



# Perspective on completing natural inflation

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We present a perspective on the inflation paths in 2-, 3-, ...,  $N$ -flation models based on the ultraviolet completion in heterotic string theory, where a number of grand unification scale axions are used. The number of non-Abelian gauge groups for a natural inflation is restricted in string compactification, and we argue that the most plausible completion of natural inflation from a theory perspective is the 2- and 3-flation.

**Keywords:** natural inflation, 2-flation,  $N$ -flation, high scale inflation, GUT scale groups

## 1. INTRODUCTION

After the recent report by the BICEP2 group on a large tensor-to-scalar ratio  $r$  [1], natural inflation [2] and the method of completing it [3] have attracted a great deal of attention [4–9, 11]. Completing natural inflation employs at least two grand unification (GUT) scale non-Abelian gauge groups, which can be called 2-, 3-, ...,  $N$ -flation models. In this paper, we present a perspective on completing natural inflation.

Cosmic inflation is a paradigm for a solution of the homogeneity and flatness problems [12–14]. For a sufficient inflation with the  $e$ -fold number  $e > (50 - 60)$ , one needs small inflation-parameters,  $\epsilon (\equiv \frac{1}{2} M_{\text{P}}^2 (V'/V)^2)$  and  $\eta (\equiv M_{\text{P}}^2 V''/V)$  [15, 16]. Single bubble inflation were proposed with the initial condition near the origin in the Coleman-Weinberg type logarithmically-flat hilltop potential [17], or at a large field value for a chaotic type potential [18]. With the slow-roll conditions satisfied, the local non-Gaussianities  $|f_{\text{NL}}^{\text{local}}|$  are much smaller than 1 for a single field inflation [19], which was reported by the Planck 2013 group [20]. In addition, if the BICEP2 result with a large  $r$  is confirmed, the hybrid inflation predicting  $n_s > 1$  (arising from the hilltop inflation) [21] and the  $\lambda\phi^4$  chaotic inflation are disfavored from the data [20]. Even though the size of dust contribution is still an ongoing issue [22], here we assume a large  $r$  of order 0.1.

The negligible non-Gaussianity pins down the inflation models to the single field  $m^2\phi^2$  chaotic inflation [4–9, 11] or the multi-field hilltop inflation [21]. The  $m^2\phi^2$  chaotic inflation needs a fine-tuning of order  $m^2 \approx 10^{-10}$  in units of the reduced Planck mass,  $M_{\text{P}} \simeq 2.44 \times 10^{18}$  GeV. For the predictability of the Einstein equation, we need that the potential  $V$  during inflation must be much smaller than  $M_{\text{P}}^4$ . In fact, this can be easily realized in *natural inflation* where there exists a GUT scale heavy axion coupling to a GUT scale confining force [2]. With the heavy axion potential at the GUT scale ( $\approx \Lambda_{\text{GUT}} \approx M_{\text{GUT}}$ ), the explicit breaking potential of the Peccei-Quinn (PQ) symmetry is given by

$\propto \frac{1}{2} \Lambda_{\text{GUT}}^4 (1 - \cos(a/f))$ , where  $a$  and  $f$  denote the axion and its decay constant, respectively; thus the potential energy is bounded by  $\Lambda_{\text{GUT}}^4$ .

The  $m^2\phi^2$  chaotic inflation has a problem, “Why does one keep only the quadratic term?” It is known that a large trans-Planckian field value is needed in the  $m^2\phi^2$  chaotic inflation for a large tensor-to-scalar ratio  $r$ , which is known as the Lyth bound ( $\langle \phi \rangle > 15 M_{\text{P}}$  [23]). In particular, with the large trans-Planckian field value higher order terms might be more important [21]. To reconcile the trans-Planckian field value with the natural inflation idea, Kim, Nilles, and Peloso (KNP) introduced two axions and two confining forces at the GUT scale. It has been generalized to  $N$ -flation [24]. An ultra-violet completed theory, in particular the heterotic string theory, may not allow a large number of non-Abelian gauge groups. We scrutinize the inflaton path, arising from the limited rank of the total gauge group, and present an argument that 2-flation (possibly 3-flation also), i.e., the KNP type, is an easily realizable one.

In Section 2, we briefly review the KNP scenario and its  $N$ -flation extension. In Section 3, we discuss the maximum rank of the heterotic string, which is argued for a limitation of the number of GUT scale confining gauge groups. Section 4 is a conclusion.

