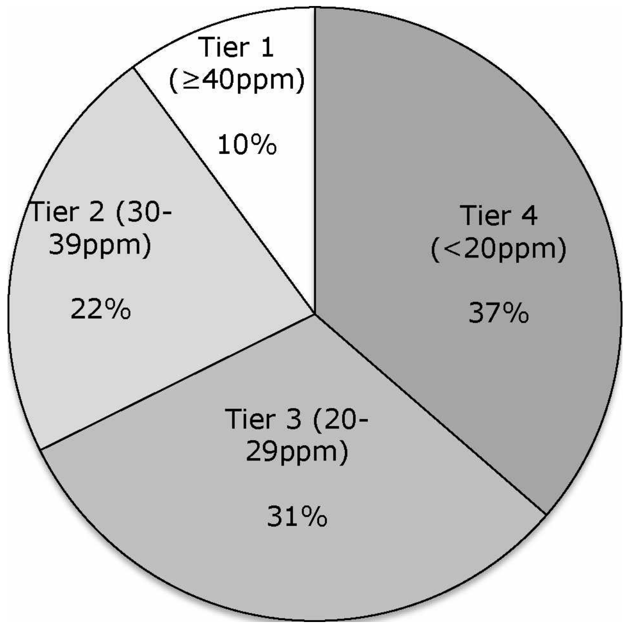
Contacts and Questions:

The researcher conducting this study is Carmel DeMaioribus. You may ask any questions you have now. If you have questions later, you are encouraged to contact Carmel DeMaioribus at dema0160@d.umn.edu, or the faculty advisor, Randy Hyman, Department of Education, University of Minnesota Duluth, 218-726-8505, rhyman@d.umn.edu.

If you have any questions or concerns regarding this study and would like to talk to someone other than the researcher or faculty advisor, you are encouraged to contact the Research Subjects' Advocate Line, D528 Mayo, 420 Delaware St. Southeast, Minneapolis, Minnesota 55455; (612) 625-1650. *You may keep this copy for your records.*

Appendix D

Percentage of Students in Each Fluency Tier



Timed math probe showed 10% of students demonstrated automaticity of basic multiplication facts (> 40ppm).

(ppm = (correct) problems per minute)

