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Comment on: "New exact solutions for the Kawahara equation using Exp-function method" [J. Comput. Appl. Math. 233 (2009) 97–102]

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ABSTRACT

Assas [Laila M.B. Assas, New exact solutions for the Kawahara equation using Exp-function method, J. Comput. Appl. Math. 233 (2009) 97–102] found some supposedly new exact solutions to the Kawahara equation by means of the Exp-function method. Unfortunately, they are incorrect. We emphasize that the article contains erroneous formulas and resulting relations. In fact, no numerical method was used.

Contents

In a recent study, Assas [1] implemented the Exp-function method [2] to the Kawahara equation

$$u_t + \alpha u u_x + \beta u_{xxx} - \gamma u_{xxxxx} = 0,$$

where α, β, and γ are nonzero arbitrary constants. This equation, firstly studied in [3], occurs in the theory of magneto-acoustic waves in plasmas. Besides, it describes water waves with surface tension. The Kawahara equation (or fifth-order KdV-type equation) has been the subject of extensive research work during the past four decades or so.

On the other hand, there are some precedents when "solutions" derived by the Exp-function method do not satisfy the original differential equation, for instance, see [4]. Hence, one must give an extra care to the calculations when applying the Exp-function method with the aid of a computer algebra system (CAS), see [5] for a nice discussion. When working with a CAS, it is our belief that we should not blindly trust our computer output, we should always be able to justify why the computer output is a believable answer. It is unfortunate that Assas [1] made one of the common errors from the list of [6], *Sixth error: Some authors do not check the obtained solutions of nonlinear differential equations*. After making a thorough and detailed analysis of [1], we have found numerous mistakes as stated below.

(1) In the Abstract, the author states that, "It is shown that the Exp-function method, with the help of symbolic computation, provides a very effective and powerful (mathematical tools) for discrete nonlinear evolution equations in mathematical physics". However, the author does not study any "discrete" nonlinear equation at all.

(2) In Section 2, the basic idea of the Exp-function method is stated with obvious typographical errors which might confuse the reader. First, u_1 in the expression (1) should be u_t. Second, in the transformation (2) and the ansatz (4), the same constant "c" is involved. Doing so is not allowed. Moreover, we observe, in Section 3, that the constant "w" is taken into account in the wave variable (2) instead of the constant "c". Thus, the wave transformation (2) must be $u = u(\xi)$, $\xi = kx + wt$.

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(3) In Section 3, the author considers the Kawahara equation (5) with two errors. The second term in Eq. (5) must be $\alpha u u_x$ (the constant "k" should not appear in the coefficient) and the third parameter in Eq. (5) is "γ", not "ν". We have additional observations in this section:

(a) For the Case 1 ($p = c = 1, d = q = 1$), the author obtained the solution (17) which is stated as

$$u(x,t) = \frac{\frac{a_{-1}}{b_{-1}} e^{kx+wt} + \frac{a_{-1}b_0}{b_{-1}} + a_{-1}e^{-kx-wt}}{e^{kx+wt} + b_0 + b_{-1}e^{-kx-wt}}$$

where $w = k\left(-4\beta k^2 b_{-1} + 16\gamma k^4 b_{-1} - \alpha a_{-1}\right)/b_{-1}$. By a careful inspection, we observe that this solution can be simplified to the constant a_{-1}/b_{-1} as follows:

$$\begin{aligned} u(x,t) &= \frac{\frac{a_{-1}}{b_{-1}} e^{kx+wt} + \frac{a_{-1}b_0}{b_{-1}} + a_{-1}e^{-kx-wt}}{e^{kx+wt} + b_0 + b_{-1}e^{-kx-wt}} \\ &= \frac{\frac{a_{-1}}{b_{-1}} e^{kx+wt} + \frac{a_{-1}b_0}{b_{-1}} + \frac{a_{-1}b_{-1}}{b_{-1}}e^{-kx-wt}}{e^{kx+wt} + b_0 + b_{-1}e^{-kx-wt}} \\ &= \frac{\frac{a_{-1}}{b_{-1}} \left(e^{kx+wt} + b_0 + b_{-1}e^{-kx-wt}\right)}{e^{kx+wt} + b_0 + b_{-1}e^{-kx-wt}} \\ &= \frac{a_{-1}}{b_{-1}}, \end{aligned}$$

which is useless. It is clear that the expressions (18)–(20) derived from (17) have no any values since they are actually derived from this constant solution. So, Case 1 reveals nothing new.