## 2. THE 2-FLATION

A large vacuum expectation value (VEV) of a scalar field is possible with a small mass parameter if a very small coupling constant  $\lambda$  is assumed,

$$V = \frac{1}{4} \lambda (|\phi|^2 - f^2)^2. \quad (1)$$

The mass parameter in this theory is  $m^2 = \lambda f^2$ . With a GUT scale  $m, f$  can be trans-Planckian of order  $> 10 M_{\text{P}}$  for  $\lambda < 10^{-6}$ . However, the potential (1) with the small  $\lambda$  describes inflation

starting from near the *convex* hilltop point (due to the high temperature effect before inflation) and hence it is not favored by the BICEP2 data [21]. This has led to the recent surge of studies on *concave* potentials near the origin of the field space in case of single field inflations [4–9, 11]. The concave potentials give positive  $\eta$ 's.

The simplest concave potential is the  $m^2\phi^2$  chaotic potential. Since this potential is not bounded from above, the natural inflation with a GUT scale confining force has been introduced so that the potential is bounded from above [2] where the GUT scale axion is the inflaton and the inflaton potential is

$$V = \Lambda_{GUT}^4 \left( 1 - \cos \frac{a_N}{f_N} \right). \quad (2)$$

With  $O(1)$  parameters at the GUT scale,  $f_N$  is  $O(M_{GUT})$ . This potential arises from  $V = -\frac{1}{2}(\Lambda_{GUT}^4 + A)e^{ia/f} + \text{h.c.}$  where  $A$  is a function of a real scalar field  $\rho$ . So, the potential (2) mainly depends on the axion field  $a$ , and  $f_N$  is not determined by Equation (2) but by the potential breaking the PQ symmetry spontaneously, i.e., by Equation (1). For the radial direction to roll down quickly, we need  $\lambda \gg 10^{-6}$  and  $f_N$  is of order  $M_{GUT}$ .

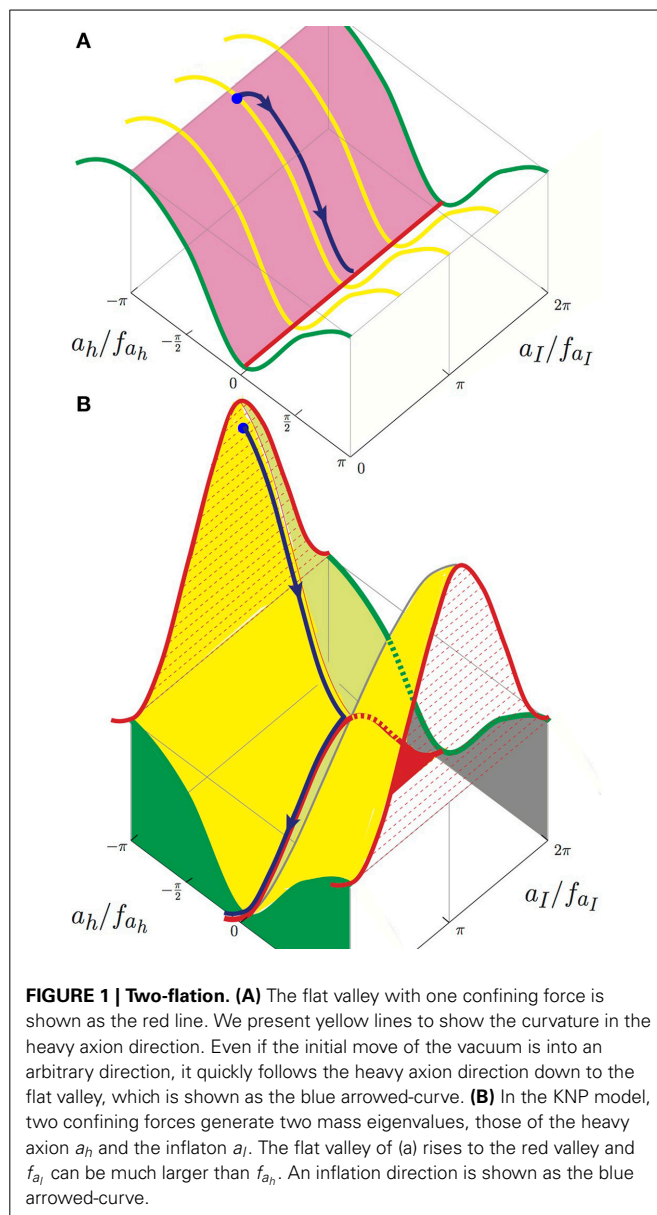
Since we need a trans-Planckian value for the decay constant of the GUT axion, the KNP model has been proposed with two axions  $a_1$  and  $a_2$  and two GUT scale ( $\Lambda_1$  and  $\Lambda_2$ ) confining forces, resulting in the following minus-cosine potential