Appendix E

Data Sorted by Number of Correct Answers

Test M	At	In	Co	City	Stat	Math Effort	Math	HS	Highest HS Math
24	54	1	53	Cobles	NY	difficult	E lot	4	Calculus
22	51	1	50	Dulut	MN	fairly easy	neutral	4	Calculus
99	52	5	47	Virgin	MN	fairly easy	a lot	4	Precalculus
69	47		47	Lanca	PA	easy	not much	4	AP Calculus
95	47	1	46	Hager	WI	easy	a lot	4	Precalculus
5	48	2	46	St. Fra	MN	fairly easy	a little	4	Calculus
101	46	2	44	Lakevi	MN	fairly easy	a little	4	Precalculus
79	44		44	Superi	WI	easy	a lot	4	Calculus
2	44	1	43	Spring	WI	easy	a lot	4+	Calculus
26	43		43	Dulut	MN	fairly easy	a little	4	Calculus
97	44	2	42	Cham	MN	easy	a little	4	AP Statistics
32	42		42	Coon	MN	fairly easy	neutral	4	Calculus BC
45	40		40	Claris	MN	difficult	a little	4	Precalculus
10	40	1	39	St. Pa	MN	fairly easy	neutral	3	Algebra I
91	40	1	39	Minne	MN	fairly easy	not at all	4	Calculus I
117	41	2	39	Dulut	MN	fairly easy	neutral	3	Precalculus
3	39		39	Coon	MN	fairly easy	a little	4	Trig
4	39		39	Owato	MN	fairly easy	a little	3	Calculus I
30	40	2	38	Crysta	MN	fairly easy	a lot	4	Precalculus
18	38		38	Dulut	MN	easy	a lot	4	AP Calculus
60	41	4	37	Braine	MN	fairly easy	neutral	4	Functions, Stats,
62	37		37	Eyota	MN	difficult	not at all	4	College Algebra
28	37	1	36	River	WI	fairly easy	a lot	4	AP Calculus
41	37	1	36	Waub	MN	very	not much	4	Precalculus
73	39	3	36	Lakevi	MN	difficult	neutral	4	Precalculus
80	40	4	36	Parke	MN	fairly easy	a little	4	Precalculus
48	36		36	Beiker	MN	easy	a lot	4	AP Calculus III
56	36		36	Coon	MN	easy	neutral	3	Honors Alg II
64	36	2	34	Moose	MN	fairly easy	a little	4	College Precalcu
78	37	3	34	Mank	MN	fairly easy	neutral	4	AP Statistics
19	34		34	Golde	MN	fairly easy	not much	4	AP Statistics
70	34	1	33	Zimm	MN	easy	a lot	4	AP Calculus
6	32	1	31	St. Fra	MN	fairly easy	a lot	2	College Algebra
94	33	2	31	St. Mic	MN	fairly easy	neutral	4	AP Calculus
68	31		31	Dulut	MN	easy	a lot	4	Calculus I
9	31		31	West	WI	fairly easy	neutral	4	CalculusI/II
40	31		31	Robbi	MN	easy	a lot	4	Calculus II
38	31	1	30	Seoul		difficult	a little	3	Trig
96	32	2	30	Dulut	MN	fairly easy	neutral	4	Precalculus
85	30		30	Brook	MN	fairly easy	not much	4	Precalculus
107	30		30	Golde	MN	fairly easy	not much	4	AP Statistics
88	33	4	29	Maple	MN	difficult	not at all	3	Algebra II
125	34	5	29	Wood	MN	difficult	not much	3	Precalculus
12	29		29	Trum	MN	fairly easy	a lot	4	Calculus I

