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Modified non-local-$F(R)$ gravity as the key for the inflation and dark energy

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Abstract

We consider FRW cosmology in non-local modified gravity. Its local scalar-tensor formulation is developed. It is explicitly demonstrated that such theory may lead to the unification of early-time inflation with late-time cosmic acceleration. The quintessence or phantom era may emerge for specific form of the action. The coupled non-local-$F(R)$ gravity is also investigated. It is shown that such theory being consistent with Solar System tests may lead to the known universe history sequence: inflation, radiation/matter dominance and dark epoch.

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1. Introduction

Modified gravity [1] suggests serious alternative for dark energy origin. Indeed, it may be naturally expected that gravitational action contains some extra terms which became relevant recently with the significant decrease of the universe curvature. The number of metric formulation modified $F(R)$ gravities has been proposed [1–5] with the purpose to explain the origin of cosmic acceleration. Special attention is paid to $F(R)$ models [6–9] with the effective cosmological constant phase because such theories may easily reproduce the well-known ΛCDM cosmology. Such models subclass which does not violate Solar System tests represents the real alternative for standard general relativity.

From another point, it is expected that due to string/M-theory corrections the early-time universe may be also governed by modified gravity which initiates the inflationary epoch. Some (phenomenological) modified gravities which may unify the inflation and dark energy have been proposed [3,8]. The search for viable gravitational dark energy/inflation continues. Recently, it was suggested to consider non-local gravity [10] which may be produced by quantum effects as the source of cosmic acceleration.

In the present Letter we propose the class of non-local modified gravities with the effective cosmological constant epoch. The local scalar-tensor formulation of such theory is developed. Several explicit examples are considered where it is shown that such non-local gravity may lead to the unification of the early-time inflation with late-time acceleration being consistent with the known universe expansion history and local tests. For other examples, non-local modified gravity may lead to quintessence/phantom cosmology. Non-local gravity coupled with $F(R)$ theory is considered. It is demonstrated that it is even easier to achieve the unification of the early-time inflation with late-time acceleration (with radiation/matter dominance phases between them) in such coupled theory which seems to be consistent with local tests. In fact, it is shown that it is natural to unify the early-time inflation with late-time acceleration in modified gravity with two different types of terms: first being relevant at the early universe and second being relevant at late universe.

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2. Unifying inflation with dark energy in the non-local modified gravity

In this section we present local (scalars-tensor) formulation of non-local gravity. The explicit example of such theory which naturally leads to the unification of inflation with cosmic acceleration is worked out.

The starting action of the non-local gravity is given by

$$S=\int d^4x\sqrt{-g}\left\{\frac{1}{2\kappa^2}R\left(1+f\left(\Box^{-1}R\right)\right)+\mathcal{L}_{\rm matter}\right\}. \tag{1}$$

Here f is some function and $\Box$ is the d'Almbertian for scalar field. Note that our approach is purely phenomenological. Generally speaking, such non-local effective action may be induced by quantum effects (for instance, via RG improving [11]). The above action can be rewritten by introducing two scalar fields ϕ and ξ in the following form:

$$\begin{aligned}S&=\int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left\{R\left(1+f(\phi)\right)+\xi(\Box\phi-R)\right\}\right.\\&\quad\left.+\mathcal{L}_{\rm matter}\right]\\&=\int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left\{R\left(1+f(\phi)\right)-\partial_\mu\xi\partial^\mu\phi-\xi R\right\}\right.\\&\quad\left.+\mathcal{L}_{\rm matter}\right].\end{aligned} \tag{2}$$

By the variation over ξ, we obtain $\Box\phi=R$ or $\phi=\Box^{-1}R$. Substituting the above equation into (2), one re-obtains (1).

