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On the low-energy limit of one-loop photon–graviton amplitudes

F. Bastianelli [a,b], O. Corradini [c], J.M. Dávila [d], C. Schubert [b,d,*]

[a] Dipartimento di Fisica, Università di Bologna and INFN Sezione di Bologna, Via Irnerio 46, I-40126 Bologna, Italy
[b] Max-Planck-Institut für Gravitationsphysik, Albert-Einstein-Institut, Mühlenberg 1, D-14476 Potsdam, Germany
[c] Centro de Estudios en Física y Matemáticas Básicas y Aplicadas, Universidad Autónoma de Chiapas, C.P. 29000, Tuxtla Gutiérrez, Mexico
[d] Instituto de Física y Matemáticas, Universidad Michoacana de San Nicolás de Hidalgo, Edificio C-3, Apdo. Postal 2-82, C.P. 58040, Morelia, Michoacán, Mexico

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ABSTRACT

We present first results of a systematic study of the structure of the low-energy limit of the one-loop photon–graviton amplitudes induced by massive scalars and spinors. Our main objective is the search of KLT-type relations where effectively two photons merge into a graviton. We find such a relation at the graviton–photon–photon level. We also derive the diffeomorphism Ward identity for the 1PI one-graviton–N-photon amplitudes.

1. Introduction

Although graviton amplitudes are presently not of phenomenological relevance, their structure has been studied in parallel with the more important Yang–Mills amplitudes. The powerful methods that have been developed during the last two decades for the computation of on-shell amplitudes (see, e.g., [1–8]) essentially apply equally to both cases. Moreover, the famous relations found in 1986 by Kawai, Lewellen and Tye (KLT) between closed and open string amplitudes [9] in the field theory limit imply relations between amplitudes in gravity and Yang–Mills theory that are not at all obvious from the field theory Lagrangians or Feynman rules [10–12]. The effort to make these relations transparent also at the field theory level is still ongoing [13–17]. The KLT relations express gravity amplitudes as sums of squares of Yang–Mills amplitudes. They hold at the tree level, but can be used together with unitarity methods for the construction of loop amplitudes in gravity. This is very interesting considering the different UV behaviour of loop amplitudes in gravity vs. Yang–Mills theory, and has been used as a tool in the study of the possible finiteness of $N=8$ supergravity (see [8,18] and refs. therein).

More recently, a different kind of KLT-like relations has been found where the same type of factorization is made manifest even at the integrand level. Reversing the original flow of information, these relations were first conjectured for the n-graviton tree amplitudes in field theory [19], and later extended to and proven in string theory [20,21]. A multiloop generalization in field theory has been conjectured in [22].

Most of this work concerned the case of massless on-shell amplitudes, for which particularly efficient computation methods are available. Much less has been done on amplitudes involving the interaction of gravitons with massive matter. At the tree level, there are some classical results on amplitudes involving gravitons [23,24]. More recently, the tree-level Compton-type amplitudes involving gravitons and spin zero, half and one particles were computed in [25], leading to another remarkable factorization property [26] of the graviton–graviton scattering amplitudes in terms of the photonic Compton amplitudes.

From the point of view of (non-)renormalizability, one-loop gravity–matter amplitudes were studied in [27,28].

There are also a number of results for mixed graviton–gauge boson amplitudes involving a matter loop, namely the graviton–photon–photon vertex [29–33], its non-abelian generalization [34] and the related amplitude for photon–graviton conversion in an external field [35–39].

Here we will present first results of a systematic study of the structure of the mixed photon–graviton amplitudes with a massive loop in the low-energy limit, and of the search for KLT-like relations for such amplitudes. The great advantage of this limit is that it is accessible through the effective action; let us discuss this first for the purely photonic case. As is well known, the information on the low-energy limit of the QED N-photon amplitudes is contained in the Euler–Heisenberg Lagrangian ("EHL") [40]. We recall the standard representation of this effective Lagrangian:

$$\mathcal{L}_{\text{spin}}^{(\text{EH})} = -\frac{1}{8\pi^2}\int_0^\infty \frac{dT}{T^3}e^{-m^2T} \times \left[\frac{(eaT)(ebT)}{\tanh(eaT)\tan(ebT)} - \frac{e^2}{3}(a^2-b^2)T^2 - 1\right]. \tag{1.1}$$

* Corresponding author at: Instituto de Física y Matemáticas, Universidad Michoacana de San Nicolás de Hidalgo, Edificio C-3, Apdo. Postal 2-82, C.P. 58040, Morelia, Michoacán, Mexico.
E-mail address: schubert@ifm.umich.mx (C. Schubert).