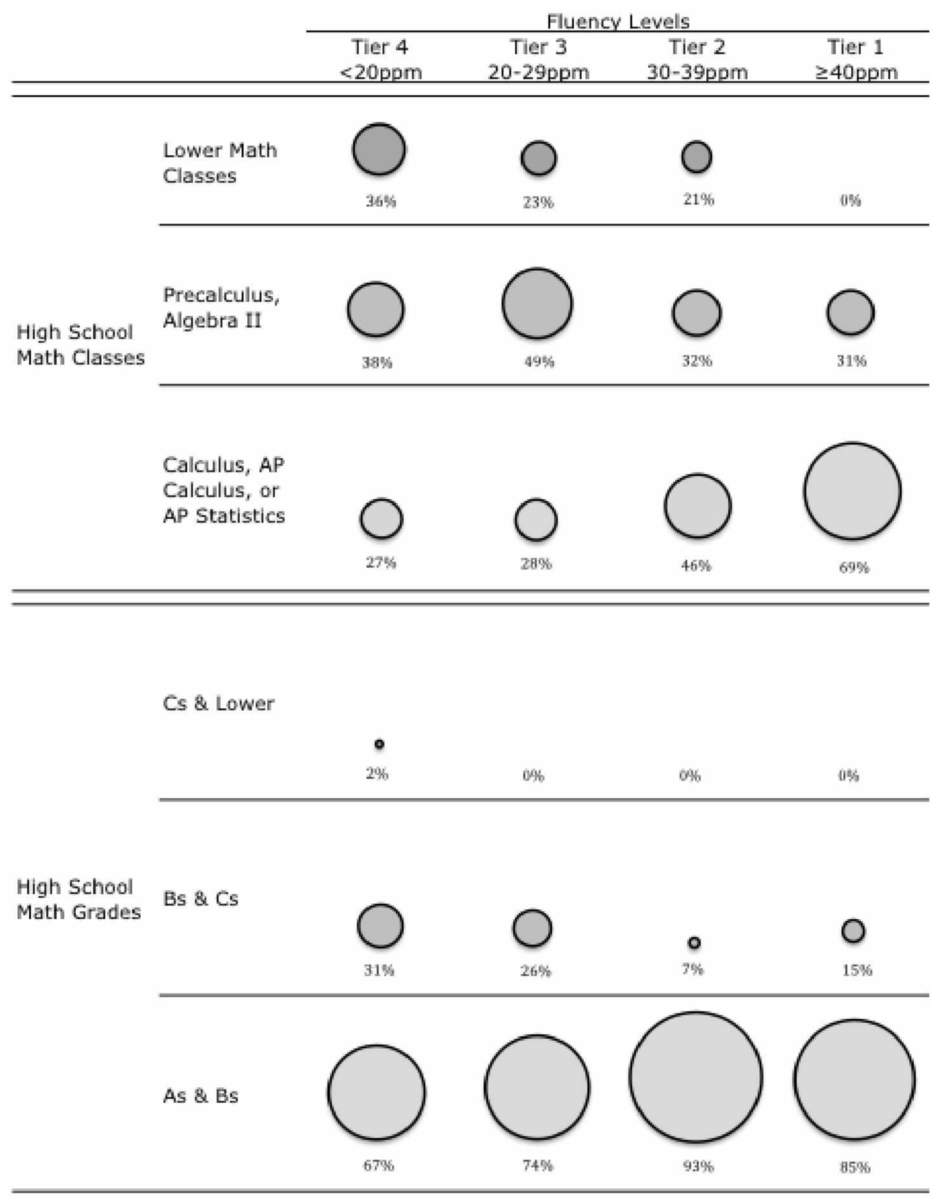
65	29	1	28	Plymo	MN	fairly easy	a little	2	Probability & Sta
77	29	1	28	Excels	MN	fairly easy	not much	4	Consumer Math
98	28		28	Brook	MN	fairly easy	not much	3	Honors Algebra
116	28		28	Andov	MN	fairly easy	not much	4	Trig
53	28	1	27	New	MN	fairly easy	not much	4	Precalculus
103	29	2	27	Apple	MN	fairly easy	a little	4	Precalculus
74	33	6	27	Green	WI	difficult	neutral	4	Precalculus
72	27	1	26	Moun	MN	difficult	not at all	3	Precalculus
82	28	2	26	South	MN	fairly easy	a little	4	Trig
51	28	3	25	Eden	MN	fairly easy	not at all	4	Algebra II/Statis
54	25		25	Fargo	ND	difficult	not much	4	Advanced Math
39	24		24	Dulut	MN	easy	a lot	3	Calculus
75	24		24	Strum	WI	fairly easy	a lot	4	AP Calculus
83	24		24	South	MN	fairly easy	a little	4	Calculus
119	24		24	Mahto	MN	fairly easy	not much	4	Algebra II
50	25	2	23	Albert	MN	fairly easy	neutral	4	AP Calculus
71	25	2	23	Herm	MN	fairly easy	a little	3	Algebra II/Trig
61	28	5	23	Maple	MN	difficult	not at all	4	Statistics
20	23		23	Esko	MN	fairly easy	a lot	4	AP Calculus
63	23		23	Eagan	MN	fairly easy	a lot	4	Precalculus
104	23		23	Zimm	MN	difficult	not much	4	Precalculus
109	23		23	New P	MN	fairly easy	a little	4	AP Calculus
110	23		23	New P	MN	difficult	not at all	3	Precalculus
86	23	1	22	Eagan	MN	difficult	not at all	3	Precalculus
55	24	2	22	Shore	MN	difficult	not at all	4	AP Statistics
92	24	2	22	Minne	MN	difficult	not much	3	Trig
87	25	3	22	Maple	MN	difficult	not at all	4	Statistics
121	29	7	22	Plymo	MN	fairly easy	neutral	4	Precalculus
89	22	1	21	Cham	MN	difficult	not much	4	Probability & Sta
123	22	1	21	St. Lo	MN	easy	neutral	4	AP Calculus
23	23	2	21	Long	MN	fairly easy	a little	4	Calculus I
29	28	7	21	Oakda	MN	fairly easy	a little	4	Precalculus
47	21		21	Monte	MN	easy	a little	4	Calculus I
14	23	3	20	Slayto	MN	difficult	a little	4	Precalculus
113	23	3	20	Brook	MN	fairly easy	a little	4	Precalculus
76	20		20	Apple	MN	easy	a lot	4	Precalculus
37	21	2	19	Minne	MN	very	not at all	3	Functions, Stats,
21	23	4	19	Orono	MN	fairly easy	not much	4	College Algebra
57	26	7	19	White	MN	difficult	not much	4	AP Calculus
46	27	8	19	Water	MN	fairly easy	a little	4	Calculus I
111	19		19	Cottag	MN	very	not at all	4	Precalculus
112	19		19	Brook	MN	very	not at all	4	Statistics
100	20	2	18	Luck	WI	very	not at all	3	Core III ?
16	24	6	18	Wrens	MN	fairly easy	a little	4	Calculus
1	18		18	Waya	MN	difficult	not much	4	Integrated Math
17	18		18	Rams	MN	difficult	a lot	4	Trig
7	18	1	17	Mahto	MN	difficult	not much	3	Algebra II
42	18	1	17	Staple	MN	difficult	a little	2	Trig
124	19	2	17	Wood	MN	fairly easy	not much	3	College Algebra
31	20	3	17	Coon	MN	fairly easy	a little	4	Trig

