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A note on functional equivalence between intertemporal and multisectoral investment adjustment costs

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Abstract

Kim (2003, JEDC) shows functional equivalence between intertemporal and multisectoral investment adjustments costs in a linearized RBC model. From an identification point of view, two parameters are not separately distinguishable, they enter as a sum into the linearized solution. We demonstrate that estimating the quadratic approximation of the model provides means to extract more information on the structural parameters from data and thus estimate both parameters that are unidentiable under the log-linearized model.

Keywords: identification, quadratic approximation, pruning, nonlinear DSGE, investment adjustment costs

JEL: C13, C51, E22, E32, O41

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1. Introduction

Current macroeconomic models commonly incorporate investment adjustment costs (IAC from now on) in order to make investment in physical capital costly. Intuitively, a firm can neither instantly change its capital stock nor immediately produce automobiles instead of books without some costs of adjustments. Also, it takes time and resources to change the composition of goods produced, e.g. goods previously used in the consumption sector cannot directly be transferred one-to-one in the investment-good sector. As Kim (2003a, p. 533f) notes:

> Two types of adjustment costs specifications coexist in the macroeconomics literature on investment. One type specifies intertemporal adjustment costs in terms of a nonlinear substitution between capital and investment in capital accumulation, as in Lucas & Prescott (1971), Hayashi (1982), and Abel & Blanchard (1983). The other specification captures multisectoral adjustment costs by incorporating a nonlinear transformation between consumption and investment, which is used by Sims (1989), Vallès (1997), and many other papers adopting multisector models.

Intertemporal IAC, are most commonly used in state-of-the-art dynamic stochastic general equilibrium (DSGE) models as they untangle the linkage to the marginal product of capital, therefore, explaining the acyclic behavior of the real interest rate. The papers mentioned above propose a specification, in which the IAC are based on the first derivative of capital or, in other words, on the current level of investment. It finds use in current DSGE models developed by e.g. the Czech National Bank (Andrle et al., 2009) or the Council for Budget Responsibility for Slovakia (Mucka & Horvath, 2015). However, due to the popularity of models in the fashion of Christiano et al. (2005) and Smets & Wouters (2003), it is now common to introduce IAC which depend on the current growth rate of investment. Even though Christiano et al. (2005) note that this specification successfully generates persistent, hump-shaped responses of aggregate investment and output to monetary policy shocks, Groth & Khan (2010) find no empirical evidence for this kind of specification.[1] The second type of costs, multisectoral IAC, provide

[1] Groth & Khan (2010) use single equations in their empirical analysis, not a full-fledged model with

models with a potentially strong propagation mechanism and can successfully explain co-movements between sectors without relying upon any extra features or frictions, see e.g. Greenwood et al. (2000) and Huffman & Wynne (1999).

Both types of specifications provide interesting model dynamics in different strands of literature. The theoretical relationship between macroeconomic (in)stability and IAC has been studied by (among others) Chin et al. (2012), Kim (2003b) and Herrendorf & Valentinyi (2003). The influence of IAC on news-driven cycles and co-movements has produced a large literature strand both for intertemporal as well as multisectoral IAC: Guo et al. (2015) and Jaimovich & Rebelo (2009) use intertemporal IAC to generate news-driven business cycles, whereas Beaudry & Portier (2007) argue that multisectoral IAC can support positive co-movements between consumption, investment and employment due to changes in expectations in a perfect market environment with variable labor supply. Dupor & Mehkari (2014) and Qureshi (2014) confirm that multisectoral IAC lead to positive sectoral and aggregate co-movement in response to news shocks. The regained interest in using multisectoral IAC is also evident in the residential investment literature, see Kydland et al. (2012) and Garriga et al. (2013). Similar specifications of multisectoral IAC are used to model imperfect labor mobility between the consumption-sector and the investment-sector, e.g. Nadeau (2009). Cassou & Lansing (2006) and Guo & Lansing (2003) also analyze fiscal policy in the presence of intertemporal IAC. Lastly, there is some discussion whether financial frictions and IAC yield almost observational equivalent models, see Bayer (2008), Casalin & Dia (2014) or Ikeda (2011).

As this short review of the literature indicates, the combination of both specifications is rather sparsely found in macroeconomic models.[2]. This is mainly due to the functional equivalence result of Kim (2003a):

> [W]hen a model already has a free parameter for intertemporal adjustment costs, adding another parameter for multisectoral adjustment costs does not

cross-restrictions. Also the variables for the marginal product of capital may be misspecified, since these typically underestimate the nonlinearities due to factor complementarities and time-varying markups, see Linnemann (2016).

[2]Moura (2015) is a recent exception who uses both IAC to study investment price rigidities in a multisector DSGE model. It is argued that the specification of intersectoral frictions solves the functional equivalence of Kim (2003a).

enrich the model dynamics (Kim, 2003a, p. 534).

From an identification point of view this relates to two parameters being collinear, and thus not separately identifiable. Specifically, in the log-linearized version of Kim (2003a)'s small RBC model the individual parameters governing intertemporal and multisectoral IACs enter as a sum into the solution, and are hence not separately identifiable no matter what estimation method one uses. Mutschler (2015), however, has shown that this is due to the log-linearization of the model, a quadratic approximation provides enough restrictions on the mean, i.e. breaking with certainty equivalence, to identify both parameters separately.

Using this theoretical insight and the motivation on intertemporal IAC, we extend the functional equivalence result in the log-linearized model to intertemporal IAC which are based on the growth rate of investment. We show theoretically that a quadratic approximation provides again means to identify both parameters separately. We then demonstrate that the original model of Kim (2003a) can also be estimated in finite samples when using a quadratic approximation to the solution of the model. To this end, we simulate data for different values of parameters and compare the estimation performance of two different extended Kalman filters within a Bayesian estimation framework, namely the *Central Difference Kalman Filter* (Andreasen, 2011) and the *Quadratic Kalman Filter* (Ivashchenko, 2014). Furthermore, we provide additional results on the use of pruning for the estimation of DSGE models, as we specifically account for the effect of pruning within both filters. Accordingly, we simulate data both from the pruned as well as unpruned quadratic approximation. Our estimation strategy is similar to An & Schorfheide (2007) who likewise estimate a small-scale DSGE model solved by a quadratic approximation, however, using a particle filter to evaluate the likelihood. We provide further evidence for their result, that estimating the quadratic approximation of a DSGE model provides means to extract more information on the structural parameters from data. In our case this enables us to estimate both parameters for IAC separately that are unidentiable under the log-linearized model.