Varying (2) with respect to the metric tensor $g_{\mu\nu}$ gives

$$\begin{aligned}0&=\frac{1}{2}g_{\mu\nu}\left\{R\left(1+f(\phi)-\xi\right)-\partial_\rho\xi\partial^\rho\phi\right\}\\&\quad-R_{\mu\nu}\left(1+f(\phi)-\xi\right)+\frac{1}{2}(\partial_\mu\xi\partial_\nu\phi+\partial_\mu\phi\partial_\nu\xi)\\&\quad-(g_{\mu\nu}\Box-\nabla_\mu\nabla_\nu)\left(f(\phi)-\xi\right)+\kappa^2T_{\mu\nu}.\end{aligned} \tag{3}$$

On the other hand, the variation with respect to ϕ gives

$$0=\Box\xi+f'(\phi)R. \tag{4}$$

Now we assume the FRW metric

$$ds^2=-dt^2+a(t)^2\sum_{i=1,2,3}\left(dx^i\right)^2, \tag{5}$$

and the scalar fields ϕ and ξ only depend on time. Then Eq. (3) has the following form:

$$\begin{aligned}0&=-3H^2\left(1+f(\phi)-\xi\right)+\frac{1}{2}\dot\xi\dot\phi\\&\quad-3H\left(f'(\phi)\dot\phi-\dot\xi\right)+\kappa^2\rho,\end{aligned} \tag{6}$$

$$\begin{aligned}0&=\left(2\dot H+3H^2\right)\left(1+f(\phi)-\xi\right)+\frac{1}{2}\dot\xi\dot\phi\\&\quad+\left(\frac{d^2}{dt^2}+2H\frac{d}{dt}\right)\left(f(\phi)-\xi\right)+\kappa^2p.\end{aligned} \tag{7}$$

On the other hand, scalar equations are:

$$0=\ddot\phi+3H\dot\phi+6\dot H+12H^2, \tag{8}$$

$$0=\ddot\xi+3H\dot\xi-\left(6\dot H+12H^2\right)f'(\phi). \tag{9}$$

The remark is in order. We may consider more general action:

$$\begin{aligned}S&=\int d^4x\sqrt{-g}\\&\quad\times\left[F\left(R,\Box R,\Box^2R,\ldots,\Box^mR,\Box^{-1}R,\Box^{-2}R,\ldots,\Box^{-n}R\right)\right.\\&\quad\left.+\mathcal{L}_{\rm matter}\right].\end{aligned} \tag{10}$$

Here m and n are positive integers. Again, for the action (10), by introducing $2n$-scalars, one can rewrite the action (10) in a local form:

$$\begin{aligned}S&=\int d^4x\sqrt{-g}\left[F\left(R,\Box R,\Box^2R,\ldots,\Box^mR,\phi_1,\phi_2,\ldots,\phi_n\right)\right.\\&\quad\left.+\sum_{k=1}^n\xi_k\left(\Box^k\phi_k-R\right)+\mathcal{L}_{\rm matter}\right].\end{aligned} \tag{11}$$

The generalization for non-integer m is also possible.

We now assume de Sitter solution $H=H_0$, then Eq. (8) can be solved as

$$\phi=-4H_0-\phi_0e^{-3H_0t}+\phi_1, \tag{12}$$

with constants of integration, ϕ_0 and ϕ_1. For simplicity, we only consider the case that $\phi_0=\phi_1=0$. We also assume $f(\phi)$ is given by

$$f(\phi)=f_0e^{b\phi}=f_0e^{-4bH_0\phi}. \tag{13}$$

Then Eq. (9) can be solved as follows,

$$\xi=-\frac{3f_0}{3-4b}e^{-4bH_0t}+\frac{\xi_0}{3H_0}e^{-3H_0t}-\xi_1. \tag{14}$$

Here ξ_0 and ξ_1 are constants. For the de Sitter space a behaves as $a=a_0e^{H_0t}$. Then for the matter with constant equation of state w, we find

$$\rho=\rho_0e^{-3(w+1)H_0t}. \tag{15}$$

Then by substiruting (12), (14), and (15) into (6), we obtain

$$\begin{aligned}0&=-3H_0^2(1+\xi_1)+6H_0^2f_0(2b-1)e^{-4H_0bt}\\&\quad+\kappa^2\rho_0e^{-3(w+1)H_0t}.\end{aligned} \tag{16}$$

When $\rho_0=0$, if we choose

$$b=\frac{1}{2},\qquad\xi_1=-1, \tag{17}$$

de Sitter space can be a solution. Even if $\rho\neq0$, if we choose

$$b=\frac{3}{4}(1+w),\qquad f_0=\frac{\kappa^2\rho_0}{3H_0^2(1+3w)},\qquad\xi_1=-1, \tag{18}$$

there is a de Sitter solution.