$$V = \Lambda_1^4 \left( 1 - \cos \left[ \alpha \frac{a_1}{f_1} + \beta \frac{a_2}{f_2} \right] \right) + \Lambda_2^4 \left( 1 - \cos \left[ \gamma \frac{a_1}{f_1} + \delta \frac{a_2}{f_2} \right] \right), \quad (3)$$

where  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\delta$  are determined by two  $U(1)$  quantum numbers. Of course,  $f_1$  and  $f_2$  are  $O(M_{GUT})$ . Let us comment on a few issues related to the above potential.

If there is only one confining force at the GUT scale, we can set  $\Lambda_2 = 0$  in Equation (3). In this case, there exists a flat Goldstone boson direction as shown with the red valley in **Figure 1A**. The blue bullet field point of **Figure 1A** quickly rolls along the blue path down to the red line vacuum. This flat red line direction cannot work as an inflation direction, but it corresponds to an axion potential with an infinite axion decay constant. Thus, with two axions and two nonvanishing axion potentials of Equation (3), there is a possibility of obtaining a trans-Planckian decay constant.

If there are two confining forces with nonvanishing  $\Lambda_1^4$  and  $\Lambda_2^4$ , the situation is shown in **Figure 1B** [3], with no Goldstone boson direction. The red line direction of **Figure 1A** is lifted. To mimic the large  $f$  limit of **Figure 1A**, we choose a direction such that the PQ quantum numbers are aligned, and some people use the terminology ‘aligned inflation’ instead of natural inflation. The condition for the alignment is  $\alpha\delta \approx \beta\gamma$ . This is because two axion masses, the heavy axion  $a_h$  and the light axion  $a_l$ , are



**FIGURE 1 | Two-flation. (A)** The flat valley with one confining force is shown as the red line. We present yellow lines to show the curvature in the heavy axion direction. Even if the initial move of the vacuum is into an arbitrary direction, it quickly follows the heavy axion direction down to the flat valley, which is shown as the blue arrowed-curve. **(B)** In the KNP model, two confining forces generate two mass eigenvalues, those of the heavy axion  $a_h$  and the inflaton  $a_l$ . The flat valley of (a) rises to the red valley and  $f_{a_l}$  can be much larger than  $f_{a_h}$ . An inflation direction is shown as the blue arrowed-curve.

$$m_{a_h}^2 = \frac{1}{2}(A + B), \quad (4)$$

$$m_{a_l}^2 = \frac{1}{2}(A - B), \quad (5)$$

where

$$A = \frac{\alpha^2 \Lambda_1^4 + \gamma^2 \Lambda_2^4}{f_1^2} + \frac{\beta^2 \Lambda_1^4 + \delta^2 \Lambda_2^4}{f_2^2}, \quad (6)$$

$$B = \sqrt{A^2 - 4(\alpha\delta - \beta\gamma)^2 \frac{\Lambda_1^4 \Lambda_2^4}{f_1^2 f_2^2}}. \quad (7)$$

Let the approximation,  $\alpha\delta \approx \beta\gamma$ , is described by a small number  $\Delta$ , i.e.,  $\alpha\delta = \beta\gamma + \Delta$ . Then, the heavy axion and inflaton masses are

$$m_{a_h}^2 \simeq \frac{\alpha^2 \Lambda_1^4 + \gamma^2 \Lambda_2^4}{f_1^2} + \frac{\beta^2 \Lambda_1^4 + \delta^2 \Lambda_2^4}{f_2^2},$$

$$m_{a_l}^2 \simeq \frac{\Delta^2 \Lambda_1^4 \Lambda_2^4}{D} \quad (8)$$

where  $D = f_2^2(\alpha^2 \Lambda_1^4 + \gamma^2 \Lambda_2^4) + f_1^2(\beta^2 \Lambda_1^4 + \delta^2 \Lambda_2^4)$ . For simplicity of discussion, we will set  $\Lambda_1 = \Lambda_2 = \Lambda$  and  $f_1 = f_2 \equiv f$ . Then, the heavy and light masses are

$$m_{a_h}^2 \simeq (\alpha^2 + \beta^2 + \gamma^2 + \delta^2) \frac{\Lambda^4}{f^2}, \quad (9)$$

$$m_{a_l}^2 \simeq \frac{\Lambda^4}{(\alpha^2 + \beta^2 + \gamma^2 + \delta^2) f^2 / \Delta^2}, \quad (10)$$

from which we obtain

$$f_{a_l} = \frac{\sqrt{\alpha^2 + \beta^2 + \gamma^2 + \delta^2} f}{|\Delta|}. \quad (11)$$