2. The Kim (2003) model

The Kim (2003a) model builds upon the canonical neoclassical growth model (see for example Schmitt-Grohé & Uribe (2004)), however, augmenting it with two kinds of IAC. First, intertemporal adjustment costs are introduced into the capital accumulation equation governed by a parameter ϕ, which involve a nonlinear substitution between capital k_t and investment i_t:

$$k_t = \left[\delta\left(\frac{i_t}{\delta}\right)^{1-\phi} + (1-\delta)\left(k_{t-1}\right)^{1-\phi}\right]^{\frac{1}{1-\phi}} \tag{1}$$

with δ denoting the depreciation rate. Note that $\phi = 0$ implies the usual linear capital accumulation specification. Second, we introduce multisectoral adjustment costs into the national budget constraint given a parameter θ, which are captured by a nonlinear transformation between consumption c_t and investment i_t:

$$y_t = a_{t-1}k_{t-1}^{\alpha} = \left[(1-s)\left(\frac{c_t}{1-s}\right)^{1+\theta} + s\left(\frac{i_t}{s}\right)^{1+\theta}\right]^{\frac{1}{1+\theta}} \tag{2}$$

with a_t denoting the level of technology. The average savings rate $s = \frac{c}{y} = \frac{\beta\delta\alpha}{1-\beta+\delta\beta}$ consists of the depreciation rate δ, the discount factor β and the share of capital in production α. Similar to Huffman & Wynne (1999) we focus on $\theta > 1$, i.e. a *reverse CES* technology, in order for the production possibilities set to be convex. Thus ,it becomes more difficult to alter the composition of goods produced in the two sectors. Note that for $\theta = 0$ the transformation is linear.

The representative agent maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \ln c_t$$

subject to the budget constraint (2) and the capital accumulation equation (1). The corresponding Euler equation is

$$\lambda_t(1+\theta)\left(\frac{i_t}{s}\right)^{\theta}\left(\frac{i_t}{\delta k_t}\right)^{\phi} = \beta E_t \lambda_{t+1}\left[\alpha(1+\theta)a_t^{1+\theta}k_t^{\alpha(1+\theta)-1} \right.$$
$$\left. + (1-\delta)(1+\theta)\left(\frac{E_t i_{t+1}}{\delta k_t}\right)^{\phi}\left(\frac{E_t i_{t+1}}{s}\right)^{\theta}\right] \tag{3}$$

with auxiliary variable $\lambda_t = \frac{(1-s)^\theta}{(1+\theta)c_t^{1+\theta}}$. Note that for $\phi = \theta = 0$ this simplifies to the canonical Euler equation. To close the model, technology evolves according to

$$log(a_t) = \rho_a log(a_{t-1}) + \varepsilon_{a,t} \tag{4}$$

with ρ_a measuring persistence and $\varepsilon_{a,t} \sim iid(0, \sigma_a^2)$. The steady state of the model is given by

$$a = 1,\ k = \left(\frac{\delta}{sa}\right)^{\frac{1}{\alpha-1}},\ i = \delta k, c = (1-s)\left[\frac{(\alpha k^\alpha)^{1+\theta} - s\left(\frac{i}{s}\right)^{1+\theta}}{1-s}\right]^{\frac{1}{1+\theta}}.$$

There are two exogenous (k_t and a_t) and no endogenous states. The controls are c_t and i_t and are both assumed to be observable and measured with error $\varepsilon_{c,t} \sim iid(0, \sigma_c^2)$ and $\varepsilon_{i,t} \sim iid(0, \sigma_i^2)$. Since we are only interested in the estimation of two parameters, we fix $\beta = 0.99$ and $\delta = 0.0125$ at standard values and consider the parameter vector at local point and prior specification given in Table 1.

3. Identification analysis

In the original paper Kim (2003a) log-linearizes the model around the non-stochastic steady-state and shows analytically that there is observational equivalence between the two specifications: θ and ϕ enter as a ratio $\frac{\phi+\theta}{1+\theta}$ into the log-linearized solution; hence, they are not distinguishable. This can also be shown via a formal identification analysis using the rank criteria of Iskrev (2010) and Qu & Tkachenko (2012), see also Ratto & Iskrev (2011). In a nutshell, Iskrev (2010)'s approach checks whether the mean, variance and autocovariogram of the observables are sensitive to changes of the deep parameters, whereas Qu & Tkachenko (2012)'s approach focuses on the mean and spectrum of the observables. These changes are measured by Jacobian matrices which are required to have full rank. If we have rank shortages we can analyze the null space to pinpoint the problematic parameters. Columns two and three of table 6 summarize the ranks for the log-linearized model. For all used tolerance levels (which we need to specify for the rank computations) the rank is short by one. Analyzing the nullspace indicates that indeed one has to fix either θ or ϕ to identify the model. Mutschler (2015) extends these criteria for higher-order approximations. Columns four and five in table 6 all display full rank in the quadratic approximation of the model. In other words, the second-order approximation provides additional restrictions to identify both parameters separately. As an additional result we change the specification of the intertemporal IAC (1) with the following specification based on the growth rate of investment:

$$k_t = (1-\delta)k_{t-1} + i_t\left[1 - \frac{\phi}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2\right] \tag{5}$$

The ranks are the same as in table 6, thus we omit the corresponding table. We conclude that the specification of intertemporal IAC in (5) yields the same functional equivalence in the log-linearized model, but can be identified when using a quadratic approximation.

4. Monte-Carlo study

4.1. Solution method and data-generating-process

The exact solution of our nonlinear model is given by a set of decision rules g and h for state variables $x_t = (k_{t-1}, a_{t-1})'$ and control variables $y_t = (c_t, i_t)'$, that is:

$$y_{t+1} = g(x_t, u_{t+1}, \sigma), \qquad x_{t+1} = h(x_t, u_{t+1}, \sigma).$$

and $u_t = (\varepsilon_{a,t}, \varepsilon_{c,t}, \varepsilon_{i,t})$ as we assume that all control variables are observable and subject to measurement errors $\varepsilon_{c,t}$ for consumption and $\varepsilon_{i,t}$ for investment. Furthermore, we introduce the perturbation parameter σ and approximate the functions g and h using a quadratic Taylor approximation around the non-stochastic steady-state ($\sigma = 0$) following e.g. Schmitt-Grohé & Uribe (2004). Therefore our first data-generating-process is given by:

DGP 1 (Unpruned solution)**.**

$$\begin{aligned}
\hat{x}_{t+1} &= h_x\hat{x}_t + h_u u_{t+1} + \frac{1}{2}H_{xx}\left(\hat{x}_t \otimes \hat{x}_t\right) + \frac{1}{2}H_{uu}\left(u_{t+1} \otimes u_{t+1}\right) \\
&\quad + \frac{1}{2}H_{xu}\left(\hat{x}_t \otimes u_{t+1}\right) + \frac{1}{2}H_{ux}\left(u_{t+1} \otimes \hat{x}_t\right) + \frac{1}{2}h_{\sigma\sigma}\sigma^2 \\
\hat{y}_{t+1} &= g_x\hat{x}_t + g_u u_{t+1} + \frac{1}{2}G_{xx}\left(\hat{x}_t \otimes \hat{x}_t\right) + \frac{1}{2}G_{uu}\left(u_{t+1} \otimes u_{t+1}\right) \\
&\quad + \frac{1}{2}G_{xu}\left(\hat{x}_t \otimes u_{t+1}\right) + \frac{1}{2}G_{ux}\left(u_{t+1} \otimes \hat{x}_t\right) + \frac{1}{2}g_{\sigma\sigma}\sigma^2
\end{aligned}$$

A hat denotes deviations from steady-state, e.g. $\hat{y}_t = y_t - \bar{y}$. h_x and g_x denote the solution matrices of the first-order approximation, H_{xx} is a 2×2^2 matrix containing all second-order terms for the i-th state variable in the i-th row, whereas G_{xx} is a 2×2^2 matrix containing all second-order terms for the i-th control variable in the i-th row. H_{xu}, H_{ux}, G_{xu} and G_{ux} are accordingly shaped for the cross terms of states and shocks, and H_{uu} and G_{uu} contain the second-order terms for the product of shocks.