In the presence of matter with $w\neq0$, we may have a de Sitter solution $H=H_0$ even if $f(\phi)$ given by

$$f(\phi)=f_0e^{\phi/2}+f_1e^{3(w+1)\phi/4}. \tag{19}$$

Then the following solution exists:

$$\phi = -4H_0 t, \qquad \xi = 1 + 3f_0 e^{-2H_0 t} + \frac{f_1}{w} e^{-3(w+1)H_0 t},$$
$$\rho = -\frac{3(3w+1)H_0^2 f_1}{\kappa^2} e^{-3(1+w)H_0 t}. \tag{20}$$

Note that H_0 in (12) can be arbitrary and can be determined by an initial condition. Since H_0 can be small or large, the theory with function (13) with $b = 1/2$ could describe the early-time inflation or current cosmic acceleration. Motivated by this, we may propose the following model:

$$f(\phi) = \begin{cases} f_0 e^{\phi/2} & 0 > \phi > \phi_1, \\ f_0 e^{\phi_1/2} & \phi_1 > \phi > \phi_2, \\ f_0 e^{(\phi-\phi_2+\phi_1)/2} & \phi < \phi_2. \end{cases} \tag{21}$$

Here ϕ_1 and ϕ_2 are constants. We also assume that matter could be neglected when $0 > \phi > \phi_1$ or $\phi < \phi_2$. Since the above function $f(\phi)$ is not smooth around $\phi = \phi_1$ and ϕ_2, one may replace the above $f(\phi)$ with a more smooth function. When $0 > \phi > \phi_1$ or $\phi < \phi_2$, the universe is described by the solution (13) although corresponding H_0 might be different. When $\phi_1 > \phi > \phi_2$, since $f(\phi)$ is a constant, the universe is described by the Einstein gravity, where effective gravitational constant $\kappa_{\rm eff}$ is given by

$$\frac{1}{\kappa_{\rm eff}^2} = \frac{1}{\kappa^2}\left(1 + f_0 e^{\phi_1/2}\right). \tag{22}$$

(Note that in non-local gravity when auxiliary scalar is not constant, the Newton coupling constant is defined ambiguously.) Then due to the matter contribution there could occur matter dominated phase. In this phase, the Hubble rate H behaves as $H = \frac{2}{3(t_0+t)}$ with a constant t_0 and the scalar curvature is given by $R = \frac{4}{3(t_0+t)^2}$. Now we assume that the universe started at $t = 0$ with a rather big but constant curvature $R = R_I = 12H_I^2$ with a constant H_I, that is, the universe is in de Sitter phase. Then in the model (21), by following (12), ϕ behaves as $\phi = -4H_I t$. Subsequently, at $t = t_1 \equiv -\phi_1/4H_I$, we have $\phi = \phi_1$ and the universe enters into the matter dominated phase. If the curvature is continuous at $t = t_1$, t_0 can be found by solving

$$R = \frac{4}{3(t_0+t_1)^2} = 12H_I^2. \tag{23}$$

If ϕ and $\dot\phi$ are also continuous, when $\phi_1 > \phi > \phi_2$, ϕ is given by solving (8) as

$$\phi = -\frac{4}{3}\ln\left(\frac{t}{t_1}\right) - \tilde\phi(t - t_1) + \phi_1,$$
$$\tilde\phi \equiv -4H_I(t_0+t_1)^2 + \frac{4}{3}(t_0+t_1). \tag{24}$$

When $\phi = \phi_2$, the de Sitter phase, which corresponds to the accelerating expansion of the present universe, could have started. The solution corresponds to de Sitter space (with some shifts of parameters) and $H_0 = H_L$ could be given by solving

$$12H_L^2 = \frac{4}{3(t_0+t_2)^2}, \tag{25}$$

if the curvature is continuous at $\phi = \phi_2$. In (25), t_2 is defined by $\phi(t_2) = \phi_2$. Thus, we got the cosmological FRW model with inflation, radiation/matter dominated phase, and current accelerating expansion.