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Here T is the proper-time of the loop fermion, m its mass, and a, b are the two Maxwell field invariants, related to $\mathbf{E}$, $\mathbf{B}$ by $a^2 - b^2 = B^2 - E^2$, $ab = \mathbf{E}\cdot\mathbf{B}$. The subtraction terms implement the renormalization of charge and vacuum energy. The analogous representation for scalar QED was obtained by Weisskopf [41]:

$$\mathcal{L}^{(\mathrm{EH})}_{\mathrm{scal}}(F) = \frac{1}{16\pi^2}\int_0^\infty \frac{dT}{T^3} e^{-m^2 T} \times \left[\frac{(eaT)(ebT)}{\sinh(eaT)\sin(ebT)} + \frac{1}{6}\left(a^2 - b^2\right)T^2 - 1\right]. \quad (1.2)$$

Obtaining the low-energy (= large-mass) limit of the N-photon amplitudes from the effective Lagrangians (1.1), (1.2) is a standard procedure (see, e.g., [42]), and the result can be expressed quite concisely [43]:

$$\begin{aligned}
&A_{\mathrm{spin}}\left[\varepsilon_1^+; \ldots; \varepsilon_K^+; \varepsilon_{K+1}^-; \ldots; \varepsilon_N^-\right] \\
&\quad = -\frac{m^4}{8\pi^2}\left(\frac{2ie}{m^2}\right)^N (N-3)! \sum_{k=0}^{K}\sum_{l=0}^{N-K}(-1)^{N-K-l} \\
&\quad\quad \times \frac{\mathcal{B}_{k+l}\mathcal{B}_{N-k-l}}{k!l!(K-k)!(N-K-l)!}\chi_K^+\chi_{N-K}^-, \\
&A_{\mathrm{scal}}\left[\varepsilon_1^+; \ldots; \varepsilon_K^+; \varepsilon_{K+1}^-; \ldots; \varepsilon_N^-\right] \\
&\quad = \frac{m^4}{16\pi^2}\left(\frac{2ie}{m^2}\right)^N (N-3)! \sum_{k=0}^{K}\sum_{l=0}^{N-K}(-1)^{N-K-l} \\
&\quad\quad \times \frac{(1-2^{1-k-l})(1-2^{1-N+k+l})B_{k+l}B_{N-k-l}}{k!l!(K-k)!(N-K-l)!}\chi_K^+\chi_{N-K}^-. \quad (1.3)
\end{aligned}$$

Here the superscripts $\pm$ refer to circular polarizations, and the $\mathcal{B}_k$ are Bernoulli numbers. The invariants $\chi_K^\pm$ are written, in spinor helicity notation (our spinor helicity conventions follow [44]),

$$\begin{aligned}
\chi_K^+ &= \frac{(\frac{K}{2})!}{2^{\frac{K}{2}}}\left\{[12]^2[34]^2\cdots\left[(K-1)K\right]^2 + \text{all permutations}\right\}, \\
\chi_K^- &= \frac{(\frac{K}{2})!}{2^{\frac{K}{2}}}\left\{\langle 12\rangle^2\langle 34\rangle^2\cdots\left\langle(K-1)K\right\rangle^2 + \text{all permutations}\right\}. \quad (1.4)
\end{aligned}$$

For the case of the "maximally helicity-violating" (MHV) amplitudes, which have "all +" or "all −" helicities, Eqs. (1.3) simplify (using Bernoulli number identities) to

$$A_{\mathrm{scal}}\left[k_1, \varepsilon_1^\pm; \ldots; k_N, \varepsilon_N^\pm\right] = -\frac{(2e)^N}{(4\pi)^2 m^{2N-4}}\frac{\mathcal{B}_N}{N(N-2)}\chi_N^\pm, \quad (1.5)$$

$$A_{\mathrm{spin}}\left[k_1, \varepsilon_1^\pm; \ldots; k_N, \varepsilon_N^\pm\right] = -2A_{\mathrm{scal}}\left[k_1, \varepsilon_1^\pm; \ldots; k_N, \varepsilon_N^\pm\right]. \quad (1.6)$$

This relation (1.6) is actually true also away from the low-energy limit, and can be explained by the fact that the MHV amplitudes correspond to a self-dual background, in which the Dirac operator has a quantum-mechanical supersymmetry [45]. For this MHV case Eqs. (1.3) have also been generalized to the two-loop level [46].