Various simulation studies show that Taylor approximations of an order higher than one may generate explosive time paths, even though the first-order approximation is stable. This is due to artificial fixed points of the approximation, see Kim et al. (2008, p. 3408) for a univariate example. Thus, the model may be neither stationary nor imply an ergodic probability distribution, both of which assumptions are essential for identification and estimation. Thus, Kim et al. (2008) propose the pruning scheme, in which one omits terms from the policy functions that have higher-order effects than the approximation order.[3] For instance, given a second-order approximation, we decompose the state vector into first-order ($\hat{x}_t^f$) and second-order ($\hat{x}_t^s$) effects ($\hat{x}_t = \hat{x}_t^f + \hat{x}_t^s$), and set up the law of motions for these variables, preserving only effects up to second-order (see the technical appendix of Andreasen et al. (2016) for details). Our second data-generating-process is hence given by:

DGP 2 (Pruned solution)**.**

$$\hat{x}_{t+1}^f = h_x \hat{x}_t^f + h_u u_{t+1} \tag{6}$$

$$\begin{aligned} \hat{x}_{t+1}^s = h_x \hat{x}_t^s &+ \frac{1}{2} H_{xx} \left(\hat{x}_t^f \otimes \hat{x}_t^f\right) + \frac{1}{2} H_{uu} \left(u_{t+1} \otimes u_{t+1}\right) \\ &+ \frac{1}{2} H_{xu} \left(\hat{x}_t^f \otimes u_{t+1}\right) + \frac{1}{2} H_{ux} \left(u_{t+1} \otimes \hat{x}_t^f\right) + \frac{1}{2} h_{\sigma\sigma} \sigma^2 \end{aligned} \tag{7}$$

$$\begin{aligned} \hat{y}_{t+1} = g_x(\hat{x}_t^f + \hat{x}_t^s) &+ g_u u_{t+1} + \frac{1}{2} G_{xx} \left(\hat{x}_t^f \otimes \hat{x}_t^f\right) + \frac{1}{2} G_{uu} \left(u_{t+1} \otimes u_{t+1}\right) \\ &+ \frac{1}{2} G_{xu} \left(\hat{x}_t^f \otimes u_{t+1}\right) + \frac{1}{2} G_{ux} \left(u_{t+1} \otimes \hat{x}_t^f\right) + \frac{1}{2} g_{\sigma\sigma} \sigma^2 \end{aligned} \tag{8}$$

Thus, terms containing $\hat{x}_t^f \otimes \hat{x}_t^s$ and $\hat{x}_t^s \otimes \hat{x}_t^s$ are omitted, since they reflect third-order and fourth-order effects which are higher than the approximation order. Also, there are

[3] This may seem an ad hoc procedure, but pruning can also be founded theoretically as a Taylor expansion in the perturbation parameter (Lombardo & Uhlig, 2014) or on an infinite moving average representation (Lan & Meyer-Gohde, 2013).

no second-order effects in u_{t+1}.

4.2. Estimation method

Due to the quadratic approximation we are faced with nonlinearities such that we cannot use the standard Kalman filter to evaluate the likelihood. There is, however, a growing literature on estimating nonlinear solutions to DSGE models, including Quasi-Maximum-Likelihood (QML) estimation (Andreasen, 2011; Ivashchenko, 2014; Kollmann, 2015) and Bayesian Sequential Monte Carlo methods (An & Schorfheide, 2007; Fernández-Villaverde & Rubio-Ramírez, 2007; Herbst & Schorfheide, 2014). We follow this literature and estimate our model parameters both with QML as well as Bayesian MCMC methods. The QML and MCMC algorithms both require a filtering step to evaluate the likelihood, for which we use four different approaches: (1) Quadratic Kalman Filter (QKF from now on), (2) Quadratic Kalman Filter taking specifically the pruned solution into account (QKFP from now on), (3) Central Difference Kalman Filter (CDKF from now on) and (4) Central Difference Kalman Filter taking specifically the pruned solution into account (CDKFP from now on). Therefore we extend results of Andreasen (2011) and Ivashchenko (2014) and tune the filters to account for the stabilizing effect of pruning. The obtained likelihood is, however, often badly shaped, multimodal and has discontinuities. The evaluation of first-order and second-order derivatives is intractable and gradient based optimization methods perform quite poorly. Therefore, we use an optimization routine that is based on simulations, namely, the evolutionary algorithm CMA-ES, see Andreasen (2010) for an application to DSGE models. The rest of the Bayesian framework is standard, as we use a random walk Metropolis-Hastings algorithm as in Schorfheide (2000) and DYNARE. That is, we run two chains, each with 15000 draws, which are initialized at the posterior mode and using the inverse hessian for the initial proposal covariance matrix.

4.3. Estimation results

For our Monte-Carlo study we draw 50 values from the prior domain in table 1 that yield a determinate solution. For each of these draws we simulate paths of the control variables of $T = 100$ using both the (possibly) explosive DGP 1 and stable DGP 2. We then estimate the parameters of the model using each of the four different Kalman filters within a Bayesian framework.[4] First, we present the bias (posterior mean - true value) and posterior standard deviation for the well-identified parameter α in table 3. Here it is evident that all filters are perfectly capable to pinpoint α precisely. Tables 4 and 5 depict the bias and standard errors for θ and ϕ, respectively. Our Monte-Carlo results confirm that all approaches are able to extract information to provide meaningful estimates for both intertemporal as well as multisectoral adjustment costs parameters. The bias and standard errors, however, are not negligible, indicating that these parameters are rather weakly identified. This is not surprising as our sample size is rather small with just 100 data points. Nevertheless, for each MC run at least one of the filters provides estimates within a reasonable credibility set. Moreover, there is apparently more learning from data for ϕ than for θ. Regarding stability we find that accounting for pruning in the filters eases the estimation regardless whether our data is generated by the explosive DGP 1 or the stable DGP 2. This is evident as in some instances we have a standard error of 0.000, which does not indicate very high precision, but rather that something went wrong in the estimation. These instances are much more frequent when we do not account for pruning in the filtering step. Lastly, we comment on estimation speed on a standard desktop computer: the computation of the posterior with 2 chains and 25000 draws each took about 40 min, whereas the computation of the mode using the CMA-ES took about 5 min.