11		17		17	Glenc	MN	difficult	not much	4	Calculus I
34		18	2	16	Beeke	MN	fairly easy	neutral	3	Algebra II
43		20	4	16	St. Mic	MN	difficult	not at all	4	Statistics
52		20	4	16	Shako	MN	difficult	a little	3	Algebra II
58		20	4	16	Eagan	MN	fairly easy	neutral	4	Precalculus
93		16	1	15	Fridle	MN	difficult	not much	3	Precalculus
105		16	1	15	Milwa	WI	very	not at all	3	Statistics
118		18	3	15	Bloom	MN	difficult	not much	4	Precalculus
115		15		15	Harle	PA	difficult	not at all	4	Precalculus
27		16	2	14	Rockf	MN	easy	a lot	4	Calculus I
90		14	1	13	Intern	MN	fairly easy	not at all	3	Algebra II
66		15	2	13	St. Mic	MN	difficult	not much	4	Probability & Sta
81		13		13	Maple	MN	fairly easy	a little	4	Precalculus
120		13		13	Andov	MN	fairly easy	a lot	4	Calculus AB
44		13	1	12	Braine	MN	fairly easy	not at all	4	Precalculus
25		14	2	12	Baxter	MN	fairly easy	a little	4	AP Statistics
35		14	2	12	Virgin	MN	easy	a lot	4	Precalculus
33		12		12	White	MN	easy	a little	4	AP Statistics
102		12	1	11	Gilber	MN	difficult	not much	3	College Algebra
15		14	3	11	Wrens	MN	fairly easy	not much	4	Calculus
36		14	3	11	Lakevi	MN	difficult	not at all	4	Algebra II
59		11		11	Onala	WI	difficult	neutral	4	AP Statistics
106		11		11	East U	MN	difficult	neutral	4	Precalculus
122		12	2	10	Maple	MN	easy	a little	4	Precalculus
8		12	3	9	St. Pa	MN	easy	a little	4+	AP Calculus
84		15	6	9	Chask	MN	fairly easy	neutral	4	Algebra I
49		10	2	8	Eagan	MN	difficult	not much	3	Algebra I
108		8	1	7	Sartell	MN	easy	a little	4	Calculus
114		11	5	6	Crook	MN	difficult	not much	3	Algebra II
13		6	2	4	Minne	MN	difficult	not much	4	Statistics
67		6	2	4	St. Pa	MN	difficult	not much	4	Algebra II