One of the long-term goals of the line of work presented here is to obtain a generalization of (1.3) to the case of the mixed N-photon–M-graviton amplitudes. As a first step, in [47] the EHL (1.1) and its scalar analogue were generalized to the effective actions corresponding to the low-energy one-graviton–N-photon amplitudes. Those were obtained in [47] in terms of two-parameter integrals (a similar result was found in [48]). Expanding out the spinor loop Lagrangian in powers of field invariants one finds, up to total derivative terms, the Lagrangian obtained in the seminal work of Drummond and Hathrell [29],

$$\mathcal{L}^{(\mathrm{DH})}_{\mathrm{spin}} = \frac{e^2}{180(4\pi)^2 m^2}\left(5RF_{\mu\nu}^2 - 26R_{\mu\nu}F^{\mu\alpha}F^\nu{}_\alpha + 2R_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta} + 24\left(\nabla^\alpha F_{\alpha\mu}\right)^2\right) \quad (1.7)$$

(see [47] for our gravity conventions). The corresponding form of the effective Lagrangian for the scalar loop (using the same operator basis) is [47]

$$\mathcal{L}^{(\mathrm{DH})}_{\mathrm{scal}} = \frac{e^2}{180(4\pi)^2 m^2}\left[15\left(\xi - \frac{1}{6}\right)RF_{\mu\nu}^2 - 2R_{\mu\nu}F^{\mu\alpha}F^\nu{}_\alpha - R_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta} + 3\left(\nabla^\alpha F_{\alpha\mu}\right)^2\right] \quad (1.8)$$

(the parameter ξ refers to a non-minimal coupling of the scalar). In [49] two of the present authors presented the next order in the expansion of the effective Lagrangians obtained in [47] in powers of the field strength, i.e. the terms of order RF^4 (there are no order RF^3 terms for parity reasons).

The purpose of the present Letter is twofold. First, we will use the above effective actions at the RF^2 level to compute the low-energy limits of the one-graviton–two-photon amplitudes with a scalar and spinor loop, and show that they relate to the four-photon amplitudes in a KLT-like way. Second, as a preparation for the study of the higher-point cases we will derive the Ward identities for the one-graviton–N-photon 1PI amplitudes in general.

2. Ward identities for the 1PI one-graviton–N-photon amplitudes

We derive the relevant Ward identities, generalizing the discussion in [37]. There are two types of Ward identities, those derived from gauge invariance and those that follow from general coordinate invariance.

Gauge transformations are defined by

$$\delta_G A_\mu = \partial_\mu\lambda, \qquad \delta_G g_{\mu\nu} = 0 \quad (2.1)$$

with an arbitrary local parameter λ. Then gauge invariance of the effective action

$$\delta_G\Gamma[g, A] = 0 \quad (2.2)$$

implies that

$$\nabla_\mu\left(\frac{1}{\sqrt{g}}\frac{\delta\Gamma}{\delta A_\mu}\right) = 0. \quad (2.3)$$

Similarly, infinitesimal reparametrizations are given by

$$\delta_R A_\mu = \xi^\nu\partial_\nu A_\mu + \partial_\mu\xi^\nu A_\nu, \qquad \delta_R g_{\mu\nu} = \nabla_\mu\xi_\nu + \nabla_\nu\xi_\mu \quad (2.4)$$

with arbitrary local parameters ξ^μ. The invariance of the effective action

$$\delta_R\Gamma[g, A] = 0 \quad (2.5)$$

now implies

$$\nabla_\mu\left(\frac{2}{\sqrt{g}}\frac{\delta\Gamma}{\delta g_{\mu\nu}} + \frac{1}{\sqrt{g}}\frac{\delta\Gamma}{\delta A_\mu}A^\nu\right) - \frac{1}{\sqrt{g}}\frac{\delta\Gamma}{\delta A_\mu}\nabla^\nu A_\mu = 0. \quad (2.6)$$

The Ward identities thus obtained can be combined and written more conveniently using standard tensor calculus as follows

$$\partial_\mu\frac{\delta\Gamma}{\delta A_\mu} = 0, \quad (2.7)$$

$$2\partial_\mu\frac{\delta\Gamma}{\delta g_{\mu\nu}} + \frac{\delta\Gamma}{\delta A_\mu}\partial_\mu A^\nu + \Gamma^\nu_{\mu\lambda}\left(2\frac{\delta\Gamma}{\delta g_{\mu\lambda}} + \frac{\delta\Gamma}{\delta A_\mu}A^\lambda\right) - \frac{\delta\Gamma}{\delta A_\mu}\nabla^\nu A_\mu = 0. \quad (2.8)$$