5. Conclusion

We show that both the *Central-Difference Kalman Filter* as well as the *Quadratic Kalman Filter* are very powerful tools to estimate pruned as well as unpruned nonlinear DSGE models, even when the likelihood is badly shaped and we are faced with weakly identified parameters. We are able to identify structural parameters that are unidentifiable under the log-linearized model; thus, confirming the findings of Mutschler (2015) empirically. Economically, we extend the functional equivalence result of Kim (2003a) given intertemporal investment adjustment costs that are based on the growth rate of investment. The quadratic approximation, again, provides means to solve the observational equivalence to multisectoral investment adjustment costs.

[4]The QML estimation is available on request.

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Table 1: Parameters, priors and bounds for Kim (2003)

Parameters		*Prior specification*			*Bounds*	
Parameter	Local Point	Density	Para (1)	Para (2)	Lower	Upper
α	0.60	Gamma	0.60	0.30	1e-5	1
θ	1	Normal	1.00	0.50	-5	5
ρ_a	0.7	Beta	0.50	0.20	1e-5	0.99999
ϕ	2	Normal	2.00	0.50	-5	5
σ_a	0.5	InvGamma	0.50	4.00	1e-8	5
σ_c	0.5	InvGamma	0.50	4.00	1e-8	5
σ_i	0.5	InvGamma	0.50	4.00	1e-8	5

Notes: Para (1) and (2) list the means and the standard deviations for Beta, Gamma, and Normal distributions; the upper and lower bound of the support for the Uniform distribution; s and v for the Inverse Gamma distribution, where $\wp_{IG}(\sigma|v,s) \propto \sigma^{-v-1}e^{-vs^2/2\sigma^2}$.

Table 2: Identification analysis of the Kim (2003) model

	Log-linearized		*Quadratic*	
tol	*Iskrev*	*Qu/Tkachenko*	*Iskrev*	*Qu/Tkachenko*
1e-05	6	6	7	7
1e-09	6	6	7	7
1e-13	6	6	7	7
rob	6	6	7	7
required	7	7	7	7

Table 3: Bias for α

	DGP UNPRUNED				DGP PRUNED			
MC run	CDKF	CDKFP	QKF	QKFP	CDKF	CDKFP	QKF	QKFP
1	0.190 (0.010)	0.014 (0.006)	0.013 (0.007)	0.014 (0.006)	0.110 (0.000)	0.034 (0.006)	0.059 (0.010)	0.060 (0.005)
2	−0.003 (0.000)	−0.004 (0.000)	0.000 (0.000)	−0.003 (0.000)	−0.003 (0.000)	−0.004 (0.000)	0.000 (0.000)	−0.007 (0.000)
3	−0.005 (0.001)	−0.005 (0.000)	0.010 (0.001)	−0.008 (0.000)	−0.004 (0.001)	−0.007 (0.001)	0.000 (0.000)	−0.007 (0.000)
4	0.003 (0.003)	0.001 (0.002)	0.211 (0.001)	−0.000 (0.002)	−0.004 (0.004)	−0.004 (0.002)	0.001 (0.002)	0.001 (0.002)
5	−0.010 (0.001)	−0.007 (0.000)	−0.005 (0.000)	−0.009 (0.000)	−0.008 (0.000)	−0.000 (0.000)	0.003 (0.001)	−0.007 (0.000)
6	0.011 (0.004)	0.003 (0.003)	−0.001 (0.004)	0.006 (0.004)	0.012 (0.002)	−0.016 (0.009)	0.006 (0.002)	0.000 (0.004)
7	−0.015 (0.001)	−0.017 (0.000)	0.000 (0.000)	−0.015 (0.000)	−0.007 (0.001)	−0.005 (0.000)	−0.000 (0.000)	−0.008 (0.000)
8	−0.005 (0.003)	−0.021 (0.002)	−0.015 (0.003)	−0.002 (0.000)	−0.006 (0.002)	−0.006 (0.000)	−0.048 (0.001)	0.001 (0.001)
9	−0.009 (0.001)	−0.009 (0.001)	0.000 (0.000)	−0.006 (0.000)	−0.007 (0.001)	−0.004 (0.001)	−0.010 (0.002)	−0.010 (0.002)
10	0.126 (0.008)	−0.034 (0.005)	−0.039 (0.006)	−0.025 (0.013)	0.110 (0.007)	−0.005 (0.006)	0.006 (0.002)	−0.010 (0.006)
11	−0.009 (0.001)	−0.008 (0.000)	−0.005 (0.000)	−0.009 (0.000)	−0.007 (0.001)	−0.016 (0.000)	−0.010 (0.000)	0.004 (0.000)
12	−0.001 (0.002)	0.003 (0.002)	0.004 (0.002)	−0.001 (0.002)	−0.002 (0.002)	0.002 (0.002)	0.004 (0.002)	0.001 (0.002)
13	0.021 (0.007)	−0.016 (0.001)	−0.008 (0.002)	−0.011 (0.003)	−0.013 (0.006)	−0.017 (0.002)	−0.010 (0.002)	−0.016 (0.003)
14	0.010 (0.003)	−0.012 (0.003)	−0.023 (0.003)	−0.015 (0.004)	−0.008 (0.003)	−0.008 (0.002)	−0.008 (0.003)	−0.006 (0.003)
15	0.005 (0.002)	0.001 (0.001)	0.030 (0.000)	−0.005 (0.001)	0.001 (0.000)	−0.007 (0.001)	0.028 (0.003)	−0.013 (0.001)
16	−0.009 (0.002)	−0.012 (0.001)	−0.010 (0.000)	−0.013 (0.002)	−0.010 (0.001)	−0.011 (0.001)	−0.001 (0.000)	−0.008 (0.000)
17	−0.007 (0.004)	−0.004 (0.002)	−0.009 (0.002)	−0.006 (0.002)	−0.006 (0.003)	0.003 (0.001)	−0.003 (0.002)	−0.016 (0.001)
18	−0.011 (0.002)	−0.021 (0.000)	−0.001 (0.000)	−0.011 (0.001)	−0.021 (0.002)	−0.007 (0.000)	−0.022 (0.000)	−0.027 (0.001)
19	0.030 (0.003)	−0.013 (0.006)	0.362 (0.000)	0.002 (0.004)	0.002 (0.001)	−0.003 (0.002)	0.006 (0.004)	0.005 (0.004)
20	−0.002 (0.003)	−0.013 (0.001)	−0.006 (0.002)	−0.012 (0.002)	0.023 (0.005)	−0.006 (0.003)	−0.007 (0.003)	−0.005 (0.004)
21	−0.002 (0.003)	−0.008 (0.001)	−0.005 (0.001)	−0.007 (0.001)	−0.001 (0.003)	−0.007 (0.001)	0.001 (0.001)	−0.005 (0.002)