3. Phantom cosmology in the non-local gravity

Let us demonstrate that non-local gravity may also lead to the effective phantom cosmology without need to introduce the non-physical scalar with negative kinetic energy. The quintessence cosmology may also emerge.

We now investigate if there could be power law solution corresponding to quintessence or phantom cosmology, where

$$H = \frac{h_0}{t} \quad \left(a \propto t^{h_0}\right). \tag{26}$$

Then a solution of Eq. (8) is given by

$$\phi = \phi_0 \ln\frac{t}{t_0}, \qquad \phi_0 = \frac{-6h_0 + 12h_0^2}{1 - 3h_0}. \tag{27}$$

Here t_0 is a constant. We now assume, as in (13), $f(\phi)$ is given by

$$f(\phi) = f_0 e^{b\phi} = f_0\left(\frac{t}{t_0}\right)^{bh_0}. \tag{28}$$

Then by solving Eq. (9) as

$$\xi = \frac{(-6h_0 + 12h_0^2) f_0}{(b\phi_0 + 3h_0 - 1)\phi_0}\left(\frac{t}{t_0}\right)^{bh_0} + \frac{t_0\tilde\xi_0}{-3h_0+1}\left(\frac{t}{t_0}\right)^{-3h_0+1} + \tilde\xi_1. \tag{29}$$

Here $\tilde\xi_0$ and $\tilde\xi_1$ are constants of integration. We also find

$$\rho = \tilde\rho_0\left(\frac{t}{t_0}\right)^{-3(1+w)h_0}. \tag{30}$$

When $\rho_0 = 0$, Eq. (6) can be satisfied if

$$\tilde\xi_1 = 1,$$
$$0 = -6h_0^2\left(-6h_0 + 12h_0^2\right)^2 b^2 + \left(-42h_0^2 + 9h_0 + 1\right)\left(-6h_0 + 12h_0^2\right)(1 - 3h_0)b + 6h_0^2(1 - 3h_0)^3. \tag{31}$$

$\tilde\xi_0$ and f_0 could be arbitrary. Then if we give b satisfying the second equation in (31), there could be a power law solution. Even if $\rho_0 \neq 0$, there could be a solution if

$$\tilde\xi_1 = 1, \qquad b = \frac{\{2 - 3(1+w)h_0\}(1 - 3h_0)}{-6h_0^2 + 12h_0^2},$$
$$f_0 = -\frac{\kappa^2\rho_0}{-3h_0^2 + 3h_0 + 9h_0 w + \frac{(-6h_0^2 - \frac{9}{2}h_0 - 3(3h_0 + \frac{1}{2})h_0 w)(1-3h_0)}{1-3h_0 w}}. \tag{32}$$

In other words, if we start with the theory where b and f_0 are given by (32), we obtain the cosmological solution given by (26) with (27) and (29). In the above formulation h_0 is an arbitrary. Since the effective equation of state (EoS) parameter $w_{\rm eff}$

is given by

$$w_{\text{eff}} \equiv -1 - \frac{2\dot{H}}{3H^2} = -1 + \frac{2}{3h_0}, \tag{33}$$

any w_{eff} corresponding to quintessence or phantom can be realized in this non-local gravity. Especially if h_0 is negative, the effective phantom cosmology occurs. When $h_0 < 0$, we may shift t as $t - t_s$ and assume $t < t_s$ in the present universe. Then we have

$$H = -\frac{h_0}{t_s - t}, \tag{34}$$

and $t = t_s$ corresponds to the Big Rip singularity.

4. Unification of the inflation with cosmic acceleration in the non-local-$F(R)$ gravity

Let us discuss the accelerating early-time and late-time cosmology in the non-local gravity where $F(R)$-term [1] is added. The starting action is:

$$S = \int d^4x \sqrt{-g}\left\{\frac{1}{2\kappa^2}R\left(1 + f\left(\Box^{-1}R\right)\right) + F(R) + \mathcal{L}_{\text{matter}}\right\}. \tag{35}$$