With the same order of  $\alpha$ ,  $\beta$ ,  $\gamma$ , and  $\delta$ , the small number  $\Delta$  can be  $O(1)$  to realize  $f_{a_l} \approx 100f$  if  $\alpha, \beta, \gamma, \delta = O(50)$ . Thus, the probability for  $\Delta \approx 1$  to be realized is 1 out of  $50 \times 50$ , i.e., the large  $f_{a_l} \approx 100f_{a_h}$  is possible in 0.04% of random PQ quantum numbers  $\alpha, \beta, \gamma$ , and  $\delta$  of  $O(50)$ . However, note that  $\alpha, \beta, \gamma$ , and  $\delta$  are integer PQ quantum numbers. So, realizing the needed 2-flation is assigning  $\alpha, \beta, \gamma$ , and  $\delta$  such that they are  $O(50)$ . They are not random numbers but quantum numbers of singlet scalar fields. The question is how one can assign these large PQ quantum numbers. It belongs to a question on discrete symmetries [25]. If it is difficult to obtain such discrete symmetries, then one can say that the probability is small. If we do N-flation with a large  $N$ , then the PQ quantum numbers of  $O(2-10)$  are needed as briefly commented below, which may be easier to construct with discrete symmetries. However, the main perspective here is not on the PQ quantum numbers but are on the feasibility of obtaining a large number of non-Abelian gauge groups.

Suppose that we have a  $\mathbf{Z}_{120}$  discrete symmetry. Then, singlet scalar fields can have quantum numbers of  $0, 1, \dots, 119$ . If this discrete symmetry is embedded in the  $U(1)$  PQ symmetry, the PQ domain wall number  $\alpha, \beta, \gamma$ , and  $\delta$  can be  $0, 1, \dots, 119$ . If the singlets having the PQ quantum numbers of  $0, 1, \dots, 49$  and  $71, \dots, 119$  are not allowed to obtain VEVs, then  $\alpha, \beta, \gamma$ , and  $\delta$  can be  $50, \dots, 70$ . In this way, we obtain a desirable quantum numbers such that  $\alpha\delta - \beta\gamma = O(1)$ . This is not an issue of fine tuning but an issue on the VEVs of scalar fields.

We commented above that the flat valley of **Figure 1A** rises to the red valley of **Figure 1B** and  $f_{a_l}$  can be  $\approx 100f_{a_h}$  in a small region of the PQ quantum number space and the inflaton is the blue curve on top of the red valley.

### 3. NUMBER AND SIZES OF NON-ABELIAN GAUGE GROUPS

The KNP 2-flation model has been generalized to N-flation models [24]. The N-flation has adopted two merits of 2-flation, one that the decay constant is  $\approx \sqrt{2}$  times larger and the other that the maximum height of the potential is  $\approx 2$  times larger, as depicted in **Figure 1B**. Namely, in the N-flation we expect that the decay

constant can be  $\approx \sqrt{N}$  times larger and the maximum height of the potential is  $\approx N$  times larger. Then, from the highest point of the potential the denominators in the  $\epsilon$  and  $\eta$  calculation become  $N$  times larger, making  $\epsilon$  and  $\eta$   $N$  times smaller, and the decay constant is about  $\sqrt{N}$  times larger. These merits are gradually diminished as the heavy axion paths shift directions as they roll down the hill<sup>1</sup>.