22	-0.002 (0.000)	-0.001 (0.000)	-0.001 (0.000)	-0.002 (0.000)	-0.003 (0.001)	-0.001 (0.000)	-0.007 (0.000)	-0.003 (0.000)
23	-0.019 (0.000)	-0.016 (0.003)	0.004 (0.003)	0.002 (0.004)	0.000 (0.000)	-0.003 (0.004)	-0.000 (0.003)	0.003 (0.004)
24	-0.003 (0.001)	-0.002 (0.001)	0.003 (0.000)	-0.003 (0.000)	-0.005 (0.001)	-0.006 (0.001)	-0.002 (0.000)	-0.006 (0.001)
25	0.002 (0.000)	0.002 (0.001)	0.009 (0.000)	0.001 (0.001)	0.000 (0.000)	-0.001 (0.001)	0.013 (0.000)	0.001 (0.002)
26	-0.006 (0.003)	-0.008 (0.002)	0.001 (0.001)	-0.006 (0.002)	-0.002 (0.003)	-0.007 (0.001)	-0.003 (0.001)	-0.008 (0.002)
27	-0.001 (0.002)	-0.007 (0.001)	-0.003 (0.001)	-0.006 (0.001)	-0.006 (0.002)	-0.004 (0.001)	-0.006 (0.001)	-0.004 (0.002)
28	0.030 (0.005)	0.017 (0.003)	0.002 (0.000)	0.026 (0.001)	0.041 (0.003)	0.032 (0.002)	0.007 (0.000)	0.069 (0.000)
29	-0.004 (0.000)	-0.003 (0.000)	0.006 (0.000)	-0.004 (0.000)	-0.002 (0.000)	-0.002 (0.000)	0.001 (0.000)	-0.003 (0.000)
30	0.021 (0.003)	-0.016 (0.003)	-0.001 (0.004)	-0.007 (0.005)	0.008 (0.003)	-0.006 (0.002)	-0.008 (0.003)	-0.007 (0.004)
31	-0.007 (0.003)	-0.031 (0.004)	-0.035 (0.005)	-0.033 (0.006)	-0.000 (0.000)	-0.054 (0.005)	-0.048 (0.005)	-0.061 (0.006)
32	-0.010 (0.001)	-0.010 (0.001)	-0.010 (0.001)	-0.008 (0.001)	-0.009 (0.001)	-0.008 (0.001)	-0.008 (0.001)	-0.005 (0.001)
33	-0.010 (0.003)	-0.013 (0.003)	-0.010 (0.001)	-0.009 (0.001)	-0.016 (0.002)	-0.019 (0.002)	0.038 (0.000)	-0.020 (0.004)
34	-0.017 (0.002)	-0.003 (0.000)	-0.002 (0.000)	-0.004 (0.002)	-0.025 (0.002)	-0.038 (0.001)	-0.019 (0.000)	-0.032 (0.001)
35	-0.010 (0.002)	-0.013 (0.001)	-0.000 (0.000)	-0.012 (0.001)	-0.007 (0.002)	-0.014 (0.001)	-0.013 (0.001)	-0.014 (0.001)
36	0.012 (0.000)	-0.007 (0.004)	0.003 (0.002)	-0.002 (0.003)	0.015 (0.004)	-0.005 (0.004)	0.008 (0.002)	0.001 (0.002)
37	-0.013 (0.001)	-0.015 (0.001)	-0.014 (0.001)	-0.013 (0.001)	-0.007 (0.001)	-0.009 (0.000)	-0.009 (0.001)	-0.008 (0.001)
38	-0.018 (0.006)	-0.110 (0.007)	-0.004 (0.001)	-0.107 (0.000)	-0.054 (0.006)	-0.045 (0.002)	-0.032 (0.000)	-0.060 (0.002)
39	-0.000 (0.000)	-0.023 (0.009)	-0.069 (0.024)	-0.034 (0.009)	0.003 (0.000)	-0.028 (0.002)	-0.012 (0.008)	-0.037 (0.006)
40	-0.013 (0.002)	-0.036 (0.002)	0.003 (0.000)	-0.010 (0.000)	-0.004 (0.003)	-0.030 (0.006)	0.005 (0.000)	-0.010 (0.000)
41	0.002 (0.000)	-0.032 (0.004)	-0.037 (0.010)	0.055 (0.000)	0.001 (0.000)	-0.027 (0.008)	-0.090 (0.012)	0.160 (0.000)
42	0.282 (0.000)	0.050 (0.017)	0.049 (0.011)	0.054 (0.012)	0.029 (0.000)	0.001 (0.007)	0.058 (0.007)	0.034 (0.030)
43	0.054 (0.002)	-0.056 (0.003)	0.013 (0.000)	-0.002 (0.003)	-0.015 (0.003)	-0.001 (0.000)	-0.001 (0.000)	-0.029 (0.002)
44	-0.005 (0.004)	-0.011 (0.002)	-0.021 (0.003)	-0.015 (0.003)	-0.013 (0.002)	-0.016 (0.001)	-0.017 (0.001)	-0.014 (0.002)

45	-0.006 (0.001)	-0.007 (0.001)	-0.007 (0.001)	-0.005 (0.000)	-0.008 (0.001)	-0.009 (0.000)	-0.006 (0.001)	-0.008 (0.001)
46	-0.012 (0.002)	-0.012 (0.001)	-0.009 (0.001)	-0.004 (0.000)	-0.008 (0.002)	-0.015 (0.002)	-0.004 (0.000)	-0.004 (0.000)
47	-0.002 (0.000)	-0.003 (0.000)	0.003 (0.000)	0.001 (0.000)	-0.000 (0.001)	0.001 (0.000)	0.000 (0.000)	-0.006 (0.000)
48	-0.000 (0.000)	-0.060 (0.014)	-0.120 (0.004)	0.000 (0.000)	-0.021 (0.000)	-0.043 (0.014)	-0.182 (0.013)	-0.013 (0.000)
49	-0.006 (0.001)	-0.000 (0.000)	-0.007 (0.001)	-0.006 (0.000)	-0.006 (0.001)	-0.008 (0.000)	-0.007 (0.001)	-0.000 (0.000)
50	-0.020 (0.002)	-0.026 (0.001)	0.006 (0.000)	-0.036 (0.000)	-0.010 (0.005)	-0.004 (0.000)	-0.004 (0.000)	-0.033 (0.002)

Standard deviation of posterior in parenthesis.