126	X	40	3	37	Apple	MN	fairly easy	neutral	4	Precalculus
127	X	22	4	18	Dayto	MN	very	not at all	4	Statistics
128	X	27	6	21	Sauk C	MN	easy	a little	4	Calculus I
129	X	13		13	Dulut	MN	very	not at all	4	Integrated
130	X	7		7	Austin	MN	easy	a lot	4	Honors Calculus
131	X	11		11	Cloqu	MN	difficult	not at all	3	?
132	X	42	1	41	Braine	MN	easy	a lot	4	Precalculus
133	X	13	1	12	Merril	WI	very	not at all	4	Algebra II
134	X	22	3	19	Mank	MN	difficult	not much	4	Precalculus
135	X	38	2	36	Fariba	MN	fairly easy	a lot	4	Calculus I
136	X	21		15			very	not at all	4	?
137	X	29	4	25	Lakevi	MN	fairly easy	a little	4	AP Calculus

Appendix F

Math Achievement by Fluency Tier

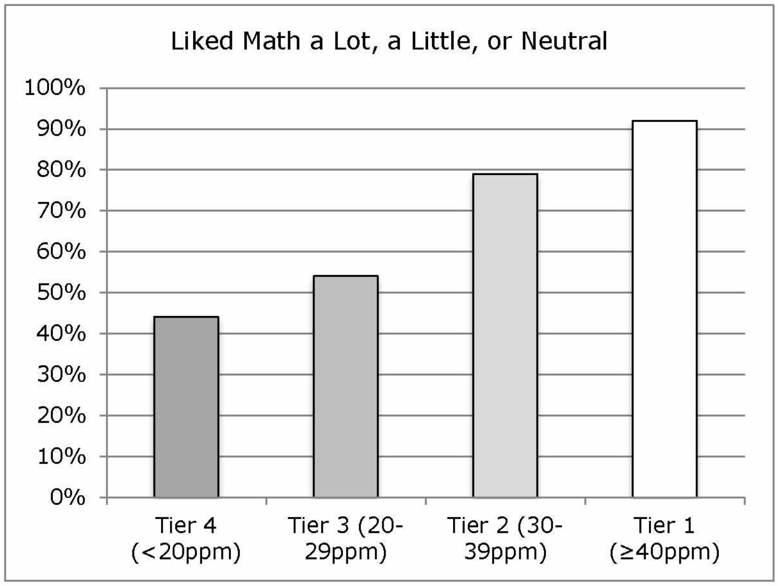


Self-reported high school math achievement within each fluency tier.
ppm = (correct) problems per minute.

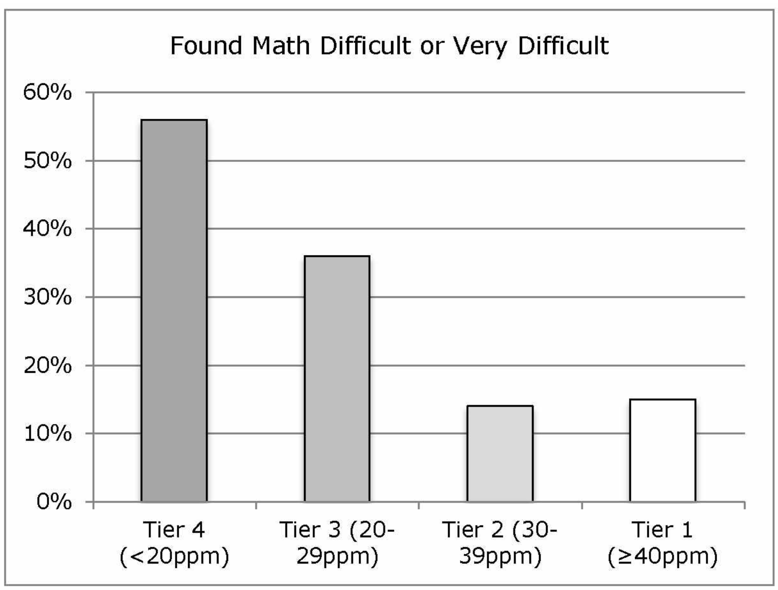
Appendix G

Math Attitude by Fluency Tier

a.