(b) For the Case 2 ($p = c = 2, d = q = 1$), the expression (21) is constructed from the ansatz (4). Then, following the procedure, the solution set (22) is obtained. However, we see that the arbitrary parameters do not match in the expression (21) and the solution set (22). In the solution set (22), there are two extra arbitrary parameters, namely, a_{-2} and b_{-2}. This is impossible since these parameters are not assumed in the expression (21). At this point, we would like to mention that the expression (21) is not written in a correct manner. According to the ansatz (4), it should be

$$u(\xi) = \frac{a_{-2}\exp(-2\xi) + a_{-1}\exp(-\xi) + a_0 + a_1\exp(\xi)}{b_{-2}\exp(-2\xi) + b_{-1}\exp(-\xi) + b_0 + b_1\exp(\xi)}.$$

Let us, for a while, suppose that this correct expression were substituted into the reduced equation (6). Then we would get the equation

$$(b_{-2} + b_{-1}\exp(\xi) + b_0\exp(2\xi) + b_1\exp(3\xi))^{-6}\sum_{j=1}^{17} C_j \exp(j\xi) = 0.$$

Equating the coefficients C_j ($1 \le j \le 17$) to zero, we obtain a system of nonlinear algebraic equations for $a_1, a_0, a_{-1}, a_{-2}, b_1, b_0, b_{-1}, b_{-2}, k, w, \alpha, \beta$, and γ. Since the system is too large, we demonstrate just one of the equations, corresponding to $j = 1$, as

$$\begin{aligned} &k\alpha a_{-2}a_{-1}b_{-2}^4 + wa_{-1}b_{-2}^5 + k^3\beta a_{-1}b_{-2}^5 - k^5\gamma a_{-1}b_{-2}^5 - k\alpha a_{-2}^2 b_{-2}^3 b_{-1} - wa_{-2}b_{-2}^4 b_{-1} - k^3\beta a_{-2}b_{-2}^4 b_{-1} \\ &\quad + k^5\gamma a_{-2}b_{-2}^4 b_{-1} = 0. \end{aligned}$$

But, the author's solution set (22) still does not satisfy this correctly derived algebraic equation. As a result, the expressions (23)–(26) cannot be the solutions of the Kawahara equation (we verified this fact by a direct substitution with the aid a computer algebra system, as well). Besides, contrary to the author's belief, the expression (26) is not periodic. So, Case 2 provides nothing new.

(c) For the Case 3 ($p = c = 1, d = q = 2$), the expression (27) is constructed from the ansatz (4) with $b_2 = 1$. Then, following the procedure, the solution set (28) is obtained. It is interesting that the solution set (28) of the nonlinear algebraic system is free of the Kawahara equation's parameters α, β, and γ. This situation gives us a clue that something went wrong here. To be more precise, let us substitute the expression (27) with $b_2 = 1$ into the reduced equation (6). Then we obtain the equation

$$(b_1\exp(2\xi) + b_{-1} + b_0\exp(\xi) + \exp(3\xi))^{-6}\sum_{j=1}^{17} C_j \exp(j\xi) = 0.$$

Equating the coefficients C_j ($1 \le j \le 17$) to zero, we obtain a system of nonlinear algebraic equations for $a_1, a_0, a_{-1}, a_2, b_1, b_0, b_{-1}, k, w, \alpha, \beta$, and γ. Since the system is too large, we illustrate just one of the equations, corresponding to $j = 17$, as

$$-wa_1 - k^3\beta a_1 + k^5\gamma a_1 - k\alpha a_1 a_2 + wa_2 b_1 + k^3\beta a_2 b_1 - k^5\gamma a_2 b_1 + k\alpha a_2^2 b_1 = 0.$$

However, the author's solution set (28) does not satisfy this correctly derived algebraic equation. As a result, the expressions (29), (30), and (32) cannot be the solutions of the Kawahara equation (we verified this fact by a direct substitution with the aid a computer algebra system, as well). Our claim is true for another reason that the expressions (29), (30), and (32) do not involve the original equation's parameters α, β, and γ, which is impossible. Moreover, contrary to the author's view, the expression (32) is not periodic. So, Case 3 asserts nothing new.

(4) In Section 4, the author compares the expressions (18), (25), and (30) with the previously known exact solution (33) by using tables. Comparison of exact solutions in this manner seems meaningless. On the other hand, there is no any evidence for the inclusion of a numerical method in the paper. If a numerical method is used, then the issue of comparing exact solutions by such a method is open to dispute. It is a well known fact that a numerical method only provides an approximate solution. It is clear that the author does not find an approximate solution so that a comparison can be made with a previously known exact solution.

(5) In Section 5, the author announces that, "The Exp-function method was successfully used to obtain the exact solutions of Kawahara equation. As a result, some new generalized solitary solutions with parameters are obtained. The obtained solutions are new". Obviously, our analysis refutes the author's argument.

Remark. One must be aware of the following equivalent cases when using the Exp-function method, namely, when dealing with the ansatz (4) as stated in [1]:

$$(p = c = 2, d = q = 1) \equiv (p = c = 1, d = q = 2),$$
$$(p = c = 3, d = q = 1) \equiv (p = c = 2, d = q = 2) \equiv (p = c = 1, d = q = 3),$$
$$(p = c = 4, d = q = 1) \equiv (p = c = 3, d = q = 2) \equiv (p = c = 2, d = q = 3) \equiv (p = c = 1, d = q = 4),$$

and so forth. So, the author [1] discussed one of the Cases 2–3 redundantly.

Conclusion. The effort made by Assas [1] to obtain some new exact solutions to the Kawahara equation failed. As far as we can see, the analytical treatment of the Kawahara equation using the ansatz (4) in [1] is still an open problem, no one achieved so far, including the author of this paper.

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