Table 4: Bias for θ

	DGP UNPRUNED				DGP PRUNED			
MC run	CDKF	CDKFP	QKF	QKFP	CDKF	CDKFP	QKF	QKFP
1	−0.189 (0.549)	−1.036 (0.478)	−1.247 (0.503)	−0.490 (0.452)	0.047 (0.000)	−0.464 (0.485)	−0.651 (0.449)	−0.573 (0.456)
2	0.103 (0.378)	−1.282 (0.166)	0.000 (0.000)	0.336 (0.344)	0.129 (0.390)	−0.256 (0.009)	0.101 (0.000)	−0.065 (0.035)
3	0.679 (0.398)	0.197 (0.023)	1.277 (0.209)	−0.130 (0.442)	0.621 (0.403)	−1.180 (0.131)	0.000 (0.000)	0.760 (0.399)
4	0.407 (0.413)	0.371 (0.396)	3.212 (0.327)	0.318 (0.404)	0.339 (0.414)	0.272 (0.339)	0.587 (0.369)	0.412 (0.393)
5	−0.622 (0.356)	−0.623 (0.045)	0.638 (0.000)	−1.155 (0.011)	−1.012 (0.321)	0.001 (0.000)	−0.947 (0.235)	2.615 (0.103)
6	−0.086 (0.333)	0.324 (0.451)	0.630 (0.286)	0.430 (0.434)	0.137 (0.238)	0.296 (0.445)	0.408 (0.287)	0.415 (0.424)
7	−0.909 (0.280)	2.419 (0.207)	0.000 (0.000)	−0.707 (0.241)	0.314 (0.255)	0.006 (0.000)	0.000 (0.000)	0.572 (0.174)
8	0.699 (0.117)	0.121 (0.341)	−1.471 (0.051)	−0.062 (0.000)	1.097 (0.173)	0.430 (0.014)	−0.298 (0.004)	0.605 (0.075)
9	0.427 (0.457)	−0.242 (0.081)	−0.575 (0.000)	0.129 (0.066)	0.135 (0.475)	0.409 (0.325)	−0.748 (0.226)	−0.047 (0.478)
10	−0.556 (0.066)	0.455 (0.415)	0.586 (0.483)	0.521 (0.397)	0.059 (0.272)	−0.151 (0.303)	−0.634 (0.280)	−0.434 (0.307)
11	0.056 (0.359)	−1.176 (0.013)	0.337 (0.000)	−0.032 (0.055)	0.679 (0.290)	1.131 (0.182)	0.563 (0.000)	−0.088 (0.015)
12	−0.737 (0.449)	−0.530 (0.469)	−1.212 (0.320)	−0.877 (0.443)	−0.610 (0.449)	−0.715 (0.435)	−0.472 (0.476)	−0.691 (0.441)
13	0.521 (0.241)	−0.154 (0.089)	0.972 (0.336)	0.178 (0.373)	0.162 (0.061)	−0.255 (0.169)	0.863 (0.293)	−0.360 (0.353)
14	−0.434 (0.302)	−0.738 (0.314)	0.651 (0.302)	−0.304 (0.379)	−0.449 (0.405)	−0.406 (0.437)	0.483 (0.358)	−0.468 (0.412)
15	1.239 (0.470)	0.772 (0.438)	−0.362 (0.008)	1.233 (0.448)	0.297 (0.000)	−0.043 (0.011)	−1.033 (0.044)	0.802 (0.507)
16	−0.400 (0.383)	−1.725 (0.325)	−1.199 (0.000)	0.205 (0.422)	−0.815 (0.433)	−1.195 (0.363)	0.155 (0.000)	−0.231 (0.205)
17	0.565 (0.428)	0.684 (0.355)	1.976 (0.323)	0.810 (0.367)	0.098 (0.270)	0.151 (0.180)	0.979 (0.182)	−0.424 (0.251)
18	−0.307 (0.306)	−1.109 (0.007)	−0.543 (0.001)	1.068 (0.380)	−0.276 (0.345)	−0.381 (0.001)	−0.075 (0.000)	1.022 (0.311)
19	0.776 (0.144)	−0.425 (0.677)	0.624 (0.000)	−0.239 (0.449)	−0.313 (0.026)	−0.519 (0.079)	0.230 (0.458)	−0.351 (0.414)
20	0.468 (0.381)	−0.319 (0.277)	0.388 (0.365)	−0.082 (0.309)	0.441 (0.086)	0.110 (0.329)	0.418 (0.344)	0.128 (0.334)
21	0.492 (0.451)	0.149 (0.412)	1.484 (0.381)	0.404 (0.459)	0.487 (0.446)	0.156 (0.407)	0.838 (0.419)	0.503 (0.443)

22	0.233 (0.383)	0.161 (0.011)	−0.590 (0.043)	0.576 (0.041)	0.377 (0.387)	0.651 (0.091)	0.510 (0.000)	0.532 (0.056)
23	−0.615 (0.000)	−0.589 (0.487)	−0.682 (0.443)	−0.469 (0.440)	−0.084 (0.000)	−0.368 (0.459)	−0.384 (0.414)	−0.331 (0.480)
24	1.110 (0.437)	0.187 (0.330)	0.148 (0.000)	1.195 (0.214)	0.700 (0.418)	0.934 (0.425)	−0.414 (0.002)	1.320 (0.470)
25	0.016 (0.000)	−0.264 (0.494)	−0.781 (0.270)	−0.255 (0.464)	0.000 (0.000)	−0.543 (0.525)	2.134 (0.022)	−0.193 (0.482)
26	−0.289 (0.435)	−0.484 (0.348)	−0.481 (0.359)	−0.220 (0.432)	−0.386 (0.443)	−0.289 (0.364)	0.696 (0.393)	−0.457 (0.391)
27	0.887 (0.406)	0.555 (0.407)	2.258 (0.331)	0.786 (0.411)	0.740 (0.430)	0.175 (0.211)	2.541 (0.319)	0.639 (0.401)
28	0.066 (0.442)	0.737 (0.504)	−0.728 (0.053)	−0.293 (0.169)	0.909 (0.561)	0.430 (0.202)	0.039 (0.000)	−0.746 (0.264)
29	−0.087 (0.458)	−0.088 (0.039)	−0.369 (0.024)	−0.218 (0.271)	−0.012 (0.454)	−0.362 (0.270)	0.293 (0.009)	−0.059 (0.022)
30	−0.134 (0.208)	0.583 (0.365)	1.793 (0.348)	0.904 (0.391)	0.011 (0.003)	1.282 (0.252)	1.381 (0.446)	1.011 (0.459)
31	−0.396 (0.142)	−0.249 (0.220)	0.713 (0.274)	−0.276 (0.291)	0.297 (0.000)	−0.676 (0.150)	0.488 (0.199)	−0.707 (0.213)
32	−0.432 (0.398)	−0.859 (0.429)	0.118 (0.337)	−0.316 (0.383)	−0.526 (0.422)	−0.965 (0.403)	0.075 (0.281)	−0.281 (0.380)
33	0.403 (0.276)	−0.074 (0.177)	1.170 (0.243)	0.535 (0.207)	−0.164 (0.113)	−0.554 (0.136)	−0.349 (0.061)	0.032 (0.031)
34	−0.067 (0.241)	−0.806 (0.001)	0.393 (0.000)	−0.265 (0.011)	0.008 (0.272)	−1.302 (0.098)	−0.086 (0.000)	0.156 (0.088)
35	−0.172 (0.324)	−0.935 (0.172)	0.817 (0.000)	−0.441 (0.396)	0.205 (0.035)	−2.114 (0.127)	2.285 (0.212)	−1.553 (0.188)
36	0.648 (0.001)	−0.121 (0.516)	−0.414 (0.425)	−0.150 (0.447)	−0.092 (0.421)	−0.388 (0.459)	−0.471 (0.481)	−0.363 (0.450)
37	0.218 (0.296)	−1.011 (0.185)	0.525 (0.252)	0.045 (0.267)	0.766 (0.316)	−0.401 (0.149)	0.417 (0.185)	0.484 (0.219)
38	0.990 (0.217)	−0.832 (0.116)	−0.658 (0.025)	0.041 (0.022)	−0.528 (0.187)	0.092 (0.037)	−1.038 (0.000)	0.074 (0.172)
39	−0.471 (0.000)	−0.537 (0.234)	−1.404 (0.079)	0.724 (0.320)	−0.231 (0.001)	−0.680 (0.235)	−1.770 (0.036)	0.536 (0.393)
40	0.129 (0.027)	−0.633 (0.145)	−0.485 (0.000)	1.220 (0.071)	−0.011 (0.250)	−0.508 (0.106)	−0.044 (0.000)	0.621 (0.024)
41	0.890 (0.000)	−0.006 (0.140)	−1.698 (0.019)	−0.739 (0.001)	−0.000 (0.000)	0.400 (0.213)	−1.761 (0.022)	−0.312 (0.023)
42	−0.046 (0.011)	−1.421 (0.206)	−0.091 (0.443)	−0.577 (0.466)	0.309 (0.000)	−1.871 (0.028)	−0.496 (0.464)	−0.199 (0.507)
43	2.461 (0.309)	−1.607 (0.072)	0.178 (0.000)	−1.040 (0.358)	0.649 (0.340)	−0.234 (0.006)	−0.064 (0.000)	0.545 (0.323)
44	−0.255 (0.109)	−1.160 (0.171)	0.932 (0.283)	−1.244 (0.230)	−0.656 (0.419)	−0.744 (0.422)	0.042 (0.385)	−0.693 (0.434)