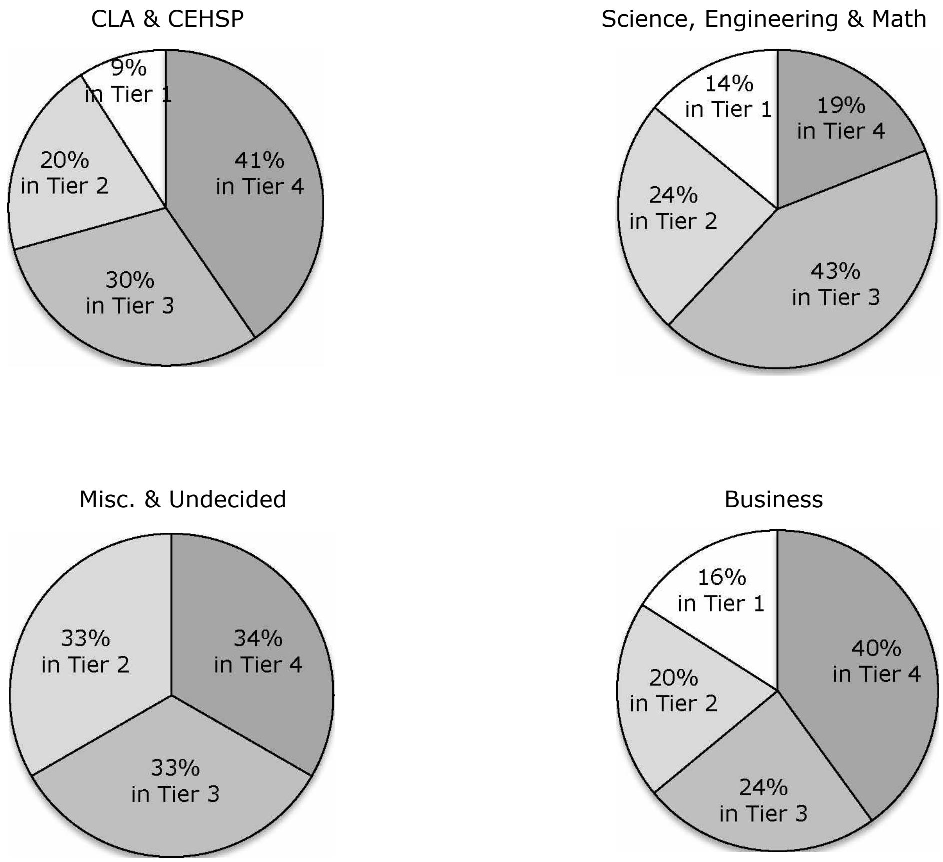
b.



Self-reported math attitudes within each fluency tier.
ppm= (correct) problems per minute. More students reported liking math (or at least being neutral to it) in higher levels of fluency (a). Conversely, more students reported math being difficult (or very difficult) in lower levels of fluency (b).

Appendix H

Fluency Levels Within Majors



Fluency levels within each major.
Fluency Tier 1 = >40ppm (correct problems per minute); Fluency Tier 2 = 30-39ppm; Fluency Tier 3 = 20-29ppm; Fluency Tier 4 = < 20ppm. CLA & CEHSP = College of Liberal Arts & College of Education and Human Service Professions. There were no students in Tier 1 in the Misc. and Undecided category.