Here $F(R)$ is some function of R. FRW equations look like

$$0 = -3H^2\left(1 + f(\phi) - \xi\right) + \frac{1}{2}\dot{\xi}\dot{\phi} - 3H\left(f'(\phi)\dot{\phi} - \dot{\xi}\right) - F(R) + 6\left(H^2 + \dot{H}\right)F'(R) - 36\left(4H^2\dot{H} + H\ddot{H}\right)F''(R) + \kappa^2\rho, \tag{36}$$

$$0 = \left(2\dot{H} + 3H^2\right)\left(1 + f(\phi) - \xi\right) + \frac{1}{2}\dot{\xi}\dot{\phi} + \left(\frac{d^2}{dt^2} + 2H\frac{d}{dt}\right)\left(f(\phi) - \xi\right)F(R) - 2\left(\dot{H} + 3H^2\right)F'(R) + \kappa^2 p. \tag{37}$$

Here $R = 12H^2 + 6\dot{H}$.

We may propose several scenarios. One is that the inflation at the early universe is generated mainly by $F(R)$ part but the current acceleration is defined mainly by $f(\Box^{-1}R)$ part. One may consider the inverse, that is, the inflation is generated by $f(\Box^{-1}R)$ part but the late-time acceleration by $F(R)$.

For instance, for the first scenario one can take: $F(R) = \beta R^2$. Here β is a constant. We choose $f(\Box^{-1}R)$ part as in (13) but f_0 is taken to be very small and ϕ starts with $\phi = 0$. Hence, at the early universe $f(\Box^{-1}R)$ is very small and could be neglected. Then due to the $F(R)$-term (37), there occurs (slightly modified) R^2-inflation. After the end of the inflation, there occurs the radiation/matter dominance era. In this phase, ϕ behaves as in (24): $\phi = -\frac{4}{3}\ln(\frac{t}{\hat{t}_0}) - \hat{\phi}_1(t - \hat{t}_0) + \hat{\phi}_2$. However, the constants $\hat{t}_0$, $\hat{\phi}_1$, and $\hat{\phi}_2$ should be determined by the proper initial conditions, which may differ from that in (24). We now assume $\hat{\phi}_1$ is very small but negative. From the expression of (13) it follows $f(\phi)$ becomes large as time goes by and finally this term dominates. As a result, de Sitter expansion (13) occurs at the present universe.

For the second scenario, the early-time inflation is generated by $f(\Box^{-1}R)$ part but the cosmic acceleration is generated by $F(R)$. As an $F(R)$-term, one can take the model [7]:

$$F_{\text{HS}}(R) = -\frac{m^2c_1(R/m^2)^n}{c_2(R/m^2)^n + 1}, \tag{38}$$

which has the following properties

$$\lim_{R\to\infty} F_{\text{HS}}(R) = \text{const}, \qquad \lim_{R\to 0} F_{\text{HS}}(R) = 0. \tag{39}$$

The second condition means that there is a flat spacetime solution (vanishing cosmological constant). The estimation of Ref. [7] suggests that R/m^2 is not so small but rather large even at the present universe and $R/m^2 \sim 41$. Hence, $F_{\text{HS}}(R) \sim -\frac{m^2c_1}{c_2} + \frac{m^2c_1}{c_2^2}(\frac{R}{m^2})^{-n}$, which gives an "effective" cosmological constant $-m^2c_1/c_2$ and generates the late-time accelerating expansion. One can show that

$$H^2 \sim \frac{m^2c_1\kappa^2}{c_2} \sim (70\ \text{km/s pc})^2 \sim \left(10^{-33}\ \text{eV}\right)^2. \tag{40}$$

At the intermediate epoch, where the matter density ρ is larger than the effective cosmological constant, $\rho > m^2c_1/c_2$, there appears the matter dominated phase and the universe expands with deceleration. Hence, above model describes the effective ΛCDM cosmology.

As a $f(\Box^{-1}R)$ part, we consider theory (13), again. It is assumed f_0 is large and $f(\Box^{-1}R)$ term could be dominant at the early universe. Hence, following (12), ϕ becomes negative and large as time goes by and therefore $f(\phi)$ becomes small and could be neglected at late universe. Then there appears naturally the radiation/matter dominated phase. After that due to $F_{\text{HS}}(R)$-term (38), the late-time acceleration occurs.