45	0.151 (0.386)	−0.579 (0.332)	−0.045 (0.187)	0.755 (0.329)	0.229 (0.407)	−0.669 (0.252)	0.126 (0.159)	0.552 (0.330)
46	0.077 (0.408)	−0.388 (0.032)	−1.398 (0.123)	0.043 (0.004)	0.109 (0.698)	−0.705 (0.221)	−0.441 (0.000)	0.130 (0.000)
47	−0.429 (0.484)	−0.081 (0.049)	0.963 (0.097)	−0.653 (0.097)	−0.299 (0.472)	0.366 (0.053)	0.000 (0.000)	−0.712 (0.080)
48	0.000 (0.000)	0.217 (0.231)	2.336 (0.222)	0.000 (0.000)	0.400 (0.000)	1.778 (0.347)	−1.343 (0.243)	0.186 (0.000)
49	−0.244 (0.283)	−0.432 (0.000)	0.320 (0.148)	−0.264 (0.193)	−0.345 (0.266)	−1.483 (0.153)	−0.664 (0.112)	0.615 (0.013)
50	0.692 (0.381)	−0.323 (0.176)	−0.035 (0.000)	−0.107 (0.054)	0.178 (0.433)	−0.197 (0.001)	−1.816 (0.017)	0.301 (0.149)

Standard deviation of posterior in parenthesis.

Table 5: Bias for ϕ

	DGP UNPRUNED				DGP PRUNED			
MC run	CDKF	CDKFP	QKF	QKFP	CDKF	CDKFP	QKF	QKFP
1	0.748 (0.403)	−2.368 (0.347)	−0.885 (1.023)	−0.282 (0.359)	−0.020 (0.000)	−1.021 (0.383)	−0.926 (0.232)	−0.272 (0.388)
2	−0.068 (0.128)	−0.508 (0.063)	−0.000 (0.000)	0.225 (0.156)	−0.051 (0.134)	−0.111 (0.006)	0.014 (0.000)	−0.102 (0.013)
3	0.212 (0.124)	0.082 (0.008)	1.019 (0.092)	−0.033 (0.149)	0.210 (0.127)	−0.402 (0.043)	0.000 (0.000)	0.295 (0.136)
4	0.700 (0.355)	0.472 (0.319)	0.086 (0.003)	0.437 (0.343)	0.334 (0.289)	0.317 (0.305)	0.782 (0.311)	0.641 (0.345)
5	−0.548 (0.211)	−0.432 (0.035)	−0.001 (0.000)	−0.723 (0.024)	−0.811 (0.180)	−0.000 (0.000)	−0.111 (0.071)	2.075 (0.153)
6	0.496 (0.171)	0.562 (0.245)	1.657 (0.218)	0.575 (0.257)	0.958 (0.150)	0.002 (0.251)	1.482 (0.220)	0.606 (0.241)
7	−1.061 (0.191)	1.419 (0.174)	0.000 (0.000)	−0.817 (0.163)	−0.135 (0.178)	−0.037 (0.000)	−0.001 (0.000)	0.024 (0.116)
8	0.226 (0.022)	−0.094 (0.088)	0.640 (0.252)	−0.057 (0.000)	0.356 (0.052)	0.101 (0.001)	0.264 (0.001)	0.115 (0.015)
9	0.033 (0.091)	−0.102 (0.016)	0.671 (0.000)	0.016 (0.011)	−0.054 (0.089)	0.078 (0.080)	1.308 (0.165)	−0.182 (0.071)
10	0.235 (0.047)	−0.312 (0.206)	−0.435 (0.197)	0.019 (0.422)	0.998 (0.294)	0.387 (0.310)	−0.024 (0.195)	0.077 (0.274)
11	−0.186 (0.225)	−0.882 (0.012)	1.101 (0.000)	−0.043 (0.035)	0.248 (0.172)	0.645 (0.129)	−0.058 (0.000)	−0.003 (0.000)
12	−0.312 (0.261)	−0.142 (0.319)	0.400 (0.273)	−0.230 (0.298)	−0.531 (0.249)	−0.076 (0.308)	−0.218 (0.298)	−0.053 (0.287)
13	0.307 (0.172)	−0.354 (0.081)	0.401 (0.265)	−0.064 (0.305)	0.009 (0.122)	−0.245 (0.145)	0.584 (0.260)	−0.296 (0.327)
14	0.283 (0.284)	0.125 (0.340)	0.256 (0.188)	−0.148 (0.310)	−0.513 (0.240)	−0.393 (0.298)	0.416 (0.257)	−0.277 (0.318)
15	0.265 (0.093)	0.194 (0.098)	0.888 (0.010)	0.136 (0.074)	0.049 (0.000)	−0.034 (0.006)	−0.172 (0.005)	0.041 (0.069)
16	−0.209 (0.199)	−0.909 (0.155)	0.008 (0.000)	−0.241 (0.244)	−0.607 (0.168)	−0.701 (0.152)	0.655 (0.000)	−0.367 (0.065)
17	0.388 (0.243)	0.362 (0.222)	1.130 (0.180)	0.432 (0.224)	0.205 (0.161)	0.345 (0.131)	1.547 (0.190)	−0.359 (0.100)
18	−0.239 (0.134)	−0.617 (0.011)	0.038 (0.000)	0.677 (0.211)	−0.198 (0.157)	−0.233 (0.002)	0.026 (0.000)	0.485 (0.187)
19	1.335 (0.145)	−0.079 (0.504)	−0.444 (0.000)	0.011 (0.394)	−0.634 (0.019)	−0.256 (0.221)	−0.998 (0.221)	−0.021 (0.401)
20	−0.125 (0.327)	−0.310 (0.353)	0.138 (0.364)	−0.202 (0.348)	0.046 (0.075)	0.004 (0.361)	−0.232 (0.285)	−0.001 (0.379)
21	0.246 (0.199)	0.012 (0.146)	0.611 (0.192)	0.147 (0.183)	0.189 (0.196)	−0.060 (0.146)	0.498 (0.224)	0.146 (0.217)

