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BOOK REVIEW
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Book Review: Mathematics for Physicists

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A book review on
Mathematics for Physicists

By B. R. Martin and G. Shaw, Chichester: Wiley, 2015.

This new book by B. R. Martin and G. Shaw [1] is part of the Manchester Physics Series, a long-established set of elementary texbooks aimed primarily at providing undergraduate physics students (but also students in other physical sciences, such as astronomy, chemistry, earth and planetary science, etc.) with relatively short, self-contained courses on a number of subjects (including waves, electromagnetism, statistical physics, relativity, quantum mechanics, computer programming and nuclear physics) which are usually covered, at least at an introductory level, in the first 2 years of a typical modern physics degree. Specifically, the book aims to fill the gap between most contemporary high-school mathematics syllabuses and the mathematics needed for a typical undergraduate degree in physics.

The book is well written and organized and it does a good job in the first two chapters of reviewing the basic elements of Euclidean geometry, trigonometry, real numbers, algebra, real variables, and elementary functions which are needed in the sequel. The authors rightly recognize that students often enter university with a variety of mathematical backgrounds. Moreover, they are clearly well aware of the fact that, in the experience of most instructors, even students with good mathematical prerequisites usually need experience using the mathematics to handle it efficiently and to develop useful intuition. The authors do achieve a good balance of formal development and pedagogic illustration by providing many worked out examples throughout the chapters, which both clarify and concretize the concepts or techniques introduced in the main body of the text and highlight useful applications in physics or engineering. In addition, a set of problems is given at the end of each chapter, which mostly provide concrete applications of the material covered in that chapter, but sometimes also introduce new concepts or topics which are left out of the main presentation to avoid digressions (examples include the first law of thermodynamics in problem 7.6, and Gaussian distributions in problem 13.22). Solutions to all the problems are provided in compact form at the end of the book. The treatment will be attractive and useful for lecturers looking for instructive and stimulating material to include, and it will be accessible to most students, including undergraduate students enrolled for an undergraduate university degree and those studying on their own.

The equally competent treatment of the remaining material, which will be new to most students embarking on the course, is conventional and historical rather than abstract and mathematically general. In particular, no formal definitions of the quantities introduced are given, no theorems are formally stated and no formal proofs of results used in the text are worked out. The "pedestrian" physics approach to the material is followed throughout the book, rather than a mathematically more rigorous approach or one which would aim at more generality. Although this approach is fairly solid for the the purposes of most physicists and engineers, and is well-tested in the classrooms of Manchester University and many other institutions offering degrees in physics, it is lacking in some respects. For example, it oftens fails to abstract the concepts and methods which have a much broader range of applicability, in both physics and mathematics, than the historically derived material (e.g. elementary group

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theory, tensors, distributions). Sometimes it also obscures the underlying conceptual and logical structure of a given subject (e.g. linear algebra). It many cases it would have been possible to give more precise and general definitions of the objects introduced, e.g. of fields and algebraic structures, of vector spaces, of linear maps, etc. It would also have been possible to state a number of important theorems and give more formal proofs rather than merely sketch the main ideas of the proofs. In a book on mathematics, albeit one aimed at physicists, this would have introduced the student to the logic and methodology of mathematics and mathematicians, which every serious student of modern physics will encounter in the course of his undergraduate and graduate studies.

The differential and integral calculus are presented in chapters 3 and 4, while series and expansions are treated in chapter 5. Partial derivatives, functions of several variables and changes of variables are discussed in chapter 7. Complex numbers and variables are not introduced before chapter 6, although the material included in this short chapter really belongs to the elementary discussion in chapters 1 and 2. Although complex variables, the complex plane, the elementary complex series and analytic functions are briefly introduced, the book does not give a satisfactory treatment of complex analysis. In particular key notions and results such as the integral of a function along a path in the complex plane are not defined, Cauchy's Theorem is not discussed, singularities and residues are omitted altogether and so are analytic continuation and the important subject of harmonic functions on the complex plane. Apart from constituting pillars of mathematics, these topics have far ranging applications in physics, including many of the subjects for which the book is a preparation.

The remaining chapters are devoted to vector calculus (chapter 8 and 12), matrices and determinants (chapter 9), eigenvalues and eigenvectors (chapter 10), multiple integrals and changes of coordinates (chapter 11), Fourier analysis (chapter 13), ODEs and their series solutions (chapter 14 and 15) and PDEs with applications in physics (chapter 16). The presentation is very clear throughout and the many examples worked out in detail in these chapters, along with the many applications to interesting physical problems which are covered (ranging from electrostatics in chapters 8 and 12, to rotation matrices in three dimensions in chapter 9, to the computation of the normal modes of oscillation of mechanical systems in chapter 10, to the calculation of the volumes of various solids in chapter 11, to signal processing in chapter 13, and to the solution of heat diffusion problems in chapter 16), will help fix the ideas of the students, who often have difficulties with these computationally somewhat more advanced subjects. The chapters on ODEs and PDEs include good elementary discussions of the properties of special mathematical functions (including Legendre polynomials, gamma functions, Bessel functions and spherical harmonics), a knowledge of which is necessary for an understanding of many modern developments in theoretical physics and applied science.

Further mathematical subjects and topics which are missing from the book, but belong to a solid undergraduate physics education, include numerical analysis, statistics (which however is covered in another monograph in the Manchester Physics Series, by R. J. Barlow), an introduction to the calculus of variations, an introduction to densities and distributions, an introduction to tensors and an introduction to group theory. Of course each one of these subjects would easily fill one or two chapters by itself, so that the resulting book would no doubt be significantly longer (and perhaps require a second volume), however there is no easy shortcut through the considerable mathematical background which is needed for a good modern physics degree. This may well be the price to pay if the declared aim in the authors' preface to cover "all the essential mathematics needed for a typical first degree in physics" is to be attained.

The book includes a good index. Moreover an associated website maintained by the editors lists a number of corrections and misprints, provides the student with more detailed solutions to a subset of the many problems set at the end of each chapter and provides registered instructors with both solutions of all problems as well as electronic versions of the figures appearing in the text. Some systematic typos deserve to be corrected in future editions, including all instances of the name of Gottfried Wilhelm Leibniz (the spelling of e.g. the "Leibnitz formula" on p. 100 and of the "Leibnitz rule" on p. 212 are incorrect).

Although some pointers to the literature are given in the text and in a number of footnotes (some of which are duplicated or redundant, however, as on pp. 491, 499, and 531), the book lacks a proper bibliography. This shortcoming could easily be rectified by the authors in subsequent editions and doing so would help the more inquisitive undergraduate student fill in some of the gaps in the text mentioned above. For those using the book for self-study, a good bibliography would provide a first guide to the vast literature on linear algebra, complex analysis, functional analysis, ODEs and PDEs, special functions, and the mathematical methods of physics and of the physical sciences.

In summary, despite some shortcomings this book provides a good basic introduction to selected topics in undergraduate mathematical physics, written by physicists for students in physics and the physical sciences, and can be recommended for its clarity and pedagogic quality. However, achieving its more ambitious aim of covering all the necessary mathematical prerequisites for a good modern physics degree will require substantial additions and revisions in any subsequent editions.

REFERENCES

1. Martin BR, Shaw G. *Mathematics for Physicists*. Chichester: Wiley (2015).

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