Although the model [7] is quite successful, the early time inflation is not included there. In [8], it was suggested the modified gravity model to treat the inflation and the late-time accelerating expansion in a unified way. In order to generate the inflation, one may require $\lim_{R\to\infty} f(R) = -\Lambda_i$. Here Λ_i is an effective cosmological constant at the early universe and therefore we assume $\Lambda_i \gg (10^{-33}\ \text{eV})^2$. One may assume $\Lambda_i \sim 10^{20-38}$. In order that the current accelerating expansion could be generated, let us consider that $f(R)$ is a small constant at present universe, that is, $f(R_0) = -2R_0$, $f'(R_0) \sim 0$. Here R_0 is current curvature $R_0 \sim (10^{-33}\ \text{eV})^2$. The next condition corresponding to the second one in (39) is: $\lim_{R\to 0} f(R) = 0$. In the above class of models, the early universe starts from the inflation driven by the effective cosmological constant. As curvature becomes smaller, the effective cosmological constant also becomes smaller. After that the radiation/matter dominates. When the density of the radiation and the matter becomes small and the curvature goes to the value R_0, there appears the small effective cosmological constant. Hence, the current cosmic expansion starts.

In [8], two examples have been proposed. The first model is given by

$$F(R) = -\frac{(R-R_0)^{2n+1} + R_0^{2n+1}}{f_0 + f_1\{(R-R_0)^{2n+1} + R_0^{2n+1}\}} = -\frac{1}{f_1} + \frac{f_0/f_1}{f_0 + f_1\{(R-R_0)^{2n+1} + R_0^{2n+1}\}}. \tag{41}$$

Here n is a positive integer, $n = 1, 2, 3, \ldots$ and

$$\frac{R_0^{2n+1}}{f_0 + f_1 R_0^{2n+1}} = 2R_0, \qquad \frac{1}{f_1} = \Lambda_i, \tag{42}$$

that is

$$f_0 = \frac{R_0^{2n}}{2} - \frac{R_0^{2n+1}}{\Lambda_i} \sim \frac{R_0^{2n}}{2}, \qquad f_1 = \frac{1}{\Lambda_i}. \tag{43}$$

The second model is

$$F(R) = -f_0 \int_0^R dR\, \mathrm{e}^{-\frac{\alpha R_1^{2n}}{(R-R_1)^{2n}} - \frac{R}{\beta\Lambda_i}}. \tag{44}$$

Here α, β, f_0, and R_1 are constants. Then by construction, as long as $0 < f_0 < 1$, $F'(R) > -1$, which shows that there is no anti-gravity regime. Since

$$f(R_1) \sim -f_0 \int_0^{R_1} dR\, \mathrm{e}^{-\frac{\alpha R_1^{2n}}{(R-R_1)^{2n}}} = -f_0 A_n(\alpha) R_1,$$
$$A_n(\alpha) \equiv \int_0^1 dx\, \mathrm{e}^{-\frac{\alpha}{x^{2n}}}, \tag{45}$$

and $-f(R_1)$ could be identified with the effective cosmological constant $2R_0$, we find

$$f_0 A_n(\alpha) R_1 = R_0. \tag{46}$$

Note that $A_n(0) = 1$, $A_n(+\infty) = 0$, and $A'(x) < 0$. On the other hand, since

$$f(+\infty) \sim \int_0^\infty dR\, \mathrm{e}^{-R\beta\Lambda_i} = -f_0\beta\Lambda_i, \tag{47}$$

and $-f(+\infty)$ could be identified with the effective cosmological constant at the inflationary epoch, Λ_i, one gets $f_0\beta = 1$.

As a $f(\Box^{-1}R)$ part, we consider (13), again and assume f_0 is large. Thus, $f(\Box^{-1}R)$ term could be dominant at the early universe. Then the inflation is generated by the combination of $F(R)$ with non-local term. Following (12), ϕ becomes negative and large as time goes by and therefore $f(\phi)$ becomes small and could be neglected at late universe. After that, there occurs the radiation/matter dominated phase. After that due to $F_{\mathrm{HS}}(R)$-term (38), the accelerating expansion starts.