22	0.028 (0.232)	0.087 (0.028)	1.401 (0.104)	0.345 (0.031)	0.059 (0.220)	0.456 (0.029)	1.554 (0.000)	0.461 (0.044)
23	0.069 (0.001)	−0.316 (0.206)	−0.250 (0.161)	0.304 (0.318)	−0.041 (0.000)	−0.330 (0.455)	0.433 (0.307)	0.058 (0.310)
24	0.429 (0.194)	0.124 (0.173)	1.888 (0.000)	0.401 (0.098)	0.207 (0.184)	0.362 (0.203)	−0.041 (0.000)	0.540 (0.230)
25	0.012 (0.000)	−0.029 (0.032)	−0.077 (0.020)	−0.076 (0.037)	−0.000 (0.000)	−0.030 (0.062)	−0.215 (0.000)	−0.125 (0.047)
26	−0.164 (0.186)	−0.304 (0.165)	−0.093 (0.175)	−0.139 (0.217)	−0.154 (0.209)	−0.109 (0.183)	0.372 (0.208)	−0.176 (0.202)
27	0.440 (0.206)	0.188 (0.193)	1.338 (0.197)	0.307 (0.197)	0.261 (0.205)	0.046 (0.115)	1.101 (0.194)	0.252 (0.221)
28	0.075 (0.061)	−0.046 (0.066)	0.142 (0.005)	0.132 (0.016)	0.021 (0.063)	0.060 (0.020)	−0.003 (0.000)	0.222 (0.015)
29	−0.088 (0.102)	−0.050 (0.010)	0.701 (0.040)	−0.044 (0.068)	−0.057 (0.102)	−0.097 (0.066)	0.816 (0.005)	0.076 (0.010)
30	0.142 (0.099)	−0.070 (0.089)	0.602 (0.159)	0.248 (0.174)	0.008 (0.031)	0.434 (0.128)	0.260 (0.159)	0.296 (0.233)
31	−0.365 (0.118)	−0.560 (0.177)	−0.179 (0.165)	−0.539 (0.245)	0.485 (0.001)	−0.921 (0.105)	−0.044 (0.147)	−0.978 (0.126)
32	−0.430 (0.211)	−0.547 (0.259)	1.077 (0.269)	−0.132 (0.247)	−0.465 (0.225)	−0.513 (0.236)	1.377 (0.277)	−0.146 (0.241)
33	0.088 (0.067)	−0.062 (0.044)	−0.117 (0.006)	0.100 (0.026)	−0.007 (0.034)	−0.159 (0.025)	0.063 (0.001)	−0.095 (0.025)
34	−0.083 (0.212)	−0.786 (0.003)	0.719 (0.000)	−0.263 (0.069)	−0.242 (0.211)	−1.326 (0.081)	0.050 (0.000)	−0.097 (0.045)
35	−0.271 (0.176)	−0.694 (0.096)	0.894 (0.000)	−0.379 (0.242)	0.032 (0.033)	−1.325 (0.072)	0.987 (0.139)	−1.024 (0.102)
36	0.249 (0.001)	−1.018 (0.414)	0.236 (0.341)	−0.227 (0.324)	0.005 (0.311)	−0.076 (0.335)	−0.085 (0.368)	0.028 (0.351)
37	−0.153 (0.216)	−1.009 (0.157)	1.945 (0.260)	−0.190 (0.221)	0.414 (0.257)	−0.442 (0.138)	2.269 (0.242)	0.289 (0.206)
38	0.362 (0.106)	−1.488 (0.085)	−0.346 (0.085)	−0.139 (0.010)	−0.994 (0.181)	−0.131 (0.133)	−1.302 (0.000)	0.306 (0.276)
39	−0.169 (0.000)	−0.254 (0.126)	0.763 (0.188)	−0.078 (0.118)	0.073 (0.001)	−0.380 (0.074)	0.013 (0.148)	−0.158 (0.102)
40	0.257 (0.024)	−0.383 (0.059)	−0.356 (0.000)	0.500 (0.023)	0.073 (0.127)	−0.342 (0.052)	0.081 (0.000)	−0.002 (0.008)
41	0.250 (0.000)	−0.249 (0.060)	−0.386 (0.153)	0.067 (0.001)	0.000 (0.000)	0.044 (0.086)	−0.992 (0.101)	0.511 (0.035)
42	0.047 (0.005)	0.368 (0.448)	−0.524 (0.174)	0.537 (0.380)	0.021 (0.000)	0.043 (0.505)	−0.793 (0.148)	−0.951 (1.002)
43	0.933 (0.102)	−0.438 (0.011)	0.021 (0.000)	−0.189 (0.268)	0.123 (0.083)	0.344 (0.002)	0.085 (0.000)	−0.164 (0.039)
44	−0.279 (0.056)	−0.876 (0.128)	0.167 (0.193)	−1.001 (0.161)	−0.691 (0.246)	−0.692 (0.269)	−0.504 (0.216)	−0.481 (0.348)

45	−0.082 (0.204)	−0.394 (0.199)	1.872 (0.265)	0.443 (0.202)	−0.009 (0.221)	−0.451 (0.158)	2.386 (0.233)	0.226 (0.199)
46	−0.238 (0.183)	−0.368 (0.021)	1.324 (0.320)	0.018 (0.004)	0.045 (0.400)	−0.445 (0.128)	−0.140 (0.000)	0.067 (0.000)
47	−0.039 (0.025)	−0.015 (0.004)	0.092 (0.005)	−0.033 (0.006)	−0.017 (0.028)	0.031 (0.005)	−0.000 (0.000)	−0.128 (0.001)
48	−0.000 (0.000)	−0.216 (0.068)	−0.671 (0.018)	0.000 (0.000)	0.132 (0.000)	1.126 (0.346)	−0.653 (0.131)	0.023 (0.000)
49	−0.548 (0.298)	−0.375 (0.000)	1.851 (0.150)	−0.356 (0.256)	−0.639 (0.283)	−1.842 (0.185)	1.522 (0.277)	0.530 (0.017)
50	0.140 (0.176)	−0.319 (0.082)	−0.161 (0.000)	−0.329 (0.023)	0.114 (0.268)	−0.182 (0.001)	−0.960 (0.008)	0.427 (0.120)

Standard deviation of posterior in parenthesis.