We remark that, alternatively, the Ward identities from gauge invariance can be used to simplify the Ward identities from reparametrizations. In fact, an infinitesimal reparametrization can be written as

$$\delta_R A_\mu = \xi^\nu \partial_\nu A_\mu + \partial_\mu \xi^\nu A_\nu = \partial_\mu\big(\xi^\nu A_\nu\big) + \xi^\nu F_{\nu\mu}, \tag{2.9}$$

$$\delta_R g_{\mu\nu} = \nabla_\mu \xi_\nu + \nabla_\nu \xi_\mu \tag{2.10}$$

where the first term in the last rule for A_μ can be interpreted as a gauge transformation. The invariance of the effective action $\delta_R \Gamma[g,A] = 0$ now implies (making use of $\delta_G \Gamma[g,A] = 0$ as well)

$$\nabla_\mu\left(\frac{2}{\sqrt{g}}\frac{\delta\Gamma}{\delta g_{\mu\nu}}\right) + \frac{1}{\sqrt{g}}\frac{\delta\Gamma}{\delta A_\mu} F_\mu{}^\nu = 0, \tag{2.11}$$

i.e.

$$2\partial_\mu \frac{\delta\Gamma}{\delta g_{\mu\nu}} + 2\Gamma^\nu_{\mu\lambda}\frac{\delta\Gamma}{\delta g_{\mu\lambda}} + \frac{\delta\Gamma}{\delta A_\mu} F_\mu{}^\nu = 0 \tag{2.12}$$

which of course is equivalent to (2.8).

Now we consider the special case of the correlation function of one graviton and N photons in flat space.

$$\Gamma^{\mu\nu,\alpha_1\ldots\alpha_N}_{(x_0,x_1,\ldots,x_N)} \equiv \frac{\delta^{N+1}\Gamma}{\delta g_{\mu\nu}(x_0)\delta A_{\alpha_1}(x_1)\cdots\delta A_{\alpha_N}(x_N)}\bigg|_{g_{\mu\nu}=\eta_{\mu\nu}, A_{\alpha_1}=A_{\alpha_2}=\cdots=A_{\alpha_N}=0}. \tag{2.13}$$

Taking appropriate functional derivatives on the general Ward identities (2.7), (2.12) we obtain the gauge Ward identities

$$\partial^{(x_i)}_{\alpha_i}\Gamma^{\mu\nu,\alpha_1\ldots\alpha_N}_{(x_0,x_1,\ldots,x_N)} = 0, \quad i = 1,\ldots,N, \tag{2.14}$$

and the gravitational Ward identities

$$\sum_{i=1}^N \frac{\delta^N\Gamma}{\delta A_\mu(x_0)\delta A_{\alpha_1}(x_1)\cdots\widehat{\delta A_{\alpha_i}}(x_i)\cdots\delta A_{\alpha_N}(x_N)}\bigg| \times \big(\delta^{\alpha_i}_\mu \partial^\nu_{(x_i)} - \eta^{\alpha_i\nu}\partial^{(x_i)}_\mu\big)\delta^D(x_0 - x_i) + 2\partial^{(x_0)}_\mu \Gamma^{\mu\nu,\alpha_1\ldots\alpha_N}_{(x_0,x_1,\ldots,x_N)} = 0, \tag{2.15}$$

where the "hat" means omission.

Fourier transforming the identities (2.14), (2.15) to momentum space

$$\int dx_0\, dx_1 \cdots dx_N\, e^{ik_0x_0+\cdots+ik_Nx_N}\Gamma_{(x_0,\ldots,x_N)} = (2\pi)^D \delta(k_0 + \cdots + k_N)\Gamma[k_0,\ldots,k_N], \tag{2.16}$$

they turn into

$$k_{i\alpha_i}\Gamma^{\mu\nu,\alpha_1\ldots\alpha_N}[k_0,\ldots,k_N] = 0, \quad i = 1,\ldots,N, \tag{2.17}$$

$$2k_{0\mu}\Gamma^{\mu\nu,\alpha_1\ldots\alpha_N}[k_0,\ldots,k_N] + \sum_{i=1}^N \Gamma^{\mu\alpha_1\ldots\widehat{\alpha_i}\ldots\alpha_N} \times [k_0 + k_i, k_1,\ldots,\widehat{k_i},\ldots,k_N]\big(\delta^{\alpha_i}_\mu k^\nu_i - \eta^{\alpha_i\nu}k_{i\mu}\big) = 0. \tag{2.18}$$

Thus the gauge Ward identity is transversal as in QED, while the gravitational Ward identity relates the one-graviton–N-photon amplitude to the pure N-photon amplitudes.