Some nice features of above scenario are related with Newton law which should be respected at current universe. For the $F(R)$-models (38), (41), and (44), the Newton law corrections have been found in [8]. It was shown that currently the corrections are very small. In the above scenario, where the inflation occurs due to $f(\Box^{-1}R)$ part, its contribution to FRW dynamics becomes very small at late universe. Hence, the contribution from $f(\Box^{-1}R)$ part to the Newton law correction is negligible. The same is true for so-called matter instability [3,12,13], which happens in some models of $F(R)$-gravity. It has been shown that such instability is absent in the above $F(R)$-models (38), (41), and (44) [8]. Then since the contribution from $f(\Box^{-1}R)$ is very small at the late time universe, there is no such instability in above non-local $F(R)$ gravity at late universe. Thus, we demonstrated that non-local gravity coupled with $F(R)$-term may naturally predict the known universe expansion sequence: inflation, radiation/matter dominance and dark energy.

5. Discussion

In summary, we demonstrated that non-local gravity may be a key for the origin of the inflation and dark energy being consistent with simplest local tests and the known universe expansion history. The known universe epochs sequence is easier to realize when modified gravity is given as some combination of terms where one term generates the inflation while other, qualitatively different term pushes the late universe to accelerate. In addition, such combined modified gravity easily passes the simplest local tests. The corresponding example of non-local-$F(R)$ gravity is discussed in detail.

More complicated versions of non-local gravity may be considered in similar scalar-tensor formulation with auxiliary scalars. It is important that even if more precise observational data define the EoS parameter w to be slightly different from -1, there exists the possibility to realize such scenario in non-local gravity as the effective quintessence or phantom cosmology. Moreover, the cosmological perturbations (for a review, see [14]) should be investigated there. This will be discussed elsewhere.

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Appendix A. Stability in de Sitter background

We now check the (in)stability in the de Sitter solution (12)–(17). By defining

$$X \equiv -\frac{\dot{\phi}}{4H}, \qquad Y \equiv \frac{1-\xi}{3f(\phi)}, \qquad W = \frac{\dot{\xi}}{6Hf(\phi)},$$
$$\frac{d}{dN} \equiv a\frac{d}{da} = \frac{1}{H}\frac{d}{dt}, \tag{A.1}$$

Eqs. (6), (8), (9) could be rewritten as

$$0 = -(1+3Y) - 4XW + 2X + 6W, \tag{A.2}$$
$$\frac{dX}{dN} = -\left(\frac{1}{H}\frac{dH}{dN} + 3\right)X + \frac{3}{2}\left(\frac{1}{H}\frac{dH}{dN} + 2\right), \tag{A.3}$$
$$\frac{dW}{dN} = -\left(\frac{1}{H}\frac{dH}{dN} + 3\right)W + \frac{1}{2}\left(\frac{1}{H}\frac{dH}{dN} + 2\right) + 2WX. \tag{A.4}$$

For the de Sitter solution (12)–(17), we have

$$X = Y = W = 1. \tag{A.5}$$

By multiplying d/dN with (A.2) and using (A.3) and (A.4) and deleting Y by (A.2), we obtain

$$0 = (2 - X - 6W + 4WX)\frac{1}{H}\frac{dH}{dN} + 6 - 12W - 4X - 2X^2 + 12WX. \tag{A.6}$$

We now consider the perturbation from the de Sitter solution (13) or (A.5):

$$X = 1 + \delta X, \qquad W = 1 + \delta W, \qquad H = H_0(1 + \delta h). \tag{A.7}$$

Here we assume $|\delta X|, |\delta W|, |\delta h| \ll 1$. Then from (A.3), (A.4), and (A.6), we obtain

$$\frac{d\delta X}{dN} = \frac{1}{2}\frac{d\delta h}{dN} - 3\delta X, \tag{A.8}$$
$$\frac{d\delta W}{dN} = -\frac{1}{2}\frac{d\delta h}{dN} - \delta W + 2\delta X, \tag{A.9}$$
$$\frac{d\delta h}{dN} = 4\delta X. \tag{A.10}$$

By deleting $d\delta h/dN$ by using (A.10) from (A.8) and (A.9), we obtain

$$\frac{d\delta X}{dN} = -\delta X, \qquad \frac{d\delta W}{dN} = -\delta W, \tag{A.11}$$

which tells δX, δW and also $\delta h \propto e^{-N} \propto 1/a$. Therefore the perturbation decreases as the universe expands, which tells that the de Sitter solution could be stable.

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