In addition, in the N-flation the PQ quantum numbers are not tuned to large values of  $O(50)$ . However, an N-flation with a large  $N$  suffers from the theoretical requirement of introducing  $N$  ( $\gg 2$ ) GUT scale non-Abelian gauge groups. In obtaining  $N$ , we must satisfy the SM phenomenology also. After realizing the weak mixing angle,  $\theta_W$  should be  $\sin^2 \theta_W = \frac{3}{8}$  at the GUT scale [27], non-prime orbifold compactification became popular since 2004 [28, 29], and  $\sin^2 \theta_W = \frac{3}{8}$  is possible in many non-prime orbifold GUTs. In general,  $\sin^2 \theta_W = \frac{3}{8}$  was not easy to be realized in earlier orbifold models [30–33]. Successful SM construction have been obtained in the  $\mathbf{Z}_{12-I}$  [34–36] and  $\mathbf{Z}_{6-II}$  orbifold compactifications [37–39]. However, heterotic string models have not provided a useful moduli stabilization program, even though there exist some suggestions on stabilization of some moduli [40–42]<sup>2</sup>. Dynamical supersymmetry breaking [45] would also be another issue in a 2-flation model with a rank 16 gauge group. In the heterotic string theory with level 1 construction, the sum of the ranks of gauge groups is 16 [or 22 in the Narain compactification [46]<sup>3</sup>]. Out of rank 16, the SM uses 4 and rank 12 is left for the GUT scale confining gauge groups. If we use  $SU(4)$ 's for the N-flation, the maximal  $N$  is 4. This is not a case that the N-flation, motivated to have a large  $N$ , is aiming at. For  $N = 3$ , the sum of the ranks of the GUT gauge groups is barely acceptable, taking into account another non-Abelian group for breaking supersymmetry. As commented before, cases of  $N \geq 3$  reduce the PQ quantum number condition of the  $N = 2$  case. For example, to obtain  $f_{a_l} \approx 100f$ , we need an approximate  $10 \times 10 \times 10$  PQ quantum numbers (as we obtained Equation (11) for the  $N = 2$  case). Thus, the probability to obtain  $f_{a_l} \approx 100f$  is about 0.1%. But note that this is the PQ quantum numbers which are not really random priors. It is the problem of vacuum expectation values of singlet scalars with an appropriate discrete symmetry.

In the D-brane construction of string theory, Ramond-Ramond (RR) charges of D-branes should be canceled with proper orientifold p-planes ( $O_p$ -planes), which can be regarded as the fixed planes under a  $\mathbf{Z}_2$  symmetry: anti-D branes can also compensate the RR charge of D-branes, but they hardly break SUSY, making the system unstable. The RR charge of an  $O_p$ -plane ( $p = 0, 1, 2, \dots, 9$ ),  $Q_{O_p}$  is given by  $Q_{O_p} = -2 \cdot 2^{p-5} \times Q_{D_p}$ , where  $Q_{D_p}$  denotes the RR charge of a  $Dp$  brane<sup>4</sup>. [The maximum

<sup>1</sup>See, for example McDonald [26].

<sup>2</sup>For moduli stabilization in other frameworks, see e.g., Kachru et al. [43], Goldberger et al. [44].

<sup>3</sup>For level greater than 1, it is possible to go beyond rank 16. We are aware of one example of construction at level 3 [47], containing the SM gauge group (with suitable Higgs fields for breaking the gauge group) with three families of quarks and leptons. In principle, other string theories dual to the heterotic one with higher levels would also allow gauge groups whose rank is larger than 16. For instance, see [48].

<sup>4</sup>See, for example, Equation (15) of Giveon and Kutasov [49].

$Q_{Op}$  is  $-32 Q_{Dp}$ .] It can be canceled by a stack of  $N_c$   $Dp$  branes (the maximum number is  $32 Q_{Dp}$ ) parallel to an  $Op$ -plane, which yield a rank  $N_c/2$  (the maximum number is 16) gauge group. Therefore, even in the D-brane construction, it is quite hard to obtain a gauge group whose rank is larger than 16.

#### 4. CONCLUSION

The idea of natural inflation, using a GUT scale axion, has been extended to include 2, 3, . . . ,  $N$  axions. For the 2-flation, the PQ quantum numbers are almost degenerate, e.g. differing 1 out of 50. This almost degeneracy of the PQ quantum numbers can be relaxed by increasing  $N$ . In addition, the slow-roll parameters  $\epsilon$  and  $\eta$  can be reduced by a factor  $1/N$ . Models along this line can be constructed at field theory level.

However, in string compactification the number of non-Abelian gauge groups are restricted, which makes the realization of N-flation very difficult. Most SM constructions from string compactification used the level 1 construction in which case the rank is 16. Even if higher levels are assumed, the rank is 22. In any case, the rank cannot be of order 100. Because of this difficulty of obtaining a large number of non-Abelian GUT scale gauge groups, the easiest realization of the trans-Planckian decay constant is the 2- and 3-flation. Nevertheless, it will be interesting to find out  $N \geq 4$  non-Abelian GUT scale gauge groups from string compactification with the features satisfying the low energy SM phenomenology.

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