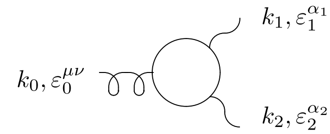

Fig. 1. Graviton–photon–photon diagram.

3. The graviton–photon–photon amplitude

We proceed to the study of the on-shell graviton–photon–photon amplitude induced by a scalar or spinor loop (see Fig. 1).

First, let us remark that this amplitude does not exist at tree level for the fully on-shell case. The covariantized Maxwell term in the action of Einstein–Maxwell theory,

$$S[g,A] = \int d^Dx\sqrt{g}\left(\frac{1}{\kappa^2}R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right), \tag{3.1}$$

contains a graviton–photon–photon vertex, and this vertex with one-photon leg off-shell is responsible for the well-known process of photon–graviton conversion in an electromagnetic field [35–39]. However, it vanishes with all legs on-shell (let us also remark that, fully off-shell, this vertex provides already an example for the non-trivialness of the gravitational Ward identity (2.18)).

The low-energy limit of the one-loop amplitudes is readily obtained from the Drummond–Hathrell form of the effective Lagrangians, (1.7) resp. (1.8). In the on-shell case, the R, $R_{\mu\nu}$ and $(\nabla^\alpha F_{\alpha\mu})^2$ terms all vanish. The remaining term can, after the usual procedure of taking the Fourier transform and then truncating to lowest order in momenta [37], be written as

$$\begin{aligned} R_{\alpha\beta\mu\nu}F^{\alpha\beta}F^{\mu\nu} &= \varepsilon_{0\mu\nu}\Gamma^{\mu\nu;\alpha\beta}\varepsilon_{1\alpha}\varepsilon_{2\beta}, \\ \Gamma^{\mu\nu;\alpha\beta} &= 4\big[-k_1^{(\mu}k_2^{\nu)}k_0^\alpha k_0^\beta + \eta^{\alpha(\mu}k_2^{\nu)}k_0^\beta k_0\cdot k_1 \\ &\quad + \eta^{\beta(\mu}k_1^{\nu)}k_0^\alpha k_0\cdot k_2 - \eta^{\alpha(\mu}\eta^{\nu)\beta}k_0\cdot k_1 k_0\cdot k_2\big]. \end{aligned} \tag{3.2}$$

At the $N = 2$ level the purely photonic terms in the gravitational Ward identity (2.18) vanish on-shell. Thus this identity holds in its usual form $k_{0\mu}\Gamma^{\mu\nu;\alpha\beta} = 0$, as can be checked with (3.2).

Proceeding to the helicity decomposition of the amplitude, using a factorized graviton polarization tensor as usual,

$$\begin{aligned} \varepsilon^{++}_{0\mu\nu}(k_0) &= \varepsilon^+_{0\mu}(k_0)\varepsilon^+_{0\nu}(k_0), \\ \varepsilon^{--}_{0\mu\nu}(k_0) &= \varepsilon^-_{0\mu}(k_0)\varepsilon^-_{0\nu}(k_0), \end{aligned} \tag{3.3}$$

together with a judicious choice of the reference momenta one can easily show that, of the six components of this amplitude, only the MHV ones are non-vanishing[1]:

$$\begin{aligned} \varepsilon^{++}_{0\mu\nu}\Gamma^{\mu\nu;\alpha\beta}\varepsilon^+_{1\alpha}\varepsilon^+_{2\beta} &= -[01]^2[02]^2, \\ \varepsilon^{--}_{0\mu\nu}\Gamma^{\mu\nu;\alpha\beta}\varepsilon^-_{1\alpha}\varepsilon^-_{2\beta} &= -\langle 01\rangle^2\langle 02\rangle^2. \end{aligned} \tag{3.4}$$

Including the prefactors in (1.7), (1.8), and restoring the coupling constants, we obtain the final result,

$$\begin{aligned} A^{(++;++)}_{\text{spin}} &= -\frac{\kappa e^2}{90(4\pi)^2m^2}[01]^2[02]^2, \\ A^{(--;--)}_{\text{spin}} &= -\frac{\kappa e^2}{90(4\pi)^2m^2}\langle 01\rangle^2\langle 02\rangle^2. \end{aligned} \tag{3.5}$$

[1] It should be mentioned that even for these components the right-hand sides will vanish after taking into account that for a massless three-point amplitude energy–momentum conservation forces collinearity of the three momenta. However, this is a low-point kinematic accident and not relevant for our structural investigation.

Here the first upper index pair refers to the graviton polarization, and κ is the gravitational coupling constant. Moreover, those components fulfill the MHV relation (1.6),

$$A_{\text{spin}}^{(++;++)} = (-2)A_{\text{scal}}^{(++;++)},$$
$$A_{\text{spin}}^{(--;--)} = (-2)A_{\text{scal}}^{(--;--)}. \tag{3.6}$$

Also, these graviton–photon–photon amplitudes relate to the (low-energy) four-photon amplitudes in the following way: From (1.3), (1.4) the only non-vanishing components of those are

$$A^{++++}[k_1,k_2,k_3,k_4] \sim [12]^2[34]^2 + [13]^2[24]^2 + [14]^2[23]^2,$$
$$A^{++--}[k_1,k_2,k_3,k_4] \sim [12]^2\langle 34\rangle^2,$$
$$A^{----}[k_1,k_2,k_3,k_4] \sim \langle 12\rangle^2\langle 34\rangle^2 + \langle 13\rangle^2\langle 24\rangle^2 + \langle 14\rangle^2\langle 23\rangle^2. \tag{3.7}$$

Replacing $k_1 \to k_0$, $k_2 \to k_0$ in the 4-photon amplitudes, the middle one of these three components becomes zero, and the remaining ones become proportional to the corresponding components of (3.5):

$$A^{++++}[k_0,k_0,k_3,k_4] \sim 2[03]^2[04]^2 \sim A^{++;++}[k_0,k_3,k_4],$$
$$A^{----}[k_0,k_0,k_3,k_4] \sim 2\langle 03\rangle^2\langle 04\rangle^2 \sim A^{--;--}[k_0,k_3,k_4]. \tag{3.8}$$

Thus in all cases one finds the same proportionality, namely

$$A_{\text{scal,spin}}^{\pm\pm;\pm\pm}[k_0,k_1,k_2] = \frac{1}{12}\frac{\kappa}{e^2}m^2 A_{\text{scal,spin}}^{\pm\pm\pm\pm}[k_0,k_0,k_1,k_2]. \tag{3.9}$$

Effectively two photons have merged to form a graviton, clearly a result in the spirit of the new KLT-like relations. In [21] the same (except for the proportionality constant) relation was found for the graviton–photon–photon amplitude in superstring theory at the tree level.

We have derived our results using the low-energy effective action, but it is straightforward to reproduce them using worldline methods, where the gravitational amplitude $A^{\pm\pm;\pm\pm}$ is obtained by computing, in the low-energy limit, a correlation function of the corresponding vertex operators of the form

$$\int_0^\infty \frac{dT}{T}\frac{e^{-m^2T}}{(4\pi T)^2}\left(-\frac{\kappa}{4T}\right)(-ie)^2\left\langle V_{grav}(k_0)V_{ph}(k_1)V_{ph}(k_2)\right\rangle. \tag{3.10}$$

Indeed, we hope to use such methods to investigate higher-point amplitudes.

4. Conclusions

We have shown here that, in the low-energy limit, the one-loop graviton–photon–photon amplitudes in Einstein–Maxwell theory coupled to scalars or spinors relate to a coincidence limit of the QED four-photon amplitudes. This provides a new example of a KLT-like factorization in field theory at the loop level, and agrees with an identity found in [21] at the tree level in superstring theory. It also raises the possibility that, at least in the low-energy limit, the M-graviton–N-photon amplitudes may be derivable from the $(N+2M)$-photon amplitudes. However, it must be emphasized that the three-point amplitude is rather special in this context due to the absence of one-particle reducible contributions. The inhomogeneity of the gravitational Ward identity (2.18) leads one to expect that, starting from the one-graviton–four-photon level, the 1PI one-graviton–N-photon amplitudes will not be transversal in the graviton indices, so that a relation with the purely photonic amplitudes can exist only for the full amplitudes. The calculation of the one-graviton–four-photon amplitude is in progress.

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